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Analytical Solutions for Rumor Spreading Dynamical Model in a Social Network

Abstract: In this paper, Laplace Adomian decomposition method is utilized for evaluating of spreading model of rumor. Firstly, a succinct review is constructed on the subject of using analytical methods such as Adomian decomposion method, Variational iteration method and Homotopy Analysis method for epidemic models and biomathematics. In continue a spreading model of rumor with consideration of forgetting mechanism is assumed and subsequently LADM is exerted for solving of it. By means of the aforementioned method, a general solution is achieved for this problem which can be readily employed for assessing of rumor model without exerting any computer program. In addition, obtained consequences for this problem are discussed for different cases and parameters. Furthermore, it is shown the method is so straightforward and fruitful for analyzing equations which have complicated terms same as rumor model. By employing numerical methods, it is revealed LADM is so powerful and accurate for eliciting solutions of this model. Eventually, it is concluded that this method is so appropriate for this problem and it can provide researchers a very powerful vehicle for scrutinizing rumor models in diverse kinds of social networks such as Facebook, YouTube, Flickr, LinkedIn and Tuitor.

Keywords: Laplace Adomian decomposition method; rumor spreading model; social networks; analytical methods

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1 Introduction

Analyzing of nonlinear problems is one the most crucial fields in the applied mathematics. In fact, nonlinear equations have an important position in the leading-edge science and in minds of scientists. Unavoidably, understanding of nonlinear problems gives us an open window for better studying and examining of our world. Recently, new analytical methods disclosed solutions of a number of equations in different fields. Perturbation methods [1, 2] are the most familiar approaches for analyzing of nonlinear problems. Homotopy Analysis method [3–5] is another technique which is so strong for analyzing of nonlinear phenomena and it has been invented by Professor S.J.Laio. Homotopy analysis method is a very potent approach in comparison with lots of analytical techniques and a number of approaches are actually special cases of this method. Abilities of the method motivate researchers to use it for different nonlinear problems [6–8]. Variational Iteration method [9] is a further example of analytical techniques for solving nonlinear equations. This approach has been so far implemented for a number of systems in fluid mechanics [10], structural dynamics [11] and applied physics [12, 13]. Adomian decomposition method [14, 15] can be cited as another instance of analytical approaches which has been suggested by G. Adomain. In addition, Non-perturbative approaches such as Energy Balance Method [16–19], Hamiltonian approach [20–25], Variational approach [26], Max-Min approach [27] and Harmonic balance method [28] can be mentioned as novel techniques that have been used for investigating a number of nonlinear equations. As mentioned in abstract of this paper, firstly, applications of Homotopy analysis method and its derivatives, variational iteration method and Adomian decomposition method are briefly outlined for analyzing epidemic models and biomathematics. As first case, SIR epidemic model with constant vaccination strategy has been studied by Yildirim and Cherruault [29]. Homotopy perturbation method has been used in their works for analyzing of their objective model [29]. Furthermore, the aforementioned model has been analyzed by O.D. Makinde using Adomian decomposition method [30]. Merdan et al. [31] have used variational iteration method and modified variational iteration method for finding analytical solutions of a dynamical system which describes HIV infection of $CD4^+$ T cells. A multistage Homotopy perturbation method has been employed for solving Human T-Cell Lymphotropic Virus I (HTLV-I) infection of $CD4^+$ T-

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Cells Model by Gokdogan and Merdan [32]. Besides, a fractional order differential equation model of human T-cell lymphotropic virus I (HTLV-I) infection of CD4+ has been explained and analyzed exerting multi-step generalized differential transform method by Ertürk et al. [33]. SIR and SIS epidemic models have been investigated by Khan et al. [34]. Homotopy Analysis method has been utilized by them for solving of their models [34].

Table 1. Biomathematics and epidemic models that have been investigated by analytical methods such as VIM, ADM, ETDM, HPM and HAM.

Models
SIR Model [29, 30] $\frac{dS}{dt} = (1-P)\pi N - \beta\frac{SI}{N} - \mu S,$ $\frac{dI}{dt} = \beta\frac{SI}{N} - (y+\mu)I$ $\frac{dR}{dt} = P\pi N + yI - \pi R$ S, I and R describe susceptible group, infected group and removed group, respectively [29, 30].
Model of HIV infection of CD4⁺ T cells [31] $\frac{dT}{dt} = q - \alpha T + yT\left(1 - \frac{T+1}{T_{\max}}\right) - kVT$ $\frac{dI}{dt} = kVT - \beta I$ $\frac{dV}{dt} = N\beta I - yV$ $T(t)$, $I(t)$ and $V(t)$represent concentration of susceptible CD4+ T – cells, CD4+ T cells infected by the HIV Viruses and free HIV virus particles, respectively [31].
Model for Humen T-Cell Lymphotropic Virus I infection of CD4⁺ Cells [32] $\frac{dT}{dt} = \lambda - \mu_T T - kT_A T$ $\frac{dT_L}{dt} = k_1 T_A T - (\mu_L + \alpha)T_L$ $\frac{dT_A}{dt} = \alpha T_L - (\mu_A + \rho)T_A$ $\frac{dT_M}{dt} = \rho T_A + \beta T_M(1 - \frac{T_M}{T\max}) - \mu_M T_M$ $T(t)$, $T_L(t)$, $T_A(t)$ and $T_M(t)$ illustrate the concentration of healthy CD4+ T-cells at time t, the concentration of latently infected CD4+ T-cells at time t, the concentration of actively infected CD4+ T-cells and the concentration of leukemic cells at time t, respectively [32].

Kelleci and Yildirım have applied Homotopy perturbation method for finding solutions of nonlinear ordinary differential equations arising in kinetic modeling of lactic acid fermentation and epidemic model [35]. Enhanced multistage differential transform method has been presented by Do and Jang [36] for evaluating dynamical model of prey-predator systems. Fractional order model of HIV infection of CD4⁺ T cells and Hanta virus infection model has been studied by Gokdogan et al. [37, 38] utilizing multistep differential transform method. Nonlinear equations of above-mentioned papers are tabulated for interested readers in Tables 1 and 2.This paper aims to analyze a spreading model of rumor with consideration of forgetting mechanism [39]. Obviously, rumor is an essential part of social communication and therefore spreading of it has striking influences on the societies. Present study focuses on model of spreading rumor in the social networks [39]. Inevitably, social networks such as Facebook, Tuitor and so on are the most straightforward ways for spreading rumors. In this paper, Laplace Adomain decomposition method [41, 42] is applied for solving of spreading rumor in the social networks with consideration of forgetting mechanism. The objective model of this paper is briefly introduced in the next section.

Table 2. Biomathematics and epidemic models that have been investigated by analytical methods such as VIM, ADM, ETDM, HPM and HAM.

Models
Fractional order differential equation model of human T-cell lymphotropic virus I [33] $D^\alpha T = \lambda - \mu_T T - kT_A T$ $D^\alpha T_L = k_1 T_A T - (\mu_L + \alpha)T_L$ $D^\alpha T_A = \alpha T_L - (\mu_A + \rho)T_A$ $D^\alpha T_M = \rho T_A + \beta T_M\left(1 - \frac{T_M}{T\max}\right) - \mu_M T_M$ $T(t)$, $T_L(t)$, $T_A(t)$ and $T_M(t)$ illustrate the concentration of healthy CD4+ T-cells at time t, the concentration of latently infected CD4+ T-cells at time t, the concentration of actively infected CD4+ T-cells and the concentration of leukemic cells at time t, respectively [33].
Hantavirus infection model [38] $\frac{dS_m}{dt} = b(S_m + I_m) - cS_m - \frac{S_m(S_m + I_m)}{k} - aS_m I_m$ $\frac{dI_m}{dt} = -CI_m - \frac{I_m(S_m + I_m)}{k} + aS_m I_m$ $S_m(t)$ and $I_m(t)$ denote susceptible and infected mice, respectively [38].
Prey and Predator [36] $\frac{dx(t)}{dt} = rx(t)\left(1 - \frac{x(t)}{k}\right) - \frac{mx^p(t)}{1+x^p(t)}y(t)$ $\frac{dy(t)}{dt} = y(t)\left(\mu\frac{mx^p(t)}{1+x^p(t)} - D\right)$ $x(t)$ and $y(t)$ are the population size for the prey and the predator model, respectively [36].

2 Mathematical Modeling

Figure 1 represents dynamical modeling of spreading rumor with consideration of forgetting mechanism in social networks schematically [39, 43].

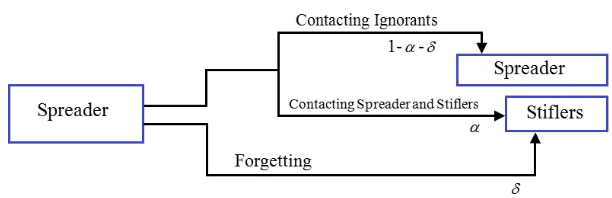

Fig. 1. Schematic of rumor spreading procedure [39].

This system can be mathematically described by nonlinear differential equations as [39]:

$$\begin{aligned}\frac{dI(t)}{dt} &= -\lambda\bar{k}\, I(t)\, S(t)\\ \frac{dS(t)}{dt} &= \lambda\bar{k}I(t)S(t) - \alpha\bar{k}S(t)(S(t)+R(t)) - \delta S(t)\\ \frac{dR(t)}{dt} &= \alpha\bar{k}S(t)(S(t)+R(t)) + \delta S(t)\end{aligned} \tag{1}$$

with the initial conditions :

$$\begin{aligned}I(0) &= \frac{N-1}{N}\\ S(0) &= \frac{1}{N}\\ R(0) &= 0\end{aligned} \tag{2}$$

The components of Eq. (1) are the density of ignorant, spreaders and stiflers that are correspondingly denoted by I(t), S(t) and R(t). Furthermore, $\bar{k}$ represents the average degree of social network. In Eq. (1), λ, δ and α are spreading, forgetting and stifling rates, respectively. In addition, the following conditions are presumed for this model:

Condition. 1

$\alpha+\delta \le 1$ This condition makes the model of rumor spreading more fitting to the actual situation [39].

Condition. 2 $R(t) + S(t) + I(t) = 1$ Normalization condition [39].

3 Solution Procedure

Firstly, $A = \lambda\bar{k}$, $B = \alpha\bar{k}$, $C = \delta$ are considered as parameters of Eq. (1) for simplicity of it. Subsequently, Laplace Adomian decomposition method is applied to Eq. (1) and next we have:

$$\begin{aligned}L\left\{\frac{dI(t)}{dt}\right\} &= -L\lambda\bar{k}\, I(t)\, S(t)\\ L\left\{\frac{dS(t)}{dt}\right\} &= L\lambda\bar{k}I(t)S(t) - L\alpha\bar{k}S(t)(S(t)+R(t)) - L\delta S(t)\\ L\left\{\frac{dR(t)}{dt}\right\} &= L\alpha\bar{k}S(t)(S(t)+R(t)) + L\delta S(t)\end{aligned} \tag{3}$$

Consequently we reach to Eq. (4):

$$\begin{aligned}L\{I\} &= \frac{r_1}{s} - \frac{A}{s}L\{IS\}\\ L\{S\} &= \frac{r_2}{s} + \frac{A}{s}L\{IS\} - \frac{B}{s}L\left\{S^2\right\} - \frac{B}{s}L\{SR\} - \frac{C}{s}L\{S\}\\ L\{R\} &= \frac{B}{s}L\left\{S^2\right\} + \frac{B}{s}L\{SR\} + \frac{C}{s}L\{S\}\end{aligned} \tag{4}$$

$E = IS$, $F = SR$ and $G = S^2$ are illustrated as new variables and Eq. (4) is altered to a new form.

As standard form of Adomian decomposition method the solutions of Eq. (4) are supposed to be as the sum of the following series:

$$I = \sum_{n=0}^{\infty} I_n, \quad S = \sum_{n=0}^{\infty} S_n, \quad R = \sum_{n=0}^{\infty} R_n \tag{5}$$

Afterwards, nonlinear terms in Eq. (4) are approximated as follows:

$$E = \sum_{n=0}^{\infty} E_n \tag{6}$$

$$F = \sum_{n=0}^{\infty} F_n \tag{7}$$

$$G = \sum_{n=0}^{\infty} G_n \tag{8}$$

then Eq. (9) is defined as:

$$\begin{aligned}IS &= \sum_{n=0}^{\infty} E_n(I_0,\dots,I_n,S_0,\dots,S_n)\\ E_n &= \frac{1}{n!}\left[\frac{d^n(\sum_{j=0}^{\infty} E_j\eta^j)(\sum_{j=0}^{\infty} S_j\eta^j)}{d\eta^n}\right]_{\eta=0}\end{aligned} \tag{9}$$

and we have

$$E_n = \phi_1(M_n, t) \quad M_n = \sum_{i=0}^{n} A_i B_{n-i} \tag{10}$$

Similarly for other nonlinear terms, we define the following function

$$SR = \sum_{n=0}^{\infty} F_n(S_0,\dots,S_n,R_0,\dots,R_n) \tag{11}$$

$$F_n = \frac{1}{n!}\left[\frac{d^n(\sum_{j=0}^{\infty} S_j\eta^j)(\sum_{j=0}^{\infty} R_j\eta^j)}{d\eta^n}\right]_{\eta=0},$$

$$F_n = \phi_2(N_n, t) \quad \text{and } N_n = \sum_{i=0}^{n} B_i C_{n-i} \tag{12}$$

and eventually for last nonlinear term of Eq. (3), Eq. (13) is considered:

$$S^2 = \sum_{n=0}^{\infty} G_n(S_0, \ldots, S_n, S_0, \ldots, S_n) \tag{13}$$

$$G_n = \frac{1}{n!}\left[\frac{d^n(\sum_{j=0}^{\infty} S_j\eta^j)(\sum_{j=0}^{\infty} S_j\eta^j)}{d\eta^n}\right]_{\eta=0},$$

$$G_n = \phi_3(P_n, t) \quad , \quad P_n = \sum_{i=0}^{n} B_i B_{n-i} \tag{14}$$

Using the abovemntioned functions, we find the Adomian polynomials as follows:

$$\begin{aligned}
A_0 &= I_0 = I(0) = r_1,\\
A_1 &= -Ar_1r_2,\\
A_2 &= -\frac{AM_1}{2},\\
A_3 &= -\frac{AM_2}{3},\\
B_0 &= S_0 = S(0) = r_2,\\
B_1 &= Ar_1r_2 - Br_2^2 - Cr_2,\\
B_2 &= \frac{1}{2}(AM_1 - BP_1 - BN_1 - CB_1),\\
B_3 &= \frac{1}{3}(AM_2 - BP_2 - BN_2 - CB_2),\\
C_0 &= R_0 = R(0) = r_3,\\
C_1 &= Br_2^2 + Cr_2,\\
C_2 &= \frac{1}{2}(BP_1 + BN_1 + CB_1),\\
C_3 &= \frac{1}{3}(BP_2 + BN_2 + CB_2)
\end{aligned} \tag{15}$$

Easily the following functions can be obtained or the Eq. (3)

$$A_n = -\frac{AM_{n-1}}{n} \tag{16}$$

$$B_n = \frac{1}{n}(AM_{n-1} - BP_{n-1} - BN_{n-1} - CB_{n-1}) \tag{17}$$

$$C_n = \frac{1}{n}(BP_{n-1} + BN_{n-1} + CB_{n-1}) \tag{18}$$

and furthermore, we obtain Eqs. (19-21) as solution of Eq. (3).

$$I_n = A_n t^n \tag{19}$$

$$S_n = B_n t^n \tag{20}$$

$$R_n = C_n t^n \tag{21}$$

Using Eqs. (16-18, 19-21) and inserting of them in Eq (5), the solution of Eq. (3) is obtained.

4 Numerical Discussion

In this section, accuracy of achieved results are examined by means of numerical methods. For two different cases, the analytical results are studied.

Case 1.

$N = 10^8, \alpha = 0.5\ , \lambda = 0.3, \delta = 0.1,\ \bar{k} = 5$

Case 2.

$N = 10^8, \alpha = 0.5\ , \lambda = 0.5, \delta = 0.5,\ \bar{k} = 5$

Firstly, I(t), R(t) and S(t) are plotted for case 1. Figures 2-4 show behavior of I(t), S(t) and R(t), respectively. As seen, Laplace Adomian decomposition method is a strong method for analyzing of this problem even for a long time.Furthermore, as parameters of case 1 have been illustarated, this method has this ability to analyze this dynamical model for strong nonlinearity. Likewise, Figures 5-7 show aptitudes of LADM for analyzing of 'Objective' problem of this paper.These figures demonstrate this method can predict dynamical behavior of the rumor spreading for a long period of time.

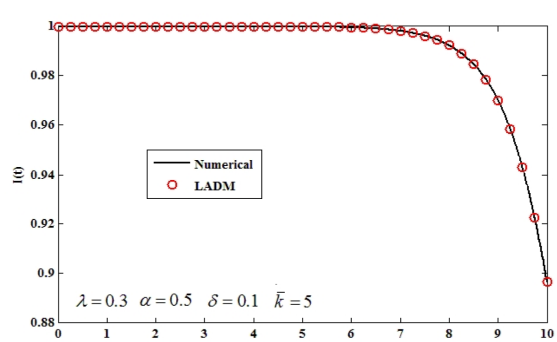

Fig. 2. Comparison of the LADM solutions of I(t) with numerical method.

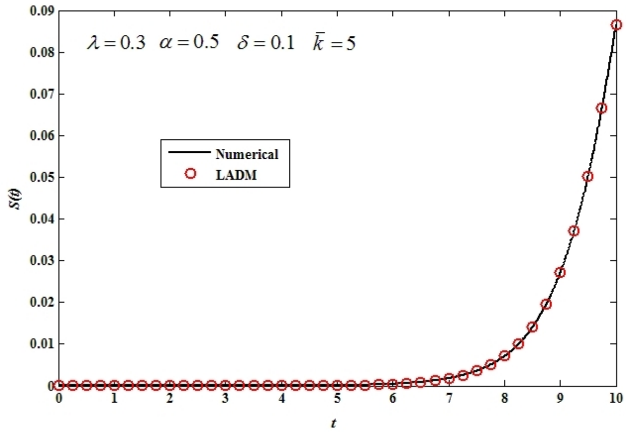

Fig. 3. Comparison of the LADM solutions of S(t) with numerical method.

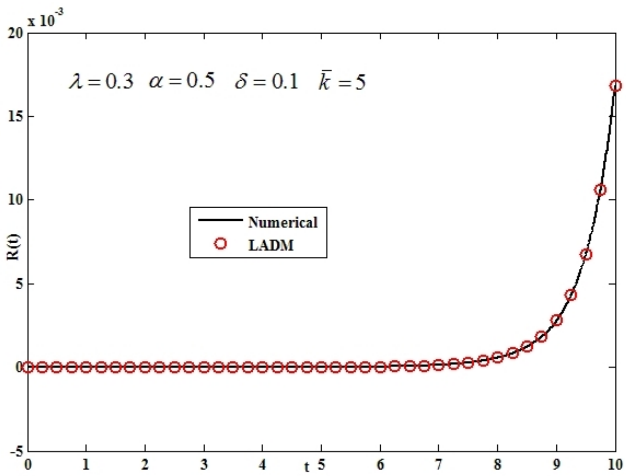

Fig. 4. Comparison of the LADM solutions of R(t) with numerical method.

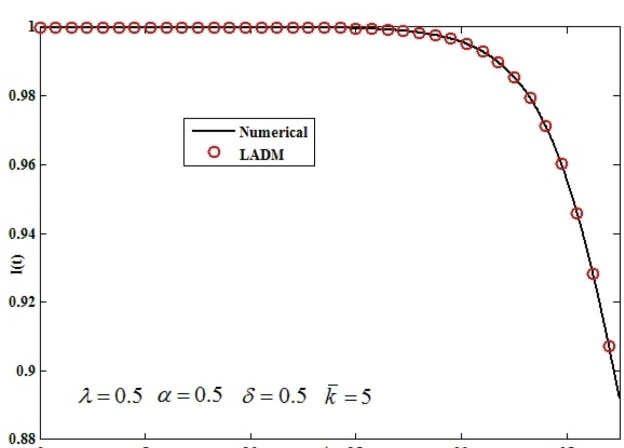

Fig. 5. Comparison of the LADM solutions of I(t) with numerical method.

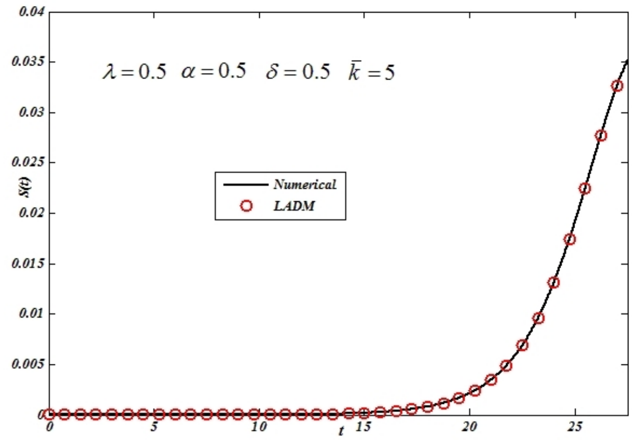

Fig. 6. Comparison of the LADM solutions of S(t) with numerical method.

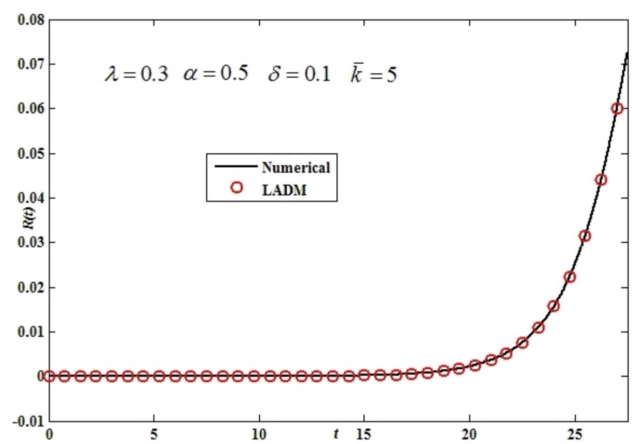

Fig. 7. Comparison of the LADM solutions of R(t) with numerical method.

5 Conclusion

In this paper, Laplace Adomian decomposition method was used for analyzing solution of a spreading model of rumor with consideration of forgetting mechanism. Firstly, recent applications of analytical methods such Homotopy Analysis method, Variational Iteration method and Adomian decomposition method in analyzing of epidemic models were discussed. Furthermore, a number of epidemic and biomathematics models that had been investigated by researchers were tabulated and illustrated. Afterwards, the objective model of this paper was schematically and mathematically revealed. In continue, LADM was employed for finding solutions of abovementioned model.

Two numerical cases were considered and next, the obtained analytical results were compared with numerical method. This comparison provides us a very outstanding conclusion. According to that, this method is so powerful for analyzing of this model even for strong nonlinearity. Furthermore, it can be very effective for predicting dynamical performance of this system for a long period of time. In addition, this approach presents us an explicit series solution without using any computer program and this feature of LADM can facilitate to understand sophisticated and vague parts of this model.

References

[1] A.H. Nayfeh, D.T. Mook, Nonlinear Oscillations, Wiley, New York, 1979.

[2] A.H. Nayfeh, Perturbation methods. Wiley, New York, 1973.

[3] S.J. Liao, Beyond Perturbation-Introduction to the Homotopy Analysis Method, Chapman & Hall/CRC, Boca Raton, 2003.

[4] S.J. Liao, On the homotopy analysis method for nonlinear problems, Applied Mathematics and Computation 147 (2004) 499–513.

[5] S.J. Liao, A kind of approximate solution technique which does not depend upon small parameters: a special example, International Journal of Non-Linear Mechanics 30 (1995) 371–380.

[6] S.J. Liao, An explicit analytic solution to the Thomas-Fermi equation, Applied Mathematics and Computation 144(2003) 495–506.

[7] S.J. Liao, An explicit totally analytic approximation of Blasius viscous flow problems, International Journal of Non-Linear Mechanics 34(1999)759–778.

[8] H. Chen, J. Shen, Z. Zhou, On the application of the homotopy analysis method to limit cycles approximation in planar self-excited systems, Communications in Nonlinear Science and Numerical Simulation 17(2012) 4494–4507.

[9] J.H. He, G. Ch. Wu, and F. Austin, The variational iteration method which should be followed, Nonlinear Science Letter A 1 (2010) 1–30.

[10] D. Younesian, H. Askari, Z. Saadatnia, A. Yıldırım, Analytical solution for nonlinear wave propagation in shallow media using the variational iteration method, Waves in Random and Complex Media (2011) doi:10.1080/17455030.2011.633578.

[11] D.Younesian, Z. Saadatnia, H. Askari, Analytical solutions for free oscillations of beams on nonlinear elastic foundation using the variational iteration method, Journal of Theoretical and Applied Mechanics 50(2012) 639-652.

[12] A.M. Wazwaz, The variational iteration method for solving linear and nonlinear Volterra integral and integro-differential equations, International Journal of Computer Mathematics 87(2010) 1131–1141.

[13] A. Yildirim, Variational iteration method for modified Camassa–Holmand and Degasperis Procesi equations, International Journal of Numerical Methods in Biomedical Engineering 26 (2010) 266–272.

[14] G. Adomian, Solving Frontier Problems of Physics: The Decomposition Method, Kluwer, Dordrecht, 1994.

[15] G. Adomian, A review of the decomposition method in applied mathematics, Journal of Mathematical Analysis and Applications135 (1988) 501–544.

[16] J.H. He, Preliminary report on the energy balance for nonlinear oscillations, Mechanics Research Communications 29 (2002) 107-111.

[17] H. Askari, M.K. Yazdi, Z. Saadatnia, Frequency analysis of nonlinear oscillators with rational restoring force via He's energy balance method and He's variational approach, Nonlinear Science Letter A 1 (2010) 425–430.

[18] H. Askari, Z. Saadatnia, D. Younesian, A. Yildirim, M. KalamiYazdi, Approximate periodic solutions for the Helmholtz–Duffing equation, Computers & Mathematics with Applications 62 (2011) 3894–3901.

[19] L. Cveticanin, M. KalamiYazdi, H.Askari, Analytical approximations to the solutions for a generalized oscillator with strong nonlinear terms, Journal of Engineering Mathematics DOI 10.1007/s10665-012-9542-4

[20] J.H. He, Hamiltonian approach to nonlinear oscillators, Physics Letters A 374 (2010) 2312–2314.

[21] A. Yildirim, H. Askari, Z. Saadatnia, M. KalamiYazdi, Y. Khan, Analysis of nonlinear oscillations of a punctual charge in the electric field of a charged ring via a Hamiltonian approach and the energy balance method, Computers & Mathematics with Applications 62 (2011) 486–490.

[22] L. Cveticanin, M. KalamiYazdi, Z. Saadatnia, H. Askari, Application of Hamiltonian approach to the generalized nonlinear oscillator with fractional power, International Journal of Nonlinear Sciences and Numerical Simulation 11 (2010) 997–1002.

[23] A. Yildirim, H. Askari, M. Kalami Yazdi, Y. Khan,A relationship between three analytical approaches to nonlinear problems, Applied Mathematics letters doi:10.1016/j.aml.2012.02.001

[24] A. Yildirim, Z. Saadatnia, H. Askari, Application of the Hamiltonian approach to nonlinear oscillators with rational and irrational elastic terms, Mathematical and Computer Modelling 54 (2011) 697–703.

[25] Y. Khan, Q. Wu, H. Askari, Z. Saadatnia, M. Kalami-Yazdi, Nonlinear vibration analysis of a rigid rod on a circular surface via Hamiltonian approach, Mathematical and Computational Applications 15 (2010) 974–977.

[26] J.H. He, Variational approach for nonlinear oscillators, Chaos, Solitons & Fractals 34 (2007) 1430–1439.

[27] J.H. He, Max-Min Approach to Nonlinear Oscillators, International Journal of Nonlinear Sciences and Numerical Simulation 9(2008)207-210.

[28] L. Cveticanin, Analysis techniques for the various forms of the Duffing equation. In: Kovacic I and BrennanM (eds) Duffing's equation: non-linear oscillators and their behavior. Wiley, Singapore, 2011.

[29] A. Yildirim, Y. Cherruault, Analytical approximate solution of a SIR epidemic model with constant vaccination strategy by homotopy perturbation method, Kybernetes 38(2009)1566-1575.

[30] O.D. Makinde, Adomian decomposition approach to a SIR epidemic model with constant vaccination strategy, Applied Mathematics and Computation 184 (2007) 842–848.

[31] M. Merdan, A. Gökdoğan, A. Yildirim, On the numerical solution of the model for HIV infection of $CD4^{+}$ T cells, Computers

& Mathematics with Applications 62(2011)118-123.
[32] A. Gokdogan,M.Merdan, A Multistage Homotopy perturbation method for solving human T-cell lymphotropic virus I(HTLV-1) infection of CD4+ T-cells model, Middle-east journal of scientific research 9(2011)503-509.
[33] V. S. Ertürk, Z. M. Odibat, S. Momani, An approximate solution of a fractional order differential equation model of human T-cell lymphotropic virus I (HTLV-I) infection of CD4+ T-cells, Computers & Mathematics with Applications 62 (2011)996-1002.
[34] H. Khan, R. N. Mohapatra, K. Vajravelu, S.J. Liao, The explicit series solution of SIR and SIS epidemic models, Applied Mathematics and Computation 215(2009)653-669.
[35] A. Kelleci, A. Yildirim, Numerical solution of the system of nonlinear ordinary differential equations arising in kinetic modeling of lactic acid fermentation and epidemic model, International Journal of Numerical Methods in Biomedical Engineering 27(2011)585-594.
[36] Y. Do, B. Jang, Enhanced Multistage Differential Transform Method: Application to the Population Models, Abstract and Applied Analysis doi:10.1155/2012/253890 (2012).
[37] A. Gökdogan, A. Yildirim, M. Merdan, Solving a fractional order model of HIV infection of CD4 + T cells, Mathematical and Computer Modelling 54 (2011) 2132–2138.
[38] A. Gökdogan, M. Merdan, A. Yildirim, A multistage differential transformation method for approximate solution of Hantavirus infection model, Communication in Nonlinear Science and Numerical Simulation 17 (2012) 1–8.
[39] L. Zhao, Q. Wang, J. Cheng, Y. Chen, J. Wang, W. Huang, Rumor spreading model with consideration of forgetting mechanism: A case of online blogging LiveJournal, Physica A 390(2011) 2619–2625.
[40] M.Y. Ongun, The Laplace Adomian Decomposition Method for solving a model for HIV infection of CD4+T cells, Mathematical and Computer Modelling 53 (2011) 597–603.
[41] E. Yusufoglu, Numerical solution of Duffing equation by the Laplace decomposition algorithm, Applied Mathematics and Computations 177 (2) (2006) 572–580.
[42] S.A. Khuri, A new approach to Bratu's problem, Applied Mathematics and Computation 147 (2004) 131–136.
[43] V. Isham, S. Harden, M. Nekovee, Stochastic epidemics and rumours on finite random networks, Physica A 389(2010) 561–576.