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Dynamic Analysis of Coupled Vehicle-Bridge System with Uniformly Variable Speed

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Abstract: In this paper, a planar biaxial vehicle model with four degrees of freedom is presented based on spring-damping-mass system theory. By using Runge-Kutta method, the dynamic characteristics of a simply support bridge acting by moving vehicle with uniform variable speed are analyzed, and the effects of inertia force, relative acceleration and initial velocity are taken into consideration in the present research. The time-deflection response curves of the bridge under the variation of initial speed and acceleration are analyzed. Some valuable results are found which can provide a theoretical direction for the consideration of dynamical characteristics in design of bridge system.

Keywords: Dynamic analysis; Dynamic characteristics; Coupled vehicle-bridge system; Uniform variable speed; Spring-damping-mass system

1 Introduction

With appearance of railway in the early nineteenth century, the study of influence of the train induced moving loads on bridges has been increasingly significant and the interactions between vehicles and structures have been investigated by numerous researchers. With the development of vehicle-bridge coupled system, the problems of moving loads have gained a lot of attention of researchers [1–4].

Numerous researchers focused on the investigation of vehicle-bridge coupled system recently, among which, the influence of initial speed and acceleration on considered system are analyzed by Yu etc [5–9]. Results showed that initial speed and acceleration may generally affect the system and the characteristics of considered vehicle-bridge coupled system should be taken into consideration. In Peng's study, the vehicle was simplified as a moving mass and the effect of initial speed and acceleration on system. However, such an assumption was unable to show the real effect of vehicles on the bridge [10]. Chen investigated the influence of initial speed and acceleration on the displacement of bridge by establishing a vehicle-bridge coupled system. However, the deceleration was not considered which was also demonstrated significant [11]. Above researches showed that a complete model for vehicle-bridge coupled system should consider many aspects to reflect reality in investigation of vehicle-bridge coupled system. Thus, taking the effects of inertia force, relative acceleration and initial velocity into consideration, dynamical characteristics of a moving vehicle with uniform varying speed on a simply supported bridge are numerically studied in the paper by using Runge-Kutta method. The time-response properties are presented to show the influence of initial speed and acceleration of the vehicle on the deflection of the bridge.

2 Equations of vehicle-bridge system

A 1/2 vehicle-bridge model can be simplified as a spring damping-mass system as shown in Figure 1. Assume that the vehicle is a rigid body with two degrees of freedom. For a vehicle running on the bridge, it can be composed of vehicle body, suspension device, axle and tire. When the vehicle model is established, the mass of the vehicle can be concentrated on the vehicle body and axle, the suspension device and the tire are simulated by the spring and damping. When considering the effect of spring and damping of the vehicle system, which can reflect more fully the effect of vehicle-bridge coupled system, and the mass model and the single axis model is conservative. In the figure, I and m_3 represent the rotational inertia and respective mass of the vehicle.x_2 and x_1 indicate the position of front and rear axle of the vehicle respectively.$Y(x, t)$ is the transient de-

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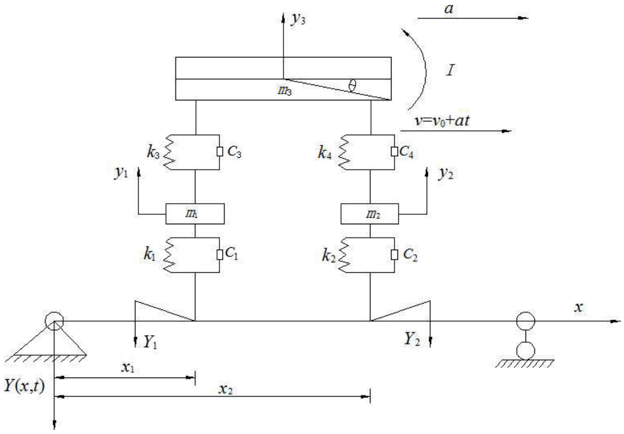

Fig. 1: Simplified mass-spring model of coupled vehicle-bridge system.

flection of the beam. m, c, EI , L are the distributed mass, damping coefficient, flexural stiffness and bridge span respectively; m_2 and m_1 are the total mass of front and rear axle load. Y_2 and Y_1 indicate the vertical displacement of front and rear wheel. k_4 and k_3 represent the stiffness of spring, and c_4 and c_3 represent the damping coefficient of spring. The front and rear damping coefficient of spring is c_2 and c_1. The front and rear equivalent stiffness is k_2 and k_1. y_1, y_2, y_3 and $\theta$are generalized coordinates of the vehicle. Considering the barycentre of the mass is along the same direction, λ and α are distance between front and rear wheels and acceleration of the vehicle. Basing on above simplification, the four equilibrium equations can be give as following,

$$EI\frac{\partial^4 Y(x,t)}{\partial x^4} + m\frac{\partial^2 Y}{\partial t^2} + c\frac{\partial Y}{\partial t} = \left\{\left(m_1 + \frac{m_3}{2}\right)g + m_1\frac{\partial^2 y_1}{\partial t^2}\right.$$
$$\left. + m_3\frac{\partial^2 y_3}{2\partial t^2} - I\frac{\partial^2\theta}{\lambda\partial t^2} - m_1 a\frac{\partial Y_1}{\partial x} - m_3 a\frac{\partial Y_1}{2\partial x}\right\}\delta(x - x_1)$$
$$+\left\{\left(m_2 + \frac{m_3}{2}\right)g + m_2\frac{\partial^2 y_2}{\partial t^2} + m_3\frac{\partial^2 y_3}{2\partial t^2} + I\frac{\partial^2\theta}{\lambda\partial t^2}\right.$$
$$\left. - m_2 a\frac{\partial Y_2}{\partial x} - m_3 a\frac{\partial Y_2}{2\partial x}\right\}\delta(x - x_2) \tag{1}$$

$$m_i\frac{\partial^2 y_i}{\partial t^2} + k_i(y_i + Y_i) + c_i\left(\frac{\partial y_i}{\partial t} + \frac{\partial Y_i}{\partial t}\right) - k_j\left(y_3 - \frac{\theta\lambda}{2} - y_i\right)$$
$$- c_j\left(\frac{\partial y_3}{\partial t} - \lambda\frac{\partial\theta}{2\partial t} - \frac{\partial y_i}{\partial t}\right) = m_i a\frac{\partial Y_i}{\partial x} + m_3 a\frac{\partial Y_i}{2\partial x} \tag{2}$$

$$m_3\frac{\partial^2 y_3}{\partial t^2} + k_3\left(y_3 - \frac{\theta\lambda}{2} - y_1\right) + c_3\left(\frac{\partial y_3}{\partial t} - \lambda\frac{\partial\theta}{2\partial t} - \frac{\partial y_1}{\partial t}\right)$$
$$+ k_4\left(y_3 + \frac{\theta\lambda}{2} - y_2\right) + c_4\left(\frac{\partial y_3}{\partial t} + \lambda\frac{\partial\theta}{2\partial t} - \frac{\partial y_2}{\partial t}\right)$$
$$= m_3 a\frac{\partial Y[\frac{x_1+x_2}{2}]}{\partial x} \tag{3}$$

$$I\frac{\partial^2\theta}{\partial t^2} - \frac{k_3\lambda}{2}\left(y_3 - \frac{\theta\lambda}{2} - y_1\right)$$
$$- \frac{c_3\lambda}{2}\left(\frac{\partial^2 y_3}{\partial t^2} - \lambda\frac{\partial^2\theta}{2\partial t^2} - \frac{\partial^2 y_1}{\partial t^2}\right) + \frac{k_4\lambda}{2}\left(y_3 + \frac{\theta\lambda}{2} - y_2\right)$$
$$+ \frac{c_4\lambda}{2}\left(\frac{\partial y_3}{\partial t} + \lambda\frac{\partial\theta}{2\partial t} - \frac{\partial y_2}{\partial t}\right) = 0 \tag{4}$$

By variables separation method, the variable functions in Eqs. (1–4) can be written as the following formation,

$$Y(x,t) = \sum_{i=1}^{N}\varphi_i(x)q_i(t) \tag{5}$$

where $\varphi_i(x) = \sin\frac{i\pi x}{L}(i = 1, 2\cdots N)$ is the *ith* modal function of simply supported beam. Substituting equation (5) into Eq. (1) and to Eq. (4), and then multiplying both ends by $\varphi_j(x)$, integration from 0 to l gives following equations by mode orthogonal method and mode superposition method.

$$w_i^2 M_i q_i(t) - M_i\frac{\partial^2 q_i(t)}{\partial t^2} + \frac{c}{m}M_i\frac{\partial q_i(t)}{\partial t} - \left\{\left(m_1 + \frac{m_3}{2}\right)g\right.$$
$$\left. m_1\frac{\partial^2 y_1}{\partial t^2} + m_3\frac{\partial^2 y_3}{2\partial t^2} - I\frac{\partial^2\theta}{\lambda\partial t^2} - m_1 a\frac{\partial Y_1}{\partial x} - m_3 a\frac{\partial Y_1}{2\partial x}\right\}\phi_j(x_1)$$
$$-\left\{\left(m_2 + \frac{m_3}{2}\right)g + m_2\frac{\partial^2 y_2}{\partial t^2} + m_3\frac{\partial^2 y_3}{2\partial t^2} + I\frac{\partial^2\theta}{\lambda\partial t^2}\right.$$
$$\left. - m_2 a\frac{\partial Y_2}{\partial x} - m_3 a\frac{\partial Y_2}{2\partial x}\right\}\phi_j(x_2) = 0 \tag{6}$$

$$m_i\frac{\partial^2 y_i}{\partial t^2}k_i\left(y_i + \sum_{i=1}^{N}\phi_i(x_1)q_i(t)\right)$$
$$+ c_i\left(\frac{\partial y_i}{\partial t} + \sum_{i=1}^{N}\phi_i(x_1)\frac{\partial q_i(t)}{\partial t} + (v + at)\sum_{i=1}^{N}\frac{\partial\phi_i(x_1)}{\partial x}q_i(t)\right)$$
$$- k_j\left(y_3 - \frac{\theta\lambda}{2} - y_i\right) - c_j\left(\frac{\partial y_3}{\partial t} - \lambda\frac{\partial\theta}{2\partial t} - \frac{\partial y_i}{\partial t}\right)$$
$$- m_i a\sum_{i=1}^{N}\frac{\partial\phi_i(x_1)}{\partial x}q_i(t) - \frac{m_3 a}{2}\sum_{i=1}^{N}\frac{\partial\phi_i(x_1)}{\partial x}q_i(t) = 0 \tag{7}$$

$$m_3\frac{\partial^2 y_3}{\partial t^2} + k_3\left(y_3 - \frac{\theta\lambda}{2} - y_1\right) + c_3\left(\frac{\partial y_3}{\partial t} - \frac{\theta\lambda}{2} - \frac{\partial y_1}{\partial t}\right)$$
$$+ k_4\left(y_3 + \frac{\theta\lambda}{2} - y_2\right) + c_4\left(\frac{\partial y_3}{\partial t} + \lambda\frac{\partial\theta}{2\partial t} - \frac{\partial y_2}{\partial t}\right)$$
$$- \frac{m_3\alpha}{2}\left[\sum_{i=1}^{N}\frac{\partial\phi_i(x_1)}{\partial x}q_i(t) + \sum_{i=1}^{N}\frac{\partial\phi_i(x_2)}{\partial x}q_i(t)\right] = 0 \tag{8}$$

$$I\frac{\partial^2\theta}{\partial t^2} - \frac{k_3\lambda}{2}\left(y_3 - \frac{\theta\lambda}{2} - y_1\right) - \frac{c_3\lambda}{2}\left(\frac{\partial y_3}{\partial t} - \lambda\frac{\partial\theta}{2\partial t} - \frac{\partial y_1}{\partial t}\right)$$
$$+ \frac{k_4\lambda}{2}\left(y_3 + \frac{\theta\lambda}{2} - y_2\right) + \frac{c_4\lambda}{2}\left(\frac{\partial y_3}{\partial t} + \lambda\frac{\partial\theta}{2\partial t} - \frac{\partial y_2}{\partial t}\right) = 0 \tag{9}$$

The above equations are general partial differential equation, and the natural frequency of the vehicle-bridge system can be described as [12]

$$w_i = (\frac{i\pi}{L})^2 \sqrt{\frac{EI}{m}} (i = 1, 2 \cdots N) \tag{10}$$

when$i = 1$ and $j = 3$,it is the same as that $i = 2$ and $j = 4$ in equation (2) and equation (7), respectively. The differential equations (6)~(8) can be described in the form of matrix as:

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{P\} \tag{11}$$

where

$$q = [q_1, q_2, q_3, y_1, y_2, y_3, \theta]^T \tag{12}$$

here, q represents the generalized coordinates of vehicles and the bridge. The generalized matrixes $[M]$, $[C]$, $[K]$ vary with the position of vehicles on the continuous bridge. With the vehicles moving on the different span of bridge, the generalized load matrix $[P]$ is generalized load and it changes accordingly when the vehicles move on the different part of bridge. In order to reduce the calculation of the first three order natural frequency of the bridge (6) to (9) q_1, q_2, q_3, y_1, y_2, y_3, θ is a variable coefficient differential equations with variable coefficients for a total of seven equations, and a simple numerical calculation method is adopted in this paper which has been verified in [13]. Introducing the variables $u = [q^T, \dot{q}^T]$, equation (11) can be translated to the following state equation,

$$\dot{u} = Au + B \tag{13}$$

And E represents unit matrixes in which A and

$$A = \begin{bmatrix} 0 & E \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1}P \end{bmatrix} \tag{14}$$

Finally, using Runge-Kutta method the equation (14) can be solved by coding in Matlab, and the numerical examples are carried out in the following section.

3 Numerical Simulation

Taking model parameters from work [14] as $m = 15.3 \times 10^3$ kg/m, $EI = 1.72 \times 10^{11}$ N·m^2, $L = 100$ m, $c = 0$ is vehicle parameters; v_0 is initial velocity of the vehicle, $k_1 = 1.9 \times 10^6$ N/m, $k_2 = 9.5 \times 10^5$ N/m, $k_3 = 4.8 \times 10^5$ N/m, $k_4 = 1.7 \times 10^5$ N/m, $I = 82615.67$ kg·m^2, $c_1 = 0$, $c_2 = 0$, $c_3 = 1.4 \times 10^4$ kg/s, $c_4 = 1.7 \times 10^5$ kg/s, $\lambda = 3.624$ m, $m_3 = 32025$ kg, $m_1 = 945$ kg ,$m_2 = 480$ kg.

The responses of mid-span with different accelerations of moving vehicle are shown in Figure 2–5. From the figures, it can be seen that when the acceleration are 9 m/s^2, 12 m/s^2, 15 m/s^2 and 18 m/s^2, the response of mid-span varies significantly when the velocities are low, for example, $v_0 = 10$ m/s and v_0 =20 m/s. When the velocities are comparably high ($v_0 = 30$ m/s and $v_0 = 40$ m/s) the effect of acceleration is not significant.

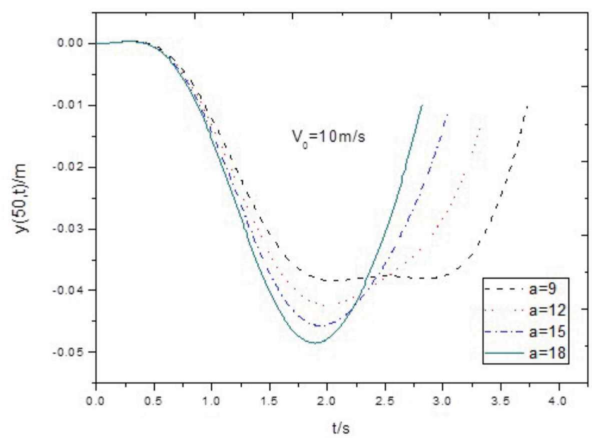

Fig. 2: Effect of acceleration on deflection of 1/2 span ($v_0 = 10$ m/s).

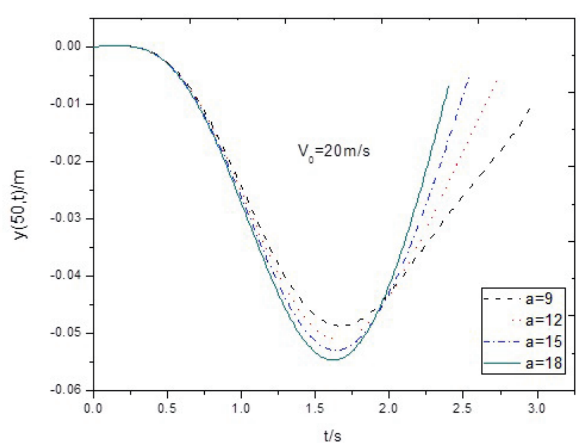

Fig. 3: Effect of acceleration on deflection central part of 1/2 span ($v_0 = 20$ m/s).

Figures 2–5 show the deflections of the mid-span under different moving loads. It can be found that with the same initial speed, the response increases with the increase of the acceleration, which can be seen obviously when $v_0 = 10$ m/s and $v_0 = 20$ m/s. However, with the same initial speed, the deflection of mid-span decrease with a larger deceleration as shown in Figures 6–8. The influence

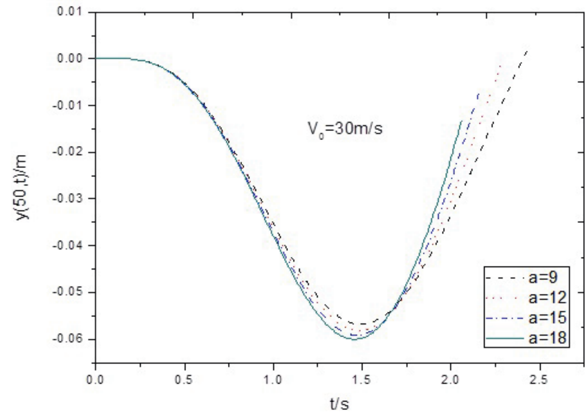

Fig. 4: Effect of acceleration on deflection of 1/2 span (v_0 = 30 m/s).

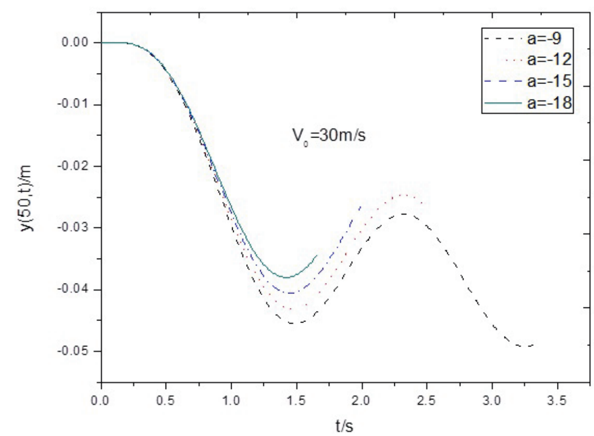

Fig. 7: Effect of deceleration on deflection of 1/2 span (v_0 = 30 m/s).

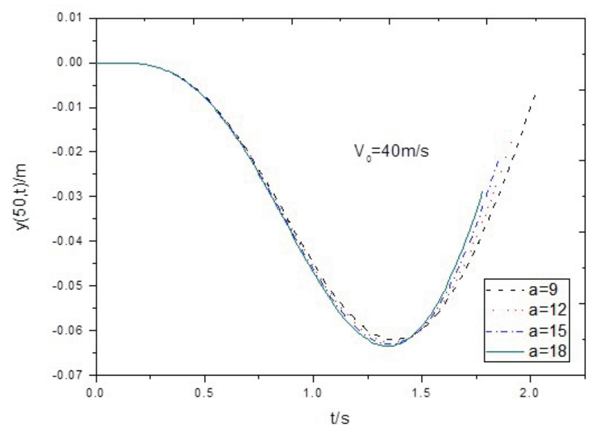

Fig. 5: Effect of acceleration on deflection of 1/2 span (v_0 = 40 m/s).

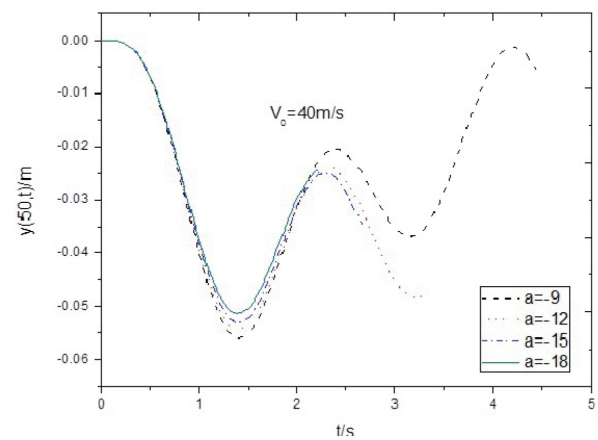

Fig. 8: Effect of deceleration on deflection of 1/2 span (v_0 = 40 m/s).

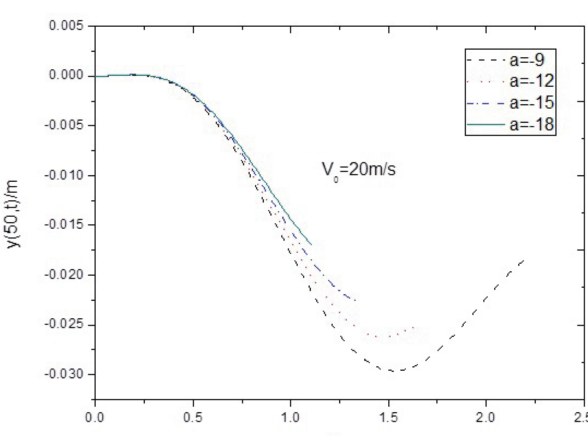

Fig. 6: Effect of deceleration on deflection of 1/2 span (v_0 = 20 m/s).

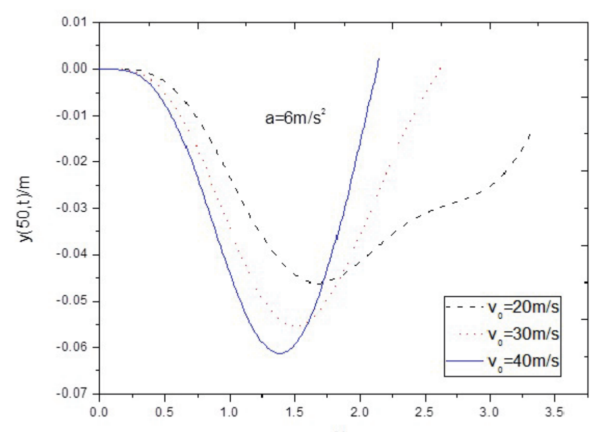

Fig. 9: Effect of initial speed on deflection of 1/2 span (a = 6 m/s^2).

of the initial velocity of the moving vehicle on the deflection of the beam at the same acceleration (a = 6 m/s^2) in Figure 9 and (a = −6 m/s^2) in Figure 10. From the figure,

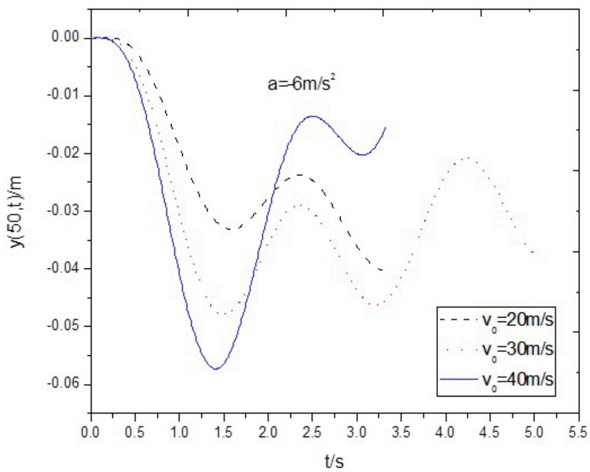

Fig. 10: Effect of initial speed on deflection of 1/2 span (a=−6 m/s^2).

the higher the initial velocity, the greater the deflection of the mid-span.

4 Conclusions and discussion

The coupled vibration of the vehicle bridge system can be attributed to the dynamic response of the system under the action of moving loads. In the establishment of the differential equations of the vibration differential equations of the coupled system with uniform and variable speed, the dynamic effects of the inertia force and acceleration of a uniform transmission vehicle are considered, and a simple numerical calculation method is used to reduce the computation time and the time. In this paper, when the initial velocity of the uniform speed vehicle is invariant, the great acceleration, the bigger deflection of the mid-span. When the initial velocity of uniform speed shift is constant, the absolute value of acceleration is greater, and the deflection of beam is smaller. The deflection of the beam increases with the increase of the initial velocity when the vehicle acceleration is unchanged. The effect of acceleration on the beam's lateral vibration is not negligible. The analysis method and conclusion of this paper can provide a reference for the dynamic characteristics of the bridge.

Nomenclature

I	rotational inertia
m_3	mass of the vehicle
x_2 and x_1	the position of front and rear axle of the vehicle
m	distributed mass
c	damping coefficient
EI	flexural stiffness
L	bridge span
m_2 and m_1	total mass of front and rear axle load
Y_2 and Y_1	vertical displacement of front and rear wheel
k_4, k_3, k_2 and k_1	stiffness of front and rear spring respectively
c_4, c_3, c_2 and c_1	damping coefficient of front and rear spring respectively
y_3, y_2, y_1 and θ	generalized coordinates of the vehicle
λ and a	distance between front and rear wheels and acceleration of the vehicle

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