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Chaos Suppression in Fractional Order Permanent Magnet Synchronous Motor and PI controlled Induction motor by Extended Back stepping Control

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Abstract: In this paper we investigate the control of three-dimensional non-autonomous fractional-order model of a permanent magnet synchronous motor (PMSM) and PI controlled fractional order Induction motor via recursive extended back stepping control technique. A robust generalized weighted controllers are derived to suppress the chaotic oscillations of the fractional order model. As the direct Lyapunov stability analysis of the controller is difficult for a fractional order first derivative, we have derived a new lemma to analyze the stability of the system. Numerical simulations of the proposed chaos suppression methodology are given to prove the analytical results.

Keywords: Permanent Magnet Synchronous Motor; Induction motor; Fractional order systems; Chaos suppression; Extended back stepping control

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1 Introduction

Permanent magnet synchronous motor (PMSM) is increasingly used in efficient AC servo driving control system due to its simple dynamics, high efficiency, high power density and high torque-current ratio. The investigation of chaos in PMSM is a field of active research due to its direct applications in many areas especially for industrial applications in low-medium power range. However, the performance of the PMSM is sensitive to system parameter and external load disturbance in the plant. Some investigations, for example, by Li et al. [1] and Jing et al. [2] show that with certain parameter values, the PMSM displays chaotic behavior. This chaotic behavior of PMSM can lead to performance degradation by causing torque ripples, low frequency oscillations and low performance to speed control. Ataei et al. [3] characterized the complex dynamics of the permanent-magnet synchronous motor (PMSM) model with a non-smooth-air-gap. Harb and Zaher [4] studied chaotic behaviors in PMSM for a certain range of its parameters, and it was eliminated by using optimal Lyapunov exponent methodology. Zribi et al. [5] proposed to use a Lyapunov exponent control algorithm to control the PMSM. Dynamical equations of three time scale brushless DC motor system were presented by Ge and Cheng [6].

In the recent years, the research on fractional order dynamical systems has been receiving increasing attention. It is found that with the help of fractional derivatives, many systems in interdisciplinary fields can be elegantly described [7–9]. Furthermore many integer order chaotic systems of fractional order have been studied widely [10–14]. All the physical phenomena in nature exist in the form of fractional order[15], integer order (classical) differential equation is just a special case of fractional differential equation. The importance of fractional-order models is that they can yield a more accurate description and give a deeper insight into the physical processes underlying a long range memory behavior.

Chaos modelling have applications in many areas in science and engineering [15–17]. Some of the common applications of chaotic systems in science and engineering are chemical reactors, Brusselators, Dynamos, Tokamak systems, biology models, neurology, ecology models, memristive devices, etc. An analysis of saddle-node and Hopf bifurcations in indirect field-oriented control (IFOC) drives due to errors in the estimate of the rotor time constant provides a guideline for setting the gains of PI speed controller in order to avoid Hopf bifurcation [18]. An appropriate setting of the PI speed loop controller permits to

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keep the bifurcations far enough from the operating conditions in the parameter space [8]. It has been proven the occurrence of either co-dimension one bifurcation such as saddle node bifurcation and Hopf bifurcation or co-dimension two such as Bogdanov-Takens and zero-Hopf bifurcation in IFOC induction motors [19–21].

2 Problem formulation and preliminaries

2.1 Fractional order PMSM model

The Non-Linear dynamical dimensionless model of the Permanent Magnet Synchronous Motor (PMSM) is given in [2, 3].

$$\begin{aligned} \dot{x}(t) &= -x(t) + y(t)z(t) \\ \dot{y}(t) &= -y(t) - x(t)z(t) + az(t) \\ \dot{z}(t) &= b \cdot \left[y(t) - z(t)\right] \end{aligned} \tag{1}$$

The system shown in (1) shows chaotic behavior when the parameters are $a = 20; b = 5.46$.

The fractional order model of the PMSM dimension less model shown in (1) can be defined as

$$\begin{aligned} D^{q_1} \cdot x(t) &= -x(t) + y(t)z(t) \\ D^{q_2} \cdot y(t) &= -y(t) - x(t)z(t) + az(t) \\ D^{q_3} \cdot z(t) &= b\left[y(t) - z(t)\right] \end{aligned} \tag{2}$$

where q_1, q_2 *and* q_3 are the fractional orders of the respective states.

For studying the state portraits of the fractional order system (2), the system parameters are chosen as $a = 20$ & $b = 5.46$. Figure 1 shows the 3D state portrait of system (2).

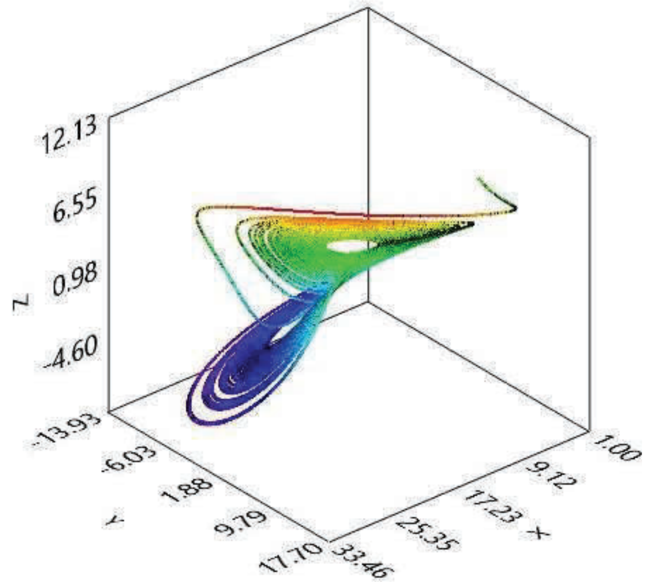

Fig. 1: 3D State portrait of the Fractional order System (2).

2.2 Fractional order induction motor model

In this section we will derive the dimensionless fractional order model of the induction motor. The dimensionless integer order model of a PI speed regulated Current Driven induction motor is given by [18],

$$\begin{aligned} \dot{x} &= -c_1 x + c_2 w - \tfrac{kc_1}{u_2^0} yw \\ \dot{y} &= -c_1 y + c_2 u_2^0 + \tfrac{kc_1}{u_2^0} xw \\ \dot{z} &= -c_3 z - c_4 \left[c_5(yw - zu_2^0) - T_L - \tfrac{c_3}{c_4} w_{ref}\right] \\ \dot{w} &= (k_i - k_p c_3)z - k_p c_4 \left[c_5(yw - xu_2^0) - T_L - \tfrac{c_3}{c_4} w_{ref}\right] \end{aligned} \tag{3}$$

where $c_1 = 13.67; c_2 = 1.56; c_3 = 0.59; c_4 = 1.76; c_5 = 2.86; u_2^0 = 4; k_p = 0.001; k_i = 1; k = 1.5; T_L = 0.5; w_{ref} = 181.1$ and initial conditions are $x_1 = 0; x_2 = 0.4; x_3 = -200; x_4 = 6$.

The fractional order model of the PI controlled induction motor (3) can be given as,

$$\begin{aligned} D^{q_1} x &= -c_1 x + c_2 w - \tfrac{kc_1}{u_2^0} yw \\ D^{q_2} y &= -c_1 y + c_2 u_2^0 + \tfrac{kc_1}{u_2^0} xw \\ D^{q_3} z &= -c_3 z - c_4 \left[c_5(yw - xu_2^0) - T_L - \tfrac{c_3}{c_4} w_{ref}\right] \\ D^{q_4} w &= (k_i - k_p c_3)z - k_p c_4 \left[c_5(yw - x_1 u_2^0) - T_L - \tfrac{c_3}{c_4} w_{ref}\right] \end{aligned} \tag{4}$$

Where q_1, q_2, q_3 *and* q_4 are the fractional orders of the respective states. Figure 2 shows the 3d state portraits of the system (4).

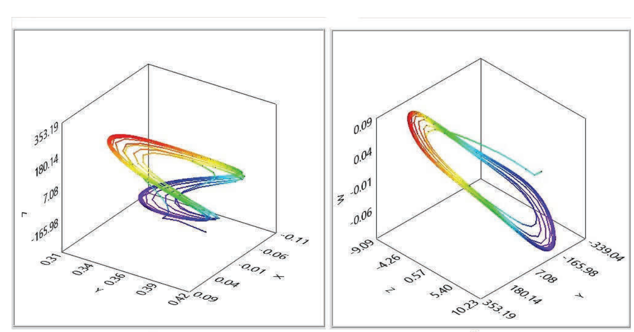

Fig. 2: 3D state portraits of the fractional order induction motor (A-XYZ Plane, B-YZW Plane).

3 Chaos suppression of the fractional order systems using extended back stepping

In this section we propose a recursive extended back stepping controller for controlling the chaotic oscillations of a PMSM system (2) and PI controlled induction motor system(4).

3.1 Fractional order chaos suppression of PMSM system

First, Let us define the fractional order PMSM model (2) with the proposed controllers as,

$$\begin{aligned} D^{q_1}x(t) &= -x(t) + y(t)z(t) + u_x(t) \\ D^{q_2}y(t) &= -y(t) - x(t)z(t) + az(t) + u_y(t) \\ D^{q_3}z(t) &= b\left[y(t) - z(t)\right] + u_z(t) \end{aligned} \tag{5}$$

where $u_x(t), u_y(t), u_z(t)$ are the controllers.

The control errors for the chaos suppression of PMSM system(2) are defined as,

$$\begin{aligned} e_x &= x - x_d \\ e_y &= y - y_d \\ e_z &= z - z_d \end{aligned} \tag{6}$$

where $x_d = f_x(t), y_d = k_x e_x, z_d = k_y e_x + k_z e_y$ and $f(t)$ is a smooth periodic function of time.

Therefore

$$\begin{aligned} e_x &= x - f_x(t) \\ e_y &= y - k_x e_x \\ e_z &= z - k_y e_x - k_z e_y \end{aligned} \tag{7}$$

The fractional derivatives of (7) gives the fractional order error dynamics,

$$\begin{aligned} D^{q_1}e_x &= D^{q_1}x - D^{q_1}f_x(t) \\ D^{q_2}e_y &= D^{q_2}y - k_x D^{q_1}e_x \\ D^{q_3}e_z &= D^{q_3}z - k_y D^{q_1}e_x - k_z D^{q_2}e_y \end{aligned} \tag{8}$$

Substitute (2) in (6)

$$\begin{aligned} D^{q_1}e_x &= -x(t) + y(t)z(t) + u_x(t) - D^{q_1}f_x(t) \\ D^{q_2}e_y &= -y(t) - x(t)z(t) + az(t) + u_y(t) - k_x D^{q_1}e_x \\ D^{q_3}e_z &= b\left[y(t) - z(t)\right] + u_z(t) - k_y D^{q_1}e_x - k_z D^{q_2}e_y \end{aligned} \tag{9}$$

Let us define the controllers for chaos suppression of the PMSM model (2) as,

$$\begin{aligned} u_x(t) &= x(t) - y(t)z(t) - D^{q_1}f_x(t) - k_1 e_x \\ u_y(t) &= y(t) + x(t)z(t) - az(t) - k_2 e_y \\ u_z(t) &= -b\left[y(t) - z(t)\right] - k_3 e_z \end{aligned} \tag{10}$$

3.1.1 Stability of the controller

In order to analyze the stability of the designed control algorithm we use Lyapunov stability theory. The Lyapunov function for the controller (10) and system (2) can be given by (11)

$$V = \frac{1}{2}\left(e_x^2 + e_y^2 + e_z^2\right) \tag{11}$$

Differentiating (11) along the trajectories of (2) we will get the Lyapunov first derivative (12).

$$\frac{dV}{dt} = \frac{1}{2}[e_x(t)\dot{e}_x(t) + e_y(t)\dot{e}_y(t) + e_z(t)\dot{e}_z(t)] \tag{12}$$

By definition of fractional calculus [22, 23],

$$\dot{x}(t) = D_t^{1-q} \cdot D_t^q x(t) \tag{13}$$

By solving (12) with respect to (13) and (2), we get (14).

$$\begin{aligned} \tfrac{dV}{dt} = {} & e_x(t){}_a^cD_t^{1-q1}{}_a^cD_t^{q1}[x - f_x(t)] + \\ & e_y(t){}_a^cD_t^{1-q2}{}_a^cD_t^{q2}[y - k_x e_x] + \\ & e_z(t){}_a^cD_t^{1-q3}{}_a^cD_t^{q3}[z - k_y e_x - k_z e_y] \end{aligned} \tag{14}$$

From (14) it is clear that the calculation of the sign of the first Lyapunov derivative is very difficult. Hence we derive a new lemma to find the sign of the Lyapunov first derivative.

3.1.2 Lemma-1

As defined by if $e(t)$ be a time continuous and derivable function. Then for any time instant $t \geq t_0$,

$$\mathbf{\frac{1}{2}D_t^\alpha e^2(t) \leq e(t) \times D_t^\alpha e(t)} \quad \forall\alpha \in (\mathbf{0}, \mathbf{1}) \tag{15}$$

Proof. To prove expression (15) is true we start with,

$$e(t)D_t^\alpha e(t) - \frac{1}{2}D_t^\alpha e^2(t) \geq 0 \;\forall\alpha \in (0, 1) \tag{16}$$

By definition

$$D_t^\alpha e(t) = \frac{1}{\Gamma(1-\alpha)}\int_{t_0}^{t} \frac{\dot{e}(\tau)}{(t-\tau)^\alpha}d\tau \tag{17}$$

$$\frac{1}{2}D_t^\alpha e^2(t) = \frac{1}{\Gamma(1-\alpha)}\int_{t_0}^{t} \frac{e(\tau)\cdot\dot{e}(\tau)}{(t-\tau)^\alpha}d\tau \tag{18}$$

Modifying (18),

$$\frac{1}{\Gamma(1-\alpha)}\int_{t_0}^{t} \frac{e(t)\cdot\dot{e}(\tau) - e(\tau)\dot{e}(\tau)}{(t-\tau)^\alpha}d\tau \geq 0 \tag{19}$$

Let us assume,

$$E(\tau) = e(t) - e(\tau) \;\&\; \dot{E}(\tau) = -\dot{e}(\tau) \tag{20}$$

Substitute (20) in (19)

$$\frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{E(\tau)\dot{E}(\tau)}{(t-\tau)^{\alpha}} d\tau \geq 0 \tag{21}$$

Integration (21) by parts

$$\frac{1}{\Gamma(1-\alpha)}(t-\tau)^{-\alpha} \cdot \frac{1}{2}E^2(\tau) - \int_{t_0}^{t} \frac{1}{2}E^2(\tau) \cdot \left(\frac{\alpha(t-\tau)^{-\alpha-1}}{\Gamma(1-\alpha)}\right) \leq 0 \tag{22}$$

$$\left[\frac{E^2(\tau)}{2\Gamma(1-\alpha)(t-\tau)^{\alpha}}\right]_{\tau=t} - \left[\frac{E^2(t_0)}{2\Gamma(1-\alpha)(t-t_0)^{\alpha}}\right] - \frac{1}{2}\frac{\alpha}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{E^2(\tau)}{(t-\tau)^{\alpha+1}} d\tau \leq 0 \tag{23}$$

Solving first term of (23) for $\tau = t$

$$\lim_{\tau \to t} \frac{E^2(\tau)}{2\Gamma(1-\alpha)(t-\tau)^{\alpha}} = \frac{1}{2\Gamma(1-\alpha)} \lim_{\tau \to t} \frac{\left[\begin{array}{c} e^2(t) + e^2(\tau) \\ -2e(t)\cdot e(\tau) \end{array}\right]^2}{(t-\tau)^{\alpha}} = \frac{1}{2\Gamma(1-\alpha)} \lim_{\tau \to t} \left[\frac{\begin{array}{c} -2e(t)\dot{e}(\tau)+ \\ 2e(\tau)\cdot\dot{e}(\tau) \end{array}}{-\alpha(t-\tau)^{\alpha-1}}\right] = 0 \tag{24}$$

Equation (24) can be rewritten as

$$\frac{E^2(t_0)}{2\Gamma(1-\alpha)(t-t_0)^{\alpha}} + \frac{\alpha}{2\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{E^2(\tau)}{(t-\tau)^{\alpha+1}} d\tau \geq 0 \tag{25}$$

which clearly holds as α lies between $0 \leq \quad \tau \quad \leq 1$, the r.h.s of the equation (25) will always be a positive value and hence Proved. □

3.1.3 Lyapunov First Derivative using Lemma-1

Applying Lemma-1(15) in equation (10) we get,

$$V \leq -k_1 e_x^2(t) - k_2 e_y^2(t) - k_3 e_z^2(t) \tag{26}$$

Hence (26) is a negative definite function which infers that the system is stable and is valid for any bounded initial conditions.

3.1.4 Numerical Simulations using LabVIEW

The Fractional order PMSM system (2) with the extended back stepping controller (10) are implemented in LabVIEW for numerical analysis and validation. The initial values of the fractional order system (2) are taken as $x(t) = 1, y(t) = 2 \;\& z(t) = 4$. The state trajectories of the controlled fractional-order chaotic system (2) are shown Figure 3, where the controller is switched at t =70 s. The Smooth function $f(t) = 35\sin 0.57t$. It can be clearly observed that the state trajectories follow the smooth periodic function as soon as the controller is introduced which clearly shows that the fractional order system (2) is controlled by the extended back stepping controller. Fig. 3 also shows the evolution of the states of the system (2) with controller (10), with the fractional orders $q_1 = 0.95, q_2 = 0.9\,5 \& q_3 = 0.95$.

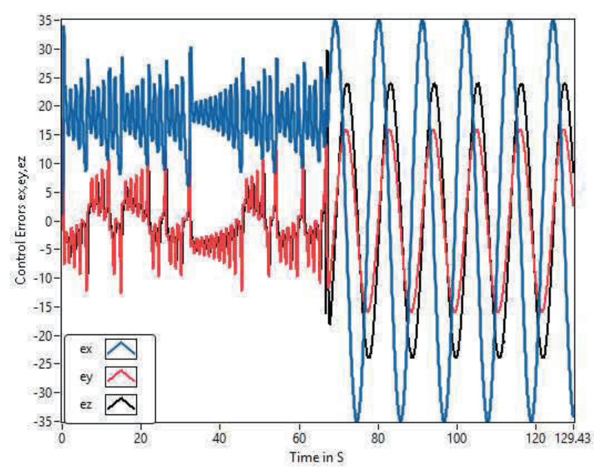

Fig. 3: State trajectories with control in action at $t = 70$ s.

3.2 Fractional order chaos suppression of PI controlled induction motor

Let us define the fractional order PI controlled Induction motor (4) with recursive extended back stepping controllers as,

$$\begin{aligned}
D^{q_1}x &= -c_1 x + c_2 w - \tfrac{kc_1}{u_2^0} yw + u_x(t) \\
D^{q_2}y &= -c_1 y + c_2 u_2^0 + \tfrac{kc_1}{u_2^0} xw + u_y(t) \\
D^{q_3}z &= -c_3 z - c_4\left[c_5(yw - xu_2^0) - T_L - \tfrac{c_3}{c_4} w_{ref}\right] + u_z(t) \\
D^{q_4}w &= (k_i - k_p c_3)z - k_p c_4\left[c_5(yw - x_1 u_2^0) - T_L - \tfrac{c_3}{c_4} w_{ref}\right] \\
&\quad + u_w(t)
\end{aligned} \tag{27}$$

where $u_x(t), u_y(t), u_z(t)$ are the controllers.

The chaos control errors of the system (4) are defined as,

$$\begin{aligned} e_x &= x - x_d \\ e_y &= y - y_d \\ e_z &= z - z_d \\ e_w &= w - w_d \end{aligned} \tag{28}$$

where $x_d = f_x(t), y_d = k_x e_x, z_d = k_y e_x + k_z e_y, w_d = k_w e_x + k_u e_y + k_v e_y$.

The fractional derivatives of (28) yields the fractional order error dynamics (29),

$$\begin{aligned} D^{q_1} e_x &= D^{q_1} x - D^{q_1} f_x(t) \\ D^{q_2} e_y &= D^{q_2} y - k_x D^{q_1} e_x \\ D^{q_3} e_z &= D^{q_3} z - k_y D^{q_1} e_x - k_z D^{q_2} e_y \\ D^{q_4} e_w &= D^{q_4} w - k_w D^{q_1} e_x - k_u D^{q_2} e_y - k_v D^{q_3} e_z \end{aligned} \tag{29}$$

Substitute (27) in (29),

$$\begin{aligned} D^{q_1} e_x &= -c_1 x + c_2 w - \tfrac{kc_1}{u_2^0} yw + u_x(t) - D^{q_1} f_x(t) \\ D^{q_2} e_y &= -c_1 y + c_2 u_2^0 + \tfrac{kc_1}{u_2^0} xw + u_y(t) - k_x D^{q_1} e_x \\ D^{q_3} e_z &= -c_3 z - c_4 \left[c_5 (yw - x u_2^0) - T_L - \tfrac{c_3}{c_4} w_{ref} \right] + u_z(t) \\ &\quad - k_y D^{q_1} e_x - k_z D^{q_2} e_y \\ D^{q_4} e_w &= (k_i - k_p c_3) z - k_p c_4 \left[c_5 (yw - x_1 u_2^0) - T_L - \tfrac{c_3}{c_4} w_{ref} \right] \\ &\quad + u_w(t) - k_w D^{q_1} e_x - k_u D^{q_2} e_y - k_v D^{q_3} e_z \end{aligned} \tag{30}$$

Let us define the recursive extended back stepping controllers as,

$$\begin{aligned} u_x(t) &= c_1 x - c_2 w + \tfrac{kc_1}{u_2^0} yw + D^{q_1} f_x(t) - k_1 e_x \\ u_y(t) &= c_1 y - c_2 u_2^0 - \tfrac{kc_1}{u_2^0} xw - k_2 e_y \\ u_z(t) &= c_3 z + c_4 \left[c_5 (yw - x u_2^0) - T_L - \tfrac{c_3}{c_4} w_{ref} \right] - k_3 e_z \\ u_w(t) &= -(k_i - k_p c_3) z + k_p c_4 \left[c_5 (yw - x_1 u_2^0) - T_L - \tfrac{c_3}{c_4} w_{ref} \right] \\ &\quad - k_4 e_w \end{aligned} \tag{31}$$

3.2.1 Stability of the controller

In order to analyze the stability of the designed control algorithm we use Lyapunov stability theory. The Lyapunov function for the controller (31) and system (4) can be given by (32),

$$V = \frac{1}{2} \left(e_x^2 + e_y^2 + e_z^2 + e_w^2 \right) \tag{32}$$

Differentiating (32) along the trajectories of (4) we will get the Lyapunov first derivative (33).

$$\frac{dV}{dt} = [e_x(t)\dot{e}_x(t) + e_y(t)\dot{e}_y(t) + e_z(t)\dot{e}_z(t) + e_w(t)\dot{e}_w(t)] \tag{33}$$

By definition of fractional calculus [22, 23],

$$\dot{x}(t) = D_t^{1-q} \cdot D_t^q x(t) \tag{34}$$

By solving (33) with respect to (34),

$$\begin{aligned} \tfrac{dV}{dt} &= [e_x(t) D_t^{1-q} \cdot D_t^q e_x(t) + e_y(t) D_t^{1-q} \cdot D_t^q e_y(t) \\ &\quad + e_z(t) D_t^{1-q} \cdot D_t^q e_z(t) + e_w(t) D_t^{1-q} \cdot D_t^q e_w(t)] \end{aligned} \tag{35}$$

From (35) it is clear that the calculation of the sign of the first Lyapunov derivative is very difficult. Hence we use (15) to find the sign of the Lyapunov first derivative.

Using (15) to solve (32),

$$\frac{dV}{dt} = -k_1 e_x^2 - k_2 e_y^2 - k_3 e_z^2 - k_4 e_w^2 \tag{36}$$

Hence (36) is a negative definite function which infers that the system is stable and is valid for any bounded initial conditions.

3.2.2 Numerical simulations using LabVIEW

The fractional order induction motor (4) with the robust adaptive controller (31) is implemented in LabVIEW for numerical analysis and validation. The initial conditions are chosen as $x_1 = 0; x_2 = 0.4; x_3 = -200; x_4 = 6$ and the parameter values are chosen as $c_1 = 13.67; c_2 = 1.56; c_3 = 0.59; c_4 = 1.76; c_5 = 2.86; u_2^0 = 4; k_p = 0.001; k_i = 1; k = 1.5; w_{ref} = 181.1$ and the Fractional orders of the system q_1, q_2, q_3 and q_4 are taken as 0.9.The initial condition for the uncertain load is selected as $T_L = 0.1$. The state trajectories of the controlled fractional-order chaotic system (5) are shown Figure 9, where the controller is switched at $t = 85$ s . It can be clearly observed that the state trajectories converges to zero as soon as the controller is introduced which clearly shows that the fractional order system (4) is controlled by the controller and the states follow the smooth trajectorty of the function $f(t) = 35 \cos 0.57t$. Fig. 4 also shows the evolution of the states of the system (4) with controller (31). As proved from the analytical analysis already presented, the origin of the system for any bounded initial conditions is asymptotically stable.

4 Conclusion and Discussions

This paper investigates control of three-dimensional nonautonomous fractional-order uncertain model of a PMSM and PI controlled fractional order induction motor system using recursive extended back stepping controllers. The chaotic oscillations in both the systems are shown with the respective 3D state portraits. To suppress such chaotic oscillations, we have derived an extended back stepping control technique. The direct Lyapunov stability analysis

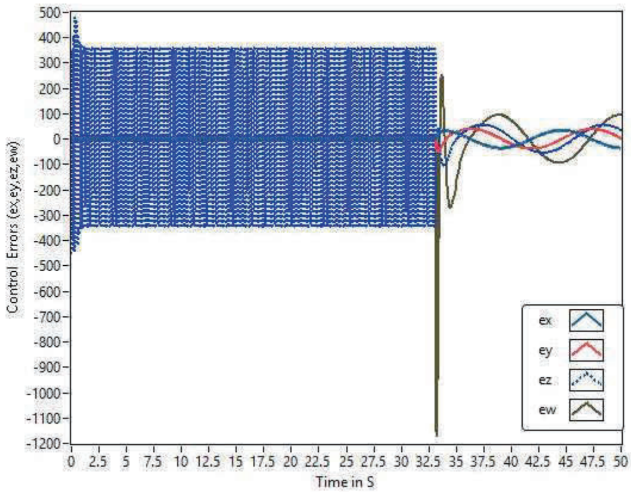

Fig. 4: State trajectories with control in action at $t = 32.5$ s.

of the robust controller is difficult and hence we have derived a new lemma to analyze the stability of the system. The proposed lemma is introduced in the Lyapunov first derivative and thus the parameter estimates are derived. We have also proved with numerical simulations that for the derived controller is asymptotically stable about the origin of the system for any bounded initial conditions.

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