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Solitons and other solutions to the coupled nonlinear Schrödinger type equations

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Abstract: Nonlinear Schrödinger type equations arise from a wide variety of fields, such as fluids, nonlinear optics, the theory of deep water waves, plasma physics, and so on. In this paper, two integration schemes are employed to obtain solitons, periodic waves and other forms of solutions of the coupled nonlinear Schrödinger type equations. The two schemes that are studied in this paper are the Bäcklund transformation of Riccati equation and the trial solution method.

Keywords: Nonlinear evolution equations, Exact solution, The Bäcklund transformation of Riccati equation, Trial solution method

1 Introduction

The Nonlinear Schrödinger (NLS) equation has been derived as a model for weakly nonlinear wave packets in a wide variety of physical systems by means of the perturbative algorithm known as the method of multiple scales. This equation is widely used in many branches of physics and dynamics that it forms a separate class of equations investigated thoroughly by many researchers. Inoué [1] derived coupled nonlinear Schrödinger equations with the same group velocities for the interaction of two wave-packets in an isotropic dielectric material. In hydrodynamics, the coupled NLS equations describe the propagation of two wave packets along a direction in which the group velocity projections overlap. In optics, the propagation of short pulses in birefringent fibers. In atmosphere, the pressure pulses in artery vessels and nonlinear Rossby waves. In a plasma, for instance, the interaction between the high-frequency Langmuir and low-frequency ion-acoustic waves is governed by the coupled NLS equation of the following form

$$iu_t + p_1 u_{xx} = \left(q_1|u|^2 + q_{12}|v|^2\right) u, \tag{1}$$

$$iv_t + p_2 v_{xx} = \left(q_2|v|^2 + q_{21}|u|^2\right) v. \tag{2}$$

These equations also appear in nonlinear optics and geophysical flows [2–11]. In Eqs. (1) and (2), u and v are the complex amplitudes or envelopes of wave packets in two different degrees of freedom of the underlying physical systems, x and t are the spatial and temporal variables, p_1 and p_2 are the dispersion coefficients, q_1 and q_2 are the Landau constants which describe the self-modulation of the wave packets, while q_{12} and q_{21} are the wave-wave interaction coefficients which describe the cross-modulations of the wave packets. The cross-modulation terms are the glue that holds coupled-mode solitary waves together. When both cross-modulation coefficients q_{12} and q_{21} are positive, however, the coupled-mode solitary wave is stable. But when both or even just one of the cross-modulation coefficients is negative, however, the solitary waves fission and separate into two single-mode solitons travelling independently in different modes at different speeds. They are all real parameters and their values vary for different polarizations in nonlinear optics or for different kinds of basic flows in geophysical fluid dynamics.

Recently, a new coupled NLS equation

$$u_{xt} = u_{xx} + \frac{2}{1-\beta^2}|u|^2 u + u(v - w), \tag{3}$$

$$v_t = -\frac{\left(|u|^2\right)_t}{1+\beta} + (1+\beta)\, v_x, \tag{4}$$

$$w_t = \frac{\left(|u|^2\right)_t}{1-\beta} + (1-\beta)\, w_x, \tag{5}$$

was proposed by Ma and Geng via a spectral problem and its auxiliary one [12], where v and w are the real functions

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of spatial variable x and temporal variable t , while u is a complex one and β is a real constant with $|\beta| \neq 1$.

In order to understand the mechanisms of those phenomena, it is necessary to explore their solutions and properties. Solutions for these equations can not only describe the designated problems, but also give more insights on the physical aspects of the problems in the related fields. In recent years, various powerful methods have been presented for finding exact solutions of the NLEEs in mathematical physics, such as transformed rational function method [13, 14], multiple exp-function method [15], extended tanh method [16], trial equation method [17–22], (G'/G) -expansion method [23–25], modified simple equation method [26–28], Q-function method [29, 30] and so on. Lu [31] has introduced a reliable and effective method called the Bäcklund transformation method of Riccati equation to look for new exact solutions of nonlinear fractional PDEs. The Bäcklund transformation method of Riccati equation [20] is based on the assumptions that the exact solutions of NLEEs can be expressed by a polynomial in ψ, such that $\psi = \psi(\xi)$ satisfies the Bäcklund transformation of Riccati equation. In this paper, we apply the Bäcklund transformation of Riccati equation and the trial function approach to obtain exact and soliton solutions of Eqs. (3)–(5).

2 The governing equation

In order to solve Eqs. (3)–(5), the following solution structure is selected [32]

$$u(x,t) = U(\xi)e^{i\phi(x,t)}, \quad v(x,t) = V(\xi), \quad w(x,t) = W(\xi), \tag{6}$$

where the wave variable ξ is given by

$$\xi = k(x - vt), \tag{7}$$

and the phase component $\phi(x,t)$ is defined as

$$\phi(x,t) = \kappa x - \omega t + \theta \tag{8}$$

where κ is the frequency of the solitons while ω represents the wave number and θ is the phase constant. Substituting Eq. (6) into Eqs. (3)–(5), we obtain the following equations

$$(1+c)U'' - \kappa(\kappa+\omega)U + \frac{2}{1-\beta^2}U^3 + U(V - W) = 0, \tag{9}$$

$$V = \frac{-c}{(\beta+1)(\beta+c+1)}U^2, \tag{10}$$

$$W = \frac{c}{(\beta-1)(\beta-c-1)}U^2, \tag{11}$$

where

$$\omega = -\kappa(c+2). \tag{12}$$

Substituting Eqs. (10),(11) and (12) into Eq. (9), we obtain the following ordinary equation

$$U'' + \kappa^2 U - \frac{2}{\beta^2 - (c+1)^2}U^3 = 0, \tag{13}$$

3 Integration Schemes:

In this section, we outline the description of the Bäcklund transformation of Riccati equation method and the trial solution method.

3.1 The Bäcklund transformation of Riccati equation

3.1.1 Description of the method

Recall that the Riccati equation:

$$\phi'(\xi) = \sigma + \phi^2(\xi), \tag{14}$$

which has the following exact solutions

$$\phi = \begin{cases} -\sqrt{-\sigma}\tanh\left(\sqrt{-\sigma}\,\xi\right), & \sigma < 0, \\ -\sqrt{-\sigma}\coth\left(\sqrt{-\sigma}\,\xi\right), & \sigma < 0, \\ \sqrt{\sigma}\tan\left(\sqrt{\sigma}\,\xi\right), & \sigma > 0, \\ -\sqrt{\sigma}\cot\left(\sqrt{\sigma}\,\xi\right), & \sigma > 0, \\ -\frac{1}{\xi+\varpi},\ \varpi = const. & \sigma = 0. \end{cases} \tag{15}$$

Next, let us consider the nonlinear evolution equation (NLEE):

$$F(u, u_t, u_x, u_{xx}, u_{xt}, \ldots) = 0, \tag{16}$$

where $u = u(x,t)$ is an unknown function, F is a polynomial in u and its various partial derivatives u_t, u_x with respect to t, x respectively, in which the highest order derivatives and nonlinear terms are involved.

By using the traveling wave transformation

$$u(x,t) = U(\xi), \quad \xi = k(x - ct), \tag{17}$$

where k, c are constant to be determined later, we can reduce Eq. (16) to a nonlinear ordinary differential equation (NLODE) of the form

$$P\left(U, U', U'', \ldots\right) = 0. \tag{18}$$

Step 1: Suppose that Eq. (18) has the following solution

$$u(\xi) = \sum_{l=0}^{N} a_l \psi^l(\xi), \tag{19}$$

where $a_l(l = 0, \dots, N)$ are constants to be determined and $\psi(\xi)$ comes from the following Bäcklund transformation for the Riccati equation:

$$\psi(\xi) = \frac{-\sigma B + D\,\phi(\xi)}{D + B\,\phi(\xi)}, \tag{20}$$

that is $\psi(\xi)$ satisfies the Riccati equation

$$\psi^{'}(\xi) = \sigma + \psi^2(\xi), \tag{21}$$

where B, D are arbitrary parameters, σ is a constant to be determined and $B \neq 0$, $\phi(\xi)$ are the well-known solutions (15).

Step2 : Balancing the highest order derivatives and nonlinear term in (18) to determine the positive integer N in (19).

Step3 : Substituting the explicit formal solution (19) with (20) into Eq. (18) and setting the coefficients of the powers of $\phi(\xi)$ to be zero, we obtain a system of algebraic equations which can be solved by the Maple or Mathematica to get the unknown constants $a_l(l = 0, \dots, N)$,σ, k and c. Consequently, we obtain the exact solutions of Eq. (16).

3.1.2 Application to the governing equation

In this subsection, the Bäcklund transformation of the Riccati equation is applied to obtain the exact solutions of the governing equation.

Balancing $U^{''}$ with U^3 in Eq. (13), we obtain $N = 1$. Then the solution has the form

$$U(\xi) = a_0 + a_1\left(\frac{-\sigma B + D\,\phi(\xi)}{D + B\,\phi(\xi)}\right). \tag{22}$$

Substituting (22) along with (21) into (13) and then setting the coefficients of $\phi(\xi)$ to be zero, we can obtain a set of algebraic equations

$$\begin{aligned}&\frac{2a_0^3B^3}{-\beta^2+c^2+2c+1} + \frac{a_0B^3c^2k^2}{-\beta^2+c^2+2c+1} + \frac{2a_0B^3ck^2}{-\beta^2+c^2+2c+1}\\ &+ \frac{a_0B^3k^2}{-\beta^2+c^2+2c+1} - \frac{a_0\beta^2B^3k^2}{-\beta^2+c^2+2c+1}\\ &+ \frac{6a_0^2a_1B^2D}{-\beta^2+c^2+2c+1} + a_1B^2Dk^2 + 2a_1B^2D\sigma\\ &+ \frac{6a_0a_1^2BD^2}{-\beta^2+c^2+2c+1} + \frac{2a_1^3D^3}{-\beta^2+c^2+2c+1} + 2a_1D^3 = 0,\end{aligned} \tag{23}$$

$$\begin{aligned}&-\frac{6a_0^2a_1B^3\sigma}{-\beta^2+c^2+2c+1} - a_1B^3k^2\sigma\\ &- 2a_1B^3\sigma^2 - \frac{12a_0a_1^2B^2D\sigma}{-\beta^2+c^2+2c+1} + \frac{6a_0^3B^2D}{-\beta^2+c^2+2c+1}\\ &+ \frac{3a_0B^2c^2Dk^2}{-\beta^2+c^2+2c+1} + \frac{6a_0B^2cDk^2}{-\beta^2+c^2+2c+1} + \frac{3a_0B^2Dk^2}{-\beta^2+c^2+2c+1}\\ &- \frac{3a_0\beta^2B^2Dk^2}{-\beta^2+c^2+2c+1} - \frac{6a_1^3BD^2\sigma}{-\beta^2+c^2+2c+1} + \frac{12a_0^2a_1BD^2}{-\beta^2+c^2+2c+1}\\ &+ 2a_1BD^2k^2 - 2a_1BD^2\sigma + \frac{6a_0a_1^2D^3}{-\beta^2+c^2+2c+1} = 0,\end{aligned} \tag{24}$$

$$\begin{aligned}&\frac{3a_0BD^2k^2}{-\beta^2+c^2+2c+1} + \frac{6a_0BcD^2k^2}{-\beta^2+c^2+2c+1} + \frac{3a_0Bc^2D^2k^2}{-\beta^2+c^2+2c+1}\\ &- \frac{3a_0\beta^2BD^2k^2}{-\beta^2+c^2+2c+1} + \frac{6a_0^3BD^2}{-\beta^2+c^2+2c+1} + a_1D^3k^2\\ &+ 2a_1D^3\sigma - 2a_1B^2Dk^2\sigma + 2a_1B^2D\sigma^2 + \frac{6a_0^2a_1D^3}{-\beta^2+c^2+2c+1}\\ &- \frac{12a_0^2a_1B^2D\sigma}{-\beta^2+c^2+2c+1} - \frac{12a_0a_1^2BD^2\sigma}{-\beta^2+c^2+2c+1}\\ &+ \frac{6a_0a_1^2B^3\sigma^2}{-\beta^2+c^2+2c+1} + \frac{6a_1^3B^2D\sigma^2}{-\beta^2+c^2+2c+1} = 0,\end{aligned} \tag{25}$$

$$\begin{aligned}&\frac{a_0D^3k^2}{-\beta^2+c^2+2c+1} + \frac{2a_0cD^3k^2}{-\beta^2+c^2+2c+1} + \frac{a_0c^2D^3k^2}{-\beta^2+c^2+2c+1}\\ &- \frac{a_0\beta^2D^3k^2}{-\beta^2+c^2+2c+1} + \frac{2a_0^3D^3}{-\beta^2+c^2+2c+1} - a_1BD^2k^2\sigma\\ &- 2a_1BD^2\sigma^2 - 2a_1B^3\sigma^3 - \frac{6a_0^2a_1BD^2\sigma}{-\beta^2+c^2+2c+1}\\ &+ \frac{6a_0a_1^2B^2D\sigma^2}{-\beta^2+c^2+2c+1} - \frac{2a_1^3B^3\sigma^3}{-\beta^2+c^2+2c+1} = 0.\end{aligned} \tag{26}$$

Solving this system with the aid of Mathemtica, we obtain

$$a_0 = 0,\quad a_1 = \pm\sqrt{\beta^2 - (c+1)^2},\quad \sigma = -\frac{\kappa^2}{2}, \tag{27}$$

where κ, k, ω, B, D are arbitrary real constants. The solutions of Eqs. (3)–(5) corresponding to Eq. (27) are

$$u_1(x,t) = \pm\kappa\sqrt{\frac{\beta^2-(c+1)^2}{2}}\left(\frac{\frac{\kappa}{\sqrt{2}}B - D\tanh\left[\frac{\kappa}{\sqrt{2}}\left(x + \left(\frac{\omega}{\kappa}+2\right)t\right)\right]}{D - \frac{\kappa}{\sqrt{2}}B\tanh\left[\frac{\kappa}{\sqrt{2}}\left(x + \left(\frac{\omega}{\kappa}+2\right)t\right)\right]}\right) e^{i(\kappa x - \omega t + \theta)}, \tag{28}$$

$$v_1(x,t) = \frac{-c\kappa^2(\beta - c - 1)}{2(\beta+1)}\left(\frac{\frac{\kappa}{\sqrt{2}}B - D\tanh\left[\frac{\kappa}{\sqrt{2}}\left(x + \left(\frac{\omega}{\kappa}+2\right)t\right)\right]}{D - \frac{\kappa}{\sqrt{2}}B\tanh\left[\frac{\kappa}{\sqrt{2}}\left(x + \left(\frac{\omega}{\kappa}+2\right)t\right)\right]}\right)^2, \tag{29}$$

$$w_1(x,t) = \frac{c\kappa^2(\beta + c + 1)}{2(\beta-1)}\left(\frac{\frac{\kappa}{\sqrt{2}}B - D\tanh\left[\frac{\kappa}{\sqrt{2}}\left(x + \left(\frac{\omega}{\kappa}+2\right)t\right)\right]}{D - \frac{\kappa}{\sqrt{2}}B\tanh\left[\frac{\kappa}{\sqrt{2}}\left(x + \left(\frac{\omega}{\kappa}+2\right)t\right)\right]}\right)^2, \tag{30}$$

and

$$u_2(x,t) = \pm\kappa\sqrt{\frac{\beta^2-(c+1)^2}{2}}\left(\frac{\frac{\kappa}{\sqrt{2}}B - D\coth\left[\frac{\kappa}{\sqrt{2}}\left(x+\left(\frac{\omega}{\kappa}+2\right)t\right)\right]}{D-\frac{\kappa}{\sqrt{2}}B\coth\left[\frac{\kappa}{\sqrt{2}}\left(x+\left(\frac{\omega}{\kappa}+2\right)t\right)\right]}\right) e^{i(\kappa x-\omega t+\theta)}, \tag{31}$$

$$v_2(x,t) = \frac{-c\kappa^2(\beta-c-1)}{2(\beta+1)}\left(\frac{\frac{\kappa}{\sqrt{2}}B - D\coth\left[\frac{\kappa}{\sqrt{2}}\left(x+\left(\frac{\omega}{\kappa}+2\right)t\right)\right]}{D-\frac{\kappa}{\sqrt{2}}B\coth\left[\frac{\kappa}{\sqrt{2}}\left(x+\left(\frac{\omega}{\kappa}+2\right)t\right)\right]}\right)^2, \tag{32}$$

$$w_2(x,t) = \frac{c\kappa^2(\beta+c+1)}{2(\beta-1)}\left(\frac{\frac{\kappa}{\sqrt{2}}B - D\coth\left[\frac{\kappa}{\sqrt{2}}\left(x+\left(\frac{\omega}{\kappa}+2\right)t\right)\right]}{D-\frac{\kappa}{\sqrt{2}}B\coth\left[\frac{\kappa}{\sqrt{2}}\left(x+\left(\frac{\omega}{\kappa}+2\right)t\right)\right]}\right)^2. \tag{33}$$

3.2 Trial equation approach

3.2.1 Description of the method

In this subsection we outline the main steps of the trial equation method as following

Step 1. Take the trial equation

$$\left(u'\right)^2 = F(u) = \sum_{l=0}^{s} a_l u^l, \tag{34}$$

where a_l, $(l = 0, 1, \ldots, s)$ are constants to be determined. Substituting Eq. (34) and other derivative terms such as u'' or u''' and so on into Eq. (18) yields a polynomial $G(u)$ of u . According to the balance principle we can determine the value of s . Setting the coefficients of $G(u)$ to zero, we get a system of algebraic equations. Solving this system, we shall determine c, k and values of $a_0, a_1, \ldots, a_s$.

Step 2. Rewrite Eq. (34) by the integral form

$$\pm(\xi-\xi_0) = \int \frac{1}{\sqrt{F(u)}}du. \tag{35}$$

According to the complete discrimination system of the polynomial, we classify the roots of $F(u)$, and solve the integral equation (35). Thus we obtain the exact solutions to Eq. (16).

3.2.2 Application to the governing equation

The trial equation method is applied to obtain the exact solutions of the governing equation. Balancing U'' with U^3 in Eq. (13), then we get $s = 4$. Using the solution procedure of the trial equation method, we obtain the system of algebraic equations as follows:

$$\frac{a_1}{2} = 0, \tag{36}$$

$$a_2 + \kappa^2 = 0, \tag{37}$$

$$\frac{3a_3}{2} = 0, \tag{38}$$

$$2a_4 - \frac{2}{\beta^2-(c+1)^2} = 0. \tag{39}$$

Solving the above system of algebraic equations, we obtain the following results:

$$a_1 = 0, \quad a_2 = -\kappa^2, \quad a_3 = 0, \quad a_4 = \frac{1}{\beta^2-(c+1)^2}. \tag{40}$$

Substituting these results into Eqs. (34) and (35), we get

$$\pm(\xi-\xi_0) = \int \frac{dU}{\sqrt{a_0 - \kappa^2U^2 + \frac{1}{\beta^2-(c+1)^2}U^4}}. \tag{41}$$

where a_0 is an arbitrary real constant. Now, we discuss two cases as following:

Case1. If we set $a_0 = 0$ in Eq. (41) and integrating with respect to u, we get the following solutions of Eqs. (3)–(5)

$$u_1(x,t) = \pm\kappa\sqrt{\beta^2-(c+1)^2}\,\sec\left[\kappa\left(x+\left(\frac{\omega}{\kappa}+2\right)t-\xi_0\right)\right] e^{i(\kappa x-\omega t+\theta)}, \tag{42}$$

$$v_1(x,t) = \frac{-c\kappa^2(\beta-c-1)}{\beta+1}\sec^2\left[\kappa\left(x+\left(\frac{\omega}{\kappa}+2\right)t-\xi_0\right)\right], \tag{43}$$

$$w_1(x,t) = \frac{c\kappa^2(\beta+c+1)}{\beta-1}\sec^2\left[\kappa\left(x+\left(\frac{\omega}{\kappa}+2\right)t-\xi_0\right)\right], \tag{44}$$

and

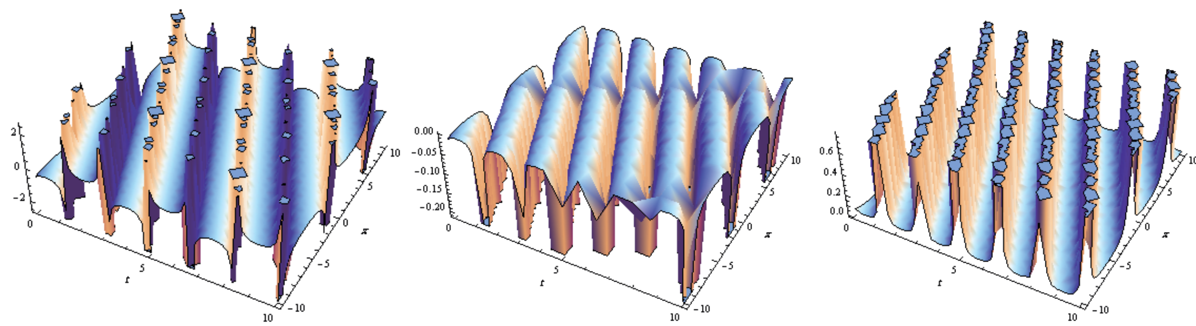
$$u_2(x,t) = \mp\kappa\sqrt{\beta^2-(c+1)^2}\,\csc\left[\kappa\left(x+\left(\frac{\omega}{\kappa}+2\right)t-\xi_0\right)\right] e^{i(\kappa x-\omega t+\theta)}, \tag{45}$$

$$v_2(x,t) = \frac{-c\kappa^2(\beta-c-1)}{\beta+1}\csc^2\left[\kappa\left(x+\left(\frac{\omega}{\kappa}+2\right)t-\xi_0\right)\right], \tag{46}$$

$$w_2(x,t) = \frac{c\kappa^2(\beta+c+1)}{\beta-1}\csc^2\left[\kappa\left(x+\left(\frac{\omega}{\kappa}+2\right)t-\xi_0\right)\right]. \tag{47}$$

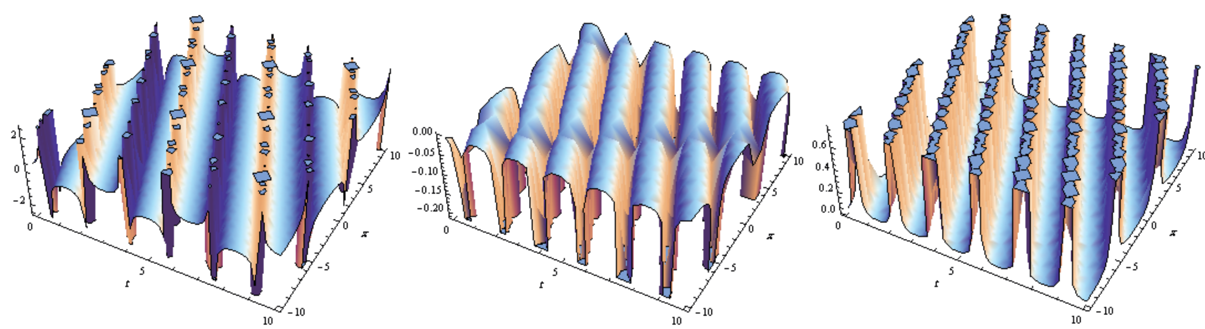
These solutions are singular periodic wave solutions.

Case2. If we set $a_0 = \frac{k^4(\beta^2-(c+1)^2)}{4}$ in Eq. (41) and integrating with resect to u, we get the following solutions of Eqs. (3)–(5)



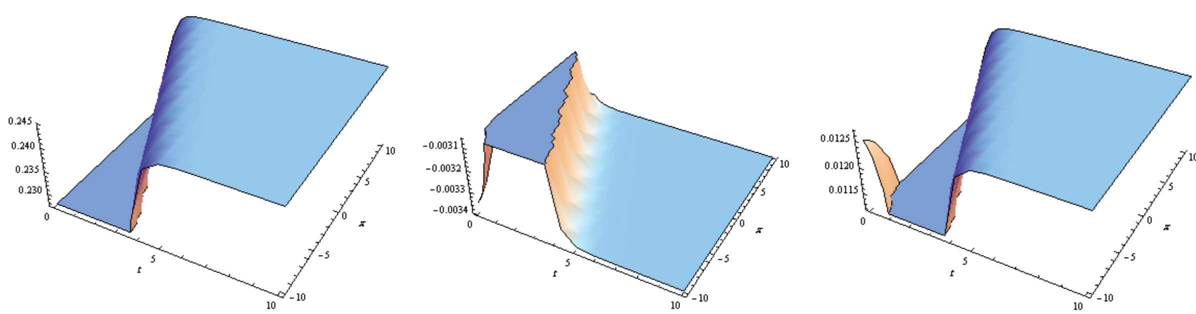
(a) (b) (c)

Fig. 1: Plot of the singular periodic wave solutions (a) $|u_1|$, (b) v_1, (c) w_1, parameters $\beta = \sqrt{3}$, $c = 0.5$, $\kappa = 0.4$, $\xi_0 = 0$, $\omega = 1$.



(a) (b) (c)

Fig. 2: Plot of the singular periodic wave solutions (a) $|u_2|$, (b) v_2, (c) w_2, parameters $\beta = \sqrt{3}$, $c = 0.5$, $\kappa = 0.4$, $\xi_0 = 0$, $\omega = 1$.



(a) (b) (c)

Fig. 3: Plot of the topological (kink-shaped) solutions (a) $|u_3|$, (b) v_3, (c) w_3, parameters $\beta = \sqrt{3}$, $c = 0.5$, $\kappa = 0.4$, $\xi_0 = 0$, $\omega = 1$.

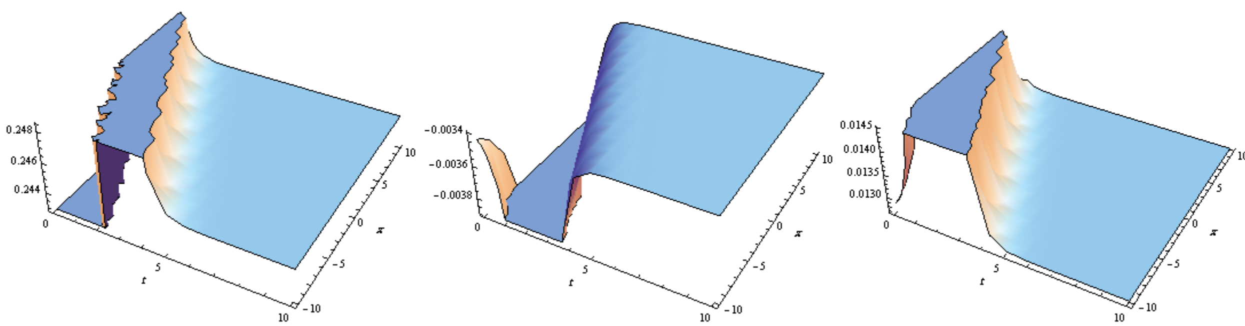
$$u_3(x,t) = \pm\kappa\sqrt{\frac{\beta^2-(c+1)^2}{2}}\tanh\left[\frac{\kappa}{\sqrt{2}}\left(x+\left(\frac{\omega}{\kappa}+2\right)t-\xi_0\right)\right] e^{i(\kappa x-\omega t+\theta)}, \tag{48}$$

$$v_3(x,t) = \frac{-c\kappa^2(\beta-c-1)}{2(\beta+1)}\tanh^2\left[\frac{\kappa}{\sqrt{2}}\left(x+\left(\frac{\omega}{\kappa}+2\right)t-\xi_0\right)\right], \tag{49}$$

$$w_3(x,t) = \frac{c\kappa^2(\beta+c+1)}{2(\beta-1)}\tanh^2\left[\frac{\kappa}{\sqrt{2}}\left(x+\left(\frac{\omega}{\kappa}+2\right)t-\xi_0\right)\right], \tag{50}$$

and

$$u_4(x,t) = \pm\kappa\sqrt{\frac{\beta^2-(c+1)^2}{2}}\coth\left[\frac{\kappa}{\sqrt{2}}\left(x+\left(\frac{\omega}{\kappa}+2\right)t-\xi_0\right)\right] e^{i(\kappa x-\omega t+\theta)}, \tag{51}$$



(a) (b) (c)

Fig. 4: Plot of the singular soliton solutions (a) $|u_4|$, (b) v_4, (c) w_4, parameters $\beta = \sqrt{3}$, $c = 0.5$, $\kappa = 0.4$, $\xi_0 = 0$, $\omega = 1$.

$$v_4(x,t) = \frac{-c\kappa^2(\beta - c - 1)}{2(\beta + 1)} \coth^2\left[\frac{\kappa}{\sqrt{2}}\left(x + \left(\frac{\omega}{\kappa} + 2\right)t - \xi_0\right)\right], \quad (52)$$

$$w_4(x,t) = \frac{c\kappa^2(\beta + c + 1)}{2(\beta - 1)} \coth^2\left[\frac{\kappa}{\sqrt{2}}\left(x + \left(\frac{\omega}{\kappa} + 2\right)t - \xi_0\right)\right]. \quad (53)$$

Theses solutions are topological and singular soliton solutions.

Remark-1
We can derive solutions (48)–(50) and (51)–(53) from solutions (28)–(30) and (31)–(33) by putting $B = 0$ and $D = 0$ into (28)–(30) or putting $D = 0$ and $B = 0$ into (31)–(33).

Remark-2
The Figures 1,2,3 and 4 show that the coupled NLS equation has singular periodic wave solutions, topological soliton solutions and singular soliton solutions. It is clear that it has no solitary wave solutions.

4 Conclusions

Many powerful methods are used in solitary waves theory to examine exact soliton solutions for NLEEs. In this paper, we studied the new application of the the Bäcklund transformation of Riccati equation and the trial function approach to derive new soliton solutions of the coupled nonlinear Schrödinger type equations. These methods are not only efficient, but also have the merit of being widely applicable. The obtained results are new and show that the proposed methods are direct, effective and can be applied to many other NLEEs in mathematical physics.

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