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Thermal Analysis of porous fin with uniform magnetic field using Adomian decomposition Sumudu transform method

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Abstract: In this paper, we consider a Roseland approximation to radiate heat transfer, Darcy's model to simulate the flow in porous media and finite-length fin with insulated tip to study the thermal performance and to predict the temperature distribution in a vertical isothermal surface. The energy balance equations of the porous fin with several temperature dependent properties are solved using the Adomian Decomposition Sumudu Transform Method (ADSTM). The effects of various thermophysical parameters, such as the convection-conduction parameter, Surface-ambient radiation parameter, Rayleigh numbers and Hartman number are determined. The results obtained from the ADSTM are further compared with the fourth–fifth order Runge– Kutta–Fehlberg method and Least Square Method(LSM) (Hoshyar et al. 2016 [1])) to determine the accuracy of the solution.

Keywords: Adomian Decomposition Sumudu Transform Method, Porous fin, Magneto hydrodynamic, Fractional Differential Equation

1 Introduction

The rate of heat transfer is mostly depends upon the temperature variation involving the surface and the surrounding fluids, an existing surface area and heat transfer coefficient. However, this necessity is frequently justified through the high cost of the high thermal conductivity metals. Magnetohydrodynamics (MHD) is the study of the magnetic properties of electrically conducting fluids in different porous geometries that is of considerable attention due to its frequent occurrence in geothermal, industrial and technological applications. The theoretical MHD's study has been a subject of large interest owing to its widespread applications, such as petroleum industries, plasma studies, the boundary layer control in aerodynamics, MHD power generators, crystal growth, and cooling of nuclear reactors.

Numerous studies concerned with the problem of MHD free convection flow in nonporous and porous media have been published by several authors. Takhar et al. [2] discussed the various effects of suction, buoyancy forces, magnetic field, and localized heating for the mixed convection flow on a heated plate in vertical direction. Abbas and Hayat [3] studied the radiation effects on MHD flow in a pore space and viscous fluid with heat transfer. Taklifi et al. [4] studied the effect of MHD on the entire heat transfer from a porous fin connected to a vertical isothermal surface. Kundu and Bhanja [5] considered different models of predictions for determination of the performance and optimum dimensions of porous fins and developed an analytical model. Das and Ooi [6] studied unknown and possible combination of parameters in a naturally convective porous fin subjected to a given temperature requirement. They have estimated different parameters of fin for solving an inverse problem involving the simulated annealing. Bhanja et al. [7] studied the temperature distribution, performance parameters and heat transfer rate through a porous pin fin in natural convection condition by using Adomian decomposition method (ADM). Kundu and Lee [8] developed an analysis for ASFs that considered radiative heat transfer and heat generated by a nuclear reactor through linearization of the radiation terms and solved by exact and approximate analytical methods. Das [9] studied forward and inverse solutions of a conductive, convective and radiative cylindrical porous fin. Ravikumar et al. [10] investigated the heat and mass transfer effects on MHD flow of viscous incompressible and electrically conducting fluid through a nonhomogeneous porous medium in the presence of oscillatory suction velocity and heat source. Das and Prasad [11] studied simultaneous inverse prediction of two parameters such as the porosity and thermal diffusivity of the fluid in a porous fin and used the differential evolution (DE)-based optimization technique

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for prediction of the parameters. Singh et al. [12] studied thermal investigation of a porous stepped fin which made from different ceramic porous materials (Al and SiC) having temperature-dependent internal heat generation. Hatami et al. [13] studied response surface methodology (RSM) based on central composite design (CCD) and applied to find an optimization design of finned type heat exchangers (HEX) to recover waste heat from the exhaust of a diesel engine. Hatami et al. [14] studied two cases of heat exchangers (HEXs) to recover the exhaust waste heat. Rahimi-Gorji et al. used [15] the porous media approach and the Galerkin method to investigate the heat transfer for the microchannel heat sink (MCHS) cooled by different nanofluids. Ghasemi at al. [16] used Collocation Method (CM) and Optimal Homotopy Asymptotic Method (OHAM) to simulate flow analysis for a third grade non-Newtonian blood in porous arteries in presence of magnetic field. Patel and Meher [17–19] discussed the heat transfer, temperature distribution and efficiency of different types of fins with or without internal heat generation.

Now a days nonlinear fractional order types of problems and phenomenon plays an essential role in engineering, physics, applied mathematics, and new branches of science specially heat transfer problems. Several approximate analytical techniques such as Variational iteration method [20], Homotopy perturbation method [20, 21], Least Square Method [1, 22–24], Differential Transform Method [25–27], Adomian decomposition method [28], have been used to solve such type of problems.

In this paper, the energy balance equation is modeled through a nonlinear fractional order differential equation and ADSTM is applied to find the series solution for temperature field of a rectangular porous fin with multiple nonlinearities.

2 Mathematical Formulation

Here, a rectangular porous fin is considered (Fig. 1) having constant cross sectional area, width W, length L and thickness t with the following assumptions

- The fin is made of porous material that allows the flow of infiltrate through it.
- The porous medium is isotropic, homogeneous and saturated with a single-phase fluid.
- A uniform magnetic field is applied in the direction of $y-$ axis having the temperature inside the fin is a function of x only.
- The Darcy's model is used to study the flow velocity in a porous medium with negligible effect of the imposed and induced magnetic field, and induced electrical field due to polarization effect.

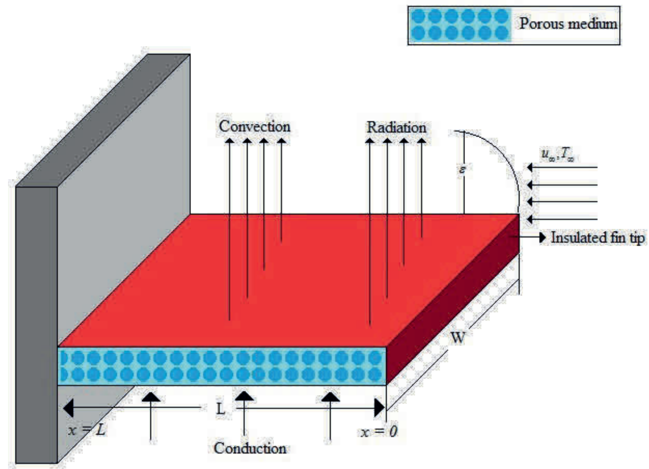


Fig. 1: The geometry of a rectangular fin profile.

The one-dimensional energy balance equation at steady state condition to the slice segment of the fin thickness ΔX is given by [1],

$$\begin{aligned} q(x) - q(x+\Delta x) =& \bar{m} c_p \left(T(x) - T(\infty)\right) \\ &+ hP\Delta x(1-\varepsilon)\left(T(x) - T(\infty)\right) + \frac{(J_c \times J_c)}{\sigma} \\ &+ P\Delta x \sigma_{st} \bar{\varepsilon}\left(T(x)^4 - \frac{y}{\bar{\varepsilon}} T(\infty)^4\right) \end{aligned} \tag{1}$$

where J_c is conduction current intensity, and it is defined by

$$J_c = \sigma\left(V \times B + E\right) \tag{2}$$

and, J is total current intensity defined by

$$J = J_c + \rho_{\bar{\varepsilon}} V$$

The mass flow rate of the fluid $\bar{m}$ passing through the porous material which is stated as

$$\bar{m} = \rho w \bar{v}_w \Delta x \tag{3}$$

The passage velocity $\bar{v}_w$ be supposed to estimated from the consideration of the flow in porous medium, then Darcy's model yields,

$$\bar{v}_w = \frac{k\beta g(T(x) - T_\infty)}{v} \tag{4}$$

The relation between conduction and radiation at the base of fin can be defined as

$$q_{fin\ base} = q_{radiation} + q_{Conduction} \tag{5}$$

using Fourier's law of conduction and the radiation heat flux term, the Rosseland diffusion approximation can be defined as

$$q_{Conduction} = -k_{eff} A_b \frac{dT}{dx}, \quad q_{radiation} = -\frac{4\sigma_{st}}{3\beta_r}\frac{dT^4}{dx} \tag{6}$$

Substituting Eq. (2)–(6) in Eq. (1), it gives

$$\begin{aligned} \frac{d}{dx}\left[\frac{dT}{dx} + \frac{4\sigma}{3\beta_r k_{eff}}\frac{dT^4}{dx}\right] = & \frac{\rho c_p g k \beta}{b v k_{eff}}\left(T(x) - T(\infty)\right)^2 \\ & + \frac{hP(1-\varepsilon)}{k_{eff}}\left(T(x) - T(\infty)\right) \\ & + \frac{(J_c \times J_c)}{\sigma k_{eff} A_b} \\ & + \frac{P\sigma_{st}\bar{\varepsilon}}{k_{eff} A_b}\left(T(x)^4 - T(\infty)^4\right) \end{aligned} \tag{7}$$

With the neglect of magnetic field and the induced current, the electromagnetic force in Eq. (1) takes the form $\frac{(J_c \times J_c)}{\sigma} = \sigma B_0^2 u^2$.

Using the dimensionless parameters

$$\theta = \frac{(T(x) - T_\infty)}{(T_b - T_\infty)}, \ \zeta = \frac{x}{L}, \ \theta_b = \frac{T_b}{T_\infty}, R_a = \frac{gk\beta b(T_b - T_\infty)}{yvk_r},$$
$$N_c = \frac{Pbh}{k_{eff} A}, \ N_r = \frac{4\sigma_{st} L T_\infty^3}{k_{eff}} H = \frac{\sigma B_0^2 u^2}{k_0 A}, \ R_d = \frac{4\sigma_{st} T_\infty^3}{3\beta_r k_{eff}} \tag{8}$$

Eq. (7) gives

$$\frac{d^2\theta}{d\zeta^2} - \frac{R_a}{(1+4R_d)}\theta^2 - \frac{(N_c(1-\varepsilon) + N_r + H)}{(1+4R_d)}\theta = 0 \tag{9}$$

with the boundary conditions

$$\left.\frac{d\theta}{d\zeta}\right|_{\zeta=1} = 0 \ and \ \theta|_{\zeta=0} = 1. \tag{10}$$

where R_a, is a modified Rayleigh number, N_c, is a convection–conduction parameter, N_r, is a Surface-ambient radiation parameter, H, is Hartman parameters, R_d, is Radiation–conduction parameter and and ε, is porosity.

In this paper, we considered and studied finite-length fin with insulated tip, so that there won't be any heat transfer at the insulated tip.

3 Mathematical Formulation of ADSTM

Consider a fractional order nonlinear nonhomogeneous differential equation as

$$D_\zeta^\alpha \theta(\zeta) + R\theta(\zeta) + N\theta(\zeta) = g(\zeta), \quad \text{with} \ n-1 < \alpha \le n \tag{11}$$

with initial condition

$$\theta(0) = K \tag{12}$$

where D_ζ^α is the Caputo fractional derivative of the function $\theta(\zeta)$, R is reminder term or linear differential operator, N represents the general nonlinear differential operator, and $g(\zeta)$ is the source term.

Taking Sumudu Transform (denoted by S) on both sides of Eq. (11), it obtain

$$S[D_\zeta^\alpha \theta(\zeta)] + S[R\theta(\zeta)] + S[N\theta(\zeta)] = S[g(\zeta)] \tag{13}$$

By using Sumudu transform of Caputo fractional derivative [33, 34], Eq. (11) can be written as

$$S[\theta(\zeta)] = \theta(0) + u^\alpha S[g(\zeta)] - u^\alpha S[R\theta(\zeta) + N\theta(\zeta)] \tag{14}$$

By taking inverse Sumudu Transform on both sides of Eq. (14), it gives

$$\theta(\zeta) = G(\zeta) - S^{-1}\left[u^\alpha S[R\theta(\zeta) + N\theta(\zeta)]\right] \tag{15}$$

where $G(\zeta) = S^{-1}\left[\theta(0) + u^\alpha S\left[g(\zeta)\right]\right]$ represents the term arising from the source term and the prescribed initial conditions.

Now by applying the ADM [28], the approximate series solution of Eq. (15) can be written as

$$\theta(\zeta) = \sum_{n=0}^{\infty} \lambda^n \theta_n(\zeta), \tag{16}$$

and the nonlinear term in Eq. (15) can be written as the summation of Adomian polynomials as

$$N\theta(\zeta) = \sum_{n=0}^{\infty} \lambda^n A_n(\theta), \tag{17}$$

where the Adomian polynomials $A_n(\theta)$ arising in differential equation can be defined as

$$A_n(\theta_0, \theta_1, \theta_2, \ldots, \theta_n) = \frac{1}{n!}\frac{\partial^n}{\partial\lambda^n}\left[N\left(\sum_{i=0}^{\infty}\lambda^i \theta_i\right)\right]_{\lambda=0}, \tag{18}$$

for $n = 0, 1, 2, \ldots$

Substituting Eqs. (16) and (17) in Eq. (15) and applying the inverse Sumudu Transform, we find

$$\sum_{n=0}^{\infty}\lambda^n \theta_n(\zeta) = G(\zeta) - \lambda\left[S^{-1}\left[u^\alpha S\left[R\sum_{n=0}^{\infty}\lambda^n\theta_n(\zeta) + \sum_{n=0}^{\infty}\lambda^n A_n(\theta)\right]\right]\right] \tag{19}$$

The resulting Eq. (19) is the coupling of the Sumudu Transform and the Adomian Decomposition Method.
Now by equating the coefficients of like powers of λ, in Eq. (19), the following iterated terms can be obtained as

$$\begin{aligned} \lambda^0 : \theta_0(\zeta) &= G(\zeta) \\ \lambda^1 : \theta_1(\zeta) &= -S^{-1}\left[u^\alpha S\left[R\theta_0(\zeta) + A_0(\theta)\right]\right] \\ \lambda^2 : \theta_2(\zeta) &= -S^{-1}\left[u^\alpha S\left[R\theta_1(\zeta) + A_1(\theta)\right]\right] \\ \lambda^3 : \theta_3(\zeta) &= -S^{-1}\left[u^\alpha S\left[R\theta_2(\zeta) + A_2(\theta)\right]\right] \\ &\vdots \end{aligned} \tag{20}$$

By considering this above procedure, $\theta_n(\zeta)$ can be completely obtained implies the series solution can be determined subsequently and hence the resulted approximate analytical solution $\sum_{n=0}^{M} \theta_n(\zeta)$ converges to the exact solution $\theta(\zeta)$ as $M \to \infty$.

4 Application of ADSTM for finding solution for porous fin having uniform magnetic field

In the last two decades, fractional calculus starts to intervene significantly in engineering, physics, economics, etc [29]. It is due to new possibilities in which fractional calculus brings into the modelling of various problems [30, 31]. Therefore, In this work, to understand the anomalous behavior of this system, the energy balance Eq. (9) fractionalize into fractional order ($\alpha > 0$), and applied Adomian decomposition Sumudu transform method in order to find the fin temperature distribution in a rectangular porous fin as,

$$\frac{d^\alpha \theta}{d\zeta^\alpha} - \frac{R_a}{(1+4R_d)}\theta^2 - \frac{(N_c(1-\varepsilon)+N_r+H)}{(1+4R_d)}\theta = 0 \tag{21}$$
$$1 < \alpha \le 2 \text{ and } 0 \le \zeta \le 1$$

and the boundary conditions given in Eq. (10).
Now, by applying Sumudu transform (denoted by S)[32] on both sides of Eq. (21), it obtains

$$S\left[\frac{d^\alpha \theta}{d\zeta^\alpha}\right] = S\left[\frac{R_a}{(1+4R_d)}\theta^2\right] + S\left[\frac{(N_c(1-\varepsilon)+N_r+H)}{(1+4R_d)}\theta\right] \tag{22}$$

By using Sumudu transform of Caputo fractional derivative [33, 34], we can write

$$\begin{aligned} \frac{S\left[\theta(\zeta)\right]}{u^\alpha} - \frac{\theta(0)}{u^\alpha} - \frac{\theta'(0)}{u^{\alpha-1}} =& S\left[\frac{R_a}{(1+4R_d)}\theta^2\right] \\ &+ S\left[\frac{(N_c(1-\varepsilon)+N_r+H)}{(1+4R_d)}\theta\right] \end{aligned} \tag{23}$$

Using initial and boundary conditions, and Inverse Sumudu transform, we get

$$\begin{aligned} \theta(\zeta) =& 1 + K\zeta + S^{-1}\left[u^\alpha S\left[\frac{R_a}{(1+4R_d)}\theta^2\right]\right] \\ &+ \left[u^\alpha S\left[\frac{(N_c(1-\varepsilon)+N_r+H)}{(1+4R_d)}\theta\right]\right] \end{aligned} \tag{24}$$

By using the Adomian decomposition method in Eq. (24), it gives

$$\begin{aligned} &\sum_{n=0}^{\infty} \lambda^n \theta_n(\zeta) = \\ &1 + K\zeta + \left[S^{-1}\left[u^\alpha S\left[\frac{R_a}{(1+4R_d)}\sum_{n=0}^{\infty}\lambda^n A_n(\theta)\right]\right]\right] \\ &+ \left[S^{-1}\left[u^\alpha S\left[\frac{(N_c(1-\varepsilon)+N_r+H)}{(1+4R_d)}\sum_{n=0}^{\infty}\lambda^n \theta_n(\zeta)\right]\right]\right] \end{aligned} \tag{25}$$

where $A_n(\theta)$'s are the Adomian's polynomial that represents nonlinear terms to be determined. The first few Adomian polynomials are obtained by using Eq. (18),

$$\begin{aligned} A_0 &= (\theta_0)^2 \\ A_1 &= 2\theta_0\theta_1 \\ A_2 &= 2\theta_0\theta_2 + (\theta_1)^2 \\ A_3 &= 2\theta_0\theta_3 + 2\theta_1\theta_2 \\ &\vdots \end{aligned}$$

On comparing the coefficients of like power of λ in Eq. (25), the iterated components can be expressed as,

$$\begin{aligned} &\theta_0(\zeta) = 1 + K\zeta \\ &\theta_{n+1}(\zeta) = S^{-1}\left[u^\alpha S\left[\frac{R_a}{(1+4R_d)}A_n(\theta)\right.\right. \\ &\left.\left.+\frac{(N_c(1-\varepsilon)+N_r+H)}{(1+4R_d)}\theta_n(\zeta)\right]\right], \quad n > 1. \end{aligned} \tag{26}$$

and it gives,

$$\begin{aligned} \theta_0(\zeta) =& 1 + K\zeta \\ \theta_1(\zeta) =& \frac{R_a}{(1+4R_d)}\left(\frac{\zeta^\alpha}{\Gamma(\alpha+1)} + \frac{2K\zeta^{\alpha+1}}{\Gamma(\alpha+2)} + \frac{\Gamma(3)K^2\zeta^{\alpha+2}}{\Gamma(\alpha+2)}\right) \\ &+ \frac{(N_c(1-E)+N_r+H)K}{(1+4R_d)}\left(\frac{\zeta^\alpha}{\Gamma(\alpha+1)} + \frac{K\zeta^{\alpha+1}}{\Gamma(\alpha+2)}\right) \\ &\vdots \end{aligned}$$

Hence, the temperature field for the rectangular porous profile fin is obtained in expressions of finite series,

$$\theta = \sum_{k=0}^{m} \theta_k(\zeta) = \theta_0(\zeta) + \theta_1(\zeta) + \theta_2(\zeta) + \ldots \tag{27}$$

Table 1: Comparison of ADSTM solution, LSM [1] solution and the numerical method for dimensionless temperature $\theta(\zeta)$ and for dimensionless parameters $N_r = 0.3,\ N_c = 0.4,\ H = 0.9,\ R_d = 0.5$ and $R_a = 0.1$.

X	ADSTM	LSM [1]	NM	Absolute error between ADSTM and NM	Absolute error between LSM [1] and NM
0.0	1.000000000	1.000000000	1.000000000	0.000000000	0.000000000
0.1	0.956987943	0.956987665	0.956988020	0.000000077	0.000000355
0.2	0.919132442	0.919132060	0.919132513	0.000000071	0.000000453
0.3	0.886217887	0.886217828	0.886217964	0.000000077	0.000000136
0.4	0.858057590	0.858057970	0.858057690	0.000000100	0.000000280
0.5	0.834492402	0.834492969	0.834492509	0.000000107	0.000000460
0.6	0.815389550	0.815389906	0.815389668	0.000000118	0.000000238
0.7	0.800641676	0.800641589	0.800641805	0.000000129	0.000000216
0.8	0.790166063	0.790165668	0.790166187	0.000000124	0.000000519
0.9	0.783904049	0.783903760	0.783904154	0.000000105	0.000000394
1.0	0.781820594	0.781820569	0.781820729	0.000000135	0.000000160

and the value of θ can be evaluated if the temperature at the fin tip K is known, and it must lie in the interval [0, 1]. The constant, K can be determined by applying Newton-Raphson method.

5 Fin Efficiency

Fin performance can also be characterized by fin efficiency and it can be defined as

$$\eta = \frac{q_f}{q_{max}} = \frac{(1+4R_d)\left.\frac{d\theta}{d\zeta}\right|_{\zeta=1}}{(N_c(1-\varepsilon)+N_r)+H+R_a} \tag{28}$$

6 Results and Discussion

Here, we discusses the temperature distribution in a rectangular porous fin with the consideration of uniform magnetic field. The effects of different dimensionless parameters such as modified Rayleigh number, R_a, convection–conduction parameter, N_c, Surface-ambient radiation parameter, N_r, Hartman parameters, H and Radiation–conduction parameter, R_d, on temperature distribution are investigated during heat transfer in a rectangular porous fin.

The parameter R_a measures the ratio of thermal convection to diffusion, when there is a balance between buoyancy and Lorentz forces, and it is the determining parameter for the flow. The parameter N_c indicates that the strength of convection versus conduction and the parameter N_r, measures the strength of surface radiation versus conduction. The values of N_c and N_r range between 0 (ideal fin of infinite thermal conductivity) and 2 for most of the fins. The parameter H measures the ratio of electromagnetic force to the viscous force.

Table 1 discusses the numerical results obtained for temperature distribution by using ADSTM and the obtained results has been compared with the standard numerical results obtained by Runge–Kutta method and Least Square Method (LSM)[1]. The absolute error has been discussed to test the accuracy of the present method and the obtained results shows that the ADSTM results is very close to the numerical results obtained by Runge–Kutta method and LSM [1].

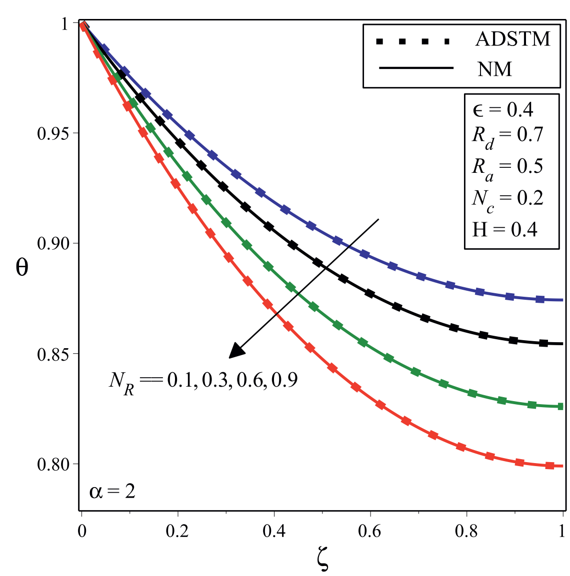


Fig. 2: Effect of surface-ambient radiation parameter, N_r and comparison between the ADSTM with the numerical solution for classical order $\alpha = 2.0$.

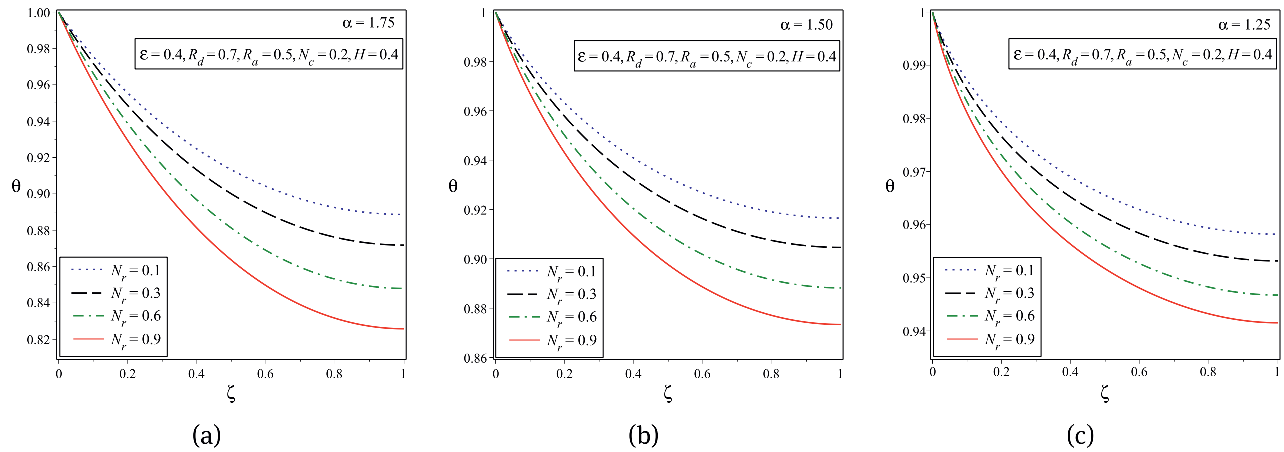


Fig. 3: The ADSTM solutions for a different value of N_r and for (a) $\alpha = 1.75$ (b) $\alpha = 1.5$ (c) $\alpha = 1.25$.

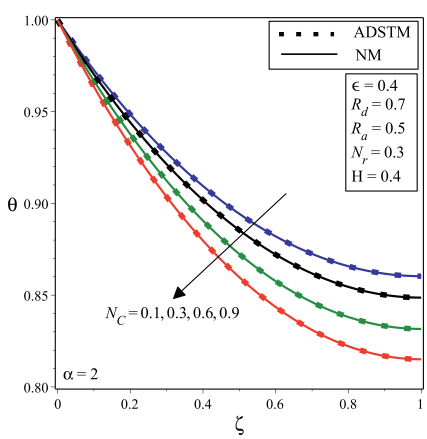


Fig. 4: Effect of convection–conduction parameter, N_c and comparison between the ADSTM with the numerical solution for classical order $\alpha = 2.0$.

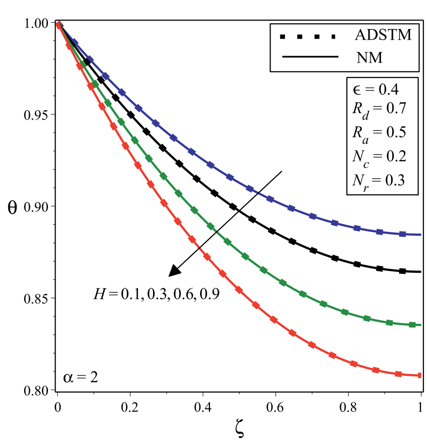
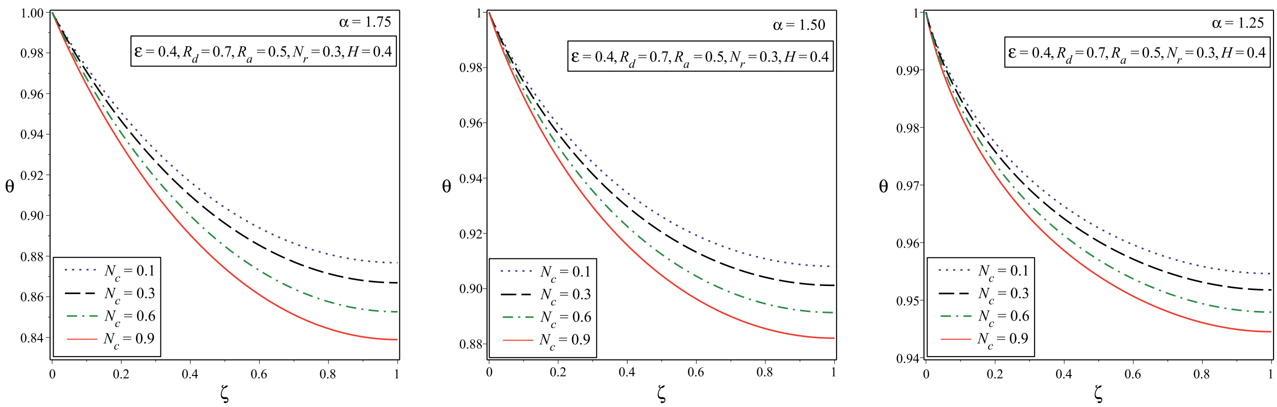


Fig. 5: Effect of Hartman parameters, H and comparison between the ADSTM with the numerical solution for classical order $\alpha = 2.0$.

Fig. 2 discusses the variation of temperature distribution for $\varepsilon = 0.4$, $R_d = 0.7, R_a = 0.5, N_c = 0.2, H = 0.4$ and $\alpha = 2$ with different values of N_r. It shows that the fin temperature decreases with N_r results, strong cooling implies lesser the radiant temperature distribution within the fin. Similarly, Fig. 3 discusses the variation of temperature distribution with different N_r and for different fractional values $\alpha = 1.75,\ 1.5,\ 1.25$ results the behavior of the solution curve is closer to the integer order solution implies the solution curve is valid for the fractional order energy balance equation.

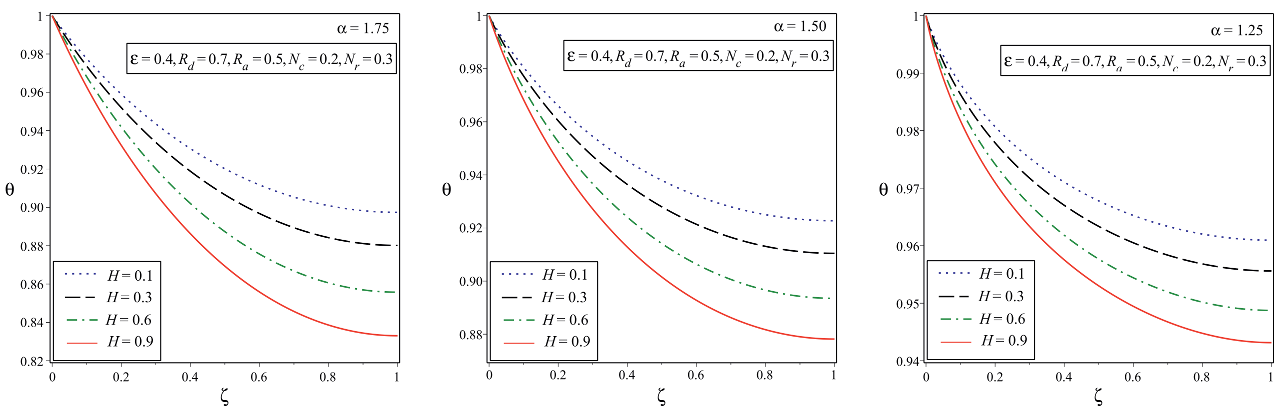
To study the effect of varying the convection-conduction parameter N_c on the performance of the fins, Fig. 4 and 6 was plotted. Fig. 4 discusses the variation of temperature distribution for $\varepsilon = 0.4$, $R_d = 0.7, R_a = 0.5, N_r = 0.3, H = 0.4$ and for classical $\alpha = 2$ with different values of N_c. It shows that the fin temperature decreases with N_c increases results, a decline in fin temperature causes a stronger decrease in local temperature of insulated tip fin. Similarly, Fig. 6 discusses the variation of temperature distribution with different N_c and for different fractional values $\alpha = 1.75, 1.5, 1.25$ results the behavior of the solution curve is closer to the integer order solution implies the solution curve is valid for the fractional order energy balance equation.

Figs. 5 and 7 depicts the results of dimensionless temperature variation for parameter H and for classical order solution $\alpha = 2$. The values of H are varied from $0.1, 0.3, 0.6$ and 0.9, while remaining parameters are kept at, $\varepsilon = 0.4$, $R_d = 0.7, R_a = 0.5, N_c = 0.3$ and $N_r = 0.3$. It shows that the fin temperature be decreases with the increase of the parametric values of H and for fractional order $\alpha = 1.75, 1.5, 1.25$ respectively.



(a) (b) (c)

Fig. 6: The ADSTM solutions for a different value of N_c and for (a) $\alpha = 1.75$ (b) $\alpha = 1.5$ (c) $\alpha = 1.25$.



(a) (b) (c)

Fig. 7: The ADSTM solutions for a different value of H and for (a) $\alpha = 1.75$ (b) $\alpha = 1.5$ (c) $\alpha = 1.25$.

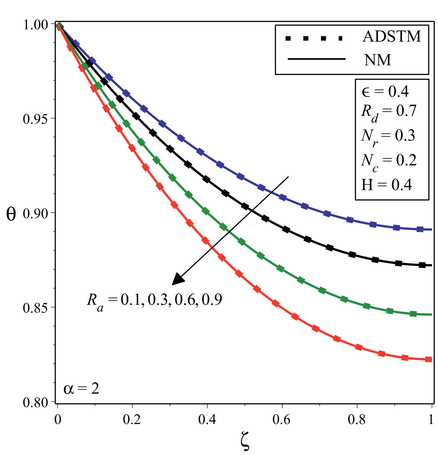


Fig. 8: Effect of Radiation–conduction parameter, R_a and comparison between the ADSTM with the numerical solution for classical order $\alpha = 2.0$.

Figs. 8 and 9 depicts the results of dimensionless temperature variation for different parameters R_a and for classical order solution $\alpha = 2$. The values of R_a are varied from 0.1, 0.3, 0.6 and 0.9, while remaining parameters are kept at, $\varepsilon = 0.4$, $R_d = 0.7$, $H = 0.4$, $N_c = 0.3$ and $N_r = 0.3$. It shows that the fin temperature be decreases with the increase of the parametric values of Ra results stronger thermal convection implies decline in fin temperature and for fractional order $\alpha = 1.75, 1.5, 1.25$ respectively. Accordingly, Fig. 8 clearly demonstrates that the radiation–conduction parameter has a minimum effect on the fins surface temperature for the rectangular porous fin.

Figs. 10(a) illustrate the variation of fin efficiency with R_a with the effect of different parameters R_d, H, N_r and N_c by keeping its parametric values fixed which shows that the fin efficiency be decreases as R_a increases and porosity decreases. In Fig. 10(b), we have plotted the fin tip temperature as a function of the radiation-conduction parameter R_d for fixed parametric values R_a, H, N_r and N_c and for the

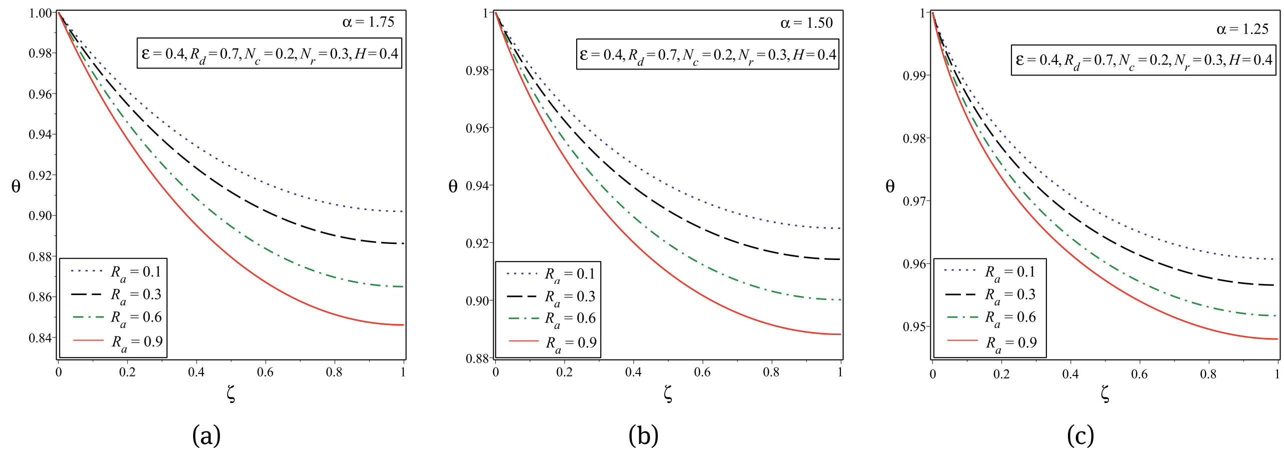


Fig. 9: The ADSTM solutions for a different value of R_a and for (a) $\alpha = 1.75$ (b) $\alpha = 1.5$ (c) $\alpha = 1.25$.

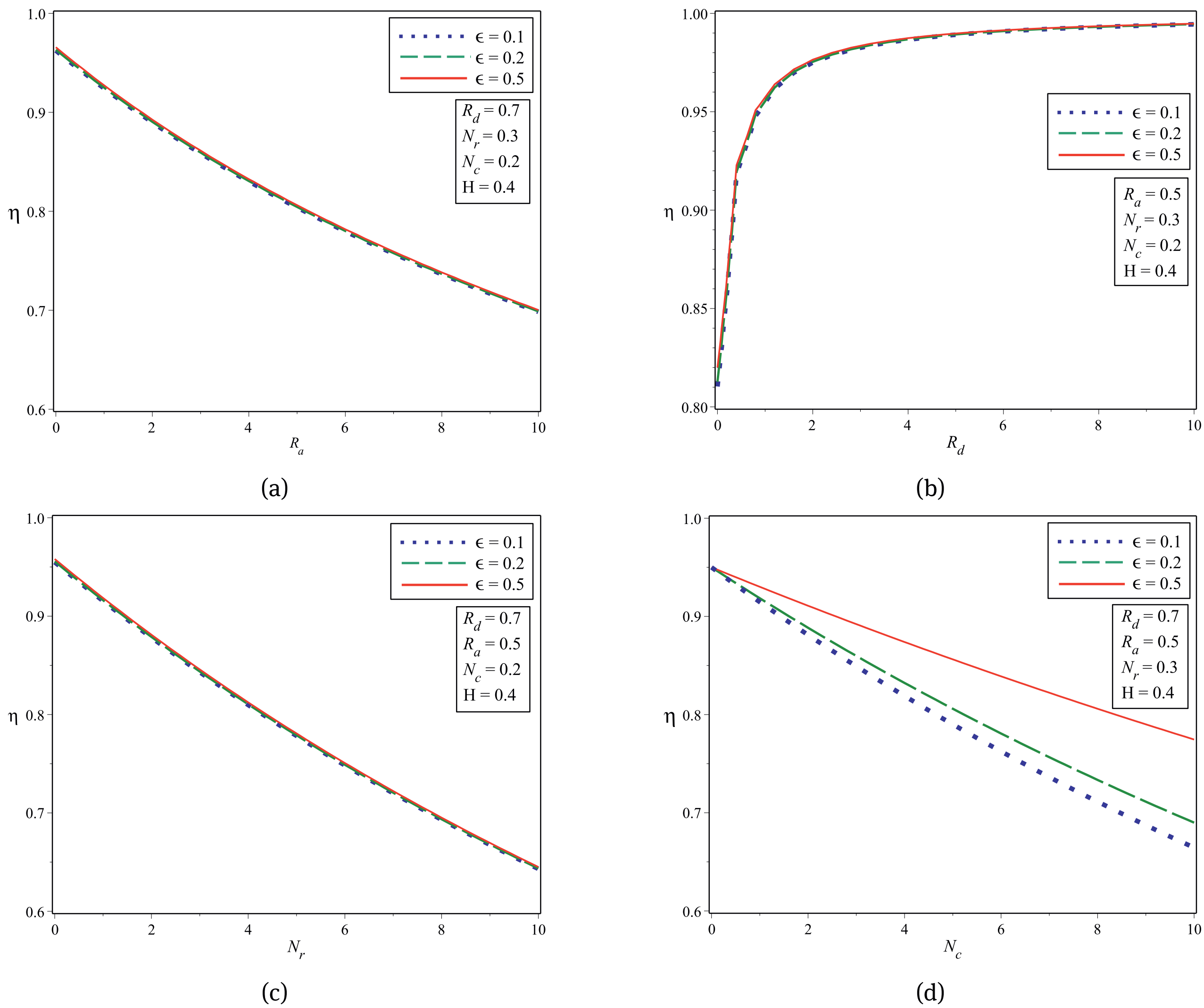


Fig. 10: The variation of efficiency with variation of (a)R_a (b)R_d (c)N_r (d)N_c for different porosity ϵ.

different porosity parameter ϵ. When radiation is stronger, it results in lower fin tip temperatures. Figs. 10(c) illustrate the variation of fin efficiency with N_r with the effect of different parameters R_a, R_d H and N_c by keeping its parametric values fixed which shows that the fin efficiency be decreases as N_r increases and porosity decreases. Figs. 10(d) illustrate the variation of fin efficiency with N_c with the effect of different parameters R_a, R_d H and N_r by keeping its parametric values fixed which shows that the fin efficiency be decreases as N_c increases and porosity decreases.

7 Conclusions

In this study, the solution of a rectangular porous fin with a uniform magnetic field in a vertical isothermal surface obtained by using the ADSTM. A dimensionless expression for the temperature distribution and fin efficiency has been derived and discussed the effects of different parameters on porous fin. Analytical and numerical results for the temperature distribution are presented through the graphs and the table in various values of the parameter. Finally, the ADSTM results has been compared with the fourth–fifth order Runge–Kutta–Fehlberg method and Least Square Method(LSM).

References

[1] H. A. Hoshyar, D. D. Ganji and A. R. Majidian, Least Square Method for Porous Fin in the Presence of Uniform Magnetic Field, Journal of Applied Fluid Mechanics, 2016, vol. 9, pp. 661–668.

[2] A. J. Chamkha, H. S. Takhar, G. Nat, Effect of thermophysical quantities on the natural convection flow of gases over a vertical cone, Heat Mass Transfer, 2004, vol. 40, pp. 835–841.

[3] Z. Abbas, T. Hayat, Radiation effects on MHD flow in poros space, Heat and Mass Transfer, 2008, vol. 51, pp.1024–1033.

[4] A. Taklifi, C. Aghanajafi and H. Akrami, The Effect of MHD on a Porous Fin Attached to a Vertical Isothermal Surface, Transport in Porous Media, 2010, vol. 85(1), pp. 215–231.

[5] Balaram Kundu, Dipankar Bhanja, An analytical prediction for performance and optimum design analysis of porous fins, International Journal of Refrigeration, 2011, vol. 34, pp. 337–352.

[6] Ranjan Das, K.T. Ooi, Predicting multiple combination of parameters for designing a porous fin subjected to a given temperature requirement, Energy Conversion and Management, 2013, vol. 66, pp. 211–219.

[7] Dipankar Bhanja, Balaram Kundu, Pabitra Kumar Mandal, Thermal analysis of porous pin fin used for electronic cooling, Procedia Engineering, 2013, vol. 64, pp. 956–965.

[8] Balaram Kundu, Kwan-Soo Lee, Analytical tools for calculating the maximum heat transfer of annular stepped fins with internal heat generation and radiation effects, Energy, 2014, vol. 76, 733–748.

[9] Ranjan Das, Forward and inverse solutions of a conductive, convective and radiative cylindrical porous fin, Energy Conversion and Management, 2014, vol. 87, pp. 96–106.

[10] V. Ravikumar, M. C. Raju and G. S. S. Raju, Combined effects of heat absorption and MHD on convective Rivlin-Ericksen flow past a semi-infinite vertical porous plate with variable temperature and suction, 2014, vol. 5(3), 867–875.

[11] Ranjan Das, Dilip K. Prasad, Prediction of porosity and thermal diffusivity in a porous fin using differential evolution algorithm, Swarm and Evolutionary Computation, 2015, vol. 23, pp. 27–39.

[12] Kuljeet Singh, Ranjan Das, Balaram Kundu, Approximate Analytical Method for Porous Stepped Fins with Temperature-Dependent Heat Transfer Parameters. Journal of Thermo physics and Heat Transfer, 2016, vol. 30, pp. 661–672.

[13] M. Hatami, M. Jafaryar, D. D. Ganji, M. Gorji-Bandpy, Optimization of finned-tube heat exchangers for diesel exhaust waste heat recovery using CFD and CCD techniques, International Communications in Heat and Mass Transfer, 2014, vol. 57, pp. 254–263.

[14] M. Hatami, D. D. Ganji, M. Gorji-Bandpy, Numerical study of finned type heat exchangers for ICEs exhaust waste heat recovery, Case Studies in Thermal Engineering, 2014, vol. 4, pp. 53–64.

[15] M. Rahimi-Gorji, O. Pourmehran, M. Hatami, D.D. Ganji, Statistical optimization of microchannel heat sink (MCHS) geometry cooled by different nanofluids using RSM analysis, The European Physical Journal Plus, 2015, vol. 130(2), pp. 1–21.

[16] S. E. Ghasemi, M. Hatami, A. K. Sarokolaie, D. D. Ganji, Study on blood flow containing nanoparticles through porous arteries in presence of magnetic field using analytical methods, Physica E: Low-dimensional Systems and Nanostructures, 2015, vol. 70, pp. 146–156.

[17] T. Patel, R. Meher, A study on temperature distribution, efficiency and effectiveness of longitudinal porous fins by using adomian decomposition sumudu transform method. Procedia Engineering, 2015, vol. 127, pp.751–758.

[18] T. Patel, R. Meher, Adomian Decomposition Sumudu Transform Method for Solving Fully Nonlinear Fractional Order Power-Law Fin-Type Problems, International Journal of Mathematics and Computation, 2015, vol. 27, pp. 7–16.

[19] T. Patel, R. Meher, Adomian decomposition sumudu transform method for solving a solid and porous fin with temperature dependent internal heat generation, SpringerPlus, 2016, vol. 5, pp. 1–18.

[20] D. D. Ganji, Y. Rostamiyan, I. Rahimi Petroudi and M. Khazayi Nejad, Analytical investigation of nonlinear model arising in heat transfer through the porous fin, Thermal sciences, 2014, vol. 18, pp. 409-417.

[21] J. Singh, D. Kumar, A. Kilicman, Homotopy perturbation method for fractional gas dynamics equation using Sumudu transform, Abstract and Applied Analysis, 2013, vol. 2013, pp. 1–8.

[22] M. Hatami, D. D. Ganji, Investigation of refrigeration efficiency for fully wet circular porous fins with variable sections by combined heat and mass transfer analysis, International Journal of

Refrigeration, 2014, vol. 40 pp. 140–151.
[23] M. Hatami, D. D. Ganji, Thermal behavior of longitudinal convective–radiative porous fins with different section shapes and ceramic materials (SiC and Si3N4), Ceramics International, 2014, vol. 40(5), pp. 6765–6775.
[24] M. Hatami, G. R. M. Ahangar, D. D. Ganji, K. Boubaker, Refrigeration efficiency analysis for fully wet semi-spherical porous fins. Energy Conversion and Management, 2014, vol. 84, pp. 533–540.
[25] S. E. Ghasemi, M. Hatami, D. D. Ganji, Thermal analysis of convective fin with temperature-dependent thermal conductivity and heat generation, Case Studies in Thermal Engineering, 2014, vol. 4, pp. 1–8
[26] S. E. Ghasemi, P. Valipour, M. Hatami, D. D. Ganji, Heat transfer study on solid and porous convective fins with temperature-dependent heat generation using efficient analytical method, Journal of Central South University, 2014, vol. 21(12), pp. 4592–4598.
[27] A.R. Ahmadi, A. Zahmatkesh, M. Hatami, D.D. Ganji, A comprehensive analysis of the flow and heat transfer for a nanofluid over an unsteady stretching flat plate, Powder Technology, 2014, vol. 258, pp. 125–133.
[28] G. Adomian, Solving Frontier Problems in Physics: The Decomposition Method, Kluwer Academic Publishers, 1988, Dordrecht.
[29] K. S. Miller, B. Ross, An introduction to the fractional calculus and fractional differential equations, 1993, Wiley, New York.
[30] H. Eltayeb, A. Kilicman, Application of Sumudu decomposition method to solve nonlinear system of partial differential equations, Abstract and Applied Analysis, 2012, vol. 2012, pp. 1–6.
[31] H. Eltayeb, A. Kilicman, S. Mesloub, Application of Sumudu decomposition method to solve nonlinear system Volterra integro-differential equations, Abstract and Applied Analysis, 2014, vol. 2014, pp. 1–6.
[32] G. K. Watugala, Sumudu transform a new integral transform to solve differential equations and control engineering problems, Mathematical Engineering in Industry, 1998, vol. 6, pp. 319–329.
[33] F. B. M. Belgacem, A. A. Karabali and S. L. Kalla, Analytical investigations of the sumudu transform and applications to integral production equations, Mathematical Problems in Engineering, 2003, vol. 2003, pp. 103–108.
[34] F. Belgacem and A. Karaballi, Sumudu transform fundamental properties investigations and applications, Journal of Applied Mathematics and Stochastic Analysis, 2005, vol. 2006, 1–23.