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Semitrailer Steering Control for Improved Articulated Vehicle Manoeuvrability and Stability

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Abstract: Articulated heavy vehicles have some specific performance limitations and safety risks due to their special dynamic characteristics. They show poor manoeuvrability at low speeds and may lose their stability in different manners at high speeds. In this study, the potential of active steering control of the semitrailer on manoeuvrability and stability of tractor-semitrailer combinations is investigated. A linear bicycle model and a nonlinear version are used for controller design and vehicle dynamic simulation in MATLAB environment. The Linear Quadratic Regulator optimal state feedback control is used to minimise the tracking error at low-speed, and regulate Rearward Amplification ratio and roll at high-speed. Quantum Particle Swarm Optimisation is used for optimising the weighting factors. Three different control algorithms are introduced and it is demonstrated through simulations that the vehicle with the proposed steering control exhibits desirable improvements compared to the baseline vehicle.

Keywords: Articulated Vehicle, Tractor-Semitrailer, Active Steering, Optimal Control, Vehicle Safety

1 Introduction

Being cost effective due to reduction in labour requirements and fuel consumption, and large load carrying capacity compared to single unit vehicles, Articulated Heavy Vehicles (AHVs) are among the most widely used means of road transport [1, 2]. On the other hand, AHVs impose higher highway safety concerns which are different from single unit vehicles; heavy weights, large dimensions, geometrical configuration, high centres of gravity, and inner forces acting on their articulation points are the main sources for these differences [3].

The presence of the articulation joint helps to reduce the effective wheelbase of the vehicle in order to improve low-speed turning ability, while large weights and high Centre of Gravity (CG) of AHVs reduce the high-speed stability in three different potentially unstable motions: jackknifing, trailer swing, and rollover [4]. Trailer swing and rollover are directly related to the amplification of lateral acceleration experienced at semitrailer's centre of gravity which is known as the Rearward Amplification (RA) ratio. The RA ratio is defined as the ratio of peak lateral acceleration of the semitrailer CG to that of the tractor unit during a high-speed single lane change manoeuvre [1, 5]. This happens due to the generation of semitrailer tyre lateral forces with a time delay with respect to those of the tractor, resulting in larger yaw motion and higher lateral acceleration of the semitrailer [6].

There have been many attempts to minimise the excessive yaw and roll behaviours of the AHVs including the use of variable damping ratio for the fifth wheel, differential braking, active and passive semitrailer steering systems, and roll control. Among such control strategies, the semitrailer steering system seems to be the most effective approach to regulate the lateral acceleration of the semitrailer resulting in more desirable yaw and roll motions with the relatively small amount of energy requirement. Another reason that makes the semitrailer steering systems interesting is that they could also be used to improve low-speed manoeuvrability of the vehicle by improving the tracking ability of the towed unit by means of proper steering strategies.

Passive semitrailer steering systems have been developed at the earlier stages of AHV researches and studied in detail by Jujnovich and Cebon [7]. Command steering, self-steering axle, and pivotal bogie systems are among the most common passive steering systems. Studies on the passive semitrailer steering systems have concluded

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that the passive steering systems improve low-speed manoeuvrability and tyre-road scrub; however they appear to diminish the high-speed lateral performance of the vehicle [1, 4, 8].

Active steering systems enable the designer to eliminate complex mechanisms required for passive systems and, if tuned properly, are also capable of providing proper steering for different purposes regardless of vehicle speed. However, the low and high-speed requirements of the AHVs naturally conflict with each other due to different side-slip behaviour of the vehicle at different speeds [9, 10].

In a recent study [11], kinematics of the turning vehicle in low speed is studied; a two-layer control strategy using fuzzy control and PID control is introduced to calculate desired steering angles of tractor and semitrailer axles as well as adjusting angular velocities of individually driven wheels. The method works in a limited range of vehicle speeds when the side-slip is small.

In a novel approach, Jujnovich and Cebon [4] provided a perspective by which both low and high-speed dynamics could be investigated in the same manner. They suggested that the tracking ability of the towed unit may be considered as the main objective for the active steering control design. This approach, despite its great level of accuracy, requires a complicated set up to achieve the control objective.

As far as the high-speed manoeuvre is concerned, it has been shown by Islam [1, 12], that the regulation of RA ratio could be used to minimise the high-speed tracking error and the roll stability at the same time. During a high-speed transient manoeuvre, the time delay between generation of tyre forces for the tractor and the semitrailer causes the semitrailer to travel slightly outwards due to its large inertia. This, in turn, results in larger tyre slip angles for the semitrailer trying to bring it back to the steady-state configuration. Consequently, larger lateral forces are generated at the semitrailer tyres causing larger lateral acceleration. In steady-state high-speed turning, the vehicle units travel with equal yaw velocities and the semitrailer experiences larger radius of curvature due to its large mass resulting in larger lateral acceleration. So, in both transient and steady-state conditions, the RA ratio is deeply related to the tracking behaviour of the vehicle. On the other hand, the roll motion of the semitrailer unit is directly related to its lateral acceleration as a consequence of the centripetal force. Thus, the excessive high-speed oscillation of the semitrailer along with its excessive roll motion, as the most important high-speeds concerns, may be treated by regulation of the RA ratio as a single criterion.

Linear Quadratic Regulator (LQR) technique has been used in the design of the active semitrailer steering systems as the optimal feedback structure in a number of researches such as [1, 12–16] with different minimisation objectives. This method has been proved to be a simple yet effective approach in dealing with active control of AHV dynamics.

Based on the discussion related to the articulated vehicle behaviour to understand how tracking ability, lateral accelerations, and roll motion are correlated; a new approach to semitrailer active steering strategy is introduced. The control strategy is expected to provide a simpler yet sufficiently accurate approach to semitrailer steering, in comparison to the more sophisticated methods requiring intensive mathematical calculations and hardware requirements. The LQR control together with Quantum Particle Swarm Optimisation (QPSO) method is used to ensure the optimal solution avoiding manual tuning of design parameters.

Current studies on the subject have mostly concentrated on a specific vehicle speed for low or high speed operation of the articulated vehicles. The proposed control strategy is developed for a wide range of vehicle speeds and a table of feedback coefficients is obtained, so that different requirements in varying travel speeds can be met smoothly in an adaptive manner. Gain-scheduling for PID control of trailer steering has been used in [17] with manual tuning of gains. The objective in this study is to avoid manual tuning of parameters as much as possible and ensure optimal performance of the control strategy at the same time. A couple of variants of the controller to improve the roll and the yaw stability of the tractor-semitrailer combinations at high-speed manoeuvres, as well as the low-speed cornering ability of the vehicle are also investigated.

2 Vehicle modelling

Selection and development of a vehicle model for a specific vehicle dynamics study is a crucial task. An over-complex model will increase the simulation time and the risk of unwanted errors. On the other hand, an excessively simple model causes information loss. Thus, the most important factors in modelling include the level of complexity (assumptions), Degrees of Freedom (DOF), linearity, and the expandability of the model. There are many AHV modelling approaches introduced in the literature, each of which has its own advantages and disadvantages. Nevertheless, a very comprehensive modelling approach for articulated vehicles, which takes into account the key characteristics of the AHVs and yet makes the most of possible simplifications, has been introduced by Sampson [18] and cited by many researchers.

Vehicle modelling in this study is also based on the general equations of motions given by Sampson [18] which are modified to meet the needs of the current research. The model includes lateral, yaw, and roll motions for both vehicle units and a kinematic constraint equation at the coupling point generating a total of 5 DOF along with linear equations of motion. The linear model is, in fact, a linear bicycle model assuming that the articulation and tyre slip angles are small; it is extended to include the roll motion which can be used for low to moderate lateral accelerations up to about 0.4 g. In order to be able to simulate low-speed manoeuvres, a nonlinear version of the proposed model is also generated due to large articulation and steering angles generated in low-speed turning manoeuvres.

2.1 Equations of motion

The vehicle model used in the study is illustrated in Figure 1.

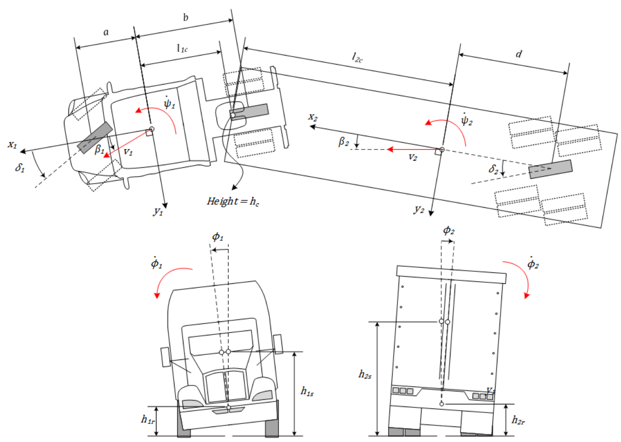


Fig. 1: AHV Linear Vehicle Model

The equations of motion are given as:

$$m_1 u_1\left(\dot{\beta}_1+\dot{\psi}_1\right)-m_{1s}\left(h_{1s}-h_{1r}\right)\ddot{\phi}_1=Y_{\beta_1}\beta_1 + Y_{\dot{\psi}_1}\dot{\psi}_1+Y_{\delta_1}\delta_1+F_{cy} \tag{1}$$

$$-I_{1xz}\ddot{\phi}_1+I_{1zz}\ddot{\psi}_1=N_{\beta_1}\beta_1+N_{\dot{\psi}_1}\dot{\psi}_1+N_{\delta_{1f}}\delta_{1f}-F_{cy}l_{1c} \tag{2}$$

$$\begin{aligned}I_{1x'x'}\ddot{\phi}_1-I_{1x'z'}\ddot{\psi}_1&=m_{1s}g\left(h_{1s}-h_{1r}\right)\phi_1\\&+m_{1s}u_1\left(h_{1s}-h_{1r}\right)\left(\dot{\beta}_1+\dot{\psi}_1\right)-\left(K_{1f}^{\star}+K_{1r}^{\star}\right)\phi_1\\&-\left(C_{1f}+C_{1r}\right)\dot{\phi}_1+K_{12}\left(\phi_2-\phi_1\right)-F_{cy}h_{1cr}\end{aligned} \tag{3}$$

Equations (1) to (3) correspond to the tractor's lateral, yaw, and roll motions, respectively.

$$m_2u_2\left(\dot{\beta}_2+\dot{\psi}_2\right)-m_{2s}\left(h_{2s}-h_{2r}\right)\ddot{\phi}_2=Y_{\beta_2}\beta_2 + Y_{\dot{\psi}_2}\dot{\psi}_2+Y_{\delta_{2r}}\delta_{2r}-F_{cy} \tag{4}$$

$$-I_{2xz}\ddot{\phi}_2+I_{2zz}\ddot{\psi}_2=N_{\beta_2}\beta_2+N_{\dot{\psi}_2}\dot{\psi}_2+N_{\delta_{2r}}\delta_{2r}-F_{cy}l_{2c} \tag{5}$$

$$\begin{aligned}I_{2x'x'}\ddot{\phi}_2-I_{2x'z'}\ddot{\psi}_2&=m_{2s}g\left(h_{2s}-h_{2r}\right)\phi_2\\&+m_{2s}u_2\left(h_{2s}-h_{2r}\right)\left(\dot{\beta}_2+\dot{\psi}_2\right)-K_2^{\star}\phi_2-C_2\dot{\phi}_2\\&-K_{12}\left(\phi_2-\phi_1\right)+F_{cy}h_{2cr}+T_{AR2}\end{aligned} \tag{6}$$

Equations (4) to (6) correspond to the semitrailer's lateral, yaw, and roll motions, respectively.

The kinematic constraint equation at the coupling point is derived by equating the velocities at the fifth wheel with respect to both vehicle units:

$$\dot{\beta}_2=\dot{\beta}_1+\frac{h_{1r}-h_{1c}}{u_1}\ddot{\phi}_1-\frac{h_{2r}-h_{2c}}{u_2}\ddot{\phi}_2-\frac{l_{1c}}{u_1}\ddot{\psi}_1 -\frac{l_{2c}}{u_2}\ddot{\psi}_2+\dot{\psi}_1-\dot{\psi}_2 \tag{7}$$

Note that for small articulation angles, we may assume equal forward velocities for vehicle units ($u_1 = u_2$).

One can construct a state-space representation of the linear system by defining the following state variables; details can be found in [19]:

$$\{x\}=\left[\begin{array}{c}\phi_1\dot{\phi}_1\beta_1\dot{\psi}_1\phi_2\dot{\phi}_2\beta_2\dot{\psi}_2\end{array}\right]^T \tag{8}$$

$$\{\dot{x}\}=[A]\{x\}+[B]\{u\} \tag{9}$$

$$\{u\}=\left[\begin{array}{c}\delta_1\delta_2T_{AR2}\end{array}\right]^T \tag{10}$$

In equation (10), δ_1 is the reference input imposed by the driver's steering action and are the control inputs corresponding to semitrailer steering and roll control torque.

The assumptions in developing the model are as listed below.

- Effect of longitudinal forces are neglected (Constant forward speed),
- Pitch motions are neglected,
- Tyre and unsprung mass dynamics are neglected,
- Articulation, tyre slip, roll and steering angles are assumed to be small (linear equations),
- Vehicle parameters are time invariant (constant),

- Fifth wheel is assumed to allow free yawing without any stiffness or damping,
- Rigid sprung masses are assumed for vehicle units,
- Effect of lateral load transfer is neglected (single track/bicycle model),
- Tyres on semitrailer axles are assumed to be lumped in an equivalent single tyre.

In order to be able to simulate low-speed manoeuvres, a nonlinear version of the proposed model is also generated by relaxing the small angle assumption (keeping the trigonometric relations) due to large articulation and steering angles generated in low-speed turning manoeuvres.

2.2 Model validation

In order to validate the vehicle model, a standard single lane change manoeuvre defined in SAE J2179 [20] is performed with vehicle data presented in the Appendix. Simulation results from the linear model and the commercial software TruckSim model for the vehicle trajectory, lateral acceleration response, yaw rate response, and roll angle response are shown in Figure 2. It can be seen that there is, in general, good agreement between the results. The tractor's roll motion has been accurately predicted by the linear model, but the roll response of the semitrailer experiences less oscillations in the linear approach which is possibly due to larger levels of simplification in the roll-plane model by ignoring the unsprung mass dynamics. But the linear model still gives acceptable overall values within the scope of this study which enables us to partially investigate the effect of active semitrailer steering on the roll motion.

As a result, the linear model is shown to provide acceptable dynamic behaviour.

3 control system design

3.1 control algorithms

LQR technique is applied with the objective of RA ratio regulation at high-speed for two purposes, namely the reduction of the semitrailer roll angle and high speed off-tracking. In the literature, there is no agreement on the desired value of the RA ratio. Islam [1] suggests that any deviation of the RA ratio from unity will correspond to a tracking error at high-speeds. This conclusion is supported by the physical explanation of the RA phenomenon given in the introduction section. With this understanding, the designer must try to keep the RA ratio as close to 1.0 as possible. Otherwise, tracking errors come to existence in the form of outwards semitrailer motion for RA ratio values larger than unity and inwards semitrailer motion for RA ratio values smaller than unity. On the other hand, it is desirable to reduce RA ratio as much as possible in order to decrease roll motion of the semitrailer unit.

Since the inwards motion of the semitrailer is possibly less destructive and more expectable than its outwards motion, it might be desirable to reduce RA ratio to values below unity to some extent, as a trade-off, in order to provide better roll stability as an important factor. This approach has been preferred in some previous works [13, 14].

In order to avoid such a compromise of the tracking ability, it is suggested to introduce another control input to the system to take care of the roll motion. In other words, it is possible to combine the active semitrailer steering with active roll control torque to achieve both targets. This is likely to come at a higher cost from both the hardware and energy requirement perspectives. Explicit discussion on the evaluation of relative merits of the two approaches is yet to appear in the literature. This study makes an attempt to fulfil the gap and suggests a general design approach which is based on the operational task and the vehicle of interest.

As a result of the aforementioned considerations, three different control algorithms are developed and suggested in this study and the results from simulations are compared to each other in order to provide a metric to be used in each individual case.

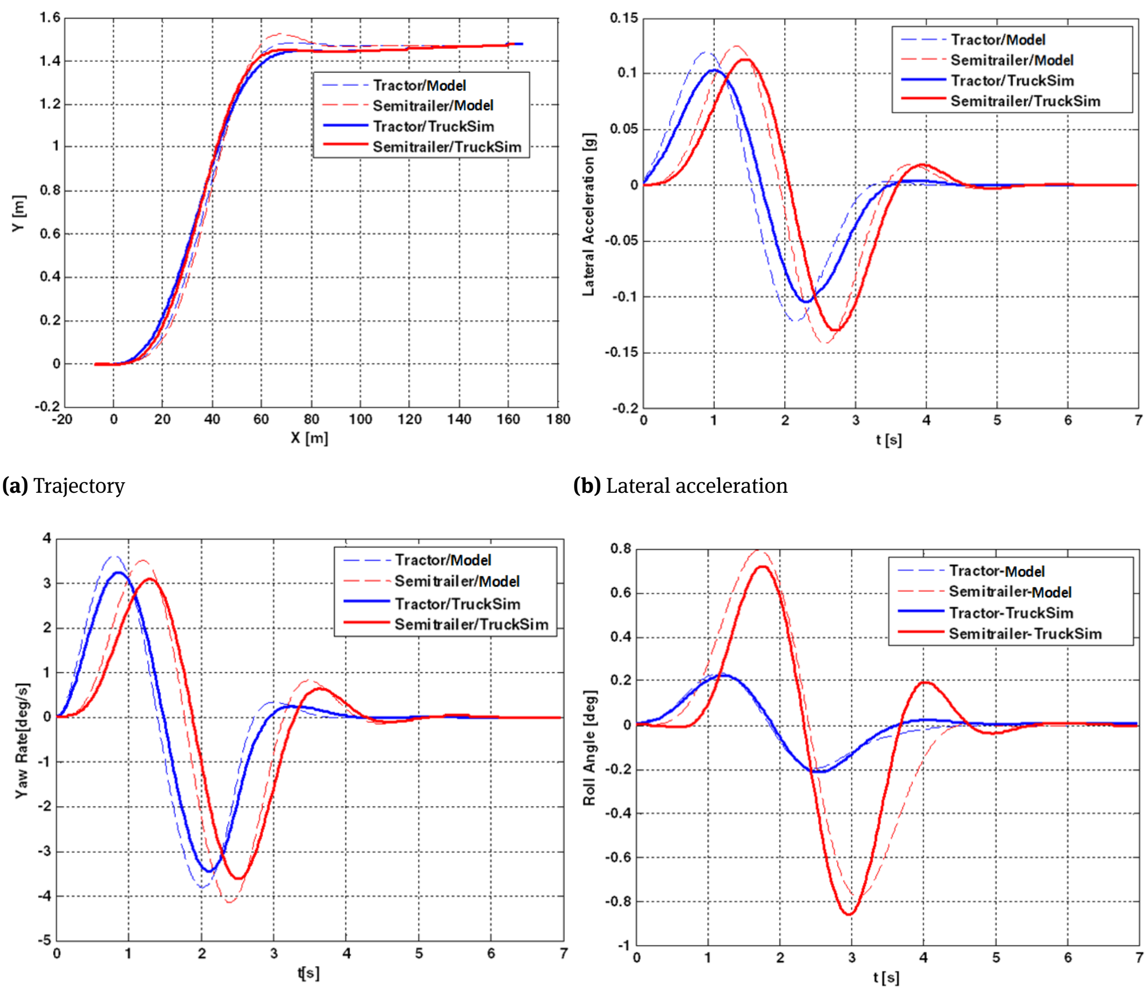
Control strategy 1: The algorithm used for this controller is developed such that it minimises the amount of High-Speed Transient Off-Tracking (HSTO) by providing the RA ratio of 1.0 without letting the ratio to go down further.

Control strategy 2: This controller is based on the algorithm which reduces the RA ratio below unity resulting in further reduction in the semitrailer's roll motion, at the expense of larger HSTO values.

Control strategy 3: Making use of an additional roll control torque input, this controller keeps the RA ratio as close to 1.0 as possible, while simultaneously constraining semitrailer roll motions.

3.2 Linear quadratic regulator

The tractor-semitrailer system has a reference steering input from the driver δ_1 and two control inputs δ_2 and T_{AR2} as the semitrailer steering angle and the roll con-



(a) Trajectory

(b) Lateral acceleration

(c) Yaw rate

(d) Roll angle

Fig. 2: Model and TruckSim Responses for SAE J2179

trol torque. Adding the roll control torque would have the advantage that the Active Steering Control (ASC) system would take the responsibility for HSTO minimisation while the Active Roll Control (ARC) torque takes care of further roll motion limitation. The LQR problem is defined as following:

$$\{u\} = \left[\ \delta_1 \delta_2 T_{AR2}\ \right]^T = \left[\ u_d \{u_c\}^T\ \right]^T \tag{11}$$

$$[B] = \left[\ [B_d][B_c]\ \right] \tag{12}$$

$$\begin{aligned}\{\dot{x}\} &= [A]\{x\} + [B]\{u\}\\ &= [A]\{x\} + \left[\ [B_d][B_c]\ \right]\begin{bmatrix} u_d \\ \{u_c\} \end{bmatrix}\\ &= [A]\{x\} + [B_d]\,u_d + [B_c]\{u_c\}\end{aligned} \tag{13}$$

$$\{u_c\} = \left[\delta_2 \quad T_{AR2}\right]^T = -[K]\{x\} \tag{14}$$

Substituting (14) into (13), the new state-space is given by:

$$\{\dot{x}\} = ([A] - [B_c][K])\{x\} + [B_d]\,u_d \tag{15}$$

$$J = \int_0^\infty \left(\{x\}^T [Q]\{x\} + u_c [R]\ u_c\right)\ dt \tag{16}$$

Selection of the weighting matrices $[Q]$ *and* $[R]$ in (16) is the designer's task in order to provide a suitable performance index for the control purposes. However, for the problem in hand, it would be very difficult to define proper quadratic cost functions expressing the Path-Following Off-Tracking (PFOT) at low-speeds and High-Speed Transient Off-tracking (HSTO) or RA at high-speeds. Specifically at high-speed, there is a phase difference between generations of lateral accelerations for the vehicle units.

Defining a single mathematical relationship for calculation of RA ratio would require dependency of the cost function to parameters such as vehicle forward speed. An alternative approach is to define the lateral accelerations of the vehicle units separately or simultaneously as the cost function which also increases the complexity in mathematical derivations as well as decreasing the accuracy level.

In order to eliminate such complexities and also increase the efficiency level of the LQR design, it is suggested to make use of the QPSO optimisation method in determining the weighting factors which are, in fact, the elements of $[Q]$ and $[R]$ matrices. In this approach, the designer assumes the general form of the weighting matrices and tries to find applicable values for their elements based on an iterative optimisation method. During the optimisation, another cost function called the fitness function is going to be calculated repeatedly in each loop in order to evaluate the overall performance of the controller and the level of fitness for each control parameter set. Then, the designer is free to choose any type of cost function even by looking at the overall performance of the system during its operation and/or looking at some specific times.

3.2.1 Low-speed

At low-speeds, the state variables are relatively small and some of them, such as the roll angles and their rates, become in fact negligible. Additionally, the yaw behaviour and level of PFOT of the vehicle is mainly determined by the yaw-rate state variables. Consequently, it is suggested to eliminate the weighting coefficients corresponding to other state variables and only keep the ones corresponding to yaw rates of vehicle units. The weighting matrices $[Q]$ and [R] take the following forms:

$$[Q] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & q_1 & 0 & 0 & 0 & q_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & q_2 & 0 & 0 & 0 & q_3 \end{bmatrix} \tag{17}$$

$$[R] = r_1 \tag{18}$$

Thus the LQR cost function is given as:

$$J_{lowspeed} = \int_0^\infty \left(q_1\dot{\psi}_1^2 + 2q_2\dot{\psi}_1\dot{\psi}_2 + q_3\dot{\psi}_2^2 + r_1\delta_2^2 \right) dt \tag{19}$$

The problem is thus reduced to the determination of $q_1, q_2, q_3,$ *and* r_1 by means of an optimisation method to meet the design requirements.

3.2.2 High-speed

Since the full weighting matrix $[Q]_{8\times 8}$ consists of 64 elements, 36 of which are independent, the selection of the weighting factors either manually or by computer optimisation would be quite a difficult task. In order to simplify the problem without losing accuracy, the following considerations have been made for the high speed case: The weighting matrices are assumed to be diagonal, resulting in minimisation of the system states. Since we are looking for settling the semitrailer excessive motions, this assumption turns out to be quite reasonable. On the other hand, the tractor's dynamic behaviour is mainly determined by the steering input from the driver. The only interaction between two vehicle units is due to the forces at the hitch point. These forces are not significantly affected by applying the active semitrailer steering. In other words, active semitrailer steering has much more significant effect on the yawing behaviour of the towed unit rather than on the tractor. As a result, it is also suggested to eliminate the state variables corresponding to the tractor unit and consider the four remaining diagonal elements of $[Q]$ which are related to semitrailer state variables.

The weighting matrix $[R]$ is also assumed to be diagonal in a similar manner, resulting in the following forms of weighting matrices:

$$[Q] = diag\,(0, 0, 0, 0, q_1, q_2, q_3, q_4) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & q_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & q_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & q_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & q_4 \end{bmatrix} \tag{20}$$

$$[R] = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \tag{21}$$

The cost function of the LQR controller is then given as:

$$J_{highspeed} = \int_0^\infty \left(q_1\phi_2^2 + q_2\dot{\phi}_2^2 + q_3\beta_2^2 + q_4\dot{\psi}_2^2 + r_1\delta_2^2 + r_2T_{AR2}^2 \right) dt \tag{22}$$

There are a total of 6 parameters to be optimised in the next stage which are: q_1, q_2, q_3, q_4, r_1, *and* r_2. Note that in the design of controllers 1 and 2, parameter r_2 automatically becomes zero due to the absence of the roll control torque, reducing the number of optimisation parameters to 5.

The additional control input weighting factor r_2 is manually selected as small as possible in order to have the maximum possible roll minimisation while needing reasonable control effort.

3.3 Optimisation of weighting factors

A relatively quick and simple method of optimisation, for numerical problems presented in the previous section, is introduced by Eberhart and Kennedy [21] as the Particle Swarm Optimisation (PSO) which is based on movement of a swarm of particles obeying the Newtonian mechanics. It is concluded that the PSO method has advantages over other methods such as faster convergence and the need for fewer iterations, which makes it suitable for application in LQR weighting factor optimisation. There has been an extension to the PSO, presented more recently in [22], which assumes that the particles follow the rules of quantum mechanics. QPSO has the advantage over PSO that it is less likely to get stuck in the local optima. The QPSO-based LQR has shown more desirable results in terms of overshoot, steady-state error, etc. compared to LQR which is tuned by PSO, genetic algorithm, and manual adjustment techniques [23].

Formulation of the general QPSO algorithm in a simple computer algorithm can be found in [22]. For the application, the design objective needs to be converted into a mathematical form, i.e. the fitness function, to be used in the optimisation procedure. Optimisation parameters as described in the previous section are to be tuned as well. Tuning of the optimisation parameters is quite a direct, sensible, and simple task compared to tuning of the LQR weighting factors, so the selection of LQR weighting factors is left to the optimisation process and the problem is reduced to the tuning of a couple of unambiguous coefficients. QPSO method is applied to both low and high-speed cases according to the suitable vehicle models.

At low-speed, the only design criterion is to minimise the PFOT. Since the calculation of exact PFOT for the low-speed case in a computer code could complicate the programming, a new function is defined to be used as an approximation for the value of PFOT:

$$E = \sqrt{\left(X_1' - X_2\right)^2 + \left(Y_1' - Y_2\right)^2} \tag{23}$$

The value of E function is, in fact, the distance between current position of semitrailer's axle denoted by X_2, Y_2 and that of the tractor's front axle at the corresponding previous time denoted by X_1', Y_1' calculated according to vehicle forward speed.

The value of error E is calculated at the points of interest on the trajectory of the vehicle during the manoeuvre in order to constitute the fitness function for low-speed optimisation. The value of the steering input may also be added to the description of fitness function to limit the control energy:

$$\begin{aligned} f_{p,lowspeed} &= c_1 |E_1| + c_2 |E_2| + \ldots + c_n |E_n| \\ &+ d_1 sum(|E|) + d_2 \max(|\delta_2|) \end{aligned} \tag{24}$$

Here c_i's and d_j's are weighting factors to give proper importance to each term.

At high-speed, the design criteria are to regulate RA and yaw rate response as well as HSTO improvement, and roll motion reduction. For the high-speed case, the designer only needs to compare the peak lateral accelerations of the vehicle units during the lane-change manoeuvre. This observation then results in the following fitness function:

$$\begin{aligned} f_{p,highspeed} &= c'_1 \left| \max\left(a_{y2}\right) - \max\left(a_{y1}\right) \right| \\ &+ c'_2 \left| \min\left(a_{y2}\right) - \min\left(a_{y1}\right) \right| + c'_3 \max(|\delta_2|) \\ &+ c_4' \max(|\phi_2|) \end{aligned} \tag{25}$$

which is calculated based on the results of each simulation at every iteration. It must be noted that the term $c_4' \max(|\phi_2|)$ is only used in Controller 2 in order to give more priority to semitrailer's roll motion compared to RA regulation which consequently results is RA values below unity. Since the controller must be able to perform properly at any vehicle speed within the selected range, the same procedure is applied to several forward speeds. More details for implementation of QPSO on the AHV system can be found in [19]. Once the weighting coefficients are determined, the state feedback matrix is calculated from:

$$[K] = [R]^{-1}[B]^T[P] \tag{26}$$

In which, $[P]$ is the solution to algebraic Riccati equation:

$$[A]^T[P] + [P][A] - [P][B][R]^{-1}[B]^T[P] + [Q] = 0 \tag{27}$$

In this study, MATLAB's "lqr" function from Control System Toolbox is used to solve for $[K]$ directly by knowing $[Q]$, $[R]$. The calculated state feedback matrices for different controllers are given in Table 1 to Table 3.

Based on iterative simulations, for the current vehicle parameters, it is suggested that at some speeds interval

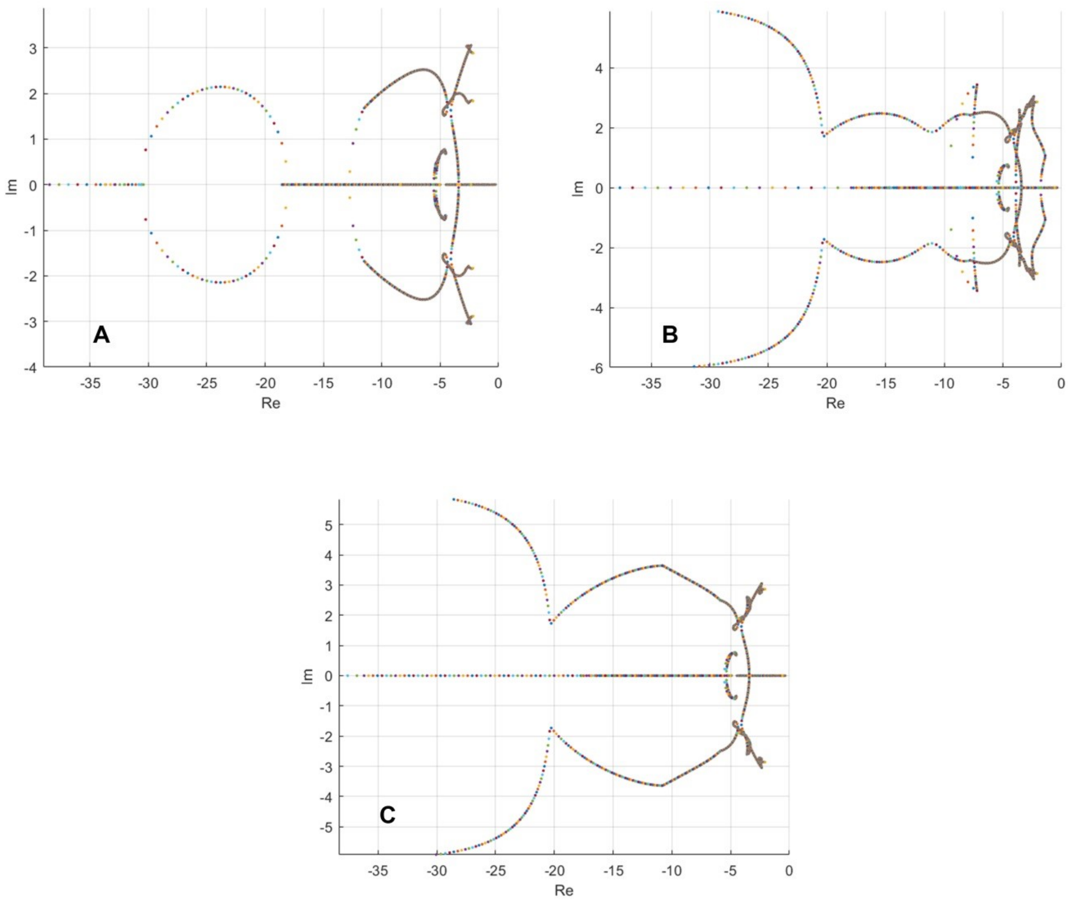


Fig. 3: Eigenvalues of Closed-Loop System (A, B, C for Controllers 1, 2, 3 respectively)

there is no need for steering control neither for PFOT enhancement nor RA reduction. Thus the high-speed active steering may be applied at speeds above a certain speed due to the presence of RA at these speeds. The low-speed active steering is suggested to be applied at speeds below a specific due to significance of PFOT. The margins may be identified by looking at elements of $[K]$ for each controller.

Within the 40 – 70 km/h range, all the weighting factors are set to zero. Interpolation is applied to determine weighting factors for mid-speeds, and speeds below 10 km/h, are treated the same as 10 km/h.

The QPSO method is applied to all LQR designs with different objectives as described earlier and the results from the simulations are represented and compared in the next section.

3.4 Stability analysis

Since a linear model is used for high-speed case, a simple eigenvalue analysis provides stability status of the closed-loop system at a specific reference input from the driver's command. On the other hand, for low-speed case, the stability of the nonlinear closed-loop system may be assessed using Lyapunov's linearization method [24]. Based on this method, if the linearised system around the equilibrium point is strictly stable, the actual nonlinear system is concluded to be asymptotically stable, which is desirable in case of the articulated vehicle. The linearised closed loop system for both high and low-speed cases is in the form of (15). Thus, the new system matrix is given by:

$$\left[A^{\star}\right] = [A] - [B_c][K] \tag{28}$$

Since several state feedback matrices have been introduced for a set of forward speeds, the eigenvalue analysis of $[A^{\star}]$ must be done at every speed interval. Based on the adopted vehicle parameters, matrices $[K]$, $[A]$, $[B_c]$

Table 1: Feedback Matrix Elements for Controller 1

Speed [km/h]	Elements of $[K]$							
10	0	0	0.2402	1.8521	0	0	0	-1.2810
20	0	0	0	0	0	0	0	0
70	0	0	0	0	0	0	0	0
80	-0.0164	-0.0028	0.0461	0.0154	0.0123	0.0034	0.0100	-0.0741
90	-0.0294	-0.0056	0.1121	0.0246	0.0568	0.0248	0.0546	-0.1512
100	-0.0330	-0.0068	0.1817	0.0161	0.1339	0.0451	0.0501	-0.1807
110	-0.0378	-0.0086	0.2803	0.0101	0.2227	0.1081	0.0550	-0.2346
120	-0.0401	-0.0096	0.3554	0.0028	0.2796	0.1363	0.0667	-0.2655

Table 2: Feedback Matrix Elements for Controller 2

Speed [km/h]	Elements of $[K]$							
10	0	0	0.3922	1.1554	0	0	0	-0.2106
20	0	0	0.3089	1.3967	0	0	0	-0.9536
30	0	0	0	0	0	0	0	0
70	0	0	0	0	0	0	0	0
80	-0.0146	-0.0026	0.0446	0.0122	0.0182	0.0050	0.0292	-0.0677
90	-0.1340	-0.0139	0.7327	-0.2213	1.3038	0.0869	0.1594	-0.3078
100	-0.0355	-0.0073	0.1952	0.0177	0.1479	0.0697	0.1153	-0.2023
110	-0.0426	-0.0088	0.2758	0.0110	0.2241	0.1034	0.2327	-0.2539
120	-0.0528	-0.0101	0.3682	-0.0021	0.3294	0.1534	0.5668	-0.3106

are calculated as functions of forward speed for every controller within a selected speed interval ($v_L < u_1 < v_H$):

$$[K]_{u_1} = \frac{u_1 - v_L}{v_H - v_L} \times [K]_L \tag{29}$$

Note that $[A]$, $[B_c]$ are directly dependant on u_1. Finally, using (29) the total system matrix $[A^\star]$ is obtained as a function of forward speed:

$$\left[A^\star(u_1)\right] = [A\,(u_1)] - \left(\frac{u_1 - v_L}{v_H - v_L}\right)[B_c\,(u_1)]\,[K]_L \tag{30}$$

Eigenvalues of $[A^\star]$ for the three proposed controllers are plotted in Figure 3 within the speed range (10 – 120 km/h). All eigenvalues have negative real parts justifying the stability of the closed-loop control system in all three cases.

Note that for speeds lower than around 10 km/h, due to very small tyre slip angles and vehicle speed, a kinematic vehicle model is needed for simulating vehicle motion and the dynamic vehicle model will lose accuracy due to division by small values of u_1, u_2. At those speeds, stability of the vehicle is automatically realised with a kinematic model.

4 Simulation results

4.1 High-speed

The high-speed simulation results for the SAE J2179 manoeuvre are presented in this section in order to make a comparison between the introduced controllers. A sinusoidal steering input is applied to the vehicle's front axle to generate the single 1.46 m lane change manoeuvre at speed of 88 km/h during 2.5 sec according to SAE J2179.

4.1.1 Baseline (passive) vehicle

The baseline vehicle's response is given in Figure 4. The vehicle experiences RA especially at the end of the manoeuvre when the driver steers back to the starting position. This is followed by a deviation of the semitrailer from its expected path and oscillations in both trajectory and lateral acceleration response of the semitrailer at that time.

4.1.2 Controller 1 (Ra=1.0)

The vehicle equipped with Controller 1 responds to the steering input as depicted in Figure 5. It can be seen that

Table 3: Feedback Matrix Elements for Controller 3

Speed [km/h]	Elements of $[K]$							
10	0	0	0.3922	1.1554	0	0	0	-0.2106
	0	0	0	0	0	0	0	0
20	0	0	0.3089	1.3967	0	0	0	-0.9536
	0	0	0	0	0	0	0	0
30	0	0	0.2299	0.5982	0	0	0	-0.2635
	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0
70	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0
80	-0.0146	-0.0026	0.0446	0.0122	0.0182	0.005	0.0292	-0.0677
	77200	-7600	565300	-116600	1106900	402800	-891000	33900
90	-0.0308	-0.0058	0.1119	0.0279	0.0469	0.0176	0.0221	-0.155
	116400	-2100	455700	-81000	2841900	1215500	-915000	20600
100	-0.0355	-0.0073	0.1952	0.0177	0.1479	0.0697	0.1153	-0.2023
	153100	8000	144700	-45100	1862200	2746000	-681700	35200
110	-0.0426	-0.0088	0.2758	0.011	0.2241	0.1034	0.2327	-0.2539
	156400	9300	87000	-31600	2619900	3081800	-624900	31300
120	-0.0528	-0.0101	0.3682	-0.0021	0.3294	0.1534	0.5668	-0.3106
	148400	9800	-9000	-16700	1162000	2641200	-510500	30500

Controller 1 has resulted in accurate tracking of the semitrailer while the peak lateral acceleration of the semitrailer has been almost equal to that of the tractor resulting in RA ratio of almost 1.0. The roll motion, however, has not shown any significant improvement with respect to the results from baseline vehicle.

4.1.3 Controller 2 (Ra<1.0)

Application of Controller 2 results in the vehicle response as shown in Figure 6. Controller 2 reduced the RA ratio to less than unity to the extent that the HSTO and the control input do not exceed a predefined threshold; here, the RA ratio is allowed to be reduced by maximum of 50% to demonstrate the effect of this control strategy and compare it against other strategies. The roll angle of the semitrailer is reduced significantly, but at the expense of introducing tracking errors and increased semitrailer steering input values. The settling time is also increased as a result of this control strategy. It must be noted that the responses provided here are based on an extreme case with high concentration on semitrailer's roll reduction, in order to show the effect of control strategy explicitly. More priority could be given to the tracking error minimisation, to achieve a compromise between tracking errors and roll motion response and mitigating the undesired motion.

4.1.4 Controller 3 (Ra=1 & Rc torque application)

Controller 3, as the most complicated strategy among others, results in dynamic behaviour as shown in Figure 7. It can be seen that controller 3 results in the most desirable responses both from tracking perspective and the roll management. Obviously, this comes at a higher level of complexity for the system and larger amount of energy consumption necessary for the roll control torque.

Further simulations and discussion on the effects of ASC on other vehicle responses can be found in [19].

4.2 Low-speed

For the low-speed case, the objective of LQR is to reduce the PFOT of the vehicle which is defined as the radial offset between the front axle path and that of the rear axle during a 360 degree steady-state turn.

The performance of the controller at low-speed is evaluated by simulating a 360-degree manoeuvre at 10 km/h as presented in Figure 8. It can be observed that the LQR active semitrailer steering have successfully reduced the PFOT significantly with the proposed method. It is noted, however, that a small outward swing of the semitrailer in the transient cornering phase has appeared.

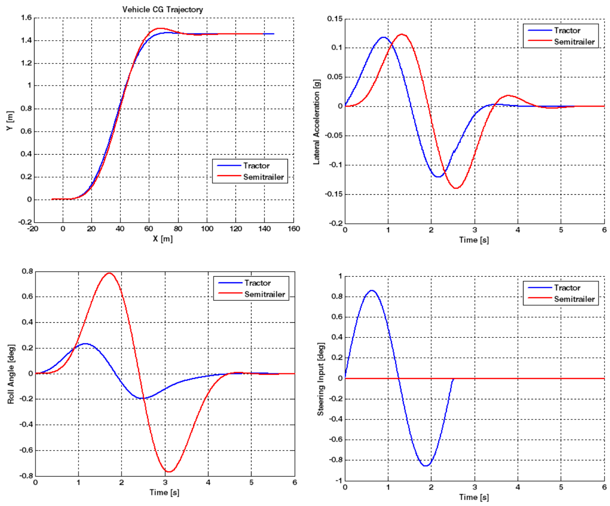


Fig. 4: Baseline Vehicle Response for SAE J2179

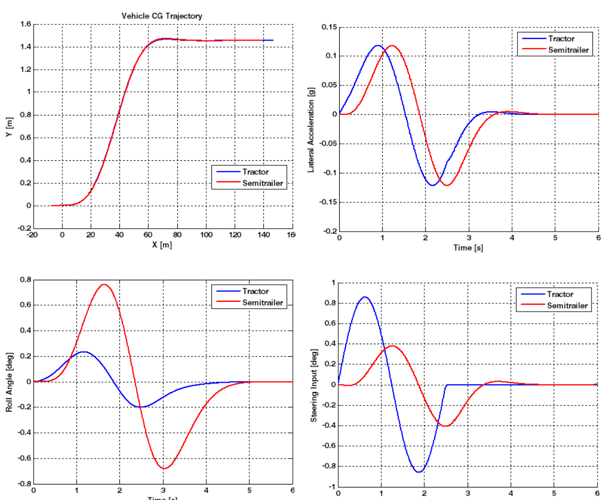


Fig. 5: Controller 1 Performance for SAE J2179

5 Conclusions

The proposed controllers are applied to the tractor-semitrailer model and the results from standard test runs are obtained. The results from the simulations show that the dynamic behaviour of the tractor-semitrailer combinations can be improved to perfectly acceptable levels with a simple state-feedback approach. For the high-speed case, results from the application of the SAE J2179 test illustrated that each controller has its own advantage over the others and the area of application depends on the specific vehicle and the specific operating conditions in which the tractor-semitrailer is assigned. Low-speed performance of the active semitrailer steering is assessed by simulating a 360-degree turn manoeuvre resulting in an improved per-

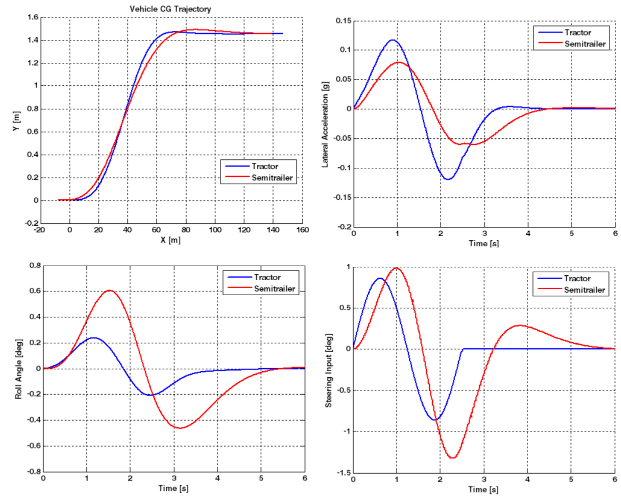


Fig. 6: Controller 2 Performance for SAE J2179

formance. In view of the results obtained from the simulations, the applications of the proposed controllers to different classes of operations are summarised as follows.

Controller 1 may be applied to a vehicle which is not likely to be subjected to large semitrailer roll motion. This may be the case for applications with moderate loading or semitrailer having large suspension roll stiffness. The semitrailer is not expected to exceed the critical roll threshold. The controller will then provide the best possible tracking ability at high-speed without much hardware complication and energy consumption.

Controller 2 may be applied to a vehicle with larger loading which makes the roll motion moderately more critical than the previous case. In this case, the best economical solution to the problem is to tune Controller 2 such that it provides the required roll management while not exceeding the tracking error target defined by standards or the designer. It must be noted that this controller might not be desirable for excessively long semitrailers for which even small tracking errors are amplified due to the length and impose severe danger to the surrounding traffic.

Controller 3 is applicable for any of the conditions mentioned above, but at expense of hardware/software complexity and higher energy consumption. It is suggested that this strategy is more suitable for super duty cases or in highly sensitive transport cases in which the cost of implementing active control imposes less concern compared to previous cases.

Unlike Controller 3 which requires extra equipment for roll control, Controllers 1 and 2 have the same hardware requirements, but slightly different objective functions as mentioned earlier. On the other hand, articulated vehicles are expected to perform under different loading and cargo

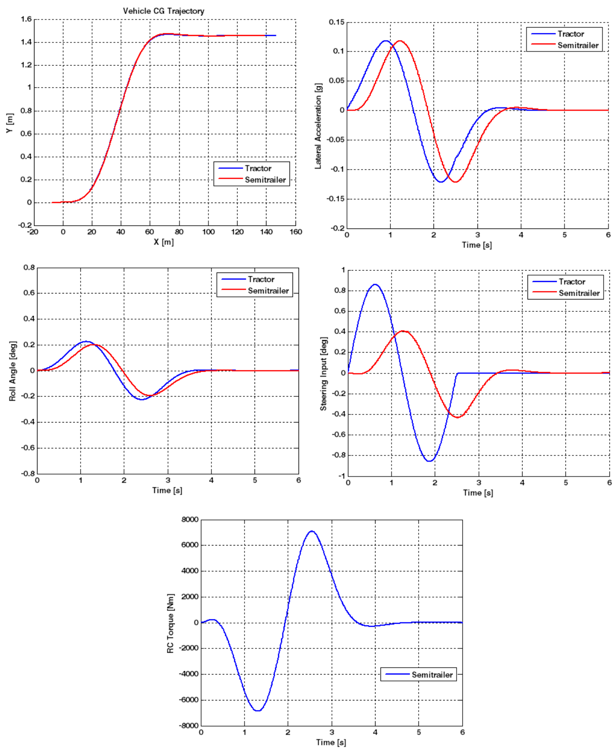


Fig. 7: Controller 3 Performance for SAE J2179

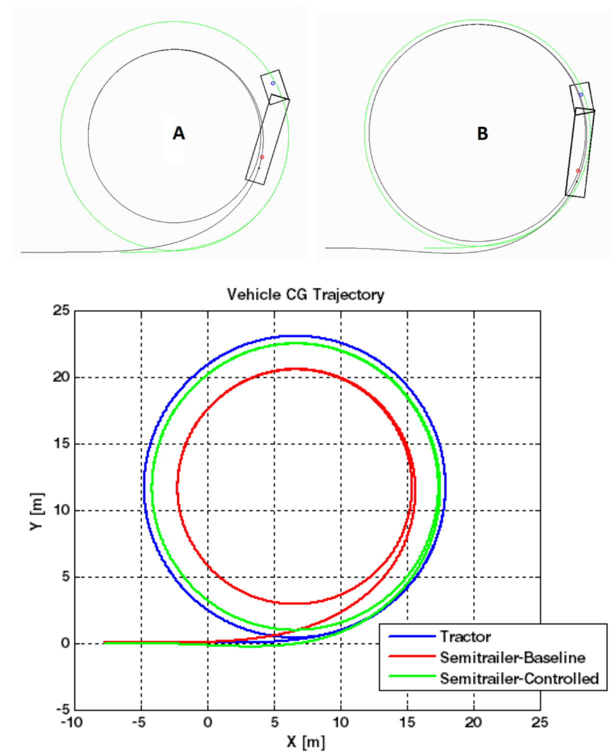


Fig. 8: Low-Speed 360-degree Turn

conditions. Hence, it is worth to mention that a more robust performance of the semitrailer steering control system with respect to vehicle loading condition and semitrailer length may be achieved by defining an activation function which manages the contribution of control commands from Controller 1 and 2. Assuming a linear relationship between roll inertia $I_{2x'x'}$ and sprung mass m_{2s}, a simple form of such an activation function may be defined as follows:

$$f_1 = \frac{d + l_{2c}}{m_{2s}} \tag{31}$$

$$\delta_2 = f_1\delta_{21} + (1 - f_1)\delta_{22} \tag{32}$$

In which δ_{21} and δ_{22} are steering commands from Controller 1 and 2 respectively. Such an activation function adjusts the contribution of Controller 1 and 2 based on severity of vehicle conditions in terms of loading and semitrailer's length. More investigation on smooth operation of the system by means of such activation functions and their requirements may be the subject of future studies in the field.

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A Appendix

Vehicle Parameters (Based on [12])

Parameter	Description	Value	Parameter	Description	Value
m_{1s} [kg]	Tractor's Sprung Mass	4819	I_{1xz} [kg.m^2]	Tractor's Total Yaw/Roll Product of Inertia	2176
m_{2s} [kg]	Semitrailer's Sprung Mass	30821	I_{2xz} [kg.m^2]	Tractor's Total Yaw/Roll Product of Inertia	18497
m_{1uf} [kg]	Tractor's Front Unsprung Mass	650	$I_{1x'x'}$ [kg.m^2]	Tractor's Sprung Mass Roll Moment of Inertia	4348
m_{1ur} [kg]	Tractor's Rear Unsprung Mass	1300	$I_{2x'x'}$ [kg.m^2]	Semitrailer's Sprung Mass Roll Moment of Inertia	42025
m_{2u} [kg]	Semitrailer's Unsprung Mass	1330	$I_{1x'z'}$ [kg.m^2]	Tractor's Sprung Mass Yaw/Roll Product of Inertia	2176
h_{1s} [m]	Tractor's Sprung Mass CG Height	1.058	$I_{2x'z'}$ [kg.m^2]	Semitrailer's Sprung Mass Yaw/Roll Product of Inertia	18497
h_{2s} [m]	Semitrailer's Sprung Mass CG Height	1.000	K_{12} [N.m/rad]	Coupling's Roll Stiffness	114590
h_{1r} [m]	Tractor's Roll Axis Height	0.558	$K^{\star}_{1f}$ [N.m/rad]	Tractor's Front Roll Stiffness	815220
h_{2r} [m]	Semitrailer's Roll Axis Height	0.723	$K^{\star}_{1r}$ [N.m/rad]	Tractor's Rear Roll Stiffness	655024
h_c [m]	Hitch Point's Height	1.100	$K^{\star}_{2}$ [N.m/rad]	Semitrailer's Roll Stiffness	409960
l_{1c} [m]	Distance from Tractor's CG to Hitch Point	1.959	C_{1f} [N.m/rad]	Tractor's Front Roll Damping Ratio	160000
l_{2c} [m]	Distance from Semitrailer's CG to Hitch Point	5.853	C_{1r} [N.m/rad]	Tractor's Rear Roll Damping Ratio	160000
a [m]	Distance from Tractor's Front Axle to its CG	1.115	C_2 [N.m/rad]	Semitrailer's Roll Damping Ratio	270000
b [m]	Distance from Tractor's Rear Axle to its CG	1.959	C_{t1f} [N/rad]	Tractor's Front Tyre Cornering Stiffness	277200
d [m]	Distance from Semitrailer's Axle to its CG	1.147	C_{t1r} [N/rad]	Tractor's Rear Tyre Cornering Stiffness	740280
I_{1zz} [kg.m^2]	Tractor's Yaw Moment of Inertia	20606	C_{t2} [N/rad]	Semitrailer Tyre Cornering Stiffness	2646000
I_{2zz} [kg.m^2]	Semitrailer's Yaw Moment of Inertia	226272			