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Heat transfer effect on MHD flow of a micropolar fluid through porous medium with uniform heat source and radiation

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Abstract: The present study examines the effect of heat transfer on electrically conducting MHD micropolar fluid flow along a semi-infinite horizontal plate with radiation and heat source. The uniform magnetic field has applied along the principal flow direction. The obtained governing equations have been converted into a set of dimensionless differential equations and then numerically solved by using a well-known Runge-Kutta method with shooting technique. The velocity, microrotation, and temperature distribution are presented for various physical parameters. The numerical values of skin friction and Nusselt numbers at the plates are shown in tabular form, and the obtained results are compared with the results of a previous study. It has been found that the magnetic parameter increases the velocity profile whereas the boundary layer thickness reduces due to the inclusion of coupling parameter and inertia effect. The presence/absence of magnetic parameter and coupling parameter enable to enhance the angular velocity profile while it is worth to note that the backflow has generated in the vicinity of the plate.

Keywords: MHD, micropolar fluid, porous medium, heat transfer, radiation

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1 Introduction

The knowledge of micropolar fluids past a porous medium has significant practical applications across a wide range of areas namely polymer blends, porous rocks, alloys, foams and aerogels, microemulsions etc. One such application is in the field of lubrication, since the gap or the clearance in a bearing may be comparable to the average grain or molecular size of a non-Newtonian lubricant. For example, in batch mixers, sodium, which displays non-Newtonian characteristics, may be used both as a heat transfer agent and as a lubricant in the journal bearing supporting the mixing screw. The theory of micropolar fluid consisting of two different effects such as microrotational and micro-inertia, first proposed by Eringen [1]. The proposal led to the attention of the researchers in the magneto-hydrodynamic (MHD) micropolar fluid past in a porous medium due to its vast applications in engineering problems namely geothermal, energy extractions, oil exploration and the boundary layer control in the field of aerodynamics.

Many studies have examined the effect of heat transfer and magnetic field MHD micropolar fluid through a porous medium with different flow geometry such as a vertical plate, semi-infinite plate, channels etc. [2–6]. Raptis and Kafousias [7] and Kim [8] investigated the influence of magnetic field on two-dimensional steady incompressible flow confined in an infinite and moving vertical porous plate, respectively. Kim [8] reported that the velocity distributions of micropolar fluids are comparable with the Newtonian fluids. Additionally, it has shown that the surface skin friction decreased with increase in the moving vertical plate velocity. Beg et al. [9] studied the effect of heat generation/absorption on MHD free convection flow from a sphere to non-Darcian porous medium wherein the mathematical problem has been solved using network simulation method. There are some studies available in the open literature where the non-Newtonian fluid in a porous medium have investigated with different flow geometry [10–16]. Takhar et al. [17] have studied the unsteady three-dimensional MHD-boundary-layer flow due to the impulsive motion of a stretching surface. Baag et al. [18] investigated the MHD boundary layer flow over an exponentially stretching sheet past a porous medium with the uniform heat source. Mixed convection from a vertical surface embedded in a porous medium saturated with a non-Newtonian nanofluid has studied by Gorla et al. [19]. Chamkha et al. [20] has presented the non-similar solutions for mixed convection along a wedge embedded in a porous medium saturated by a non-Newtonian nanofluid. Similarity solutions for MHD thermosolutal Marangoni convection over a flat surface in the presence of heat generation or absorption effects have observed by Mudhaf and Chamkha [21]. RamReddy et al. [22] has examined soret impact on mixed convection flow in a nanofluid under convective boundary condition. Ramesh et al. [23] has presented the magnetohydrodynamic flow of a non-newtonian nanofluid over an impermeable surface with heat generation/absorption. Chamkha and Rashad [24] have observed the MHD forced convection flow of a nanofluid adjacent to a non-isothermal wedge. Recently, Raju et al. [25, 26] studied the radiation effect of Casson fluid over a moving wedge filled with gyrotactic microorganism and Carreau nanofluid over a cone packed with alloy nanoparticles, respectively. A molecular dynamics study on transient non-Newtonian MHD Casson fluid flow dispersion over a vertical radiative cylinder with entropy heat generation has studied by Reddy et al. [27]. The studies mentioned above concluded that the uniform magnetic field enables to control the heat generation of electrically conducting fluid.

The viscous dissipation is one of the significant parameters in fluid dynamics and usually very challenging to incorporate into the mathematical model. However, many researchers studied the effect of viscous dissipation on hydromagnetic flow through a saturated porous medium and proposed a different mathematical model based on many ways [28–34]. Chamkha et al. [35] have presented non-Darcy natural convection flow for non-Newtonian nanofluid over cone saturated in a porous medium with uniform heat and volume fraction fluxes. Gorla and Chamkha [36] has observed natural convective boundary layer flow over a horizontal plate embedded in a porous medium saturated with a nanofluid. Al-Hadhrami et al. [37] proposed a new viscous dissipation model for a porous medium, which is most suitable for most practical applications. It has found that the porous medium enables to enhance the thermal performance when the aspect ratio of the channel or the thermal conductivity of the channel matrix increased.

In the last few years, many researchers are devoted to investigating the unsteady micropolar fluid through a porous medium in the presence or absence of a uniform magnetic field [38, 39]. Mohanty et al. [40] numerically studied the effect of heat and mass transfer on MHD micropolar fluid using Runge-Kutta fourth order method followed by shooting technique. It has found that the Lorentz force produced by the magnetic field which enables the resistance to momentum field in the presence/absence of porous matrix. Using the similar numerical techniques (Mohanty et al. [40]), the effect of heat source parameter on electrically conducting micropolar fluid has investigated by Mishra et al. [41]. Due to the presence/absence of heat source parameter, the two layers variation has observed in the thermal boundary layer. Later, Tripathy et al. [42] studied the effect of chemical reaction on MHD micropolar fluid considering moving the vertical porous plate. The thermal radiation significantly influences the temperature distribution and heat transfer of electrically conducting micropolar fluid. But here, it is noted that there are very few publications available in the literature which analysed the effect of thermal radiation and heat transfer of micropolar fluid and demands more systematic investigation in future. Rashedi et al. [43] proposed analytical solutions based on homotopy analysis method (HAM) for micropolar fluid past through a porous medium.

In the analysis of flow through a porous medium, Darcy's law usually has been considered to be the fundamental equation (Muskat [44]; Collins [45]). The principle of Darcy's law states that the velocity components are directly proportional to the pressure gradient where the convective acceleration term of fluid does not exist. Hence, this law is only valid for low-speed flows. The force of the fluid which is also proportional to the velocity components may deviate from the Darcy drag force (Bear [46]). The generalisation proposed by Brinkman considered the convective force. To study the flow through a highly porous medium (such as fibres) the generalised Darcy law is recommended. Hence, in the present study, generalised Darcy law has been used to account for the porosity of the medium. Consequently, the novelties of the present study placed down as follows:

1. The momentum transport equation has been modified by inclusion of two terms i.e. $\frac{\sigma {B_0}^2 \varphi}{\rho}(U_0 - u)$ magnetic parameter and $C\varphi({U_0}^2 - u^2)$ (Non-Darcian) which account for the effect of permeability of the medium on the flow phenomena.
2. The heat energy equation has been generalised by considering the uniform heat source/sink, not taken care of by Rashidi et al. [43] which affect the heat transport phenomena.

Motivated by these applications the present study explores the effects of the magnetic parameter, heat source/sink parameter and inertia effect of micropolar fluid by modifying the momentum and energy equations, which primarily constitutive the flow model of any liquid. To achieve this aim, we considered the electrically conducting MHD micropolar flow past through a semi-infinite horizontal plate. The porous medium with variable suction velocity and viscous dissipation are discussed in the analysis. The effect of viscous dissipation is modelled by following the suggestion proposed by Al-Hadhrami et al. [37]. It is worth to mention that, the proposed mathematical model is highly non-linear, hence only approximate numerical solution is possible in contrast to the analytical solution. Finally, the obtained results are compared with the analytical results reported by Rashidi et al. [43] as a particular case.

2 Mathematical formulation

A steady two-dimensional MHD flow of a micropolar fluid through a porous medium past a semi-infinite horizontal plate in the presence of uniform heat source was considered in the present study. The effect of radiation was analysed by modifying the energy equation. The semi-infinite horizontal plate is placed along the x-direction, and y-axis is perpendicular to it. We assumed that a uniform transverse magnetic field of strength B_0 employed to the principle direction of flow. The transverse magnetic field can be negligible due to the small magnetic Reynold number. A schematic of the flow geometry along with coordinate system is shown in Fig. 1.

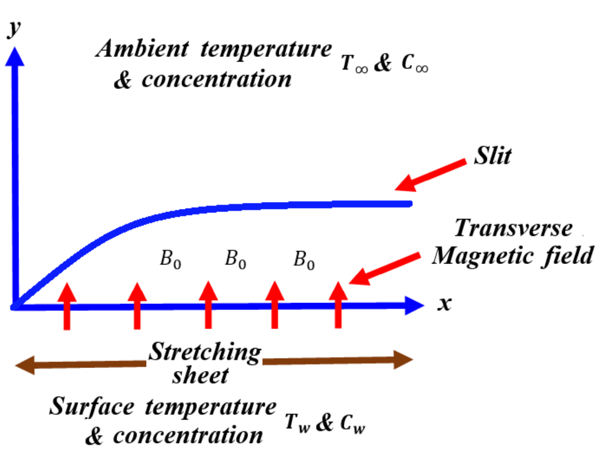

Fig. 1: Schematic of flow geometry

All the physical parameters in the proposed model have been independent along the x-direction as we considered an infinite plate along the vertical direction (x-direction). To investigate the effects of heat absorption, a heat source parameter has been applied to the flow. Hence, the set of governing equations of the problem can be expressed as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + k_1\frac{\partial N}{\partial y} + \frac{\sigma {B_0}^2\varphi}{\rho}(U_0 - u) + \frac{\upsilon\varphi}{{K_p}^*}(U_0 - u) + C\varphi({U_0}^2 - u^2) \tag{2}$$

$$G_1\frac{\partial^2 N}{\partial y^2} - 2N - \frac{\partial u}{\partial y} = 0 \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} + \frac{S'}{\rho C_p}(T - T_\infty) \tag{4}$$

By assuming Rosseland approximation, the radiative heat flux is taken as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y},$$

where σ^* is the Stefan–Boltzmann constant and k^* is the mean absorption coefficient. Assuming that the temperature differences within the fluid in the boundary layer are sufficiently small, we can express the term T^4 as a linear function of temperature. Hence, using expanding T^4 in a Taylor series about T_w and neglecting higher-order terms, we obtain. Then, q_r can be expressed as

$$q_r = -\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial T}{\partial y}$$

The associated boundary conditions of the present mathematical problem can be defined as follows:

$$\begin{array}{ll} u = 0,\ v = 0,\ N = 0,\ T = T_w & \text{at } y = 0 \\ u = U_0,\ N = 0,\ T = T_\infty & \text{as } y \to \infty \end{array} \tag{5}$$

3 Solution of the problem

The following non-dimensional variables have been used to achieve the dimensionless form of the governing equations:

$$\left.\begin{array}{l} \psi(x,y) = (2\upsilon U_0 x)^{\frac{1}{2}} f(\eta) \\ N = \left(\frac{U_0}{2\upsilon x}\right)^{\frac{1}{2}} U_0\, g(\eta) \\ \eta = \left(\frac{U_0}{2\upsilon x}\right)^{\frac{1}{2}} y \\ \theta = \frac{T - T_\infty}{T_w - T_\infty} \end{array}\right\} \tag{6}$$

Using Eq. (6) into the Eqs. (2) – (4), the non-dimensional governing equations can be written as:

$$f''' + ff'' + \Delta g' + \left(M + \frac{1}{K_p}\right)(1 - f') + F(1 - f'^2) = 0 \quad (7)$$

$$Gg'' - 2(2g + f'') = 0 \quad (8)$$

$$(3R + 4)\theta'' + 3R\mathrm{P}_r f\theta' + S\mathrm{P}_r\theta = 0 \quad (9)$$

Hence, the boundary conditions (Eq. 5) can be expressed as:

$$\begin{aligned} &f(\eta) = 0,\ f'(\eta) = 0,\ g(\eta) = 0,\ \theta(\eta) = 1 \quad && at\,\eta = 0 \\ &f'(\eta) \to 1,\ g(\eta) \to 0,\ \theta(\eta) \to 0 && \text{as}\,\eta \to \infty \end{aligned} \quad (10)$$

where,

$$\mathrm{P}_r = \frac{\mu C_p}{k},\ \Delta = \frac{k_1}{\upsilon},\ G = \frac{G_1 U_0}{\upsilon x},\ R = \frac{k^* k}{4\sigma^* T_\infty^3},\ K_p = \frac{2\varphi \upsilon x}{K_p{}^* U_0},$$
$$F = 2\varphi Cx,\ M = \frac{\sigma B_0{}^2 \varphi x}{\rho} k_1 = \rho\kappa,\ \upsilon = (\mu + \kappa)/\rho,\ S = \frac{2S' x}{\rho C_p U_0},$$

In the present study, the local skin friction coefficient (C_f), the local wall couple stress (M_{wx}) and the local heat transfer coefficient (Nu_x) can be defined by the following relation:

$$C_f = \frac{\tau_w}{\rho U_0{}^2/2},\quad \tau_w = \left[(\mu + \kappa)\frac{\partial u}{\partial y} + \kappa N\right]_{y=0} = \frac{2U_0 x}{\upsilon} f''(0) \quad (11)$$

$$M_{wx} = G_1\left(\frac{\partial N}{\partial y}\right) = \frac{U_0 x}{\upsilon} g'(0) \quad (12)$$

$$Nu_x = \frac{x q_w}{k(T_w - T_\infty)} = -\frac{\nu}{U_0 x}\theta'(0),\ q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (13)$$

4 Numerical solution

The set of coupled non-linear differential Eqs. (7) – (9) have been solved numerically using robust shooting method with Runge-Kutta scheme and considering the boundary conditions mentioned in Eq. (10). By following the way of superposition the Eq. (7) – (9) were expressed as 1st order simultaneous equations of seven unknown variables. We considered that $y_1 = f, y_2 = f', y + 3 = f'', y_4 = g, y_5 = g', y_6 = \theta, y_7 = \theta'$. Hence, the Eqs. (7) – (9) can be written as a set of ordinary differential equations. i.e.

$$y_3' = -y_1 y_3 - \Delta y_5 - \left(M + \frac{1}{K_p}\right)(1 - y_2) - F(1 - y_2^2) \quad (14)$$

$$y_5' = \frac{2}{G}(2y_4 + y_3) \quad (15)$$

$$y_7' = \frac{-P_r}{4 + 3R}[3Ry_1 y_7 + Sy_6] \quad (16)$$

with the boundary conditions

$$\left.\begin{aligned} y_1(0) = 0, y_2(0) = 0, y_2(\infty) = 1, y_4(0) = 0, \\ y_4(\infty) = 0, y_6(0) = 1, y_6(\infty) = 0 \end{aligned}\right\} \quad (17)$$

The simulation has been repeated for a considerable value of η_∞ up to two consecutive values of $f''(0), f'''(0), g''(0)$ and $\theta'(0)$ only diverse when the digit indicates the limit of the boundary towards η. For a particular set of parameters, the final value of η_∞ is chosen based on the limit $\eta \to \infty$. The simulation has repeated till the accuracy of 10^{-6} is attended.

5 Results and discussion

A numerical approach such as Runge-Kutta method with shooting technique has been used to solve the non-linear self-similar governing equations (2) to (4), where Eq. (5) denote the two-point boundary value problem (BVP). To convert the BVP into IVP, we assign some guessed value with unknown initial conditions and initiate the process of computation. The step size of the current simulation has fixed at $\eta = 0.001$. Here it is noted that the present solution affected by two additional parameters namely magnetic parameter (M) in the momentum equation and uniform heat source/sink parameter in the energy equation (not considered by Rashidi et al. [43]).

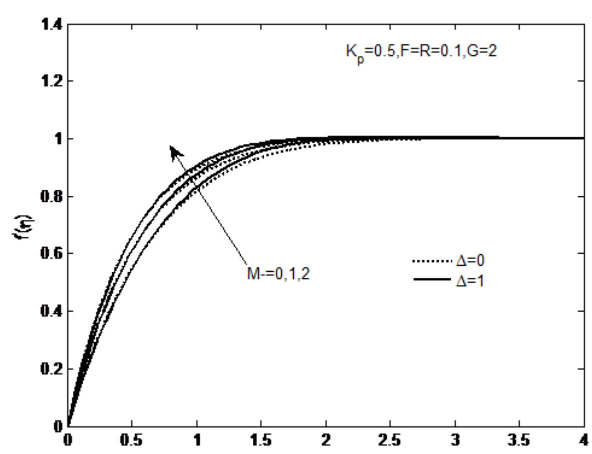

Fig. 2: Velocity profile for different values of M and Δ

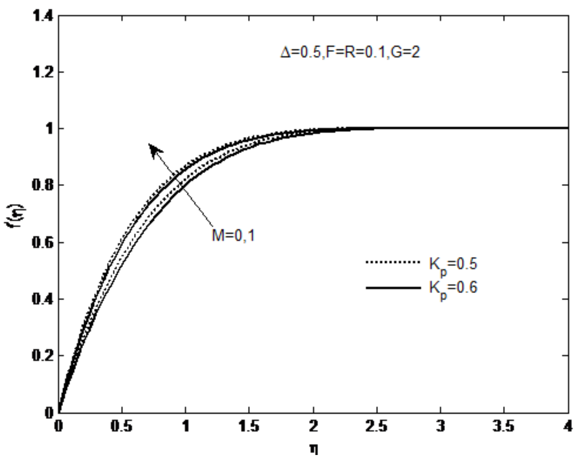

Fig. 3: Velocity profile for different values of M and K_p

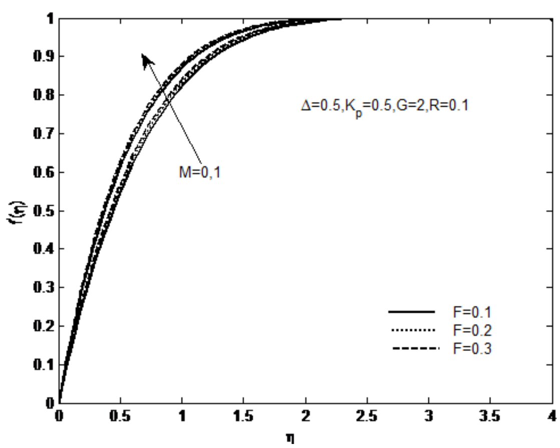

Fig. 4: Velocity profile for different values of M and F

Figs. 2 – 4 reveal the velocity profile for different physical parameters such as magnetic parameter (M), the porous parameter (K_p), coupling parameter (Δ), radiation parameter (R) and microrotation parameter (G). The illustration of microrotation profile has shown in Figs. 5 - 7 with the inclusion of above physical parameter. The effect of heat source/sink in the absence/presence of and with radiation parameter have been discussed in Figs. 8 – 10. Fig. 2 exhibits the impact of the magnetic parameter, M on the velocity distribution in the absence/presence of coupling parameter and the fixed values of $K_p = 0.5$, $F = R = 0.1$ and $G = 2$, respectively. It is observed that the velocity profile enhanced with the increase of magnetic parameter in the absence/presence of coupling parameter ($\Delta = 0$ and $\Delta = 1$). The higher value of the magnetic parameter ($M = 2$) increases the velocity distribution, hence, the velocity boundary layer thickness decreased. It is because the magnetic field applied along to the normal of flow direction. This magnetic field gives rise to a resistive force and slows down the movement of the fluid. The velocity profile was remaining invariant from $\eta \geq 2$.

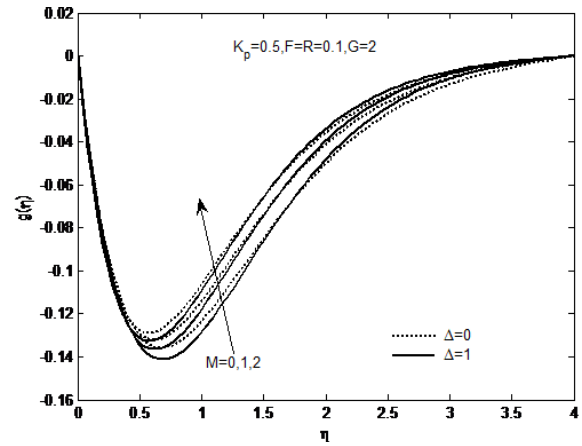

Fig. 5: Angular velocity profile for different values of M and Δ

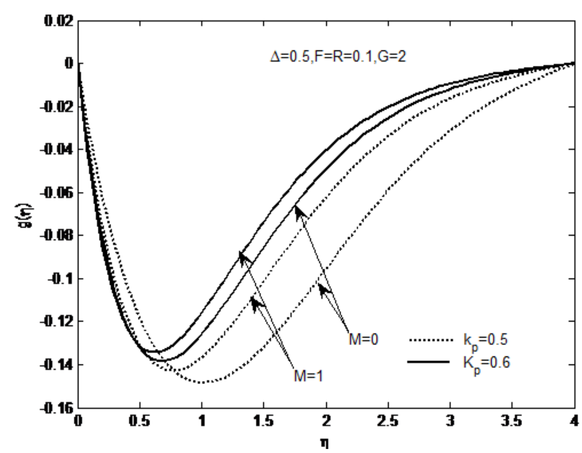

Fig. 6: Angular velocity profile for different values of M and K_p

The velocity profile for the dimensionless parameters M and K_p is shown in Fig. 3 with the fixed values of $\Delta = 0.5$, $F = R = 0.1$ and $G = 2$. The velocity profile observed to decelerates in the boundary layer in the presence/absence of magnetic parameters due to the porous matrix. It is also noted that the reverse effect appears to be right in the case of the magnetic parameter. Fig. 4 illustrates the effect of M and F on velocity profile. It is clear that the increase in microinertia coefficient increases the velocity profile at all points in the flow domain for both presence/absence of magnetic parameter. The present result is in good agreement with the outcome of Rashidi et al. [43] by withdrawing the magnetic parameter, i.e. $M = 0$ from the velocity profile.

Fig. 5 exhibits the effect ofand coupling parameter (Δ) on angular velocity profile with fixed values of other parameters such as $K_p = 0.5, F = R = 0.1$ and $G = 2$. It is clear that in the presence/absence of M and Δ, angular velocity profile increases and the backflow is generated in the vicinity of the plate after this it meets the boundary conditions. It has appeared that the magnetic field retards the angular velocity profile with coupling parameter. The effect of M and K_p on angular velocity is exhibited in Fig. 6. It is interesting to note that magnetic field enhances the profile whereas, K_p retards it. A similar observation has well marked that the backflow occurs in the boundary layer.

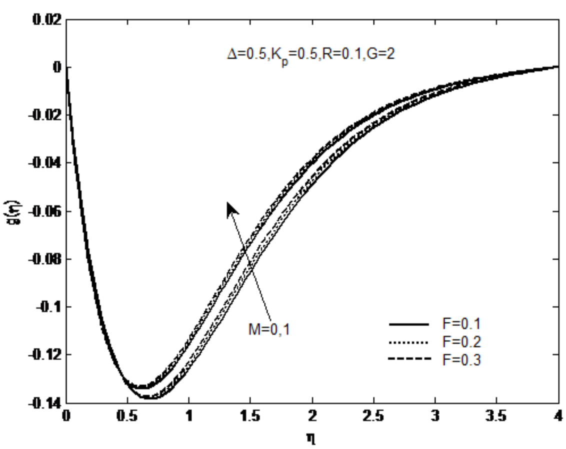

Fig. 7: Angular velocity profile for different values of M and F

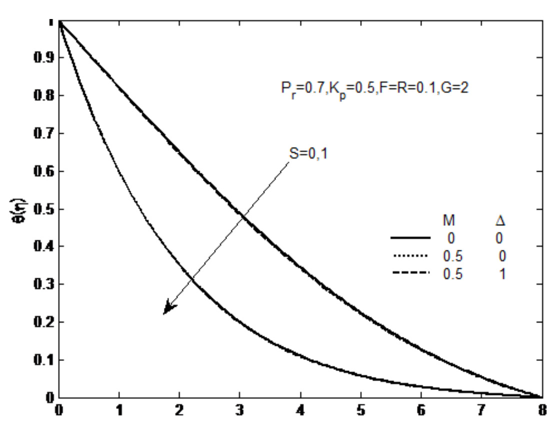

Fig. 8: Temperature profile for different values of S, Δ and M

Fig. 7 illustrates the effect of M and F on angular velocity profile with the fixed values of the pertinent parameters $K_p = 0.5 = \Delta, R = 0.1$ and $G = 2$. It is remarked that the angular velocity has linear behaviour within the flow domain $\eta < 0.5$ and after that, it rises to meet the boundary conditions. We observed that the increase in microinertia coefficient had enhanced the profile at all points in the boundary layer. Due to the Lorentz force (often treated as a resistive force), a reduction in the velocity observed whereas the reverse effect has encountered with the higher values of M. The critical aspect of the present study is the effect of uniform heat source on the temperature profile in the presence/absence of the M, Δ, K_p and R parameters with the fixed values of other pertinent parameters.

From Fig. 8, it is remarked that increase in heat source parameter retards the temperature profile at all points in the presence of M and Δ ($M = 0.5$ and $\Delta = 1$). In contrast for and, the maximum temperature has been observed at the boundary-layer which is parabolic in nature. It is because the resistive lorenze force which retards a certain amount of energy stored up in the thermal boundary layer, therefore, the temperature profile increased.

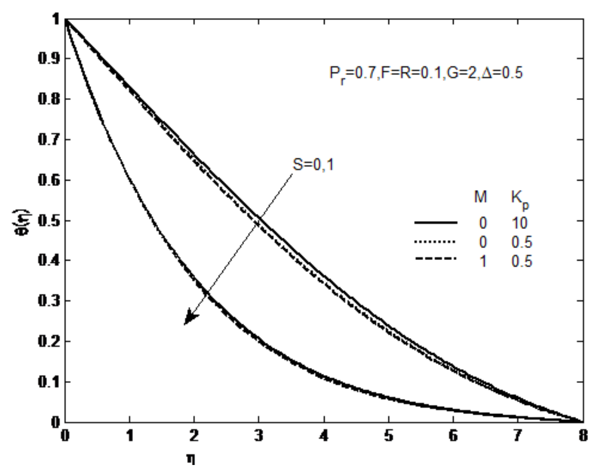

Fig. 9: Temperature profile for various values of S, M and K_p

Fig. 9 illustrates the effect of the uniform heat source in the presence/absence of M and K_p. It is evident that growth in M and K_p the temperature profile irrespectively decelerates for different values of the heat source. When $M = 0, S = 0$ and K_p (without porous matrix) the profile attaining the maximum value and tends to meet the boundary conditions for $\eta \to \infty$. The thermal radiation effect has been observed in both the presence/absence of heat source and shown in Fig. 10. There two-layer variations in temperature profile have been remarked for $S = 0$ and $S = 1$ at different values of radiation parameter (R). From Fig. 10, it can be observed that the temperature decreases in the thermal boundary layer with an increase in R and S. The present results make a good agreement with the outcome of Rashidi et al. [23] by withdrawing the parame-

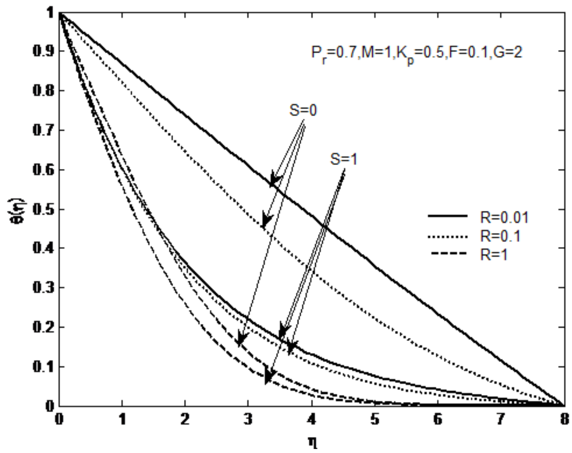

Fig. 10: Temperature profile for different values of S and R

ter $S(S = 0)$ from the energy equation. Thermal radiation emitted by a hot body depends not only on its temperature but also on the material of the body, its shape and the nature of its surface.

The flow characteristics at the boundary surface are vital in determining the flow stability and hence calculation of skin friction, wall couple stress and Nusselt number are essential. The numerical values of aforesaid parameters have been shown in Table 1 for different pertinent parameters. It can appear that the skin friction and rate of heat transfer increases with the increase of magnetic parameter while the reverse effect has been observed for couple stress. The inclusion of porous matrix opposes the flow and heat transfer aspect resulting in retardation in skin friction coefficient and Nusselt number, but couple stresses enhanced it. In the unsteady case, the presence of heat source parameter increases the values of $f''(0)$, $g'(0)$ and $-\theta'(0)$.

6 Conclusions

In the present study, the effect of heat transfer on an electrically conducting MHD micropolar fluid through a porous medium has been analysed considering radiation and uniform heat source. The present simulations results highlighted the following facts:

1. The velocity profile enhances at all points in the boundary layer with an increase in the magnetic parameter for the absence/presence of coupling parameter.
2. The velocity distribution increases with increase in microinertia coefficient for both presence/absence of magnetic parameter.
3. The temperature distributions in the thermal boundary layer follow a declining trend as the radiation and source parameter increases.
4. The increase in magnetic parameter and porous parameter decelerates the temperature profile irrespectively for the values of the heat source parameter.
5. The presence/absence of magnetic parameter and coupling parameter enable to enhance the angular velocity profile while it is worth to note that the backflow has generated in the vicinity of the plate.
6. In the absence of heat source, i.e. $S = 0$, present result is in good agreement with Rashidi et al. [43].

Nomenclature

B_0	applied magnetic field
C_{fx}	local skin friction co-efficient
C_p	specific heat at constant pressure
R	radiation parameter
F	Inertia coefficient parameter
f	dimensionless stream function
G	microrotation parameter
K_p	local porous parameter
k	thermal conductivity
$K_p{}^*$	permeability of the porous medium
M	magnetic field parameter
M_{sx}	local wall couple stress
N_{ux}	local Nusselt number
P_r	Prandtl number
Q_0	heat source coefficient
R_{ex}	local Reynold number
S	heat source parameter
T	non-dimensional temperature
t	non-dimensional time
T_ω	wall temperature of the fluid
T_∞	temperature of the fluid far away from the sheet
U_ω	sheet velocity
(u, v)	velocity components
(x, y)	cartesian coordinates

Greek symbol

υ	kinematics coefficient of viscosity
μ	coefficient of viscosity
σ	electrical conductivity
Δ	coupling constant
ρ	density of the fluid
τ_w	wall shear stress
θ	non-dimensional temperature
η	similarity variable

subscripts

ω	condition at wall
∞	condition at free stream

Table 1: Skin friction, wall couple stress, Nusselt number for the values of $R = 0.1$, $G = 2$.

M	K_p	F	S	Δ	$f''(0)$	$g'(0)$	$-\theta'(0)$
0	0.5	0.1	0	0	1.520775	−0.53477	0.271877
1	0.5	0.1	0	0	1.819147	−0.57523	0.273121
1	0.6	0.1	0	0	1.725367	−0.56327	0.272758
1	0.5	0.2	0	0	1.855385	−0.58043	0.27328
1	0.5	0.1	1	0	1.819147	−0.57523	0.517112
1	0.5	0.1	1	1	1.728685	−0.57578	0.517155
2	0.5	0.1	1	1	1.987426	−0.60527	0.517801
2	0.6	0.1	1	1	1.904827	−0.5963	0.517605
2	0.5	0.2	1	1	2.019429	−0.60923	0.517887

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