



## Opinionated article

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# Metamaterials with high degrees of freedom: space, time, and more

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**Abstract:** In this brief opinionated article, I present a personal perspective on metamaterials with high degrees of freedom and dimensionality and discuss their potential roles in enriching light–matter interaction in photonics and related fields.

**Keywords:** dimensionality; light–matter interaction; metamaterial; photonics.

## 1 Main text

To control and manipulate photons we often use materials. The light–matter interaction, which is at the heart of optics and photonics, is governed by the laws of electrodynamics, both classical as well as quantum electrodynamics. In dealing with electromagnetic wave and field interaction with materials, one can always start at the quantum level where photons interact with atoms. However, we often prefer to parameterize such interaction and consequently we commonly use the notion of *macroscopic* light–matter interaction, in which we assign certain material parameters to media with which waves and fields interact. Such electromagnetic parameters are well known quantities, such as permittivity ( $\epsilon$ ), permeability ( $\mu$ ), conductivity ( $\sigma$ ), chirality ( $\xi$ ), nonlinear susceptibilities ( $\chi^{(2)}$ ,  $\chi^{(3)}$ , ...), etc., which are based on electric and magnetic dipolar terms, an approximation from the formal treatment of multipole expansion in electromagnetic interaction [1]. (The higher multipolar terms, such as quadrupoles, octopoles, etc. can also be added and augmented, but often dipolar terms suffice.) In Nature, there are numerous materials, all derived from the elements of the periodic table, that

interact with electromagnetic waves. Such materials are made of atoms and molecules with specific arrangements. Their chemical compositions determine their various properties, ranging from mechanical, thermal, acoustical, optical, and electromagnetic properties, just to name a few. For these materials, the material parameters listed above attain certain ranges of values, which depend on various factors including frequency of operation. However, specially designed material structures, often termed *metamaterials* and *metastructures*, have provided scientists and engineers with the opportunities to go beyond this limitation by considering structural arrangements, instead of (and in addition to) natural chemical arrangements, as collections of tiny structures with man-made patterns embedded in host media [2–5]. Depending on the compositions, arrangements, alignments, densities, shapes and geometries of these inclusions, and their host media, such metamaterials may offer unusual wave–matter features and their effective parameters may attain values beyond what Nature has given us. This has provided unprecedented opportunities for innovations in material-based devices and systems.

To achieve desired functionality from light–matter interaction, we often utilize spatial inhomogeneities in material parameters. For example, a simple convex lens operates based on the light propagation from air to the lens, formed by a material with  $\epsilon(x, y, z)$  different from that of air and with a proper shape. Waveguides are another common example of spatial inhomogeneity, in which the core has higher relative permittivity than that of the cladding, thus confining the guided waves within and along the guide. In general, based on spatial distributions of material parameters, one can categorize the spatial dimensionality of electromagnetic problems as three-dimensional (3D) such as general 3D scattering, two-dimensional (2D) such as parallel-plate waveguides, one-dimensional (1D) such as transmission lines, and ‘zero-dimensional’ (0D) as lumped circuit elements [6]. This categorization has also been utilized in the field of metamaterials, leading to the notions of 3D volumetric metamaterials [2–5], 2D metasurfaces [7–14], 1D left-handed transmission lines [4, 5], and 0D optical metatronics [15, 16]. However, a question

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may be naturally raised here: Can we have higher dimensional metamaterials? Can we have metastructures with four dimensions? How about higher degrees of freedom? This indeed may be possible. But why should one be interested in such increase in material dimensionality? On the one hand, in most cases the higher the dimensions, the more degrees of freedom we can use to get to our goals of desired functionalities in wave-based devices and systems. On the other hand, frequently the complexity also increases with the dimensionality. As one approach to increasing the dimensionality of metamaterials, one can consider bringing the dimension of “time” into this platform. This means that one would desire to have parameters such as  $\varepsilon$  also change with time, instead of (or in addition to) its variation in space, i.e.,  $\varepsilon(t)$  or  $\varepsilon(x, y, z, t)$ . Such time-varying, or time and space inhomogeneous platforms can certainly increase the number of “knobs” we can “turn” to control and achieve desired functionalities in light–matter interaction. The research on time-varying electromagnetic platforms has a long history dating back to 1950s, when one of the early works, by Morgenthaler [17], explored theoretically what would happen to a monochromatic electromagnetic wave in a medium when the medium’s phase velocity is rapidly changed in time. In the past decades, there have been numerous efforts in investigating various phenomena related to the temporal variation of electromagnetic media, providing temporal boundaries [18], temporal holography [19], Doppler cloaking [20], temporal band gaps [21–23], temporal effective medium concept [24], temporal impedance matching [25, 26], Fresnel drag in spatiotemporal metamaterials [27], spacetime cloaks [28, 29], modeling time and causality with metamaterials [30, 31], rapidly growing plasma and accelerating reference frame [32], and exceptional points in time-varying media [33], just to name a few.

Although in the Maxwellian electrodynamics there are symmetries between space  $(x, y, z)$  and time  $(t)$ , such as the time-reversal and space-reversal phenomena, and consequently numerous features have similar and analogous spatial and temporal characteristics, there are of course some fundamental differences. For instance, we have three dimensions of  $x, y, z$  in the Euclidean space, whereas in time  $(t)$  we have only one. For the spatial dimensions, we have no spatial “causality”, i.e., we can have positive and negative values for  $x, y, z$ , without any special orders. However, we have causality in time  $(t)$ , so events always move forward in time, consistent with the second law of thermodynamics. Such differences between the space  $(x, y, z)$  and time  $(t)$  provide additional richness to wave–matter interaction in spatiotemporal metamaterials. For

example, such four-dimensional (4D) structures can be used to break the reciprocity without using any biasing DC magnetic field, as demonstrated by several groups [34–43]. Moreover, temporal media exhibit wave features that are more complex than those of their time-invariant counterparts, such as time-varying polarizabilities [44] and a generalization of the Kramers–Kronigs relations based on the mathematics of linear time-variant systems [45].

Clearly, 4D metamaterials have brought, and continue to bring, various exciting new phenomena into the realm of light–matter interaction. But can we have 4D metamaterials with more variables and higher degrees of freedom? Adding additional degrees of freedom can bring more novelty in controlling the wave phenomena using such structures. One possible way to do so is by merging temporal variation of the materials parameters, e.g.,  $\varepsilon(t)$  or  $\varepsilon(x, y, z, t)$  with the notion of anisotropy of these parameters. In other words, instead of changing isotropic values of  $\varepsilon$  in time, can we consider a temporal change of  $\varepsilon$  from an isotropic value to an anisotropic tensoral value, which implies that one may want to change a single scalar value of  $\varepsilon$  to different values for each element of anisotropic permittivity tensors? In this way, for example, can we change a wave platform temporally from a simple isotropic medium to a uniaxial or a biaxial crystal? This approach, which we can call “temporal anisotropy” can bring a new set of degrees of freedom into the electromagnetic platforms. Since wave propagation in anisotropic media has specific features, distinct from that in isotropic media, such temporal anisotropy can bring novel wave phenomena. For example, it is well known that in anisotropic media waves can experience birefringence, i.e., refractive index is different for different directions of propagation and/or different wave polarizations [1]. Moreover, it is also known that in anisotropic media the direction of phase flow (i.e., the wave vector) is not necessarily the same as the direction of energy flow (i.e., the Poynting vector). These two features have been recently utilized in developing new phenomena of the inverse prism [46], in which after the rapid change of refractive index from isotropic to an anisotropic set of values the waves experience different frequencies in different directions of propagation, and the temporal aiming [47], in which the direction of energy propagation for a wave packet can change midstream by rapid change of permittivity from an isotropic to an anisotropic case. Together with my collaborators, we are currently investigating various aspects of such 4D metamaterials with high degree of freedom.

Although in the above discussion, we mentioned the 4D structures with high degrees of freedom based only on

one of the material parameters, namely the permittivity, obviously this methodology can also be applied to other material parameters such as, permeability ( $\mu$ ), conductivity ( $\sigma$ ), chirality ( $\xi$ ), nonlinear susceptibilities ( $\chi^{(2)}, \chi^{(3)}, \dots$ ), etc. Therefore, one can imagine such high degree-of-freedom media for several of these parameters concurrently. This would bring interesting curiosity-driven research possibilities. For example, what would happen if the permittivity of a structure can be engineered spatiotemporally with given spatial and temporal distributions, while at the same time and in the same place, its permeability and/or its chirality can change with space and time with a different set of spatiotemporal distributions? What if in a layered structure, each layer consists of spatiotemporal structures with a set of space–time modulated parameters different from those of other layers, some isotropic and some anisotropic? What about the applications of mathematical optimization and inverse design tools to such metamaterials? One can of course think about many other possibilities for increasing and merging various degrees of freedom. This will certainly provide far richer (and admittedly more complex) platforms for wave–matter interaction, which may provide novel material-based devices and components.

To summarize, in my opinion it seems that increasing degrees of freedom in spatiotemporal metamaterials, while adds more complexity to the structures, can open new directions in research in light–matter interaction, with the goals towards devices and systems with new functionalities. Such possibilities are limitless.

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