Xiangling Zhu

WEIGHTED COMPOSITION OPERATORS
FROM WEIGHTED HARDY SPACES
TO WEIGHTED-TYPE SPACES

Abstract. The boundedness and compactness of the weighted composition operator from weighted Hardy spaces to weighted-type spaces are studied in this paper.

1. Introduction

Let $\mathbb{D}$ be the open unit disk in the complex plane $\mathbb{C}$. We denote by $H(\mathbb{D})$ the class of all holomorphic functions on $\mathbb{D}$. Let φ be an analytic self-map of $\mathbb{D}$. The composition operator C_φ is defined by

$$(C_\varphi f)(z) = f(\varphi(z)), \quad z \in \mathbb{D}, \ \ f \in H(\mathbb{D}).$$

It is interesting to provide a function theoretic characterization of when φ induces a bounded or compact composition operator between spaces of analytic functions. The book [2] contains plenty of information on this topic.

Let φ be an analytic self-map of $\mathbb{D}$ and $u \in H(\mathbb{D})$. The weighted composition operator uC_φ on $H(\mathbb{D})$ is given by $(uC_\varphi f)(z) = u(z)f(\varphi(z))$, $z \in \mathbb{D}$, $f \in H(\mathbb{D})$.

Throughout this paper, we assume that $\{\beta(n)\}_{n=0}^{\infty}$ is a sequence of positive numbers such that

$$\beta(0) = 1, \quad \liminf_{n \to \infty} \beta(n)^{1/n} = 1 \ \ \text{and} \ \ \sum_{n=0}^{\infty} 1/(\beta(n))^2 = \infty.$$

The weighted Hardy space, denoted by $H^2(\beta)$, is defined to be the set of all $f(z) = \sum_{n=0}^{\infty} a_n z^n \in H(\mathbb{D})$ such that

$$\|f\|_{H^2(\beta)}^2 = \sum_{n=0}^{\infty} |a_n|^2 (\beta(n))^2 < \infty.$$

It is clear that $H^2(\beta)$ is a Hilbert space on $\mathbb{D}$ with the inner product given by

$$\langle f, g \rangle = \sum_{n=0}^{\infty} a_n \overline{b_n} (\beta(n))^2,$$

where $f(z) = \sum_{n=0}^{\infty} a_n z^n$ and $g(z) = \sum_{n=0}^{\infty} b_n z^n$ are in $H^2(\beta)$. Some well-known special cases of this type of Hilbert space are: the Hardy space H^2 with weights $\beta(n) \equiv 1$, the Bergman space A^2 with weights $\beta(n) = (n+1)^{-1/2}$, and the Dirichlet space $\mathcal{D}^2$ with weights $\beta(n) = (n+1)^{1/2}$ for all n. See [2] for more details of the weighted Hardy space.

A positive continuous function μ on $[0,1)$ is called normal, if there exist positive numbers α and β, $0 < \alpha < \beta$, and $\delta \in [0,1)$ such that (see [11])

$$\frac{\mu(r)}{(1-r)^{\alpha}} \text{ is decreasing on } [\delta, 1) \text{ and } \lim_{r \to 1} \frac{\mu(r)}{(1-r)^{\alpha}} = 0;$$

$$\frac{\mu(r)}{(1-r)^{\beta}} \text{ is increasing on } [\delta, 1) \text{ and } \lim_{r \to 1} \frac{\mu(r)}{(1-r)^{\beta}} = \infty.$$

An $f \in H(\mathbb{D})$ is said to belong to the weighted-type space, denoted by $H_{\mu}^{\infty} = H_{\mu}^{\infty}(\mathbb{D})$, if $\|f\|_{H_{\mu}^{\infty}} = \sup_{z \in \mathbb{D}} \mu(|z|) |f(z)| < \infty$, where μ is normal on $[0,1)$. H_{μ}^{∞} is a Banach space with the norm $\|\cdot\|_{H_{\mu}^{\infty}}$. The little weighted-type space, denoted by $H_{\mu,0}^{\infty}$, is the subspace of H_{μ}^{∞} consisting of those $f \in H_{\mu}^{\infty}$ such that $\lim_{|z| \to 1} \mu(|z|)|f(z)| = 0$. When $\mu(r) = (1 - r^2)^{\alpha}$, the induced spaces H_{μ}^{∞} and $H_{\mu,0}^{\infty}$ become the Bers space H_{α}^{∞} and little Bers space $H_{\alpha,0}^{\infty}$, respectively (see [6]).

Composition operators and weighted composition operators on weighted-type spaces have been studied, for example, in [1, 3, 6, 9, 10]. For the case of the unit ball, the problem has already been treated in [4, 22, 24]. Weighted composition operators from some function spaces (such as generalized weighted Bergman space, mixed norm space, $F(p, q, s)$ space and Bloch-type space) to weighted type space H_{μ}^{∞} in the unit ball were studied in [5, 7, 8, 12–16, 18, 23].

In this paper, we study the weighted composition operator from the weighted Hardy space $H^2(\beta)$ to spaces H_{μ}^{∞} and $H_{\mu,0}^{\infty}$. Some necessary and sufficient conditions, for the weighted composition operator uC_{φ} to be bounded and compact, are given. As corollaries, we obtain the characterizations of the weighted composition operator from the Hardy space, the Bergman space and the Dirichlet space to spaces H_{μ}^{∞} and $H_{\mu,0}^{\infty}$.

Throughout the paper, constants are denoted by C, they are positive and may not be the same in every occurrence.

2. Main results and proofs

In this section, we give our main results and their proofs. Before stating these results, we need some auxiliary results, which are incorporated in the lemmas which follow.

Lemma 1. *Assume that $u \in H(\mathbb{D})$, φ is an analytic self-map of $\mathbb{D}$ and μ is a normal function on $[0,1)$. Then $uC_\varphi : H^2(\beta) \to H^\infty_\mu$ is compact if and only if $uC_\varphi : H^2(\beta) \to H^\infty_\mu$ is bounded and for any bounded sequence $(f_k)_{k\in\mathbb{N}}$ in $H^2(\beta)$ which converges to zero uniformly on compact subsets of $\mathbb{D}$ as $k \to \infty$, we have $\|uC_\varphi f_k\|_{H^\infty_\mu} \to 0$ as $k \to \infty$.*

The proof of Lemma 1 follows by standard arguments (see, for example, Proposition 3.11 of [2]). Hence, we omit the details.

Lemma 2. [9] *Assume that μ is normal. A closed set K in $H^\infty_{\mu,0}$ is compact if and only if it is bounded and satisfies*

$$\lim_{|z|\to 1} \sup_{f\in K} \mu(|z|)|f(z)| = 0.$$

Lemma 3. *Let $f \in H^2(\beta)$. Then*

$$|f(z)| \leq \|f\|_{H^2(\beta)} \sqrt{\sum_{n=0}^\infty \frac{|z|^{2n}}{\beta^2(n)}}.$$

Proof. For $w \in \mathbb{D}$, define $K_w(z) = \sum_{n=0}^\infty \frac{\overline{w}^n}{\beta^2(n)} z^n$. Then $K_w \in H^2(\beta)$. Let $f(z) = \sum_{n=0}^\infty a_n z^n$. From [2, page 16], we see that

$$(1) \qquad f(w) = \langle f, K_w \rangle = \sum_{n=0}^\infty \frac{a_n w^n}{\beta^2(n)} \beta^2(n) = \sum_{n=0}^\infty a_n w^n$$

and

$$(2) \qquad \|K_w\|_{H^2(\beta)} = \sqrt{\sum_{n=0}^\infty \frac{|w|^{2n}}{\beta^4(n)} (\beta^2(n))} = \sqrt{\sum_{n=0}^\infty \frac{|w|^{2n}}{\beta^2(n)}} < \infty.$$

Then the desired result follows from (1) and (2). ∎

Now we are in a position to state and prove our main results.

Theorem 1. *Assume that $u \in H(\mathbb{D})$, φ is an analytic self-map of $\mathbb{D}$ and μ is a normal function on $[0,1)$. Then $uC_\varphi : H^2(\beta) \to H^\infty_\mu$ is bounded if and only if*

$$(3) \qquad M := \sup_{z\in\mathbb{D}} \mu(|z|)|u(z)| \sqrt{\sum_{n=0}^\infty \frac{|\varphi(z)|^{2n}}{\beta^2(n)}} < \infty.$$

Proof. Assume that $uC_\varphi : H^2(\beta) \to H^\infty_\mu$ is bounded. For $a \in \mathbb{D}$, set

(4)
$$f_a(z) = \sum_{n=0}^{\infty} \frac{\overline{a}^n z^n}{\beta^2(n)} \left(\sum_{n=0}^{\infty} \frac{|a|^{2n}}{\beta^2(n)} \right)^{-1/2}.$$

It is easy to see that $f_a \in H^2(\beta)$ and $\sup_{a \in \mathbb{D}} \|f_a\|_{H^2(\beta)} = 1$. For any $b \in \mathbb{D}$, we have

(5)
$$\infty > \|uC_\varphi f_{\varphi(b)}\|_{H^\infty_\mu} = \sup_{z \in \mathbb{D}} \mu(|z|) |(uC_\varphi f_{\varphi(b)})(z)|$$

$$= \sup_{z \in \mathbb{D}} \mu(|z|) |u(z)| |f_{\varphi(b)}(\varphi(z))|$$

$$\geq \mu(|b|) |u(b)| \sqrt{\sum_{n=0}^{\infty} \frac{|\varphi(b)|^{2n}}{\beta^2(n)}},$$

which implies (3).

Conversely, assume that (3) holds. Then, for any $f \in H^2(\beta)$,

(6)
$$\mu(|z|) |(uC_\varphi f)(z)| = \mu(|z|) |u(z)| |f(\varphi(z))|$$

$$\leq \mu(|z|) |u(z)| \sqrt{\sum_{n=0}^{\infty} \frac{|\varphi(z)|^{2n}}{\beta^2(n)}} \|f\|_{H^2(\beta)}.$$

Taking the supremum in (6) over $\mathbb{D}$ and using the condition (3), the boundedness of the operator $uC_\varphi : H^2(\beta) \to H^\infty_\mu$ follows, as desired. ∎

THEOREM 2. *Assume that $u \in H(\mathbb{D})$, φ is an analytic self-map of $\mathbb{D}$ and μ is a normal function on $[0,1)$. Then $uC_\varphi : H^2(\beta) \to H^\infty_\mu$ is compact if and only if $u \in H^\infty_\mu$ and*

(7)
$$\lim_{|\varphi(z)| \to 1} \mu(|z|) |u(z)| \sqrt{\sum_{n=0}^{\infty} \frac{|\varphi(z)|^{2n}}{\beta^2(n)}} = 0.$$

Proof. Assume that $uC_\varphi : H^2(\beta) \to H^\infty_\mu$ is compact. Then it is obvious that $uC_\varphi : H^2(\beta) \to H^\infty_\mu$ is bounded. Taking the function $f(z) = 1 \in H^2(\beta)$, we see that $u \in H^\infty_\mu$. Let $(\varphi(z_k))_{k \in \mathbb{N}}$ be a sequence in $\mathbb{D}$ such that $\lim_{k \to \infty} |\varphi(z_k)| = 1$ (if such a sequence does not exist (7) is automatically satisfied). Set

(8)
$$f_k(z) = \sum_{n=0}^{\infty} \frac{\overline{\varphi(z_k)}^n z^n}{\beta^2(n)} \left(\sum_{n=0}^{\infty} \frac{|\varphi(z_k)|^{2n}}{\beta^2(n)} \right)^{-1/2}, \quad k \in \mathbb{N}.$$

It is easy to see that $\sup_{k \in \mathbb{N}} \|f_k\|_{H^2(\beta)} < \infty$. Moreover, $f_k \to 0$ uniformly on compact subsets of $\mathbb{D}$ as $k \to \infty$. By Lemma 1,

(9)
$$\lim_{k \to \infty} \|uC_\varphi f_k\|_{H^\infty_\mu} = 0.$$

We have

$$(10) \qquad \|uC_\varphi f_k\|_{H_\mu^\infty} = \sup_{z\in\mathbb{D}} \mu(|z|)|u(z)f_k(\varphi(z))|$$

$$\geq \mu(|z_k|)|u(z_k)| \sqrt{\sum_{n=0}^{\infty} \frac{|\varphi(z_k)|^{2n}}{\beta^2(n)}},$$

which together with (9) implies that

$$\lim_{k\to\infty} \mu(|z_k|)|u(z_k)| \sqrt{\sum_{n=0}^{\infty} \frac{|\varphi(z_k)|^{2n}}{\beta^2(n)}} = 0.$$

This proves that (7) holds.

Conversely, assume that $u \in H_\mu^\infty$ and (7) holds. From this it follows that (3) holds, hence $uC_\varphi : H^2(\beta) \to H_\mu^\infty$ is bounded. In order to prove that $uC_\varphi : H^2(\beta) \to H_\mu^\infty$ is compact, according to Lemma 1, it suffices to show that if $\{f_k\}_{k\in\mathbb{N}}$ is a bounded sequence in $H^2(\beta)$ converging to 0 uniformly on compact subsets of $\mathbb{D}$, then $\lim_{k\to\infty} \|uC_\varphi f_k\|_{H_\mu^\infty} = 0$.

Let $\{f_k\}_{k\in\mathbb{N}}$ be a bounded sequence in $H^2(\beta)$ such that $f_k \to 0$ uniformly on compact subsets of $\mathbb{D}$ as $k \to \infty$. By (7), we have that for any $\varepsilon > 0$, there is a constant $\delta \in (0,1)$, such that

$$(11) \qquad \mu(|z|)|u(z)| \sqrt{\sum_{n=0}^{\infty} \frac{|\varphi(z)|^{2n}}{\beta^2(n)}} < \varepsilon$$

whenever $\delta < |\varphi(z)| < 1$. Let $K = \{w \in \mathbb{D} : |w| \leq \delta\}$. Inequality (11) along with the fact that $u \in H_\mu^\infty$ implies

$$\|uC_\varphi f_k\|_{H_\mu^\infty} = \sup_{z\in\mathbb{D}} \mu(|z|)|(uC_\varphi f_k)(z)| = \sup_{z\in\mathbb{D}} \mu(|z|)|u(z)f_k(\varphi(z))|$$

$$\leq \left(\sup_{\{z\in\mathbb{D}:\, |\varphi(z)|\leq\delta\}} + \sup_{\{z\in\mathbb{D}:\, \delta\leq|\varphi(z)|<1\}} \right) \mu(|z|)|u(z)||f_k(\varphi(z))|$$

$$\leq \|u\|_{H_\mu^\infty} \sup_{w\in K} |f_k(w)|$$

$$+ \sup_{\{z\in\mathbb{D}:\, \delta\leq|\varphi(z)|<1\}} \mu(|z|)|u(z)| \sqrt{\sum_{n=0}^{\infty} \frac{|\varphi(z)|^{2n}}{\beta^2(n)}} \|f_k\|_{H^2(\beta)}$$

$$\leq \|u\|_{H_\mu^\infty} \sup_{w\in K} |f_k(w)| + C\varepsilon.$$

Observe that K is a compact subset of $\mathbb{D}$, so that $\lim_{k\to\infty} \sup_{w\in K} |f_k(w)| = 0$. Hence

$$\limsup_{k\to\infty} \|uC_\varphi f_k\|_{H_\mu^\infty} \leq C\varepsilon.$$

Since ε is an arbitrary positive number, it follows that the last limit is equal to zero. Therefore, $uC_\varphi : H^2(\beta) \to H_\mu^\infty$ is compact. The proof is completed. ∎

THEOREM 3. *Assume that $u \in H(\mathbb{D})$, φ is an analytic self-map of $\mathbb{D}$ and μ is a normal function on $[0,1)$. Then $uC_\varphi : H^2(\beta) \to H_{\mu,0}^\infty$ is bounded if and only if $uC_\varphi : H^2(\beta) \to H_\mu^\infty$ is bounded and $u \in H_{\mu,0}^\infty$.*

Proof. First assume that $uC_\varphi : H^2(\beta) \to H_{\mu,0}^\infty$ is bounded. Then, it is clear that $uC_\varphi : H^2(\beta) \to H_\mu^\infty$ is bounded. Taking the function $f(z) = 1$, we obtain that $u \in H_{\mu,0}^\infty$.

Conversely, assume that $uC_\varphi : H^2(\beta) \to H_\mu^\infty$ is bounded and $u \in H_{\mu,0}^\infty$. Then, for each polynomial $p(z)$, we have that

$$\mu(|z|)|(uC_\varphi p)(z)| \le \mu(|z|)|u(z)p(\varphi(z))|,$$

from which it follows that $uC_\varphi p \in H_{\mu,0}^\infty$. Since the set of all polynomials is dense in $H^2(\beta)$ (see [2]), we have that for every $f \in H^2(\beta)$ there is a sequence of polynomials $\{p_k\}_{k\in\mathbb{N}}$ such that $\|f - p_k\|_{H^2(\beta)} \to 0$, as $k \to \infty$. Hence

$$\|uC_\varphi f - uC_\varphi p_k\|_{H_\mu^\infty} \le \|uC_\varphi\|_{H^2(\beta)\to H_\mu^\infty}\|f - p_k\|_{H^2(\beta)} \to 0$$

as $k \to \infty$, since the operator $uC_\varphi : H^2(\beta) \to H_\mu^\infty$ is bounded. Since $H_{\mu,0}^\infty$ is a closed subset of H_μ^∞, we obtain $uC_\varphi(H^2(\beta)) \subset H_{\mu,0}^\infty$. Therefore $uC_\varphi : H^2(\beta) \to H_{\mu,0}^\infty$ is bounded. ∎

THEOREM 4. *Assume that $u \in H(\mathbb{D})$, φ is an analytic self-map of $\mathbb{D}$ and μ is a normal function on $[0,1)$. Then the following statements are equivalent.*

(i) $uC_\varphi : H^2(\beta) \to H_{\mu,0}^\infty$ is compact;
(ii) $uC_\varphi : H^2(\beta) \to H_\mu^\infty$ is compact and $u \in H_{\mu,0}^\infty$;
(iii) $u \in H_{\mu,0}^\infty$ and

$$(12) \qquad \lim_{|\varphi(z)|\to 1} \mu(|z|)|u(z)|\sqrt{\sum_{n=0}^\infty \frac{|\varphi(z)|^{2n}}{\beta^2(n)}} = 0;$$

(iv) (13)
$$\lim_{|z|\to 1} \mu(|z|)|u(z)|\sqrt{\sum_{n=0}^\infty \frac{|\varphi(z)|^{2n}}{\beta^2(n)}} = 0.$$

Proof. (i)⇒(ii). Assume that $uC_\varphi : H^2(\beta) \to H_{\mu,0}^\infty$ is compact. Then it is clear that $uC_\varphi : H^2(\beta) \to H_\mu^\infty$ is compact. In addition, taking the function $f(z) = 1$ we get $u \in H_{\mu,0}^\infty$.

(ii)⇒(iii). This implication follows from Theorem 2.

(iii)⇒(iv). Suppose that $u \in H_{\mu,0}^{\infty}$ and (12) holds. From (12) we see that for every $\varepsilon > 0$, there exists a $\delta \in (0,1)$, such that

$$\mu(|z|)|u(z)| \sqrt{\sum_{n=0}^{\infty} \frac{|\varphi(z)|^{2n}}{\beta^2(n)}} < \varepsilon$$

when $\delta < |\varphi(z)| < 1$. From the assumption $u \in H_{\mu,0}^{\infty}$, we have that for the above ε, there exists an $r \in (0,1)$ such that

$$\mu(|z|)|u(z)| < \frac{\varepsilon}{\sqrt{\sum_{n=0}^{\infty} \frac{\delta^{2n}}{\beta^2(n)}}}$$

when $r < |z| < 1$. Therefore, if $r < |z| < 1$ and $\delta < |\varphi(z)| < 1$, we obtain

$$(14) \qquad \mu(|z|)|u(z)| \sqrt{\sum_{n=0}^{\infty} \frac{|\varphi(z)|^{2n}}{\beta^2(n)}} < \varepsilon.$$

If $|\varphi(z)| \leq \delta$ and $r < |z| < 1$, we have that

$$(15) \qquad \mu(|z|)|u(z)| \sqrt{\sum_{n=0}^{\infty} \frac{|\varphi(z)|^{2n}}{\beta^2(n)}} \leq \mu(|z|)|u(z)| \sqrt{\sum_{n=0}^{\infty} \frac{\delta^{2n}}{\beta^2(n)}} < \varepsilon.$$

Combing (14) with (15) we get (13).

(iv)⇒(i). Let $f \in H^2(\beta)$. From the proof of Theorem 1, we have that

$$\mu(|z|)|(uC_\varphi f)(z)| \leq \mu(|z|)|u(z)| \sqrt{\sum_{n=0}^{\infty} \frac{|\varphi(z)|^{2n}}{\beta^2(n)}} \|f\|_{H^2(\beta)}.$$

Taking the supremum in the above inequality over all $f \in H^2(\beta)$ such that $\|f\|_{H^2(\beta)} \leq 1$, then letting $|z| \to 1$, by (13) it follows that

$$\lim_{|z| \to 1} \sup_{\|f\|_{H^2(\beta)} \leq 1} \mu(|z|)|(uC_\varphi(f))(z)| = 0.$$

From this and by employing Lemma 2, we see that $uC_\varphi : H^2(\beta) \to H_{\mu,0}^{\infty}$ is compact. The proof is completed. ∎

Let $\beta(n) \equiv 1$. Then

$$\sum_{n=0}^{\infty} \frac{|\varphi(z)|^{2n}}{\beta^2(n)} = \sum_{n=0}^{\infty} |\varphi(z)|^{2n} = \frac{1}{1 - |\varphi(z)|^2}.$$

From Theorems 1–4, we have the following corollary.

COROLLARY 1. *Assume that $u \in H(\mathbb{D})$, φ is an analytic self-map of $\mathbb{D}$ and μ is a normal function on $[0,1)$. Then the following statements hold.*

Let $\beta(n) = (n+1)^{1/2}$. Then

$$\sum_{n=0}^{\infty} \frac{|\varphi(z)|^{2n}}{\beta^2(n)} = \sum_{n=0}^{\infty} \frac{|\varphi(z)|^{2n}}{n+1} \asymp \ln \frac{e}{1 - |\varphi(z)|^2}.$$

From Theorems 1–4, we have the following corollary.

COROLLARY 3. *Assume that* $u \in H(\mathbb{D})$, φ *is an analytic self-map of* $\mathbb{D}$ *and* μ *is a normal function on* $[0,1)$. *Then the following statements hold.*

(i) *The operator* $uC_\varphi : \mathcal{D}^2 \to H_\mu^\infty$ *is bounded if and only if*

$$\sup_{z \in \mathbb{D}} \mu(|z|)|u(z)| \sqrt{\ln \frac{e}{1 - |\varphi(z)|^2}} < \infty.$$

(ii) *The operator* $uC_\varphi : \mathcal{D}^2 \to H_{\mu,0}^\infty$ *is bounded if and only if* $uC_\varphi : \mathcal{D}^2 \to H_\mu^\infty$ *is bounded and* $u \in H_{\mu,0}^\infty$.

(iii) *The operator* $uC_\varphi : \mathcal{D}^2 \to H_\mu^\infty$ *is compact if and only if* $u \in H_\mu^\infty$ *and*

$$\lim_{|\varphi(z)| \to 1} \mu(|z|)|u(z)| \sqrt{\ln \frac{e}{1 - |\varphi(z)|^2}} = 0.$$

(iv) *The operator* $uC_\varphi : \mathcal{D}^2 \to H_{\mu,0}^\infty$ *is compact if and only if*

$$\lim_{|z| \to 1} \mu(|z|)|u(z)| \sqrt{\ln \frac{e}{1 - |\varphi(z)|^2}} = 0.$$

REMARK 3. Corollary 3(i) is a particular case of Theorem 1 in [20], as well as of its generalization in [21].

References

[1] J. Bonet, P. Domański, M. Lindström, J. Taskinen, *Composition operators between weighted Banach spaces of analytic functions*, J. Austral. Math. Soc. (Ser. A) 64 (1998), 101–118.

[2] C. C. Cowen, B. D. MacCluer, *Composition Operators on Spaces of Analytic Functions*, Studies in Advanced Math., CRC Press, Boca Raton, 1995.

[3] P. Galindo, M. Lindström, *Essential norm of operators on weighted Bergman spaces of infinite order*, J. Operator Theory 64 (2010), 387–399.

[4] P. Galindo, M. Lindström, S. Stević, *Essential norm of operators into weighted-type spaces on the unit ball*, Abstr. Appl. Anal., Vol. 2011, Article ID 939873, (2011), 13 pp.

[5] D. Gu, *Weighted composition operators from generalized weighted Bergman spaces to weighted-type space*, J. Inequal. Appl., Vol. 2008, Article ID 619525, (2008), 14 pp.

[6] W. He, L. Jiang, *Composition operator on Bers-type spaces*, Acta Math. Sci. 22B(3) (2002), 404–412.

[7] S. Li, S. Stević, *Weighted composition operators from α-Bloch space to H^∞ on the polydisk*, Numer. Funct. Anal. Opt. 28(7) (2007), 911–925.

[8] S. Li, S. Stević, *Weighted composition operators between H^∞ and α-Bloch spaces in the unit ball*, Taiwanese J. Math. 12(7) (2008), 1625–1639.

[9] A. Montes-Rodríguez, *Weighted composition operators on weighted Banach spaces of analytic functions*, J. London Math. Soc. 61(3) (2000), 872–884.

[10] M. Wang, Y. Liu, *Weighted composition operator between Bers-type spaces*, Acta Math. Sci. 27A (2007), 665–671.

[11] A. L. Shields, D. L. Williams, *Bounded projections, duality, and multipliers in spaces of analytic functions*, Trans. Amer. Math. Soc. 162 (1971), 287–302.

[12] S. Stević, *Weighted composition operators between mixed norm spaces and H^∞_α spaces in the unit ball*, J. Inequal. Appl., Vol 2007, Article ID 28629, (2007), 9 pp.

[13] S. Stević, *Norm of weighted composition operators from Bloch space to H^∞_μ on the unit ball*, Ars Combin. 88 (2008), 125–127.

[14] S. Stević, *Norms of some operators from Bergman spaces to weighted and Bloch-type space*, Utilitas Math. 76 (2008), 59–64.

[15] S. Stević, *Norm of weighted composition operators from α-Bloch spaces to weighted-type spaces*, Appl. Math. Comput. 215 (2009), 818–820.

[16] S. Stević, *On a new integral-type operator from the Bloch space to Bloch-type spaces on the unit ball*, J. Math. Anal. Appl. 354 (2009), 426–434

[17] S. Stević, *Norms of some operators on the Bergman and the Hardy space in the unit polydisk and the unit ball*, Appl. Math. Comput. 215 (2009), 2199–2205.

[18] S. Stević, *Norms of multiplication operators on Hardy spaces and weighted composition operators from Hardy spaces to weighted-type spaces on bounded symmetric domains*, Appl. Math. Comput. 217 (2010), 2870–2876.

[19] S. Stević, *Norms of some operators on bounded symmetric domains*, Appl. Math. Comput. 216 (2010), 187–191.

[20] S. Stević, *Norm of an integral-type operator from Dirichlet to Bloch space on the unit disk*, Utilitas Math. 83 (2010), 301–303.

[21] S. Stević, *Norm and essential norm of an integral-type operator from the Dirichlet space to the Bloch-type space on the unit ball*, Abstr. Appl. Anal., Vol. 2010, Article ID 134969, (2010), 9 pp.

[22] S. Stević, E. Wolf, *Differences of composition operators between weighted-type spaces of holomorphic functions on the unit ball of $\mathbb{C}^n$*, Appl. Math. Comput. 215 (2009), 1752–1760.

[23] X. Zhu, *Weighted composition operators from $F(p, q, s)$ spaces to H^∞_μ spaces*, Abstr. Appl. Anal., Vol. 2009, Article ID 290978, (2009), 12 pp.

[24] X. Zhu, D. Gu, *Weighted composition operators on Bers-type space in the unit ball*, Ars Combin., to appear.

DEPARTMENT OF MATHEMATICS
JIAYING UNIVERSITY
514015, MEIZHOU, GUANGDONG, CHINA
E-mail: xiangling-zhu@163.com