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Vincenzo De Filippis and Basudeb Dhara

COCENTRALIZING GENERALIZED DERIVATIONS ON MULTILINEAR POLYNOMIAL ON RIGHT IDEALS OF PRIME RINGS

Abstract. Let R be a prime ring with Utumi quotient ring U and with extended centroid C, I a non-zero right ideal of R, $f(x_1, \ldots, x_n)$ a multilinear polynomial over C which is not central valued on R and G, H two generalized derivations of R. Suppose that $G(f(r))f(r) - f(r)H(f(r)) \in C$, for all $r = (r_1, \ldots, r_n) \in I^n$. Then one of the following holds:

1. there exist $a, b, p \in U$ and $\alpha \in C$ such that $G(x) = ax + [p, x]$ and $H(x) = bx$, for all $x \in R$, and $(a - b)I = (0) = (a + p - \alpha)I$;
2. R satisfies s_4, the standard identity of degree 4, and there exist $a, a' \in U$, $\alpha, \beta \in C$ such that $G(x) = ax + xa' + \alpha x$ and $H(x) = a'x - xa + \beta x$, for all $x \in R$;
3. R satisfies s_4 and there exist $a, a' \in U$, and $d : R \to R$, a derivation of R, such that $G(x) = ax + d(x)$ and $H(x) = xa' - d(x)$, for all $x \in R$, with $a + a' \in C$;
4. R satisfies s_4 and there exist $a, a' \in U$, and $d : R \to R$, a derivation of R, such that $G(x) = xa + d(x)$ and $H(x) = ax' - d(x)$, for all $x \in R$, with $a - a' \in C$;
5. there exists $e^2 = e \in Soc(RC)$ such that $I = eR$ and one of the following holds:
   (a) $[f(x_1, \ldots, x_n), x_{n+1}]x_{n+2}$ is an identity for I;
   (b) $char(R) = 2$ and $s_4(x_1, x_2, x_3, x_4)x_5$ is an identity for I;
   (c) $[f(x_1, \ldots, x_n)^2, x_{n+1}]x_{n+2}$ is an identity for I and there exist $a, a', b, b' \in U$, $\alpha \in C$ and $d : R \to R$, a derivation of R, such that $G(x) = ax + xa' + d(x)$, $H(x) = bx + xb' - d(x)$, for all $x \in R$, with $(a - b' - \alpha)I = (0) = (b - a' - \alpha)I$.

1. Introduction

Throughout this article R always denotes an associative prime ring with center $Z(R)$, extended centroid C and U its Utumi quotient ring. The commutator of x and y is denoted by $[x, y]$ and defined by $[x, y] = xy - yx$, for $x, y \in R$. An additive mapping $d : R \to R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ holds, for all $x, y \in R$. A derivation d is inner if there exists $a \in R$ such that $d(x) = [a, x]$ holds, for all $x \in R$. If the derivation d is

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not inner, then it is said to be outer derivation in R. An additive mapping $F : R \to R$ is called a generalized derivation if there exists a derivation $d : R \to R$ such that $F(xy) = F(x)y + xd(y)$ holds for all $x, y \in R$. If for some $a, b \in R$, $F(x) = ax + xb$ holds for all $x \in R$, then F is called as inner generalized derivation of R. Evidently, any derivation is a generalized derivation. We denote by s_4, the standard identity in four variables.

A well known result of Posner [26] states that if d is a derivation of R such that $d(x)x - xd(x) \in Z(R)$ for all $x \in R$, then either $d = 0$ or R is commutative. Several authors generalized Posner's theorem. For instance, Brešar proved in [3] that if d and δ are two derivations of R such that $d(x)x - x\delta(x) \in Z(R)$ for all $x \in R$, then either $d = \delta = 0$ or R is commutative. Later Lee and Wong [23] consider the situation $d(x)x - x\delta(x) \in Z(R)$, for all x in some noncentral Lie ideal L of R and obtained the result that either $d = \delta = 0$ or R satisfies s_4. In [22], Lee and Shiue consider the situation $d(x)x - x\delta(x) \in C$, for all $x \in \{f(x_1, \ldots, x_n) | x_1, \ldots, x_n \in R\}$, where $f(x_1, \ldots, x_n)$ is any polynomial over C and obtained that either $d = \delta = 0$, or $\delta = -d$ and $f(x_1, \ldots, x_n)^2$ is central valued on RC, except when char $(R) = 2$ and $dim_C RC = 4$.

In [2], the first author and Argac studied the same situation of Lee and Shiue, replacing derivations d and δ with two generalized derivations G and H in prime ring R. More precisely in [2] it is proved the following:

Theorem A. *Let R be a prime ring, U its Utumi quotient ring, $C = Z(U)$ its extended centroid, I a non-zero two-sided ideal of R, H and G non-zero generalized derivations of R. Suppose that $f(x_1, \ldots, x_n)$ is a non-central valued multilinear polynomial over C such that $H(f(X))f(X) - f(X)G(f(X)) = 0$, for all $X = (x_1, \ldots, x_n) \in I^n$, then one of the following holds:*

1. *there exists $a \in U$ such that, $H(x) = xa$ and $G(x) = ax$, for all $x \in R$;*
2. *$f(x_1, \ldots, x_n)^2$ is central valued on R and there exist $a, b \in U$ such that $H(x) = ax + xb$, $G(x) = bx + xa$, for all $x \in R$;*
3. *$char(R) = 2$ and R satisfies s_4, the standard identity of degree 4.*

In the present paper, our aim is to extend the previous cited result to the one-sided case, more precisely our main result will be:

Theorem. *Let R be a prime ring with Utumi quotient ring U and with extended centroid C, I a non-zero right ideal of R, $f(x_1, \ldots, x_n)$ a multilinear polynomial over C which is not central valued on R and G, H two generalized derivations of R. Suppose that $G(f(r))f(r) - f(r)H(f(r)) \in C$, for all $r = (r_1, \ldots, r_n) \in I^n$. Then one of the following holds:*

1. *there exist $a, b, p \in U$ and $\alpha \in C$ such that $G(x) = ax + [p, x]$ and $H(x) = bx$, for all $x \in R$, and $(a - b)I = (0) = (a + p - \alpha)I$;*

2. *R satisfies* s_4*, the standard identity of degree* 4*, and there exist* $a, a' \in U$, $\alpha, \beta \in C$ *such that* $G(x) = ax + xa' + \alpha x$ *and* $H(x) = a'x - xa + \beta x$*, for all* $x \in R$*;*
3. *R satisfies* s_4 *and there exist* $a, a' \in U$*, and* $d : R \to R$*, a derivation of R such that* $G(x) = ax + d(x)$ *and* $H(x) = xa' - d(x)$*, for all* $x \in R$*, with* $a + a' \in C$*;*
4. *R satisfies* s_4 *and there exist* $a, a' \in U$*, and* $d : R \to R$*, a derivation of R such that* $G(x) = xa + d(x)$ *and* $H(x) = ax' - d(x)$*, for all* $x \in R$*, with* $a - a' \in C$*;*
5. *there exists* $e^2 = e \in Soc(RC)$ *such that* $I = eR$ *and one of the following holds:*
   (a) $[f(x_1, \ldots, x_n), x_{n+1}]x_{n+2}$ *is an identity for I;*
   (b) $char(R) = 2$ *and* $s_4(x_1, x_2, x_3, x_4)x_5$ *is an identity for I;*
   (c) $[f(x_1, \ldots, x_n)^2, x_{n+1}]x_{n+2}$ *is an identity for I and there exist* $a, a', b, b' \in U$*,* $\alpha \in C$ *and* $d : R \to R$*, a derivation of R such that* $G(x) = ax + xa' + d(x)$*,* $H(x) = bx + xb' - d(x)$*, for all* $x \in R$*, with* $(a - b' - \alpha)I = (0) = (b - a' - \alpha)I$*.*

In order to prove our result, in the next sections we will make use of the following well known facts:

Fact 1. *Let R be a prime ring and L a non-central Lie ideal of R. Then either* $char(R) = 2$ *and R satisfies* s_4*, the standard identity of degree* 4*, or there exists a non-central ideal I of R such that* $0 \neq [I, R] \subseteq L$*.*

Proof. See [12] (pages 4–5), Lemma 2 and Proposition 1 in [9], Theorem 4 in [16]. ∎

Fact 2. *In* [21, Theorem 3]*, Lee proved that every generalized derivation H on a dense right ideal of R can be uniquely extended to a generalized derivation of U and assume the form* $H(x) = ax + d(x)$*, for all* $x \in U$*, for some* $a \in U$ *and a derivation d of U.*

2. The result for Inner Generalized Derivations

We begin with some lemmas:

Lemma 3. *Let R be a non-commutative prime ring,* $a, b \in U$*,* $p(x_1, \ldots, x_n)$ *be any polynomial over C, which is of nonzero value on R. If* $ap(r) - p(r)b \in C$*, for all* $r = (r_1, \ldots, r_n) \in R^n$ *then one of the following holds:*

1. $a = b \in C$*;*
2. $a - b \in C$ *and* $p(x_1, \ldots, x_n)$ *is central valued on R;*
3. $char(R) = 2$ *and R satisfies* s_4*;*
4. $char(R) \neq 2$*, R satisfies* s_4 *and* $a + b \in C$*.*

Proof. If $p(x_1, \ldots, x_n)$ is central valued on R then our assumption $ap(r) - p(r)b \in C$ yields $(a - b)p(r) \in C$, for all $r = (r_1, \ldots, r_n) \in R^n$. Since $p(x_1, \ldots, x_n)$ is nonzero valued on R, $a - b \in C$ and hence we obtain our conclusion (b).

Next assume that $p(x_1, \ldots, x_n)$ is not central valued on R. Let G be the additive subgroup of R generated by the set $S = \{p(x_1, \ldots, x_n) | x_1, \ldots, x_n \in R\}$. Then $S \neq \{0\}$, since $p(x_1, \ldots, x_n)$ is nonzero valued on R. By our assumption we get $ax - xb \in C$, for any $x \in G$. By [5], either $G \subseteq Z(R)$ or char $(R) = 2$ and R satisfies s_4, except when G contains a noncentral Lie ideal L of R. Since $p(x_1, \ldots, x_n)$ is not central valued on R, the first case can not occur. If char $(R) = 2$ and R satisfies s_4 then we obtain our conclusion (c). So, let either char $(R) \neq 2$ or R does not satisfy s_4. Then G contains a noncentral Lie ideal L of R. By Fact 1, there exists a noncentral two sided ideal I of R such that $[I, R] \subseteq L$. In particular, $a[x_1, x_2] - [x_1, x_2]b \in C$, for all $x_1, x_2 \in I$. As a reduction of Theorem 1 in [1], we have that either R satisfies s_4 and $a + b \in C$ or $a, b \in C$. In this last case $(a - b)[x_1, x_2] \in C$, for all $x_1, x_2 \in I$ implies either $a = b$ (and we are done) or $[I, I] \subseteq C$, i.e. R is commutative. In any case we obtain one of Lemma's conclusions. ∎

Fact 4. *Let R be a prime ring with Utumi quotient ring U and extended centroid C, I be a non-zero ideal of R and $f(x_1, \ldots, x_n)$ be a multilinear polynomial over C which is not central valued on R. If for some $a, b, w \in U$, $af(r)^2 + f(r)wf(r) + f(r)^2 b = 0$, for all $r = (r_1, \ldots, r_n) \in I^n$, then one of the following holds:*

1. *$a, b, w \in C$ with $a + w + b = 0$;*
2. *$w = -a - b \in C$ and $f(x_1, \ldots, x_n)^2$ is central valued on R;*
3. *char $(R) = 2$ and R satisfies s_4.*

Proof. Since by [6], R and I satisfies the same generalized polynomial identities with coefficients in U, the result follows directly from Lemma 3 in [2]. ∎

Lemma 5. *Let $R = M_k(F)$ be the set of all $k \times k$ matrices over a field F and $f(x_1, \ldots, x_n)$ be a multilinear polynomial over F with noncentral value on R. If for some $a, b, w \in R$, $af(x_1, \ldots, x_n)^2 + f(x_1, \ldots, x_n)wf(x_1, \ldots, x_n) + f(x_1, \ldots, x_n)^2 b \in F \cdot I_k$, for all $x_1, \ldots, x_n \in R$, then one of the following holds:*

1. *$a, b, w \in F \cdot I_k$ with $a + w + b = 0$;*
2. *$w, a + b \in F \cdot I_k$ and $f(x_1, \ldots, x_n)^2$ is central valued on R;*
3. *$char(R) = 2$ and $k = 2$;*
4. *$k = 2$, $a - b \in F \cdot I_k$ and $w \in F \cdot I_k$.*

Proof. Let $a = (a_{ij})_{k\times k}$, $b = (b_{ij})_{k\times k}$ and $w = (w_{ij})_{k\times k}$. Since $f(x_1, \dots, x_n)$ is not central on R, by [24, Lemma 2, Proof of Lemma 3] there exists a sequence of matrices $r = (r_1, \dots, r_n)$ in R such that $f(r_1, \dots, r_n) = \gamma e_{ij}$ with $0 \neq \gamma \in F$ and $i \neq j$. Since the set $f(R) = \{f(x_1, \dots, x_n), x_i \in R\}$ is invariant under the action of all inner automorphisms of R, for all $i \neq j$ there exists a sequence of matrices $r = (r_1, \dots, r_n)$ such that $f(r) = \gamma e_{ij}$. Thus

$$af(r_1, \dots, r_n)^2 + f(r_1, \dots, r_n)wf(r_1, \dots, r_n) + f(r_1, \dots, r_n)^2 b$$
$$= \gamma e_{ij} w \gamma e_{ij} = \gamma^2 w_{ji} e_{ij} \in F \cdot I_k$$

implying $w_{ji} = 0$, for any $i \neq j$. Thus w is a diagonal matrix. Now for any F-automorphism θ of R, w^θ satisfies the same property as w does, namely,

$$a^\theta f(r_1, \dots, r_n)^2 + f(r_1, \dots, r_n)w^\theta f(r_1, \dots, r_n) + f(r_1, \dots, r_n)^2 b^\theta \in F \cdot I_k,$$

for all $r_1, \dots, r_n \in R$. Hence, w^θ must be diagonal. Write, $w = \sum_{i=0}^{k} w_{ii} e_{ii}$; then for $s \neq t$, we have that

$$(1 + e_{ts})w(1 - e_{ts}) = \sum_{i=0}^{k} w_{ii} e_{ii} + (w_{ss} - w_{tt})e_{ts} \tag{2.1}$$

is diagonal. Hence, $w_{ss} = w_{tt}$ and so $w \in F \cdot I_k$. Therefore, R satisfies $af(x_1, \dots, x_n)^2 + f(x_1, \dots, x_n)^2(b + w) \in F \cdot I_k$. Then by Lemma 3, one of the following holds:

- $a = -(b + w) \in F \cdot I_k$, that is $a, b, w \in F \cdot I_k$ with $a + w + b = 0$.
- $a+b+w \in F \cdot I_k$ and $f(x_1, \dots, x_n)^2$ is central valued on R. Since $w \in F \cdot I_k$, $a + b + w \in F \cdot I_k$ implies $w, a + b \in F \cdot I_k$.
- char $(R) = 2$ and $k = 2$.
- $k = 2$, $a - b - w \in F \cdot I_k$, that is both $a - b \in F \cdot I_k$ and $w \in F \cdot I_k$.

Thus the lemma is proved. ∎

Lemma 6. *Let R be a prime ring with Utumi quotient ring U and extended centroid C, and $f(r_1, \dots, r_n)$ be a multilinear polynomial over C which is not central valued on R. If for some $a, b, w \in U$, $af(r)^2 + f(r)wf(r) + f(r)^2 b \in C$, for all $r = (r_1, \dots, r_n) \in R^n$, then one of the following holds:*

1. *$a, b, w \in C$ with $a + w + b = 0$;*
2. *$w, a + b \in C$ and $f(x_1, \dots, x_n)^2$ is central valued on R;*
3. *$char(R) = 2$ and R satisfies s_4;*
4. *R satisfies s_4, $a - b \in C$ and $w \in C$.*

Proof. Since R and U satisfy same generalized polynomial identity (see [6]), U satisfies

$$g(x_1,\dots,x_{n+1})$$
$$= [af(x_1,\dots,x_n)^2 + f(x_1,\dots,x_n)wf(x_1,\dots,x_n) + f(x_1,\dots,x_n)^2b, x_{n+1}].$$

Suppose first that $g(x_1\dots,x_{n+1})$ is a trivial generalized polynomial identity for R. Let $T = U *_C C\{x_1,\dots,x_{n+1}\}$ be the free product of U and $C\{x_1,\dots,x_{n+1}\}$, the free C-algebra in non-commuting indeterminates $x_1,\dots,x_{n+1}$.

Then,

$$[af(x_1,\dots,x_n)^2 + f(x_1,\dots,x_n)wf(x_1,\dots,x_n) + f(x_1,\dots,x_n)^2b, x_{n+1}]$$

is zero element in T. If $a \notin C$ then a and 1 are linearly independent over C. Thus $af(x_1,\dots,x_n)^2x_{n+1} = 0$ implying $a = 0$, a contradiction. Therefore, $a \in C$. Similarly, we can show that $w, b \in C$. Then the GPI becomes $(a+b+w)f(x_1,\dots,x_n)^2 \in C$. Since $a+b+w \in C$, $a+b+w \neq 0$ implies that $f(x_1,\dots,x_n)^2 \in C$ contradicting the fact that R does not satisfy any nontrivial GPI. Thus, $a+b+w = 0$. The conclusion (a) is obtained.

Thus we assume that $g(x_1\dots,x_{n+1})$ is a non-trivial generalized polynomial identity for R and so also for U. Let I be a two-sided ideal of U. In case

$$af(x_1,\dots,x_n)^2 + f(x_1,\dots,x_n)wf(x_1,\dots,x_n) + f(x_1,\dots,x_n)^2b$$

is satisfied by I, the conclusion follows from Fact 4.

Hence we assume that there exist $r_1,\dots,r_n \in I$ such that

$$af(r_1,\dots,r_n)^2 + f(r_1,\dots,r_n)wf(r_1,\dots,r_n) + f(r_1,\dots,r_n)^2b \neq 0$$

so that

$$af(x_1,\dots,x_n)^2 + f(x_1,\dots,x_n)wf(x_1,\dots,x_n) + f(x_1,\dots,x_n)^2b \in C$$

is a non-zero central generalized identity for I. As in Theorem 1 in [4], it follows that R is a PI-ring, therefore $RC = Q = U$ is a non-trivial GPI-ring simple with 1. By Lemma 2 in [15] and Theorem 2.3.29 in [27], there exists a field E such that $U \subseteq M_k(E)$, the ring of all $k \times k$ matrices over E; moreover U and $M_k(E)$ satisfy the same generalized identities. Therefore $M_k(E)$ satisfies $g(x_1,\dots,x_{n+1})$ and the result follows from Lemma 5. ■

Proposition 7. *Let R be a non-commutative prime ring, I be a non-zero right ideal of R and $f(x_1,\dots,x_n)$ be a multilinear polynomial over C. If for some $a, b, w \in U$, $af(r)^2 + f(r)wf(r) + f(r)^2b \in C$, for all $r = (r_1,\dots,r_n) \in I^n$, then one of the following holds:*

1. *$[f(x_1,\dots,x_n), x_{n+1}]x_{n+2}$ is an identity for I;*
2. *$b \in C$ and there exist $\alpha, \beta \in C$ such that $(a-\alpha)I = 0$, $(w-\beta)I = 0$ and $(b+\alpha+\beta) = 0$;*
3. *$char(R) = 2$ and I satisfies $s_4(x_1,\dots,x_4)x_5$;*

4. $[f(x_1,\dots,x_n)^2, x_{n+1}]x_{n+2}$ *is an identity for* I *and there exists* $\gamma \in C$ *such that* $(w-\gamma)I = 0$ *and* $(a+b+\gamma)I = 0$;
5. R *satisfies* s_4, $a-b \in C$ *and* $w \in C$.

Proof. Let $0 \neq u \in I$, then

$$(2.2) \qquad \Big[af(ux_1,\dots,ux_n)^2 + f(ux_1,\dots,ux_n)wf(ux_1,\dots,ux_n) + f(ux_1,\dots,ux_n)^2 b, X\Big]$$

is a generalized polynomial identity for R. Our first aim is to show that R satisfies some non-trivial generalized polynomial identity, unless when the conclusion (b) occurs. To do this, we suppose R does not satisfy any non-trivial generalized polynomial identities. Then (2.2) is a trivial GPI for R. This happens when $\{b, 1\}$ are linearly C-depending (see [6]), that is, $b \in C$. Thus (2.2) becomes

$$(2.3) \qquad \Big[af(ux_1,\dots,ux_n)^2 + f(ux_1,\dots,ux_n)(w+b)f(ux_1,\dots,ux_n), X\Big],$$

which is a trivial GPI for R. Once again by [6], it follows that $\{au, u\}$ are linearly C-depending, for all $0 \neq u \in I$, that is, there exists $\alpha \in C$ such that $(a-\alpha)I = 0$. In light of this, (2.3) becomes

$$(2.4) \qquad \Big[f(ux_1,\dots,ux_n)(\alpha+w+b)f(ux_1,\dots,ux_n), X\Big],$$

which must be trivial for R. This happens when:

- either $(\alpha+w+b)u = 0$, for all $0 \neq u \in I$, that is, $(w-\beta)I = 0$, where $\beta = -\alpha - b \in C$ (which is the conclusion (b) of the Proposition);
- or there exists $\gamma \in C$ such that $0 \neq (\alpha+w+b)u = \gamma u$ and $f(ux_1,\dots,ux_n)^2$ is central valued in R, that is, $[f(ux_1,\dots,ux_n)^2, x_{n+1}]$ is a non-trivial generalized polynomial identity for R, a contradiction.

Thus in all that follows, we assume that R is a non-trivial GPI-ring.

Notice that if $I = R$ then by Lemma 6 we are done. Thus we assume that $I \neq R$. Moreover, by contradiction suppose that the conclusions (a) to (d) don't hold, that is, we assume the following hold simultaneously:

- there exist $t_1,\dots,t_{n+2} \in I$ such that $[f(t_1,\dots,t_n), t_{n+1}]t_{n+2} \neq 0$;
- if $char(R) = 2$, there exist $v_1,\dots,v_5 \in I$ such that $s_4(v_1,\dots,v_4)v_5 \neq 0$;
- if $b \in C$ then there exist $c_1, c_2 \in I$ such that, for all $\lambda, \mu \in C$, either $(a-\lambda)c_1 \neq 0$, or $(w-\mu)c_2 \neq 0$;
- either there exist $u_1,\dots,u_{n+2} \in I$ such that $[f(u_1,\dots,u_n)^2, u_{u+1}]u_{u+2} \neq 0$ or there exists $c_3 \in I$ such that $(w-\lambda)c_3 \neq 0$, for all $\lambda \in C$;
- either there exist $u_1,\dots,u_{n+2} \in I$ such that $[f(u_1,\dots,u_n)^2, u_{u+1}]u_{u+2} \neq 0$ or there exists $c_4 \in I$ such that $(a+b+\mu)c_4 \neq 0$, for all $\mu \in C$.

We will prove that the previous assumptions lead to a number of contradictions.

Without loss of generality, R is simple and equals to its own socle, $IR = I$. In fact, R is GPI and so RC has non-zero socle $S = Soc(RC)$ with non-zero right ideal $J = IS$ (Theorem 3 in [25]). Note that S is simple, $J = JS$ and J satisfies the same basic conditions as I. Now just replace R by S, I by J and we are done.

Since $R = S$ is a regular ring then for any $a_1, \ldots, a_n \in I$ there exists $h = h^2 \in R$ such that $\sum_{i=1}^{n} a_i R = hR$. Then $h \in IR = I$ and $a_i = ha_i$, for each $i = 1, \ldots, n$. In particular, there exists $e = e^2 \in R$ such that $eR = \sum_{i=1}^{4} c_i R + \sum_{j=1}^{n+2} u_j R + \sum_{k=1}^{n+2} t_k R + \sum_{l=1}^{5} v_l R$, and $c_i = ec_i$, $u_j = eu_j$, $t_k = et_k$, $v_l = ev_l$, for each $i = 1, \ldots, 4$, $l = 1, \ldots, 5$ and $j, k = 1, \ldots, n+2$. Then we have $f(eRe) = f(eR)e \neq 0$.

Since

$$af(ex_1, \ldots, ex_n)^2 + f(ex_1, \ldots, ex_n)wf(ex_1, \ldots, ex_n) + f(ex_1, \ldots, ex_n)^2 b \in C \tag{2.5}$$

is satisfied by R, then commuting (2.5) with $x(1-e) \in R(1-e)$ and right multiplying by e, we obtain that $(1-e)af(ex_1, \ldots, ex_n)^2 e$ is an identity for R. By [7], it follows that $(1-e)ae = 0$, that is, $ae = eae \in eR$, since $f(eRe) \neq (0)$. This implies that

$$af(er_1, \ldots, er_n)^2 + f(er_1, \ldots, er_n)wf(er_1, \ldots, er_n) + f(er_1, \ldots, er_n)^2 b \in I \cap C,$$

for all $r_1, \ldots, r_n \in R$. In case there exist $t_1, \ldots, t_n \in eR$ such that

$$0 \neq af(t_1, \ldots, t_n)^2 + f(t_1, \ldots, t_n)wf(t_1, \ldots, t_n) + f(t_1, \ldots, t_n)^2 b \in eR \cap C$$

then eR possesses a central generalized polynomial identity and from Theorem 1 in [4], R is a PI-ring and RC is a finite-dimensional simple C-algebra by Posner's Theorem. By the Wedderburn-Artin Theorem, there exists a division ring D such that $RC \cong M_k(D)$ where D is a finite-dimensional C-algebra. Replacing R with RC, we may assume that $R = M_k(D)$. Choose a maximal subfield K of D. Then $M_k(K) \otimes_C K \cong M_l(K)$ and

$$af(x_1, \ldots, x_n)^2 + f(x_1, \ldots, x_n)wf(x_1, \ldots, x_n) + f(x_1, \ldots, x_n)^2 b \in Z(M_l(K))$$

is satisfied by $eR \otimes_C K$ (Lemma 2 in [18] and Proposition in [23]). Thus without loss of generality, we may assume $R = M_l(K)$ and $I = eR \otimes_C K$. So $I \cap Z(M_l(K)) \neq 0$, that is I contains some invertible element of R and $I = R$, a contradiction.

Therefore in the following, we assume that for all $x_1, \dots, x_n \in R$

$$af(ex_1, \dots, ex_n)^2 + f(ex_1, \dots, ex_n)wf(ex_1, \dots, ex_n) + f(ex_1, \dots, ex_n)^2 b = 0 \tag{2.6}$$

with $ae = eae$ and also

$$af(ex_1e, \dots, ex_ne)^2 + f(ex_1e, \dots, ex_ne)wf(ex_1e, \dots, ex_ne) + f(ex_1e, \dots, ex_ne)^2 b = 0. \tag{2.7}$$

Right multiplying by $(1-e)$ in (2.7), we have that $f(ex_1, \dots, ex_n)^2 eb(1-e) = 0$, for all $x_1, \dots, x_n \in R$, which implies by [7] that $eb(1-e) = 0$, that is, $eb = ebe$, since $f(eR)e \neq (0)$.

Here we write

$$f(x_i, \dots, x_n) = \sum_i t_i(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)x_i,$$

where each $t_i(y_1, \dots, y_{n-1})$ is a multilinear polynomial in $n-1$ variables and x_i never appears in any monomial of $t_i(y_1, \dots, y_{n-1})$. In particular, there exists at least one $t_i(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)$, which is not an identity for eRe, if not $f(x_1, \dots, x_n)$ should be an identity for eRe, which is a contradiction. Assume that $t_n(x_1, \dots, x_{n-1})$ is not an identity for eRe. Then $t_n(eRe) \neq 0$. Now let $x \in R$. We replace x_n with $x(1-e)$ in (2.7) and then obtain that

$$t_n(ex_1, \dots, ex_{n-1})ex(1-e))wt_n(ex_1, \dots, ex_{n-1})ex(1-e)) = 0,$$

which implies $((1-e)wet_n(ex_1, \dots, ex_{n-1})ex)^3 = 0$, for all $x \in R$. By Levitzki's Lemma [12, Lemma 1.1], $(1-e)wet_n(ex_1e, \dots, ex_{n-1}e) = 0$, for all $x_1, \dots, x_{n-1} \in R$. Since $t_n(eRe) \neq 0$, it follows that $(1-e)we = 0$, that is, $we = ewe$.

Now, left and right multiplying by e in (2.7), we have that eRe satisfies

$$eaef(x_1, \dots, x_n)^2 + f(x_1, \dots, x_n)ewef(x_1, \dots, x_n) + f(x_1, \dots, x_n)^2 ebe = 0. \tag{2.8}$$

Now, we may apply the result contained in Theorem A to the prime ring eRe. Thus either $char(R) = 2$ and eRe satisfies s_4 (which contradicts with the choices of $v_1, \dots, v_5$) or one of the following cases occurs:

1. There exist $\alpha, \beta, \gamma \in C$ such that $(a-\alpha)e = 0$, $(w-\beta)e = 0$ and $eb = e\gamma$. Hence, since by (2.6) R satisfies

$$af(ex_1, \dots, ex_n)^2 + f(ex_1, \dots, ex_n)wf(ex_1, \dots, ex_n) + f(ex_1, \dots, ex_n)^2 b,$$

it follows that $f(eR)^2(\alpha + \beta + b) = (0)$. Since $f(eR)e \neq (0)$, by [7] it follows $\alpha + \beta + b = 0$, that is, $b \in C$ and $(a-\alpha)c_1 = (w-\beta)c_2 = 0$, a contradiction.

2. $f(x_1,\dots,x_n)^2$ is central valued on eRe and there exists $\gamma \in C$ such that $we = -ae - eb = \gamma e$. In this case, right multiplying (2.6) by e and using $[f(eR)^2, eR]e = (0)$, we have that $f(eR)^2(a+\gamma+b)e = (0)$, that is $(a+\gamma+b)e = 0$ (see again [7]), which contradicts with the choices of $u_1,\dots,u_{n+2}$, c_3 and c_4. ■

Finally we are ready to prove our main result:

3. The Proof of Main Theorem

By Fact 2, we may assume that for all $x \in U$, $G(x) = ax + d(x)$ and $H(x) = bx + \delta(x)$, where $a, b \in U$ and d, δ are derivations of U. By the hypothesis I satisfies

$$(3.1)\quad (af(x_1,\dots,x_n)^2 + d(f(x_1,\dots,x_n))f(x_1,\dots,x_n) \\ - f(x_1,\dots,x_n)(bf(x_1,\dots,x_n) + \delta(f(x_1,\dots,x_n))) \in C.$$

Since I and IU satisfy the same generalized polynomial identities (see [6]) as well as the same differential identities (see [17]), we may assume, for $u_1,\dots,u_n \in I$, that U satisfies

$$(3.2)\quad (af(u_1x_1,\dots,u_nx_n)^2 + d(f(u_1x_1,\dots,u_nx_n))f(u_1x_1,\dots,u_nx_n) \\ - f(u_1x_1,\dots,u_nx_n)(bf(u_1x_1,\dots,u_nx_n) + \delta(f(u_1x_1,\dots,u_nx_n))) \in C.$$

Now, we divide the proof into two cases:

3.1. CASE 1: Let $d(x) = [p, x]$ for all $x \in U$ and $\delta(x) = [q, x]$ for all $x \in U$ i.e., d and δ are both inner derivations of U. Then from (3.1), we obtain that I satisfies

$$(3.3)\quad ((a+p)f(x_1,\dots,x_n)^2 - f(x_1,\dots,x_n)(b+p+q)f(x_1,\dots,x_n) \\ + f(x_1,\dots,x_n)^2q) \in C.$$

By Proposition 7, one of the following holds:

1. $[f(x_1,\dots,x_n), x_{n+1}]x_{n+2}$ is an identity for I;
2. $q \in C$, $(a+p-\alpha)I = 0$, $(b+p+q+\beta)I = 0$, $(q+\alpha+\beta)I = 0$, for some $\alpha, \beta \in C$. So that $(a-b)I = 0$, $(a+p)I = \alpha I$ and $G(x) = ax + [p, x]$, $H(x) = bx$, for all $x \in R$. This is the conclusion of the Theorem in the case I is not PI.
3. $char(R) = 2$ and $s_4(x_1,x_2,x_3,x_4)x_5$ is an identity for I;
4. $[f(x_1,\dots,x_n)^2, x_{n+1}]x_{n+2}$ is an identity for I, $(b+p+q+\gamma)I = 0$, $(a+p+q+\gamma)I = 0$ for some $\gamma \in C$ and $G(x) = (a+p)x + x(-p)$, $H(x) = (b+q)x - xq = (b+q+\gamma)x + x(-q-\gamma)$, for all $x \in R$ (which is a particular case of conclusion (5c) when $d = 0$ and $\alpha = 0$).
5. R satisfies s_4, $a+p-q = \alpha \in C$, $b+p+q = \beta \in C$, that is $G(x) = qx - xp + \alpha x$ and $H(x) = -px - xq + \beta x$, for all $x \in R$.

3.2. CASE 2: Let d and δ are not both inner derivations of U.

Assume that d and δ are C-dependent modulo inner derivations of U, say $\delta = \lambda d + ad_p$, where $\lambda \in C$, $p \in U$ and $ad_p(x) = [p, x]$, for all $x \in U$. Then d can not be inner derivation of U. From (3.1), we obtain that I satisfies

$$\begin{aligned}(3.4)\qquad &(af(x_1,\dots,x_n) + d(f(x_1,\dots,x_n))f(x_1,\dots,x_n)\\ &- f(x_1,\dots,x_n)(bf(x_1,\dots,x_n) + \lambda d(f(x_1,\dots,x_n)) + [p, f(x_1,\dots,x_n)]) \in C.\end{aligned}$$

that is, as in (3.2), for $u_1,\dots,u_n \in I$, U satisfies

$$\begin{aligned}(3.5)\qquad &\Big(af(u_1x_1,\dots,u_nx_n) + f^d(u_1x_1,\dots,u_nx_n)\Big)f(u_1x_1,\dots,u_nx_n)\\ &+ \Big(\sum_i f(u_1x_1,\dots,d(u_i)x_i + u_id(x_i),\dots,u_nx_n)\Big)f(u_1x_1,\dots,u_nx_n)\\ &- f(u_1x_1,\dots,u_nx_n)\Big(bf(u_1x_1,\dots,u_nx_n) + \lambda f^d(u_1x_1,\dots,u_nx_n)\\ &+\lambda\sum_i f(u_1x_1,\dots,d(u_i)x_i+u_id(x_i),\dots,u_nx_n)+[p, f(u_1x_1,\dots,u_nx_n)]\Big) \in C.\end{aligned}$$

Then by Kharchenko's theorem [14], we have that U satisfies

$$\begin{aligned}(3.6)\qquad &\Big(af(u_1x_1,\dots,u_nx_n) + f^d(u_1x_1,\dots,u_nx_n)\Big)f(u_1x_1,\dots,u_nx_n)\\ &+ \Big(\sum_i f(u_1x_1,\dots,d(u_i)x_i + u_iy_i,\dots,u_nx_n)\Big)f(u_1x_1,\dots,u_nx_n)\\ &- f(u_1x_1,\dots,u_nx_n)\Big(bf(u_1x_1,\dots,u_nx_n) + \lambda f^d(u_1x_1,\dots,u_nx_n)\Big)\\ &- f(u_1x_1,\dots,u_nx_n)\Big(\lambda\sum_i f(u_1x_1,\dots,d(u_i)x_i + u_iy_i,\dots,u_nx_n)\\ &+ [p, f(u_1x_1,\dots,u_nx_n)]\Big) \in C.\end{aligned}$$

In particular, U satisfies the blended component

$$\begin{aligned}(3.7)\qquad &\Big(\sum_i f(u_1x_1,\dots,u_iy_i,\dots,u_nx_n)\Big)f(u_1x_1,\dots,u_nx_n)\\ &- f(u_1x_1,\dots,u_nx_n)\Big(\lambda\sum_i f(u_1x_1,\dots,u_iy_i,\dots,u_nx_n)\Big) \in C.\end{aligned}$$

Since I and IU satisfy the same polynomial identities, we have that I satisfies

$$\begin{aligned}(3.8)\qquad &\Big(\sum_i f(x_1,\dots,y_i,\dots,x_n)\Big)f(x_1,\dots,x_n)\\ &- f(x_1,\dots,x_n)\Big(\lambda\sum_i f(x_1,\dots,y_i,\dots,x_n)\Big) \in C.\end{aligned}$$

Thus I is a PI-right ideal of R, then by Proposition in [19], there exists an idempotent element $e^2 = e \in Soc(RC)$ such that $I = eR$. Hence

$$(3.9)\qquad \Big(\sum_i f(ex_1,\dots,ey_i,\dots,ex_n)\Big)f(ex_1,\dots,ex_n) - f(ex_1,\dots,ex_n)\Big(\lambda\sum_i f(ex_1,\dots,ey_i,\dots,ex_n)\Big) \in C$$

is satisfied by U. If $[I,I]I = (0)$, then a fortiori $[f(x_1,\dots,x_n),x_{n+1}]x_{n+2}$ is an identity for I and we are done. So, let $[I,I]I \neq (0)$. Therefore, there exists $v \in R$ such that $ev \in I = eR$ and $[ev,I]I \neq (0)$. This implies $(ev-\mu)I \neq (0)$ for all $\mu \in C$. Then $ev \notin C$, otherwise for $\mu = ev \in C$, $(ev-\mu)I = (0)$, a contradiction.

Now replacing in (3.9) each ey_i with $[ev,ex_i]$, we get

$$(3.10)\qquad [ev,f(ex_1,\dots,ex_n)]f(ex_1,\dots,ex_n) - \lambda f(ex_1,\dots,ex_n)[ev,f(ex_1,\dots,ex_n)] \in C$$

is satisfied by U, that is,

$$(3.11)\qquad evf(ex_1,\dots,ex_n)^2 + f(ex_1,\dots,ex_n)(-ev-\lambda ev)f(ex_1,\dots,ex_n) + f(ex_1,\dots,ex_n)^2(\lambda ev) \in C.$$

Now we may apply Proposition 7. Since $(ev-\mu)I \neq (0)$ for all $\mu \in C$, we conclude that either I satisfies $[f(x_1,\dots,x_n),x_{n+1}]x_{n+2}$ or $char(R) = 2$ and I satisfies $s_4(x_1,\dots,x_4)x_5$, unless $[f(x_1,\dots,x_n)^2,x_{n+1}]x_{n+2}$ is an identity for I and $((1+\lambda)ev+\mu)I = (0)$ for some $\mu \in C$. In the following, we consider just the last case (if not we are done).

In this case, if $1+\lambda \neq 0$, then $(ev+\mu(1+\lambda)^{-1})I = (0)$, contradicting the fact $(ev-\gamma)I \neq (0)$ for all $\gamma \in C$. Hence, $\lambda = -1$. Thus $G(x) = ax + d(x)$ and $H(x) = bx - d(x) + [p,x]$ for all $x \in U$. Moreover, starting by (3.6) for $\lambda = -1$, we have that U satisfies the blended component

$$af(u_1x_1,\dots,u_nx_n)^2 - f(u_1x_1,\dots,u_iy_i,\dots,u_nx_n)(b+p)f(u_1x_1,\dots,u_nx_n) + f(u_1x_1,\dots,u_nx_n)^2p \in C$$

and in particular U satisfies

$$af(ex_1,\dots,ex_n)^2 - f(ex_1,\dots,ex_n)(b+p)f(ex_1,\dots,ex_n) + f(ex_1,\dots,ex_n)^2p \in C.$$

Applying again Proposition 7, it follows that either $[f(x_1,\dots,x_n),x_{n+1}]x_{n+2}$ is a polynomial identity for I or $char(R) = 2$ and I satisfies $s_4(x_1,\dots,x_4)x_5$ unless when one of the following holds:

(i) $p \in C$ and there exist $\alpha,\beta \in C$ such that $(a-\alpha)I = (0)$, $(b+p+\beta)I = (0)$, $(p+\alpha+\beta) = (0)$, that is, $G(x) = ax + d(x)$, $H(x) = bx - d(x)$, for all $x \in R$, with $(a-\alpha)I = (b-\alpha)I = (0)$ (a particular case of conclusion (5c)).

(ii) There exists $\gamma \in C$ such that $(b+p+\gamma)I = (0)$ and $(a+p+\gamma)I = (0)$, that is, $G(x) = ax + d(x)$ and $H(x) = b'x - xp - d(x)$, with $b' = b + p$, $(b' + \gamma)I = (0) = (a + p + \gamma)I$ (again a particular case of conclusion (5c)).

(iii) R satisfies s_4, $a - p \in c$ and $b + p \in C$, that is $G(x) = ax + d(x)$ and $H(x) = xb - d(x)$, for all $x \in R$, with $a + b \in C$.

Similarly, from the symmetry of the expression $d + \delta = ad_p$, we can say that $d = \mu\delta + ad_p$, and by similar above argument, it follows that either $[f(x_1, \ldots, x_n), x_{n+1}]x_{n+2}$ is a polynomial identity for I or $char(R) = 2$ and I satisfies $s_4(x_1, \ldots, x_4)x_5$ unless when $\mu = -1$, $[f(x_1, \ldots, x_n)^2, x_{n+1}]x_{n+2}$ is an identity for I and one of the following holds:

(i) There exist $a' \in U$ and $\alpha \in C$ such that $G(x) = a'x - \delta(x)$ and $H(x) = bx + \delta(x)$, for all $x \in R$, with $(a' - \alpha)I = (0) = (b - \alpha)I$.

(ii) There exist $a', a'' \in U$ and $\alpha \in C$ such that $G(x) = a'x + xa'' - \delta(x)$ and $H(x) = bx + \delta(x)$, for all $x \in R$, with $(a' - \alpha)I = (0) = (b - a'' - \alpha)I$.

(iii) R satisfies s_4, $G(x) = xa - \delta(x)$ and $H(x) = bx + \delta(x)$, for all $x \in R$, with $a - b \in C$.

Here also we get particular cases of conclusion (c).

Next assume that d and δ are C-independent modulo inner derivations of U. As in (3.2), for $u_1, \ldots, u_n \in I$, we have that U satisfies

$$\begin{aligned}(3.12)\qquad &\Big(af(u_1x_1, \ldots, u_nx_n) + f^d(u_1x_1, \ldots, u_nx_n)\Big)f(u_1x_1, \ldots, u_nx_n)\\ &+ \Big(\sum_i f(u_1x_1, \ldots, d(u_i)x_i + u_id(x_i), \ldots, u_nx_n)\Big)f(u_1x_1, \ldots, u_nx_n)\\ &- f(u_1x_1, \ldots, u_nx_n)\Big(bf(u_1x_1, \ldots, u_nx_n) + f^\delta(u_1x_1, \ldots, u_nx_n)\Big)\\ &- f(u_1x_1, \ldots, u_nx_n)\Big(\lambda\sum_i f(u_1x_1, \ldots, \delta(u_i)x_i + u_i\delta(x_i), \ldots, u_nx_n)\Big) \in C.\end{aligned}$$

Then by Kharchenko's theorem [14], we have that U satisfies

$$\begin{aligned}(3.13)\qquad &\Big(af(u_1x_1, \ldots, u_nx_n) + f^d(u_1x_1, \ldots, u_nx_n)\Big)f(u_1x_1, \ldots, u_nx_n)\\ &\Big(\sum_i f(u_1x_1, \ldots, d(u_i)x_i + u_iy_i, \ldots, u_nx_n)\Big)f(u_1x_1, \ldots, u_nx_n)\\ &- f(u_1x_1, \ldots, u_nx_n)\Big(bf(u_1x_1, \ldots, u_nx_n) + f^\delta(u_1x_1, \ldots, u_nx_n)\Big)\\ &- f(u_1x_1, \ldots, u_nx_n)\Big(\lambda\sum_i f(u_1x_1, \ldots, \delta(u_i)x_i + u_iz_i, \ldots, u_nx_n)\Big) \in C.\end{aligned}$$

In particular, U satisfies the blended component

$$\Big(\sum_i f(u_1x_1,\ldots,u_iy_i,\ldots,u_nx_n)\Big)f(u_1x_1,\ldots,u_nx_n)\in C. \tag{3.14}$$

Since I and IU satisfy the same polynomial identities, we have that I satisfies

$$\Big(\sum_i f(x_1,\ldots,y_i,\ldots,x_n)\Big)f(x_1,\ldots,x_n)\in C. \tag{3.15}$$

Thus I is a PI-right ideal of R, then as above $I = eR$ for some idempotent element $e^2 = e \in soc(RC)$. Also, here we replace any y_i with $[ev, ex_i]$ with $v \in R$ such that $[ev, I]I \neq (0)$. Thus $I = eR$ satisfies $[v, f(x_1,\ldots,x_n)]f(x_1,\ldots,x_n) \in C$. In this case, since $ev \notin C$, by Proposition 7 we conclude that either I satisfies $[f(x_1,\ldots,x_n), x_{n+1}]x_{n+2}$ or $char(R) = 2$ and I satisfies $s_4(x_1,\ldots,x_4)x_5$. ∎

References

[1] A. Argac, L. Carini, V. De Filippis, *An Engel condition with generalized derivations on Lie ideals*, Taiwanese J. Math. 12(2) (2008), 419–433.

[2] A. Argac, V. De Filippis, *Actions of Generalized Derivations on Multilinear Polynomials in Prime Rings*, Algebra Colloquium 18 (spec. 1) 2011, 955–964.

[3] M. Brešar, *Centralizing mappings and derivations in prime rings*, J. Algebra 156 (1993), 385–394.

[4] C. M. Chang, T. K. Lee, *Annihilators of power values of derivations in prime rings*, Comm. Algebra 26(7) (1998), 2091–2113.

[5] C. L. Chuang, *The additive subgroup generated by a polynomial*, Israel J. Math. 59(1) (1987), 98–106.

[6] C. L. Chuang, *GPI's having coefficients in Utumi quotient rings*, Proc. Amer. Math. Soc. 103(3) (1988), 723–728.

[7] C. L. Chuang, T. K. Lee, *Rings with annihilator conditions on multilinear polynomials*, Chinese J. Math. 24(2) (1996), 177–185.

[8] B. Dhara, V. De Filippis, *Notes on generalized derivations on Lie ideals in prime rings*, Bull. Korean Math. Soc. 46(3) (2009), 599–605.

[9] O. M. Di Vincenzo, *On the n-th centralizer of a Lie ideal*, Boll. Un. Mat. Ital. A (7) 3 (1989), 77–85.

[10] T. S. Erickson, W. S. Martindale III, J. M. Osborn, *Prime nonassociative algebras*, Pacific J. Math. 60 (1975), 49–63.

[11] C. Faith, Y. Utumi, *On a new proof of Litoff's theorem*, Acta Math. Acad. Sci. Hung. 14 (1963), 369–371.

[12] I. N. Herstein, *Topics in Ring Theory*, Univ. of Chicago Press, Chicago, IL, 1969.

[13] N. Jacobson, *Structure of Rings*, Amer. Math. Soc. Colloq. Pub., 37, Amer. Math. Soc., Providence, RI, 1964.

[14] V. K. Kharchenko, *Differential identity of prime rings*, Algebra Logic 17 (1978), 155–168.

[15] C. Lanski, *An Engel condition with derivation*, Proc. Amer. Math. Soc. 118(3) (1993), 731–734.

[16] C. Lanski, S. Montgomery, *Lie structure of prime rings of characteristic 2*, Pacific J. Math. 42(1) (1972), 117–135.

[17] T. K. Lee, *Semiprime rings with differential identities*, Bull. Inst. Math. Acad. Sinica 20(1) (1992), 27–38.

[18] T. K. Lee, *Left annihilators characterized by GPIs*, Trans. Amer. Math. Soc. 347 (1995), 3159–3165.

[19] T. K. Lee, *Power reduction property for generalized identities of one-sided ideals*, Algebra Colloq. 3 (1996), 19–24.

[20] T. K. Lee, *Derivations with Engel conditions on polynomials*, Algebra Colloq. 5(1) (1998), 13–24.

[21] T. K. Lee, *Generalized derivations of left faithful rings*, Comm. Algebra 27(8) (1999), 4057–4073.

[22] T. K. Lee, W. K. Shiue, *Derivations cocentralizing polynomials*, Taiwanese J. Math. 2(4) (1998), 457–467.

[23] P. H. Lee, T. L. Wong, *Derivations cocentralizing Lie ideals*, Bull. Inst. Math. Acad. Sinica 23 (1995), 1–5.

[24] U. Leron, *Nil and power central valued polynomials in rings*, Trans. Amer. Math. Soc. 202 (1975), 97–103.

[25] W. S. Martindale III, *Prime rings satisfying a generalized polynomial identity*, J. Algebra 12 (1969), 576–584.

[26] E. C. Posner, *Derivations in prime rings*, Proc. Amer. Math. Soc. 8 (1957), 1093–1100.

[27] L. M. Rowen, *Polynomial Identities in Ring Theory*, Pure and Applied Mathematics 84, Academic Press, New York, 1980.

[28] T. L. Wong, *Derivations with cocentralizing multilinear polynomials*, Taiwanese J. Math. 1 (1997), 31–37.

V. De Filippis
UNIVERSITY OF MESSINA
98166, MESSINA, ITALY
E-mail: defilippis@unime.it

B. Dhara
DEPARTMENT OF MATHEMATICS
BELDA COLLEGE
BELDA, PASCHIM MEDINIPUR-721424, INDIA
E-mail: basu_dhara@yahoo.com

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