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Mariusz Jurkiewicz, Bogdan Przeradzki

EXISTENCE OF SOLUTIONS FOR HIGHER ORDER BVP WITH PARAMETERS VIA CRITICAL POINT THEORY

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Abstract. This paper is concerned with the existence of at least one solution of the nonlinear $2k$-th order BVP. We use the Mountain Pass Lemma to get an existence result for the problem, whose linear part depends on several parameters.

1. Introduction

There has recently been an increased interest in studying the existence of solutions for boundary value problems (BVPs) of higher order differential equations (cf. [1, 4–6, 9]). Most of the earlier discussions were devoted to the fourth order BVPs, for example see [3, 7]. In this paper, we consider the depending on real parameters family of $2k$th order ($k \geq 2$) BVP,

$$(1)\qquad \begin{cases} (-1)^k x^{(2k)} + \sum_{j=1}^{k} \lambda_j x^{(2k-2j)} = (-1)^{i-1} f\left(t, x^{(2i-2)}\right), \\ x^{(2j)}(0) = x^{(2j)}(1) = 0, \quad j = 0, \ldots, k-1, \end{cases},$$

where $f : [0,1] \to \mathbb{R}$ is a continuous function. Here, i is a fixed integer $1 \leq i \leq k$. Notice that the nonlinear term f depends only on the $2i-2$-th derivative of unknown function. It is seen that for some unions of lambdas, the differential operator that corresponds to the left-hand side of (1) is invertible and is not for others (see [4]). The second case, commonly called a resonance one, needs additional conditions of Landesman–Lazer type and was examined in [3], [4]. Here, we focus on the the nonresonant case. Jurkiewicz [6] established the existence of infinitely many solution for (1), by applying the Rabinowitz's theorem about unbounded sequence of critical points.

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Theorem 1. [6] *Assume that* $\lambda \in \Delta_+$ (*see p.* 55) *and let* f *be odd with respect to the second variable, i.e.* $f(t,-u) = -f(t,u)$ *for all* $t \in [0,1]$ *and* $u \in \mathbb{R}$. *If* f *satisfies the following conditions*

(i) $\limsup_{u \to 0} \frac{f(t,u)}{u} < \frac{\pi^{2k} + \Lambda(1)}{\pi^{2i-2}}$ (*definition of* Λ – (2)),

(ii) $\liminf_{u \to +\infty} \frac{f(t,u)}{u} = +\infty$,

(iii) *there exist* $k \in [0, 1/2)$ *and* $N > 0$, *such that* $\int_0^w f(t,u)\,du \leq kwf(t,w)$, *for* $|w| \geq N$, $t \in [0,1]$,

then the problem (1) *possesses infinitely many solutions.*

It is well known that in a lot of problems of the form $Lu = N(u)$, where L is a linear operator and N – a nonlinear one, the existence of at least m solution is obtained if an asymptotic behaviour of N near 0 and near ∞ is similar to $\mu_0 I$ and $\mu_\infty I$, respectively (I – the identity operator), and there are exactly m eigenvalues of L in the interval (μ_0, μ_∞). Assumptions (i) and (ii) of Theorem 1 mean that in this interval sit infinitely many eigenvalues, although we generalize the notion of eigenvalue as you will see in Section 2: the eigenvalues are k-dimensional vectors, which forms a sequence (H_n) $k-1$-dimensional hyperplanes and the segment in $\mathbb{R}^k$ joining points with all coordinates 0 except $2(k+1-i)$-th, where there are limits μ_0 from condition (i) and μ_∞ from (ii), respectively, intersects all these hyperplanes. Thus, it is natural to ask if there is at least m solution to (1), if in assumption (ii), the limit is a number μ_∞ such that the above mentioned segment intersect exactly $H_1, \ldots, H_m$. The present paper is an answer to this question for $m = 1$. This enables us to drop the assumption of oddness of f.

2. Preliminaries

Assume that $k \geq 2$ and $i = 1, \ldots k-1$ are fixed positive integer numbers and let

$$\Lambda(x) = \sum_{j=1}^{k} (-1)^{k-j} \lambda_j \left(x^2 \pi^2\right)^{k-j} . \tag{2}$$

Definition 1. (see [4], [5]) A point $\lambda = (\lambda_1, \ldots \lambda_k) \in \mathbb{R}^k$ will be called a *k-dimensional eigenvalue* iff the homogeneous problem

$$\begin{cases} (-1)^k x^{(2k)}(t) + \lambda_1 x^{(2k-2)}(t) + \lambda_2 x^{(2k-4)}(t) \\ \qquad\qquad + \ldots + \lambda_{k-1} x''(t) + \lambda_k x(t) = 0, \\ x^{(2j)}(0) = x^{(2j)}(1) = 0, \quad \text{for } j = 0 \ldots k-1, \end{cases} \tag{3}$$

has a nonzero solution. The set of all such k-tuples will be denoted by σ^k.

It can be proved that the set σ^k has the form

$$\sigma^k = \bigcup_{n \in \mathbb{N}} H_n,$$

where H_n is a hyperplane of the form $H_n := \left\{\lambda \in \mathbb{R}^k \mid \left(n^2\pi^2\right)^k + \Lambda\left(n\right) = 0\right\}$ (see [4], [5]).

Theorem 2. (see [4]) *Let $s \in \mathbb{N}$ and $n_1, n_2, \dots, n_s \in \mathbb{N}$ be such that $n_i \neq n_j$, $i \neq j$.*

(1) *If $s \leq k$, then $W = \bigcap_{i=1}^{s} H_{n_i} \neq \emptyset$ is an affine subspace in $\mathbb{R}^k$ and* $\dim W = k - s$.
(2) *If $s > k$, then $\bigcap_{i=1}^{s} H_{n_i} = \emptyset$.*

Shortly, hyperplanes H_n are in a general position.

Let us put $\Delta_+ := \bigcap_{n \in \mathbb{N}} \{\lambda \in \mathbb{R}^k \mid \Lambda(n) \geq 0\}$. It is easily seen that $\Delta_+ \neq \emptyset$. Indeed, putting

$$D_+ := \begin{cases} \{\lambda \in \mathbb{R}^k \mid \lambda^s \leq 0 \text{ if } s \text{ is odd and } \lambda^s \geq 0 \text{ if } s \text{ is even}\}, \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{if } k \text{ is an even number}, \\ \{\lambda \in \mathbb{R}^k \mid \lambda^s \geq 0 \text{ if } s \text{ is odd and } \lambda^s \leq 0 \text{ if } s \text{ is even}\}, \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{if } k \text{ is an odd number}, \end{cases}$$

we see that $D_+ \subset \Delta_+$. Furthermore, due to the inequality $(n^2\pi^2)^k + \Lambda(n) > 0$, $n = 1, 2, \dots,$ we get $\Delta_+ \cap \sigma^k = \emptyset$.

Definition 2. (see [8]) A sequence $\{y_n\} \subset X$ is a *Palais–Smale sequence* for $\varphi \in C^1\left(X, \mathbb{R}\right)$, if $\{\varphi\left(y_n\right)\}$ is bounded while $\varphi'\left(y_n\right) \to 0$ as $n \to \infty$.

Definition 3. (see [8]) We say $\varphi \in C^1\left(X, \mathbb{R}\right)$ satisfies (P.-S.) condition, if any Palais–Smale sequence has a (strongly) convergent subsequence.

Theorem 3. (Mountain Pass Lemma (MPL), [8], Theorem 6.1) *Let X be a real Banach space and $\varphi \in C^1\left(X, \mathbb{R}\right)$ such that φ satisfies (P.-S.) condition and $\varphi(0) = 0$. Furthermore*

(1) *there exist $\alpha > 0$ and $r > 0$ such that $\varphi\left(y\right) \geq \alpha$ for all $y \in \partial B\left(0, r\right)$;*
(2) *there exists $y_0 \in X$ such that $\varphi\left(y_0\right) < \alpha$ with $\|y_0\| \geq r$.*

Then φ possesses a critical value $c \geq \alpha$.

If x is a solution to (1) and $y = x^{(2i-2)}$, then we have (see [4], [5] and [6])

$$y\left(t\right) = \left(-1\right)^{i-1} \int_0^1 \mathcal{H}_i\left(t, s\right) f\left(s, y\left(s\right)\right) ds,$$

where

$$\mathcal{H}_i(t,s) = \sum_{n=1}^{\infty} \frac{(-1)^{i-1}(n\pi)^{2i-2}}{(n^2\pi^2)^k + \Lambda(n)} \cdot \sin(n\pi s)\sin(n\pi t). \tag{4}$$

Putting

$$(T_i y)(t) = (-1)^{i-1}\int_0^1 \mathcal{H}_i(t,s) f(s, y(s))\, ds,$$

we see that T_i maps C – the space of real continuous functions on $[0,1]$ – into itself.

Let $(H_i z)(t) = (-1)^{i-1}\int_0^1 \mathcal{H}_i(t,s)z(s)ds$ and $(\mathbf{f}y)(s) = f(s, y(s))$ (both operators act in C), then T_i can be described as a composition of H_i and $\mathbf{f}$. Consider the unique extension of the operator H_i to the Hilbert space L^2 denoted also H_i for simplicity. The operator H_i is continuous, self-adjoint and $\sigma(H_i) = \sigma_p(H_i) \cup \sigma_c(H_i)$, where $\sigma_p(H_i) = \{(n\pi)^{2i-2} \cdot [(n^2\pi^2)^k + \Lambda(n)]^{-1} \mid p = 1, 2, \ldots\}$ and $\sigma_c(H_i) = \{0\}$ (see [6]). Moreover, if $\lambda \in \Delta_+$ then $\sigma(H_i) \subset [0, +\infty)$ and H_i is a positive operator. This implies that there exists a unique positive and self-adjoint S_i such that $S_i^2 = H_i$ (see [2], Theorem 2.2.10). Below, the norm without subscript stands for the norm in the space L^2; the supremum norm in the space of continuous functions will be denoted by $\|\cdot\|_C$.

Remark. Hereinafter, it will be assumed that $\lambda \in \Delta_+$.

If we reason in the similar way as in [4], we can deduce that

$$(S_i z)(t) = \sum_{n=1}^{\infty} \frac{(n\pi)^{i-1}}{\sqrt{(n\pi)^{2k} + \Lambda(n)}} \cdot \int_0^1 \sin(n\pi s) z(s)\, ds \cdot \sin(n\pi t).$$

(i) The function

$$\mathcal{S}_i(t,s) = \sum_{n=1}^{\infty} \frac{(n\pi)^{i-1}}{\sqrt{(n\pi)^{2k} + \Lambda(n)}} \sin(n\pi s)\sin(n\pi t)$$

is continuous.

(ii) Operator S_i maps L^2 into C and it has the form

$$(S_i z)(t) = \int_0^1 \mathcal{S}_i(t,s) z(s)\, ds.$$

It easy to see that the sequence $(n\pi)^{2i-2} \cdot [(n^2\pi^2)^k + \Lambda(n)]^{-1}$ is decreasing, thus by Theorem 2.2.5 [2], it is easy to calculate the norms of H_i and S_i, treating them as operators from L^2 into itself

$$\|H_i\| = \pi^{2i-2} \cdot \left(\pi^{2k} + \Lambda(1)\right)^{-1} \quad \text{and} \quad \|S_i\| = \pi^{i-1} \cdot \left(\pi^{2k} + \Lambda(1)\right)^{-\frac{1}{2}}. \tag{5}$$

After [6], one can notice that

- (1) has a solution if $T_i = H_i \circ \mathbf{f}$ has a fixed point;
- fixed points of the operator $S_i \circ \mathbf{f} \circ S_i$ in L^2 are also fixed points of $T_i = H_i \circ \mathbf{f}$ in C;
- if $\psi : C \to \mathbb{R}$ is defined by the formula

$$\psi y = \int_0^1 \int_0^{y(t)} f(t, u)\, du dt,$$

then functionals ψ and $\psi \circ S_i$ are Fréchet differentiable on C and L^2, respectively and $(\psi'(y))\, h = \langle \mathbf{f} y, h \rangle_{L^2}$, $(\psi \circ S_i)' y = (S_i \circ \mathbf{f} \circ S_i)\, y$.

Let $\varphi_i : L^2 \to \mathbb{R}$, be the functional of the form

$$\varphi_i(y) = \frac{1}{2} \langle y, y \rangle_{L^2} - (\psi \circ S_i)\, y. \tag{6}$$

We have

$$\varphi_i'(y) = y - (S_i \circ \mathbf{f} \circ S_i)\, y, \tag{7}$$

for $y \in L^2$.

Due to the above arguments, solutions to (1) are critical points of φ_i.

3. Main result

Lemma 1. [6] *Assume that there are $k \in [0, 1/2)$ and $N > 0$, such that for $|w| \geq N$, we have*

$$\int_0^w f(t, u)\, du \leq kwf(t, w)$$

then the functional φ_i satisfies the (P.S.) condition.

Remark. If f is odd then functions $\mathbb{R} \ni w \mapsto \int_0^w f(t, u)\, du$ and $\mathbb{R} \ni w \mapsto w \cdot f(t, w)$ are even for any $t \in [0, 1]$. Therefore, to verify assumption of Lemma 1, it is sufficient to check the inequality for $w \geq N > 0$.

Now we can prove the main theorem.

Theorem 4. *Assume that $\lambda \in \Delta_+$ and let $f : [0, 1] \times \mathbb{R} \to \mathbb{R}$ be a continuous function that satisfies the following conditions*

(i) $\limsup\limits_{u \to 0} \dfrac{f(t, u)}{u} < \dfrac{\pi^{2k} + \Lambda(1)}{\pi^{2i-2}}$,

(ii') $\liminf\limits_{u \to +\infty} \dfrac{f(t, u)}{u} > \dfrac{\pi^{2k} + \Lambda(1)}{\pi^{2i-2}}$,

(iii) *there exist $k \in [0, 1/2)$ and $N > 0$, such that for $|w| \geq N$, we have*

$$\int_0^w f(t, u)\, du \leq kwf(t, w),$$

for each $t \in [0, 1]$. Then problem (1) possesses at least one nonzero solution.

Proof. We shall use the Mountain Pass Lemma. The considered functional has the form

$$\varphi_i(y)(t) = \frac{1}{2}\|y\|^2 - \int_0^1 \int_0^{(S_i y)(t)} f(t,u)\,du dt.$$

It is obvious that $\varphi_i(0) = 0$, furthermore we explained on the page 56, that

$$\frac{\pi^{2k} + \Lambda(1)}{\pi^{2i-2}} = \|H_i\|^{-1}.$$

Due to assumption, there exist $\delta > 0$ and $\varepsilon_1 \in (0, \|H_i\|^{-1})$ such that

$$\frac{f(t,u)}{u} \leq \|H_i\|^{-1} - \varepsilon_1, \quad \text{for } |u| \leq \delta.$$

This implies that

$$f(t,u) \leq (\|H_i\|^{-1} - \varepsilon_1)u, \quad \text{for } u \in [0,\delta],$$

and

$$f(t,u) \geq (\|H_i\|^{-1} - \varepsilon_1)u, \quad \text{for } u \in [-\delta, 0).$$

Thus we have, for $w \in [0,\delta]$,

$$\int_0^w f(t,u)\,du \leq (\|H_i\|^{-1} - \varepsilon_1)\int_0^w u du = (\|H_i\|^{-1} - \varepsilon_1)\frac{1}{2}w^2,$$

and, for $w \in [-\delta, 0)$,

$$\int_0^w f(t,u)du = -\int_w^0 f(t,u)du \leq -(\|H_i\|^{-1} - \varepsilon_1)\int_w^0 u du = (\|H_i\|^{-1} - \varepsilon_1)\frac{1}{2}w^2.$$

The above calculations imply that

$$\int_0^w f(t,u)\,du \leq (\|H_i\|^{-1} - \varepsilon_1)\frac{1}{2}w^2, \tag{8}$$

for $|w| \leq \delta$.

Let $y \in L^2$, the kernel $\mathcal{S}_i$ of S_i is a positive continuous function. Therefore, let $B_i > 0$ be its maximal value. This and the Hölder's inequality imply that

$$\|S_i y\|_C \leq B_i \int_0^1 |y(s)|\,ds \leq B_i \left(\int_0^1 |y(s)|^2\,ds\right)^{\frac{1}{2}} = B_i\|y\|.$$

Therefore, if $\|y\| = r$, where $r = B^{-1}\delta$, we get

$$\|S_i y\|_C \leq \delta.$$

Now, the last estimate together with (8), give us the following condition

$$\begin{aligned}
\varphi_i(y) &= \frac{1}{2}\|y\|^2 - \int_0^1\int_0^{(S_iy)(t)} f(t,u)\,dudt \\
&\geq \frac{1}{2}\|y\|^2 - (\|H_i\|^{-1} - \varepsilon_1)\frac{1}{2}\int_0^1 ((S_iy)(t))^2\,dt \\
&= \frac{1}{2}\|y\|^2 - (\|H_i\|^{-1} - \varepsilon_1)\frac{1}{2}\langle S_iy, S_iy\rangle_{L^2} \\
&= \frac{1}{2}\|y\|^2 - (\|H_i\|^{-1} - \varepsilon_1)\frac{1}{2}\langle H_iy, y\rangle_{L^2} \\
&\geq \frac{1}{2}\|y\|^2 - (\|H_i\|^{-1} - \varepsilon_1)\frac{1}{2}\|H_i\|\,\|y\|^2 = \frac{1}{2}\|H_i\|\,\|y\|^2\varepsilon_1 \\
&= \frac{1}{2}\|H_i\|\,r^2\varepsilon_1.
\end{aligned}$$

Thus $\varphi_i(y) \geq \alpha$, $\alpha = \frac{1}{2}\|H_i\|\,r^2\varepsilon_1$, for $y \in \partial B(0,r)$, $r = B_i^{-1}\delta$, and assumption (1) of MPL holds. Assumption (ii) implies the existence of such $N > 0$ and $\varepsilon_2 > 0$, that

$$\frac{f(t,u)}{u} \geq \|H_i\|^{-1}(1+\varepsilon_2), \quad \text{for } u > N \text{ and } t \in [0,1].$$

Then

$$f(t,u) \geq \|H_i\|^{-1}(1+\varepsilon_2)\,u, \quad \text{for } u > N \text{ and } t \in [0,1]. \tag{9}$$

Because the function $f(t,u) - \|H_i\|^{-1}(1+\varepsilon_2)\,u$ is continuous on $[0,1] \times [0,N]$, there exists $M > 0$ such that

$$f(t,u) \geq \|H_i\|^{-1}(1+\varepsilon_2)\,u - M, \quad \text{for } u \in [0,N] \text{ and } t \in [0,1]. \tag{10}$$

The inequalities (9) and (10) led us to the following conclusion

$$f(t,u) \geq \|H_i\|^{-1}(1+\varepsilon_2)\,u - M, \quad \text{for } u \geq 0,\ t \in [0,1].$$

Integrating the above inequality, we get

$$\int_0^w f(t,u)\,du \geq \frac{1}{2}\|H_i\|^{-1}(1+\varepsilon_2)\,w^2 - Mw, \tag{11}$$

for $w \geq 0$ and uniformly for $t \in [0,1]$.

We have explained that

$$H_iu = \sum_{n=1}^{\infty}\xi_n^2 \cdot \langle e_n, u\rangle_{L_2} e_n \quad \text{and} \quad S_iu = \sum_{n=1}^{\infty}\xi_n \cdot \langle e_n, u\rangle_{L_2} e_n,$$

where

$$\xi_n^2 = \frac{(n\pi)^{2i-2}}{(n\pi)^{2k} + \Lambda(n)} \quad \text{and} \quad e_n(t) = \sqrt{2}\sin(n\pi t).$$

Furthermore

$$\xi_1^2 = \frac{\pi^{2i-2}}{\pi^{2k} + \Lambda(1)} = \|H_i\|.$$

It easily seen that

$$S_i e_1 = \xi_1 e_1 = \sqrt{\|H_i\|} e_1 \geq 0. \tag{12}$$

For any real τ, we have from (11) and (12)

$$\begin{aligned}
\varphi_i(\tau e_1) &= \frac{1}{2}\langle \tau e_1, \tau e_1\rangle_{L^2} - \int_0^1 \int_0^{(S_i(\tau e_1))(t)} f(t,u)\, du dt \\
&\leq \frac{1}{2}\tau^2 - \frac{1}{2}\|H_i\|^{-1}(1+\varepsilon_2)\int_0^1 (S_i(\tau e_1))^2(t)\, dt \\
&\quad + M\int_0^1 (S_i(\tau e_1))(t)\, dt \\
&= \frac{1}{2}\tau^2 - \frac{1}{2}\tau^2\|H_i\|^{-1}(1+\varepsilon_2)\langle S_i e_1, S_i e_1\rangle_{L^2} \\
&\quad + M\tau\int_0^1 (S_i(e_1))(t)\, dt \\
&= \frac{1}{2}\tau^2 - \frac{1}{2}\tau^2\|H_i\|^{-1}(1+\varepsilon_2)\|H_i\| + M\tau\sqrt{\|H_i\|}\int_0^1 e_1(t)\, dt \\
&= -\frac{1}{2}\varepsilon_2\tau^2 + M\sqrt{\|H_i\|}\frac{2\sqrt{2}}{\pi}\tau.
\end{aligned}$$

Let $\tau > 0$. The above estimate implies that $\varphi_i(\tau e_1) \to -\infty$ and $\|\tau e_1\| = \tau \to +\infty$ as $\tau \to +\infty$.

Therefore, there exists $\tau_0 > r$ such that $\varphi_i(y_0) = \varphi_i(\tau_0 e_1) < 0 < \alpha$ and assumption (2) of MPL is satisfied. Due to MPL, we get the assertion. ∎

It has to be emphasized that we do not assume oddness of a nonlinear part.

Example. Let us consider the following problem

$$\begin{cases} u^{(8)} - \pi^2 u^{(6)} + \pi^4 u^{(4)} - \pi^6 u'' + \pi^8 u = f\left(t, u^{(4)}\right), \\ \qquad u^{(2j)}(0) = u^{(2j)}(1) = 0, \quad \text{for } j = 0\ldots 3, \end{cases}$$

where

$$f(t,w) = \begin{cases} w^3 + 5\pi^4 + 1, & \text{for } \ w < -1, \\ w^2 + 5\pi^4 - 1, & \text{for } \ w \in [-1,1], \\ w^2 \arctan(w) + 5\pi^4 - \frac{1}{4}\pi, & \text{for } \ w > 1. \end{cases}$$

It is seen that $\|H_3\|^{-1} = 5\pi^4$, furthermore it is easy to verify that f is continuous, satisfies conditions (i)–(iii) of Theorem 4 and does not satisfy assumptions of Theorem 1.

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M. Jurkiewicz
INSTITUTE OF MATHEMATICS AND CRYPTOLOGY
MILITARY UNIVERSITY OF TECHNOLOGY
Gen. Sylwestra Kaliskiego 2
00-908 WARSZAWA 49, POLAND
E-mail: mjurkiewicz@wat.edu.pl

B. Przeradzki
INSTITUTE OF MATHEMATICS
LODZ UNIVERSITY OF TECHNOLOGY
Wólczańska 215
90-924 ŁÓDŹ, POLAND
E-mail: bogdan.przeradzki@p.lodz.pl

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