

Analytical studies on amplitude change enhancement in coupled aluminium nitride coated single crystal silicon oscillator pair applicable to ultra-sensitive resonating microfluidic flowmeters

Dong F. Wang¹, Keisuke Chatani^{1,3}, Kenji Kozuka¹, Tsuyoshi Ikehara², Ryutaro Maeda²

¹Micro Engineering and Micro Systems Laboratory, Department of Mechanical Engineering, Faculty of Engineering, Ibaraki University, Hitachi, Ibaraki 316-8511, Japan

²Research Center for Ubiquitous MEMS & Micro Engineering (UMEMSME), National Institute of Advanced Industrial Science and Technology (AIST), Tsukuba, Ibaraki 305-8564, Japan

³Present address: Toyota Motor Corporation, Aichi 471-8571, Japan
E-mail: dfwang@mx.ibaraki.ac.jp

Published in Micro & Nano Letters; Received on 24th May 2013; Revised on 18th July 2013; Accepted on 26th July 2013

The use of vibration mode localisation in arrays of micromechanically coupled, nearly identical beam-shaped resonators has been analytically studied for ultra-sensitive resonating-based flowmeters. Eigenstate shifts (amplitude change in this Letter) that are about two times (compared with the single resonator), and orders (compared with the resonator array) of magnitude greater than corresponding shifts in resonant frequency for an induced fluid flow (corresponding to an induced small mass perturbation) are preliminarily obtained by theoretical analysis. When an external force of 100 Pa, corresponding to an estimated fluid flow velocity of about 3.15 m/s, is applied to any one (because of the symmetrical design) of the two coupled aluminium nitride coated single crystal silicon resonators, two orders of the amplitude enhancement can be observed for both the resonators at the higher (second) resonance frequency because of vibration mode localisation, which implies the application possibility to highly sensitive resonating-based flow sensors.

1. Micromachined or MEMS flow sensors: The measurement of fluid flow rates is an essential requirement in both industrial and commercial applications. It has been estimated by Hayward [1] that there are more than one hundred different types of flow sensors with a mode of operation based on almost any physical domain. Although several large-scale flowmeter types are commercially available, the continuous development of the three-dimensional techniques of microfabrication and MEMS, with consequent cost reduction and quality improvement, has rapidly extended the market of micromachined or MEMS flow sensors. In particular, applications of micromachined or MEMS sensors to monitor gas and liquid flows hold immense potential because of their valuable characteristics, such as low-energy consumption, relatively good accuracy, the ability to measure very small flow, small size and so on.

After the first integrated silicon-based sensor for gas flow measurement [2] appeared in 1974, a higher growth of research works in the field of micromachined or MEMS flow sensors took place in the eighties and about a decade was needed to develop the integration of many microfluidic devices into a single chip [3, 4].

Based on the working principles, the micromachined or MEMS flow sensors can be distinguished into two groups [5]. The first group contains flow sensors based on heat exchange, named ‘thermal flow sensors’; whereas all the other flowmeters based on different working principles rather than thermal exchange, are thus named as ‘non-thermal flow sensors’, which are also divided into several subgroups depending on the principle of measurement used like cantilever flow sensors, differential pressure-based flow sensors, resonating flow sensors and laser Doppler flowmeters.

2. Amplitude change enhancement by vibration mode localisation: The micro resonator sensor [6–10] is used to detect external stimulus, that is, force, mass, molecular as well as atomic adsorption and so on. The amount of relative change in resonant frequency or corresponding amplitude, or any other related eigenstate (such as resonance frequencies, phases, peak vibration amplitudes etc.) changes is measured to detect external stimulus.

If the amount of changes can be enhanced by a certain amplification mechanism, ultimate sensing can be achieved and small stimulus can be detected.

In resonant frequency-based sensors, the output corresponds to a shift in the resonant frequency of a vibrating micromechanical structure when subjected to small perturbations in either its stiffness or mass. The most sensitive microcantilever-based mass detection experiments using the frequency-shift approach have reported attogram level detection in an ultra-high vacuum environment [6, 8] and femtogram level detection under ambient conditions [9, 10].

In contrast, the concept of using Anderson or vibration mode localisation [11–18] in any array of nearly identical coupled resonators has also been proposed as an eigenstate-shift-based sensing mechanism in recent years in coupled microcantilevers under ambient conditions [17, 19, 20].

Some advantages of mode localised sensing can be listed below. Firstly, times or orders of magnitude in the parametric sensitivity of micromechanical mass detection compared with the conventional frequency-shift approach can be obtained. Secondly, such sensors can offer important advantages to intrinsic common-mode rejection that renders it less susceptible to false-positive readings than frequency-shift-based sensors. Finally, both the ultra-sensitive detection and analyte identification of small perturbation can be achieved at the same time with a single coupled resonator array.

This kind of relative changes in eigenstates was experimentally studied in [17] using a coupled single crystal silicon (abbreviated as SCS hereafter) two-resonator array and concluded that the relative change in eigenstates because of the added mass could be two orders of magnitude greater than the relative change in resonance frequency.

In our previous Letter, the effect of different geometrical designs of the coupling overhang (coupling spring) [21, 22] on the relative change (%) in the amplitude was theoretically studied using a coupled SCS 15-resonator array for each vibration mode before and after a small mass perturbation of 10 pg. However, using a SCS five-resonator array without a small mass perturbation [21, 22], the vibration characteristics were complicated and the

analytical results were also found to be not clearly corresponding to the experimental ones.

Recently, a mechanically-coupled SCS three-resonator array was further selected and prepared for vibration localisation evaluation [23–25], because three is the large integer next to two. It is concluded that the amount of amplitude change of coupled three-resonator is higher than that of the frequency shift of a single-resonator, and the analytical results are clearly corresponding to the experimental ones.

In this Letter, the vibration mode localisation-based amplification mechanism is studied using a simplified two-beam oscillator pair. The consequent amplitude change in coupled aluminium nitride coated SCS (abbreviated as AlN/SCS hereafter) microresonators is thus preliminarily considered as a possible resonating flow sensor for further real application.

3. Geometrical design and physics of amplitude change enhancement in coupled two-resonator array: Since the discretised model of identical resonators coupled by overhangs in a large array is too complicated to be mathematically tackled, a two-resonator array with two overhangs for each resonator is therefore hypothetically introduced here as our analysis subject using CoventorWare™ software.

Fig. 1 shows the schematic of a single weakly coupled array, corresponding to a perfect array of two identical spring-mass beams with each beam connected to its neighbour by an overhang. The geometrical size of the overhang is also detailed in Fig. 1, where the gap b between the beams is sized $600\ \mu\text{m}$ and the contact length a of the overhang adjacent to the beam is sized $550\ \mu\text{m}$. Two identical beams are nominally $2500\ \mu\text{m}$ long, $300\ \mu\text{m}$ wide, $2.7\ \mu\text{m}$ thick ($0.5\ \mu\text{m}$ for AlN coating and $2.2\ \mu\text{m}$ for SCS) and are made of SOI wafer.

As shown in Fig. 2, two eigenstate resonance frequencies (810.27 and $876.84\ \text{Hz}$) can be analytically observed for both resonators. The lower resonance frequency (mode 1) is related to a mode where both resonators vibrate in phase. At the higher resonance frequency (mode 2), two resonators numbers 1 and 2 vibrate in opposite-phase mutually.

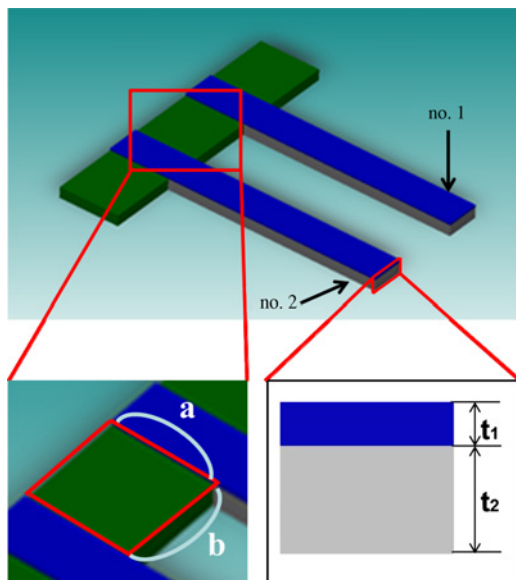


Figure 1 Schematic of the coupled beam-shaped AlN/SCS two-resonator array, and the size definition of coupling overhang and cross-section. Resonator is sized $2500 \times 300 \times 2.7\ \mu\text{m}^3$, where the thickness of AlN coating (blue) is $0.5\ \mu\text{m}$ and the thickness of SCS (grey) is $2.2\ \mu\text{m}$. For the coupling overhang (green), the gap between the resonators is sized $600\ \mu\text{m}$, and the contact length of the overhang adjacent to the resonator is sized $550\ \mu\text{m}$.

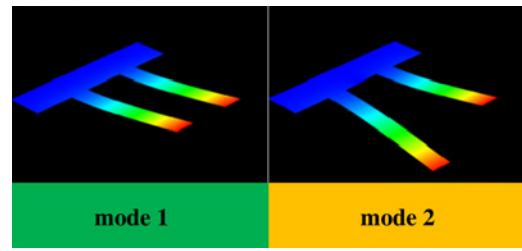


Figure 2 Analytical results by using CoventorWare™ software for two resonators under two different vibration states of mode 1 and mode 2. Mode 1 shows that both resonators vibrate in phase, whereas mode 2 shows that the two resonators vibrate in opposite-phase mutually.

Considering the case of two initially identical resonators, the relative change in normalised eigenstate u is given by the authors [17, 21–25]

$$\frac{|u_i - u_i^0|}{|u_i^0|} = \left(\frac{1}{4} + \frac{1}{4Kc/K} \right) \delta, \quad i = 1, 2 \quad (1)$$

where K , Kc are, respectively, the bending stiffnesses of the two resonators and the overhang coupling of the two resonators, whereas δ represents the ratio of the effect mass (δM) being detected to the single resonator mass (M). The relative change in the eigenvalue λ or resonance frequency of a single resonator is given by

$$\frac{\lambda - \lambda_0}{\lambda_0} = \frac{-\delta}{2} \quad (2)$$

Note that the perturbed eigenstates U_i , $i = 1, 2$ start becoming localised in the sense that in each eigenstate one resonator oscillates more (large magnitude) than the other.

Equation (1), which defines the sensed quantity in this sensing paradigm, suggests that simply by decreasing the scaled coupling between the two resonators Kc/K , the relative changes in eigenstates u can be made orders of magnitude greater than the relative change in eigenvalue λ of a single resonator.

4. Estimating reference flow velocity by applying a small mass perturbation: Considering the case of a rectangle beam in a flowing fluid, if C_D and ρ (kg/m^3) are, respectively, the drag coefficient and the density of the fluid, the corresponding pressure P (kg/m^3) acting on the beam surface, as shown in Fig. 3, as a function of the relative flow velocity of V (m/s) can be written as follows

$$p = \frac{1}{2} C_D \rho V^2 \quad (3)$$

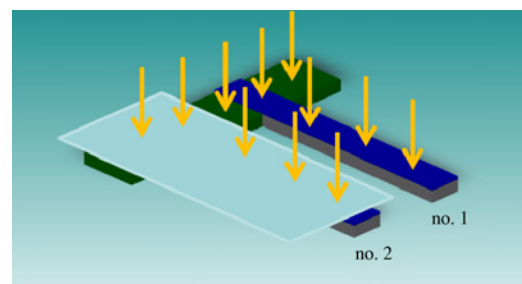


Figure 3 Model of the coupled AlN/SCS two-resonator array in analysis, where an external pressure of $100\ \text{Pa}$ is supposed to be applied to resonator no. 1.

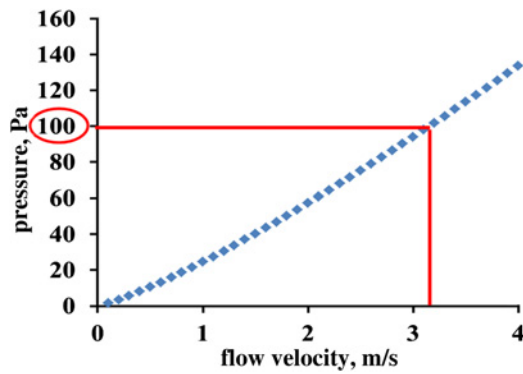


Figure 4 Applied pressure against relative flow velocity, where an applied pressure of 100 Pa is found to be corresponding to a reference flow velocity of about 3.15 m/s

whereas the Reynolds number as a function of the relative velocity of V can be expressed as

$$R_e = \frac{Vd}{\nu} \quad (4)$$

where d (m) and ν (m^2/s) are, respectively, the typical flow distance and the coefficient of kinematic viscosity. In the case of $0.1 < R_e < 10$, the relation between the drag coefficient of C_D and the Reynolds number can be written as (5), and the acting pressure P in Fig. 3 as a function of the relative flow velocity V can be further simplified as (6). As analytically drawn in Fig. 4, in the present Letter, an applied pressure of 100 Pa is found to be corresponding to a reference relative flow velocity of about 3.15 m/s

$$C_D = 10R_e^{-0.778} = 10\left(\frac{Vd}{\nu}\right)^{-0.778} \quad (5)$$

$$P \propto V^{1.23} \quad (6)$$

5. Vibration analysis of beam-shaped AlN/SCS two-resonator array: As shown in Fig. 5, a typical amplitude–frequency spectrum has been obtained for the mechanically-coupled two-resonator array, under an exciting force of 0.01 kg mm/s^2 and a damping of 0.001 kg/s .

Taking off couplings from the model in Fig. 1, two resonators turn out to be separated from the neighbouring one by the size of a coupling overhang. In the analytical level, however, the two resonators are supposed to be far away from each other. Hence, it is necessary to analytically combine two resonators via a common base to ensure a separating distance corresponding to the coupling overhang. As reported in our previous Letter [23–25], in the case of overhang coupling, only one resonance frequency can be obtained for all the resonators. This indicates that all the resonators vibrate individually so that vibration mode localisation cannot be observed.

In the case of coupling overhang, as shown in Fig. 5, two resonance frequencies (810.27 and 876.84 Hz) can be obtained for both the resonators. At the two resonance frequencies, the amplitudes of resonator no. 1 and no. 2 are exactly the same but very different from that of the resonators without couplings. It means that vibration mode localisation happens because of the resonator interacting under vibration via coupling overhang.

The analytical results show that higher amplitudes at a frequency of 810.27 Hz, almost reaching $450 \mu\text{m}$ as shown in Fig. 5 (upper), could be obtained for both the resonators (no. 1 and no. 2) because of the symmetrical design. In contrast, the amplitudes at the frequency of 876.84 Hz are found to be slightly over $2.5 \mu\text{m}$, as shown in Fig. 5 (lower).

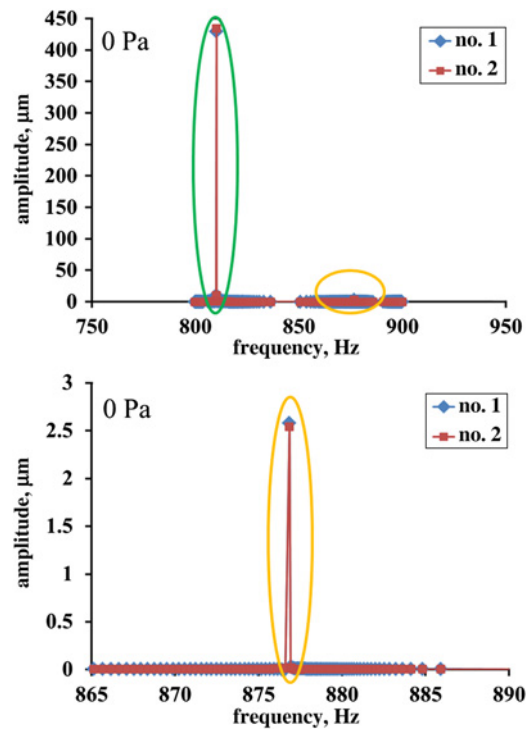


Figure 5 Analytical results of the micromechanically coupled beam-shaped AlN/SCS two-resonator array simulated by using CoventorWare™ software, where the lower figure shows the scaled up part of the initial graph around the resonance point of 876.84 Hz

6. Applicability to highly sensitive resonating-based flowmeters: Fig. 3 shows the analytical model of the coupled AlN/SCS two-resonator array, where an external pressure of 100 Pa, corresponding to an estimated reference fluid flow velocity of 3 m/s, is supposed to be applied to resonator no. 1. It is noted that two resonance frequencies can be observed for two resonators in Fig. 6, which are similar to those of the analytical results without small mass perturbation in Fig. 5. However, the observed resonance frequencies are shifted to a higher range from those of the results in Fig. 5, because of the applied pressure of 100 Pa generated from the reference fluid flow of about 3 m/s.

As summarised in Table 1, for resonator no. 1, at the lower resonance point, the frequency increased from 810.27 to 828.74 Hz, whereas the amplitude ratio ($\delta A/A_0$) decreased to 0.67. At the higher resonance point, the frequency increased from 876.84 to 999.23 Hz, and the amplitude ratio further increased to 63.30.

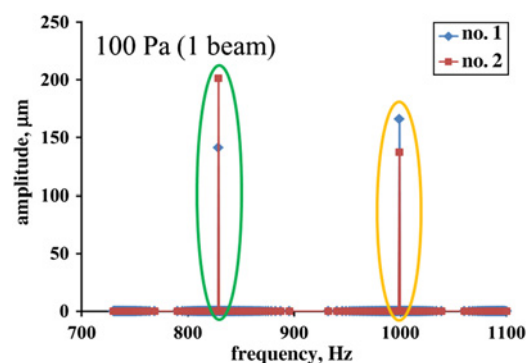


Figure 6 Analytical results of the coupled AlN/SCS two-resonator array by applying a small mass perturbation (100 Pa) to resonator no. 1, where the amplitude change enhancement can be observed at both the vibration modes

Table 1 Analytical results showing the amplitude ratio ($\delta A/A_0$) at different vibration modes for the two resonators, respectively, when an external pressure of 100 Pa is supposed to be applied to resonator no. 1

No. 1	Frequency [Hz]		$ \Delta A/A_0 $	
	100 pa	No Pressure	No. 1	No. 2
mode 1	828.74	810.27	0.67	0.54
mode 2	999.23	876.84	63.30	52.99

It is also noted from Table 1, that even for resonator no. 2, when 100 Pa was applied to the coupled resonator no. 1, the amplitude ratio at the lower resonance frequency decreased to 0.54, whereas the amplitude ratio at the higher resonance frequency increased conversely to 52.99, because of the vibration mode localisation of the coupling overhang.

As reported in our previous Letter [23–25], although the measured resonance frequencies might shift to a higher level compared with those of the analytical results because of the geometrical size changing caused by the fabrication error, the mutual relationships among the changed amplitudes and phase differences of each resonator at each resonance frequency are almost equivalent for both analysis and experiment.

The above preliminary results imply that the amplitude enhancement observed at the higher (second) resonance frequency for both the resonators might be used to measure the fluid flow. For achieving a higher amplitude change because of a small mass perturbation (corresponding to flow velocity), both beams are suitable as detecting (loaded) resonators or as sensing (unloaded) ones, if both beams are geometrically identical.

It can be further noted from Fig. 7, that two resonance frequencies can also be observed and are also shifted to a higher range, if an external pressure of 100 Pa, corresponding to an estimated

Table 2 Analytical results showing the amplitude ratio ($\delta A/A_0$) at different vibration modes for the two resonators, respectively, when an external pressure of 100 Pa is supposed to be applied to both the resonators

No. 1	Frequency [Hz]		$ \Delta A/A_0 $	
	100 pa	No Pressure	No. 1	No. 2
mode 1	851.75	810.27	0.17	0.17
mode 2	958.10	876.84	0.84	0.84

reference fluid flow velocity of 3 m/s, is supposed to be applied to both the resonators. As summarised in Table 2, even at the higher resonance frequency, the frequency increased from 876.84 to 958.10 Hz, whereas the amplitude ratio ($\delta A/A_0$) decreased to 0.84 for both the resonators.

7. Conclusions: A micromechanically-coupled beam-shaped AlN/SCS two-resonator array, applicable to ultra-sensitive resonating flowmeters, has been geometrically designed and preliminarily analysed.

It is verified from the analytical results that vibration mode localisation happens because of coupling overhang in the proposed nearly identical AlN/SCS two-resonator array. In the present Letter, there are two-resonance points in the coupled two-resonator array, which proves again that the number of resonance points is the same with the number of coupled resonators.

When an external force of 100 Pa, corresponding to an estimated reference fluid flow velocity of 3.15 m/s, is applied to any one (because of the symmetrical design) of the two coupled AlN/SCS resonators, two orders of the amplitude enhancement (the amplitude ratio ($\delta A/A_0$) varying from 52.99 to 63.30) can be observed for both the resonators at the higher (second) resonance frequency because of vibration mode localisation.

Since the mutual relationships among the changed amplitudes and phase differences of each resonator at each resonance frequency are observed to be almost equivalent for both analysis and experiment [23–25], the above analytical results further imply the application possibility to highly sensitive resonating-based flow sensors, which needs to be experimentally specified.

8. Acknowledgment: This research is granted by the Japan Society for the Promotion of Science (JSPS) through the ‘Funding Program for World-Leading Innovative R&D on Science and Technology (FIRST Program)’, initiated by the Council for Science and Technology Policy (CSTP). This work is also managed and supported by MEMS/Nanotech Inter-University Networking (MIN) steered by the Research Center for Ubiquitous MEMS and Micro Engineering (UMEMSME), National Institute of Advanced Industrial Science and Technology (AIST).

9 References

- [1] Miller R.W.: ‘Introduction to the differential producer’ in Miller R. W., (Ed.): ‘Flow measurement engineering handbook’ (McGraw-Hill, New York, 1996, 3rd edn) Sections 7.1–7.5
- [2] Putten A.F.P., van Middlehoek S.: ‘Integrated silicon anemometer’, *Electron. Lett.*, 1974, **10**, pp. 425–426
- [3] Nguyen N.T.: ‘Micromachined flow sensors – a review’, *Flow Meas. Instrum.*, 1997, **8**, pp. 7–16
- [4] Petersen K., Brown J.: ‘High-precision, high-performance mass-flow sensor with integrated laminar flow micro-channels’. Proc. Tech. Dig. of 3rd Int. Conf. on Solid-State Sensors and Actuators (Transducers’85), Philadelphia, PA, USA, June 1985, pp. 361–363
- [5] Silvestri S., Schena E.: ‘Review – micromachined flow sensors in biomedical applications’, *Micromachines*, 2012, **3**, pp. 225–243
- [6] Ekinci K.L., Huang X.M.H., Roukes M.L.: ‘Ultrasensitive nanoelectromechanical mass detection’, *Appl. Phys. Lett.*, 2004, **84**, pp. 4469–4471

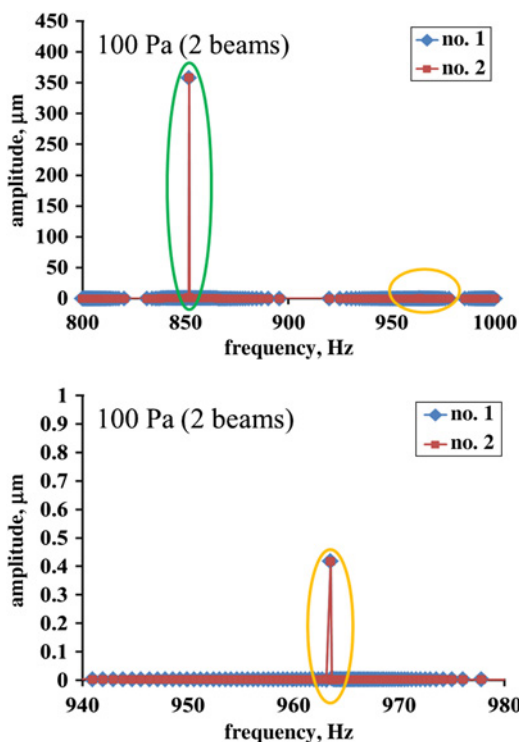


Figure 7 Analytical results of the coupled AlN/SCS two-resonator array by applying a small mass perturbation (100 Pa) to both the resonators, where the lower figure shows the scaled up part of the initial graph around the resonance point of 958.10 Hz

- [7] Ono T., Wang D.F., Esashi M.: 'Mass sensing with resonating ultra-thin double beams'. Proc. 2nd IEEE Int. Conf. Sensors (IEEE Sensors 2003), Toronto, Canada, October 2003, pp. 825–829
- [8] Davis Z.J., Boisen A.: 'Aluminum nanocantilevers for high sensitivity mass sensors', *Appl. Phys. Lett.*, 2005, **87**, article id 013102
- [9] Hosaka S., Chiyoma T., Ikeuchi A., Okano H., Sone H., Izumi T.: 'Possibility of a femtogram mass biosensor using a self-sensing cantilever', *Curr. Appl. Phys.*, 2006, **6**, pp. 384–388
- [10] Ilic B., Czaplewski D., Zalalutdinov M., Craighead H.G.: 'Single cell detection with micromechanical oscillators', *J. Vac. Sci. Technol. B*, 2001, **19**, pp. 2825–2828
- [11] Anderson P.W.: 'Absence of diffusion in certain random lattices', *Phys. Rev.*, 1958, **109**, pp. 1492–1505
- [12] Hodges C.H.: 'Confinement of vibration by structural irregularity', *J. Sound Vib.*, 1982, **82**, pp. 411–424
- [13] Pierre C., Dowell E.H.: 'Localization of vibrations by structural irregularity', *J. Sound Vib.*, 1987, **114**, pp. 549–564
- [14] Buks E., Roukes M.L.: 'Electrically tunable collective response in a coupled micromechanical array', *J. Microelectromech. Syst.*, 2002, **11**, pp. 802–807
- [15] Sato M., Hubbard B.E., Sievers A.J., Ilic B., Czaplewski D.A., Craighead H.G.: 'Observation of locked intrinsic localized vibrational modes in a micromechanical oscillator array', *Phys. Rev. Lett.*, 2003, **90**, article id 044102
- [16] Napoli M., Zhang W.H., Turner K., Bamieh B.: 'Characterization of electrostatically coupled microcantilevers', *J. Microelectromech. Syst.*, 2005, **14**, pp. 295–304
- [17] Spletzer M., Raman A., Wu A.Q., Xu X., Reifenberger R.: 'Ultrasensitive mass sensing using mode localization in coupled cantilever', *Appl. Phys. Lett.*, 2006, **88**, article id 254102
- [18] Spletzer M., Raman A., Sumali H., Sullivan J.P.: 'Highly sensitive mass detection and identification using vibration localization in coupled microcantilever arrays', *Appl. Phys. Lett.*, 2008, **92**, article id 114102
- [19] Nicu L., Bergaud C.: 'Modeling of a tuning fork biosensor based on the excitation of one particular resonance mode', *J. Micromech. Microeng.*, 2004, **14**, pp. 727–736
- [20] Qazi A., Nonis D., Pozzato A., *ET AL.*: 'Asymmetrical twin cantilevers for single molecule detection', *Appl. Phys. Lett.*, 2007, **90**, article id 173118
- [21] Chatani K., Wang D.F., Ikehara T., Maeda R.: 'Amplitude enhancement using vibration mode localization with a single micromechanically coupled beam-shaped resonator array'. Proc. 13th Int. Conf. Design, Integration and Packaging of MEMS/MOEMS, Aix-en-Provence, France, 2011, pp. 339–343
- [22] Wang D.F., Chatani K., Ikehara T., Maeda R.: 'Mode localization analysis and characterization in a 5-beam array of coupled nearly identical micromechanical resonators for ultra-sensitive mass detection and analyte identification', *Microsyst. Technol.*, 2012, **18**, pp. 1923–1929
- [23] Chatani K., Wang D.F., Ikehara T., Maeda R.: 'Investigation of vibration mode localization using micromechanically-coupled beam-shaped 3-resonator array for ultra-sensitive mass detection and analyte identification'. Dig. 2nd Japan-China-Korea Joint Conf. MEMS/NEMS for Green and Life Innovation, Jeju Island, Korea, 2011, pp. 137–138
- [24] Chatani K., Wang D.F., Ikehara T., Maeda R.: 'Vibration mode localization in coupled beam-shaped resonator array'. Proc. 7th IEEE Int. Conf. Nano/Micro Engineered and Molecular Systems, Kyoto, Japan, 2012, pp. 94–97
- [25] Wang D.F., Chatani K., Ikehara T., Maeda R.: 'Observation of localized vibration modes in a micromechanical oscillator trio with coupling overhang for highly sensitive mass sensing', *Micro Nano Lett.*, 2012, **7**, pp. 713–716