

Analysis of nonlinear dynamic stability of single-walled carbon nanotubes in thermal environments

Yiming Fu, Jun Zhong

College of Mechanical and Vehicle Engineering, Hunan University, Changsha 410082, People's Republic of China
E-mail: zhongjun320@126.com

Published in Micro & Nano Letters; Received on 9th October 2013; Accepted on 11th February 2014

Based on the non-local Euler beam theory, the nonlinear dynamic stability of single-walled carbon nanotubes (SWCNTs) embedded in an elastic medium including the thermal effects is presented. The nonlinear dynamic equations and the boundary conditions of the SWCNTs are obtained by using the Hamilton variation principle. By adopting the Galerkin procedure, the governing nonlinear partial differential equation is converted into a nonlinear ordinary differential equation, and then the incremental harmonic balance method is applied to obtain the principal unstable regions of the SWCNTs. In the numerical examples, the effects of the thermal loads, the non-local parameters and the elastic medium on the nonlinear dynamic stability, respectively, are discussed.

1. Introduction: Carbon nanotubes (CNTs) have attracted much attention since they were discovered by Iijima [1]. Owing to their excellent properties in electricity, optics and mechanics, CNTs have potential usage in the areas of nanocomposites, nanoelectronic devices and nanosensors, respectively. To improve the service quality and the life of the CNTs, the mechanical properties need to be studied in detail.

Much research has been conducted on the mechanical behaviours of CNTs by experiments [2], molecular dynamic (MD) simulation [3–4] and theoretical analysis, based on continuum elastic modelling. Ansari *et al.* [5] studied the nonlinear vibration of single-walled CNTs (SWCNTs) with the arbitrary boundary conditions including the thermal effects based on the Euler beam model. Fu *et al.* [6] investigated the nonlinear vibration of the embedded CNTs by using the incremental harmonic balance method (IHB). On the basis of a continuum cylindrical shell model, He *et al.* [7] analysed the buckling of multi-walled CNTs (MWCNTs). Shen [8] presents an elastic double-shell model to study the postbuckling of double-walled CNTs (DWCNTs) under a hydrostatic pressure by using a singular perturbation technique. However, both the experiments and the atomic simulations have shown that the nanostructures present significant size effects. Although the classic elastic theory is efficient, it cannot reveal or predict the size effects. To characterise the property, some higher elasticity theories [9–11] are introduced in which the non-local elasticity theory is widely used. Reddy [12] establishes various available beam theories, including the Euler, the Timoshenko, the Reddy and the Levinson beam models, based on the non-local elasticity theory, and the analytical solutions for bending and buckling were presented. Wang *et al.* [13] discussed the propagation of DWCNTs based on the non-local Euler and Timoshenko beam models. By applying the differential quadrature method, Ke *et al.* [14] and Yang *et al.* [15] studied the nonlinear vibrations of SWCNTs and DWCNTs modelled by the non-local Timoshenko beam theory. Hu *et al.* [16] investigated the transverse and the torsional waves in SWCNTs and DWCNTs on the basis of the non-local elastic cylindrical shell models and provided verifications of these models by using MD simulation. Shen and Zhang [17] dealt with the buckling and the postbuckling of DWCNTs subjected to torsion in thermal environments.

This Letter reports an investigation of the nonlinear dynamic stability of SWCNTs resting on an elastic medium in thermal environments modelled by the non-local Euler beam theory. The Hamilton variation principle is adopted to obtain the nonlinear governing equations and the boundary conditions. The principal unstable

regions are obtained by the Galerkin method and the IHB, respectively. The numerical examples reveal the significance of the thermal loads, the non-local parameters and the elastic medium on the nonlinear dynamic stability of SWCNTs.

2. Non-local elasticity theory: Eringen's non-local elasticity theory [9], different from the classical elasticity theory, states that the stress at the arbitrary point x depends on the strains of all the points in a body. As a result, the non-local stress tensor σ at the point x can be expressed as

$$\sigma = \int_V \alpha(|x' - x|, \mu) T(x') dx' \quad (1)$$

where $\alpha(|x' - x|, \mu)$ is the kernel function which represents the non-local modulus. $|x' - x|$ is the distance in the Euclidean norm and $\mu = e_0 a / l$ is a small scale factor where e_0 is a material constant, and a, l are the internal and the external characteristic lengths, respectively. $T(x)$ is the classical macroscopic stress tensor at the point x which can be expressed in the following form according to the generalised Hooke's law

$$T(x) = C(x) : \varepsilon(x) \quad (2)$$

where $C(x)$ and $\varepsilon(x)$ are the fourth-order elasticity tensor and the macroscopic stress tensor, respectively. $\cdot :$ denotes the double dot product.

Since the integral-form non-local constitutive relation in (1) is not convenient to use, it is usually represented by the following equivalent differential form, as

$$\sigma - (e_0 a)^2 \nabla^2 \sigma = S \quad (3)$$

For the Euler beam theory, (3) can be simplified as

$$\sigma_x - (e_0 a)^2 \sigma_{xx} = E \varepsilon_x \quad (4)$$

where σ_x and ε_x are the axial stress and the strain, respectively. E is the Young's modulus of the material. The subscript comma represents the derivative with respect to the coordinate variable.

3. Basic equations: Consider a hinged-hinged SWCNT with the length l embedded in the elastic medium as shown in Fig. 1 and

assume that one of the ends is fixed and the other one is subjected to a certain axial harmonic motion

$$u_l = \bar{u}_l \cos 2\omega t \quad (5)$$

The displacement fields (u_1, u_3) of the arbitrary point are assumed to have the following forms on the basis of the Euler beam theory

$$\begin{aligned} u_1 &= u - zw_{xx} \\ u_3 &= w \end{aligned} \quad (6)$$

where u, w are the displacements on the corresponding point of the midplane along the x and z axes.

The Von Karman type geometrically nonlinear strain-displacement relation of the SWNT is

$$\varepsilon_x = u_1 + \frac{1}{2}w_x^2 = u_x + \frac{1}{2}w_x^2 - zw_{xx} \quad (7)$$

The total energy of the SWNT can be expressed as

$$\begin{aligned} \Pi &= \int_t \int_V \frac{1}{2} \sigma_x \varepsilon_x dV dt \\ &\quad - \int_t \int_V \frac{1}{2} \rho \dot{w}^2 dA dt + \int_t \int_L \frac{1}{2} kw^2 dx dt \end{aligned} \quad (8)$$

where the subscripts t, V, L represent the time, the volume and the length, respectively. ρ is the mass density of the SWNT. The dot upon the characters represents the derivative with respect to the time variable t . The three terms on the right-hand side of (8) are the strain and the kinetic energy of the SWNT and the elastic potential energy of the elastic medium modelled as a Winkler-like model, respectively. By applying the Hamilton variational principle, that is, $\delta \Pi = 0$, the equations of motion can be obtained as

$$\begin{aligned} N_{x,x} &= 0 \\ M_{x,xx} + (N_x w_x)_x - kw &= \rho A \ddot{w} \end{aligned} \quad (9)$$

in which A is the area of the cross-section, and the internal force and the internal couple are defined as

$$(N_x, M_x) = \int_A \sigma_x(1, z) dA \quad (10)$$

respectively. When the thermal effects are taken into consideration, the non-local constitutive relations in (4) become

$$\begin{aligned} N_x - (e_0 a)^2 N_{x,xx} &= EA \left(u_x + \frac{1}{2} w_x^2 \right) - EA \alpha \Delta T \\ M_x - (e_0 a)^2 M_{x,xx} &= -EI w_{xx} \end{aligned} \quad (11)$$

in which α is the thermal expansion coefficient and ΔT is the temperature change, respectively. I is the inertia moment of the cross-section.

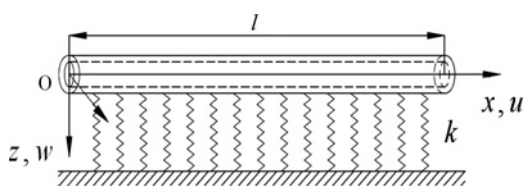


Figure 1 Sketch of an SWCNT

It could be known from the first equation in (9) that the axial force N_x is a constant along the x axis. Thus, by combining with the first equation in (11), one can obtain

$$N_x = EA \left(u_x + \frac{1}{2} w_x^2 \right) - EA \alpha \Delta T \quad (12)$$

Integrating (12) along the x axis leads to

$$N_x = p \cos 2\omega t - EA \alpha \Delta T + \frac{EA}{2l} \int_L w_x^2 dx \quad (13)$$

where $p = (EA \bar{u}_l / l)$.

By substituting (13) into the second equation of (9), and combining the second equation of (11), the internal couple can be written as

$$\begin{aligned} M_x &= (e_0 a)^2 \left[- \left(p \cos 2\omega t - EA \alpha \Delta T + \frac{EA}{2l} \int_L w_x^2 dx \right) w_{xx} \right. \\ &\quad \left. + kw + \rho A \ddot{w} \right] - EI w_{xx} \end{aligned} \quad (14)$$

By inserting (14) into the second equation of (9), the governing equations can be given as

$$\begin{aligned} &\rho A \ddot{w} + EI w_{xxxx} + kw \\ &\quad - \left(p \cos 2\omega t - EA \alpha \Delta T + \frac{EA}{2l} \int_L w_x^2 dx \right) w_{xx} \\ &\quad - (e_0 a)^2 \left[\rho A \ddot{w}_{xx} + kw_{xx} - \left(p \cos 2\omega t - EA \alpha \Delta T + \frac{EA}{2l} \int_L w_x^2 dx \right) w_{xxxx} \right] = 0 \end{aligned} \quad (15)$$

The boundary conditions of the simply supported SWCNTs with the movable axial are

$$x = 0, l: \quad w = M_x = 0 \quad (16)$$

The displacement w which satisfies the above boundary conditions is taken as

$$w = w_0(t) \sin \frac{\pi x}{l} \quad (n = 1, 2) \quad (17)$$

By substituting (17) into (15), it becomes

$$\begin{aligned} &\rho A (1 + \mu^2) \ddot{w} + \left[k(1 + \mu^2) + \frac{EI \pi^4}{l^4} - \frac{EA \alpha \Delta T \pi^2}{l^2} (1 + \mu^2) \right. \\ &\quad \left. + \frac{p \pi^2 \cos 2\theta t}{l^2} (1 + \mu^2) \right] w \\ &\quad + \frac{EA \pi^4}{4l^4} (1 + \mu^2) w^3 = 0 \end{aligned} \quad (18)$$

Introduce the following dimensionless parameters

$$\begin{aligned} r &= \sqrt{\frac{I}{A}}, \quad W = \frac{w}{r}, \quad \tau = \omega t; \\ \Omega &= \frac{\omega}{\omega_0}, \quad \omega_L = \frac{1}{\omega_0} \sqrt{\frac{EI \pi^4}{\rho A l^4}} \\ \omega_k &= \frac{1}{\omega_0} \sqrt{\frac{k}{\rho A}}, \quad p_{cr} = \frac{EI \pi^2}{l^2}, \\ N_n^T &= \frac{E \alpha \Delta T \pi^2}{\rho l^2 \omega_0^2}, \quad \lambda = \frac{p}{2 p_{cr}} \end{aligned} \quad (19)$$

where ω_0 is the natural frequency of the SWCNTs and $\omega_0^2 = EI \pi^4 / [(1 + \mu^2) \rho A l^4] + k / \rho A - E \alpha \Delta T \pi^2 / \rho l^2$. Then, by substituting (19) into (18), the nonlinear Mathieu type equation of the SWCNTs under a periodic excitation can be obtained as

$$\begin{aligned} \Omega^2 \ddot{W} + \left[\frac{\omega_L^2}{1 + \mu^2} + \omega_k^2 - N_1^T + 2\lambda \omega_L \cos 2\tau \right] W \\ + \frac{1}{4} \omega_L^2 W^3 = 0 \end{aligned} \quad (20)$$

4. Solution methodology: Generally, it is hard to obtain the analytical solution of (20). The IHB method [18] is employed to seek the numerical solution of (20). Assume (λ_0, Ω_0) to be a known instability boundary point corresponding to a periodic solution $W_0(\tau)$, then a neighbouring boundary point can be written as

$$\begin{aligned} \lambda &= \lambda_0 + \Delta\lambda \\ \Omega &= \Omega_0 + \Delta\Omega \\ W(\tau) &= W_0(\tau) + \Delta W(\tau) \end{aligned} \quad (21)$$

By inserting (21) into (20) and neglecting all the high-order microcontents, a linearised increment equation can be expressed as

$$\begin{aligned} \Omega_0^2 \Delta \ddot{W} + \left[\frac{\omega_L^2}{1 + \mu^2} + \omega_k^2 - N_1^T + 2\lambda \omega_L \cos 2\tau \right] \Delta W \\ + \frac{3}{4} \omega_L^2 W_0^2 \Delta W \\ = R - 2\Delta\lambda \cos 2\tau W_0 - 2\Delta\Omega \Omega_0 \ddot{W}_0 \end{aligned} \quad (22)$$

where

$$R = - \left[\Omega_0^2 \ddot{W}_0 + \left(\frac{\omega_L^2}{1 + \mu^2} + \omega_k^2 - N_1^T + 2\lambda_0 \omega_L \cos 2\tau \right) W_0 + \frac{1}{4} \omega_L^2 W_0^3 \right] \quad (23)$$

and it is a correction term, and it will become zero if $(\lambda_0, \Omega_0, W_0)$ are the exact solutions of (20). $W_0(\tau)$, $\Delta W(\tau)$ are the periodic functions.

According to the dynamic stability theory of the elastic systems [19], the unstable regions are surrounded by the solutions with the same period. Thus, the functions $W_0(\tau)$ and $\Delta W(\tau)$ with the period 2π can be expressed as the following forms

$$\begin{aligned} W_0(\tau) &= \sum_{k=1,3,5,\dots} (a_k \sin k\tau + b_k \cos k\tau) = CA \\ \Delta W(\tau) &= \sum_{k=1,3,5,\dots} (\Delta a_k \sin k\tau + \Delta b_k \cos k\tau) = C\Delta A \end{aligned} \quad (24)$$

By substituting (24) into (22), and applying the Galerkin method, a

set of linear equations containing the increments $\Delta\lambda$, $\Delta\Omega$ and ΔA can be obtained in the following form

$$M \Delta A = R + \Delta\Omega S + \Delta\lambda T \quad (25)$$

where

$$\begin{aligned} M &= \Omega_0^2 \cdot H_1 \\ &+ \left(\frac{\omega_L^2}{1 + \mu^2} + \omega_k^2 - N_1^T + 2\lambda_0 \omega_L \cos 2\tau \right) H_2 + \frac{3}{4} \omega_L^2 H_3 \\ R &= - \left[\Omega_0^2 H_1 + \left(\frac{\omega_L^2}{1 + \mu^2} + \omega_k^2 - N_1^T + 2\lambda_0 \omega_L \cos 2\tau \right) \right. \\ &\quad \left. H_2 + \frac{1}{4} \omega_L^2 H_3 \right] A \\ H_1 &= \int_0^{2\pi} C^T \frac{d^2 C}{d\tau^2} d\tau, \quad H_2 = \int_0^{2\pi} C^T C d\tau, \quad H_3 = \int_0^{2\pi} C^T C A C A d\tau \\ S &= -2\Omega_0 H_1 A, \quad T = -2\omega_L^2 \cos 2\tau H_2 A \end{aligned} \quad (26)$$

In the solution procedure of (25), either $\Delta\lambda$ or $\Delta\Omega$ can be chosen as the active increment, and if any one of the coefficients a_n and b_n are given, the corresponding increments are taken as zero, for example, $a_1 = 1$ and $\Delta a_1 = 0$. Thus, the rest of the increments can be obtained from (25). By proceeding with the increments iteration procedure, the dynamic unstable regions of the SWCNTs can be determined.

5. Numerical results and discussion: To ensure the validity and the accuracy of the method presented in this Letter, do not take the thermal and the non-local effects into consideration, and by setting $\bar{u}_l = 0$, the present problem degenerates into the nonlinear free vibration problem of the simply supported SWCNTs with the axial immovable. In this case, λ , a_n and the corresponding increments $\Delta\lambda$, Δa_n in (25) should be removed in the IHB method [6, 20]. If the values of b_n and Ω_0 are given, the corresponding increments can be solved. For example, by choosing $\Delta\Omega$ as the active increment, Δb_n can be obtained. Consequently, the next vibration statuses $b_n + \Delta b_n$ and $\Omega_0 + \Delta\Omega$ can be calculated. Then, by taking these solutions as a new vibration status and by proceeding with the former solving process, the nonlinear amplitude frequency response curves will be determined. Fig. 2 shows the comparison of the nonlinear amplitude frequency responses between the present method and Ansari *et al.* [5]. The close agreements demonstrate that the present method is accurate and effective.

In the numerical results, the material and the geometry parameters are taken as $E = 1.1$ Tpa, $\rho = 1.3 \times 10^3$ kg/m³, $l = 45$ nm, the inner diameter as $d_0 = 2.32$ nm and the outer diameter as $d_1 = 3$ nm, respectively. As indicated by Jiang *et al.* [21], the thermal expansion coefficients of the CNTs are negative at a low or room temperature and become positive at a high temperature. For the low or room temperature $\alpha = -1.6 \times 10^{-6}$ K⁻¹ and for the high temperature $\alpha = 1.1 \times 10^{-6}$ K⁻¹ [22, 23].

Fig. 3 shows the effect of the nonlinear factor on the principal unstable regions of the SWNTs in the case of the high temperature ($\Delta T = 100$ K, $\mu = 0.15$ and $k = 10^7$ N/m², respectively). It can be found from Fig. 3 that the principal unstable regions of the SWCNTs obtained by the geometric nonlinear theory are close to those obtained by the linear theory when the nonlinear parametric vibration amplitude is small ($a_1 = b_1 = 0.5$). When the amplitude is larger, the effect of the geometric nonlinearity on the principal unstable regions becomes significant. The boundaries of the principal unstable regions move upwards when the nonlinear parametric

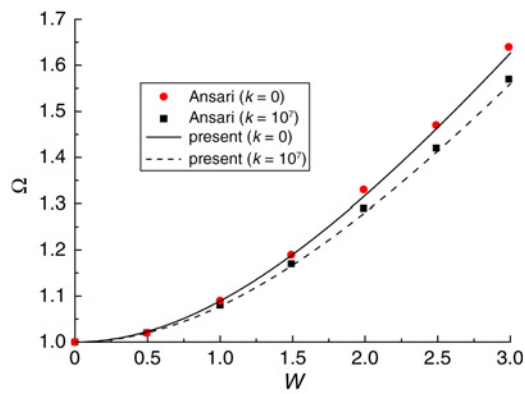


Figure 2 Comparison of the nonlinear amplitude frequency response of the SWCNTs

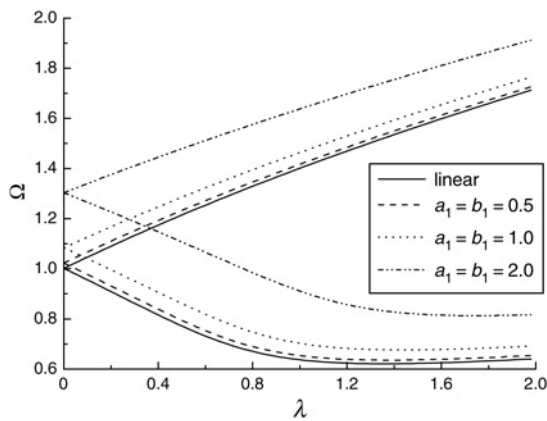


Figure 3 Effect of the nonlinear factor on the principal unstable regions of the SWCNTs

vibration amplitude increases, and the frequency of the nonlinear parametric resonance increases, consequently.

Fig. 4 presents the effect of the non-local parameter μ on the principal unstable regions of the SWCNTs in the high temperature environments ($\Delta T = 100$ K, $a_1 = b_1 = 1$ and $k = 10^7$ N/m², respectively). When the non-local parameter $\mu = 0$, it represents the classical Euler beam model. The results show that when the non-local parameter μ increases, the principal unstable regions become larger, and the boundaries of the principal unstable regions move up.

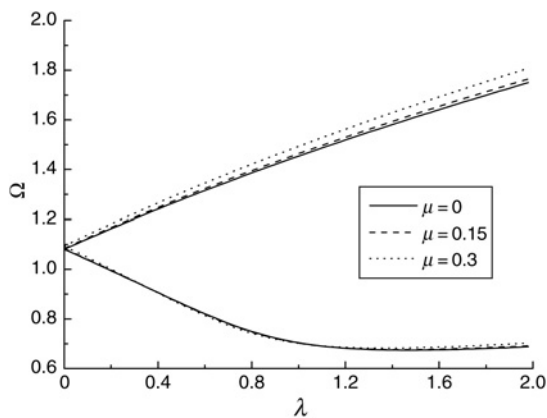


Figure 4 Effect of the non-local parameter μ on the principal unstable regions of the SWCNTs

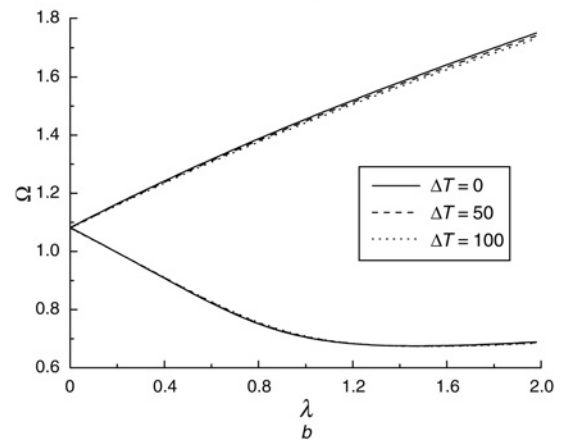
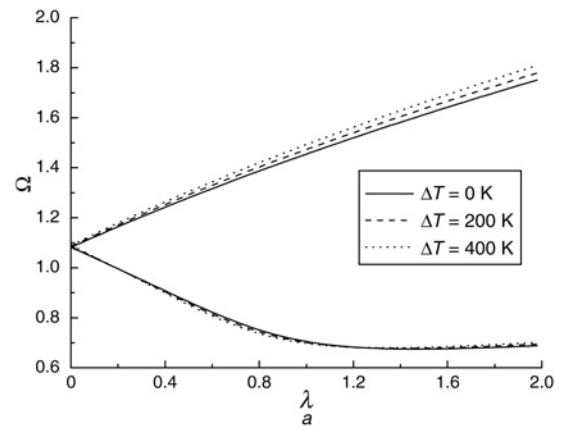


Figure 5 Thermal effects on the principal unstable regions of the SWCNTs
a In the case of a high temperature
b In the case of a low or room temperature

The thermal effects on the principal unstable regions of the SWCNTs are shown in Fig. 5, where Fig. 5a stands for the case of the high temperature and Fig. 5b stands for that of the low or room temperature. As can be observed, when the SWCNTs are under the high temperature environments, the principal unstable regions expand and the boundaries are shifted upward with the increase of the temperature. The principal unstable regions shrink and the boundaries move down when the temperature goes up under the low or room temperature environments. The main reasons for the differences between the two environments are that the thermal expand coefficients are diverse in different environments.

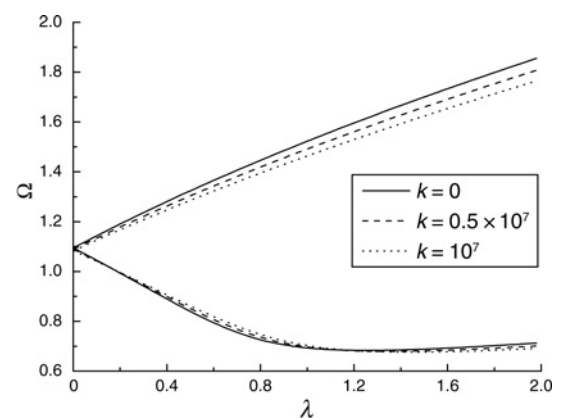


Figure 6 Effect of the elastic medium on the principal unstable regions of the SWCNTs

Fig. 6 gives the effect of the elastic medium on the principal unstable regions of the SWCNTs in the case of the high temperature ($\mu = 0.15$, $a_1 = b_1 = 1$ and $k = 10^7 \text{ N/m}^2$, respectively). It is clear that the area of the principal unstable regions is smaller and the boundaries move down as the stiffness of the elastic medium increases. It illustrates that the greater the elastic medium's stiffness, the more superior the dynamic stability.

6. Conclusions: In this Letter, a Euler beam model based on the non-local elastic theory is established to study the dynamic stability behaviours of SWCNTs embedded in an elastic medium with the thermal effects. The nonlinear governing equations are obtained by employing the Hamilton variation principle. The whole problem is solved by using the Galerkin method and the IHBM.

The main conclusions are given as follows. The results of the principal unstable regions of the SWCNTs obtained by the geometric nonlinear theory are approximate to that obtained by the linear theory when the amplitude is small, and the principal unstable regions move up with the increase of the amplitude of the nonlinear parametric vibration. When the non-local parameter μ increases, it will expand the principal unstable regions and raise the boundaries. The effect of the elastic medium, however, is opposite to that of the non-local parameter μ . The principal unstable regions become larger and the boundaries move up as the temperature increases in the high temperature environments, which is opposite to that in the low or room temperature.

7. Acknowledgment: This Letter is supported by the National Natural Science Foundation of China under grant no. 11272117.

8 References

- [1] Iijima S.: 'Helical microtubules of graphitic carbon', *Nature*, 1991, **354**, pp. 56–58
- [2] Yu M.-F., Lourie O., Dyer M.J., Moloni K., Kelly T.F., Ruoff R.S.: 'Strength and breaking mechanism of multiwalled carbon nanotubes under tensile load', *Science*, 2000, **287**, pp. 637–640
- [3] Liew K., He X., Wong C.: 'On the study of elastic and plastic properties of multi-walled carbon nanotubes under axial tension using molecular dynamics simulation', *Acta Mater.*, 2004, **52**, pp. 2521–2527
- [4] Frankland S., Harik V., Odegard G., Brenner D., Gates T.: 'The stress-strain behavior of polymer-nanotube composites from molecular dynamics simulation', *Comput. Sci. Technol.*, 2003, **63**, pp. 1655–1661
- [5] Ansari R., Hemmatnezhad M., Rezapour J.: 'The thermal effect on nonlinear oscillations of carbon nanotubes with arbitrary boundary conditions', *Curr. Appl. Phys.*, 2011, **11**, pp. 692–697
- [6] Fu Y., Hong J., Wang X.: 'Analysis of nonlinear vibration for embedded carbon nanotubes', *J. Sound Vib.*, 2006, **296**, pp. 746–756
- [7] He X., Kitipornchai S., Liew K.: 'Buckling analysis of multi-walled carbon nanotubes: a continuum model accounting for van der Waals interaction', *J. Mech. Phys. Solids*, 2005, **53**, pp. 303–326
- [8] Shen H.-S.: 'Postbuckling prediction of double-walled carbon nanotubes under hydrostatic pressure', *Int. J. Solids Struct.*, 2004, **41**, pp. 2643–2657
- [9] Eringen A.C.: 'On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves', *J. Appl. Phys.*, 1983, **54**, pp. 4703–4710
- [10] Yang F., Chong A., Lam D., Tong P.: 'Couple stress based strain gradient theory for elasticity', *Int. J. Solids Struct.*, 2002, **39**, pp. 2731–2743
- [11] Lam D., Yang F., Chong A., Wang J., Tong P.: 'Experiments and theory in strain gradient elasticity', *J. Mech. Phys. Solids*, 2003, **51**, pp. 1477–1508
- [12] Reddy J.: 'Nonlocal theories for bending, buckling and vibration of beams', *Int. J. Eng. Sci.*, 2007, **45**, pp. 288–307
- [13] Wang Q., Zhou G., Lin K.: 'Scale effect on wave propagation of double-walled carbon nanotubes', *Int. J. Solids Struct.*, 2006, **43**, pp. 6071–6084
- [14] Ke L., Xiang Y., Yang J., Kitipornchai S.: 'Nonlinear free vibration of embedded double-walled carbon nanotubes based on nonlocal Timoshenko beam theory', *Comp. Mater. Sci.*, 2009, **47**, pp. 409–417
- [15] Yang J., Ke L., Kitipornchai S.: 'Nonlinear free vibration of single-walled carbon nanotubes using nonlocal Timoshenko beam theory', *Phys. E*, 2010, **42**, pp. 1727–1735
- [16] Hu Y.-G., Liew K., Wang Q., He X., Yakobson B.: 'Nonlocal shell model for elastic wave propagation in single- and double-walled carbon nanotubes', *J. Mech. Phys. Solids*, 2008, **56**, pp. 3475–3485
- [17] Shen H.-S., Zhang C.-L.: 'Torsional buckling and postbuckling of double-walled carbon nanotubes by nonlocal shear deformable shell model', *Comput. Struct.*, 2010, **92**, pp. 1073–1084
- [18] Lau S., Cheung Y., Wu S.: 'Variable parameter incrementation method for dynamic instability of linear and nonlinear elastic systems', *J. Appl. Mech.*, 1982, **49**, pp. 849–853
- [19] Bolotin V.V.: 'The dynamic stability of elastic systems' (Holden-Day, San Francisco, 1964)
- [20] Lau S., Cheung Y.: 'Amplitude incremental variational principle for nonlinear vibration of elastic systems', *J. Appl. Mech.*, 1981, **48**, pp. 959–964
- [21] Jiang H., Liu B., Huang Y., Hwang K.C.: 'Thermal expansion of single wall carbon nanotubes', *J. Eng. Mater. Technol.*, 2004, **126**, pp. 265–270
- [22] Benzair A., Tounsi A., Besseghier A., Heireche H., Moulay N., Boumia L.: 'The thermal effect on vibration of single-walled carbon nanotubes using nonlocal Timoshenko beam theory', *J. Phys. D, Appl. Phys.*, 2008, **41**, pp. 225–404
- [23] Yan Y., Wang W., Zhang L.: 'Nonlocal effect on axially compressed buckling of triple-walled carbon nanotubes under temperature field', *Appl. Math. Model.*, 2010, **34**, pp. 3422–3429