Vibration characteristics of double-piezoelectric-nanoplate-systems

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In this reported work, the vibration response of a double-piezoelectric-nanoplate-system (DPNPS) under an external electric voltage is investigated. The two piezoelectric nanoplates are coupled by a polymer matrix. Small scale effects are taken into consideration using the non-local elasticity theory. Hamilton's principle is employed to derive the differential equations of motion. Explicit closed-form expressions are derived for the natural frequencies and critical electric voltages of the DPNPS. The numerical results are presented for both in-phase and out-of-phase vibrational modes. It is shown that the natural frequencies of the DPNPS are quite sensitive to the vibrational mode and non-local parameter. The present work is likely to prove very useful in the designing of micro-/nanodevices using smart nanocomposite.

1. Introduction: After the invention of ZnO piezoelectric nanowires [1], many research works have been carried out focusing on the mechanical, electrical and thermal properties of piezoelectric nanostructures. It has been reported that piezoelectric nanomaterials have potential applications as building blocks for microelectromechanical systems and nanoelectromechanical systems including nanogenerators [2], piezoelectric gated diodes [3], nanoresonators and nano-oscillators [4] because of their superior properties. Furthermore, recently, Lian *et al.* [5] have shown that the $(ZnO)_x(MgO)_{1-x}$ nanoplates have high photocatalytic performance and thus would be promising candidates for polluted water treatment. Owing to these potential applications, understanding the vibration response of piezoelectric nanostructures under thermo-electromechanical loading is an important problem.

In recent years, the non-local elasticity theory [6] has been widely used in the theoretical investigations of nanostructural elements such as carbon nanotubes [7, 8], nanorods [9, 10] and graphene sheets [11–14]. The non-local elasticity theory is based on the assumption that the stress tensors at an arbitrary point in the domain of nanomaterial depends not only on the strain tensor at that point but also on strain tensors at all other points in the domain. Both atomistic simulation results and experimental observations on phonon dispersion have shown the accuracy of this observation [6, 14]. A review of the literature shows that compared with the carbon nanotubes and graphene sheets, few research works have been reported on the continuum based analysis of piezoelectric nanostructures. Based on the Euler-Bernoulli beam model, Wang and Feng [15] studied the surface effect on the vibration and buckling of piezoelectric nanowires. Furthermore, the influences of surface elastic modulus, residual surface stress and surface piezoelectricity on the electromechanical response of a curved piezoelectric nanobeam were investigated [16]. The non-local effects on the vibration characteristics of piezoelectric nanobeams were also studied using the non-local Timoshenko beam theory [17]. Arani *et al.* [18] presented the buckling analysis of double-walled boron nitride nanotubes surrounded by a bundle of carbon nanotubes. Yan and Jiang [19] studied the electroelastic response of a thin piezoelectric plate with nanoscale thickness considering surface effects. In another work, Liu *et al.* [20] investigated the thermo-electro-mechanical free vibration of a single-piezoelectric nanoplate (PNP) without an elastic medium based on the non-local theory.

In a composite nanostructure, PNPs may be coupled to each other by bonding resins to form a complex-piezoelectric-system. A simple example of these systems is made of two PNPs which are bonded by a polymer matrix. Double-PNP-systems (DPNPS) are important in view of practical design applications. Recently, Murmu and Adhikari [21] investigated the vibration of bonded double-nanoplate-systems without piezoelectric properties using the non-local elasticity theory.

In the present Letter, an attempt is made to study the transverse vibration of a DPNPS under an external electric voltage. Using non-local elasticity theory and Hamilton's principle, the differential equations of motion of the DPNPS are derived. The influence of the coupling polymer matrix is taken into consideration employing the Winkler foundation model. Exact solutions for the natural frequencies and critical electric voltages of a simply supported DPNPS are obtained. The results are presented for both in-phase and out-of-phase vibration modes. The effect of small size and the elastic foundation parameter on the natural frequencies of the system through considering various parameters such as the non-local parameter, mode number, thickness and aspect ratio are examined and discussed.

2. Non-local plate model for DPNPS: A rectangular DPNPS coupled by a polymer matrix is shown in Fig. 1. The Cartesian coordinate frame with axes x, y and z used for the DPNPS is also shown in the Figure. It is assumed that the upper and lower piezoelectric nanoplates are subjected to the same external

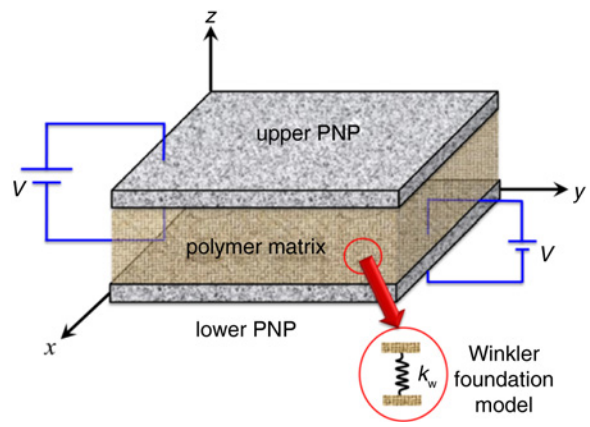

Figure 1 *Continuum model for DPNPS*

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electric voltage. The length and width of each nanofilm are denoted by ℓ_x and ℓ_y, respectively.

The mechanical behaviour of the nanostructures depends considerably on their size. The non-local elasticity theory of Eringen [6] will be used to account for the effect of small scale. This theory is based on a simple physical concept that the components of a stress tensor at a given point are a function not only of the strain tensor at that point but also are a function of strain tensors at all other points in the domain. According to the non-local elasticity theory, the basic equations for Hookean piezoelectric solids neglecting the body force are expressed by the following relationships

$$\sigma_{ij} = \iiint_V \psi(|x' - x|, \chi)\left[c_{ijkl}\varepsilon_{kl}(x') - e_{kij}E_k(x')\right]\mathrm{d}x' \tag{1}$$

$$D_i = \iiint_V \psi(|x' - x|, \chi)\left[e_{ikl}\varepsilon_{kl}(x') + \kappa_{kij}E_k(x')\right]\mathrm{d}x' \tag{2}$$

$$\sigma_{ij,j} = \rho\ddot{u}_i, \quad D_{i,j} = 0 \tag{3a, b}$$

$$\varepsilon_{ij} = \frac{1}{2}(U_{i,j} + U_{j,i}), \quad E_i = -\Phi_{,i} \tag{4a, b}$$

where σ_{ij}, ε_{ij}, D_i, E_i, U_i and Φ are the components of the non-local stress tensor, strain tensor, electric displacement vector, electric field vector, displacement vector and electric potential, respectively. Also, the terms c_{ijkl}, e_{kij}, κ_{kij} and ρ are the components of a fourth-order elasticity tensor, piezoelectric constants, dielectric constants and mass density, respectively. The function ψ is the non-local modulus, which contains the small scale effects. $|x - x'|$ is the distance between points x and x' and $\chi = e_0 l_i / l_e$ is defined where l_i is an internal characteristic length and l_e is an external characteristic length. Choice of the value of parameter e_0 is vital for the validity of non-local models. This parameter can be determined by matching the dispersion curves based on the atomic models. It is difficult to apply (1) and (2) for solving the non-local elasticity problems. Therefore the following differential equations are often used [6]

$$\begin{aligned} \sigma_{ij} - (e_0 l_i)^2\nabla^2\sigma_{ij} &= c_{ijkl}\varepsilon_{kl} - e_{kij}E_k \\ D_i - (e_0 l_i)^2\nabla^2 D_i &= e_{ikl}\varepsilon_{kl} + \kappa_{kij}E_k \end{aligned} \tag{5a, b}$$

where ∇^2 is the Laplacian operator. $e_0 l_i$ is the non-local parameter incorporating the small scale effects into the constitutive equations. Let u, v and w be the components of the displacement vector of material point $(x, y, 0)$ in the middle surface of the plate (mid-plane) along the x, y and z directions, respectively. Applying the strain–displacement relation (4a) and using the Kirchhoff plate theory, one obtains the following equations

$$\begin{aligned} &\varepsilon_{xx} = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2}, \quad \varepsilon_{yy} = \frac{\partial v}{\partial y} - z\frac{\partial^2 w}{\partial y^2}, \\ &\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - z\frac{\partial^2 w}{\partial x \partial y} \end{aligned} \tag{6}$$

Now, we need to know the distribution of electric potential through the thickness of the PNP. Recently, Yan and Jiang [16] investigated the influence of surface energy on the electromechanical response of a curved piezoelectric nanobeam. They obtained results for the linear distribution of electric potential along the thickness direction of nanobeams. This assumption does not satisfy the Maxwell equation. To satisfy the Maxwell equation, Ke and Wang [17] studied the thermoelectric-mechanical vibration of piezoelectric nanobeams using the non-local theory based on the assumption that the electric potential distribution is a combination of a cosine and linear functions. Thus, following Ke and Wang [17], the electric potential can be expressed as follows

$$\Phi(x, y, z, t) = -\cos\left(\frac{\pi z}{h}\right)\phi(x, y, t) + \frac{2zV_0}{h}e^{i\omega t} \tag{7}$$

where $\phi(x, y, t)$ is the electric potential of point $(x, y, 0)$ in the mid-plane at time t; V_0 is the external electric voltage; and ω represents the natural frequency of the system. Using (4b) and (7), the components of the electric field can be written as

$$\begin{aligned} &E_x = \cos\left(\frac{\pi z}{h}\right)\frac{\partial \phi}{\partial x}, \quad E_y = \cos\left(\frac{\pi z}{h}\right)\frac{\partial \phi}{\partial y} \\ &E_z = -\frac{\pi}{h}\sin\left(\frac{\pi z}{h}\right)\phi - \frac{2V_0}{h}e^{i\omega t} \end{aligned} \tag{8}$$

Using (5), the non-local constitutive relations of thin PNP in the Cartesian coordinates can be approximated as

$$\begin{aligned} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} - (e_0 l_i)^2\nabla^2 \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} &= \begin{bmatrix} \tilde{c}_{11} & \tilde{c}_{12} & 0 \\ \tilde{c}_{12} & \tilde{c}_{11} & 0 \\ 0 & 0 & \tilde{c}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ 2\varepsilon_{xy} \end{Bmatrix} \\ &- \begin{bmatrix} 0 & 0 & \tilde{e}_{31} \\ 0 & 0 & \tilde{e}_{31} \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} \end{aligned} \tag{9}$$

$$\begin{aligned} \begin{Bmatrix} D_x \\ D_y \\ D_z \end{Bmatrix} - (e_0 l_i)^2\nabla^2 \begin{Bmatrix} D_x \\ D_y \\ D_z \end{Bmatrix} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \tilde{e}_{31} & \tilde{e}_{31} & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ 2\varepsilon_{xy} \end{Bmatrix} \\ &+ \begin{bmatrix} \tilde{\kappa}_{11} & 0 & 0 \\ 0 & \tilde{\kappa}_{11} & 0 \\ 0 & 0 & \tilde{\kappa}_{33} \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} \end{aligned} \tag{10}$$

where

$$\begin{aligned} &\tilde{c}_{11} = c_{11} - \frac{c_{13}^2}{c_{33}}, \quad \tilde{c}_{12} = c_{12} - \frac{c_{13}^2}{c_{33}}, \quad \tilde{e}_{31} = e_{31} - \frac{c_{13}e_{33}}{c_{33}} \\ &\tilde{c}_{66} = c_{66}, \quad \tilde{\kappa}_{11} = \kappa_{11}, \quad \tilde{\kappa}_{33} = \kappa_{33} - \frac{e_{33}^2}{c_{33}} \end{aligned} \tag{11}$$

Using Hamilton's principle, the following differential equations of motion can be obtained [20]

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = \rho h\frac{\partial^2 u}{\partial t^2} \tag{12}$$

$$\frac{\partial N_{yy}}{\partial y} + \frac{\partial N_{xy}}{\partial x} = \rho h\frac{\partial^2 v}{\partial t^2} \tag{13}$$

$$\frac{\partial^2 M_{xx}}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} + q + N_{xx}^{el}\frac{\partial^2 w}{\partial x^2} + N_{yy}^{el}\frac{\partial^2 w}{\partial y^2} = \rho h\frac{\partial^2 w}{\partial t^2} \tag{14}$$

$$\int_{-h/2}^{h/2}\left[\cos\left(\frac{\pi z}{h}\right)\frac{\partial D_x}{\partial x} + \cos\left(\frac{\pi z}{h}\right)\frac{\partial D_y}{\partial y} + \frac{\pi}{h}\sin\left(\frac{\pi z}{h}\right)D_z\right]dz = 0 \tag{15}$$

where $N_{xx}^{el} = N_{yy}^{el}(= 2\tilde{e}_{31}V_0)$ are the in-plane loads caused by the applied electric voltage and q is the transverse distributed load. N_{ij} and M_{ij} are, respectively, the stress resultants and stress couples and are defined as follows

$$\langle N_{xx}, N_{yy}, N_{xy}\rangle = \int_{-h/2}^{h/2} \langle \sigma_{xx}, \sigma_{yy}, \sigma_{xy}\rangle \mathrm{d}z$$
$$\langle M_{xx}, M_{yy}, M_{xy}\rangle = \int_{-h/2}^{h/2} \langle \sigma_{xx}, \sigma_{yy}, \sigma_{xy}\rangle z\mathrm{d}z \tag{16}$$

Multiplying (9) by $z\mathrm{d}z$, integrating from $-h/2$ to $h/2$ and using (16), one can obtain

$$M_{xx} - (e_0 l_i)^2 \nabla^2 M_{xx} = -\tilde{D}_{11}\frac{\partial^2 w}{\partial x^2} - \tilde{D}_{12}\frac{\partial^2 w}{\partial y^2} + \tilde{F}_{31}\phi$$
$$M_{yy} - (e_0 l_i)^2 \nabla^2 M_{yy} = -\tilde{D}_{12}\frac{\partial^2 w}{\partial x^2} - \tilde{D}_{11}\frac{\partial^2 w}{\partial y^2} + \tilde{F}_{31}\phi \tag{17a–c}$$
$$M_{xy} - (e_0 l_i)^2 \nabla^2 M_{xy} = -2\tilde{D}_{66}\frac{\partial^2 w}{\partial x \partial y}$$

Similarly, from (10), we have

$$\int_{-h/2}^{h/2} \cos\left(\frac{\pi z}{h}\right)\left[\frac{\partial D_x}{\partial x} - (e_0 l_i)^2 \nabla^2 \left(\frac{\partial D_x}{\partial x}\right)\right]\mathrm{d}z = \tilde{X}_{11}\frac{\partial^2 \phi}{\partial x^2}$$
$$\int_{-h/2}^{h/2} \cos\left(\frac{\pi z}{h}\right)\left[\frac{\partial D_y}{\partial y} - (e_0 l_i)^2 \nabla^2 \left(\frac{\partial D_y}{\partial y}\right)\right]\mathrm{d}z = \tilde{X}_{11}\frac{\partial^2 \phi}{\partial y^2}$$
$$\int_{-h/2}^{h/2} \frac{\pi}{h}\sin\left(\frac{\pi z}{h}\right)\left[D_z - (e_0 l_i)^2 \nabla^2 D_z\right]\mathrm{d}z \tag{18a–c}$$
$$= -\tilde{F}_{31}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) - \tilde{X}_{33}\phi$$

where

$$\langle \tilde{D}_{11}, \tilde{D}_{12}, \tilde{D}_{66}\rangle = \frac{h^3}{12}\langle \tilde{c}_{11}, \tilde{c}_{12}, \tilde{c}_{66}\rangle$$
$$\langle \tilde{F}_{31}, \tilde{X}_{11}, \tilde{X}_{33}\rangle = \left\langle \frac{2}{\pi}\tilde{e}_{31}h, \frac{\tilde{\kappa}_{11}h}{2}, \frac{\pi^2 \tilde{\kappa}_{33}}{2h}\right\rangle \tag{19a, b}$$

Here $\tilde{D}_{11}$ and $\tilde{D}_{12}$ are the flexural rigidities of the PNP. $\tilde{D}_{66}$ is called the torsional stiffness of the PNP. Substituting (17) and (18) into (14) and (15), one can obtain the non-local governing differential equations of motion for the transverse vibration of piezoelectric nanoplates under electrical loading

$$-\tilde{D}_{11}\frac{\partial^4 w}{\partial x^4} - 2(\tilde{D}_{12} + 2\tilde{D}_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} - \tilde{D}_{11}\frac{\partial^4 w}{\partial y^4} + \tilde{F}_{31}\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right)$$
$$+ q - (e_0 l_i)^2 \nabla^2 q + 2\tilde{e}_{31}V_0\left(1 - (e_0 l_i)^2 \nabla^2\right)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$
$$= \rho h\left(1 - (e_0 l_i)^2 \nabla^2\right)\frac{\partial^2 w}{\partial t^2} \tag{20}$$

$$\tilde{X}_{11}\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) - \tilde{F}_{31}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) - \tilde{X}_{33}\phi = 0 \tag{21}$$

Now, let us consider a DPNPS consisting of two piezoelectric layers which are coupled by a polymer matrix (Fig. 1). The Winkler model can be used to describe the effects of the polymer matrix on the vibration characteristics of the DPNPS. According to the Winkler model, the foundation consists of a system of vertical closely spaced springs that resist normal pressure. Based on this model, the resisting force of the polymer matrix acting on each nanoplate, that is, $q_i^{pf}\,(i = 1, 2)$ can be expressed as

$$q_i^{pf} = -(-1)^{i+1}k_\mathrm{w}(w_1 - w_2) \tag{22}$$

where k_w denotes the Winkler modulus parameter of the elastic medium. It is assumed that the two piezoelectric layers are flat and have the same thickness, width, length and material properties. Substituting for the transverse load in the governing (20) from relation (22), the non-local governing equations of the rectangular DPNPS can be obtained as follows

$$\begin{aligned}
&\tilde{D}_{11}\frac{\partial^4 w_1}{\partial x^4} + 2(\tilde{D}_{12} + 2\tilde{D}_{66})\frac{\partial^4 w_1}{\partial x^2 \partial y^2} + \tilde{D}_{11}\frac{\partial^4 w_1}{\partial y^4} \\
&+ (2\tilde{e}_{31}V_0)\left[(e_0 l_i)^2\left(\frac{\partial^4 w_1}{\partial x^4} + 2\frac{\partial^4 w_1}{\partial x^2 \partial y^2} + \frac{\partial^4 w_1}{\partial y^4}\right)\right. \\
&\left.-\left(\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2}\right)\right] + k_w\left[w_1 - (e_0 l_i)^2\left(\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2}\right)\right] \\
&- \tilde{F}_{31}\left(\frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial^2 \phi_1}{\partial y^2}\right) \\
&- k_w\left[w_2 - (e_0 l_i)^2\left(\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2}\right)\right] \\
&+ \rho h\left[\frac{\partial^2 w_1}{\partial t^2} - (e_0 l_i)^2\left(\frac{\partial^4 w_1}{\partial x^2 \partial t^2} + \frac{\partial^4 w_1}{\partial y^2 \partial t^2}\right)\right] = 0
\end{aligned} \tag{23}$$

$$\tilde{F}_{31}\left(\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2}\right) - \tilde{X}_{11}\left(\frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial^2 \phi_1}{\partial y^2}\right) + \tilde{X}_{33}\phi_1 = 0 \tag{24}$$

$$\begin{aligned}
&\tilde{D}_{11}\frac{\partial^4 w_2}{\partial x^4} + 2(\tilde{D}_{12} + 2\tilde{D}_{66})\frac{\partial^4 w_2}{\partial x^2 \partial y^2} + \tilde{D}_{11}\frac{\partial^4 w_2}{\partial y^4} \\
&+ (2\tilde{e}_{31}V_0)\left[(e_0 l_i)^2\left(\frac{\partial^4 w_2}{\partial x^4} + 2\frac{\partial^4 w_2}{\partial x^2 \partial y^2} + \frac{\partial^4 w_2}{\partial y^4}\right)\right. \\
&\left.-\left(\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2}\right)\right] + k_w\left[w_2 - (e_0 l_i)^2\left(\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2}\right)\right] \\
&- \tilde{F}_{31}\left(\frac{\partial^2 \phi_2}{\partial y^2} + \frac{\partial^2 \phi_2}{\partial x^2}\right) \\
&- k_w\left[w_1 - (e_0 l_i)^2\left(\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2}\right)\right] \\
&+ \rho h\left[\frac{\partial^2 w_2}{\partial t^2} - (e_0 l_i)^2\left(\frac{\partial^4 w_2}{\partial x^2 \partial t^2} + \frac{\partial^4 w_2}{\partial y^2 \partial t^2}\right)\right] = 0
\end{aligned} \tag{25}$$

$$\tilde{F}_{31}\left(\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2}\right) - \tilde{X}_{11}\left(\frac{\partial^2 \phi_2}{\partial x^2} + \frac{\partial^2 \phi_2}{\partial y^2}\right) + \tilde{X}_{33}\phi_2 = 0 \tag{26}$$

As seen from the above relations, the thermo-electromechanical vibration of a DPNPS is described by a system of four coupled differential equations.

3. Out-of-phase vibration of DPNPS: In this Section, the explicit expressions of natural frequencies are presented for the out-of-phase vibration of DPNPSs. Similar to double-layer graphene nanoribbons [13], the vibrational mode of the DPNPS is divided into the in-phase and out-of-phase modes (OPMs). In the case of the OPM, that is, $w_1 - w_2 \neq 0$, the non-local governing

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differential equations can be written as

$$\tilde{D}_{11}\frac{\partial^4 w'}{\partial x^4}+2\left(\tilde{D}_{12}+2\tilde{D}_{66}\right)\frac{\partial^4 w'}{\partial x^2\partial y^2}+\tilde{D}_{11}\frac{\partial^4 w'}{\partial y^4}$$
$$+\left(2\tilde{e}_{31}V_0\right)\left[\left(e_0l_i\right)^2\left(\frac{\partial^4 w'}{\partial x^4}+2\frac{\partial^4 w'}{\partial x^2\partial y^2}+\frac{\partial^4 w'}{\partial y^4}\right)\right.$$
$$\left.-\left(\frac{\partial^2 w'}{\partial x^2}+\frac{\partial^2 w'}{\partial y^2}\right)\right]$$
$$+2k_w\left[w'-\left(e_0l_i\right)^2\left(\frac{\partial^2 w'}{\partial x^2}+\frac{\partial^2 w'}{\partial y^2}\right)\right]-\tilde{F}_{31}\left(\frac{\partial^2\phi'}{\partial x^2}+\frac{\partial^2\phi'}{\partial y^2}\right)$$
$$+\rho h\left[\frac{\partial^2 w'}{\partial t^2}-\left(e_0l_i\right)^2\left(\frac{\partial^4 w'}{\partial x^2\partial t^2}+\frac{\partial^4 w'}{\partial y^2\partial t^2}\right)\right]=0 \tag{27}$$

$$\tilde{F}_{31}\left(\frac{\partial^2 w'}{\partial x^2}+\frac{\partial^2 w'}{\partial y^2}\right)-\tilde{X}_{11}\left(\frac{\partial^2\phi'}{\partial x^2}+\frac{\partial^2\phi'}{\partial y^2}\right)+\tilde{X}_{33}\phi'=0 \tag{28}$$

where

$$w'=w_1-w_2,\quad \phi'=\phi_1-\phi_2 \tag{29}$$

Without losing the generality, we assume that the boundary conditions are simply supported at all edges of the DPNPS. In addition, the value of the electric potential is equal to zero at the edges. In order to satisfy these boundary conditions, the solutions of (27) and (28) can be expressed as

$$w'(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}W'_{nm}\sin\left(\frac{m\pi}{\ell_x}x\right)\sin\left(\frac{n\pi}{\ell_y}y\right)\mathrm{e}^{i\omega t}$$
$$\phi'(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\Phi'_{nm}\sin\left(\frac{m\pi}{\ell_x}x\right)\sin\left(\frac{n\pi}{\ell_y}y\right)\mathrm{e}^{i\omega t} \tag{30}$$

where m and n are the half-wave number in the x and y directions, respectively, and ω is the natural frequency of the DPNPS. Substituting (30) into (27) and (28) leads to

$$\Omega_{\mathrm{OP}}^2=\frac{1}{1+\pi^2\chi^2\left(m^2+\beta^2n^2\right)}$$
$$\times\left\{\pi^4\left[m^4+2\left(\tilde{D}^*_{12}+2\tilde{D}^*_{66}\right)\beta^2m^2n^2+\beta^4n^4\right]\right.$$
$$+\left[1+\pi^2\chi^2\left(m^2+\beta^2n^2\right)\right]\left[2\bar{k}_w+\pi^2\hat{N}_{el}\left(m^2+\beta^2n^2\right)\right]$$
$$\left.+\frac{12\pi^4\left(\tilde{F}^*_{31}\right)^2\left(m^2+\beta^2n^2\right)^2}{\tilde{X}^*_{33}+\pi^2\hbar^2\tilde{X}^*_{11}\left(m^2+\beta^2n^2\right)}\right\} \tag{31}$$

Here the non-dimensional parameters are defined as

$$\beta=\frac{\ell_x}{\ell_y},\quad \hbar=\frac{h}{\ell_x},\quad \chi=\frac{e_0l_i}{\ell_x},$$
$$\bar{k}_w=\frac{k_w\ell_x^4}{\tilde{D}_{11}},\quad \hat{N}_{el}=\frac{2\tilde{e}_{31}V_0\ell_x^2}{\tilde{D}_{11}}$$
$$\phi_0=\sqrt{\frac{\tilde{c}_{11}h}{\tilde{X}_{33}}},\quad \tilde{D}^*_{12}=\frac{\tilde{D}_{12}}{\tilde{D}_{11}},\quad \tilde{D}^*_{66}=\frac{\tilde{D}_{66}}{\tilde{D}_{11}},\quad \tilde{F}^*_{31}=\frac{\tilde{F}_{31}\phi_0}{\tilde{c}_{11}h^2}$$
$$\tilde{X}^*_{11}=\frac{\tilde{X}_{11}\phi_0^2}{\tilde{c}_{11}h^3},\quad \tilde{X}^*_{33}=\frac{\tilde{X}_{33}\phi_0^2}{\tilde{c}_{11}h},\quad \Omega=\omega\ell_x^2\sqrt{\frac{\rho h}{\tilde{D}_{11}}} \tag{32}$$

It should be noted that the natural frequencies given by (31) reduce to those obtained by Murmu and Adhikari [22] for the out-of-phase vibration of double-nanoplate-systems when the piezoelectric effects are neglected. By setting $\Omega_{\mathrm{OP}}=0$ in (31), the following expression is obtained for the critical buckling voltages of the DPNPS for out-of-phase vibration

$$(V_0)^{\mathrm{OP}}_{cr}=\frac{\tilde{D}_{11}}{2|\tilde{e}_{31}|\ell_x^2\left[1+\pi^2\chi^2\left(m^2+\beta^2n^2\right)\right]}\left\{\frac{\pi^2}{\left(m^2+\beta^2n^2\right)}\right.$$
$$\times\left[m^4+2\left(\tilde{D}^*_{12}+2\tilde{D}^*_{66}\right)\beta^2m^2n^2+\beta^4n^4\right]$$
$$\left.+\frac{12\pi^2\left(\tilde{F}^*_{31}\right)^2\left(m^2+\beta^2n^2\right)}{\tilde{X}^*_{33}+\pi^2\hbar^2\tilde{X}^*_{11}\left(m^2+\beta^2n^2\right)}\right\}+\frac{\bar{k}_w\tilde{D}_{11}}{\pi^2|\tilde{e}_{31}|\ell_x^2\left(m^2+\beta^2n^2\right)} \tag{33}$$

4. In-phase vibration of DPNPS: Similarly, a closed-form solution can be obtained for the in-phase mode (IPM) in which the relative displacement between two piezoelectric nanofilms is ignored ($w_1-w_2=0$). Using the Navier method, the natural frequencies of the DPNPS can be found for the synchronous vibration

$$\Omega_{\mathrm{IP}}^2=\frac{1}{1+\pi^2\chi^2\left(m^2+\beta^2n^2\right)}$$
$$\times\left\{\pi^4\left[m^4+2\beta^2m^2n^2\left(\tilde{D}^*_{12}+2\tilde{D}^*_{66}\right)+\beta^4n^4\right]\right.$$
$$+\pi^2\hat{N}_{el}\left(m^2+\beta^2n^2\right)\left[1+\pi^2\chi^2\left(m^2+\beta^2n^2\right)\right]$$
$$\left.+\frac{12\pi^4\left(\tilde{F}^*_{31}\right)^2\left(m^2+\beta^2n^2\right)^2}{\tilde{X}^*_{33}+\pi^2\hbar^2\tilde{X}^*_{11}\left(m^2+\beta^2n^2\right)}\right\} \tag{34}$$

Furthermore, from the above relation, we can derive the critical voltage of the DPNPS

$$(V_0)^{\mathrm{IP}}_{cr}=\frac{\tilde{D}_{11}}{2|\tilde{e}_{31}|\ell_x^2\left[1+\pi^2\chi^2\left(m^2+\beta^2n^2\right)\right]}\left\{\left(\frac{\pi^2}{m^2+\beta^2n^2}\right)\right.$$
$$\times\left[m^4+2\left(\beta mn\right)^2\left(\tilde{D}^*_{12}+2\tilde{D}^*_{66}\right)+\left(\beta n\right)^4\right]$$
$$\left.+\frac{12\pi^2\left(\tilde{F}^*_{31}\right)^2\left(m^2+\beta^2n^2\right)}{\tilde{X}^*_{33}+\pi^2\hbar^2\tilde{X}^*_{11}\left(m^2+\beta^2n^2\right)}\right\} \tag{35}$$

It can be seen that the natural frequencies and critical electric voltages of the DPNPS are independent of the Winkler modulus parameter when the relative distance is equal to zero in the IPM.

5. Results and discussion: In the presented analysis, it is assumed that the DPNPS is made of PZT-4 with the following material properties [20]

$$c_{11}=132\ \text{GPa},\quad c_{12}=71\ \text{GPa}, c_{13}=73\ \text{GPa},\quad c_{33}=115\ \text{GPa}$$
$$c_{66}=30.5\ \text{GPa},\quad e_{31}=-4.1\ \text{C/m}^2,\quad e_{33}=14.1\ \text{C/m}^2$$
$$\kappa_{11}=5.841\times10^{-9}\ \text{C/Vm},\quad \kappa_{33}=7.124\times10^{-9}\ \text{C/Vm}$$

Unless noted otherwise, the length, width and thickness of the piezoelectric nanoplates are taken as $\ell_x=\ell_y=50$ nm and $h=5$ nm, respectively. To illustrate the influence of the Winkler modulus parameter of the coupling elastic medium on the natural frequencies of the DPNPS, the variation of the fundamental frequency parameter with the external electric voltage is plotted in Fig. 2 for various values of stiffness of the connecting springs. The small scale coefficient is set to $\chi=0.1$. It is observed that the

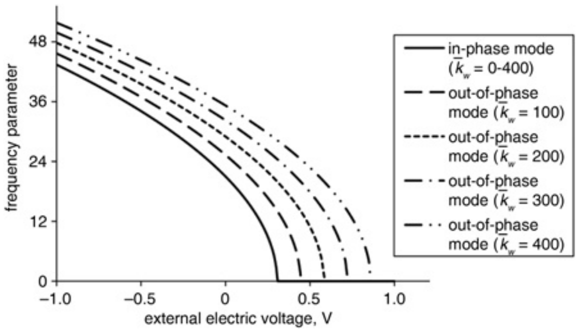

Figure 2 *Variation of fundamental frequency with applied voltage for different Winkler parameters (χ = 0.1)*

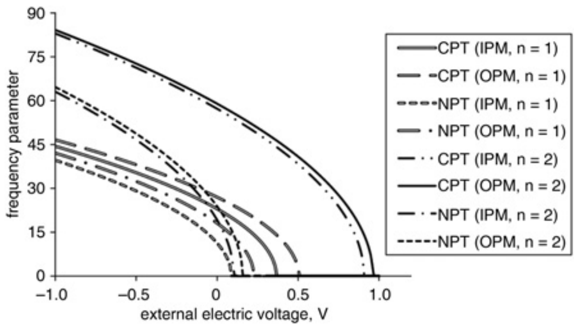

Figure 3 *Variation of frequency parameter with applied voltage for the first two modes of oscillation ($\chi = 0.4$, $\bar{k}_w = 100$)*

fundamental natural frequencies of the DPNPS depend strongly on the elastic medium parameter in the OPM, whereas the natural frequencies of the IPM are independent of $\bar{k}_w$. The frequency parameter increases with increasing the stiffness of the coupling springs from 0 to 400. Furthermore, the fundamental frequencies decrease by increasing the applied voltage and become zero (buckling state) for a critical value of the external electric voltage. These critical values increase as the Winkler elastic modulus increases.

A comparison between the results of non-local and classical plate theories (NPT and CPT) for in-phase and out-of-phase modes is presented in Fig. 3. The results are plotted for the first two vibration modes (i.e. $m = n = 1$ and $m = 1$, $n = 2$). From the Figure, it is found that the non-local parameter has a decreasing effect on the dimensionless frequencies of the DPNPS. In addition, non-local effects are higher at higher vibration modes. The difference between the IPM and OPM is lower at higher mode numbers. Another interesting result is that the non-local critical voltages in which the buckling occurs are always smaller than their local counterparts. Fig. 4 shows the effect of the nanoplate's thickness on the non-dimensional natural frequencies of the DNPS for the OPM. It can be concluded that the frequency parameter decreases with increasing thickness for negative values of the external electric voltage, but the thickness of the piezoelectric nanofilm has an increasing effect on the frequency parameter for positive values of the applied voltage.

Fig. 5 depicts the variation of the critical electric voltage with small scale coefficient ($\chi = e_0 l_i/\ell_x$) for in-phase and out-of-phase modes. Computations have been performed considering different values of aspect ratio. The dimensionless stiffness of the connecting springs is assumed to be $\bar{k}_w = 200$. From Fig. 5, it can be seen that the critical electric voltage decreases with increasing small scale coefficient from 0 to 0.5. A greater value of aspect ratio leads to higher small scale effects. This means that aspect ratio has an increasing effect on the non-local effects when the length of each PNP is

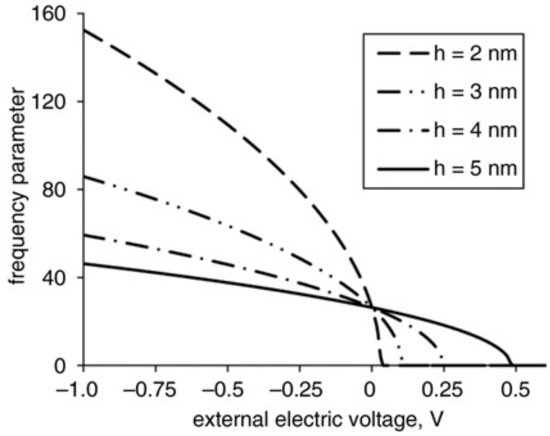

Figure 4 *Variation of dimensionless fundamental frequency with applied voltage for different values of thickness ($\chi = 0.2$, $\bar{k}_w = 200$)*

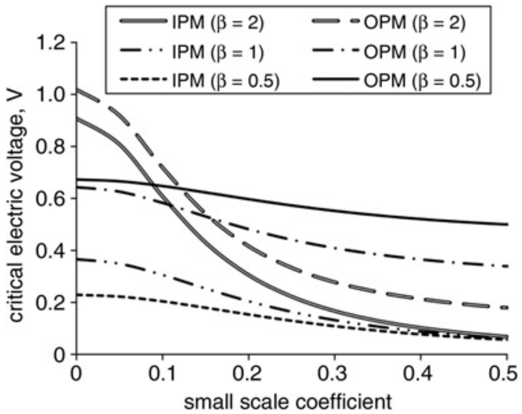

Figure 5 *Variation of critical electric voltage with scale coefficient for different aspect ratios ($h = 5$ nm, $\ell_x = 50$ nm, $\bar{k}_w = 200$)*

constant. By increasing the aspect ratio, at constant length, the width of the DPNPS decreases and thus non-local effects increase. Furthermore, the difference in critical electric voltage between the IPM and OPM is lower for greater values of the aspect ratio.

6. Conclusions: Based on the non-local plate theory, the small scale effects on the transverse vibration of rectangular double-PNPs under external electric voltage are studied. Explicit expressions are derived for natural frequencies and critical buckling voltages in the in-phase and out-of-phase modes. It is found that the small scale effect has a significant role in the vibration behaviour of a DPNPS and cannot be neglected. The non-local parameter has a decreasing effect on the frequency parameter. Furthermore, the effect of small scale is higher for higher mode numbers. The natural frequencies of the OPM are always greater than their IPM counterparts.

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