

Size-dependent free vibration of nano/microbeams with piezo-layered actuators

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The size-dependent free vibration of nano/microbeams with piezo-layered actuators is analytically investigated. The size-dependent dynamic modelling of the piezo-layered beam is presented on the basis of the non-local continuum theory. Equations of the motion and boundary conditions of the beam are obtained by implementation of Hamilton's principle. Analytical solutions for natural frequencies and mode shapes are obtained as a function of the piezo-layered beam characteristic size and non-local size scale parameter. The size effects on the vibration behaviour of the beam are studied and it is found that the non-local parameter, length ratio and thickness ratio have significant effects on the free vibration of systems.

1. Introduction: During the past decades, beams have been one of the most important structures used extensively in engineering applications. With respect to recent developments in science and technology, the implementation of small beams whose characteristic dimensions and behaviour are in terms of micrometres and nanometres has been developed in the past two decades [1–3]. Currently, nano/microbeams are known as the key components in nano/micro-scale systems, because of their novel features such as low weight, small size, simple fabrication and high-frequency operation [4]. Another important feature of nano/microbeams relates to the motion of these small systems by means of elasticity of beams and their reflection. This feature originates from many problems in fabricating joints in nano-to-micro scale [5]. Therefore, nano/microbeams have obtained a very important position and have been utilised in many applications, such as atomic force microscopy, nano/microresonators, biological sensors, microactuators and so on [6, 7]. Firdaus *et al.* [6] presented an experimental study on a piezo-actuated microbeam as a mass sensor and showed that the piezo layers can enhance the sensor sensitivity. Baglio *et al.* [8] developed a hybrid system composed of a piezo layer actuator bonded on a microbeam as a highly sensitive sensor for various applications. Fu *et al.* [9] presented the fabrication and testing of a piezoelectric microbeam applied as a microactuator.

According to the literature, the study of nano/microbeams has attracted much research attention. Demir *et al.* [10] studied the vibration behaviour of a nanotube considering the shear effects of the beam. They used a numerical method to solve the free vibration problem. Jiang and co-workers [11] investigated the nonlinear vibration of a nanobeam aroused by interaction forces from the surrounding medium. They found the natural frequency was sensitive to different parameters of the beam.

Although the above-mentioned papers have focused on the mechanical modelling of nano/microbeams based on classical continuum theory, the capability of this theory to describe the mechanics of such systems is a relevant question because of several experimental results in the nano-to-microscale [12, 13]. Moreover, experiments at the nano/microscale are extremely difficult to conduct and atomistic dynamic modelling is strongly restricted by computational capacities.

Therefore, high-order continuum theories have emerged as an applicable approach to analyse the dynamic behaviour of nano/microsystems, which have attracted much interest in recent years. Jiang and Yan [14] employed the surface elasticity theory to study the static behaviour of nanowires. They presented explicit solutions to analyse surface effects on the deflection of the beam.

Moreover, Eringen [15, 16] proposed non-local continuum mechanics formulation which has been extensively used to investigate the size effect on small-scale systems [17, 18]. Pang and co-workers [19] implemented non-local elasticity theory to analyse the buckling of a nanobeam with axial force. They discussed the small length scale effects on the buckling force of the nanobeam. Janghorban [20] studied the deflection of microbeams based on non-local beam models, using two types of differential quadrature method to analyse the effect of small parameters. Moreover, the non-local theory is applied to different classical beams in [21] and governing equations of non-local beams are derived; however, a simply supported beam is presented only as a case study. Demir and co-workers [22] presented the free vibration of cantilever microtubules modelled by non-local theory. They presented some numerical results to show the effect of small-scale parameters on the static and dynamic behaviour of microbeams. The presented literature review shows that numerous papers have studied the static and dynamic behaviour of nano/microbeams based on classical and non-local elasticity theories. However, the modelling of piezo-layered beams has not been taken into account.

This Letter studies the size-dependent dynamic analysis of small beams with piezo-layered actuators. To this end, the size-dependent dynamic modelling of both the beams and the piezo layers have been derived based on non-local elasticity theory. Then, a combined version of non-local elasticity is developed with respect to the properties of the beam and the piezo layer. Thus, by obtaining the governing equation of the system, the closed-form solution of the system is presented. The natural frequency and mode shape functions of such a system are presented, and the small-size effect on them is investigated. Finally, by developing various non-local boundary conditions, the effects of small size on the vibration behaviour of various boundary conditions are compared.

2. Non-local elasticity theory: Despite classical elasticity theory, the non-local elasticity theory considers information of adjacent points to present stress–strain relationships of a body. The differential form of the non-local elasticity theory is stated by Eringen and Edelen [15] as

$$(1 - \mu^2 \nabla^2) \sigma_{kl} = \tau_{kl} \quad (1)$$

where ∇^2 is the Laplacian operator, σ_{kl} is the non-local stress tensor, τ_{kl} indicates the classical stress tensor and μ is a parameter that represents small size effects.

As non-local continuum mechanics has recently presented good agreement in atomic simulations and experiments, non-local

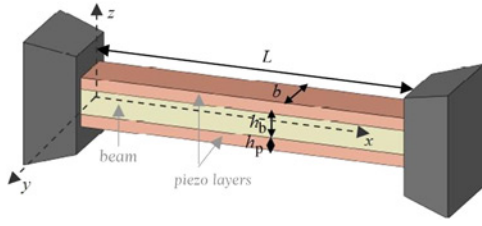


Figure 1 Piezo-layered micro/nanobeam

continuum mechanics is extended to the piezoelectric materials [23]. Therefore, the non-local equations for a piezoelectric material can be written in differential forms as [23]

$$(1 - \mu^2 \nabla^2) \sigma_{kl} = C_{klmn} \varepsilon_{mn} - e_{mkl} E_m \quad (2)$$

$$(1 - \mu^2 \nabla^2) D_i = e_{ikl} \varepsilon_{kl} + \xi_{ik} E_k \quad (3)$$

where D_i , E_i , C_{klmn} , e_{mkl} and ξ_{ik} are the electric displacement, electric field, elastic constants, piezoelectric constants and dielectric constants, respectively.

3. Non-local model of piezo-layered small beam: As shown in Fig. 1, there is a micro/nanobeam with a pair of piezoelectric layers bounded at the top and bottom surfaces. The depth and length of the beam and the piezo layers are represented by b and L , respectively. The height of the piezo layer and beam are shown by h_p and h_b , respectively. It is assumed that the piezo layers are polarised along the z -direction. As two piezo layers are actuators with opposite direction of polarisation, the transverse vibration of the system is considered.

The displacement of a point in the beam and the piezo layers represented by U and W in the x - and z -direction are given as

$$U = -z \frac{\partial w(x, t)}{\partial x} \quad (4)$$

$$W = w(x, t)$$

where $w(x, t)$ is the transverse deflection in the centre line of the system and t represents the time of motion. The non-zero strain component of the system is expressed as

$$\varepsilon_{xx} = -z \frac{\partial^2 w(x, t)}{\partial x^2} \quad (5)$$

To derive the governing equation of the system, Hamilton's principle is employed. So, the elastic strain energy of the system, Π , is a combination of the strain energy of the beam, Π_b , and the strain energy of the piezo layers, Π_p , as

$$\begin{aligned} \Pi &= \Pi_b + \Pi_p \\ &= \frac{1}{2} \iiint_{V_b} \sigma_{xx,b} \varepsilon_{xx} dV_b + \frac{1}{2} \iiint_{V_p} (\sigma_{xx,p} \varepsilon_{xx} - D_z E_z) dV_p \end{aligned} \quad (6)$$

where $\sigma_{xx,b}$, V_b , V_p , $\sigma_{xx,p}$, D_z and E_z are the axial stress in the beam, the volume occupied by the beam, the volume occupied by the piezo layers, the axial stress in the piezo layers, the electric displacement and the electric field in the piezo layers, respectively.

Substituting (5) into (6), yields the following relation

$$\begin{aligned} \Pi &= \frac{1}{2} \int_0^L \left[- \iiint_{A_b} \sigma_{xx,b} z \frac{\partial^2 w}{\partial x^2} dA_b - \iiint_{A_p} (\sigma_{xx,p} z \frac{\partial^2 w}{\partial x^2} + D_z E_z) dA_p \right] dx \\ &= \frac{1}{2} \int_0^L \left(-M_{x,b} \frac{\partial^2 w}{\partial x^2} - M_{x,p} \frac{\partial^2 w}{\partial x^2} - \bar{\phi} E_z \right) dx \end{aligned} \quad (7)$$

where the bending moments $M_{x,b}$ and $M_{x,p}$, and $\bar{\phi}$ are considered as

$$\begin{aligned} M_{x,b} &= \int \int_{A_b} \sigma_{xx,b} z dA_b \\ M_{x,p} &= \int \int_{A_p} \sigma_{xx,p} z dA_p \\ \bar{\phi} &= \int \int_{A_p} D_z dA_p \end{aligned} \quad (8)$$

Moreover, the kinetic energy of the system, T , is written as

$$\begin{aligned} T &= \frac{1}{2} \iiint_{V_b} \rho_b \left(\frac{\partial w}{\partial t} \right)^2 dV_b + \frac{1}{2} \iiint_{V_p} \rho_p \left(\frac{\partial w}{\partial t} \right)^2 dV_p \\ &= \frac{1}{2} \int_0^L (\rho_b A_b + \rho_p A_p) \left(\frac{\partial w}{\partial t} \right)^2 dx \end{aligned} \quad (9)$$

where ρ_b , ρ_p , A_b and A_p are the mass density of the beam, the density of the piezo layers, the beam cross-sectional area and the piezo cross-sectional area, respectively.

The work $W_{n.c.}$, because of the external damping, can be calculated from

$$W_{n.c.} = \frac{1}{2} \int_0^L (-f_{vis} w) dx \quad (10)$$

where $f_{vis} = B(\partial w / \partial t)$ is considered as the damping force and B is the viscous damping constant of the system. Hamilton's principle states that

$$\int_0^t (\delta T - \delta \Pi + \delta W_{n.c.}) dt = 0 \quad (11)$$

Then, submitting (7), (9) and (10) into (11), integrating by parts and setting the coefficients of δw to zero, the equations of vibration and the boundary conditions are achieved as

$$(\rho A)_{eq} \frac{\partial^2 w(x, t)}{\partial t^2} + B \frac{\partial w(x, t)}{\partial t} = \frac{\partial^2 M_{eq}}{\partial x^2} \quad (12)$$

$$M_{eq} \delta \left(\frac{\partial w(x, t)}{\partial t} \right) \Big|_0^L = 0 \quad (13)$$

$$\frac{\partial M_{eq}}{\partial x} \delta w(x, t) \Big|_0^L = 0$$

where $(\rho A)_{eq} = \rho_b A_b + \rho_p A_p$ is the effective inertia and $M_{eq} = M_b + M_p$ is the effective moment of the system.

For the beam with the piezo layers, the non-local constitutive relationships (1)–(3) can be stated in a one-dimensional (1D) form as

$$\sigma_{xx,b} - \mu^2 \frac{\partial^2 \sigma_{xx,b}}{\partial x^2} = c_b \varepsilon_{xx} \quad (14)$$

$$\sigma_{xx,p} - \mu^2 \frac{\partial^2 \sigma_{xx,p}}{\partial x^2} = c_p \varepsilon_{xx} - e_p E_z \quad (15)$$

$$D_z - \mu^2 \frac{\partial^2 D_z}{\partial x^2} = e_p \varepsilon_{xx} + \xi_p E_z \quad (16)$$

where c_b , c_p , e_p and ξ_p are Young's modulus of the beam, the Young's modulus of the piezo layers, the piezoelectric constant and the dielectric constant, respectively. By integrating (14)–(16) with (8) and combining them, we can obtain

$$M_{eq} - \mu^2 \frac{\partial^2 M_{eq}}{\partial x^2} = -(cI)_{eq} \frac{\partial^2 w}{\partial x^2} - e_p Q_p E_z \quad (17)$$

where the following relationships are considered

$$\begin{aligned} (cI)_{eq} &= c_b I_b + c_p I_p \\ \{I_b, I_p\} &= \iint_{\{A_b, A_p\}} z^2 \{dA_b, dA_p\} \\ Q_p &= \iint_{A_p} z dA_p \end{aligned} \quad (18)$$

Moreover, submitting (17) into (12) gives

$$M_{eq} = -(cI)_{eq} \frac{\partial^2 w}{\partial x^2} - e_p Q_p E_z + \mu^2 \left[(\rho A)_{eq} \frac{\partial^2 w(x, t)}{\partial t^2} + B \frac{\partial w(x, t)}{\partial t} \right] \quad (19)$$

Also, by substituting (19) into (12) and (13), the equation of the motion and boundary conditions of the piezo-layered micro/nano-beam can be concluded as

$$\begin{aligned} (\rho A)_{eq} \left(1 - \mu^2 \frac{\partial^2}{\partial x^2} \right) \frac{\partial^2 w(x, t)}{\partial t^2} + (cI)_{eq} \frac{\partial^4 w(x, t)}{\partial x^4} \\ + B \left(1 - \mu^2 \frac{\partial^2}{\partial x^2} \right) \frac{\partial w(x, t)}{\partial t} = K_p \frac{\partial^2 V_p}{\partial x^2} \end{aligned} \quad (20)$$

$$\begin{aligned} \left[-(cI)_{eq} \frac{\partial^2 w}{\partial x^2} + \mu^2 (\rho A)_{eq} \frac{\partial^2 w(x, t)}{\partial t^2} \right] \delta \left(\frac{\partial w(x, t)}{\partial x} \right) \Big|_0^L = 0 \\ \frac{\partial}{\partial x} \left[-(cI)_{eq} \frac{\partial^2 w}{\partial x^2} + \mu^2 (\rho A)_{eq} \frac{\partial^2 w(x, t)}{\partial t^2} \right] \delta w(x, t) \Big|_0^L = 0 \end{aligned} \quad (21)$$

where V_p is the piezoelectric applied voltage and $K_p = -e_p Q_p / h_p$ is a constant coefficient. Furthermore, it should be noted that the constitutive equation for a classical piezo-layered cantilever beam can be recovered by setting the non-local parameter to zero, which is in agreement with [24].

4. Problem solution: In the previous Section, (20) and (21) present completely the flexural vibration equations and boundary conditions of a micro/nanobeam with piezo layer actuation. To solve the free vibration problem and study the size-dependant behaviour of the system, a harmonic solution for the undamped transverse displacement of the form of $w(x, t) = \tilde{W}(x) e^{i\omega t}$ is assumed, in which ω denotes the natural frequencies of the system. Submitting the harmonic solution into (20) leads to

$$(cI)_{eq} \frac{d^4 \tilde{W}(x)}{dx^4} + \mu^2 (\rho A)_{eq} \omega^2 \frac{d^2 \tilde{W}(x)}{dx^2} - (\rho A)_{eq} \omega^2 \tilde{W}(x) = 0 \quad (22)$$

The solution of the foregoing ordinary differential equation can be stated as

$$\tilde{W}(x) = k_1 \cos(\chi_1 x) + k_2 \sin(\chi_1 x) + k_3 \cosh(\chi_2 x) + k_4 \sinh(\chi_2 x) \quad (23)$$

where the unknown coefficients k_1 through k_4 are obtained by implementation of the boundary conditions. Moreover, α and β are functions of the natural frequency and geometrical characteristic of the system as (see (24))

By substituting the solution given by (23) into the boundary conditions given by (21), the characteristic equation can be calculated which determines the natural frequency of the system.

For the simply supported beam, the characteristic equation is calculated as (see (25));

For the clamped–clamped beam the characteristic equation is (see (26))

and, for the clamped–hinged beam, the characteristic equation is (see (27))

5. Simulation results: In this Section, the size-dependent vibration of a micro/nanobeam is simulated. The geometry and material properties of the beam and the piezo actuator are listed in Table 1.

The width of the beam and the piezo layer are given as $b = 1200$ nm.

To verify the correctness of the modelling of the piezo-actuated non-local microbeam, we compared the current model with a non-local beam presented in [19].

As can be seen from Table 2, the present model is in good agreement with previous studies.

For the first simulation, it is desired to investigate the size effects on natural frequencies of the piezo-actuated beam. Therefore, Table 3 lists the fundamental frequency against different values of the non-local parameter (μ) and the slenderness ratio of the beam (L/h_b) with consideration of various boundary conditions.

$$\begin{aligned} \chi_1^2 &= \left\{ (\rho A)_{eq} \mu \omega^2 + \left[\left((\rho A)_{eq} \mu \omega^2 \right)^2 + 4(\rho A)_{eq} (cI)_{eq} \omega^2 \right]^{(1/2)} \right\} / 2(cI)_{eq} \\ \chi_2^2 &= \left\{ -(\rho A)_{eq} \mu \omega^2 + \left[\left((\rho A)_{eq} \mu \omega^2 \right)^2 + 4(\rho A)_{eq} (cI)_{eq} \omega^2 \right]^{(1/2)} \right\} / 2(cI)_{eq} \end{aligned} \quad (24)$$

$$f_1(\omega) = (cI)_{eq} \left[-\chi_1^2 \sin(L\chi_1) + \chi_2^2 \sinh(L\chi_2) + (cI)_{eq} \chi_1^2 \chi_2^2 \sin(L\chi_1) \sinh(L\chi_2) \right] \quad (25)$$

$$f_2(\omega) = 2\chi_1 \chi_2 [1 - \cos(L\chi_1) \cosh(L\chi_2)] - (\chi_1^2 - \chi_2^2) \sin(L\chi_1) \sinh(L\chi_2) \quad (26)$$

$$f_3(\omega) = -(cI)_{eq} (\chi_1^2 + \chi_2^2) [\chi_1 \cos(L\chi_1) \sinh(L\chi_2) - \chi_2 \sin(L\chi_1) \cosh(L\chi_2)] \quad (27)$$

Table 1 Material and geometric properties of micro/nanobeams and piezoelectric actuator

	Material	Density, kg/m ³	Young's modulus, Gpa	Piezoelectric constant, m/V	Thickness, nm
beam	SiO ₂	2330	107	–	600
piezo actuator	PZT	7500	139	123 × 10 ^{–12}	200

From Table 3, it can be seen that the first natural frequency of the system decreases as the non-local term μ increases. This reduction is related to reduction of the stiffness of the beam and may be explained as the non-local elasticity theory, which assumes that atoms are linked by an elastic matrix while the classic continuums assume the spring constant to take on an infinite value [25]. Moreover, as the slenderness ratio or length increases, the frequency decreases. When the beam length is sufficiently large and approaches macroscale, the non-local frequency approaches the local one. In other words, the size effects diminish by increasing the slenderness ratio. Moreover, for a given slenderness ratio, the frequency reduces more by increasing the non-local coefficient. Moreover, the effect of the non-local parameter is more significant for the C–C boundary conditions. For example, for a given slenderness ratio $L/h_b = 10$, by increasing the non-local term from $\mu = 0$ to $\mu = 0.1$, the frequency of the C–C beam decreases by about 13.7%, this reduction being about 11.4% for the H–H boundary conditions.

Figs. 2 and 3 depict the fundamental and second natural frequencies of the H–H beam against the slenderness ratio of the beam, respectively. It should be noted that in these figures the non-local parameter μ is represented by μ and divided by 10 μ m.

As seen from Figs. 2 and 3, decreasing the slenderness ratio increases the natural frequencies. Moreover, increasing the non-local parameter decreases the natural frequencies. This reduction is more obvious for the second natural frequency.

For another simulation, the effect of the piezoelectric thickness ratio ($\tilde{h} = h_p/h_b$) on the first natural frequency of the system

was investigated. So, Table 4 represents the first natural frequency of the piezo-actuated beam.

Table 4 Comparison of fundamental frequency (MHz) with consideration of $L/h_b = 20$

	μ	$\tilde{h} = 0.1$	$\tilde{h} = 0.2$	$\tilde{h} = 0.6$	$\tilde{h} = 1$	$\tilde{h} = 2$
H–H	0	13.9301	15.2978	21.3600	27.6855	43.7604
	0.1	13.4762	14.7993	20.6640	26.7834	42.3345
	0.2	12.3410	13.5527	18.9235	24.5274	38.7686
C–C	0	31.5779	34.6783	48.4215	62.7597	99.2014
	0.1	30.3083	33.2842	46.4740	60.2366	95.2115
	0.2	27.2391	29.9135	41.7677	54.1366	85.5696

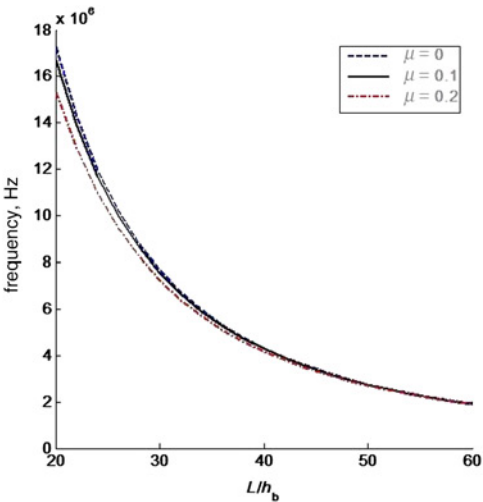


Figure 2 Fundamental natural frequency of the system

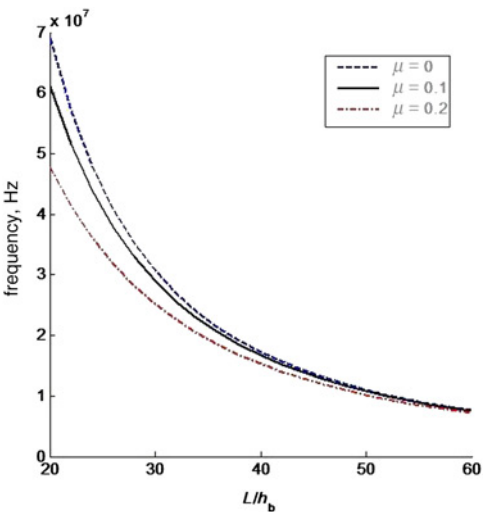


Figure 3 Second natural frequency of the system

Table 2 Comparison of non-dimensional first natural frequency of an H–H beam

L/h_b	μ	Present study	Ref. [19]
10	0	9.8695	9.8696
	0.1	9.4156	9.4159
	0.2	8.3570	8.3569

Table 3 Comparison of fundamental frequency (MHz) with consideration of different boundary conditions

L/h_b	μ	Boundary conditions		
		H–H	C–H	C–C
10	0	69.0295	107.8383	156.4815
	0.1	61.1552	93.8180	134.9812
	0.2	47.6736	71.2037	101.2393
20	0	17.2574	26.9596	39.1204
	0.1	16.6951	25.9417	37.7547
	0.2	15.2889	23.4545	33.7453
50	0	2.7612	4.3135	6.2593
	0.1	2.7462	4.2862	6.2171
	0.2	2.7026	4.2071	6.0948
100	0	0.6903	1.0784	1.5648
	0.1	0.6837	1.0766	1.5612
	0.2	0.6865	1.0715	1.5502

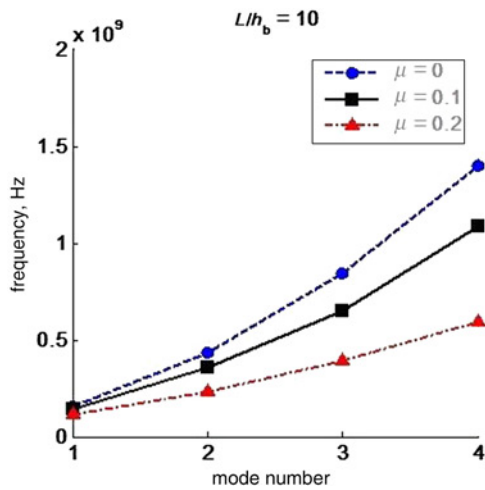


Figure 4 First four frequencies of the C-C beam for $L/h_b = 10$

As seen from Table 4, increasing the piezoelectric thickness ratio increases the first natural frequency of the piezo-layered beam. This can be explained by the fact that this increasing raises the stiffness of the beam. However, it must be kept in mind that increasing the piezoelectric thickness demands higher voltage [26]. Moreover, it can be seen that the first natural frequency increases as the non-local term decreases. Furthermore, the size effect on the higher modes is considered. Figs. 4 and 5 plot the size effect on the first four frequencies of the C-C beam.

As can be observed from Figs. 4 and 5, the effect of the non-local term on the higher natural frequencies is more prominent. This may be explained by the fact that wavelengths are decreased for higher modes and the stronger interactions between atoms lead to increasing of the non-local elasticity. Moreover, increasing the non-local parameter decreases the frequency more in higher modes. Moreover, as the slenderness ratio is increased, the effect of non-local continuum theory is reduced.

In another case study, the size effect on the shape functions of the piezo-actuated beam was investigated. Figs. 6 and 7, respectively, show the first mode shape of the H-H and the C-C piezo-actuated beam.

As can be seen from Fig. 6, the non-local term has almost no influence on the first mode shape of the simply supported beam, but has a significant effect for the clamped-clamped beam. So the mode shapes of the classic H-H beam can be used for the non-local beam in the microscale to nanoscale, but the non-local mode shape must be taken into account for the C-C beam.

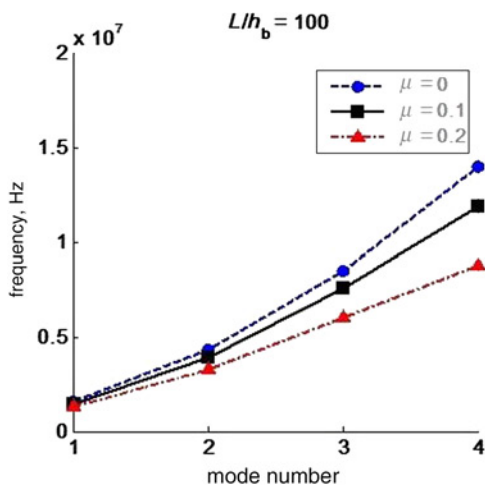


Figure 5 First four frequencies of the C-C beam for $L/h_b = 100$

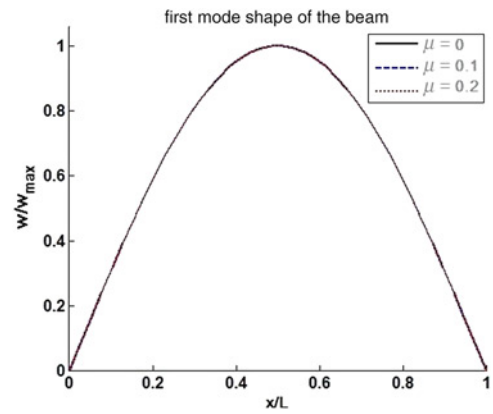


Figure 6 First mode shape of the H-H beam

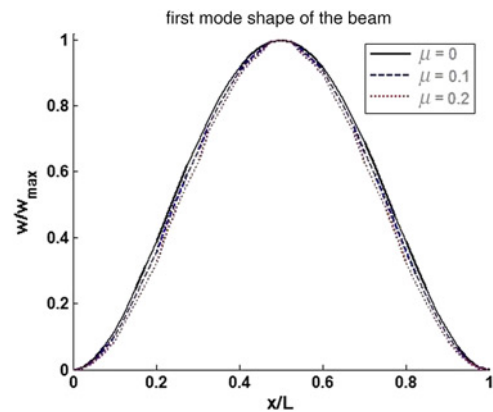


Figure 7 First mode shape of the C-C beam

6. Conclusion: In this Letter, the size-dependent free vibration behaviour of nano/microbeams with piezo-layered actuators has been studied. The dynamic modelling of the system has been performed considering the non-local model of the piezo layers and the beam. The governing equations of the motion and boundary conditions of the system have been derived and analytical solutions for the natural frequencies and mode shapes have been obtained. The size-dependent effects of the piezo-layered beam's characteristic of size and the non-local length scale parameter have been studied. It was seen that natural frequencies decrease as the non-local parameter increases and this reduction is more prominent for higher natural frequencies. Besides, increasing the length ratio decreased the frequency, and for a large length, the non-local frequency approached the local one. It is also shown that the effect of the non-local parameter is more significant for the natural frequency and mode shape of the C-C beam. The size effects on the vibration behaviour of the beam have been studied and it was found that the non-local parameter, length ratio and thickness ratio have significant effects on the free vibration of the system.

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