Nonlinear circuit analysis technique for microelectromechanical systems with a time-variant capacitor and an AC power source

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Published in Micro & Nano Letters; Received on 8th July 2015; Revised on 1st September 2015; Accepted on 2nd September 2015

Microelectromechanical systems (MEMS) utilise time-variant capacitors as transducers in many applications. However, this kind of component can introduce harmonics and disturbances into the circuit with an AC power source, which is difficult to evaluate through closed-form solutions. This Letter proposes an iterative solution to analyse the dynamics of MEMS devices which contain a time-variant capacitor and an AC source. First, the expressions of the time-variant capacitor, AC source and their derivatives with respect to time are determined. Then, an initial solution that is sufficiently close to the actual solution is determined using linear circuit analysis. On the basis of the previous steps and the principles of the iterative method, an approximated solution combining the initial solution and its iteratively-derived higher-order terms is reached. Adding additional higher-order terms can improve the accuracy of the solution. A case study considering a MEMS device which has an AC power source and sinusoidal motion was performed using MATLAB Simulink. The simulation study further demonstrated that: (i) this iterative solution can effectively analyse the dynamics of MEMS devices with a time-variant capacitor and an AC power source; and (ii) computing additional higher-order terms derived from the initial solution can further improve the solution's accuracy.

1. Introduction: Time-variant capacitors exist in many microelectromechanical systems (MEMS) devices. They could be designed as sensing structures where the variation in capacitance represents different external physical parameters; examples include humidity sensors [1], vibration sensors [2], strain sensors [3], pressure sensors [4], gyroscopes [5] and accelerometers [6]. On the other hand, electrostatic actuators also include time-variant capacitors, which are the key components in many applications such as resonators [7, 8], micro mirrors [8], series switches [9], compliant structures [10] and RF devices [11, 12].

Thus, it is important to accurately analyse systems possessing time-variant capacitors. For a time-variant capacitor, $C(t)$, the current, $I_c(t)$, through it is

$$I_c(t) = \dot{V}(t)C(t) + V(t)\dot{C}(t). \tag{1}$$

Since (1) results in a nonlinear circuit model, it is difficult to obtain a closed-form solution. In some circumstances, this problem can be simplified by considering the variable capacitance as a constant, especially if it is powered by a DC voltage source or the AC source's frequency is much higher than the MEMS device's bandwidth. Then, linear circuit analysis can be applied to obtain a reasonably accurate solution. However, the time variation cannot be ignored in many applications. For example, consider an MEMS resonator vibrating at 2 kHz resonant frequency. In order to accomplish the feedback control, a 100 kHz voltage signal is applied to the device to detect the capacitance to measure the proof mass motion, which is a common configuration [13, 14]. Typically, the capacitance is treated as time invariant in this situation. Since the detection signal and the mechanical vibration should not affect each other, because the detection signal's frequency is much greater than the device's bandwidth, it is commonly considered that no additional components from the mechanical motion will be introduced [15]. In reality, however, the time-variant capacitor caused by the mechanical motion will introduce more harmonics into the detection signal, with frequency spacing equal to the frequency of the fundamental component. The spectrum of this example is given in Fig. 1 from a MATLAB SIMULINK simulation of the system. This phenomenon, though, has been experimentally documented in MEMS devices [16, 17]. In this figure, it is clear that harmonics with 2 kHz intervals are introduced, where each interval is equal to the vibration frequency. These harmonics can corrupt the measurement readings and introduce additional noise into the electrical system. Linear circuit analysis does not account for this very real and observable effect.

Note that the result contains both harmonics and intermodulation products of the AC source and the resonant frequency. Linear circuit analysis considers neither of these components. Dean and Wilson [18] proposed a nonlinear circuit analysis method for a time-variant MEMS capacitor system driven with a DC source, but the case with an AC voltage source was not investigated. To solve this problem, a standard circuit model needs to be considered.

Consider a time-variant capacitor with a series resistor, R, which is used to protect the capacitive element and prevent the power source from shorting to ground in case the MEMS device's electrodes physically contact each other. The circuit's schematic is given in Fig. 2. In addition, different configurations can be transformed into this model using the Thevenin equivalent circuit method.

The circuit's behaviour is described by

$$I_c(t) = (V_s(t) - V_c(t))/R = \dot{V}_c C(t) + V_c \dot{C}(t), \tag{2}$$

where V_s is the AC power source, V_c is the voltage across the variable capacitor and I_c is the current through it. Although (2) fully characterises the circuit's behaviour, it is difficult to solve this nonlinear differential equation in practice and obtain a closed-form solution.

2. Analysis approach: Since (2) is difficult to solve, an alternative iterative approach can be applied to obtain an approximate solution [18]. The first step is deriving an initial approximate solution $V_{c0}(t)$, which is sufficiently close to the $V_c(t)$ [19]. To achieve this initial solution, the variable capacitor in an MEMS device can be modelled as

$$C(t) = C_0 + C_1(t), \tag{3}$$

where C_0 is time invariant, and $C_1(t)$ is the time-variant part that

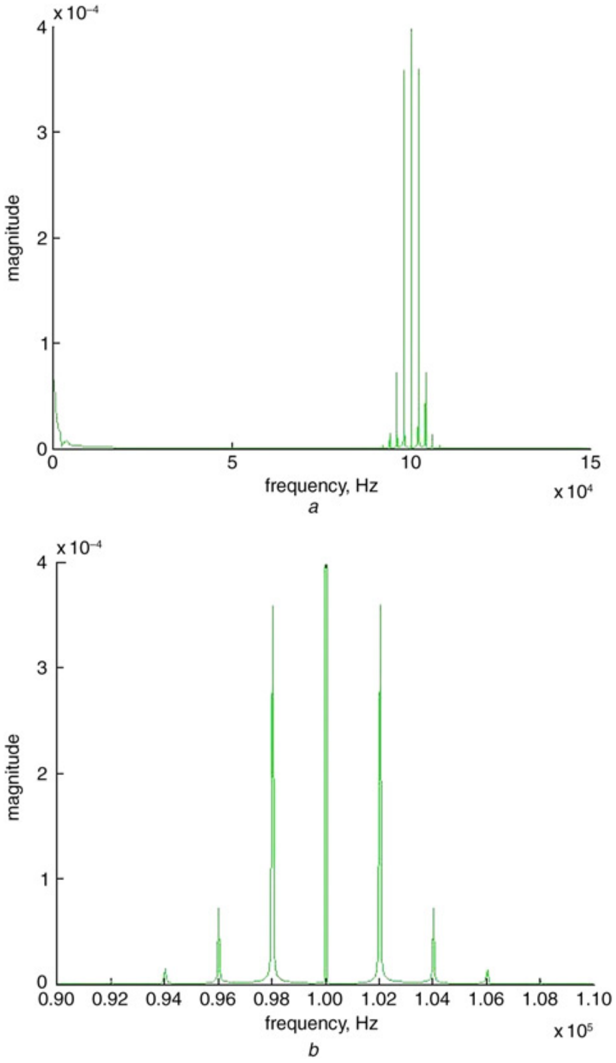

Fig. 1 *Spectral analysis of a MEMS resonator with detection signals is the*
a Overall spectrum
b Spectrum around the detection signal's fundamental frequency

must be less than C_0 to ensure that $C(t)$ is always positive. The derivative of (3) is

$$\dot{C}(t) = \dot{C}_1(t). \tag{4}$$

Ignoring the time-variant part and considering this circuit as a linear circuit, linear circuit analysis can be applied to obtain the initial solution $V_{c0}(t)$. The approximated circuit's transfer function is

$$\frac{V_c(S)}{V_s(S)} = \frac{1}{RC_0 s + 1}. \tag{5}$$

Then the first step is the analysis of this circuit ignoring the effect of $C_1(t)$ and considering it as a linear circuit. The purpose of this step is to obtain the steady state of $V_c(t)$, denoted by $V_{c0}(t)$. Then, $V_{c1}(t)$ can be calculated by (2) and (3). Correspondingly, the $I_{c1}(t)$ term is

$$I_{c1}(t) = \dot{V}_{c0} C(t) + V_{c0} \dot{C}(t). \tag{6}$$

Thus, using a small signal analysis to obtain V_{c1}

$$V_{c1}(t) = -I_{c1}(t) R. \tag{7}$$

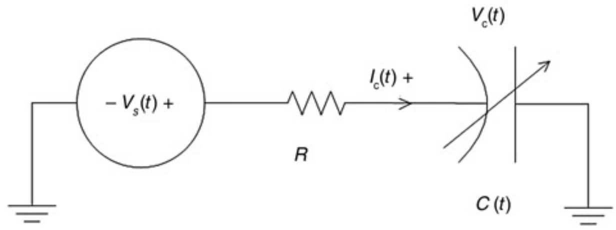

Fig. 2 *Thevenin equivalent schematic diagram of a time-variant capacitor with a series resistor and an AC voltage source*

Additional terms can be calculated recursively

$$I_{ck+1}(t) = \dot{V}_{ck} C(t) + V_{ck} \dot{C}(t). \tag{8}$$

The $I_{lk}(t)$ is then solved using as many terms as required to obtain sufficient accuracy. The overall equation for $V_c(t)$ is

$$V_c(t) = V_{c0}(t) + \sum_{k=0}^{\infty} (\dot{V}_{ck}(t) C(t) + V_{ck}(t) \dot{C}(t)) R^{k+1}. \tag{9}$$

3. Case study

3.1. System modelling and analysis: To verify the proposed technique, the following case study is considered. Consider an MEMS resonator oscillating at ω Hz and a ϕ Hz frequency signal is used to detect its capacitance. The capacitance's expression is

$$C(x) = \frac{\varepsilon_0 \varepsilon_r A}{x_0 - x}, \tag{10}$$

where x is the displacement of the movable electrode, ε_0 is the permittivity of free space, ε_r is the relative permittivity of the dielectric material between the two electrodes, A is the overlapping surface area of the electrodes and x_o is the initial gap between the two electrodes.

Assuming that the MEMS device is stimulated by an external mechanical sinusoidal displacement input

$$x = y_1 \sin(\omega t), \tag{11}$$

where y_1 is a small magnitude and ω is the stimulating frequency. The capacitance $C(t)$ is

$$C(t) = \frac{\varepsilon_0 \varepsilon_r A}{x_0 - y_1 \sin(\omega t)}, \tag{12}$$

If $x \ll x_0$, the capacitance's expression can be approximated as [17]

$$C(t) = y_0 + y_1 \sin(\omega t). \tag{13}$$

The derivative of $C(t)$ is

$$\dot{C}(t) = y_1 \omega \cos(\omega t), \tag{14}$$

where y_0 is the time-invariant part when $x = 0$

$$y_0 = \frac{\varepsilon_0 \varepsilon_r A}{x_0}. \tag{15}$$

Let the AC power source, V_s, be equal to

$$V_s(t) = A_s \sin(\phi t). \tag{16}$$

doi: 10.1049/mnl.2015.0276

The derivative of $V_s(t)$ with respect to time is

$$\dot{V}_s(t) = A_s \phi \cos(\phi t). \tag{17}$$

To obtain the initial solution, V_{c0}, the linear circuit's steady-state solution is used

$$V_{c0}(t) = A_s \sin(\omega t). \tag{18}$$

On the basis of (6) and (14), I_{c1} can be obtained

$$\begin{aligned} I_{c1}(t) = {} & \phi A_s \cos(\phi t)(y_0 + y_1 \sin(\omega t)) \\ & + A_s \sin(\phi t) y_1 \omega \cos(\omega t). \end{aligned} \tag{19}$$

Then, V_{c1} can be found using (7)

$$\begin{aligned} V_{c1}(t) &= -I_{c1}(t)R \\ &= -R(\phi A_s \cos(\phi t) y_1 \sin(\omega t) \\ &\quad + A_s \sin(\phi t) y_1 \omega \cos(\omega t)). \end{aligned} \tag{20}$$

Similiarly, V_{c2} can be calculated recursively

$$\begin{aligned} V_{c2}(t) = {} & R^2 A_s(-y_0 \phi^2 \sin(\phi t) - y_1 \phi^2 \sin(\phi t) \sin(\omega t) \\ & + y_1 \phi\omega \cos(\phi t) \cos(\omega t) + y_1 \phi\omega \cos(\phi t) \cos(\omega t) \\ & - y_1 \omega^2 \sin(\phi t) \sin(\omega t))(y_0 + y_1 \sin(\omega t)) \\ & - R^2 A_s(\phi \cos(\phi t) y_1 \sin(\omega t) \\ & + \sin(\phi t) y_1 \omega \cos(\omega t)) \omega y_1 \cos(\omega t) \end{aligned} \tag{21}$$

3.2. Simulation study: To verify the feasibility of the proposed technique, a MATLAB Simulink model was built and analysed. The values of the series resistance and the capacitance are typically <1 MΩ and 20 pF, respectively [20, 21]. The excitation signal applied to the MEMS devices tends to be about ten times greater than the device's resonant frequency, where typical values could be up to 100–200 kHz. Thus, the time invariant y_0 was 20 pF, while y_1 was 5 pF, the series resistor, R, was 10 kΩ, the mechanical vibration frequency was 2 kHz, the AC voltage source's amplitude was 1 V and its frequency, ϕ, was 100 kHz in this simulation study.

The system (13) was solved using a numerical method (Bogacki–Shampine) with a fixed time step of 1×10^{-8} s. The iterative solution with up to two high-order terms was simultaneously computed for comparison. Fig. 3 demonstrates the Simulink model. The left side is the system (13) and the right side calculates the iterative solutions with different orders.

In Figs. 4–6 the waveforms with the caption 'Simulink' present the output of the Simulink model based on (2) and (13), the waveforms with caption V_{c0}, V_{c1} and V_{c2} indicate the iterative solutions

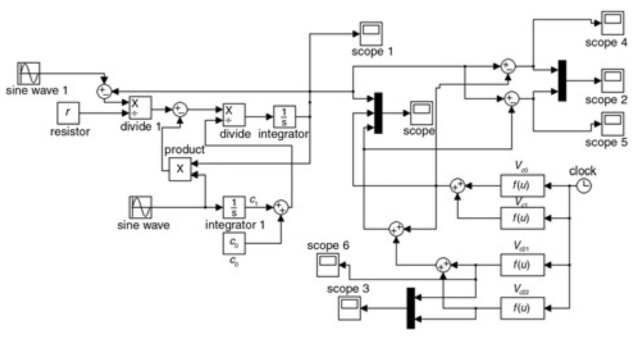

Fig. 3 *Simulink model*

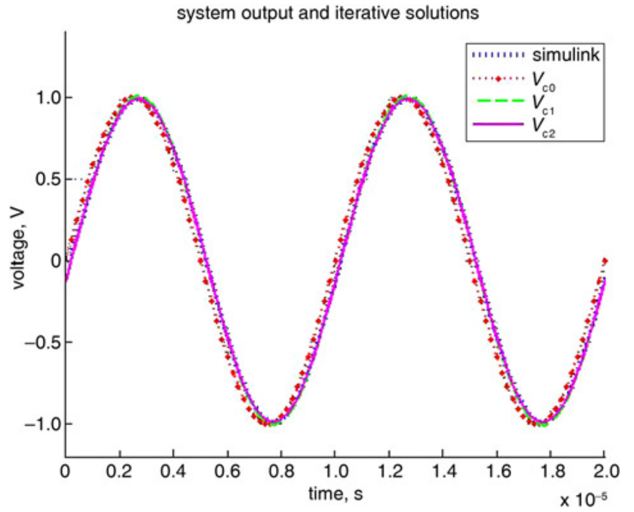

Fig. 4 *Simulation results of the Simulink solution and the iterative solution zoomed in on higher-order terms*

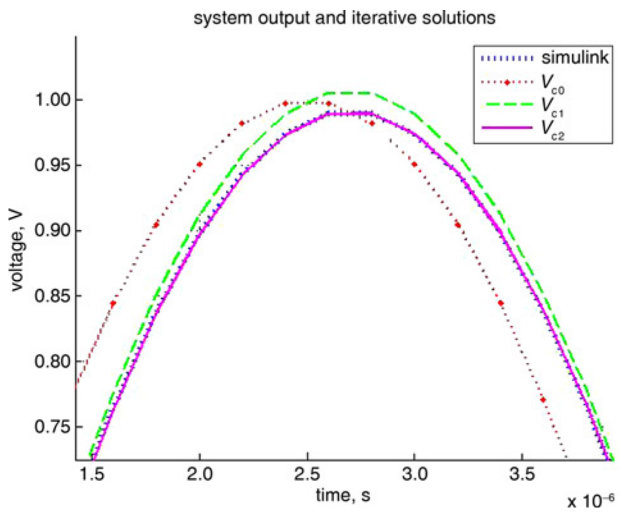

Fig. 5 *Zoomed in simulation results of the Simulink solution and the iterative solution*

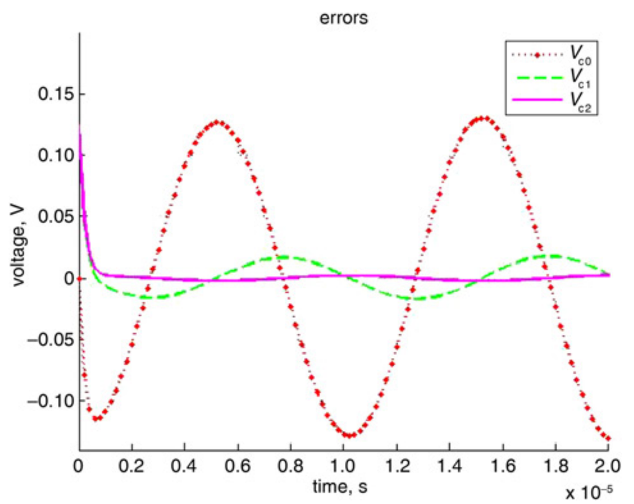

Fig. 6 *Simulation errors between the Simulink solution and the iterative solution zoomed in on higher-order terms*

doi: 10.1049/mnl.2015.0276

with different higher-order terms. The waveform V_{c0} is the linear circuit solution.

Fig. 4 shows the time response of the system and the analytical solutions with up to two higher-order terms. The results generated by the proposed method match the theoretical result well in the steady state. For the linear circuit solution, it appears that its amplitude was more precise than the solution with V_{c1}, but it has an obvious phase shift which generated more errors. It is clear that the iterative solution with just one term has more error than the result with two terms. Fig. 5 presents the comparison zoomed in on different higher-order terms. This figure further demonstrates that adding additional high-order terms can increase the solution's accuracy. Fig. 6 shows the errors using different high-order terms. It is shown that the linear circuit analysis yielded the greatest errors, with an amplitude that was more than 0.1 V. The solution with one higher-order term produced less error, with an amplitude that was 0.03 V. In contrast, the error using two higher-order terms was the smallest, which was <0.001 V.

4. Conclusions: An iterative analysis technique was proposed to solve an MEMS device's nonlinear circuit consisting of a time-variant capacitor and an AC power source connected to the MEMS device through a resistive network. A simulation study demonstrated that this method provides a more accurate solution compared with regular linear circuit analysis. The solution's accuracy can be increased by adding additional higher-order terms.

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doi: 10.1049/mnl.2015.0276