

Energy dissipation of quantum-dot cellular automata logic gates

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Quantum-dot cellular automata (QCA) is a new paradigm in nanoelectronics, where binary information is represented by charge configuration in cells. Ideal QCA logic gates are thought to be dissipationless, since there is no intercell charge transfer and no current flows out of cells. This work presents that these gates dissipate energy and compare energy consumptions of conventional QCA logic gates in electrostatic and thermodynamic approaches. The results show that increasing the number of inputs, concentration of the geometry and the unbalanced numbers of '0' and '1' output states in the gate's truth table add to the energy dissipation of a QCA gate.

1. Introduction: Quantum-dot cellular automata (QCA) [1] is an emerging technology to design nanoscale electronic devices that are likely to meet the requirements of the modern digital systems. Compared with microfabricated devices in complementary metal-oxide-semiconductor (CMOS) technology, the QCA devices are fabricated at the molecular size, allowing integration of 10^{11} – 10^{12} devices per square centimetre, besides lower power dissipation and terahertz frequency operation capability [2]. In QCA technology, binary information is not represented by the on/off states of current switches, rather it is encoded in the charge configuration of cells. Each cell consists of four quantum dots and two electrons. All quantum dots are able to localise electric charge; however, Coulomb repulsive forces make electrons localise in two antipodal quantum dots. These distinctive electronic configurations represent two different binary digits as shown in Fig. 1 [1]. QCA logic gates are constructed by arranging the cells near each other. Fig. 2 illustrates the structure of the inverter (Inv) and the majority voter (Maj) gates [3]. These two gates provide a complete set to implement any Boolean logic function. Ideal QCA devices are thought to be dissipationless, since electric charges do not leave any cells and no current flows out as a consequence [4, 5]. We believe electron transport between dots is a dissipative phenomenon, though the net current is zero. In this Letter, it is presented that the total energy of the device's cells changes after the logic decision is made, i.e. the information is processed. We employ basic electrostatic model and the Landauer formula to calculate and compare the energy dissipations of a number of conventional QCA logic gates.

2. Energy dissipation: We adopt two approaches for calculation of the energy dissipation in QCA logic gates. The first approach is based on the electrostatic energy of the physical device, which incorporates device geometry in calculations. The second approach ignores the geometric features of the device and calculates the computational energy dissipation based on the minimum energy needed to process a bit of information thermodynamically, as presented by the Landauer formula.

2.1. Electrostatic approach: The electrostatic energy of a QCA system depends on: (i) the positions of the electrons, which determine the electric potential of each dot, and (ii) the amount of localised charge at each dot. When the states of some cells alter, from '0' to '1' or vice versa, the electrons reorganise and cause the energy of the QCA gate to change. Energy dissipation is attributed to the change in the total energy of the QCA system. The electrostatic energy, W , of any structure composed of electric

charges is given by

$$W = \frac{1}{2} \sum_i \sum_{j \neq i} \frac{1}{4\pi\epsilon R_{ij}} q_i q_j \quad (1)$$

where q_i and q_j signify charges at the dot i and dot j , R_{ij} is the distance between them and ϵ is the electric permittivity of the medium. The electrostatic energy dissipation, ΔW , of a QCA logic gate can be approximated as

$$\Delta W = W_b - W_a \quad (2)$$

W_b and W_a in (2) denote the electrostatic energies of the gate before and after the logic decision is made, respectively. Data flow through QCA devices is controlled by clock pulses. The logic gate finds several states before the final output becomes stable. Detailed calculation of the energy dissipation requires incorporation of the clocking schemes and all the states, which we ignore here for simplicity. Thus, only energies corresponding to two critical states, the states of the logic gate before and after the logic decision, are taken into account. Energy dissipations of five common QCA logic gates are studied. These gates include the inverter gate [3], the three-input majority voter gate [3], the five-input majority voter gate [6], the and-or-inverter (AOI) gate and the nand-nor-inverter (NNI) gate [7–9]. The logic function of each gate is shown in Table 1.

Fig. 3 depicts the electronic structure of these gates before and after the information is processed. The energy dissipations of the studied gates have been calculated from (2), and the results are listed in Table 2. In this table, N_c represents the number of quantum cells in each gate. In all calculations, the size of each QCA cell is 10 nm. It is important to note that the energy dissipation strongly depends on the logic states of the inputs. The results shown in Table 2 are maximum energy dissipations of each gate.

2.2. Thermodynamic approach: For any computer system with N possible states, each with probability P_i , the Shannon entropy H ,

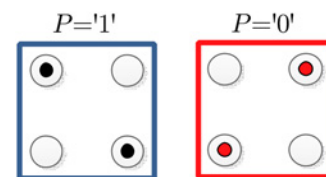


Fig. 1 Representation of binary digits by charge configuration
Right and left cells encode '0' and '1' binary digits, respectively

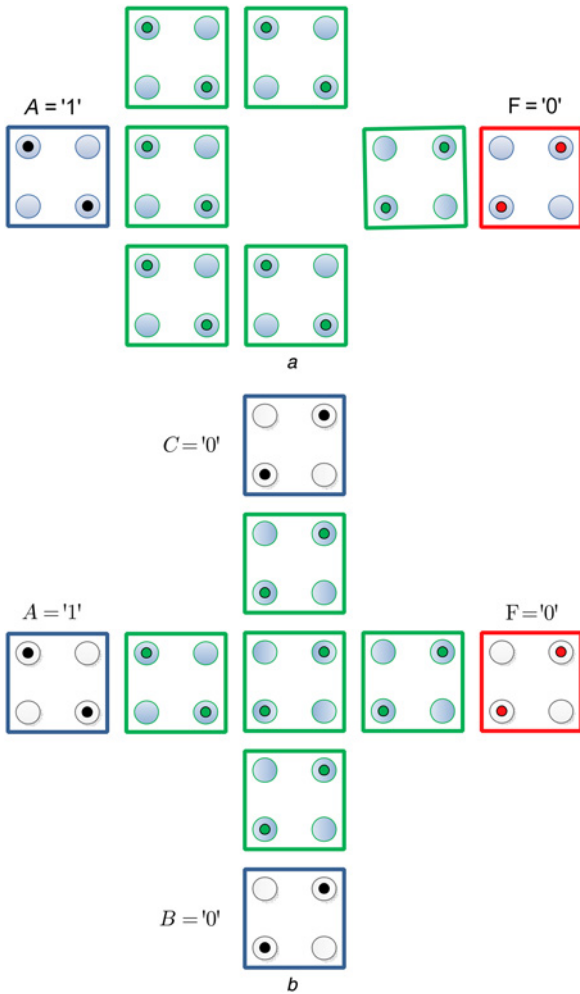


Fig. 2 Structure of basic QCA gates
a Inverter gate (Inv)
b Three-input majority gate (3Maj)

Table 1 Boolean functions of QCA gates

Gate	Boolean function
Inv	$F = \bar{A}$
3Maj	$F = A.B + A.C + B.C$
5Maj	$F = A.B.C + B.C.D + C.D.E + \dots$
AOI	$F = \bar{D}.\bar{E} + (\bar{D} + \bar{E})(A.B + B.C + C.A)$
NNI	$F = \bar{A}.B + B.\bar{C} + \bar{C}.\bar{A}$

Table 2 Electrostatic energy dissipations of QCA gates

Gate	N_c	ΔW , eV
Inv	8	0.01
3Maj	9	0.08
5Maj	10	0.20
AOI	8	0.08
NNI	4	0.01

is defined as

$$H = - \sum_{i=1}^{i=N} p_i \log_2 p_i \quad (3)$$

From the Landauer theory, the minimum energy dissipated per bit is $K_B T \ln(2)$, where K_B denotes the Boltzmann constant and T is the temperature in Kelvin. The thermodynamic energy dissipation, ΔE , of the computer is given by [10]

$$\Delta E = -k_B T \ln(2)(H_{\text{out}} - H_{\text{in}}) \quad (4)$$

where H_{out} and H_{in} represent the output and the input entropy of the computer, respectively. The difference between the entropies of a gate before and after the logic decision represents the thermodynamic energy dissipation. In order to take all the input and output states into account, the gate's truth table is used. The input/output entropy and the energy dissipations of the five QCA

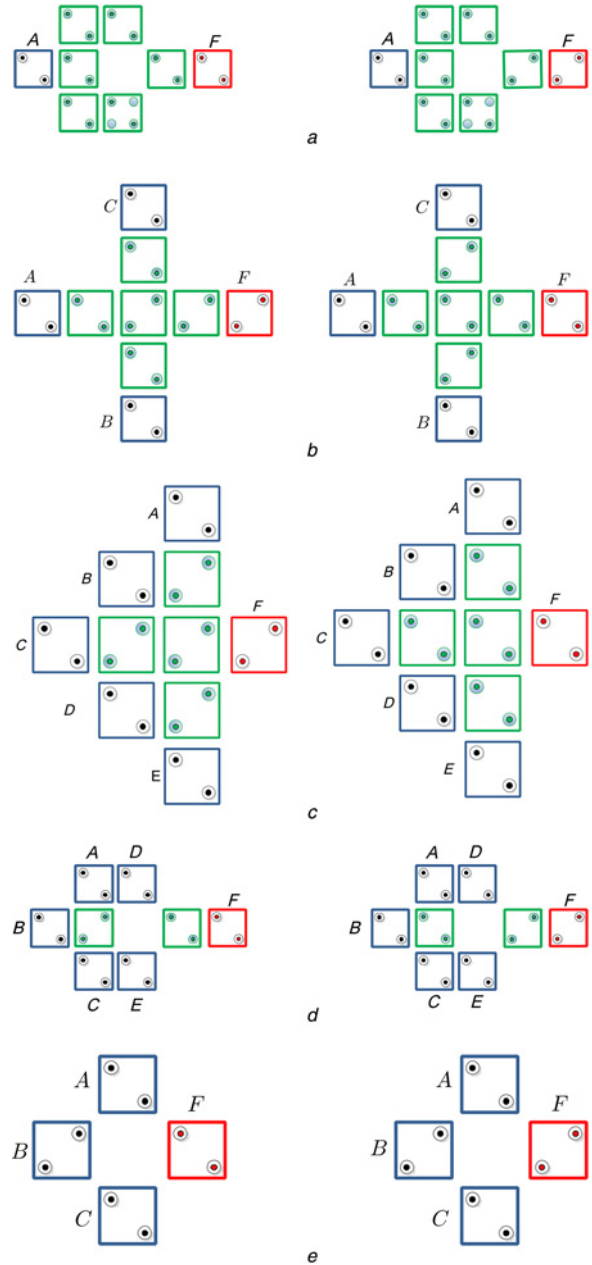
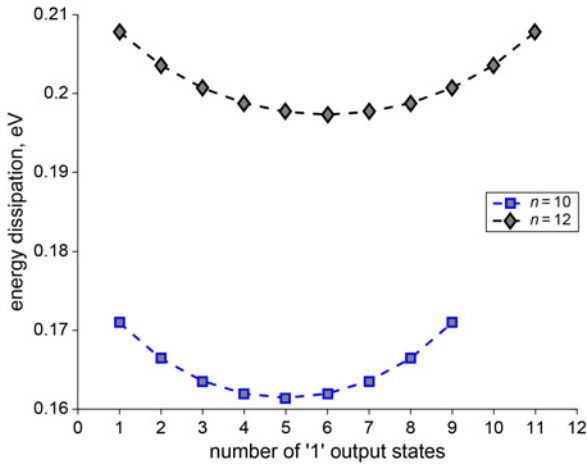


Fig. 3 Electronic structure of QCA logic gates before (left side) and after (right side) logic decision
a Inverter gate (Inv)
b Three-input majority gate (3Maj)
c Five-input majority gate (5Maj)
d AOI gate
e NNI gate

Table 3 Thermodynamic energy dissipations of QCA gates

Gate	H_{in}	H_{out}	ΔE , eV
Inv	1	1	0
3Maj	3	1	0.03
5Maj	5	1	0.07
AOI	5	1	0.07
NNI	3	1	0.03

**Fig. 4** Energy dissipation of 10-input and 12-input model gates. Horizontal axis corresponds to the number of '1' states, p , in the output of each gate

gates have been calculated according to their truth tables by using (4). The results are presented in Table 3.

3. Discussion: Although in QCA gates electric charges do not leave the cells and no current flows out of cells, our recent results present that these devices are not energy dissipationless. The results obtained from electrostatic analysis suggest that two factors strongly influence the electrostatic energy dissipation: (i) the number of inputs and (ii) the concentration of the geometry of the QCA gate. Likewise, the thermodynamic energy dissipation of a gate is affected by: (i) the number of inputs and (ii) the unbalanced numbers of '0' and '1' output states in its truth table.

Electrostatic and thermodynamic energy dissipations of the studied gates, as listed in Tables 2 and 3, show the energy dissipations of 5Maj and AOI gates are significantly larger than the 3Maj and the Inv gates, respectively, though their numbers of cells are almost the same. Thus, in similar gates, the energy dissipation does not generally depend on the number of cells; rather it depends on the number of inputs. In fact, multi-input gates are designed to reduce the area of the circuits and increase the overall performance; however, they dissipate more energy. It is interesting to note that in (1), the R_{ij} parameter implies the geometry or electronic structure dependence of the energy. In concentrated geometries, the R_{ij} factors are smaller, which cause the stored energy to increase. As a consequence, a change in the state of a cell within a concentrated geometry may lead to considerable energy dissipation.

Thermodynamic approach also presents that when the number of inputs is increased, the entropy difference between input and output is raised, which adds to the energy dissipation. The thermodynamic energy dissipation of the Inv gate is zero, since it is a reversible gate. In a reversible gate, the inputs can be determined from the outputs.

The thermodynamic approach reveals that unbalanced numbers of '0' and '1' output states in the truth table of a gate increases

its thermodynamic energy dissipation. Consider an arbitrary logic gate with n inputs and one output, where the number of '1' output states in its truth table is p , ($n > p \geq 1$). From (3) and (4) the energy dissipation of this gate is

$$\Delta E = k_B T \ln(2) \left(n - \frac{1}{n} \log_2 \frac{n^n (n-p)^{p-1}}{p^p} \right) \quad (5)$$

The minimum and the maximum energy dissipation, ΔE_{\min} and ΔE_{\max} , correspond to $p = n/2$ and $p = 1$ (or $p = n-1$), respectively. These energies are

$$\Delta E_{\min} = k_B T \ln(2) \left(n - \frac{1}{n} \log_2 2n^{n-1} \right) \quad (6)$$

$$\Delta E_{\max} = k_B T \ln(2) (n - \log_2 n) \quad (7)$$

Fig. 4 depicts the energy dissipation versus the number of '1' output states in the truth tables of two arbitrary 10-input single-output and 12-input single-output model gates. Obviously, the 12-input gate consumes more energy than the 10-input gate. The minimum energy dissipations of these gates correspond to the case where the numbers of '1' and '0' output states in their truth tables are equal, i.e. $p = 5$ for the 10-input and $p = 6$ for the 12-input model gates. Fig. 4 also shows that the more unbalanced numbers of '0' and '1' output states, the more energy is dissipated.

4. Conclusion: The QCA paradigm employs the charge configuration to construct transistorless circuits as alternatives to CMOS circuits. Although in QCA technology, no current flows out of cells, the energy dissipation is inevitable. We calculated and compared electrostatic and thermodynamic energy dissipations of five common QCA logic gates. The results show that the number of inputs, the geometry concentration and the unbalanced numbers of '0' and '1' output states in a gate's truth table are important factors that determine energy dissipation of a QCA logic gate.

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