

Wave propagation in fluid-conveying nanotubes under multi-physical fields based on non-local higher-order strain gradient model

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This work studies the wave propagation in embedded single-walled carbon nanotubes (CNTs) conveying fluid and placed in multi-physical fields based on the non-local higher-order strain gradient model with surface effect considered. The nanotubes are modelled as Timoshenko beams. Utilising Hamilton's principle, the governing equations of wave motion in CNTs are derived. The solution for the phase velocity of wave motion is obtained, which can not only consider the stiffness softening effect but also reflect the stiffness enhancement effect of scale parameter observed by experiments. The numerical simulations are conducted to compare the results on the basis of different order non-local strain gradient models. It is shown that the behaviours of wave propagation based on the non-local higher-order strain gradient models are quite different from those based on the classical continuum model. In addition, the scale and surface effects and influences of various external factors including inner flow, surrounding medium, temperature field, and magnetic field on the wave dispersion are investigated.

1. Introduction: As known, carbon nanotubes (CNTs) have drawn huge attention as they have great advantages on thermal conductivity, chemical and thermal stability, mechanical strength and stiffness, and electrical and optical properties. On account of the excellent behaviours mentioned above, CNTs have been widely applied in nanosensors, nanorobots, nanomotors, nanopipes, nanocontainers and nanofluid storage devices [1–4]. Thus, it is of much importance to study the mechanical and physical properties of CNTs. Among all the researches, wave propagation in CNTs has become one of the hot issues.

Generally, there are mainly three approaches to investigate the wave propagation in CNTs, which include molecular dynamics simulations, experimental studies, and continuum mechanics methods. A great number of research results have shown that the small-scale effects of CNTs are significant and should not be ignored, which means that the classical continuum mechanics theories cannot accurately evaluate the static and dynamic mechanical behaviours of CNTs. Therefore, a wide range of non-classical continuum theories including non-local elasticity theory and strain gradient elasticity theory are extended to take into account the scale effects [5–11].

However, it should be pointed out that the non-local elasticity theory developed by Eringen [12] can only consider the stiffness softening effects but cannot take account of the stiffness enhancement effects found from experimental investigations and the strain gradient theory [13]. Thus, Lim [14] extended the non-local elasticity theory and derived a non-local higher-order strain gradient theory in which the higher-order terms are retained. Based on this non-local strain gradient theory, Eltaher *et al.* [15] studied the mechanical properties of higher-order gradient nanobeams. Mohammad and Moslem [16] conducted the vibration and instability analyses of nanotubes conveying fluid subjected to a longitudinal magnetic field. Yakaiah and Rao [17] investigated the wave dispersion characteristics of a carbon nanorod through second, fourth, sixth, and eighth-order non-local strain gradient models. For the beam theories, the Euler–Bernoulli beam theory and Timoshenko beam theory are widely utilised in the analyses of mechanical properties of CNTs [18–26].

So far, nanotubes have played important roles in many potential applications to hold fluid [27–30]. Hence, there are many models proposed to study the properties of vibration and wave propagation in fluid-conveying CNTs. Ghavanloo and Fazelzadeh [31]

investigated flexural vibration of viscoelastic CNTs conveying fluid and embedded in viscous fluid by the non-local Timoshenko beam model. The relationship between the critical flow velocity and the non-local parameter was analysed. Rashidi *et al.* [32] proposed a novel model for coupled vibrations of fluid-conveying nanotubes by taking into account the small-size effects on the flow field. It is demonstrated that neglecting the small-size effects on flow field in a fluid-structure interaction problem at nanoscale may result in erroneous results. Wang *et al.* [33] studied the wave behaviour of embedded fluid-conveying CNTs in thermal and magnetic fields. The influence of fluid velocity on the flexural wave frequency was discussed. Kaviani and Mirdamadi [34] analysed wave propagation in CNTs conveying fluid with slip boundary condition and strain/inertial gradient theory. The wave dispersion relations and corresponding characteristic equations were derived. Zeighampour and Beni [35] conducted an investigation on the vibrations of fluid-conveying double-walled CNTs by the use of a modified couple stress shell theory. It is shown that the effects of fluid velocity and size parameter on vibrational properties in the modified couple stress theory are stronger than in the classical theory. Liu *et al.* [36] developed an iterative algorithm-based interval analysis method to cope with wave propagation of fluid-conveying CNTs with system uncertainties. The influences of the system uncertainties and conveyed fluid on the wave propagation behaviours of CNTs are clarified. Based on non-local strain gradient theory, Zeighampour *et al.* [37] performed an analysis of wave propagation properties in a double-walled CNT conveying fluid with slip boundary condition and shear deformable shell model. The effects of different parameters such as fluid velocity, non-local constant, and wave number were evaluated and contrasted to those in the non-classical and classical theories.

Owing to the high surface-to-volume ratio of nanostructures, the surface effects on the mechanical properties of nanostructures should not be ignored. Wang [38] explored the surface effect on the buckling behaviour of nanobeams containing internal flowing fluid. Arani and Roudbari [39] found that the wave frequencies of fluid-conveying single-walled nanotubes with surface effect are higher as compared to those without surface effect. Reza and Shahrokh [40] investigated the impacts of surface density on the nonlinear vibration of Timoshenko and Euler–Bernoulli nanobeams. Ansari *et al.* [41] carried out an investigation of the surface effect on the vibration characteristics and instability of

fluid-conveying nanoscale pipes. They summarised that surface modulus and surface residual stress have great impacts on the frequencies. Pang *et al.* [42] studied the properties of wave propagation in viscoelastic single-walled CNTs (SWCNTs) considering scale and surface effects. Taking account of surface stress effects and slip boundary conditions, Amiri *et al.* [43] analysed the wave dispersion behaviour of piezoelectric nanotubes conveying viscous fluid.

In addition, it is verified that temperature change and magnetic field could also impact upon the vibration of CNTs [44–56]. Considering the thermal and surface effects, Zhen and Zhou [57] utilised the Euler–Bernoulli beam model and non-local strain gradient theory to analyse the properties of wave propagation in fluid-conveying SWCNTs under magnetic field.

This Letter aims to study the wave propagation in an embedded fluid-conveying SWCNT subjected to multi-physical fields. The multi-physical fields include the magnetic and temperature fields. The SWCNT is assumed to be embedded in a Pasternak elastic medium. The model is developed based on combination of Timoshenko beam theory and the non-local higher-order strain gradient model proposed by Lim [14]. Utilising Hamilton's principle, the governing equations of non-local second, fourth and sixth-order strain gradient models are obtained, and these non-local higher-order strain gradient models are compared with the classical continuum model. Through the numerical simulations, the influences of inner flow, temperature change, magnetic flux vector, Pasternak medium, and small-scale and surface effects on the properties of wave propagation are discussed.

2. Constitutive equation based on the non-local higher-order strain gradient theory: As the non-local behaviours can be overlooked in the transverse direction for beam-like structures, the one-dimensional constitutive equation of the non-local elasticity can be defined by [58]

$$\sigma_{xx} - (e_0 a)^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E \varepsilon_{xx}, \quad \sigma_{xz} - (e_0 a)^2 \frac{\partial^2 \sigma_{xz}}{\partial x^2} = G \gamma_{xz} \quad (1)$$

where σ_{xx} , ε_{xx} , σ_{xz} , γ_{xz} , E and G denote the axial stress and strain (along the x -axis), the shear stress and strain, and the Young's modulus and shear modulus, respectively. $e_0 a$ represents the non-local parameter. By setting $e_0 a = 0$, the non-local constitutive relations can be reduced to the classical (local) ones.

According to the higher-order strain gradient theory [14], the above non-local stress in terms of strain gradient can be expressed as

$$\begin{aligned} \sigma_{xx} &= E \sum_{n=1}^{\infty} (e_0 a)^{2(n-1)} \frac{\partial^{2(n-1)} \varepsilon_{xx}}{\partial x^{2(n-1)}}, \\ \sigma_{xz} &= G \sum_{n=1}^{\infty} (e_0 a)^{2(n-1)} \frac{\partial^{2(n-1)} \gamma_{xz}}{\partial x^{2(n-1)}} \end{aligned} \quad (2)$$

It should be mentioned that the higher-order strain gradient theory looks like the non-local stress field theory but it fundamentally does not. This strain gradient theory is based on the strain energy density which depends on both the symmetric part of the first-order deformation gradient (the classical strain) and on the second-order deformation gradient, while the non-local stress field theory holds that the stress at a reference point is affected by the stress throughout the region of interest in an integral sense.

Equation (2) can be simplified with the first two terms, three terms or four terms retained when $e_0 a/L \ll 1$. Respectively, the non-local stress based on second-order strain gradient model can be obtained as

$$\sigma_{xx} = E \nabla^2 \varepsilon_{xx}, \quad \sigma_{xz} = G \nabla^2 \gamma_{xz} \quad (3a)$$

with $\nabla^2 = 1 + (e_0 a)^2 (\partial^2 / \partial x^2)$, the non-local stress of fourth-order strain gradient model can be given by

$$\sigma_{xx} = E \nabla^4 \varepsilon_{xx}, \quad \sigma_{xz} = G \nabla^4 \gamma_{xz} \quad (3b)$$

with $\nabla^4 = 1 + (e_0 a)^2 (\partial^2 / \partial x^2) + (e_0 a)^4 (\partial^4 / \partial x^4)$, and the non-local stress of sixth-order strain gradient model can be expressed as

$$\sigma_{xx} = E \nabla^6 \varepsilon_{xx}, \quad \sigma_{xz} = G \nabla^6 \gamma_{xz} \quad (3c)$$

with $\nabla^6 = 1 + (e_0 a)^2 (\partial^2 / \partial x^2) + (e_0 a)^4 (\partial^4 / \partial x^4) + (e_0 a)^6 (\partial^6 / \partial x^6)$. Thus, to make the equations simpler, ∇^n ($n = 2, 4, 6$) is used to represent the partial differential terms in the n th-order strain gradient model.

3. Equation of wave motion: Consider a fluid-conveying SWCNT subjected to multi-physical fields, which is illustrated in Fig. 1. The fluid is assumed to be inviscid and in uniform flow. The gravity effect of inner flow is neglected. The SWCNT with inner radius R_i , outer radius R_o and thickness h is simply supported and embedded in a Pasternak elastic medium with elastic stiffness K_w and shear stiffness K_g . It is also assumed that the SWCNT and the surrounding medium have the same magnetic permeability.

Based on the Timoshenko beam theory, the axial and transverse displacement components of the beam can be expressed as

$$U(x, z, t) = u(x, t) + z\varphi(x, t), \quad W(x, z, t) = w(x, t) \quad (4)$$

where $u(x, t)$ and $w(x, t)$ denote the axial and transverse displacement components in the mid-plane, respectively, $\varphi(x, t)$ represents the rotation of beam cross-section, and t is the time. In addition, the axial and shear strains are given by

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + z \frac{\partial \varphi}{\partial x}, \quad \gamma_{xz} = \frac{\partial w}{\partial x} + \varphi \quad (5)$$

In order to set up the dynamic equation of the tube, the energy principle and the variational approach are herein adopted. The kinetic energy of the SWCNT is given by

$$T_c = \frac{1}{2} m_c \int_0^L \left[\left(\frac{\partial u}{\partial t} \right)^2 + \left(\frac{\partial w}{\partial t} \right)^2 \right] dx + \frac{1}{2} J_c \int_0^L \left(\frac{\partial \varphi}{\partial t} \right)^2 dx \quad (6)$$

where m_c denotes the mass of the tube per unit length, and J_c is the mass moment of inertia for the tube.

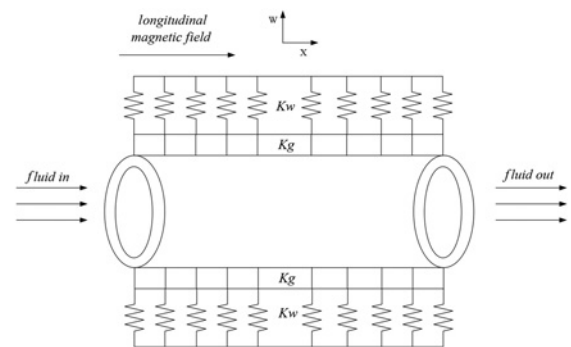


Fig. 1 Configuration of an embedded fluid-conveying SWCNT under multi-physical fields

The kinetic energy of the internal fluid is denoted by

$$T_f = \frac{1}{2} m_f \int_0^L \left[\left(\frac{\partial u}{\partial t} \right)^2 + \left(\frac{\partial w}{\partial t} + V \frac{\partial w}{\partial x} \right)^2 \right] dx + \frac{1}{2} J_f \int_0^L \left(\frac{\partial \varphi}{\partial t} \right)^2 dx \quad (7)$$

where m_f represents the mass of the fluid per unit length, J_f is the mass moment of inertia for fluid, and V is the mean flow velocity.

The strain energy of the SWCNT can be written as

$$U_s = \int_0^L \int_A (\sigma_{xx} \varepsilon_{xx} + \sigma_{xz} \varepsilon_{xz}) dA dx = \int_0^L \left(N_x \frac{\partial u}{\partial x} + M_x \frac{\partial \varphi}{\partial x} \right) dx + \int_0^L Q_x \left(\frac{\partial w}{\partial x} + \varphi \right) dx \quad (8)$$

where A is the area of cross-section of the tube. The normal resultant force N_x , transverse shear force Q_x , and bending moment M_x can be obtained by

$$N_x = \int_A \sigma_{xx} dA, Q_x = \int_A \sigma_{xz} dA, M_x = \int_A z \sigma_{xx} dA \quad (9)$$

In view of (3) and (5), (9) can be rewritten as

$$N_x = EA \nabla^n u', Q_x = \beta GA \nabla^n (w' + \varphi), M_x = EI \nabla^n \varphi' \quad (10)$$

where I is the moment of inertia of cross-section, and β is the shear correction factor which is used to compensate for the error due to the assumption of constant shear stress on the cross-section. In addition, it should be mentioned that the superscript $'$ denotes $\partial/\partial x$.

As analysed by Arani and Roudbari [39], to take into account the surface elasticity, the flexural rigidity can be obtained by $(EI)^* = EI + \pi E_s R_0^3$ with E_s the surface Young's modulus of the tube. Due to the longitudinal magnetic field, the transverse magnetic force per unit length is obtained as $F_m = \eta A H_x^2 (\partial^2 w / \partial x^2)$ with η the magnetic permeability and H_x the component of the longitudinal magnetic field vector in the x -direction. In addition, the distributed transverse force due to the effect of surface tension can be determined by $F_s = H_s (\partial^2 w / \partial x^2)$, where H_s is a constant defined as $H_s = 4\tau R_0$ with τ the residual surface tension of nanotube.

Eventually, the external work due to magnetic force, thermal force, and surrounding elastic medium can be obtained by

$$W = \frac{1}{2} \int_0^L \left[-K_w w^2 - K_g \left(\frac{\partial w}{\partial x} \right)^2 - N_\alpha \left(\frac{\partial w}{\partial x} \right)^2 \right] dx + \int_0^L (F_m + F_s) w dx \quad (11)$$

where N_α is the axial thermal force upon a beam constrained at two ends, which can be expressed as $N_\alpha = -(EA/(1-\nu))\alpha_x T$ with ν the Poisson's ratio of the tube, α_x the thermal expansion coefficient in the x direction, and T the temperature change [45].

Hamilton's principle can be expressed as

$$\delta \int_0^t (W - U_s + T_c + T_f) dt = 0 \quad (12)$$

Inserting (6)–(8), (10), and (11) into (12), taking the variation of u , w , and φ , integrating by parts and letting the coefficients of δu , δw ,

and $\delta \varphi$ vanish, the differential equations of wave motion can be derived as

$$\delta u: -EA \nabla^n u'' + (m_c + m_f) \ddot{u} + m_f V^2 w'' + m_f V \dot{w}' = 0 \quad (13a)$$

$$\delta w: -\beta GA \nabla^n (w'' + \varphi') + K_w w - K_g w'' - N_\alpha w'' - (F_m + F_s) + (m_c + m_f) \ddot{w} + m_f V^2 w'' + 2m_f V \dot{w}' = 0 \quad (13b)$$

$$\delta \varphi: -EI \nabla^n \varphi'' + \beta GA \nabla^n (w' + \varphi) + (J_c + J_f) \ddot{\varphi} = 0 \quad (13c)$$

where the symbols $'$ and $'\cdot'$ above the variables denote $\partial/\partial t$ and $\partial^2/\partial t^2$, respectively, and the superscript $'\cdot'$ denotes $\partial^2/\partial x^2$.

4. Wave propagation analysis: For the wave propagation, the displacement of SWCNTs can be expressed as

$$\{u, w, \varphi\} = \{U, W, \Phi\} e^{i(kx - \omega t)} \quad (14)$$

where U, W, Φ are the amplitude of harmonic waves, k is the wave number, ω is the excitation frequency, and $i = \sqrt{-1}$.

In this study, the longitudinal displacement is ignored. Substituting (14) into (13), a linear matrix with two unknown variables W and Φ is obtained as

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} W \\ \Phi \end{bmatrix} = 0 \quad (15)$$

where

$$\begin{aligned} a_{11} &= -(m_c + m_f) \omega^2 + 2m_f V k \omega \\ &\quad + (\beta G A k^2 \Delta^n + K_w - m_f V^2 k^2 + 4\tau r_0 k^2 + d k^2), \\ a_{12} &= -\beta G A k \Delta^n, a_{21} = \beta G A k \Delta^n i, a_{22} = -(J_c + J_f) \omega^2 \\ &\quad + (EI k^2 + \beta G A) \Delta^n, d = K_g + N_\alpha + \eta H_x^2 A \end{aligned}$$

To obtain non-singular solution of (15), the determinant of the coefficient matrix should vanish. Thus, the characteristic equation is given as

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = 0 \quad (16)$$

Substituting the expressions of a_{11} , a_{12} , a_{21} and a_{22} into (16), one can get the characteristic equation of frequency ω . Moreover, the solution of this quartic equation is complex, and its real part represents the wave frequency. According to Lu *et al.* [58], the wave mode with lower frequency is related to flexural waves, while the wave mode with higher frequency is related to shear waves. Once the circular frequency of the wave motion is calculated, the phase velocity can be given as

$$c = \omega/k \quad (17)$$

5. Numerical results and discussions: In this section, the flexural waves are discussed. The wave through a fluid-conveying SWCNT subjected to multi-physical fields is simulated considering the surface effect. The material and geometric properties of the nanotube are presented as follows:

$$\begin{aligned} E &= 1.0 \times 10^{12} \text{ Pa}, R_i = 2.16 \text{ nm}, R_o = 2.5 \text{ nm}, h = 0.34 \text{ nm}, \\ \alpha_x &= 1.6 \times 10^{-6} \text{ K}^{-1}, \nu = 0.3, \rho_c = 2300 \text{ kg/m}^3, \\ \rho_f &= 1000 \text{ kg/m}^3, \beta = 5/6, \eta = 4\pi \times 10^{-7} \end{aligned}$$

The wave propagation in SWCNTs is investigated based on the non-local higher-order strain gradient model is discussed in this section. In comparison, the behaviour of wave propagation based on classical continuum model is also given. The relationship between the phase velocity c and wave number k is illustrated in Fig. 2. The values of small-scale parameter e_0a are set to be 0.5 nm (the red lines), 1 nm (the blue lines) and 2 nm (the green lines). For the classical continuum model $e_0a=0$ nm.

The dispersion curves of classical continuum model in Fig. 2 show that the phase velocity increases first then remains constant as the wave number grows. However, it is seen from Fig. 2 that the behaviours of wave propagation based on the non-local strain gradient models are quite different from that based on the classical continuum model. It is noticed that the phase velocity of wave motion may become zero when wave numbers exceed certain values in the second and sixth-order non-local strain gradient models. These values are called critical wave numbers. In addition, these critical wave numbers k_{cr} are valued as $1/e_0a$. In other words, k_{cr} is $2 \times 10^9/\text{m}$ when $e_0a = 0.5$ nm, k_{cr} is $1 \times 10^9/\text{m}$ when $e_0a = 1$ nm, and k_{cr} is $5 \times 10^8/\text{m}$ when $e_0a = 2$ nm. Actually, as the phase velocity becomes zero, the wave stops propagating in the SWCNT. However, this does not happen in fourth-order non-local strain gradient model and the phase velocity grows sharply with the wave number increasing.

The effect of e_0a on the wave dispersion can be found in Fig. 2. At the range of smaller wave number, the bigger the small-scale parameter e_0a is, the smaller phase velocity will be. This stiffness softening effect of small-scale parameter is always found in studies using the non-local elasticity theory. However, at the range of larger wave number, the phase velocity of wave motion will increase as the value of e_0a increases. This stiffness enhancement effect cannot be reflected using the non-local elasticity theory. Hence, the non-local higher-order strain gradient model can help to find stiffness enhancement effect of small-scale parameter tested by experiments.

In what follows, the influences of flow velocity, temperature change, magnetic flux, surrounding elastic medium, and surface effect on the wave propagation through SWCNTs will be analysed based on sixth-order strain gradient model with $e_0a=2$ nm.

Fig. 3 plots the phase velocity of wave motion versus the wave number for various flow speeds. In this figure, the effect of inner flow velocity on the phase velocity is investigated. Figs. 4 and 5 show the effects of temperature increase and magnetic flux on the phase velocity of wave propagation, respectively. The influences of the shear stiffness K_g and elastic stiffness K_w of surrounding elastic medium on the phase velocity of wave motion are illustrated in Figs. 6 and 7, respectively. It should be pointed out that when considering one external factor, other external factors mentioned

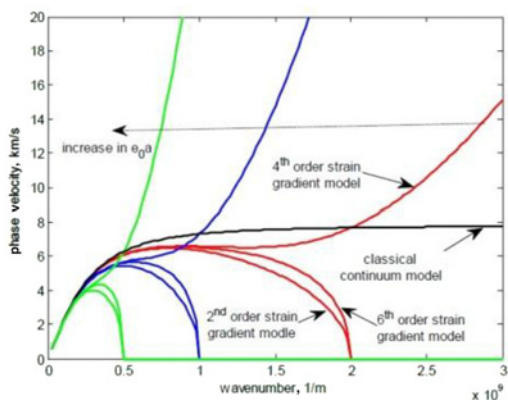


Fig. 2 Wave phase velocity based on non-local different order strain gradient models and classical continuum model ($V=T=H_x=K_w=K_g=E_s=\tau=0$)

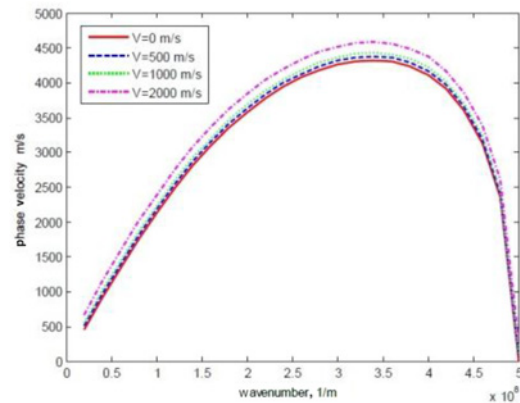


Fig. 3 Phase velocity versus wavenumber for various inner flow velocity ($e_0a = 2$ nm)

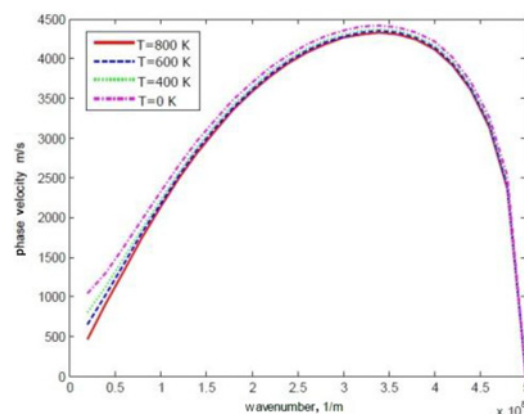


Fig. 4 Phase velocity versus wavenumber for different values of temperature change ($e_0a = 2$ nm)

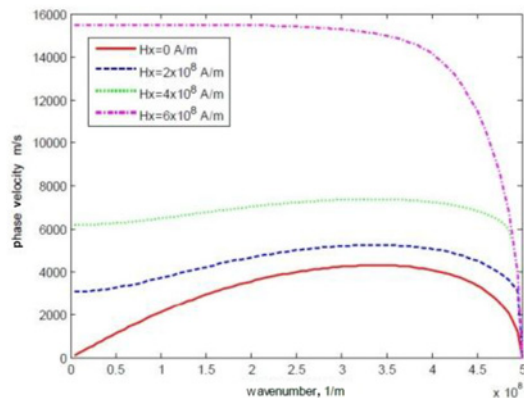


Fig. 5 Phase velocity versus wavenumber for various magnetic field flux ($e_0a = 2$ nm)

above are neglected. For example, when investigating the effect of temperature change on wave propagation of SWCNTs, $V=H_x=K_w=K_g=E_s=\tau=0$ is taken.

It is found from Figs. 3–7 that in addition to the temperature increase, the influences of inner flow velocity, magnetic field flux, and shear stiffness and elastic stiffness of surrounding elastic medium on the phase velocity of wave propagation are similar. The phase velocity becomes larger as the values of inner flow velocity, magnetic field flux, and shear stiffness and elastic stiffness of surrounding medium increase, whereas it gets smaller

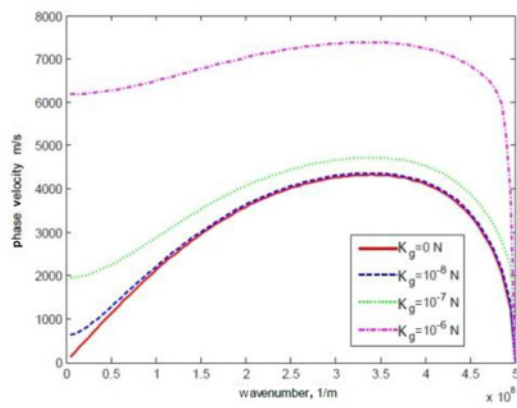


Fig. 6 Phase velocity versus wavenumber for various shear stiffness of surrounding medium ($e_0a = 2 \text{ nm}$)

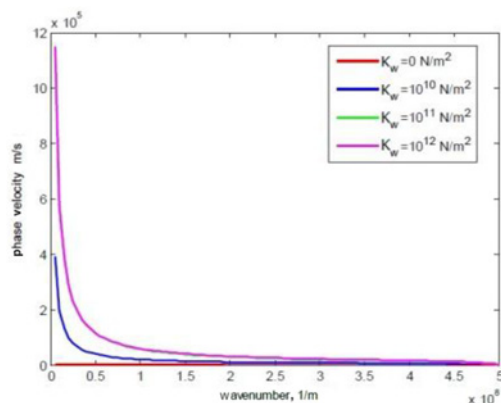


Fig. 7 Phase velocity versus wavenumber for various elastic stiffness of surrounding medium ($e_0a = 2 \text{ nm}$)

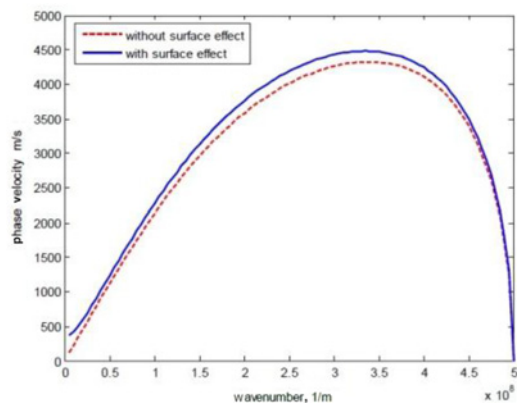


Fig. 8 Phase velocities with and without surface effect ($e_0a = 2 \text{ nm}$)

as the value of temperature increase becomes larger. In other words, except the temperature increase, the increases of magnetic field flux, and shear stiffness and elastic stiffness of the surrounding medium can make the nanotube stiffer. In addition, it is noted that the influence of temperature change on wave propagation is much less significant than other factors.

With $E_s = 35.3 \text{ N/m}$ and $\tau = 0.31 \text{ N/m}$, Fig. 8 illustrates the surface effect on the phase velocity of wave propagation in the SWCNT. It is seen that the phase velocity is larger with surface effect than without surface effect at the same wave number, which implies that the surface effect may stiffen SWCNTs.

6. Conclusions: This Letter investigates the wave propagation in embedded fluid-conveying SWCNTs under multi-physical fields based on the non-local higher-order strain gradient model with surface effect included. Utilising Timoshenko beam theory and Hamilton's principle, the governing equations of wave motion in CNTs are derived. The solution for the phase velocity of wave motion is obtained. The numerical simulations are conducted to compare the results on the basis of different order non-local strain gradient models. The scale and surface effects and influences of variable external factors including inner flow, surrounding medium, temperature field, and magnetic field on the wave dispersion are investigated.

It is concluded that the behaviours of wave propagation based on the non-local higher-order strain gradient models are quite different from those based on the classical continuum model. The solutions for wave propagation obtained by these non-local higher-order strain gradient models can not only consider the stiffness softening effect but also reflect the stiffness enhancement effect of scale parameter observed by experiments. Compared with the scale and surface effects and impacts of inner flow, magnetic field and surrounding medium on the wave dispersion, the influence of temperature change is very limited.

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