Nonlinear vibration of fluid conveying cantilever nanotube resting on visco-pasternak foundation using non-local strain gradient theory

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Published in Micro & Nano Letters; Received on 15th July 2019; Revised on 25th September 2019; Accepted on 9th December 2019

Frequency analysis and forced vibration response of fluid conveying viscoelastic nanotubes that resting on nonlinear visco-pasternak foundation under magnetic field using size-dependent non-local strain gradient theory are considered in this study. It is supposed that the nanotube is modelled as cantilever type beam and subjected to a harmonic load. The material property of the nanotube is modelled by Kelvin–Voigt viscoelastic constitutive relation and slip boundary conditions of nanotube conveying fluid are taken into account. Extended Galerkin method is used to obtain the nonlinear differential equation of the motion and the multiple time-scales method is utilised to investigate the primary vibration resonance of the nanotube. Firstly, the frequency analysis is performed on the linear system and the effects of foundation coefficients on the natural frequency are investigated at several flow velocities. Moreover, the resonance properties of the system are solved in closed form and analysed from the frequency-response curves, and then the effects of the non-local parameter, length scale parameter and magnetic field are fully investigated. In this case, non-local parameter, length scale parameter and foundation coefficients are highly influential on the frequency response of the considered system.

1. Introduction: Application of nanotubes is broadly detected in mechanical and biological structures in last decade, considering specific geometrical, mechanical and natural features [1, 2]. One of the major efficient and decisive usage of such structures is known as nano-transporter for fluid flows which are found in medical science likewise mechanical systems. Conveying fluid nano-system exist in numerous devices [3] and have various requests as sensors, resonators, drug delivery, filtration devices, nano-systems for tumour targeting and nanodevices for diagnosis of serious diseases. Thus, vibration and dynamic analysis of fluid conveying nanotubes is a crucial subject and has involved excessive attention of scientists.

As instances of applying classic continuum theories, Yoon and Ru [4] investigated the effect of moving fluid on free vibrations and stability of carbon nanotubes (CNTs) based on classical beam model. They showed that fluid velocity affects resonant frequencies of CNTs, they also evaluated role of the elastic substrate on results. Chang and Lee [5] employed Timoshenko beam modelling for analysis of CNT having fluid which freely vibrates. Ghavanloo *et al.* [6] analysed both vibration and stability of fluid conveying nanotube rested on linear visco-elastic foundation. Discounting the small-size effects in a nano-scale problem might cause inaccurate results. To resolve it, several size-dependent non-classical continuum theories such as non-local elasticity theory (NET) [7], modified couple stress theory (MCST) [8] and modified strain gradient theory (MSGT) [9] have been anticipated.

Vibration of a viscous fluid-conveying single-walled CNT (SWCNT) surrounded by an elastic medium was considered by Lee and Chang [10]. It has been reported that the effect of non-local parameter on the frequency becomes significant as the flow velocity of the viscous fluid decreases. Also, Lee and Chang [11] studied the small-scale effects on the vibration characteristics of fluid-conveying CNTs. The NET was applied to nonlinear vibration analysis of fluid-conveying CNTs by Soltani *et al.* [12]. Structural instability of CNTs, including static and dynamic states, resting on the viscoelastic foundation which is affected by distributed tangential load based on the NET was studied by Kazemi-Lari *et al.* [13]. Vibration and stability of fluid-conveying SWCNTs embedded in biological soft tissue were considered based on the NET by Hosseini *et al.* [14]. Arani *et al.* [15] studied vibration and instability analysis of double-walled (DW)-CNT with following fluid subjected to a magnetic field using NET. Bahaadini and Hosseini [16] presented free vibration and instability investigation of cantilever viscoelastic CNTs conveying fluid by consideration of slip condition using NST. Chang [17] investigated the dynamic behaviour and vibration of fluid-conveying DW-CNTs embedded in a biological soft and viscous tissue and subjected to a moving load by considering effects of the geometric nonlinearity and the van der Waals force [18]. Closed-form expressions were obtained for the large-amplitude vibration by Askari *et al.* [19] using Galerkin method. Besides effects of moving nanoparticle on dynamic response of nanotubes were analysed by Arani and Roudbari [20] and Roudbari *et al.* [21–23].

In micro-scale structure, Tang *et al.* [24] studied three-dimensional nonlinear vibration behaviour of curved micro-tubes conveying fluid via MCST. Yin *et al.* [25] developed a size-dependent model based on MSGT to investigate the vibration of microscale pipes conveying fluid. Nonlinear vibration analysis of functionally graded (FG) micropipes conveying fluid via SGT has been carried out by Setoodeh and Afrahim [26]. Hosseini and Bahaadini [27] analysed the vibration and flutter instability of micropipe conveying fluid based on the MSGT and showed that the predicted frequencies and flutter velocities are size-dependent. The NET and SGT describe two entirely different physical features of materials and structures at small scale; the NET does not contain non-locality of higher-order stress while the SGT only considers local higher-order strain gradients without non-local effects. Though, researches show that the individual NET or SGT has some limits on identifying the size-dependent stiffness of nano-beams [28, 29]. Lim *et al.* [30] combined the NET with SGT and resulted in a higher-order non-local strain gradient theory (NSGT), to evaluate the true effect of the two length scale parameters on the responses of small-scale structures, the result is consistent with results of molecular dynamics [31]. Several studies considered the mechanical behaviour of small-scaled structures based on the NSGT, for instance nonlinear free vibration of FG nanobeam has been studied by Simcsek [32].

This work analytically investigates the vibration of CNT conveying nanoflow and resting on nonlinear foundation under magnetic field and external harmonic load via NSGT. Governing equations of system is developed using Hamilton principle. Then extended Galerkin (EG) method and multiple scale method is utilised to find the nonlinear frequency response.

2. Modelling of flow-induced vibration in nanotube using NSGT: According to the NSGT presented in [30], the constitutive relation for the normal stress σ_{xx} and strain ε_{xx} in nanobeams is written as

$$\left(1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right)\sigma_{xx}=\left(1-l^2\frac{\partial^2}{\partial x^2}\right)E\varepsilon_{xx} \tag{1}$$

where e_0a and l are the non-local and strain gradient length scale parameters, respectively. They show the significance of the non-local elastic stress and strain gradient field, respectively. Considering the internal damping for viscoelastic nanotube, based on the Kelvin-Voigt viscoelastic model the part $E\varepsilon_{xx}$ can be substituted with $E\left(\varepsilon_{xx}+g\partial\varepsilon_{xx}/\partial t\right)$ where g is the material damping coefficient. Consecutively, (1) is rewritten as [20–23, 30, 33]

$$\left(1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right)\sigma_{xx}=E\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\left(\varepsilon_{xx}+g\frac{\partial\varepsilon_{xx}}{\partial t}\right) \tag{2}$$

Fig. 1 represents the system of fluid conveying nanotube resting on visco-pasternak substrate, exposed to excitation in magnetic field.

The governing equation of motion for this system can be stated as

$$\begin{aligned}&EI\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\left(1+g\frac{\partial}{\partial t}\right)\frac{\partial^4 w(x,t)}{\partial x^2}\\&+\left(1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right)[m_c\ddot{w}+F_f-(F_l+R)-F_{\text{exc}}]=0\end{aligned} \tag{3}$$

where $w(x, t)$ represents lateral vibration of nanobeam, m_c is the mass per length of the nanotube, F_f, F_l, R and F_{exc} are the external force induced by the flow, Lorentz force due to magnetic field, external force due to substrate and harmonic excitation, respectively

$$F_f=m_f\left(\ddot{w}(x,t)+2\,v\,\frac{\partial\dot{w}(x,t)}{\partial x}+\hat{\alpha}v^2\frac{\partial^2 w(x,t)}{\partial x^2}\right) \tag{4}$$

In which m_f is the density of fluid, v is the mean flow velocity of fluid and $\hat{\alpha}=1$ for a turbulent regime with uniform flow profile. Besides assuming continuously connected foundation, the external force due to surrounding visco-Pasternak foundation can be written as

$$R=K_Lw(x,t)+K_{NL}w(x,t)^3+C_D\dot{w}(x,t)-K_G\frac{\partial^2 w(x,t)}{\partial x^2} \tag{5}$$

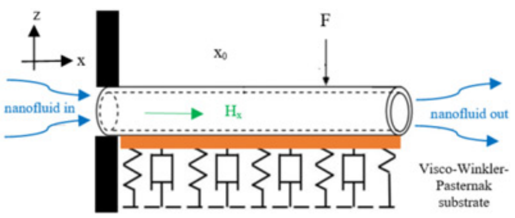

Fig. 1 *Schematic model of system*

where K_L, K_{NL}, C_D and are the linear and nonlinear stiffness, damping coefficients and shear stiffness of foundation, respectively. Also Lorentz force per length can be defined as [33]

$$F_l=\eta_0\,H_x^2A\,\frac{\partial^2 w(x,t)}{\partial x^2}=H\frac{\partial^2 w(x,t)}{\partial x^2} \tag{6}$$

where η_0 denotes the magnetic permeability, H_x is the component of the longitudinal magnetic field vector exerted on the nanotube in the x-direction. Replacing (4)–(6) in main (3), we have

$$\begin{aligned}&\left(1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right)\begin{bmatrix}(m_c+m_f)\ddot{\overset{+}{w}}2m_f\,\text{VCF}\,(V_{ns})+m_f\,\text{VCF}^2\,(V_{ns})^2\dfrac{\partial^2 w}{\partial x^2}\\+K_Lw+K_{NL}w^3+C_D\dot{w}-(K_G+H)\dfrac{\partial^2 w}{\partial x^2}\end{bmatrix}\\&+EI\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\left(1+g\frac{\partial}{\partial t}\right)\frac{\partial^4 w}{\partial x^4}\\&=\left(1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right)F\,\delta(x-x0)\text{Cos}\Omega t\end{aligned} \tag{7}$$

For consideration of slip effect, the relative $v=\text{VCF}\times V_{\text{ns}}$ has been used in above where VCF is the average velocity correction factor which can be assumed as $\text{VCF}=V/V_{ns}=(1+a_k\,Kn)\times[1+4((2-\chi)/\chi)(Kn/(1+Kn))]$ [34], where V_{ns} is fluid velocity with non-slip assumption, Kn is the Knudsen number and $\chi=0.7$. In order to simplify the parametric analysis, the following dimensionless parameters are defined:

$$\begin{aligned}&X=\frac{x}{L};\; W=\frac{w}{L};\; T=\frac{t}{L^2}\sqrt{\frac{EI}{(m_c+m_f)}};\; \tau=\frac{g}{EL^2}\sqrt{\frac{EI}{(m_c+m_f)}};\; \beta=\frac{m_f}{(m_c+m_f)}\\&u=L\,V_{ns}\sqrt{m_f/EI};\; C=\frac{C_DL^2}{\sqrt{EI(m_c+m_f)}};\; \mu^2=\frac{(e_0a)^2}{L^2};\; \theta^2=\frac{l^2}{L^2}\\&f=\frac{F}{EI};\; \bar{H}=\frac{H\,L^4}{EI};\; \overline{K_G}=\frac{K_G\,L^4}{EI};\; K=\frac{K_LL^4}{EI};\; \overline{K_{NL}}=\frac{K_{NL}L^6}{EI}\end{aligned} \tag{8}$$

3. Solution procedure: By employing EG approximate solution method, (7) is converted into a set of ordinary differential equations. The key property of the EG is selecting a finite dimension subspace of the Hilbert space (trial function space) for approximating the solution and imposing orthogonality relation of the obtained error to a finite dimension space (test function space) which is similar to the trial space. The following expansion which is expanded in a series of modes is assumed for the dimensionless lateral deflection of nanotube

$$W(X,T)=\sum_{r=1}^{N}\varphi_r(X)\eta_r(T) \tag{9}$$

In which $\varphi_r(X)$ are the unknown generalised coordinates and $\eta_r(T)$ are the eigenmodes of nanotube in EG method. Considering a cantilever nanotube, one can use the resulting orthogonal functions

$$\begin{aligned}\varphi_r(X)&=\text{Cosh}(\lambda_rX)-\text{Cos}(\lambda_rX)\\&-\left(\frac{\text{Sinh}\lambda_r-\text{Sin}\lambda_r}{\text{Cosh}\lambda_r+\text{Cos}\lambda_r}\right)(\text{Sinh}(\lambda_rX)-\text{Sin}(\lambda_rX))\end{aligned} \tag{10}$$

The value of λ_r can be obtained by solving the equation: $\cosh\lambda_r\times\cos\lambda_r=-1$. Substituting (9) and (10) into non-dimensional governing equations (7) multiplying the resultant

doi: 10.1049/mnl.2019.0420

equations by the corresponding eigenfunctions, and integrating with respect to X from 0 to 1, leads to the following discretised form for the equation:

$$
\begin{aligned}
&[m]\{\ddot{\eta}(T)\}+[c]\{\dot{\eta}(T)\}+[k]\eta(T)+[r]\eta(T)^3=\{f(T)\}\\
&[m]=\left[\int_0^1\varphi(X)^2dX-\mu^2\int_0^1\varphi(X)''\varphi(X)\mathrm{d}X\right].\\
&[c]=\tau\left[\int_0^1\varphi(X)^{(4)}\,\varphi(X)dX-\theta^2\int_0^1\varphi(X)^{(6)}\,\varphi(X)\mathrm{d}X\right]\\
&\quad+C\left[\int_0^1\varphi(X)^2\mathrm{d}X-\mu^2\int_0^1\varphi(X)''\varphi(X)\mathrm{d}X\right]\\
&\quad+2\sqrt{\beta}(vcf\times u)\left[\int_0^1\varphi(X)'\,\varphi(X)\mathrm{d}X-\mu^2\int_0^1\varphi(X)^{(3)}\varphi(X)\mathrm{d}X\right]\\
&[k]=\left[\int_0^1\varphi(X)^{(4)}\,\varphi(X)\mathrm{d}X-\theta^2\int_0^1\varphi(X)^{(6)}\,\varphi(X)\mathrm{d}X\right]\\
&\quad+K\left[\int_0^1\varphi(X)^2\mathrm{d}X-\mu^2\int_0^1\varphi(X)''\varphi(X)\mathrm{d}X\right]\\
&\quad+(vcf\times u)^2\left[\int_0^1\varphi(X)''\varphi(X)\mathrm{d}X-\mu^2\int_0^1\varphi(X)^{(4)}\varphi(X)\mathrm{d}X\right]\\
&\quad-(\overline{K_G}+\overline{H})\left[\int_0^1\varphi(X)''\,\varphi(X)\mathrm{d}X-\mu^2\int_0^1\varphi(X)^{(4)}\varphi(X)\mathrm{d}X\right]\\
&[r]=\overline{K_{NL}}\left[\int_0^1\varphi(X)^4\mathrm{d}X-6\mu^2\int_0^1[\varphi(X)'']^2\varphi(X)^2\mathrm{d}X-3\int_0^1\varphi(X)^3\varphi(X)'\mathrm{d}X\right]
\end{aligned}\tag{11}
$$

For the frequency analysis, the nonlinear and excitation part of the main relation is ignored. Considering time harmonic motion, where $\eta(t)=\eta e^{i\omega t}$ and substituting this form in above relation, one can obtain an equation $Det(-[m]\omega^2+i[c]\omega+[k])=0$ that generally have two complex conjugate roots, real part of roots are natural frequency.

The method of multiple time scales is employed to study the nonlinear forced vibration (7). A small parameter $\grave{o}$ is presented and an approximate solution can be represented by an extension in terms of different time scales in the subsequent formula

$$\eta(t)=\eta_0(T_0,T_1,\ldots)+\grave{o}\eta_1(T_0,T_1,\ldots)+\mathrm{O}(\grave{o}^2)\tag{12}$$

where $T_0=t$, $T_1=\varepsilon t$. Besides for fundamental mode, (7) can be rewritten in the following form:

$$\ddot{\eta}+\omega_0^2\eta=-2\grave{o}\bar{\mu}\dot{\eta}-\bar{\alpha}\grave{o}\eta^3+\bar{f}\grave{o}\cos\Omega t\tag{13}$$

Also it follows that the derivatives with respect to t become expansions in terms of the partial derivatives with respect to the T_n, where $\partial/\partial T_n=D_n$ $(n=0,1,2,\ldots)$, according to

$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}=(D_0+\grave{o}D_1+\grave{o}^2D_2+\ldots);\\
&\frac{\mathrm{d}^2}{\mathrm{d}t^2}=(D_0^2+2\grave{o}D_0D_1+\grave{o}^2D_1^2+\ldots)
\end{aligned}\tag{14}
$$

One can write $\Omega=\omega_0+\grave{o}O$ that O is detuning parameter and gives the nearness of Ω to ω_0. Substituting (14) into (13) and comparing the coefficients of $\grave{o}^i$ to zero

$$
\begin{aligned}
&D_0^2\,\eta_0+\omega_0^2\eta_0=0,\\
&D_0^2\,\eta_1+\omega_0^2\eta_1=-2D_0D_1\eta_0-2\bar{\mu}D_0\eta_0-\bar{\alpha}\eta_0^3+\bar{f}\cos\Omega t
\end{aligned}\tag{15}
$$

We shall seek the general solution of (15) in the formula

$$\eta_0=Ae^{i\omega_0T_0}+\bar{A}e^{i\omega_0T_0}+C.C\tag{16}$$

where A is a function of T_1 and $\bar{A}$ is the complex conjugate of A. Substituting (16) into (14), and setting all the resonant terms to zero to avoid secular terms in the result offers the solvability state

$$-2i\omega_0D_1Ae^{i\omega_0T_0}-2i\omega_0\bar{\mu}Ae^{i\omega_0T_0}-3\bar{\alpha}A^2\bar{A}e^{i\omega_0T_0}+1/2\bar{f}e^{i\omega_0T_0}e^{iOT_1}=0\tag{17}$$

Equation (16) is solved in introducing the polar form $A=0.5\,\alpha\,(T_1)e^{i\beta(T_1)}$, where the α is amplitudes of oscillation. Replacing this polar form into (17), identifying real and imaginary parts, one obtains a dynamical system. This system can be converted to an independent one when defining $OT_1-\beta=\gamma$. Separating the real and the imaginary parts gives the frequency–response relation $O-\alpha$ for the steady-state response as

$$\bar{\mu}^2+\left(O-\frac{3\bar{\alpha}}{8\omega_0}\alpha^2\right)^2=\frac{\bar{f}^2}{4\alpha^2\omega_0^2}\tag{18}$$

4. Results, validation and discussions

4.1. Free vibration, validation: In order to ensure about the accuracy of calculations, we compare the obtained frequencies of vibration by the proposed model for a viscous cantilever nanotube conveying fluid flow with those of other researchers. For this purpose, the obtained frequencies in the linear model are verified with those of [16] when $K=0$, $C=0$ and $l=0$. Variation of real and imaginary parts of fundamental eigenvalue by fluid velocity has been validated by Bahaadini and Hosseini [16] in Fig. 2. All curves are according to [16] exactly. Variation of real and imaginary parts of fundamental eigenvalue by velocity has been plotted in Fig. 3 for different values of K and C.

By increasing K, the total stiffness of system increased so the imaginary part (vibration frequency) and u_{cr} increased which u_{cr}

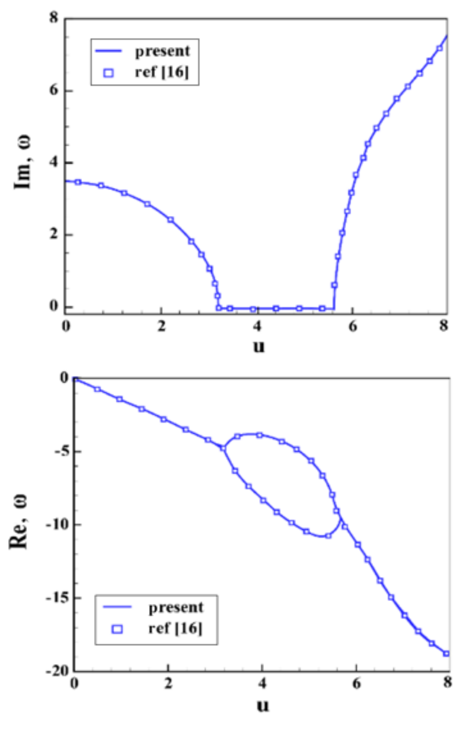

Fig. 2 *Validation of real and imaginary parts of eigenvalue by [16]*

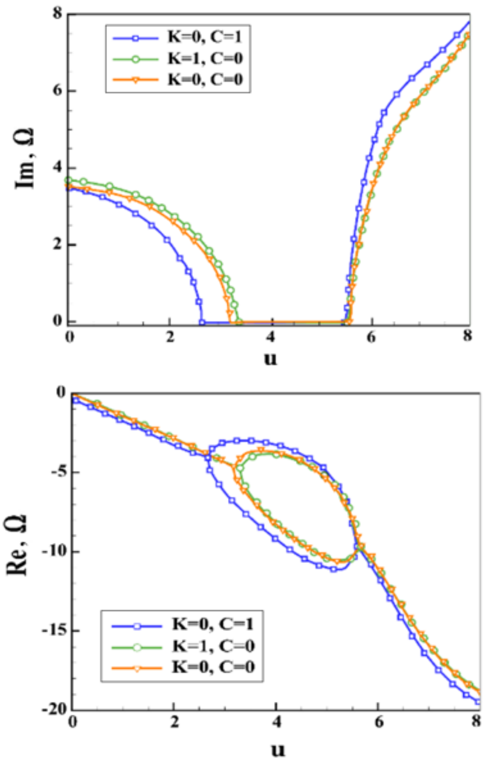

Fig. 3 *Variation of eigenvalue by velocity for different value of K, C*

denotes the critical dimensionless flow velocity at which $\mathrm{IM}(\Omega) = 0$ and the system becomes unstable through a bifurcation. Also, the real part (damping ratio) curve did not change at lower fluid speed but divergence at higher speeds. Besides by increasing C, total damping of system increased, the imaginary part and u_{cr} decreased. Also, the real part increased and divergence at lower speeds. Variation of imaginary part of first and second eigenvalues of nanotube has been plotted in Fig. 4 ($\theta = 0$, $\tau = 0.001$, $Kn = 0.1$, $\mu = 0.1$, $H = 0$).

4.2. Forced vibration: In this section, the nonlinear size-dependent forced vibration behaviour of a nano-pipe conveying fluid is investigated in vicinity of fundamental frequency and in subcritical domain, i.e. $u \le u_{cr}$. The origin of nonlinear equations is the foundation medium, which causes cubic nonlinearities in the differential equation. The influence of main parameters on the frequency response of system is taken into account and a parametric sensitivity analysis has been performed. As seen in Figs. 5–10, the nonlinear behaviour of the vibration is hardening.

Fig. 5 represents the effects of nonlinear coefficient of foundation on the frequency response of the system. The numerical results show that for each curve there are two limit point bifurcations in

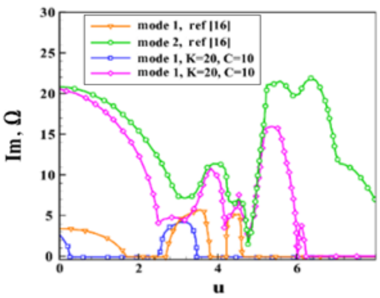

Fig. 4 *Effect of viscoelastic foundation on first and second frequencies*

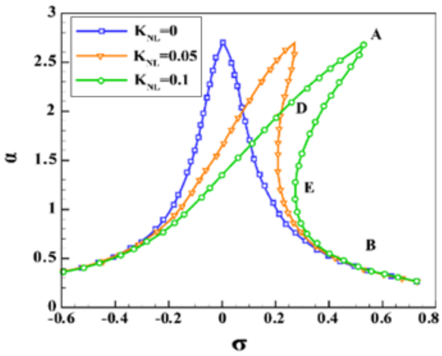

Fig. 5 *Effect of nonlinear coefficient on the frequency response*

the nonlinear resonant behaviour of system; the first one is related to a jump from the high amplitude motion to the lower amplitude one (for ex. A-B) and the second one for a reverse state (for ex. E-D). It is obvious that increasing K_{NL} causes higher range of frequency, the response becomes harder and nonlinearity increases, though for small values of K_{NL} the system tends to be linear, but this parameter does not change the max. value of vibration amplitude known as α_{peak}.

According to Fig. 6, α_{peak} increases with increasing the non-local parameter. Furthermore, an increase in the value of μ would create a higher nonlinearity in the system.

It could be seen from Fig.7 that the α_{peak} would decrease by increasing the magnetic field. In general, the effect of the magnetic field is to make the nanotube treats stiffer. It was found that increasing the H would result into an increase of the stiffness, so including this parameter in the system can provide more capacity for the nanotubes to convey fluid with a very high velocity. Also, it can be highlighted that for higher magnetic fields, the frequency range in response would be wider.

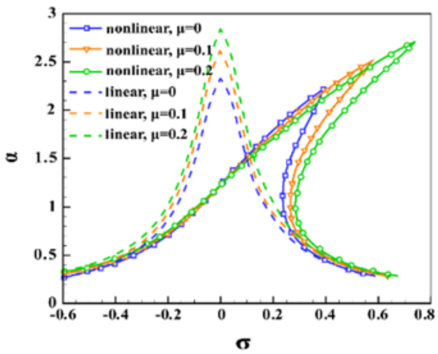

Fig. 6 *Effect of non-local parameter on the frequency response*

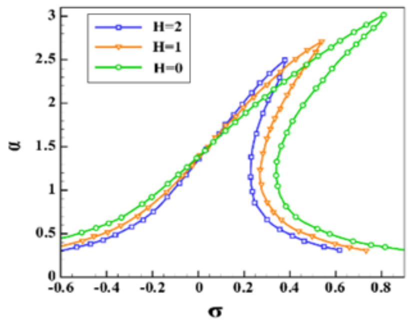

Fig. 7 *Effect of magnetic field on the frequency response of nanotube*

Micro & Nano Letters, 2020, Vol. 15, Iss. 3, pp. 181–186
doi: 10.1049/mnl.2019.0420

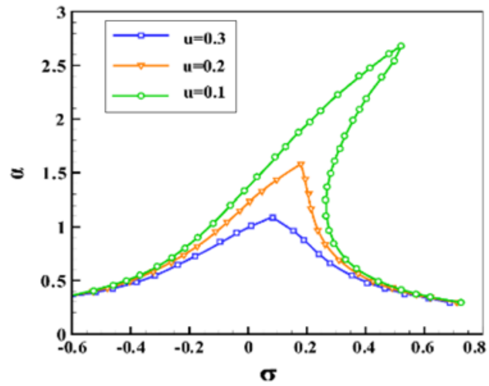

Fig. 8 *Effect of flow velocity on the frequency response of nanotube*

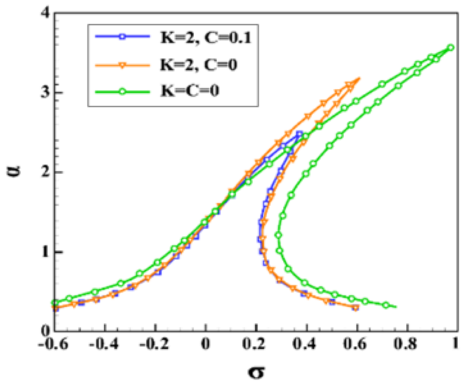

Fig. 9 *Effect of visco-pasternak coefficient on the frequency response*

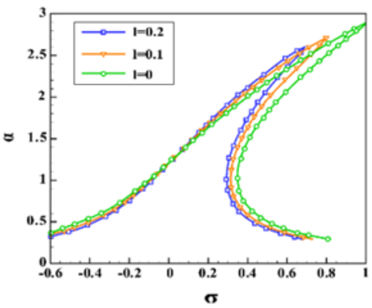

Fig. 10 *Effect of small-scale parameter on the frequency response*

Fig.8 shows increase of the velocity has opposite influence on the amplitude of vibration and has dampening effect on system but this parameter has no impact on the nonlinearity.

According to Fig. 9, the viscosity parameter leading to losses more energy and the α_{peak} decreased greatly and occurs at lower frequency. Besides, higher values of linear stiffness result into a lower softening nonlinearity as well as lower amplitude of oscillation.

As can be seen in Fig. 10, by decreasing the small-scale parameter (l) as the flexural rigidity is decreased, α_{peak} and hardening nonlinearity behaviour increased, so the hardening behaviour of system has a descending trend with respect to the size dependency.

5. Conclusions: Using NSGT, the free and forced vibration of a fluid conveying cantilever nanotube in magnetic field resting on nonlinear foundation subjected to periodic excitation are discussed. Hence, a parametric study has been carried out to explore the influence of different parameters such as strain gradients and stress nonlocality on the vibration frequency and also the primary resonance of the system.

It is found that the nonlinear resonant behaviour of the nanotube with and without size dependency is a hardening type due to nonlinear foundation (K_{NL}). Furthermore, the hardening behaviour of system has a descending trend with respect to the small-scale parameter. The obtained results indicate that the linear stiffness K increased the system frequency while the viscosity C had an inverse effect. The value of u_{cr} is raised with increasing K or decreasing C. It can be deduced that for a prescribed non-local parameter, the α_{peak} increases when the velocity parameter is raised, and also the rate of increase has an ascending trend with respect to the small-scale parameter. Also, in the nonlinear vibration analysis, it has been revealed that: (i) existence of magnetic field would result into a decrease in nonlinearity and α_{peak}, (ii) by increasing the small-scale parameter, α_{peak} and hardening behaviour of the vibration, decreased, (iii) increase of fluid velocity has dampening effect, (iv) the viscosity parameter of nanotube and substrate leading to losses of more energy so the α_{peak} and the sharpness of curves decreased significantly. It was confirmed that the fluid velocity, magnetic field and substrate parameters can be considered as control parameters for the vibration of the fluid flow conveying CNT. Present results could be useful for the fabrication of nanomechanical devices using fluid-conveying CNT as a future work. Also, vibration behaviour of the system and max. response within the frequency band can be predicted to avoid from failure.

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doi: 10.1049/mnl.2019.0420