

# Propagation of longitudinal waves in a single-walled carbon nanotube under thermoelastic damping

Mahmoud Mohamed Selim<sup>1,2</sup> ✉

<sup>1</sup>Department of Mathematics, College of Science and Humanities in Al-Aflaj, Prince Sattam bin Abdul-Aziz University, Saudi Arabia, Al-Aflaj 710-11912, Saudi Arabia

<sup>2</sup>Department of Mathematics, Suez Faculty of Science, Suez University, Egypt

✉ E-mail: m.selim@psau.edu.sa

Published in Micro & Nano Letters; Received on 25th December 2019; Revised on 14th May 2020; Accepted on 19th May 2020

The propagation of the longitudinal waves in a single-walled carbon nanotube (SWCNT) considering the effects of thermoelastic damping is investigated in this Letter. The cylindrical shell is considered and the Donnell–Mushtari–Vlasov approach is utilised. The thermoelasticity governing equations are derived based on thin shell theory. Analytical expressions for the quality factor (Q-factor) of thermoelastic damping are presented for plane stress and strain conditions. The numerical results are presented to investigate the influence of some parameters on Q-factor of thermoelastic damping, such as the length of a nanotube, nanotube thickness and the ambient temperature. In addition, the dispersion curve of the longitudinal waves propagation in the SWCNT under the effects of thermoelastic damping is plotted. It can be seen that the Q-factor is proportional to the length and thickness of the SWCNT. It means that the increase in both length and radial thickness of the nanotube makes the Q-factor increase. For the final observation, it should be noted that the ambient temperature has an inverse effect on the Q-factor for the SWCNT. It means that the increase in ambient temperature makes the Q-factor decrease. These results can be helpful in the design of resonators and nanodevices.

**1. Introduction:** Nowadays, advances in science and technology have made the nanoresonators widely used in nanoelectromechanical systems (NEMSs) and some devices such as nanosensors, atomic force microscopy and nanoactuators. Therefore, due to the lack of permanent access to energy sources and to reduce the energy consumption of such systems and accordingly enhance their performance to the highest quality, surveying the sources of energy dissipation in nanoscales is essentially important. The quality performance of resonators is generally measured by the quality factor (Q-factor) as a well-known dimensionless parameter, which defined as the ratio of the kinetic and potential energy of the system to dissipated energy by various damping mechanics. In general, the energy dissipation mechanisms can be categorised into two groups, intrinsic and extrinsic [1]. Thermoelastic damping is a dominant source of intrinsic damping in microelectromechanical systems [2] and NEMSs [3] working under vacuum condition. In contrast to intrinsic loss, extrinsic loss such as the surrounding medium damping can be weakened by appropriate design and operating conditions [4]. Zener [5–7] has done the first study of the effects of thermoelastic damping. He identified the existence of thermoelastic damping as a significant dissipation mechanism in flexural resonators. Since Zener's work, many studies relating to thermoelastic vibration analysis have been reported [8–18]. Landau and Lifshitz [19] studied the attenuation of thermoelastic vibration and provided an exact expression for the attenuation coefficient. Evoy *et al.* [20] and Duwel *et al.* [21] showed experimentally that the thermoelastic damping is a dominant source of damping in nanoelectromechanical devices. Hamidi *et al.* [22] presented the exact solution on gold microbeam with thermoelastic damping via generalised Green–Naghdi and modified couple stress theories. Yasumura *et al.* [23] presented the effects of the thermomechanical noise on the quality factor for a single-crystal silicon cantilever.

Although the thermoelastic damping in structures is not a new topic, the problems have been re-attracted increasing attention more recently as the rapid development of micro- and nanotechnology. However, only a limited portion of the literature is concerned with the vibration and buckling analysis of carbon

nanotubes considering the thermal effects [24–28]. Zhang *et al.* [29] presented the thermal effects on the vibration of double-walled carbon nanotubes using thermal elasticity. Wang *et al.* [30] presented the thermal effects on the vibration and instability of conveying fluid carbon nanotubes (CNTs) based on thermal elasticity mechanics. Hsu *et al.* [31] analysed the frequency of chiral single-walled CNT (SWCNT) subjected to thermal vibration and using Timoshenko beam model. Ni *et al.* [32] conducted an analysis of buckling behaviour of SWCNTs subjected to axial compression under a thermal environment. Yao and Han [33] performed a buckling analysis of multi-walled CNTs (MWNTs) subjected to torsional load under temperature field. Based on the Euler–Bernoulli beam, Tang and Yang [34] presented a novel model of fluid-conveying nanotubes made of bi-directional functionally graded materials to investigate the dynamic behaviours and stability of the nanotube. To study the non-linear free vibration of the bi-directional (2D) functionally graded materials (FGMs), Tang *et al.* [35] presented a novel model of Euler–Bernoulli beams. Based on thermal elasticity mechanics, Zhang *et al.* [36] developed elastic multiple column model for column buckling of MWNTs with large aspect ratios under axial compression coupling with temperature change. They concluded that at low or room temperature, the buckling strain, including thermal effect is larger than that excluding the thermal effect and increases with the increase of temperature change.

Non-linear bending, buckling and vibration of bi-directional functionally graded nanobeams have been studied by Tang *et al.* [37, 38]. A novel model of fluid-conveying nanotubes made of bi-directional functionally graded materials was presented by Tang and Yang [39] for investigating the dynamic behaviours and stability of the graded nanotubes. The post-buckling behaviour and non-linear vibration of a fluid-conveying pipe composed of a functionally graded material were analytically studied by Tang and Yang [40]. The buckling and free vibration of a Euler–Bernoulli beam composed of 2D FGMs in the thermal environment were analysed by Tang *et al.* [41]. Alibeigloo *et al.* [42] investigated the thermoelastic analysis of functionally graded CNT-reinforced composite plate using the theory of elasticity.

Hoseinzadeh and Khadem [43, 44] studied the thermoelastic vibration and damping analysis of double-walled CNTs based on shell theory. Tang and Ding [45] presented the non-linear hygrothermal dynamics of a bi-directionally functionally graded beam with coupled transverse and longitudinal displacements. The effects of thermoelastic damping on transverse waves propagating in an SWCNT have been investigated by Selim [46]. To the best of the author's knowledge, the effects of the temperature and thickness on the vibration of the SWCNT using thermoelasticity theory neglecting the nanotube length accomplished in previous works, while the experimental researches reveal that the effect of the length of the nanotube is important for vibrational analysis. To fill this gap, the vibrational behaviour of the SWCNT considering the effects of temperature and thickness as well as nanotube length is investigated in this work. Firstly, the effect of the ambient temperature on the Q-factor of thermoelastic damping was included. Secondly, the influence of the radial thickness of the nanotube based on the thermoelasticity along with Donnell–Muskhvishvili–Vlasov (DMV) approach was studied. Finally, the influence of nanotube length based on the Q-factor of thermoelastic damping was discussed in the context. The governing equation of the longitudinal wave propagation based on thin shell theory considering the thermoelasticity theory was derived. The effects of various parameters such as the length of CNT, the radial thickness of the nanotube, as well as ambient temperature on the Q-factor of thermoelastic damping were scrutinised.

**2. Shell theory of the thermoelastic equation:** Assuming small strains and displacements, and considering the thin shell theory, Fig. 1 illustrates the cylindrical coordinate system ( $x, \phi, z$ ) and the geometry of the problem. In the figure,  $x, \phi$  and  $z$ -axis are longitudinal, circumferential and transverse directions, respectively. Furthermore, the dimensions of the model are presented in terms of the mean radius  $R$ , radial thickness  $h$ , the mass density  $\rho$  and tube length  $L$ .

The thermoelastic linear equations of the cylindrical shell in  $u_1$ ,  $u_2$  and  $u_3$  displacements can be written as follows [47]:

$$\frac{\partial(N_x - N_T)}{\partial x} + \frac{1}{R} \frac{\partial N_{x\phi}}{\partial \phi} = \rho h \frac{\partial^2 u_1}{\partial t^2}, \quad (1)$$

$$\frac{\partial N_{x\phi}}{\partial x} + \frac{1}{R} \frac{\partial(N_\phi - N_T)}{\partial \phi} + \frac{1}{R} \frac{\partial M_{x\phi}}{\partial x} + \frac{1}{R^2} \frac{\partial(M_\phi - M_T)}{\partial \phi} = \rho h \frac{\partial^2 u_2}{\partial t^2}, \quad (2)$$

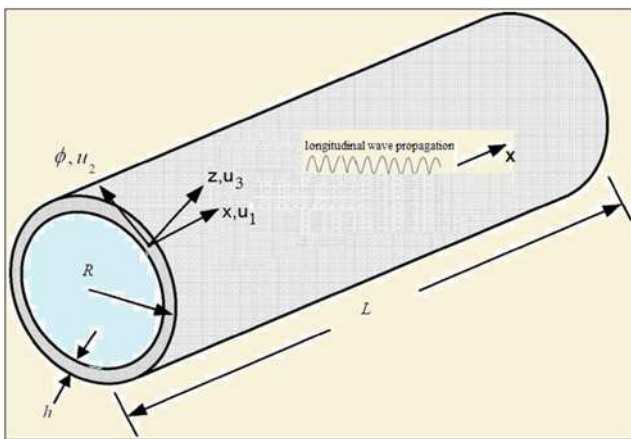


Fig. 1 Geometry of the tube with coordinate system

$$\frac{\partial^2(M_\phi - M_T)}{\partial x^2} + \frac{2}{R} \frac{\partial^2 M_{x\phi}}{\partial x \partial \phi} + \frac{1}{R^2} \frac{\partial^2(M_\phi - M_T)}{\partial \phi^2} - \frac{N_\phi - N_T}{R} = \rho h \frac{\partial^2 u_3}{\partial t^2}. \quad (3)$$

The strain–displacements relations in our case are

$$\varepsilon_x = \varepsilon_x^0 + z\kappa_x, \quad \varepsilon_\phi = \varepsilon_\phi^0 + z\kappa_\phi, \quad \varepsilon_{x\phi} = \varepsilon_{x\phi}^0 + z\kappa_{x\phi}, \quad (4)$$

where  $\varepsilon_x$ ,  $\varepsilon_\phi$  and  $\varepsilon_z$  are the longitudinal circumferential and transverse components of strains, respectively,

$$\varepsilon_x^0 = \frac{\partial u_1}{\partial x}, \quad \varepsilon_\phi^0 = \frac{1}{R} \left( \frac{\partial u_2}{\partial \phi} + u_3 \right), \quad (5)$$

$$\varepsilon_{x\phi}^0 = \frac{\partial u_2}{\partial x} + \frac{1}{R} \frac{\partial u_1}{\partial \phi},$$

and

$$\kappa_x = \frac{\partial u_1}{\partial x}, \quad \kappa_\phi = \frac{1}{R} \left( \frac{\partial u_2}{\partial \phi} + u_3 \right), \quad (6)$$

$$\kappa_{x\phi} = \frac{1}{R} \frac{\partial u_1}{\partial \phi} + \frac{\partial u_2}{\partial x},$$

are the curvatures.

The bending moments and membrane forces can be written as

$$M_x = D(\kappa_x + \Omega\kappa_\phi), \quad M_\phi = D(\kappa_\phi + \Omega\kappa_x), \quad (7)$$

$$M_{x\phi} = \frac{D(1 - \Omega)}{2} \kappa_{x\phi},$$

$$N_x = \Delta(\varepsilon_x^0 + \Omega\varepsilon_\phi^0), \quad N_\phi = \Delta(\varepsilon_\phi^0 + \Omega\varepsilon_x^0), \quad (8)$$

$$N_{x\phi} = \frac{\Delta(1 - \Omega)}{2} \varepsilon_{x\phi}^0,$$

where  $\Omega$  and  $E$  are Poisson's ratio and elasticity constant, respectively.  $\Delta = Eh/(1 - \Omega^2)$  and  $D = Eh^3/[12(1 - \Omega^2)]$  are the membrane and bending stiffness.

Due to the thermal effects we can write the bending moment  $M_T$  and the membrane force  $N_T$  as

$$N_T = \frac{E\alpha_t}{1 - \Omega} \int_{-h/2}^{h/2} (T - T_0) dz, \quad M_T = \frac{E\alpha_t}{1 - \Omega} \int_{-h/2}^{h/2} (T - T_0) z dz, \quad (9)$$

where  $\alpha_t$  expresses the thermal expansion coefficient.

The equation of the heat conduction in the present case is [48]

$$\chi \hat{\nabla}^2 T = \rho C_p \frac{\partial T}{\partial t} + \frac{E\alpha_t T}{(1 - 2\Omega)} \frac{\partial e}{\partial t}, \quad (10)$$

where  $\chi$  and  $C_p$  are the thermal conductivity and heat capacity coefficient at constant pressure, respectively. The dilatation strain  $e$  due to the thermal effects is defined as follows:

$$e = \varepsilon_x + \varepsilon_\phi + \varepsilon_z. \quad (11)$$

Using the thin shell theory, we could neglect the normal stress  $\sigma_z$ , along the thickness direction, this mean that the relation between stress and strain with thermoelastic effects can be written in

the following form:

$$\begin{aligned}\varepsilon_z &= -\frac{\Omega}{E}(\sigma_\phi + \sigma_x) + \alpha_t(T - T_0), \\ \varepsilon_\phi &= \frac{1}{E}(\sigma_\phi - \Omega\sigma_x) + \alpha_t(T - T_0), \\ \varepsilon_x &= \frac{1}{E}(\sigma_x - \Omega\sigma_\phi) + \alpha_t(T - T_0).\end{aligned}\quad (12)$$

Solving the above equation, we get the transverse strain as

$$\varepsilon_x = \left[ \frac{(\Omega - 1)}{\Omega} \varepsilon_z - \varepsilon_\phi \right] + \frac{(\Omega + 1)}{\Omega} \alpha_t(T - T_0). \quad (13)$$

Using the DMV method [49], (6) and the first two equations in (1) in can be rewritten as

$$\begin{aligned}\kappa_x &= -\frac{\partial^2 u_3}{\partial x^2}, \quad \kappa_\phi = -\frac{1}{R^2} \left( \frac{\partial^2 u_3}{\partial \phi^2} \right), \\ \kappa_{x\phi} &= -\frac{2}{R} \left( \frac{\partial^2 u_3}{\partial x \partial \phi} \right),\end{aligned}\quad (14)$$

$$\frac{\partial(N_x - N_T)}{\partial x} + \frac{1}{R} \frac{\partial N_{x\phi}}{\partial \phi} = 0, \quad \frac{1}{R} \frac{\partial(N_\phi - N_T)}{\partial \phi} + \frac{\partial N_{x\phi}}{\partial x} = 0 \quad (15)$$

With relations (14) above, the compatibility equation is given as follows [43]:

$$\kappa_x + \frac{\partial}{\partial x} \left( R \frac{\partial \varepsilon_\phi^0}{\partial x} - \frac{1}{2} \frac{\partial \varepsilon_{x\phi}^0}{\partial \phi} \right) + \frac{1}{R} \frac{\partial}{\partial \phi} \left( \frac{\partial \varepsilon_x^0}{\partial \phi} - \frac{R}{2} \frac{\partial \varepsilon_{x\phi}^0}{\partial x} \right) = 0. \quad (16)$$

The membrane strains (5) can be written in terms of the stress function (15) and thermal membrane force (7) as

$$\begin{aligned}\varepsilon_x^0 &= \frac{1}{\Delta(1 - \Omega^2)} \left( \frac{1}{R^2} \frac{\partial^2 \eta}{\partial \phi^2} - \Omega \frac{\partial^2 \eta}{\partial x^2} \right) \\ &+ \frac{1}{\Delta(1 + \Omega)} N_T \varepsilon_\phi^0 = \frac{1}{\Delta(1 - \Omega^2)} \left( \frac{\partial^2 \eta}{\partial x^2} - \frac{\Omega}{R^2} \frac{\partial^2 \eta}{\partial \phi^2} \right) \\ &+ \frac{1}{\Delta(1 + \Omega)} N_T \varepsilon_{x\phi}^0 = \frac{2}{\Delta(1 - \Omega)} \left( \frac{1}{R} \frac{\partial^2 \eta}{\partial x \partial \phi} \right).\end{aligned}\quad (17)$$

By substituting (14) and (17) into (16), it gives

$$\Delta(1 - \Omega^2) \nabla_q^2 u_z - \nabla^4 \eta - (1 - \Omega) \nabla^2 N_T = 0, \quad (18)$$

where

$$\begin{aligned}\nabla_q^2 &= \frac{1}{R} \frac{\partial^2}{\partial x^2}, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{1}{R^2} \frac{\partial^2}{\partial \phi^2}, \\ \nabla^4 &= \frac{\partial^4}{\partial x^4} + \frac{2}{R^2} \frac{\partial^4}{\partial x^2 \partial \phi^2} + \frac{1}{R^4} \frac{\partial^4}{\partial \phi^4}.\end{aligned}\quad (19)$$

Furthermore, the dilation strain can be expressed by substituting (4), (13), (17) and (14) into (11), will get

$$\begin{aligned}e &= \frac{1 - 2\Omega}{1 - \Omega} \left[ \frac{1}{\Delta(1 + \Omega)} (\nabla^2 \eta + 2N_T) - z \nabla^2 u_3 \right] \\ &+ \frac{1 + \Omega}{1 - \Omega} \alpha_t(T - T_0).\end{aligned}\quad (20)$$

Thus, (10) can be rewritten as

$$\begin{aligned}\chi \hat{\nabla}^2 T &= \rho C_p \frac{\partial T}{\partial t} \\ &+ \frac{E \alpha_t T}{(1 - \Omega)} \left( \frac{1 - \Omega}{Eh} \frac{\partial}{\partial t} (\nabla^2 \eta + 2N_T) \right. \\ &\quad \left. - \frac{\partial}{\partial t} (z \nabla^2 u_3) + \alpha_t \frac{1 + \Omega}{1 - 2\Omega} \frac{\partial T}{\partial t} \right).\end{aligned}\quad (21)$$

The thermal gradient  $\hat{\nabla}^2 T$  for cylindrical shell is

$$\hat{\nabla}^2 T = \frac{1}{R + z} \frac{\partial T}{\partial z} + \frac{\partial^2 T}{\partial z^2} + \frac{1}{(R + z)^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial x^2}. \quad (22)$$

Further, (22) can be reduced by considering the first two terms on right-hand side only because thermal gradients in the thickness direction  $z$  are much larger than the other directions [50]. Moreover,  $T$  in the second term on the right-hand side of (22) can be replaced by  $T_0$ , and  $2NT$  can be neglected [49]. Therefore, (22) can be simplified as

$$\begin{aligned}\chi \frac{\partial^2 T}{\partial z^2} + \frac{\chi}{R + z} \frac{\partial T}{\partial z} &= \left[ \rho C_p + \frac{E \alpha_t T_0 (1 + \Omega)}{(1 - \Omega)(1 - 2\Omega)} \right] \frac{\partial T}{\partial t} \\ &+ \frac{\alpha_t T_0}{h} \frac{\partial (\nabla^2 \eta)}{\partial t} - \frac{E \alpha_t T_0}{(1 - \Omega)} \frac{\partial}{\partial t} (z \nabla^2 u_3).\end{aligned}\quad (23)$$

**3. Formulation of the problem:** In this study, the bending moments can be naturally neglected for simplicity and the dynamical equations (1)–(3) in this case become

$$\frac{\partial(N_x - N_T)}{\partial x} + \frac{1}{R} \frac{\partial N_{x\phi}}{\partial \phi} = \rho h \frac{\partial^2 u_1}{\partial t^2}, \quad (24)$$

$$\frac{\partial N_{x\phi}}{\partial x} + \frac{1}{R} \frac{\partial(N_\phi - N_T)}{\partial \phi} = \rho h \frac{\partial^2 u_2}{\partial t^2}, \quad (25)$$

$$\frac{N_\phi - N_T}{R} = -\rho h \frac{\partial^2 u_3}{\partial t^2}. \quad (26)$$

The geometric condition of the longitudinal wave propagation in SWCNT is

$$\frac{\partial}{\partial \phi} = 0, \quad \varepsilon_x^0 = \frac{\partial u_1}{\partial x}, \quad \varepsilon_\phi^0 = \frac{u_3}{R} \text{ and } \varepsilon_{x\phi}^0 = \frac{\partial u_2}{\partial x}. \quad (27)$$

For the longitudinal wave propagation in the SWCNT, the temperature change only in the  $x$ -direction. In this case, for simplicity we assume  $T$  is constant in the  $z$ -direction and then

$$N_T = \frac{Eh\alpha_t T}{1 - \Omega}. \quad (28)$$

Substituting (27) into the membrane forces equation (7) one get

$$\begin{aligned}N_x &= \Delta \left( \frac{\partial u_1}{\partial x} + \Omega \frac{u_3}{R} \right), \quad N_\phi = \Delta \left( \frac{u_3}{R} + \Omega \frac{\partial u_1}{\partial x} \right), \\ N_{x\phi} &= \frac{\Delta(1 - \Omega)}{2} \frac{\partial u_2}{\partial x}.\end{aligned}\quad (29)$$

Substituting (27)–(29) into the dynamical equations (24)–(26) one get

$$\Delta \frac{\partial^2 u_1}{\partial x^2} + \frac{\Omega}{R} \frac{\partial u_3}{\partial x} - \left( \frac{Eh\alpha_t}{(1 - \Omega)} \right) \frac{\partial T}{\partial x} = \rho h \frac{\partial^2 u_1}{\partial t^2}, \quad (30)$$

$$\left(\frac{\Delta(1-\Omega)}{2}\right)\frac{\partial^2 u_2}{\partial x^2} = \rho h \frac{\partial^2 u_2}{\partial t^2}, \quad (31)$$

$$\left(\frac{\Delta}{R^2}\right)u_3 + \left(\frac{\Omega}{R}\right)\frac{\partial u_1}{\partial x} - \left(\frac{Eh\alpha_t}{R(1-\Omega)}\right)(T - T_0) = -\rho h \frac{\partial^2 u_3}{\partial t^2}. \quad (32)$$

Furthermore, in this section we concentrated our studies for the wave propagation along the longitudinal direction. Then, the  $z$ -directional displacement  $u_3$  is assumed to be zero and the motion equation becomes:

$$\Delta \frac{\partial^2 u_1}{\partial x^2} - \left(\frac{Eh\alpha_t}{(1-\Omega)}\right)\frac{\partial T}{\partial x} = \rho h \frac{\partial^2 u_1}{\partial t^2}. \quad (33)$$

The dilatation strain due to the thermal effect in this case become

$$e = \frac{\partial u_1}{\partial x}. \quad (34)$$

Substituting (33) into (10), will get

$$\chi \hat{\nabla}^2 T = \rho C_p \frac{\partial T}{\partial t} + \frac{\alpha_t T}{(1-2\Omega)h} \frac{\partial}{\partial t} \left[ \frac{\partial u_1}{\partial x} \right]. \quad (35)$$

The thermal gradient  $\hat{\nabla}^2 T$  becomes

$$\hat{\nabla}^2 T = \frac{\partial^2 T}{\partial x^2}. \quad (36)$$

Replacing  $T$  by  $T_0$ , (33), can be simplified as

$$\chi \frac{\partial^2 T}{\partial x^2} = \rho C_p \frac{\partial T}{\partial t} + \frac{\alpha_t T_0}{(1-2\Omega)h} \frac{\partial}{\partial t} \left( \frac{\partial u_1}{\partial x} \right). \quad (37)$$

Equations (33) and (37) are the simplified thermoelastic equations which, can be used to show the effects of thermoelastic damping on propagation of longitudinal wave's propagation in a SWCNT.

**4. Harmonic solution of dilatation waves:** In this study, we use the harmonic solution of the displacement in the form [51]

$$\begin{aligned} u_1(x, t) &= \hat{u}_1 e^{i\hat{k}(x-ct)}, \\ T(x, t) &= \hat{T} e^{i\hat{k}(x-ct)}, \end{aligned} \quad (38)$$

where  $i = \sqrt{-1}$ ,  $c$  is the phase velocity and  $\hat{k}$  is the wave number ( $\hat{k} = (2\pi/\lambda)$ ).

Using expressions (38) into (33) and (37) one get

$$\left(\rho h \omega^2 - \Delta \hat{k}^2\right) \hat{u}_1 - \left(\frac{i h E \hat{k} \alpha_t}{(1-\Omega)}\right) \hat{T} = 0, \quad (39)$$

$$-\left[\frac{\hat{k} \omega \alpha_t T_0}{(1-2\Omega)h}\right] \hat{u}_1 + \left[i \rho \omega C_p - \chi \hat{k}^2\right] \hat{T} = 0, \quad (40)$$

where  $\omega = \hat{k}c$  is the circular frequency of the dilatation waves.

Using matrix form (39) and (40) become

$$\begin{bmatrix} \left(\rho h \omega^2 - \frac{h E \hat{k}^2}{(1-\Omega^2)}\right) & -\left(\frac{i h E \hat{k} \alpha_t}{(1-\Omega)}\right) \\ -\left(\frac{\hat{k} \omega \alpha_t T_0}{(1-2\Omega)h}\right) & [i \rho \omega C_p - \chi \hat{k}^2] \end{bmatrix} \begin{bmatrix} \hat{u}_1 \\ \hat{T} \end{bmatrix} = 0. \quad (41)$$

The existence of non-zero solution of the above equation requires

$$\begin{bmatrix} \left(\rho h \omega^2 - \frac{h E \hat{k}^2}{(1-\Omega^2)}\right) & -\left(\frac{i h E \hat{k} \alpha_t}{(1-\Omega)}\right) \\ -\left(\frac{\hat{k} \omega \alpha_t T_0}{(1-2\Omega)h}\right) & [i \rho \omega C_p - \chi \hat{k}^2] \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (42)$$

Then, the real and imaginary parts of the natural frequencies of the dilatation wave propagation in the SWCNT under thermoelastic damping can be written in the following expressions:

$$\text{Re}(\omega) = \hat{k} \sqrt{\frac{E}{\rho(1-\Omega^2)}}, \quad (43)$$

$$\text{Im}(\omega) = \hat{k} \sqrt{\frac{E}{\rho(1-\Omega^2)} + \frac{L T_0 \alpha_t^2 E}{h \rho^2 C_p (1-\Omega)(1-2\Omega)}}. \quad (44)$$

Finally, the Q-factor of thermoelastic damping can be calculated using the following definition:

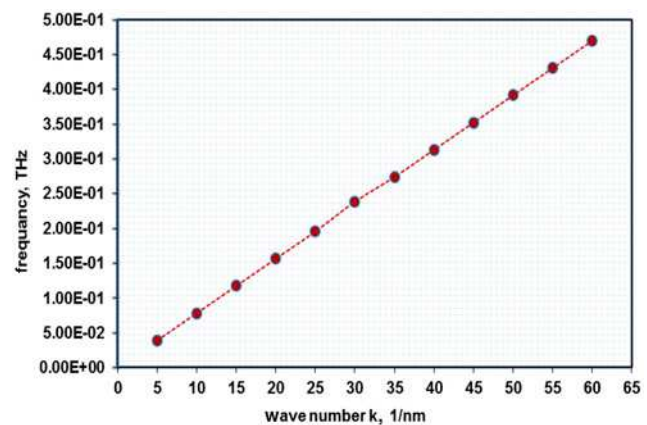
$$Q^{-1} = 2 \left| \frac{\text{Im}(\omega)}{\text{Re}(\omega)} \right|. \quad (45)$$

**5. Numerical results:** This section shows the verification of the analytical work, considering the numerical calculation to show the effect of thermoelastic damping on propagation of the longitudinal in SWCNTs.

The material constants are Young's modulus  $E$  (Gpa) = 1060, the mass density  $\rho$  ( $\text{kgm}^{-3}$ ) = 2270, the Poisson's ratio  $\Omega = 0.25$ , heat capacity coefficient  $\rho C_p$  ( $10^6$ ) ( $\text{Jm}^3\text{k}^{-1}$ ) = 1.36 and the thermal expansion coefficient  $\alpha_t$  ( $10^{-6}$ ) ( $\text{k}^{-1}$ ) = 7 [50].

Fig. 2 shows the dispersion curve of the longitudinal wave's propagation in the SWCNT under the effects of thermoelastic damping. The dispersion curve is plotted for the natural frequency of the longitudinal wave versus the wave number. This figure shows that the increase of the wave number increasing of the natural frequency of the longitudinal wave's propagation in a SWCNT.

Figs. 3 and 4 show the variation of Q-factor of thermoelastic damping with the change of the radial thickness ( $h$ ) and the length ( $L$ ) of the nanotube, respectively. The results for Q-factor of thermoelastic damping with different radial thicknesses of nanotube are shown in Fig. 3. Fig. 4 shows the variation of the Q-factor



**Fig. 2** Flexural dispersion curve of longitudinal wave's propagation in a SWCNT



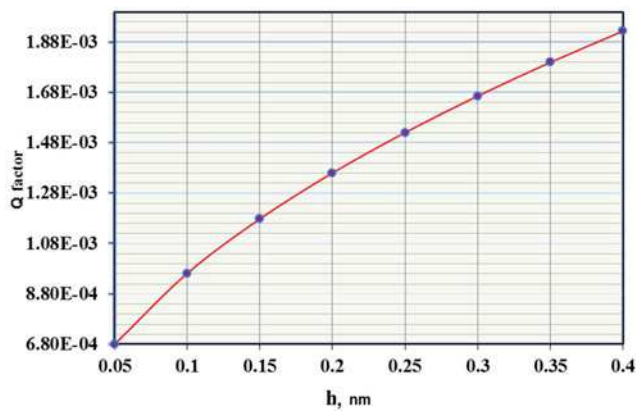


Fig. 3 Variation of the Q-factor with radial thickness ( $h$ )

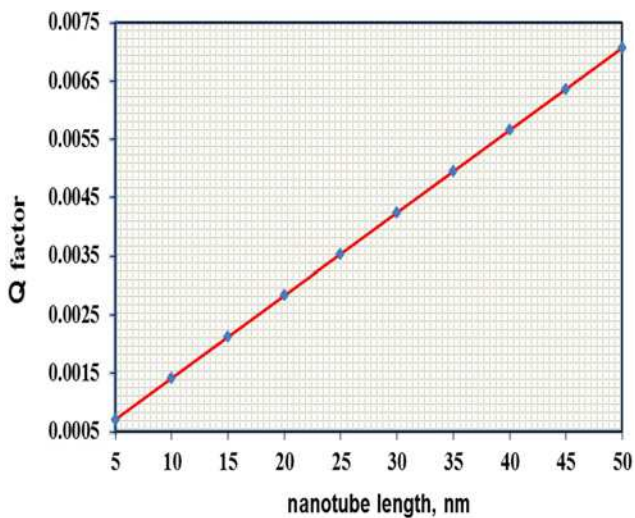


Fig. 4 Variation of the Q-factor with nanotube length ( $L$ )

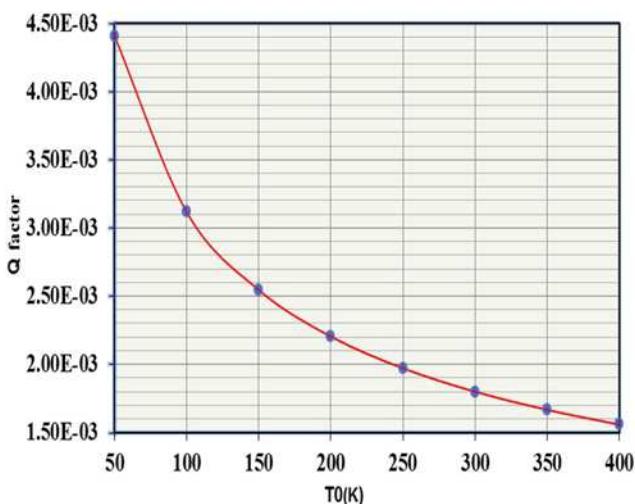


Fig. 5 Variation of the Q-factor with ambient temperature  $T_0$

of thermoelastic damping with different nanotube lengths. These figures show that the Q-factor of thermoelastic damping is proportional to the length and thickness of the SWCNT. It means that the increase in both length and radial thickness of the nanotube makes the Q-factor increase.

Fig. 5 shows the variation of Q-factor of thermoelastic damping with the change of the ambient temperature ( $T_0$ ). The temperature changes from  $T = 50$  to 400 K. For the final observation, it should be noted that the ambient temperature has an inverse effect on the Q-factor for the SWCNT. It means that, the increase in ambient temperature makes the Q-factor decrease, and has a direct effect on the longitudinal wave's propagation in the time domain.

**6. Conclusion:** In this work, we showed the effects of the thermoelastic damping on the longitudinal wave's propagation in the SWCNTs. The thin shell theory was used for the vibration analysis and DMV approach is applied as well. A new analytical expression for the natural frequency of a longitudinal waves propagating in a SWCNT was obtained. The analysis of the quality factor (Q-factor) of thermoelastic damping was investigated. The numerical results were presented to investigate the influence of some parameters on Q-factor of thermoelastic damping, such as the length of nanotube, nanotube thickness, nanotube length and the ambient temperature. In addition, the dispersion curve of the longitudinal waves propagation in the SWCNT under the effects of thermoelastic damping was plotted. It can be concluded that, the Q-factor is proportional to the length and thickness of the SWCNT. It means that the increase in both length and radial thicknesses of the nanotube make the Q-factor increase. Moreover, the inverse observation, it be noted with the ambient temperature. It means that the increase in ambient temperature makes the Q-factor decrease, and has a direct effect on the longitudinal wave's propagation in the time domain. These results can be helpful in the design of resonators and nanodevices.

**7. Acknowledgment::** The author acknowledges the supports have been received from the Deanship of Scientific Research of Prince Sattam bin Abdulaziz university during this work. Also, the author acknowledges the supports provided by the King Abdul-Aziz City for Science and Technology (KACST) during this work.

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