Size effects on stability and self-instability of non-uniform nanobeams with consideration of surface effects

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In this work, size-dependent stability and self-instability of non-uniform nanobeams are studied with consideration of the surface elasticity theory. The nanobeams are modelled as step-wise beams and the governing equation of the nanobeam is solved for different boundary conditions. At first, a backward formulation is developed to calculate the effective Young's modulus to compare it with experimental results. Wide ranges of numerical results are presented for buckling load and self-instability of Al and Si nanobeams with positive and negative surface properties. The obtained results indicate that the selection of the number of steps in the presence of surface effects has a considerable impact on the mechanical behaviour of non-uniform nanobeams. In addition, the aspect ratio of one section of a step-wise nanobeam with negative surface residual stresses can make the whole structure self-instable. The results for effective Young's modulus are compared with previous experiments and satisfactory agreement is illustrated.

1. Introduction: Recent advances in manufacturing technologies have made it possible to develop small-scale systems at micron/submicron scales [1–3]. In recent decades, nanoscale beams are known as a basic component of micro/nanosystems due to their novel features, such as easy manufacturing and high-frequency operation [2–4]. Therefore, the mechanical analysis of nanobeams has attracted the attention of many researchers in the field of nanotechnology. Yan and Jiang [5] extended the Euler–Bernoulli beam model for bending and buckling analysis of a nanobeam with different boundary conditions. They presented an analytical solution for the bending response of the nanobeam subjected to both electrical and mechanical loads. Korayem *et al.* [6] presented modelling for an atomic force nanobeam adjacent to a surface considering tip–sample interaction forces based on classical elasticity theory.

On the other hand, the capability of classical continuum theory to model micro/nanosystems is strongly doubted through conducting experimental tests and molecular simulations. Feng and Jiang [7] employed molecular dynamic simulation to investigate the size-dependent mechanical behaviour of nanobeams and showed the size effects are excluded in classical analytical models. Therefore, higher order elasticity theories have emerged in micro/nano researches. Bakhtiari-Nejad and Nazemizadeh [8] studied size-dependent vibrations of piezoelectric nanobeams based on the non-local elasticity theory. Jalali *et al.* [9] developed a modified couple stress theory to study the size-dependent mechanical behaviour of functionally graded nanobeams. They employed the Rayleigh–Ritz method to obtain the size-dependent behaviour of the beam.

Furthermore, among the higher-order mechanics' theories, the theory of surface elasticity has attracted great interest of researchers. Jiang and Yan [10] employed the surface elasticity theory for static bending of shear deformable nanobeams. They derived the governing equation of the nanobeam and analytically solved the problem. Assadi and Farshi [11] studied the size-dependent dynamics of nanotubes with surface effects. Also, Farshi *et al.* [12] presented the size effects of vibration of the nanobeams taken into account the surface elasticity theory. They illustrated that the mechanical behaviour of the nanobeams with consideration of surface effects deviates from the results obtained by classical theories considerably. Liu *et al.* [13] investigated the deflection behaviour of a size-dependent nanobeam under static bending. They considered the surface stresses as an external load and derived equilibrium equations of the nanobeam. Also, in [14], size-dependent static bending of nanobeams is presented based on the surface elasticity theory. They developed the Euler nanobeam model to derive the differential equation and used a theoretical solution for the static behaviour of the nanobeam.

In addition, these higher order theories are considered together to obtain their combined size-scale effects. For example, Keivani *et al.* [15] studied the static and dynamic stability of conductive nanotweezers under Casimir force based on couples stress theory together with consideration of the surface stresses. They discussed the concepts related to size effects of pull-in voltage, pull-in gap and -detachment length of the system. Yekrangisendi *et al.* [16] studied the dynamic stability of a nanowire subject to the acceleration field based on strain gradient elasticity together with Gurtin–Murdoch surface stress model. They presented useful results for changes in the pull-in voltage of the nanobeam with respect to angular velocity and the size-scale factors. In addition, Hashemian *et al.* [17] comprehensively discussed the static bending and buckling of simply supported nanobeams for different types of higher order beam models based on non-local strain gradient elasticity considering the surface stresses. They discussed the stiffening-softening effect of the dominant size-scale factors in the problem. Sourani *et al.* [18] studied the size effects related to non-local strain gradient and surface stresses in dynamic stability of nanobeams and discussed the shift in the dynamic instability region to lower and higher frequencies due to the considered basic parameters in the model. Although the above papers studied surface effects on the mechanical behaviour of the nanobeam, there is a need to study the size-dependent characteristics of non-uniform nanobeam.

According to the introduced approaches and concepts, here it is tried to observe, the majority of the effect of surface stresses as real physical parameters in non-uniform nanobeams and evaluate its effectiveness in the prediction of the size-scale effects that were observed in previous experiments. In this Letter, a backward formulation is developed to calculate the surface parameters in good agreement with experimental results. Along this line, the non-uniform nanobeams are modelled as step-wise beams and the governing equation of the nanobeam is solved for different boundary conditions. A wide range of numerical results are presented for buckling load, and self-instability of the nanobeam. The obtained numerical results demonstrate the effectiveness of the proposed method.

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2. Problem formulation: In this section, the governing equation of the size-dependent buckling of non-uniform nanobeam is presented. Consider a nanobeam with non-uniform cross-section over its length, as shown in Fig. 1.

In this figure, E, ρ and υ are represented for Young's Modulus, mass density and Poisson's ratio of the nanobeams' bulk material, respectively. Additional surface properties, including surface elasticity E^s and surface residual stress τ^s are considered over the external boundaries of the nanobeams. Each section of the beam in Fig. 1 is defined by its cross-sectional diameter D and length L.

Here a step-wise model is suggested for smoothly tapered nanobeams with a non-uniform diameter through its length. It must be noted that this will be true when the number of steps is chosen properly due to the intensity of cross-section variation over the real nanobeam's length. Besides, if the calculated mode shape of the buckling is nearly similar to that of the main structure, the model will predict the true buckling loads. Therefore, for any non-uniform nanobeam, the selection procedure of the number of steps must be done. Here the problem will be solved for one-step and two-step problems to illustrate the general trend for nanowires with a moderate variation of the cross-section and the results are compared for many cases. Now, the governing differential equation for each step can be obtained based on surface-stress models. According to the Euler–Bernoulli beam theory, the strain field of the nanobeam is given as follows:

$$\varepsilon_{xx} = -z\frac{\partial^2 w}{\partial x^2}, \quad \varepsilon_{xz} = 0 \tag{1}$$

In this relation, w is the transverse deflection of the nanobeam. Moreover, x and z are the coordinates across the nanobeams' length and thickness, respectively. It is to be noted that the origin of z is the nanobeams' neutral axes. As a major point, the strain field is considered to be continuous over the nanobeam's cross-section without any jumps over the material surfaces according to fundamentals of continuum mechanics. Simply, from these relations, the stress field of the deflected nanobeams can be given as follows:

$$\sigma_{xx} = \sigma^{o}_{xx} - Ez\frac{\partial^2 w}{\partial x^2} \tag{2}$$

In this equation, σ^o is the residual normal stress-induced in nanobeams by surface residual stresses. On the other hand, according to generalised Gurtin–Murdoch relation, the following stress–strain relation ($\sigma - \varepsilon$) is written for elastic solid surfaces in three-dimensional situations [15]:

$$\sigma^s_{\alpha\beta} = \tau^s\delta_{\alpha\beta} + \left(\frac{E^s}{1+\upsilon} - 2\tau^s\right)\varepsilon^s_{\alpha\alpha} + \frac{E^s}{2(1+\upsilon)(1-2\upsilon)}u^s_{\gamma,\gamma} + \tau^s u^s_{\alpha,\beta}$$
$$\sigma^s_{\alpha z} = \tau^s u_{z,\alpha} \tag{3}$$

In these relations, subscripts α, β and γ change from 1 to 2 for any coordinate parallel to the surface materials. Simplifying the relations for one-dimensional structures and according to the given strain field in (1), the following relations will be obtained for the Euler–Bernoulli beam model:

$$\sigma^s_{xx} = \tau^s - E^s z\frac{\partial^2 w}{\partial x^2}\bigg|_S$$
$$\sigma^s_{xz} = \tau^s\frac{\partial w}{\partial x} \tag{4}$$

Accordingly, the bending moment of the cross-section will be obtained from the following integral equation:

$$M = \iint_A \sigma_{xx} z\,\mathrm{d}A + \int_z \sigma^s_{xx} z\,\mathrm{d}S$$
$$= \left(\iint_A z\sigma^o_{xx}\,\mathrm{d}A + \int_z \tau^s z\,\mathrm{d}S\right) - EI_{\text{eff}}\frac{\partial^2 w}{\partial x^2} \tag{5}$$

while the effective bending rigidity EI^{eff} is obtained from the following relation:

$$EI_{\text{eff}} = \iint_A Ez^2\,\mathrm{d}A + \int_z E^s z^2\,\mathrm{d}S \tag{6}$$

On the one hand and according to the self-equilibrating condition, the integral relations in the right-hand side of (5) cancel each other and the bending moment is obtained as follows:

$$M = -EI_{\text{eff}}\frac{\partial^2 w}{\partial x^2} \tag{7}$$

On the other hand, considering the additional term of (4), the updated equilibrium equation for a beam which is subject to axial loading F is given as follows:

$$\frac{\partial^2 M_x}{\partial^2 x} + \frac{\partial}{\partial x}\left(\int_S \sigma_{xz}\boldsymbol{n}\cdot\mathrm{d}\boldsymbol{S}\right) + F\frac{\partial^2 w}{\partial^2 x} = \rho A\frac{\partial^2 w}{\partial t^2} \tag{8}$$

After doing some mathematical process, another reformulation of this equation in a shorter form is given as follows in which $N = F - 2\tau^s b$:

$$EI_{\text{eff}}\frac{\partial^4 w}{\partial x^4} + N\frac{\partial^2 w}{\partial x^2} + \rho A\frac{\partial^2 w}{\partial t^2} = 0 \tag{9}$$

In (8), N is the effective longitudinal force applied to the nanowire derived from the model. It is the summation of the external load and those made by internal residual stresses related to surface stresses. Besides the Gurtin–Murdoch model and using (3), N can be obtained from Laplace–Young's model for surface stress that considers the effect of surface residual stress as a σ_{zz} stress component over the deflected nanobeam surface [12]. Confirming discussion is presented in [19, 20] for proper using the Gurtin–Murdoch surface-stress model in the nanostructure.

3. Problem solution: Consider a nanobeam with total length L, which is divided into n sections with different lengths of L^i and circular cross-section of diameter D^i in which $i = 1, 2, \ldots, n$. First, it is intended to find the governing differential equation for that. Also, N is obtained equal to $F - 2\tau^s D^i$ for the ith section. For buckling of Euler–Bernoulli nanobeams with surface effects, the following differential equation must be solved:

$$\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2}{\partial x^2} - \beta_i^2\right)w^i = 0, \quad \beta_i^2 = -\frac{N}{EI^i_{\text{eff}}}, \quad 0 < x < L^i \tag{10}$$

where the new parameter β is defined as follows:

$$\beta_i^2 = \frac{64\left(F - 2\tau^s D^i\right)}{\pi E(D^i)^4}\left[1 + \frac{8E^s}{ED^i}\right]^{-1} \tag{11}$$

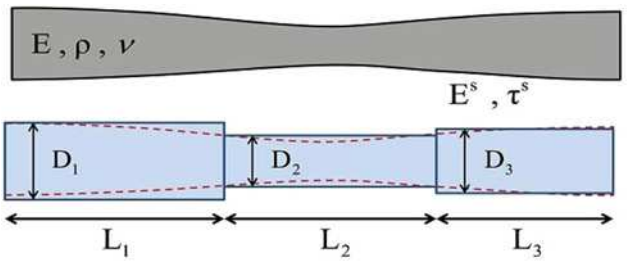

Fig. 1 *Non-uniform tapered nanobeam modelled as a step-wise beam*

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Consequently, the general solution for (9) will be given as follows:

$$w^i = C_1^i + C_2^i x + C_3^i \sinh(\beta_i x) + C_4^i \cosh(\beta_i x) \tag{12}$$

Furthermore, according to the definition of a local coordinate for each section, the following boundary conditions must be satisfied between two adjacent nanobeams:

$$\begin{aligned} w^i\big|_{x=L_i} &= w^{i+1}\big|_{x=0}, \quad \left.\frac{\mathrm{d}w^i}{\mathrm{d}x}\right|_{x=L_i} = \left.\frac{\mathrm{d}w^{i+1}}{\mathrm{d}x}\right|_{x=0} \\ EI_{\text{eff}}^i \left.\frac{\mathrm{d}^2 w^i}{\mathrm{d}x^2}\right|_{x=L_i} &= EI_{\text{eff}}^{i+1} \left.\frac{\mathrm{d}^2 w^{i+1}}{\mathrm{d}x^2}\right|_{x=0} \\ EI_{\text{eff}}^i \left.\frac{\mathrm{d}^3 w^i}{\mathrm{d}x^3}\right|_{x=L_i} &= EI_{\text{eff}}^{i+1} \left.\frac{\mathrm{d}^3 w^{i+1}}{\mathrm{d}x^3}\right|_{x=0} + 2\tau^s (D^i - D^{i+1}) \end{aligned} \tag{13}$$

For each section, substituting from (13) into (12) gives the following matrix equation for the characteristic parameters β_i and β_{i+1} as follows:

$$\begin{bmatrix} 1 & L_i & h_i & l_i & -1 & 0 & 0 & -1 \\ 0 & 1 & \beta_i l_i & \beta_i h_i & 0 & 1 & \beta_{i+1} & 1 \\ 0 & 0 & \beta_i^2 h_i & \beta_i^2 l_i & 0 & 0 & 0 & \phi_i \beta_{i+1}^2 \\ 0 & 0 & \beta_i^3 l_i & \beta_i^3 h_i & 0 & 0 & \phi_i \beta_{i+1}^3 & 0 \end{bmatrix} [C] = 0$$

$$h_i = \sinh(\beta_i L_i), \quad \phi_i = \frac{EI_{\text{eff}}^{i+1}}{EI_{\text{eff}}^{i}}, \quad l_i = \cosh(\beta_i L_i)$$

$$C = \begin{bmatrix} C_1^i & \dots & C_4^i & C_1^{i+1} & \dots & C_4^{i+1} \end{bmatrix}^{\mathrm{T}} \tag{14}$$

A combination of (14) for all sections gives a general matrix relation of order $n+1$ with the right-hand side of zero. On the other hand, the coefficient matrix in this equation contains the parameters $\beta_1, \beta_2, \dots, \beta_n$. For the given geometric parameters and material properties, the determinant roots of the coefficient matrix as an eigenvalue problem gives the critical buckling load of non-uniform nanobeams with consideration of surface effects.

Also, in the case of self-buckling, F must be set zero and therefore (11) is rewritten as follows:

$$\beta_i^2 = \frac{-128\tau^s}{\pi E (D^i)^3} \left[1 + \frac{8E^s}{ED^i}\right]^{-1} \tag{15}$$

Substituting β from (15) into the coefficient matrix of (14) gives the eigenvalue problem of self-instability analysis. If all the geometric parameters except than one of them are given, then the value of this remaining parameter determines the self-instability condition of the structure.

3.1. Determination of effective modulus: In a backward procedure, it is assumed that the nanobeams are made of an elastic material with Young's modulus of E^{eff}. On the other hand, the buckling loads of the nanobeam are obtained from the developed framework in this work in the previous sections. Then regardless of the effect of surface properties or any other additional properties, all the obtained numerical results for a given geometry are assumed to be related to the value of E^{eff}. Therefore, for each section of the equivalent nanobeam, the following classical differential equation must be written:

$$\begin{bmatrix} 1 & L_i & \sinh(\delta_i) & \cosh(\delta_i) & -1 & 0 & 0 & -1 \\ 0 & E^{\text{eff}} I_i & F\cosh(\delta_i) & F\sinh(\delta_i) & 0 & E^{\text{eff}} I_i & F\xi_i & E^{\text{eff}} I_i \\ 0 & 0 & \sinh(\delta_i) & \cosh(\delta_i) & 0 & 0 & 0 & 1 \\ 0 & 0 & \cosh(\delta_i) & \sinh(\delta_i) & 0 & 0 & \xi_i & 0 \end{bmatrix}$$

$$\delta_i = \frac{FL_i}{E^{\text{eff}} I_i}, \quad \xi_i = \frac{I_i}{I_{i+1}} \tag{16}$$

Writing this equation for other sections of the nanobeam, the total coefficient matrix will be obtained for the whole nanobeam. Next, the determinant of this matrix must be set equal to zero. On the other hand, in the backward formulation of this section, it is assumed that F is obtained from the previous section and the only unknown of the above matrix is E^{eff} that must be determined here. In this manner, we have found the effective elastic modulus of the nanobeam with an inverse formulation.

4. Simulation results: In this section, a wide range of simulation results are given for aluminium and silicon [100] with the following material parameters given in Table 1.

To verify the proposed method, the obtained results for buckling analysis of a uniform nanobeam are compared with experimental results presented in [22]. The results are given in Fig. 2.

From this figure, it is observed that satisfactory agreement is achieved between the results of this work and those of the experiment in [22]. It is seen that the results buckling analyses are exactly the same for uniform nanobeams, as explained as a strategy of verifying the results. Therefore, the general trend of reported size dependencies is reliable and can be used in engineering designs.

In order to show the surface effects on the buckling problems, a new parameter *NBL* is introduced, which is the ratio of the buckling load with consideration of surface effects to that one without surface effects. Fig. 3 shows the variation of *NBL* for different geometrical aspect ratios of one-step nanobeams made of aluminium and silicon. The diameter of the first section is given as $D_1 = 10$ nm.

It is seen that the surface effects on the buckling load are higher at longer nanobeams. The positive surface residual stress increases the

Table 1 Physical characteristics of the nanobeam [21]

Material	E, GPa	υ	E^s, N/m	τ^s, N/m
Al	68.50	0.35	6.090	0.910
Si [100]	130.0	0.24	−11.50	−0.505

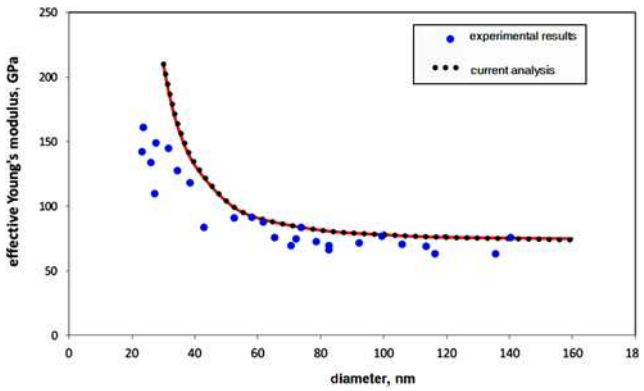

Fig. 2 *Comparison of effective Young's modulus for different studies*

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buckling load, but the negative one decreases it. Moreover, the sensitivity of the problem to D_2/D_1 is higher for longer nanobeams. For example, for the length of $L = 100$ nm, NBL changes from 2.25 to 1.60, i.e. 0.65 units, but for $L = 40$ nm, it changes from 1.27 to 1.17, i.e. 0.10 units. Therefore, consideration of the non-uniformity into the analysis becomes more important for longer nanobeams, while in shorter cases, the problem can simply be treated as uniform nanobeam with a mean diameter.

Also, Fig. 4 depicts the results for a two-step nanobeam for different diameter parameters.

In this figure, for silicon nanobeams with negative surface residual stress, reduction of the diameter of one section makes *NBL* equal to zero in which the nanobeam becomes self-instable. On the other hand, for a constant length, the sensitivity of the problem to D_2 is the same for different values of D_3 with an approximate variation of 1.7 units in *NBL*. This demonstrates that the consideration of variations in the cross-section plays a significant role in the size-dependent behaviour of the buckling load.

Another important concept in step-wise modelling of non-uniform nanobeams is given in Fig. 5. In this figure, the selected nanobeam is first modelled as one-step nanobeam and then as two-step one and the results for NBL are compared with that of uniform nanobeam with a mean diameter.

From this figure, it is observed that proper modelling of the structure for size-dependent behaviour is very important in the analyses. In this figure, as the parameter of the vertical axis deviates from unity, the impact of non-uniform modelling gets more observable. For example, comparing the red and black lines, respectively, for one-step and two-step aluminium nanobeams, for $D_2 = 7$ nm, the *NBL* (*non-uniform*)/*NBL* (*uniform*) for one-step is 1.18 and for two-step is obtained equal to 1.52. Therefore, how to model the structure with the assumed parameters can enter errors into the results up to 30%. On the other hand, obviously, since the length of the nanobeam is long enough to follow the Euler–Bernoulli beam model, two-step modelling is here preferred. This error even gets higher for silicon nanobeams up to 70%. As a general conclusion, it is seen that incorrect modelling of the structure may give some results that prevent the self-instability of the structure in spite of the physical counterparts of the problem.

In another simulation, the self-instability limitation of the non-uniform nanobeam is investigated. Fig. 6 illustrates the critical value of the nanobeam diameter for self-instability occurrence.

In this figure, the upper side of each curve shows the geometric parameters for which the non-uniform nanobeam is stable. It is seen that if one section of a step-wise nanobeam gets longer and thinner, the whole nanobeam goes toward the self-instability region.

Also, Fig. 7 shows the instability region for the two-step nanobeams.

As shown in Fig. 3, similar conclusions are obtained to the previous case of one-step nanobeams. Moreover, the sensitivity of the problem to the diameter of this section is too higher in one-step

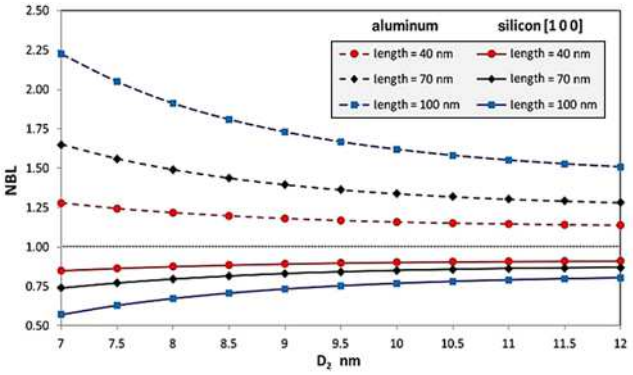

Fig. 3 *Size-dependent buckling factor of one-step nanobeams*

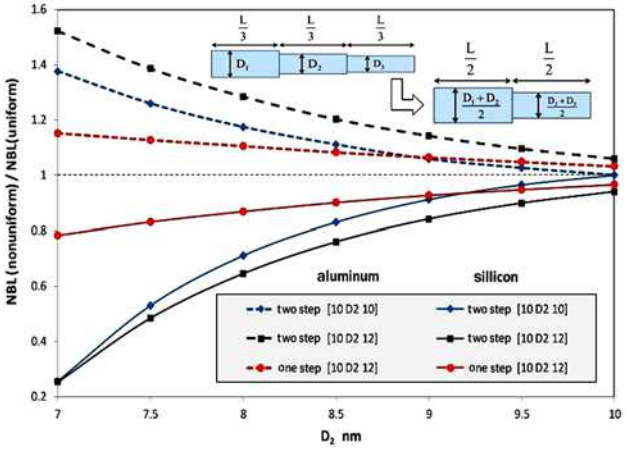

Fig. 5 *Size-dependent buckling factor for different steps modelling*

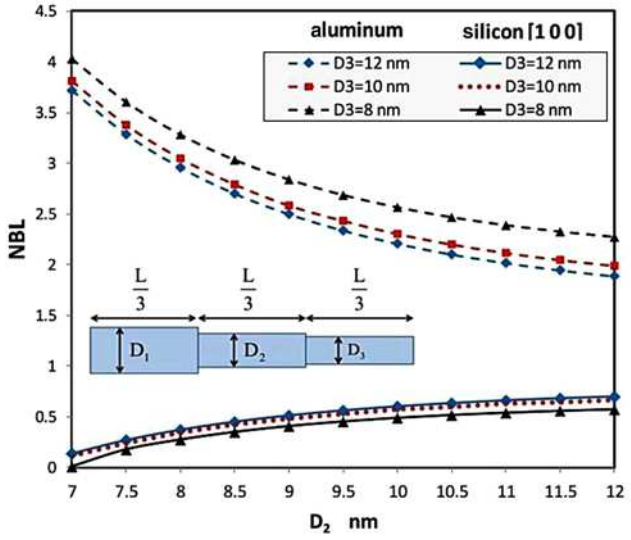

Fig. 4 *Size-dependent buckling factor of two-step nanobeams for different diameters*

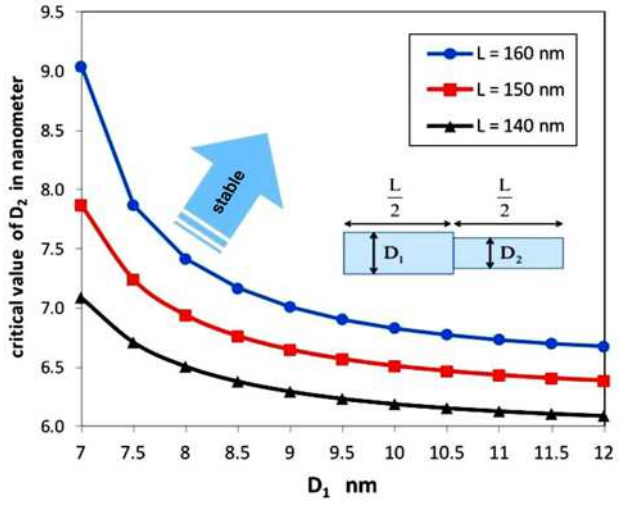

Fig. 6 *Self-instability of the one-step nanobeam for different values of the diameter*

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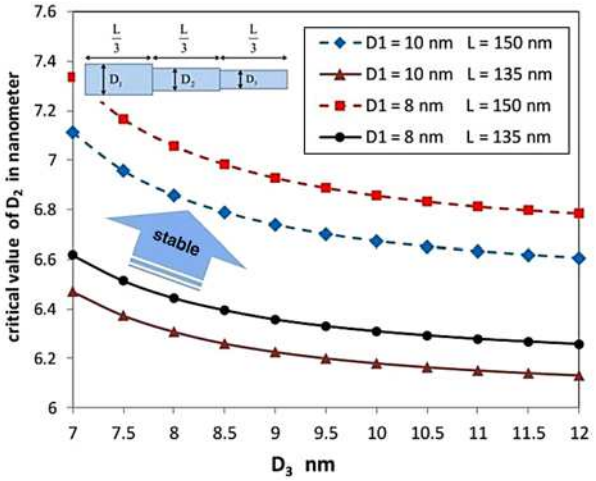

Fig. 7 *Self-instability of the two-step nanobeam for different values of the diameter*

nanobeams. This obviously is related to the relative length of the chosen section to the whole nanobeam. As another important conclusion, it is seen that attaching a thicker nanobeam to a thin self-instable one, can reinforce it considerably and prevent its self-buckling problem. This is based on the basic definition of buckling, which is defined for the whole structure.

The results in Fig. 6 for one-step nanobeam can give the self-instability limitation for uniform nanobeams with $D_1 = D_2$. For example, when $L = 140$ nm, the critical diameter for self-instability occurrence is evaluated equally to $D_1 = D_2 = 7.05$ nm. Similarly, for $L = 150$ nm, the diameter is obtained equal to $D_1 = D_2 = 7.35$ nm. On the other hand, from a mathematical viewpoint, there is no problem to set a constant value for diameters of different sections and evaluate the results for uniform nanobeams.

5. Conclusions: In summary, size-dependent stability and self-instability of non-uniform nanobeams are studied with consideration of surface effects. The general trend of the size dependencies is explored and a wide range of numerical results is given for nanobeams with positive and negative surface parameters. The following major conclusions are made from the analysis:

- As one section of a step-wise nanobeam gets shorter and thicker in a meaningful range, the effect of surface properties on the buckling load of the whole nanobeam reduces considerably.

- Proper modelling and choosing the number of steps play important roles in the appropriate determination of the mechanical behaviour of non-uniform nanobeams. In this case, the sensitivity of the problem gets higher for nanobeams with negative surface residual stresses with errors in the range of 70%.

- In the case of self-instability, as one section gets thinner and elongated, the whole nanobeam goes to the self-instable region and conversely attaching a thicker section to the nanobeams reinforce them again self-instability problems.

- For shorter and thicker nanobeams, NBL of non-uniform nanobeams can be approximately taken equal to that of uniform nanobeams with a mean diameter with satisfactory errors.

- The presented results for the effective Young's modulus are verified with satisfactory agreements with those of experiments of [22].

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