CAAI Transactions on Intelligence Technology

Special Issue: Neutrosophic Decision Making and Applications in Knowledge Management

Complex neutrosophic generalised dice similarity measures and their application to decision making

ISSN 2468-2322
Received on 5th December 2019
Revised on 22nd January 2020
Accepted on 18th February 2020
doi: 10.1049/trit.2019.0084
www.ietdl.org

Zeeshan Ali, Tahir Mahmood ✉

Department of Mathematics and Statistics, International Islamic University, Islamabad, Pakistan
✉ *E-mail: tahirbakhat@iiu.edu.pk*

Abstract: Complex neutrosophic set (CNS) is a modified version of the complex fuzzy set, to cope with complicated and inconsistent information in the environment of fuzzy set theory. The CNS is characterised by three functions expressing the degree of complex-valued membership, complex-valued abstinence and degree of complex-valued non-membership. The aim of this manuscript is to initiate the novel dice similarity measures and generalised dice similarity using CNS. The special cases of the investigated measures are discussed with the help of some remarks. Moreover, some distance measures based on CNS are also proposed in this manuscript. Then, the authors applied the generalised dice similarity measures and weighted generalised dice similarity measures using CNS to the pattern recognition model to examine the reliability and superiority of the established approaches. The advantages and comparative analysis of the proposed measures with existing measures are also discussed in detail. At last, a numerical example is provided to illustrate the validity and applicability of the presented measures.

1 Introduction

Multi-attribute group decision making (MAGDM) problems are an important part of modern decision theory. In real decision making, because the decision-making problems are fuzzy and uncertain, the attribute values are not always expressed as real numbers, and some of them are more suitable to be denoted by fuzzy numbers. So, Zadeh [1] developed the notion of a fuzzy set (FS) to cope with complexity. The FS only characterised by membership function, whose range is interval [0, 1]. FS is successfully applied in the environment of aggregation operators [2, 3], medical diagnosis [4, 5] and MAGDM [6] problems. Further, Zadeh [7] proposed the interval-valued FS, which is an extension of FS and characterised by membership grade, whose rang is some closed interval of interval [0, 1]. Rosenfeld [8] investigated fuzzy group, Chang [9] found fuzzy topological spaces. But some decision maker arise a question, what will be the result when we change the range of FS, which is [0, 1] instead of unit disc in a complex plane. Therefore, Ramot *et al.* [10] developed the notion of complex FS (CFS), which consists of complex-valued membership grade, whose range is unit disc in a complex plane. Further, Ramot *et al.* [11] again initiated the notion of complex fuzzy logic. The notion of CFS, which is proposed by Ramot, is totally different from complex fuzzy number that was proposed by Buckley [12].

Further, Atanassov [13] found the notion of intuitionistic FS (IFS), characterised by membership and non-membership grades. The boundedness of IFS is that the sum of membership and non-membership grades is belonging to [0, 1]. IFS is a useful tool to cope with complicated and difficult information in real-decision problems. IFS overcomes a disadvantage of the FS which can only have a membership grade. The IFS is successfully applied in the environment of pattern recognition [14, 15], medical diagnosis [16, 17], aggregation operators [18, 19], distance and similarity measures [20, 21] and MAGDM [22, 23] problems. Moreover, the concept of complex IFS (CIFS) was investigated by Alkouri and Salleh [24]. CIFS is an extension of CFS and FS to cope with uncertain and unpredictable information. The CIFS contains two functions so-called complex-valued membership and complex-valued non-membership grades, with a condition that is the sum of real-part (also for imaginary part) of membership degree and real-part (also for imaginary part) of non-membership degree is less than or equal to 1. The CIFS is modified version of CFS, which contain two dimensions information in a single set. The membership degree and non-membership degree represents the polar coordinates in CIFS. Moreover, Kumar and Bajaj [25] found complex intuitionistic fuzzy soft sets with distance measures and entropies. Garg and Rani [26, 27] robust correlation coefficient measure of CIFS and their applications in decision-making and some generalized complex intuitionistic fuzzy aggregation operators and their application in multicriteria decision making process. Further, Rani and Garg [28, 29] again initiated distance measures between the CIFSs and their applications in decision making process and complex intuitionistic fuzzy power aggregation operators and their application in multicriteria decision making.

Smarandache [30] generalised the idea of IFS is to propose the framework of a neutrosophic set (NS) to deal with indeterminate and inconsistence information. The NS is characterised by three functions expressing the degree of membership (MS), abstinence and non-membership (NMS). The NS is successfully applied in different areas such as distance measures [31], aggregation operators [32] and multi-attribute decision making (MADM) [33]. Further, interval NS was pioneered by Wang *et al.* [34]. Broumi *et al.* [35] initiated the notion of rough NS. Single-valued NS (SVNS) was found by Haibin *et al.* [36]. The constraint of NS is that the sum of MS, abstinence and NM grades are restricted to $]0^{-}, 3^{+}[$, but the constraint of SVNS is less than or equal to 3. For more work on NS and SVNS, we may refer to [37–39]. The concept of complex NS (CNS), proposed by Ali and Smarandache [40], is a generalisation of NS and CIFS to deal with two-dimensional information in a single element. The CNS is characterised by complex-valued MS, complex-valued abstinence and complex-valued NMS grades with a condition that the sum of real-valued MS (imaginary-valued MS), real-valued abstinence (imaginary-valued abstinence) and real-valued NMS (imaginary-valued NMS) grades is less than or equal to 3^{+}. Recently, Ali *et al.* [41] initiated interval complex NS. Further, the generalised dice similarity measures (GDSMs) for picture FS is originally proposed by Wei and Gao [42]. The similarity and distance measures of CNSs are defined to discriminant the information conveyed by different CNSs. The notion of distance and similarity measures is complementary. They can be regarded as two different aspects of the discrimination measure. The similarity measure quantifies the closeness degree between CNSs, while the measure of distance is defined to depict the difference between CNSs.

Basically, complex neutrosophic set (CNS) is an extension of complex IFS to deal with uncertain and unpredictable information in FS theory. The constraint of the CNS is that the sum of positive, abstinence and negative grades is less than or equal to three. They provide a wide range to cope with uncertain and vagueness. Keeping the advantages of the generalised dice similarity and CNS, firstly we reviewed the notion of CNS and their basic operational law such as union, intersection and so on. Then the novel dice similarity measures and generalised dice similarity for CNS are developed. The special cases of the investigated methods are discussed with the help of some remarks. Moreover, the distance measures for CNS are also proposed in this manuscript. Then, we applied the GDSMs and WGDSMs between CNSs to pattern recognition. The advantages of found approaches and the compression between proposed methods with existing methods are initiated. The proposed measures are compared with the following existing approaches. At last, an illustrative numerical example is provided to demonstrate the efficiency and effectiveness of the proposed approaches.

The reminder of this manuscript is set as follows. In Section 2, we briefly review the background of IFSs, CIFSs, NSs, CNSs and their properties. In Section 3, the novel dice similarity measures and generalised dice similarity for CNS are developed. The special cases of the investigated measures are discussed with the help of some remarks. Moreover, the distance measures for CNS are also proposed in this manuscript. In Section 4, we applied the GDSMs and WGDSMs between CNSs to the pattern recognition. The advantages of found approaches and the compression between proposed measures with existing measures are initiated. At last, a numerical example is provided to illustrate the validity and applicability of the presented distance measures. The conclusion is discussed in Section 5. The graphical interpretation of the explored work in this manuscript is discussed in Fig. 1.

2 Preliminaries

In this section, we discuss basic notions of IFSs, CIFSs, NSs, CNSs, DSMs and their properties.

Definition 1: The notion of IFS is taken from [13] and given by

$$S = \{(\alpha'_S(y), \beta'_S(y)) : y \in X\} \tag{1}$$

where $\alpha'_S, \beta'_S : X \to [0, 1]$ represent the degree of membership (MS) and the degree of non-membership (NMS), with a condition $0 \le \alpha'_S + \beta'_S \le 1$.

Definition 2: The notion of CIFS is taken from [24] and given by

$$S = \{(\alpha'_S(y),\ \beta'_S(y)) : y \in X\} \tag{2}$$

where $\alpha'_S = \alpha_S e^{i2\pi(\delta_{\alpha_S})}$ and $\beta'_S = \beta_S e^{i2\pi(\delta_{\beta_S})}$ represent the degree of complex-valued MS and the degree of complex-valued NM, with conditions $0 \le \alpha_S + \beta_S \le 1$ and $0 \le \delta_{\alpha_S} + \delta_{\beta_S} \le 1$. The CIFS is denoted by CIFS(X).

Definition 3: The notion of NS is taken from [30] and given by

$$S = \{(\alpha'_S(y), \gamma'_S(y), \beta'_S(y)) : y \in X\} \tag{3}$$

where $\alpha'_S, \gamma'_S, \beta'_S : X \to [0^-, 1^+]$ represents the degree of MS, abstinence (AB) and the degree of NM, with a condition $0^- \le \alpha'_S + \gamma'_S + \beta'_S \le 3^+$.

Definition 4: The notion of CNS is taken from [40] and given by

$$S = \{(\alpha'_S(y), \gamma'_S(y), \beta'_S(y)) : y \in X\} \tag{4}$$

where $\alpha'_S = \alpha_S e^{i2\pi(\delta_{\alpha_S})}$, $\gamma'_S(y) = \gamma_S e^{i2\pi(\delta_{\gamma_S})}$ and $\beta'_S = \beta_S e^{i2\pi(\delta_{\beta_S})}$ represented the degree of complex-valued MS, complex-valued AB and the degree of complex-valued NM, with conditions $0^- \le \alpha_S + \gamma_S + \beta_S \le 3^+$ and $0^- \le \delta_{\alpha_S} + \delta_{\gamma_S} + \delta_{\beta_S} \le 3^+$. The CNS is denoted by CNS(X). Where $S = \left(\alpha_S(y_i)e^{i2\pi(\delta_{\alpha_S}(y_i))}, \gamma_S(y_i)e^{i2\pi(\delta_{\gamma_S}(y_i))}, \beta_S(y_i)e^{i2\pi(\delta_{\beta_S}(y_i))}\right)$ represents the complex neutrosophic number (CNN).

Definition 5: These operational laws are taken from [40]. Let $S, \mathcal{T} \in$ CNS(X), then the following hold:

(i) $S^c = \left(\beta_S(y_i)e^{i2\pi(\delta_{\beta_S}(y_i))}, \gamma_S(y_i)e^{i2\pi(\delta_{\gamma_S}(y_i))}, \alpha_S(y_i)e^{i2\pi(\delta_{\alpha_S}(y_i))}\right)$;

(ii) $S \subseteq \mathcal{T}$ iff $\alpha_S(y_i) \le \alpha_{\mathcal{T}}(y_i), \delta_{\alpha_S}(y_i) \le \delta_{\alpha_{\mathcal{T}}}(y_i)$, $\gamma_S(y_i) \le \gamma_{\mathcal{T}}(y_i), \delta_{\gamma_S}(y_i) \le \delta_{\gamma_{\mathcal{T}}}(y_i), \beta_S(y_i) \ge \beta_{\mathcal{T}}(y_i)$ and $\delta_{\beta_S}(y_i) \ge \delta_{\beta_{\mathcal{T}}}(y_i)$;

(iii) $S = \mathcal{T}$ iff $S \subseteq \mathcal{T}$ and $S \supseteq \mathcal{T}$

Definition 6: The notion of DSM is taken from [42]. Considered the two vectors $\boldsymbol{X} = (y_1, y_2, \ldots, y_n)$ and $\boldsymbol{Y} = (\mathcal{Y}_1, \mathcal{Y}_2, \ldots, \mathcal{Y}_n)$, then the DSM is denoted and defined by

$$\mathcal{dM}(\boldsymbol{X}, \boldsymbol{Y}) = \frac{2\boldsymbol{X}.\boldsymbol{Y}}{\|\boldsymbol{X}\|_2^2 + \|\boldsymbol{Y}\|_2^2} = \frac{2\sum_{i=1}^n y_i \mathcal{Y}_i}{\sum_{i=1}^n y_i^2 + \sum_{i=1}^n \mathcal{Y}_i^2} \tag{5}$$

where $\boldsymbol{X}.\boldsymbol{Y} = \sum_{i=1}^n y_i \mathcal{Y}_i$ represents the inner products and $\|\boldsymbol{X}\|_2^2 = \sum_{i=1}^n y_i^2$, $\|\boldsymbol{Y}\|_2^2 = \sum_{i=1}^n \mathcal{Y}_i^2$ represents the Euclidean norm of vectors $\boldsymbol{X}$ and $\boldsymbol{Y}$. If $y_i = \mathcal{Y}_i = 0$, so the DSM is undefined.

3 Dice similarity measures for complex *q*-rung orthopair fuzzy sets

In this section, we propose the notion of DSM and WDSM for CNSs. The special cases of the proposed approaches are also discussed in detail. The distance measures for CNSs are also discussed in some remarks. Throughout this paper, the weight vector is given by: $\boldsymbol{\omega} = \{\omega_1, \omega_2, \ldots, \omega_n\}$, $\omega_i \in [0, 1]$ with a condition $\sum_{i=1}^n \omega_i = 1$. Similarly, the CNN is denoted and defined by: $S = \left(\alpha_S(y_i)e^{i2\pi(\delta_{\alpha_S})},\ \gamma_S(y_i)e^{i2\pi(\delta_{\gamma_S})},\ \beta_S(y_i)e^{i2\pi(\delta_{\beta_S})}\right)$ on a finite universal set X.

Definition 7: A DSM $\mathcal{dM}^1_{\mathrm{Cq-ROF}}(S, \mathcal{T})$ for CNSs is given by

(see (6))

which satisfy the following conditions:

(i) $0 \le \mathcal{dM}^1_{\mathrm{CNS}}(S, \mathcal{T}) \le 1$

(ii) $\mathcal{dM}^1_{\mathrm{CNS}}(S, \mathcal{T}) = \mathcal{dM}^1_{\mathrm{CNS}}(\mathcal{T}, S)$

(iii) $\mathcal{dM}^1_{\mathrm{CNS}}(S, \mathcal{T}) = 1 \Leftrightarrow S = \mathcal{T}$

$$\mathcal{dM}^1_{\mathrm{CNS}}(S, \mathcal{T}) = \frac{1}{n}\sum_{i=1}^n \left(\frac{2\begin{pmatrix} \alpha_S(y_i)\alpha_{\mathcal{T}}(y_i) + \gamma_S(y_i)\gamma_{\mathcal{T}}(y_i) + \beta_S(y_i)\beta_{\mathcal{T}}(y_i) + \\ \delta_{\alpha_S}(y_i)\delta_{\alpha_{\mathcal{T}}}(y_i) + \delta_{\gamma_S}(y_i)\delta_{\gamma_{\mathcal{T}}}(y_i) + \delta_{\beta_S}(y_i)\delta_{\beta_{\mathcal{T}}}(y_i) \end{pmatrix}}{\begin{pmatrix} \alpha_S^2(y_i) + \gamma_S^2(y_i) + \beta_S^2(y_i) + \\ \delta_{\alpha_S}^2(y_i) + \delta_{\gamma_S}^2(y_i) + \delta_{\beta_S}^2(y_i) \end{pmatrix} + \begin{pmatrix} \alpha_{\mathcal{T}}^2(y_i) + \gamma_{\mathcal{T}}^2(y_i) + \beta_{\mathcal{T}}^2(y_i) + \\ \delta_{\alpha_{\mathcal{T}}}^2(y_i) + \delta_{\gamma_{\mathcal{T}}}^2(y_i) + \delta_{\beta_{\mathcal{T}}}^2(y_i) \end{pmatrix}} \right) \tag{6}$$

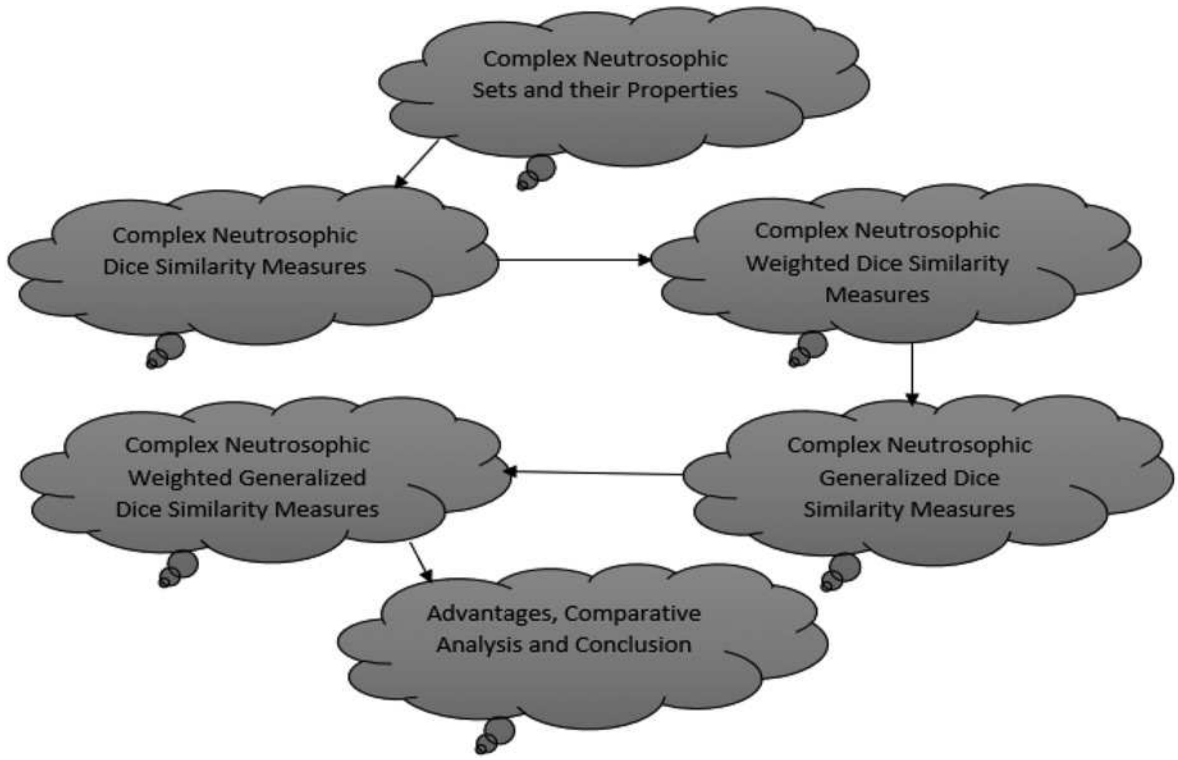

Fig. 1 *Structure of this paper in the form of flowchart*

Proof: We will consider (6), and prove the following conditions:

(i) Let

$$\begin{aligned}&d\mathcal{M}^{1}_{\text{CNS}}(S,\mathcal{T})\\&=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{2\begin{pmatrix}\alpha_S(y_i)\alpha_{\mathcal{T}}(y_i)+\gamma_S(y_i)\gamma_{\mathcal{T}}(y_i)+\beta_S(y_i)\beta_{\mathcal{T}}(y_i)+\\ \delta_{\alpha_S}(y_i)\delta_{\alpha_{\mathcal{T}}}(y_i)+\delta_{\gamma_S}(y_i)\delta_{\gamma_{\mathcal{T}}}(y_i)+\delta_{\beta_S}(y_i)\delta_{\beta_{\mathcal{T}}}(y_i)\end{pmatrix}}{\begin{pmatrix}\alpha^2_S(y_i)+\gamma^2_S(y_i)+\beta^2_S(y_i)+\\ \delta^2_{\alpha_S}(y_i)+\delta^2_{\gamma_S}(y_i)+\delta^2_{\beta_S}(y_i)\end{pmatrix}+\begin{pmatrix}\alpha^2_{\mathcal{T}}(y_i)+\gamma^2_{\mathcal{T}}(y_i)+\beta^2_{\mathcal{T}}(y_i)+\\ \delta^2_{\alpha_{\mathcal{T}}}(y_i)+\delta^2_{\gamma_{\mathcal{T}}}(y_i)+\delta^2_{\beta_{\mathcal{T}}}(y_i)\end{pmatrix}}\right)\end{aligned}$$

It is clear that $d\mathcal{M}^{1}_{\text{CNS}}(S,\mathcal{T})\geq 0$, and

$$\begin{aligned}&\left(\begin{array}{l}\begin{pmatrix}\alpha^2_S(y_i)+\gamma^2_S(y_i)+\beta^2_S(y_i)+\\ \delta^2_{\alpha_S}(y_i)+\delta^2_{\gamma_S}(y_i)+\delta^2_{\beta_S}(y_i)\end{pmatrix}+\\ \begin{pmatrix}\alpha^2_{\mathcal{T}}(y_i)+\gamma^2_{\mathcal{T}}(y_i)+\beta^2_{\mathcal{T}}(y_i)+\\ \delta^2_{\alpha_{\mathcal{T}}}(y_i)+\delta^2_{\gamma_{\mathcal{T}}}(y_i)+\delta^2_{\beta_{\mathcal{T}}}(y_i)\end{pmatrix}\end{array}\right)\\&\geq 2\begin{pmatrix}\alpha_S(y_i)\alpha_{\mathcal{T}}(y_i)+\gamma_S(y_i)\gamma_{\mathcal{T}}(y_i)+\beta_S(y_i)\beta_{\mathcal{T}}(y_i)+\\ \delta_{\alpha_S}(y_i)\delta_{\alpha_{\mathcal{T}}}(y_i)+\delta_{\gamma_S}(y_i)\delta_{\gamma_{\mathcal{T}}}(y_i)+\delta_{\beta_S}(y_i)\delta_{\beta_{\mathcal{T}}}(y_i)\end{pmatrix}\end{aligned}$$

According to the inequality $a^2+b^2\geq 2ab$. Thus, $0\leq d\mathcal{M}^{1}_{\text{CNS}}(S_i,\mathcal{T}_i)\leq 1$, so from (6), we get $0\leq d\mathcal{M}^{1}_{\text{CNS}}(S,\mathcal{T})\leq 1$.
(ii) Equation (6) easily verified condition (ii).
(iii) If $S=\mathcal{T}$, i.e. $\alpha_S(y_i)=\alpha_{\mathcal{T}}(y_i), \delta_{\alpha_S}(y_i)=\delta_{\alpha_{\mathcal{T}}}(y_i), \gamma_S(y_i)=\gamma_{\mathcal{T}}(y_i), \delta_{\gamma_S}(y_i)=\delta_{\gamma_{\mathcal{T}}}(y_i), \beta_S(y_i)=\beta_{\mathcal{T}}(y_i)$ and $\delta_{\beta_S}(y_i)=\delta_{\beta_{\mathcal{T}}}(y_i)$, we have

$$\begin{aligned}&d\mathcal{M}^{1}_{\text{CNS}}(S,\mathcal{T})\\&=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{2\begin{pmatrix}\alpha_S(y_i)\alpha_{\mathcal{T}}(y_i)+\gamma_S(y_i)\gamma_{\mathcal{T}}(y_i)+\beta_S(y_i)\beta_{\mathcal{T}}(y_i)+\\ \delta_{\alpha_S}(y_i)\delta_{\alpha_{\mathcal{T}}}(y_i)+\delta_{\gamma_S}(y_i)\delta_{\gamma_{\mathcal{T}}}(y_i)+\delta_{\beta_S}(y_i)\delta_{\beta_{\mathcal{T}}}(y_i)\end{pmatrix}}{\begin{pmatrix}\alpha^2_S(y_i)+\gamma^2_S(y_i)+\beta^2_S(y_i)+\\ \delta^2_{\alpha_S}(y_i)+\delta^2_{\gamma_S}(y_i)+\delta^2_{\beta_S}(y_i)\end{pmatrix}+\begin{pmatrix}\alpha^2_{\mathcal{T}}(y_i)+\gamma^2_{\mathcal{T}}(y_i)+\beta^2_{\mathcal{T}}(y_i)+\\ \delta^2_{\alpha_{\mathcal{T}}}(y_i)+\delta^2_{\gamma_{\mathcal{T}}}(y_i)+\delta^2_{\beta_{\mathcal{T}}}(y_i)\end{pmatrix}}\right)\\&=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{2\begin{pmatrix}\alpha_S(y_i)\alpha_S(y_i)+\gamma_S(y_i)\gamma_S(y_i)+\beta_S(y_i)\beta_S(y_i)+\\ \delta_{\alpha_S}(y_i)\delta_{\alpha_S}(y_i)+\delta_{\gamma_S}(y_i)\delta_{\gamma_S}(y_i)+\delta_{\beta_S}(y_i)\delta_{\beta_S}(y_i)\end{pmatrix}}{\begin{pmatrix}\alpha^2_S(y_i)+\gamma^2_S(y_i)+\beta^2_S(y_i)+\\ \delta^2_{\alpha_S}(y_i)+\delta^2_{\gamma_S}(y_i)+\delta^2_{\beta_S}(y_i)\end{pmatrix}+\begin{pmatrix}\alpha^2_S(y_i)+\gamma^2_S(y_i)+\beta^2_S(y_i)+\\ \delta^2_{\alpha_S}(y_i)+\delta^2_{\gamma_S}(y_i)+\delta^2_{\beta_S}(y_i)\end{pmatrix}}\right)\\&=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{2\left(\alpha^2_S(y_i)+\gamma^2_S(y_i)+\beta^2_S(y_i)+\delta^2_{\alpha_S}(y_i)+\delta^2_{\gamma_S}(y_i)+\delta^2_{\beta_S}(y_i)\right)}{2\left(\alpha^2_S(y_i)+\gamma^2_S(y_i)+\beta^2_S(y_i)+\delta^2_{\alpha_S}(y_i)+\delta^2_{\gamma_S}(y_i)+\delta^2_{\beta_S}(y_i)\right)}\right)\\&=1\end{aligned}$$

The proof is completed. □

Definition 8: A WDSM $d\mathcal{M}^{w1}_{\text{CNS}}(S,\mathcal{T})$ for CNSs is given by (see (7))

$$d\mathcal{M}^{w1}_{\text{CNS}}(S,\mathcal{T})=\sum_{i=1}^{n}\omega_i\left(\frac{2\begin{pmatrix}\alpha_S(y_i)\alpha_{\mathcal{T}}(y_i)+\gamma_S(y_i)\gamma_{\mathcal{T}}(y_i)+\beta_S(y_i)\beta_{\mathcal{T}}(y_i)+\\ \delta_{\alpha_S}(y_i)\delta_{\alpha_{\mathcal{T}}}(y_i)+\delta_{\gamma_S}(y_i)\delta_{\gamma_{\mathcal{T}}}(y_i)+\delta_{\beta_S}(y_i)\delta_{\beta_{\mathcal{T}}}(y_i)\end{pmatrix}}{\begin{pmatrix}\alpha^2_S(y_i)+\gamma^2_S(y_i)+\beta^2_S(y_i)+\\ \delta^2_{\alpha_S}(y_i)+\delta^2_{\gamma_S}(y_i)+\delta^2_{\beta_S}(y_i)\end{pmatrix}+\begin{pmatrix}\alpha^2_{\mathcal{T}}(y_i)+\gamma^2_{\mathcal{T}}(y_i)+\beta^2_{\mathcal{T}}(y_i)+\\ \delta^2_{\alpha_{\mathcal{T}}}(y_i)+\delta^2_{\gamma_{\mathcal{T}}}(y_i)+\delta^2_{\beta_{\mathcal{T}}}(y_i)\end{pmatrix}}\right) \tag{7}$$

which satisfy the following conditions:

(i) $0 \le d\mathcal{M}^{w1}_{\mathrm{CNS}}(S, \mathcal{T}) \le 1$
(ii) $d\mathcal{M}^{w1}_{\mathrm{CNS}}(S, \mathcal{T}) = d\mathcal{M}^{w1}_{\mathrm{CNS}}(\mathcal{T}, S)$
(iii) $d\mathcal{M}^{w1}_{\mathrm{CNS}}(S, \mathcal{T}) = 1 \Leftrightarrow S = \mathcal{T}$

Example 1: Let $S = \left(y_1, \left\langle \begin{matrix} 0.3e^{i2\pi(0.33)}, \\ 0.6e^{i2\pi(0.45)}, 0.3e^{i2\pi(0.56)} \end{matrix} \right\rangle\right)$ and $S = \left(y_1, \left\langle \begin{matrix} 0.34e^{i2\pi(0.39)}, \\ 0.22e^{i2\pi(0.13)}, 0.13e^{i2\pi(0.22)} \end{matrix} \right\rangle\right)$ be two CNNs. Then, we use (6) such that

(see (equation below))

Remarks 1: If we take $\omega = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)^{\mathrm{T}}$ then the WDSM for CNS reduced to DSM for CNS, i.e. $d\mathcal{M}^{w1}_{\mathrm{CNS}}(S, \mathcal{T}) = d\mathcal{M}^{1}_{\mathrm{CNS}}(S, \mathcal{T})$. Moreover, we proposed the other form of DSM and WDSM for CNS.

Remarks 2: The dice distance measure (DDM) and weighted dice distance measure (WDDM) for CNSs are as follows:

$$d\mathcal{M}d\mathcal{M}^{1}_{\mathrm{CNS}}(S, \mathcal{T}) = 1 - d\mathcal{M}^{1}_{\mathrm{CNS}}(S, \mathcal{T}) \tag{8}$$

$$d\mathcal{M}d\mathcal{M}^{w1}_{\mathrm{CNS}}(S, \mathcal{T}) = 1 - d\mathcal{M}^{w1}_{\mathrm{CNS}}(S, \mathcal{T}) \tag{9}$$

Definition 9: A DSM $d\mathcal{M}^{3}_{\mathrm{CNS}}(S, \mathcal{T})$ for CNSs is given by

(see (10))

which satisfy the following conditions:

(i) $0 \le d\mathcal{M}^{3}_{\mathrm{CNS}}(S, \mathcal{T}) \le 1$
(ii) $d\mathcal{M}^{3}_{\mathrm{CNS}}(S, \mathcal{T}) = d\mathcal{M}^{3}_{\mathrm{CNS}}(\mathcal{T}, S)$
(iii) $d\mathcal{M}^{3}_{\mathrm{CNS}}(S, \mathcal{T}) = 1 \Leftrightarrow S = \mathcal{T}$

Definition 10: A WDSM $d\mathcal{M}^{w3}_{\mathrm{CNS}}(S, \mathcal{T})$ for CNSs is given by

(see (11))

which satisfy the following conditions:

(i) $0 \le d\mathcal{M}^{w3}_{\mathrm{CNS}}(S, \mathcal{T}) \le 1$
(ii) $d\mathcal{M}^{w3}_{\mathrm{CNS}}(S, \mathcal{T}) = d\mathcal{M}^{w3}_{\mathrm{CNS}}(\mathcal{T}, S)$
(iii) $d\mathcal{M}^{w3}_{\mathrm{CNS}}(S, \mathcal{T}) = 1 \Leftrightarrow S = \mathcal{T}$

Remarks 3: If we take $\omega = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)^{\mathrm{T}}$ then the WDSM for CNS reduced to DSM for CNS, i.e. $d\mathcal{M}^{w3}_{\mathrm{CNS}}(S, \mathcal{T}) = d\mathcal{M}^{3}_{\mathrm{CNS}}(S, \mathcal{T})$.

3.1 Generalised dice similarity measures for CNSs

In this section, we propose the notion of GDSM and WGDSM for CNSs. The special cases of the proposed approaches are also discussed in detail. The distance measures for CNSs are also discussed in some remarks.

Definition 11: A GDSM $d\mathcal{M}^{G1}_{\mathrm{CNS}}(S, \mathcal{T})$ for CNSs is given by

(see (12))

which satisfy the following conditions:
(i) $0 \le d\mathcal{M}^{G1}_{\mathrm{CNS}}(S, \mathcal{T}) \le 1$
(ii) $d\mathcal{M}^{G1}_{\mathrm{CNS}}(S, \mathcal{T}) = d\mathcal{M}^{G1}_{\mathrm{CNS}}(\mathcal{T}, S)$
(iii) $d\mathcal{M}^{G1}_{\mathrm{CNS}}(S, \mathcal{T}) = 1 \Leftrightarrow S = \mathcal{T}$.

If we consider the value of $\Upsilon = 0$, then

$$d\mathcal{M}^{G1}_{\mathrm{CNS}}(S, \mathcal{T}) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\begin{pmatrix} \alpha_S(y_i)\alpha_{\mathcal{T}}(y_i) + \gamma_S(y_i)\gamma_{\mathcal{T}}(y_i) + \beta_S(y_i)\beta_{\mathcal{T}}(y_i) + \\ \delta_{\alpha_S}(y_i)\delta_{\alpha_{\mathcal{T}}}(y_i) + \delta_{\gamma_S}(y_i)\delta_{\gamma_{\mathcal{T}}}(y_i) + \delta_{\beta_S}(y_i)\delta_{\beta_{\mathcal{T}}}(y_i) \end{pmatrix}}{\begin{pmatrix} \alpha^2_{\mathcal{T}}(y_i) + \gamma^2_{\mathcal{T}}(y_i) + \beta^2_{\mathcal{T}}(y_i) + \\ \delta^2_{\alpha_{\mathcal{T}}}(y_i) + \delta^2_{\gamma_{\mathcal{T}}}(y_i) + \delta^2_{\beta_{\mathcal{T}}}(y_i) \end{pmatrix}}\right) \tag{13}$$

$$\begin{aligned} d\mathcal{M}^{w1}_{\mathrm{CNS}}(S, \mathcal{T}) &= \left(\frac{2(0.3 \times 0.34 + 0.6 \times 0.22 + 0.3 \times 0.13 + 0.33 \times 0.39 + 0.45 \times 0.13 + 0.56 \times 0.22)}{(0.3^2 + 0.6^2 + 0.3^2 + 0.33^2 + 0.45^2 + 0.56^2) + (0.34^2 + 0.22^2 + 0.13^2 + 0.39^2 + 0.13^2 + 0.22^2)}\right) \\ &= \left(\frac{1.167}{1.156 + 0.40}\right) = 0.75 \end{aligned}$$

$$d\mathcal{M}^{3}_{\mathrm{CNS}}(S, \mathcal{T}) = \left(\frac{\sum_{i=1}^{n} 2\begin{pmatrix} \alpha_S(y_i)\alpha_{\mathcal{T}}(y_i) + \gamma_S(y_i)\gamma_{\mathcal{T}}(y_i) + \beta_S(y_i)\beta_{\mathcal{T}}(y_i) + \\ \delta_{\alpha_S}(y_i)\delta_{\alpha_{\mathcal{T}}}(y_i) + \delta_{\gamma_S}(y_i)\delta_{\gamma_{\mathcal{T}}}(y_i) + \delta_{\beta_S}(y_i)\delta_{\beta_{\mathcal{T}}}(y_i) \end{pmatrix}}{\sum_{i=1}^{n}\begin{pmatrix} \alpha^2_S(y_i) + \gamma^2_S(y_i) + \beta^2_S(y_i) + \\ \delta^2_{\alpha_S}(y_i) + \delta^2_{\gamma_S}(y_i) + \delta^2_{\beta_S}(y_i) \end{pmatrix} + \sum_{i=1}^{n}\begin{pmatrix} \alpha^2_{\mathcal{T}}(y_i) + \gamma^2_{\mathcal{T}}(y_i) + \beta^2_{\mathcal{T}}(y_i) + \\ \delta^2_{\alpha_{\mathcal{T}}}(y_i) + \delta^2_{\gamma_{\mathcal{T}}}(y_i) + \delta^2_{\beta_{\mathcal{T}}}(y_i) \end{pmatrix}}\right) \tag{10}$$

$$d\mathcal{M}^{w3}_{\mathrm{CNS}}(S, \mathcal{T}) = \left(\frac{2\sum_{i=1}^{n} \omega_i^2 \begin{pmatrix} \alpha_S(y_i)\alpha_{\mathcal{T}}(y_i) + \gamma_S(y_i)\gamma_{\mathcal{T}}(y_i) + \beta_S(y_i)\beta_{\mathcal{T}}(y_i) + \\ \delta_{\alpha_S}(y_i)\delta_{\alpha_{\mathcal{T}}}(y_i) + \delta_{\gamma_S}(y_i)\delta_{\gamma_{\mathcal{T}}}(y_i) + \delta_{\beta_S}(y_i)\delta_{\beta_{\mathcal{T}}}(y_i) \end{pmatrix}}{\sum_{i=1}^{n} \omega_i^2 \begin{pmatrix} \alpha^2_S(y_i) + \gamma^2_S(y_i) + \beta^2_S(y_i) + \\ \delta^2_{\alpha_S}(y_i) + \delta^2_{\gamma_S}(y_i) + \delta^2_{\beta_S}(y_i) \end{pmatrix} + \sum_{i=1}^{n} \omega_i^2 \begin{pmatrix} \alpha^2_{\mathcal{T}}(y_i) + \gamma^2_{\mathcal{T}}(y_i) + \beta^2_{\mathcal{T}}(y_i) + \\ \delta^2_{\alpha_{\mathcal{T}}}(y_i) + \delta^2_{\gamma_{\mathcal{T}}}(y_i) + \delta^2_{\beta_{\mathcal{T}}}(y_i) \end{pmatrix}}\right) \tag{11}$$

$$d\mathcal{M}^{G1}_{\mathrm{CNS}}(S, \mathcal{T}) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\begin{pmatrix} \alpha_S(y_i)\alpha_{\mathcal{T}}(y_i) + \gamma_S(y_i)\gamma_{\mathcal{T}}(y_i) + \beta_S(y_i)\beta_{\mathcal{T}}(y_i) + \\ \delta_{\alpha_S}(y_i)\delta_{\alpha_{\mathcal{T}}}(y_i) + \delta_{\gamma_S}(y_i)\delta_{\gamma_{\mathcal{T}}}(y_i) + \delta_{\beta_S}(y_i)\delta_{\beta_{\mathcal{T}}}(y_i) \end{pmatrix}}{\Upsilon\begin{pmatrix} \alpha^2_S(y_i) + \gamma^2_S(y_i) + \beta^2_S(y_i) + \\ \delta^2_{\alpha_S}(y_i) + \delta^2_{\gamma_S}(y_i) + \delta^2_{\beta_S}(y_i) \end{pmatrix} + (1 - \Upsilon)\begin{pmatrix} \alpha^2_{\mathcal{T}}(y_i) + \gamma^2_{\mathcal{T}}(y_i) + \beta^2_{\mathcal{T}}(y_i) + \\ \delta^2_{\alpha_{\mathcal{T}}}(y_i) + \delta^2_{\gamma_{\mathcal{T}}}(y_i) + \delta^2_{\beta_{\mathcal{T}}}(y_i) \end{pmatrix}}\right) \tag{12}$$

If we consider the value of $\Upsilon = 1$, then

$$d\mathcal{M}_{\mathrm{CNS}}^{G1}(S,\mathcal{T}) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\left(\begin{array}{c}\alpha_S(y_i)\alpha_{\mathcal{T}}(y_i)+\gamma_S(y_i)\gamma_{\mathcal{T}}(y_i)+\beta_S(y_i)\beta_{\mathcal{T}}(y_i)+\\ \delta_{\alpha_S}(y_i)\delta_{\alpha_{\mathcal{T}}}(y_i)+\delta_{\gamma_S}(y_i)\delta_{\gamma_{\mathcal{T}}}(y_i)+\delta_{\beta_S}(y_i)\delta_{\beta_{\mathcal{T}}}(y_i)\end{array}\right)}{\left(\begin{array}{c}\alpha_S^2(y_i)+\gamma_S^2(y_i)+\beta_S^2(y_i)+\\ \delta_{\alpha_S}^2(y_i)+\delta_{\gamma_S}^2(y_i)+\delta_{\beta_S}^2(y_i)\end{array}\right)}\right) \tag{14}$$

is called asymmetric similarity measures or projection similarity measures.

Example 2: Let $S = \left(y_1, \left\langle \begin{array}{c}0.3e^{i2\pi(0.33)},\\ 0.6e^{i2\pi(0.45)}, 0.3e^{i2\pi(0.56)}\end{array}\right\rangle\right)$ and $S = \left(y_1, \left\langle \begin{array}{c}0.34e^{i2\pi(0.39)},\\ 0.22e^{i2\pi(0.13)}, 0.13e^{i2\pi(0.22)}\end{array}\right\rangle\right)$ be two CNNs with parameter $\Upsilon = 0.3$. Then, we use (12) such that

(see (equation below))

Definition 12: A WGDSM $d\mathcal{M}_{\mathrm{CNS}}^{wG1}(S,\mathcal{T})$ for CNSs is given by

(see (15))

which satisfy the following conditions:

(i) $0 \le d\mathcal{M}_{\mathrm{CNS}}^{wG1}(S,\mathcal{T}) \le 1$
(ii) $d\mathcal{M}_{\mathrm{CNS}}^{wG1}(S,\mathcal{T}) = d\mathcal{M}_{\mathrm{CNS}}^{wG1}(\mathcal{T},S)$
(iii) $d\mathcal{M}_{\mathrm{CNS}}^{wG1}(S,\mathcal{T}) = 1 \Leftrightarrow S = \mathcal{T}$

If we consider the value of $\Upsilon = 0$, then

$$d\mathcal{M}_{\mathrm{CNS}}^{wG1}(S,\mathcal{T}) = \sum_{i=1}^{n}\omega_i\left(\frac{\left(\begin{array}{c}\alpha_S(y_i)\alpha_{\mathcal{T}}(y_i)+\gamma_S(y_i)\gamma_{\mathcal{T}}(y_i)+\beta_S(y_i)\beta_{\mathcal{T}}(y_i)+\\ \delta_{\alpha_S}(y_i)\delta_{\alpha_{\mathcal{T}}}(y_i)+\delta_{\gamma_S}(y_i)\delta_{\gamma_{\mathcal{T}}}(y_i)+\delta_{\beta_S}(y_i)\delta_{\beta_{\mathcal{T}}}(y_i)\end{array}\right)}{\left(\begin{array}{c}\alpha_{\mathcal{T}}^2(y_i)+\gamma_{\mathcal{T}}^2(y_i)+\beta_{\mathcal{T}}^2(y_i)+\\ \delta_{\alpha_{\mathcal{T}}}^2(y_i)+\delta_{\gamma_{\mathcal{T}}}^2(y_i)+\delta_{\beta_{\mathcal{T}}}^2(y_i)\end{array}\right)}\right) \tag{16}$$

If we consider the value of $\Upsilon = 1$, then

$$d\mathcal{M}_{\mathrm{CNS}}^{wG1}(S,\mathcal{T}) = \sum_{i=1}^{n}\omega_i\left(\frac{\left(\begin{array}{c}\alpha_S(y_i)\alpha_{\mathcal{T}}(y_i)+\gamma_S(y_i)\gamma_{\mathcal{T}}(y_i)+\beta_S(y_i)\beta_{\mathcal{T}}(y_i)+\\ \delta_{\alpha_S}(y_i)\delta_{\alpha_{\mathcal{T}}}(y_i)+\delta_{\gamma_S}(y_i)\delta_{\gamma_{\mathcal{T}}}(y_i)+\delta_{\beta_S}(y_i)\delta_{\beta_{\mathcal{T}}}(y_i)\end{array}\right)}{\left(\begin{array}{c}\alpha_S^2(y_i)+\gamma_S^2(y_i)+\beta_S^2(y_i)+\\ \delta_{\alpha_S}^2(y_i)+\delta_{\gamma_S}^2(y_i)+\delta_{\beta_S}^2(y_i)\end{array}\right)}\right) \tag{17}$$

is called asymmetric similarity measures or projection similarity measures.

Remarks 4: Where the value of $0 \le \Upsilon \le 1$. By changing $\Upsilon = 0.5$, (12) and (15) are reduced into (6) and (7).

Remarks 5: If we take $\omega = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)^{\mathrm{T}}$, then the WGDSM for CNS reduced to GDSM for CNS, i.e. $d\mathcal{M}_{\mathrm{CNS}}^{wG1}(S,\mathcal{T}) = d\mathcal{M}_{\mathrm{CNS}}^{G1}(S,\mathcal{T})$.

Remarks 6: The generalised dice distance measure (GDDM) and weighted generalised dice distance measure (WGDDM) for Cq-ROFSs are as follows:

$$d\mathcal{M}d\mathcal{M}_{\mathrm{CNS}}^{G1}(S,\mathcal{T}) = 1 - d\mathcal{M}_{\mathrm{CNS}}^{G1}(S,\mathcal{T}) \tag{18}$$

$$d\mathcal{M}d\mathcal{M}_{\mathrm{CNS}}^{wG1}(S,\mathcal{T}) = 1 - d\mathcal{M}_{\mathrm{CNS}}^{wG1}(S,\mathcal{T}) \tag{19}$$

Definition 13: A GDSM $d\mathcal{M}_{\mathrm{CNS}}^{G3}(S,\mathcal{T})$ for Cq-ROFSs is given by

(see (20))

which satisfy the following conditions:

(i) $0 \le d\mathcal{M}_{\mathrm{CNS}}^{G3}(S,\mathcal{T}) \le 1$
(ii) $d\mathcal{M}_{\mathrm{CNS}}^{G3}(S,\mathcal{T}) = d\mathcal{M}_{\mathrm{CNS}}^{G3}(\mathcal{T},S)$
(iii) $d\mathcal{M}_{\mathrm{CNS}}^{G3}(S,\mathcal{T}) = 1 \Leftrightarrow S = \mathcal{T}$

$$\begin{aligned} d\mathcal{M}_{\mathrm{CNS}}^{G1}(S,\mathcal{T}) &= \left(\frac{(0.3\times 0.34 + 0.6\times 0.22 + 0.3\times 0.13 + 0.33\times 0.39 + 0.45\times 0.13 + 0.56\times 0.22)}{(0.3)\left(0.3^2+0.6^2+0.3^2+0.33^2+0.45^2+0.56^2\right)+(1-0.3)\left(0.34^2+0.22^2+0.13^2+0.39^2+0.13^2+0.22^2\right)}\right)\\ &= \left(\frac{0.58}{0.3\times 1.156 + 0.7\times 0.40}\right) = 0.93 \end{aligned}$$

$$d\mathcal{M}_{\mathrm{CNS}}^{wG1}(S,\mathcal{T}) = \sum_{i=1}^{n}\omega_i\left(\frac{\left(\begin{array}{c}\alpha_S(y_i)\alpha_{\mathcal{T}}(y_i)+\gamma_S(y_i)\gamma_{\mathcal{T}}(y_i)+\beta_S(y_i)\beta_{\mathcal{T}}(y_i)+\\ \delta_{\alpha_S}(y_i)\delta_{\alpha_{\mathcal{T}}}(y_i)+\delta_{\gamma_S}(y_i)\delta_{\gamma_{\mathcal{T}}}(y_i)+\delta_{\beta_S}(y_i)\delta_{\beta_{\mathcal{T}}}(y_i)\end{array}\right)}{\Upsilon\left(\begin{array}{c}\alpha_S^2(y_i)+\gamma_S^2(y_i)+\beta_S^2(y_i)+\\ \delta_{\alpha_S}^2(y_i)+\delta_{\gamma_S}^2(y_i)+\delta_{\beta_S}^2(y_i)\end{array}\right)+(1-\Upsilon)\left(\begin{array}{c}\alpha_{\mathcal{T}}^2(y_i)+\gamma_{\mathcal{T}}^2(y_i)+\beta_{\mathcal{T}}^2(y_i)+\\ \delta_{\alpha_{\mathcal{T}}}^2(y_i)+\delta_{\gamma_{\mathcal{T}}}^2(y_i)+\delta_{\beta_{\mathcal{T}}}^2(y_i)\end{array}\right)}\right) \tag{15}$$

$$d\mathcal{M}_{\mathrm{CNS}}^{G3}(S,\mathcal{T}) = \left(\frac{\sum_{i=1}^{n}\left(\begin{array}{c}\alpha_S(y_i)\alpha_{\mathcal{T}}(y_i)+\gamma_S(y_i)\gamma_{\mathcal{T}}(y_i)+\beta_S(y_i)\beta_{\mathcal{T}}(y_i)+\\ \delta_{\alpha_S}(y_i)\delta_{\alpha_{\mathcal{T}}}(y_i)+\delta_{\gamma_S}(y_i)\delta_{\gamma_{\mathcal{T}}}(y_i)+\delta_{\beta_S}(y_i)\delta_{\beta_{\mathcal{T}}}(y_i)\end{array}\right)}{\Upsilon\sum_{i=1}^{n}\left(\begin{array}{c}\alpha_S^2(y_i)+\gamma_S^2(y_i)+\beta_S^2(y_i)+\\ \delta_{\alpha_S}^2(y_i)+\delta_{\gamma_S}^2(y_i)+\delta_{\beta_S}^2(y_i)\end{array}\right)+(1-\Upsilon)\sum_{i=1}^{n}\left(\begin{array}{c}\alpha_{\mathcal{T}}^2(y_i)+\gamma_{\mathcal{T}}^2(y_i)+\beta_{\mathcal{T}}^2(y_i)+\\ \delta_{\alpha_{\mathcal{T}}}^2(y_i)+\delta_{\gamma_{\mathcal{T}}}^2(y_i)+\delta_{\beta_{\mathcal{T}}}^2(y_i)\end{array}\right)}\right) \tag{20}$$

If we consider the value of $\Upsilon = 0$, then

$$d\mathcal{M}_{\mathrm{CNS}}^{G3}(S, \mathcal{T}) = \left(\frac{\sum_{i=1}^{n} \left(\begin{array}{c} \alpha_S(y_i)\alpha_{\mathcal{T}}(y_i) + \gamma_S(y_i)\gamma_{\mathcal{T}}(y_i) + \beta_S(y_i)\beta_{\mathcal{T}}(y_i) + \\ \delta_{\alpha_S}(y_i)\delta_{\alpha_{\mathcal{T}}}(y_i) + \delta_{\gamma_S}(y_i)\delta_{\gamma_{\mathcal{T}}}(y_i) + \delta_{\beta_S}(y_i)\delta_{\beta_{\mathcal{T}}}(y_i) \end{array} \right)}{\sum_{i=1}^{n} \left(\begin{array}{c} \alpha_{\mathcal{T}}^2(y_i) + \gamma_{\mathcal{T}}^2(y_i) + \beta_{\mathcal{T}}^2(y_i) + \\ \delta_{\alpha_{\mathcal{T}}}^2(y_i) + \delta_{\gamma_{\mathcal{T}}}^2(y_i) + \delta_{\beta_{\mathcal{T}}}^2(y_i) \end{array} \right)} \right) \tag{21}$$

If we consider the value of $\Upsilon = 1$, then

$$d\mathcal{M}_{\mathrm{CNS}}^{G3}(S, \mathcal{T}) = \left(\frac{\sum_{i=1}^{n} \left(\begin{array}{c} \alpha_S(y_i)\alpha_{\mathcal{T}}(y_i) + \gamma_S(y_i)\gamma_{\mathcal{T}}(y_i) + \beta_S(y_i)\beta_{\mathcal{T}}(y_i) + \\ \delta_{\alpha_S}(y_i)\delta_{\alpha_{\mathcal{T}}}(y_i) + \delta_{\gamma_S}(y_i)\delta_{\gamma_{\mathcal{T}}}(y_i) + \delta_{\beta_S}(y_i)\delta_{\beta_{\mathcal{T}}}(y_i) \end{array} \right)}{\sum_{i=1}^{n} \left(\begin{array}{c} \alpha_S^2(y_i) + \gamma_S^2(y_i) + \beta_S^2(y_i) + \\ \delta_{\alpha_S}^2(y_i) + \delta_{\gamma_S}^2(y_i) + \delta_{\beta_S}^2(y_i) \end{array} \right)} \right) \tag{22}$$

is called asymmetric similarity measures or projection similarity measures.

Definition 14: A WGDSM $d\mathcal{M}_{\mathrm{CNS}}^{wG3}(S, \mathcal{T})$ for CNSs is given by

(see (23))

which satisfy the following conditions:

(i) $0 \le d\mathcal{M}_{\mathrm{CNS}}^{wG3}(S, \mathcal{T}) \le 1$
(ii) $d\mathcal{M}_{\mathrm{CNS}}^{wG3}(S, \mathcal{T}) = d\mathcal{M}_{\mathrm{CNS}}^{wG3}(\mathcal{T}, S)$
(iii) $d\mathcal{M}_{\mathrm{CNS}}^{wG3}(S, \mathcal{T}) = 1 \Leftrightarrow S = \mathcal{T}$

If we consider the value of $\Upsilon = 0$, then

$$d\mathcal{M}_{\mathrm{CNS}}^{wG3}(S, \mathcal{T}) = \left(\frac{\sum_{i=1}^{n} \omega_i^2 \left(\begin{array}{c} \alpha_S(y_i)\alpha_{\mathcal{T}}(y_i) + \gamma_S(y_i)\gamma_{\mathcal{T}}(y_i) + \beta_S(y_i)\beta_{\mathcal{T}}(y_i) + \\ \delta_{\alpha_S}(y_i)\delta_{\alpha_{\mathcal{T}}}(y_i) + \delta_{\gamma_S}(y_i)\delta_{\gamma_{\mathcal{T}}}(y_i) + \delta_{\beta_S}(y_i)\delta_{\beta_{\mathcal{T}}}(y_i) \end{array} \right)}{\sum_{i=1}^{n} \omega_i^2 \left(\begin{array}{c} \alpha_{\mathcal{T}}^2(y_i) + \gamma_{\mathcal{T}}^2(y_i) + \beta_{\mathcal{T}}^2(y_i) + \\ \delta_{\alpha_{\mathcal{T}}}^2(y_i) + \delta_{\gamma_{\mathcal{T}}}^2(y_i) + \delta_{\beta_{\mathcal{T}}}^2(y_i) \end{array} \right)} \right) \tag{24}$$

If we consider the value of $\Upsilon = 1$, then

$$d\mathcal{M}_{\mathrm{CNS}}^{wG3}(S, \mathcal{T}) = \left(\frac{\sum_{i=1}^{n} \omega_i^2 \left(\begin{array}{c} \alpha_S(y_i)\alpha_{\mathcal{T}}(y_i) + \gamma_S(y_i)\gamma_{\mathcal{T}}(y_i) + \beta_S(y_i)\beta_{\mathcal{T}}(y_i) + \\ \delta_{\alpha_S}(y_i)\delta_{\alpha_{\mathcal{T}}}(y_i) + \delta_{\gamma_S}(y_i)\delta_{\gamma_{\mathcal{T}}}(y_i) + \delta_{\beta_S}(y_i)\delta_{\beta_{\mathcal{T}}}(y_i) \end{array} \right)}{\sum_{i=1}^{n} \omega_i^2 \left(\begin{array}{c} \alpha_S^2(y_i) + \gamma_S^2(y_i) + \beta_S^2(y_i) + \\ \delta_{\alpha_S}^2(y_i) + \delta_{\gamma_S}^2(y_i) + \delta_{\beta_S}^2(y_i) \end{array} \right)} \right) \tag{25}$$

is called asymmetric similarity measures or projection similarity measures.

Remarks 7: Where the value of $0 \le \Upsilon \le 1$. By changing $\Upsilon = 0.5$, (20) and (23) are reduced into (10) and (11).

Remarks 8: If we take $\omega = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)^{\mathrm{T}}$ then the WGDSM for CNS reduced to GDSM for CNS, i.e. $d\mathcal{M}_{\mathrm{CNS}}^{wG3}(S, \mathcal{T}) = d\mathcal{M}_{\mathrm{CNS}}^{G3}(S, \mathcal{T})$.

In this section, if we consider the abstinence part will be zero, then the proposed work is converted for IFS. The proposed work is more generalised than existing drawbacks due to its condition. Basically, CNS deals with two-dimensional information in a single element. The CNS is characterised by complex-valued MS, complex-valued abstinence and complex-valued NMS grades with a condition that the sum of real-valued MS (imaginary-valued MS), real-valued abstinence (imaginary-valued abstinence) and real-valued NMS (imaginary-valued NMS) grades is less than or equal to 3^+.

4 Apply the generalised dice similarity measures between CNSs to pattern recognition

To examine the reliability and effectiveness of the pioneered work, we compared the established work with some existing works which is investigated by Ali and Samarandache [40], applied to pattern recognition with the help of numerical example. The proposed measures are also compared with the following notions, which is explained in [43].

Example 3: For any four known patterns S_1, S_2, S_3 and S_4 with respect to unknown pattern $\mathcal{T}$ in the form of CNNs based on finite universal sets $X = \{y_1, y_2, y_3, y_4\}$ defined below:

$$S_1 = \left\{ \begin{array}{l} (y_1, 0.8e^{i2\pi(0.77)}, 0.78e^{i2\pi(0.56)}, 0.56e^{i2\pi(0.55)}), \\ (y_2, 0.9e^{i2\pi(0.87)}, 0.88e^{i2\pi(0.76)}, 0.76e^{i2\pi(0.65)}), \\ (y_3, 0.6e^{i2\pi(0.67)}, 0.58e^{i2\pi(0.86)}, 0.46e^{i2\pi(0.75)}), \\ (y_4, 0.7e^{i2\pi(0.57)}, 0.68e^{i2\pi(0.93)}, 0.36e^{i2\pi(0.85)}) \end{array} \right\}$$

$$S_2 = \left\{ \begin{array}{l} (y_1, 0.8e^{i2\pi(0.77)}, 0.78e^{i2\pi(0.56)}, 0.56e^{i2\pi(0.55)}), \\ (y_2, 0.9e^{i2\pi(0.87)}, 0.88e^{i2\pi(0.76)}, 0.76e^{i2\pi(0.65)}), \\ (y_3, 0.6e^{i2\pi(0.67)}, 0.58e^{i2\pi(0.86)}, 0.46e^{i2\pi(0.75)}), \\ (y_4, 0.7e^{i2\pi(0.57)}, 0.68e^{i2\pi(0.93)}, 0.36e^{i2\pi(0.85)}) \end{array} \right\}$$

$$S_3 = \left\{ \begin{array}{l} (y_1, 0.8e^{i2\pi(0.77)}, 0.78e^{i2\pi(0.56)}, 0.56e^{i2\pi(0.55)}), \\ (y_2, 0.9e^{i2\pi(0.87)}, 0.88e^{i2\pi(0.76)}, 0.76e^{i2\pi(0.65)}), \\ (y_3, 0.6e^{i2\pi(0.67)}, 0.58e^{i2\pi(0.86)}, 0.46e^{i2\pi(0.75)}), \\ (y_4, 0.7e^{i2\pi(0.57)}, 0.68e^{i2\pi(0.93)}, 0.36e^{i2\pi(0.85)}) \end{array} \right\}$$

$$S_4 = \left\{ \begin{array}{l} (y_1, 0.8e^{i2\pi(0.77)}, 0.78e^{i2\pi(0.56)}, 0.56e^{i2\pi(0.55)}), \\ (y_2, 0.9e^{i2\pi(0.87)}, 0.88e^{i2\pi(0.76)}, 0.76e^{i2\pi(0.65)}), \\ (y_3, 0.6e^{i2\pi(0.67)}, 0.58e^{i2\pi(0.86)}, 0.46e^{i2\pi(0.75)}), \\ (y_4, 0.7e^{i2\pi(0.57)}, 0.68e^{i2\pi(0.93)}, 0.36e^{i2\pi(0.85)}) \end{array} \right\}$$

$$\mathcal{T} = \left\{ \begin{array}{l} (y_1, 0.8e^{i2\pi(0.77)}, 0.78e^{i2\pi(0.56)}, 0.56e^{i2\pi(0.55)}), \\ (y_2, 0.9e^{i2\pi(0.87)}, 0.88e^{i2\pi(0.76)}, 0.76e^{i2\pi(0.65)}), \\ (y_3, 0.6e^{i2\pi(0.67)}, 0.58e^{i2\pi(0.86)}, 0.46e^{i2\pi(0.75)}), \\ (y_4, 0.7e^{i2\pi(0.57)}, 0.68e^{i2\pi(0.93)}, 0.36e^{i2\pi(0.85)}) \end{array} \right\}$$

$$d\mathcal{M}_{\mathrm{CNS}}^{wG3}(S, \mathcal{T}) = \left(\frac{\sum_{i=1}^{n} \omega_i^2 \left(\begin{array}{c} \alpha_S(y_i)\alpha_{\mathcal{T}}(y_i) + \gamma_S(y_i)\gamma_{\mathcal{T}}(y_i) + \beta_S(y_i)\beta_{\mathcal{T}}(y_i) + \\ \delta_{\alpha_S}(y_i)\delta_{\alpha_{\mathcal{T}}}(y_i) + \delta_{\gamma_S}(y_i)\delta_{\gamma_{\mathcal{T}}}(y_i) + \delta_{\beta_S}(y_i)\delta_{\beta_{\mathcal{T}}}(y_i) \end{array} \right)}{\Upsilon \sum_{i=1}^{n} \omega_i^2 \left(\begin{array}{c} \alpha_S^2(y_i) + \gamma_S^2(y_i) + \beta_S^2(y_i) + \\ \delta_{\alpha_S}^2(y_i) + \delta_{\gamma_S}^2(y_i) + \delta_{\beta_S}^2(y_i) \end{array} \right) + (1 - \Upsilon) \sum_{i=1}^{n} \omega_i^2 \left(\begin{array}{c} \alpha_{\mathcal{T}}^2(y_i) + \gamma_{\mathcal{T}}^2(y_i) + \beta_{\mathcal{T}}^2(y_i) + \\ \delta_{\alpha_{\mathcal{T}}}^2(y_i) + \delta_{\gamma_{\mathcal{T}}}^2(y_i) + \delta_{\beta_{\mathcal{T}}}^2(y_i) \end{array} \right)} \right) \tag{23}$$

We compute the GDSM between known and unknown patterns by using (20). The comparison of the proposed measures with existing measures, whose detail is that the notion introduced by Li *et al.* [44] based on similarity measures between IFS, and the notion initiated by Chen [45] based on similarity measures between vague set and between elements, and the concept pioneered by Chen *et al.* [15] based on similarity measures between IFS, and the idea found by Hung and Yang [21] based on similarity measures between IFS using Hausdroff distance, and the idea found by Hong and Kim [46] based on similarity measures between vague set and their elements, and the idea found by Li and Cheng [14] based on similarity measures for IFS and their application to pattern recognition, and the idea found by Li and Xu [47] based on measures of similarity between vague sets, and the idea found by Liang and Shi [48] based on similarity measures on IFS, and the idea found by Mitchell [49] based on similarity measures and its application to pattern recognition, and the idea found by Ye [50] based on cosine similarity measures for IFS and their applications, and the idea found by Wei and Wei [51] based on similarity measures based on PFS, and the idea found by Zhang [52] based on similarity measures for pythagorean fuzzy multi-criteria group decision making, and the idea found by Peng *et al.* [53] based on information measures for PFS, and the idea found by Boran and Akay [54] based on parametric similarity measures for IFS, and the idea found by Peng and Liu [43] based on information measures for q-rung orthopair FSs are discussed in Table 1 for the values of weight vectors is $(0.3, 0.4, 0.1, 0.2)^{T}$.

The graphical representation of the explored work with existing work which is mention in Table 1 is discussed in Fig. 2.

4.1 Advantages and comparative analysis of the CNSs

In the following, some comparisons of the initiated methods with drawback ideas are discussed to examine the validity and superiority of the proposed methods. Further, we will compare our proposed dice similarity measures to 21 other existing measures, and we will consider the following drawbacks to solve with the help of example, including the notion introduced by Li *et al.* [44] based on similarity measures between IFS, and the notion initiated by Chen [45] based on similarity measures between vague set and between elements, and the concept pioneered by Chen *et al.* [15] based on similarity measures between IFS, and the idea found by Hung and Yang [21] based on similarity measures between IFS using Hausdroff distance, and the idea found by Hong and Kim [46] based on similarity measures between vague set and their elements, and the idea found by Dengfeng and Chuntian [14] based on similarity measures for IFS and their application to pattern recognition, and the idea found by Li and Xu [47] based on measures of similarity between vague sets, and the idea found by Liang and Shi [48] based on similarity measures on IFS, and the idea found by Mitchell [49] based on similarity measures and its application to pattern recognition, and the idea found by Ye [50] based on cosine similarity measures for IFS and their applications, and the idea found by Wei and Wei [51] based on similarity measures based on PFS, and the idea found by Zhang [52] based on similarity measures for pythagorean fuzzy multi-criteria group decision making, and the idea found by Peng *et al.* [53] based on information measures for PFS, and the idea found by Boran and Akay [54] based on parametric similarity measures for IFS, and the idea found by Peng and Liu [43] based on information measures for q-rung orthopair FSs. However, all the existing drawbacks are failed to deal with problems that involve two-dimensional information/date, i.e. two different types of information/data pertaining to the problem parameters. The existing methods (discussed in advantages section below) and proposed methods are compared with the help of Example 4 and see the final results in Table 2.

To examine the reliability and effectiveness of the explored work we solve another example which is taken from [43].

Example 4: [43]: For any four known patterns $\mathcal{S}_1$, $\mathcal{S}_2$ and $\mathcal{S}_3$ with respect to unknown pattern $\mathcal{T}$ in the form of CNNs based on

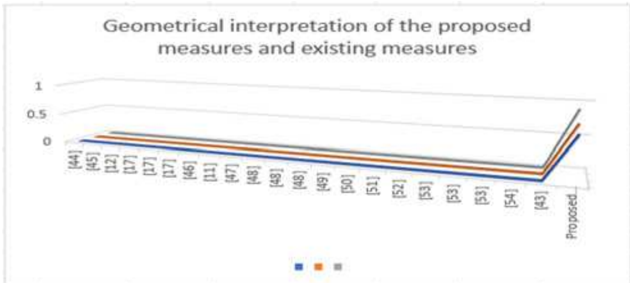

Fig. 2 *Comparison of the proposed measure with existing measures, using graphical interpretation*

Table 1 Comparison of the proposed measure with existing measures

Methods	Similarity measures	Ranking
S_L [44]	cannot be classified	cannot be classified
S_C [45]	cannot be classified	cannot be classified
S_{CC} [15]	cannot be classified	cannot be classified
S_{HY1} [21]	cannot be classified	cannot be classified
S_{HY2} [21]	cannot be classified	cannot be classified
S_{HY3} [21]	cannot be classified	cannot be classified
S_{HK} [46]	cannot be classified	cannot be classified
S_{LC} [14]	cannot be classified	cannot be classified
S_{LX} [47]	cannot be classified	cannot be classified
S_{LS1} [48]	cannot be classified	cannot be classified
S_{LS2} [48]	cannot be classified	cannot be classified
S_{LS3} [48]	cannot be classified	cannot be classified
S_M [49]	cannot be classified	cannot be classified
S_Y [50]	cannot be classified	cannot be classified
S_W [51]	cannot be classified	cannot be classified
S_Z [52]	cannot be classified	cannot be classified
S_{P1} [53]	cannot be classified	cannot be classified
S_{P2} [53]	cannot be classified	cannot be classified
S_{P3} [53]	cannot be classified	cannot be classified
S_{BA} [54]	cannot be classified	cannot be classified
S_{13} [43]	cannot be classified	cannot be classified
proposed method in this paper	$\lceil\mathcal{M}(\mathcal{S}_1,\mathcal{T})=0.6723$, $\lceil\mathcal{M}(\mathcal{S}_2,\mathcal{T})=0.7467$, $\lceil\mathcal{M}(\mathcal{S}_3,\mathcal{T})=0.892$	$\mathcal{S}_3 \geq \mathcal{S}_2 \geq \mathcal{S}_1$

Table 2 Comparison of the proposed measure with existing measures

Methods	Similarity measures	Ranking
S_L [44]	$\lceil\mathcal{M}(\mathcal{S}_1, \mathcal{T}) = 0.86$, $\lceil\mathcal{M}(\mathcal{S}_2, \mathcal{T}) = 0.72$, $\lceil\mathcal{M}(\mathcal{S}_3, \mathcal{T}) = 0.91$	$\mathcal{S}_3 \geq \mathcal{S}_1 \geq \mathcal{S}_2$
S_C [45]	$\lceil\mathcal{M}(\mathcal{S}_1, \mathcal{T}) = 1$, $\lceil\mathcal{M}(\mathcal{S}_2, \mathcal{T}) = 1$, $\lceil\mathcal{M}(\mathcal{S}_3, \mathcal{T}) = 0.97$	$\mathcal{S}_1 \geq \mathcal{S}_2 \geq \mathcal{S}_3$
S_{CC} [15]	$\lceil\mathcal{M}(\mathcal{S}_1, \mathcal{T}) = 0.86$, $\lceil\mathcal{M}(\mathcal{S}_2, \mathcal{T}) = 0.74$, $\lceil\mathcal{M}(\mathcal{S}_3, \mathcal{T}) = 0.89$	$\mathcal{S}_3 \geq \mathcal{S}_1 \geq \mathcal{S}_2$
S_{HY1} [21]	$\lceil\mathcal{M}(\mathcal{S}_1, \mathcal{T}) = 0.87$, $\lceil\mathcal{M}(\mathcal{S}_2, \mathcal{T}) = 0.75$, $\lceil\mathcal{M}(\mathcal{S}_3, \mathcal{T}) = 0.90$	$\mathcal{S}_3 \geq \mathcal{S}_2 \geq \mathcal{S}_1$
S_{HY2} [21]	$\lceil\mathcal{M}(\mathcal{S}_1, \mathcal{T}) = 0.81$, $\lceil\mathcal{M}(\mathcal{S}_2, \mathcal{T}) = 0.65$, $\lceil\mathcal{M}(\mathcal{S}_3, \mathcal{T}) = 0.84$	$\mathcal{S}_3 \geq \mathcal{S}_2 \geq \mathcal{S}_1$
S_{HY3} [21]	$\lceil\mathcal{M}(\mathcal{S}_1, \mathcal{T}) = 0.78$, $\lceil\mathcal{M}(\mathcal{S}_2, \mathcal{T}) = 0.60$, $\lceil\mathcal{M}(\mathcal{S}_3, \mathcal{T}) = 0.82$	$\mathcal{S}_3 \geq \mathcal{S}_2 \geq \mathcal{S}_1$
S_{HK} [46]	$\lceil\mathcal{M}(\mathcal{S}_1, \mathcal{T}) = 0.87$, $\lceil\mathcal{M}(\mathcal{S}_2, \mathcal{T}) = 0.75$, $\lceil\mathcal{M}(\mathcal{S}_3, \mathcal{T}) = 0.92$	$\mathcal{S}_3 \geq \mathcal{S}_2 \geq \mathcal{S}_1$
S_{LC} [14]	$\lceil\mathcal{M}(\mathcal{S}_1, \mathcal{T}) = 1$, $\lceil\mathcal{M}(\mathcal{S}_2, \mathcal{T}) = 1$, $\lceil\mathcal{M}(\mathcal{S}_3, \mathcal{T}) = 0.97$	$\mathcal{S}_1 \geq \mathcal{S}_2 \geq \mathcal{S}_3$
S_{LX} [47]	$\lceil\mathcal{M}(\mathcal{S}_1, \mathcal{T}) = 0.94$, $\lceil\mathcal{M}(\mathcal{S}_2, \mathcal{T}) = 0.87$, $\lceil\mathcal{M}(\mathcal{S}_3, \mathcal{T}) = 0.95$	$\mathcal{S}_3 \geq \mathcal{S}_2 \geq \mathcal{S}_1$
S_{LS1} [48]	$\lceil\mathcal{M}(\mathcal{S}_1, \mathcal{T}) = 0.87$, $\lceil\mathcal{M}(\mathcal{S}_2, \mathcal{T}) = 0.75$, $\lceil\mathcal{M}(\mathcal{S}_3, \mathcal{T}) = 0.92$	$\mathcal{S}_3 \geq \mathcal{S}_2 \geq \mathcal{S}_1$
S_{LS2} [48]	$\lceil\mathcal{M}(\mathcal{S}_1, \mathcal{T}) = 0.94$, $\lceil\mathcal{M}(\mathcal{S}_2, \mathcal{T}) = 0.87$, $\lceil\mathcal{M}(\mathcal{S}_3, \mathcal{T}) = 0.95$	$\mathcal{S}_3 \geq \mathcal{S}_2 \geq \mathcal{S}_1$
S_{LS3} [48]	$\lceil\mathcal{M}(\mathcal{S}_1, \mathcal{T}) = 0.92$, $\lceil\mathcal{M}(\mathcal{S}_2, \mathcal{T}) = 0.83$, $\lceil\mathcal{M}(\mathcal{S}_3, \mathcal{T}) = 0.94$	$\mathcal{S}_3 \geq \mathcal{S}_2 \geq \mathcal{S}_1$
S_M [49]	$\lceil\mathcal{M}(\mathcal{S}_1, \mathcal{T}) = 0.87$, $\lceil\mathcal{M}(\mathcal{S}_2, \mathcal{T}) = 0.75$, $\lceil\mathcal{M}(\mathcal{S}_3, \mathcal{T}) = 0.92$	$\mathcal{S}_3 \geq \mathcal{S}_2 \geq \mathcal{S}_1$
S_Y [50]	$\lceil\mathcal{M}(\mathcal{S}_1, \mathcal{T}) = 1$, $\lceil\mathcal{M}(\mathcal{S}_2, \mathcal{T}) = 1$, $\lceil\mathcal{M}(\mathcal{S}_3, \mathcal{T}) = 0.99$	$\mathcal{S}_1 \geq \mathcal{S}_2 \geq \mathcal{S}_3$
S_W [51]	$\lceil\mathcal{M}(\mathcal{S}_1, \mathcal{T}) = 1$, $\lceil\mathcal{M}(\mathcal{S}_2, \mathcal{T}) = 1$, $\lceil\mathcal{M}(\mathcal{S}_3, \mathcal{T}) = 0.99$	$\mathcal{S}_1 \geq \mathcal{S}_2 \geq \mathcal{S}_3$
S_Z [52]	$\lceil\mathcal{M}(\mathcal{S}_1, \mathcal{T}) = 1$, $\lceil\mathcal{M}(\mathcal{S}_2, \mathcal{T}) = 1$, $\lceil\mathcal{M}(\mathcal{S}_3, \mathcal{T}) = 0.98$	$\mathcal{S}_1 \geq \mathcal{S}_2 \geq \mathcal{S}_3$
S_{P1} [53]	$\lceil\mathcal{M}(\mathcal{S}_1, \mathcal{T}) = 0.5$, $\lceil\mathcal{M}(\mathcal{S}_2, \mathcal{T}) = 0.5$, $\lceil\mathcal{M}(\mathcal{S}_3, \mathcal{T}) = 0.5$	$\mathcal{S}_3 \geq \mathcal{S}_2 \geq \mathcal{S}_1$
S_{P2} [53]	$\lceil\mathcal{M}(\mathcal{S}_1, \mathcal{T}) = 0.5$, $\lceil\mathcal{M}(\mathcal{S}_2, \mathcal{T}) = 0.24$, $\lceil\mathcal{M}(\mathcal{S}_3, \mathcal{T}) = 0.62$	$\mathcal{S}_3 \geq \mathcal{S}_2 \geq \mathcal{S}_1$
S_{P3} [53]	$\lceil\mathcal{M}(\mathcal{S}_1, \mathcal{T}) = 0.87$, $\lceil\mathcal{M}(\mathcal{S}_2, \mathcal{T}) = 0.79$, $\lceil\mathcal{M}(\mathcal{S}_3, \mathcal{T}) = 0.92$	$\mathcal{S}_3 \geq \mathcal{S}_2 \geq \mathcal{S}_1$
S_{BA} [54]	$\lceil\mathcal{M}(\mathcal{S}_1, \mathcal{T}) = 0.96$, $\lceil\mathcal{M}(\mathcal{S}_2, \mathcal{T}) = 0.91$, $\lceil\mathcal{M}(\mathcal{S}_3, \mathcal{T}) = 0.96$	$\mathcal{S}_3 \geq \mathcal{S}_2 \geq \mathcal{S}_1$
S_{13} [43]	$\lceil\mathcal{M}(\mathcal{S}_1, \mathcal{T}) = 0.98$, $\lceil\mathcal{M}(\mathcal{S}_2, \mathcal{T}) = 0.97$, $\lceil\mathcal{M}(\mathcal{S}_3, \mathcal{T}) = 0.99$	$\mathcal{S}_3 \geq \mathcal{S}_2 \geq \mathcal{S}_1$
proposed method in this paper	$\lceil\mathcal{M}(\mathcal{S}_1, \mathcal{T}) = 0.858$, $\lceil\mathcal{M}(\mathcal{S}_2, \mathcal{T}) = 0.8972$, $\lceil\mathcal{M}(\mathcal{S}_3, \mathcal{T}) = 0.923$	$\mathcal{S}_3 \geq \mathcal{S}_2 \geq \mathcal{S}_1$

finite universal sets $X = \{y_1, y_2, y_3, y_4\}$ are defined below:

$$S_1 = \left\{ \begin{array}{c} (y_1, 0.3e^{i2\pi(0.0)}, 0.0e^{i2\pi(0.0)}, 0.3e^{i2\pi(0.0)}), \\ (y_2, 0.4e^{i2\pi(0.0)}, 0.0e^{i2\pi(0.0)}, 0.4e^{i2\pi(0.0)}), \\ (y_3, 0.4e^{i2\pi(0.0)}, 0.0e^{i2\pi(0.0)}, 0.4e^{i2\pi(0.0)}), \\ (y_4, 0.4e^{i2\pi(0.0)}, 0.0e^{i2\pi(0.0)}, 0.4e^{i2\pi(0.0)}) \end{array} \right\}$$

$$S_2 = \left\{ \begin{array}{c} (y_1, 0.5e^{i2\pi(0.0)}, 0.0e^{i2\pi(0.0)}, 0.5e^{i2\pi(0.0)}), \\ (y_2, 0.1e^{i2\pi(0.0)}, 0.0e^{i2\pi(0.0)}, 0.1e^{i2\pi(0.0)}), \\ (y_3, 0.5e^{i2\pi(0.0)}, 0.0e^{i2\pi(0.0)}, 0.5e^{i2\pi(0.0)}), \\ (y_4, 0.1e^{i2\pi(0.0)}, 0.0e^{i2\pi(0.0)}, 0.1e^{i2\pi(0.0)}) \end{array} \right\}$$

$$S_3 = \left\{ \begin{array}{c} (y_1, 0.5e^{i2\pi(0.0)}, 0.0e^{i2\pi(0.0)}, 0.4e^{i2\pi(0.0)}), \\ (y_2, 0.4e^{i2\pi(0.0)}, 0.0e^{i2\pi(0.0)}, 0.5e^{i2\pi(0.0)}), \\ (y_3, 0.3e^{i2\pi(0.0)}, 0.0e^{i2\pi(0.0)}, 0.3e^{i2\pi(0.0)}), \\ (y_4, 0.2e^{i2\pi(0.0)}, 0.0e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)}) \end{array} \right\}$$

$$\mathcal{T} = \left\{ \begin{array}{c} (y_1, 0.4e^{i2\pi(0.0)}, 0.0e^{i2\pi(0.0)}, 0.4e^{i2\pi(0.0)}), \\ (y_2, 0.5e^{i2\pi(0.0)}, 0.0e^{i2\pi(0.0)}, 0.5e^{i2\pi(0.0)}), \\ (y_3, 0.2e^{i2\pi(0.0)}, 0.0e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)}), \\ (y_4, 0.3e^{i2\pi(0.0)}, 0.0e^{i2\pi(0.0)}, 0.3e^{i2\pi(0.0)}) \end{array} \right\}$$

We compute the GDSM between known and unknown patterns by using (20). The comparison of the proposed measures with existing measures, whose detail is that the notion introduced by Li *et al.* [44] based on similarity measures between IFS, and the notion initiated by Chen [45] based on similarity measures between vague set and between elements, and the concept pioneered by Chen *et al.* [15] based on similarity measures between IFS, and the idea found by Hung and Yang [21] based on similarity measures between IFS using Hausdroff distance, and the idea found by Hong and Kim [46] based on similarity measures between vague set and their elements, and the idea found by Dengfeng and Chuntian [14] based on similarity measures for IFS and their application to pattern recognition, and the idea found by Li and Xu [47] based on measures of similarity between vague sets, and the idea found by Liang and Shi [48] based on similarity measures on IFS, and the idea found by Mitchell [49] based on similarity measures and its application to pattern recognition, and the idea found by Ye [50] based on cosine similarity measures for IFS and their applications, and the idea found by Wei and Wei [51] based on similarity measures based on PFS, and the idea found by Zhang [52] based on similarity measures for pythagorean fuzzy multi-criteria group decision making, and the idea found by Peng *et al.* [53] based on information measures for PFS, and the idea found by Boran and Akay [54] based on parametric similarity measures for IFS, and the idea found by Peng and Liu [43] based on information measures for q-rung orthopair FSs are discussed in Table 2.

The graphical representation of the explored work with existing work which is mention in Table 2 is discussed in Fig. 3.

From Example 3, it can be seen that existing drawbacks are not able to solve the decision making problem presented, which

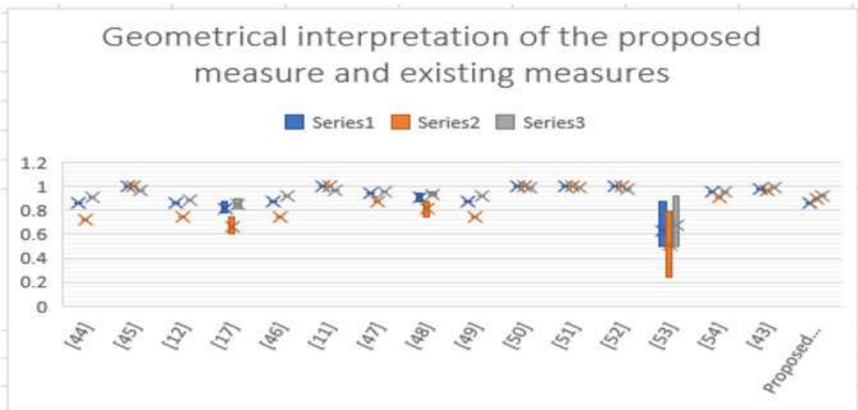

Fig. 3 *The graphical interpretation of the explored work with existing works*

involves two types of complex neutrosophic information (the degree of the influence and the total time of the influence) since established measures lacks the phase terms which represent the time frame of this problem. But when we choose the intuitionistic fuzzy types of information, the established work can solve easily; see in Example 4.

4.2 Sensitive analysis

The GDSM under the CIFS [34] environment can only handle situations in which the degree of complex-valued membership and complex-valued non-membership is provided to the decision maker. The constraint of CIFS is that the sum of real part (also for imaginary part) of membership and real part (also for imaginary part) of non-membership grades are bounded to [0, 1]. This kind of measure is unable to deal with such kind of information, whose sum is greater than 1, which commonly occurs in real-life applications. As CNSs [40] are a successful tool to handle such kinds of information, which cannot deal effectively by CIFS, the proposed dice similarity measure in the CNS can effectively be used in many real applications in decision making. The special cases of the proposed approaches are discussed below.

(i) By ignoring the imaginary parts in the triplet $(\alpha_S(y_i)e^{i2\pi(\delta_{\alpha_S}(y_i))}, \gamma_S(y_i)e^{i2\pi(\delta_{\gamma_S}(y_i))}, \beta_S(y_i)e^{i2\pi(\delta_{\beta_S}(y_i))})$, then the proposed approaches based on CNS [40] are reduced for the NSs [30].
(ii) By ignoring the imaginary parts and abstinence degree in the triplet $(\alpha_S(y_i)e^{i2\pi(\delta_{\alpha_S}(y_i))}, \gamma_S(y_i)e^{i2\pi(\delta_{\gamma_S}(y_i))}, \beta_S(y_i)e^{i2\pi(\delta_{\beta_S}(y_i))})$, then the proposed approaches based on CNS [40] are reduced for the IFSs [13].
(iii) By ignoring the abstinence degree in the triplet $(\alpha_S(y_i)e^{i2\pi(\delta_{\alpha_S}(y_i))}, \gamma_S(y_i)e^{i2\pi(\delta_{\gamma_S}(y_i))}, \beta_S(y_i)e^{i2\pi(\delta_{\beta_S}(y_i))})$, then the proposed approaches based on CNS [40] are reduced for the CIFSs [24].

The proposed measures based on CNSs are more powerful and more general than existing methods discussed in [43–54]. We are currently working on developing a more in-depth theoretical framework concerning the similarity measures, and have plans to extend this to other types of similarity measures in the future. We are also motivated by the works presented in [48–50], and look forward to extend our work to other generalisations of NSs, such as interval CNSs, and apply the work in medical imaging problems and recommender systems.

5 Conclusion

CNS is an extension of CFS, to cope with complicated and inconsistence information in the environment of FS theory. The CNS is characterised by three functions expressing the degree of complex-valued membership, complex-valued abstinence and degree of complex-valued non-membership. The aim of this manuscript is to initiate the novel dice similarity measures and generalised dice similarity for CNS. The special cases of the investigated methods are discussed with the help of some remarks. Moreover, the distance measures for CNS are also proposed in this manuscript. Then, we applied the GDSMs and WGDSMs between CNSs to pattern recognition. The advantages of found approaches and the compression between proposed methods with existing methods are initiated. At last, an illustrative numerical example is provided to demonstrate the efficiency and effectiveness of the proposed approaches. In future, we use the GDSMs in the environment of [55–58], neutrosophic generalisations [59–62], complex q-rung orthopair FSs [63, 64] and decision making [65–70].

6 References

[1] Zadeh, L.A.: 'Fuzzy sets', *Inf. Control*, 1965, **8**, (3), pp. 338–353
[2] Yager, R.R.: 'Aggregation operators and fuzzy systems modeling', *Fuzzy Sets Syst.*, 1994, **67**, (2), pp. 129–145
[3] Zimmermann, H.J.: 'Fuzzy set theory—and its applications' (Springer Science & Business Media, New York, NY, USA, 2011)
[4] Steimann, F.: 'Fuzzy set theory in medicine', *Artif. Intell. Med.*, 1997, **11**, (1), pp. 1–7
[5] Szczepaniak, P.S., Lisboa, P.J. (Eds.): 'Fuzzy systems in medicine' (Physica-Verlag, Heidelberg, Berlin, 2012, vol. 41)
[6] Kahraman, C. (Ed.): 'Fuzzy multi-criteria decision making: theory and applications with recent developments', vol. 16 (Springer Science & Business Media, New York, NY, USA, 2008)
[7] Zadeh, L.A.: 'The concept of a linguistic variable and its application to approximate reasoning—I', *Inf. Sci.*, 1975, **8**, (3), pp. 199–249
[8] Rosenfeld, A.: 'Fuzzy groups', *J. Math. Anal. Appl.*, 1971, **35**, (3), pp. 512–517
[9] Chang, C.L.: 'Fuzzy topological spaces', *J. Math. Anal. Appl.*, 1968, **24**, (1), pp. 182–190
[10] Ramot, D., Milo, R., Friedman, M., *et al.*: 'Complex fuzzy sets', *IEEE Trans. Fuzzy Syst.*, 2002, **10**, (2), pp. 171–186
[11] Ramot, D., Friedman, M., Langholz, G., *et al.*: 'Complex fuzzy logic', *IEEE Trans. Fuzzy Syst.*, 2003, **11**, (4), pp. 450–461
[12] Buckley, J.J.: 'Fuzzy complex numbers', *Fuzzy Sets Syst.*, 1989, **33**, (3), pp. 333–345
[13] Atanassov, K.T.: 'Intuitionistic fuzzy sets', in 'Intuitionistic fuzzy sets' (Physica, Heidelberg, Germany, 1999), pp. 1–137
[14] Dengfeng, L., Chuntian, C.: 'New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions', *Pattern Recognit. Lett.*, 2002, **23**, (1–3), pp. 221–225
[15] Chen, S.M., Cheng, S.H., Lan, T.C.: 'A novel similarity measure between intuitionistic fuzzy sets based on the centroid points of transformed fuzzy numbers with applications to pattern recognition', *Inf. Sci.*, 2016, **343**, pp. 15–40
[16] De, S.K., Biswas, R., Roy, A.R.: 'An application of intuitionistic fuzzy sets in medical diagnosis', *Fuzzy Sets Syst.*, 2001, **117**, (2), pp. 209–213
[17] Szmidt, E., Kacprzyk, J.: 'A similarity measure for intuitionistic fuzzy sets and its application in supporting medical diagnostic reasoning'. Int. Conf. on Artificial Intelligence and Soft Computing, Zakopane, Poland, June 2004, pp. 388–393
[18] Liu, P.: 'Some hamacher aggregation operators based on the interval-valued intuitionistic fuzzy numbers and their application to group decision making', *IEEE Trans. Fuzzy Syst.*, 2013, **22**, (1), pp. 83–97
[19] Liu, P.: 'Some generalized dependent aggregation operators with intuitionistic linguistic numbers and their application to group decision making', *J. Comput. Syst. Sci.*, 2013, **79**, (1), pp. 131–143
[20] Xu, Z.S., Chen, J.: 'An overview of distance and similarity measures of intuitionistic fuzzy sets', *Int. J. Uncertain. Fuzziness Knowl.-Based Syst.*, 2008, **16**, (4), pp. 529–555
[21] Hung, W.L., Yang, M.S.: 'Similarity measures of intuitionistic fuzzy sets based on hausdorff distance', *Pattern Recognit. Lett.*, 2004, **25**, (14), pp. 1603–1611
[22] Liu, P., Chen, S.M.: 'Multiattribute group decision making based on intuitionistic 2-tuple linguistic information', *Inf. Sci.*, 2018, **430**, pp. 599–619
[23] Liu, P., Chen, S.M., Liu, J.: 'Multiple attribute group decision making based on intuitionistic fuzzy interaction partitioned Bonferroni mean operators', *Inf. Sci.*, 2017, **411**, pp. 98–121
[24] Alkouri, A.M.D.J.S., Salleh, A.R.: 'Complex intuitionistic fuzzy sets'. AIP Conf. Proc., Kuala Lumpur, Malaysia, September 2012, vol. 1482, no. 1, pp. 464–470
[25] Kumar, T., Bajaj, R.K.: 'On complex intuitionistic fuzzy soft sets with distance measures and entropies', *J. Math.*, 2014, **2014**, pp. 1–12
[26] Garg, H., Rani, D.: 'A robust correlation coefficient measure of complex intuitionistic fuzzy sets and their applications in decision-making', *Appl. Intell.*, 2019, **49**, (2), pp. 496–512
[27] Garg, H., Rani, D.: 'Some generalized complex intuitionistic fuzzy aggregation operators and their application to multicriteria decision-making process', *Arab. J. Sci. Eng.*, 2019, **44**, (3), pp. 2679–2698
[28] Rani, D., Garg, H.: 'Distance measures between the complex intuitionistic fuzzy sets and their applications to the decision-making process', *Int. J. Uncertain. Quantif.*, 2017, **7**, (5), pp. 423–439,
[29] Rani, D., Garg, H.: 'Complex intuitionistic fuzzy power aggregation operators and their applications in multicriteria decision-making', *Expert Syst.*, 2018, **35**, (6), p. e12325
[30] Smarandache, F.: 'Neutrosophic set–a generalization of the intuitionistic fuzzy set', *J. Def. Resour. Manag. (JoDRM)*, 2010, **1**, pp. 107–116
[31] Broumi, S., Smarandache, F.: 'New distance and similarity measures of interval neutrosophic sets', *Neutrosophic Theor. Appl. Collected Papers*, 2014, **1**, pp. 1–7
[32] Ye, J.: 'A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets', *J. Intel. Fuzzy Syst.*, 2014, **26**, pp. 2459–2466
[33] Ye, J.: 'Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making', *J. Intel. Fuzzy Syst.*, 2014, **26**, pp. 165–172
[34] Wang, H., Smarandache, F., Sunderraman, R., *et al.*: 'Interval neutrosophic sets and logic: theory and applications in computing' (Hexis, Phoenix, AZ, USA, 2005, vol. 5), pp. 1–16
[35] Broumi, S., Smarandache, F., Dhar, M.: 'Rough neutrosophic sets', *IJPAM*, 2014, pp. 493–502
[36] Haibin, W.A.N.G., Smarandache, F., Zhang, Y., *et al.*: 'Single valued neutrosophic sets', in 'Infinite study' (Hexis, Phoenix, AZ, USA, 2010), pp. 1–4
[37] Garg, H.: 'A novel divergence measure and its based TOPSIS method for multi criteria decision-making under single-valued neutrosophic environment', *J. Intell. Fuzzy Syst.*, 2019, **36**, (1), pp. 101–115
[38] Garg, H.: 'Some hybrid weighted aggregation operators under neutrosophic set environment and their applications to multicriteria decision-making', *Appl. Intell.*, 2018, **48**, (12), pp. 4871–4888

[39] Garg, H.: ‘An improved score function for ranking neutrosophic sets and its application to decision-making process’, *Int. J. Uncertain. Quantif.*, 2016, **6**, (5), pp. 377–385,

[40] Ali, M., Smarandache, F.: ‘Complex neutrosophic set’, *Neural Comput. Appl.*, 2017, **28**, (7), pp. 1817–1834

[41] Ali, M., Dat, L.Q., Smarandache, F.: ‘Interval complex neutrosophic set: formulation and applications in decision-making’, *Int. J. Fuzzy Syst.*, 2018, **20**, (3), pp. 986–999

[42] Wei, G., Gao, H.: ‘The generalized dice similarity measures for picture fuzzy sets and their applications’, *Informatica*, 2018, **29**, (1), pp. 107–124

[43] Peng, X., Liu, L.: ‘Information measures for q-rung orthopair fuzzy sets’, *Int. J. Intell. Syst.*, 2019, **34**, (8), pp. 1795–1834

[44] Li, Y., Olson, D.L., Qin, Z.: ‘Similarity measures between intuitionistic fuzzy (vague) sets: A comparative analysis’, *Pattern Recognit. Lett.*, 2007, **28**, (2), pp. 278–285

[45] Chen, S.M.: ‘Similarity measures between vague sets and between elements’, *IEEE Trans. Syst., Man, Cybern., B (Cybernetics)*, 1997, **27**, (1), pp. 153–158

[46] Hong, D.H., Kim, C.: ‘A note on similarity measures between vague sets and between elements’, *Inf. Sci.*, 1999, **115**, (1–4), pp. 83–96

[47] Li, F., Xu, Z.Y.: ‘Measures of similarity between vague sets’, *J. Softw.*, 2001, **12**, (6), pp. 922–927

[48] Liang, Z., Shi, P.: ‘Similarity measures on intuitionistic fuzzy sets’, *Pattern Recognit. Lett.*, 2003, **24**, (15), pp. 2687–2693

[49] Mitchell, H.B.: ‘On the dengfeng–chuntian similarity measure and its application to pattern recognition’, *Pattern Recognit. Lett.*, 2003, **24**, (16), pp. 3101–3104

[50] Ye, J.: ‘Cosine similarity measures for intuitionistic fuzzy sets and their applications’, *Math. Comput. Model.*, 2011, **53**, (1–2), pp. 91–97

[51] Wei, G., Wei, Y.: ‘Similarity measures of pythagorean fuzzy sets based on the cosine function and their applications’, *Int. J. Intell. Syst.*, 2018, **33**, (3), pp. 634–652

[52] Zhang, X.: ‘A novel approach based on similarity measure for pythagorean fuzzy multiple criteria group decision making’, *Int. J. Intell. Syst.*, 2016, **31**, (6), pp. 593–611

[53] Peng, X., Yuan, H., Yang, Y.: ‘Pythagorean fuzzy information measures and their applications’, *Int. J. Intell. Syst.*, 2017, **32**, (10), pp. 991–1029

[54] Boran, F.E., Akay, D.: ‘A biparametric similarity measure on intuitionistic fuzzy sets with applications to pattern recognition’, *Inf. Sci.*, 2014, **255**, pp. 45–57

[55] Riaz, M., Raza Hashmi, M.: ‘Linear diophantine fuzzy set and its applications towards multi-attribute decision-making problems’, *J. Intell.Fuzzy Syst., (Preprint)*, 2019, **37**, (4), pp. 5417–5439

[56] Riaz, M., Tehrim, S.T.: ‘Bipolar fuzzy soft mappings with application to bipolar disorders’, *Int. J. Biomath.*, 2019, **12**, (7), 1950080

[57] Tehrim, S.T., Riaz, M.: ‘A novel extension of TOPSIS to MCGDM with bipolar neutrosophic soft topology’, *J. Intell. Fuzzy Syst., (Preprint)*, 2019, **37**, (4), pp. 5531–5549

[58] Riaz, M., Davvaz, B., Firdous, A., *et al.*: ‘Novel concepts of soft rough set topology with applications’, *J. Intell. Fuzzy Syst.*, 2019, **36**, (4), pp. 3579–3590

[59] Smarandache, F.: ‘Neutrosophy’. Neutrosophic Probability, Set, and Logic, ProQuest Information & Learning’, Ann Arbor, Michigan, USA, 1998, vol. 105, pp. 118–123

[60] Smarandache, F.: ‘Plithogeny, plithogenic set, logic, probability, and statistics’, in ‘Infinite study’ (Pons, Brussels, Belgium, 2017)), pp. 1–143

[61] Smarandache, F.: ‘Extension of soft set to hypersoft set, and then to plithogenic hypersoft set’, in ‘Infinite study’ (University of New Mexico, Mexico, 2018), vol. 22, pp. 1–4

[62] Smarandache, F.: ‘A unifying field in Logics: Neutrosophic Logic’, in ‘Philosophy’ (American Research Press, Rehoboth, DE, USA, 1999), pp. 1–141

[63] Liu, P., Ali, Z., Mahmood, T.: ‘A method to multi-attribute group decision-making problem with Complex q-rung orthopair linguistic information based on heronian mean operators’, *Int. J. Comput. Intell. Syst.*, 2019, **12**, (2), pp. 1465–1496

[64] Liu, P., Mahmood, T., Ali, Z.: ‘Complex q-rung orthopair fuzzy aggregation operators and their applications in multi-attribute group decision making’, *Information*, 2020, **11**, (1), p. 5

[65] Ullah, K., Ali, Z., Jan, N., *et al.*: ‘Multi-attribute decision making based on averaging aggregation operators for picture hesitant fuzzy sets’, *Tech. J.*, 2018, **23**, (4), pp. 84–95

[66] Ullah, K., Garg, H., Mahmood, T., *et al.*: ‘Correlation coefficients for T-spherical fuzzy sets and their applications in clustering and multi-attribute decision making’, *Soft Comput.*, 2019, **24**, (3), pp. 1647–1659

[67] Ullah, K., Mahmood, T., Ali, Z., *et al.*: ‘On some distance measures of complex pythagorean fuzzy sets and their applications in pattern recognition’, *Complex Intell. Syst.*, 2020, **6**, pp. 15–27

[68] Garg, H.: ‘Novel neutrality aggregation operator-based multiattribute group decision-making method for single-valued neutrosophic numbers’, *Soft Comput.*, 2019, pp. 1–23, https://doi.org/10.1007/s00500-019-04535-w

[69] Rani, D., Garg, H.: ‘Some modified results of the subtraction and division operations on interval neutrosophic sets’, *J. Exp. Theor. Artif. Intell.*, 2019, **31**, (4), pp. 677–698

[70] Garg, H., Nancy: ‘Multiple criteria decision making based on frank choquet heronian mean operator for single-valued neutrosophic sets’, *Appl. Comput. Math.*, 2019, **18**, (2), pp. 163–188