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Decision framework of group consensus with hesitant fuzzy linguistic preference relations

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Abstract: Group decision making typically requires eliciting and coordinating the preferences of group members. In this study, the authors develop an overall framework of group consensus in context of hesitant fuzzy linguistic preference relations (HFLPRs). A new notion of distance measure between two HFLPRs is defined, based on which a consensual framework consists of two mechanisms, the automatic consensus reaching and the interactive consensus reaching is presented. Accordingly, they, respectively, design two algorithms to implement these mechanisms. Finally, a group decision problem of green supplier selection is provided to exemplify and verify both of the proposed methods, and a comparative analysis with some related approaches is performed.

1 Introduction

Group decision making (GDM) is an attractive scheme as every member's opinion can be pooled and exchanged, but no one has veto power over panel decision [1]. GDM typically consists of two processes: the consensus process (CP) and the selection process. When opinions are not sharply divergent, the whole group's judgments can be aggregated and yield an outcome that less likely to be repudiated. But such aggregation will be unaccepted without an implicit homogeneity premise [2]. As a democratic principle, consensual result is developed with the goal of unanimous agreement, which is hard to achieve in practice. In response to this, a flexible notion of consensus, i.e. 'soft consensus' has since been suggested [3].

Consensus building pursues a maximal degree of agreement among group members. As a way of updating decision maker (DM)'s judgment, the feedback mechanism is a critical step in CP. There roughly exist two strategies [4], the automatic strategy and the interactive strategy in feedback course. By using the automatic strategy, a negotiation can be carried out without DMs' participations; conversely, the latter requires DMs to manually coordinate their efforts in securing an agreement. Despite the extra workloads, the interactive strategy is more liable to absorb DMs' authentic attitudes. However, not all the members are willing to change their opinions when a divergence is appeared.

It is common that judgments can only be stated linguistically owing to human's natural characteristics [5–17]. Many researches have been conducted on 'soft' consensus with linguistic preference relations (LPRs). Herrera *et al.* [5] initiated a consensus model in GDM with linguistic assessment, where human consistency can be incorporated. Herrera-Viedma *et al.* [6] built a consensus aid system to assist DMs in improving consensual level with multi-granularity linguistic preferences. Xu *et al.* [11] constructed a consensus-reaching process with uncertain linguistic decision matrix in achieving a satisfactory agreement. Dong *et al.* [13] explored a consensus-based GMD model under multi-granular unbalanced 2-tuple LPRs with minimum information loss, considering non-cooperative behaviours. However, classic linguistic computational models are invalid if a DM has several linguistic terms for the membership degree to a given element [18]. For example, expressions like 'pretty good', 'better than good' or 'not bad', may be float in a DM's mind if he/she wants to give a positive evaluation. To overcome this flaw, Rodríguez *et al.* [19] initiated the concept of hesitant fuzzy linguistic term set (HFLTS), and later Zhu and Xu [20] introduced the notion of hesitant fuzzy linguistic preference relation (HFLPR) based on HFLTSs. For consensus building with HFLPRs, Dong *et al.* [16] addressed an optimisation-based two-stage procedure which optimises the solutions to the proposed consensual model. Xu *et al.* [21] devised a consistency and consensus reaching model with HFLPRs by local adjustment strategy. Combined with 2-tuple linguistic model, Wu and Xu [17] proposed a consensus improving procedure, where the feedback system in CP was established directly on the calculated consensus degrees; besides, they also concerned an interactive CP in multi-attribute GDM with HFLTSs [22]. Xiao *et al.* [23] suggested a framework to handle personalised individual semantics and consensus using linguistic distribution preference relations; based on the multiplicative consistency of fuzzy LPRs, Zhang *et al.* [24] defined a new consensus index in measuring the agreement among individuals for GDM. Wu *et al.* [25] propose some novel approaches to manage consensus measure and priority weights of HFLPRs using local feedback strategy. Song and Li [26] developed a mathematical programming with dual functionality to process with consensus building under incomplete HFLPRs. In addition, Garg [9] proposed the linguistic Pythagorean fuzzy set, which is characterised by linguistic membership and non-membership degrees, to deal with uncertain and imprecise information.

From the stated works above, CP should be managed to avoid a misleading result in GDM. Yet a comprehensive analysis of consensus building with hesitant qualitative preferences is still needed to be further studied since the attitudes or activeness of members are quite different and unpredictable. The motivation of this study is to provide an integrated, unified framework consisting both of automatic consensus reaching (ACR) and interactive consensus reaching (ICR), to facilitate group negotiation under the context of HFLTSs. Despite the different mechanisms, the two proposed approaches are still quite connected with each other. As will be seen later, we present an example of green supplier selection for illustration that the idea of consensus improving in ACR can be implanted into ICR when the DMs refused to change their opinions.

To do so, some relevant issues should be answered: (i) how to measure the deviation of group members with HFLPRs; (ii) how to take the advantages of HFLTSs in preference modification; (iii) how to conduct an action if someone stick to his/her previous opinions in negotiation? These questions motivate us to conduct a comprehensive group consensual study. The objective of this paper is to present a useful way for consensus support based on HFLTSs, where an integrated framework consisting of ACR and ICR is sequentially developed. Despite the different mechanisms, the two proposed methods are still quite connected with each other in some cases. As will be seen later, the feedback mechanism of consensus improving in ACR can be implanted into ICR when a DM is reluctant to change opinions. The rest of the paper is structured as follows. Section 2 briefly reviews some related concepts. Section 3 develops two methods of group consensus reaching. An application of the developed approaches is explained in Section 4. Some comparisons are discussed in Section 5. Section 6 concludes the paper.

2 Preliminaries

2.1 Hesitant fuzzy linguistic term set

Let $S = \{s_t | t = 0, \ldots, \tau\}$ be a LTS with odd granularity $\tau + 1$, where s_t depicts a possible value for a linguistic variable. S satisfies the following conditions:

(i) The set is order: $s_\alpha \geq s_\beta$ if $\alpha \geq \beta$;
(ii) Negation operator is defined: $\text{neg}(s_\alpha) = s_{\tau-\alpha}$.

Motivated by the hesitant fuzzy sets [27], Rodríguez *et al.* introduced the concept of HFLTSs as follows.

Definition 1: Let $S = \{s_0, \ldots, s_\tau\}$ be a LTS, a HFLTS h is an ordered finite subset of the consecutive linguistic term of S. Denote $h = \{h^{\sigma(s)} | s = 1, 2, \ldots, |h|\}$, where $h^{\sigma(s)}$ and $|h|$ are the sth smallest element and the length of h, respectively [19].

Remark 1: Liao *et al.* [28, 29] pointed out some problems will arise under the asymmetric subscripts $S = \{s_0, \ldots, s_\tau\}$, and suggested a symmetric subscripts $S = \{s_t | t = -\tau, \ldots, 0, \ldots \tau\}$ for instead. Xu and Wang [30] extended S into a continuous form $\overline{S} = \{s_\alpha | \alpha \in [-\tau, \tau]\}$ to prevent information loss. Given $s_\alpha \in \overline{S}$, if $s_\alpha \in S$, then s_α is called an original linguistic term; otherwise, it is called a virtual linguistic term (VLT).

Let $s \in \overline{S}$, let $I(S)$ be the position index of s, if $s = s_\alpha$, then $I(S) = \alpha$. Via the position index, linguistic variables can be calculated in a quasi-numerical way.

2.2 Hesitant fuzzy linguistic preference relation

Definition 2: Let $X = (x_1, x_2, \ldots, x_n)$ be a set of alternatives. An HFLPR is represented by a matrix $\boldsymbol{H} = (h_{ij})_{n\times n}$ on $X \times X$ where $h_{ij} = \left\{h_{ij}^{\sigma(s)} \middle| s = 1, 2, \ldots, \left|h_{ij}\right|\right\}$ is an HFLTS, implying the hesitant preference degrees of x_i over x_j, with conditions (i, j = 1, 2,...,n, i<j) [20]

$$\begin{aligned} & h_{ij}^{\sigma(s)} \oplus h_{ji}^{\sigma(s)} = \{s_0\}, h_{ii} = \{s_0\}, \left|h_{ij}\right| = \left|h_{ji}\right| \\ & I(h_{ij}^{\sigma(s)}) \leq I(h_{ij}^{\sigma(s+1)}), I(h_{ji}^{\sigma(s)}) \geq I(h_{ji}^{\sigma(s+1)}) \end{aligned} \quad (1)$$

An HFLPR H can be transformed into a normalised HFLPR (NHFLPR) $\tilde{H} = (\tilde{h}_{ij})_{n\times n}$ when all its elements have been normalised.

Definition 3: For two HFLTSs $h_i = \left\{h_i^{\sigma(s)} \middle| s = 1, 2, \ldots, \left|h_i\right|\right\}$ (i = 1, 2) with $\left|h_1\right| = \left|h_2\right|$, then [20]

$$\begin{cases} h_1 \oplus h_2 = \cup_{h_1^{\sigma(s)} \in h_1, h_2^{\sigma(s)} \in h_2} \left\{h_1^{\sigma(s)} \oplus h_2^{\sigma(s)}\right\} \\ \lambda h_1 = \cup_{h_1^{\sigma(s)} \in h_1} \left\{\lambda h_1^{\sigma(s)}\right\} (\lambda \geq 0) \\ h_1 \oplus t = \cup_{h_1^{\sigma(s)} \in h_1} \left\{h_1^{\sigma(s)} \oplus t\right\} \end{cases} \quad (2)$$

where t is a linguistic term. Accordingly, the hesitant fuzzy linguistic weighted averaging (HFLWA) operator is redefined as follows.

Definition 4: Let h_i (i = 1, 2,..., n) be a collection of HFLTSs, an HFLWA operator is a mapping $\tilde{H}^n \to \tilde{H}$ such that [28]

$$\text{HFLWA}(h_1, h_2, \ldots, h_n) = w_1 \tilde{h}_1^{\sigma(s)} \oplus w_2 \tilde{h}_2^{\sigma(s)} \oplus \cdots \oplus w_n \tilde{h}_n^{\sigma(s)} \quad (3)$$

where $\boldsymbol{W} = (w_1, w_2, \ldots, w_n)^{\text{T}}$ is the weight vector of h_i (i = 1, 2,..., n) with $\sum_{i=1}^n w_i = 1$, $w_i \geq 0$.

Definition 5: Given two NHFLTSs $\tilde{h}_1$ and $\tilde{h}_2$ with $|\tilde{h}_1| = |\tilde{h}_2|$, the Hamming distance between them is [28]

$$D(\tilde{h}_1, \tilde{h}_2) = \frac{1}{|\tilde{h}|} \sum_{\tilde{h}_1^{\sigma(s)} \in \tilde{h}_1, \tilde{h}_2^{\sigma(s)} \in \tilde{h}_2} \left| \frac{I(\tilde{h}_1^{\sigma(s)}) - I(\tilde{h}_2^{\sigma(s)})}{2\tau + 1} \right| \quad (4)$$

where $\tilde{h}_1^{\sigma(s)}$ and $\tilde{h}_2^{\sigma(s)}$ are the sth smallest elements in $\tilde{h}_1$ and $\tilde{h}_2$, respectively.

3 Consensus reaching procedures

We present both the mechanisms of ACR and ICR in this section. The advantage of ACR is it facilitates the group members to reach an agreement without a complex negotiation, while the advantage of ICR is that a feedback mechanism is used to address the consensual procedure. Therefore, DMs can easily choose one of them according to their practical decision-making scenario.

3.1 Automatic consensus reaching

Suppose $\tilde{H}_k = (\tilde{h}_{ij,k})_{n\times n}$ be a NHLFPR given by the kth DM e_k (k = 1, 2,...m), whose relative weight λ_k satisfying $\sum_{k=1}^m \lambda_k = 1$, $\lambda_k \geq 0$. Consensus building is a burdensome work that requires effort and time; sometimes a DM may feel boring to rectify their assessments frequently [31]. Therefore, an automatic optimisation method of consensus improving caters for this requirement.

To facilitate our studies, we introduce a new distance metric between two NHFLPRs.

Definition 6: Given two NHFLPRs, $\tilde{H}_1 = (\tilde{h}_{ij,1})_{n\times n}$ and $\tilde{H}_2 = (\tilde{h}_{ij,2})_{n\times n}$, the distance between $\tilde{H}_1$ and $\tilde{H}_2$ is defined as

$$\Gamma(\tilde{H}_1, \tilde{H}_2) = \frac{2}{n(n-1)} \sum_{i<j}^n D(\tilde{h}_{ij,1}, \tilde{h}_{ij,2}) \quad (5)$$

where $D(\tilde{h}_{ij,1}, \tilde{h}_{ij,2})$ is the Hamming distance between $\tilde{h}_{ij,1}$ and $\tilde{h}_{ij,2}$ according to (1).

For a soft consensus model, the consensual level of group must be determined. We achieve it by measuring deviation between the individual preferences and the collective preference of group members. Based on the HFLWA operator, the collective NHFLPR

$\tilde{G} = (\tilde{g}_{ij})_{n\times n}$ of e_k ($k = 1, 2, \ldots, m$) can be derived

$$\tilde{g}_{ij} = \overset{m}{\underset{k=1}{\oplus}} \lambda_k \tilde{h}_{ij,k}, \quad i, j = 1, 2, \ldots, n. \tag{6}$$

Then, we put forward the following group consensus index (GCI) of $\tilde{H}_k$:

$$\begin{aligned} \mathrm{GCI}(\tilde{H}_k) = \Gamma(\tilde{H}_k, \tilde{G}) &= \frac{2}{n(n-1)} \cdot \sum\nolimits_{i<j}^{n} D(\tilde{h}_{ij}, \tilde{g}_{ij}) \\ &= \frac{2}{n(n-1)} \cdot \sum\nolimits_{i<j}^{n} D\left(\tilde{h}_{ij}, \overset{m}{\underset{k=1}{\oplus}} (\lambda_k \tilde{h}_{ij,k})\right) \end{aligned} \tag{7}$$

From (7), GCI can be interpreted as a divergence between personal opinion and group collective opinion. If $\mathrm{GCI}(\tilde{H}_k) = 0$, it implies the e_k is fully concord with the collective opinion.

Definition 7: Let $\overline{\mathrm{GCI}}$ be a predefine threshold, $\tilde{H}_k$ is considered to be acceptable if satisfies

$$\mathrm{GCI}(\tilde{H}_k) \le \overline{\mathrm{GCI}} \tag{8}$$

In (8), the value of $\overline{\mathrm{GCI}}$ can be determined by moderator in practical situations. If (8) holds for any $\tilde{H}_k$, $k = 1, 2, \ldots, m$, we conclude that the group $\{e_1, e_2, \ldots, e_m\}$ reach a consensus. Conversely, if there exists $\mathrm{GCI}(\tilde{H}_k) > \overline{\mathrm{GCI}}$, then $\tilde{H}_k$ is said to be unacceptable and should be revised until the threshold level is hold. We improve the GCI of $\tilde{H}_k$ by the following iterative algorithm.

Algorithm 1:
Input: original HFLPR H_k, relative weight λ_k, $k = 1, 2, \ldots, m$, and threshold $\overline{\mathrm{GCI}}$;
Output: adjusted NHFLPR $\tilde{H}_k^{(l)} = (\tilde{h}_{ij,k}^{(l)})_{n\times n}$, $\mathrm{GCI}(\tilde{H}_k^{(l)})$, $k = 1, 2, \ldots m$;

Step 1. Convert H_k into $\tilde{H}_k$ ($k = 1, 2, \ldots, m$), and set $\tilde{H}_k^{(l)} = \tilde{H}_k$ ($l = 0$).
Step 2. Establish the collective HFLPR $\tilde{G}^{(l)} = (\tilde{g}_{ij}^{(l)})_{n\times n}$ according to (6), where

$$\tilde{g}_{ij}^{(l)} = \overset{m}{\underset{k=1}{\oplus}} \lambda_k \tilde{h}_{ij,k}^{(l)}, \quad i, j = 1, 2, \ldots, n. \tag{9}$$

Step 3. Compute each member's GCI, i.e. $\mathrm{GCI}(H_k)$;
If $\mathrm{GCI}(H_k) \le \overline{\mathrm{GCI}}$ for $k = 1, 2, \ldots, m$, then go to step 5; otherwise, go to the next step.
Step 4. Let $l = l + 1$, and update the $\tilde{H}_k^{(l+1)} = (\tilde{h}_{ij,k}^{(l+1)})_{n\times n}$, where

$$\begin{aligned} I\left(\tilde{h}_{ij,k}^{\sigma(s)}\right)^{(l+1)} &= \delta I\left(\left(\tilde{h}_{ij,k}^{\sigma(s)}\right)^{(l)}\right) + (1-\delta) I\left(\left(\tilde{g}_{ij,k}^{\sigma(s)}\right)^{(l)}\right), \\ s &= 1, 2, \ldots, \left|\tilde{h}_{ij,k}\right|, i, j = 1, 2, \ldots, n \end{aligned} \tag{10}$$

then go back to step 2.
Step 5. Output the $\tilde{H}_k^{(l)}$ and $\mathrm{GCI}(\tilde{H}_k^{(l)})$, $k = 1, 2, \ldots, m$.
Step 6. End.

Remark 2: The δ ($0 \le \delta \le 1$) in step 4 is a regulating parameter, which determines the times of iterations in updating $\tilde{H}_k$, $k = 1, 2, \ldots, m$.

Theorem 1: Given original NHFLPR $\tilde{H}_k = (\tilde{h}_{ij,k})_{n\times n}$, $k = 1, 2, \ldots, m$, let $\{\tilde{H}_k^{(0)}, \tilde{H}_k^{(1)}, \tilde{H}_k^{(l)} \ldots\ldots\}$ be a sequence of NHFLPRs ($l \ge 1$) that obtained by Algorithm 1, then $\mathrm{GCI}(\tilde{H}_k^{(l+1)}) \le \mathrm{GCI}(\tilde{H}_k^{(l)})$ and $\lim\limits_{l\to\infty}\left\{\mathrm{GCI}(\tilde{H}_k^{(l)})\right\} = 0$.

Table 1 Number of iterations under Algorithm 1

n	m	p	$\overline{\mathrm{GCI}}$	l		
				$\delta = 0.2$	$\delta = 0.5$	$\delta = 0.8$
3	4	3	0.30	1.92	3.31	10.72
			0.10	2.97	4.69	12.53
4	4	2	0.30	2.08	4.48	14.43
			0.10	2.66	5.22	15.71
4	5	3	0.30	2.41	4.87	16.85
			0.10	3.07	6.34	18.53
5	5	4	0.30	2.59	4.54	13.52
			0.10	2.84	6.16	15.84
6	4	3	0.30	2.63	5.89	14.55
			0.10	2.37	6.31	18.36
6	5	5	0.30	3.16	8.24	16.91
			0.10	3.77	9.95	20.68

The proof of Theorem 1 is included in Appendix.

After the consensus improving process, we use the score function of HFLTSs, denoted as $b(h)$, to rank alternatives as given below:

$$b(h) = \frac{1}{|h|} \sum\nolimits_{s_\alpha \in h} \left(\frac{\alpha}{\tau}\right) \tag{11}$$

To analyse the computational complexity of Algorithm 1, we figure out the number of iterations by assigning various values to parameters: the dimension of $\tilde{H}_k$, n, the number of DMs, m, the length of HFLTSs, p, the threshold value, $\overline{\mathrm{GCI}}$ and the regulating parameter δ. We randomly generate 500 HFLPRs and compute the average number of iterations l, and some results are shown in Table 1.

It can be seen from Table 1 that ACR approach guides the group to a consensus after several rounds. The average number of iterations grow when the parameters, i.e. n, m, p and $\overline{\mathrm{GCI}}$ increase. Also, we find a higher number of iterations is associated with a bigger value of δ.

3.2 Interactive consensus reaching

Despite achievement of group consensus by ACR, the preferences modified without supervision may not accord with DMs' inner thoughts. In one aspect, lacking the personally involvements of DMs, their feedbacks cannot be well integrated into consensus reaching process. Another aspect lies in that almost all elements (except diagonal) stored in a NHFLPR are updated by new ones iteratively. Under certain conditions, however, a DM is ready to amend his/her judgments when realising what he/she had made are fairly various from those of other DMs. Probably he/she will have a close examination again and makes some changes.

As mentioned before, a premise to ICR is that DMs are joined in negotiation and collaborated to rectification gradually. To begin with, we find out the DMs who fail to satisfy the minimum consensus condition. According to (8), for any HFLPR $H_k^{(l)}$ in the lth iteration ($k = 1, 2, \ldots, m$; $l = 1, 2, \ldots$), if $\mathrm{GCI}(H_k^{(l)}) > \overline{\mathrm{GCI}}$, thus e_k is unqualified in current round and should be elected for further adjustment. Let $\vec{E} = \{\vec{e}_1, \vec{e}_2, \ldots, \vec{e}_{\vec{m}}\}$ be the set of DMs who have been selected, which can be mathematically expressed as

$$\begin{cases} \vec{E} = \left\{\vec{e}_t \middle| \exists e_k, \text{s.t. } \mathrm{GCI}(H_k^{(l)}) > \overline{\mathrm{GCI}}\right\} \\ \vec{E} \subseteq E \end{cases}, \quad l = 1, 2, \ldots \tag{12}$$

After $\vec{E}$ is identified, we attempt to find out an alternative $x_{t,y}^*$ for $\vec{e}_t$ ($t = 1, 2, \ldots, \vec{m}$), where $\vec{e}_t$'s judgments on $x_{t,y}^*$ is most deviated from the collective preference. This is a critical step of the ICR procedure. In mathematical phrases, the yth alternative $x_{t,y}^*$

satisfies the condition

$$\left\{x_{t,y}^{*}\middle| y=i,\quad s.t.\quad \underset{i,j=1,2,\ldots,n,i<j}{\arg\max}\ D\left(\tilde{h}_{ij,t}^{(l)},\tilde{g}_{ij}^{(l)}\right)\right\},\qquad t=1,2,\ldots,\vec{m},\quad l=1,2,\ldots \tag{13}$$

where $D\left(\tilde{h}_{i\ j,t}^{(l)},\ \tilde{g}_{ij}^{(l)}\right)$ is the Hamming distance between HFLTSs $\tilde{h}_{ij,t}^{(l)}$ and $\tilde{g}_{ij}^{(l)}$. It is emphasised that one and only one alternative was picked out from an HFLPR. It is applicable and makes sense because a DM may feel uncomfortable when total denial and modification of his/her previous judgments [32]. Actually, group consensus reaching is a step-by-step negotiation requiring continual efforts to find a result that is attractive to all participants.

In what follows, $\vec{e}_t$ ($t=1, 2,\ldots, \vec{m}$) is invited to check and rectify his/her previous assessment about $x_{t,y}^{*}$ by referring to the collective HFLPR $\tilde{G}^{(l)}$. More specifically:

(i) if $I(\tilde{h}_{ij,t}^{(l)}) > I(\tilde{g}_{ij}^{(l)})$, then assign smaller VLTs to $\tilde{h}_{ij,t}^{(l+1)}$;
(ii) if $I(\tilde{h}_{ij,t}^{(l)}) < I(\tilde{g}_{ij}^{(l)})$, then assign larger VLTs to $\tilde{h}_{ij,t}^{(l+1)}$.

This judgment updating principle is similar to the well-known direction rules in [17]. Then $\tilde{H}_t^{(l)}$ is evolved into a new NHFLPR $\tilde{H}_t^{(l+1)}$, and $\tilde{G}^{(l+1)}$ is acquired from $\tilde{G}^{(l)}$ as well. It is found that $\tilde{h}_{ij,t}^{(l+1)}$ meets the condition

$$\left\{\tilde{h}_{ij,t}^{(l+1)}\middle|\min\left[I\left(\tilde{h}_{ij,t}^{(l)}\right),I\left(\tilde{g}_{ij}^{(l)}\right)\right]\le I\left(\tilde{h}_{ij,t}^{(l+1)}\right)\le\max\left[I\left(\tilde{h}_{ij,t}^{(l)}\right),I\left(\tilde{g}_{ij}^{(l)}\right)\right]\right\} \tag{14}$$

Remark 3: As indicated in [17], a challenge may appear due to the revised linguistic terms are VLTs rather than LTSs. Future endeavours are needed to facilitate the modification.

By now, the first iteration is finished. We recalculate the GCIs and check whether they are acceptable or not. If not, a same process will be performed again until the group reach a consensus. As can be seen, achievement of ICR requires the whole collaboration which induces DMs' opinions close to each other step by step. Let us consider a more complicated and tricky situation. After finding out $x_{t,y}^{*}$, assume $\vec{e}_t$ asserts his/her judgments are reasonable and he/she will still adhere to the previous assessments. Under such case, the process of ICR will be stuck without $\vec{e}_t$'s active improvements. Liao *et al.* [32] presented a solution that removing $\vec{e}_t$ ($t=1, 2,\ldots, \vec{m}$) from the decision group since $\vec{e}_t$'s assessment is much different from the others. However, simply removal of a DM may result in losing much useful information.

To overcome this obstacle, we turn back and utilise the updating strategy of ACR. Enlighten by (10), the following equation can be formulated to revise $x_{t,y}^{*}$:

$$I\left(\tilde{h}_{yj,t}^{(l+1)}\right)=\delta I\left(\tilde{h}_{yj,t}^{(l)}\right)+(1-\delta)I\left(\tilde{g}_{yj}^{(l)}\right),\quad y\ne j \tag{15}$$

where $\tilde{h}_{yj,t}^{(l)}$ and $\tilde{h}_{yj,t}^{(l+1)}$ are the NHFLTSs of $x_{t,y}^{*}$ in the lth time and $(l+1)$th iterations, respectively. Compare to (10), (15) updates the information of $x_{t,y}^{*}$ only, whereas all information of an NHFLPR are modified in (10). After doing so, the current GCI of $\vec{e}_t$, $t=1, 2,\ldots, \vec{m}$, is improved.

On the basis of the above analysis, the algorithm of ICR can be summarised as:

Algorithm 2:
Input: original HFLPR H_k, relative weight λ_k, $k=1, 2,\ldots,m$, and threshold value $\overline{\text{GCI}}$;
Output: adjusted NHFLPR $\tilde{H}_k^{(l)}=(\tilde{h}_{ij,k}^{(l)})_{n\times n}$ and $\text{GCI}(\tilde{H}_k^{(l)})$, $k=1, 2,\ldots m$.
Step s 1-2: Same as the steps in Algorithm 1.
Step 3. Calculate each DM's GCI, if $\text{GCI}(H_k)\le\overline{\text{GCI}}$, $k=1, 2,\ldots, m$, then go to step 7; otherwise, go to step 4.
Step 4. Find out the $\vec{e}_t$, $t=1, 2,\ldots, \vec{m}$. For any $k=1, 2,\ldots,m$, if $\text{GCI}(H_k)>\overline{\text{GCI}}$, denoted e_k as $\vec{e}_t$.
Step 5. Pick out an alternative $x_{t,y}^{*}$ who has the largest deviation to others according to (4).
Step 6. Let $l=l+1$, ask $\vec{e}_t$ ($t=1, 2,\ldots \vec{m}$) to revise the preference about $x_{t,y}^{*}$. If $\vec{e}_t$ agrees to do that, then $\tilde{H}_t^{(l+1)}$ is obtained directly. If no, using (15) to generate $\tilde{H}_t^{(l+1)}$, and go back to step 2.
Step 7. Output the $\tilde{H}_k^{(l)}$ and $\text{GCI}(\tilde{H}_k^{(l)})$, $k=1, 2,\ldots,m$.
Step 8. End.

As both the ACR and ICR methods can assist the DMs in achieving a predefined consensus level, we intend to draw the reader's attention to the differences between them: (i) Generally, ICR is executed under the DMs' supervision, while ACR can work well without DMs' active participation. (ii) ACR has a wider scope of preference adjustment, where all pairs of comparisons of an NHFLPR are updated in iterative step. Yet in ICR one and only one pair of comparison is chosen and rectified. (iii) The regulating parameter is a necessity in ACR but is dispensable in ICR. (iv) An identical result is not guaranteed since subjective activities were involved in the process of ICR.

4 Illustrative example

4.1 Problem description

Fujian Motor Industry Group (FMIG), a flagship automobile manufacture in Fuzhou, China, intends to select a green supplier to improve the environmental performance. After preliminary scanning, four candidates (denoted as x_i, $i=1, 2, 3, 4$) are invited for further assessment. Suppose that these candidates are evaluated by a group of four experts e_k ($k=1, 2, 3, 4$) whose relative weights are equal to 0.25. Two main green criteria, the performances of pollution control (air emission, water waste) and resource consumption (renewable and non-renewable resources etc.) are considered in this evaluation. Owing to the complexity and uncertainty of multi-criteria judgments, it is inconvenient for experts to assign exact data directly. In fact, experts prefer to express their judgments by linguistic terms since much qualitative information may appear on the assessment process. The following LTS S ($\tau=4$) is employed to depict the experts' preferences.

$S=\{s_{-4}$: extremely inferior, s_{-3}: very inferior, s_{-2}: inferior, s_{-1}: slightly inferior, s_0: indifference, s_1: slight superior, s_2: superior, s_3: very superior, s_4: extremely superior$\}$Besides, it is feasible to express preferences of pairs of candidates in terms of HFLPR in order to preserve the original information. To achieve a convincing result, FMIG requires the minimum consensus, i.e. $\overline{\text{GCI}}$, should be less than 0.11. The original comparisons in the form of HFLPRs are given as follows:

$$H_1=\begin{Bmatrix}\{s_0\}&\{s_0,s_1\}&\{s_1,s_2\}&\{s_2,s_3\}\\\{s_0,s_{-1}\}&\{s_0\}&\{s_0,s_1\}&\{s_1\}\\\{s_{-1},s_{-2}\}&\{s_0,s_{-1}\}&\{s_0\}&\{s_0,s_1\}\\\{s_{-2},s_{-3}\}&\{s_{-1}\}&\{s_0,s_{-1}\}&\{s_0\}\end{Bmatrix},$$

$$H_2=\begin{Bmatrix}\{s_0\}&\{s_1,s_2,s_3\}&\{s_{-2},s_{-1}\}&\{s_2\}\\\{s_{-1},s_{-2},s_{-3}\}&\{s_0\}&\{s_{-2}\}&\{s_0,s_1\}\\\{s_2,s_1\}&\{s_2\}&\{s_0\}&\{s_2,s_3\}\\\{s_{-2}\}&\{s_0,s_{-1}\}&\{s_{-2},s_{-3}\}&\{s_0\}\end{Bmatrix}$$

$$H_3=\begin{Bmatrix}\{s_0\}&\{s_1\}&\{s_2,s_3\}&\{s_2,s_3\}\\\{s_{-1}\}&\{s_0\}&\{s_0,s_1\}&\{s_1,s_2\}\\\{s_{-2},s_{-3}\}&\{s_0,s_{-1}\}&\{s_0\}&\{s_1\}\\\{s_{-2},s_{-3}\}&\{s_{-1},s_{-2}\}&\{s_{-1}\}&\{s_0\}\end{Bmatrix},$$

$$H_4=\begin{Bmatrix} \{s_0\} & \{s_0, s_1\} & \{s_{-1}, s_0\} & \{s_{-3}, s_{-2}\} \\ \{s_0, s_{-1}\} & \{s_0\} & \{s_{-3}, s_{-2}\} & \{s_{-4}, s_{-3}\} \\ \{s_1, s_0\} & \{s_3, s_2\} & \{s_0\} & \{s_{-1}, s_0\} \\ \{s_3, s_2\} & \{s_4, s_3\} & \{s_1, s_0\} & \{s_0\} \end{Bmatrix}$$

4.2 Procedure of consensus reaching

(i) The ACR approach:

Step 1. Let $l=0$; transforms the HFLPRs H_k into $\tilde{H}_k$ ($k=1$–4) and set $\tilde{H}_k^{(0)} = \tilde{H}_k$.

Step 2. Build the collective NHFLPR $\tilde{G}^{(0)} = (\tilde{g}_{ij}^{(0)})_{4\times 4}$ by the HFLWA operator

(see (first equation below))

Step 3. Check $\mathrm{GCI}(\tilde{H}_k^{(0)})$, $k=1, 2, 3, 4$. Through (7), we have $\mathrm{GCI}(\tilde{H}_1^{(0)}) = 0.1065$, $\mathrm{GCI}(\tilde{H}_2^{(0)}) = 0.1343$, $\mathrm{GCI}(\tilde{H}_3^{(0)}) = 0.1296$ and $\mathrm{GCI}(\tilde{H}_4^{(0)}) = 0.2129$; since $\overline{\mathrm{GCI}} = 0.11$, so only e_1 satisfies the consensus requirement, while experts $\{e_2, e_3, e_4\}$ need to rectify their judgments.

Step 4. Let $l=l+1$. Set $\delta = 0.3$, utilise (10) to update $\tilde{H}_k^{(0)}$ and obtain $\tilde{H}_k^{(1)}$ ($k=2, 3, 4$) and $\tilde{G}^{(1)}$. The $\tilde{G}^{(1)}$ is updated as follows:

(see (second equation below))

Step 5. Check the $\mathrm{GCI}(\tilde{H}_k^{(1)})$ ($k=2, 3, 4$), which yield $\mathrm{GCI}(\tilde{H}_2^{(1)}) = 0.0456$, $\mathrm{GCI}(\tilde{H}_3^{(1)}) = 0.0243$, $\mathrm{GCI}(\tilde{H}_4^{(1)}) = 0.0768$. Obviously, the group reaches a consensus after this update.
Step 6. Aggregate $\tilde{g}_{ij}^{(1)}$ ($j=1$–4) in $\tilde{G}^{(1)}$ and obtain the overall preference $\tilde{g}_i^{(1)}$ for the ith candidate, then rank the candidates according to their score functions $b(\tilde{g}_i^{(1)})$, $i=1$–4. The results are listed in Table 2.

Note that the parameter δ in step 4 can vary depending on the solution implementation. If we take δ as 0.8, we have $\mathrm{GCI}(\tilde{H}_2^{(1)}) = 0.1098$, $\mathrm{GCI}(\tilde{H}_3^{(1)}) = 0.0891$, $\mathrm{GCI}(\tilde{H}_4^{(1)}) = 0.1833$. Obviously, the convergent rate is slowing down with δ increasing, which is consistent with the conclusions drawn from Table 1.

Table 2 Ranking results of ACR

	Overall preference degree	Score function	Rank
$\tilde{g}_1^{(1)}$	$\{s_{0.389}, s_{0.738}, s_{1.088}\}$	0.7383	1
$\tilde{g}_2^{(1)}$	$\{s_{-0.450}, s_{-0.369}, s_{-0.317}\}$	−0.3688	3
$\tilde{g}_3^{(1)}$	$\{s_{0.317}, s_{0.192}, s_{0.067}\}$	−0.1922	2
$\tilde{g}_4^{(1)}$	$\{s_{-0.286}, s_{-0.562}, s_{-0.838}\}$	−0.5617	4

ii) The ICR approach

We begin with step 4 as steps 1–3 are the same in ACR approach.

Step 4. Denote $\vec{E} = \{\vec{e}_2, \vec{e}_3, \vec{e}_4\}$ since the GCIs of experts $\{e_2, e_3, e_4\}$ are not acceptable.
Step 5. Compute the Hamming distances $D\left(\tilde{h}_{ij,k}^{(0)}, \tilde{g}_{ij}^{(0)}\right)$ between HFLTSs $\tilde{g}_{ij}^{(0)}$ and $\tilde{h}_{ij,k}^{(0)}$ ($i, j=1, 2, 3, 4, i<j$; $k=2, 3, 4$), respectively, which are shown in Table 3.
Step 6. Let $l=l+1=1$. Pick out an alternative with maximum deviation for $\vec{e}_k$ ($k=2, 3, 4$). From Table 3, we should ask $\vec{e}_2$, $\vec{e}_3$ and $\vec{e}_4$ to revise the HFLTSs of $\tilde{h}_{13,2}^{(0)}$, $\tilde{h}_{13,3}^{(0)}$ and $\tilde{h}_{14,4}^{(0)}$, respectively (values with underline). Without loss of generality, assume both $\vec{e}_3$ and $\vec{e}_4$ agree with our suggestions, while $\vec{e}_2$ is unwilling to change his/her mind. Under such case, we utilise (17) to update the linguistic terms in $\tilde{h}_{13,2}^{(0)}$ as below (δ is randomly assume as 0.3):

(see (third equation below))

In addition, let $\vec{e}_3$ and $\vec{e}_4$ modify their preferences as $h_{13,3}^{(0)} = \{s_0, s_1\}$, $h_{14,4}^{(0)} = \{s_{-1}, s_0, s_1\}$. After normalisation we have

(see (fourth equation below))

$$\tilde{G}^{(0)}=\begin{Bmatrix} \{s_0\} & \{s_{0.5}, s_1, s_{1.5}\} & \{s_0, s_{0.5}, s_1\} & \{s_{0.75}, s_{1.125}, s_{1.5}\} \\ \{s_{-0.5}, s_{-1}, s_{-1.5}\} & \{s_0\} & \{s_{-1.25}, s_{-0.875}, s_{-0.5}\} & \{s_{-0.5}, s_{-0.125}, s_{0.25}\} \\ \{s_0, s_{-0.5}, s_{-1}\} & \{s_{1.25}, s_{0.875}, s_{0.5}\} & \{s_0\} & \{s_{0.5}, s_{0.875}, s_{1.25}\} \\ \{s_{-0.75}, s_{-1.125}, s_{-1.5}\} & \{s_{0.5}, s_{0.125}, s_{-0.25}\} & \{s_{-0.5}, s_{-0.875}, s_{-1.25}\} & \{s_0\} \end{Bmatrix}$$

$$\tilde{G}^{(1)}=\begin{Bmatrix} \{s_0\} & \{s_{0.412}, s_{0.913}, s_{1.412}\} & \{s_{0.175}, s_{0.675}, s_{1.175}\} & \{s_{0.968}, s_{1.365}, s_{1.762}\} \\ \{s_{-0.412}, s_{-0.913}, s_{-1.412}\} & \{s_0\} & \{s_{-1.031}, s_{-0.634}, s_{-0.237}\} & \{s_{-0.237}, s_{0.072}, s_{0.381}\} \\ \{s_{-0.175}, s_{-0.675}, s_{-1.175}\} & \{s_{1.031}, s_{0.634}, s_{0.237}\} & \{s_0\} & \{s_{0.412}, s_{0.809}, s_{1.206}\} \\ \{s_{-0.968}, s_{-1.365}, s_{-1.762}\} & \{s_{0.237}, s_{0.072}, s_{-0.381}\} & \{s_{-0.412}, s_{-0.809}, s_{-1.206}\} & \{s_0\} \end{Bmatrix}$$

$$\tilde{H}_2^{(1)}=\begin{Bmatrix} \{s_0\} & \{s_1, s_2, s_3\} & \{\underline{s_{-0.6}, s_{-0.1}, s_{0.4}}\} & \{s_2, s_2, s_2\} \\ \{s_{-1}, s_{-2}, s_{-3}\} & \{s_0\} & \{s_{-2}, s_{-2}, s_{-2}\} & \{s_0, s_{0.5}, s_1\} \\ \{\underline{s_{0.6}, s_{0.1}, s_{-0.4}}\} & \{s_2, s_2, s_2\} & \{s_0\} & \{s_2, s_{2.5}, s_3\} \\ \{s_{-2}, s_{-2}, s_{-2}\} & \{s_0, s_{-0.5}, s_{-1}\} & \{s_{-2}, s_{-2.5}, s_{-3}\} & \{s_0\} \end{Bmatrix}$$

$$\tilde{H}_3^{(1)}=\begin{Bmatrix} \{s_0\} & \{s_1, s_1, s_1\} & \{\underline{s_0, s_{0.5}, s_1}\} & \{s_2, s_{2.5}, s_3\} \\ \{s_{-1}, s_{-1}, s_{-1}\} & \{s_0\} & \{s_0, s_{0.5}, s_1\} & \{s_1, s_{1.5}, s_2\} \\ \{\underline{s_0, s_{-0.5}, s_{-1}}\} & \{s_0, s_{-0.5}, s_{-1}\} & \{s_0\} & \{s_1, s_1, s_1\} \\ \{s_{-2}, s_{-2.5}, s_{-3}\} & \{s_{-1}, s_{-1.5}, s_{-2}\} & \{s_{-1}, s_{-1}, s_{-1}\} & \{s_0\} \end{Bmatrix}$$

Table 3 Computation results of Hamming distances

$D\left(\tilde{h}_{ij,k}^{(0)},\ \tilde{g}_{ij}^{(0)}\right)$	Individual NHFLPR								
	$\tilde{H}_2^{(0)}$			$\tilde{H}_3^{(0)}$			$\tilde{H}_4^{(0)}$		
Collective NHFLPR $\tilde{G}^{(0)}$	0.111	$\underline{0.222}$ 0.125	0.097 0.069 0.181	0.037	$\underline{0.222}$ 0.153	0.153 0.181 0.032	0.056	0.111 0.181	$\underline{0.403}$ 0.375 0.153

(see (first equation below))

The new collective NHFLPR is

(see (second equation below))

Step 7. Check the consensus level of $\mathrm{GCI}(\tilde{H}_k^{(1)})$ $(k=2, 3, 4)$. Using (7), we have $\mathrm{GCI}(\tilde{H}_2^{(1)}) = 0.0970$, $\mathrm{GCI}(\tilde{H}_3^{(1)}) = 0.0961$, $\mathrm{GCI}(\tilde{H}_4^{(1)}) = 0.1785$. Since only $\mathrm{GCI}(\tilde{H}_4^{(1)}) > \overline{GCI}$, so $\vec{E} = \{\vec{e}_4\}$.

Step 8. Continue the looping process of steps 5–7. This process terminates after three iterations, which is depicted in Table 4. For sake of illustration, we assume $h_{24,4}^{(1)} = \{s_{-1}, s_0\}$ and $h_{34,4}^{(3)} = \{s_0, s_2\}$ are given by $\vec{e}_4$ directly, while $h_{24,4}^{(2)}$ is updated by (15) (values with underline).

Step 9. Utilise HFLWA operator to fuse the ith row of HFLTSs in $\tilde{G}^{(4)}$, then acquire the overall preference degree $\tilde{g}_i^{(4)}$ of the candidate x_i, $i=1, 2, 3, 4$, as below, $\tilde{g}_1^{(4)} = \{s_{0.530}, s_{0.868}, s_{1.267}\}$, $\tilde{g}_2^{(4)} = \{s_{-0.375}, s_{-0.313}, s_{-0.250}\}$, $\tilde{g}_3^{(4)} = \{s_{0.506}, s_{0.413}, s_{0.288}\}$, $\tilde{g}_4^{(4)} = \{s_{-0.500}, s_{-0.781}, s_{-1.125}\}$ and their score functions are $b(\tilde{g}_1^{(4)}) = 0.8883$, $b(\tilde{g}_2^{(4)}) = -0.3125$, $b(\tilde{g}_3^{(4)}) = 0.4021$, $b(\tilde{g}_4^{(4)}) = -0.8021$, i.e. $x_1 \succ x_3 \succ x_2 \succ x_4$, which is the same as the results of ACR.

Table 4 Iterative process for $\vec{e}_4$

l	$\mathrm{GCI}(\tilde{H}_4^{(l)})$	$D\left(\tilde{h}_{ij,4}^{(l)},\ \tilde{g}_{ij}^{(l)}\right)$			Revised NHFLTS
1	0.1785	0.056	0.094 0.181	0.213 $\underline{0.042}$ 0.153	$\tilde{h}_{24,4}^{(1)} \to \{s_{-1}, s_{-0.5}, s_0\}$
2	0.1229	0.056	0.113 0.181	$\underline{0.213}$ 0.042 0.153	$\tilde{h}_{14,4}^{(2)} \to \{s_{0.575}, s_{1.075}, s_{1.875}\}$
3	0.1149	0.056	0.113 0.181	0.063 0.125 $\underline{0.153}$	$\tilde{h}_{34,4}^{(3)} \to \{s_0, s_1, s_2\}$
4	0.1040	0.056	0.113 0.181	0.099 0.125 $\underline{0.051}$	

5 Comparative analysis and discussions

In this section, we compare the proposed approaches with some existing relevant consensus methods from research focus, which are summarised in Table 5.

(i) Wu and Xu [17] presented a method of consistency and consensus improving with HFLPRs. In CP, a feedback mechanism is set up directly on the calculated consensus degrees, and an agreement among the DMs can be reached efficiently. As revealed in Table 5, Wu and Xu employed the interactive strategy as the ICR method does, but overall their working mechanisms are quite different. For measuring consensus degrees, they utilised the similarity between individual preference and collective preference, where a higher value is associated with a higher consensus level. After two iterations, it follows that $\mathrm{GCI}(\tilde{H}_1^{(2)}) = 0.889$, $\mathrm{GCI}(\tilde{H}_2^{(2)}) = 0.840$, $\mathrm{GCI}(\tilde{H}_3^{(2)}) = 0.802$, $\mathrm{GCI}(\tilde{H}_4^{(2)}) = 0.864$; on the contrary, the ICR method is built on the distance metric, so the smaller the GCI value, the higher the consensual level. However, the adjustment of preferences in ICR method is similar to the direction rules in [17], as discussed before. For the illustrative

Table 5 Comparison on the different methods

Methods	Research focus		
	Linguistic description	Consensus measurement	Regulate strategies
Wu and Xu [17]	HFLTS	similarity to collective preference	interactive
Dong *et al.* [16]	HFLTS	distance to collective preference	automatic
Zhang *et al.* [33]	VLT	distance between members	automatic
Xu *et al.* [11]	ULTS	distance to collective preference	automatic
Herrera-Viedma *et al.* [6]	VLT	similarity to collective preference	interactive
proposed methods	HFLTS	distance to collective preference	automatic and interactive

$$\tilde{H}_4^{(1)} = \begin{Bmatrix} \{s_0\} & \{s_0, s_{0.5}, s_1\} & \{s_{-1}, s_{-0.5}, s_0\} & \{\underline{s_{-1}, s_0, s_1}\} \\ \{s_0, s_{-0.5}, s_{-1}\} & \{s_0\} & \{s_{-3}, s_{-2.5}, s_{-2}\} & \{s_{-4}, s_{-3.5}, s_{-3}\} \\ \{s_1, s_{0.5}, s_0\} & \{s_3, s_{2.5}, s_2\} & \{s_0\} & \{s_{-1}, s_{-0.5}, s_0\} \\ \{\underline{s_1, s_0, s_{-1}}\} & \{s_4, s_{3.5}, s_3\} & \{s_1, s_{0.5}, s_0\} & \{s_0\} \end{Bmatrix}$$

$$\tilde{G}^{(1)} = \begin{Bmatrix} \{s_0\} & \{s_{0.5}, s_1, s_{1.5}\} & \{s_{-0.15}, s_{0.35}, s_{0.85}\} & \{s_{1.25}, s_{1.75}, s_{2.25}\} \\ \{s_{-0.5}, s_{-1}, s_{-1.5}\} & \{s_0\} & \{s_{-1.25}, s_{-0.875}, s_{-0.5}\} & \{s_{-0.5}, s_{-0.125}, s_{0.25}\} \\ \{s_{0.15}, s_{-0.35}, s_{-0.85}\} & \{s_{1.25}, s_{0.875}, s_{0.5}\} & \{s_0\} & \{s_{0.5}, s_{0.875}, s_{1.25}\} \\ \{s_{-1.25}, s_{-1.75}, s_{-2.25}\} & \{s_{0.5}, s_{0.125}, s_{-0.25}\} & \{s_{-0.5}, s_{-0.875}, s_{-1.25}\} & \{s_0\} \end{Bmatrix}$$

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example, a same ranking order $x_1 \succ x_3 \succ x_2 \succ x_4$ is obtained in spite of various consensual mechanisms, which verify the feasibility of the proposed methods.

(ii) Dong *et al.* [16] constructed an optimisation-based consensus model to help DMs in reaching a consensus under hesitant fuzzy linguistic environment. This model includes a two-stage procedure, which can be transformed and solved by the mixed 0-1 linear programming models. Analogous to the proposed ACR method, Dong *et al.*'s method employed the automatic strategy, where all DMs' preferences will be updated in each round. Both Dong *et al.*'s method and the ACR method adopt the distance between the individual preferences to the collective preference for consensus measure. Nevertheless, Dong *et al.* [16] adopt a new distance metric, which computes the number of different linguistic terms between two HFLTSs. It is claimed that the number of adjusted linguistic terms can be minimised in preference adjustment. By contrast, the ACR method utilises the Hamming distance between two HFLTSs, and further applied it for consensus measure. When group consensus is reached, and using (11) to compute the score functions of alternatives, we have $b(\tilde{g}_1^{(3)}) = 0.6083$, $b(\tilde{g}_2^{(3)}) = 0.2025$, $b(\tilde{g}_3^{(3)}) = 0.4288$, $b(\tilde{g}_4^{(3)}) = 0.1958$. Clearly, the x_1 is the most preferred one.

(iii) The consensus models in [6, 34, 35] are designed for LRPs yet cannot deal with HFLPRs. The proposed methods remedy this drawback as two distinct mechanisms of consensus reaching were constructed for HFLPRs. Our methods have broader applications in practical group decision analysis especially when someone is irresolute or uncertain about his/her evaluations over alternatives.

6 Concluding remarks

A consensus building of opinions should be carried out since divergent viewpoints may be spreading in GDM. The contributions of this paper can be highlighted as below: (i) an unified scheme of consensus building is formulated; (ii) the complete algorithms for both methods were given; (iii) a compensative technique in ICR was designed for exceptional cases. The two complementary approaches of ACR and ICR are not exclusive but complementary. In essence, the former is suited to a situation where the DMs cannot afford to stressful and time-consuming negotiation process, while the latter is more applicable when DMs can fully exchange and collaborate with each other. A case study of green supplier selection in FMIG was presented to show the effectiveness of the proposed methods. In addition, we conducted a comparative analysis with some existing consensus methods. However, a limitation of this paper is that we focused on the issue of group consensus only, while another step of consistency analysis with HFLPRs did not address yet. It is also an interesting topic to adopt some other fuzzy sets, such as dual HFSs [36], probabilistic dual HFSs [37] in group consensus building, or extend the proposed method to solve the problem of brain haemorrhage in practice [38].

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9 Appendix: The proof of Theorem 1

Proof: From (7), we get

$$\mathrm{GCI}(\tilde{H}_k^{(l+1)}) = \Gamma(\tilde{H}_k^{(l+1)}, \tilde{G}_k^{(l+1)}) = \frac{2}{n(n-1)} \cdot \sum_{i<j}^{n} D(\tilde{h}_{ij,k}^{(l+1)}, \tilde{g}_{ij,k}^{(l+1)})$$

That is

$$\mathrm{GCI}(\tilde{H}_k^{(l+1)}) = \frac{2}{n(n-1)|h|} \cdot \sum_{i<j}^{n} \sum_{s=1}^{|h|} \left| I(\tilde{h}_{ij,k}^{(l+1)})^{\sigma(s)} - I(\bigoplus_{q=1}^{m} \lambda_q \tilde{h}_{ij,q}^{(l+1)})^{\sigma(s)} \right| \tag{16}$$

Due to the fact that $\sum_{q=1}^{m} \lambda_q = 1$, (16) can be equivalently rewritten as

$$\mathrm{GCI}(\tilde{H}_k^{(l+1)}) = \frac{2}{n(n-1)|h|} \cdot \sum_{i<j}^{n} \left(\sum_{s=1}^{|h|} \left| \sum_{q=1}^{m} \lambda_q \left(I(\tilde{h}_{ij,k}^{(l+1)})^{\sigma(s)} - I(\tilde{h}_{ij,q}^{(l+1)})^{\sigma(s)} \right) \right| \right)$$

According to (10), it follows that

$$\mathrm{GCI}(\tilde{H}_k^{(l+1)}) = \frac{2\delta}{n(n-1)|h|} \cdot \sum_{i<j}^{n} \left(\sum_{s=1}^{|h|} \left| \sum_{q=1}^{m} \lambda_q \left(I(\tilde{h}_{ij,k}^{(l)})^{\sigma(s)} - I(\tilde{h}_{ij,q}^{(l)})^{\sigma(s)} \right) \right| \right)$$

By further normalisation, we have

$$\begin{aligned} \mathrm{GCI}(\tilde{H}_k^{(l+1)}) &= \frac{2\delta}{n(n-1)|h|} \cdot \sum_{i<j}^{n} \left(\sum_{s=1}^{|h|} \left| I(\tilde{h}_{ij,k}^{(l)})^{\sigma(s)} - \sum_{q=1}^{m} \lambda_q \left(I(\tilde{h}_{ij,q}^{(l)})^{\sigma(s)} \right) \right| \right) \\ &= \frac{2\delta}{n(n-1)} \cdot \sum_{i<j}^{n} \left(D(\tilde{h}_{ij,k}^{(l)}, \tilde{g}_{ij,k}^{(l)}) \right) = \delta \Gamma(\tilde{H}_k^{(l)}, \tilde{G}_k^{(l)}) = \delta \mathrm{GCI}(\tilde{H}_k^{(l)}) \end{aligned} \tag{17}$$

Since $0 \le \delta \le 1$, we have $\mathrm{GCI}(\tilde{H}_k^{(l+1)}) \le \mathrm{GCI}(\tilde{H}_k^{(l)})$. Furthermore, we conclude $\mathrm{GCI}(\tilde{H}_k^{(l)}) = \delta^2 \mathrm{GCI}(\tilde{H}_k^{(l-1)}) = \cdots = \delta^{2l} \mathrm{GCI}(\tilde{H}_k^{(0)})$. So, $\lim_{l\to\infty} \left\{ \mathrm{GCI}(\tilde{H}_k^{(l)}) \right\} = \lim_{l\to\infty} \left\{ \delta^{2l} \mathrm{GCI}(\tilde{H}_k^{(0)}) \right\} = 0$, which completes the proof. □