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이학석사 학위논문

Value at Risk and Other Risk measure

(Value at Risk와 Risk measure에 대한 고찰)

2012년 6월

서울대학교 대학원

수리과학부

백 승 택

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Value at Risk and Other Risk measure

by

Seungtaik Paik

A DISSERTATION

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Abstract

This paper is survey on Risk measure. In this paper we study Value at Risk and Conditional Value at Risk.

Key words : Value at Risk, Risk measure, Conditional Value at Risk
Student number : 2010-23067

Contents

1 Introduction

This paper is a summary of the investigation of Value at Risk.
As financial markets evolve and change, preparing for the risk measure is one of the important issues. The bank may not expect how much should prepare for the loss of amount. Because the bank does not expect what will happen when owe it requires the ability to handle this, the need to measure comes in.
The most commonly used risk measurement is a Value at Risk(VaR). The concept of VaR has now been incorporated in the Basel II Capital Accord.
Here is basic concept of VaR.
VaR answers to the question :
"How much can one lose with $\alpha\%$ probability over a pre-set horizon"
That is, VaR measures the amount of risk in currency. So, investors can calculate VaR and get the answer "How bad things can we get?"
It is very useful and convenient for investor.
Thus the purpose of this paper is to provide the measurement and applications of Value at Risk. Then we study some methods for the computation of VaR and limitations of VaR will be discussed. Finally, we'll talk about properties of risk measure and introduce Conditional Value at Risk(CVaR) which solves the problems of VaR.

2 What is the VaR?

2.1 Concept of VaR

2.1.1 Definition of VaR

VaR summarizes the expected maximum loss(or worst loss) over a target horizon within a given confidence interval or a lower α-percentile of random variable X.

Definition 2.1 *Let X be a random variable with the c.d.f $F_X(x) = P[X \geq x]$, the Value at Risk of X with a confidence level $c \in (0,1)$ is*

$$VaR(X) = -inf\{x|F_X(x) \geq c\}$$

2.2 Computing VaR

2.2.1 Mathematical Representation of VaR

To compute VaR, it is much better to know the mathematical expression of VaR. So, this chapter show how to represent VaR mathematically. Let's show previous definition by formula.
Define W_0 as the initial investment and R as W_0's rate of return.
The portfolio value at the end of the target horizon is $W = W_0(1+R)$.
μ, σ : the expected return and volatility of R.
Define now the lowest portfolio value at the given confidence level c as $W^* = W_0(1+R^*)$.

$$VaR = E(W) - W^* = E[W_0(1+R)] - W_0(1+R^*)$$

$$= W_0(1+\mu) - W_0(1+R^*) = -W_0(R^* - \mu)$$

If $\mu = 0$ then $VaR = -W_0R^*$.

Moreover, if R^* following normal distribution then we can associate $R^* < 0$ with standart normal deviate α

$$-\alpha = \frac{-|R^*| - \mu}{\sigma}$$

For example, confidence level on the vertical axis, say 5%. This corresponds to $\alpha = 1.65$. From the previous equation, the cutoff return is

$$R^* = -\alpha\sigma + \mu$$

For more generality, assume now that the parameters μ and σ are expressed on annual basis. The time interval considered is Δt, in years.

Then we find the VaR represnetation.

$$VaR = E(W) - W^* = -W_0(R^* - \mu)$$

$$= -W_0(-\alpha\sigma\sqrt{\Delta t} + \mu - \mu) = W_0\alpha\sigma\sqrt{\Delta t}$$

2.2.2 Steps in Constructing VaR

We need to measure the VaR of 100million won equity portfolio over 10days ate the 99% confidence level.

The following steps are required to compute VaR(Philippe Jorion)

○ Mark-to-market of the current portfolio.(100mil won)

○ Measure the variability of the risk factors.(15% per annum)

○ Set the time horizon, or the holding period.(10 business day)

○ Set the confidence level.(99%, 2.33 factor assuming a normal distribution)

○ Report the worst loss by processing all the preceding information.(a 7mil won)

$$\textbf{Sample computation : 100M won} \times \textbf{15\%} \times \sqrt{\tfrac{10}{252}} \times \textbf{2.33} = \textbf{7M won}$$

3 Portfolio VaR

3.1 Representaion of Portfolio VaR

In the previous section, we have focoused on single finacial instruments. But VaR is more than useful measurment the risk of the portfolio. Portfoilio VaR is smaller than sum of individual VaR because of the diversification.

Define the portfolio rate of return as

$$R_P = w_1 R_1 + w_2 R_2 + \cdots + w_N R_N = [w_1 w_2 \cdots w_n] \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_3 \end{bmatrix} = w^T R$$

The portfolio expected return is

$$E(R_P) = \mu_P = w_1 E(R_1) + w_2 E(R_2) + \cdots + w_N E(R_N) = \sum_{i=1}^{N} w_i \mu_i$$

The variance is

$$\begin{aligned} V(R_p) =& \sigma_p^2 = V(w_1 R_1 + w_2 R_2 + \cdots + w_N R_N) \\ =& w_1^2 V(R_1) + w_2^2 V(R_2) + \cdots + w_N^2 V(R_N) \\ &+ w_1 w_2 Cov(R_1, R_2) + \cdots + w_{N-1} w_N Cov(R_{N-1}, R_N) \\ =& \sum_{i=1}^{N} w_i \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} w_i w_j \sigma_{ij} \end{aligned}$$

The sum component of variance grows as the number of assets increases, thus using matrix notation is more convenient.

$$\sigma_P^2 = [w_1 w_2 \cdots w_n] \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1N} \\ \vdots & \vdots & \vdots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_{NN} \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}$$

Defining $\sum$ as the covariance matrix, the variance of the portfolio rate of return can be written more compactly as

$$\sigma_p^2 = w^T \sum w$$

This also can be expressed in terms of dollar amounts as

$$\sigma_p^2 W^2 = x^T \sum x$$

Defining W as the initial portfolio value, the portfolio VaR is

$$\text{Portfolio VaR} = VaR_p = \alpha \sigma_p W = \alpha \sqrt{x^T \sum x}$$

Until now, the portfolio VaR calculation methods have been investigated.

3.2 Diversification of Portfolio VaR

From now, We observe what happens to diversification of portfolio depending on the correlation between individual elements of portfolio through a simple case.

If there is two assets whose correlation is 0 ($\rho = 0$)

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2\rho_{12}w_1w_2\sigma_1\sigma_2$$

$$VaR_p = \alpha\sigma_p W = \alpha\sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2\rho_{12}w_1w_2\sigma_1\sigma_2}W$$

$$= \sqrt{\alpha^2 w_1^2\sigma_1^2 + \alpha^2 w_2^2\sigma_2^2}W = \sqrt{VaR_1^2 + VaR_2^2}$$

The portfolio risk must be lower than the sum of individual VaRs

$$VaR_p < VaR_1 + VaR_2$$

This means that asset moves independently so that a portfolio will be less risky than either asset.

Now observe the case when two asset's correlation is 1 ($\rho = 1$)

$$VaR_p = \alpha\sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2\rho_{12}w_1w_2\sigma_1\sigma_2}W$$

$$= \sqrt{\alpha^2w_1^2\sigma_1^2 + \alpha^2w_2^2\sigma_2^2 + 2\alpha^2\rho_{12}w_1w_2\sigma_1\sigma_2}W$$

$$= \sqrt{VaR_1^2 + VaR_2^2 + 2VaR_1VaR_2} = VaR_1 + VaR_2$$

In this case, the portfolio VaR is equal to the sum of individual VaR measures if the two assets are perfectly correlated.

3.3 Simplfying Covariance Matrix

We have shown that correlations are essential driving force behind portfolio risk. When the number of assets is large, the measurement of the covariance matrix becomes increasingly difficult. So in this section we study how to simplfying the covariance matrix.

3.3.1 Diagonal Model

As the number of assets increses, the measurement of correlations is likely to have estimation error. So we use single-factor model to simplify the structure for the covariance matrix.
The assumption is that common movement in all assets is due to one common factor only. That is market factor. Formally, the model is

$$R_i = \alpha_i + \beta_i R_m + \varepsilon_i$$

$$E[\varepsilon_i] = 0,\ E[\varepsilon_i R_m] = 0,\ E[\varepsilon_i\varepsilon_j] = 0,\ E[\varepsilon_i] = \sigma_{\varepsilon,i}^2$$

α_i is expected return on security, R_m is Return on market factor, ε_i is called idiosyncratic risk that occur as a result of the company's own risk, and β_i is sensitivity of the security. Here is the method of computing beta

$$\beta_i = \frac{cov(R_i, R_p)}{\sigma_p^2} = \frac{\sigma_{ip}}{\sigma_p^2} = \frac{\rho_{ip}\sigma_i\sigma_p}{\sigma_p^2} = \rho_{ip}\frac{\sigma_i}{\sigma_p}$$

As a result, we get the expectation, variance and covariance

$$
\begin{aligned}
E[R_i] =& E[R_i = \alpha_i + \beta_i R_m + \varepsilon_i] = E[\alpha_i] + E[\beta_i R_m] + E[\varepsilon_i] \\
=& \alpha_i + \beta_i E[R_m] \\
V(R_i) =& \sigma_i^2 = E[R_i^2] - \{E[R_i]^2\} \\
=& E[\alpha_i^2 + \beta_i^2 R_m^2 + \varepsilon_i^2 + 2(\alpha_i \beta_i R_m + \alpha_i \varepsilon_i + \beta_i R_m \varepsilon_i)] - (\alpha_i + \beta_i E[R_m])^2 \\
=& \alpha_i^2 + \beta_i^2 E[R_m^2] + \sigma_{\varepsilon_i,i}^2 + 2(\alpha_i \beta_i E[R_m] + \alpha_i E[\varepsilon_i] + \beta_i E[R_m \varepsilon_i]) \\
& - (\alpha_i^2 + \beta_i^2 E[R_m]^2 + 2\alpha_i \beta_i E[R_m]^2) \\
=& \beta_i^2 \sigma_m^2 + \sigma_{\varepsilon,i}^2
\end{aligned}
$$

$$
\begin{aligned}
Cov(R_i, R_j) =& E[R_i R_j] - E[R_i] - E[R_j] \\
=& E[(\alpha_i + \beta_i R_m + \varepsilon_i)(\alpha_j + \beta_j R_m + \varepsilon_j)] \\
& - (\alpha_i + \beta_i E[R_m])(\alpha_j + \beta_j E[R_m]) \\
=& \beta_i \beta_j (E[R_m^2] - E[R_m]^2) \\
=& \beta_i \beta_j \sigma_m^2
\end{aligned}
$$

Thus the full matrix is

$$
\sum = \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_N \end{bmatrix} [\beta_1 \cdots \beta_2] \sigma_m^2 + \begin{bmatrix} \sigma_{\varepsilon,1}^2 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & \sigma_{\varepsilon,N}^2 \end{bmatrix}
$$

Written in vector notation, the covariance matrix is

$$
\sum = \beta \beta^T \sigma_m^2 + D_\varepsilon
$$

and the number of parameters is reduced from $N(N+1)/2$ to $2N+1$(β_i : N, σ_m : 1, $\sigma_{\varepsilon,i}$: N)

Futhermore, the variance of large, well-diversified portfolios can be simplifies even futher, reflecting only exposure to common factor. The variance of portfolio is

$$V(R_p) = V(w^T R) = w^T \sum w = (w^T \beta\beta^T w)\sigma_m^2 + w^T D_\varepsilon w$$

$w^T D_\varepsilon w = \sum_{i=1}^N w_i^2 \sigma_{\varepsilon,i}^2$ becomes very small as the number of assets in the portfolio increases.

The diversification of idosyncartic risk is

$$V(\sum_{i=1}^{N} w_i \varepsilon_i) = \sum w_i^2 V(\varepsilon_i)$$

If $w_i = \frac{1}{n}$ then

$$\begin{aligned} V(\varepsilon_p) =& \sum \frac{1}{n^2} V(\varepsilon_i) = \frac{1}{n}\left[\frac{\sum_{i=1}^n V(\varepsilon)}{n}\right] \\ =& \frac{1}{n}(\text{average of Idiosyncratic variance}) \end{aligned}$$

It converges to 0 as N increases.
So the variance of it converges to

$$V(R_p) = (w^T \beta\beta^T w)\sigma_m^2 = (\beta_p \sigma_m)^2$$

Thus, if the number of asset of the portfolio is large then specific risk becomes unimportant for the purpose of measuring VaR.

3.3.2 Multi Factor Model

If single-factor model couldn't give sufficient solution, Multi-factor model is a better substitute.

$$R_i = \alpha_i + \beta_{i1} R_{f1} + \cdots + \beta_{ik} R_{fk} + \varepsilon_i$$

In the same way as above, the full matrix structure $\sum$ is

$$\sum = \begin{bmatrix} \beta_{11} \\ \vdots \\ \beta_{1N} \end{bmatrix} [\beta_{11} \cdots \beta_{N1}]\sigma_1^2 + \cdots + \begin{bmatrix} \beta_{k1} \\ \vdots \\ \beta_{kN} \end{bmatrix} [\beta_{k1} \cdots \beta_{Nk}]\sigma_1^2 + \begin{bmatrix} \sigma_{\varepsilon,1}^2 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & \sigma_{\varepsilon,N}^2 \end{bmatrix}$$

Written in vector notation, the covariance matrix is

$$\sum = \beta_1 \beta_1^T \sigma_1^2 + \cdots + \beta_k \beta_k^T \sigma_k^2 + D_\varepsilon$$

The number of parameter is $(N \times K + K + N)$

4 VaR approach

The method to calculate the VaR has Variance-Covariance, Historical Simulation, Monte Carlo Simulation. Estimated VaR by means of these three method, under normal market conditions during the holding period can occur in a given confidence level means the maximum amount of loss. The estimated VaR assuming normal market conditions, so a more accurate and precise for risk management in the event of an extreme case to estimate the amount of the loss analysis is needed.

4.1 Variance-Covariance

The first method to calculate VaR is Analytic-variance-covariance method.
This method uses historical data to estimate variance and covariance and using these values is a way to calculate VaR. In particular, variance-covariance method is called delta-normal method when using Morgan's RiskMetrics. This approach assumes all of the assets are following the normal distribution.
This method assumes that all asset returns are normally distributed. The portfolio return is a linear combination of normal variables, it is also normally distributed.
Portfolio risk is then generated by a linear combination of many risk factors that are assumed to be normally distributed, and by the forecast of the covariance matrix.
This methodology is well-konwn techniques and it can be done in a simple Excel spreadsheet. However, the assumption that the asset returns are normally distributed is rarely true.

4.2 Historical Simulation

Historical simulation does not assume a specific probability distribution of the historical changes in market variables on the basis of full valuation VaR is calculated by simulating the way. This method consists of past data and applying current weights to a time-series of historical asset returns.
In most cases, distribution of historical data has fat tail so the expected results are worse than when we using normal distribution. Using the historical simulation, we can accomodate the actual price , non-linear and non-normal distirbution. This methodology is very simple to implement. It does not assume a norml distribution of asset returns.But the method requries a large database containing historical date, the quality of data management must be high.

4.3 Monte Carlo Simulation

Monte Carlo simulation to calculate VaR in the most efficient manner is a way. This nonlinearity, changes in volatility, fat tails, and all can be considered in extreme circumstances. The disadvantage of this method is that the computation is expensive. Monte

Carlo simulation consists of the following two steps.
First, financial Risk Manager determine the stochastic process and process parameter. Here, such as risk and correlation coefficients calculated from historical data or the optional data. The most commonly used model to represent the probability distribution of the option pricing model is the basis of the geometric Brownian motion.
Second, hypothetical price changes are simulated for all variables. During a given target period of the portfolio market value is calculated by the full valuation model. VaR is calculated directly by obtained from this hypothetical distribution of the price and obtained from this distribution . Number of simulations, the more empirical approach to the distribution of a continuous distribution and access to the actual distribution will be.

5 Problem of VaR

VaR is commonly used risk measurement in finacial risk management. However, VaR has some problems.

Here is the reason why VaR is not an acceptable risk measure

- ∘ It does not measure losses exceeding VaR
- ∘ A reduction of VaR may lead to stretch the tail exceeding VaR
- ∘ Non-subadditivity implies that portfolio diversification may lead to an increasse of risk and prevents to add up the VaR of different risk sources
- ∘ Nonconvexity make it impossible to use VaR in optimization problems

The Academic problem of VaR is that is not subadditive.
That is. it's possible to construct two portfolio A and B then

$$VaR(A+B) > VaR(A) + VaR(B)$$

Counter Example of VaR is not subadditive(Avellaneda)

Let B(i), i=1, $\cdots$, 100, be i.i.d defaultable bonds.
B(i) defaults with probability 1% at the end of the period, and has a period end coupon of 2%, when is does not default.
Suppose each bond has face value \$100 then

$$B(i) = \begin{cases} -100 & \text{with prob } 0.01 \\ 2 & \text{with prob } 0.99 \end{cases}$$

Consider the following two portfolios

$$\begin{aligned} &P_1 : B(1) + \cdots + B(100)(Diversified) \\ &P_2 : 100 \times B(1)(Non-diversified) \end{aligned}$$

It is easy to check at 95%

$$VaR(P_1) > VaR(P_2) = 100 \times VaR(B(1))$$

Hence VaR is non-subadditive.

6 Risk mesure

We have seen the problem of VaR as a Risk measure. This section we study some features which risk measure should have.
These conditions typically think investors should have a risk measure that would set forth conditions.

- If you multiply you portfolio, you should multiply you risk capital
- If you allocate to 2 subunits, you have enough for the whole dest
- If portfolio A is bigger than B, risk capital of A is more than B
- If you add cash, you should take that off your risk capital

By considering preceding conditions, let express the characteristics mathematically.
Let X and Y denote the future loss of two portfolios, then a risk measure ρ is coherent if it adheres to the four axioms

$A1 : \rho(\lambda x) = \lambda\rho(x)$ for $\lambda > 0$(positive homogeneity)

$A2 : \rho(x + y) \leq \rho(x) + \rho(y)$(subadditivity)

$A3 : \rho(x) \leq \rho(y)$ when $x \leq y$(monotonicity)

$A4 : \rho(x + \alpha r_0) = \rho(x) - \alpha$ where r_0 : riskless rates(Transitional invariance)

Sometimes conditions A1 and A2 have repalced other condition that ρ be convex.

$$\rho((1-\lambda)X + \lambda y) \leq (1-\lambda)\rho(X) + \lambda\rho(y) \text{ for } \lambda \in (0, 1]) \text{ convexity}$$

Convexity is impotant condition for the risk measure. Because convexity property that allow to slove the optimizaion problem.

7 Conditional Value at Risk

In the previous section, we dicuss the problems of VaR and properties of risk measure. In this section, we talk about Coditional Value at Risk(CVaR) which satisfies the aforementioned properties.

An alternative percentile measure of risk is Conditional Value at Risk(CVaR). For random variables with continuous distrubition funcions, CVaR(X) equals the conditional expection of X subject to $X \geq VaR(X)$. This definition is the basis for the name of Conditional Value at Risk. The general definition of CVaR for random variables with a possibly discontinuous distribution function is as follows.

Definition 7.1 *The CVaR of X with confidence level $\alpha \in$ [0,1] is the mean of the generalized α-tail distribution*

$$CVaR(X) = \int_{-\infty}^{\infty} z dF_X^{\alpha}(z),$$

where

$$F_X^{\alpha}(X) = \begin{cases} 0, & \text{when } z < VaR(X) \\ \dfrac{F_X(z) - \alpha}{1 - \alpha}, & \text{when } z \geq VaR_{\alpha}(X) \end{cases}$$

Contrary to popular belief, in the general case, CVaR(X) is not equal to an average of outcomes greater than VaR(X). To explain this idea in more detail, we need alternative definitions of CVaR.

Let $CVaR^{+}(X)$, called upper CVaR and $CVaR^{-}(X)$, called lower CVaR.

$$CVaR^{+}(X) = E[X|X > VaR(X)]$$

expected losses strictly exceeding VaR or it is called Mean Excess loss and Expected Shortfall.

$$CVaR^{-}(X) = E[X|X \geq VaR(X)]$$

expected losses which are equal to or exceed VaR or it is called Tail VaR.

$CVaR^{-}(X)$ coincides with $CVaR(X)$ for continuous distributions, however, for general distributions it is discontinuous and is not convex.

Now we define CVaR as the weighted average of $VaR(X)$ and $CVaR^{+}(X)$.
If $F_X(VaR(X)) < 1$, then there is a chance of a loss greater than $VaR(X)$, then

$$CVaR(X) = \lambda(X)VaR(X) + (1 - \lambda(X))CVaR^{+}(X)$$

where

$$\lambda(X) = \frac{F_X(VaR(X)) - \alpha}{1 - \alpha}$$

whereas if $F_X(VaR(X)) = 1$, that is $VaR(X)$ is the highest loss that can occur, then

$$CVaR(X) = VaR(X)$$

We define $CVaR(X)$ as the average of $VaR(X)$ and $CVaR^{+}(X)$. It is very meaningful structure. Neither $VaR(X)$ nor $CVaR^{+}(X)$ are convex, but CVaR is convex. Thus we can do CVaR optimization.

Features of CVaR

- CVaR is convex, but VaR CVaR^{-}CVaR^{+}may be non-convex
- Inequalities are valid : $VaR \leq CVaR^{-} \leq CVaR \leq CVaR^{+}$
- CVaR is continuous with repect to confidence level α consistent at different confidence levels compared to VaR
- CVaR accounts for risks beyond VaR

8 Conclusion

As financial crisis continues, predicting the risk is important to us. So investors want to know how to mesure risk. They have questions how much money they need, how much the stock market will decline, etc. Then due to the need we mentioned earlier, the risk measure is finally created.
This paper has surveyed the basic risk measures in Financial Mathematics. We have studied what is the VaR and how to calculate it. We also discussed problems of VaR and its variants(Conditional Value at Risk).

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국문초록

이 논문은 Risk measure에 대한 조사이다. 이 논문에서 Value at Risk 와 Conditional Value at Risk에 대하여 이야기해본다.

주요 어휘 : 위험의 가치, 위첨 측도, 조건부 위험의 가치
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항성 저를 아끼고 사랑으로 보살펴주신 부모님께 감사드립니다. 그리고 항상 챙겨주고 도와준 누나와 매형 조카 호준이, 예원이에게도 고마움을 전합니다. 가까이 계신 큰아버지, 큰어머니, 승희누나, 승현누나, 승호와 작은할아버지 철현아저씨와 숙모님 승천, 승엽에게도 감사의 마음을 전합니다. 항상 응원을 해주시는 할아버지, 할머니, 큰이모부, 작은이모부, 큰이모, 작은이모와 교진, 민엽, 준엽, 교일에게도 고마움을 전합니다.

대학원에 함께 들어와서 서로 격려와 충고를 하며 의지를 하게 된 동기 최동헌, 배한울, 정남규, 이승훈, 정지인, 이아란, 홍영순, 최태림, 정희원 모두에게 감사의 말을 전합니다. 또한 함께 유년기를 보내며 지금까지 의지가 되는 친구 황주호, 황시내, 김동철, 김명우, 이동윤, 이재우와 대학시절을 함께했던 하태영, 서유태, 안정욱, 신진호, 김학래, 백창인, 김수영, 전채석, 김경조 그리고 학창시절을 함께한 정홍래, 이윤기, 홍성우, 임선영, 김명성, 전용재, 성영준, 이대명, 박인범, 서명원, 배영한, 이대성, 김동은, 윤종식, 음준상, 정지영, 송창섭에게 고마움을 전합니다. 자주 만나지는 못하지만 상담을 자주 해주는 김수하, 양유나, 김진희에게도 감사함을 표현합니다. 그리고 옆에서 힘이 되고 응원해준 희연이에게 고마운 마음을 전합니다.

마지막으로 제가 표혆하지 못한 다른 모든 분들께도 감사드리며 앞으로 더욱 믿음직한 모습을 보여드릴 수 있는 제가 되겠습니다. 감사합니다.