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Constrained College Admission Problem: Manipulability of Truncated College-optimal Stable Mechanism

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Constrained College Admission Problem: Manipulability of Truncated College-optimal Stable Mechanism

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Abstract

A central issue in college admission problems is their vulnerability to manipulation by students. Students can often falsely report their preferences and get into more preferred colleges. This paper first shows that the college admission mechanisms in South Korea and the US, where students are limited in the number of schools that they can apply to, are equivalent to truncated college-optimal stable mechanism (TCOSM). Then, by adopting the framework proposed by Pathak and Sönmez (2013), it proves that the type space which is vulnerable under TCOSM is equivalent to that incurs different matching under TCOSM and student-optimal stable mechanism (SOSM). The result implies that TCOSM becomes less manipulable as the truncation quota (the limited number of each student's applications) increases, considering the type space which is vulnerable under TCOSM with the lower truncation quota. The analysis on manipulability of TCOSM supports policy reforms that increase the truncation quota to enhance students' satisfaction in college admission.

Keywords: college admission, matching, manipulability, truncated mechanism, college-optimal stable mechanism

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Introduction

College admission and school choice problems are widely studied subjects in the field of market design.[1] There has been significant theoretical development in economics that helped mitigate real world issues, such as instability and manipulability. Most notably, Gale and Shapley (1962) proposed a Deferred Acceptance (DA, henceforth) mechanism. There are two types of the mechanism: student-proposing and college-proposing DA mechanisms. Student (college)-proposing DA mechanism satisfies stability and student (college)-optimality.[2] Hence, student-proposing and college-proposing DA mechanisms are called student-optimal and college-optimal stable mechanisms (SOSM and COSM, henceforth), respectively. Unlike COSM, SOSM offers one more nice property: strategy-proofness on students' side. Due to the three desirable properties (stability, student-optimality, and strategy-proofness) of SOSM, many school districts in the US have adopted modified versions of SOSM. For example, New York City in 2003 changed its student assignment system to truncated SOSM, where students can apply up to five schools (Abdulkadiroglu, Pathak and Roth 2005).

Since most college admission mechanisms are not based on SOSM, vulnerability to manipulation by students still remains as a significant issue that needs to be resolved. Students can often achieve better outcomes in the college admission "game" by falsely reporting their preferences. In other words, the mechanisms are not strategy-proof.[3] For instance, in the early admission process of South Korea, students cannot apply to more than six colleges and most students do not apply to top six colleges in their preferences. Students and parents put enormous time and effort in choosing six colleges and there even exist consulting firms specializing in assist-

[1]The two problems differ only in that, unlike schools, colleges are strategic agents which have their own preferences over students.

[2]For the definitions of stability and optimality, see Appendix A.

[3]Manipulability and strategy-proofness are closely related notions. However, for consistency, manipulability is mostly used in this paper.

ing students in selecting colleges. More detrimentally, many students fail to get into colleges and reapply the next year just because of their bad play in the game. For these reasons, the gaming by students in college admission has been one of the core concerns of education policymakers in South Korea.

The college admission mechanism in the US is similar to that of South Korea, since students cannot apply to all colleges they want to because of the lack of their time and financial resources. For example, they have to pay application fees and write different essays for respective colleges. Therefore, the truncation quota (the limited number of each student's applications) can be understood as a vector in the US, whereas that of Korea is a scalar.

This paper aims to analyze the manipulability of college admission mechanisms based on their truncation quota. There are two papers which provide frameworks to compare manipulability of truncated mechanisms. Pathak and Sönmez (2013) provides the notion of "more manipulable." Rough definition of the notion is as follows: a mechanism A is more as manipulable as a mechanism B if the type space (the set of preference profiles) under which some player(s) can falsely report their preferences to achieve better outcome is strictly larger in A than in B. Haeringer and Klijn (2009), while exploring necessary and sufficient conditions for stability or efficiency of truncated versions of Immediate Acceptance (IA), DA, and Top Trading Circle mechanisms, proves the nested structure of Nash equilibria for the mechanisms. Since Nash equilibrium do not necessarily imply truth-telling by students, it is inadequate to analyze the manipulability of truncated mechanisms in the Nash equilibria perspective. Hence, the framework of Pathak and Sönmez (2013) is used for this paper's analysis.

Another related paper that uses the concept of "more manipulable" is Chen and Kesten (2017). They showed that China's transition from an Immediate Acceptance (IA) mechanism to the parallel mechanism, a hybrid of IA and DA mechanisms, is a shift to a less manipulable mechanism. In the companion paper, Chen and Kesten (2016) experimentally assisted their theoretical results.

This paper first shows that college admission mechanisms in South Korea and the US, where students are limited in the number of schools that they can apply to, are equivalent to truncated college-optimal stable mechanism (TCOSM, henceforth). Then, it proves that the type space which is vulnerable under TCOSM is equivalent to that incurs different matchings under TCOSM and SOSM. The result implies that TCOSM becomes less manipulable as the truncation quota increases, considering the type space which is vulnerable under TCOSM with the lower truncation quota.

Considering the strand of prior literature, the contribution of this paper is two-folds. First, it brings COSM, a less studied mechanism compared to SOSM, to academic attention. The equivalence of the South Korean and American college admission mechanisms and TCOSM implies that theoretical or empirical analysis on TCOSM could improve the college admission outcomes in the real world. Second, it is the first paper to analyze the manipulability of TCOSM. The manipulability of COSM has been proven by Gale and Sotomayor (1985). However, after it, there has been no further research on manipulability of constrained versions of COSM.

The remainder of this paper is organized as follows. In Section 1, a college admission problem is formally defined. In Section 2, college admission mechanisms in South Korea and the US are introduced. Also, by abstracting from these two mechanisms, a Real-world Mechanism is defined. In Section 3, TCOSM is formally described and its equivalence to the Real-world Mechanism is proved. In Section 4, the notion of manipulability is provided and the main theorem proves the equivalence of TCOSM's manipulability and its inequality to SOSM. As the corollary of the theorem, the paper analyzes manipulability of TCOSM based on its truncation quota. Lastly, in Section 5, concluding remarks and further research opportunities are provided. Basic concepts of matching theory necessary for this paper are defined in Appendix A, and all the proofs are relegated to Appendix B.

1 College Admission Problem

A college admission problem consists of a set of students and colleges and their preferences along with the quota (capacity) of each college. An individual (a student or a college) is assumed to have a strict preference over its counterpart and himself. Furthermore, each college has a fixed quota and it can only admit students up to the quota. The outcome of a college admission problem is called a matching, and a mechanism is a systematic way of providing a matching given a college admission problem. Formal definitions are as follows.

Definition 1.1. A *College admission problem* is a 5-tuple (S, C, q, P_S, P_C).

1. S: a set of students, $S = \{s_1, ..., s_m\}$.
2. C: a set of colleges, $C = \{c_1, ..., c_n\}$.
3. q: quota (capacity) vector, $q = \{q_1, ..., q_n\}$, q_j denotes the total number of available seats at c_j.
4. P_S: a strict preference profile of students, $P_S = \{P_{s_1}, ..., P_{s_m}\}$, P_{s_i} is a linear ordering of $C \cup \{s_i\}$.
5. P_C: a strict preference profile of colleges, $P_C = \{P_{c_1}, ..., P_{c_n}\}$, P_{c_j} is a linear ordering of $S \cup \{c_j\}$.[4]

$c_p P_{s_i} c_q$ means that s_i strictly prefers c_p to c_q. $c_r P_{s_i} s_i$ means that s_i strictly prefers being admitted to c_r to remaining unassigned (not going to any school). In this case, c_r is called acceptable. Often unacceptable colleges and student himself are omitted when stating a student's preference. For example, $P_{s_1} : c_1, c_2$ denotes s_1's preference, where his top and second choices are c_1 and c_2, respectively, and all the other colleges are unacceptable. Symmetric statement holds for P_{c_j}. Furthermore, weak preference

[4] It is implicitly assumed that colleges' preferences are responsive since they are not defined over 2^S. Roughly speaking, responsive preference means that colleges' preferences over 2^S is consistent with their preferences over S.

relationship R can be derived from P: $c_p R_{s_i} c_q$ if and only if $c_p = c_q$ or $c_p P_{s_i} c_q$. If no confusion arises, a college admission problem is often simply written by omitting some of the entries in the 5-tuple. For instance, if $S, C, q,$ *and* P_C are considered fixed, a college admission problem is written as P_S.

Definition 1.2. A *Matching* is a function $\mu : S \cup C \rightarrow 2^S \cup C$ that satisfies

1. $|\mu(s_i)| = 1$ and $\mu(s_i) \in C \cup \{s_i\}, \forall s_i \in S$
2. $|\mu(c_j)| \leq q_j$ and $\mu(c_j) \in 2^S, \forall c_j \in C$
3. $\mu(s_i) = c_j$ iff $s_i \in \mu(c_j)$.

Definition 1.3. A *Mechanism* is a systematic way that provides a matching μ given a college admission problem (S, C, q, P_S, P_C).

2 College Admission Mechanisms in South Korea and the U.S.

The college admission system in South Korea consists of two parts: early (*sooshi*) and regular admission processes (*jeongshi*). In the early admission process, each student can apply up to six programs, i.e., the truncation quota is six.[5] Then, each college offers its first round of admission up to its quota by December 15th. If a student gets admitted to multiple programs at the first round of the early admission process, he or she should choose whichever college he or she prefers the most and rejects all the other before the fixed deadline (December 21st).

Also, colleges have waitlists for admission and after the first round of admission, available seats are offered to the next best candidates in the lists. There is no nation-wide fixed date dictating the number of the subsequent rounds and their timeline.

[5] In South Korea, students usually apply to a specific program of a college. In this paper, the terms "program" and "college" are interchangeably used.

After December 21st, each student cannot retain more than one seat, so if admitted off the waitlist, a student has to choose between the previously-admitted and the newly-admitted institutions. Most colleges strive to meet their quotas, unless they are only left with students below their standards. For example, Some colleges increase the number of admission rounds to fill their quotas. Some even call students and offer them admission, when there are few seats left for admission.

If a student gets admitted to some program(s) in the early admission process, he or she cannot apply to any college in the regular admission. Hence, in this case, s_i at P_{s_i} denotes remaining unassigned at the early admission process and going to the regular one.

In the regular admission process, most post-secondary education programs are divided into three categories, and each student can apply up to one program in each category. The rest of the process is almost the same with the early admission process, except that students' outside option is to give up their post-secondary education or to apply again in the following years. Unlike its name, regular admission takes up smaller part in college admission. According to Korea Council for University Education, colleges filled up 30.1% of their capacities through regular admission in 2017. In 2018 and 2019, the numbers are expected to decrease to 26.3% and 23.8%, respectively.[6]

The college admission system in the United States consists of early and regular admission processes. There are two types of early admission process: early decision and early action. Early decision is binding, which means that if admitted, students must enroll and withdraw applications from all the other institutions. Students can only apply up to one college. In contrast, Early action is a non-binding decision. Even though a student gets admitted to a college through early action, he or she can wait for early action and regular admission results from other colleges and then

[6] http://www.kcue.or.kr/bbs/view.php?gb=high&page=2&idx=1265&kind=&culm=&word=, retrieved on June 18th, 2017.

decide. Colleges have different regulations regarding the early admission processes. However, generally a student can apply through both early decision (one college) and early action (multiple colleges). In the regular admission process in the US, students can apply to any colleges they want. As in the case of South Korea, there is a fixed deadline (May 1st) by which students must choose only one offer and reject all the others. Early action shares this same deadline.

Early decision in the US differs with the early admission process of South Korea only in that the truncation quota is one. When students are limited in the number of their applications in early action and regular admission in the US due to the lack of financial (application fees) and time (preparing for each application) resources, the two processes combined are equivalent to the early admission process of South Korea, where a different truncation quota is imposed on each student.[7]

By abstracting from the admission mechanisms of the two countries, except the regular admission mechanism in South Korea, which is losing its importance, the Real-world Mechanism is defined.

Definition 2.1. The Real-world Mechanism, $\rho(t)$, where t denotes the number of programs that each student can apply to, consists of the following steps.[8]

Step 1. Each student applies up to t programs.

Step 2. If the number of qualified students applied to c is greater than q_c, c offers admission to top q_c students and put other students on the wait-list. Otherwise, c offers admission to all qualified students. This step is completed by the fixed deadline (deadline 1).

Step 3. Admitted students choose one college by another fixed deadline (deadline 2, which can be the same with the deadline 1).

[7]In reality, P_C's are different in early action and regular admission, since students have more time to prepare for their applications in regular admission. However, for simplicity of analysis, the change in preferences of colleges is ignored.

[8]As for the combined process of early action and regular admission in the US, t can be thought as a truncation vector imposed on students. The rest of the arguments in this paper robustly holds even when t is a vector instead of a scalar.

Step 4. After the deadline 2, c offers subsequent rounds of admission from the waitlist. (The timeline of subsequent rounds varies by colleges.)

Step 5. When admitted off the waitlist, a student who is previously-admitted by another college must choose between the two institutions.

Step (termination) 4-5 are repeated until all colleges either filled up their quotas or exhausted all the waitlisted students.[9]

3 Truncated College-optimal Stable Mechanism and Its Equivalence to the Real-world Mechanism

Since TCOSM is a variant of COSM, the formal definition of COSM is first provided.

Definition 3.1. College-optimal stable mechanism (COSM) consists of the following steps.

Step 0. Each individual reports his strict preference over his counterpart and himself.

Step 1. Each college c proposes to top q_c acceptable students. (If the number of acceptable students is less than q_c, it proposes to all students.) Each student who receives offer(s) from colleges tentatively accepts one most attractive offer and rejects all the other offers.

Step k ($k \geq 2$). Each college c proposes to top q_c students from its list of acceptable students who did not rejected its offer at the steps 1 to $k-1$ (including those who tentatively accepted its offer). Each student who receives offer(s) from colleges tentatively accepts one most attractive offer and reject all the other offers.

[9]Note that $\rho(t)$ ends in finite rounds, since there are a finite number of colleges and students.

Step (termination). When no rejection occurs, the algorithm terminates and the assignment becomes final.[10]

TCOSM is the variant of COSM where only top t colleges in each student's preference is considered. Here, t denotes the truncation quota.

Definition 3.2. Truncated college-optimal stable mechanism (TCOSM) with the truncation quota t, $\gamma(t)$, consists of the following steps.

Step 0. Each individual reports his strict preference over his counterpart and himself. Each student's preference is truncated so that only top t colleges remain on it. If the number of acceptable colleges is less than or equal to t for a student, his preference is not truncated.

Step 1 to Step (termination) same with those of COSM.

By Definition 3.2, COSM automatically becomes $\gamma(\infty)$, or equivalently $\gamma(|C|)$.

Proposition 3.1. $\rho(t)$ (the Real-world Mechanism where each student can apply up to t colleges) and $\gamma(t)$ (TCOSM with the truncation quota t) are equivalent in that they produce the same matching given the same reported preference profile.

Theoretical analysis on $\rho(t)$ incurs technical difficulties since colleges have different timelines for their decisions on waitlists. Proposition 3.1 simplifies analysis of $\rho(t)$ since one can analyze $\gamma(t)$ instead. With the help of Proposition 3.1, the analysis on the manipulability of $\gamma(t)$ also applies to $\rho(t)$.

[10]Since each college proposes only to those who have never rejected them and there are a finite number of colleges and students, COSM always terminates.

4 Manipulability of Truncated College-optimal Stable Mechanism

As aforementioned, manipulability of the college admission mechanism has been one of the core concerns of education policymakers in South Korea. Pathak and Sönmez (2013) provides a general framework that can be utilized to compare manipulability of different mechanisms. Provided below is the modified form of their framework in the college admission problem setting. Only student-side of manipulability is considered here.

Definition 4.1. A mechanism ϕ is manipulable by a student $s_i \in S$ at problem P_S if there exists $\widetilde{P_{s_i}}$ such that $\phi(\widetilde{P_{s_i}}, P_{s_{-i}}) P_{s_i} \phi(P_S)$.[11]

Definition 4.2. Problem P_S is vulnerable under a mechanism ϕ if ϕ is manipulable by some student $s_i \in S$ at P_S.

Definition 4.3. A mechanism ψ is at least as manipulable as a mechanism ϕ if any problem P_S that is vulnerable under ϕ is also vulnerable under ψ.

Definition 4.4. A mechanism ψ is more as manipulable as a mechanism ϕ if

1. ψ is at least as manipulable as ϕ and
2. there exists (S, C, q, P_S, P_C) such that P_S is vulnerable under ψ but not under ϕ.

Pathak and Sönmez (2013) has shown that truncated SOSM becomes less manipulable as the truncation quota increases. However, as the three examples below illustrate, it cannot be said that TCOSM with a smaller truncation quota is more as manipulable as that with a greater truncation quota.

Definition 4.5. $\mu_c^t(P_S)$, $\mu_c(P_S)$, and $\mu_{so}(P_S)$ denote a matching of P_S through $\gamma(t)$, COSM ($\gamma(\infty)$), and SOSM, respectively.

[11] s_{-i} denotes $S - \{s_i\}$, thus $P_{s_{-i}}$ is the preference profile of all students except s_i.

Example 4.1. P_s is vulnerable under $\gamma(1)$, but not under $\gamma(2)$.

$$\begin{array}{ll} P_{c_1}\colon s_1, s_2, s_3 \ (q_{c_1} = 1) & P_{s_1} : c_1, c_2 \\ P_{c_2}\colon s_1, s_2, s_3 \ (q_{c_2} = 1) & P_{s_2} : c_1, c_2 \\ & P_{s_3} : c_1, c_2 \end{array}$$

- $\mu_c^2(P_S) = \mu_c(P_S) = \{(c_1, s_1), (c_2, s_2)\} = \mu_{so}(P_S)$. Here, no student can manipulate the mechanism.

- $\mu_c^1(P_S) = \{(c_1, s_1)\} \neq \mu_{so}(P_S)$. Let $\widetilde{P_{s_2}} : c_2$. Then, $\mu_c^1(\widetilde{P_{s_2}}, P_{s_{-2}}) = \{(c_1, s_1), (c_2, s_2)\}$. Since $c_2 P_{s_2} s_2$, $\gamma(1)$ is manipulable by s_2 at P_S.

Example 4.2. P_s is vulnerable under $\gamma(3)$, but not under $\gamma(2)$.

$$\begin{array}{ll} P_{c_1}\colon s_1, s_4, s_3, s_2 \ (q_{c_1} = 1) & P_{s_1} : c_3, c_2, c_1 \\ P_{c_2}\colon s_4, s_2, s_1, s_3 \ (q_{c_2} = 1) & P_{s_2} : c_1, c_2, c_3 \\ P_{c_3}\colon s_3, s_1, s_2, s_4 \ (q_{c_3} = 1) & P_{s_3} : c_1, c_3, c_2 \\ & P_{s_4} : c_2, c_1, c_3 \end{array}$$

- $\mu_c^3(P_S) = \mu_c(P_S) = \{(c_1, s_1), (c_2, s_4), (c_3, s_3)\} \neq \mu_{so}(P_S)$. Let $\widetilde{P_{s_1}} : c_3$. Then, $\mu_c^3(\widetilde{P_{s_1}}, P_{s_{-1}}) = \{(c_1, s_3), (c_2, s_4), (c_3, s_1)\}$. Since $c_3 P_{s_1} c_1$, $\gamma(3)$ is manipulable by s_1 at P_S.

- $\mu_c^2(P_S) = \{(c_1, s_3), (c_2, s_4), (c_3, s_1)\} = \mu_{so}(P_S)$. Here, no student can manipulate the mechanism.

Example 4.3. P_s is vulnerable under both $\gamma(1)$ and $\gamma(2)$.

$$\begin{array}{ll} P_{c_1}\colon s_1, s_3, s_4, s_2 \ (q_{c_1} = 2) & P_{s_1} : c_2, c_1 \\ P_{c_2}\colon s_2, s_1, s_3, s_4, s_5 \ (q_{c_2} = 2) & P_{s_2} : c_1, c_2 \\ & P_{s_3} : c_2, c_1 \\ & P_{s_4} : c_2, c_1 \\ & P_{s_5} : c_1, c_2 \end{array}$$

- $\mu_c^2(P_S) = \mu_c(P_S) = \{(c_1, s_3, s_4), (c_2, s_1, s_2)\} \neq \mu_{so}(P_S)$. Let $\widetilde{P_{s_2}} : c_1$. Then, $\mu_c^2(\widetilde{P_{s_2}}, P_{s_{-2}}) = \{(c_1, s_2, s_4), (c_2, s_1, s_3)\}$. Since $c_1 P_{s_2} c_2$, $\gamma(3)$ is manipulable by s_2 at P_S.

- $\mu_c^1(P_S) = \{(c_1, s_2), (c_2, s_1, s_3)\} \neq \mu_{so}(P_S)$. Let $\widetilde{P_{s_4}} : c_1$. Then, $\mu_c^1(\widetilde{P_{s_4}}, P_{s_{-4}}) = \{(c_1, s_2, s_4), (c_2, s_1, s_3)\}$. Since $c_1 P_{s_4} s_4$, $\gamma(1)$ is manipulable by s_4 at P_S.

Even though the above examples imply that the manipulability of TCOSM does not directly depend on the truncation quota, it illustrates which P_S is vulnerable under $\gamma(t)$: P_S such that $\mu_c^t(P_S) \neq \mu_{so}(P_S)$. In fact, $\mu_c^t(P_S) \neq \mu_{so}(P_S)$ is the necessary and sufficient condition for P_S to be vulnerable under $\gamma(t)$. Based on the theorem, the analysis of TCOSM's manipulability based on the truncation quota becomes possible.

Theorem 4.1. *P_S is vulnerable under mechanism $\gamma(t)$ if and only if $\mu_c^t(P_S) \neq \mu_{so}(P_S)$.*[12]

Proposition 4.1. *Only considering college admission problems, where $\mu_c^t(P_S) \neq \mu_{so}(P_S)$, $\gamma(t)$ is more as manipulable as $\gamma(t+1)$.*

The proposition states that if $\gamma(t)$ does not incur $\mu_{so}(P_S)$ at P_S, increase in the truncation quota by one decreases manipulability of a mechanism. The type space considered in the proposition is where $\mu_c^t(P_S) \neq \mu_{so}(P_S)$, and it implies that all P_S's in the space are vulnerable under $\gamma(t)$. This seems to be a strong assumption that directly leads to the conclusion. However, this is a reasonable assumption considering the reality of college admission mechanisms. In the real-world, there are a large number of colleges and students, and thus $\gamma(t)$ would not produce a student-optimal matching most of the time. For example, under TCOSM, there usually exist unfilled seats (quota) at some colleges and unassigned students that would prefer to enter that colleges, as $\gamma(1)$ in Example 4.1.

Proposition 4.1 is a theoretical support for the admission policy reforms that increase the truncation quota. For example, the introduction of Common Application

[12] The case when $t = |C|$ or ∞ was proved by Gale and Sotomayor (1985).

in the US could be theoretically supported in that it makes the admission system less manipulable. Common Application is a college admission platform that applicants can use to apply to any of its 693 member colleges. It reduces students' burden of applying to different schools on separate platforms. Even though it does not intend to deal with manipulability of admission system, it virtually increases the truncation quota, and thus contribute to the system being less manipulable.[13] Also, since manipulability is one of major issues in the South Korean admission mechanism, the policymakers can increase the truncation quota to mitigate the problem.

5 Conclusion

In college admission problems, manipulability is one of crucial problems that policymakers are striving to resolve. Failure to properly "game" leads to qualified students unjustly going to less desirable colleges or even taking an unwanted gap year. For example, there are many students in South Korea who are re-preparing for college admission not because they were unqualified but because they applied to "wrong" colleges. This paper sheds light to the solution.

First, it shows that the Real-world Mechanism, the abstraction of college admission mechanisms in South Korea and the United States, is equivalent to truncated college-optimal stable mechanism (TCOSM). The result facilitates the analysis of college admission mechanisms, since TCOSM could be utilized instead. Furthermore, the equivalence result evidences the academic importance of COSM, which has been less studied compared to SOSM.

Second, it proves that the type space (preference space) which is vulnerable under TCOSM and the space of which matching under TCOSM is different from that under SOSM are equivalent. As an implication of this result, TCOSM becomes less manipulable as the truncation quota increases considering the type space which incurs

[13] Its mission is to "promote access, equity, and integrity in the college admission process."

different matchings under SOSM and TCOSM with the less stringent quota. This result is a theoretical proof of an assertion that strategic behaviors of students will decrease when they are allowed to apply to more schools.

Further research could be conducted to examine the empirical validity of the theoretical result. For example, the change incurred by the US Common Application might provide data source for statistical analysis. Also, as in Chen and Kesten (2016), experimental methods could be applied as well.

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Appendix A: Basic Concepts of Matching Theory

Definition A.1. A matching μ is blocked by an individual i if $iP_i\mu(i)$.[14]

Definition A.2. A matching μ is individually rational if it is not blocked by any individual.

Definition A.3. A matching μ is blocked by a student-college pair (s,c) if $cP_s\mu(s)$ and sP_cs', for some $s' \in \mu(c)$.

Definition A.4. A matching μ is (pairwise) stable if it is individually rational and not blocked by any student-college pair.[15]

Definition A.5. A stable matching μ is student-optimal if for all $s_i \in S$, $\mu(s_i)R_{s_i}\mu'(s_i)$, where μ' is a stable matching.

Definition A.6. A student s is achievable for a college c if s and c can be matched in a stable matching.

Definition A.7. μ is college-optimal if for all $c_i \in C$, $\mu(c_i)$ consists of q_i top-ranked students among students achievable for c_i. If the number of students achievable for c_i is less than q_i, $\mu(c_i)$ is the set of all students achievable for c_i.

Appendix B: Proofs for the Theorem and Propositions

Definition B.1. $\mu_c^t(i|P_S)$, $\mu_c(i|P_S)$, and $\mu_{so}(i|P_S)$ denote the matching of an individual i in $\mu_c^t(P_S)$, $\mu_c(P_S)$, and $\mu_{so}(P_S)$, respectively. $P_{s_i}^t$ and P_S^t denote the t-truncation of P_{s_i} and P_S, respectively. The t-truncation means the preference profile that leaves only top t colleges in the preference profile.

[14] As for a college, μ is blocked by an individual (a college c) if cP_cs for some $s \in \mu(c)$.

[15] As for a many-to-one matching like a college admission problem, there is a stronger stability condition called group stability. However, since colleges' preferences are responsive in this paper's model, group stability and pairwise stability are equivalent.

__Proof of Proposition 3.1.__

Proposition 3.1. $\rho(t)$ (the Real-world Mechanism where each student can apply up to t colleges) and $\gamma(t)$ (TCOSM with the truncation quota t) are equivalent in that they produce the same matching given the same reported preference profile.

Proof. Let $\widetilde{P_S}, \widetilde{P_C}$ denote the reported preference profiles of students and colleges, respectively. It is enough to show that a matching μ_r incurred by $\rho(t)$ given $(S, C, q, \widetilde{P_S}^t, \widetilde{P_C})$ is stable and college-optimal. This is because COSM incurs a stable and college-optimal matching assuming that the reported preference profiles are true, and TCOSM is equivalent to COSM assuming that the truncated preference profile is true.

Claim 1. μ_r is stable.

Proof. μ_r is individually rational since students (colleges) are not matched to unacceptable colleges (students) under $\rho(t)$. Therefore, it is enough to show that μ_r is not blocked by any student-college pair. Suppose not: There exists (s_i, c_j) which blocks μ_r. Then, $c_j \widetilde{P_{s_i}}^t \mu_r(s_i)$, and there exists $s_k \in \mu_r(c_j)$ such that $s_i \widetilde{P_{c_j}} s_k$.[16] Since, s_i is strictly preferred to s_k, c_j must have offered admission to s_i, before offering it to s_k. Furthermore, since s_i strictly prefers c_j to $\mu_r(s_i)$, s_i must have chosen a college c_l such that $c_l R_{s_i} c_j$. Since $c_j \widetilde{P_{s_i}}^t \mu_r(s_i)$, $c_l \neq \mu_r(s_i)$. This is a contradiction to the fact that s_i is matched to $\mu_r(s_i)$.

Claim 2. μ_r is college-optimal.

Proof. Let a proposal step denote a step of $\rho(t)$ where some college(s) offers admission to some student(s). Then, the k^{th} proposal step means the k^{th} of all colleges' admission rounds. Thus, In this definition, a college may or may not offer admission to student(s) at a proposal step. In other words, a proposal step is an admission round of some colleges, but it may not be the case for other colleges.

[16] If c_j has not filled up its quota q_j in μ_r, then $s_k = c_j$.

College-optimality can be seen by induction. Suppose that up to the k^{th} proposal step, all achievable students of each college who received offers from it tentatively accepted them. Then it is enough to prove that at the $(k+1)^{th}$ proposal step, each college's achievable students who have tentatively accepted offers before the $(k+1)^{th}$ proposal step don't reject the offers in favor of other new offers (Claim 2.1) and achievable students who newly get offer from it at the $(k+1)^{th}$ proposal step tentatively accept its offers (Claim 2.2).

Claim 2.1. c_j's achievable students tentatively accepted c_j's offer before the $(k+1)^{th}$ proposal step do not decline c_j's offer at the $(k+1)^{th}$ proposal step.

Proof. Suppose not: A student s_a achievable for c_j who tentatively accepted c_j's offer before the $(k+1)^{th}$ step declines c_j in favor of another school c_p at the $(k+1)^{th}$ step. Then, any matching μ that assigns s_a to c_j is unstable, since $c_p\widetilde{P_{s_a}}^t c_j$ and $s_a\widetilde{P_{c_p}}s, for some s \in \mu(c_p)$. The later part is from the fact that c_p proposes to s_a at the $(k+1)^{th}$ proposal step, which in turn implies that s_a is one of q_{c_p} top-ranked students among the achievable students for c_p. Since (s_a, c_p) is a blocking pair of μ, s_a is not achievable for c_j.

Claim 2.2. c_j's achievable students who newly receive offers from c_j at the $(k+1)^{th}$ proposal step tentatively accepts it.

Proof. Let s_n denote a c_j's achievable student who newly receives offer from c_j at the $(k+1)^{th}$ proposal step.

If s_n is unassigned and c_j is the only college that offers admission to s_n at the $(k+1)^{th}$ proposal step, s_n accepts the offer since the fact that s_n is a achievable student of c_j implies that c_j is an acceptable college for s_n.

It remains to be seen that s_n who has received multiple offers before and at the $(k+1)^{th}$ proposal step accepts $c_j's$ offer. Suppose to the contrary that

s_n chooses another college c_p instead of c_j. Then, any matching μ that assigns s_n to c_j is unstable. It is because $c_p\widetilde{P_{s_n}}^t c_j$, and $s_n\widetilde{P_{c_p}}s$, $for\,some\,s \in \mu(c_p)$. The later part is from the fact that c_p proposes to s_n at the $(k+1)^{th}$ proposal step, which in turn implies that s_n is one of q_{c_p} top-ranked students among the achievable students for c_p. Since (s_n, c_p) is a blocking pair of μ, s_n is not achievable for c_j.

Claim 2.1. and Claim 2.2. also applies to $\forall c \in C$. Hence, μ_r is college-optimal.

<u>Proof of Theorem 4.1.</u>

Theorem 4.1. P_S is vulnerable under mechanism $\gamma(t)$ if and only if $\mu_c^t(P_S) \neq \mu_{so}(P_S)$.

Lemma 4.1. If $\gamma(t)$ is strategy-proof (on students' side) at P_S and $\mu_c^t(P_S)$ is stable, then $\mu_c^t(P_S) = \mu_{so}(P_S)$.

Proof. Suppose that $\mu_c^t(P_S)$ is stable. Then, it is enough to show that strategy-proofness implies student-optimality. Suppose to the contrary: $\mu_c^t(P_S) \neq \mu_{so}(P_S)$. Then, there exists $s_j \in S$ such that $\mu_{so}(s_j|P_S)P_{s_j}\mu_c^t(s_j|P_S)$, by the lattice property of stable equilibria. By the Rural Hospital Theorem (Roth 1986), the set of unassigned students is the same across $\mu_{so}(P_S)$ and $\mu_c^t(P_S)$. Furthermore, considering the lattice property and $\mu_c^t(P_S) = \mu_c(P_S^t)$, $\mu_{so}(P_S) = \mu_{so}(P_S^t)$. Let $\widetilde{P_{s_j}}$: $\mu_{so}(s_j|P_S)$. Then, $\mu_c^t(s_j|\widetilde{P_{s_j}}, P_{s_{-j}}) = \mu_c(s_j|\widetilde{P_{s_j}}, P_{s_{-j}}^t) = \mu_{so}(s_j|P_S^t) = \mu_{so}(s_j|P_S)$. This contradicts the strategy-proofness.

Proof of "if" part

(Contrapositive of "if": If P_S is not vulnerable under $\gamma(k)$, then $\mu_c^t(P_S) = \mu_{so}(P_S)$.)

Proof. The fact that P_S is not vulnerable under $\gamma(k)$ is equivalent to the strategy-proofness of $\gamma(k)$ at P_S. Therefore, $\mu_c^t(s_m|P_S)R_{s_m}\mu_c^t(s_m|\widetilde{P_{s_m}}, P_{s_{-m}})\,\forall s_m \in S\; and\; \forall \widetilde{P_{s_m}}$.

By Lemma 4.1, it is enough to show that $\mu_c^t(P_S)$ is stable. To prove the stability, suppose to the contrary that there exists (s_i, c_j) such that $c_j P_{s_i} \mu_c^t(s_i|P_S)$ and $s_i P_{c_j} s'$, $where\, s' \in \mu_c^t(c_j|P_S)$. $s_i P_{c_j} s'$ means that c_j must have offered admission to s_i before $\gamma(k)$ ends. If $\mu_c^t(s_i|P_S) \neq s_i$, then c_j is included in $P_{s_i}^t$. Hence, $c_j P_{s_i} \mu_c^t(s_i|P_S)$ cannot hold. Therefore, $\mu_c^t(s_i|P_S) = s_i$ and c_j is not in $P_{s_i}^t$. If s_i reports $\widetilde{P_{s_i}} : c_j$. Then, $\mu_c^t(s_i|\widetilde{P_{s_i}}, P_{s_{-i}}) = c_j$. This is a contradiction to the strategy-proofness.

Proof of "only if" part

(Contrapositive of "only if": If $\mu_c^t(P_S) = \mu_{so}(P_S)$, then P_S is not vulnerable under $\gamma(k)$.)

Proof. Suppose not: There exist $s_v \in S$ and $\widetilde{P_{s_v}}$ such that $\mu_c^t(s_v|\widetilde{P_{s_v}}, P_{s_{-v}}) P_{s_v} \mu_c^t(s_v|P_S)$.

Case 1. $\mu_c^t(s_v|P_S) = c_q$, for some $c_q \in C$.

Since $\mu_c^t(P_S) = \mu_{so}(P_S)$, $\mu_{so}(P_S) = \mu_{so}(P_S^t)$. Hence, $\mu_c(P_S^t) = \mu_{so}(P_S^t)$. At P_S^t. truth-telling is optimal for each student by Gale and Sotomayor (1985). Since it is optimal for s_v not to lie under $\gamma(\infty)$ at P_S^t, and s_v is assigned to some college in $P_{s_v}^t$, truth-telling is optimal for s_v.

Case 2. $\mu_c^t(s_v|P_S) = s_v$.

Let $\mu_c^t(s_v|\widetilde{P_{s_v}}, P_{s_{-v}}) = c_p$. Without loss of generality, $\widetilde{P_{s_v}} : c_p$[17]. $\mu_{so}(s_v|P_S) = s_v$ means that s_v are rejected by all colleges in P_{s_v} at SOSM. Along with the assumption that $\mu_c^t(P_S) = \mu_{so}(P_S)$, this in turn implies that, at each of these colleges, the quota is filled up with higher ranking students at $\mu_c^t(P_S) = \mu_{so}(P_S)$ or s_v was unacceptable. Hence, $|\mu_c^t(c_p|P_S)| = q_{c_p}\, and\, s' P_{c_p} s_v$, $\forall s' \in \mu_c^t(c_p|P_S)$, or $c_p P_{c_p} s_v$. Therefore, $\mu_c^t(s_v|\widetilde{P_{s_v}}, P_{s_{-v}}) \neq c_p$.

[17] By Theorem 5.34 of Roth and Sotomayor (1990), deleting colleges after c_p in $\widetilde{P_{s_v}}$ does not affect the matching. Also, since $\mu_c^t(s_v|\widetilde{P_{s_v}}, P_{s_{-v}}) = c_p$, deleting colleges before c_p does not affect the matching as well.

<u>Proof of Proposition 4.1.</u>

Proposition 4.1. Only considering college admission problems, where $\mu_c^t(P_S) \neq \mu_{so}(P_S)$, $\gamma(t)$ is more as manipulable as $\gamma(t+1)$.

Proof. $\gamma(t)$ is at least as manipulable as $\gamma(t+1)$, by Theorem 4.1 and the assumption that $\mu_c^t(P_S) \neq \mu_{so}(P_S)$. Also, there always exist (S, C, q, P_s, P_c) such that P_s is vulnerable under $\gamma(t)$, but not under $\gamma(t+1)$. This is trivial since one can construct a college admission problem as in Example 4.1, for all t.

국 문 초 록

대학입시제도의 주요 문제 중 하나는 학생들에 의한 조작가능성이다. 즉, 학생들은 거짓 선호를 밝힘으로써 더 선호하는 대학교에 입학할 수 있다. 이 논문은 각 학생의 지원 가능 학교 숫자가 제한되어 있는 한국 및 미국의 대학입시제도가 축약 학교최적 안정 메커니즘(Truncated College-optimal Stable Mechanism, TCOSM)과 동일함을 보인다. 그리고 Pathak and Sönmez (2013)가 제시한 분석틀을 활용하여 TCOSM이 조작가능한 선호집합과 TCOSM 및 학생최적 안정 메커니즘(Student-optimal Stable Mechanism, SOSM)에서 다른 매칭을 가져오는 선호집합이 동일함을 증명한다. 이 결과를 바탕으로 축약수(truncation quota, 각 학생이 지원할 수 있는 학교 수)가 낮은 TCOSM에서 조작가능한 선호집합을 고려했을 때, 축약수를 높이면 TCOSM의 조작가능성이 감소함을 보인다. 조작가능성에 대한 이 논문의 분석은 대학입시에 대한 학생들의 만족도를 높이기 위해 지원 가능 학교 수를 늘리는 정책 변화를 뒷받침한다.

주요어: 대학입시, 매칭, 조작가능성, 축약 메커니즘, 학교최적 안정 메커니즘

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