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Carryover Effect and Risk Aversion : Dynamic Incentives in Sales Force Compensation

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Abstract

I solve the discrete dynamic decision of sales agents' effort allocation under a quota bonus compensation when the carryover from the past period is introduced in sales. With the solution of dynamic programming, I generate the sales data from two segments of sales agents: one with high risk-aversion and the other with low risk-aversion. As the carryover in sales increases both the expected mean and variance of sales in the next period, the sales agent's optimal effort allocation and thus the realized sales pattern vary according to his degree of risk aversion. The highly risk-averse set the baseline of performance while the less risk averse fluctuate their sales above the highly risk-averse. Also, the frequency of achieving quotas is higher in the less risk averse group compared to the highly risk-averse group. These different patterns could be interpreted as that the highly risk averse try not to exert more effort to avoid the uncertainty from the increased sales.

Following Arcidiacono and Miller (2011), I estimate the segment-wise optimal effort functions and utility functions in two steps: calculating the conditional choice probability with nonparametric functions and then searching for parameters with EM algorithm. The estimation result shows that ignoring the carryover when it exists gives out poor estimates of the number and even the size of segments. This is because ignoring carryover results in the wrong segmenting of the sales agents from the first stage estimation and thus affects the second stage estimation subsequently. The result highlights the necessity of considering carryover when understanding sales force's performance history from the sales data if carryover exists. Neglecting carryover might lead to wrong segmentation of sales force and thus the inefficient design of segment-wise compensation plans.

Keywords: salesforce compensation, dynamic programming, carryover, risk aversion, heterogeneity, two-step CCP estimation, simulation

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1. Introduction

Personal selling is a crucial part of the marketing mix. In US economy in 2006, at least 20 million people were involved in sales (Zoltners et al. 2008). The total investment in sales force was as high as over $800 billion, which was close to three times the $285 billion spent on advertising in 2006 (Zoltners et al. 2008).

This significance has brought about the needs to design the optimal compensation scheme for the practitioners. However, to design an efficient compensation scheme that incentivizes a sales force to exert its full effort is difficult because managers can only observe the proxy of effort, the performance outcomes with noise (i.e. sales performance). This means that a sales force could be incentivized to shirk behind the performance incommensurate to effort incurring a cost to him while the efficient compensation scheme is to induce full effort.

One possible performance outcome that a sales force could shirk behind is sales carryover. The sales carryover refers to the process in which a significant portion of the given year's sales volume is not due to efforts of the sales force in the given year but is a function of the prior year's selling efforts and other factors (Madhani 2011). These factors include marketing mix variables, unique product characteristics, market competition, customer relationship strategies, regulatory requirements, government regulation, general market conditions, increased promotional and advertising expenditure, a particularly excellent product or attractive pricing (Madhani 2011). There are some industries with notably high sales carryover rates, such as pharmaceuticals, financial services, office equipment, or professional software (Zoltners et al. 2006). According to a study of 50 pharmaceutical companies in 6 countries with sales forces ranging in size from 35 to several thousand, the aggregate carryover sales from selling efforts in one year was 75% to 80% the next year, 62% to 78% in the third year and 52% to 70% in the fourth year (Sinha and Zoltners 2001). This study attributed high carryover rates in pharmaceutical industries to physicians' reluctance to switch patients from medications that are working.

As the carryover sales increase the sales agent's temptation to shirk (Rubel and Prasad 2016), it makes the compensation plans lose some efficiencies in incentivizing full effort. Due to the unobservability of the effort as well as the complexities in selling process between a sales force and its customer, the managers find it hard to estimate the exact sales carryover rate and to reflect it in the compensation contract. The result is that sales force could easily consider a portion of commissions as a hidden or free salary (Madhani 2011).

Besides the carryover sales, the non-linear compensation structure adds up the complexities and the inefficiencies in incentivizing effort. The non-linearity of compensation structure could give sales agents incentives to time the allocation of effort. A forward-looking sales agent would maximize his expected utilities by "gaming" in effort allocation considering his current decision making affects the future compensation. For example, the most commonly used compensation, which is quota-based compensation, generates a perverse incentive to the sales force who already achieve the quota to postpone additional effort to the future (Misra and Nair 2011). If the bonus payment from quota achievement is big enough and marginal income for sales beyond the quota is small enough, a sales agent might keep his effort in a given compensation cycle after earning bonus and exert the saved effort in the next compensation cycle to gain another bonus rather than exhaust all his effort every time. This gaming behavior could be reinforced if the marginal gain for sales beyond the quota is zero (i.e. ceiling in the compensation).

How does the effort allocation of a sales agent under a non-linear compensation contract change if there are significant carryover sales? This paper starts with the above question. If carryover sales are significant, a forward-looking sales agent will start to consider the longer effect of his selling effort. He might still exert some effort after achieving the quota, by reckoning that his current effort with little or even minus current income (i.e. the marginal cost of exerting effort is bigger than the marginal income earned for realizing sales) would be compensated by increased probability of gaining another bonus from carryover sales in the next compensation cycle. Without carryover sales, after achieving the quota, he might lose the motivation to put in extra effort till the next compensation cycle comes. The difference would be stark if sales beyond the quota give no marginal income and the quota is set high with a big bonus.

One variable that comes to be the fore in introducing carryover effect is sales agents' risk aversion. As carryover sales increase the mean and also the variance of the future sales, a sales agent' risk aversion acts on his optimal effort allocation decision. How much he could endure the increased variance of future sales from carryover sales would affect how he exerts his effort in every period. Thus, this paper allows the sales agent's heterogeneity over risk aversion. Rubel and Prasad (2016) found that forward-looking sales agents need different optimal compensation according to their risk aversion degrees with the restraint that the optimal compensation should be monotonically increasing. Thus, this paper is the extension of their idea with non-linear and not monotonically increasing compensation plan.

Among several combinations in the non-linear compensation schemes, this paper chooses to focus on quota and bonus combination because of its popularity. The quotas and bonuses are used in more than 75% of firms in industries (Joseph and Kalwani 1998). Also, according to the 2008 Incentive Practices Research Study by ZS Associates, 73%, 85%, and 89% of firms in pharma/biotech, medical devices, and high-tech industries, respectively, use quota-based compensation (Training 2008). Considering that the pharmaceutical industry has popularly used quota-based compensation plan and it has quite high sales carryover, analyzing dynamic effort allocation with carryover sales under the quota and bonus compensation would be practically meaningful for at least the pharmaceutical industry. And there could be a lot of areas which popularly use quota-based compensation for their sales agents and have high rates of carryover sales.

In sum, this paper deals with dynamic effort allocation of sales agents with heterogeneous risk aversion degrees under quota and bonus compensation when carryover sales are introduced. The specific questions the paper deals are 1) how does the effort allocations of forward-looking sales agents vary across their risk aversion degrees? and 2) if the managers ignore carryover sales, could they segment precisely their sales agents varying across risk aversion from seeing the realized sales performance?

I set forward-looking sales agents in quota and bonus compensation structure and predict their effort allocation decisions by solving dynamic programming. The heterogeneity in risk aversion across sales agents is reflected in segment-wise effort allocations. From the segment-wise effort allocations, I generate the simulated sales data to try estimation of main parameters and followingly segmentation of sales agents with two-stage dynamic programming estimation using EM algorithm.

In Chapter 2, I address some historical points of literature regarding sales force compensation and forward-looking agents. In Chapter 3, detailed settings and explanation of model are introduced, and in Chapter 4 I point out the reason of embracing risk aversion degree in the main model. Followingly, I address data generation by solving single agent dynamic programming in Chapter 5 and estimation of main parameters with two-stage EM based dynamic programming estimation method in Chapter 6. Then, Chapter 7 explains the result of estimation and Chapter 8 closes the paper with general discussion.

2. Literature Review

There has been a wide range of research on sales force compensation. Starting from Holmstrom (1979), the widely used "standard" framework to understand the contracting relationship between the sales manager and the sales agent has been the principal-agent framework. Holmstrom's model describes a contract between a risk-neutral principal and a risk-averse agent in a risky production process. As only the output of production is observable while the input of the agent is not, there arise the information asymmetry resulting in the second-best contract.

Following Holmstrom (1979), the dynamics in principal-agent models were first taken notice for the non-aligned interest between principal and agents and the mitigation of moral hazard was mostly examined. Rubinstein and Yaari (1983) and Radner (1981) studied an infinitely repeated problem in which neither the principal nor the agent discounts the future. In these cases, both the principal and the agent could get the same amounts of expected utilities as their first best outcomes, and therefore moral hazard is completely overcome. Lambert (1983) examined the repeated problem with discounting noting that optimal contract depends on the entire previous history of the relationship. He interpreted the intertemporal arrangements as a smoothing of incomes across periods for agents which is similar to an insurance mechanism.

These multi-period examinations in principal-agent models soon started to highlight the manipulation of inputs by sales agents and the effect of compensation scheme on it. Holmstrom and Milgrom (1987) made Mirrless (1974)'s two-wage nonlinear compensation contract, i.e. a fixed wage unless output is very low or a very low wage for very low output, precise by providing its variant in which the agent chooses his labor input over time in response to observations of how well he is doing. The authors showed that assuming the agent has an exponential utility function and controls the drift rate of a Brownian motion over the unit time interval in continuous time model, the optimal incentive scheme was derived as linear in output because the agent would choose a constant drift rate independently of the path of output. Holmstrom and Milgrom interpreted it as because the two-wage scheme leads the agent to work hard only when that appears necessary to avoid a disaster, that a linear scheme applying the same incentive pressure on the agent no matter what his past performance has been is proved optimal. Later, Lal and Srinivasan (1993) corroborated Holmstrom and Milgrom (1987) with further comparative statics results. Lal and Srinivasan (1993) additionally derived that the commission income as a fraction of total compensation goes up with an increase in the effectiveness of the sales-effort while the salary component goes up with increases in uncertainty, absolute risk aversion, marginal cost of production, perceived cost of effort, and alternative job opportunities for the sales agents.

However, the literature could not easily fill the gap between analytically optimal compensation plans and ubiquitous simple nonlinear plans in practice. The attempts to address the nonlinearity have gone deep and broad. Oyer (1998) empirically showed that discrete bonuses and other nonlinearities in compensation could lead sales agents to take actions that maximize their expected income over several pay cycles. He used the aggregate sales across different industries in different quarters and concluded that the effect of fiscal year ends combined with the nonlinear incentive contracts undermines the attempts to smooth production by leading employees to take actions that affect firm seasonality. More recently, Steenburgh (2008) showed that an aggregate analysis might have concluded in the opposite direction regarding the effect of quotas in compensation compared to that of Oyer (1998). In analytically, Oyer (2000) showed that with the strong assumption that the sales agent has a liability limitation and participation constraint does not bind, optimal compensation is derived to be a discrete bonus for meeting a quota. He interpreted this result as because sales agent’s skills are most valuable in a sales context, the agent cannot expect comparable compensation in other professions, allowing firms to select the least expensive compensation plan without concern for insuring that the salesperson will participate, while inducing the optimal level of the sales agent's effort by concentrating marginal compensation on sales that maximize the additional revenue from additional effort.

While addressing the reason of popularity of quota in practice, the sales force literature got favored from detailed real data. The later empirical research directed to new and minute focus on individual worker's productivity with the acquisition of the detailed performance outcomes associated with every processed check of each sales agent while previous literature dealt aggregate sales force productivity. Copeland and Monnet (2009) tracked a worker's productivity at a very fine level of detail within the day where bonuses are calculated on a daily basis, and a worker starts each day anew. Using these fine data, the authors modeled and estimated the worker's dynamic effort decision problem.

More recent analysis on within period dynamics in sales agent's effort allocation adopted the forward-looking behavior from solving dynamic programming. The two recent and utmost literatures are Misra and Nair (2011) and Chung, Steenburgh, and Sudhir (2014). They both showed the forward-looking behaviors of sales agents under compensation plans with quotas using dynamic programming approaches. They both dealt with compensation structures consisting of quotas, however with different focuses. Misra and Nair analyzed quotas with floors and ceilings on commissions and concluded that quotas reduce performance. According to them, two characteristics of the quotas were important: First, the quota ceiling limits the effort of the most productive salespeople, who would normally have exceeded that ceiling. Second, the company followed an explicit policy of ratcheting quotas based on past productivity. This reduced salespeople's incentives to work hard in any given period, because hard work was penalized through higher future quotas. However, Chung, Steenburgh, and Sudhir focused on quotas with bonuses and concluded that when coupled with bonuses, quotas enhance performance. Their compensation scheme included the overachievement commissions for exceeding quotas and group quota updates minimized the ratcheting effects.

In the methodological perspective, two papers followed the recent advance in dynamic programming computation: two-step conditional choice probability estimation. The two-step CCP estimation approaches have recently gained popularity because of their ease of computation relative to traditional nested fixed point approaches. The main difference in methodologies between the two papers is whether it allows heterogeneity in the model. Misra and Nair avoided the unobserved heterogeneity issue by estimating each salesperson's utility function separately, while Chung, Steenburgh, and Sudhir followed Arcidiacono and Miller (2011) to allow heterogeneity within the two-step framework.

In my setting, I followed Chung, Steenburgh, and Sudhir in methodological perspectives but with different quota-bonus plans. I set ceiling in compensation above quota similar to Misra and Nair. This is because I want to see the carryover effect drawing effort even without marginal gains from achieving more above quotas. Thus, the ceiling in compensation works as a significant factor inducing further dynamics in the model. And for simplicity, here I ignore the ratcheting effect of sales compensation.

The further layer of complexity in sales agent literature was addressed in Rubel and Prasad (2015). The authors cast light upon the unexplored problem of carryover in sales response model. According to their analytical paper, if carryover effect exists, but the compensation plan is designed without recognizing it, then the firm will lose money because it compensates sales generated through carryover as well as effort, but attributes sales only to effort. With differential equations, they discovered that the degree of risk aversion of a salesperson, relative to the noisiness of the sales response function, plays an important role in determining the effort strategy of the salesperson and the optimal contract in the presence of carryover effects. This insight manifests because the carryover effect increases both the mean and the variance of future sales. As a result, they found that the shape of optimal compensation plan is convex in sales for a low risk-aversion salesperson and concave in sales for a high risk-aversion salesperson.

The main difference between my setting and Rubel and Prasad's is that I focused on the forward-looking behavior of sales agents under quota-bonus with ceiling compensation structures while they focused on the effort allocation between in new business and existing business and derived the equilibrium with firm's optimal contracts among monotonically increasing plans. Here, I focus only on the distorted effort allocation of sales agents derived from the mixture of carryover and quota-bonus structures and see whether introducing carryover effects in sales affects the estimation of dynamic structural parameters with heterogeneity in risk aversion of sales agents.

3. Model

Consider an infinite horizon with time discounting where the sales agent is compensated every period. Given the states at the beginning of time t, the sales agent exerts his optimal selling effort, weighing the expected income from future periods against the cost of effort. The sales at time t are realized based on his level of selling effort and the market random error. This realized sales become his selling performance at time t. At the end of time t, the sales agent is compensated according to his performance at time t under a particular compensation scheme. Followingly, the states are updated in the beginning of time $(t+1)$, which affects the decision making in the next period.

Here, I assume that the sales agent participates in the infinite cycle of realizing sales and getting compensation. In my simulated model, reservation wage is set to minus infinite, which implies that once participating in, the sales agent never gets out of the cycle. This assumption is restrictive because I focus only on perpetual sales agents. While literatures have handled getting out of the cycle by normalizing the reservation wage as zero, here I ignore any chance of getting out of the sales field. Considering getting in and out of the firm, future research could extend to the function of the compensation scheme as I will discuss later.

As in common practices, I assume that the agent is risk-averse and there is no private information. Also, the firm or manager cannot observe the sales agents' effort directly, rather it could infer the unobserved effort only from the realized sales data. I first summarize the notation of variables and parameters in Table 1 and Table 2.

Table 1. Variables

name	explanation	range
State variables		
M_t	Period type at time t	{1, 2, 3}
S_t	Sales at time t	[0.10, 12.18]
lS_t	$\ln(\text{Sales}_t)$: log representation of realized sales at time t	[-2.3, 2.5]
Q_t	Percentage cumulative quota achievement at time t	[0, 3.65]
Action variables		
eff_t	Effort of sales agent at time t	[0, 1]
Utility variables		
W_t	Income of sales agent at time t	$(-\infty, +\infty)$
U_t	Utility of sales agent at time t	$(-\infty, +\infty)$
V_t	Expected future utilities under optimal effort policy at time t	$(-\infty, +\infty)$
Random variables		
ϵ_t	Market variation on sales response function	*i.i.d.* N (0, 1)

Table 2. Parameters

name	explanation	true
Sales Response Function		
λ	Carryover rate	0.5
σ	Degree of market variation	0.01
Compensation Scheme		
r	Commission rate	0.01
q	Quota	10
B	Lump-sum Bonus	0.1
Utility Function		
γ	Risk aversion factor (high risk aversion/ low risk aversion)	2.5 / 0.001
c	Unit cost of exerting effort	0.05
Value Function		
δ	Time discount factor	0.95

3.1. Sales Dynamics

I define the sales response function as the equation (1) following Rubel and Prasad (2015).

$$ln(S_{i,t}) = v(eff_{i,t}|state_{i,t}) + \lambda ln(S_{i,t-1}) + \sqrt{\sigma}\epsilon_t,$$

$$where\ \ 0 < \lambda < 1\ and\ \epsilon_t \sim i.i.d.\ N(0,1) \quad (1)$$

Rubel and Prasad adapted the canonical Nerlov-Arrow model (1962) to define the continuous sales rate. While Nerlov-Arrow model addressed the decay in advertising with the factor of $(1-\lambda)$ using the differential equation in a deterministic way, Rubel and Prasad added a stochastic term in the differential equation. Rubel and Prasad defined the sales at time *t* as $x(t)$ and the change of sales at time *t* as $\frac{dx(t)}{dt} = v(t) - (1-\lambda)x(t) + \sqrt{\sigma}\epsilon(t),\ where\ \ x(0) = x_0$. In their model, $(1-\lambda)x(t)$ is the decayed sales from the previous period as in Nerlov-Arrow model. And $v(t)$ represents the sales agent's selling effort. For the stochastic term, $\epsilon(t)$ is the demand shock, and $\sigma = Var(\frac{dx(t)}{dt})$ is the noisiness of the sales response. Thus, the change of sales at time t consists of sales agent's effort at time t, decayed sales from the previous period and unspecified demand shock. Since subtracting the amount of decay is the same as adding the amount of carryover, the second term in Rubel and Prasad's also could be interpreted as the carryover from the previous period.

Here as shown in (1), I adapt Rubel and Prasad's model in a discretized way with the log-transformed sales, $ln(S)$ rather than the realized sales, S itself. If we take the logarithmic transformation of sales in the sales response equation (1), $ln(S)$ frees us from the truncation issue in calculating conditional probabilities and still is realistic as the realized sales S are restricted above zero.

The three components in the sales response function are the same as in Rubel and Prasad but in a discretized way. First, the optimal selling effort is the function of effort the sales agent exerts in time t and is conditional on the state variables. The agent first looks at the state variables at time t and decides which degree of effort to exert considering the cost of effort and the expected income. This part represents each sales agent's decision-making process regarding the degree of effort to exert. Second, the carryover from previous sales is restricted with the factor of λ ranging in (0, 1). Following Rubel and Prasad, the carryover factor λ is a constant and only one time lagged sales are considered for carryover. The constant carryover factor might ignore the heterogeneity in buyer-sales agent relationships or in the deals of different time periods. Moreover in reality, it would be more plausible to consider that some sales contracts have longer effects than just one period. However, here we put the strong assumption of constant λ and attain the simplest form of sales response model. Thus λ could be interpreted as the market average carryover rate. Lastly, the market variation follows the identically independent normal distribution with zero mean and variance of σ. The sales agents share the same market variation at time t, which rules out any geographical variation in sales or any private information of market. Also, as the market variation is independent at the different time, the model also rules out any market seasonality. Thus, besides the same market error unknown to all sales agents and individually expected carryover from the previous sales, the realized sales in the next period should be explained only with the function v of unobserved effort term eff_{it} by individual i.

3.2. Compensation Contract

At the end of time t, the sales agent i realizes $S_{i,t}$ and earns $W_{i,t}$ based on his $S_{i,t}$ and the compensation plan which he agrees to work under. I design the compensation plan following Chung, Steenburgh, and Sudhir (2014). Chung et al (2014) empirically argued the role of a quota as a pacemaker for the sales agent with middle achievement and the incentive of an overachievement commission for the sales agent with high achievement. The compensation scheme in their data is comprised of a linear commission, a lumpsum bonus for sales above a quota and an overachievement commission. I basically follow their compensation structure but omit overachievement commission. The modification is to distinguish the carryover effect on sales agent's forward-looking behavior from the incentives of overachievement commission. With no overachievement commission, only the carryover interprets why the sales agent exerts effort even after accomplishing the quota while he earns no marginal gain for sales above the quota at that period: the sales agent is expecting the increased gain in the next cycle due to the carryover. Therefore, by omitting the overachievement commission, I could identify the forward-looking behavior of sales agents in their performance after achieving the quota.

To observe the dynamic effort allocation of sales agents, I design three-period cycle of compensation structure. In the start of every first period, the cumulative quota achievement is renewed as zero and the sales agent earns linear commission for the sales he realizes. He earns the linear commission in every second period, also. In every third period, the bonus payment is given if the cumulative achievement during the first, second and third period is above the quota, or he earns only the linear commission for the sales he makes in the third period.

Below is the specific earning $W_{i,t}$ under the compensation contract I describe above.

$$W_{i,t}(S_{i,t}) = \begin{cases} rS_{i,t} & period\ 1,2 \\ I(Q_t \geq 1)(rq + B) + I(Q_t < 1)rS_{i,t} & period\ 3 \end{cases} \qquad (2)$$

The sales agent i earns $rS_{i,t}$ for period 1 and period 2 where r is the linear commission rate. And in period 3, the sales agent earns $rS_{i,t}$ if he does not achieve the quota or he earns a lumpsum bonus, $rq + B$. $Q_{i,t}$ is the sales agent *i*'s percentage of quota achievement till the end of period t in a cycle.

Different from Misra and Nair (2011), ratcheting effect from updating the compensation scheme based on previous performance is not considered. The compensation contract never changes, thus there is no uncertainty on the compensation contract itself.

3.3. Sales agent's Per-Period Utility

I define the utility function of sales agent *i* at time *t* as below.

$$U_{i,t}(eff_{i,t}, S_{i,t}; \boldsymbol{\theta_i}) = W_{i,t} - \gamma_i W_{i,t}^2 - C\ eff_{i,t}^2 \qquad (3)$$

The sales agent *i*'s utility at time *t*, U_{it} is derived from his compensation, W_{it} which is assumed to be equal to his consumption. As shown in equation (3), the utility function is the quadratic form of W_{it} conditioning on eff_{it} and thus S_{it}, following the equation (1) and (2). And from now on, the sales response function (1) and the compensation scheme (2) for the sales agent *i* are parameterized as $\boldsymbol{\theta_i} = \{\gamma_i, C, \lambda, \sigma\}$. Here, γ_i is a nonnegative risk aversion parameter for the sales agent *i*. I assume that the risk aversion degree differs in sales agents but is constant across the time. And I add the disutility from exerting effort, adopting the common specification of cost as $C\ eff_{i,t}^2$ where C is a nonnegative scalar.

Because the sales agent does not control the market variation ϵ_t , his decision on which degree of selling effort to exert, eff_{it} is solely based on the expected utility over the market variation as below.

$$E_{\epsilon_t}\ (U_{i,t}(eff_{i,t}, S_{i,t}; \boldsymbol{\theta})) = E(W_{i,t}) - \gamma_i E(W_{i,t}^2) - C\ eff_{i,t}^2 \qquad (4)$$

The sales agent has the uncertainty over the realized sales $S_{i,t}$ because of the market variation ϵ_t. But once the sales are realized, the earning $W_{i,t}$ and followingly the utility $U_{i,t}$ is calculated exactly. And as the sales agent decides his effort level before the market variation realizes, he cares about the expected utility in (4) not the realized utility in (3).

Note that in the above concave utility function, the utility has a maximum point after which it decreases with increasing earning. Here, I assume that the utility is monotonically increasing with increasing earnings and ignore any satiation of utility. Thus, I confine the relationship between the utility and the earning before reaching the maximum point by setting the risk aversion, γ_i in the range of $(0, \frac{1}{2W_{i,t}})$.

3.4. State Transitions

There are two sources of dynamics in the model. First is the nonlinearity in the compensation scheme in period 3. The sales agent's effort in period 1 and period 2 affects the probability to earn a bonus in period 3. Thus, the sales agent cannot choose the optimal effort independently across the time. Second is the carryover term in the sales response function. As the current sales affect future sales by depreciating carryover terms, each period is not independent nor is each cycle of three time periods. Specifically, without carryover, there is no incentive to exert effort after getting the bonus in period 3 since the marginal utility is negative. However, with the carryover in sales introduced, the sales agent considers the investment for future sales by exerting more effort than the single period optimal effort level. The sales agent thus has an incentive to exert effort even after achieving the quota in period 3 because he wants to make the probability to get a bonus in the next cycle higher. Hence, the sales agent needs to take into account how current decision on effort affects his expected future compensation.

These dynamics are embedded in the transition of the three state variables: period type, M_{it}, sales from previous month, $S_{i,t-1}$, and cumulative quota achievement up to the previous period, $Q_{i,t-1}$ which is in [0,1]. At the beginning of time *t*, the sales agent chooses his optimal effort level in time *t* considering the expected income based on the state variables, $state_{it}$.

$$state_{it} = \{M_t, S_{i,t-1}, Q_{i,t-1}\} \tag{5}$$

The first state variable M_t, period type (or month type) in one cycle, is deterministic. M_t rotates as $1 \rightarrow 2 \rightarrow 3 \rightarrow 1 \rightarrow \cdots$. As I assume all sales agents face the same period type at the same time *t*, I delete the subscript for individual sales agent *i*.

$$M_t = \begin{cases} M_{t-1} + 1 & if\ M_{t-1} = 1,2 \\ 1 & otherwise \end{cases} \tag{6}$$

The second state variable $S_{i,t-1}$, sales in the previous period, is to consider the carryover effect. In previous month, the realized sales following the equation (1) is saved for the second state variable in the next month.

The third state variable $Q_{i,(t-1)}$, cumulative quota achievement up to previous period in one cycle, is a measure for how close the accumulation of sales to the quota. It is augmented by the realized sales each period, except at the end of period 3 when the sales agent gets into a new cycle of accumulating achievement rate.

$$Q_{i,t} = \begin{cases} Q_{i,t-1} + \dfrac{S_{i,t}}{q} & if\ M_{i,t-1} = 1,2, \\ 0 & otherwise \end{cases} \quad (7)$$

Whereas the first variable, period type transits in a purely deterministic way, the latter two evolve in a stochastic way as they involve market random error term in sales response function (1).

3.5. Optimal Choice of Effort

Each sales agent chooses an effort level based on the state variables in the beginning of time t to maximize the discounted stream of expected utility flows, given the compensation plan $\boldsymbol{\Psi}$, both the sales response function and the utility function parameterized with $\boldsymbol{\theta}$. The present-discounted utility under the optimal effort policy can be represented by a value function that satisfies the following Bellman equation (8). The Bellman equation solves the optimal effort function ***eff*** conditional on $state_{i,t}$. Here, δ is the time discount factor.

$$\begin{aligned} &V\left(state_{i,t}; \boldsymbol{\theta_i}, \boldsymbol{\Psi}\right) \\ &\quad = \max_{\boldsymbol{eff}} \{ U\left(state_{i,t}, eff_{i,t}; \boldsymbol{\theta_i}, \boldsymbol{\Psi}\right) \\ &\quad + \delta V\left(state'_{i,(t+1)} \middle| state_{i,t}, eff_{i,t}; \boldsymbol{\theta_i}, \boldsymbol{\Psi}\right) \} \end{aligned} \quad (8)$$

4. Existence of Carryover

In this chapter, I address the effect of carryover in sales on sales agents' decision making under my setting. When carryover is introduced in sales response function, both expectation and variance of future sales increase compared to the model without carryover, given that the states and effort level are the same (i.e. $state_{it} = \{M_t, S_{i,t-1}, Q_{i,t-1}\}\ and\ eff_t$ are the same for the models with carryover or without carryover). This is the same argument as in Rubel and Prasad (2015). Let's consider the two sales response functions with and without sales carryover effect. Table 4 compares the expectation and variance of future sales for two different models.

As shown in Table 4, the bigger the last period sales $S_{i,t-1}$ or the higher the carryover effect rate λ, the more the sales agent expects for the next period's sales and the more he should bear the uncertainty from the market variation. Therefore, for the sales agent, exerting effort and proportionally increasing realized sales can be explained as participating in the risky gambling. With higher risk but higher return from the carryover effect, the sales agents with different risk aversion degrees differ in decision making over optimal effort levels. The caveat here is that the above argument holds only when $S_{t-1} > 1$. Otherwise, if $0 < S_{t-1} < 1$, the carryover effect only decreases the expectation and the variance of future sales compared to the case without carryover effect. I allow S_{t-1} to be between the range of [0.10, 12.18], however, the optimal effort functions from solving the dynamic programming compute the simulated sales data above 1 for most of the cases for each of the different risk aversion degrees. For details, following Chapter 5 will demonstrate the specific data generation process and the result of simulated sales data for different risk aversion degrees.

Table 3. Expectation and variance of future sales with/without carryover effect

	With carryover effect $S_t = S_{t-1}^{\lambda}\, exp(eff_t)\, exp(\sqrt{\sigma}\epsilon_t)$	**Without carryover effect** $S_t = exp(eff_t)\, exp(\sqrt{\sigma}\epsilon_t)$
$E(S_t)$	$E(S_t)$ $= E\left(S_{t-1}^{\lambda}\, exp(eff_t)\, exp(\sqrt{\sigma}\epsilon_t)\right)$ $= S_{t-1}^{\lambda}\, exp(eff_t)\, exp(\frac{1}{2}\sigma)$	$E(S_t)$ $= E\left(exp(eff_t)\, exp(\sqrt{\sigma}\epsilon_t)\right)$ $= exp(eff_t)\, exp(\frac{1}{2}\sigma)$
$Var(S_t)$	$Var(S_t)$ $= Var\left(S_{t-1}^{\lambda}\, exp(eff_t)\, exp(\sqrt{\sigma}\epsilon_t)\right)$ $= S_{t-1}^{2\lambda}\, exp(eff_t)^2\, exp(\sigma)\,(exp(\sigma) - 1)$	$Var(S_t)$ $= Var\left(exp(eff_t)\, exp(\sqrt{\sigma}\epsilon_t)\right)$ $= exp(eff_t)^2\, exp(\sigma)\,(exp(\sigma) - 1)$

5. Data Generation

To generate the data, I build the hypothetical sales environment by setting parameters and variables as in Table 1 and Table 2. Then, I numerically solve dynamic programming in a discretized space and interpolate the space with Chebyshev polynomials of state variables. Using the optimal effort policy that I attain from solving dynamic programming, I forward-estimate the actions, here the effort levels, of 100 sales agents and get 60 periods of sales data which includes 20 cycles of bonus payment.

5.1. Parameter Setting and Discretization of Variables

First step to generate the simulated data is to define the sales setting. To build the sales response function, compensation scheme, utility and value function, I set the true values of 8 parameters as in Table 2.

With the market variation term ϵ following normal distribution in the sales response function (1), realized sales S are naturally continuous values and the cumulative quota achievements Q are followingly continuous. However, to solve dynamic programming with the numerical approach of simple approximation, one needs to discretize the state and action variables.

Among the three state variables, period type M_t is already discrete; $M_t \in \{1, 2, 3\}$. The time t can be infinite by repeating the cycles of three discrete period types infinitely. As t goes to infinite, M_t changes as $1 \to 2 \to 3 \to 1 \to 2 \to \cdots$. The second state variable $S_{i,t}$ is followingly discretized as I generate $ln(S_{i,t})$ first by equally spacing 100 points within the range of [-2, 2.5] and putting these points in the simple inverse function. Thus, $S_{i,t}$ is restricted in (0.13, 12.19) with 100 points. As $S_{i,t}$ is calculated from $exp(ln(S_{i,t}))$, almost all possible points of $S_{i,t}$ could not be represented in less than 6 decimal places, such as $0.13535353 \ldots$ So here, the open parenthesis in the range of $S_{i,t}$ reflects the lengthy decimal places or irrational numbers of $S_{i,t}$. The third state variable $Q_{i,t}$ is generated by calculating all possible combination of $S_{i,t}$ in one cycle and adding zero point. In my setting, the quota q is 10 and the realized sales are bound below around 12.18. Thus, the maximum accumulated sales for three periods are theoretically above 36, which results in maximum $Q_{i,t}$ above 7 in the third period. However, as I discretize sales through equally spaced log transformed sales, sales below 5 are much denser. There are only 20 points in (5, 12.19) among 100 points in $S_{i,t}$. Thus, $Q_{i,t}$ is arranged much denser below 1.5 and sparser above 1.5 and below 7.

The finite grids in the 3-dimensional state space are all possible 5275 combinations of three state variables. The finite points in the 1-dimensional action space are 30 possible effort values. I design action space with 30 equally spaced points in the range of [0, 1] and for the simplicity I use $v(eff_{i,t}|state_{i,t}) = eff_{i,t}(state_{i,t})$. So, the sales response function is represented hereafter as the below function (1)'.

$$ln(S_{i,t}) = eff_{i,t}(state_{i,t}) + \lambda ln(S_{i,t-1}) + \sqrt{\sigma}\epsilon_t \quad (1)'$$

5.2. Solving the Dynamic Programming

After discretizing the state and action variables, I solve the dynamic programming with a numerical approach of successive approximation following Rust (1996). Solving the dynamic programming is to find the optimal action policy given a state. The optimal effort policy for 5275 states is the function of a state which given a state, chooses the level of effort giving the maximum expected utility among 30 different effort levels. Hence, the sales agent's optimal effort policy function has the domain of 5275 states and the range of 30 effort levels.

Remember that the state variables at the beginning of time t, $\{M_t, S_{i,t-1}, Q_{i,t-1}\}$ are the period type at time t, the lagged sales from the previous time $(t-1)$ and the percentage of quota achievement till the last time $(t-1)$. Given a state at the beginning of time t, $\{M_t, S_{i,t-1}, Q_{i,t-1}\}$, the sales agent could expect the probability of state transition to another state $\{M_{t+1}, S_{i,t}, Q_{i,t}\}$ based on the distribution of market variation in sales response function (1). This is because M_t changes in a deterministic way and $Q_{i,t}$ has a one-to-one relation with $S_{i,t}$, and henceforth, the market variation in the sales response function (1) explains all probabilities in the state transition.

I construct 30 state transition matrices which sizes are 5275 by 5275 for 30 different effort levels. With one effort level fixed, I could build a 5275 by 5275 state transition matrix which (i, j)-element represents the probability of transition from state i to state j. For example, with the effort level at $eff_{i,t}$ and given a state $\{M_t, S_{i,t-1}, Q_{i,t-1}\}$, the log transformed next time realized sales $S_{i,t}$ follows the normal distribution with the mean, $eff_{i,t} + \lambda ln(S_{i,t-1})$ and the variance of σ, which are all known with known parameters λ and σ. Based on this distribution of sales, I calculate 5275 discrete probabilities for each effort fixed given a state and normalize them to make them conform to the axioms of probabilities. After building the transition matrices, the optimal effort policy comes out followingly. With 30 transition matrices, given a state, I could calculate 30 expected utilities from (4) in a probabilistic way and could choose one optimal effort level which gives the highest expected utility among 30 effort levels.

I solve the optimal effort policy by successive approximation. As shown in Rust (1996), the solution to infinite horizon Markov-Chain dynamic programming problems is mathematically equivalent to computing a fixed point of the Bellman operator. Guaranteed by the contraction mapping theorem, I approach the solution starting with an arbitrary initial guess. I set the tolerance level at 1e-5 and iterate until the solution converges within the tolerance level. As a result, I attain a vector of optimal effort policy for 5275 different states.

5.3. Interpolation

After solving the dynamic programming, I interpolate the solution space to predict the optimal effort level from any continuous states other than 5275 discrete states. This step is necessary because while generating the sales data with random market errors, the state variables, specifically the sales and the quota achievement could be any continuous values other than 5275 discrete values.

For interpolation, I first try using the simple linear regression with the orthogonal polynomials of state variables. As the main purpose here is to predict the optimal effort level in the neighborhood of discrete space, I could expand the regressor set sufficiently enough to make the R-square close to 1. However, the three state variables show high multicollinearity, especially sales quantity and cumulative quota achievement in percentage have a highly positive correlation. And this condition requires so many high-ordered terms if I expand the regressor set only with the standard $1, x, x^2$ ... polynomials and even they do not contribute to large marginal increases in R-square. Thus, I follow a hint from Chung et al (2014) which links unobservable effort and observable states with a nonparametric model of effort function in their estimation stage with the real data. They model nonparametric effort function with Chebyshev polynomials of state variables to estimate conditional choice probabilities. I follow using Chebyshev polynomials of state variables and use these orthogonal basis functions to expand the regressor set with fewer regressors so as to relieve concern for multicollinearity among state variables.

In order to restrict the predicted effort values between 0 and 1, I re-parameterize the effort values x as $y = \ln\left(\frac{x}{1-x}\right)$. Thus, the set of the Chebyshev polynomials of sales and cumulative quota achievement, dummy variables for period types and their interaction terms explains re-parameterized y in a linear regression, restricting the effort level x in the range of (0, 1). The challenge here is that the optimal solutions for 5275 discrete states are not smooth, and thus even the orthogonal polynomials could not make R-square reach above 0.7. Therefore, I need to add up additive dummy variables to split the data points with eye measurement, seeing the data points on the graph and comparing them with the interpolated surface. After some adjustments, the resulting R-square for two different risk aversion cases (high/low) becomes 0.9090319 and 0.8611435, respectively.

I also try using the nearest neighborhood interpolation. I first match the month type then I find the nearest neighbor of 2-dimensional vector (i.e. sales and quota achievement rate) with the Euclidean distance measure. As I use 1-nearest neighborhood, in sample explanation of original 5275 data increases compared to the linear regression with basis functions. The two data sets generated by different interpolation methods show some differences. Specifically, sales performance generated from 1-nearest neighborhood interpolation shows that sales agents perform the best in every third period while sales performance generated from the linear regression shows that sales agents perform the best in every second period. This could be interpreted as the identification in the data generation process is approved. However, the focal interest in my simulation is the differences in mean and variance of sales performance among two types of sales agents, the highly risk averse or the less risk averse. And the interpolation methods change little between the two groups. The less risk averse show higher variance and higher mean while the highly risk averse show lower mean and lower variance. Thus, I keep the data generated by the linear regression with orthogonal polynomials and use this data for estimation later.

5.4. Sales data with the Heterogeneity in Risk Aversion factors

Given the continuous optimal policy function from interpolation, I generate sales data of 60 time periods by 100 sales agents with one of two risk aversion degrees; 60 sales agents have high risk aversion of 3 and the other 40 sales agents have low risk aversion of 0.01. Since I only use the first half of the quadratic utility function before reaching the maximum point, the risk aversion degree should be below $\frac{1}{2W_t}$ as I explain earlier. The maximum value of W_t is 0.15 when achieving the bonus in month 3. Therefore, the risk aversion degree should be below 3.33 to keep the utility function monotonically increasing. As the sales agents are risk averse following the literatures, the risk aversion factor should always be above zero. And to make segregation of one group from another clear and easy, I set the degree of less risk aversion at 0.01 which seems close to the risk neutrality.

I forward simulate the actions of the sales agents up to 160 times using the interpolated value functions starting with the initial state (1, 0, 0): in period 1, with no lagged sales and zero accumulated quota achievement. The first 100 sales data are burned in and the latter 60-period sales data for 100 sales agents are saved. The plotting of sales data shows that only 5-period burn-in is enough to confirm the stability.

5.5. Summary Statistics of Data

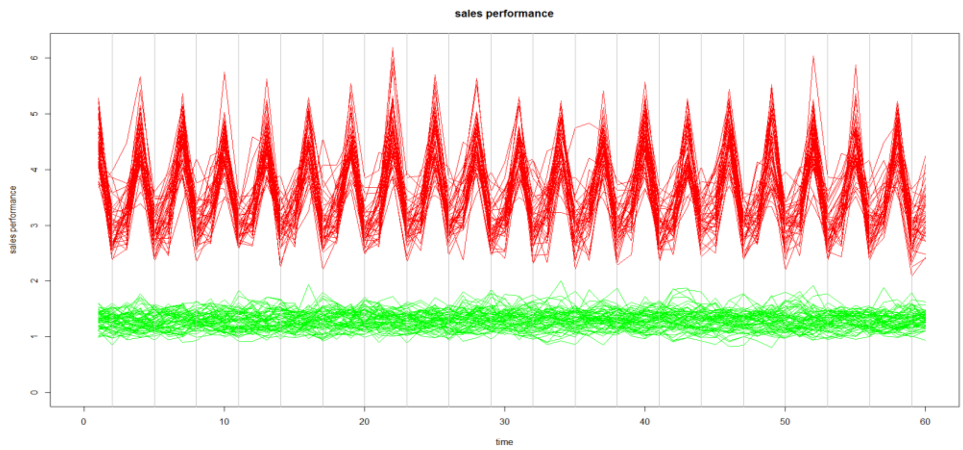
The simulated sales data for 100 sales agents are shown in Figure 3. The sales agents with low risk aversion perform better in average in terms of making sales. However, they have higher variance of performance compared to the counterpart with low risk aversion. The sales are steadier for the highly risk averse than for the less risk averse.

The introduction of sales carryover results in different sales patterns compared to the previous literature. In Misra and Nair (2011) without sales carryover, the sales pattern has a spike at the end of quarters suggesting that agents tend to increase effort as they reach closer to quota. However, in my simulated data with the sales carryover introduced, the sales pattern has a spike rather in the beginning of quarters in month 1. Across all risk aversion degrees, the sales agent in average makes the biggest performance in month 1 while realizing the least amount of sales in month 2. In month 3 when the bonus is to be given out based on the quota achievement, a little increase of sales from previous month is shown in the graph.

Also, as in Table 4, the sales agent makes sales even after achieving quota in month 3. In the end of month 3, a new cycle of sales starts with zero cumulative quota achievement. However, as the sales agent expects the carryover of sales to the next cycle which affects the chance to reach the quota in the next quarter, he still exerts effort to build sales.

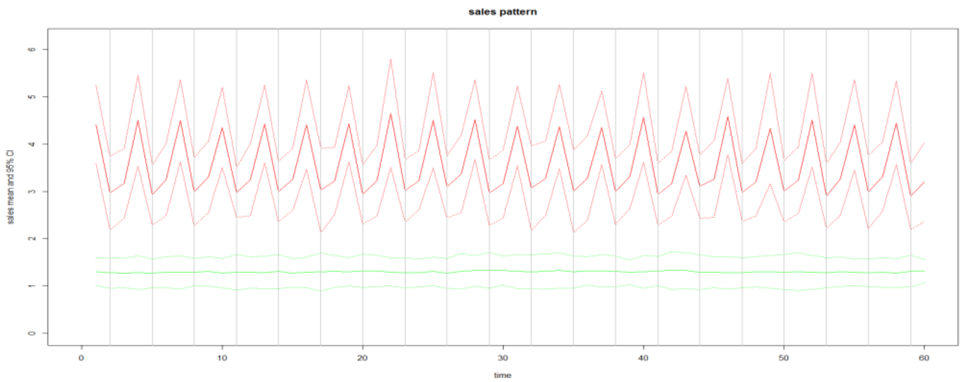
Figure 1. Sales performance by sales agents with high/low risk aversion

(a) Sales data from 100 sales agents



1. The grey vertical lines indicate period type 3 when the bonus payment is given according to the quota achievement.
2. The red lines on the upper level indicates the sales performance by 40 less risk averse sales agents.
3. The green lines on the bottom indicates the sales performance by 60 highly risk averse sales agents.

(b) 95% range of sales for high/low risk aversion



1. The grey vertical lines indicate period type 3 when the bonus payment is given according to the quota achievement.
2. The red lines on the upper level indicates the sales performance by 40 less risk averse sales agents.
3. The green lines on the bottom indicates the sales performance by 60 highly risk averse sales agents.
4. The bold lines are the means for sales by the same group of sales agents with respect to the risk aversion degree.
5. The dotted lines are upper/lower bound for 95% confidence interval of the sales by the same group of sales agents with respect to the risk aversion degree.

Table 4. Descriptive statistics of the simulated data

Variable	Mean	SD	Min	Max
Number of sales agents	100			
Periods of time	60			
All sales agents	**100**			
Wealth for one quarter (end of quarter)	0.0295	0.0520	0.0065	0.2000
Number of times achieving bonus	1.38	5.4335	0	11
Sales in month 1 (end of month)	2.2470	2.0030	0.6605	10.780
Sales in month 2 (end of month)	1.8130	1.4482	0.5562	8.1660
Sales in month 3 (end of month)	1.7980	1.3385	0.6535	5.8340
Cumulative sales in month 3 (end of month)	3.611	1.3304	1.357	12.420
Cumulative quota achievement in month 2 (beginning of month)	0.2244	0.2002	0.0660	1.0780
Cumulative quota achievement in month 3 (beginning of month)	0.4057	0.3387	0.1217	1.7940
Sales agents with high risk aversion	**60**			
Wealth for one quarter (end of quarter)	0.0150	0.0094	0.0065	0.0342
Number of times achieving bonus	0	0	0	0
Sales in month 1 (end of month)	1.7230	1.1822	0.6605	4.0070
Sales in month 2 (end of month)	1.5090	1.0352	0.5562	3.2590
Cumulative sales in month 3 (end of month)	3.016	0.7238	1.357	6.401
Cumulative quota achievement in month 2 (beginning of month)	0.1719	0.1177	0.0660	0.3913
Cumulative quota achievement in month 3 (beginning of month)	0.3228	0.2170	0.1217	0.6670
Sales agents with low risk aversion	**40**			
Wealth for one quarter (end of quarter)	0.0513	0.0814	0.0067	0.2
Number of times achieving bonus	3.45	0.0814	0	11
Sales in month 1 (end of month)	3.0320	2.7328	0.7606	10.78
Sales in month 2 (end of month)	2.2680	1.7423	0.7417	8.1660
Cumulative sales in month 3 (end of month)	4.503	1.5202	1.496	12.420
Cumulative quota achievement in month 2 (beginning of month)	0.3032	0.2735	0.0760	1.0780
Cumulative quota achievement in month 3 (beginning of month)	0.530	0.4292	0.1630	1.7940

6. Estimation

Using the generated sales data with heterogeneous degrees of risk aversion, I estimated the dynamic model following the recent two-step conditional choice probabilities (CCP) approach with unobserved heterogeneity of Arcidiacono and Miller (2011). The two-step CCP approach in the dynamic model is first introduced by Hotz and Miller (1993) and is extended by Bajari et al (2007), overcoming the computational burden in the nested fixed-point algorithm of Rust (1987). Notably, Arcidiacono and Miller (2011) utilized expectation-maximization algorithm to accommodate the unobserved heterogeneity in the first step of estimation. Here, I follow Arcidiacono and Miller to estimate the segment-wise structural models. Chung, Steenburgh, and Sudhir (2014) extended the empirical validity of Arcidiacono and Miller (2011) under the context of the effort allocation of sales agents as in my case, but with different compensation scheme and without sales carryover effect. I first estimate the parameters acknowledging the existence of carryover effect and then compare the results with those estimated ignoring the carryover effect.

6.1 The first step: effort and sales response functions for each segment

The unobserved heterogeneity in risk aversion affects the optimal effort policies and thus the realized sales. Using the EM algorithm, I segment the sales agents with respect to their optimal effort policies in sales response functions.

Below is the sales response function for sales agent *i* in segment *s* at time *t*.

$$ln(S_{i,t}) = eff_{i,t,s}(\boldsymbol{state_{i,t}}) + \lambda ln(S_{i,t-1}) + \sqrt{\sigma}\epsilon_{i,t} \qquad (1)''$$

Only the optimal effort function differs across segments while other parameters λ, σ remain the same for all sales agents regardless of the segment. Considering the effort is a decision by sales agents given the current state variables, a nonparametric model of effort function using the combination of Chebyshev polynomial basis functions from the state variables is possible. Chung et al (2014) employs Chebyshev basis functions to map between observable states and actions including unobservable effort functions. Here, I follow Chung et al (2014) to represent the unobservable effort with observable states nonparametrically. Note that I already use the orthogonal basis function in data generation process with interpolation. Here, in the estimation stage, the estimated effort function could be different from the linear regression model in interpolation. The nonparametric effort function for sales agent *i* in segment *s* at time *t* is as below where $\rho_{s,m}$ is the Chebyshev polynomial of degree *m* in segment *s*.

$$eff_{i,t,s}(\boldsymbol{state_{i,t}}) = \sum_{m=1}^{M} \gamma_{s,m}\rho_{s,m}\big(\boldsymbol{state_{i,t}}\big) \qquad (9)$$

Therefore, the sales response function for segment *s* becomes,

$$ln(S_{i,t}) = \sum_{m=1}^{M} \gamma_{s,m}\rho_{s,m}(\boldsymbol{state_{it}}) + \lambda ln(S_{i,t-1}) + \sqrt{\sigma}\epsilon_{i,t} \qquad (1)'''$$

and $\boldsymbol{\Theta_s} = \{\boldsymbol{\gamma_s}, \lambda, \sigma\}$, the set of parameters given the order of Chebyshev polynomial basis *m* and the segment *s* for sales agent *i* is to be estimated. To be clear, $\boldsymbol{\gamma_s}$ is the vector of coefficients of Chebyshev polynomial basis and is the only parameter varying across the segment *s*. Thus, heterogeneity across segments comes in the sales response function only through $\boldsymbol{\gamma_s}$.

Assume that sales agent *i* belongs to one of S segments, $s \in \{1, \dots, S\}$ with segment probabilities $q_i = \{q_{i1}, \dots, q_{iS}\}$. Let the population probability of being in segment *s* be π_s. Then, the likelihood of individual *i* making $ln(S_{i,t})$ at time *t*, conditional on the observed states, $\boldsymbol{state_{it}}$ and unobservable segment *s* is,

$$L_{ist} = L\big(ln(S_{i,t})\ \big|\ \boldsymbol{state_{it}}, s;\ \boldsymbol{\Theta_s}\big) \qquad (10)$$

And the likelihood of observing sales history $ln(S_i)$ over the time period ($t = 1, \ldots, T$), given the observable state history $state_i$ and the unobservable segment s, is given by,

$$L_{is} = L(\ln(S_i)|\boldsymbol{state_i}; \boldsymbol{\Theta_s}, \pi_s) = \pi_s\left(\prod_{t=1}^{T} L_{ist}\right) \qquad (11)$$

Overall, the likelihood of individual i is obtained by summing over all the unobserved segments $s \in \{1, \ldots, S\}$.

$$L_i = \ L(ln(S_i)\ |\boldsymbol{state_i}; \boldsymbol{\Theta}, \boldsymbol{\pi}) = \sum_{s=1}^{S} L_{is} \qquad (12)$$

Hence, the log-likelihood over the N sample of individuals becomes,

$$log(L) = \sum_{i=1}^{N} \log(L_i) \qquad (13)$$

Since maximizing the above exact log likelihood is computationally infeasible, I follow Arcidiacono and Miller (2011) to maximize the alternative, the expected log-likelihood, A:

$$A = \sum_{i=1}^{N}\sum_{s=1}^{S}\sum_{t=1}^{T} q_{is}\log(L_{ist}),$$

$$q_{is}(ln(S_i)\ |\boldsymbol{state_i}; \boldsymbol{\Theta}, \boldsymbol{\pi}) = \frac{L_{is}(ln(S_i)\ |\boldsymbol{state_i}; \boldsymbol{\Theta_s}, \pi_s)}{L_i(ln(S_i)\ |\boldsymbol{state_i}; \boldsymbol{\Theta}, \boldsymbol{\pi})}$$

Given λ, and model specification of effort function, I iteratively search for σ and $\boldsymbol{\gamma}$ by updating $\boldsymbol{\pi}$ with $\boldsymbol{q}$ until the loglikelihood converges. The process at the $(m+1)^{th}$ iteration after getting parameters $\{\boldsymbol{\Theta_s^m}, \boldsymbol{\pi^m}\}$ from m^{th} iteration is as follows:

(1) With $\{\boldsymbol{\Theta_s^m}, \boldsymbol{\pi^m}\}$, compute q_{is}^{m+1} from the equation below.

$$q_{is}^{m+1}(ln(S_i)\ |\boldsymbol{state_i}; \boldsymbol{\Theta^m}, \boldsymbol{\pi^m}) = \frac{L_{is}(ln(S_i)\ |\boldsymbol{state_i}; \boldsymbol{\Theta_s^m}, \pi_s^m)}{L_i(ln(S_i)\ |\boldsymbol{state_i}; \boldsymbol{\Theta^m}, \boldsymbol{\pi^m})}$$

(2) With q_{is}^{m+1}, obtain $\boldsymbol{\Theta}^{\boldsymbol{m+1}}$ by maximizing A where $\boldsymbol{\Theta}^{\boldsymbol{m+1}} = \{\boldsymbol{\gamma}^{\boldsymbol{m+1}}, \lambda, \sigma^{m+1}\}$.

Note that λ is given in this algorithm, which means λ never updates but is fixed.

1) With q_{is}^{m+1}, compute coefficients $\boldsymbol{\gamma}^{\mathbf{m+1}}$ from the weighted least squares.

2) With $\boldsymbol{\gamma}^{\mathbf{m+1}}$**,** update σ^{m+1} from minimizing sum of squares of residuals in linear regression.

(3) Update $\boldsymbol{\pi}^{\boldsymbol{m+1}}$ by taking the average of q_{is}^{m+1} over the sample.

$$\pi_s^{m+1} = \frac{1}{N}\sum_{i=1}^{N} q_{is}^{m+1}$$

I iterate (1) to (3) till the loglikelihood converges with the tolerance level at 1e-5. The initial value needed to start the iteration is only $\boldsymbol{q}$. This is because I use the residuals as new regressand by subtracting carryover term from the sales performance (note that λ is given) and compute weighted least squares with $\boldsymbol{q}$ being weights. And the initial values of the segment probabilities $\boldsymbol{q}$ are set equally across segments and sales agents. After convergence, I get the final estimate of $\widehat{\boldsymbol{\Theta}_s}$ and $\widehat{\boldsymbol{\pi}_s}$ for all segments. Hence, from $\widehat{\boldsymbol{\Theta}_s}$, I can set the effort function for all segments $\widehat{eff}_s(\boldsymbol{state})$, and thus complete the sales response function for all segments $\hat{S}_s(\boldsymbol{state})$. Also from $\widehat{\boldsymbol{\pi}}_{\mathbf{s}}$, I now know the population probabilities of all segments. The probability of individual i in segment s, q_{is} is no longer in use: it just helps calculate the main parameters in the sales response function and the population probabilities. Again, the segment-wise parameters are only the coefficients of effort function, $\boldsymbol{\gamma}_{\mathbf{s}}$.

The next step is to find the optimal λ given model specification of effort function. The above iterative process is the function of λ and I optimize λ by maximizing the converged loglikelihood resulting from above EM algorithm. I use basic optim function in R with BFGS method.

Lastly, the model specification of effort function should be chosen. I try 47988 combinations of Chebyshev polynomials and the number of segments. I build the regressor set with Chebyshev polynomials of state variables whose degrees vary from one to 6 and whose combinations with month type dummies vary from case 1 to case 6. Thus, for each segment, I build 36 cases of effort function and as I try the number of segments from one to 3, I build $36 + 36^2 + 36^3 = 47988$ numbers of effort function specifications. This does not cover every possible combination for effort function model specification. However, I believe that this number of trials in model selection procedure is large enough. Among 47988 candidate model specifications, I choose one with the lowest BIC and continue to estimate segment-wise utility functions with chosen effort function specification.

6.2 The second step: utility functions for each segment

The second stage is to find the structural parameters that rationalize the optimal actions estimated in the first stage (i.e. estimated segment-wise effort functions and estimated segment-wise sales response functions). Below is the utility function of a representative agent in segment *s* at time *t* with $\boldsymbol{state_t}$ who conforms to the estimated optimal effort $\widehat{eff}_{st}(\boldsymbol{state_t})$ and thus the estimated sales response function $\hat{S}_{st}(\boldsymbol{state_t})$, both parameterized with the set $\widehat{\boldsymbol{\Theta}}_{\boldsymbol{s}}$.

$$U_{st} = U_{st}\left(S_{st}(\boldsymbol{state_t}); \widehat{\boldsymbol{\Theta}}_{\boldsymbol{s}}\right) = E(W_t) - \boldsymbol{\gamma_s} Var(W_t) - c\widehat{eff}_{st}(\boldsymbol{state_t}), \forall s = \{1, \ldots, S\} \quad (14)$$

The value function is the expected sum of utility flows over infinite time periods. The expectation operator is over the sales shock $\epsilon_t \sim i.i.d.N(0,1)$. The value function for a representative agent in segment *s* at time *t* with $\boldsymbol{state_t}$ is,

$$V_s = V_s(\boldsymbol{state_t}; \widehat{\boldsymbol{\Theta}}_s, \boldsymbol{\Lambda_s}) = E\{\sum_{t=0}^{\infty} \delta^t U_{st}\} \qquad (15),$$

where δ is the time discount factor and $\boldsymbol{\Lambda_s}$ is the parameter set for the utility function of segment *s*. Specifically, $\boldsymbol{\Lambda_s} = \{\boldsymbol{\gamma_s}, c\}$. Here, I assume that the cost of exerting effort *c* is known and thus focus on the estimation of risk aversion factors $\boldsymbol{\gamma_s}$ among different segments. As segment probabilities $\widehat{\boldsymbol{\pi}}$ is estimated in the first stage, I can simplify the value function for segments as below.

$$V = V(\boldsymbol{state_t}; \widehat{\boldsymbol{\theta}}, \boldsymbol{\Lambda}) = \sum_{s=1}^{S} \hat{\pi}_s\, E\{\sum_{t=0}^{\infty} \delta^t U_{st}\} \qquad (15)',$$

As in the conventional dynamic estimation, I assume that the time discount factor δ as 0.95 which here is the same as the true parameter for data generation.

I first construct the optimal value function for each segment. Using the estimated policy functions and the sales response function from the first stage and with the distribution of the sales shock ϵ known, I carry out the forward-simulation of the actions of sales agents and construct the value function of agents. In my setting, sales agents are in the infinite cycle of time horizon. However, in the estimation, I believe that the finite time up to 60 is enough to retain the value function.

The detailed forward simulation is as follows.

(1) From initial state $\boldsymbol{state_t}$, calculate the optimal actions as $\widehat{eff_{st}}(\boldsymbol{state_t})$.

(2) Draw sales shock ϵ_t from the standard normal distribution.

(3) Update state $\boldsymbol{state_{t+1}}$, using the realized sales $\hat{S}\left(\widehat{eff_{st}}(\boldsymbol{state_t})\right) + \epsilon_t$

I iterate from (1) to (3) till *t=60*. Then I average the sum of discounted utility flows over 60 periods, which becomes the estimate of the value function $\widehat{V_s}(\boldsymbol{state}; \widehat{eff_s}(\boldsymbol{state}); \widehat{\boldsymbol{\Theta}}_s, \boldsymbol{\Lambda_S})$. Then with the estimated segment probabilities, I could calculate $\hat{V}(\boldsymbol{state})$ with the optimal effort policy. For the state variables in the value functions $\hat{V}$, I choose 5275 discretized states used in data generation, and hence could derive 5275 value function outputs.

To estimate the utility parameters $\boldsymbol{\Lambda}$, I perturb the optimal effort policies in 200 different ways which could be parameterized as $\boldsymbol{\theta}'_i$ for i = 1, …, 200. Then I derive 200 perturbed value functions for the same 5275 states but with the perturbed effort policies up to t=60. Then the difference in the optimal value function and one of perturbed value function is retained as below.

$$Q_i = V\left(\boldsymbol{state}_t; \widehat{\boldsymbol{\theta}}, \boldsymbol{\Lambda}\right) - V(\boldsymbol{state}_t; \boldsymbol{\theta}'_i, \boldsymbol{\Lambda}) \quad \forall i = \{1, \dots, 200\} \tag{16}$$

As the value function with optimal action function is never less than that with deviated action function, Q_i is always greater than or equal to zero. Thus, I finally obtain the estimates of the structural parameters by minimizing the objective function below:

$$\frac{1}{200}\sum_{i=1}^{200}(min\{Q_i, 0\})^2 \tag{17}$$

The above objective function minimizes the case where the deviated value function is greater than the optimal value function based on the first stage estimates. Here, I set the same market errors ϵ_t at each t between the optimal and the deviated value function to minimize the effect from random errors on forward simulation.

To find the standard errors of second stage parameters, I follow the two-stage dynamic estimation of Bajari, Benkard and Levin (2007). I numerically find the gradients and the hessians of the objective functions in the first stage and in the second stage and then multiply adequate matrices for the standard errors of the structural parameters.

7. Results

I estimate the sales response functions and the utility functions in segment-wise. Especially, to check the effect of carryover in the estimation, I first conduct the estimation knowing the existence of carryover in equation $(1)''$. And then I conduct the estimation ignoring the existence of carryover in equation $(1)''$. (i.e. I set $\lambda = 0$.) Thus, the number of parameters in the first approach knowing the carryover is one larger than that in the second approach ignoring the carryover. And note that to minimize the effect from random errors on forward simulation in the second stage estimation, I give the same market shock ϵ_t at the same time t for both of the two approaches.

The results from the two approaches are in Table 5 and Table 6. The critical difference of the two approaches is in describing the heterogeneity of sales agents. When ignoring the carryover, I could not get the true numbers of segments in sales agents while acknowledging the carryover leads to the right segmentation of sales agents. Ignoring the carryover while it exists concludes that the number of segments is three while the true number is two. It seems that ignoring the carryover divides the less risk averse group in another two segments. However, acknowledging the carryover concludes that there are two segments, which is correct and moreover, the estimated segment sizes are close to true sizes based on the three-sigma rule.

Besides the segmentation, both the two approaches estimate the first stage parameters for sales response function well. The market variation σ is estimated close to true sizes based on the three -sigma rule. Also, estimating carryover factor is successful in the case of not ignoring carryover.

The focused heterogeneous parameters, risk aversion factors γ are estimated in segment-wise. The estimates when the carryover is ignored are poorer than those when the carryover is not ignored. This is because ignoring carryover results in wrong segmenting of the sales agents in the first stage estimation and thus affects the second stage estimation. However, the problem with the true approach of not ignoring the carryover is represented with too small standard errors in the second stage to include the true value based on three-sigma rule. I could not confirm that the estimated risk aversion factors are in the 95% bounds roughly calculated by the three-sigma rule. The plausible reason of this poor estimate could be in the interpolation while generating the data. With linear regression of orthogonal polynomials, I could make the R-square high as 0.87 but this would not be enough to interpolate well for generating the simulated data. I admit that the simulated data itself could lack accuracy for continuous values. But still, the different degrees of risk aversion result in the different patterns in the sales performance and also affect the estimation as to finding the right segmentation of sales agents.

Table 5. Parameter Estimates when ignoring the carryover

(a) Effort Policy Function

Variable	Segment 1		variable	Segment 2		variable	Segment 3	
	estimate	*s.e.*		*estimate*	*s.e.*		*estimate*	*s.e.*
Segment Size	0.59658	0.008201		0.21640	0.039836		0.18701	-
Intercept	1234.813	67.0126	*Intercept*	-398.129	77.9950	*Intercept*	111.510	203.2984
$\rho_1(QQ)$	-2399.200	195.7339	$\rho_1(QQ)$	-1535.39	5537.7350	$\rho_1(QQ)$	-158.894	292.6718
$\rho_2(QQ)$	1597.203	62.6342	$\rho_2(QQ)$	-596.463	52.9197	$\rho_2(QQ)$	138.196	252.3204
$\rho_3(QQ)$	-1011.740	83.7153	$\rho_3(QQ)$	-506.958	1840.3720	$\rho_3(QQ)$	-52.501	93.5652
$\rho_4(QQ)$	363.184	94.1653	$\rho_4(QQ)$	-198.329	114.2758	$\rho_4(QQ)$	27.500	49.53079
$\rho_5(QQ)$	-130.452	80.7166	$\rho_1(QQ)D2$	2045.539	5622.4610	$\rho_1(QQ)D3$	116.300	2551.2880
$\rho_1(QQ)D2$	21023.850	1767.6270	$\rho_2(QQ)D2$	99.018	163.4155	$\rho_2(QQ)D3$	19.540	266.0624
$\rho_2(QQ)D2$	-157.279	176.4500	$\rho_3(QQ)D2$	668.131	1864.0100	$\rho_3(QQ)D3$	32.136	781.2688
$\rho_3(QQ)D2$	11148.310	140.9823	$\rho_4(QQ)D2$	97.688	161.9900	$\rho_4(QQ)D3$	18.797	256.9963
$\rho_4(QQ)D2$	-156.125	195.9997						
$\rho_5(QQ)D2$	2473.325	389.0482						

1. $\rho_i(x)$ is a Chebyshev polynomial of x in degree i.
2. *D2* is a dummy for month 2 and *D3* is a dummy for month 3.
3. For the segment size, I re-parameterized it to restrict each segment probability set between 0 and 1 and to restrict the sum of all probabilities to be 1. In the parenthesis, the numbers represent the original 0 to 1 scaled probabilities. The standard errors are estimated from the re-parameterized estimates. For segment size s, the degree of freedom in the segment size parameters is (s-1).

(b) Sales Response Function

Variable		*True*	*Estimates*	*S.E.*
Market variation	$ln(\sigma)$	-4.6051701	-4.4290700	0.247632

(c) Utility Function

Variable			*True*	*Estimates*	*S.E.*
Degree of risk aversion	$\ln(\gamma)$	Segment 1	0.9162907	0.8979649	149281.22
		Segment 2	0.8979649	-6.7696002	50200.55
		-	-	0.8979649	113864.30

Table 6. Parameter Estimates when the carryover is considered

(a) Effort Policy Function

Variable	Segment 1		Variable	Segment 2	
	estimate	*s.e.*		*estimate*	*s.e.*
Segment size	0.600073	3.2298326	**Segment size**	0.399926	-
Intercept	-0.509415	7.2236407	***Intercept***	10710.120000	2561.7985191
$\boldsymbol{\rho_1(QQ)}$	0.068084	2.4367443	$\boldsymbol{\rho_2(lS)}$	-54.184310	79.3302963
$\boldsymbol{\rho_2(QQ)}$	0.116181	1.4478691	$\boldsymbol{\rho_3(lS)}$	80.228980	119.2318990
$\boldsymbol{\rho_3(QQ)}$	-0.579343	6.4755045	$\boldsymbol{\rho_4(lS)}$	-66.648260	95.9185972
			$\boldsymbol{\rho_5(lS)}$	33.538480	44.7543617
			$\boldsymbol{\rho_6(lS)}$	-8.923251	10.5407782
			$\boldsymbol{\rho_1(QQ)}$	-43239.860000	4260.7158659
			$\boldsymbol{\rho_2(QQ)}$	13734.970000	819.7258859
			$\boldsymbol{\rho_3(QQ)}$	-20610.630000	1771.4441268
			$\boldsymbol{\rho_4(QQ)}$	2844.845000	5420.6278698
			$\boldsymbol{\rho_5(QQ)}$	-3736.519000	1854.5409511
			$\boldsymbol{\rho_6(QQ)}$	-193.953400	2035.0883214
			$\boldsymbol{\rho_1(lS)D3}$	113.225000	3927.5961629
			$\boldsymbol{\rho_2(lS)D3}$	-75.167370	4592.0007876
			$\boldsymbol{\rho_3(lS)D3}$	27.480220	4879.2056869
			$\boldsymbol{\rho_4(lS)D3}$	9.374336	3227.2296625
			$\boldsymbol{\rho_5(lS)D3}$	-16.051850	1238.6402853
			$\boldsymbol{\rho_6(lS)D3}$	7.049693	225.3571797
			$\boldsymbol{\rho_1(QQ)D3}$	24114.700000	6929.6557366
			$\boldsymbol{\rho_2(QQ)D3}$	933.646800	2271.6124171
			$\boldsymbol{\rho_3(QQ)D3}$	12003.410000	871.4757618
			$\boldsymbol{\rho_4(QQ)D3}$	1420.884000	2107.7915178
			$\boldsymbol{\rho_5(QQ)D3}$	2366.065000	1077.9405320
			$\boldsymbol{\rho_6(QQ)D3}$	543.462100	436.8480284

1. $\rho_i(x)$ is a Chebyshev polynomial of x in degree i.
2. *D2* is a dummy for month 2 and *D3* is a dummy for month 3.
3. For the segment size, I re-parameterized it to restrict each segment probability set between 0 and 1 and to restrict the sum of all probabilities to be 1. In the parenthesis, the numbers represent the original 0 to 1 scaled probabilities. The standard errors are estimated from the re-parameterized estimates. For segment size s, the degree of freedom in the segment size parameters is (s-1).

(b) Sales Response Function

Variable		***True***	***Estimates***	***S.E.***
Carryover factor	$ln(\frac{\lambda}{1-\lambda})$	0	-0.0219450	0.6234153
Market variation	$ln(\sigma)$	-4.6051701	-4.5803730	0.2043847

(c) Utility Function

Variable			***True***	***Estimates***	***S.E.***
Degree of risk aversion	$ln(\gamma)$	Segment 1	0.9162907	0.8560872	0.0003319
		Segment 2	-6.9077553	-6.7304993	0.0002666

8. CONCLUSION

Personal selling is a primary marketing mix tool for research on how the compensation plan motivates the sales force and affects performance. But literatures have focused mainly on principal-agent framework or certain compositions of compensation features to discuss the effect of it. However, this paper allows the existence of carryover in sales, which is pervasive in industries. In addition of forward-looking behavior derived from quota-based compensation, carryover in sales adds up dynamics of sales agent's effort allocation varying across the level of risk aversion of sales agents.

From the simulation, I show that the different levels of risk aversion resulted in the different optimal effort policy functions and thus in different patterns of realized sales of sales agents. The highly risk averse set the base line of performance while the less risk averse fluctuate their sales above the highly risk averse. The frequency of achieving quotas is higher in the less risk averse group compared to the highly risk averse group. As the variance and the mean of future sales increase because of the presence of carryover, the highly risk averse try not to exert more effort to avoid the uncertainty from the increased sales.

Moreover, from the two-step estimation of structural parameters in the simulated data, I confirm that ignoring the carryover factor in the sales results in the wrong segment of sales agents in the first stage and thus the wrong estimates of risk aversion in the second stage. Thus, the presence of carryover in sales affects the estimation results significantly.

As shown in the simulated data, the degree of risk aversion derives the different patterns of performance in sales. It is shown that the less risk averse endure the uncertainty of increased future sales and exert more effort trying to achieve quotas for bonuses. Thus, from the sales data, the managers can easily conclude that the less risk averse are high-performers in the firm. For the future research, I want to show that in the presence of carryover, if the firm's quota-bonus plan might function as a filter through which only the less risk-averse are let in. By letting the sales agents getting in and out of sales agent pool freely, we can check whether the carryover affects the function of sales compensation as a filter. Using the contraction theory, we could expand our knowledge of carryover effect on sales dynamics.

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APPENDIX. Computing the expected utility function

Expectation and Variance of Wealth in Utility Function

$$W_t(S_t) = \begin{cases} rS_t & month\ 1,2 \\ I(Q_t \geq 1)(rq + B) + I(Q_t < 1)rS_t & month\ 3 \end{cases}$$

$$Q_t = Q_{t-1} + \frac{S_{t-1}^{\lambda} * exp(eff_t) * exp(\sqrt{\sigma}\epsilon_t)}{q}$$

let, $C_1 = r * S_{t-1}^{\lambda} * exp(eff_t)$, $C_2 = r * q + B$ *and* $C_3 = q * \frac{1-Q_{t-1}}{\frac{C_1}{r}}$

Expectation and variance of wealth over market variation, given eff_t

<u>*Month 3*</u>

$E(W_t)$

$$= E\left(I(Q_t < 1) * C_1 * exp(\sqrt{\sigma}\epsilon_t) + I(Q_t \geq 1) * C_2\right)$$

$$= C_1 * E\left(I(Q_t < 1) * exp(\sqrt{\sigma}\epsilon_t)\right) + C_2 * P(Q_t \geq 1)$$

$$= C_1 * E\left(I\left(\left(Q_{t-1} + \frac{S_{t-1}^{\lambda} * exp(eff_t) * exp(\sqrt{\sigma}\epsilon_t)}{q}\right) < 1\right) * exp(\sqrt{\sigma}\epsilon_t)\right)$$

$$+ C_2 * Pr\left(\left(Q_{t-1} + \frac{S_{t-1}^{\lambda} * exp(eff_t) * exp(\sqrt{\sigma}\epsilon_t)}{q} \geq 1\right)\right)$$

$$= C_1 * E\left(I\left(exp(\sqrt{\sigma}\epsilon_t) < C_3\right) * exp(\sqrt{\sigma}\epsilon_t)\right) + C_2 * Pr\left(exp(\sqrt{\sigma}\epsilon_t) \geq C_3\right)$$

$$if, Q_{t-1} < 1$$

$$E(W_t) = C_1 * E\left(exp(\sqrt{\sigma}\epsilon_t) \,|\epsilon_t < \frac{1}{\sqrt{\sigma}} log(C_3)\right) + C_2 * (1 - \Phi\left(\frac{1}{\sqrt{\sigma}} log(C_3)\right))$$

$$= C_1 * \int_{-\infty}^{\frac{1}{\sqrt{\sigma}} log(C_3)} \frac{1}{\sqrt{2\pi}} exp\left(\sqrt{\sigma}\epsilon_t - \frac{1}{2}\epsilon_t^2\right) d\epsilon_t + C_2 * (1 - \Phi\left(\frac{1}{\sqrt{\sigma}} log(C_3)\right))$$

$$= C_1 * exp(\frac{\sigma}{2}) * \int_{-\infty}^{\frac{1}{\sqrt{\sigma}}log(C_3)} \frac{1}{\sqrt{2\pi}} exp(-\frac{1}{2}\ (\epsilon_t - \sqrt{\sigma})^2) d\epsilon_t + C_2 * (1 - \Phi\left(\frac{1}{\sqrt{\sigma}} log(C_3)\right))$$

$$= C_1 * exp(\frac{\sigma}{2}) * \int_{-\infty}^{\frac{1}{\sqrt{\sigma}}log(C_3) - \sqrt{\sigma}} \frac{1}{\sqrt{2\pi}} exp(-\frac{1}{2}\ (u_t)^2) du_t + C_2 * (1 - \Phi\left(\frac{1}{\sqrt{\sigma}} log(C_3)\right))$$

$$= C_1 * exp(\frac{\sigma}{2}) * \Phi\left(\frac{1}{\sqrt{\sigma}} log(C_3) - \sqrt{\sigma}\right) + C_2 * (1 - \Phi\left(\frac{1}{\sqrt{\sigma}} log(C_3)\right))$$

$$if, Q_{t-1} \geq 1$$

$$E(W_t) = C_2$$

$$E(W_t^2)$$

$$= E\left(I(Q_t < 1) * C_1^2 * exp\left(2\sqrt{\sigma}\epsilon_t\right) + I(Q_t \geq 1) * C_2^2\right)$$

$$= C_1^2 * E\left(I(Q_t < 1) * exp\left(\sqrt{\sigma}\epsilon_t\right)\right) + C_2^2 * P(Q_t \geq 1)$$

$$= C_1^2 * E\left(I\left(\left(Q_{t-1} + \frac{S_{t-1}^{\lambda} * exp(eff_t) * exp\left(\sqrt{\sigma}\epsilon_t\right)}{q}\right) < 1\right) * exp\left(\sqrt{\sigma}\epsilon_t\right)\right)$$

$$+ C_2^2 * Pr\left(\left(Q_{t-1} + \frac{S_{t-1}^{\lambda} * exp(eff_t) * exp\left(\sqrt{\sigma}\epsilon_t\right)}{q} \geq 1\right)\right)$$

$$= C_1^2 * E\left(I\left(exp\left(\sqrt{\sigma}\epsilon_t\right) < C_3\right) * exp\left(\sqrt{\sigma}\epsilon_t\right)\right) + C_2^2 * Pr\left(exp\left(\sqrt{\sigma}\epsilon_t\right) \geq C_3\right)$$

$$if, Q_{t-1} < 1$$

$$E(W_t^2) = C_1^2 * E\left(exp\left(\sqrt{\sigma}\epsilon_t\right) | \epsilon_t < \frac{1}{\sqrt{\sigma}} log(C_3)\right) + C_2^2 * (1 - \Phi\left(\frac{1}{\sqrt{\sigma}} log(C_3)\right))$$

$$= C_1^2 * \int_{-\infty}^{\frac{1}{\sqrt{\sigma}}log(C_3)} \frac{1}{\sqrt{2\pi}} exp\left(\sqrt{\sigma}\epsilon_t - \frac{1}{2}\epsilon_t^2\right) d\epsilon_t + C_2^2 * (1 - \Phi\left(\frac{1}{\sqrt{\sigma}} log(C_3)\right))$$

$$= C_1^2 * exp(\frac{\sigma}{2}) * \int_{-\infty}^{\frac{1}{\sqrt{\sigma}}log(C_3)} \frac{1}{\sqrt{2\pi}} exp(-\frac{1}{2}\ (\epsilon_t - \sqrt{\sigma})^2) d\epsilon_t + C_2^2 * (1 - \Phi\left(\frac{1}{\sqrt{\sigma}} log(C_3)\right))$$

$$= C_1^2 * exp(\frac{\sigma}{2}) * \int_{-\infty}^{\frac{1}{\sqrt{\sigma}}log(C_3)-\sqrt{\sigma}} \frac{1}{\sqrt{2\pi}} exp(-\frac{1}{2}\ (u_t)^2) du_t + C_2^2 * (1 - \Phi\left(\frac{1}{\sqrt{\sigma}} log(C_3)\right))$$

$$= C_1^2 * exp(\frac{\sigma}{2}) * \Phi\left(\frac{1}{\sqrt{\sigma}} log(C_3) - \sqrt{\sigma}\right) + C_2^2 * (1 - \Phi\left(\frac{1}{\sqrt{\sigma}} log(C_3)\right))$$

$$if, Q_{t-1} \geq 1$$

$$E(W_t) = C_2^2$$

$$Var(W_t) = E(W_t^2) - E(W_t)^2$$

$$if, Q_{t-1} < 1$$

$$Var(W_t) = C_1^2 * exp(\frac{\sigma}{2}) * \Phi\left(\frac{1}{\sqrt{\sigma}} log(C_3) - \sqrt{\sigma}\right) + C_2^2 * (1 - \Phi\left(\frac{1}{\sqrt{\sigma}} log(C_3)\right))$$

$$-\left\{C_1 * exp\left(\frac{\sigma}{2}\right) * \Phi\left(\frac{1}{\sqrt{\sigma}} log(C_3) - \sqrt{\sigma}\right) + C_2 * \left(1 - \Phi\left(\frac{1}{\sqrt{\sigma}} log(C_3)\right)\right)\right\}^2$$

$$if, Q_{t-1} \geq 1$$

$$Var(W_t) = C_2 - C_2^4$$

Month 1,2

$$E(W_t)$$

$$= E\left(r * S_{t-1}^{\lambda} * exp(eff_t) * exp\left(\sqrt{\sigma}\epsilon_t\right)\right)$$

$$= E\left(C_1 * exp\left(\sqrt{\sigma}\epsilon_t\right)\right)$$

$$= C_1 * E\left(exp\left(\sqrt{\sigma}\epsilon_t\right)\right)$$

$$= C_1 * exp(\frac{\sigma}{2})$$

$E(W_t^2)$

$= E\left(C_1^2 * exp\left(2\sqrt{\sigma}\epsilon_t\right)\right)$

$= C_1^2 * E\left(exp\left(2\sqrt{\sigma}\epsilon_t\right)\right)$

$= C_1^2 * exp(2\sigma) * \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} exp(-\frac{1}{2}\left(\epsilon_t - 2\sqrt{\sigma}\right)^2) d\epsilon_t$

$= C_1^2 * exp(2\sigma)$

$Var(W_t) = E(W_t^2) - E(W_t)^2$

$= C_1^2 * exp(2\sigma) - \left\{C_1 * ex\,p\left(\frac{\sigma}{2}\right)\right\}^2$

국 문 초 록

판매 이월 효과와 위험 회피도
: 영업사원 보상 체계에서의 동적 인센티브

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지난 시기로부터 판매의 이월 효과가 있는 경우, 쿼터-보너스 결합의 보상체계 하에서 영업사원이 본인의 영업 노력 배분을 이산적 (discrete), 동태적 (dynamic)으로 어떻게 결정하는지 풀어 보았다. 다이나믹 프로그래밍 (dynamic programming)을 통해 얻은 해를 바탕으로 영업 사원을 두 가지 그룹으로 구분하여 영업 데이터를 만들었다: 한 그룹은 위험 회피 정도가 높고, 다른 한 그룹은 위험 회피 정도가 낮게 설계하였다. 판매의 이월효과가 다음 시기 판매량의 기대값 뿐만 아니라 그 분산도 높이기 때문에, 영업사원에게 최적의 영업 노력 배분과 이에 따른 판매량 형태는 위험 회피 정도에 따라 달라진다. 위험 회피 정도가 높은 그룹의 영업 사원이 판매량의 기본을 맞추고, 위험 회피 정도가 낮은 그룹의 영업사원이 전자의 판매량을 넘어 요동치는 판매량을 기록한다. 또한 쿼터 달성 빈도도 위험 회피 정도가 낮은 그룹이 그렇지 않은 그룹에 비해 더 높았다. 이렇게 다른 판매량 추이는 위험 회피 정도가 높은 집단이 판매량을 늘릴 때 증가하는 불확실성

(uncertainty)를 피하기 위해 노력을 더 많이 투입하지 않도록 조절하는 것으로 설명 가능하다.

Arcidiacono and Miller (2011)를 따라 그룹마다 최적 노력 배분 함수와 효용 함수를 두 단계에 걸쳐 추정해 보았다: 비모수 함수를 통해 조건부 선택 확률 (conditional choice probability)를 계산하고, EM 알고리즘을 통해 구조적 모수 (structural parameters)를 추정하였다. 추정 결과는 판매의 이월 효과가 존재하는데 이를 무시하고 추정했을 경우 영업사원의 그룹을 그 수와 크기 모두에 있어서 잘 추정하지 못한다는 것이다. 그 이유는 판매의 이월 효과를 무시하면 첫 번째 추정 단계에서 영업사원을 제대로 그룹화하지 못하고 따라서 이어지는 두 번째 추정에도 영향을 미치기 때문이다. 추정 결과는 판매의 이월 효과가 존재하는 경우에 영업부의 판매량 추이를 제대로 이해하기 위해선 판매의 이월 효과를 충분히 검토해야 한다는 것을 강조한다. 판매의 이월 효과를 무시하게 되면, 영업 사원을 세분화 하지 못해서 세분 그룹별 보상 체계를 설계할 때 비효율성을 낳게 된다.

주요어: 영업사원 보상체계, 다이나믹 프로그래밍, 이월효과, 위험회피도, 이질성, 2 단계 CCP 측정, 시뮬레이션

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