

**DESIGN OF SINGLE HUB CROSSDOCKING NETWORKS:
GEOMETRIC RELATIONSHIPS AND CASE STUDY**

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**DESIGN OF SINGLE HUB CROSSDOCKING NETWORKS:
GEOMETRIC RELATIONSHIPS AND CASE STUDY**

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To my parents,

Pichai and Suchada Kittithreerapronchai.

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SUMMARY

In the distribution network of a large retailer, shipments can either be transported by the retailer's own trucks or outsourced to third-party logistics (3PL) companies. In the former case, shipments are consolidated and transported from their origins through an intermediate facility, namely a crossdock. At a crossdock, shipments are unloaded, sorted, re-consolidated, loaded and transported to their destinations. The consolidation process offers economies of scale that reduce the transportation costs. At the same time, it increases travel distances and incurs handling costs at a crossdock. For this reason, consolidation is uneconomic for a shipment in which origin and destination are located close to one other, especially through a distant crossdock. It is cheaper to outsource transportation of such a shipment to 3PL companies.

This shipping decision raises a series of questions. Should a shipment be consolidated through a crossdock or outsourced to 3PL companies? How do facility locations, the operational cost of a crossdock and mode of shipments influence the shipping decision? Can the robustness and potential growth of a crossdock be measured? How does outsourcing affect the robustness and potential growth of a crossdock?

We formulate a strategic model of a retailer's distribution network as an economic trade-off between consolidated shipments through a crossdock and outsourced shipments to 3PL companies. We study the locus of facility locations where the costs of a consolidated shipment and an outsourced shipment are equal and discover that the trade-off can be modeled by classical geometric curves, particularly an ellipse, a hyperbola, a limaçon and a Cartesian oval. These curves can be developed into a preliminary routing and locating tool. We also observe interesting connections between the single hub crossdocking network and other fields of geometric study, such as Voronoi diagrams and geometric inversion.

In addition, the area bounded by these curves represents the likelihood in which a particular shipment is consolidated through a crossdock. We expand this concept to multiple

vendor-store pairs and suggest an index that measures robustness and potential growth of a particular crossdock. This asymptotic-probability index explains economic driving factors of consolidation and outsourcing. Although the derivation of the index is limited by the dimension and spatial distribution of facilities, its numerical value can be determined by a computer simulation. Therefore, we use Monte Carlo simulation to compute the proposed index to explain the outsourcing and the interaction between TL threshold[†] and mode of shipments. The analysis and computer simulation suggest that outsourcing may cause an adverse effect in a single hub crossdocking network, resulting in the abrupt reduction of consolidated shipments in the network. Furthermore, we propose transportation planning to alleviate this effect and compare them to the optimal allocation.

The routing and locating application of the model is illustrated using the Home Depot distribution network. Our model predicts 5.5% and additional 1.0% savings in transportation cost by re-allocation of shipments and re-location of crossdocks, respectively. The empirical study shows that the adverse effect of outsourcing can be eliminated by limiting the number of crossdocks used by each store.

[†]TL threshold is a minimal freight requirement for a TL shipment (see Section 5.1 for the definition).

CHAPTER I

TRANSPORTATION IN DISTRIBUTION NETWORKS

Transportation plays an important role in a distribution network, as it influences the price, the availability, the quality and the inventory of products. One primary means of reducing transportation costs is to implement a suitable transportation network design that exploits the freight characteristics, such as the size and the direction of shipments.

1.1 Transportation Network Design

In the US distribution network, the majority of freight is moved by trucks because of the accessibility of motor ways and the ability to control arriving shipments. In general, freight in trucking transportation can be classified by the size of a shipment as either a *full truckload* (TL) shipment or a *less-than truckload* (LTL) shipment. Typically, a carrier classifies freight by a weight threshold. If freight, weighs more than 10,000 pounds, for example, it is considered a TL shipment. Otherwise, it is categorized as an LTL shipment. A TL shipment is less expensive than an LTL shipment because typically the truck has higher utilization and freight can be carried directly to its destination.

The difference in shipping rate affects the design of a transportation network, which can generally be grouped into the following three types:

- Direct Shipment Network
- Hub-and-Spoke Network
- Single Hub Crossdocking Network.

1.1.1 Direct Shipment Network

The simplest design of a transportation network is a direct shipment network because it requires neither complex operation nor an intermediate facility, as shown in Figure 1.1.

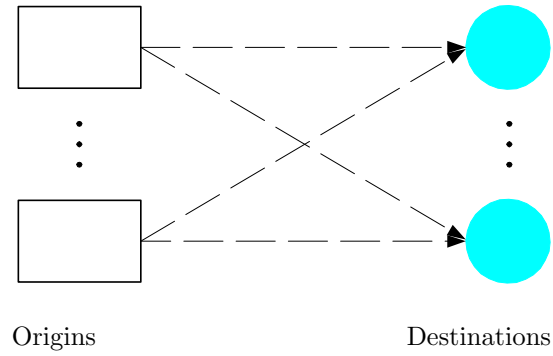


Figure 1.1: An example of the direct shipment network from origins to destinations

Each shipment in Figure 1.1 is directly transported from an origin to a destination. Without transit to any intermediate facility, the direct shipment network incurs little or no handling cost. Moreover, the transit time of the shipment depends only on the distance between an origin and a destination. An efficient direct shipment network should consist of a small number of origin-destination pairs, and the majority of shipments should be TL.

However, the distribution network of a large retailer usually has many origin-destination pairs. In 2000, for example, the Home Depot transportation network consisted of more than 200,000 origin-destination pairs. The majority of which were LTL, as shown in Figure 1.2.

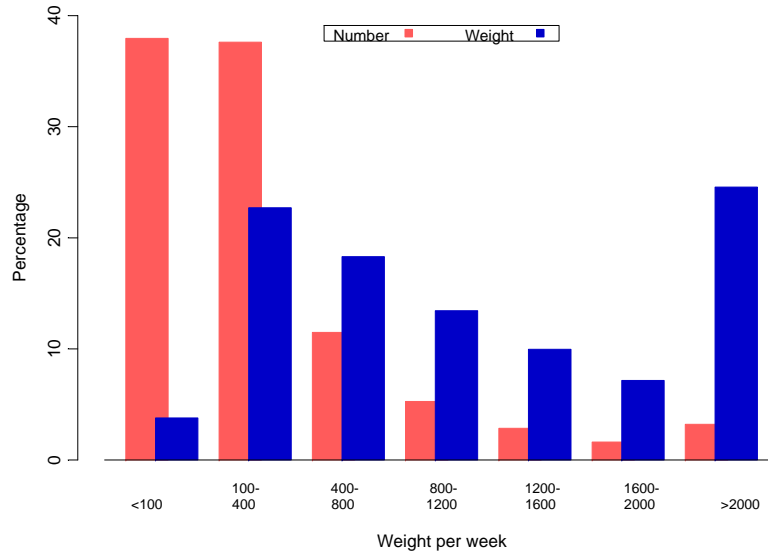


Figure 1.2: Over 70% of origin-destination pairs account for less than 30% of weight

In Figure 1.2, 70% of origin-destination pairs account for less than 30% of total annual weight in the network. For the Home Depot network, a direct shipment network would lead to unnecessarily high transportation costs. Other disadvantages of a direct shipment network for the Home Depot are low truck utilization and high number of total trucks. One approach to counter these disadvantages is to implement a *hub-and-spoke network*.

1.1.2 Hub-and-Spoke Network

An important concept in the modern transportation network is the hub-and-spoke network. This network requires an intermediate facility, called a *hub*. In the network, a hub is assigned to *terminals*, which are the origin and the destination of freight. Hubs, which are often located near big cities, connect and form the connected arcs from one terminal to another. Freight is shipped through a series of assigned hubs and arcs in the network from its origin to its destination. In a typical hub-and-spoke network, each hub usually directly connects to all other hubs. Therefore, a shipment needs at most to pass through two hubs that are assigned its origin and its destination, as shown in Figure 1.3.

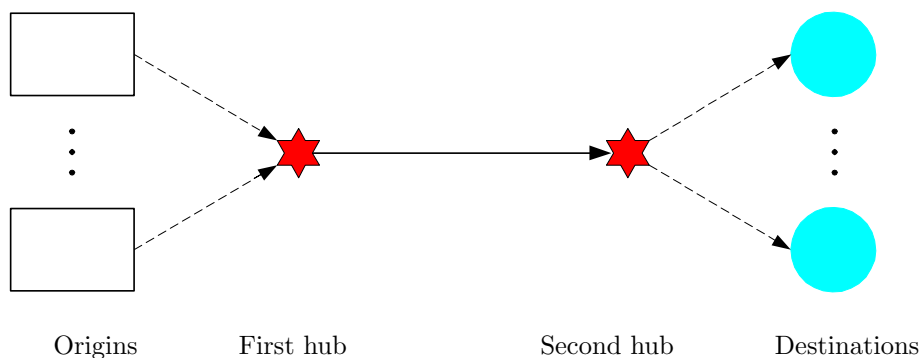


Figure 1.3: Example of the hub-and-spoke network from origins to destinations

In Figure 1.3, each origin ships an LTL shipment to the first hub. At the hub, shipments that share the same second hub are sorted, consolidated and shipped to the hub assigned to destinations. Because of the many terminals that are connected to the hub, shipments between hubs gain an advantage of the size of a shipment and become TL. At the second hub, the sorting and consolidation processes are repeated before an LTL shipment reaches a destination. The main advantages of the hub-and-spoke network are that the number of

shipments is reduced and economies of scale are realized.

An example of the hub-and-spoke network is the parcel delivery business in which a parcel station serves as a terminal of the network. Each parcel must be shipped to the assigned hub of its origin (the first hub), where a high-capability sorting facility is installed. Once sorted, a parcel is shipped to the associated hub of the destination (the second hub) and then delivered by the local delivery system of each destination. The shipment may pass through more than two hubs if an arc that connects the origin hub to the destination hub is unavailable.

In some cases, the traffic in a distribution network is high enough that shipments to and from a hub are usually TL. The shipments need transit only the first hub and can bypass the second hub. We refer such a network as a *single hub crossdocking network*.^{1.1}

1.1.3 Single Hub Crossdocking Network

The single hub crossdocking network is a variation of the hub-and-spoke network in which an origin and a destination are always assigned to the same hub, referred to as the *crossdock*.

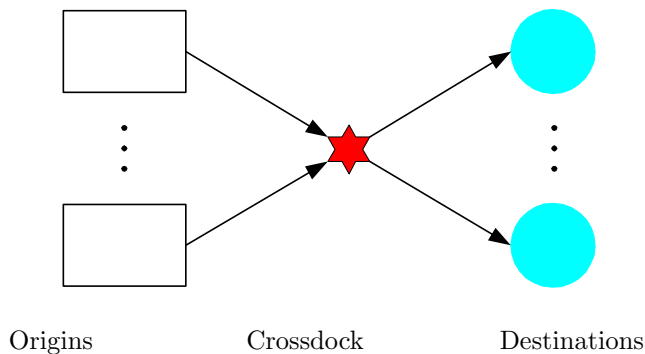


Figure 1.4: An example of the single hub crossdocking network from origins to destinations

Figure 1.4 shows a single hub crossdocking network in which shipments to or from a crossdock are TL. An efficient single hub crossdocking network should have a level of traffic flow between the direct shipment network and the hub-and-spoke network. An example of the single hub crossdocking network is the distribution network of a national retailer

^{1.1}The similar network is called *all shipments via central DC with crossdock* by Chopra and Meindl [20].

that consists of a large number of stores (destination) and vendors (origin). In such a network, freight can be shipped only through a single hub because of high demand and variety of products. Upstream of the hub, the total demand for a product is sufficient to generate a weekly TL shipment from each vendor because each store carries similar products. Downstream, each store carries high varieties of products such that a weekly shipment to each store is TL. Compared with the hub-and-spoke network, the single hub crossdocking network not only reduces the handling costs and the transit time at the hub, but also achieves economy of scale from the per-unit TL shipment transportation costs. This same economy of scale could be realized in the regular hub-and-spoke. However, additional handling costs and transit time are incurred at the second hub.

In North America, the single hub crossdocking network has been adopted and implemented by many large distribution networks, such as Home Depot and Costco [58]. However, the most well-known example of crossdocking is Wal-Mart [9]. The original purpose of a crossdock in Wal-Mart was to gain an advantage of discounted prices by purchasing large quantities of products. Wal-Mart is capable of knowing when each store needs a product and in what quantity as the information technology is embedded into its supply chain. As a result, Wal-Mart not only consolidates products from many vendors and many stores into few shipments, but also coordinates them such that the transit time of shipments to re-consolidate at the facilities is minimized. This removes two of the most labor-intensive activities of the traditional warehouse: storage and order picking. The implementation of a crossdock also increases the inventory turnover because of the short transit time. Although the terms “hub” and “crossdock” can be used synonymously, this dissertation uses the term “crossdock” to emphasize the short transit time of the process within the facility, usually less than a day.

1.2 Motivation

In the distribution network of a large retailer, shipments can either be transported by the retailer's own trucks or outsourced to third-party logistics (3PL) companies. In the former case, shipments are consolidated and transported from their origins through a crossdock. Freight that is consolidated through a crossdock is referred to as a *consolidated shipment*. At a crossdock, shipments are unloaded, sorted, re-consolidated, loaded and transported to their destinations. The consolidation offers economies of scale that reduce the transportation costs. At the same time, it increases travel distances and incurs handling costs at a crossdock. For this reason, consolidation is uneconomic for a shipment in which its origin and destination are located close to one other, especially through a distant crossdock (Figure 1.5). Such a shipment, thus, should be excluded from the network and outsourced to 3PL companies that transport the shipment from a vendor to a store. This *outsourced shipment* incurs a higher shipping-cost rate. Outsourced shipments are transported by the 3PL's distribution network. Hence, the shipments appear in the retailer's network as if they were shipped directly to stores.

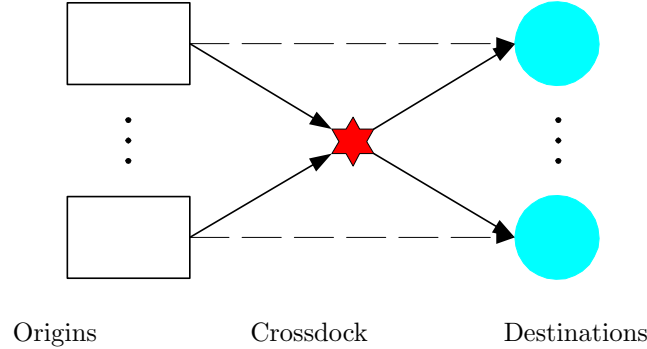


Figure 1.5: An example of the single hub crossdocking network from origins to destinations with outsourced shipments

The crossdock location influences the decision of whether the shipment should be an outsourced shipment or a consolidated shipment. Nevertheless, we argue that the transportation decision is also affected by the traffic of the network. In an extreme case, a retailer's distribution network has high traffic flows, and both the incoming shipment and

the outgoing shipments of a crossdock are TL. For example, the Home Depot, the world's largest home improvement retailer and the second largest retailer in the United States [59], can provide sufficient sales to generate TL shipments to or from a crossdock. A supplier ships TL shipments to a crossdock that carries products for many of Home Depot's stores. At the crossdock, the products are re-consolidated and shipped in TL shipment(s) to destinations (Home Depot stores).

In some cases, consolidated shipments are economically justified for a low traffic network that has mixed shipments between TL and LTL. For example, a drill-bit vendor probably does not send TL shipments to a Home Depot crossdock because of the size of the products. However, at the crossdock, the shipment of drill bits can be consolidated with other products and shipped in TL to Home Depot stores. One may dispute this use of a crossdock as theoretical because the frequency of shipments can be adjusted to accumulate more freight such that all shipments through a crossdock are TL. However, we argue that the mixed shipments between TL and LTL can be applied to an expedited shipment. It also increases the service level of a network and reduces inventory. In addition, a shipment planner should be aware of a trade-off between an LTL shipment and inventory in a crossdock. As a result, the consolidated shipment can be categorized based on the size of the shipment in each leg into three *mode of shipments*, as follows:

(TL, TL) is the most efficient mode of shipments, in which assumes the presence of sufficient other product for a crossdock to receive and ship freight via TL. An example of this mode of shipments is a shipment from a national manufacturer to a Home Depot store.

(LTL, TL) refers to shipments, for which the incoming and the outgoing modes of transportation are LTL and TL, respectively, such as a shipment from a small vendor to a Home Depot store.

(TL, LTL) refers to shipments, for which the incoming and the outgoing modes of transportation are TL and LTL. A situation in which this mode of shipments may arise is a shipment from Black and Decker, a national manufacturer of electronic tools, to a

local neighborhood hardware store.

The mode of shipments may be viewed as a decentralized perspective with a potential to be developed into a routing and locating tool. Obviously, the lack of a central perspective may result in a sub-optimal network or a myopic decision, yet we insist that such a decentralized routing approach is necessary and practical. In a retailer's distribution network, vendors and stores are constantly added in and/or removed. Therefore, it is difficult^{1,2} to update the changes and to re-optimize the network every period. To keep up with the dynamics, a shipment planner may have to decide without central information whether a shipment should be an outsourced shipment that is handled by 3PL companies, or a consolidated shipment that transits to a crossdock. In addition, the planner has sufficient information to practically decide whether the shipment should be an outsourced shipment or a consolidated shipment. He typically knows locations of all facilities (i.e., vendors, stores and crossdocks). For modes of shipments, the shipping planner can foresee the quantities of incoming and outgoing shipments of each crossdock or the relative freight of each route. This freight information is used to predict traffic flow each route whether a shipment is TL or LTL and estimate modes of shipments.

1.2.1 Research Questions

- Which shipments be outsourced to 3PL companies and which should be consolidated through a crossdock?
- How do facility locations, the operational cost of a crossdock, and mode of shipments influence the shipping decision?
- How might one measure the robustness and potential growth of a crossdock?
- How does outsourcing affect the robustness and potential growth of a crossdock?

The answers to these questions provide insights into how to achieve maximum efficiency from a single hub crossdocking network.

^{1,2}If not, impossible

1.3 Purpose

- We formulate a strategic model of a retailer’s distribution network as an economic trade-off between outsourced shipments to 3PL companies and consolidated shipments through a crossdock. We study the locus of facility locations where the costs of an outsourced shipment and a consolidated shipment are equal and discover that the trade-off can be explained by classical geometric curves.
- We show that the shape of curves is influenced by the modes of shipments, the relationships of facilities, and the operational costs of the crossdock. Based on transportation costs, the curves partition an area into an outsourced-shipping region and a consolidated-shipping region. The shipping decision can be determined by such regions and facility locations. In addition, these curves can be developed into a preliminary routing and locating tool.
- We explore interesting connections between the curves and other fields of geometric study, such as Voronoi diagrams and geometric inversion. In particular, some of the economic trade-off curves are special cases of Voronoi diagrams. In addition, a curve formed by one group of facilities can be re-stated as other related curves by geometric inversion.
- We observe that the area of a consolidated-shipping region represents the likelihood that a particular shipment is consolidated through a crossdock. We expand this concept to multiple vendor-store pairs and propose an index that measures robustness and potential growth of a particular crossdock. This asymptotic-probability index explains economic driving factors of consolidation and outsourcing.
- We incorporate the interaction between TL threshold and mode of shipments into the model and study the asymptotic-probability index. The results suggest that outsourcing may cause an adverse effect in a single hub crossdocking network, resulting in the abrupt reduction of consolidated shipments in the network.
- We analyze the data of the Home Depot distribution network and embed them into

the model, which predicts 5.5% and additional 1.0% savings in transportation cost by re-allocation of shipments and re-location of crossdocks, respectively. The empirical study shows that the adverse effect of outsourcing can be eliminated by limiting the number of crossdocks used by each store.

CHAPTER II

LITERATURE REVIEW

Academic literature on the single hub crossdocking network can be categorized into three streams as follows:

- Crossdocking Operations
- Crossdocking Routings
- Crossdocking Locations

2.1 Crossdocking Operations

In the supply chain, a crossdock can be considered as a high-speed warehouse that facilitates the direct movement of freight from receiving docks to shipping docks in minimal transit time. In the crossdocking operation, researchers have focused on operating activities inside a crossdock (e.g., congestion reduction, scheduling, queuing and layout), while ignoring the interaction between a crossdock and other facilities. Labor-cost reductions and the operating activities of a crossdock were first addressed by Gue [30] and studied intensively by Bartholdi and Gue [5], who discussed three types of congestion inside a crossdock and ways to reduce the congestion. They proposed the arrangement of docks for both incoming and outgoing shipments by simulations and found that “look-ahead” information and assigning shipments to minimize workload can prevent congestion and reduce labor costs.

Typically, trucks operate on time window; therefore, several studies have addressed truck scheduling. The incoming and outgoing truck scheduling of a crossdock was addressed by Li *et al.* [42], who extended the concept of the machine-scheduling problem and formulated a mixed integer-programming (MIP) model of the problem. The complexity of this problem was found to be a strongly \mathcal{NP} -complete by Chen *et al.* [19], who mathematically reduced the 3-Partition problem. They also proposed heuristics and compared the computational time and objective function with the MIP model. In general, freight is shipped through a

crossdock by trucks and moved across docks by material handling equipments. However, the trailer of a truck can be utilized as a material handling equipment, as some freight is left inside an incoming trailer. The trailer and freight then are moved around a crossdock to an outgoing dock. The use trailers as the material handling equipments was formulated into a mathematical model by Yu [66], who showed that the congestion within a crossdock could be reduced because a partial shipment remains inside trailers. A variation of this scheduling problem with a temporary storage was considered by Yu and Egbelu [67], who presented a MIP model to minimize makespan (the maximum completed time) and proposed a heuristic method that independently computes the scheduling of incoming and outgoing trailers. The complexity of the minimizing makespan problem is found to be a strongly \mathcal{NP} -complete by Chen and Lee [17], who reduced the minimizing makespan problem in a flow shop with precedence constraints. They also suggested a lower bound and a branching scheme to enumerate its linear programming (LP) relaxation. Recently, Chen and Song [18] extended the minimizing makespan problem by considering multiple incoming and outgoing trailers and proposed several heuristics to determine the incoming trailer scheduling.

When freight is unloaded, it is stacked in an outgoing area, creating a queuing system within a crossdock. Unlike in a conventional queuing system, freight cannot move forward autonomously to an available space in the queue. This unconventional queuing system is called a *staging queue*. The staging queue was addressed and modeled by Gue and Kang [31] and recently by Bartholdi *et al.* [6], who analyzed various operating policies and their characteristics in the queuing system.

In addition to the dock arrangement and the scheduling, researchers found that the number of dock doors also impacts labor costs in a crossdock because they determine the travel distance and affect the congestion. The relationship between the shape of a crossdock and the number of dock doors was illustrated by Bartholdi and Gue [7], who examined the similarities and differences between a crossdock and an airport terminal. They also suggested the shape of a crossdock according to the number of dock doors. For a crossdocking background and general information, we recommend a book [46] by Napolitano, which elaborates on history, development, types of crossdock, product selection, equipments and

operations.

2.2 *Crossdocking Routings*

Transporting freight accounts for the majority of costs in a single hub crossdocking network. The reduction of transportation costs requires an understanding of the network and the optimization of routing, especially the transportation network optimization and the vehicle routing problem. Early studies analyzed the properties of a small simplified transportation network. One of the most important analyses of transportation and inventory costs was addressed by Blumenfeld *et al.* [8], who extended the economic-order-quantity (EOQ) model to study the interaction among costs involved in both an outsourced shipment and a consolidated shipment and to determine each optimal shipment size. They also discussed the trade-off between an outsourced shipment and a consolidated shipment. The comparison between an outsourced shipment and a consolidated shipment in the single intermediate facility environment was first introduced by Hall [35], who modeled the total cost function as a concave function of traffic flow. The general convex cost function of this problem was studied by Klinecicz [37], who suggested that the decomposition of the network by the origin-destination pair could reduce the computation time. The constant flows in the single hub crossdocking network was discussed by Bramel and Simchi-Levi [10], who analyzed the asymptotic properties and order policy for a single crossdock. They assumed that no inventory is held in a crossdock. This assumption was later relaxed and the inventory benefit of decentralized single hub crossdocking network was analyzed by Waller *et al.* [61], who compared the lower and upper bounds of inventory level and the total transportation costs. The results suggest that although crossdocking reduces the inventory in a distribution center, stores require additional inventories to prevent stock out and accommodate increase in lead time. Therefore, the inventory benefit of crossdocking, which is defined as the inventory difference between a distribution center and stores, decreases asymptotically as number of store increases. They also discovered that a crossdock also reduces the bullwhip effect because of the reduction of inventory buffer and the centralized information. A review of the properties of a simplified network can be found in Daganzo [23].

In general, a single hub crossdocking network consists of many crossdocks, the interactions of which cannot be fully captured by a simplified network. The practical setting of the single hub crossdocking network often involves a large scale transshipment problem. Researchers have emphasized quality of solutions and computational times, which usually involve optimization and heuristic techniques. For example, Gümüş and Bookbinder [32] considered the multi-echelon network of the problems and constructed an MIP model. The routing problem was incorporated in the single hub crossdocking network by Liu *et al.* [43], who formulated an integer-programming (IP) model that integrates a consolidated milk-run shipment and an outsourced shipment. Teo and Shu [57] studied a similar problem and solved it with the column generation technique. Many numerical and case studies of the assignment and transshipment problems were reported, such as the US Postal Service (USPS) network by Donaldson *et al.* [25], the automobile network by Ratliff *et al.* [54], and the third-party logistics company by Lapierre *et al.* [39]. In the USPS study, Donaldson *et al.* formulated the network problem as an IP model. Due to the size of the problem, the instance could not be solved by the branch and bound method or the Bender's cut. As a result, they implemented a Lagrangian-based heuristic algorithm that returns a solution within 5% of the lower bound of the instance. Ratliff *et al.* reported the implementation of Ford railway crossdocks, called *mixing centers*, across the United States. Instead of an outsourced shipment from Ford plants, newly assembled automobiles are consolidated and shipped through mixing centers by railway network. At the mixing center, the automobiles are sorted and shipped to the dealers. The implementation of the mixing centers has reduced transportation time and inventory cost. Recently, Lapierre *et al.* analyzed a Canadian third-party logistics company that provides four types of shipping services and obtained the solution of the problem with a tabu search.

Researchers have recently addressed game-theory aspects of a distribution network in which the network is decentralized and a party in the network attempts to minimize his own transportation costs but overlooks the total transportation costs. The decentralized shipping decisions lead to economic conflict and transportation planning between parties. For example, the tradeoff between a direct shipment and a consolidated shipment was presented

by Hsu and Hsieh [36], who proposed a two-objective (shipping and inventory cost) model in the maritime network. In the Pareto equilibrium, they found that an outsourced shipment is preferred when the origin-destination pairs increase. The review of routing and operation issues of rail, motor and intermodal can be found in Crainic [22].

2.3 *Crossdocking Locations*

One of the important aspects of a single hub crossdocking network is the crossdock locations since the spatial distribution of facilities affects the travel distances and the ability to consolidate freight of the network. Locating crossdocks can be considered as a facility location problem, one of the classical problems in operations research literature. An overview of the facility location problem can be found in [24, 29, 34, 38, 55]. In particular, this dissertation overlaps with several fields in the facility location problem as follows:

- The Weber Problem and the Location-Allocation Problem
- Hub Location

2.3.1 **Weber Problem and Location-Allocation Problem**

One of the first problems of the facility-location community is the Weber problem [27]. Given general fixed-location facilities (source), each of which has different sizes of shipments, the goal of the Weber problem is to locate a single central facility (facility) in a continuous space among others such that the sum of the product between the weight and the distance is minimal. The early Weber problem was solved by a mechanical method in which the source locations, the distances and the sizes of shipments were physically represented by holes in a frame, the string lengths and relative attached weights, respectively. The optimal location of a facility is the equilibrium of this mechanical system. The physical interpretation of the method has inspired the study of the geometric relationship of locations, which allows the interpretation between a mathematical model and a relationship of locations. Another important computational geometry concept is the *Voronoi diagram*, which can be applied to solve the continuous covering location problem [53]. The Voronoi diagram also overlaps with main results of this dissertation; therefore, the terminology of the Voronoi diagram is

discussed in Section 3.7.1. The concept of the Voronoi diagram is applied to solve non-convex facility location problems by Drezner and Suzuki [28], who proposed an approximation global search algorithm, called a *big triangle small triangle* algorithm, that divides feasible regions into triangles by Voronoi diagram and Triangulation. In each iteration, a triangle is partitioned into four smaller triangles, and the upper and lower bounds are updated until an ϵ -optimal solution is found.

The most effective way to solve the Weber problem, however, is the Weiszfeld heuristic, an iterative gradient-based approach proposed by Weiszfeld [65]. Successful applications of the Weiszfeld heuristic in Euclidian space and a sphere space can be found in Aykin [2] and Aykin [3], respectively. Recently, Drezner *et al.* [26] compared the Weiszfeld heuristic with the big triangle small triangle search algorithm in a location model in which traveling time is not necessarily linear proportional to the distance. They found that both algorithms are efficient because the Weiszfeld heuristic requires less computational time but needs to perform many times with different starting points to achieve the same solution quality as the big triangle small triangle algorithm does. The comprehensive details of the Weiszfeld heuristic are discussed in Section 6.3.1, in which the heuristic is implemented for the Home Depot data. An alternative to the Weiszfeld heuristic is an algorithm for solving a large-scale multi-facility location problem, called *Newton blanketing method*, proposed by Levin and Ben-Israel [41]. The Newton blanketing method extends the Taylor series expansion to update the upper bound and the lower bound. A computational comparison shows that the Newton blanketing method performs slightly better than the Weiszfeld heuristic. The comparison between heuristics is investigated by Brimberg [13], who implements traditional heuristics into empirical data. He shows that heuristics give poor results when the number of facilities to locate is large. Recently, Brimberg and Salhi [12] have introduced a generalization of the multi-facility Weber problem, called *zone-dependent fixed cost location problem*, which can be considered as an approximation of the discrete facility location problem.

In the multi-facility network, each facility may connect to only a few sources because of the costs of establishing and operating connections. For example, a store may limit a number of crossdocks in which it receives shipments to reduce a number of transactions

and increase sizes of each shipment. This shipment allocation problem between facilities and sources is referred to as a location-allocation problem because the solution of the problem involves two simultaneous questions: Where are the optimal locations of facilities? What is the optimal allocation? The classical literature in the location-allocation problem is Cooper [21], who suggested that the problem can be considered as two relatively easy-to-solve sub-problems: location and allocation. In the location sub-problem, the allocation of facility-source pairs is pre-determined, while the location of facilities is fixed in the allocation sub-problem. Each sub-problem is solved until the stopping conditions are met. The concept of two alternative sub-problems is similar to the idea behind the Weiszfeld heuristic (see Section 6.3.1). As a result, this algorithm is sometimes referred to as the Weiszfeld-Cooper algorithm [3, 48]. O’Kelly [48] applied this algorithm to solve the single hub crossdocking network and hub-and-spoke network in the Euclidian space. He argued that the single hub crossdocking network is a variant of the Weber problem, and the location-allocation problem can be seen as a sub-problem of the hub location problem, which is discussed in Section 2.3.2.

Since 1909, the Weber problem has captured the interest of researchers, who have proposed many algorithms and variations of the problem. A review of the Weber problem and its extensions can be found in [24, 27, 44].

2.3.2 Hub Location

The problem of locating transit facilities in a hub-and-spoke network is known as the hub location problem. In a typical setting of a hub location problem, the inter-hub shipments receive a discounted shipping rate because of quantity of shipments. In addition, researchers have assumed that every hub is connected to other hubs; therefore, a shipment needs to transit only two hubs. The majority of early studies were dedicated to the airline industry and the trucking industry. Campbell [14] formulated the problem as a 0-1 IP problem for the p -hub median problem, the un-capacitated hub location problem, the p -hub center problem and the hub covering problem. He also considered the flow threshold on spoke. The empirical procedures of these problems were improved by O’Kelly *et al.* [51], who incorporated a

heuristic solution to strengthen the lower bound and presented exact and heuristic solution procedures for the design of the hub-and-spoke network. O’Kelly and Bryan [49] analyzed the sensitivity of the hub location with respect to the discounted shipping rate. The economy of scale of inter-hub flows was addressed by O’Kelly and Bryan [50], who showed that the flow-independent assumption affects actual transportation costs and optimal hub locations. Recently, Campbell and Ernst [15] proposed a generalization of the hub location problem, called the *hub arc problem*, in which a hub is connected to only selected hubs and some of the inter-hub shipments may not receive a discounted shipping rate. Hence, an inter-hub shipment may transit more than two hubs before reach its destination.

In general, location and routing problems are the important parts of the distribution network design. The integrated model of the distribution network design was introduced by Perl and Daskin [52], who studied warehouse location-routing problem. They proposed a three-indexed formulation to evaluate number, size and central hub locations in a regional distribution network. This model was later extended by Ambrosino and Scutella [1], who incorporated a *transit point*, a small local hub that facilitates shipments between a central hub and destinations in the distribution network. Shipments in this four-echelon network can reach to a destination by two channels: a direct shipment from a central hub or milk-run from a transit point.

The location and routing problem of a crossdock was also studied by Lee [40], who formulated an optimization model with multiple flow scenarios and proposed a continuous approximation method to determine the number of crossdocks that minimizes total transportation costs. By randomized numbers and locations of crossdocks in simulation, he argued that the optimal numbers of crossdocks can be determined without their optimal locations. In addition, he asserted that the crossdock locations depends on data set. A comprehensive review of the Hub location problem can found in Campbell *et al.* [16].

2.4 *Organization of Topics*

The remaining chapters are organized as follows. In Chapter 3, we formulate a strategic model of a retailer’s distribution network as an economic trade-off between consolidated shipments through a crossdock and outsourced shipments to 3PL companies. In addition, the properties of this decentralized single hub crossdocking network are analyzed. We study the locus of facility locations where the costs of a consolidated shipment and an outsourced shipment are equal and discover that the trade-off can be explained by classical geometric curves. We investigate the properties of these curves and related mathematical transformation. These curves can be developed into a preliminary routing and locating tool. We also discover interesting connections between the single hub crossdocking network and other fields of geometric study, such as Voronoi diagrams and geometric inversion.

In addition, the area bounded by these curves represents the likelihood that a particular shipment is consolidated through a crossdock. In Chapter 4, we expand this concept to multiple vendor-store pairs and suggest an index that measures robustness and potential growth of a particular crossdock. This asymptotic-probability index explains economic driving factors of consolidation and outsourcing. Although the derivation of the index is limited by the dimension and spatial distribution of facilities, its numerical value can be determined by a computer simulation.

In Chapter 5, we introduce a concept of TL threshold, an interaction between amounts of freight and type of shipment. We use Monte Carlo simulation to compute the proposed index and explain the outsourcing and interaction between TL threshold and mode of shipments in the unit-interval single hub crossdocking network. The simulation results suggest that outsourcing may cause an adverse effect in the network, resulting in an abrupt reduction of consolidated shipments in the network. Furthermore, we propose three levels of transportation planning at a network to alleviate this effect and compare them with the optimal shipment allocation.

In Chapter 6, we discuss a case study of a large-scale network. In particular, we embed the transportation data of the Home Depot distribution network into the mathematical model and suggest incremental improvements in shipping policies. The model predicts

5.5% and additional 1.0% savings in transportation cost by re-allocation of shipments and re-location of crossdocks, respectively. The empirical study of the Home Depot data shows that the adverse effect of outsourcing can be eliminated by limiting the number of crossdocks employed by each store.

CHAPTER III

GEOMETRIC RELATIONSHIP OF FACILITIES

In this chapter, we consider shipment decisions in the single hub crossdocking network and discover that the tradeoff between an outsourced shipment and a consolidated shipment can be explained by geometric relationships. We also establish the linkage between such geometric relationships and other fields of study.

3.1 Problem Description

Freight is shipped either as an outsourced shipment from its origin (vendor) to its destination (store) by 3PL companies or as a consolidated shipment through a crossdock. The consolidated shipment offers economies of scale that reduce the transportation costs. However, it increases travel distances and incurs handling costs at a crossdock. For this reason, the consolidated shipment is uneconomic for a shipment in which a vendor and a store are close to one another. Such a shipment should be outsourced to 3PL companies. This raises the following questions: What is the economic tradeoff between an outsourced shipment and a consolidated shipment? How do shipping parameters interact with the shipping decision? How do the parameters influence the spatial pattern? We formulate a mathematical model to explore the economic tradeoff between an outsourced shipment and a consolidated shipment through a single crossdock.

The problem and its notation can be described as follows. Vendor V_i must ship freight that weighs f_{ij} pounds to store S_j , which is located d_{ij} miles from the vendor. To minimize the total transportation costs, a shipment planner must decide whether to outsource freight to a 3PL company or to consolidate it to crossdock XD_p , for consolidation with other shipments. A successfully consolidated shipment may reduce transportation costs by a factor e , but the shipment must travel an additional $d_{ip} + d_{pj} - d_{ij}$ miles and incur handling costs h_p dollars per pound.

Throughout the chapter, we are interested in the set of facility points where an outsourced shipment and a consolidated shipment cost the same.

Definition 3.1.1. *A point is called an **iso-cost point** if the transportation cost of the outsourced shipment via an LTL shipment and the consolidated shipment through a crossdock at that point are equal. The collection of the iso-cost points is called an **iso-cost curve**.*

An iso-cost curve of a particular facility is the locations of the facility in which the shipping decision does not affect the total transportation costs. For example, if a store is located on an iso-cost curve, the freight designated to the store can be shipped directly from the originating vendor, or it can be consolidated through the associated crossdock with the same total transportation costs. If a store, however, is located slightly off the iso-cost curve, the total transportation cost of each shipment decision is different.

3.2 Assumptions

To facilitate a mathematical model, we assume several characteristics of the single hub crossdocking network.

Assumption 1. *An outsourced shipment between a store and a vendor is always an LTL shipment.*

Assumption 2. *The transportation cost is a linear function of the freight amount, the shipping rate and the travel distance (i.e., $\text{freight} \times \text{shipping rate} \times \text{distance}$).*

Assumption 2 is consistent with the fact that the travel distance and the amount of freight are correlated with the amount of gas consumed and number of trucks. Though we concede that the transportation cost depends on an origin-destination pair and a type of freight, we still insist that the linear cost function serves as an estimation of the transportation costs because retailers are usually capable of customizing an exact contract and negotiating the price based on the total weight-miles and the minimum quantities.

Assumption 3. *A shipment of each vendor-store pair is independent of the other pairs.*

Justification of Assumption 3 is predicated upon the number of facilities in the retailer's network. In the network, the number of crossdocks is far less than the numbers of vendors

and stores. For example, the number of the Home Depot (THD) crossdocks is 140 times fewer than the number of THD stores and 13 times fewer than the number of THD vendors. A small number of crossdocks affects the relative amounts of freight in the network. In the upstream, a crossdock receives shipments from many vendors, and a shipment from a vendor accounts for small amounts of freight compared with the total incoming freight of the crossdock. Therefore, an incoming individual shipment has little or no effect on other shipments, as if it were independent of other vendor-store pairs. We can apply this argument to the downstream and assert that the shipment from a crossdock to an individual store is independent of other vendor-store pairs. Assumption 3 is the most important assumption in our model. We re-visit, validate and relax this assumption in Section 6.5.

The total transportation cost of outsourced shipment from vendor V_i to store S_j is $f_{ij}d_{ij}$. A consolidated shipment, however, depends on the mode of shipments of freight to and from a crossdock. Given the mode of shipments of the freight from vendor V_i to crossdock XD_p and from crossdock XD_p to store S_j and the handling costs of crossdock h_p , then the expression of the total transportation and handling costs of the consolidated shipment is:

$$f_{ij}(c_{ip}(e) + c_{pj}(e) + h_p)$$

where,

- f_{ij} = amounts of freight originating at vendor V_i and ending at store S_j
- $c_{lk}(e)$ = shipping cost per unit of freight on leg l - k
- $c_{lk}(e) = \begin{cases} e d_{lk} & \text{if the shipment is TL} \\ d_{lk} & \text{if the shipment is LTL} \end{cases}$
- d_{lk} = distance on \mathbb{R}^2 of facility l , located at (x_l, y_l) , and facility k , located at (x_k, y_k)
- $d_{lk} = \sqrt{(x_l - x_k)^2 + (y_l - y_k)^2}$
- e = shipping cost ratio between TL and LTL
- ζ_{lk} = mode of shipments of the freight on leg l - k
- h_p = handling costs of crossdock XD_p .

In a consolidated shipment, the transportation costs depend on the mode of shipments of a crossdock. If crossdock XD_p , for example, always receives and ships TL, then the transportation costs of the freight originating at vendor V_i , consolidated at crossdock XD_p , and ending at store S_j is $f_{ij}(e d_{ip} + e d_{pj} + h_p)$. If the crossdock receives TL shipments but ships LTL, the transportation costs become $f_{ij}(e d_{ip} + d_{pj} + h_p)$. Similarly, if the crossdock receives LTL shipments but ships TL, the transportation costs become $f_{ij}(d_{ip} + e d_{pj} + h_p)$. For vendor-store pair (V_i, S_j) , the total transportation cost (TC_{ij}) is the smaller of an outsourced shipment and a consolidated shipment through the cheapest crossdock, as shown in Expression 3.1.

$$TC_{ij}(e, \mathbb{XD}) = f_{ij} \min \left(d_{ij}, \min_{p \in \mathbb{XD}} (c_{ip}(e) + c_{pj}(e) + h_p) \right) \quad (3.1)$$

where, \mathbb{XD} is the set of crossdock locations. Note also that a potential crossdock must have a total transportation per weight cost less than d_{ij} ; otherwise, an outsourced shipment is preferred.

3.3 Properties

Based on Expression 3.1, we study the geometric relationship of an iso-cost curve and its properties.

Proposition 3.3.1. *If the handling costs are zero, the iso-cost curve of a crossdock when a crossdock always receives and ships TL, denoted by $XD(TL, TL)$, is an ellipse. In addition, the eccentricity of the ellipse is the cost-per-distance ratio between a TL shipment and an LTL shipment.*

Proof. the consolidated-shipping cost = the outsourced-shipping cost

$$\begin{aligned} f_{ij}(e d_{ip} + e d_{pj} + h) &= f_{ij} d_{ij} \\ d_{ip} + d_{pj} + 0 &= d_{ij}/e. \end{aligned} \quad (3.2)$$

Given locations of vendor V_i and store S_j , the set of crossdocks satisfies the two-foci equation of an ellipse [56]. In addition, the locations of the vendor and the store are the

focus points of the ellipse, and the eccentricity of the ellipse equals $\frac{d_{ij}}{d_{ij}/e} = e$. ■

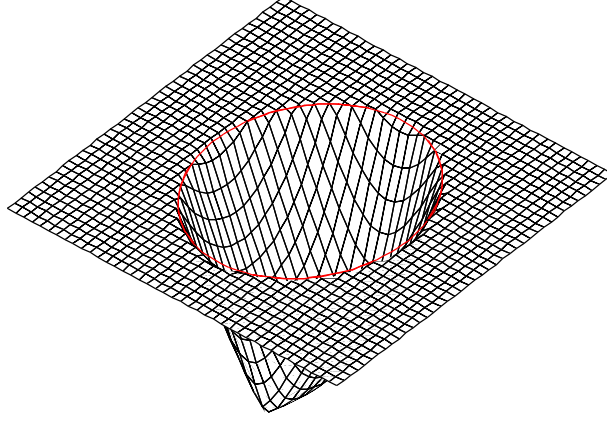


Figure 3.1: The cost contour and iso-cost curve (a red curve) of a crossdock when a crossdock always receives and ships TL, denoted by $XD(TL, TL)$, are shown. Within the boundary of the iso-cost curve, the transportation-cost contour (z -axis) forms an ellipse, while the transportation cost is constant outside the boundary.

In Figure 3.1, the total transportation costs for each crossdock location (a point in x - y plane) is represented by the height (z -axis) of that point. The ellipse in Figure 3.1 shows the boundary that separates a consolidated-shipment decision and an outsourced-shipment decision. Within the boundary of the iso-cost curve, a consolidated shipment is preferred, and the cost contour is an ellipse with respect to the crossdock location; on the contrary, an outsourced shipment is preferred outside the boundary, and transportation costs become independent of the crossdock location. Transportation costs are minimal when a crossdock location is within the line connecting a vendor and a store, and they gradually increase as a crossdock moves away from the vendor-store line until they reach the boundary.

Proposition 3.3.2. *As eccentricity increases, an outsourced shipment through LTL becomes more attractive.*

Proof. By the triangle inequality, travel distance of an outsourced shipment is less than or equal to travel distances of a consolidated shipment. Since higher eccentricity implies the economic indifference between TL and LTL shipments, an LTL outsourced shipment that travels a shorter distance becomes more attractive. ■

Next, we show that a rule-of-thumb in the network. That is, a distant vendor should ship its freight to a crossdock.

Theorem 3.3.1. *If a crossdock receives and ships every shipment through TL, then all shipments from vendors sufficiently far away will go through the crossdock to its nearest stores. That is, all stores located less than R from the crossdock always receive freight from the vendor located more than $\frac{R(1+e)+h}{1-e}$ from the crossdock.*

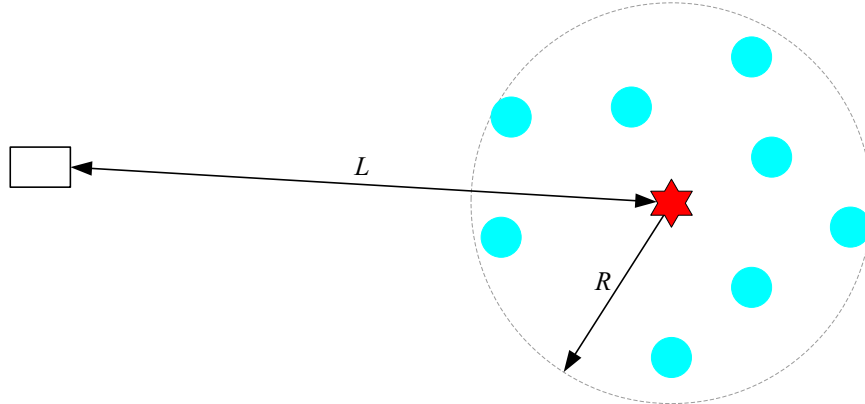


Figure 3.2: A vendor (a square) ships all freight to stores (blue circles) within a cluster radius R through a crossdock (a red star) located at the center of the cluster.

Proof. Let L be the distance between vendor V_i and the crossdock (see Figure 3.2). By the triangle inequality, $R - d_{ij} \geq L$ for any V_i and S_j . The total transportation costs of the outsourced shipments from vendor V_i to all stores are $\sum_j f_{ij} d_{ij} \geq \sum_j f_{ij} (L - R)$. With the consolidated shipments through the crossdock, the total transportation costs of the same

stores are $\sum_j f_{ij} (ed_{ip} + ed_{pj} + h) \leq \sum_j f_{ij} (eL + eR + h)$. We use the crossdock only when the total transportation costs of an outsourced shipment are higher than the total transportation costs of the consolidated shipment (i.e., $\sum_j f_{ij} (ed_{ip} + ed_{pj} + h) \leq \sum_j f_{ij} d_{ij}$). Using two previous inequalities and re-arranging terms, we have $L \geq \frac{R(1+e)+h}{1-e}$. ■

Given a cluster of stores, a consolidated shipment through the crossdock located at the center of the cluster becomes more attractive to a distant vendor as the distance between a vendor and a crossdock increases. When the distance reaches the threshold of mode of shipments (TL, TL) (i.e., $\frac{R(1+e)+h}{1-e}$), all freight shipped to the stores inside the cluster travels more cheaply than consolidated shipments. This threshold is a linear function of the radius of the cluster (R) and the handling costs (h).

Observation 3.3.1. *The handling costs have less effect on the threshold on than on the radius of the cluster because the coefficient of R is never less than the coefficient of h (i.e., $\frac{1+e}{1-e} \geq \frac{1}{1-e}$).*

The threshold in Theorem 3.3.1 can be written as follows:

$$L \geq R + h + \frac{e}{1-e} (2R + h). \quad (3.3)$$

Expression 3.3 consists of two terms: $R + h$ and $\frac{e}{1-e} (2R + h)$. For a given cluster of stores and handling costs, the first term of Expression 3.3 is constant, while the second term is a function of the eccentricity. Coefficient $\frac{e}{1-e}$ of the second term may be viewed as the inversion of the savings from using a TL shipment. Since $0 \leq e \leq 1$, the value of the second term is always non-negative and $L \geq R$. Therefore, the threshold is always greater than the radius of cluster and cannot be extended to the vendor within the cluster.

Observation 3.3.2. *The greater the advantage of TL over LTL (i.e., the smaller e), the closer L is to R and the more consolidated shipments. On the contrary, the value of the coefficient can be a large number when the eccentricity approaches one (i.e., $0 \leq \frac{e}{1-e} < \infty$).*

We can derive a similar expression of Theorem 3.3.1 for small stores located within a certain radius from a crossdock that ships LTL shipment, as shown in Corollary 3.3.1.

Corollary 3.3.1. *If a crossdock receives every shipment from vendors through TL but ships it to stores through LTL, then all stores located less than R from the crossdock always receive freight through the crossdock from vendors located more than $\frac{2R+h}{1-e}$ from the crossdock.*

Proof. The threshold in Corollary 3.3.1 can be derived from Theorem 3.3.1. Since all stores in the cluster receive LTL shipments from the crossdock, the total transportation costs of the consolidated shipment become $\sum_j f_{ij}(ed_{ip} + d_{pj} + h) \leq \sum_j f_{ij}(eL + R + h)$. ■

The threshold in Corollary 3.3.1 is greater than the threshold in Theorem 3.3.1 (i.e., $\frac{2R+h}{1-e} \geq \frac{R(1+e)+h}{1-e}$), and the difference is equal to the radius R of the cluster. In other words, if total freight is suddenly decreased and a crossdock ships LTL to every store, a distant vendor must be located at least $\frac{2R+h}{1-e}$ away from the crossdock or additional R from the case when the crossdock always ships and receives TL. The increasing of the threshold also implies that the number of distant vendors shipping through the crossdock remains the same or decreases. As a result, the attractiveness of crossdock decreases as shipments from a crossdock to stores become LTL.

3.4 Zero Handling Cost Case

When a crossdock always receives and ships TL, the iso-cost relationship between a vendor and a store is an ellipse. In the single hub crossdocking network, there are other pairs of facilities and other modes of shipments. This raises an interesting question: What are the shapes of other iso-cost relationships? With the methodology similar to Proposition 3.3.1, we analyze iso-cost relationships by assuming zero handling cost. That is, every unit of a consolidated shipment incurs only the transportation cost, while the handling cost at the selected crossdock is ignored.

3.4.1 Properties of Iso-cost Curves

Based on Expression 3.1, we derive the closed-form equations of iso-cost curves of three types of facilities and three possible modes of shipments. A derivation of each iso-cost curve requires the locations of two given facilities and its mode of shipments. The iso-cost curves are well-known *quadratic* (second degree) and *quartic* (fourth degree) curves that are *conic* and *limaçon*. Both curves are usually seen in the polar coordinate system, in which each point is referred to as the distance from the origin, denoted by r , and the degree from the x -axis in the counter-clockwise direction, denoted by θ . The closed-form equation of iso-cost curves presented in the polar coordinate system is shown in Table 3.1, while the shapes of the iso-cost curves are depicted in Figures 3.3 and 3.4.

The first column of Table 3.1 shows a facility of interest, referred to as an *interested facility*, and properties of iso-cost curves are listed in the next column. The last three columns represent the mode of shipments, an order pair (ζ_1, ζ_2) . ζ_1 and ζ_2 denote the shipping scheme from a vendor to a crossdock and from a crossdock to a store, respectively. The first three rows of each case show the x - y coordinate system of three facilities. They are standardized^{3.1} such that the distance between two known facilities is equal to 1. These coordinates are inputs for Expression 3.1 to algebraically derive the next row. The “Equation” row represents the equation of the iso-cost curves. These equations, which are the key of the table, are used to derive the remaining rows and to construct Figures 3.3 and 3.4. In the fifth row, we describe the shape of iso-cost curves, which is discussed later. The next two rows are “Condition” and “ θ ”. The “Condition” is derived from the non-negativity condition of a distance, and the “ θ ” is the interval that satisfies this restriction.

^{3.1}Coordinates are standardized by shifting, scaling and rotating the axes.

Table 3.1: In zero handling cost case, iso-cost curves can be presented as polar-form equations

Interested Facility	Properties	mode of shipments(ζ_1, ζ_2)		
		(TL, TL)	(LTL, TL)	(TL, LTL)
Crossdock (XD)	V	$(0, 0)$	$(-1, 0)$	$(0, 0)$
	S	$(1, 0)$	$(0, 0)$	$(1, 0)$
	XD	(x, y)	(x, y)	(x, y)
	Equation	$r = \frac{1-e^2}{2e(1-e \cos \theta)}$	$r = -\frac{2e}{1-e^2} + \frac{2}{1-e^2} \cos \theta$	$r = -\frac{2e}{1-e^2} + \frac{2}{1-e^2} \cos \theta$
	Boundary Shape Condition	ellipse	limaçon	limaçon
Store (S)	θ	$r > 0$ $[0, 2\pi]$	$r \geq 0$ $[\pi - \arccos e, \pi + \arccos e]$	$r \geq 0$ $[-\arccos e, \arccos e]$
	V	$(0, 0)$	$(-1, 0)$	$(0, 0)$
	S	(x, y)	(x, y)	(x, y)
	XD	$(1, 0)$	$(0, 0)$	$(1, 0)$
	Equation	$r = \frac{2e}{1-e^2} - \frac{2e^2}{1-e^2} \cos \theta$	$r = \frac{2e}{1-e^2} - \frac{2}{1-e^2} \cos \theta$	$r = \frac{1-e^2}{2(\cos \theta - e)}$
Vendor (V)	Boundary Shape Condition	limaçon	limaçon	half-hyperbola
	θ	$r \geq 0$ $[0, 2\pi]$	$r \geq 0$ $(\arccos e, 2\pi - \arccos e)$	$r > 0$ $(-\arccos e, \arccos e)$
	V	(x, y)	(x, y)	(x, y)
	S	$(0, 0)$	$(1, 0)$	$(1, 0)$
	XD	$(-1, 0)$	$(0, 0)$	$(0, 0)$
Equation	Equation	$r = \frac{2e}{1-e^2} + \frac{2e^2}{1-e^2} \cos \theta$	$r = \frac{1-e^2}{2(\cos \theta + e)}$	$r = \frac{2e}{1-e^2} + \frac{2}{1-e^2} \cos \theta$
	Boundary Shape Condition	limaçon	half-hyperbola	limaçon
	θ	$r \geq 0$ $[0, 2\pi]$	$r > 0$ $(\arccos e, 2\pi - \arccos e)$	$r \geq 0$ $[-\pi + \arccos e, \pi - \arccos e]$

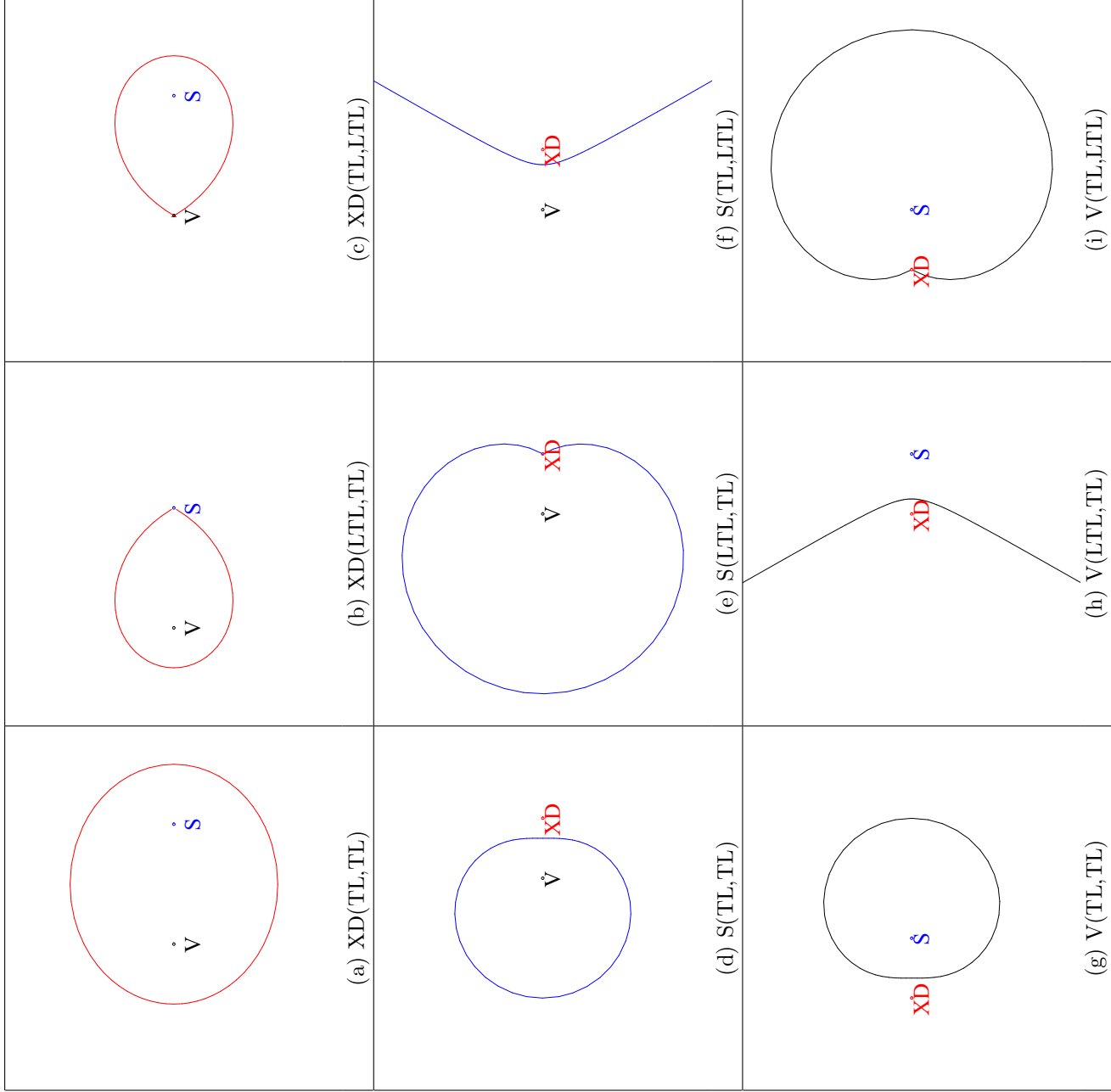


Figure 3.3: The shapes of an iso-cost curve depends on the interested facility (letter) and the mode of shipments (order pair). In the zero handling cost case, iso-cost curve are an ellipse (Figure 3.3.a), a limaçon (Figures 3.3.b, 3.3.c, 3.3.d, 3.3.e, 3.3.g and 3.3.i) and a half-hyperbola (Figures 3.3.f and 3.3.h)

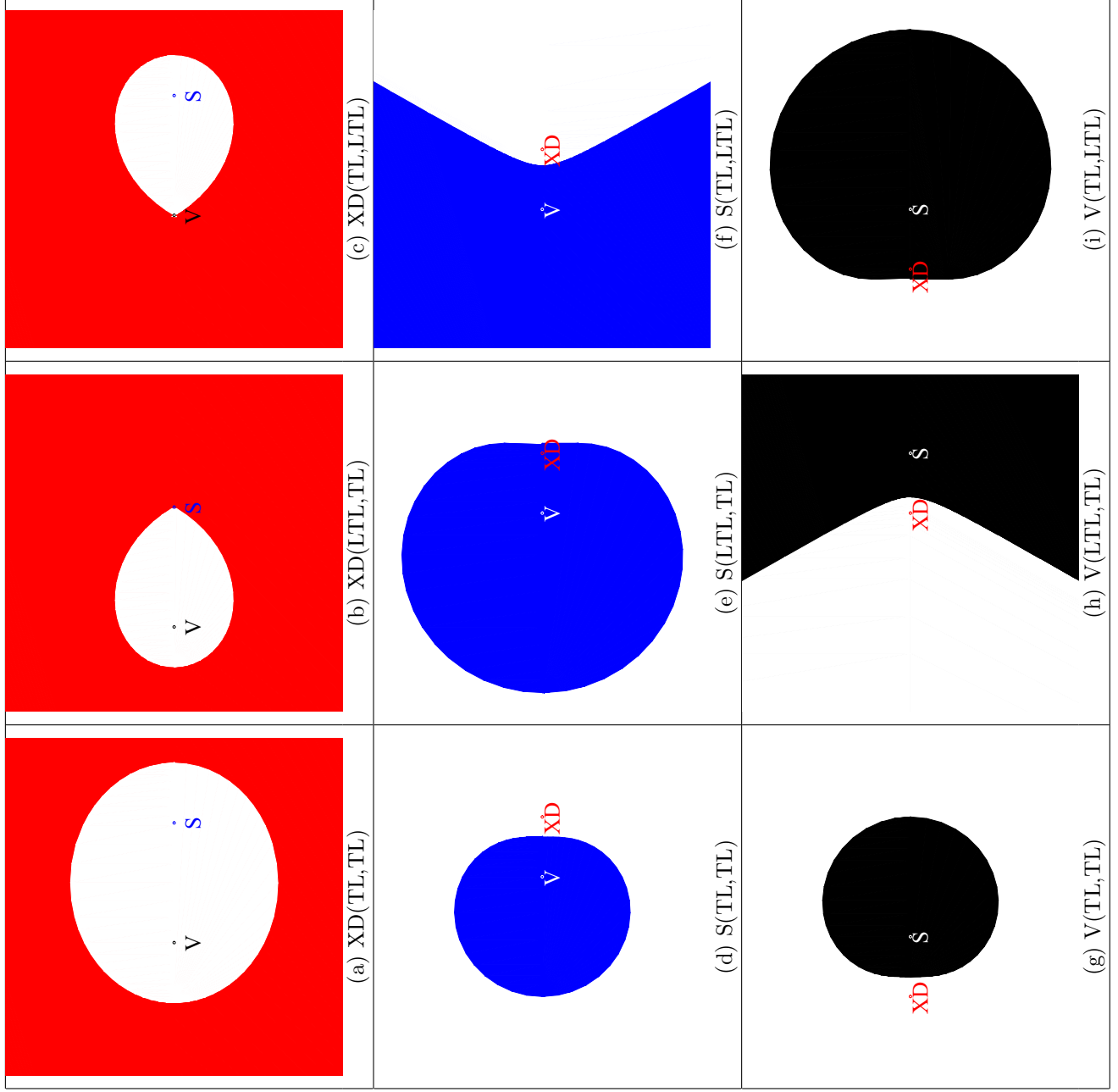


Figure 3.4: A consolidated shipment is preference in the white area, while an outsourced shipment is preference in the colored area

3.4.2 Shapes of Iso-cost Curves

Figure 3.3 shows the curves listed in Table 3.1. This figure can help a shipment planner decide on either an outsourced shipment or a consolidated shipment. An area where an outsourced shipment is cheaper is called an *outsourced-shipping region*, while an area where a consolidated shipment is cheaper is referred to as a *consolidated-shipping region*. The outsourced-shipping region and the consolidated-shipping region are illustrated in Figure 3.4, in which the outsourced-shipping region is the colored area and the consolidated-shipping region is depicted by the white area. A shipment through a crossdock is preferred when an interested facility is located within the consolidated-shipping region. These regions are either interior or exterior of an iso-cost curve. If the outsourced-shipping region is interior of an iso-cost curve, then the consolidated-shipping region is exterior of the same iso-cost curve. Hence, the outsourced-shipping region is the *conjugate* of the consolidated-shipping region and vice-versa. In particular, the outsourced-shipping (consolidated-shipping) region of a crossdock of interest is the exterior (interior) of an iso-cost curve (Figures 3.4.a-3.4.c), while the outsourced-shipping (consolidated-shipping) region of a vendor of interest or a store of interest is the interior (exterior) of an iso-cost curve (Figures 3.4.d-3.4.i). It should be noted that the eccentricity value has a negative correlation with the size of the consolidated-shipping region. A higher eccentricity value implies less difference between a TL shipment and an LTL shipment. Therefore, an LTL shipment becomes more economically attractive, and the consolidated-shipping region gets smaller. The consolidated-shipping region of (LTL, TL) and (TL, LTL) are completely contained inside the consolidated-shipping region of (TL, TL) since the use of mode of shipments (TL, TL) is more cost efficient than both (TL, LTL) and (LTL, TL) for the same set of facilities. The area of the region and this relationship connect the iso-cost curve of the same facility.

Figure 3.3 shows nine iso-cost curves, corresponding to the “Shape” row in Table 3.1. The unique annotation of each case represents an interested facility (letter) and a mode of shipments to and from a crossdock (an order pair). A (black) square, a (blue) circle and a (red) cross represent locations of a vendor, a store and a crossdock, respectively. Among

nine iso-cost curves in Figure 3.3, five curves are geometrically unique: two are conic and the three others are limaçon. The shape in Figure 3.3.a, an ellipse (conic), is the only one without its reflecting pair. The reflecting pairs in Figures 3.3.b, 3.3.d, 3.3.e and 3.3.f are Figures 3.3.c, 3.3.g, 3.3.i and 3.3.h, respectively. As discussed in Proposition 3.3.1, the eccentricity of an ellipse is e , where e is the ratio of a TL shipping rate to an LTL shipping rate. Another conic shape is a half-hyperbola, as shown in Figures 3.3.f and 3.3.h. Note that the non-negativity condition of a distance yields only half of the generic hyperbola curve. The conic eccentricity of the half-hyperbola is $1/e$. The focus points of both conics are the location of two given facilities.

A notable feature of limaçon is that the curve may have an inner loop. This feature of a limaçon depends on the parameter ratio [56]. This ratio ($\frac{a}{b}$) is the ratio of the constant term to the coefficient term of a limaçon in the standard polar form ($r = a + b \cos \theta$). If the ratio is less than one, the limaçon has both outer and inner loops, as shown in Figure 3.5.

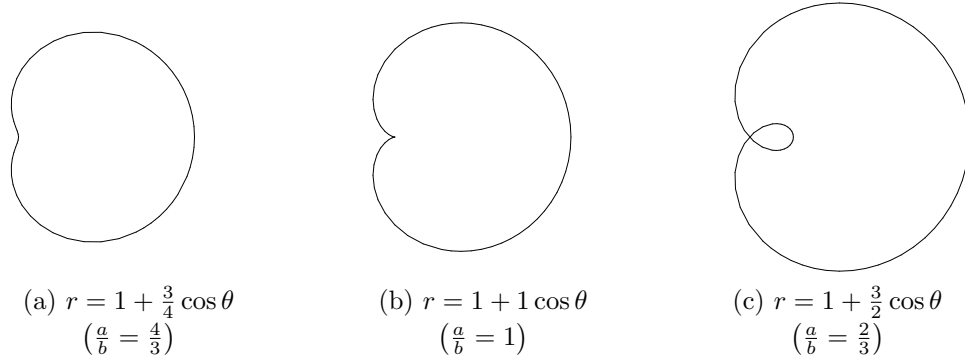


Figure 3.5: If the ratio $\frac{a}{b}$ is less than one, the limaçon has both outer and inner loops.

Although the limaçon can develop an inner loop as its parameter ratio changes, we assure that the shapes of all the iso-cost curves are consistent throughout the practical range of the eccentricity^{3.2}. In Figures 3.3.d and 3.3.g, the parameter ratio is $1/e$. Since $0 \leq e \leq 1$, these figures have no inner loop. The parameter ratio in Figures 3.3.b, 3.3.c, 3.3.e and 3.3.i are e . Nevertheless, they are portrayed as either an inner loop or an outer loop because of the negativity condition of distances. We note that Table 3.1 contains the

^{3.2}If eccentricity were allowed to be greater than one, shapes of the iso-cost curves would not be consistent. For example, the iso-cost curve XD(TL, TL) would become a hyperbola, instead of an ellipse.

normalized iso-cost curves. Therefore, the distance between two known facilities in a polar-form equation is scaled to one. In general, the distance affects the size of the iso-cost curves. The longer the distance of interested facilities, the larger the size of the iso-cost curves. The polar-form equation of a limaçon also suggests that the iso-cost curve is a linear function of $\cos \theta$. The coefficient of $\cos \theta$ is the maximized and minimized value of r when the interested facility is aligned on the axis of two known facilities ($\cos \theta = 1$ or $\cos \theta = -1$), while it becomes zero when the interested facility is located on the perpendicular of the axis ($\cos \theta = 0$). This explains why an interested facility is more likely to receive and/or ship a consolidated shipment along the axis of known facilities. For example, a shipment is always consolidated through a crossdock when the crossdock is located between a vendor and a store (Figures 3.3.b and 3.3.c).

In Figure 3.3, we notice two interesting features: an unbounded curve (in Figures 3.3.f and 3.3.h) and a sharp tail (in Figures 3.3.b and 3.3.c). The unbounded curve implies that the consolidated shipment is still attractive even a facility is located very far, while the sharp tail suggests that a small neighborhood area of an interested facility may have different shipping decisions. These significant properties are examined through Figures 3.3.f (the unbounded curve) and 3.3.b (the sharp tail). As a store is moved away in Figure 3.3.f, both the consolidated-shipping and the outsourced-shipping cost increase. Through a certain direction (a contour of the half-hyperbola), these costs increase at the same rate, which causes the unbounded property. To explain the sharp tail in Figure 3.3.b, we observe that the outsourced-shipping cost is a constant since the locations of a vendor and a store are fixed, and the TL shipment from a crossdock to a store is a dominating term in the consolidated-shipping cost. The closer a crossdock to the store, the less different between the consolidated shipment and the outsourced shipment. They are equal when the crossdock and the store are located on the same spot. This causes the iso-cost curve to pass through the store, creating the sharp tail.

As eccentricity increases, an outsourced-shipping region expands, while a consolidated-shipping region shrinks. This raises a question: How does the region change as eccentricity changes from its upper bound to its lower bounds?

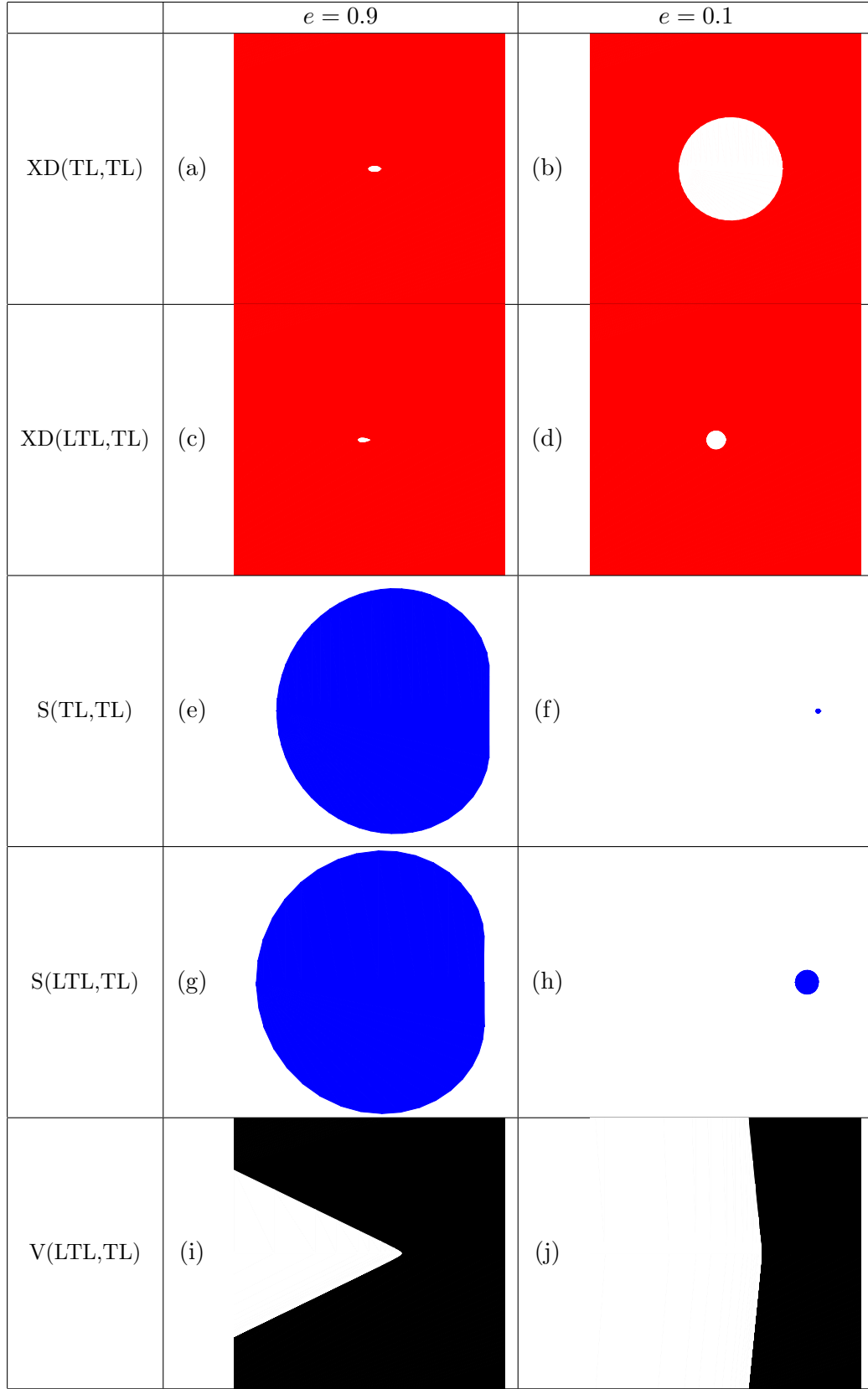


Figure 3.6: The consolidated-shipping region (white area) as eccentricity reaches the upper and lower bounds are illustrated by $e = 0.9$ and $e = 0.1$, respectively.

To answer this question, we consider only a consolidated-shipping region since an outsourced-shipping region is the conjugate of a consolidated-shipping region. We illustrate the consolidated-shipping regions as eccentricity reaches the upper and lower bounds by using $e = 0.9$ and $e = 0.1$, as shown in Figure 3.6. The figure omits iso-cost curves $XD(TL, LTL)$, $S(TL, LTL)$, $V(TL, TL)$ and $V(TL, LTL)$ as they are the reflecting curves of iso-cost curves $XD(LTL, TL)$, $V(LTL, TL)$, $S(TL, TL)$ and $S(TL, LTL)$, respectively.

As eccentricity reaches its upper bound ($e \rightarrow 1$), an LTL outsourced shipment dominates a consolidated shipment because of triangle inequality of distance. Therefore, a consolidated-shipping region disappears, as shown in Figure 3.6.a, 3.6.c, 3.6.e, 3.6.g and 3.6.i. The disappearance of a consolidated-shipping region when eccentricity approaches its upper bound is independent of both modes of shipments and an interested facility. Interestingly, the consolidated-shipping regions of iso-cost curves $XD(TL, TL)$, $XD(LTL, TL)$ and $XD(TL, LTL)$ eventually become the line between locations of a vendor and a store.

On the contrary, the property of a consolidated-shipping region when eccentricity approaches its lower bound depends on a mode of shipments and/or an interested facility. It is important to note that a TL shipment is virtually “free” because its transportation cost approaches zero as eccentricity reaches its lower bound ($e \rightarrow 0$). Of three modes of shipments, the consolidated-shipping region of mode of shipments (TL, TL) is simple because both inbound and outbound shipments are virtually “free”. Hence, the consolidated-shipping region is the space itself (\mathbb{R}^2) regardless of the type of interested facility, as shown in Figure 3.6.b and 3.6.f. In modes of shipments (LTL, TL) and (TL, LTL) , the consolidated-shipping regions depend on a type of an interested facility. Because of the reflecting pair of iso-cost curves, we can discuss only mode of shipments (LTL, TL) and apply similar argument to mode of shipments (TL, LTL) . In mode of shipments (LTL, TL) , the shipment from a crossdock to a store can be ignored because it is TL shipment. Nevertheless, the property of iso-cost curves relies on a type of an interested facility. For iso-cost curve $XD(LTL, TL)$, the consolidated-shipping region is the interior of the circle centering at a vendor. The radius of the circle is the distance between a vendor and a store, as shown in Figure 3.6.d. For iso-cost curve $S(LTL, TL)$, the consolidated-shipping region is the exterior

of the circle centering at a vendor, the radius of which is the distance between a vendor and a crossdock, as shown in Figure 3.6.h. In addition, the consolidated shipment is unbounded, which means that a distant store always receives shipments from a crossdock. For iso-cost curve $V(LTL, TL)$, the consolidated-shipping region is the crossdock-side of the half-space partitioned by the line that perpendicularly and equally divides the distance between a store and a crossdock, Figure 3.6.i. Hence, both the consolidated-shipping and outsourced-shipping regions are unbounded. Next, we discuss applications and insights of iso-cost curves.

3.4.3 Insights of Iso-Cost Curves

Table 3.1 contains comprehensive information about each iso-cost curve. For example, it can be used to derive Theorem 3.3.1 when the handling costs are zero, as presented in Corollary 3.4.1. This corollary can be directly derived from Theorem 3.3.1 by substituting the zero handling cost, $h = 0$. However, we show the alternative method using the concept of an iso-cost curve and the information from Table 3.1.

Corollary 3.4.1. *If a crossdock receives and ships every shipment through TL and the handling costs are zero, then all stores, located less than R from the crossdock, always receive freight from the vendor located more than $\frac{R(1+e)}{1-e}$ from the crossdock.*

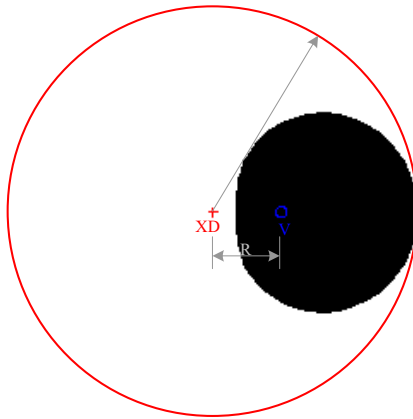


Figure 3.7: The radius of the smallest circle centering at a crossdock (XD) and covering the outsourced-shipping region (black area) of $V(TL, TL)$ iso-cost curve is $\frac{R(1+e)}{1-e}$. A distant vendor located outside of this circle should ship through the crossdock.

Proof. Since a crossdock always receives and ships TL shipments and a vendor location is a location of interest, the setting of this corollary associates with iso-cost curve $V(TL, TL)$. Such the iso-cost curve can be constructed by locations of a crossdock and a store located exactly R away from the crossdock, as shown in Figure 3.7. All stores, located less than R from the crossdock, receive freight through the crossdock if a vendor is located in the consolidated-shipment region of the iso-curve. Since the consolidated-shipping region (white area of the figure) is the exterior of the iso-cost curve, we want to compute the radius of the smallest circle centering at a crossdock that completely covers the outsourced-shipping region (black area of the figure). From Table 3.1, the equation of this case is $r(d, \theta) = \frac{2de}{1-e^2} (1 + e \cos \theta)$. The distance between a crossdock and vendor must be $R + r(R, 0) = R + \frac{2R}{1-e} = \frac{R(1+e)}{1-e}$ ■

Theorem 3.3.1 and Corollary 3.4.1 suggest that freight from a distant vendor is more likely to ship through a crossdock. The reverse implication of Theorem 3.3.1 and Corollary 3.4.1 is that stores are more likely to receive consolidated shipments if they are located near a crossdock.

Next, we derive a statement regarding an outsourced shipment from Table 3.1. Particularly, an outsourced shipment is preferred if a vendor and a store are located within a certain threshold of one another regardless of a mode of shipments, as shown in Theorem 3.4.1.

Theorem 3.4.1. *If the distance between a store and its nearest crossdock is d , then a vendor located within $\frac{2ed}{1+e}$ of that store should use an outsourced shipment.*

Proof. Given the distance between a store and its nearest crossdock, we can compute the minimal radius of the biggest circle centering at the store that is contained in the outsourced-shipping region, or touches an iso-cost curve. The radii of the tangent circle of $V(TL, TL)$, $V(LTL, TL)$ and $V(TL, LTL)$ cases are $\frac{2ed}{1+e}$, $\frac{d(1+e)}{2}$ and d , respectively. The minimal radius of the tangent circle is $\frac{2ed}{1+e}$. ■

Theorem 3.4.1 follows our intuition because the expression is a function of distance between a store and a crossdock. As the distance between a crossdock and a store increases, the threshold value of Theorem 3.4.1 increases and the store receives outsourced shipments from more vendors. Because Theorem 3.4.1 is derived from Table 3.1, which corresponds to the zero handling-cost case, the derived threshold value is a lower bound of the non-zero handling-cost case. That is, the threshold value of non-zero handling-cost case is greater than $\frac{2ed}{1+e}$ because handling costs promotes outsourced shipments.

3.5 Non-Zero Handling Cost Case

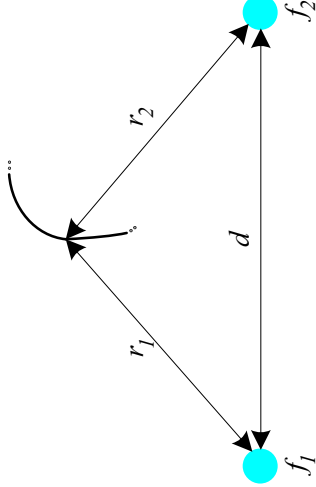
We extend the previous results and generalize iso-cost curves by incorporating handling costs. A unit of the handling costs is incurred when a unit of freight passes through a crossdock. These generalized iso-cost curves are two-foci curves, as shown in Table 3.2. Given locations of known facilities (f_1 and f_2), the two-foci equations describe relationships between distances of an interested facility to the first focus (r_1) and to the second focus (r_2). In Table 3.2, the set of solutions that satisfies non-negativity of distance (“Condition” row) is called *locus points* and listed in the “Boundary Shape” row. By the similar methodology to Proposition 3.3.1, we can derive the polar-form expressions of the generalized iso-cost curves. However, the polar-form equations are cumbersome (as shown in Section A.1).

3.5.1 Properties of Generalized Iso-Cost Curves

The analysis shows that both the zero-handling-cost case and the non-zero-handling-cost case share three general properties: a consolidated-shipping region, a dynamic of eccentricity and a reflecting pair. The consolidated-shipping region of a crossdock is the interior of the iso-cost curves, while the consolidated-shipping region of a vendor and a store are the exterior of the curves. As the second property, a consolidated shipment becomes less attractive when the eccentricity increases. In fact, a consolidated shipment is a function of eccentricity and handling costs. The last property shared by both cases is the reflecting pair, as shown in Figure 3.8. Of the nine curves, four reflecting pairs (eight curves) are found. The only one without the reflecting pair curve is iso-cost curve XD(TL,TL)(Figure 3.8.a), which is an ellipse. Similar to Figure 3.3.a, Figure 3.8.a is also the iso-cost curve that

is symmetrical on both the horizontal and the vertical axis. Note that the other iso-cost curves are symmetrical only on the horizontal axis. In addition, the shapes of $XD(TL,TL)$, $S(TL,LTL)$ and $V(LTL,TL)$ remain conics, while the others change from the limaçon to another quartic curve.

Table 3.2: The generalized iso-cost curves, the shape of which is defined by the focus points (f_1 and f_2), the equation corresponding to distances between locus points and focus points (r_1 and r_2) and the listed conditions.



Interested Facility	Properties	mode of shipments(ζ_1, ζ_2)	
		(TL, TL)	(LTL, TL)
Crossdock (XD)	First Focus (f_1) Second Focus (f_2) Equation Boundary Shape Condition	vendor $(0, 0)$ store $(d_{ij}, 0)$ $e r_1 + e r_2 = d_{ij} - h$ ellipse $0 \leq r_1 \leq d_{ij}/e - h/e$ $0 \leq r_2 \leq d_{ij}/e - h/e$	vendor $(0, 0)$ store $(d_{ij}, 0)$ $e r_1 + r_2 = d_{ij} - h$ inner Cartesian oval $0 \leq r_1 \leq d_{ij}/e - h/e$ $0 \leq r_2 \leq d_{ij} - h$
Store (S)	First Focus (f_1) Second Focus (f_2) Equation Boundary Shape Condition	vendor $(0, 0)$ crossdock $(d_{ip}, 0)$ $r_1 - e r_2 = e d_{ip} + h$ outer Cartesian oval $e d_{ip} + h \leq r_1 \leq \infty$ $0 \leq r_2 \leq r_1/e$	vendor $(0, 0)$ crossdock $(d_{ip}, 0)$ $r_1 - r_2 = e d_{ip} + h$ half-hyperbola $e d_{ip} + h \leq r_1 \leq \infty$ $0 \leq r_2 \leq r_1$
Vendor (V)	First Focus (f_1) Second Focus (f_2) Equation Boundary Shape Condition	crossdock $(-d_{pj}, 0)$ store $(0, 0)$ $-e r_1 + r_2 = e d_{pj} + h$ outer Cartesian oval $0 \leq r_1 \leq r_2/e$ $e d_{pj} + h \leq r_2 \leq \infty$	crossdock $(0, 0)$ store $(d_{pj}, 0)$ $-e r_1 + r_2 = d_{pj} + h$ outer Cartesian oval $0 \leq r_1 \leq r_2/e$ $d_{pj} + h \leq r_2 \leq \infty$

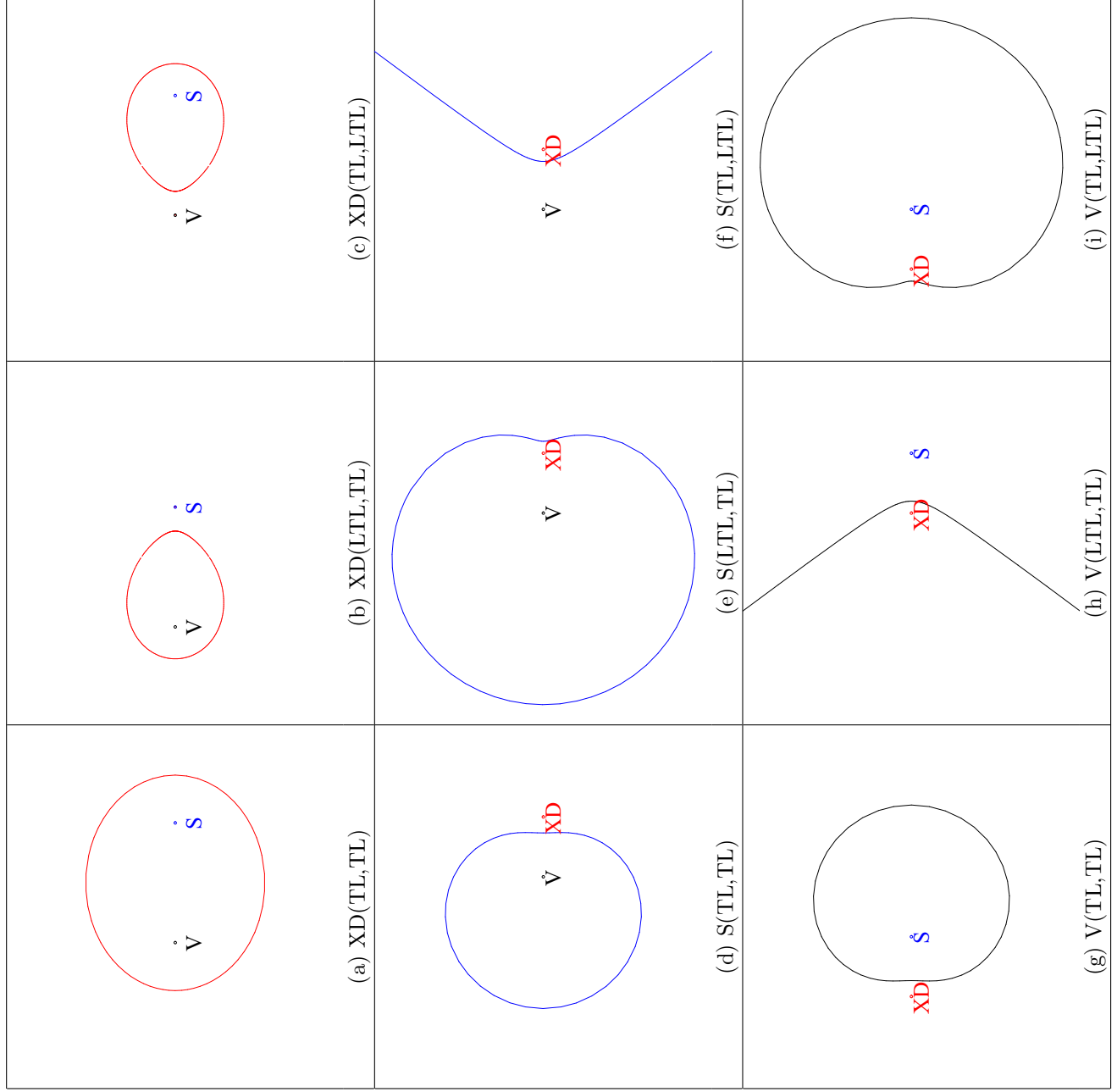


Figure 3.8: In non-zero handling cost, iso-cost curves are an ellipse (Figure 3.8.a), Cartesian oval (Figures 3.8.b, 3.8.c, 3.8.d, 3.8.e, 3.8.g and 3.8.i) and a half-hyperbola (Figures 3.8.f and 3.8.h)

3.5.2 Features of Generalized Iso-Cost Curves

Handling cost affects two features of iso-cost curves: the shape of curves and a valid value of eccentricity. The first and the most notable feature is that, the shape of an iso-cost curve, which is a limaçon in the zero-handling-cost case, is generalized to a *Cartesian oval* [63], as shown in Figure 3.8. Similar to an limaçon, a Cartesian oval typically consists of both an inner loop and an outer loop. However, an iso-cost curve can have only one loop—either inner and outer loops—because of the non-negativity of distance. The sharp tail feature in the zero-handling-cost case disappears (see Figures 3.8.b and 3.8.c). In addition, the inner Cartesian oval curve no longer passes the focus point, as shown in Figures 3.8.b, 3.8.c, 3.8.e and 3.8.i.

The final feature of the generalization is a value of eccentricity. In the zero-handling-cost case, the valid value of eccentricity is $[0, 1]$. In the generalized iso-cost curve, the valid value is functions of distance of focus points and handling costs. For example, the valid value of iso-cost curve XD(TL, TL) can be determined by the equation in Table 3.2 (i.e., $r_1 + r_2 = \frac{d_{ij}}{e} - h$) and the triangle inequality ($r_1 + r_2 \leq d_{ij}$). Hence, the valid eccentricity is

$$e \leq \frac{d_{ij}}{d_{ij} + h} \quad (3.4)$$

where, d_{ij} is the distance between a vendor and a store.

It is worth reminding that the equation in Table 3.2 is normalized and expressed in terms of the distance between a vendor and a store. Expression 3.4 provides an upper bound on the eccentricity value of iso-cost curve XD(TL, TL). For other iso-cost curves, the valid value of eccentricity is difficult to compute because the triangle inequality is not an effective inequality of the equations in Table 3.2. In addition, the exact expression, which varies case-by-case, depends on the direction and the type of the interested facility as well as the mode of shipments. For simplicity, we adopt Expression 3.4 as the domain of eccentricity. Hence, $e \in [0, \frac{d_{ij}}{d_{ij}+h}] \subseteq [0, 1]$. This expression can be re-written as:

$$h \leq \frac{d_{ij}(1 - e)}{e}. \quad (3.5)$$

Expression 3.5 provides a threshold for the handling costs. That means, if the handling costs exceed this value, the freight of a particular vendor-store pair should be shipped directly because the handling costs overshadow the benefits of a consolidated shipment. For example, if the handling costs of a crossdock exceed $\frac{d_{ij}(1-e)}{e}$, vendor-store pair (V_i, S_j) will ship its freight directly via an LTL shipment from vendor V_i to store S_j regardless of the crossdock location because the handling costs alone are the same as the transportation costs of an outsourced shipment. Expression 3.4 also implies that increasing handling costs make the consolidated shipment less attractive. That is, the consolidated-shipping region of the handling-cost case (Figure 3.8) is smaller than that of the zero handling cost case (Figure 3.3). Therefore, the effect of increasing handling costs and the eccentricity leads to the shrinking of the consolidated-shipping region.

3.6 Geometric Inversion

In this section, we introduce a geometric transformation called an *inversion* and use it to explore relationships among iso-cost curves.

3.6.1 Background

Given a circle, an inversion systematically maps points outside the circle onto points inside the circle, and vice versa. The transformation preserves the angular relationship of mapped and original points. For example, the center point of the circle is transformed into the point at infinity. The circle, its center and its radius are called an *inversion circle*, a *center of inversion* and a *radius of inversion*, respectively [64]. Given inversion circle C centering at point o with radius R , we can transform point p to its *inversion point* or *image*, denoted by p' , that satisfies the following property:

$$\begin{aligned} \frac{op}{R} &= \frac{R}{op'}, \\ \text{or} \\ op \cdot op' &= R^2 \end{aligned}$$

where, lines op and op' are the distances from the center of inversion to point p and to point p' , as shown in Figure 3.9.

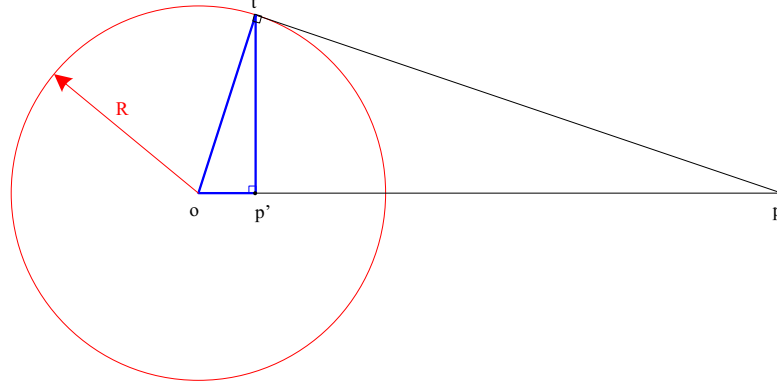


Figure 3.9: An example of geometric inversion. Point p' , the inversion of point p with respect to circle radius R centering at o , can be computed by $\frac{op}{R} = \frac{R}{op'}$.

Figure 3.9 shows that point p' is the inversion point of point p with respect to circle C centering at point o . Point t is a tangent of the circle and serves as the vertex of congruent triangles $\triangle op't$ and $\triangle opt$, each of which has one right angle. In addition, line op' is a part of line op , or points p' , p and o are collinear, because the inversion preserves an angle with respect to a center of inversion. Note that the inversion of an inversion point with respect to the same circle is the original point, i.e., $(p')' = p$. In addition, the inversion can be considered as a geometric transformation that maps a point in Euclidean space onto another point in non-Euclidean space with respect to an inversion circle.

The transformation can be applied to a curve, and a transformed curve is referred to as an *inversion curve*. Interestingly, the shape of an inversion curve depends on only a center of inversion, while the size of the curve depends on both a center and a radius of inversion (an inversion circle). An inversion curve with respect to a certain inversion circle is unique, and it can be transformed back into the original curve by the inversion with respect to the same circle. The inversion is also considered as a generalization of the geometric reflection in which the ordinary reflection is the inversion with the infinite radius. If a curve remain unchange under the inversion, it is called an *anallagmatic curve* [62]. The examples include

circle, ellipse, limaçon and Cartesian oval.

In general, equations describing an original curve and its inversion curve have little in common. However, the polar-form equation of an inversion curve, denoted by $r'(\theta)$, can be derived from the polar-form equation of its an original curve, denoted by $r(\theta)$, when the inversion circle centering at the origin with radius R [4] as:

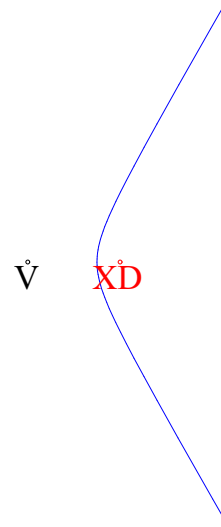
$$r'(\theta) = \frac{R^2}{r(\theta)}. \quad (3.6)$$

For the comprehensive background of the inversion, we suggest that the readers consult the book by Bakel'man [4].

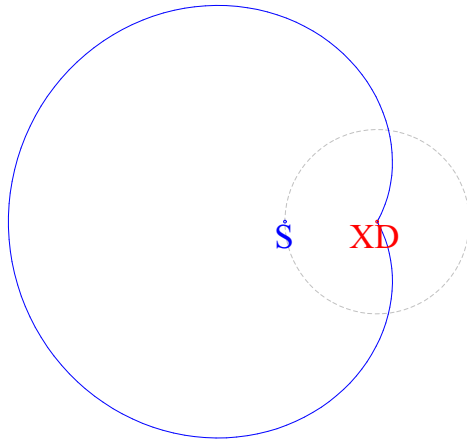
3.6.2 Inversion Curves of Iso-Cost Curves

With the appropriate center of inversion, we observe that an iso-cost curve can be transformed to the other iso-cost curves in the same mode of shipments. For example, the inversion curves of iso-cost curve $S(TL, LTL)$ with respect to the circle centering at its focus points are iso-cost curves, as shown in Figure 3.10.

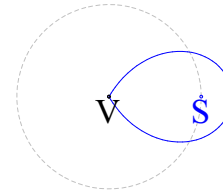
Figure 3.10 shows that the inversion curves of iso-cost curve $S(TL, LTL)$ (Figure 3.10.a) with respect to the circle centering at a crossdock location and a vendor location are iso-cost curves $V(TL, LTL)$ (Figure 3.10.b) and $XD(TL, LTL)$ (Figure 3.10.c), respectively. Based on this observation, the inversion provides an alternative method to construct iso-cost curves using another iso-cost curve of the same modes of shipments. Rather than using locations of a crossdock and a store, the iso-cost curve $V(TL, LTL)$ may be constructed as the inversion curve of iso-cost curve $S(TL, LTL)$. Constructing an iso-cost curve with the inversion geometric is intriguing because the inversion curves of an iso-cost curve, in general, may not be iso-cost curves. The successful transformation of an iso-cost curve, referred to as a *valid inversion curve*, requires an appropriate circle. Figure 3.10 shows that the center of the appropriate circle must be focus points of an original iso-cost curve.



(a) Iso-cost curve $S(TL, LTL)$



(b) Inversion circle centering at a crossdock



(c) Inversion circle centering at a vendor

Figure 3.10: The inversion curves of iso-cost curve $S(TL, LTL)$ with respect to circles centering at the crossdock and the vendor are iso-cost curves $V(TL, LTL)$ (Figure 3.10.b) and $XD(TL, LTL)$ (Figure 3.10.c), respectively

In addition, the radius of the appropriate circle must be a function of the focus points of an original iso-cost curve and its valid inversion curve as follows:

$$d_{ol} d_{ok} = R^2 \quad (3.7)$$

where,

o = the center of inversion

R = the radius of inversion

d_{ok} = the distance from the center of inversion to point k

d_{ol} = the distance from the center of inversion to point l .

Expression 3.7 extends from a basis inversion equation, describing point k as the inversion point of point l with respect to the circle centering at point o with radius R . The expression also implies that points o , l and k are collinear.

Next, we address locations and types of known facilities of a valid iso-cost curve in terms of focus points of the original curve. The first focus point of a valid iso-cost curve is located at the center of inversion, which is also a focus point and serves as the same type of facility of the original iso-cost curve. For example, the first focus point in Figure 3.10.b is a crossdock located at the center of inversion, the same location as the crossdock in Figure 3.10.a. The second focus point is located at the inversion point of the remaining focus point of an original iso-cost curve. Interestingly, the type of facility is switched into the interested facility corresponded with locus points of an original iso-cost curve. For example, the second focus point in Figure 3.10.b is a store (the interested facility in the original curve) located at the inversion point of the vendor location in Figure 3.10.a. It is important to note that the vendor location in Figure 3.10.a and the store location in Figure 3.10.b (i.e., the inversion point of the original vendor) appear to be the same points because the distances between a crossdock to a vendor and a store are equal ($d_{ok} = d_{ol}$), or points k and l are located on the inversion circle. The relationship of facilities of an iso-cost curve and its valid inversion curve leads the following three statements:

- A vendor is the inversion point of a store with respect to a circle centering at a crossdock
- A crossdock is the inversion point of a vendor with respect to a circle centering at a store
- A store is the inversion point of a crossdock with respect to a circle centering at a vendor.

These statements are derived from the focus points of an iso-cost curve and its valid inversion curve. Another way to derive the same statement is from the locus points of an iso-cost curve and its valid inversion curve. In this version, the statement provides the details about shipping decision as locus points partition consolidated-shipping and outsourced-shipping regions. For example, a vendor that receives consolidated shipments (outsourced shipments) is the inversion point of a store that ships receives consolidated shipments (outsourced shipments) with respect to the circle centering at a crossdock. The shipping decision of this statement shows the inversion property to preserve the grouping relationship.

We can summarize these statements as well as locations of focus points, as described in Observation 3.6.1.

Observation 3.6.1. *Given an iso-cost curve of facility l (locus points) and its focus points are located at facilities o and k , the valid inversion curve of this iso-cost curve with respect to the circle centering facilities o is the iso-cost curve of facility k with the same mode of shipments. In addition, the focus points of the valid inversion curve facility o located at the center of inversion and facility l located at the inversion point of facility k in original iso-cost curve.*

Observation 3.6.1 presents a connection between an iso-cost curve and its valid inversion curve. That is, an inversion curve is an iso-cost curve when its original curve and itself share the location of their first focus points and their second focus points are the inversion point of each other. Observation 3.6.1 emphasizes the condition of valid inversion curves and also raises our curiosity about higher order of an inversion curve, as shown in Figure 3.11.

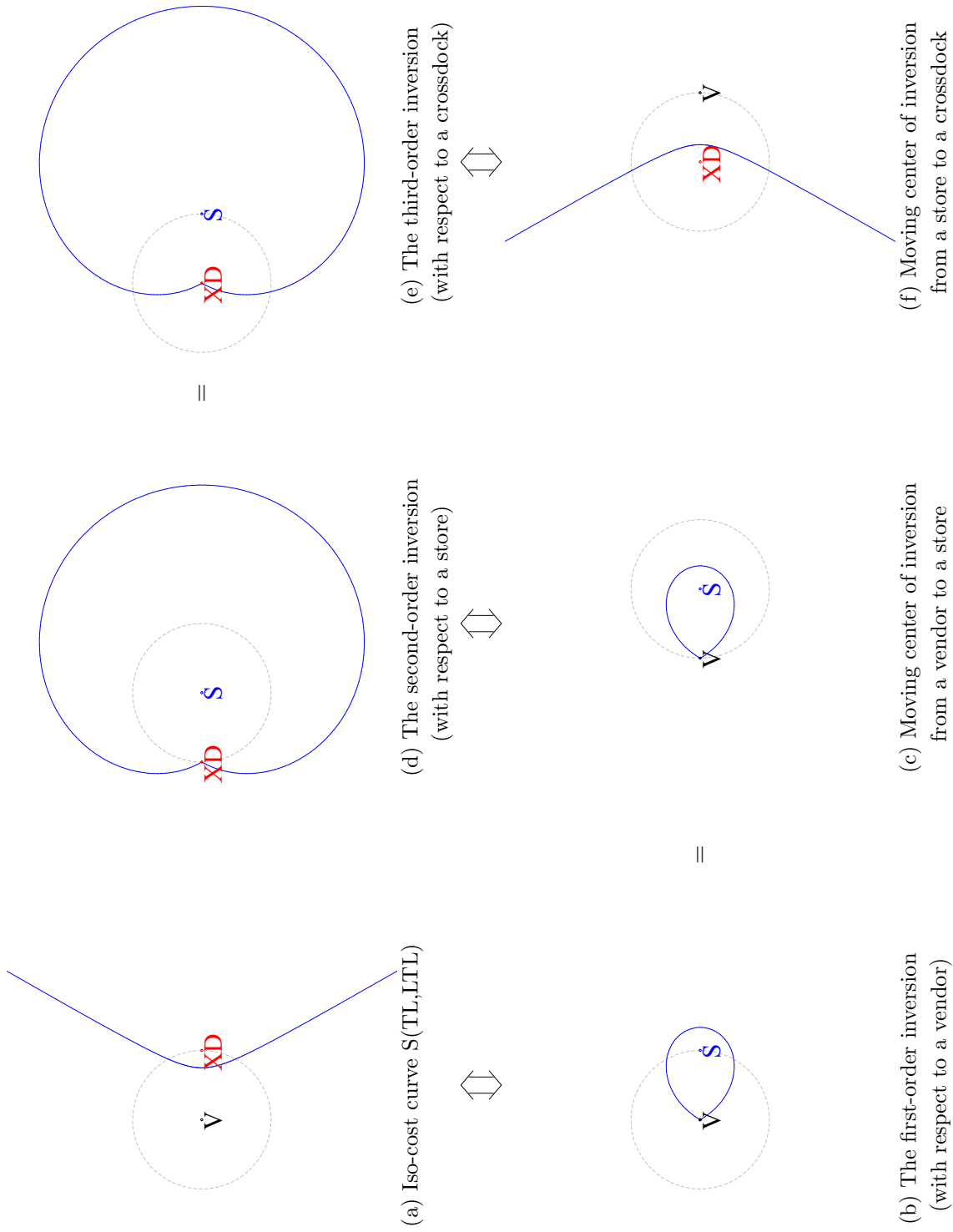


Figure 3.11: Higher orders of inversion of iso-cost curve $S(TL, LTL)$

Figures 3.11.b, 3.11.d and 3.11.f show the first-, second- and third-order inversion curves of iso-cost curve $S(TL, LTL)$ with respect to circles centering at a vendor, a store and a crossdock with radii equal to the square of distance between focus points, respectively. Figures 3.11.b and 3.11.c as well as Figures 3.11.d and 3.11.e are the same iso-cost curve with different inversion circles. Figures 3.11.a and 3.11.f are mirror images of each other. This suggests a non-trivial way to reverse the inversion curve back into the original curve. That is to inverse an iso-cost curve six times with respect to different inversion circles. Observation 3.6.1 also implies that the inversion is independent of eccentricity in the sense that an inversion circle is unaffected by change of eccentricity. A simple verification of this implication is to apply Expression 3.6 to an iso-cost curve, as shown in Figure 3.12.

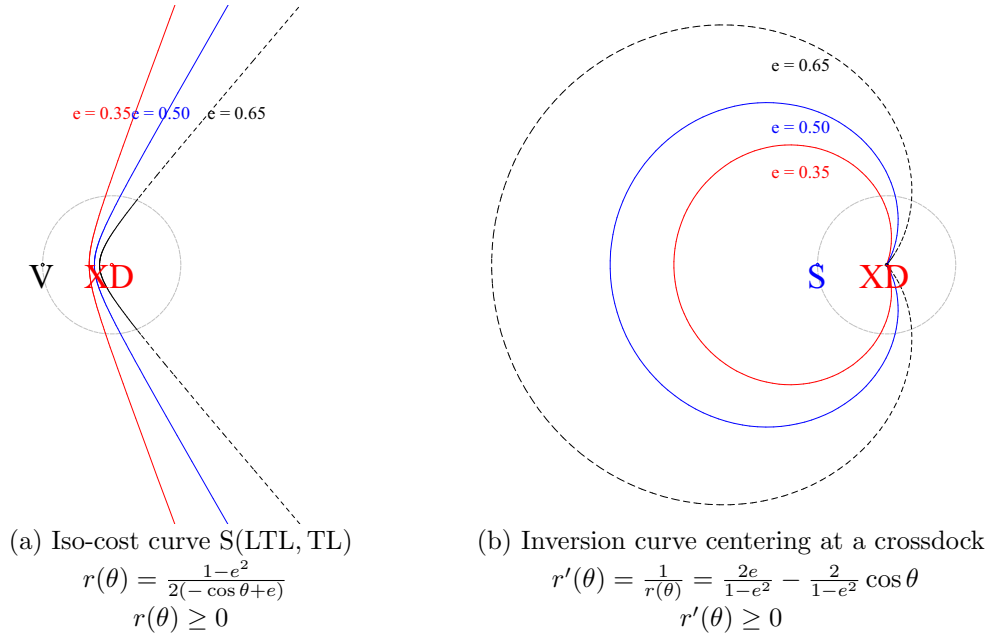


Figure 3.12: The polar-form equation of a valid inversion curve, denoted by $r'(\theta)$, with respect to an inversion circle radius R centering at the origin is the ratio of the square of radius to the polar-form equation of an original curve, denoted by $r(\theta)$, or $r'(\theta) = \frac{R^2}{r(\theta)}$. Therefore, the inversion curves of different eccentricity are valid inversion curves with respect to the same circle.

In Figure 3.12, the iso-cost curves $S(TL, LTL)$ corresponded at different eccentricity values are mapped to their valid inversion curves with respect to the same circle centering at a crossdock. In addition, the polar-form equations in Figures 3.12.a and 3.12.b follow

Table 3.1 (discussed above in Section 3.5) because a crossdock is located at the origin. Figure 3.12 also shows that Expression 3.6 is a simply way to determine the closed-form equation of an inversion curve. Before applying Expression 3.6 to a two-foci curve and discussing mechanism and limitation of inversion, we need to address the limitation of Expression 3.6. Particularly, there is no direct procedure for shifting the origin in polar-coordinated system; therefore, we must resort to x - y coordinated system to match a center of inversion with the origin point before applying the expression. Hence, the polar-form equations resulting from the same iso-cost curve with the different origins may share nothing in common. For example, the polar-form equation of XD(TL,LTL) is $r = -\frac{2e}{1-e^2} + \frac{2}{1-e^2} \cos \theta$ when the vendor is the origin. However, the equation becomes $r = \frac{e^2+e \cos \theta}{1-e^2} - \frac{e\sqrt{e^2 \cos^2 \theta + 2 \cos \theta + 2 - e^2}}{1-e^2}$ when the store is the origin. In the worst case, the polar equation of an intended original point cannot be obtained if a shifted curve has no solutions.

3.6.3 Mechanism and Limitation of Inversion

Given focus points f_1 and f_2 , an iso-cost curve with zero-handling cost can be viewed as a two-foci equation in which the distance between focus points is d and the distances from locus points to focus points are r_1 and r_2 with associated coefficients e_1 and e_2 as:

$$e_1 r_1 + e_2 r_2 = d. \quad (3.8)$$

Expression 3.8 is a generalization of iso-cost curves because the coefficients e_1 and e_2 may be negative when a given focus point is crossdock and they may be normalized by the coefficient of the distance between focus points so that the coefficient remains 1. This expression is important to establish the relationship between an original curve and its inversion curve.

Lemma 3.6.1. *If r_1 , r_2 and d are three sides of a triangle (see Figure 3.13), the inversion of side r_2 with respect to the circle centering at the vertex associated with sides r_1 and d with radius R is $\frac{R^2 r_2}{d r_1}$.*

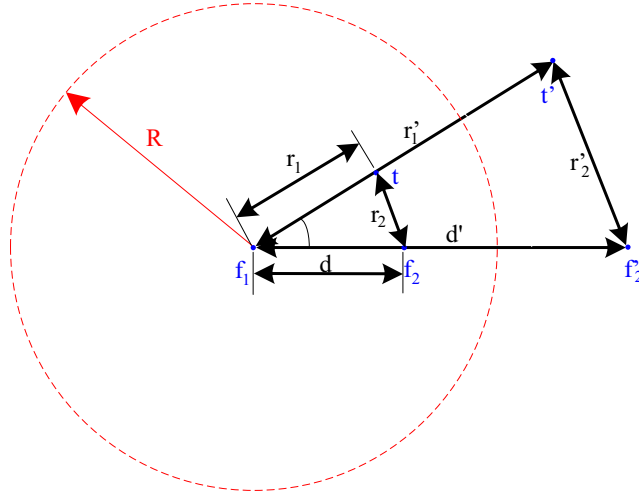


Figure 3.13: Trigonometry of inversion of two-foci curve

Proof. Consider triangle $\triangle f_1 f_2 t$, if the angle formed by sides r_1 and d is denoted by θ , one can show that $\cos \theta = -\frac{r_2^2 - r_1^2 - d^2}{2dr_1}$ by the law of cosine. Then, we re-apply the law of cosine again to triangle $\triangle f_1 f_2 t'$,

$$\begin{aligned}
 (r_2')^2 &= (r_1')^2 + (d')^2 - 2r_1' d' \cos \theta \\
 &= \left(\frac{R^2}{r_1}\right)^2 + \left(\frac{R^2}{d}\right)^2 - 2\left(\frac{R^2}{r_1}\right)\left(\frac{R^2}{d}\right)\left(-\frac{r_2^2 - r_1^2 - d^2}{2dr_1}\right) \\
 &= R^4 \left[\frac{1}{r_1^2} + \frac{1}{d^2} + \frac{r_2^2 - r_1^2 - d^2}{d^2 r_1^2} \right] \\
 &= R^4 \left[\frac{r_2^2}{d^2 r_1^2} \right].
 \end{aligned}$$

Hence, the last expression becomes $r_2' = \frac{R^2 r_2}{dr_1}$. ■

Lemma 3.6.1 shows that the distances between an inversion of locus points and the inversion of focus points (r_2') can be represented as a function of an original two-foci curve (d , r_1 and r_2) and a radius of inversion (R). The function is derived from the trigonometry and inversion condition; therefore, it also holds when handling costs are incurred at the crossdock.

Theorem 3.6.1. *Inversion transforms an iso-cost curve with zero-handling cost to another iso-cost curve with zero-handling cost within the same mode of shipments.*

Proof. Without loss of generality, we assume that an iso-cost can be described by $e_1 r_1 + e_2 r_2 = d$, and focus point f_1 is the origin. As a result, we can apply Expression 3.6 and derive the inversion of e_1 , denoted by r'_1 , with respect to the circle radius R centering at f_1 as follows:

$$\begin{aligned}
r'_1 &= \frac{R^2}{r_1} \\
&= e_1 \frac{R^2}{d - e_2 r_2} \\
&= e_1 \frac{R^2}{d} + e_1 \frac{R^2 (d - d + e_2 r_2)}{d (d - e_2 r_2)} \\
&\quad \text{substitute } d - e_2 r_2 \text{ with } e_1 r_1 \text{ as in Expression 3.8} \\
&= e_1 \frac{R^2}{d} + e_1 \frac{e_2 R^2 r_2}{d (e_1 r_1)}.
\end{aligned}$$

We can apply Lemma 3.6.1 (i.e., $r'_2 = \frac{R^2 r_2}{d r_1}$); hence, the last expression becomes $r'_1 = e_1 \frac{R^2}{d} + e_2 r'_2$, which is another iso-cost curve as two-foci equation. ■

Theorem 3.6.1 shows that the associated coefficients e_1 and e_2 are unchanged; therefore, the mode of shipments remains the same. This result further suggests that a valid inversion curve may have different eccentricity values for inbound and outbound shipments. The last expression also refers to the appropriate inversion circle, particularly the distance of the inversion focus points is $\frac{R^2}{d}$ (Expression 3.7).

Theorem 3.6.1 also reveals the mathematical mechanism of the inversion: the interchange of the constant and variable terms of a two-foci curve. Specifically, we can interpret variable term r'_1 as “equivalent” to constant term d because they have the same coefficient. Similarly, constant term $\frac{R^2}{d}$ and variable r_1 are “equivalent” as they share coefficient e_1 . Theorem 3.6.1 suggests the interchanges between variable r and constant $\frac{R^2}{d}$ as well as between constant d and variable r'_1 . This means, the inversion is a process that describes an

original iso-cost curve in terms of a particular variable by interchanging it with the constant term. This interchange also explains why the store in Figure 3.10.b is the inversion point of the vendor in Figure 3.10.a with respect to a circle centering at a crossdock.

A limitation of inversion as the concept is that it is unable to address the non-zero handling cost case. Particularly, the last expression of Theorem 3.6.1 becomes $r'_1 = e_1 \frac{R^2}{d} + e_2 r'_2 + h r_1$ when handling costs are incurred at the crossdock. Though this expression is still a two-foci curve, the additional term results in an invalid inversion curve as the inversion part of handling costs is a function of distance, rather than a constant. In fact, the constant term of a valid inversion curve must be the multiplication of the distance between two focus points. This limits Theorem 3.6.1 to a zero-handling cost case. Another important limitation of the inversion is a center of inversion. If a center of inversion is not located at a focus point, the inversion curve is a three-foci curve, the focus points of which are the center of inversion and the inversion points of two original focus points. Locating a center of inversion at a focus point causes the inversion of one focus point is the point of infinity and, hence, reduces a three-foci curve into a two-foci curve.

The inversion may be interpreted as a dual statement regarding an iso-cost curve of a different facility perspective. For example, iso-cost curve XD(TL,LTL) is a set of crossdock locations in which the total transportation costs via an outsourced shipment and a consolidated shipment in mode of shipments (TL, LTL) are equal. This curve reflects the perspective of a vendor and a store. The inversion curve of iso-cost curve XD(TL,LTL) with respect to a circle centering at the vendor is iso-cost curve S(TL,LTL), which reflects the perspective of a crossdock and a vendor.

3.7 Voronoi Diagram

We have seen that an iso-cost curve partitions a plane into two areas. For example, the store that is located outside of an iso-cost curve receives consolidated shipments from a crossdock, while the inside-curve store receives outsourced shipments from a vendor. This concept of the geometric partition of plane is similar to the Voronoi diagram that can be extended to a multiple facility network, as shown in Figure 3.14.

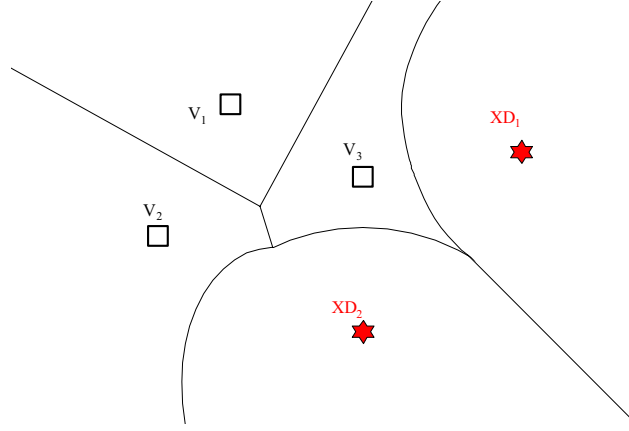


Figure 3.14: An example of a multiple-facilities iso-cost curves as Voronoi diagrams

In Figure 3.14, vendors and crossdocks partition the plane of stores into cells. A store should receive shipments from the facility located at the center of each cell.

In this section, we find that some iso-cost curves are generalizations of the Voronoi diagram. Our goals are to understand the connection between the Voronoi diagram and iso-cost curves and to apply the Voronoi diagram algorithms and insights into a multiple facility network. Hence, we will re-visit Figure 3.14 and after the explanation of terminology and the introduction of the generalizations.

3.7.1 Terminology

Given a set of points, called *vertices*, a Voronoi diagram partitions plane into cells by a set of curves, called *bisectors*. A bisector is constructed such that each cell contains only one vertex and it is the “nearest” vertex of the cell where the nearest point depends on how the distance function is defined. As a result, a cell is also known as a *dominated area* of a vertex.

In the simplest version of a Voronoi diagram, each bisector is a straight line, corresponding to two adjacent vertices, such that the distances between the bisector and the vertices are equal. In addition, the dominated areas are polygons. A classical application of Voronoi diagrams in the facility location problem is the division of area of responsibility in public facilities such as police stations and fire stations. The responsibility area of a particular fire station is a region where the fire station is closer than the other stations. In this application, vertices are locations of fire stations, and the responsibility area is a dominated area. A comprehensive review of the Voronoi diagram and its applications are discussed in Okabe *et al.* [47].

3.7.2 Compoundly Weighted Voronoi Diagram

Some iso-cost curves are the bisectors in a generalization of the Voronoi diagram, called the *compoundly weighted Voronoi diagram* (CWVD) [47]. In the CWVD, the nearness is defined as a linear expression of the Euclidian distance, and each vertex is associated with its location and two constants. Given vertex l , its location on the x - y coordinate system is denoted by x_l . The constants, which are named by the operation, are additive and multiplicative weighted constants, denoted by w_l^a and w_l^m , respectively. A physical visualization of the CWVD is the waves that are originated from different locations of a pond, generated at different times and moved with different speeds. The originated points and the collision boundaries of these wave are the vertices and the bisectors of CWVD, respectively. In the wave visualization of the CWVD, the additive weighted constants represent its generating times, whereas the multiplicative weighted constants correspond to travel speeds. The bisector between vertices l and k , denoted by B_{lk} , is defined as:

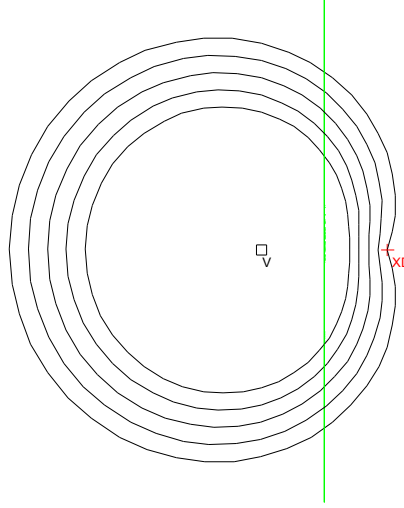
$$B_{lk} = \left\{ x \in \mathbb{R}^2 \mid w_l^m \|x - x_l\| - w_l^a = w_k^m \|x - x_k\| - w_k^a \right\}.$$

If $w_k^m \neq 0$, B_{lk} can be rewritten as:

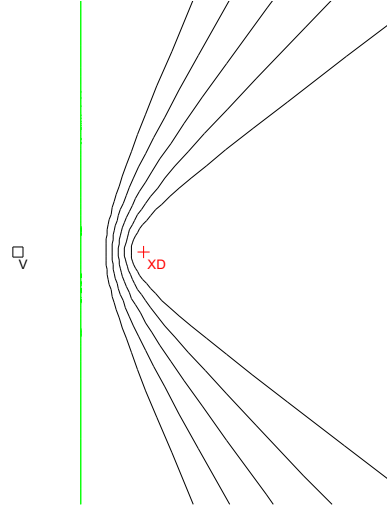
$$B_{lk} = \left\{ x \in \mathbb{R}^2 \mid \frac{w_l^m}{w_k^m} \|x - x_l\| - \|x - x_k\| = \frac{w_l^a - w_k^a}{w_k^m} \right\}. \quad (3.9)$$

In Expression 3.9, $\frac{w_l^m}{w_k^m}$ is interpreted as the ratio of moving speeds, while $\frac{w_l^a - w_k^a}{w_k^m}$ is

seen as the difference in generating time with respect to the speed of vertex k . This wave visualization is useful to determine the dominated area of CWVD [33] because the closed-form expression, in general, is difficult to derive. If $\frac{w_l^m}{w_k^m} = 1$ and $\frac{w_l^a - w_k^a}{w_k^m} = 0$, Expression 3.9 is a simple Voronoi diagram. An example of the CWVD bisectors that are generated by the two vertices is shown in Figure 3.15.



(a) CWVD when $\frac{w_l^m}{w_k^m} = \frac{1}{3}$



(b) CWVD when $\frac{w_l^m}{w_k^m} = 1$

Figure 3.15: If $\frac{w_l^m}{w_k^m} \neq 1$ (Figure 3.15.a), the bisection of the two-vertex compoundly weighted Voronoi diagram (CWVD) is a Cartesian oval. Otherwise, it is a half-hyperbola (Figure 3.15.b).

Figures 3.15.a and 3.15.b present two series of curves that have different additive weighted constants but share the same multiplicative weighted constants. In Figure 3.15.a, the ratio of moving speeds is $\frac{1}{3}$, while the two vertices have the same moving speed ($\frac{w_l^m}{w_k^m} = 1$) in Figure 3.15.b.

After introducing CWVD, we need to discuss its connection with iso-cost curves, particularly the curves in Figure 3.15 which are similar to the six iso-cost curves shown in the second and third rows in Figures 3.8.d-3.8.h and 3.3.d-3.3.h. In fact, Expression 3.9 is equivalent to the two-foci equation in Table 3.2 when either a vendor or a store is an interested facility; therefore, the bisection of CWVD is either a Cartesian oval curve (Figure 3.15.a) when $\frac{w_l^m}{w_k^m} \neq 1$ or a half hyperbola curve^{3.3} (Figure 3.15.b) when $\frac{w_l^m}{w_k^m} = 1$. It is worth remarking that a two-foci equation can be transformed into a bi-section expression. However, a bi-section expression has insufficient information to determine parameters of an iso-cost curve. A mode of shipments and handling costs are required when $\frac{w_l^m}{w_k^m} \neq 1$ and $\frac{w_l^m}{w_k^m} = 1$, respectively.

Having shown their mathematical connection, we interpret the iso-cost curve in terms of the CWVD. Nonetheless, the interpretation is not intuitive because a vertex in Voronoi diagram is assumed to be identical, while an iso-cost curve always consists of two different facilities. This raises a question how to view two different facilities as identical. We answer the question through the single vendor-crossdock network. The goal of the network is to determine whether a store should receive an outsourced shipment from a vendor or else a consolidated shipment from a crossdock. In this environment, a store views crossdocks and vendors as competitive sourcing facilities that supply an identical product. A crossdock can be seen as an *artificial vendor* that acquires the same product at a higher cost per pound. The additional cost is the sum of handling costs at the crossdock and transportation costs from the vendor to the crossdock. An advantage of an artificial vendor is that it provides a lower unit of transportation costs to a store (cost per mile-pound). On the other hand, receiving a shipment from a real vendor incurs only the transportation costs to a store, but at a higher unit cost. The competitive sourcing metaphor suggests that iso-cost curves of a

^{3.3}We consider a line as a special case of a half hyperbola

crossdock (Figures 3.3.a-3.3.c and 3.8.a-3.8.c) are unable to be described as Voronoi diagram because neither the outsourced-shipping region nor the consolidated-shipping region is a dominated region. In fact, vendors and stores never compete for shipments; they both influence the consolidated-shipping region through a crossdock. Hence, the concept of an iso-cost curve overlaps with the Voronoi diagram, but they are not interchangeable.

The fact that the concept of an iso-cost curve overlaps with the CWVD presents an opportunity to apply the CWVD algorithms and its extensions in various networks. We show an example in which the CWVD solves a shipping decision in a single hub crossdocking network.

3.7.2.1 *Network with Multiple Vendor Locations*

The CWVD can be applied to a network that consists of multiple vendors and crossdocks. A store may receive a single shipment of products from either a vendor or a crossdock. To transform the network into CWVD, we need to compute the additive and multiplicative weighted constants of each vertex. For each vendor, the additive weighted constant is zero, and the multiplicative weighted constant is equal to eccentricity as the shipment from a vendor to a store is LTL. Given a crossdock, we need to determine its cheapest sourcing location (vendor or crossdock). Then, the incoming transportation costs from the location to a crossdock and the handling costs at a crossdock become the additive weighted constant. The multiplicative weighted constant of a crossdock depends on a mode of shipments, particularly an outbound shipment from a crossdock to a store. Once both constants are determined, a bisector algorithm of the CWVD is directly applied, and the shipping dominated areas of each facility are computed.

The dominated areas and bisections of the network are shown in Figure 3.14. In Figure 3.14, the CWVD-dominated area of a vendor is the outsourced-shipping region, while the consolidated-shipping region is equivalent to the CWVD-dominated area of a crossdock. The bisections represent the locations where transportation costs between two adjacent vertices are equal. The bisection between vendors is a line because their additive and multiplicative weighted constants are always the same. That is, $\frac{w_{V_l}^m}{w_{V_k}^m} = 1$ and $\frac{w_{V_l}^a - w_{V_k}^a}{w_{V_k}^m} = 0$. The

bisection between a vendor and a crossdock is always a part of the Cartesian oval because the additive weighted constant of a crossdock is always non-zero, while that of a vendor is always zero. The bisection between crossdocks may be a line, a part of a half-hyperbola or a part of a Cartesian oval. If crossdocks have different modes of shipments, the bisection is a part of a Cartesian oval. The bisection is a line when crossdocks share the mode of shipments and the sum of inbound transportation costs and handling costs are equal. Otherwise, the bisection between adjacent crossdocks is a part of a half-hyperbola, i.e., $\frac{w_{XD_l}^m}{w_{XD_k}^m} = 1$ and $\frac{w_{XD_l}^a - w_{XD_k}^a}{w_{XD_k}^m} \neq 0$.

Given vertices, the bisections can be computed by an algorithm proposed by Hagen *et al.* [33], who exploits the circular expansions of vertices when they are represented by the wave visualization. The algorithm computes collision points between different waves at pre-determined interval period and then connects the collision points of a certain pair of vertices chronologically to determine a bisection. In general, the closed-formed derivation of a bisection is difficult because two adjacent bisections may not intersect or may intersect one or two points.

In the next chapter, the concept of iso-cost curves is applied to a multiple vendor-store pairs network, and a measurement of crossdock attractiveness is studied.

CHAPTER IV

ATTRACTIVENESS OF SINGLE HUB CROSSDOCKING NETWORK

In this chapter, we analyze consolidation in a single hub crossdocking network consisting of multiple vendor-store pairs and propose a measurement of crossdock attractiveness, namely *asymptotic probability of shipments*, that explains economic factors affecting uses of a crossdock. We also derive the closed-form expressions in selected one- and two-dimensional spaces and suggest a Monte Carlo Simulation to obtain the numerical values.

4.1 *Attractiveness of a Crossdock*

In the previous chapter, an iso-cost curve can be viewed as the boundary between consolidated-shipping and outsourced-shipping regions. The size of the consolidated-shipping region, computed by a given mode of shipments and two facility locations, may be interpreted as the likelihood of shipments being consolidated through a crossdock, or the *attractiveness* of a crossdock. For example, a distant vendor-store pair induces a large consolidated-shipping region which, in turn, increases the likelihood of a crossdock located within the region and shipments to be consolidated. The larger the size of a consolidated-shipping region, the higher likelihood of freight being shipped through the crossdock. The region size reflects shipping parameters, including eccentricity, handling costs and modes of shipments. However, the consolidated-shipping region fails to address the attractiveness of a crossdock in a multiple vendor-store pairs network because such a network has many overlapped consolidated-shipping regions. In addition, the region is unable to explain important factors that affect the attractiveness of a crossdock, such as a crossdock location as well as spatial distributions of vendors and stores.

Hence, we study consolidated shipments in a network that consists of a single fixed-location crossdock and multiple vendors and stores. Vendors and stores are, unless mentioned otherwise, uniformly distributed across given space in the network. A shipment can be transported either by an LTL outsourced shipment or by a consolidated shipment through

the crossdock. The shipping decision of each pair is independent of other pairs. However, all shipments that are consolidated through the crossdock must have the same mode of shipments. We are interested in the attractiveness of a crossdock in terms of a likelihood of shipments being consolidated and propose a measurement of such the attractiveness.

Definition 4.1.1. *Given spatial distributions of vendors and stores, a fraction of consolidated shipments to total shipments is called **asymptotic probability of shipments** if it is independent of numbers of vendors and stores in a network.*

This measurement computes the probability of consolidated shipments in the network. Specifically, the asymptotic probability of shipments determines the expected ratio of consolidated shipments for a given eccentricity and mode of shipments. In addition, the proposed measurement may be viewed as a steady-state of consolidated shipments. The assumptions on the distributions of vendors and stores and the mode of shipments differentiate this network from the single hub crossdocking network discussed in the previous chapter.

The analysis of this measurement shows that it is also independent of the order of modes of shipments. In particular, the asymptotic-probability functions of modes of shipments (LTL, TL) and (TL, LTL) are identical when spatial distributions of vendors and stores are the same, as discussed in Theorem 4.1.1.

Theorem 4.1.1. *For identical distribution of vendors and stores, the asymptotic probabilities of shipments in modes of shipments (LTL, TL) and (TL, LTL) are equal.*

Proof. Since spatial distributions of vendors and stores are identical and the definition of asymptotic probability of shipments implies infinite numbers of vendors and stores, vendors and stores are always located very close to each other. Mathematically, for any vendor V_i and $\epsilon > 0$, there exists store S_k and an ϵ -neighborhood of vendor V_i , denoted by $\mathcal{B}_\epsilon(V_i)$, such that $S_k \in \mathcal{B}_\epsilon(V_i)$. Similarly, for any store S_j and $\epsilon > 0$, there exists store V_l and an ϵ -neighborhood of store S_j , denoted by $\mathcal{B}_\epsilon(S_j)$, such that $V_l \in \mathcal{B}_\epsilon(S_j)$.

Let's consider store S_j that receives its freight from vendor V_i through a crossdock. We assume that the shipment from vendor V_i to a particular crossdock is LTL while the shipment from the crossdock to store S_j is TL (mode of shipments (LTL, TL)). From the

earlier argument, we can find a vendor that is located within an ϵ -neighborhood of store S_j , denoted by $V_\epsilon(S_j)$, and a store that is located within an ϵ -neighborhood of vendor V_i , denoted by $S_\epsilon(V_i)$, with probability 1. As ϵ approaches zero, the distances from the crossdock to vendor $V_\epsilon(S_j)$ and store $S_\epsilon(V_i)$ converge to the distances from the crossdock to store S_j and vendor V_i , respectively. Since vendor-store pair (V_i, S_j) ships through the crossdock, vendor-store pair $(V_\epsilon(S_j), S_\epsilon(V_i))$ also ships through the crossdock with the same transportation costs when the shipment from vendor $V_\epsilon(S_j)$ to the crossdock is TL and the shipment from the crossdock to store $S_\epsilon(V_i)$ is LTL (mode of shipments (TL, LTL)). Hence, the consolidation of vendor-store pairs in mode of shipments (LTL, TL) implies that in mode of shipments (TL, LTL) .

The similar arguments can be applied to mode of shipments (TL, LTL) . Hence, the consolidation of vendor-store pairs in mode of shipments (TL, LTL) implies that in mode of shipments (LTL, TL) . Thus, the asymptotic probabilities of shipments in modes of shipments (LTL, TL) and (TL, LTL) are equivalent. ■

If vendors and stores are uniformly distributed, Theorem 4.1.1 affirms that the asymptotic-probability function depends on numbers of TL shipments and is independent of the order of shipments. The asymptotic-probability functions of modes of shipments (LTL, TL) and (TL, LTL) are identical, and we referred to these modes of shipments as $LTL-TL$.

Next, we compute and discuss the asymptotic probabilities of shipments in the unit interval, the unit circle and the unit disk, respectively.

4.2 Asymptotic Probability of Shipments in the Unit Interval

Having introduced the concept of the asymptotic probability of shipments, we analyze its properties in the unit interval and derive its closed-form expression for mode of shipments (TL, TL) by a given vendor location, as shown in Table 4.1.

Table 4.1: The asymptotic-probability function in the unit interval of mode of shipments (TL, TL)

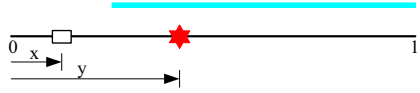
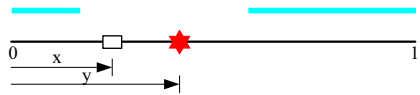
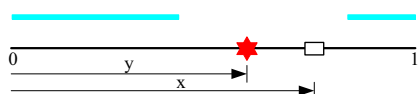
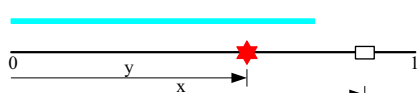
Case	Domain of integration	Integrand
1 	$0 \leq x < \frac{2ey}{1+e}$	$\int_0^{\frac{2ey}{1+e}} 1 - x - \frac{2e(y-x)}{1+e} dx$ $= \frac{2ey(1+2e+e^2-3ey-e^2y)}{(1+e)^3}$
2 	$\frac{2ey}{1+e} \leq x < y$	$\int_{\frac{2ey}{1+e}}^y 1 - \frac{4e(y-x)}{1+e^2} dx$ $= \frac{y(1-e)(1+2e+e^2-2ey)}{(1+e)^3}$
3 	$y \leq x < \frac{1-e+2ey}{1+e}$	$\int_y^{\frac{1-e+2ey}{1+e}} 1 - \frac{4e(x-y)}{1+e^2} dx$ $= \frac{(1-y)(1-e)(1+2ey+e^2)}{(1+e)^3}$
4 	$\frac{1-e+2ey}{1+e} \leq x \leq 1$	$\int_{\frac{1-e+2ey}{1+e}}^1 x - \frac{2e(x-y)}{1+e} dx$ $= \frac{2e(1-y)(1-e+3ey+e^2y)}{(1+e)^3}$
Summation of given location crossdock		$f(e, y) = \frac{1-e+4ey-4ey^2}{1+e}$
Summation of random crossdock	$0 \leq y \leq 1$	$\int_0^1 f(e, y) dy = \frac{3-e}{3(1+e)}$

Table 4.1 shows four different domains of integration, resulting from the relationship of a vendor and a crossdock. The first column depicts the relationship of a vendor (black square) and a crossdock (red star) located in the unit interval $[0, 1]$. In addition, the blue bar above the unit interval represents a consolidated-shipping region of the given vendor and crossdock, while an outsourced-shipping region is uncolored. These regions are computed by iso-cost curve $S(TL, TL)$. A store that is located in the consolidated-shipping region should receive freight from the vendor through the crossdock, i.e., the consolidated shipments in mode of shipments (TL, TL). The next column is “Domain of integration”, the mathematical description of the first column. It describes the domain of a vendor location (x), the interval used to derive the last column. “Integrand” column

summarizes the first two columns and shows the asymptotic-probability function of each case. Given crossdock (y) and eccentricity (e), the summation of these functions becomes the asymptotic-probability function, denoted by $f(e, y)$, and the integration of this function across all crossdock locations is the asymptotic-probability function of randomized crossdock (last row). We are interested in the interactions between eccentricity and a crossdock location, as shown in Figure 4.1.

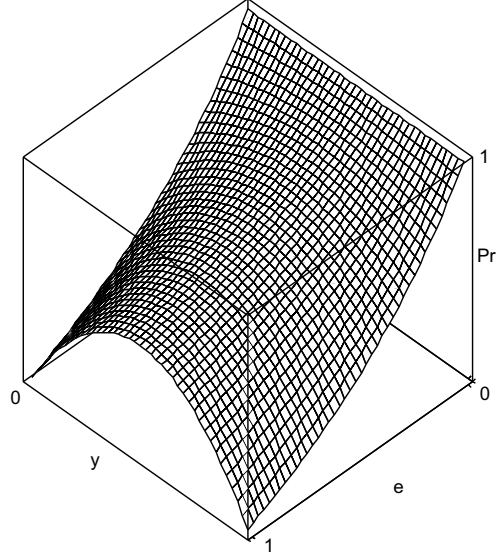


Figure 4.1: In mode of shipments (TL, TL) or when all shipments are TL, the asymptotic probability of shipments (z -axis) is a decreasing function in terms of eccentricity (e) and a parabola function in terms of a crossdock location (y).

Figure 4.1 shows the asymptotic-probability function (z -axis) as a function of eccentricity and a crossdock location. The asymptotic-probability function is constant and attains its maximum value when eccentricity is equal to zero, i.e., $f(e, y) = 1$ when $e = 0$ and $0 \leq y \leq 1$. For other values of eccentricity ($0 > e \geq 1$), the function is a parabola function of a crossdock location. The symmetric axis of the parabola is located at the center of the unit interval ($y = \frac{1}{2}$). As a result, points along the center of the unit interval are the most attractive locations of crossdocks for given eccentricity. In fact, the crossdock located at the center always consolidates at least half of all shipments in a network. In addition, two crossdocks have the same attractiveness when their distances to the center of the unit interval are equal (e.g., $y = \frac{2}{5}$ and $y = \frac{3}{5}$). For a given crossdock location, the function is a decreasing

non-linear function of eccentricity. In Figure 4.1, the asymptotic-probability function yields its minimum value when a TL shipment offers no savings (i.e., $e = 1$) and a crossdock is located at the boundary of the unit interval. This means, shipments are always consolidated through a crossdock unless a crossdock is located at the boundary, i.e., $f(e, y) > 0$ when $e = 1$ and $0 < y < 1$. The asymptotic-probability function is discontinuous when a TL shipment offers no savings ($\lim_{e \rightarrow 1^-} f(e, y) \neq \lim_{e \rightarrow 1^+} f(e, y)$, $0 < y < 1$). This suggests that a large number of shipments can be either consolidated or outsourced because the transportation costs are equal. The result is a special case that occurs only in one-dimensional space, and we discuss in Section 4.2.1 when the asymptotic-probability functions of different modes of shipments are compared. Table 4.1 also shows that the expected asymptotic-probability function when a crossdock is randomly located in the unit interval is $\frac{3-e}{3(1+e)}$. If a TL shipment offers no savings the probability becomes $\frac{3-1}{3(1+1)} = \frac{1}{3}$, which is equivalent to the probability that a crossdock is randomly located between a vendor and a store.

Table 4.1 and Figure 4.1 are derived from a given vendor location. In general, the asymptotic-probability function can be derived from the conditional probability on either a store location or a vendor location. The result of both methods is identical, as stated in Proposition 4.2.1.

Proposition 4.2.1. *The asymptotic probability of shipments can be derived from conditioning on either a vendor location or a store location.*

The proof of Proposition 4.2.1 is skipped because it follows a standard conditional probability.

After discussing the asymptotic-probability function of mode of shipments (TL, TL), we apply this methodology to modes of shipments (LTL, TL) and (TL, LTL), or $LTL-TL$. In particular, we consider the unit-interval single hub crossdocking network when one of the shipments through a crossdock is TL and the other is LTL. The mathematical properties are derived and shown in Tables 4.2 and 4.3.

Table 4.2: The asymptotic-probability function in the unit interval of mode of shipments $LTL-TL$ derived from mode of shipments (LTL, TL)

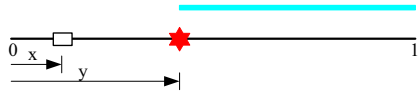
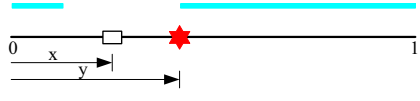
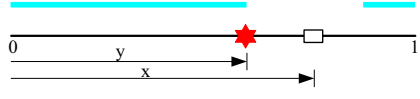
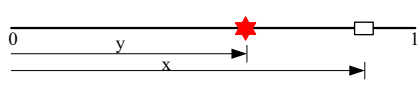
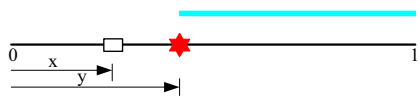
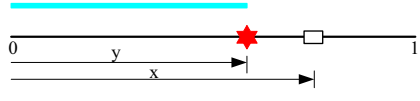
Case	Domain of integration	Integrand
1 	$0 \leq x < \frac{y(1+e)}{2}$	$\int_0^{\frac{y(1+e)}{2}} 1 - y \, dx$ $= \frac{y^2(1+e)}{2}$
2 	$\frac{y(1+e)}{2} \leq x < y$	$\int_{\frac{y(1+e)}{2}}^y 1 - \frac{2(y-x)}{1-e} \, dx$ $= \frac{(2-y)(1-e)y}{4}$
3 	$y \leq x < \frac{1-e+y+ey}{2}$	$\int_y^{\frac{1-e+y+ey}{2}} 1 - \frac{2(x-y)}{1-e} \, dx$ $= \frac{(1-y)(y+1)(1-e)}{4}$
4 	$\frac{1-e+y+ey}{2} \leq x \leq 1$	$\int_{\frac{1-e+y+ey}{2}}^1 y \, dx$ $= \frac{(y-1)^2(1+e)}{2}$
Summation of given location crossdock		$f(e, y) = \frac{1-e+2ey-2ey^2+6y-6y^2}{4}$
Summation of random crossdock	$0 \leq y \leq 1$	$\int_0^1 f(e, y) \, dy = \frac{3-e}{6}$

Table 4.3: The asymptotic-probability function in the unit interval of mode of shipments $LTL-TL$ derived from mode of shipments (TL, LTL)

Case	Domain of integration	Integrand
1 	$0 \leq x < y$	$\int_0^y 1 - \left(x + \frac{(1+e)(y-x)}{2} \right) \, dx$ $= \frac{y(4-ey-3y)}{4}$
2 	$y \leq x \leq 1$	$\int_y^1 x - \frac{(1+e)(x-y)}{2} \, dx$ $= \frac{(1-y)(1-e+ey+3y)}{4}$
Summation of given location crossdock		$f(e, y) = \frac{1-e+2ey-2ey^2+6y-6y^2}{4}$
Summation of random crossdock	$0 \leq y \leq 1$	$\int_0^1 f(e, y) \, dy = \frac{3-e}{6}$

Tables 4.2 and 4.3 show the different domains of integration. Similar to Table 4.1, their first and second columns illustrate and represent their mathematical relationship of each case, depending on a given vendor location. The domain of each case is a function of eccentricity and a crossdock location. The last column of Tables 4.2 and 4.3 shows their consolidate-shipping region and integrand. The summation of the integrand of each case becomes the asymptotic-probability function. Although the derivation procedures of modes of shipments (LTL, TL) and (TL, LTL) are different, their asymptotic-probability functions are identical because spatial distributions of vendors and stores are the same (Theorem 4.1.1). The surface of this asymptotic-probability function is illustrated in Figure 4.2.

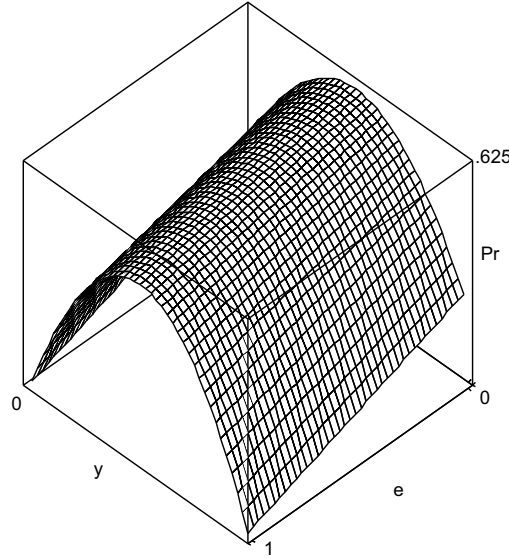


Figure 4.2: In modes of shipments (LTL, TL) and (TL, LTL) , the asymptotic probability of shipments (z -axis) is a linear function in terms of eccentricity (x -axis) and a parabola in terms of a crossdock (y -axis).

Figure 4.2 shows the asymptotic probability of shipments is a linear function in terms of eccentricity. It shares many similarities with Figure 4.1, particularly a parabola function in terms of a crossdock and discontinuity of asymptotic-probability function. The symmetric axis of the parabola is located at the center of the unit interval; therefore, two crossdocks located across the symmetric axis yield the same values. The closer a crossdock is to the boundary of the unit interval, the fewer consolidated shipments through the crossdock. Hence, the asymptotic probability of shipments (i.e., the ratio of consolidated shipments)

reaches its maximum when eccentricity is zero and a crossdock is located at the center of the unit interval, i.e., $f(e, y) = 0.625$. Similar to mode of shipments (TL, TL) , the asymptotic-probability function of modes of shipments (LTL, TL) and (TL, LTL) is discontinuous when eccentricity reaches one. As a result, a crossdock located between a vendor and a store continues to consolidate shipments even though a TL shipment offers no savings in transportation costs. If a crossdock is randomly located, the probability that this crossdock receives or sends exactly one TL and one LTL is $\frac{3-e}{6}$.

This methodology is applied to any spatial distributions of vendors and stores to derive specific asymptotic-probability functions. The domains of integration are independent of spatial distributions of vendor and/or stores; therefore, the consolidated- and outsourced-shipping regions remain intact. However, different distributions of vendors and stores result in different integrands and asymptotic-probability functions. We apply the concept to selected spatial distributions in which their densities depend on a crossdock location and derive the asymptotic-probability functions, as shown in Appendix B.1. The analysis of these functions provides no further insight regarding the attractiveness of a crossdock with exception of the preference of TL shipment in a long-haul shipment. If vendors, for example, are concentrated near a crossdock while stores are dispersed around a crossdock, mode of shipments (LTL, TL) results in a higher asymptotic probability of shipments than mode of shipments (TL, LTL) does because the long-haul outbound shipments between a crossdock and stores in mode of shipments (LTL, TL) are TL shipments.

4.2.1 Ratio of Stores Receiving Consolidated Shipments

The derivation of asymptotic-probability functions also reveals store locations that receive consolidated shipments through a crossdock from a particular vendor. A ratio of such stores to total stores in the network, referred to as a *ratio of stores receiving consolidated shipments*, is the integral part^{4.1} of each case in Tables 4.1, 4.2 and 4.3. For example, the integral part of Case 1 in Table 4.1 is $1 - x - \frac{2e(y-x)}{1+e}$. Given a mode of shipments, its closed-form expression is a function of eccentricity as well as locations of a vendor and a crossdock. We

^{4.1}Each integral part is derived from a conditional of a vendor location, whereas stores are assumed to be uniformly distribution.

are interested in the ratio of stores receiving consolidated shipments because it can be used to compare the attractiveness of a crossdock to different vendors and modes of shipments, as shown in Figure 4.3.

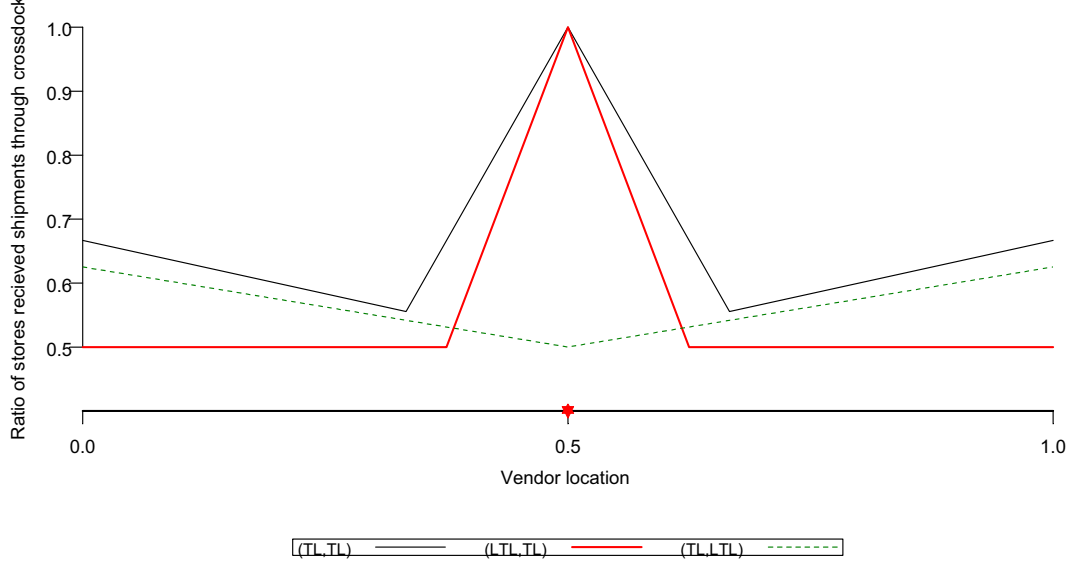


Figure 4.3: The ratio of stores receiving consolidated shipments through a crossdock, located at the center of the unit interval, at eccentricity 0.5.

Figure 4.3 shows the ratio of stores receiving consolidated shipments through a crossdock, located at the center of the unit interval, at eccentricity 0.5. Because of the crossdock location, the first and second halves of the unit interval are symmetric with respect to a vendor location (x -axis). Therefore, we need to discuss the ratio only when a vendor is located in the first half of the unit interval. In mode of shipments (TL, LTL) , the ratio of stores receiving consolidated shipments through a crossdock is $\frac{5}{8} \left(\frac{5-2x}{8} \right)$ when a vendor is located at the boundary of the interval ($x = 0$). The ratio gradually decreases as a vendor is located near a crossdock, implying decreases in benefits of consolidated shipments from vendors to the crossdock. In particular, stores located in $[0, x + \frac{2x-3}{8}]$ should receive outsourced shipments, while stores located in $[\frac{2x-3}{8}, 1]$ should receive consolidated shipments. Interestingly, every store located in the second half of the unit interval receives consolidated shipments through a crossdock because the crossdock (the center) is always located between vendors (the first half) and stores (the second half). Hence, the shipping distances

of the outsourced and consolidated shipments are always equal ($d_{ip} + d_{pj} = d_{ij}$). As a result, a half of vendor-store pairs may be either consolidated or outsourced when a TL shipment offers no savings with the same transportation costs, and a crossdock always consolidates some shipments unless it is located at the boundary of the unit interval.

In mode of shipments (LTL, TL), the ratio of stores receiving consolidated shipments is constant for a distant vendor ($0 \leq x \leq \frac{3}{8}$), and stores that receive consolidated shipments are located in the second half of the unit interval. As a vendor is located near a crossdock ($\frac{3}{8} \leq x \leq \frac{1}{2}$), additional stores that are located in $[0, \frac{8x-3}{4}]$ should receive consolidated shipments.

The last mode is mode of shipments (TL, TL) that shares features of modes of shipments (TL, LTL) and (LTL, TL). Particularly, the ratio decreases and then increases afterward. In mode of shipments (TL, TL), $\frac{2}{3}$ of stores receive consolidated shipments through a crossdock when a vendor located at the boundary of the interval. As a vendor is located in $[0, \frac{1}{3}]$, the ratio of stores receiving consolidated shipments gradually decreases until the value reaches a threshold (i.e., the ratio is $\frac{5}{9}$ at $e = \frac{1}{2}$ and $y = \frac{1}{2}$) because stores located in $[\frac{x+1}{3}, 1]$ should receive consolidated shipments. Then, the ratio increases (i.e., $\frac{8x-1}{3}$) because additional stores located in $[0, 3x-1]$ begin to receive consolidated shipments as the vendor is located closer to a crossdock, i.e., $\frac{1}{3} \leq x \leq \frac{1}{2}$. Such a threshold, denoted by τ , is a function of eccentricity and a crossdock location, as shown in Expression 4.1.

$$\tau(e, y) = 1 - \frac{4e \max(y, 1-y)}{(1+e)^2} \quad (4.1)$$

The location of the vendor corresponded to the value of Expression 4.1 depends on a crossdock location. Specifically, the vendor is located at $\frac{1-e+2ey}{1+e}$ and $\frac{2ey}{1+e}$ when a crossdock is located in the first and second halves of the unit interval, respectively. The threshold reaches its maximum when a crossdock is located at the center of the unit interval. If the crossdock location is moved from the center, the value of Expression 4.1 decreases. This expression indicates the location of a vendor that ships the least consolidated shipments through a crossdock. Such a vendor would be the first vendor to switch from mode of shipments (TL, TL) to other modes of shipments, if a certain size of shipment were required

to ship TL. Expression 4.1 also determines the minimal consolidated shipments originating at such a vendor and enables us to compute the additional shipments needed to retain mode of shipments (TL, TL) . This concept will be discussed in detail when we introduce a concept of TL threshold, in Chapter 5.

Figure 4.3 shows the sizes of consolidated shipments originating at a certain vendor in different modes of shipments. For example, a vendor located at the center of the unit interval should consolidate all of its shipments and ship them to a crossdock if stores always receive TL shipments from the crossdock (modes of shipments (TL, TL) and (LTL, TL)), whereas the same vendor should consolidate only half of its freight when stores always receive LTL shipments from the crossdock (mode of shipments (TL, LTL)). In Figure 4.3, the ratio of mode of shipments (TL, TL) dominates those of other modes of shipments, indicating that mode of shipments (TL, TL) is more attractive than modes of shipments (LTL, TL) and (TL, LTL) . However, the comparison between modes of shipments (LTL, TL) and (TL, LTL) is unclear. Figure 4.3 suggests that the attractiveness of a crossdock between these modes of shipments depends on the relative distances and a TL shipment. Specifically, a crossdock is more attractive in a particular mode of shipments if the majority of shipping distances are covered by TL shipments. For example, a crossdock appears more attractive to distant vendors when shipments from the vendors to a crossdock are TL (i.e., mode of shipments (TL, LTL)) because the majority of shipments travel from vendors to a crossdock. Conversely, vendors located near a crossdock reaps more benefits from mode of shipments (LTL, TL) because the travel distances from a crossdock to stores dominate those from vendors to a crossdock. The curves in Figure 4.3 also connect to the asymptotic probability functions as the area under each curve is the asymptotic probability of shipments of each mode of shipments when vendors and stores are uniformly distributed. Hence, the area under curve of mode of shipments (TL, TL) is the largest while the area under curve of modes of shipments (LTL, TL) and (TL, LTL) are equal.

The results of the unit interval show that the attractiveness of a crossdock is unstable when the outsourced-shipping region overlaps with boundary of the unit interval, i.e., Cases 1 and 4 of Tables 4.1 and 4.2. As a result, we consider an unbounded space, a *unit*

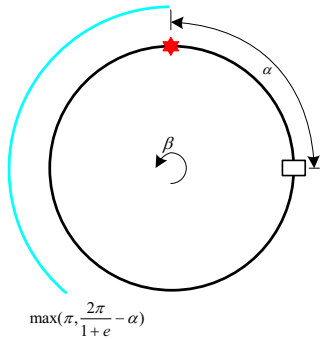
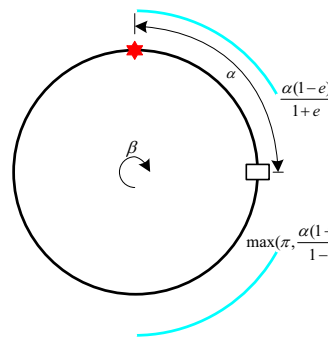
circle.

4.3 Asymptotic Probability of Shipments in the Unit Circle

In this space, facilities are located on a circle, i.e., $[0, 2\pi]$. The distance between two facilities is measured by the minimum angle between their locations, the value of which is between 0 and π . Without any loss of generality, a crossdock is assumed to be located at zero.

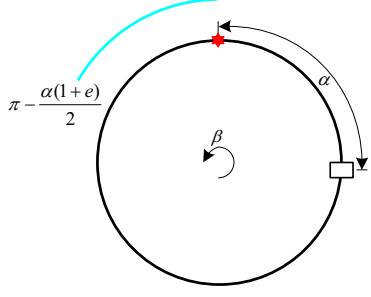
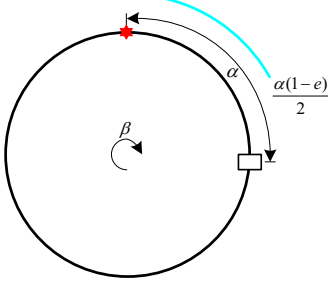
Given vendor and store locations, denoted by α and β , we derive asymptotic-probability functions of modes of shipments (TL, TL) and $LTL-TL$, as shown in Tables 4.4 and 4.5.

Table 4.4: The asymptotic-probability function in the circle of modes of shipments (TL, TL)

Case	Domain of integration	Integrand
<p>1</p>  <p>$\max(\pi, \frac{2\pi}{1+e} - \alpha)$</p>	$0 < \alpha < \pi$ $0 < \beta < \pi$ β ccw	$\frac{1}{2\pi^2} \left[\int_0^{\frac{2\pi}{1+e} - \pi} \pi d\alpha + \int_{\frac{2\pi}{1+e} - \pi}^{\frac{2\pi}{1+e}} \frac{2\pi}{1+e} - \alpha d\alpha \right]$ $= \frac{1+2e-e^2}{2(1+e)^2}$
<p>2</p>  <p>$\frac{\alpha(1-e)}{1+e}$ $\max(\pi, \frac{\alpha(1+e)}{1-e})$</p>	$0 < \alpha < \pi$ $0 < \beta < \pi$ β cw	$\frac{1}{2\pi^2} \left[\int_0^{\frac{(1-e)\pi}{1+e}} \pi - \frac{4e\alpha}{1-e^2} d\alpha + \int_{\frac{(1-e)\pi}{1+e}}^{\frac{\alpha(1+e)}{1-e}} \frac{\alpha(1-e)}{1+e} d\alpha \right]$ $= \frac{1-e}{2(1+e)}$
Summation		$f(e) = \frac{1+e-e^2}{(1+e)^2}$

Tables 4.4 and 4.5 show the derivation of the asymptotic-probability functions when a crossdock always both receives and sends TL shipments (mode of shipments (TL, TL))

Table 4.5: The asymptotic-probability function in the circle of mode of shipments *LTL-TL*

Case	Domain of integration	Integrand
<div style="display: flex; align-items: center;"> <div style="margin-right: 20px;">1</div>  </div>	$ \begin{aligned} 0 < \alpha < \pi \\ 0 < \beta < \pi \\ \beta \text{ ccw} \end{aligned} $	$ \begin{aligned} \frac{1}{2\pi^2} \left[\int_0^\pi \pi - \frac{\alpha(1+e)}{2} d\alpha \right] \\ = \frac{3-e}{8} \end{aligned} $
<div style="display: flex; align-items: center;"> <div style="margin-right: 20px;">2</div>  </div>	$ \begin{aligned} 0 < \alpha < \pi \\ 0 < \beta < \pi \\ \beta \text{ cw} \end{aligned} $	$ \begin{aligned} \frac{1}{2\pi^2} \left[\int_0^\pi \frac{\alpha(1-e)}{2} d\alpha \right] \\ = \frac{1-e}{8} \end{aligned} $
Summation		$f(e) = \frac{2-e}{4}$

and when a crossdock either receives or sends exactly one TL shipment (modes of shipments (*LTL, TL*) and (*TL, LTL*)). Each table is divided into two cases each of which represents different store locations with respect to a crossdock and a vendor. Case 1 represents stores located in the counter-clockwise (ccw) direction of a crossdock, whereas stores that are located in the clockwise (cw) direction are represented in Case 2. In both cases, the first column of the tables depicts the relationship of a vendor (black square) and a crossdock (red star) that located at the rim of a circle. In addition, the blue curves represent store locations that receive shipments from a given vendor through a crossdock, or consolidated-shipping region. Tables 4.4 and 4.5 emphasize that stores receive consolidated shipments through a crossdock when they are located near the crossdock and receive outsourced shipments directly from a vendor when they located near a vendor. The tables suggest that asymptotic probability of shipments is independent of a crossdock location because vendors and stores are uniformly distributed and the unit circle is unbounded. The mathematical

relationship is presented in the second column of the tables. The last column shows the integrand of a given direction of stores. The summation of both directions becomes the asymptotic-probability function, as shown in Figure 4.4.

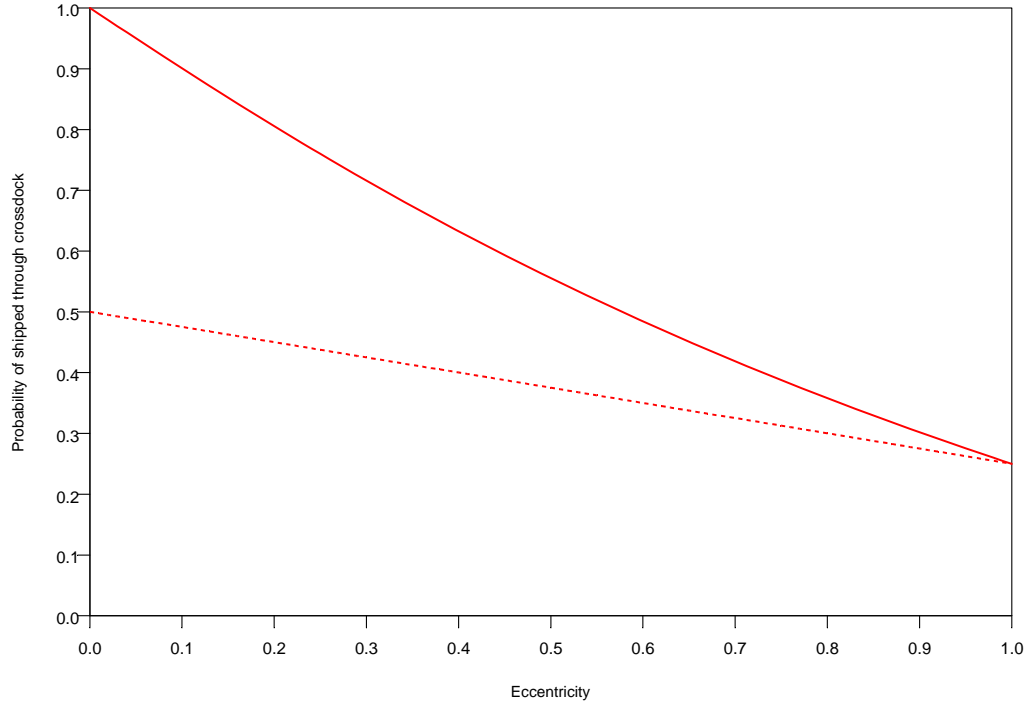


Figure 4.4: In the unit circle, the asymptotic probability of shipments (y -axis) of modes of shipments (TL, TL) , denoted by solid curve, and $LTL-TL$, denoted by a dotted curve, are decreasing functions of eccentricity.

The y - and x -axes of Figure 4.4 represent the asymptotic probability of shipments and eccentricity in the unit circle, respectively. In Figure 4.4, patterns of each curve symbolize different modes of shipments through a crossdock. The asymptotic probability of shipments in modes of shipments (TL, TL) , denoted by a solid curve, and $LTL-TL$, denoted by a dotted curve, are decreasing functions of eccentricity. We observe that both functions converge to the same value when a TL shipment offers no savings in transportation cost. Figure 4.4 reveals that the asymptotic-probability functions of the unit circle is also discontinuous. Specifically, 25% of shipments can be either consolidated through a crossdock or

outsourced directly to stores when eccentricity is one, and a crossdock always consolidates shipments as long as it is located between vendors and stores. This observation is similar to the asymptotic-probability functions of the unit interval. Therefore, the asymptotic-probability functions of the unit interval are added and compared, as shown in Figure 4.5.

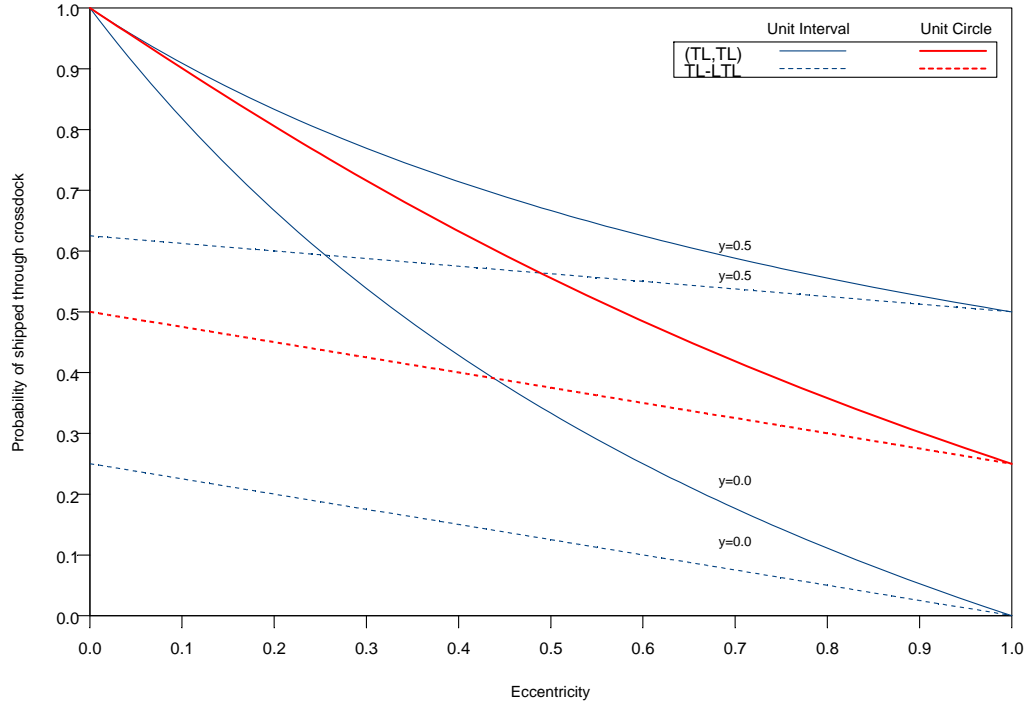


Figure 4.5: Comparison of the asymptotic probability of shipments (y -axis) shows that the value of the unit circle (red curve) is between the minimum ($y = 0$) and maximum ($y = \frac{1}{2}$) values of the unit interval (blue curve).

Figure 4.5 results from overlaying Figure 4.4 with the asymptotic-probability functions of the unit interval (blue curve). The patterns of curves represent different modes of shipments. Each mode of shipments of the unit interval consists of two curves, which correspond to the minimum ($y = 0$) and maximum ($y = \frac{1}{2}$) values of the function. These values suggest that mode of shipments (TL, TL) dominates mode of shipments $LTL-TL$. However, the decreasing rate of mode of shipments (TL, TL) is higher than that of mode of shipments $LTL-TL$ as the gap between modes of shipments (TL, TL) and $LTL-TL$ decreases

when eccentricity increases. For example, the gap changes from 75% when eccentricity is zero to 0% when eccentricity is one and a crossdock is located at the boundary of the unit interval. This suggests that changes in mode of shipments could reduce up to 75% of consolidated shipments, and a crossdock location becomes an important parameter in high eccentricity values and/or mode of shipments $LTL-TL$. Given eccentricity and mode of shipments, Figure 4.5 shows that the asymptotic probability of shipments of the unit circle is between the minimum and maximum values of the unit interval. In fact, the comparison of Tables 4.1-4.5 indicates that the asymptotic-probability functions of modes of shipments (TL, TL) and $LTL-TL$ in the unit interval are the same as the ones in the unit circle, when a crossdock is located at $\frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{e}{1+e}}$ and $\frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{1+e}{3+e}}$, respectively. The results show that the asymptotic-probability functions of the unit circle is a special case of that of the unit interval.

4.4 Asymptotic Probability of Shipments in the Unit Disk

In Section 3.4.2, we observe that an iso-cost curve can be formulated as a polar-coordinated function; therefore, the natural extension of the asymptotic probability of shipments in a two-dimensional space is the unit disk. In addition, the analysis of the unit disk is relatively simpler than other two-dimensional spaces because of its symmetry. Throughout the study, we assume that a crossdock is located at the center. The asymptotic-probability functions of modes of shipments (TL, TL) and $LTL-TL$ are reported in Tables 4.6 and 4.7, respectively.

Table 4.6: The asymptotic-probability function in the unit disk of modes of shipments (TL, TL)

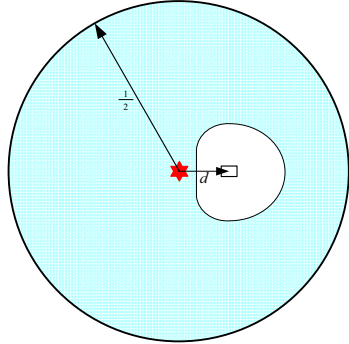
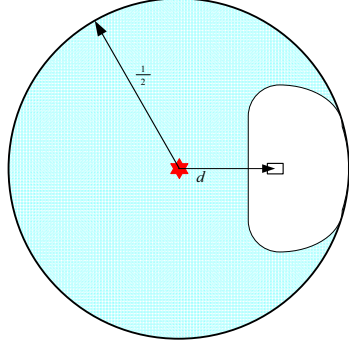
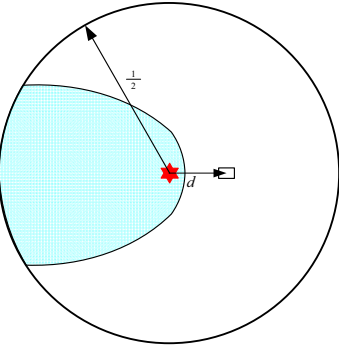
Case	Domain of integration	Integrand
<p>1</p> 	$0 \leq d < \frac{1-e}{2(1+e)}$	$\frac{3+9e+2e^2-2e^3-11e^4-e^5}{3(1+e)^5}$ see Appendix B.2.1
<p>2</p> 	$\frac{1-e}{2(1+e)} \leq d \leq \frac{1}{2}$	see Appendix B.2.2

Table 4.7: The asymptotic-probability function in the unit disk of mode of shipments $LTL-TL$

Case	Domain of integration	Integrand
<p>1</p> 	$0 \leq d \leq \frac{1}{2}$	see Appendix B.3

The first column of Tables 4.6 and 4.7 shows store locations that shipments from a given vendor (black square) either through a crossdock (red star), i.e., consolidated-shipping region (shaded area), and receive shipments directly from a given vendor, i.e., outsourced-shipping region (white area). The size of an outsourced-shipping region with respect to the unit disk may create different domain(s) of integration. Table 4.6 shows two domains of integration, while there is only one domain of Table 4.7. The first domain of Table 4.6 depicts a small outsourced-shipping region in which an iso-cost curve completely contained. In the second domain, the outsourced-shipping region results from the overlapped areas between an iso-cost curve and the unit disk. Particularly, an iso-cost curve induces a large outsourced-shipping region that exceeds the unit disk. The overlapped-area domain is the only case occurs in Table 4.7 as parts of the outsourced-shipping region of a half hyperbola curve always overlap with the unit disk. The mathematical relationships of these tables are shown in the second column. The last column describes the asymptotic probability function, a decreasing function of eccentricity. However, the functions in the unit disk are tedious and involve the Taylor's approximation of some geometric functions; therefore, their mathematical derivations of Cases 1 and 2 of Table 4.6 and Case 1 of Table 4.7 are shown in Appendix B.2.1, B.2.1 and B.3, respectively.

The complex derivation of the unit disk implies the difficulty to derive the function in other two-dimensional spaces and/or higher dimensions. A general approach to compute the asymptotic probability of shipments is Monte Carlo Simulation. The simulation evaluates each vendor-store pair individually whether it should be shipped through a given crossdock. A limitation of this approach is that the convergent rate of a simulation depends on the numbers of evaluated pairs and the dimension. We compare the function with the simulation result, as shown in Figure 4.6.

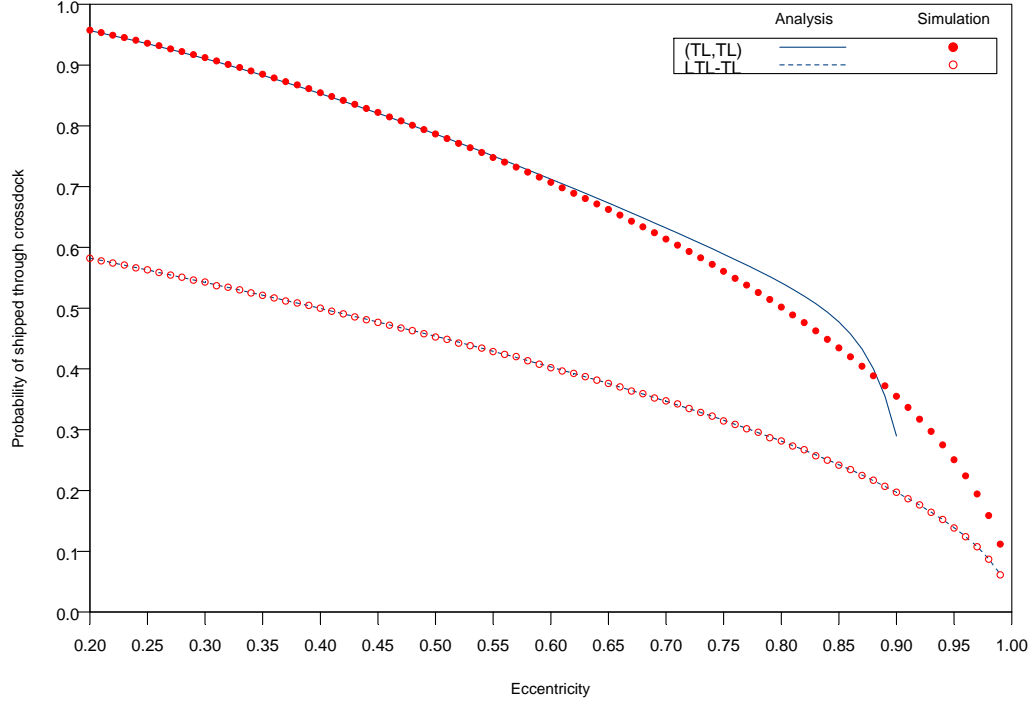


Figure 4.6: In the unit disk, the asymptotic-probability function is difficult to compute because the numerical analysis (curves) and the Monte Carlo Simulation (dots) of this probability require approximations and large samples.

The horizontal and vertical axes of Figure 4.6 show the eccentricity and the asymptotic probability of shipments. The approximated values of the asymptotic-probability functions and the mean values of the simulation are represented by curves and dots, respectively. The patterns in Figure 4.6 depict different modes of shipments. Mode of shipment (TL, TL) is represented by a solid curve or a solid dot while a dotted curve or a hollow dot shows mode of shipments $LTL-TL$. The approximation (dotted blue curve) and the simulation (hollow red dot) of mode of shipments $LTL-TL$ yield similar values. We observe a similar result in mode of shipments (TL, TL) when eccentricity is relatively low. As eccentricity increases, the approximation and simulation values deviate because of the inaccuracy value of the inverted cosine ($\arccos \theta$). As the numbers of the approximation terms increase, the deviation decreases and the expression becomes more tedious. Therefore, we suggest the derivation of the asymptotic-probability function by Monte Carlo simulation.

4.5 *Insights of the Attractiveness of a Crossdock*

Before extending the asymptotic probability of shipments to study transportation planning in the next chapter, we summarize important insights of this chapter.

- The attractiveness of a crossdock, measured by the asymptotic probability of shipments, are defined as the expected ratio of consolidated shipments to total shipments. The value of which depends on a crossdock location, mode of shipments and eccentricity, but independent of a ratio of number of vendors to number of stores. In addition, the asymptotic probability of shipments are identical in modes of shipments (LTL, TL) and (TL, LTL) if vendors and stores are identically distributed.
- Given a crossdock location, the attractiveness of a crossdock increases if the majority of shipping distances (long-haul shipments) are covered by TL shipments. Conversely, the attractiveness of a crossdock decreases if the majority of shipping distances are covered by LTL shipments. This means, a comprehensive analysis is necessary for locating crossdocks in a network consisting of multiple small vendors and stores in which modes of shipments (LTL, TL) and (TL, LTL) are more likely to be presence.
- The closed-form expressions of the asymptotic probability of shipments, namely the asymptotic-probability functions, in the unit circle are special cases of those in the unit interval.
- The asymptotic-probability functions in a one-dimensional space can be derived from an iso-cost curve. In addition, the same methodology can be applied to derive the expected transportation costs and the minimum freight independent threshold. However, the asymptotic-probability functions in a two-dimensional space are difficult to derive because of the boundary conditions of integrands and the approximation of geometric functions.
- The computer simulation shows that the asymptotic probability of shipments in a two-dimensional space is more sensitive to changes in eccentricity than those in a

one-dimensional space because a crossdock has higher opportunities to consolidation or outsourcing as shipments are transported in many directions.

- The unit-interval and unit-disk single hub crossdocking networks share consistent trends of consolidated shipments with respect to key parameters, particularly eccentricity, a crossdock location and a mode of shipments.

CHAPTER V

ANALYSIS OF TRANSPORTATION PLANNING IN THE UNIT INTERVAL CROSSDOCKING NETWORK

In this chapter, we incorporate interactions between amounts of freight and modes of shipments and embed transportation planning at different levels of a network into the model. The fraction of consolidated shipments resulted from the interactions and the transportation planning are computed by Monte Carlo simulation and compared with the optimal shipment allocation network.

5.1 *TL Threshold*

In previous chapters, we assume that a mode of shipments is predetermined and overlook its interactions with amounts of freight (f)—either weight or volume. Typically, high amounts of freight usually imply a TL shipment, which has high truck utilization and low shipping-cost rate. A TL shipment must accumulate minimum amounts of freight, referred to as *TL threshold*, to economically ship through a crossdock. TL threshold, denoted by τ , affects mode of shipments and transportation cost, as shown in Figure 5.1.

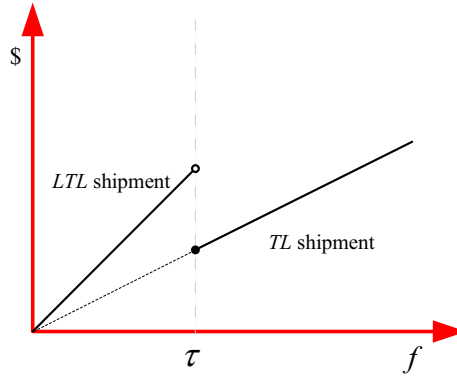


Figure 5.1: The TL threshold affects shipments, whether TL or LTL. If amounts of freight exceed this threshold, a shipment is justified as a TL shipment; otherwise, it is LTL.

In Figure 5.1, a shipment is justified as a TL shipment if amounts of freight exceed TL

threshold. Otherwise, it is LTL and incurs higher shipping-cost rate. Figure 5.1 suggests that a TL shipment through a crossdock becomes more attractive as numbers of vendor-store pairs and/or amounts of freight increase. The effects of TL threshold on consolidated shipments are studied in the following network.

We consider a single hub crossdocking network that consists of a single crossdock, n identical vendors and m identical stores in the unit interval. Locations of vendors and stores are uniformly distributed, and a crossdock is located at the center of the unit interval. The total freight is normalized to one. We assume that each store requires $\frac{1}{nm}$ units of freight from each vendor. Therefore, each vendor must supply $\frac{1}{m}$ units of freight and each store receives $\frac{1}{n}$ units of freight. Freight for each vendor-store pair can be either consolidated through a crossdock or shipped directly. Rather than the actual TL threshold, we consider the weight ratio of the actual TL threshold to minimal amounts of freight that a vendor or a store is shipped or received ($\min\left(\frac{1}{m}, \frac{1}{n}\right)$), called *standardized TL threshold*. This ratio normalizes the TL threshold and simplifies simulation results. As a result, the network is independent of the absolute numbers of vendors and stores but depends on a ratio of number of vendors to number of stores, referred to as *vendor/store ratio*. If the size of a consolidated shipment exceeds the standardized TL threshold, the shipment is classified as TL, and its transportation costs is e times less than the LTL shipment of the same distance and weight. The standardized TL threshold also indicates the possibility in mode of shipments (TL, TL) . Particularly, a crossdock is able to receive and ship TL shipments when the standardized TL threshold is less than one. When this standardized ratio exceeds one, at least one leg of a consolidated shipment through a crossdock must be LTL because the actual TL threshold is greater than either total supply of a vendor or total demands of a store.

In this network, we are interested in the fraction of consolidated shipments and total shipments, called *fraction of consolidated shipments*, which is a function of the standardized TL threshold, eccentricity and a vendor/store ratio. The fraction of consolidated shipments reflects the uses of consolidated shipments similar to the asymptotic probability of shipments (recall Section 4.1). However, the fraction depends on a vendor/store ratio, while

the asymptotic probability of shipment is independent of a vendor/store ratio in a network. This fraction also requires the Monte Carlo simulation to compute because it is mathematically difficult to derive its closed-form expression for different vendor/store ratios. For each set of parameters, 100 independent replications of simulation are performed, and their mean values are reported. The numerical study is comprised of two interdependent simulation experiments

- Experiment 1: TL threshold and eccentricity (vendor/store ratio is fixed)
- Experiment 2: TL threshold and vendor/store ratio (eccentricity is constant).

Before discussing the experiments and their numerical results, we need to address reasons why the unit-interval single hub crossdocking network is selected to explain the effects of TL threshold. We concede that the single hub crossdocking network is theoretical. There is no empirical network that vendors and stores are uniformly distributed across the unit interval and/or a crossdock is located at the center of the interval. However, the results of Chapter 4 suggest that the unit-interval network has consistent trends of consolidated shipments with respect to key parameters similar to two-dimensional single hub crossdocking networks (discussed in Section 4.5). As a threshold of consolidated shipments, the TL threshold should have consistent trends with respect to key parameters in two-dimensional networks as well. In addition, we can derive important closed-form expressions in the unit interval, particularly the minimum independent freight threshold and the expected transportation cost (as discussed below). These expressions can be used for explaining how a vendor or a store estimates and perceives the shipping decisions of other vendors or stores without central information and to develop a shipment allocation algorithm when a vendor or a store attempts to maximize its own benefits. We believe that the unit-interval network is useful to explain the interaction between TL threshold and other key parameters and to simplify the shipment allocation algorithm in vendor and store levels.

5.2 *Numerical Study of TL Threshold*

The introduction of TL threshold raises issues of transportation planning at different levels of a network. Without TL threshold, vendor-store pairs are unrelated as the consolidation of each vendor-store pair depends on the cost savings between consolidated and outsourced shipments. This means, any consolidated shipment through a crossdock always qualifies for a TL shipment, and the shipping decision of each vendor-store pair can be made independent of other pairs. The shipment allocation becomes trivial as each shipment can be independently allocated to its best mean of shipments. As a result, minimizing the total transportation costs of a network is equivalent to minimizing the total transportation costs of every vendor-store pair, and the shipment allocation of both centralized and decentralized networks are the same. As TL threshold is incorporated, vendor-store pairs that share an origin and/or a destination are connected in terms of amounts of freight to receive and/or ship through a crossdock. Consolidating more shipments implies that freight is more likely to exceed the TL threshold and qualify for a TL shipment. Therefore, transportation planning, which influences amounts of freight and shipment allocation, affects consolidated shipments and total transportation costs. For example, a vendor minimizes its own transportation cost by an outsourced shipment, which reduces total inbound freight of a crossdock. At the crossdock, fewer inbound shipments means fewer outbound shipments. Consequently, outbound consolidated shipments to stores become LTL shipments, and some shipments lose economic benefits of being consolidated. This leads to further decreases in inbound shipments and increases in the total transportation costs. In the worst case, none of shipments is consolidated through a crossdock. We discuss three transportation planning schemes in the network:

- Vendor-store pair level
- Facility level (either vendor or store level)
- Network level

These transportation planning schemes range from pure a decentralized network (vendor-store pair level) to a centralized network (network level), as summarized in Table 5.1.

Table 5.1: The transportation planning schemes in the network, their associated algorithm and results

Level of transportation planning	Vendor-store pair level (Section 5.2.1)	Facility level (Section 5.2.2)	Network level (Section 5.2.3)
Experiment 1	Figure 5.2	Figure 5.4	Figure 5.6
Experiment 2	Figure 5.3	Figure 5.5	Figure 5.7
Transportation planning algorithm	independent freight	store control	reversed greedy

Table 5.1 shows the organization of materials and sections of the dissertation in which motivations and results are discussed. In each section, a specific shipment allocation algorithm is embedded to model the corresponding transportation planning, and the fraction of consolidated shipments is computed by Monte Carlo simulation. For example, we discuss the independent freight algorithm, which is embedded to model the transportation planning at a vendor-pair level, and its simulation results in Section 5.2.1. The fraction of different eccentricity values (Experiment 1) and selected ratios of vendors and store (Experiment 2) are reported in Figure 5.2 and 5.3, respectively. After we discuss the fraction of consolidated shipments, the total transportation costs are compared with the optimal solution in Section 5.3.

5.2.1 Transportation Planning at the Vendor-Store Pair Level

A simple shipment allocation is one that allows each pair to decide its shipping mode of shipments independently of others, based on its own transportation cost. Such an allocation is intended to model how vendor-store pairs as participants can act as independent agents in arranging their shipping decisions. Hence, each vendor-store pair is consolidated through a crossdock only when a consolidated shipment provides some savings. This shipment allocation is modeled by the *independent freight* algorithm, as described in Algorithm 1.

Algorithm 1 Independent Freight

```
for all vendor  $V_i$  and store  $S_j$  in  $\mathbb{V} \times \mathbb{S}$  do
  compute the outsourced shipment cost of vendor-store pair  $(V_i, S_j)$ 
  compute the consolidated shipment cost of vendor-store pair  $(V_i, S_j)$ 
  assign pair  $(V_i, S_j)$  to the minimal cost crossdock or the LTL shipment
5: end for
repeat
  for all consolidated shipment  $l$  do
    if a consolidated shipment  $l$  is less than TL threshold  $\tau$  then
      change its mode of shipments
10:   end if
  end for
until all consolidate shipments exceed TL threshold  $\tau$ 
return assignment of all vendor-store pairs
```

The independent freight algorithm operates at the level of a vendor-store pair—the smallest component of network. In this algorithm, a consolidated shipment is initially assumed to be mode of shipments (TL, TL) . Then, the algorithm checks for TL shipments. If one does not meet the TL threshold, the vendor-store pairs associated with the violated consolidated shipment are re-allocated to a suitable mode of shipments, including an outsourced shipment.

5.2.1.1 Numerical Results of Independent Freight Algorithm

We present the fractions of consolidated shipments as the interactions between TL threshold and eccentricity while numbers of vendor and store are controlled, as shown in Figure 5.2.

The y -axis of Figure 5.2 represents the fraction of consolidated shipments, whereas the x -axis shows the standardized TL threshold—the ratio of TL threshold to the minimal of either total demand of a store or total supply of a vendor. The mean values associated with each eccentricity value are connected by a dotted blue curve. In Figure 5.2, the fraction of each curve initially is constant and is equal to the asymptotic probability of shipments in the unit interval because the TL threshold is initially too low to restrict shipments. Therefore, all consolidated shipments have mode of shipments (TL, TL) . Each curve rapidly declines as the standardized TL threshold reaches a threshold (red curve), called *minimum independent freight threshold*, denoted by τ_{min} , which is a function of eccentricity. In general, the minimum independent freight threshold is difficult to derive in

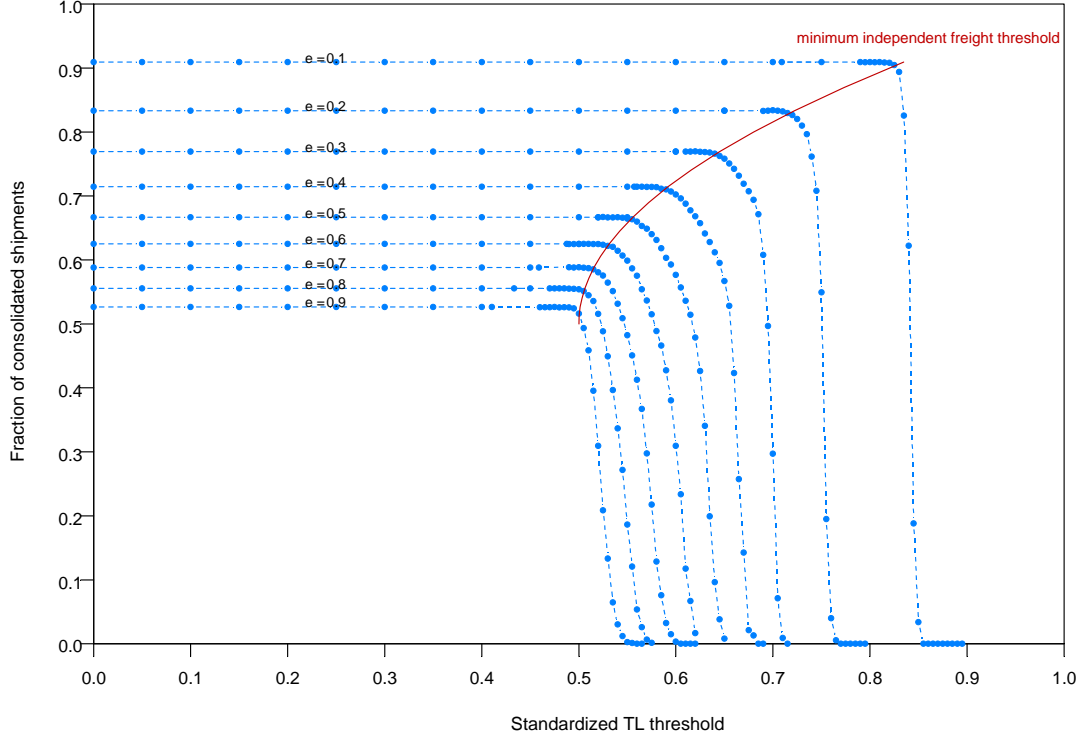


Figure 5.2: The fractions of consolidated shipments through the crossdock located at the center of the unit interval is a cascading function of eccentricity, TL threshold, number of vendors and number of stores.

higher dimensions. However, we can apply Expression 4.1 and determine the closed-form expression in which a crossdock is located at the center of the unit interval as follows:

$$\tau_{min}(e) = 1 - \frac{2e}{(1+e)^2}. \quad (5.1)$$

Expression 5.1 serves as a lower bound on mode of shipments (TL, TL) . This quantity represents the minimal amount of freight such that all consolidated shipments through a crossdock are mode of shipments (TL, TL) . As the standardized TL threshold exceeds Expression 5.1, shipments to or from a crossdock are unqualified for TL because of the insufficient amounts of freight and the independent decision of vendor-store pair. Some unqualified shipments may be economically shipped through the crossdock with modes of

shipments (LTL, TL) or (TL, LTL). Others are broken down and directly shipped to stores via LTL shipments. As a result, the number and weight of consolidated shipments through a crossdock decrease as the TL threshold increases. Figure 5.2 suggests that TL threshold can be ignored when its value is below the minimum independent freight threshold. Within this range, each vendor-store pair can be independently allocated because TL threshold is too low to affect modes of shipments.

In addition, we observe that Expression 5.1 is the tipping point of an adverse network behavior, called the *evaporation effect*, in which each vendor-store pair independently minimizes its individual piece of transportation cost but causes the total transportation cost to increase because of lack of transportation planning. The evaporation effect begins when a vendor (store) ships (receives) small shipments that are less than the TL threshold. Consequently, these shipments are outsourced, which decreases the amount of freight through the crossdock. As a result, the crossdock has less freight to consolidate and becomes unattractive. It should be noted that the evaporation effect is a positive feedback process. That is, switching of some shipments causes others to leave the crossdock and renders the crossdock more unattractive.

Next, we hold eccentricity constant at 0.35 and explore effects of numbers of vendors and stores. Our analysis suggests that the fraction of consolidated shipments depends on the vendor/store ratio. As the TL threshold changes, some legs of consolidated shipments may become LTL, and freight may be shipped through a crossdock as modes of shipments (LTL, TL) and/or (TL, LTL). This raises an interesting question: How do the vendor/store ratio and TL threshold affect modes of shipments? Hence, all three modes of shipments of consolidated shipments, including mode of shipments (TL, TL), are considered, as shown in Figure 5.3.

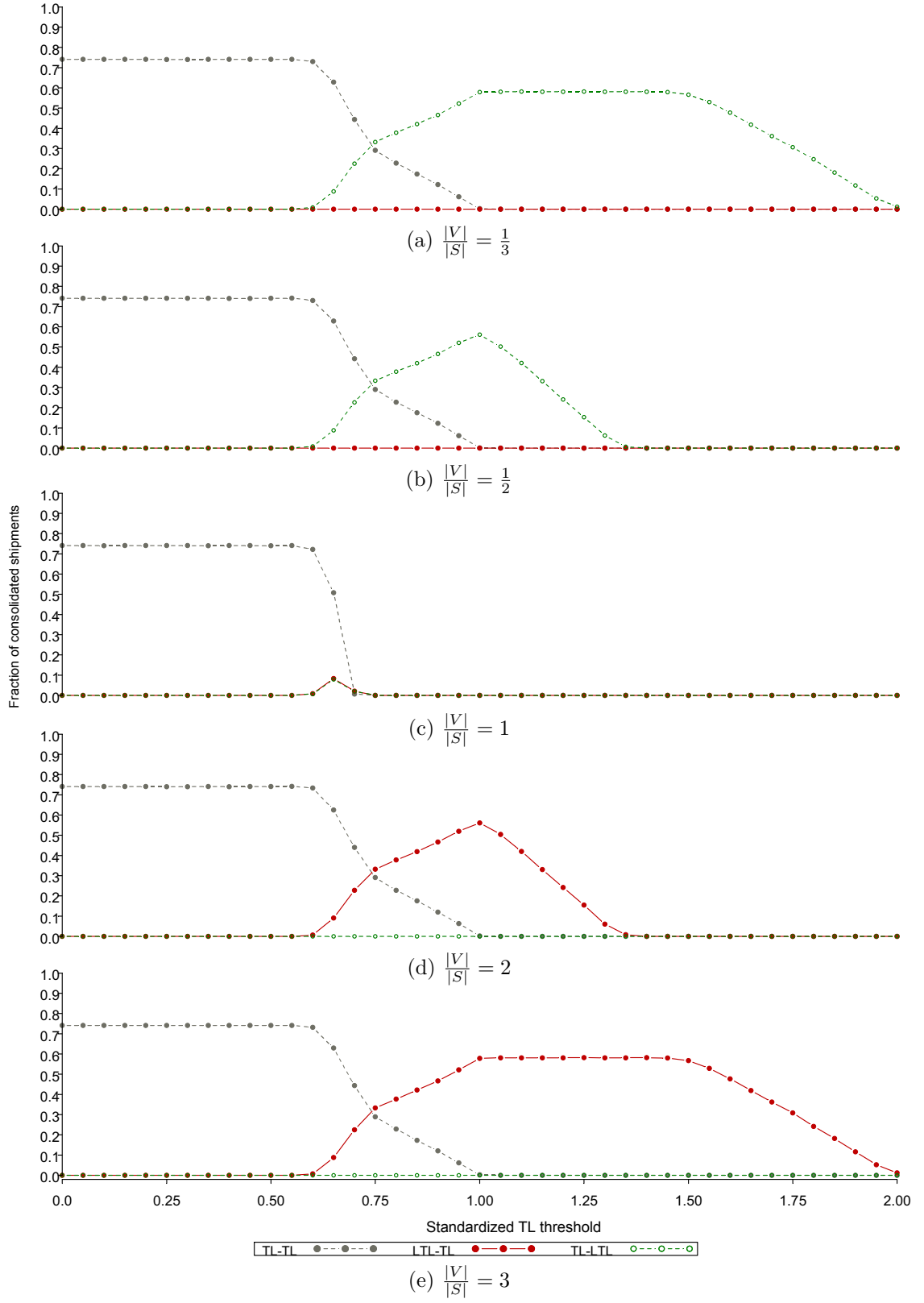


Figure 5.3: Simulation experiments on the unit-interval center crossdock show the probability of freight shipped through the crossdock by the types of shipment.

Figure 5.3 shows the fraction of consolidated shipments of $\frac{|V|}{|S|} = \frac{1}{3}, \frac{1}{2}, 1, 2$ and 3. The fractions in modes of shipments (TL, TL) , (LTL, TL) and (TL, LTL) are represented by solid grey, solid red and hollow green dots, respectively. The remaining shipments are shipped as outsourced shipment, which is not reported but can be determined by subtraction. In Figure 5.3, none of consolidated shipments is initially transhipped through a crossdock in modes of shipments (LTL, TL) and (TL, LTL) because the standardized TL threshold is too low to restrict mode of shipments (TL, TL) . As the standardized TL threshold increases and reaches the minimum independent freight threshold ($\tau_{min}(0.35) = 0.616$), the fraction in mode of shipments (TL, TL) decreases, and a vendor/store ratio begins to affect the fractions of consolidated shipments. If $\frac{|V|}{|S|} = 1$ (Figure 5.3.c), the fractions of consolidated shipments in modes of shipments (LTL, TL) and (TL, LTL) are equal and simultaneously approach zero when the standardized TL threshold is 0.75, which is the lowest value of all selected vendor/store ratios. This suggests that the fraction is sensitive to changes in TL threshold, which intensifies the evaporation effect. If $\frac{|V|}{|S|} > 1$ (Figures 5.3.d and 5.3.e), the fraction of consolidated shipments in mode of shipments (TL, LTL) is greater than mode of shipments (LTL, TL) . On the contrary, the fraction in mode of shipments (TL, LTL) is less than mode of shipments (LTL, TL) if $\frac{|V|}{|S|} < 1$ (Figures 5.3.a and 5.3.b). Figure 5.3 also shows that the rate at which the fraction in mode of shipments (TL, TL) decreases is higher than the rate at which the fractions in modes of shipments (TL, LTL) and (LTL, TL) increase in $\frac{|V|}{|S|} < 1$ and $\frac{|V|}{|S|} > 1$, respectively. Hence, the total consolidated shipments decrease as the standardized TL threshold increases.

Both $\frac{|V|}{|S|} < 1$ and $\frac{|V|}{|S|} > 1$ shares two similarities that should be discussed together. The first similarity is that the fractions in modes of shipments (TL, LTL) and (LTL, TL) of the inverse vendor/store ratio are equal. For example, mode of shipments (TL, LTL) in Figure 5.3.a ($\frac{|V|}{|S|} = \frac{1}{3}$) shares the fraction of consolidated shipments with mode of shipments (LTL, TL) in Figure 5.3.d ($\frac{|V|}{|S|} = 3$). This means, we can infer the analysis of mode of shipments (LTL, TL) of a high vendor/store ratio (Figures 5.3.d and 5.3.e) from the fraction of consolidated shipments in mode of shipments (LTL, TL) of low vendor/store

ratios (Figures 5.3.a and 5.3.b) and vice versa. The last similarity occurs between Figures 5.3.a and 5.3.b as well as Figures 5.3.d and 5.3.e. Particularly, the fractions of a vendor/store ratio that is less deviated with respect to $\frac{|V|}{|S|} = 1$ can be viewed as the parts of a more deviated vendor/store ratio. For example, the fractions in Figures 5.3.a $\left(\frac{|V|}{|S|} = \frac{1}{3}\right)$ and 5.3.b $\left(\frac{|V|}{|S|} = \frac{1}{2}\right)$ increases at the same rate from the standardized threshold between 0.60 to 1.0. Then the fraction in Figure 5.3.a is constant between the standardized threshold between 1.0 to 1.55, while the fraction in Figure 5.3.b decreases. Although the fractions in thin Figures 5.3.a and 5.3.b decrease at different standardized threshold, they decrease at the same rate. This implies that the fraction of a more deviated vendor/store ratio, say $\left(\frac{|V|}{|S|} = \frac{1}{4}\right)$, should have the same increasing and decreasing rates as the fraction of $\left(\frac{|V|}{|S|} = \frac{1}{3}\right)$ and should be constant at least between the standardized threshold between 1.0 to 1.55.

The implication of these similarities is that we can discuss only the fraction in mode of shipments (TL, LTL) at $\frac{|V|}{|S|} = \frac{1}{3}$ (Figure 5.3.a) and extend the results to the remaining settings (Figures 5.3.b, 5.3.d and 5.3.e). In Figure 5.3.a, the fraction in mode of shipments (TL, LTL) is initially zero because it is dominated by mode of shipments (TL, TL) . As the standardized TL threshold increases, shipments from a crossdock are gradually disqualified for TL, thereby encouraging outsourced shipments to 3PL companies. On the contrary, shipments to a crossdock are relatively large and remain TL because total demand of each store is one third of the total supply of each vendor. The increase in TL threshold creates economic benefits for some unqualified shipments, especially the shipments bound to stores located near the crossdock, to receive LTL consolidated shipments. Some consolidated shipments gradually change from mode of shipments (TL, TL) into mode of shipments (TL, LTL) . When the standardized TL threshold reaches one, the crossdock is unable to ship TL shipment to any store because the TL threshold exceeds total supply of each store. As a result, the fraction in mode of shipments (TL, TL) becomes zero. Despite a higher value, the standardized TL threshold is too low to restrict any inbound shipments. Therefore, the fraction in mode of shipments (TL, LTL) is constant and equal to the asymptotic-probability function of (TL, LTL) , particularly $f(0.35, 0.5) = 0.58125$.

As the standardized TL threshold further increases, the network is unable to retain consolidated shipments as inbound shipments are slowly restricted. This leads to a decrease in the fraction of consolidated shipments in mode of shipments (TL, LTL) and an increase in outsourced shipments. Eventually, all shipments become LTL because the standardized TL threshold becomes extremely high.

5.2.2 Transportation Planning at the Facility Level

In this section, we develop the transportation planning at the facility level that either stores or vendors (but not both) determine their shipment allocation for their best interests. Particularly, we assume that each store is able to control its own shipment allocation by requesting either a consolidated shipment or an outsourced shipment from each vendor independently of other stores (a similar analysis also holds for vendors). This assumption reflects the distribution network of a national retailer that allows stores to decide the way by which products from each vendor are delivered. The incoming consolidated shipment (from a vendor to a crossdock) is a TL shipment when multiple stores request the same products from a certain vendor through a crossdock. Hence, each store partially influences TL shipments to a crossdock. Conversely, the outgoing consolidated shipment (from a crossdock to a store) is a TL shipment when a store requests large quantities of products from multiple vendors through a crossdock. Hence, each store determines a TL shipment from a crossdock. Such the shipment allocation requires each store to address the following inter-related issues:

- Which vendors should a store request consolidated shipments through a crossdock?
- Are the incoming and outgoing consolidated shipments through a crossdock TL shipments?
- Does a store benefit from deciding a way by which products are delivered?

Before assembling these issues into an algorithm, we need to elaborate why these issues affect the shipment allocation of stores. The first issue is the most important and influences the remaining issues, especially in high TL threshold. For example, a store may receive

freight from a close vendor location through a crossdock, instead of outsourcing to a 3PL company, to ensure TL shipments and minimize total transportation costs. The second issue concerns sizes of incoming and outgoing consolidated shipments. Although the sizes of incoming shipments depend on every store in a network, each store can estimate amounts of freight shipped from a particular vendor based on locations of the vendor and the crossdock. For example, a distant vendor tends to consolidate freight through a crossdock and ship a TL shipment, whereas a vendor surrounded by stores is more likely to ship an LTL shipment. The last issue determines whether or not the transportation planning should be implemented. Rather than consolidating more freight and generating a TL shipment, a store may receive an LTL shipment from a crossdock if the LTL shipment results in less total transportation costs.

Addressing these issues in the unit-interval network has an advantage as the minimum freight independent threshold and the asymptotic-probability functions (Section 4.2) have closed-form expressions. The former implies that we need to consider the transportation planning when TL threshold exceeds the minimum freight independent threshold.^{5.1} This leads to a tradeoff as a store must determine sufficient vendors to ensure a TL shipment, while minimizing the increases in transportation costs. A store may address such the tradeoff by determining vendors according to the differences in transportation costs between a consolidated shipment and an outsourced shipment. Specifically, we can associate each vendor with a ratio to which its transportation costs are equal because a lower ratio implies a higher saving in transportation cost (Proposition 3.3.1). In mode of shipments (TL, TL) , this ratio is identical to eccentricity, and a store can to determine vendors by selecting the eccentricity, as shown in Expression 5.2.

$$e_{min}(\tau) = \frac{\tau - \sqrt{2\tau - 1}}{1 - \tau}. \quad (5.2)$$

Expression 5.2 is the inverse function of the minimum independent freight threshold (Expression 5.1) as it returns the eccentricity such that the TL threshold (τ) is equal to

^{5.1}If TL threshold is less than the minimum freight independent threshold, shipments are unaffected by the TL threshold. Hence, a crossdock ships and receives TL shipments.

the minimum independent freight threshold. In Expression 5.2, $\frac{1}{2} \leq \tau \leq 1$ because the TL threshold reflects mode of shipments (TL, TL) and a crossdock location. Specifically, at least half of vendors always ship consolidated shipments through the crossdock located at the center of the unit interval, and the crossdock is unable to receive and ship TL shipments when $\tau > 1$. Because a store determines vendors, each store controls whether its shipment from a crossdock is TL or LTL. Hence, we can omit an outgoing shipment and focus on whether an incoming consolidated shipment is TL or LTL. In the unit-interval network, we can estimate the amounts of freight shipped from a particular vendor (x) in mode of shipments (TL, TL) by the asymptotic-probability function, as shown Expression 5.3.

$$E_{(TL, TL)}[f(x, e)] = \begin{cases} 1 - x - \frac{e(1-2x)}{1+e} & , 0 \leq x < \frac{e}{1+e} \\ 1 - \frac{2e(1-2x)}{1-e^2} & , \frac{e}{1+e} \leq x < \frac{1}{2} \\ 1 - \frac{2e(2x-1)}{1-e^2} & , \frac{1}{2} \leq x < \frac{1}{1+e} \\ x - \frac{e(2x-1)}{1+e} & , \frac{1}{1+e} \leq x \leq 1 \end{cases} \quad (5.3)$$

Expression 5.3 determines the amounts of freight shipped from a vendor based on an assumption that a shipment of each vendor-store pair is allocated independently of other pairs (vendor-store pair level). This value never exceeds the actual amounts when each store determines its vendors because these shipments should be transshipped through the crossdock if the TL threshold is low and the transportation planning requires additional shipments to ensure mode of shipments (TL, TL) when the TL threshold restricts shipments. Therefore, the expression may serve as a lower bound when a store determines shipments from vendors. Interestingly, the expression can be used to estimate the amounts of freight bound to each store through a crossdock as well because vendors and stores are uniformly distributed and the crossdock is assumed to receive and ship TL shipments. We exploit this property in the proposed algorithm. The last issue is the saving in transportation costs resulting from the transportation planning. To compare the savings, we need to discuss the alternative means by which a store receives the shipments. Particularly, a store may forfeit full benefits of consolidation and receive an LTL shipment from a crossdock and/or an outsourced shipment from a 3PL company depending on a transportation

cost of each vendor-store pair. In this manner, some shipments are consolidated through a crossdock using mode of shipments (TL, LTL) . Using the concepts of the asymptotic-probability functions, we can derive the transportation cost of this alternative, which is a function of a vendor location (x) , as follows:

$$E_{(TL, LTL)}[TC(x, e)] = \begin{cases} \frac{4(3+2e-e^2)x^2-4(1+4e-e^2)x+3+6e-e^2}{16} & , 0 \leq x < \frac{1}{2} \\ \frac{4(3+2e-e^2)x^2-4(5-e^2)x+11-2e-e^2}{16} & , \frac{1}{2} \leq x \leq 1. \end{cases} \quad (5.4)$$

Having addressed three important issues, we assemble them into an algorithm, referred to as *store control* algorithm, as shown in Algorithm 2

Algorithm 2 Store Control Algorithm

```

for all store  $S_j$  in  $\mathbb{S}$  do
    compute expected amounts of independent freight of store  $S_j$ 
    if expected amounts of independent freight is greater than threshold  $\tau$  then
        for all vendor  $V_i$  in  $\mathbb{V}$  do
5:         compute outsourced- and consolidated-shipment costs of vendor-store pair  $(V_i, S_j)$ 
            assign vendor-store pair  $(V_i, S_j)$  to the minimal cost crossdock or the LTL shipment
        end for
    else
        determine list  $L$ , the minimal vendor-store pairs require to generate TL shipment
10:    if list  $L$  exists then
        for all vendor  $V_i$  in  $\mathbb{V}$  do
            if vendor-store pair  $(V_i, S_j)$  in list  $L$  then
                compute consolidated-shipment cost of vendor-store pair  $(V_i, S_j)$ 
            else
15:                compute outsourced-shipment cost of vendor-store pair  $(V_i, S_j)$ 
            end if
            keep cost of pair  $(V_i, S_j)$  in cooperative transportation costs of store  $S_j$ 
        end for
    else
20:        compute non-cooperative transportation costs of store  $S_j$ 
    end if
    end if
    compute transportation costs of store  $S_j$  when all shipments bypass a crossdock
    compare and select minimal transportation costs of store  $S_j$ 
25: end for

```

The store control algorithm explores the structure of the unit-interval single hub cross-docking network, which enables a store to estimate its total transportation costs and the shipping decisions of other stores. The algorithm begins by determining whether or not

the TL threshold has any effect on each store (step 2). The amounts of freight bound to a particular store through a crossdock can be estimated by Expression 5.4. If the amounts of freight are greater than TL threshold, the TL threshold has no effect on the network and the algorithm is reduced to the independent freight algorithm (Algorithm 1) in which each vendor-store pair can be allocated independently of other pairs. Otherwise, a store must choose between three alternatives to minimize its total transportation cost:

1. A store has benefit from receiving TL shipment, and some shipments are consolidated and shipped to the store by mode of shipments (TL, TL)
2. A store has benefit from receiving LTL shipment, and some shipments are consolidated and shipped to the store by mode of shipments (TL, LTL)
3. A store has **no** benefit from receiving any shipment through a crossdock, and all shipments bound to the store are outsourced to a 3PL company.

The first possible choice is to receive TL shipment by redirecting the minimal vendor-store pairs to satisfy the requirement of a TL shipment (list L in step 9). Such vendor-store pairs can be determined in terms of eccentricity by Expression 5.2. The eccentricity implies the exact number of vendor-store pairs required to generate a TL shipment and minimize the transportation costs incurred by a store. In other words, a store can determine vendors by comparing the transportation costs of consolidated and outsourced shipments using the eccentricity of Expression 5.2, instead of the actual eccentricity (step 13). Such vendors ensure TL shipment from a crossdock to the store with the minimal transportation costs. Expression 5.2 may return negative eccentricity, indicating insufficient freight. In such situation, a store cannot receive any TL shipment because the total freight bound to the store is less than the TL threshold. To compute the transportation costs (step 17), each store needs to predict whether a shipment from a particular vendor is TL or LTL. The amounts of freight shipped from each vendor through a crossdock in mode of shipments (TL, TL) can be estimated by Expression 5.3.

A store may find that the second choice—to receive an LTL shipment from a crossdock—serves its best interests (step 20). In the unit interval single hub crossdocking network, the

transportation cost of this choice can be estimated by Expression 5.4. The last choice is that all shipments bound to a store bypass a crossdock and ship by a 3PL company (step 23). The last step of the algorithm (step 24) is to determine which decision is the best for a store. The results of the embedded algorithm are shown in Figures 5.4 and 5.5, respectively

5.2.2.1 Numerical Results of Store Control Algorithm

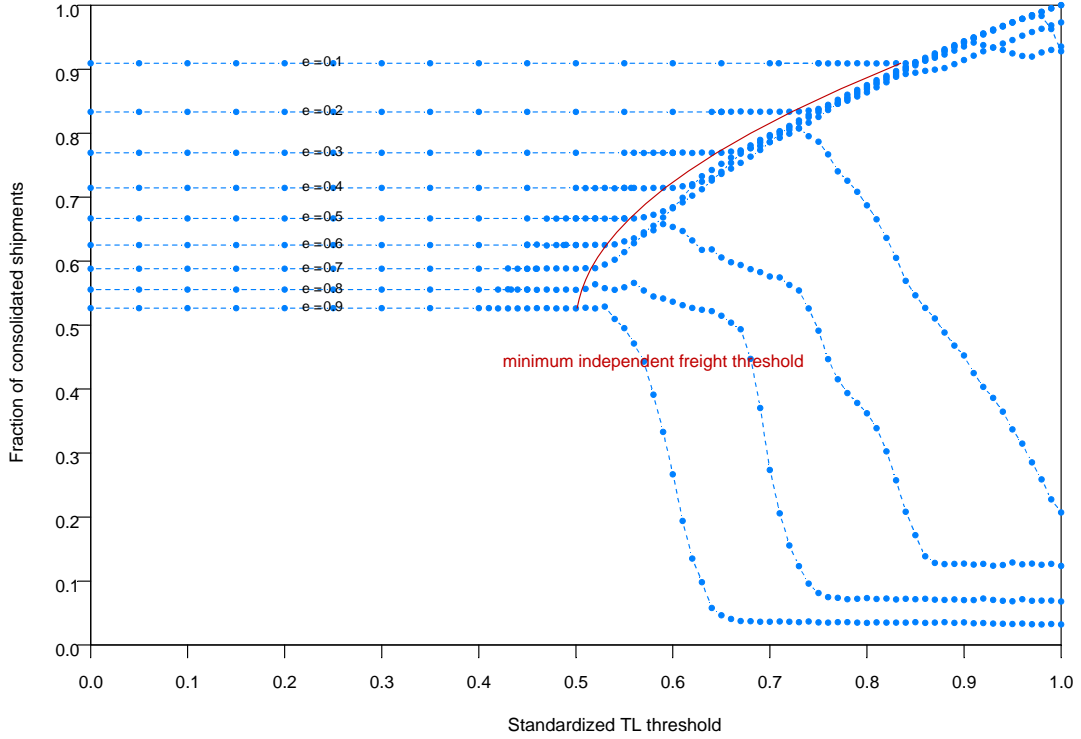


Figure 5.4: The fractions of consolidated shipments through the crossdock located at the center of the unit interval of the store control algorithm.

Figure 5.4 shows the consolidated shipments of different eccentricity while numbers of vendors and stores are equal: $\frac{|V|}{|S|} = 1$ (Experiment 1). The most interesting portion of the figure is when the standardized TL threshold exceeds the minimal independent freight threshold (red curve). In this portion, the fraction of consolidated shipments initially increases as an indication of additional shipments through a crossdock. As TL threshold further increases, Figure 5.4 shows that the fraction decreases in high eccentricity (i.e., $e = 0.7, 0.8$, and 0.9). These high eccentricity curves suggest that a store balances high transportation costs of some pairs with benefits of TL shipment. In particular, the transportation costs incurred by a store exceed the benefits of TL shipment. One important

remark regarding Figure 5.4 is that some shipments continue to be consolidated through the crossdock (i.e., $e \leq 0.6$) because this shipping decision remains the cheaper alternative for nearby stores. Next, we focus on the effect on modes of shipments, as shown in Figure 5.5.

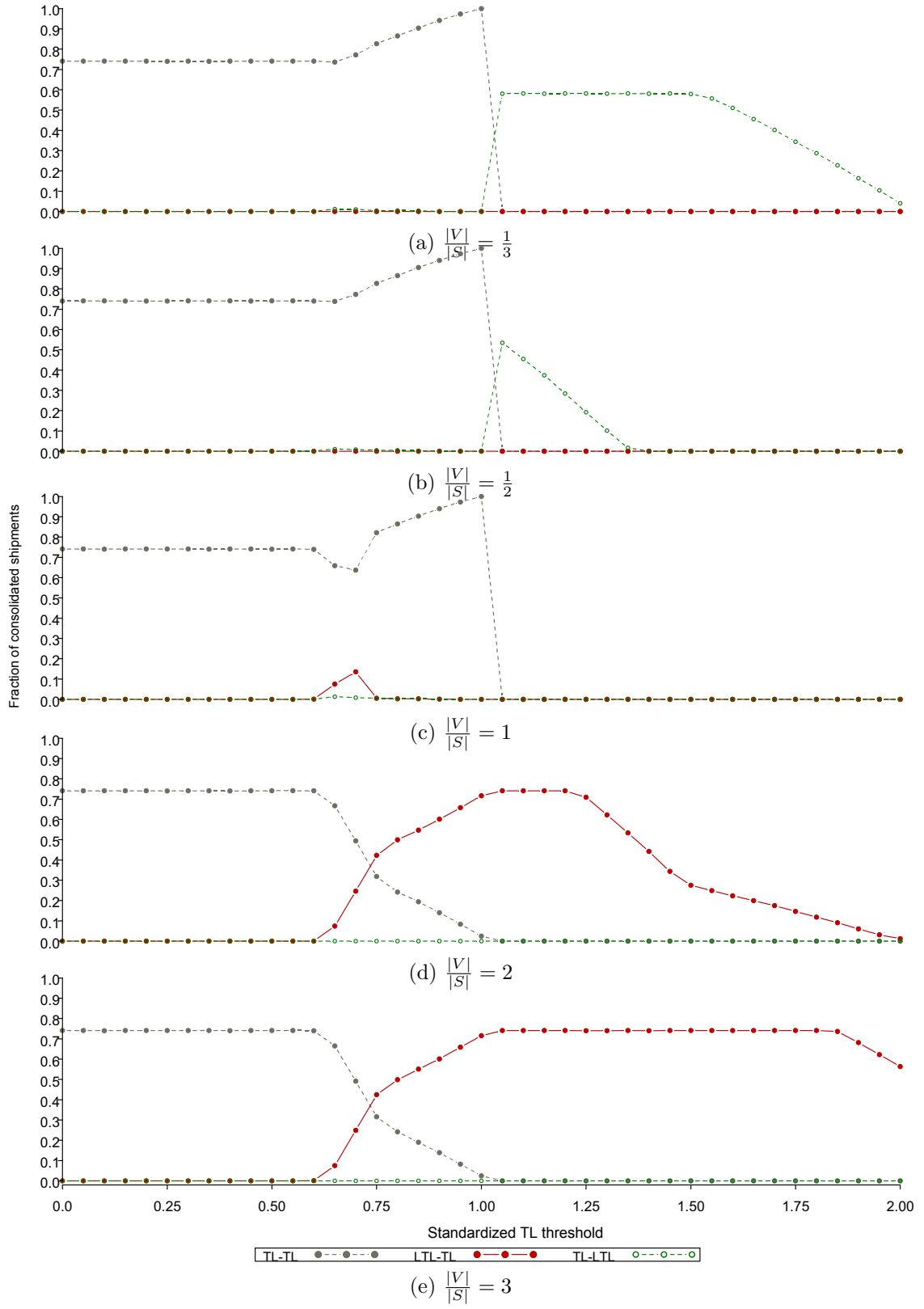


Figure 5.5: Simulation experiments on the unit-interval center crossdock of the store control algorithm.

Figure 5.5 shows the fraction of consolidated shipments while eccentricity is constant: $e = 0.35$ (Experiment 2). It suggests that the means of which shipments are consolidated through a crossdock (mode of shipments) depend highly on the vendor/store ratio. If $\frac{|V|}{|S|} < 1$, Figures 5.5.a and 5.5.b show that stores request and redirect more shipments through a crossdock so that they can receive TL shipments. If $\frac{|V|}{|S|} > 1$, Figures 5.5.d and 5.5.e suggest no transportation planning as the fractions of consolidated shipments are similar to that in Figures 5.3.d and Figures 5.3.e. As a result, the transportation planning occurs when $\frac{|V|}{|S|} \leq 1$. Among different transportation planning schemes, the store level gives a unique result as the fractions of consolidated shipments in modes of shipments (TL, LTL) and (LTL, TL) of an inverse vendor/store ratio are different.

The effects of a vendor/store ratio on the store control algorithm can be explained by the sizes of consolidated shipments. When $\frac{|V|}{|S|} < 1$, a network is comprised of more stores than vendors. An incoming consolidated shipment from each vendor is larger than an outgoing shipment to each store. Thus, incoming shipments are TL, whereas outgoing shipments are sensitive to TL threshold and more likely to become LTL. As one who determines shipments, each store observes this LTL shipment and its associated high transportation costs. Stores could minimize transportation costs by consolidating more freight and ensuring TL shipment. When $\frac{|V|}{|S|} > 1$, an incoming shipment shipped by each vendor, on the contrary, is smaller than an outgoing shipment bound to each store. Therefore, incoming shipments are easily affected by TL threshold and more likely to become LTL. A store, as an individual, has little influence on an incoming consolidated shipment because each store perceives LTL shipments from vendors to a crossdock as beyond its control and exerts no attempt to consolidate more freight because the best interests of each store lie in itself. Some stores may even outsource all shipments to a 3PL company, resulting in the evaporation effect. Hence, the performance of transportation planning at a store level (the store control algorithm) behaves like that in a vendor-store pair level (the independent freight algorithm) because there is little or no transportation planning among stores to accumulate freight upstream.

One disadvantage of transportation planning at the store level is that a store needs efficient ways to determine vendors that ship through a crossdock, to estimate whether an

incoming consolidated shipment from a particular vendor is TL or LTL, and to compute transportation costs of alternative means by which the store may receive shipments.

5.2.3 Transportation Planning at the Network Level

Since independent transportation strategies might allow evaporation, we propose transportation planning at the network level that eliminates the mal-effect of mode of shipments. In particular, an outsourced shipment reduces amounts of freight shipped through a crossdock and may change mode of shipments. A shipment should not bypass a crossdock merely because it provides a saving in transportation costs. As a result, we propose a shipment allocation algorithm that takes into account the amounts of saving and the effect of mode of shipments, called *reversed greedy* algorithm (Algorithm 3).

Algorithm 3 Reversed Greedy Algorithm

```

for all vendor  $V_i$  and store  $S_j$  in  $\mathbb{V} \times \mathbb{S}$  do
    compute the saving by use outsourced shipments
    if the saving is positive then
        record vendor-store  $V_i$ - $S_j$  and the saving in list  $L$ 
5:   end if
end for
Sort list  $L$  by increasing order of the saving
for all vendor  $V_i$  and store  $S_j$  in list  $L$  do
    if total flows vendor  $V_i$  and store  $S_j$  greater than threshold  $\tau$  then
10:   vendor-store  $V_i$ - $S_j$  is shipped directly
    end if
end for
return assignment of all vendor-store pairs

```

In Algorithm 3, all vendor-store pairs are initially shipped through a crossdock. Then, the algorithm determines the difference in transportation costs between outsourced and consolidated shipments of every pairs. Then, the positive-difference pairs (i.e., savings by the use of outsourced shipment) are collected and sorted in the incremental order (step 7). Then, each sorted vendor-store pair is verified whether it can be switched into an outsourced shipment without changes in a mode of shipments. If so, such vendor-store pair is switched. This algorithm models the transportation planning by preventing the change in modes of shipments unless TL threshold is violated. It also takes the amounts of saving into account by the incremental sorting. However, this implies that all savings in

transportation costs must be calculated and collected. Thus, the computational time and memory, required by the algorithm, increase as a number of vendors and stores increases. The reversed greedy algorithm ensures that a shipping pattern accounts for TL threshold, searches for the violated consolidated shipments and re-allocates them to the next best allocation channel.

5.2.3.1 Numerical Results of Reversed Greedy Algorithm

We implement the reversed greedy algorithm and perform the similar simulations on the effects of between TL threshold and eccentricity (Experiment 1) and between TL threshold and ratio of number of vendors to number of stores (Experiment 2). The results of both simulations are reported in Figures 5.6 and 5.7.

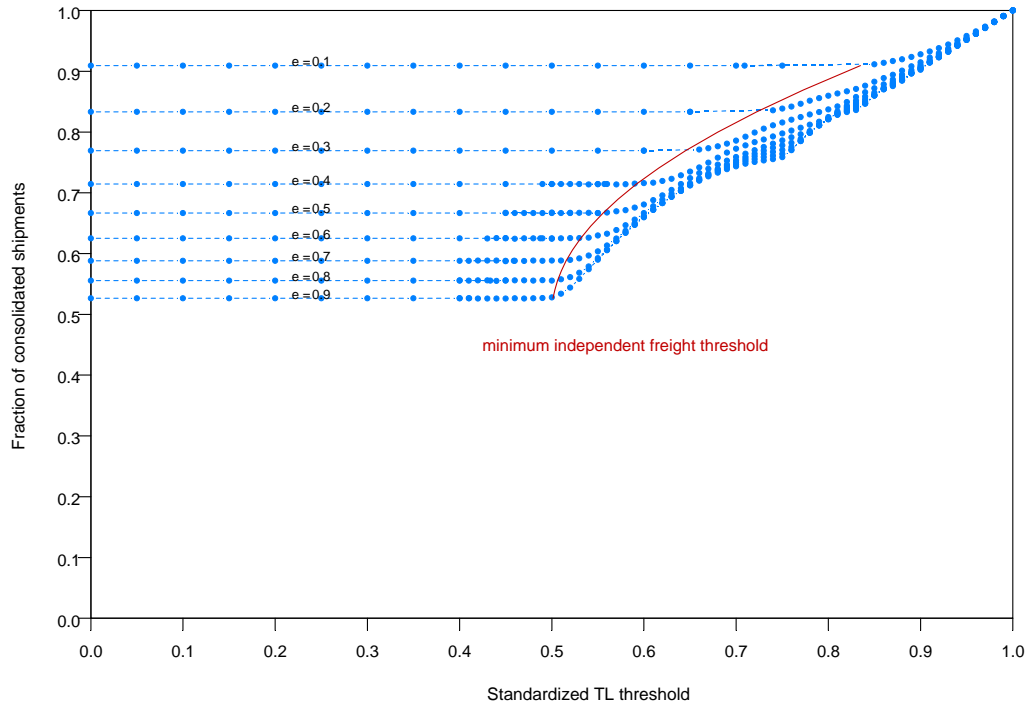


Figure 5.6: The fractions of consolidated shipments through the crossdock located at the center of the unit interval of the reversed greedy algorithm.

Figure 5.6 shows the effects of eccentricity and TL threshold on the fraction of consolidated shipments when numbers of vendors and stores are equal: $\frac{|V|}{|S|} = 1$. In Figure 5.6, the fraction of each eccentricity is initially constant as TL threshold is low. As the standardized TL threshold further exceeds the minimal independent freight threshold (red curve), the fraction increases to reflect redirection of shipments through a crossdock. In fact, a crossdock continues to receive and ship TL shipments until the standardized TL threshold reaches one when all shipments are consolidated at a crossdock, which is the opposite result of the independent freight algorithm. Algorithm 3 encourages a TL shipment and allows LTL shipment only when the TL threshold condition is not violated; therefore, a crossdock always receives and ships TL shipments (mode of shipments (TL, TL)). This raises an interesting question: What would occur if a crossdock is unable to receive or ship TL shipments as the standardized TL threshold exceeds one. We graph the fractions of consolidated shipments to this answer, as shown in Figure 5.7.

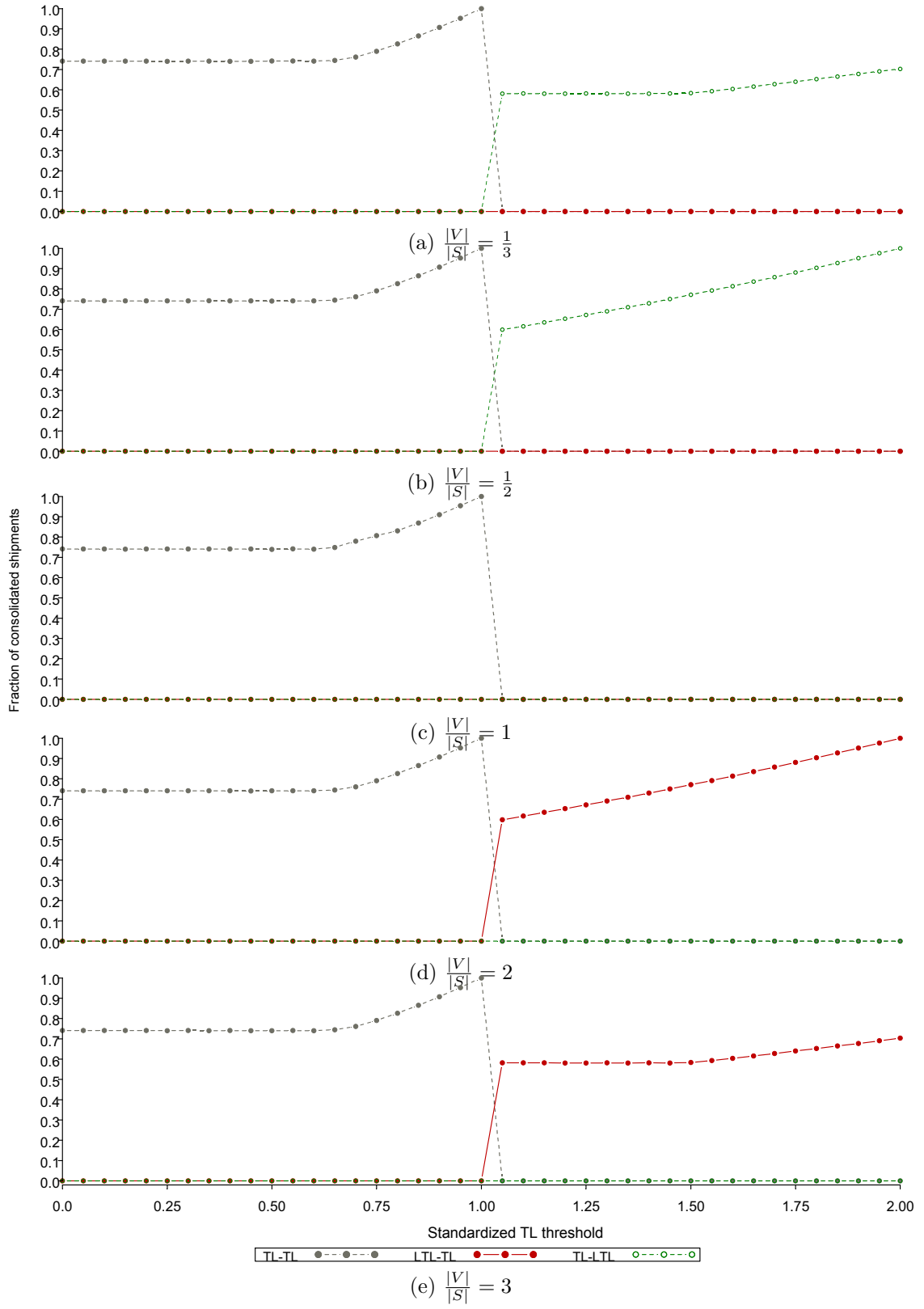


Figure 5.7: Simulation experiments on the unit-interval center crossdock of the reversed greedy algorithm.

Figure 5.7 shows the effect of vendor/store ratio $\left(\frac{|V|}{|S|}\right)$ and TL threshold on consolidated shipments when eccentricity is fixed ($e = 0.35$). When the standardized TL threshold is less than one, the consolidated shipments are independent of the vendor/store ratio. In particular, Figures 5.7.a-5.7.e are identical as the fractions in modes of shipments (LTL, TL) and (TL, LTL) remain zero until the standardized TL threshold reaches one. Specifically, the fraction in mode of shipments (TL, TL) is initially constant and increases as TL threshold gradually increases until all shipments are shipped through a crossdock when the standardized TL threshold is one. The increases in consolidated shipments indicate that a crossdock consolidates more shipments to satisfy the TL threshold.

As the standardized TL threshold exceeds one, a crossdock is unable to both receive and send TL shipments, and the fractions of consolidated shipments depend on vendor/store ratio. If $\frac{|V|}{|S|} = 1$ (Figure 5.7.a), all shipments are outsourced because the actual TL threshold exceeds total demand of each store and total supply of each vendor. If $\frac{|V|}{|S|} > 1$ (Figures 5.7.d and 5.7.e), mode of shipments (LTL, TL) dominates. On the contrary, the fraction in mode of shipments (TL, LTL) dominates if $\frac{|V|}{|S|} < 1$ (Figures 5.7.a and 5.7.b).

Both $\frac{|V|}{|S|} < 1$ and $\frac{|V|}{|S|} > 1$ in Figure 5.7 have similarities like those in Figure 5.3. These similarities are: the fractions in modes of shipments (TL, LTL) and (LTL, TL) of the inverse vendor/store ratio are equal, and the fractions of a vendor/store ratio that is less deviated with respect to $\frac{|V|}{|S|} = 1$ are the parts of a more deviated vendor/store ratio. As a result, we can analyze only Figure 5.7.a and infer our findings to the remaining figures. In Figure 5.7.a, the fraction in mode of shipments (TL, LTL) partially replaces that in mode of shipments (TL, TL) as the TL threshold restricts all outgoing TL shipments from the crossdock. The fraction of consolidated shipments is constant and equal to the asymptotic probability function of (TL, LTL) . As the standardized TL threshold further increases, the crossdock consolidates more shipments to increase the sizes of incoming shipments and to resist the TL threshold restriction, resulting increases in mode of shipments (TL, LTL) and decreases in outsourced shipments. Eventually, all shipments must be consolidated to qualify for mode of shipments (TL, LTL) . If the TL threshold slightly increases than this point, all shipments are outsourced as the TL threshold exceeds total supply of each store. The

reversed greedy algorithm may be viewed as the pure transportation planning because it prioritizes the protection of modes of shipments without considering the costs and benefits of transportation planning.

5.3 Optimization

In the previous section, three transportation planning schemes and the associated algorithms are proposed. To compare their transportation costs, we formulate a mixed-integer linear program (Appendix C.1) and compute the optimal shipment allocation of a simulated unit interval crossdock. Two numerical simulation experiments with different eccentricity and vendor/store ratio of the optimal allocation are conducted and shown in Figures 5.8 and 5.9, respectively.

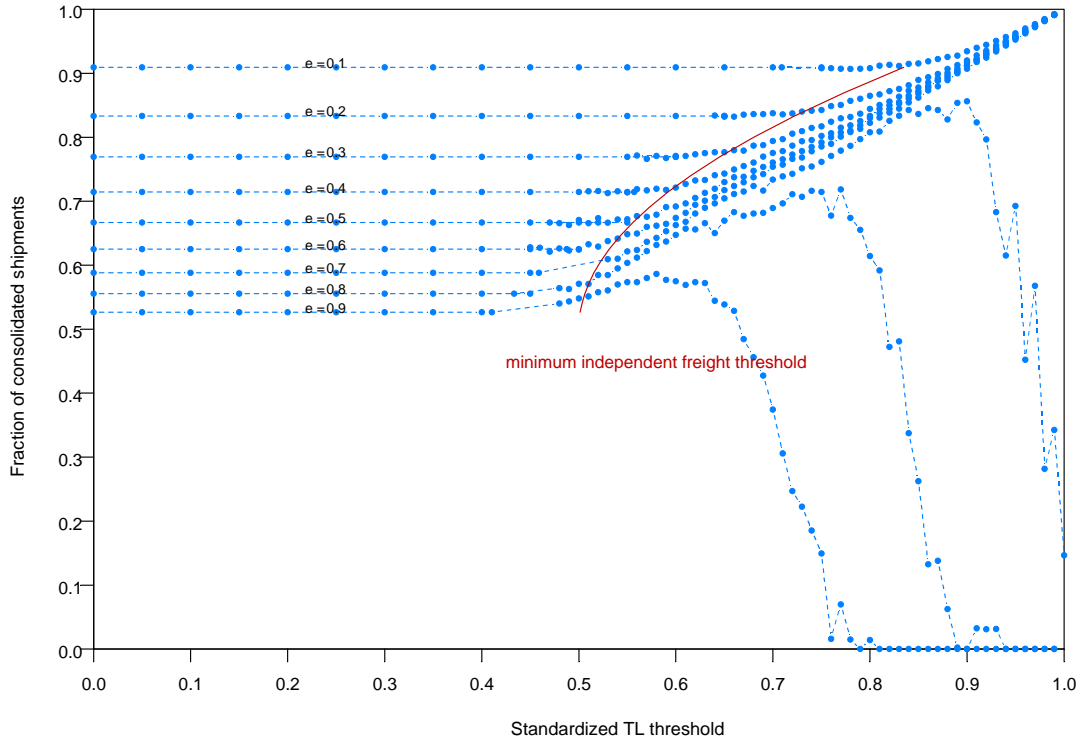


Figure 5.8: The fractions of consolidated shipments through the crossdock located at the center of the unit interval of the optimal solution.

Figure 5.8 shows the effects of eccentricity and TL threshold on the fraction of consolidated shipments when numbers of vendors and stores are equal ($\frac{|V|}{|S|} = 1$). The figure suggests a tradeoff between costs and benefits of the transportation planning. While the costs of transportation planning increase as TL threshold increases, the benefits decrease as eccentricity increases. At some point, the costs of transportation planning exceed its benefits. Specifically, the fraction initially increases as the standardized TL threshold exceeds the minimal independent freight threshold (red curve). The increases in consolidated shipment continue until the standardized TL threshold reaches one in low eccentricity. However, the fraction of consolidated shipments in high eccentricity (i.e., $e \geq 0.7$) reaches the peak and gradually decreases as the TL threshold further increases. The effect of modes of shipments or uses of crossdock of the optimal shipment allocation is shown in Figure 5.9. We also observe similarities between Figures 5.9 and 5.4 that will be discussed together with the effects of vendor/store ratios in Figure 5.9.

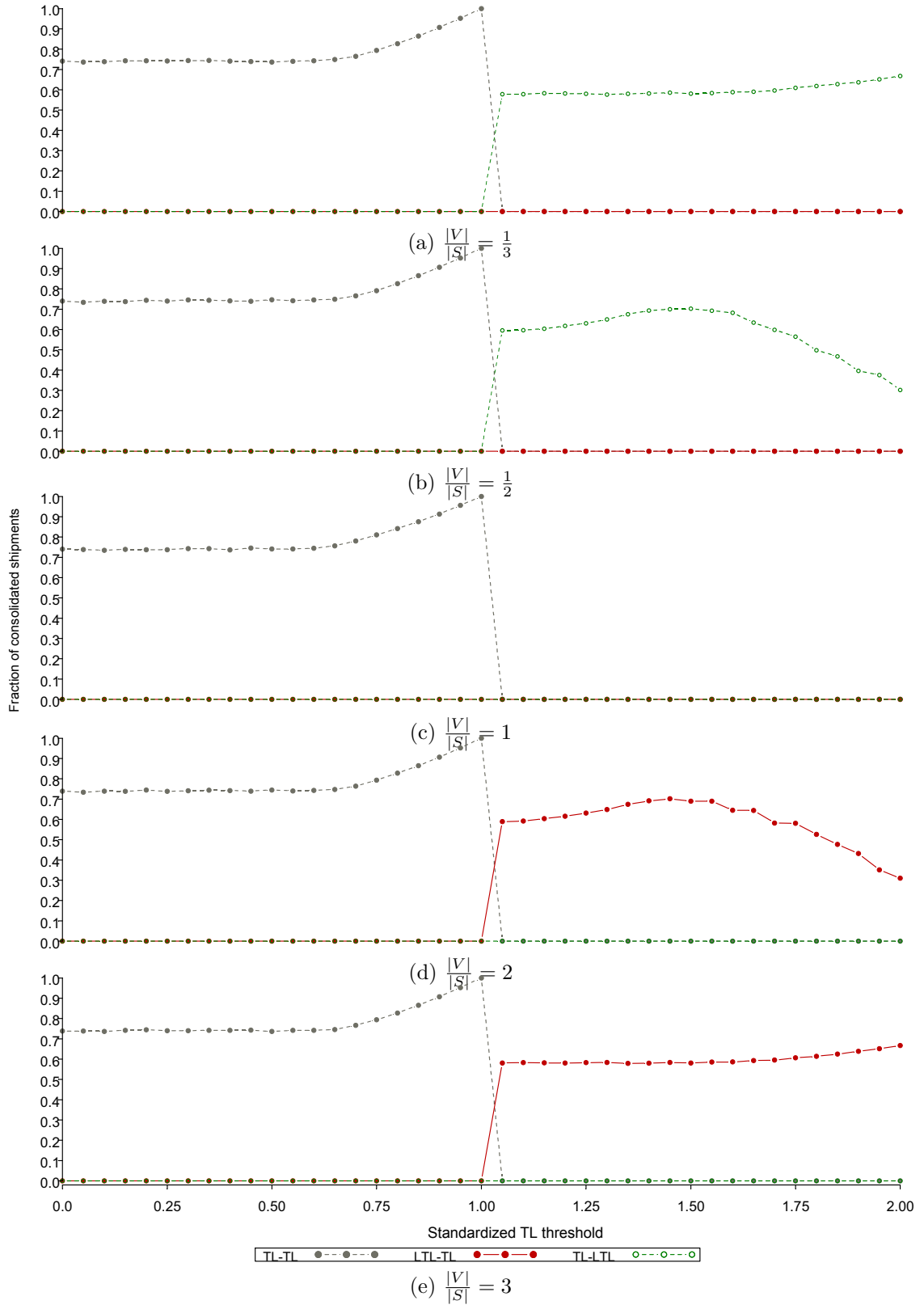


Figure 5.9: Simulation experiments on the unit-interval center crossdock of the unit interval of the optimal solution.

Figure 5.9 shows the effects of vendor/store ratio and TL threshold on fractions of consolidated shipments when eccentricity holds constant at 0.35. If the standardized TL threshold is less than or equal to one, mode of shipments (TL, TL) dominates other modes of shipments. The vendor/store ratio affects consolidated shipments only when the standardized TL threshold exceeds one. If $\frac{|V|}{|S|} = 1$ (Figures 5.9.c), all shipments are outsourced. If $\frac{|V|}{|S|} < 1$ (Figures 5.9.a and 5.9.b), shipments are transported through the crossdock as mode of shipments (TL, LTL) . If $\frac{|V|}{|S|} > 1$ (Figures 5.9.d and 5.9.e), the crossdock receives LTL shipments from vendors but ships TL shipments to stores. We observe that the fraction in mode of shipments (TL, LTL) of low eccentricity is similar to the fraction in mode of shipments (LTL, TL) of high eccentricity.

Despite differing in their peaks and decreasing rate of fractions of consolidated shipments, Figures 5.7 and 5.9 share the similarities in pattern if they are put side-by-side, especially $\frac{|V|}{|S|} \leq 1$. Particularly, they redirect additional shipments through a crossdock in low eccentricity values and forfeit the transportation planning when eccentricity and TL threshold are high. This means, if the number of vendors is less than the number of stores, allowing each store to determine its means of shipment from vendors should yield a near optimal shipment allocation. One explanation comes from the store control algorithm that is embedded into the model as the algorithm addresses the costs and benefits of the transportation planning. If $\frac{|V|}{|S|} > 1$, the algorithm performs poorly as discussed earlier. Next, we focus on the total transportation costs of each transportation planning scheme with respect to the optimal solution, as shown in Figure 5.10.

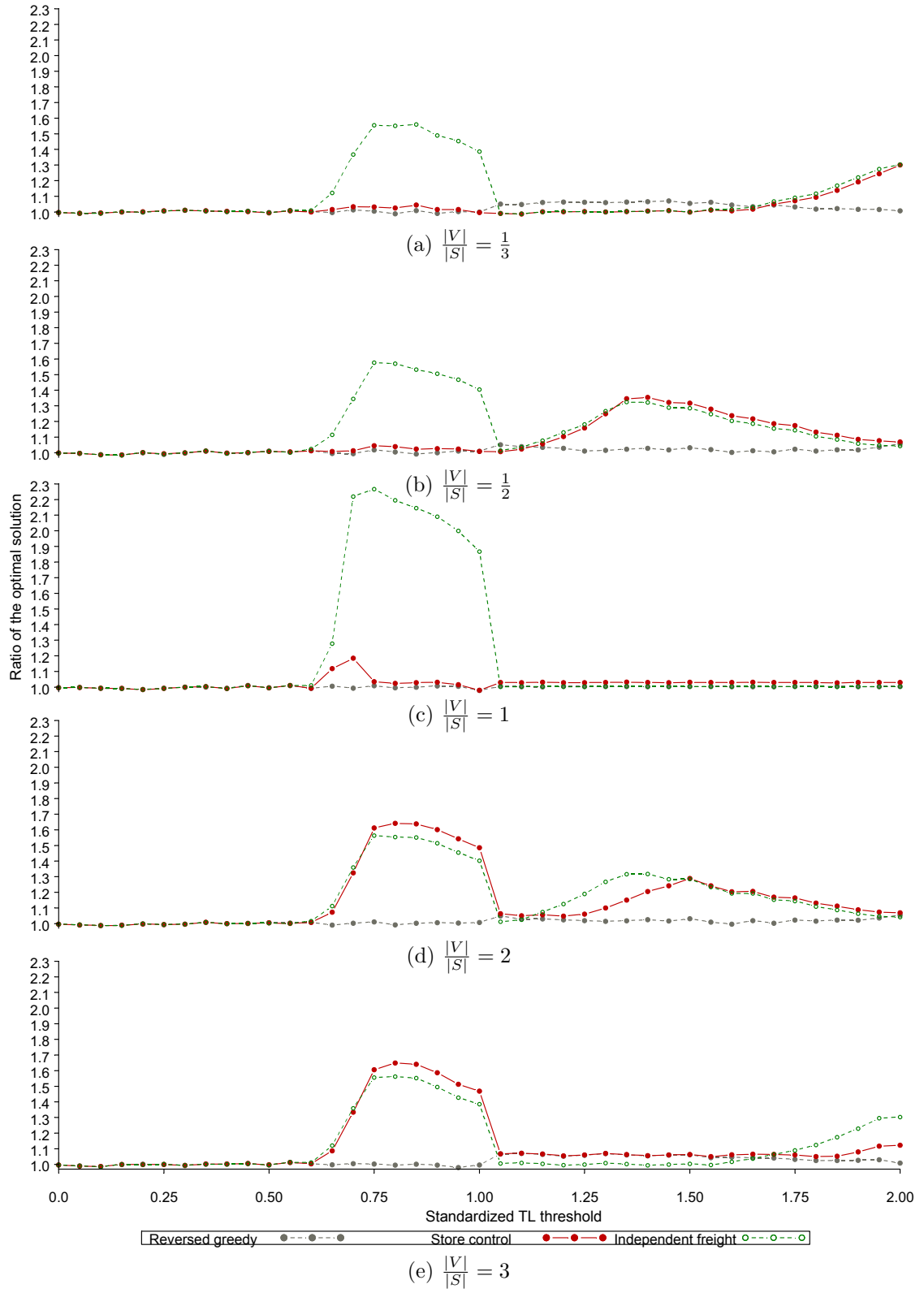


Figure 5.10: The total transportation costs of each transportation planning scheme as the ratio of optimal solution

Figure 5.10 displays the total transportation costs of selected vendor/store ratios and the standardized TL threshold as ratios of the optimal solution. Ratios of the optimal solution of each scheme are represented by the same color and connected by dotted line. In Figure 5.10, each transportation planning scheme has approximately the same transportation costs as the optimal solution when TL threshold is less than the minimal independent freight threshold. As the standardized TL threshold exceeds this threshold, each scheme begins to influence the transportation costs. We find that the implementation of the vendor-store pair level (the independent freight algorithm) causes notably high transportation costs, particularly when numbers of vendors and stores are equal, because of evaporation effects. In addition, the reversed greedy algorithm results in better total transportation costs compared to other algorithms across vendor/store ratio and TL threshold. Hence, the reversed greedy algorithm is embedded into the model when we illustrate the Home Depot distribution network.

CHAPTER VI

LOCATION-ALLOCATION IN THE CROSSDOCKING NETWORK

In this chapter, we analyze the Home Depot’s network and embed its data into the model extended from Expression 3.1. We propose shipping patterns and suggest numbers and locations of crossdocks to reduce the total transportation costs. In addition, we discuss the limitations of the model, including regional eccentricity and the evaporation effect.

6.1 Crossdocking Network Analysis

In the previous chapters, we simplify shipping decisions in a single hub crossdocking network to obtain insights. However, a practical network usually consists of multiple vendors, stores and crossdocks which are rarely located uniformly across a geographic region. This raises several important questions: What are the characteristics of a practical single hub crossdocking network? Do these characteristics support the assumption stated in the previous chapter? Can the model be efficiently implemented? If so, what are the limitations of the model? To answer these questions, we analyze the Home Depot (THD) network.

6.1.1 THD Network

In North America, THD network consists of 180 vendors at 322 locations, shipping through 10 crossdocks, to 1,451 stores, as shown in Figure 6.1.

In Figure 6.1, locations of a vendor and a store are represented by a black square and a blue circle, while a crossdock location is shown as a red star with its nearest city. Figure 6.1 suggests that the majority of THD facilities are located in the east and the west coasts where the population is concentrated. Based on their locations, we arrange vendors and stores by the nearest crossdock and then represent the number and the total weight of each arrangement in Figure 6.2.

Figure 6.2.a (Figure 6.2.b) shows the number of vendors (stores) and the total weight of vendors (stores) arranged by their nearest crossdocks and sorted by their total weight.

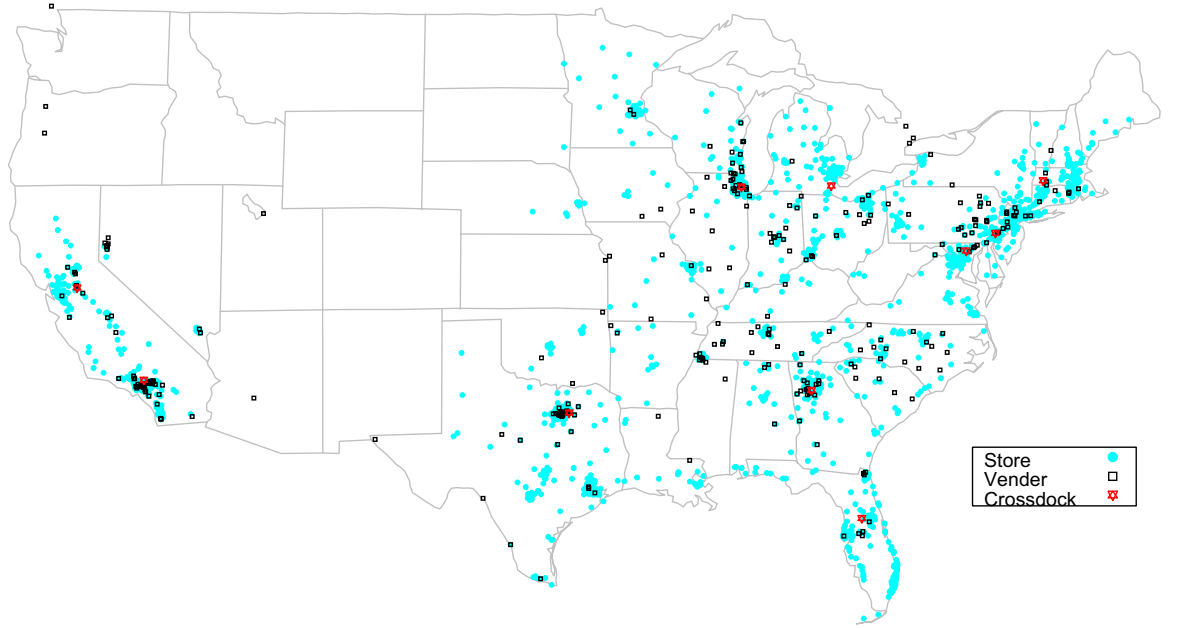


Figure 6.1: THD network consists of 180 vendors (black squares), 10 crossdocks (red stars) and 1,451 stores (blue circles). The majority of THD facilities are located in the east and the west coasts.

Figure 6.2.a emphasizes that the vendor locations are dispersed around the crossdocks. For example, the Baltimore crossdock is located near some large vendors, accounting for 10% of freight. The Westfield and Philadelphia crossdocks, however, are surrounded by smaller vendors, which account for 2% and 5% of freight. In addition, the Atlanta crossdock is expected to be a major transit facility because it is surrounded by major vendors. Unlike Figure 6.2.a, Figure 6.2.b suggests that the nearest crossdock is located at approximately the center of the stores, measured by either the weight of freight or the number of stores.

Having discussed the spatial patterns of the facilities, we analyze the shipping pattern of the current THD network, which ships more than 200,000 vendor-store (origin-destination) pairs throughout the continental US and some Canadian provinces. We discover that the majority of vendors ship their freight through every crossdock, while the majority of stores receive their freight through a single crossdock as shown in Figures 6.3.a and 6.3.b, respectively.

The y -axis of Figure 6.3.a (Figure 6.3.b) represents the percentage of weight and the

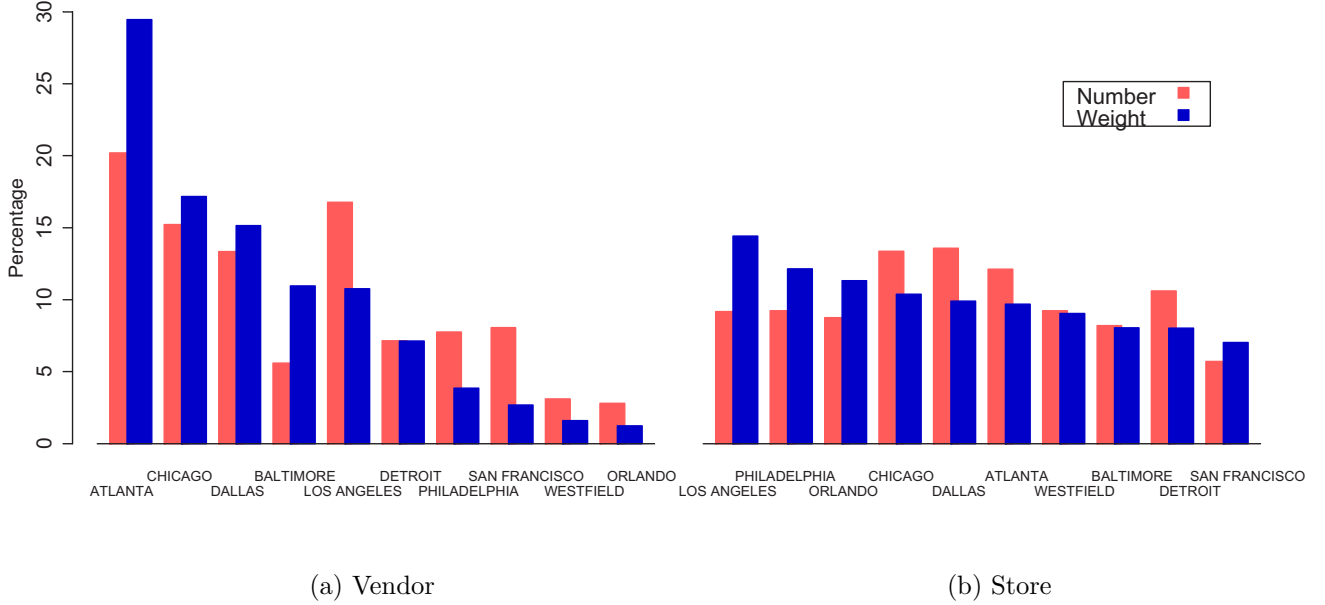


Figure 6.2: Crossdock locations are driven by store locations, not by vendor locations

number of vendors (stores), while the number of crossdocks associated with these shipments are shown in the x -axis. In Figure 6.3.a, 60% of vendors, accounting for 80% of freight by weight, ship the freight through all ten crossdocks. These vendors are mostly household names and/or market-leading manufacturers. Figure 6.3.b shows that a store mostly (approximately 95%) receives its freight from a single crossdock. Because of this unique pattern of shipment allocation, we can visualize the spatial relationship of crossdocks and their associated store, as shown in Figure 6.4.

In Figure 6.4, the stores are colored according to the crossdock from which they receive most freight. Each THD store is represented by a circle with a diameter proportional to total weight. All stores assigned to the same crossdock are colored the same. Each crossdock is represented by a red star. Figure 6.4 indicates that most crossdocks serve the store closest to them. For example, the Westfield crossdock serves New England stores.

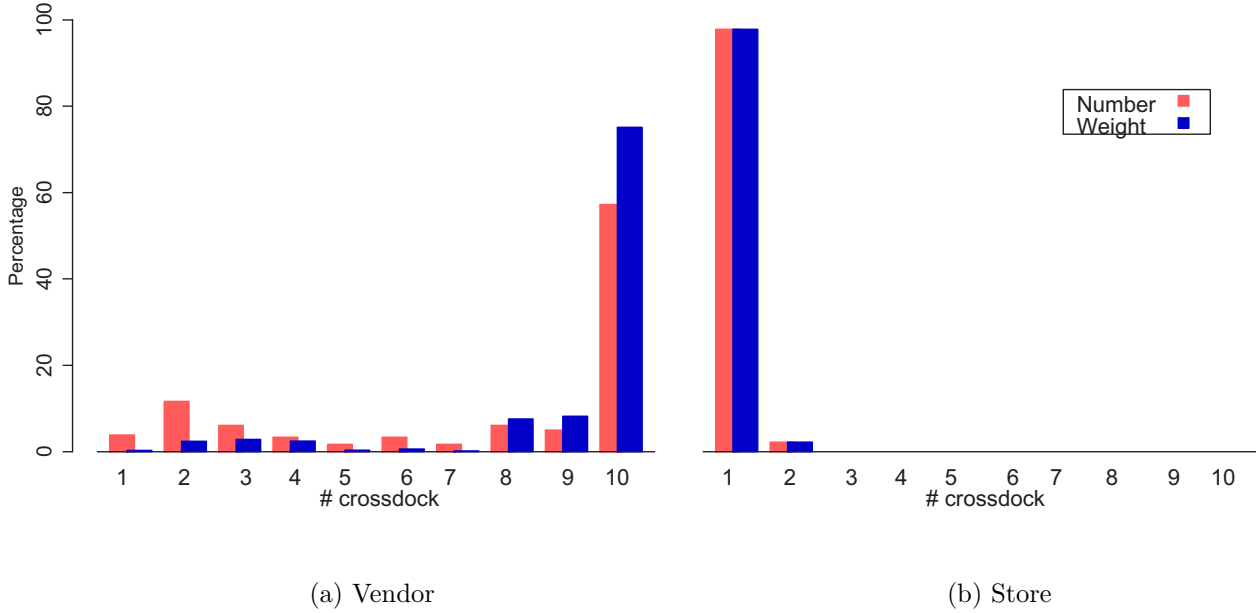


Figure 6.3: Over 55% of vendors (75% of weight) ship freight through all crossdocks, while over 95% of stores receive freight from a single crossdock

6.1.2 Independent Freight Model

In Section 3.2, we discuss Assumptions 1-3 and elaborate that characteristics of a large retailer support these assumptions. When the assumptions hold, a shipment of each vendor-store pair can be viewed as independent of the other pairs. The transportation costs of each vendor-store pair can be calculated by Expression 3.1. In general, the estimation of mode of shipments is difficult because of its interactions with amounts of freight. However, we argue that the economy of scale of THD is sufficient for its crossdocks mostly to receive and ship TL shipments.

In THD network, each store carries similar products from identical vendors. This creates a large quantity of demand in each vendor. Hence, the total demands of each vendor are sufficient to generate TL shipments through a crossdock. As a national retailer, THD network consists of hundreds of stores, and the average THD store occupies 90,000 square feet [59]. Therefore, shipments from a crossdock to each store is TL. Thus, a crossdock always receives and ships TL shipments. In other words, the mode of shipments of the network is (TL, TL) . Next, we present a mathematical model, as shown in Expression 6.1

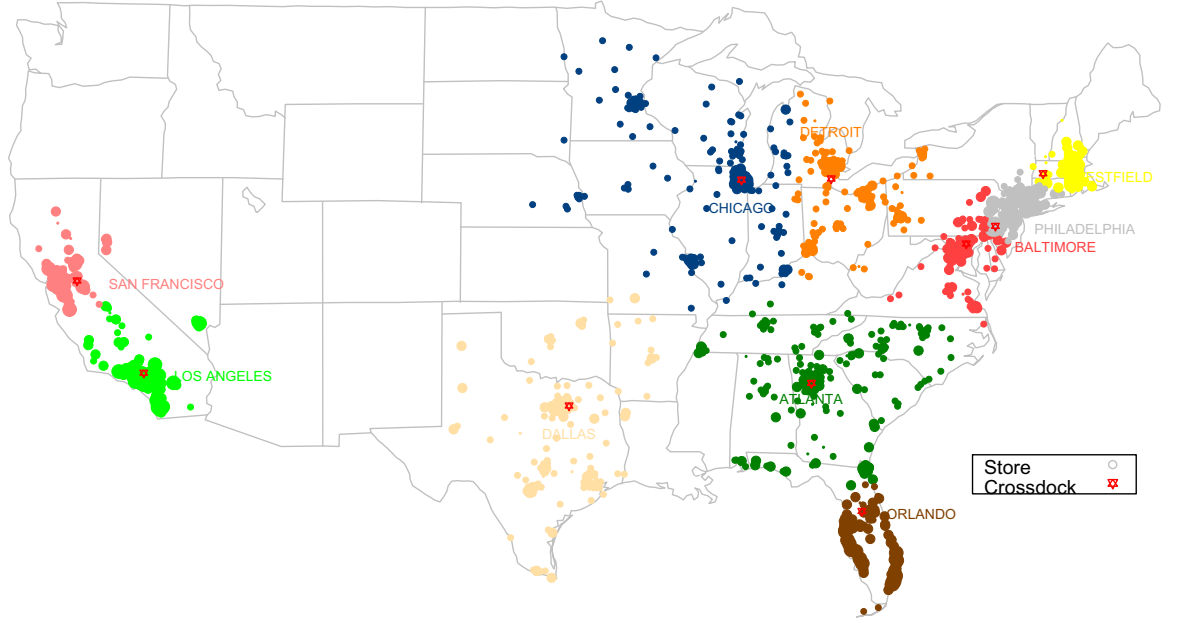


Figure 6.4: Location of the relative weight of stores (circles)

$$\begin{aligned}
 TC(e, \mathbb{X}\mathbb{D}) &= \sum_i \sum_j TC_{ij}(e, \mathbb{X}\mathbb{D}) \\
 &= \sum_i \sum_j f_{ij} \min \left(d_{ij} \min_{p \in \mathbb{X}\mathbb{D}} (e d_{ip} + e d_{pj} + h_p) \right) \quad (6.1)
 \end{aligned}$$

Expression 6.1 is the aggregated version of Expression 3.1 when the mode of shipments is (TL, TL) . We refer to the model as *independent freight model* because each vendor-store pair is assigned independently of others. Given locations of crossdocks, the distances between all facilities can be computed. Therefore, Expression 6.1 is an assignment problem or a transportation-cost comparison between the different means of shipping, which requires $O(m \cdot n \cdot p)$ iterations to solve, where m , n and p are the numbers of vendors, stores and crossdocks, respectively. If the crossdock locations are to-be-determined locations, Expression 6.1 is similar to the multiple-location Weber problem with an outsourced-shipment option. The simple version of Weber problem is known to be a convex problem when the distance is a convex function [24]. However, the outsourced-shipment option renders the problem to be non-convex, as shown in Proposition 6.1.1.

Proposition 6.1.1. *If the distance function is a strictly convex function, Expression 6.1 is neither concave nor convex with respect to crossdock locations.*

Proof. First, we show that a special case of Expression 6.1 is a convex function. Therefore, Expression 6.1 cannot be a concave function with respect to crossdock locations. Let consider a special case where a network consists of many crossdocks so that there is no outsourced shipment. In this case, each vendor-store pair is a convex function because the distance function is convex and the amount of freight and the eccentricity are non-negative. Since summation of convex functions is a convex function, a special case of Expression 6.1 in which there is no outsourced shipment is a convex function. Hence, Expression 6.1 is non-concave.

To complete this proposition, we show that $TC_{ij}(e, \mathbb{X}\mathbb{D})$ is also non-convex by deriving a contradiction. We consider two different crossdocks: crossdock XD_1 and crossdock XD_2 ($XD_1 \neq XD_2$). For vendor-store pair (V_i, S_j) , we assume that an outsourced shipment is preferable at crossdock XD_1 (i.e., $\min TC_{ij}(e, XD_1) = f_{ij}d_{ij}$), while a consolidated shipment is the better decision at crossdock XD_2 (i.e., $\min TC_{ij}(e, XD_2) = f_{ij}(e d_{ip_2} + e d_{p_2j} + h_{p_2})$). This implies $TC_{ij}(e, XD_1) > TC_{ij}(e, XD_2)$. We consider the convex combination of crossdocks XD_1 and XD_2 .

We also let λ , $0 \leq \lambda \leq 1$. Suppose that Expression 6.1 is convex. Then, its convex combination must satisfy:

$$\begin{aligned} TC_{ij}(e, \lambda XD_1 + (1 - \lambda)XD_2) &\leq (\lambda TC_{ij}(e, XD_1) + (1 - \lambda)TC_{ij}(e, XD_2)) \\ &< \lambda TC_{ij}(e, XD_1) + (1 - \lambda)TC_{ij}(e, XD_1) \\ &= TC_{ij}(e, XD_1) \\ &= f_{ij}d_{ij}. \end{aligned}$$

The last expression implies that $TC_{ij}(e, \lambda XD_1 + (1 - \lambda)XD_2) < f_{ij}d_{ij}$. However, there exists λ (e.g., $\lambda = 1$) such that $TC_{ij}(e, \lambda XD_1 + (1 - \lambda)XD_2) = f_{ij}d_{ij}$. Therefore, this contradicts to our assumption that Expression 6.1 is convex. ■

Proposition 6.1.1 implies that Expression 6.1 can be optimally solved with the classic convex-programming technique if the distance function is linear. Otherwise, the solution of Expression 6.1 is a local optimum.

Next, we embed THD data into the independent freight model to aid visualization of the network. Throughout this chapter, locations of facilities are assumed as points in the sphere in which their distance or norm are equivalent to the great-arc distances. In addition, we propose several ways to improve the performance of THD network by the re-allocation of shipments.

6.2 *Re-Allocating Shipments*

6.2.1 Pattern Generation

We propose shipping patterns that reduce the transportation costs by means of an outsourced shipment, crossdock re-assignment and multiple-crossdock allocation. The patterns are generated by algorithms 4 and 5, referred to as Independent-Shipment and TL-Shipment-Swap Algorithms.

Algorithm 4 Independent-Shipment

```

for all vendor  $V_i$  and store  $S_j$  in  $\mathbb{V} \times \mathbb{S}$  do
  compute the outsourced shipment cost of pair  $(V_i, S_j)$ 
  for all crossdock  $XD_p$  in  $\mathbb{XD}$  do
    compute the consolidated shipment cost of path  $V_i-XD_p-S_j$ 
5: end for
  assign vendor-store pair  $(V_i, S_j)$  to the minimal cost crossdock or the LTL shipment
end for
return assignment of all vendor-store pairs

```

Independent-Shipment Algorithm is a variation of a greedy algorithm. That is, each vendor-store pair is independently assigned to its cheapest mean of shipments. This algorithm may suggest that a store receives shipments from multiple crossdocks or even all crossdocks. Although the multiple shipments to a store reduce transportation costs, a company usually avoids the implementation because of managing difficulty. As a result, a company may restrict the number of crossdocks for each store. Such the restriction is enforced by Algorithm 5.

Algorithm 5 searches for the restriction violated store and reassigns them to the next

Algorithm 5 TL-Shipment-Swap

```
for all store  $S_j$  in  $\mathbb{S}$  do
    count number of crossdocks used by store  $S_j$ 
    while number of crossdocks exceeds a given integer  $k$  do
        find minimal weight leg  $XD_p-S_j$  in the open-leg list
5:    return all vendor-store pairs in leg  $XD_p-S_j$  and keep in list  $L$ 
        remove leg  $XD_p-S_j$  from the open-leg list
        for all vendor-store pair  $(V_i, S_j)$  in list  $L$  do
            re-assign pair  $(V_i, S_j)$  to the minimal cost crossdock or the LTL shipment
        end for
10:    end while
    end for
return assignment of all vendor-store pairs
```

cheapest mean of shipping. The shipping patterns generated by the both algorithms are listed by their complication as follows:

Default-network pattern: is generated by assigning each store to the crossdock for which it currently receives most freight. The LTL outsourced shipment is omitted in this pattern. The total transportation cost of this pattern is normalized to be 100%.

LTL pattern: is generated by comparing each shipment independently choose the least cost means of delivery, either directly from a vendor to a store via LTL or from a vendor to the crossdock (assigned by THD) via TL and from the crossdock to a store via TL.

Best-assignment pattern: is generated by sorting the best single crossdock (for each store) from which receives and ships every shipment via TL.

Single-crossdock pattern: is generated by implementing Independent-Shipment and TL-Shipment-Swap Algorithms when the number of crossdocks used by a store (k) equal 1.

Double-crossdock pattern: is similar to the single crossdock pattern, but $k = 2$.

Triple-crossdock pattern: is similar to the single crossdock pattern, but $k = 3$.

Ideal pattern: is generated by computing the best means of delivering freight from a vendor to a store by Independent-Shipment Algorithm.

The relative costs of the shipping patterns are shown in Figure 6.5.

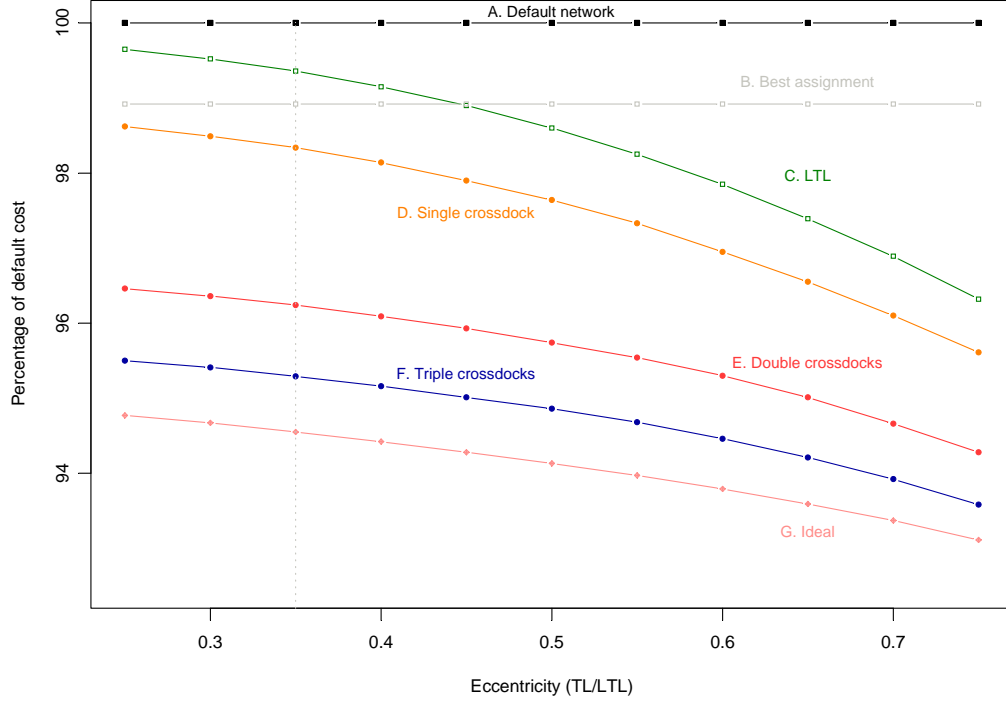


Figure 6.5: Total transportation costs a fraction of the costs in the default network

6.2.2 Results and Analysis

The y - and x -axes of Figure 6.5 show the normalized total transportation costs and the eccentricity value, the ratio between TL and LTL shipping costs per pound-mile. The vertical dotted line in Figure 6.5 represents the current eccentricity value of THD, which is approximately 0.35. At eccentricity 0.35, THD can save approximately 1% by assigning each store to its best crossdock and an additional 0.5% by choosing the cheaper transportation mode for each shipment. In the extreme case, up to 5.5% of the total transportation costs in the network can be reduced.

Figure 6.5 also shows the effect of the outsourced shipment and the number of crossdocks served by each store on the total transportation cost curve. Because the Default-network and Best-assignment patterns prohibit LTL shipments, their curves are constant and unaffected

by the eccentricity value. The curves of the other shipping patterns in Figure 6.5, however, are decreasing. This implies that an outsourced shipment becomes a significant factor at high eccentricity value because of the indifference between TL and LTL and the triangle inequality of distance. Unlike the outsourced shipment, the number of crossdocks served by a store dominates other effects at a low eccentricity value because a TL shipment becomes attractive. Figure 6.5 shows that an increasing number of crossdocks served by a store reduces the total transportation costs, but the marginal benefit of additional crossdocks decreases. As stores can receive freight from multiple crossdocks, the opportunity to consolidate increases which reduces the transportation costs. We suggest THD implements the Double-crossdock pattern because it saves approximately 4% of the total transportation costs and some THD stores experience this shipping pattern already. The implementation may affect the amounts of freight in both outsourced shipments and consolidated shipments. Therefore, a comparison of the total weight between the Default-network pattern and the suggested shipping pattern is depicted in Figure 6.6.

The red and blue bars in Figure 6.6 show the percentage of the total weights of the Default-network and Double-crossdock patterns at the eccentricity value 0.35, sorted by their differences. For example, the Westfield crossdock loses 4.31% of total weight, while the Atlanta crossdock gains 4.20%. The percentage of LTL is zero in the Default-network pattern because no LTL shipment occurs in this shipping pattern. The majority of freight from the Westfield crossdocks are outsourced and shipped LTL directly, while the Atlanta crossdock handles additional freight from the Orlando crossdock. In the Double-crossdock pattern, the total weight of each crossdock follows the spatial characteristic of the vendors in THD network, as shown in Figure 6.2.a. For example, the Atlanta crossdock, which is located near many large vendors, is one of the major crossdocks in the shipping pattern, while the Orlando crossdock is the least freight crossdock and adjacent to vendors.

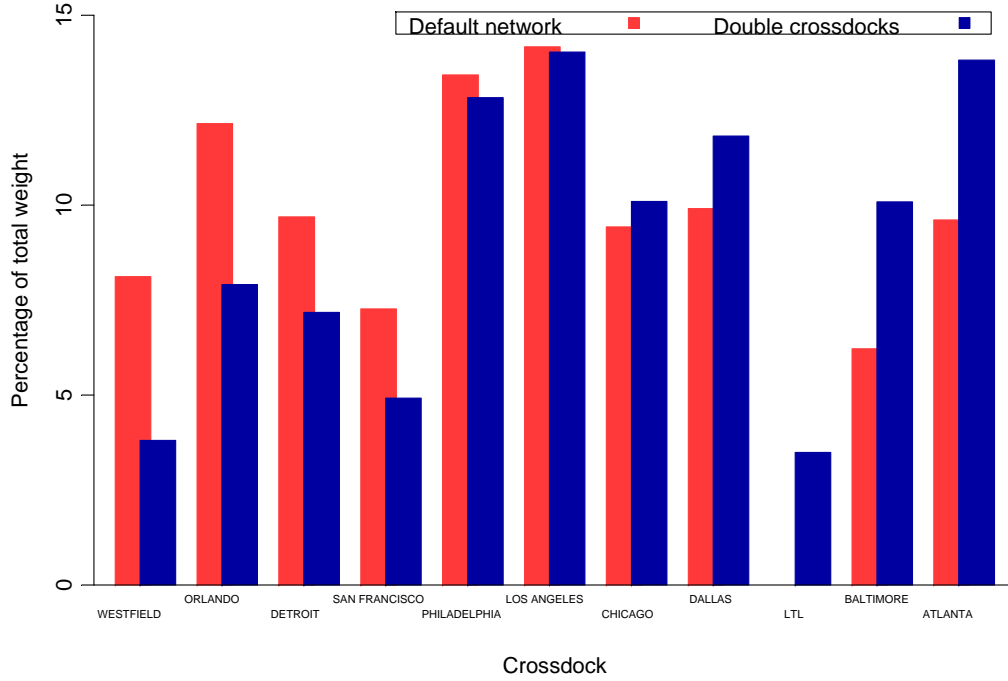


Figure 6.6: In the Double-crossdock pattern, the freight from the Westfield crossdock and the Orlando crossdock are re-allocated to the outsourced shipment via LTL and to the Atlanta crossdock, respectively

6.3 Re-Locating Crossdocks

A crossdock location is an important decision in a network. Crossdock locations are usually optimized when a retailer's distribution network is first set up. However, changes in demands and location of stores and business growth deteriorate the network. The crossdock locations, therefore, need to be re-optimized. The re-location of crossdocks also reveals several aspects of the network, such as the sensitivity of each crossdock. The re-location involves two interdependent questions: How many crossdocks should the network have? Where are the optimal locations of the crossdocks?

Throughout this section, crossdock locations are generated by the Ideal pattern (a shipment allocation in which each vendor-store pair is independently assigned without any constraint) and a standard location-allocation heuristic, called the *Weiszfeld heuristic*.

6.3.1 Weiszfeld Heuristic

The Weiszfeld heuristic (WH) is a gradient-based numerical approach to relocate facilities in a continuous space that alternates between two sub-problems: location and allocation. In the location sub-problem, the allocation of vendor-store pairs is pre-determined, and the heuristic determines a new location as a convex combination of vendors and stores. In the allocation sub-problem, the location of facilities is fixed. The convergence of the heuristic depends on a distance function. This topic was investigated by Brimberg and Love [11], who modified the l_p -norm of distance with the hyperbolic approximation and analyzed the global convergence of the approximated l_p -norm for $1 \leq p \leq 2$. The convergence of approximated l_p -norm for $p > 2$ was studied by Üster and Love [60], who showed that the convergence of the algorithm depends on the step size factor and proposed the geometric series of the convergent step size factor. The WH provides a good solution in the single facility location problem [3, 21]. We implement this heuristic in the multiple-facility location problem because we observe that the surface of the total transportation cost with respect to crossdock locations is relatively flat. The pseudo-code of implemented WH, which handles multiple crossdocks and outsourced shipments, is described as follows:

Algorithm 6 Weiszfeld heuristic

```

initialize crossdock locations and select stopping ratio  $\epsilon$ 
repeat
    assign the shipments with the Independent-Shipment Algorithm
    compute the total transportation costs
5:   compute the next crossdock locations with first order condition
until the relative improving ratio of the total transportation costs is less than  $\epsilon$ 
return assignment of all vendor-store pairs and crossdock locations.

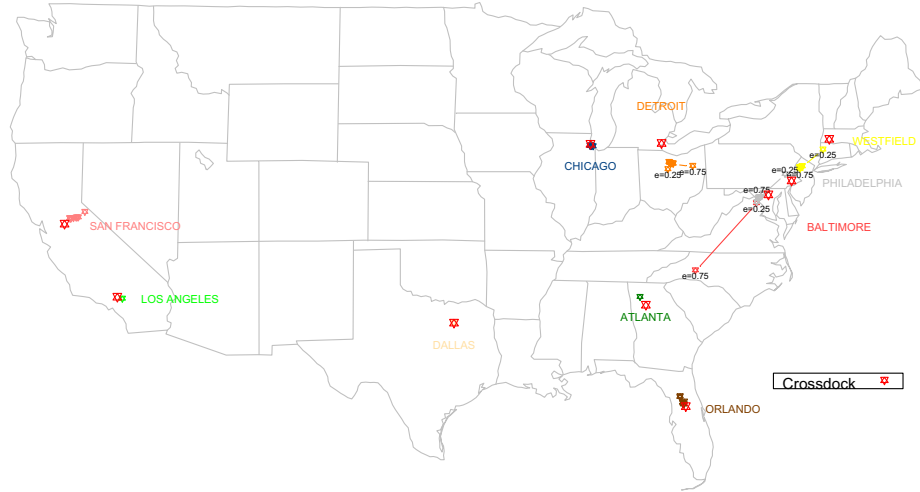
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The allocation sub-problem of Algorithm 6 is handled by Independent-Shipment Algorithm, while its location sub-problem is completed by the first order condition. The algorithm is terminated when there is no significant improvement, or the relative improvement of the total transportation costs are less than a given stopping ratio. It should be mentioned that the other shipping patterns, discussed in Section 6.2.1, can be implemented in the Weiszfeld heuristic. However, the additional information is required to track all shipments.

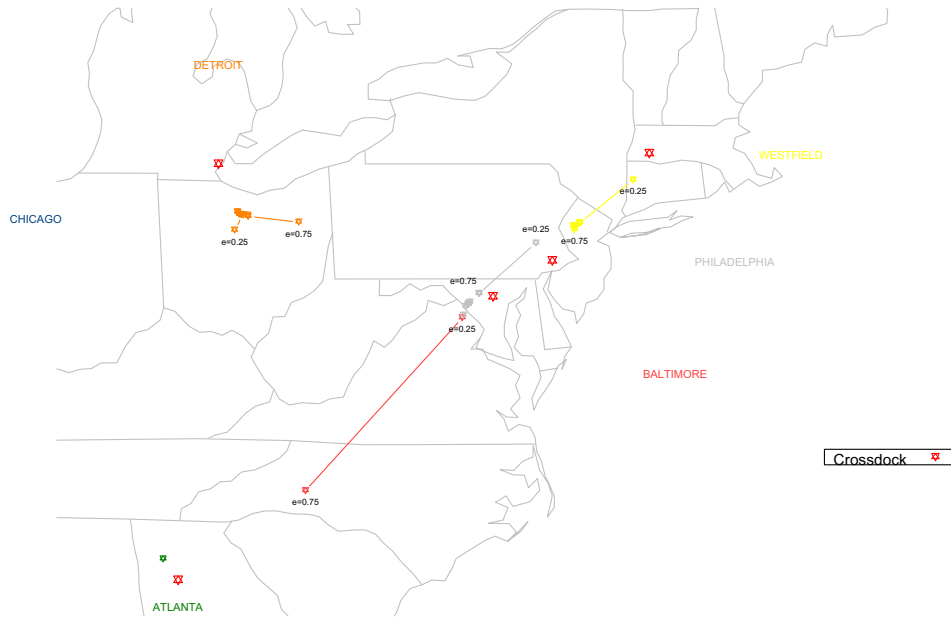
6.3.2 Results and Analysis

We implement the WH in THD network and compare crossdock locations, as shown in Figure 6.7. The current crossdocks and their nearest cities are denoted by red stars and colored texts, respectively. The WH determined series of the suggested crossdock locations, corresponding to eccentricity ($0.25 \leq e \leq 0.75$) These series are represented by color stars (other than red) and colored according to their initial cities of crossdock. For example, the yellow stars refer to a series of re-located crossdocks with different eccentricity that the initial city is Westfield, MA.

In Figure 6.7.a, we observe that the crossdock locations in the northeastern region of the United States are not optimal and are sensitive to changes in eccentricity, while the locations in other regions are near-optimal. Hence, a zoom-in version of the crossdock locations is presented in Figure 6.7.b. The figure shows that Westfield crossdock should be closed. In addition, Philadelphia, Detroit, and Baltimore crossdocks should be re-located slightly away from their current locations. We agree with the closing Westfield crossdock because the northeastern region has three existing crossdocks within a 300-miles radius and relatively low activities comparing to other regions. Two options after closing Westfield crossdock are considered. The first option is to re-open a crossdock near the city of Charlotte, NC (thus, the number of crossdocks remains ten). This option leaves the Philadelphia and Baltimore crossdocks in the New England region that slightly increases outsourced shipments and the transportation costs of the northeastern region. However, the Charlotte crossdock is near to many vendors, which promotes consolidated shipments and reduces the total transportation costs of the network.

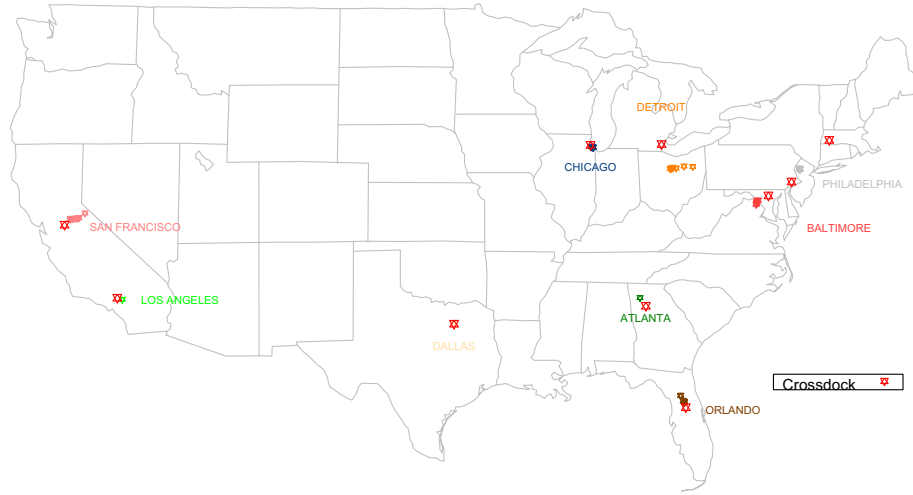


(a) United States

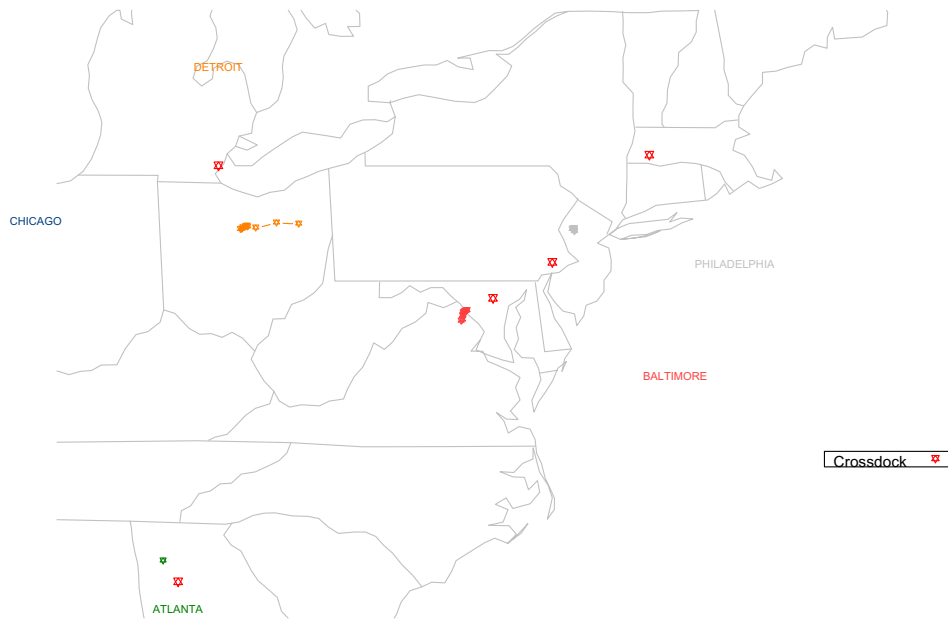


(b) Northeastern region of United Stated

Figure 6.7: Closing the Westfield crossdock and re-opening a crossdock near Charlotte, NC reduces the total transportation costs



(a) United States



(b) Northeastern region of United Stated

Figure 6.8: As the Westfield crossdock removed, the current locations of others are stable

The second option is to remove the Westfield crossdock without re-opening a new crossdock (hence, the network has only nine crossdocks), as shown in Figure 6.8. Figures 6.8.a and 6.8.b present the suggested crossdock locations determined by the WH in United States and the northeastern regions. As the Westfield crossdock is closed, the suggested crossdock locations are relatively unchanged compared with the existing crossdocks and unaffected by the practical range of eccentricity. With exception of the Philadelphia crossdock that has additional 3% of freight, all crossdocks handle the same amount of freight. The additional freight in the Philadelphia crossdock that may raise the capacity issue. The total transportation costs of both configurations are overlayed on Figure 6.5 and shown as two lower curves in Figure 6.9.

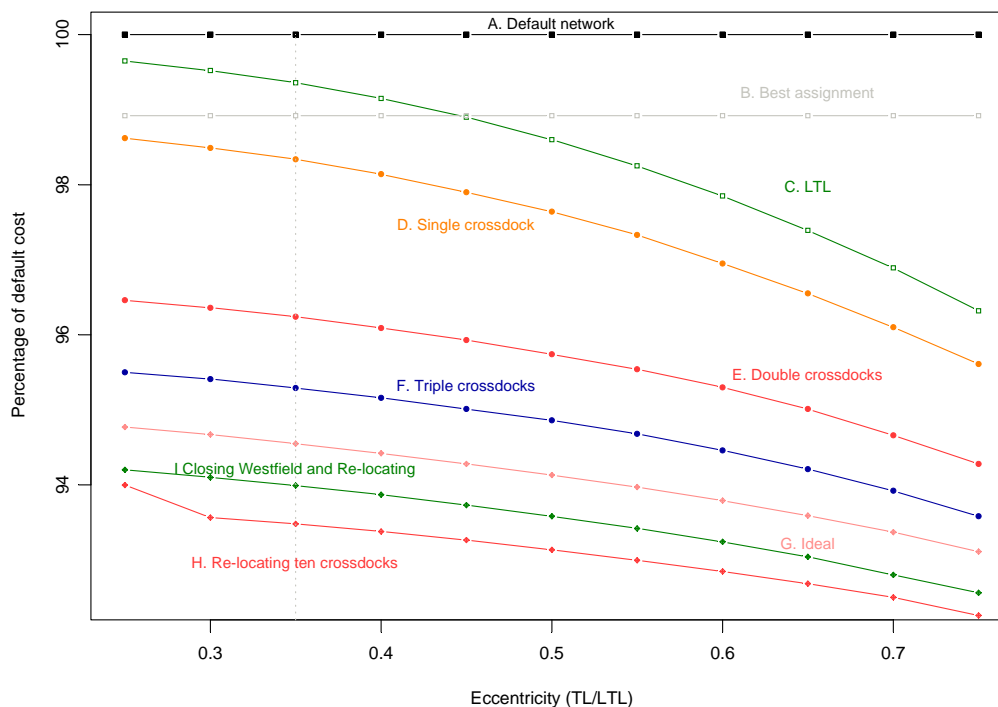


Figure 6.9: The two lower curves show the total transportation costs of moving the Westfield crossdock to Charlotte (red curve) and closing the Westfield crossdock (green curve), comparing to the total transportation costs of re-allocating the shipping patterns

Figure 6.9 shows that the re-location of crossdocks can improve the network by saving additional 1% of the total transportation costs. In the re-locating ten-crossdock curve,

we observe that the curve sharply declines when $e = 0.3$ because the algorithm (see Figure 6.7) strongly suggests moving the Westfield crossdock to the city of Charlotte at higher eccentricity.

6.4 Regional Eccentricity

In this section, we evaluate the homogeneous eccentricity assumption and consider regional effects in THD network. We show that both average and regional values of eccentricity have large variance. In addition, we demonstrate that the model can be extended to support the regional eccentricity data.

6.4.1 Problems of the Average Eccentricity

In previous sections, the eccentricity is assumed to be homogenous and represented by its average value. However, this assumption may be disputed in a large geographic area because of the demographic differences in population and economics. We concede that the homogenous eccentricity value may not be the most realistic representation of THD transportation-cost structure since the distribution has large variance, as shown in Figure 6.10.

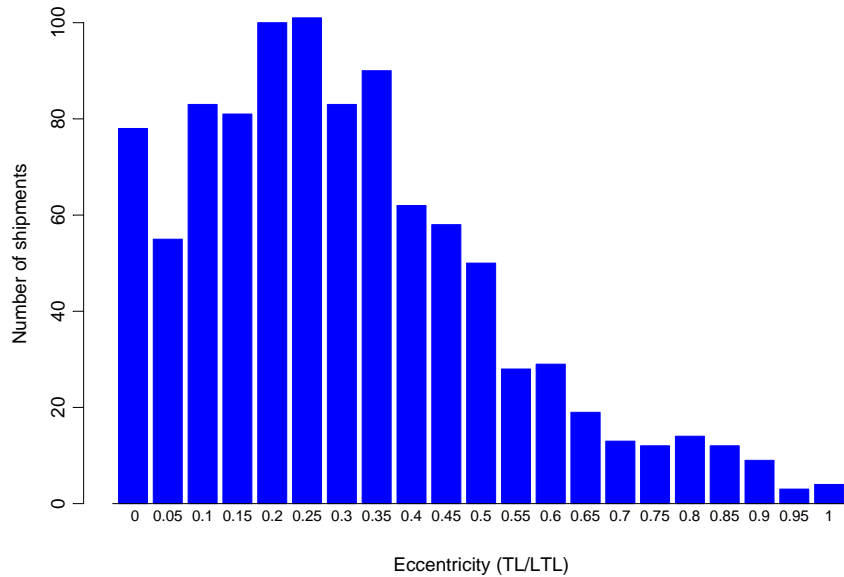
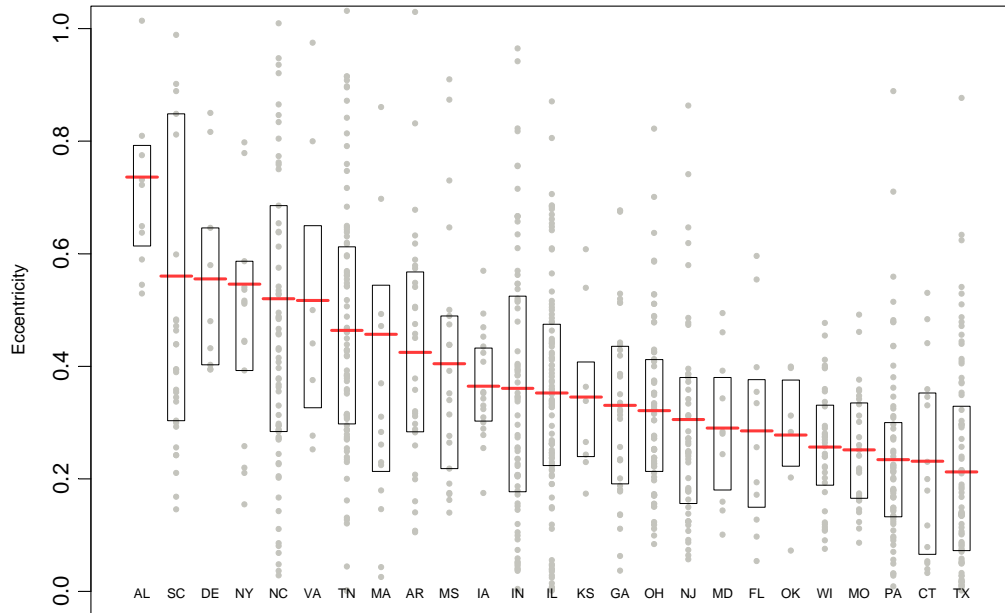
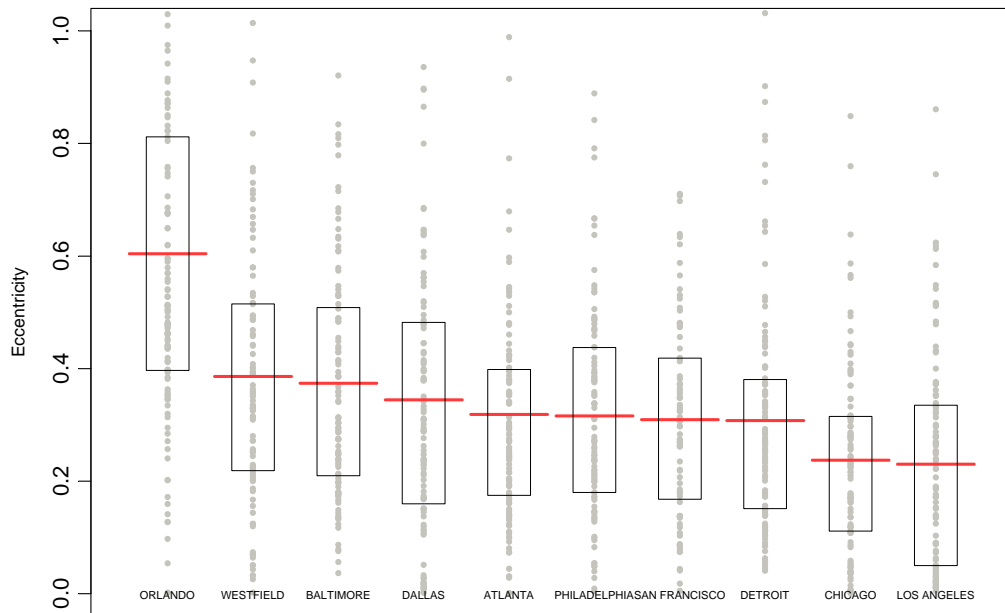


Figure 6.10: While the average-eccentricity value is 0.345, eccentricity values are distributed across the possible range and the empirical distribution is asymmetry.

Each bar in Figure 6.10 represents the frequency within 0.05 interval of THD eccentricity. While the average value is 0.345, the eccentricity values are distributed across the possible range. This implies that the average value is a poor representative of the parameter. Thus, we consider the alternative where eccentricity is heterogenous. In general, eccentricity may depend on many variables, such as distance, direction of shipment, percentage of shipment or locations. However, the majority of these variables can be captured by considering origins and/or destinations of shipments. It also represents the demographic differences in population and economic regions. For example, the shipping rate of freight from Georgia to Florida is more expensive than the reverse one (freight from Florida to Georgia) as Florida is surrounding by ocean and trucks must travel to Georgia before re-routing. Rather than move empty trucks, a trucking company is desperate to fill some capacity and charges a low shipping rate. Therefore, the eccentricity values grouped by its shipping origin and destination are depicted in descending order of its average values, as shown in Figure 6.11.a and Figure 6.11.b, respectively.



(a) State from which freight originates



(b) City where freight is sent

Figure 6.11: The eccentricity values of the location in which the Home Depot freight originate (Figure 6.11.a) and where it is sent (Figure 6.11.b) are disperse.

For each column in Figure 6.11, an average-eccentricity value of each location is represented by a horizontal dash, whereas a range of first- and the third-quartile values of each location is shown within a black box. Figure 6.11 suggests that the eccentricity values are diverse. With exception of shipments that originate from Alabama (Figure 6.11.a) and shipments that are sent to Florida crossdock (Figure 6.11.b), the average-eccentricity values of each location are approximately the same. We also consider a possibility that eccentricity could depend on origin-destination pair, as shown in the supplementary materials (Figures D.1 and D.2). However, there is no further significant finding and the eccentricity has large variance and remains diverse. Therefore, neither is average eccentricity nor regional eccentricity a good representation of the data. In addition, the implementation of the regional eccentricity in THD network requires extrapolation of the data because there is insufficient cost data. The available cost data are for approximately 10% of total origin-destination pairs because the majority of THD shipments are consolidated and transported through a crossdock by TL. As a result, the regional eccentricity covers only 33% of the network. Nevertheless, we demonstrate that the model can be extended and accounted for the regional effect of eccentricity.

6.4.2 Numerical Comparison between the Regional and Average Eccentricity

We associate each shipment with its regional eccentricity and present the numerical study. For the portion of the network that the regional eccentricity is unavailable, the eccentricity values are estimated by the average-eccentricity value. The comparison between the average and regional eccentricity suggests that eccentricity is an important parameter and strongly affects the allocation between crossdocks and stores, as shown in Figure 6.12.

Figure 6.12 shows the normalized total transportation costs. For each shipping pattern, the transportation-cost normalization of the average and regional eccentricity is represented by red and blue bars, respectively. The first two columns in Figure 6.12 represent the Default-network and LTL patterns. In both shipping patterns, the use of the regional eccentricity is approximately 2% higher in transportation costs than the average eccentricity. This is, the regional eccentricity can be ignored when consolidated shipments are shipped

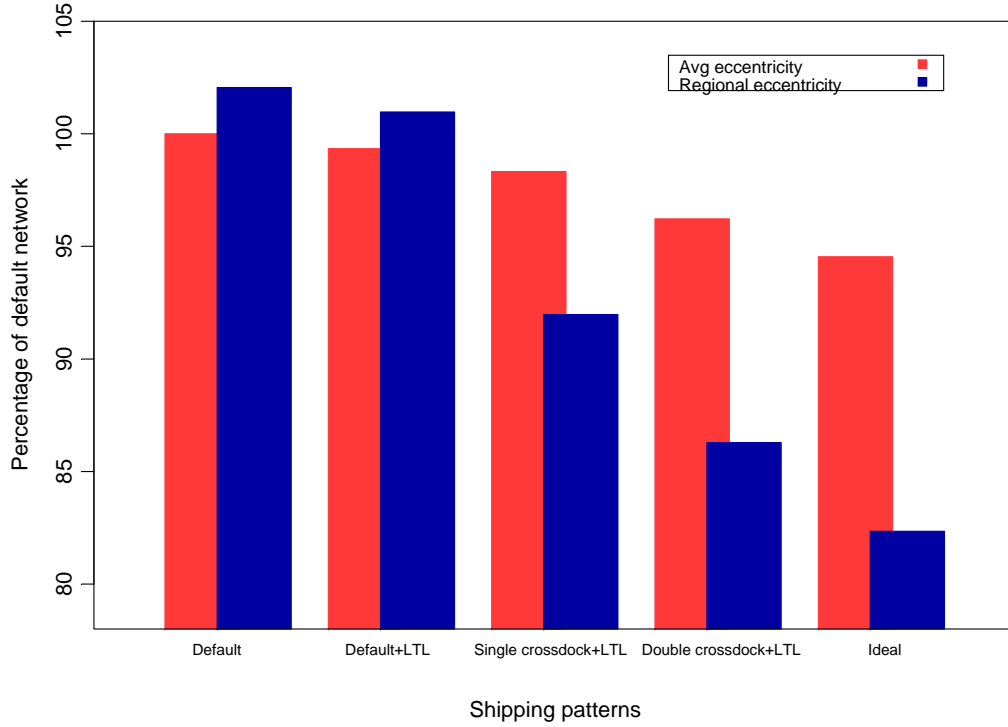
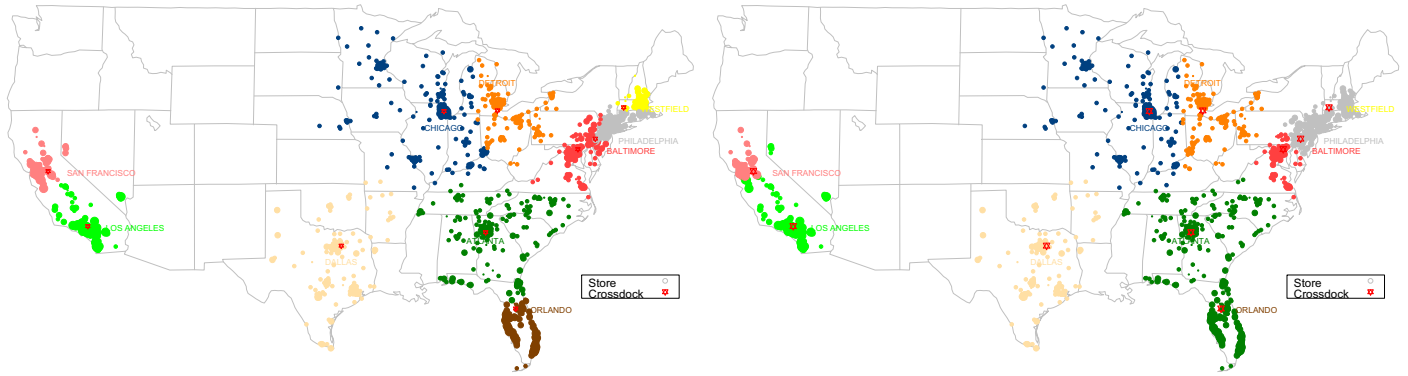


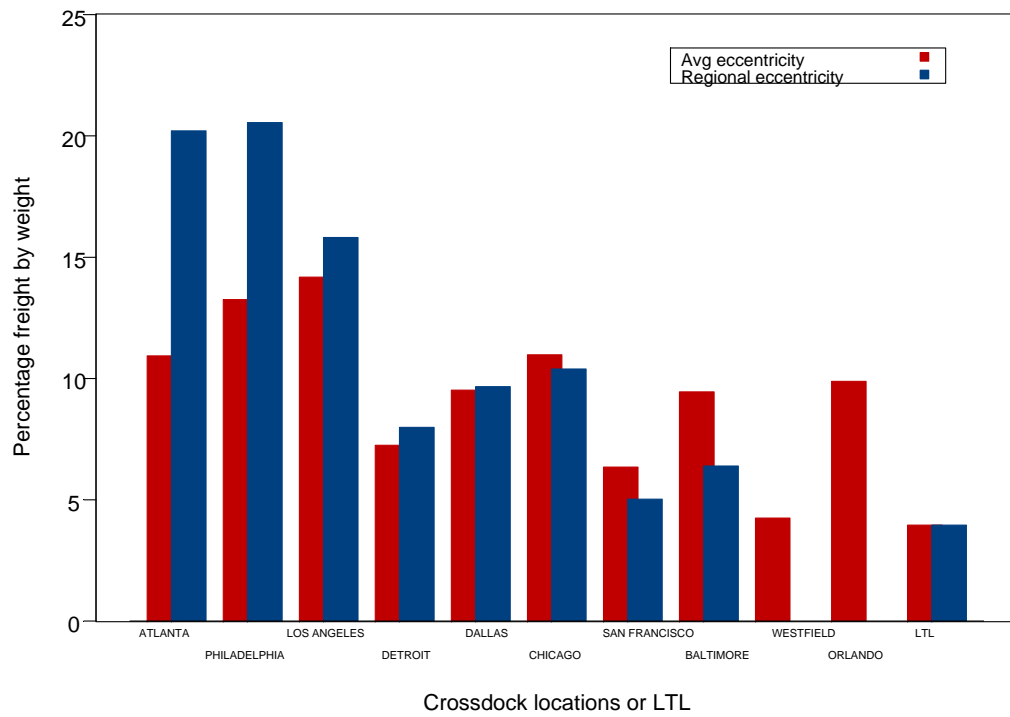
Figure 6.12: With an exception of the default crossdock-store assignment, the projected savings of total transportation costs with the proposed model increase with the regional eccentricity.

to crossdocks assigned by THD. The next three columns show the total transportation costs allocated by the Single-crossdock, Double-crossdock and, Ideal patterns. The uses of the regional eccentricity in such the shipping patterns result in approximately 6%, 10% and 15% lower in the total transportation costs than the average eccentricity, respectively. This indicates the impacts on the shipment allocation and the amounts of freight received by each crossdock. The impacts in Single-crossdock pattern is shown in Figure 6.13.



(a) Average eccentricity

(b) regional eccentricity



(c) Weight of each mean of shipment

Figure 6.13: The comparison between the average and regional eccentricity of the Single-crossdock pattern shows that some crossdocks lose their freight to adjacent crossdocks (Figures 6.13.a and 6.13.b), while others remain unchanged (Figure 6.13.c). In addition, the use of the regional eccentricity emphasizes the importance of crossdock-store assignment.

Figures 6.13.a and 6.13.b show locations of relative weight and assigned crossdock of stores in the average and regional eccentricity, respectively. The difference of total weight is sorted and shown in Figure 6.13.c. In Figure 6.13, we find that some crossdocks lose/gain their freight to adjacent crossdocks because of the regional eccentricity. For example, Atlanta and Philadelphia crossdocks gains approximately 10% and 8% of total weight from Orlando and Westfield crossdocks. Others remain unchanged, such as Chicago, Detroit and Dallas crossdocks.

6.5 Verification of the Independent Assumption

6.5.1 Consolidated Shipments in THD Network

The independent freight model, discussed in the re-allocation and re-location of THD network, assumes that the modes of shipments are given or known in advance. As a result, a shipment of each vendor-store pair is independent of the other pairs (Assumption 3). However, such a model overlooks TL threshold, an interactions between amounts of freight, and modes of shipments discussed in Chapter 5. The lack of the TL threshold in the independent freight model raises several questions. Is the model valid for THD network? If not, how is TL threshold incorporated into the model? The validity of the independent freight model for THD network can be cross-checked with shipping weights of the consolidated shipments through crossdocks, as shown in Figure 6.14.

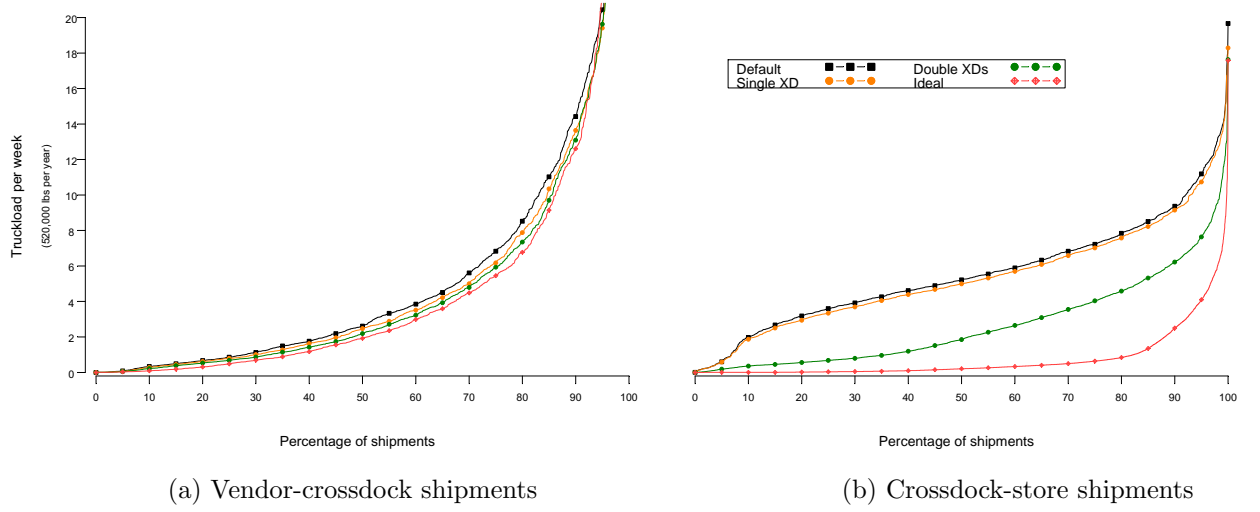


Figure 6.14: Shipments from vendors to crossdocks (Figure 6.14.a) are skewed and the profile of shipping patterns are similar, unlike the shipments from crossdocks to stores (Figure 6.14.b).

The y -axis of Figure 6.14 represents the annual shipping weight, while the percentage of incoming and outgoing consolidated shipments of a crossdock are shown in the x -axis of Figures 6.14.a and 6.14.b, respectively. We assume that each store is equally received its shipments throughout the period and TL threshold is 10,000 pounds (520,000 pounds per year). Each color of the curves in Figures 6.14.a and 6.14.b corresponds to a shipping pattern at THD eccentricity ($e = 0.35$). For example, the black curve shows that 60% of the shipments from a vendor to a crossdock and 20% of the shipments from a crossdock to a store in the Default-network pattern requires less than 4 truckloads per week (≈ 2 million pounds per year). Figure 6.14.a shows that the annual shipping weights of incoming consolidated shipments are skewed, indicating the varieties and the weight differences of products. For example, a crossdock receives both drill bits and bathroom ceramic tires that correspond to 5 percentile and 95 percentile of Figure 6.14.a, respectively. The figure suggests that the shipping patterns have a little or no effect on the incoming shipment as the profile of each curves is quite similar. In the downstream of THD network, Figure 6.14.b shows that each store receives approximately the same shipping weight from a crossdock

because of each store shares similar products. However, we observe that the shipping patterns in Figure 6.14.b greatly affect the profiles of the curves. As an additional crossdock used by each store is allowed, some vendor-store pairs have economic incentive to consolidate through the newly allowed crossdock. This reduces the average shipping weight. For example, the average outgoing weight of the Double-crossdock pattern is approximately one half of the Single-crossdock pattern. Figure 6.14.b also suggests that TL threshold becomes an important factor as each store is allowed to receive shipments from more than two crossdocks. Without TL threshold, the model may incorrectly predict transportation costs, as shown in Figure 6.15.

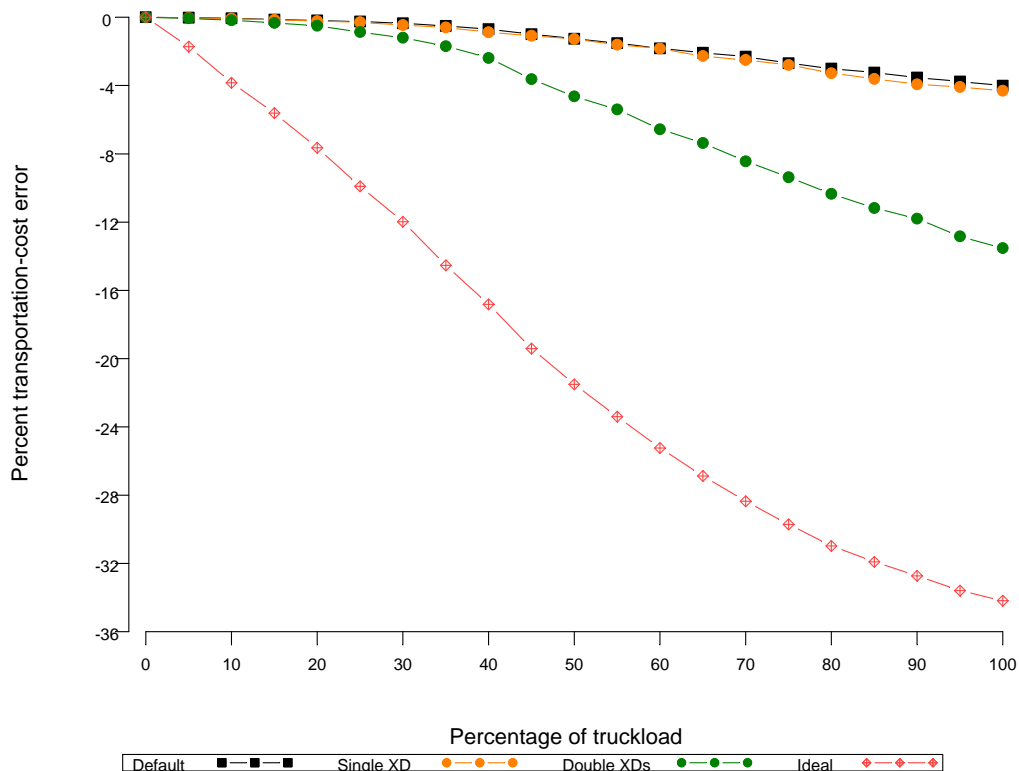


Figure 6.15: The independent freight model underestimates transportation costs. As each store is allowed to receive shipments from more two crossdocks, TL threshold becomes an important parameter and the model begins to breaks down.

Figure 6.15 shows the percentage of transportation cost error predicted by the independent freight model (y -axis) of selected shipping patterns when $e = 0.35$. The x -axis of

Figure 6.15 represents the TL threshold as the percentage of truckload. Figure 6.15 suggests that the independent freight model underestimates transportation costs and breaks down at different TL threshold depending on the shipping pattern. For example, the transportation-cost error of the model exceeds 10% in the Double-crossdock pattern when TL threshold exceeds 70% of truckload (364,000 pounds per year) and the independent freight model begins to break down. In addition, Figure 6.16 shows that the independent freight model is justified for the Single-crossdock and Default-network patterns, yet it is more likely to break down in the other shipping patterns. The figure also confirms the observation that TL threshold becomes an important parameter and the model begins to break down as each store is allowed to receive shipments from multiple crossdocks. One way to improve a quality of solutions is to incorporate TL threshold in the model with the reversed greedy algorithm— a greedy algorithm that ranks savings between outsourced and consolidated shipments discussed in Chapter 5— before re-allocation of shipments, as shown in Figure 6.16.

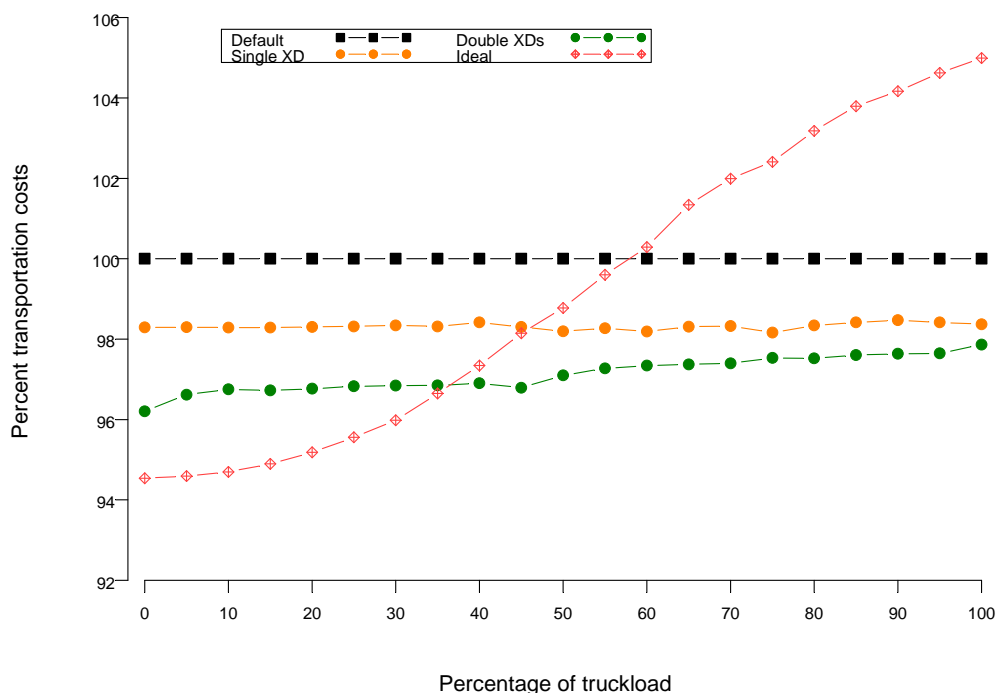


Figure 6.16: The total transportation costs as the ratio of the Default-network pattern when $e = 0.35$ with the reversed greedy algorithm

Figure 6.16 shows the total transportation costs as the ratio of the Default-network pattern when $e = 0.35$. The reversed greedy algorithm alleviates the effect of the TL threshold by accounting for TL threshold and limiting outsourced shipments. In Figure 6.16, the transportation costs of the Double-crossdock and Ideal patterns increase as TL threshold increases. This suggests that the benefits of receiving consolidated shipments from multiple crossdocks cannot be sustained in high TL threshold. We also observe that consolidated shipments in mode of shipments (TL, TL) decrease as TL threshold increases. Crossdocks are unable to generate a sufficient weight for a TL shipment to store and the shipments from crossdocks to stores are more likely to be LTL, and consolidated shipments in mode of shipments (TL, LTL) increase.

CHAPTER VII

CONCLUSIONS AND FUTURE RESEARCH

7.1 *Conclusions*

This thesis formulates the distribution network of a large retailer as the economic trade-off between consolidated shipments through a crossdock and outsourced shipments to 3PL companies, which is referred to as the independent freight model. The model shows that the trade-off can be explained geometrically by an iso-cost curve, the locus of facility locations where the costs of a consolidated shipment and an outsourced shipment are equal. Depending on shipping parameters, iso-cost curves are ellipse, hyperbola, limacon or Cartesian oval. The curve provides a managerial rule of thumb. Particularly, the shipping decision whether consolidation or outsourcing can be visualized by a facility location and an iso-cost curve. In addition, an iso-cost curve represents the transportation-costs contour of a shipment, adds a visualization into the network and serves as a preliminary routing and locating tool. Despite its simple implementation and explanation, an iso-cost curve mathematically connects with Voronoi diagram and geometric inversion. As some iso-cost curves are a generalization of Voronoi diagram, a multiple vendor-crossdock network can be converted to Voronoi diagram. The Voronoi diagram's algorithm can be used to determine the iso-cost curves of vendors and crossdocks. Apart from the connection to Voronoi diagram, we observe that an iso-cost curve can be transformed to another iso-cost curve by geometric inversion. This non-Euclidian transformation is independent of eccentricity and mode of shipments but depends on facility locations as well as an inversion circle. This transformation preserves relationships between consolidated-shipping and outsourced-shipping regions. Geometric inversion can be interpreted as an economic duality statement about an iso-cost curve of a different facility perspective.

We show that the concept of an iso-cost curve can be extended to a network with multiple vendor-store pairs and suggest an index to measure robustness and potential growth of a

crossdock, called the asymptotic probability of shipments. The index reflects the strategic significance of a crossdock and explains the economic driving factors of consolidation and outsourcing, specifically eccentricity, handling costs, distributions of vendors and stores, mode of shipments and a crossdock location. Although the derivation of this index is limited by the dimension and spatial distribution of facilities, its numerical value can be determined by Monte Carlo simulation. The results suggest that a one-dimensional single hub crossdocking network has consistent trends of consolidated shipments with respect to key parameters similar to a two-dimensional single hub crossdocking network.

The key assumption of the independent freight model is that a shipment of each vendor-store pair is independent of other pairs. We relax this assumption and introduce TL threshold, an interaction between amounts of freight and modes of shipments. A numerical study using Monte Carlo simulation shows that the evaporation effect, the cascading of consolidated shipments caused by outsourced shipments, occurs when TL threshold exceeds the minimal independent freight threshold. Given TL threshold, the evaporation effect is intensified by increases of eccentricity and the imbalance between incoming and outgoing shipments of a crossdock. To alleviate the evaporation effect, we propose several transportation planning schemes, including the reversed greedy algorithm. The algorithm is a transportation-cost improving heuristic that accounts for TL threshold and allows necessary changes in modes of shipments based on an initialized network. The comparison with the optimal solution suggests that the reversed greedy algorithm reaps benefits of both consolidated and outsourced shipments and works well for low values of eccentricity. For high values, the solution suggested by the algorithm is different from the optimal solution as the algorithm prioritizes the changes in modes of shipments over the costs and benefits of outsourced shipments. This disadvantage may be overcome by using different initialized networks.

We add the visualization of the Home Depot (THD) data and implement the independent freight model as a routing and locating tool. The analysis of THD distribution network shows that its spatial and operational data are justified for the implementation of the independent freight model. The model predicts the reduction of transportation costs by

means of outsourced shipments, re-allocation of shipments and limiting numbers of crossdocks used by each store. We discover that removal of the Westfield crossdock stabilizes the remaining crossdock locations. Furthermore, the stores located in the northeastern region of the United States require more attention than the other regions. In general, the increasing of number of crossdocks used by each store reduces a larger portion of transportation costs than outsourced shipments; however, it may lead to the evaporation effect. As an additional crossdock used by each store is allowed, the marginal benefit of a crossdock decreases and the model begins to break down from TL threshold. The independent freight model emphasizes the importance of eccentricity and its impacts on shipments and crossdock locations.

7.2 Design Rules of a Single Hub Crossdocking Network

In the spirits of the attractiveness of a crossdock (Section 4.5), we summarize the useful insights into the shipping in a single hub crossdocking network. They enable a shipping planner to design a single hub crossdocking network and to understand the limitations of the model.

- Consolidation through a crossdock increases travel distances and incurs handling cost. However, it may provide savings as the transportation rate of a consolidated shipment is cheaper than that of an outsourced shipment.
- If a network has “high” traffic flows such that the shipping decision of each vendor-store pair is independent of other pairs (Assumption 3 is held), the economic trade-off between consolidated shipments and outsourced shipments can be visualized as a geometric relationship between facilities. This geometric relationship simplify shipping decisions and can be developed into a decentralized model.
- This decentralized model serves as a primary locating and routing tool. The most important parameter in the model is the transportation-cost ratio of a TL shipment to an LTL shipment, namely eccentricity. As a result, a shipping planner should test different values of eccentricity within an expected range (sensitivity analysis) before

exercising the recommendations.

- If a crossdock is surrounded by many stores, a distant vendor will be more likely to ship consolidated shipments bounded to those stores through the crossdock than a vendor located close to the crossdock. Conversely, the closer the store to the crossdock, the higher likelihood of freight being shipped through the crossdock.
- The consolidated shipping region may be unbounded, if interested facilities are the vendor in mode of shipments (LTL, TL) and the store in mode of shipments (TL, LTL) because the transportation costs of a consolidated shipment and an outsourced shipment change as the locations of interested facilities change.
- If a network has “low” traffic flows (Assumption 3 is relaxed), a shipping planner may determine each vendor-store pair independently when TL threshold is less than the minimal independent freight threshold. The planner should incorporate the central transportation information and/or execute an appropriate transportation planning scheme when TL threshold exceeds the threshold as outsourcing may cause the evaporation effect, resulting in the abrupt reduction of consolidated shipments in the network.
- If the central transportation information is unobtainable and eccentricity is very low, a shipping planner may obtain a near-optimal result by avoiding any outsourced shipments that affect mode of shipments (the reversed greedy algorithm).
- The analysis of THD shows that the increasing number of crossdocks used by each store has decreasing marginal benefit and may result in the evaporation effect.

7.3 *Future Works*

7.3.1 Sensitivity of Eccentricity and Generalization of Model

Our experience shows that eccentricity is the most sensitive parameter that affects shipment allocation and transportation costs, yet its behaviors cannot be fully understood. In addition, the impacts of a recent fluctuation of gas price on eccentricity remain unclear. On the one hand, we observe a sign of change in eccentricity as gas price increases. For example, a retailer increases numbers of TL shipments by allowing more inventory buffer. On the other hand, one might argue that eccentricity is a transportation-cost ratio between TL and LTL shipments. The fluctuation of gas price should affect both type of shipments, thus eccentricity should remain unchanged.

Another issue that deteriorates the reliability and accuracy of the model is the labor associated with each product. For example, some products may require temperature-control trailers or a dedicated area in a crossdock. Better estimations and analysis of eccentricity and labor associated with each product could lead to a generalization of a model. The new model may have a complex transportation-cost function and require a different approach to explain the economic trade-off between consolidated and outsourced shipments.

7.3.2 Location-Routing Problem

As a preliminary routing and locating tool, our model serves as a service network model predicting crossdock locations and their allocating stores. The model overlooks the utilization of vehicles and workforce. Hence, we would to take the initial results and incorporate vehicle routing and workforce scheduling into the model. As a result, the model is extended from a strategic model into an operational model. The process is referred to as two-stage location-routing model as the model determined strategic and operational decisions sequentially. A key advantage of the two-stage model is the reduction of complexity and computational time.

One of the disadvantages of the two-stage model is that the decisions compromise the quality of solutions. This could be overcome by a single-stage location-routing model that accounts for numbers and locations of crossdocks, utilization and routing of vehicles and

workforce scheduling, simultaneously. This raises interesting questions. Can the complexity and computational time of the single-stage model be made reasonable? Do the benefits of the two-stage model justify the lower quality of solutions? What are the best options to separate decisions in each stage and estimate associated costs in the two-stage model?

7.3.3 Coordination

The use of the independent freight model for a decentralized network leads to the multi-player prisoner dilemma problem and the transportation planning among individuals (i.e., vendors and/or stores) in a network. In Section 5.2.3, several transportation planning schemes have been proposed to promote the network coordination. However, none of them satisfies Nash’s equilibrium as some vendor-store pairs gain economic benefits by outsourcing their shipments. The transportation planning scheme that is derived from the optimal shipment allocation condition would predict “game” behaviors of individuals and explain underlying motivations. Our experiences suggest that the problem can be modeled as a cooperative game [45], in which majority of vendors and/or stores forms a coalition to achieve the optimal allocation. The key research question is what is a “fair” mechanism to distribute the benefits among vendors and/or stores in the coalition?

APPENDIX A

APPENDIX FOR CHAPTER 3

A.1 Polar Equation of the Generalized Iso-Cost Curve

$$\begin{aligned}
 \text{XD(TL,TL)} \quad r &= \frac{e^2 d_{ij}^2 - d_{ij}^2 - e^2 h^2 + 2e h d_{ij}}{2e(e d_{ij} \cos \theta - d_{ij} + e h)} \\
 \text{S(TL,TL)} \quad r &= \frac{e^3 d_{ip} - e^2 d_{ip} \cos \theta + e^3 h}{1 - e^2} + e d_{ip} + e h \\
 &\quad - \frac{e \sqrt{d_{ip}^2 (1 - e \cos \theta)^2 + e^2 h^2 - 2e h d_{ip} \cos \theta + 2e^2 h d_{ip}}}{1 - e^2} \\
 \text{V(TL,TL)} \quad r &= \frac{e^3 d_{pj} + e^2 d_{pj} \cos \theta + e^3 h}{1 - e^2} + e d_{pj} + e h \\
 &\quad + \frac{e \sqrt{d_{pj}^2 (1 + e \cos \theta)^2 + e^2 h^2 + 2e h d_{pj} \cos \theta + 2e^2 h d_{pj}}}{1 - e^2} \\
 \text{XD(LTL,TL)} \quad r &= \frac{-e d_{ij} - d_{ij} \cos \theta + e^2 h}{1 - e^2} \\
 &\quad \mp \frac{\sqrt{d_{ij}^2 (e + \cos \theta)^2 + e^2 h^2 - 2e^2 h d_{ij} \cos \theta - 2e h d_{ij}}}{1 - e^2} \\
 \text{S(LTL,TL)} \quad r &= \frac{e d_{ip} - d_{ip} \cos \theta + e^2 h}{1 - e^2} \\
 &\quad + \frac{\sqrt{d_{ip}^2 (e - \cos \theta)^2 + e^2 h^2 - 2e^2 h d_{ip} \cos \theta + 2e h d_{ip}}}{1 - e^2} \\
 \text{V(LTL,TL)} \quad r &= \frac{e^2 d_{pj}^2 - d_{pj}^2 + e^2 h^2 + 2e^2 h d_{pj}}{-2(d_{pj} \cos \theta + e d_{pj} + e h)} \\
 \text{XD(TL,LTL)} \quad r &= \frac{-e d_{ij} + d_{ij} \cos \theta + e^2 h}{1 - e^2} \\
 &\quad - \frac{\sqrt{d_{ij}^2 (e - \cos \theta)^2 + e^2 h^2 + 2e^2 h d_{ij} \cos \theta - 2e h d_{ij}}}{1 - e^2} \\
 \text{S(TL,LTL)} \quad r &= \frac{e^2 d_{pj}^2 - d_{pj}^2 + e^2 h^2 + 2e^2 h d_{pj}}{2(-d_{pj} \cos \theta + e d_{pj} + e h)} \\
 \text{V(TL,LTL)} \quad r &= \frac{e d_{pj} + d_{pj} \cos \theta + e^2 h}{1 - e^2} \\
 &\quad + \frac{\sqrt{d_{pj}^2 (e + \cos \theta)^2 + e^2 h^2 + 2e^2 h d_{pj} \cos \theta + 2e h d_{pj}}}{1 - e^2}
 \end{aligned}$$

where,

r = the distance between the origin and $loci$

θ = the angle in the counter-clockwise direction from the x-axis to $loci$

e = eccentricity

h = the handling cost incurred at a crossdock

d_{ij} = the distance between vendor V_i and store S_j

d_{ip} = the distance between vendor V_i and crossdock XD_p

d_{pj} = the distance between crossdock XD_p to store S_j

APPENDIX B

APPENDIX FOR CHAPTER 4

B.1 Closed-Form Expression of Selected Spatial Distributions of Vendors and Stores

B.1.1 Notation

x = a vendor location in the unit interval

y = a crossdock location in the unit interval

e = eccentricity

$c_{\mathcal{X}}(\cdot)$ = probability distribution function of \mathcal{X}

\mathcal{U} = vendor or store is uniformly distributed

$c_{\mathcal{U}} = 1$ for valid x and y

\mathcal{T} = vendor or store is concentrated near a crossdock with a linear rate,
or a triangle distribution

$$c_{\mathcal{T}}(x, y) = \begin{cases} \frac{2x}{y} & \text{for } x < y \\ \frac{2(1-x)}{1-y} & \text{for } x \geq y \end{cases}$$

\mathcal{M} = vendor or store is dispersed around a crossdock with a linear rate,
or a reverse triangle distribution

$$c_{\mathcal{M}}(x) = \begin{cases} \frac{2(y-x)}{y} & \text{for } x < y \\ \frac{2(x-y)}{1-y} & \text{for } x \geq y \end{cases}$$

$(\mathcal{X}_1 - \zeta_1, \mathcal{X}_2 - \zeta_2)$ = order pair of spatial distribution and mode of shipments, in particular vendor locations follow \mathcal{X}_1 distribution whereas store locations follows \mathcal{X}_2 distribution, and the consolidated shipment through crossdock is mode of shipments (ζ_1, ζ_2)

$f(e, y)$ = asymptotic-probability function

B.1.2 Mode of Shipments (TL, TL)

$$\begin{aligned}
& (\mathcal{T}\text{-TL}, \mathcal{T}\text{-TL}); (\mathcal{T}\text{-TL}, \mathcal{U}\text{-TL}); (\mathcal{T}\text{-TL}, \mathcal{M}\text{-TL}) \\
f(e, y) &= -\frac{16y^2e^2 + 8y^2e - 16ye^2 - 8ye + 5e^2 - 2e - 3}{3(e+1)^2} \\
& (\mathcal{U}\text{-TL}, \mathcal{U}\text{-TL}) \\
f(e, y) &= -\frac{4y^2e - 4ye + e - 1}{e+1} \\
& (\mathcal{U}\text{-TL}, \mathcal{M}\text{-TL}) \\
f(e, y) &= -\frac{8y^2e^2 + 16y^2e - 8ye^2 - 16ye + e^2 + 2e - 3}{3(e+1)^2} \\
& (\mathcal{M}\text{-TL}, \mathcal{M}\text{-TL}) \\
f(e, y) &= \frac{-8y^2e + 8ye + e^2 - 2e + 1}{(e+1)^2}
\end{aligned}$$

B.1.3 Modes of Shipments (LTL, TL)

$$(\mathcal{U}\text{-}LTL, \mathcal{U}\text{-}TL)$$

$$f(e, y) = \frac{-2y^2e - 6y^2 + 2ye + 6y - e + 1}{4}$$

$$(\mathcal{T}\text{-}LTL, \mathcal{U}\text{-}TL)$$

$$f(e, y) = \frac{-2y^2e^2 - 8y^2e + 2ye^2 - 14y^2 + 8ye - e^2 + 14y - 4e + 5}{12}$$

$$(\mathcal{M}\text{-}LTL, \mathcal{U}\text{-}TL)$$

$$f(e, y) = \frac{2y^2e^2 - 4y^2e - 2ye^2 - 22y^2 + 4ye + e^2 + 22y - 2e + 1}{12}$$

$$(\mathcal{U}\text{-}LTL, \mathcal{T}\text{-}TL)$$

$$f(e, y) = \frac{-2y^2e - 10y^2 + 10y + 2ye - e + 1}{6}$$

$$(\mathcal{T}\text{-}LTL, \mathcal{T}\text{-}TL)$$

$$f(e, y) = \frac{-2y^2e^2 - 12y^2e + 2ye^2 - 34y^2 + 12ye - e^2 + 34y - 6e + 7}{24}$$

$$(\mathcal{M}\text{-}LTL, \mathcal{T}\text{-}TL)$$

$$f(e, y) = \frac{2y^2e^2 - 4y^2e - 2ye^2 - 46y^2 + 4ye + e^2 + 46y - 2e + 1}{24}$$

$$(\mathcal{U}\text{-}LTL, \mathcal{M}\text{-}TL)$$

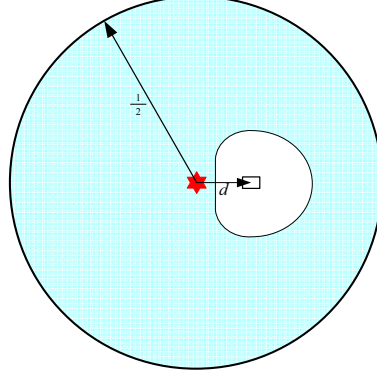
$$f(e, y) = \frac{-2y^2e - 4y^2 + 2ye + 4y - e + 1}{3}$$

$$(\mathcal{T}\text{-}LTL, \mathcal{M}\text{-}TL)$$

$$f(e, y) = \frac{-6y^2e^2 - 20y^2e + 6ye^2 - 22y^2 + 20ye - 3e^2 + 22y - 10e + 13}{24}$$

$$(\mathcal{M}\text{-}LTL, \mathcal{M}\text{-}TL)$$

$$f(e, y) = \frac{2y^2e^2 - 4y^2e - 2ye^2 - 14y^2 + 4ye + e^2 + 14y - 2e + 1}{8}$$



B.2 The asymptotic-probability function in the Unit Disk in mode of shipments (TL, TL)

B.2.1 Case 1: Direct-shipping region is full limacqn $0 \leq r < \frac{1-e}{2(1+e)}$

B.2.1.1 Notation

d = distance between crossdock and a vendor

r = polar equation of limacqn

$$\frac{2ed(1 + e \cos \theta)}{1 - e^2}$$

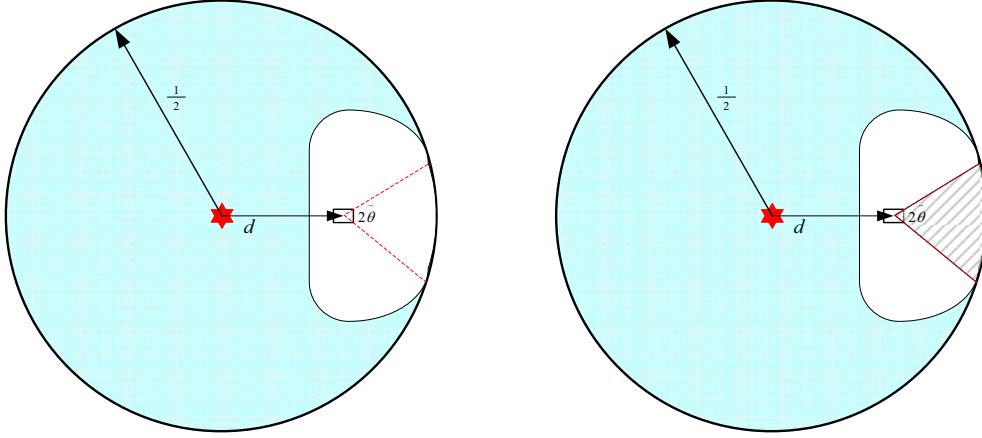
A_{limacqn} = Area of limacqn

$$A_{\text{limacqn}} = \int_0^{2\pi} \frac{1}{2} \left(\frac{2ed(1 + e \cos \theta)}{1 - e^2} \right)^2 \partial \theta = \frac{2\pi e^2 d^2 (2 + e^2)}{(1 - e^2)^2}$$

B.2.1.2 Algebra

$$\begin{aligned} & Pr. \left(\text{freight is shipped through crossdock in the unit disk space by (TL, TL) when } 0 \leq d < \frac{1-e}{2(1+e)} \right) \\ &= \frac{\text{Consolidated-shipping region of of unit-circle space by (TL, TL) when } 0 \leq d < \frac{1-e}{2(1+e)}}{\text{Total area of the unit disk space}} \\ &= \frac{2 \int_0^{\frac{1-e}{2(1+e)}} \pi \left(\frac{1}{2} \right)^2 - 2\pi e^2 d^2 (2 + e^2) \partial d}{\pi \left(\frac{1}{2} \right)^2} \\ &= \frac{8}{\pi} \left(\frac{\pi}{4} d \Big|_0^{\frac{1-e}{2(1+e)}} - \frac{2\pi e^2 d^3 (2 + e^2)}{3(1 - e^2)^2} \Big|_0^{\frac{1-e}{2(1+e)}} \right) \\ &= \frac{1-e}{1+e} - \frac{2e^2(2+e^2)(1-e)}{3(1+e)^5} = \frac{3+9e+2e^2-2e^3-11e^4-e^5}{3(1+e)^5} \end{aligned}$$

B.2.2 Case 2: Direct-shipping region is the interaction between limaçon and the unit disk space $\frac{1-e}{2(1+e)} \leq r \leq \frac{1}{2}$



B.2.2.1 Notation

d = distance between crossdock and a vendor

r = polar equation of limaçon

$$\frac{2ed(1 + e \cos \theta)}{1 - e^2}$$

$\tilde{\theta}$ = critical angle that limaçon touches the unit disk

$$\left(\frac{1}{2}\right)^2 = d^2 + r^2 + 2dr \cos \tilde{\theta}$$

$$\frac{1}{4} = d^2 + \left(\frac{2ed(1 + e \cos \tilde{\theta})}{1 - e^2}\right)^2 + 2d \left(\frac{2ed(1 + e \cos \tilde{\theta})}{1 - e^2}\right) \cos \tilde{\theta}$$

$$\cos \tilde{\theta} = \frac{1 - 2d - 2de^2 - e^2}{4ed} ; \sin \tilde{\theta} = \frac{\sqrt{(1 - e^2)(2ed + 2d + e - 1)(2ed - 2d + e11)}}{4ed}$$

note

$$\arccos x \approx \frac{\pi}{2} - x - \frac{x^3}{6} - \frac{3x^5}{40} - \frac{5x^7}{112} - \frac{35x^9}{1152}$$

A = Interaction Area between limacqn and unit disk

$$A_{\text{partial limacqn}} + A_{\text{arc}}$$

$A_{\text{partial limacqn}}$ = Area of partial limacqn from $\tilde{\theta} \leq \theta \leq 2\pi - \tilde{\theta}$

$$\begin{aligned} A_{\text{partial limacqn}} &= \int_{\tilde{\theta}}^{2\pi-\tilde{\theta}} \frac{1}{2} \left(\frac{2ed(1+e\cos\theta)}{1-e^2} \right)^2 \partial\theta \\ &= 2 \int_{\tilde{\theta}}^{\pi} \frac{1}{2} \left(\frac{2ed(1+e\cos\theta)}{1-e^2} \right)^2 \partial\theta \\ &= \frac{4e^2d^2}{(1-e^2)^2} \left(\theta + 2\sin\theta + \frac{e^2}{2} \sin\theta \cos\theta + \frac{e^2}{2} \theta \right) \Big|_{\tilde{\theta}}^{\pi} \\ &= \frac{2e^2d^2(2\pi + \pi e^2 - 2\tilde{\theta} - 4\sin\tilde{\theta} - e^2\sin\tilde{\theta}\cos\tilde{\theta} - e^2\tilde{\theta})}{(1-e^2)^2} \end{aligned}$$

A_{arc} = Area of arc $-\tilde{\theta} \leq \theta \leq \tilde{\theta}$

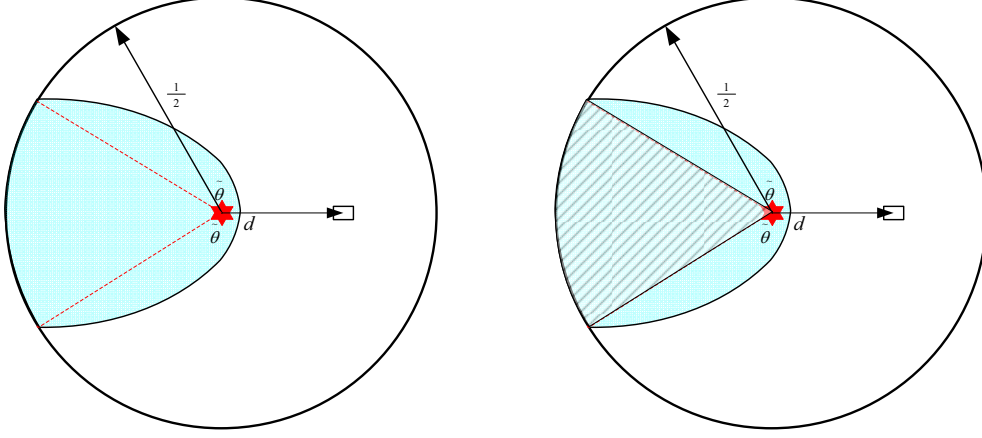
$$\begin{aligned} A_{\text{arc}} &\approx \frac{2\tilde{\theta}}{2} \left(\frac{1}{2} - d \right) r = \tilde{\theta} \left(\frac{1}{2} - d \right) \frac{2ed(1+e\cos\tilde{\theta})}{1-e^2} \\ &= \frac{1}{4}e(4d^2 - 1) \left[\arccos \left(\frac{-1 + 2d + 2e^2d + e^2}{4ed} \right) - \pi \right] \end{aligned}$$

B.2.2.2 Algebra

$$\begin{aligned} Pr. &\left(\text{freight is shipped through crossdock in the unit disk space by (TL,TL) when } \frac{1-e}{2(1+e)} \leq d < \frac{1}{2} \right) \\ &= \frac{\text{Consolidated-shipping region of of the unit disk space by (TL,TL) in } \frac{1-e}{2(1+e)} \leq d < \frac{1}{2}}{\text{Total area of the unit disk space}} \\ &= \frac{2 \int_{\frac{1-e}{2(1+e)}}^{\frac{1}{2}} \left(\pi \left(\frac{1}{2} \right)^2 - A_{\text{partial limacqn}} - A_{\text{arc}} \right) \partial d}{\pi \left(\frac{1}{2} \right)^2} \\ &= \frac{8}{\pi} \left(\left. \frac{\pi}{4} d \right|_0^{\frac{1-e}{2(1+e)}} - \left. \frac{2\pi e^2 d^3 (2+e^2)}{3(1-e^2)^2} \right|_0^{\frac{1-e}{2(1+e)}} \right) \\ &\approx \frac{1}{576e(1-e)^4(1+e)^5} \left[48e^2(1+e)(e^8 - 4e^6 + 6e^5 + 6e^4 + 12e^3 - 4e^2 + 1) \arcsin(e) \right. \\ &\quad + 48e^2(1-e)^2(1+e)^3(e^4 - 5e^2 + 1) \arctan \left(\frac{e}{\sqrt{1-e^2}} \right) + 3(1-e)^5(1+e)^6(e^2 + 2) \ln \left(\frac{1+e}{1-e} \right) \\ &\quad + 48e^3(1+e)(2e^7 - e^6 - 10e^5 + 26e^3 + 3e^2 - 2) \sqrt{1-e^2} - 20e^{13} - 14e^{12} - 30e^{11} + (48\pi - 52)e^{10} \\ &\quad - (24\pi + 148)e^9 + (216\pi + 708)e^8 - (768\pi + 1076)e^7 + (96\pi + 1696)e^6 - (792\pi + 1452)e^5 \\ &\quad \left. + (360\pi + 362)e^4 + 50e^3 - 12e^2 - 12e \right] \end{aligned}$$

B.3 *The asymptotic-probability function in the unit disk in modes of shipments (LTL, TL) and (TL, LTL)*

B.3.1 Case 1: Direct-shipping region is half-hyperbola



B.3.1.1 Notation

d = distance between crossdock and a vendor

r = polar equation of half hyperbola

$$\frac{d(1-e^2)}{2(e+\cos\theta)}; -\arccos e\theta \leq \arccos e$$

$\tilde{\theta}$ = critical angle that half hyperbola touches the unit disk

$$\frac{1}{2} = \frac{d(1-e^2)}{2(e+\cos\theta)}$$

$$\cos \tilde{\theta} = d(1-e^2) - e$$

A = Interaction Area between half hyperbola and unit disk

$$A = A_{\text{hyper}} + A_{\text{arc}}$$

A_{hyper} = Area of hyperbola from $-\tilde{\theta} \leq \theta \leq \tilde{\theta}$

$$\begin{aligned} A_{\text{hyper}} &= \int_{-\tilde{\theta}}^{\tilde{\theta}} \frac{1}{2} \left(\frac{d(1-e^2)}{2(e+\cos\theta)} \right)^2 \partial\theta = 2 \int_0^{\tilde{\theta}} \frac{1}{2} \frac{d^2(1-e^2)^2}{4} \left(\frac{1}{e+\cos\theta} \right)^2 \partial\theta \\ &= d^2(1-e^2) \left[\frac{\tan \frac{\theta}{2}}{2(1-e^2) \left(e + e \tan^2 \frac{\theta}{2} - \tan^2 \frac{\theta}{2} + 1 \right)} - \frac{(e+2) \operatorname{arctanh} \left(\frac{(e-1) \tan \frac{\theta}{2}}{\sqrt{1-e^2}} \right)}{2(1-e^2)^{3/2}} \right] \\ &= \frac{d^2}{2} \left[\frac{\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}}{\left(e + e \left(\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \right)^2 - \left(\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \right)^2 + 1 \right)} - (e+2) \sqrt{1-e^2} \operatorname{arctanh} \left(\frac{(e-1) \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}}{\sqrt{1-e^2}} \right) \right] \end{aligned}$$

A_{arc} = Area of arc $\pi - \tilde{\theta} \leq \theta \leq \pi + \tilde{\theta}$

$$A_{\text{arc}} = \frac{2(\pi - \tilde{\theta})}{2} \left(\frac{1}{2} \right)^2 = \frac{\pi}{4} - \frac{\arccos(d(1-e^2) - e)}{4}$$

$$\begin{aligned}
& Pr. \left(\text{freight is shipped through crossdock in the unit disk space by (LTL, TL) when } 0 \leq d < \frac{1}{2} \right) \\
&= \frac{\text{Consolidated-shipping region of of the unit disk space by (LTL, TL) when } 0 \leq d < \frac{1}{2}}{\text{Total area of the unit disk space}} \\
&= \frac{2 \int_0^{\frac{1}{2}} (A_{\text{hyper}} + A_{\text{arc}}) \partial d}{\pi(\frac{1}{2})^2} \\
&= \frac{8}{\pi} \left(\int_0^{\frac{1}{2}} A_{\text{hyper}} \partial d + \int_0^{\frac{1}{2}} A_{\text{arc}} \partial d \right) \\
&= \frac{8}{\pi} \left(\int_0^{\frac{1}{2}} \frac{d^2}{2} \left[\frac{\sqrt{1 - \cos^2 \theta}}{2e + \cos \theta} - (e + 2)\sqrt{1 - e^2} \operatorname{arctanh} \left(\frac{(e - 1)\sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}}{\sqrt{1 - e^2}} \right) \right] \partial d \right. \\
&\quad \left. + \int_0^{\frac{1}{2}} \frac{1}{4} \arccos(d(1 - e^2) - e) \partial d \right) \\
&= -\frac{1}{24\pi(1 - e^2)^{\frac{3}{2}}} \left[4e^5 \operatorname{arctanh} \left(\frac{\sqrt{2 + 2e}}{\sqrt{2e + 6}} \right) - e^4 \sqrt{2 + 2e} \sqrt{2e + 6} - 8e^3 \operatorname{arctanh} \left(\frac{\sqrt{2 + 2e}}{\sqrt{2e + 6}} \right) \right. \\
&\quad - 32e^2 - 2e^2 \sqrt{2 + 2e} \sqrt{2e + 6} + 12\pi e^2 \sqrt{1 - e^2} - 24e^2 \sqrt{1 - e^2} \arcsin \left(-\frac{1}{2} + \frac{1}{2}e^2 + e \right) \\
&\quad - 32e \sqrt{1 - e^2} \arcsin \left(-\frac{1}{2} + \frac{1}{2}e^2 + e \right) + 32e \sqrt{1 - e^2} \arcsin(e) + 4e \operatorname{arctanh} \left(\frac{\sqrt{2 + 2e}}{\sqrt{2e + 6}} \right) \\
&\quad - 24e \sqrt{1 - e^2} \sqrt{3 - e^2 - 2e} - 24\sqrt{1 - e^2} \sqrt{3 - e^2 - 2e} - 12\pi \sqrt{1 - e^2} + 3\sqrt{2 + 2e} \sqrt{2e + 6} \\
&\quad \left. + 24\sqrt{1 - e^2} \arcsin(-\frac{1}{2} + \frac{1}{2}e^2 + e) + 32 \right]
\end{aligned}$$

APPENDIX C

APPENDIX FOR CHAPTER 5

C.1 Mixed-Integer Programming Model

$$\begin{aligned}
 \text{Min} \quad & \sum_i \sum_p \sum_j (e d_{ip} + e d_{ip} + h_p) x_{ipj}^{TL,TL} + (d_{ip} + e d_{ip} + h_p) x_{ipj}^{LTL,TL} + (e d_{ip} + d_{ip} + h_p) x_{ipj}^{TL,LTL} \\
 & + \sum_i \sum_j d_{ij} e x_{ij}^{TL} + x_{ij}^{LTL} \quad (C.1)
 \end{aligned}$$

subject to

$$\sum_p \left(x_{ipj}^{TL,TL} + x_{ipj}^{LTL,TL} + x_{ipj}^{TL,LTL} \right) + x_{ij}^{TL} + x_{ij}^{LTL} = f_{ij} \quad \forall i, \forall j \quad (C.2)$$

$$\sum_j \left(x_{ipj}^{TL,TL} + x_{ipj}^{TL,LTL} \right) \geq \tau z_{ip} \quad \forall i, \forall p \quad (C.3)$$

$$x_{ipj}^{TL,TL} + x_{ipj}^{TL,LTL} \leq f_{ij} z_{ip} \quad \forall i, \forall p, \forall j \quad (C.4)$$

$$\sum_i \left(x_{ipj}^{TL,TL} + x_{ipj}^{TL,LTL} \right) \geq \tau z_{pj} \quad \forall p, \forall j \quad (C.5)$$

$$x_{ipj}^{TL,TL} + x_{ipj}^{TL,LTL} \leq f_{ij} z_{pj} \quad \forall i, \forall p, \forall j \quad (C.6)$$

$$x_{ij}^{TL} \geq \tau z_{ij} \quad \forall i, \forall j \quad (C.7)$$

$$x_{ij}^{TL} \leq f_{ij} z_{ij} \quad \forall i, \forall j \quad (C.8)$$

$$x_{ipj}^{TL,TL}, x_{ipj}^{LTL,TL}, x_{ipj}^{TL,LTL}, x_{ij}^{TL}, x_{ij}^{LTL} \geq 0 \quad \forall i, \forall j, \forall p \quad (C.9)$$

$$z_{ip}, z_{pj} \in \{0, 1\} \quad \forall i, \forall j, \forall p \quad (C.10)$$

where,

- $x_{ipj}^{\zeta_1, \zeta_2}$ = variable represents amounts of consolidated shipment originating at vendor i
transiting at crossdock p and ending at store j with mode of shipments (ζ_1, ζ_2)
- x_{ij}^{ζ} = variable represents amounts of outsourced shipment originating at vendor i
and ending at store j with ζ
- z_{lk} = binary variable represents mode of shipments

$$z_{lk} = \begin{cases} 1 & \text{if shipment originating at facility } l \text{ and ending at facility } k \text{ is TL} \\ 0 & \text{otherwise} \end{cases}$$
- d_{lk} = distance between facility l and facility k
- e = eccentricity or shipping cost ratio between TL and LTL
- h_p = handling costs of crossdock p
- f_{ij} = amounts of freight originating at vendor i and ending at store j
- τ = TL threshold

The mixed-integer programming model consists of Expression C.1 as the objective function and Expression C.2-C.10 as the constraints. The objective function has of two transportation-cost terms: consolidated shipping and outsourced shipping. Expression C.2 ensures that freight shipped by different modes of shipments meets a specification a vendor-store pair. Expressions C.3 and C.4 govern TL shipments of inbound consolidated shipping. Expressions C.3 asserts that binary variable z_{ip} becomes one when the shipment originating at store i and ending at crossdock p exceeds threshold τ . Expressions C.4 ensures that no modes of shipments (TL, TL) and (TL, LTL) are allowed when binary variable z_{ip} is zero. Expressions C.5-C.6 and C.7 and C.8 serve the similar purpose for outbound consolidated shipping and outsourced shipping. That is, a shipment must exceed TL requirement to become TL. Expressions C.9 and C.10 assure non-negativity of flow variables and integrity of binary variables, respectively

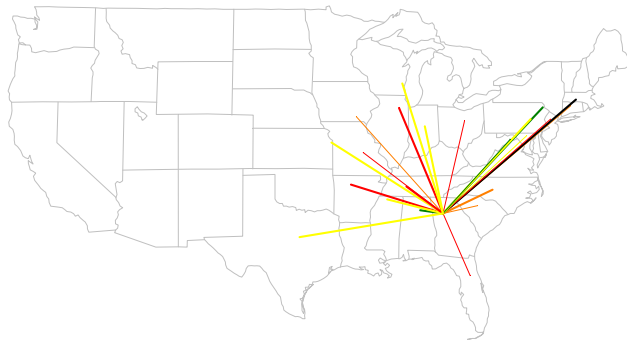
To limit number of crossdocks received by a store, we introduce Expression C.11 and

C.12.

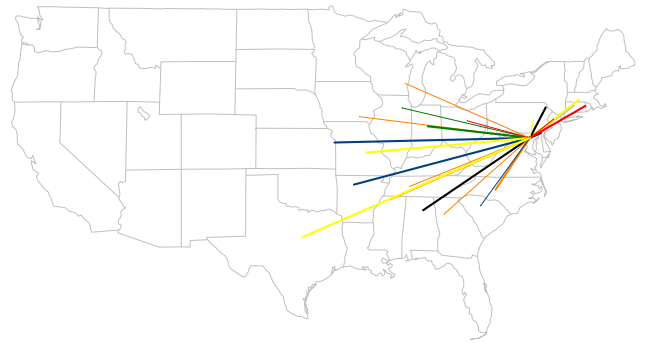
$$\sum_i \sum_p \left(x_{ipj}^{TL,TL} + x_{ipj}^{LTL,TL} + x_{ipj}^{TL,LTL} \right) \leq \sum_i f_{ij} y_{pj} \quad \forall p, \forall j \quad (\text{C.11})$$

$$\sum_p y_{pj} \leq n \quad \forall j \quad (\text{C.12})$$

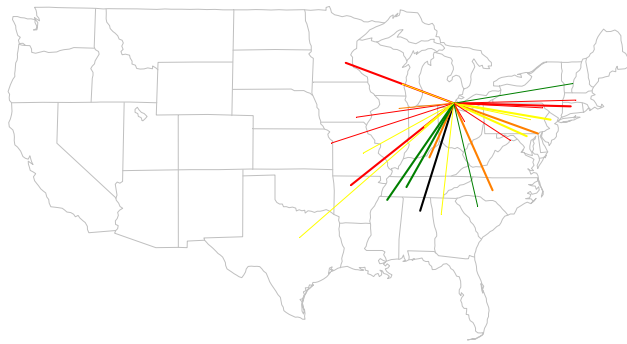
The first expression tracks number of positive flow to a crossdock. Expression C.12 is simply count number of crossdocks received by a store and assures that it is less than limit n , where y_{pj} is binary variable and n is an integer.



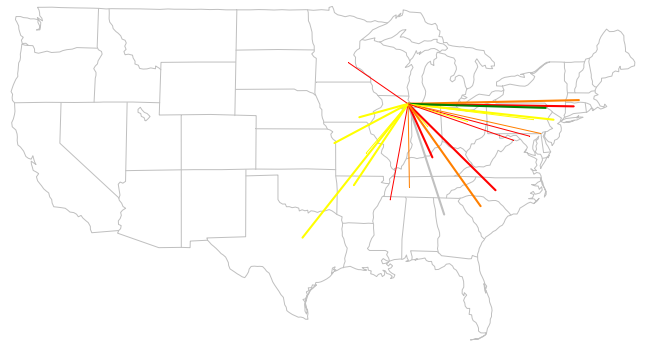
(a) Atlanta Crossdock



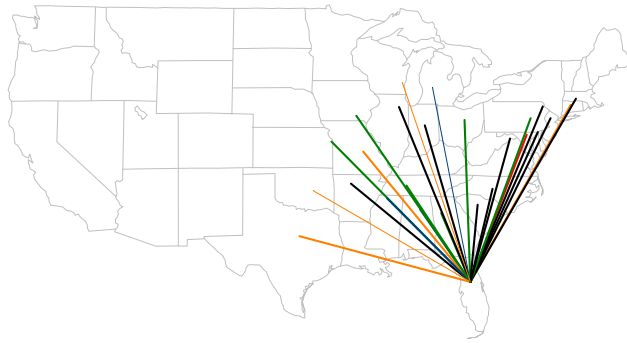
(b) Baltimore Crossdock



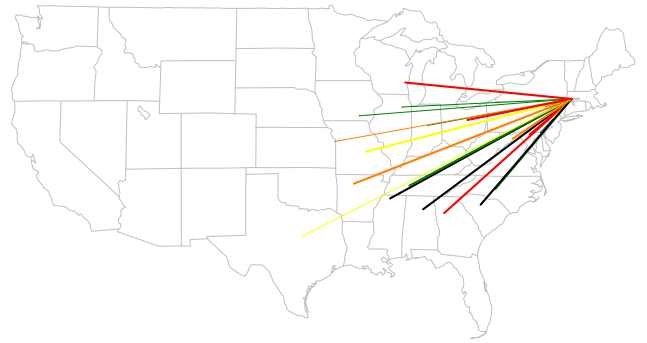
(c) Detroit Crossdock



(d) Chicago Crossdock



(e) Orlando Crossdock



(f) Westfield Crossdock



Figure D.2: Average regional eccentricity values of freight shipped to Atlanta, Baltimore, Detroit, Chicago, Orlando and Westfield Crossdocks

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VITA

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