Ranking and Selection Procedures for Bernoulli and Multinomial Data

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Ranking and Selection Procedures for Bernoulli and Multinomial Data

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For my mother,

my source of strength and inspiration,

my mentor, my role model,

my friend.

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SUMMARY

Ranking and Selection procedures have been designed to select the best system from a number of alternatives, where the best system is defined by the given problem. The primary focus of this thesis is on experiments where the data are from simulated systems. In simulation ranking and selection procedures, we focus on two classes of problems; Bernoulli and multinomial selection. In these procedures, wish to select the best system from a number of simulated alternatives where the best system is defined as either the one with the largest probability of success (Bernoulli selection) or the one with the greatest probability of being the best performer (multinomial selection). We focus on procedures that are sequential and use an indifference-zone formulation wherein the user specifies the smallest practical difference he wishes to detect between the best system and other contenders.

We apply fully sequential procedures due to Kim and Nelson (2004) to Bernoulli data for terminating simulations, employing common random numbers. We find that significant savings in total observations can be realized for two to five systems when we wish to detect small differences between competing systems. We also study the multinomial selection problem. We offer a Monte Carlo simulation of the Bechhofer and Kulkarni (1984) $\mathcal{M}_{\mathrm{BK}}$ multinomial procedure and provide extended tables of results. In addition, we introduce a multi-factor extension of the $\mathcal{M}_{\mathrm{BK}}$ procedure. This procedure allows for multiple independent factors of interest to be tested simultaneously from one data source (e.g., one person will answer multiple independent surveys) with significant savings in total observations compared to the factors being tested in independent experiments (each survey is run with separate focus groups and results are combined after the experiment). Another multi-factor multinomial procedure is also introduced, which is an extension to the $\mathcal{M}_{\mathrm{BG}}$ procedure due to Bechhofer and Goldsman (1985, 1986). This procedure performs better than any other procedure to date for the multi-factor multinomial selection problem and should always be used whenever table values for the truncation point are available.

CHAPTER I

INTRODUCTION

Ranking and Selection procedures have been designed to select the best system from a number of alternatives, where the best system is defined by the given problem. The primary focus of this thesis is on experiments where the data are from simulated systems. Stochastic simulation is often used in situations where physical experiments are not feasible and where the systems are too complex to fit mathematical models (such as many service or industrial systems).

There are two types of simulations — terminating and steady-state. In terminating simulation, due to the nature of the system, we have well-defined initial conditions and stopping times for each replication. A typical example of such a system is a branch office of a bank. The bank opens every morning at 9 am completely empty and at the end of the day it empties out once again as soon as the last customer who entered before the closing time of 4 pm has finished being served. Due to the central limit theorem, the average of results taken within a single replication are approximately normal. If an estimate of the average time a customer waits to see a teller is desired, we compute the average of the wait times from each replication, and those averages are approximately normally distributed.

For steady-state simulations, there are no clear start-up or terminating conditions. An example of such a situation would be a model of a manufacturing facility which operates 24 hours a day and we are interested in the long-run performance of a given statistic. Such simulations are typically run for very long periods (usually just one or a few very long replications) in order to remove the bias of the start-up period and get an accurate estimate of long-run performance. The raw data are typically correlated serially (e.g., the wait time of one individual or part depends on the wait times of those that came before). A typical method used in steady-state simulations is the method of batch means. Using this method, the data is grouped in batches and the average of a large number of outputs in each batch is calculated. These batch means are much less dependent and more normally distributed than the raw observations.

Simulation often involves comparison. Even when we simulate a single system, we often compare the performance of it with a standard. Situations encountered in the simulation environment often make the selection problem much more difficult when the observations are simulated data instead of data obtained from physical experiments. Some of the difficulties encountered in simulation experiments are non-normality of data, unknown and unequal variances, serial dependence of data, and dependence due to the use of Common Random Numbers (CRN).

- Raw simulation output data such as individual wait times are rarely normal. A typical technique for approximating normal data is using the batch means method for steady-state simulations. For terminating simulations we can approximate normal data by looking at the averages of within-replication values, collected over multiple independent replications of the experiment.

- The variances of simulated systems are almost always unknown and typically unequal. While there are procedures that can deal with unknown variances, many of them assume equal variances and thus are not usually applicable to the simulation environment. In order to account for unknown variances, a typical method used in simulation selection procedures is to look at standardized differences between systems.

- Dependence between responses is quite often encountered in simulation experiments. One type of dependence is serial dependence; where one data point (e.g., a person's wait time) is correlated with the points before and after it. This problem is often dealt with by looking at averages of the data over multiple replications (terminating) or using a method such as batch means (steady-state) as previously mentioned. When comparing multiple simulated systems, it is reasonable to assume that the data across the systems are independent of each other. As long as completely independent random data streams are used for each model's input data, the assumption of independence between systems is valid. However, it is potentially beneficial to intentionally correlate the between-system data using CRN. CRN is employed by using the same input data streams for all systems (to as much extent as possible given the different configurations of the systems). Procedures that employ the use of CRN often look at the differences between systems so that the variance estimate of the differences takes the induced correlation into account. If CRN is used correctly and it is possible to induce some amount of positive correlation, the result is the reduction of the variance of the differences since

$$\mathrm{Var}(X_{ij} - X_{\ell j}) = \mathrm{Var}(X_{ij}) + \mathrm{Var}(X_{\ell j}) - 2\mathrm{Cov}(X_{ij}, X_{\ell j}).$$

By reducing the variance of the difference due to removing some of the randomness of the input data, true differences between the systems are more easily recognized and thus the selection of the best system can often be made sooner.

Comparison procedures should be able to account for the above mentioned difficulties in order to be useful in a simulation environment. In simulation ranking and selection procedures, four classes of comparison problems are typically encountered. The first class of problems is concerned with selecting the system with the largest (or smallest) expected value of the performance measure of interest. This class of problems is typically known as the "selection of the best." Another class compares all systems against a standard, referred to as "comparison with a standard." The other two classes of problems are the ones we focus on in this thesis. In the class of problems known as multinomial selection, we are concerned with selecting the system with the greatest probability of being the best on any one trial. In other words, we wish to select the system i with the largest value p_i where $p_i = \Pr\{X_{im} > X_{jm}, \forall i \neq j\}$ and X_{km} is the mth observation from population k. The previous probability statement assumes that we want to select the system that will give us the largest response but it is possible to look at the smallest or more general responses as well. For this class of problems, each trial has one winner, which is given a value of 1, and the values for the other systems are zero. The last class of problems is known as Bernoulli selection. In Bernoulli selection problems, the result of each trial is either 0 or 1. For this problem we wish to select the system with the largest (or smallest) probability of success where the probability of success is defined as $p_i = \Pr(X_{ij} = 1)$, where X_{ij} is the jth observation from population i.

As mentioned, ranking and selection procedures select the best system from a number of simulated alternatives. In this thesis, the best system is defined as either the one with the largest probability of success (Bernoulli selection) or the one with the greatest probability of being the best performer (multinomial selection). We focus on procedures that are sequential and use an indifference-zone (IZ) formulation wherein the user specifies the smallest practical difference he wishes to detect between the best system and other contenders.

The first part of the thesis deals with the Bernoulli selection problem. Kim and Nelson (2001, 2004) proposed procedures $\mathcal{KN}$, $\mathcal{KN}+$, and $\mathcal{KN}{++}$ which address issues typically faced in simulation experiments such as non-normality, unknown and unequal variances, and the use of Common Random Numbers (CRN). In this thesis, we apply Kim and Nelson's procedures to problems where the responses are Bernoulli. We do not apply the $\mathcal{KN}$ procedure, which can deal with unknown and unequal variances and CRN, since it requires a normality assumption which is not valid for our problem. Instead, we first apply the $\mathcal{KN}+$ and $\mathcal{KN}{++}$ procedures with independence, which can handle non-normality and unknown and unequal variances. These procedures were designed for use with steady-state simulations where the goal is to estimate long-run performance measures and output data are stationary and dependent. However, we apply the $\mathcal{KN}+$ and $\mathcal{KN}{++}$ procedures to terminating simulations where each data point within a system is independent of the next. We find that the resulting performance is only slightly better than current procedures and only in very specific instances. We further extend the $\mathcal{KN}{++}$ variance-updating procedure to include the use of CRN and provide asymptotic validity of their use. We find that, compared to current procedures, significant savings in total observations can be realized when applying CRN to the $\mathcal{KN}{++}$ procedure for Bernoulli data in certain circumstances. We provide results and a comparison with current procedures under various scenarios as well as guidelines for implementation.

The second part of the thesis deals with the multinomial selection problem. We provide a Monte Carlo simulation of the $\mathcal{M}_{\text{BK}}$ procedure due to Bechhofer and Kulkarni (1984) as well as exact results for two systems. This simulation is used to provide extended table results for use in performing the $\mathcal{M}_{\mathrm{BK}}$ procedure as well as a multi-factor procedure to be introduced below. Bechhofer, Goldsman, and Jennison (1989) introduced a multi-factor extension to the single-stage multinomial procedure $\mathcal{M}_{\mathrm{BEM}}$ introduced by Bechhofer, Elmaghraby, and Morse (1959). $\mathcal{M}_{\mathrm{BEM}}$ is designed for use with single-factor experiments, in other words, an experiment where we wish to determine which level of a single factor is the best. For example, suppose we wish to determine which television station (factor), TBS, FOX, or the WB (levels) is the favorite among teenagers. This is an example of a single-factor multinomial experiment. A multi-factor experiment can test multiple factors of interest simultaneously. Suppose in the previous example that we plan to pass out a survey at a local high school in order to collect the desired data. Also suppose that we have another client who is interested in determining which brand of jeans, Gap, Levi's, or Old Navy, teenagers prefer. Rather than performing two separate studies, the multi-factor procedure allows both factors (TV station and jeans brand) to be studied simultaneously (asking both questions in one survey), assuming that the responses to one factor-level are completely independent of those for the other factor. This type of procedure is most easily applied to non-simulation experiments such as surveys or laboratory-type experiments, though it can also be applied to simulated data as long as we are certain that the factors of interest are indeed independent of each other.

The multi-factor procedure introduced by Bechhofer, Goldsman, and Jennison (1989) must take a specified number of observations from each factor-level and cannot stop early even if the decision is obvious. We introduce a multi-factor extension to the Bechhofer and Kulkarni (1984) multinomial procedure that improves on $\mathcal{M}_{\mathrm{BEM}}$ by allowing for early termination once an obvious decision can be reached. This procedure allows for multiple factors of interest to be tested simultaneously with significant savings in total observations compared to the factors being tested in independent experiments. It also provides the same probability of correct selection as the one introduced by Bechhofer, Goldsman, and Jennison (1989), but with fewer total observations. We provide results for this procedure under various circumstances and guidelines for use. We introduce another multi-factor multinomial procedure that is an extension of the procedure by Bechhofer and Goldsman (1985, 1986). This procedure performs better than any other procedure to date for the multi-factor multinomial selection problem and should always be used whenever table values for the truncation point are available.

The thesis is organized as follows: In Chapter 2 we introduce the Bernoulli selection problem, along with the $\mathcal{KN}$ procedures, and we discuss our initial application of the $\mathcal{KN}$ procedures to Bernoulli data. Section 2.1 introduces the Bernoulli selection problem along with relevant notation and Section 2.2 gives a review of current procedures and examples. In Section 2.3 we review the $\mathcal{KN}$ procedures designed by Kim and Nelson (2001, 2004). In Section 2.4 we discuss our initial application of the $\mathcal{KN}+$ and $\mathcal{KN}++$ procedures to independent Bernoulli responses and discuss the efficiency of these procedures relative to current procedures. In Chapter 3 we discuss the use of CRN, present a procedure for Bernoulli selection with the use of CRN, and give results for that procedure. In Section 3.1 we discuss the potential benefits of using CRN and give examples of their use. Section 3.2 presents our procedure for Bernoulli data with the use of CRN and a proof supporting its validity, and Section 3.3 gives results for the procedure and guidelines for its use. Section 3.4 presents the conclusion of our work in Bernoulli selection.

We then introduce the multinomial selection problem with examples and current procedures in Chapter 4. Section 4.1 gives notation and examples of the multinomial selection problem. In Section 4.2 we discuss the procedure introduced by Bechhofer and Kulkarni (1984) and our use of a Monte-Carlo simulation to expand their tables. Chapter 5 introduces a multi-factor multinomial selection procedure, which is a generalization of the Bechhofer and Kulkarni (1984) multinomial procedure. Section 5.1 looks at the original multi-factor multinomial procedure introduced by Bechhofer, Goldsman, and Jennison (1989). In Section 5.2 we propose a multi-factor generalization of the Bechhofer and Kulkarni (1984) multinomial procedure and present results of its application. Section 5.3 proposes a multi-factor generalization of the Bechhofer and Goldsman (1985, 1986) multinomial procedure and presents results of its use. Section 5.4 presents the conclusion of our work in multinomial selection. Chapter 6 summarizes the contributions of our research.

CHAPTER II

BERNOULLI RANKING AND SELECTION PROCEDURES

Bernoulli ranking and selection procedures are characterized by output data that are either a zero (failure) or a one (success). The binary nature of the responses make this selection problem difficult, because it is especially hard to recognize differences between systems when they all have very large or small probabilities of success. In addition, the data clearly violate the standard normality assumption for many ranking and selection procedures. Our goal is to compare a finite number of alternatives and select the alternative with the largest (or smallest) probability of success, where the probability of success is defined as $p_i = \Pr(X_{ij} = 1)$, where X_{ij} is the jth observation from population i. For the Bernoulli distribution, this corresponds to selecting the system with the largest (or smallest) expected value. It should be noted that since the sum of Bernoulli random variables has a binomial distribution, this selection problem is also known as binomial selection.

The ideal goal of any selection procedure is to maximize the probability of correctly selecting the best system (PCS) in the fewest number of observations. However, as these goals are counter to each other it is necessary to constrain each procedure by either the maximum allowable number of observations, which relates to budget constraints, or by the desired PCS, which relates to the desired accuracy. To meet the first constraint (budget), procedures will try to most efficiently allocate the specified budget of observations in order to maximize the PCS. With regard to the second constraint (accuracy), procedures attempt to minimize the number of observations such that they satisfy the PCS constraint. Although, for background purposes, we will briefly introduce procedures that meet the first constraint, our focus will be on procedures that meet a specified PCS while trying to minimize the number of observations.

We are interested in selection procedures that use the IZ formulation. An IZ is used so that experimenters can specify a "practical" difference between systems that they wish to detect. One type of IZ is simply the difference, defined as $p_k - p_{k-1} \geq \delta$, where p_k is the success probability of the best system, p_{k-1} is the success probability of the second best system, and δ is the smallest significant difference worth detecting. Thus, in a procedure that meets a PCS constraint, we want to provide the guaranteed PCS whenever the best system's probability is at least δ better than the second best. If there are two probabilities that are within δ of each other, then we are indifferent as to which system we choose.

Another type of IZ uses the odds ratio. This IZ looks at the ratio of the odds of success for the best system to the odds of success for the second best system. Thus, we wish to select the best system whenever $\frac{p_k/(1-p_k)}{p_{k-1}/(1-p_{k-1})} \geq \theta$, where p_k and p_{k-1} are the probabilities of success for the best and second best systems respectively, and θ is the user-specified odds ratio. The odds ratio IZ is sometimes preferred over the difference-based IZ, because with the difference-based IZ the significance of the difference we want to detect is not tied directly to the individual magnitudes of the success probabilities as it is with the odds ratio formulation. However, a potential drawback of using the odds ratio is that points near $(p_k,\, p_{k-1})$ = (0,0) or (1,1) (i.e., sets of probabilities which are all very high or low) may require significant sampling, making procedures that employ the odds ratio IZ inefficient in such instances. The following figures from Bechhofer, Santner, and Goldsman (1995) illustrate the differences between the two types of indifference zones. Figure 1 shows us a difference-based IZ while Figure 2 shows an odds ratio IZ. The shaded region represents the portion of the parameter space that is in the preference zone, i.e., the complement of the indifference zone.

2.1 Examples and Notation

The typical example of a Bernoulli selection problem is a drug study. Suppose we wish to test a selection of drugs to see which one performs best for treating a given ailment. For each trial, a drug either succeeds or fails to treat the ailment and the experimenter wants to select the drug that has the largest probability of success.

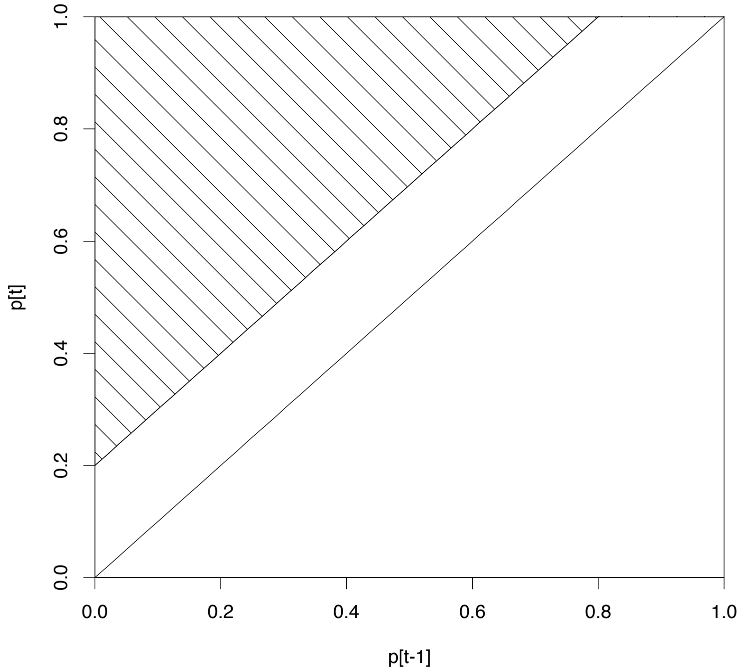

Figure 1: An Example of a Difference-Based IZ with $\delta = 0.2$

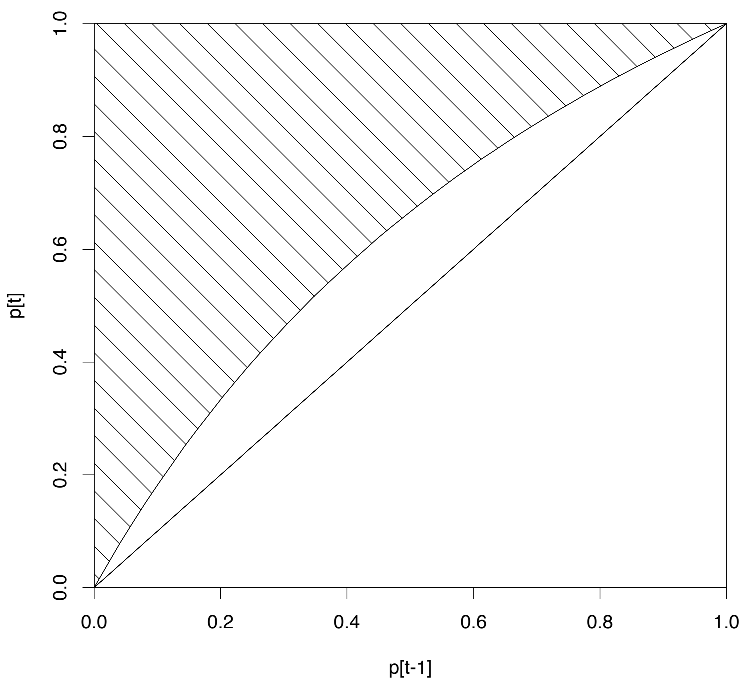

Figure 2: An Example of an Odds Ratio IZ with $\theta = 2.0$

These procedures are also applicable in simulation experiments where each observation is a run of a simulated system. For instance, looking at simulated models of multiple production facilities, we may wish to determine which facility is most likely to meet a specified goal, such as a budget constraint or a production timeline. Alternatively, looking at multiple simulated manufacturing processes, we may wish to determine which one has the highest probability of producing a non-defective item. Such simulated experiments will be the primary focus of our research.

The following notation will be used throughout the Bernoulli section of this thesis to define the procedures described below. Let k denote the number of systems or populations we wish to compare. Denote p_i as the probability of success for system i, $i = 1, \ldots, k$. We denote the ordered probabilities by $p_{[k]} \geq p_{[k-1]} \geq \cdots \geq p_{[1]}$, and thus we wish to select the system associated with $p_{[k]}$. The probabilities, their order, and their pairings with the populations are unknown. Let X_{ij} be the jth observation from system i. Let an IZ be represented by δ or θ depending on the type of IZ used, and let P^* denote the desired confidence level (PCS). The user specifies the indifference zone value (δ or θ) and P^*.

2.2 Current Procedures

The first Bernoulli selection procedure, $\mathcal{B}_{\mathrm{SH}}$, was proposed by Sobel and Huyett (1957). Their procedure is a single-stage selection procedure, meaning that all observations are taken in one stage, or setup, of experimentation. This procedure takes exactly n observations from each population and uses the difference-based indifference zone.

$\mathcal{B}_{\mathrm{SH}}$ Procedure

Setup: For the given k, select δ and P^*. Select n using the appropriate table. (A small portion of the table is shown in Table 1 as an example, and complete results can be found in Gibbons, Olkin and Sobel 1977 or Bechhofer, Santner, and Goldsman 1995.)

Sampling: Take n observations, X_{ij}, $j = 1, \ldots, n$, from each population, $i = 1, \ldots, k$, in one stage.

Table 1: Smallest sample size needed for each system when $k = 3$ for given δ and P^* (from Sobel and Huyett 1957).

	P^*						
δ	0.60	0.75	0.80	0.85	0.90	0.95	0.99
0.05	79	206	273	364	498	735	1308
0.10	20	52	69	91	125	184	327
0.15	9	23	31	41	55	82	145
0.20	5	13	17	23	31	46	81
0.25	4	9	11	15	20	29	52
0.30	3	6	8	10	14	20	35
0.35	2	5	6	8	10	15	26
0.40	2	4	5	6	8	11	20
0.45	2	3	4	5	6	9	15
0.50	2	3	3	4	5	7	12

Terminal decision rule: For each system, $i = 1, \ldots, k$, calculate $Y_{in} = \sum_{j=1}^{n} X_{ij}$. Select the system with the largest sample sum as the winner. If there is a tie for the largest sample sum, randomly select one of the tied systems as the winner.

Example: Suppose we want to determine which of three dandruff shampoos is most likely to eliminate dandruff. Each observation is a 1 (0) if the shampoo succeeds (fails) to eliminate the dandruff. We select $\delta = 0.1$ and $\mathrm{P}^* = 0.95$ because we wish to detect the best system with a probability of correct selection of at least 0.95 whenever $p_k - p_{k-1} \geq 0.1$. From Table 1, we see that 125 observations are required, so we take 125 independent observations from each shampoo. Suppose at the end of sampling we have $Y_{1,125} = 75$, $Y_{2,125} = 92$, and $Y_{3,125} = 67$. We would then select system 2 as the best. $\square$

This procedure is useful if the nature of the experiment prohibits the use of sequential sampling. For instance, if the time it takes to collect one observation is very long, it may be infeasible to wait until an observation has concluded to begin the next observation as we would in sequential sampling. In this instance, the $\mathcal{B}_{\mathrm{SH}}$ procedure could be employed. However, this procedure cannot eliminate systems that are clearly inferior nor can it stop sampling early if we can determine the obvious winner before taking all the observations.

Bechhofer and Kulkarni (1982) improved on the $\mathcal{B}_{\text{SH}}$ procedure by introducing a closed, adaptive sequential procedure that allows for early termination if a definitive decision can be made before the maximum number of observations has been taken. It is closed because there is an upper bound on the number of samples to take, adaptive because the next observation we take depends on the previous observations, and sequential because we take one observation at a time. Their procedure, $\mathcal{B}_{\text{BK}}$, uses a so-called "least failures" sampling rule in that it will continue to sample from populations that have the smallest number of failures. It also uses strong curtailment because it will stop sampling whenever a population is far enough ahead that all other populations can at best tie by the end of sampling.

$\mathcal{B}_{\text{BK}}$ Procedure

Setup: Select n, the maximum number of observations to take from each system.

Sampling: If m is the current observation and n_{im} is the current number of observations from system i, $i = 1, \ldots, k$, that have been taken, take the next observation from the system that has the smallest number of failures among all of the systems for which $n_{im} < n$. If there is a tie, take the next observation from the system with the greatest number of wins. If there is a further tie, select one at random from the tied systems.

Stopping rule: Stop sampling at the first observation m for which there exists a system i such that

$$Y_{im} \geq Y_{jm} + n - n_{jm} \text{ for all } j \neq i \quad (1 \leq i,\ j \leq k).$$

Terminal decision rule: Suppose there are q systems that satisfy the stopping rule when sampling stops. If $q = 1$, then select that system as the best. If $q > 1$, then randomize to pick the winner.

Example: Suppose $k = 3$ and $n = 4$, and we take our first observation from system 1 which is a success, denoted as S_1. Our next observation should be from the system with the fewest failures but since all of the systems have zero failures, we choose the one with the most successes, which is again system 1. If our second observation from system 1 is a failure, denoted as F_1, then the next observation should be from system 2 or 3, chosen randomly. Assume we get the following results, where S_i^k (F_i^k) means that k successes (failures) have been observed for system i:

Table 2: $\mathcal{B}_{\text{BK}}$ Example.

Observation Number	Observation	Current results: System 1	System 2	System 3
1	S_1	S_1	-	-
2	F_1	S_1, F_1	-	-
3	S_3	S_1, F_1	-	S_3
4	S_3	S_1, F_1	-	S_3^2
5	F_3	S_1, F_1	-	S_3^2, F_3
6	F_2	S_1, F_1	F_2	S_3^2, F_3
7	S_3	S_1, F_1	F_2	S_3^3, F_3

At $N = 7$ we would stop sampling and select system 3 because systems 1 and 2 could at best catch up with system 3 in their remaining observations. □

For this procedure, n, the maximum number of samples to be taken from each system, is typically set by the user due to budget constraints, and thus the procedure does not involve a desired PCS. However, if a specific PCS is desired then n could be determined by looking up the value in the appropriate table that matches the desired P^* and θ.

The $\mathcal{B}_{\text{BK}}$ procedure has potential benefits in its least failures sampling rule because due to the nature of the experiment it is often desirable to sample less frequently from inferior populations. It is also advantageous to use the $\mathcal{B}_{\text{BK}}$ procedure because it allows for early termination, and it is sequential so it may be able to stop after each observation. Actually, the $\mathcal{B}_{\text{BK}}$ procedure will achieve precisely the same PCS as the $\mathcal{B}_{\text{SH}}$ procedure with a smaller number of total observations. Therefore, as long as sequential sampling is practical due to the nature of the experiment (as is typical for simulation experiments), then the $\mathcal{B}_{\text{BK}}$ procedure is superior.

The following two procedures, $\mathcal{B}_{\text{BKS}}$ and $\mathcal{B}_{\text{P}}$ proposed by Bechhofer, Kiefer, and Sobel (1968) and Paulson (1993) respectively, use the odds ratio formulation and look at differences between pairs of observations. These procedures were designed to satisfy a PCS constraint. Procedure $\mathcal{B}_{\text{BKS}}$ is a non-eliminating open, sequential procedure.

$\mathcal{B}_{\text{BKS}}$ Procedure

Setup: For the given k, specify the indifference zone value θ and P^*.

Sampling: At the mth stage of experimentation ($m \geq 1$), observe the random Bernoulli vector $(X_{1m}, \ldots, X_{km})$.

Stopping rule: Denote $Y_{im} = \sum_{j=1}^{m} X_{ij}$ $(1 \leq i \leq k)$ as the sample sums and $Y_{[1]m} \leq \cdots \leq Y_{[k]m}$ as their ordered values. After the mth stage of experimentation, compute the statistic

$$z_m = \sum_{i=1}^{k-1} (1/\theta)^{Y_{[k]m} - Y_{[i]m}}.$$

Stop at the first value of m (referred to as N), where $z_m \leq (1 - \text{P}^*)/\text{P}^*$.

Terminal decision rule: Select the system with the largest sample sum, $Y_{[k]N}$, as the best or randomize if there is a tie.

Example: Suppose we are testing 3 systems with $\theta = 1.4$ and $\text{P}^* = 0.65$, and that we collect the following observations:

Table 3: $\mathcal{B}_{\text{BKS}}$ Example.

m	X_{1m}	X_{2m}	X_{3m}	Y_{1m}	Y_{2m}	Y_{3m}	$Y_{[3]m} - Y_{[2]m}$	$Y_{[3]m} - Y_{[1]m}$	z_m
1	0	1	0	0	1	0	1	1	1.43
2	1	1	0	1	2	0	1	2	1.22
3	1	1	1	2	3	1	1	2	1.22
4	1	0	0	3	3	1	0	2	1.51
5	0	1	1	3	4	2	1	2	1.22
6	1	1	1	4	5	3	1	2	1.22
7	0	1	0	4	6	3	2	3	0.87
8	0	1	0	4	7	3	3	4	0.62
9	0	1	0	4	8	3	4	5	0.45

We stop at stage $N = 9$ and select system 2 as the best since $z_9 \leq (1 - \mathrm{P}^*)/\mathrm{P}^* = 0.538$. $\Box$

Paulson improved on certain aspects of $\mathcal{B}_{\mathrm{BKS}}$ by proposing $\mathcal{B}_{\mathrm{P}}$, an open sequential procedure that allows for elimination of systems deemed to be inferior.

$\mathcal{B}_{\mathrm{P}}$ **Procedure**

Setup : For the given k, specify θ and P^*.

Sampling: At the mth stage of experimentation ($m \geq 1$), take a random Bernoulli observation, X_{im} for each of the systems $i \in R_m$, where R_m is the set of all systems that have not yet been eliminated from contention.

Stopping rule: Let $Y_{im} = \sum_{j=1}^{m} X_{ij}$ $(1 \leq i \leq k)$ and e_i $(1 \leq i \leq k,\ e_i$ initialized to $\infty)$ denote the stage at which system i was eliminated. Define $n_{im} = \min(m, e_i)$ as the number of observations taken from system i through the mth stage of experimentation. For each system still in contention, let

$$g_i(m) = \sum_{j=1}^{k} (\theta)^{Y_{j,n_{jm}} - Y_{i,n_{jm}}}\ , \text{ where } j \neq i,\ j = 1, 2, \ldots, k, \text{ and } i \in R_m.$$

After the mth stage of experimentation, eliminate from further consideration and sampling any system still in contention ($i \in R_m$) for which

$$g_i(m) > \frac{k-1}{1-\mathrm{P}^*}$$

and set $e_i = m$ for such i.

Stop at the first value of m for which only one system remains in contention.

Terminal decision rule: Select the one remaining system as the best.

Example: The following artificial example is taken from Bechhofer, Santner, and Goldsman (1995). For $k = 3$, $(\theta,\ \mathrm{P}^*) = (2,\ 0.75)$, suppose that the sequence of vector observations in Table 4 is obtained using procedure $\mathcal{B}_{\mathrm{P}}$. We see that system 1 is eliminated after Stage 3, since $g_1(3) = 12 > 8 = (k-1)/(1-\mathrm{P}^*)$. Similarly, system 2 is eliminated after Stage 5, because $g_2(5) = 8.25 > 8$; at that point, system 3 is the only system still in contention,

Table 4: Calculations to Implement Procedure $\mathcal{B}_{\text{P}}$ in the Example.

	stage				
m	1	2	3	4	5
x_{1m}	0	0	0	-	-
x_{2m}	0	1	1	0	0
x_{3m}	1	1	1	1	1
y_{1m}	0	0	0	-	-
y_{2m}	0	1	2	2	2
y_{3m}	1	2	3	4	5
n_{1m}	1	2	3	3	3
n_{2m}	1	2	3	4	5
n_{3m}	1	2	3	4	5
$(\theta)^{y_{2,n_{2m}}-y_{1,n_{2m}}}$	1	2	4	-	-
$(\theta)^{y_{3,n_{3m}}-y_{1,n_{3m}}}$	2	4	8	-	-
$(\theta)^{y_{1,n_{1m}}-y_{2,n_{1m}}}$	1	0.5	0.25	0.25	0.25
$(\theta)^{y_{3,n_{3m}}-y_{2,n_{3m}}}$	2	2	2	4	8
$(\theta)^{y_{1,n_{1m}}-y_{3,n_{1m}}}$	0.5	0.25	0.125	0.125	0.125
$(\theta)^{y_{2,n_{2m}}-y_{3,n_{2m}}}$	0.5	0.5	0.5	0.25	0.125
$g_1(m)$	3	6	12	-	-
$g_2(m)$	3	2.5	2.25	4.25	8.25
$g_3(m)$	1	0.75	0.625	0.375	0.25

and it is declared the winner. The entire procedure requires 5 stages and a total of 13 observations. □

When compared to each other, the $\mathcal{B}_{\text{BKS}}$ procedure usually minimizes the total number of stages of experimentation, while $\mathcal{B}_{\text{P}}$ minimizes the total number of raw observations (see Bechhofer, Santner and Goldsman 1995). A stage relates to a setup of the experiment and the duration of the study. In the simulation context, a stage equates to the initiation of a simulation (or set of simulations, one for each system still in contention) to obtain scalar (vector) observations from one (multiple) system(s), dependent on the nature of the procedure. It is beneficial to minimize the total number of stages if the cost of setting up the experiment and/or switching between setups is high. However, if the cost of each observation is high relative to the setup and switching cost, then it is beneficial to minimize the total number of observations. In simulation experiments where each observation is a simulation run which can require significant time and CPU usage, we believe these costs will typically outweigh the cost of setting up the stages. Therefore, our focus is a procedure that minimizes the total number of observations.

We propose a sequential, eliminating procedure for Bernoulli data that uses the IZ formulation, meets a desired PCS constraint, and attempts to minimize the total number of observations. Since $\mathcal{B}_{\mathrm{P}}$ usually requires the fewest observations to satisfy a common PCS constraint among those IZ procedures under study, it will be the procedure used for performance comparisons.

2.3 The Kim and Nelson $\mathcal{KN}$ Procedures

Kim and Nelson developed three procedures, $\mathcal{KN}$, $\mathcal{KN}+$, and $\mathcal{KN}++$, with the goal of addressing problems commonly encountered in a simulation environment. Such issues include non-normality of data, unknown and unequal variances, and dependence among systems due to the use of variance reduction techniques such as CRN. The first procedure, $\mathcal{KN}$ (Kim and Nelson 2001), deals with the use of CRN with data that are i.i.d. normal. Due to the normality requirement, we do not study the use of this procedure for Bernoulli data. However, the other two IZ procedures, $\mathcal{KN}+$ and $\mathcal{KN}++$ (Kim and Nelson 2004), allow for non-normal and dependent data and satisfy a PCS constraint *asymptotically*. These procedures do not use CRN and deal primarily with steady-state simulation where only one replication is made and batch means are taken as basic observations to achieve approximate independence. On the other hand, we assume that our data are typically independent from one replication to the next due to the nature of terminating simulations and are amenable to CRN. Nevertheless, we feel these procedures ($\mathcal{KN}+$ and $\mathcal{KN}++$) have potential applications to the Bernoulli problem.

In this section we will define the $\mathcal{KN}+$ and $\mathcal{KN}++$ procedures as set forth in Kim and Nelson (2004). Then in the following section we will discuss applications of these procedures to the Bernoulli problem.

2.3.1 The $\mathcal{KN}+$ Procedure

The $\mathcal{KN}+$ and $\mathcal{KN}++$ procedures are similar to the $\mathcal{B}_{\mathrm{P}}$ procedure in that they look at paired differences between the systems. The cumulative sum process of these paired differences has been shown to approximate a Brownian motion process with a drift thanks to a functional central limit theorem. This fact, along with a result from Fabian (1974), allows us to bound the probability of incorrect selection (PICS). This result defines a triangular "continuation region" for the procedure. The continuation region is simply a boundary on the cumulative sum of the differences within which sampling continues. Figures 3 and 4 and the following commentary provide insight on how the procedure is designed and how it works.

In Figure 3 the Y-axis denotes the partial sum of the differences between the response for system k and system i, $\sum_{j=1}^{r}(X_{kj} - X_{ij})$, and the X-axis plots r, the current observation. In this example the data are from a normal distribution. As we can see, the data resemble a Brownian motion process with positive drift. The triangle defines the continuation region; we continue to sample until the sum of the differences exits that region. If we exit the triangle upward, we select system k, if we exit downward, we select system i. We assume that system k is the best, so exiting upward would give us a correct selection, as we witness in this figure. Although the figure indicates that the Brownian motion exits the triangle at an earlier point, we are only observing the data at discrete times and thus do not observe an exit until a short time later. The Brownian motion process is a continuous one, but since we can only observe discrete realizations of it, it is possible for the true process to be able to exit before we actually observe an exit.

Figure 4 shows a different path of the same process we saw in Figure 3 with the same boundary. In this graph an incorrect selection is made by exiting down (although we would have exited in the correct direction if given more time). If we know how to determine the boundary so that we can control the probability of an incorrect selection, then we have a statistically valid procedure. The correct choice of bounds is critical to efficiently selecting a winner while maintaining the desired PCS. If the bounds are "too narrow", then we may be more likely to make an incorrect selection as in Figure 4. However, if the bounds are "too wide", then we will not be able to distinguish between systems — and thus eliminate inferior systems from contention — without significant sampling. Therefore, we hope to have a tight continuation region so that we can make a selection as soon as possible, while maintaining the desired PCS confidence in our result. To determine the bound of the

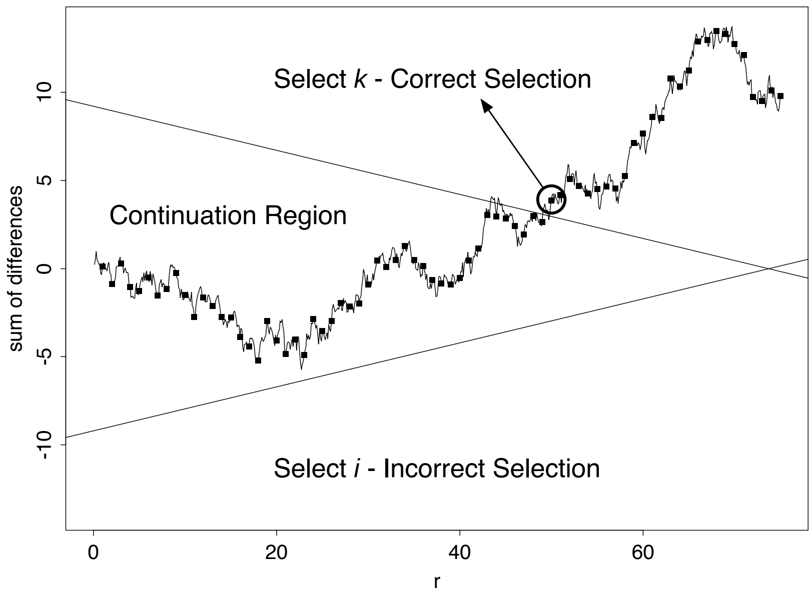

Figure 3: An Example of a Continuation Region where a Correct Selection is Made.

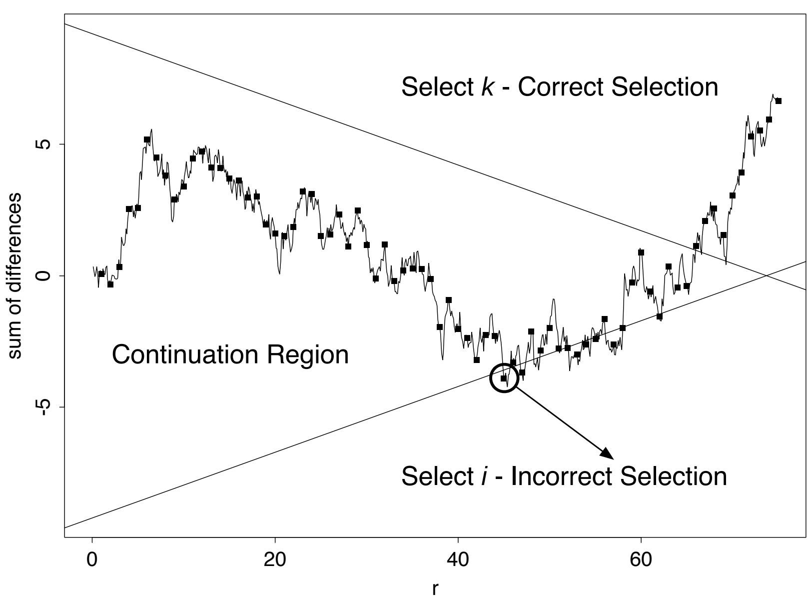

Figure 4: An Example of a Continuation Region where an Incorrect Selection is Made.

triangular region, a result due to Fabian (1974) is used by Kim and Nelson to bound the probability of incorrect selection. By doing so, Kim and Nelson show that their procedures are asymptotically valid for meeting the desired PCS when the data are non-normal, with variances unknown and unequal, and there is within-system (serial) dependence (e.g., as in steady-state simulations). Although they cannot give a PCS guarantee with finite samples, they show that their procedures asymptotically achieve the desired PCS in an appropriate limit.

The formulation for the $\mathcal{KN}+$ procedure is shown below. First, we present the following definition for asymptotic variance. The asymptotic variance of a system i is defined as

$$v_i^2 \equiv \lim_{n\to\infty} n\mathrm{Var}(\bar{X}_i(n)).$$

$\mathcal{KN}+$ Procedure

Setup: Select confidence level $1/k < 1-\alpha < 1$, indifference-zone parameter $\delta > 0$, first-stage sample size $n_0 \geq 2$, and batch size $m_0 < n_0$. Calculate the constants η and c as described below.

Initialization: Let $I = \{1, 2, \ldots, k\}$ be the set of systems still in contention, and let $h^2 = 2c\eta d$, where the degrees of freedom d is determined by which variance estimator is used.

Obtain n_0 first stage observations $X_{ij}, j = 1, 2, \ldots, n_0$, from each system $i = 1, 2, \ldots, k$.

For all $i \neq \ell$, compute the estimator $m_0 V_{i\ell}^2$, a sample asymptotic variance of the difference between systems i and ℓ. Notice that $m_0 V_{i\ell}^2$ is based only on the first n_0 observations. See the following for more details on the variance estimator.

Screening: Here we screen out systems deemed inferior. Set $I^{\mathrm{old}} = I$. Update I as follows:

$$I = \left\{ i : i \in I^{\mathrm{old}} \text{ and } \bar{X}_i(r) \geq \bar{X}_\ell(r) - W_{i\ell}(r), \forall \ell \in I^{\mathrm{old}}, \ell \neq i \right\},$$

where

$$W_{i\ell}(r) = \max\left\{0, \frac{\delta}{2cr}\left(\frac{h^2 m_0 V_{i\ell}^2}{\delta^2} - r\right)\right\},$$

and r is the current observation.

Stopping Rule: If $|I| = 1$, then stop and select the system whose index is in I as the best. Otherwise, take one additional observation $X_{i,r+1}$ from each system $i \in I$, set $r = r+1$, and go to **Screening**.

Constants: The constant c may be any nonnegative integer. The constant η is the solution to the equation

$$g(\eta) = \sum_{\ell=1}^{c} (-1)^{\ell+1} \left(1 - \frac{1}{2}\mathcal{I}(\ell = c)\right) \left(1 + \frac{2\eta(2c-\ell)\ell}{c}\right)^{-d/2} = 1 - (1-\alpha)^{1/(k-1)}, \quad (1)$$

where $\mathcal{I}$ is the indicator function. To guarantee a unique solution to the above equation, $c = 1$ is usually chosen.

In addition to making Equation (1) easy to solve, the use of $c = 1$ was also shown to be the best choice (Kim and Nelson 2001) when the experimenter knows nothing about the systems; see Goldsman, Kim, Marshall and Nelson (2000, 2002) for relative empirical results.

The term $rW_{i\ell}(r)$ defines a continuation region for the partial sum, $\sum_{j=1}^{r}(X_{ij} - X_{\ell j})$. Note that W decreases as r increases. As we collect more information, less of a difference needs to be observed to select a winner. Figure 5 shows the continuation region for $\mathcal{KN}+$. The horizontal axis represents the current sampling size r and the vertical axis represents the partial sum of the differences $\sum_{j=1}^{r}(X_{ij} - X_{\ell j})$. If $c < \infty$, this region is a triangle, as shown in the figure. As c increases the triangle becomes longer and narrower. When c is infinity it becomes two parallel lines.

Since no assumptions are made about the variances of the differences between systems, they must be estimated after an initial number of samples. In order for the procedure to be asymptotically valid, the variance estimator of the difference between the systems, $m_0 V_{il}^2$, needs to have the following property: $m_0 V_{il}^2 \rightarrow v_{il}^2 \frac{\chi^2_{(d)}}{d}$ as $m_0 \rightarrow \infty$ when b, the number of batches, is fixed, v_{il}^2 is the true value of the asymptotic variance of differences and $\chi^2_{(d)}$ denotes a chi-square random variable with d degrees of freedom. For details about variance estimators with the chi-square property, see Schruben (1983), Chien, Goldsman, and Melamed (1997), Sargent, Kang, and Goldsman (1992) and Meketon and Schmeiser (1984).

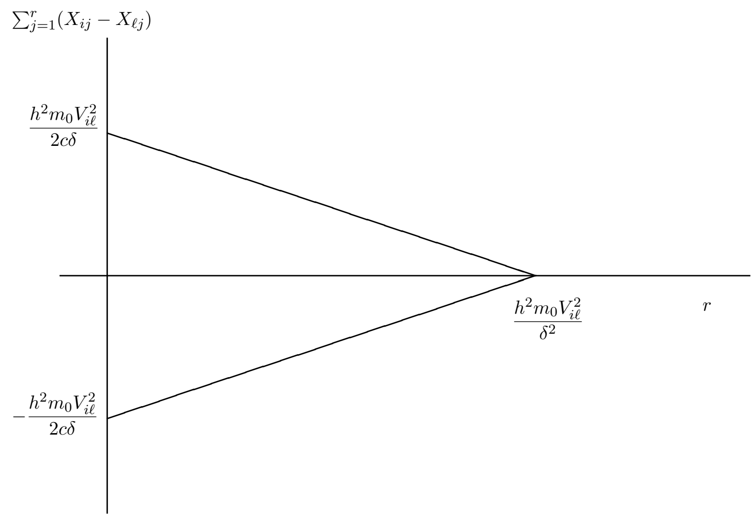

Figure 5: Continuation Region for the Fully Sequential, Indifference-Zone Procedure when $c < \infty$.

The $\mathcal{KN}+$ procedure does not update this variance estimate after the first stage of sampling, so selecting an appropriate initial sample size is key. If n_0 is too large then we may take more observations than needed and the procedure may be inefficient; but if n_0 is too small we may have a very poor variance estimate which will also make the procedure inefficient and could potentially result in poor PCS. Therefore, Kim and Nelson proposed $\mathcal{KN}$++, a procedure that updates the variance estimate throughout sampling, so that an exact choice of an appropriate initial sample size is not as critical.

2.3.2 $\mathcal{KN}$++ Procedure

There are few differences between the $\mathcal{KN}+$ and $\mathcal{KN}$++ procedures. The primary difference concerns a recalculation of the variance estimate at various update points where the continuation region is redefined. Since the degrees of freedom for the variance estimator will change with each update, we must also update the h^2, η, and W values.

For this procedure we define a batching sequence (m_r, b_r), where b_r is the number of batches and m_r is the batch size at stage r. We estimate the variance of the differences v_{il}^2 with the estimator $m_r V_{il}^2(r)$ which is based on n_0 initial observations and then updated frequently according to the update period as more observations are taken. For the asymptotic validity of the procedure to hold, the variance estimator for the $\mathcal{KN}$++ procedure,

$m_r V_{il}^2(r)$, must have the strong consistency property, i.e., $m_r V_{il}^2(r) \rightarrow v_{il}^2$ with probability one as $m_r \rightarrow \infty$ and $b_r \rightarrow \infty$. For more information about variance estimators with the strong consistency property, see Damerdji (1994, 1995), Damerdji and Goldsman (1995) and Chien, Goldsman, and Melamed (1997). The $\mathcal{KN}$++ procedure is given in Chapter 3 in a form specific to Bernoulli data.

2.4 Initial Application of $\mathcal{KN}$+ and $\mathcal{KN}$++ to Bernoulli Data with Independence

Since the $\mathcal{KN}$+ and $\mathcal{KN}$++ procedures are proven to be asymptotically valid for non-normal data and they meet the other goals we set forth for our Bernoulli procedure, we choose to first test the performance of these procedures in the case where we obtain independent observations from each Bernoulli contender. As mentioned earlier, the Bernoulli problem is also known as the binomial selection problem since the sums of Bernoulli data are distributed as binomial. We choose to exploit the normal approximation to the binomial to apply the $\mathcal{KN}$ procedures to the Bernoulli problem. With an appropriate choice of initial sample size, we believe that the sum of Bernoulli data will be approximately normal and thus these procedures should be asymptotically valid for Bernoulli data.

We first apply $\mathcal{KN}$+, the procedure without variance updates, to the Bernoulli data. As mentioned, we need a variance estimator with the chi-square property for this procedure. Since the data we are interested in are i.i.d. without batching (we are not looking at steady-state simulations), we can simply use the standard sample variance estimate, as it has an approximate chi-square property.

We first test the procedure with $c = 1$ and with various choices of n_0. When compared to Paulson's procedure, our expected number of total observations is sometimes close to Paulson's if n_0 is properly chosen, but always larger. For instance, with $\delta = 0.11$ and $k = 2$, the expected total number of observations for $\mathcal{KN}$+ is 94 when $n_0 = 30$ while it is 85 for Paulson's. See Table 22 in the Appendix for details. Paulson's procedure likely performs better because it does not require a variance estimate and thus does not need to take an initial n_0 observations from each system. Therefore, $\mathcal{B}_{\mathrm{P}}$ has the potential to eliminate

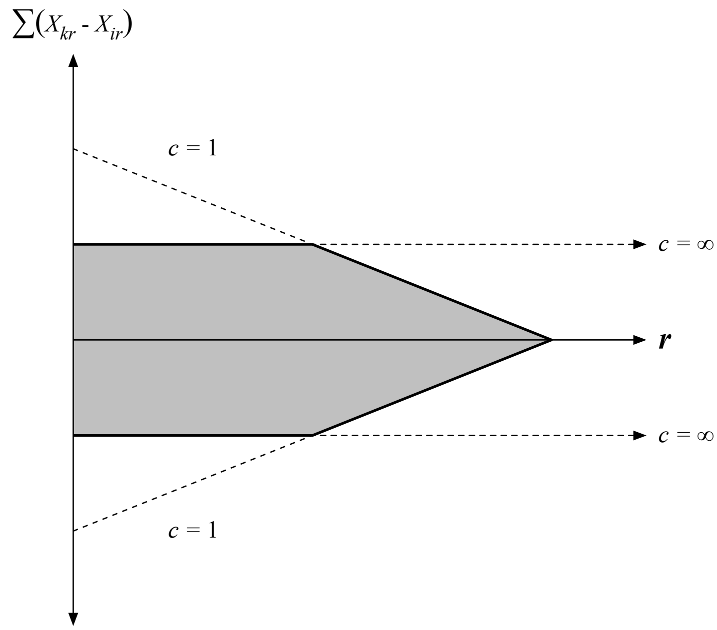

Figure 6: The Combined Region Defined by the Minimum W_{il} Value when $c = 1$ or $c = \infty$.

inferior systems earlier than our procedure. In addition, $\mathcal{B}_{\mathrm{P}}$ has a tighter continuation region during early sampling than $\mathcal{KN}+$ with $c = 1$. The reason for this is that the continuation region for $\mathcal{KN}+$ with $c = 1$ is a triangle that is short, but fat during early sampling, and therefore very hard to exit in the early stages of the sampling.

In the hope of improving the results, we look at a combined continuation region defined by the minimum region when $c = \infty$ and $c = 1$. When $c = 1$, our continuation region is a short triangle, but very fat at the beginning observations. Thus as more data are collected, the region narrows quickly, but it is hard to exit the region at the start of the experiment. When $c = \infty$, the region is two parallel lines which are much narrower than the $c = 1$ triangle at the beginning of experimentation. By looking at the minimum region defined by the two, we hope the partial sums will exit as soon as possible. See Figure 6 for an illustration.

The results for the combined continuation region are an improvement over the $c = 1$ case. In comparison to Paulson's procedure, the combined region shows results that have lower total numbers of observations when δ is small and n_0 is reasonably chosen. For instance, when $k = 2$, $\delta = 0.06$ and $n_0 = 50$, procedure $\mathcal{KN}+$ yields $\widehat{E(N)} = 236$ while Paulson's requires $\widehat{E(N)} = 250$, where $\widehat{E(N)}$ denotes our Monte Carlo estimate of the expected number of observations based on 100,000 replications. However, without precise selection of n_0 or when δ is relatively large, our procedure has slightly larger total numbers of observations. For example, when $k = 2$ and $\delta = 0.11$, the lowest value of $\widehat{E(N)}$ we obtain over all tested values of n_0 is 91 but Paulson's $\widehat{E(N)}$ is 85. See Table 5 for details, which shows $\widehat{\text{PCS}}$ and $\widehat{E(N)}$ results for each n_0 value as well as for $\mathcal{B}_{\text{P}}$. Another problem with the combined region is the asymptotic validity. Once we combine two regions, it is not clear whether asymptotic validity will still hold.

Table 5: $\mathcal{KN}+$ Independent Bernoulli Results with Combined $c = 1$ and $c = \infty$ Region.

δ	k	$n_0 = 30$	$n_0 = 40$	$n_0 = 50$	$n_0 = 60$	$n_0 = 70$	$\mathcal{B}_{\text{P}}$
0.06	2	0.944	0.947	0.946	0.950	0.949	0.963
		242	237	236	239	245	250
	3	0.942	0.943	0.946	0.947	0.947	0.961
		499	483	480	477	478	501
	4	0.942	0.942	0.945	0.946	0.945	0.959
		754	735	717	712	709	731
	5	0.940	0.941	0.944	0.943	0.944	0.956
		1013	988	965	953	948	935
0.11	2	0.955	0.960	0.963	0.966	0.967	0.970
		91	101	112	127	144	85
	3	0.953	0.956	0.959	0.962	0.964	0.965
		174	181	194	210	229	163
	4	0.952	0.954	0.959	0.961	0.963	0.958
		257	264	278	296	320	223
	5	0.949	0.953	0.959	0.961	0.962	0.964
		344	348	363	384	412	311

Hoping to reduce the results' reliance on n_0 selection, we apply the variance updating procedure, $\mathcal{KN}{+}{+}$. However, a decision still must be made on an appropriate n_0 to use. Though we hope that the exact choice will be less critical than with the $\mathcal{KN}+$ procedure, it is still important to select n_0 so that we approximate the normal distribution and obtain an accurate estimate of the variance. The normal approximation to the binomial distribution is generally valid when the number of observations (n) is large and the probability of success (p) is not too close to 0 or 1; and it improves in accuracy as n increases. The general rule of thumb is that one should choose n such that $np \geq 5$ and $n(1-p) \geq 5$. For example, if $p = 0.5$, then $n_0 = 10$ should be sufficient, but for larger or smaller p, a larger n_0 is often required. If the experimenter thinks the p values are very high or low ($p \geq 0.95$ or $p \leq 0.05$), then an n_0 of 100 or more would be needed, and thus the procedure may be inefficient. The preceding examples are for a simple single system. Our problem is more complicated because we are comparing multiple systems with multiple values of p. However, the rule of thumb can be used as a starting point for estimating n_0. If the experimenter has no knowledge about the probabilities of success for the systems, a quick experiment can be performed to estimate p and thus choose an n_0. We initially test results for both $n_0 = 30$ and 50 and then study multiple options for n_0 to evaluate performance over a wide range of values. Further discussion on guidelines for selection of n_0 is covered in later sections.

The results, where each cell shows $\widehat{\text{PCS}}$ and $\widehat{E(N)}$ for the given configuration, are given in Table 6. As hoped, the performance of the procedure has much less reliance on the choice of n_0; and a smaller n_0 is required than in the non-updating case for the procedure to be efficient in terms of meeting the desired PCS while trying to minimize the required number of observations. In fact, it appears that an n_0 of 30 is sufficient (though smaller values were not tested and therefore may a better choice) and results do not vary widely when n_0 is changed. However, the expected total number of observations is only slightly lower than Paulson's when $\delta = 0.06$ and $k = 2$ ($\widehat{E(N)} = 240$ versus Paulson's 250). In all other situations, Paulson's procedure is superior in performance.

The results indicate that Paulson's procedure is indeed quite good for Bernoulli data and is very hard to beat in the independent case. We hope that a procedure that induces correlation between systems by using CRN will be able to produce better results than Paulson's independent procedure in terms of expected total number of observations. In the following chapter, we will discuss the use of CRN and introduce a procedure employing CRN for Bernoulli data.

Table 6: $\mathcal{KN}$++ Independent Bernoulli Results.

δ	k	$n_0 = 30$	$n_0 = 50$	$\mathcal{B}_{\mathrm{P}}$
0.06	2	0.947 240	0.950 244	0.963 250
	3	0.950 503	0.952 505	0.967 502
	4	0.954 768	0.954 769	0.972 721
	5	0.954 1038	0.955 1041	0.977 916
0.11	2	0.953 89	0.960 111	0.971 86
	3	0.949 174	0.956 193	0.972 168
	4	0.950 260	0.953 279	0.968 226
	5	0.950 350	0.955 369	0.981 311

CHAPTER III

$\mathcal{KN}$++ FOR BERNOULLI WITH CRN

The benefit of using Common Random Numbers (CRN) in simulation experiments has already been discussed in multiple applications (see Law and Kelton 2000 for an in-depth look). However, to date no procedure has been introduced which incorporates CRN for use in Bernoulli data procedures without assumptions being made. Tamhane (1980, 1985) proposed a procedure for using CRN with Bernoulli data, but only for $k = 2$ systems and only when the experimenter is able to provide an upper bound on the probability that $X_{1r} \neq X_{2r}$.

Our goal is to provide a procedure that makes use of the benefits provided by CRN without making any such assumptions and which is valid for $k \geq 2$. In the next section we discuss some general pros and cons of employing CRN and detail how to incorporate them into simulation experiments. The subsequent sections give the formulation for the $\mathcal{KN}$++ procedure for Bernoulli data, apply $\mathcal{KN}$++ to Bernoulli data utilizing CRN, and discuss the results and guidelines for the selection of n_0.

3.1 The Potential Benefits of Using CRN

Similar to blocking in design of experiments, the use of CRN can help us reduce random error between compared systems so that observed differences are more likely to be actual differences in the systems rather than just differences between the random inputs in the model. When data are paired differences between outputs, then the variance of those differences is

$$\mathrm{Var}(X_{ij} - X_{\ell j}) = \mathrm{Var}(X_{ij}) + \mathrm{Var}(X_{\ell j}) - 2\mathrm{Cov}(X_{ij}, X_{\ell j})$$

Thus, if we can induce a positive correlation between the data, the covariance term is > 0 and we can reduce the variance of the difference between systems. See Banks et al. (2001) for references on implementing CRN with simulation data.

An example of the use of CRN is in inventory control. Suppose we are testing three different inventory models. If we give each model the same demands for each set of observations, then we would know that differences observed in the output are not biased by different demand streams. Conversely, if we use separate demand streams for each model, it may be that one system receives a very favorable (or unfavorable) demand input compared to the other two, making that model seem better (worse) when in fact it may not be if compared under equal conditions.

For instance, we will consider a very simple newsvendor model. Assume that we are selling daily newspapers and that anything not sold at the end of the day has no salvage value. The cost for us to buy the papers is 65 cents per paper and we will sell them for \$1.00 each. If a person comes to buy a paper and we are out of stock, we will assess a penalty of 5 cents because it may potentially generate ill will with the customer and he may permanently take his business elsewhere. We assume the demand distribution to be unknown and thus we are not sure how many papers we should buy each day in order to maximize our profit. Suppose that we wish to test two different purchasing schemes, one where we buy 55 papers daily (denoted as scheme A) and the other where we buy 60 per day (denoted as scheme B).

To determine which scheme is the best, we will collect daily profit data for both purchasing schemes and apply the following heuristic test. We will look at the differences between the profits for each scheme and calculate a 95% confidence interval for the expected value of that difference. We will continue to collect data until we can make a decision about which scheme is best, which equates to calculating a confidence interval that does not contain zero. Once the confidence interval does not contain zero, we have observed differences significant enough to recognize that the expected profits of the two systems are clearly unequal, and can then determine the best choice.

For purposes of this example, we will assume that the demand is distributed as discrete Uniform (45, 70). We will run two simulation experiments: one where we will use a single random number generator to produce the same demand streams for both schemes (CRN), and the other where we will use two random number generators to produce two independent streams of demand data. In both simulations, our data from day to day will be independent. However, when CRN is used, the data between the two systems on a single day will be correlated. Our statistic of interest is the number of observations it takes until we can make a decision. After running the experiment 50,000 times the average number of observations necessary to make a decision without CRN is 174.07, but with CRN it is only 39.45. Although this is only a heuristic procedure, it illustrates how the use of CRN can reduce the variation in a response and can help to make a decision earlier.

For a further look, let us reconsider the above example. This time, we will take a set of 100 observations from each system so we can assume normality and compute a confidence interval (CI) of the differences between profits for two order schemes A and B. We will repeat this experiment 50,000 times and count how many times out of 50,000 experiments of 100 observations each the computed CI contains zero. Roughly speaking, a CI containing zero means that we are unable to determine which scheme is the best. After 50,000 runs, the CI contains zero 11,407 times when we use CRN and 42,005 times when we do not. Therefore, there are a large number of runs for which the procedure run with CRN can detect a winner while the other procedure without CRN cannot. By using the same demand stream, we can more clearly see the actual differences in profits between the two systems without the added variability created by different demand sequences. For instance, in one run after 100 observations the variance of the difference between the daily profits was 33.62 without CRN, but was only 6.166 when the same demand stream was used. Thus we can see how CRN can be used as a variance reduction technique to enable us to make decisions with fewer total observations.

In the preceding example, both purchasing schemes have the same distribution of input data. It is important to note that it is not necessarily required for competing systems to have the same input distributions in order for CRN to be employed. Consider two simple queueing systems, A and B. System A has only one very fast server and system B has two servers whose rate of service is much slower. Suppose that each system has the same arrival distribution but that the service times for one server in system A are distributed Uniform(3,7) and the service times for the two servers in system B are Normal(10,2) for each server. In this example we have two systems inputs; arrival and service times. To employ CRN we will use the same pseudo-random numbers to generate arrival times for both systems which will result in identical arrival streams to both systems since the arrival distributions are identical. However, we can also employ CRN with the service times even though the distributions are different when we are comparing one versus two servers. Each arrival will have a service time associated with it (generated from either the uniform or normal distribution depending on the system). If we use the same pseudo-random number for generating both service times for a given arrival, then even though the numbers will not be the same, we hope that when one service time is large or small, the other one will be as well (as may be the case, for instance, when the inverse transform method is used to generate the random numbers). In that case, we are able to reduce the variance of the differences by eliminating some of the noise created by using different sets of random numbers to generate data from each distribution.

To further illustrate the benefits of CRN we look at generated Bernoulli data in a simulation. Note that this example does not represent how data are actually collected in experiments, but rather how they are generated in Monte Carlo simulations. The typical way of generating k independent Bernoulli observations is to generate k independent Uniform(0,1) random variates $U_1, \ldots, U_k$. Then

$$\text{Set } X_i = \begin{cases} 1 & \text{if } U_i \leq p \\ 0 & \text{if } U_i > p. \end{cases}$$

However, if we can somehow induce complete correlation ($\rho = 1$) between all of the systems, then we will only need to use one uniform (rather than k uniforms) to generate the k observations in a Monte Carlo simulation. For example, suppose we have 2 systems, A and B, with success probabilities 0.85 and 0.90 respectively, and we can induce complete correlation. Then we will generate an observation as follows: Generate one uniform(0,1) random variable, U.

$$\begin{array}{ll} \text{If } U \leq 0.85 \Longrightarrow & A = 1,\ B = 1 \\ \text{If } 0.85 < U \leq 0.90 \Longrightarrow & A = 0,\ B = 1 \\ \text{If } U \geq 0.90 \Longrightarrow & A = 0,\ B = 0 \end{array}$$

We see that in this example it is impossible to obtain a result of $A = 1$ and $B = 0$. So if we have a situation where we can induce complete correlation, whenever we compare two systems and obtain a result of (1,0) or (0,1) we will know which system is the winner. While this sounds advantageous, this added information can come at a cost. In the given example we will only see such a result five percent of the time, and thus it may take a long time to realize any differences, making the procedure inefficient.

Although complete correlation is not really possible in practice, the above example illustrates how we can gain added information from the use of CRN; but it can also make our procedure inefficient. In order to deal with this problem, the bounds that define the continuation region need to be narrowed so that they take into account the added information. If we are reducing the variance of the difference between the systems, it should take less of a realized difference in the experiment for us to be able to declare a winner.

3.2 The $\mathcal{KN}$++ Procedure for Bernoulli Employing CRN

As previously mentioned, there are few differences between the $\mathcal{KN}$+ and $\mathcal{KN}$++ procedures. The primary difference is $\mathcal{KN}$++'s update of the variance estimate and subsequent update of the values that depend on the degrees of freedom — the h^2, η, and W values.

We present the following procedure in a form that is appropriate for Bernoulli data. Specifically, we assume that our data are serially independent samples taken from terminating simulations. Therefore we do not need to employ batching in order to achieve independence. Since we do not need to employ batching and data are i.i.d. as the number of observations goes to ∞, we use the standard sample variance estimator, which has the strong consistency property.

We choose to update the variance every 10 samples. We think this update point is small enough to take into account new information quickly, but large enough to stabilize the variance estimates somewhat, as they can fluctuate greatly for Bernoulli data during initial observations. This also helps avoid repeated calculations of the variance which reduces CPU time.

One problem that needs to be addressed is the possibility of the variance estimate being zero. Due to the binary nature of the data, if the p values of the systems are all very large (small) then the majority of the data will be ones (zeroes) and so many of the differences will be zero. To address this, we set the procedure to continue sampling until the next updating point if a zero variance is estimated before continuing to screening.

The $\mathcal{KN}$++ procedure is presented below with the modifications described above for Bernoulli data. The procedure takes initial observations to obtain variance estimates for the partial sum of differences between pairs of systems and then uses those estimates to create continuation regions. In each stage, simulations are initiated for all competing systems to obtain additional observations, and the sums of the differences between all pairs of systems still in contention are calculated. If the sum falls within the continuation region for a given pair, then we will continue sampling from the systems in that pair. As more samples are taken, we update our variance estimates and recalculate the continuation regions. As soon as the sum of differences between a pair falls outside the continuation region, the lesser of the two in terms of wins is eliminated from further consideration. This procedure is repeated until one system remains.

$\mathcal{KN}$++ **Procedure**

Setup: Specify the desired confidence level $1/k < 1 - \alpha < 1$, indifference-zone parameter $\delta > 0$, and first-stage sample size $n_0 \geq 2$. Calculate η and c as described below.

Initialization: Let $I = \{1, 2, \ldots, k\}$ be the set of systems still in contention, and let $h^2 = 2c\eta(n_0 - 1)$.

Obtain n_0 observations X_{ij}, $j = 1, 2, \ldots, n_0$, from each system $i = 1, 2, \ldots, k$.

For all $i \neq \ell$, compute the estimator $S_{i\ell}^2(n_0)$, the sample variance of the difference between system i and ℓ, based on n_0 observations. If all variance estimates are > 0, set the observation counter $r = n_0$ and continue to screening. Otherwise, take 10 more observations and recalculate the variance estimates. Continue to do so until nonzero estimates are obtained.

Update: If we have reached the next update point, then for all $i \neq \ell$, $i, \ell \in I$ compute the estimator $S^2_{i\ell}(r)$, the sample variance of the difference between systems i and ℓ based on r observations, and $h^2 = 2c\eta(r-1)$. Let

$$N_{i\ell}(r) = \left\lfloor \frac{h^2 S^2_{i\ell}(r)}{\delta^2} \right\rfloor$$

and

$$N_i(r) = \max_{\ell \neq i} N_{i\ell}(r).$$

This is the largest possible number of observations we can take from system i.

Go to **Screening**.

Screening: Set $I^{\text{old}} = I$. Let

$$I = \left\{ i : i \in I^{\text{old}} \text{ and } \bar{X}_i(r) \geq \bar{X}_\ell(r) - W_{i\ell}(r), \forall \ell \in I^{\text{old}}, \ell \neq i \right\},$$

where

$$W_{i\ell}(r) = \max\left\{0, \frac{\delta}{2cr}\left(\frac{h^2 S^2_{i\ell}(r)}{\delta^2} - r\right)\right\}.$$

Stopping Rule: If $|I| = 1$, then stop and select the system whose index is in I as the best. Otherwise, take one additional observation $X_{i,r+1}$ from each system $i \in I$, set $r = r+1$, and go to **Update**.

Constants: The constant c may be any nonnegative integer. The constant η is the solution to the equation

$$g(\eta) = \sum_{\ell=1}^{c} (-1)^{\ell+1} \left(1 - \frac{1}{2}\mathcal{I}(\ell = c)\right) \exp\left(-\frac{\eta}{c}(2c - \ell)\ell\right) = \frac{\alpha}{k-1}, \tag{2}$$

where $\mathcal{I}$ is the indicator function. The constant $c = 1$ is suggested for use if the experimenter has no prior knowledge about the experiment.

Kim and Nelson's original $\mathcal{KN}$ procedure (2001) allowed for the use of CRN. In their paper they show that the only change needed to incorporate CRN into the $\mathcal{KN}$ procedure is a change in the function $g(\eta)$. Without CRN, $g(\eta) = 1 - (1-\alpha)^{\frac{1}{k-1}}$; but with CRN, $g(\eta) = \frac{\alpha}{k-1}$ due to the use of a Bonferroni type inequality — since now the error is shared by the systems rather than them being independent. Although this change in the function $g(\eta)$ will lead to a larger value of η, thus potentially increasing the continuation region, we hope that the decrease in the variance of the differences due to CRN will offset this increase and effectively narrow the continuation region of the procedure.

Kim and Nelson prove the validity of the use of CRN for their $\mathcal{KN}$ procedure for normal data. Here we will provide a proof of the validity of using CRN for the $\mathcal{KN}$++ procedure in the Bernoulli case. It should be noted that the majority of the proof is identical to that of the proof for the $\mathcal{KN}$++ procedure without CRN. The only changes are to the function $g(\eta)$, the variance estimator, and the final part of the proof where the Bonferroni inequalities are used. Thus, most of this proof is taken from the Kim and Nelson paper (2004) with slight modifications made for η and the variance. The only substantial change is at the end of the proof where the result is extended to more than one pair of systems. For more details and theoretical background see Kim and Nelson (2004).

The Functional Central Limit Theorem (FCLT) (see Billingsley 1968, Chapter 4) and a result due to Fabian (1974) are the foundations for proving the validity of the procedure. Additionally, we also note a few other key lemmas and an assumption needed for the proof.

First of all, we denote the sample mean of the first r observations from system i by $\bar{X}_i(r)$, and the standardized partial sum for system i, $C_i(t,r)$, is defined as

$$C_i(t,r) \equiv \frac{\sum_{j=1}^{\lfloor rt \rfloor} X_{ij} - rt\mu_i}{v_i\sqrt{r}},\ 0 \le t \le 1, \tag{3}$$

where μ_i is the steady state mean, and

$$v_i^2 \equiv \lim_{r\to\infty} r\mathrm{Var}(\bar{X}_i(r))$$

is the asymptotic variance of system i, both assumed to exist and be finite, and where $\lfloor\cdot\rfloor$ indicates truncation of any fractional part. For any $r \ge 1$, the random function $C_i(t,r)$ is an element of the Skorohod space $D[0,1]$ as defined in Chapter 3 of Billingsley (1968).

Assumption 1 (FCLT) *There exist finite constants μ_i and v_i^2 such that, as $r \to \infty$, the probability distribution of $C_i(t,r)$ over $D[0,1]$ converges to that of a standard Brownian* *motion process, $\mathcal{W}(t)$, for t on the unit interval; i.e.,*

$$C_i(\cdot, r) \Longrightarrow \mathcal{W}(\cdot)$$

where $\Longrightarrow$ denotes convergence in distribution (i.e., weak convergence). Further, we assume that for every $t \in [0,1]$, the family of random variables $\{C_i^2(t,r) : r = 1, 2, \ldots\}$ is uniformly integrable. See Billingsley (1968, Chapter 4) for more details.

Since our procedure observes the sum of differences between two systems, we need to establish that the standardized partial sum of differences between two systems behaves like $\mathcal{W}(t)$. We can show this by defining $Z_{i\ell}(j) = X_{ij} - X_{\ell j}$ for $j = 1, 2, \ldots, r$, $i \neq \ell$, and letting

$$C_{i\ell}(t,r) = \frac{\sum_{j=1}^{\lfloor rt \rfloor} Z_{i\ell}(j) - rt(\mu_i - \mu_\ell)}{v_{i\ell}\sqrt{r}}. \tag{4}$$

Lemma 1 *For $i \neq \ell$, if $\mathbf{X}_i$ and $\mathbf{X}_\ell$ ($\mathbf{X}_k = \{X_{kj}; j = 1, 2, \cdots\}$) satisfy Assumption 1 and are independent, then there exists a constant $v_{i\ell}^2$ such that*

$$C_{i\ell}(\cdot, r) \Longrightarrow \mathcal{W}(\cdot) \text{ as } r \to \infty.$$

The proof of Lemma 1 can be found in Kim and Nelson (2004).

Lemma 2 (Fabian 1974) *Let $\mathcal{W}(t,\Delta)$ be a Brownian motion process on $[0, +\infty)$, with $\mathrm{E}[\mathcal{W}(t,\Delta)] = \Delta \cdot t$ and $\mathrm{Var}[\mathcal{W}(t,\Delta)] = t$, where $\Delta > 0$. Let*

$$\begin{aligned} L(t) &= -\mathcal{A} + \mathcal{B}t \\ U(t) &= \mathcal{A} - \mathcal{B}t \end{aligned}$$

for some $\mathcal{A} > 0$ and $\mathcal{B} = \Delta/(2c)$, for some positive integer c. Let $R(t)$ denote the interval $(L(t), U(t))$ and let T^* be the first time that $\mathcal{W}(t,\Delta) \notin R(t)$. Finally, let $\mathcal{E}$ be the event that $\mathcal{W}(T^*,\Delta) \leq L(T^*)$. Then

$$\Pr\{\mathcal{E}\} = \sum_{\ell=1}^{c} (-1)^{\ell+1} \left(1 - \frac{1}{2}\mathcal{I}(\ell = c)\right) \exp\{-2\mathcal{A}\mathcal{B}(2c - \ell)\ell\}.$$

Remark: $R(t)$ is the continuation region defined by the upper bound $U(t)$ and the lower bound $L(t)$. The event $\mathcal{E}$ will correspond to an incorrect selection (incorrectly eliminating the best system from consideration by exiting the region going down).

For our procedure we also make the following assumption. Let

$$\begin{aligned} X_{kj} &= \mu_k + \varepsilon_{kj} \\ X_{ij} &= \mu_i + \varepsilon_{ij} \end{aligned} \tag{5}$$

represent the output processes from systems k and i, where $\{\varepsilon_{kj}, j = 1, 2, \ldots\}$ and $\{\varepsilon_{ij}, j = 1, 2, \ldots\}$ are independent, mean-zero, stationary stochastic processes satisfying Assumption 1 (this is the model adopted by Schruben 1983, for instance).

For proof of the validity of $\mathcal{KN}$++, we also need a result from Billingsley (Theorem 4.4, page 27, 1968).

Lemma 3 *If* $Y_n \Longrightarrow Y$ *and* $Z_n \xrightarrow{P} a$*, where* a *is a constant, then* $(Y_n,\ Z_n) \Longrightarrow (Y,\ a)$.

Without loss of generality, suppose that the true steady-state means of the systems are indexed so that $\mu_k \geq \mu_{k-1} \geq \cdots \geq \mu_1$. Now we present our main result:

Theorem 1 *If* $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X_k}$ *are independent, each satisfies Assumption 1 and Equation (5), then* $\liminf_{\delta \to 0} \Pr\{\ \mathcal{KN}{++}$ *selects* $k\ \} \geq 1 - \alpha$ *provided* $\mu_k \geq \mu_{k-1} + \delta$.

Kim and Nelson (2004) originally designed the $\mathcal{KN}$++ procedure to use an h^2 value of $2c\eta$ (used in the following proof) instead of $h^2 = 2c\eta d$ (as described in the procedure in this thesis). Due to the $\mathcal{KN}$++ procedure's use of a variance estimator with the strong consistency property, the proof of asymptotic validity is similar to the known-variances case; and the value of $h^2 = 2c\eta$ originally used in $\mathcal{KN}$++ is the same as if the asymptotic variances were known. The h^2 value used in the original designation of the $\mathcal{KN}$++ procedure is smaller than h^2 in $\mathcal{KN}$+, which treats the variances as unknown. The smaller h^2 value results in a narrower continuation region for $\mathcal{KN}$++ compared to $\mathcal{KN}$+. This narrower region has been proven to be asymptotically valid. However, for finite samples, it could potentially cause the procedure to eliminate good systems early on, decreasing performance of the procedure (see Goldsman, Kim, Marshall and Nelson 2000). To avoid this potential loss in performance, a larger value of h^2 is used than is required to prove asymptotic validity, and of course, asymptotic validity still holds for the larger value of h^2. Goldsman, Kim, Marshall and Nelson (2002) chose to use the h^2 value as computed in $\mathcal{KN}+$ with $c = 1$, and their experiments showed that $\mathcal{KN}++$ works better with this adjustment. Thus, our procedure incorporates that adjustment, although the following proof uses the original value of h^2 which is also theoretically valid.

Proof of Theorem 1: We begin by considering the case of only two systems, denoted k and i, with $\mu_k \geq \mu_i + \delta$. We also assume that v_{ik}^2 is known, so that

$$N_{ik}(r) = N_{ik} = \left\lfloor \frac{h^2 v_{ik}^2}{\delta^2} \right\rfloor$$

for all r, where we select the value of η so that $g(\eta) = \frac{\alpha}{k-1}$. We relax the assumption of known asymptotic variance later.

Let

$$T(\delta) = \min\left\{r : r \geq n_0 \text{ and } |\bar{X}_k(r) - \bar{X}_i(r)| \geq W_{ik}(r)\right\}.$$

Thus, $T(\delta)$ is the stage at which the procedure terminates by leaving the continuation region.

Let ICS denote the event that an incorrect selection is made. Then

$$\begin{aligned}
&\Pr\{\text{ICS}\} \\
&= \Pr\left\{\bar{X}_k(T(\delta)) - \bar{X}_i(T(\delta)) \leq -W_{ik}(T(\delta))\right\} \\
&= \Pr\left\{\sum_{j=1}^{T(\delta)} (X_{kj} - X_{ij}) \leq -T(\delta) W_{ik}(T(\delta))\right\} \\
&= \Pr\left\{\sum_{j=1}^{T(\delta)} (X_{kj} - X_{ij}) \leq \min\left\{0, \frac{-h^2 v_{ik}^2}{2\delta c} + \frac{\delta T(\delta)}{2c}\right\}\right\} \\
&= \Pr\left\{\frac{\sum_{j=1}^{T(\delta)} (X_{kj} - X_{ij}) - (\mu_k - \mu_i) T(\delta)}{v_{ik}\sqrt{N_{ik}+1}} + \frac{(\mu_k - \mu_i) T(\delta)}{v_{ik}\sqrt{N_{ik}+1}}\right. \\
&\qquad \left. \leq \min\left\{0, \frac{-h^2 v_{ik}^2}{2\delta c v_{ik}\sqrt{N_{ik}+1}} + \frac{\delta T(\delta)}{2 c v_{ik}\sqrt{N_{ik}+1}}\right\}\right\}
\end{aligned}$$

$$\leq \Pr\left\{ \frac{\sum_{j=1}^{T(\delta)}(X_{kj}-X_{ij})-(\mu_k-\mu_i)T(\delta)}{v_{ik}\sqrt{N_{ik}+1}} + \frac{\delta T(\delta)}{v_{ik}\sqrt{N_{ik}+1}} \right.$$

$$\left. \leq \min\left\{0, \frac{-h^2 v_{ik}^2}{2\delta c v_{ik}\sqrt{N_{ik}+1}} + \frac{\delta T(\delta)}{2 c v_{ik}\sqrt{N_{ik}+1}}\right\}\right\}. \tag{6}$$

The inequality arises because we replace $\mu_k - \mu_i$ with δ, which is no larger. To establish the result we will show that $\limsup_{\delta\to 0}\Pr\{\text{ICS}\} \leq \frac{\alpha}{k-1}$. To do so, let

$$C_{ki}(t,\delta) = \frac{\sum_{j=1}^{\lfloor (N_{ik}+1)t \rfloor}(X_{kj}-X_{ij}) - (N_{ik}+1)(\mu_k-\mu_i)t}{v_{ik}\sqrt{N_{ik}+1}},$$

for $0 \leq t \leq 1$, where we express C_{ki} as a function of δ, instead of $N_{ik}+1$, since N_{ik} is a function of δ. Further, define

$$\begin{aligned}\widehat{T}(\delta) &= \min\left\{t \in \left\{\frac{n_0}{N_{ik}+1}, \frac{n_0+1}{N_{ik}+1}, \ldots, 1\right\} : \right. \\ &\qquad \left. \left|C_{ki}(t,\delta) + \frac{(N_{ik}+1)\delta t}{v_{ik}\sqrt{N_{ik}+1}}\right| \geq \frac{h^2 v_{ik}^2}{2\delta c v_{ik}\sqrt{N_{ik}+1}} - \frac{(N_{ik}+1)\delta t}{2 c v_{ik}\sqrt{N_{ik}+1}}\right\}.\end{aligned}$$

Clearly, $\widehat{T}(\delta) = T(\delta)/(N_{ik}+1)$. Also define the stopping time of the corresponding continuous-time process as

$$\begin{aligned}\tilde{T}(\delta) &= \min\left\{t \geq \frac{n_0}{N_{ik}+1} : \right. \\ &\qquad \left. \left|C_{ki}(t,\delta) + \frac{(N_{ik}+1)\delta t}{v_{ik}\sqrt{N_{ik}+1}}\right| \geq \frac{h^2 v_{ik}^2}{2\delta c v_{ik}\sqrt{N_{ik}+1}} - \frac{(N_{ik}+1)\delta t}{2 c v_{ik}\sqrt{N_{ik}+1}}\right\}.\end{aligned}$$

Notice that for fixed δ, $C_{ki}(\widehat{T}(\delta),\delta)$ corresponds to the right-hand limit of a point of discontinuity of $C_{ki}(\cdot,\delta)$. We can show that $\widehat{T}(\delta) \to \tilde{T}(\delta)$ with probability 1 as $\delta \to 0$, making use of the fact that $1/(N_{ik}+1) \to 0$ with probability 1. Thus, in the limit, we can focus on $C_{ki}(\tilde{T}(\delta),\delta)$.

Then Assumption 1, Lemma 1, and the Continuous Mapping Theorem (CMT, Theorem 5.5 of Billingsley 1968) imply that

$$C_{ki}(t,\delta) + \frac{(N_{ik}+1)\delta t}{v_{ik}\sqrt{N_{ik}+1}} \Longrightarrow \mathcal{W}(t,\Delta)$$

as $\delta \to 0$, where

$$\Delta = \lim_{\delta\to 0}\frac{(N_{ik}+1)\delta}{v_{ik}\sqrt{N_{ik}+1}} = h.$$

Let

$$\mathcal{A}(\delta) = \frac{h^2 v_{ik}^2}{2\delta c v_{ik}\sqrt{N_{ik}+1}} \stackrel{\delta\to 0}{\longrightarrow} \frac{h}{2c} \equiv \mathcal{A}$$

$$\mathcal{B}(\delta) = \frac{(N_{ik}+1)\delta}{2c v_{ik}\sqrt{N_{ik}+1}} \stackrel{\delta\to 0}{\longrightarrow} \frac{h}{2c} \equiv \mathcal{B}. \tag{7}$$

Notice that the stopping time $\tilde{T}(\delta)$ is the first time t at which the event

$$\left\{\left|C_{ki}(t,\delta) + \frac{(N_{ik}+1)\delta t}{v_{ik}\sqrt{N_{ik}+1}}\right| - \mathcal{A}(\delta) + \mathcal{B}(\delta)t \geq 0\right\}$$

occurs. Define the mapping $s_\delta : D[0,1] \to \Re$ such that $s_\delta(Y) = Y(T_{Y,\delta})$ where

$$T_{Y,\delta} = \inf\{t : |Y(t)| - \mathcal{A}(\delta) + \mathcal{B}(\delta)t \geq 0\}$$

for every $Y \in D[0,1]$ and $\delta > 0$. Similarly, define $s(Y) = Y(T_Y)$ where

$$T_Y = \inf\{t : |Y(t)| - \mathcal{A} + \mathcal{B}t \geq 0\}$$

for every $Y \in D[0,1]$ and $\delta > 0$. Notice that

$$s_\delta\left(C_{ki}(t,\delta) + \frac{(N_{ik}+1)\delta t}{v_{ik}\sqrt{N_{ik}+1}}\right) = C_{ki}(\tilde{T}(\delta),\delta) + \frac{(N_{ik}+1)\delta\tilde{T}(\delta)}{v_{ik}\sqrt{N_{ik}+1}}$$

and

$$s\left(\mathcal{W}(\cdot,\Delta)\right) = \mathcal{W}(T_{\mathcal{W}(\cdot,\Delta)},\Delta).$$

We need to show that

$$s_\delta\left(\mathcal{G}_{ki}(\cdot,\delta)\right) \Longrightarrow s\left(\mathcal{W}(\cdot,\Delta)\right)$$

as $\delta \to 0$, where

$$\mathcal{G}_{ki}(t,\delta) \equiv C_{ki}(t,\delta) + \frac{(N_{ik}+1)\delta t}{v_{ik}\sqrt{N_{ik}+1}}$$

for $t \in [0,1]$ and $\delta > 0$. However, this follows from the CMT by noting that the only discontinuities of s correspond to sample paths of $|\mathcal{W}(t,\Delta)|$ that, when they touch the boundary $\mathcal{A} - \mathcal{B}t$ for the first time, do not cross it again until some non-zero time increment later; the collection of all such paths have probability 0 because when a Brownian motion process touches a boundary it touches it at a dense set of times neighboring that point.

Therefore, by Lemma 1 and the CMT,

$$\begin{aligned}
\limsup_{\delta\to 0} \Pr\{\text{ICS}\} &\leq \Pr\{\mathcal{W}(t,\Delta) \text{ exits continuation region through the lower boundary}\} \\
&= \sum_{\ell=1}^{c} (-1)^{\ell+1}\left(1-\frac{1}{2}\mathcal{I}(\ell=c)\right)\exp\left\{-2\frac{h^2}{(2c)^2}(2c-\ell)\ell\right\} \text{ (by Lemma 2)} \\
&= \sum_{\ell=1}^{c} (-1)^{\ell+1}\left(1-\frac{1}{2}\mathcal{I}(\ell=c)\right)\exp\left\{-\frac{\eta}{c}(2c-\ell)\ell\right\} = \frac{\alpha}{k-1},
\end{aligned}$$

where the equality follows from the way we choose η.

Now consider $k \geq 2$ systems and let CS be the event that k is selected (correct selection) and let ICS_i be the event that an incorrect selection is made when systems k and i are considered *in isolation*. Then

$$\Pr\{\text{CS}\} = 1 - \Pr\{\text{ICS}\} \geq 1 - \sum_{i=1}^{k-1} \Pr\{\text{ICS}_\text{i}\},$$

which implies that

$$\begin{aligned}
\liminf_{\delta\to 0} \Pr\{\text{CS}\} &\geq \liminf_{\delta\to 0}(1 - \sum_{i=1}^{k-1} \Pr\{\text{ICS}_\text{i}\}) \\
&= 1 - \sum_{i=1}^{k-1} \liminf_{\delta\to 0} \Pr\{\text{ICS}_\text{i}\} \\
&= 1 - \sum_{i=1}^{k-1} \frac{\alpha}{k-1} \\
&= 1-\alpha.
\end{aligned}$$

The first inequality comes from the fact that it is more difficult for each inferior system (i) to individually eliminate the best system (k) than for only some or one of them to do so.

This argument establishes the asymptotic validity of a special case of $\mathcal{KN}$++ in which v_{ik}^2 is known. To prove the validity of $\mathcal{KN}$++ in general, we replace v_{ik}^2 by a strongly consistent estimator of it, S_{ik}^2. Therefore — considering again the case $k = 2$ — at termination we have

$$N_{ik}(T(\delta)) = \left\lfloor \frac{h^2 S_{ik}^2(T(\delta))}{\delta^2} \right\rfloor .$$

To establish the result, we note the following:

1. With probability 1, $T(\delta)$ goes to infinity as $\delta \to 0$. This is because the continuation region implied by δ' contains the continuation region implied by δ if $\delta' < \delta$.

2. As a consequence of item 1, the number of sampling stages goes to infinity as $\delta \to 0$, insuring $S^2_{ik}(T(\delta)) \to v^2_{ik}$ with probability 1. Since strong consistency implies convergence in probability, Lemma 3 can be applied to $(C_{ik}(T(\delta), \delta), S^2_{ik}(T(\delta)))$.

3. As a consequence of item 2, $N_{ik}(T(\delta))$ goes to infinity with probability 1 as $\delta \to 0$.

4. As a consequence of item 3, and a straightforward application of the random-change-of-time theorem (Theorem 17.1, Billingsley 1968), the standardized difference still converges in distribution to $\mathcal{W}(t, \Delta)$ with $\Delta = h$. The terms in (7) also converge as in the known variance case.

A subtle point in the derivation is that in a finite sample we may have $T(\delta) > N_{ik}(T(\delta)) + 1$ if a variance update occurs at time $T(\delta)$. However, in the limit $\tilde{T}(\delta) \leq 1$ since it is not possible for $\mathcal{W}(t, \Delta)$ to exit the continuation region for the first time beyond the end of the region. □

The experimenter will still take independent samples in the sense that each sample — where one sample is made up of a single observation from each system still in contention — is independent from the next. In other words, the output data from an individual system are independent. The only change to be made is to make use of CRN for all input data where possible. Therefore, the data from a given sample (the set of observations taken from all systems in contention) will be correlated (i.e., between-system dependence). Figure 7 illustrates that the data, X_{ij}, within a population i are i.i.d. but the data from a given sample j are correlated with each other.

$$\begin{array}{ccccc} X_{11}, & X_{12}, & \ldots, & X_{1n} & \leftarrow \text{i.i.d.} \\ \rho \updownarrow & \rho \updownarrow & & \rho \updownarrow & \\ X_{21}, & X_{22}, & \ldots, & X_{2n} & \leftarrow \text{i.i.d.} \end{array}$$

Figure 7: How the Use of CRN Affects the Data.

We only need to be able to induce positive correlation but in practice we do not know how much correlation is actually induced. If the experimenter has concerns about the magnitude or sign of the induced correlation (ρ), a first stage sample can be used to estimate ρ. Although with finite samples the estimate will not be very precise, it can provide a general idea of the amount of correlation that is induced.

3.3 Experimental Setup and Results

In order to test the performance of $\mathcal{KN}$++ with CRN, we need to first be able to generate correlated multivariate Bernoulli data. For this purpose, we use the NORTA (NORmal To Anything) method proposed by Cario and Nelson (1997). For this method, we specify a correlation structure for the Bernoulli data we want to generate. We then transform that correlation structure into one for normal data. The transformation is done in such a way that once we convert the normal data to the desired distribution, it will have the correlation structure originally specified. Thus, once the appropriate correlation structure for the normal is established, k normals are generated and then transformed into uniforms. Those uniforms are used to generate Bernoulli data which will have the specified correlation. For more in-depth information on this procedure, see Cario and Nelson's paper or Appendix C for the necessary pieces of code and some explanation of the steps required.

In practice, the experimenter will not be able to induce a specified amount of correlation, but for Monte Carlo testing purposes we choose to induce the same amount of correlation between all the systems and test the implications over a wide range of values. This provides a chance to see how the procedure will perform if only slight correlation can be induced, and how the results will be improved if more correlation is possible. The experimenters will thus be given a lower bound on performance, but will also see the potential savings in observations if they believe they can induce a larger amount of correlation. We test values of $\rho = 0$ (no CRN), 0.05, 0.1, 0.15. 0.2, 0.25, and 0.5.

We test the procedure under two primary configurations. The first is an "unfavorable" configuration of competing systems' p_i-values known as the Slippage Configuration (SC). For purposes of procedure evaluation we assume that k is the best system and that under

SC, $p_k = p$ and $p_{k-1} = p_{k-2} = \cdots = p_1 = p - \delta$. The second configuration is a "more favorable" configuration of the p_i-values which we will denote as MFC. In this configuration we let $p_k = p$, $p_{k-1} = p - \delta$, and $p_{k-2} = \cdots = p_1 = p - 2\delta$. In the MFC, all systems are still fairly competitive with the best system, but the main competition is between the best and second best systems. This configuration will represent a situation where the experimenter feels all systems are viable options for the best (highly competitive with each other) but is potentially more likely to occur than the very restrictive SC.

To compare our results to those of Paulson's procedure, his odds ratio IZ is converted into a difference given p_k using the equation

$$\theta = \frac{p_k/(1 - p_k)}{(p_k - \delta)/(1 - p_k - \delta)} \ . \tag{8}$$

It should be noted that if we want to define the procedure by an odds ratio IZ rather than a difference based IZ, we can always convert to an odds ratio if we have some idea about the probabilities. The odds ratio and difference-based IZ's are related by Equation (8), so given a desired θ value and an estimate of p_k we can find the equivalent δ value. Even if we have no idea of the probabilities, we can do a quick experiment to get an estimate for conversion purposes.

We first test the proposed procedure under the slippage configuration for various values of k and ρ to get a general idea of performance. We find that our procedure works very well under the slippage configuration, particularly when a moderate amount of correlation ($\rho \geq 0.25$) can be induced. For instance we see in Table 23, with $k = 5$ systems, $n_0 = 10$, and $\delta = 0.06$ under the SC, once $\rho = 0.10$ we can achieve approximately the same $E(N)$ as Paulson's. However, as ρ increases, we can achieve significant savings in total observations. For instance, if we were able to induce more correlation in the data, at $\rho = 0.20$ our $E(N)$ drops to 827, a savings of over 100 observations compared to Paulson's. Even when k is larger (10 and 15) our procedure still performs well compared to Paulson's in the slippage configuration when a moderate amount of correlation can be induced. For instance, in Table 7, we see that if we can induce a correlation of $\rho \geq 0.20$ (actually a slightly smaller

Table 7: Monte Carlo Results for $k = 10$, $\delta = 0.06$ and $p_k = 0.85$ in the SC.

ρ	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	$\mathcal{B}_{\text{P}}$ $\widehat{\text{PCS}}$	$\mathcal{B}_{\text{P}}$ $\widehat{E(N)}$
0	0.951	2478	0.958	2051
0.10	0.954	2229		
0.15	0.957	2103		
0.20	0.949	1980		

amount, probably around $\rho = 0.17$ would most likely suffice only we did not test that value), then we perform better than $\mathcal{B}_{\text{P}}$ in terms of $E(N)$.

We next test our procedure under a very favorable configuration of the probabilities for selecting the winner. This configuration is designed so that each ranked probability is separated by the next largest probability by the odds-ratio indifference zone value of θ. In other words we let

$$\frac{p_i(1-p_j)}{p_j(1-p_i)} = \theta, \text{ for } i = 2, 3, \ldots, k \text{ and } j = i - 1.$$

Obviously, this configuration is designed with respect to Paulson's procedure since it uses θ; but we can easily convert our desired δ value to θ in order to define our probabilities the same way for comparison purposes. We will denote this configuration as the Equally-Spaced Configuration (EC). If $k = 5$, $p_k = 0.85$, and $\delta = 0.11$ (i.e., $\theta = 2$), the EC will give us the following probabilities: $p = [0.26, 0.42, 0.59, 0.74, 0.85]$. This configuration is obviously a very favorable configuration where only a few of the systems are competitive and the rest are clearly inferior.

We found that under the EC, our procedure does not perform well relative to Paulson's, particularly as k increases and for large δ. This is because the EC makes it very easy for Paulson's procedure to eliminate inferior systems very early on during sampling. Our procedure, however, must take a fixed number of initial samples from each system in order to estimate the variance. Thus in very favorable configurations, when we only care about detecting large differences between the best and other systems, Paulson's procedure can eliminate the inferior systems earlier than ours can. Indeed, our total number of observations is often greater than that of Paulson's, particularly for values of $k > 4$. For details, see

Table 23 in the Appendix. In addition, our procedure maps a somewhat linear function of the sums of the differences. On the other hand, Paulson's looks at θ raised to the power of the sums of the differences; hence large differences between systems are magnified in Paulson's procedure, much more so than in ours. Therefore, even though our procedure works well under the slippage configuration for scenarios when k is large and δ is small or when $k \leq 5$ and δ is relatively large, we feel we cannot recommend our procedure for use in these situations due to the decrease in performance when we are in a very favorable configuration. In other words, if we are in one of these two specified situations, we perform well under SC but not under very favorable configurations. Since we do not know the true configuration of the probabilities, we feel we cannot recommend our procedure in these scenarios since performance could potentially be poor if the configuration is favorable. Thus, we recommend that our procedure only be used for values of $k \leq 5$, values of $\delta \leq 0.1$ or when the experimenter believes all or most of the systems will be competitive. In other words, our procedure works well when the selection problem is difficult, i.e., involving less favorable configurations, and when we wish to be able to detect small differences between the best and second best system.

In order to determine which value of n_0 should be used, we tested values of $n_0 = 10$, 20, 30, and 40 and look at the resulting PCS estimate as well as how our estimates of $E(N)$ compare to those of Paulson's. We test $p_k = 0.85$, $k = 2, 3, 4, 5$, and values of $\rho = 0.05$, 0.1, 0.15. 0.2, 0.25, 0.5. Experiments were run under various configurations and δ values. An example of the results for $k = 3$ and $\delta = 0.06$ under the SC is shown in Table 8 and more extensive results are found in Table 24 in the Appendix for $\delta = 0.06$ under the SC.

These results indicate that a good choice of n_0 is 10. This value almost always results in the lowest number of total observations. However, some of the PCS values are lower than the desired 0.95 (since we set $\alpha = 0.05$), particularly for larger values of ρ. This is partly because our procedure is only asymptotically guaranteed to meet the PCS constraint, and furthermore we are using an approximation to the normal. In addition, as ρ increases, the X_{ij} values become more and more alike (particularly when the true differences between the systems are small); this makes it harder to get a good variance estimate. The PCS values

Table 8: $\mathcal{KN}$++ Bernoulli Tests of n_0 Values for $k = 3$ when $p_k = 0.85$ in the SC.

n_0	δ	ρ	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	$\mathcal{B}_{\text{P}}$ $\widehat{E(N)}$
10	0.06	0.05	0.949	475	501
10	0.06	0.10	0.948	451	
10	0.06	0.15	0.944	426	
10	0.06	0.20	0.943	398	
10	0.06	0.25	0.938	374	
10	0.06	0.50	0.913	245	
20	0.06	0.05	0.948	476	
20	0.06	0.10	0.947	450	
20	0.06	0.15	0.943	425	
20	0.06	0.20	0.941	399	
20	0.06	0.25	0.939	373	
20	0.06	0.50	0.914	245	
30	0.06	0.05	0.950	478	
30	0.06	0.10	0.948	451	
30	0.06	0.15	0.947	427	
30	0.06	0.20	0.944	400	
30	0.06	0.25	0.942	374	
30	0.06	0.50	0.916	248	
40	0.06	0.05	0.951	478	
40	0.06	0.10	0.951	454	
40	0.06	0.15	0.947	427	
40	0.06	0.20	0.947	403	
40	0.06	0.25	0.944	377	
40	0.06	0.50	0.928	253	

increase as the n_0 value increases since we have more values upon which to base our initial variance estimate. However, the gain in PCS with increased n_0 is not very large since the procedure updates the variance estimate every 10 observations. Thus the initial number of observations does not have as significant an impact because of the high frequency of variance updates. We believe the potential for significant savings in observations outweighs the potential for slight loss in PCS. Therefore we choose to use an n_0 value of 10 since it typically yields the lowest $E(N)$ values. Although there may be some loss in expected PCS, our values never drop below 0.90 and are often very close to the desired value of 0.95. If the experimenter wants a more certain PCS guarantee, a larger value of n_0 (30, for instance) can be used when δ is small that will provide larger PCS than with $n_0 = 10$ as well as lower $E(N)$ values compared to Paulson's.

Each experiment was run for 100,000 replications and results were compared to Paulson's under the same k, p_k, and IZ for $\mathrm{P}^* = 0.95$. We test values of $\delta = 0.03$, 0.06, and 0.09 under the SC and MFC for all values of $k \leq 5$ and $\rho = 0$ (no CRN), 0.05, 0.1, 0.15. 0.2, 0.25, and 0.5. Two values of $p_k = 0.35$ and 0.85 are considered. All results are for an $n_0 = 10$. A small portion of the results that illustrate some of the key findings is shown in Tables 9 and 10. $\widehat{\mathrm{PCS}}$ and $\widehat{E(N)}$ denote the Monte Carlo estimates of the PCS and expected value of total observations, respectively. For complete results see Tables 25–31 in the Appendix.

When $k = 2$, we beat Paulson's procedure over all configurations as long as $\rho \geq 0.10$. The savings when more correlation can be induced can be significant. For instance, with $k = 2$, $p_k = 0.85$ and $\delta = 0.06$, even a ρ value of 0.15 produces considerable savings, with an estimated number of observations of 201 versus 250 for Paulson's. In the case when $k = 3$, $\rho \geq 0.15$ is sufficient to produce $E(N)$ values that are less than or equal to those of Paulson's for all configurations. When $k = 4$, a ρ value of 0.20 provides results better than Paulson's for all configurations, though a value of $\rho \geq 0.15$ suffices in almost all instances. For example, when $k = 4$, $p_k = 0.85$ and $\delta = 0.06$, under the SC we achieve an $E(N) = 727$ for $\rho = 0.05$ and using Paulson's procedure we have $E(N) = 731$, but under the MFC our $E(N) = 529$ for $\rho = 0.15$ while Paulson's $E(N) = 518$. For $k = 5$, we require a $\rho \geq 0.25$ to beat Paulson's in the MFC case, though $\rho = 0.15$ is all that is necessary under the SC.

Table 9: Monte Carlo Results for $k = 2$ and $p_k = 0.85$ in the SC.

δ	ρ	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	$\mathcal{B}_{\text{P}}$ $\widehat{\text{PCS}}$	$\mathcal{B}_{\text{P}}$ $\widehat{E(N)}$
0.03	0.00	0.951	893	0.954	850
0.03	0.05	0.951	856		
0.03	0.10	0.951	804		
0.03	0.15	0.950	761		
0.03	0.20	0.949	713		
0.03	0.25	0.947	669		
0.03	0.50	0.937	430		
0.06	0.00	0.946	238	0.963	250
0.06	0.05	0.944	225		
0.06	0.10	0.943	213		
0.06	0.15	0.942	201		
0.06	0.20	0.941	187		
0.06	0.25	0.937	176		
0.06	0.50	0.923	116		
0.09	0.00	0.940	113	0.971	126
0.09	0.05	0.939	107		
0.09	0.10	0.936	101		
0.09	0.15	0.934	96		
0.09	0.20	0.933	91		
0.09	0.25	0.931	84		
0.09	0.50	0.931	60		

Table 10: Monte Carlo Results for $k = 4$ and $p_k = 0.85$ in the SC and MFC.

δ	ρ	Config.	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	$\mathcal{B}_{\text{P}}$ $\widehat{\text{PCS}}$	$\mathcal{B}_{\text{P}}$ $\widehat{E(N)}$
0.03	0.00	SC	0.957	2871	0.956	2559
0.03	0.05	SC	0.958	2736		
0.03	0.10	SC	0.955	2584		
0.03	0.15	SC	0.957	2438		
0.03	0.20	SC	0.954	2297		
0.03	0.25	SC	0.953	2155		
0.03	0.50	SC	0.946	1408		
0.03	0.00	MFC	0.983	2277	0.983	1823
0.03	0.05	MFC	0.983	2169		
0.03	0.10	MFC	0.983	2047		
0.03	0.15	MFC	0.984	1934		
0.03	0.20	MFC	0.983	1821		
0.03	0.25	MFC	0.982	1711		
0.03	0.50	MFC	0.979	1127		
0.06	0.00	SC	0.953	767	0.959	731
0.06	0.05	SC	0.949	727		
0.06	0.10	SC	0.948	690		
0.06	0.15	SC	0.947	651		
0.06	0.20	SC	0.943	613		
0.06	0.25	SC	0.939	571		
0.06	0.50	SC	0.907	373		
0.06	0.00	MFC	0.982	623	0.986	518
0.06	0.05	MFC	0.981	593		
0.06	0.10	MFC	0.980	561		
0.06	0.15	MFC	0.979	529		
0.06	0.20	MFC	0.978	498		
0.06	0.25	MFC	0.976	465		
0.06	0.50	MFC	0.961	308		
0.09	0.00	SC	0.945	363	0.962	346
0.09	0.05	SC	0.943	345		
0.09	0.10	SC	0.942	327		
0.09	0.15	SC	0.936	310		
0.09	0.20	SC	0.934	292		
0.09	0.25	SC	0.928	274		
0.09	0.50	SC	0.914	187		
0.09	0.00	MFC	0.979	300	0.987	247
0.09	0.05	MFC	0.979	285		
0.09	0.10	MFC	0.976	271		
0.09	0.15	MFC	0.976	257		
0.09	0.20	MFC	0.974	243		
0.09	0.25	MFC	0.971	228		
0.09	0.50	MFC	0.966	158		

We also performed a few tests under a very difficult problem, when $\delta = 0.01$. The results indicate that our procedure works well for difficult selection problems. A $\rho \geq 0.1$ is sufficient to provide savings in $E(N)$ when $k = 2$ or $k = 3$ in the SC. See Table 11 for results.

All of our results are compared to $\mathcal{B}_{\mathrm{P}}$ run without the use of CRN, because Paulson's procedure is not designed to be used in such a way. If we were to arbitrarily apply CRN to the procedure anyway, Paulson's procedure actually becomes less efficient in terms of $E(N)$ as more correlation is induced (see Table 32). The inefficiency occurs because larger values of ρ make it harder to differentiate between systems and thus the procedure must be able to take the correlation into account in order to maintain efficiency.

Table 11: Monte Carlo Results for $\delta = 0.01$ and $p_k = 0.85$ in the SC.

k	ρ	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	$\mathcal{B}_{\mathrm{P}}$ $\widehat{\text{PCS}}$	$\mathcal{B}_{\mathrm{P}}$ $\widehat{E(N)}$
2	0.0	0.952	7628	0.955	7289
2	0.1	0.951	6842		
3	0.0	0.955	16029	0.954	14621
3	0.1	0.955	14426		

Therefore, the user can be fairly certain to achieve savings in $E(N)$ using our procedure when $\rho \geq 0.15$ for $k \leq 4$ and $\rho \geq 0.25$ for $k = 5$. As previously mentioned, the user does not necessarily know how much correlation will be induced. However, if he is concerned about how much correlation he can expect, an estimate of the correlation can be calculated after the initial sample size to give some indication of its magnitude. As long as it appears reasonable to expect values of $\rho \geq 0.15$ (or 0.25 in the case of $k = 5$), then our procedure is recommended. However, even if we cannot estimate the correlation with any accuracy, our procedure is only slightly less efficient than Paulson's when ρ is very small (0.05 to 0.1), particularly for $k \leq 3$, so there is little risk in using it and potentially large savings in total observations.

3.4 Conclusion for Bernoulli Selection

We present a couple of new procedures for Bernoulli selection. Among these, a modified $\mathcal{KN}{+}{+}$ shows competitive performances compared to $\mathcal{B}_{\mathrm{P}}$ and there are cases where $\mathcal{KN}{+}{+}$ clearly defeats $\mathcal{B}_{\mathrm{P}}$ in terms of total number of observations. Use of the modified $\mathcal{KN}{+}{+}$ is recommended when it is important to be able to detect small differences between the best and second best system ($\delta \leq 0.1$) for the case when k is small (say, $k \leq 5$) or when all of the inferior systems are considered to be highly competitive with the best.

CHAPTER IV

THE MULTINOMIAL SELECTION PROBLEM

The multinomial selection problem is very similar to that of the Bernoulli selection problem. The main difference is that the probability of success for each system is independent of each other for the Bernoulli problem, whereas the probabilities are "dependent" for the multinomial one. The multinomial problem corresponds to finding which of a set of options is most likely to be the best. The resulting data is still binary, but now for each observation there can be only one winner (1) while the rest are losers (0). The goal of selection procedures is to select the system that has the highest probability of being the most desirable on any given trial, where the term "most desirable" is defined by the nature of the response. Let X_{km} be the mth observation from population k. We might want to select the system i with the largest value p_i, where $p_i = \Pr\{X_{im} > X_{jm}, \forall i \neq j\}$, assuming that we want to select the system that will give us the largest response (although we can look at the smallest or more general responses as well).

In the following section, we will give examples of multinomial selection scenarios and cover some current procedures. We will then focus on one particular procedure, the $\mathcal{M}_{\mathrm{BK}}$ procedure proposed by Bechhofer and Kulkarni (1984), and discuss our Monte Carlo simulation to provide more-detailed results with respect to that procedure. We will then introduce a multi-factor extension of the $\mathcal{M}_{\mathrm{BK}}$ procedure. In addition, we propose a multi-factor extension to the $\mathcal{M}_{\mathrm{BG}}$ procedure and then conclude with a summary of our results in multinomial selection.

4.1 Examples, Notation, and Background

The standard example of multinomial selection is preference surveys. For instance, we may want to find out which of a group of political candidates most people will vote for, or which brand of soft drink is the favorite among teenagers. These procedures also find application in the simulation environment. For instance, we may be interested in finding which of k simulated manufacturing layouts is most likely to produce the greatest throughput, or the highest profit.

Much of the notation is similar to that of the Bernoulli problem. Let k denote the number of systems we wish to test. Let p_i be the probability of system i being the best out of all populations in contention on any given trial, and note that $\sum_{i=1}^{k} p_i = 1$. We will assume that the ordered probabilities are $p_{[1]} \leq p_{[2]} \leq \cdots \leq p_{[k]}$, and thus we wish to select the system associated with $p_{[k]}$. The probabilities, their order, and their pairings with the alternatives are unknown. Let $X_j = (X_{1j}, X_{2j}, \ldots, X_{kj})$ be an observation taken from all alternatives at observation j, where X_{ij} =1 (0) if system i won (lost). Let $Y_{im} = \sum_{j=1}^{m} X_{ij}$ denote the total wins for system i after m observations.

The first multinomial selection procedure is due to Bechhofer, Elmaghraby, and Morse (1959). It is similar to the $\mathcal{B}_{\text{SH}}$ procedure for Bernoulli data in that it is a single-stage procedure that takes exactly n observations from each system. The number of observations taken from each system is determined either by budget constraints or by the user choosing a specific P* and odds-ratio IZ value θ and looking up the corresponding n in the table of results produced by the authors. A portion of the $\mathcal{M}_{\text{BEM}}$ table is given in Table 12 and more complete results can be found in Gibbons, Olkin and Sobel (1977) or Bechhofer, Santner, and Goldsman (1995).

$\mathcal{M}_{\text{BEM}}$ Procedure

Setup: For given k, select θ and P*, and using the table, select n, or select n based on budget constraints.

Sampling: Take n random observations, $X_j = (X_{1j}, X_{2j}, ...X_{kj})$, $j = 1, \ldots, n$, in one stage.

Terminal decision rule: For each system, calculate $Y_{in} = \sum_{j=1}^{n} X_{ij}, i = 1, \ldots, k$. Select the system with the largest sample sum as the winner. If there is a tie, randomize.

Table 12: Total Observations Needed from Each System for the $\mathcal{M}_{\text{BEM}}$ Procedure when $k = 4$ and Given θ and P*.

	P*			
θ	0.75	0.90	0.95	0.99
3.0	8	16	23	39
2.8	9	19	26	45
2.6	10	22	31	53
2.4	12	26	37	63
2.2	15	33	46	79
2.0	20	43	61	104
1.8	29	61	87	147
1.6	46	98	139	235
1.4	92	196	278	471
1.2	326	692	979	1660

Example: Suppose that we wish to test 4 different possible layouts for the assembly area in a fast food restaurant. We have simulated each layout and wish to select the one where workers will most likely be able to assemble orders the fastest. Therefore, for each observation we will run a simulation of each of the four layouts, and the one that provides the shortest assembly time will be the winner (1) and the rest will receive zeros. Assume that we want to meet a probability requirement of $\text{P}^* = 0.95$ with an odds ratio indifference parameter of $\theta = 1.6$. Table 12 shows that we need to take 139 observations. If after taking 139 random multinomial observations we obtain $Y_1 = 34$, $Y_2 = 52$, $Y_3 = 43$, and $Y_4 = 10$, then we would select layout 2 as the overall winner. $\square$

Procedure $\mathcal{M}_{\text{BEM}}$ does not allow for early termination if a decision can be reached before all samples have been taken. To improve on this procedure, Bechhofer and Kulkarni (1984) propose procedure $\mathcal{M}_{\text{BK}}$ which does allow for early termination. This procedure is the main focus of our multinomial work and will be discussed in detail in the following section. For a more in-depth discussion of multinomial selection procedures, see Bechhofer, Santner, and Goldsman (1995).

4.2 The $\mathcal{M}_{\mathrm{BK}}$ Procedure

The $\mathcal{M}_{\mathrm{BK}}$ procedure for multinomial data proposed by Bechhofer and Kulkarni (1984) is very similar to their procedure for Bernoulli data mentioned earlier. It is a closed, sequential procedure that employs the use of strong curtailment. As with their Bernoulli procedure, n, the maximum number of samples to be taken from each system, is typically set by the user due to budget constraints and thus the procedure does not require a pre-specified PCS. However, if a specific PCS is desired, then n could be found by looking up the value in the table (see Table 4.2 in the Appendix) that matches the desired P^* and θ.

M_{BK} Procedure

Setup: Select n, the maximum number of observations to take from each system for a given k.

Sampling: At the mth stage of sampling, take the random multinomial observation $X_m = (X_{1m}, X_{2m}, \ldots, X_{km})$.

Stopping rule: Calculate the sample sums Y_{im} through stage m. Stop sampling at the first stage m where there exists a system i such that

$$Y_{im} \geq Y_{jm} + n - m \text{ for all } j \neq i \ (1 \leq i, j \leq k).$$

Terminal decision rule: Let the random variable N represent the value of m when sampling terminates. If $N < n$, the procedure will terminate with a single category — the one with the largest sample sum. Select this category as the winner, i.e., the one associated with $p_{[k]}$. If $N = n$ and there are multiple categories tied for the largest sample sum, then randomize to pick the winner.

Example: Suppose we want to know which of three candidates is most likely to be selected as the next city council representative. Due to a time constraint there is only time to call 10 voters. Assume after 8 calls we have the following total number of votes for each candidate, $Y_8 = (4, 2, 2)$. At this point we would stop and select the first candidate as the winner because the other two candidates could at best catch up in the remaining two calls that we had planned to make. □

As mentioned above, the value of n for this procedure can be chosen by the experimenter to satisfy budget constraints or from a table given a desired PCS constraint and indifference-zone odds ratio. However, the current results available for the $\mathcal{M}_{\mathrm{BK}}$ procedure are very limited in scope, making it hard to use the tables to guarantee a given PCS for the procedure. Because of this and the added information a more-thorough table would provide, we use a Monte Carlo simulation of the procedure to obtain more-detailed results.

The previously available results are only for $k = 2$, 3, 4, 5, and 6. They cover values of θ = 1, 1.4, 1.8, 2.2, 2.6, and 3.0. Most importantly, the currently available results only cover values of n from 2 to 20 by two (2, 4, 6, 8, ..., 20). This is particularly limiting because for most k and θ we cannot achieve a "good" PCS (0.9 or better) in that range of n.

Using a Monte Carlo simulation, we provide results for values of k from 2 to 10. We cover values of $\theta = (1, 1.1, 1.2, \ldots, 1.6, 1.8, 2.0, 2.2, 2.6)$ and values of $n = (1, 2, 3, 4, \ldots, 149, 150, 155, 160, 165, \ldots, 195, 200, 210, 220, 230, 240, 250)$. For complete results see Tables 33–41 in the Appendix.

The simulation is validated by comparing our results to those available previously. Our results match to two decimal places those provided by Bechhofer and Kulkarni. In addition, exact results were calculated using a recursive formula for the procedure when $k = 2$; these results also match to two decimal places. Thus, we are confident that our Monte Carlo simulation provides an accurate extension to the results for the $\mathcal{M}_{\mathrm{BK}}$ multinomial procedure.

CHAPTER V

MULTI-FACTOR MULTINOMIAL SELECTION

A multi-factor extension of the $\mathcal{M}_{\text{BEM}}$ procedure was proposed by Bechhofer, Goldsman, and Jennison (1989). The procedure is a single-stage procedure that must take all observations from each factor-level with no ability to stop early if a decision can already be reached. We propose a multi-factor extension to the $\mathcal{M}_{\text{BK}}$ procedure that will allow for an early decision to be made before the termination of sampling. Our procedure provides the same PCS as that of the Bechhofer, Goldsman, and Jennison (1989) procedure with fewer total observations.

In this chapter we will review the procedure by Bechhofer, Goldsman, and Jennison (1989). We then introduce a generalized procedure for $\mathcal{M}_{\text{BK}}$ which can be used to test multiple independent factors simultaneously. We also present some results for the procedure to highlight potential savings that can be obtained with its use.

5.1 The $\mathcal{M}_{\text{BGJ}}$ Multi-Factor Multinomial Procedure

Bechhofer, Goldsman, and Jennison (1989) introduced a multi-factor multinomial procedure $\mathcal{M}_{\text{BGJ}}$ that is a generalization of the $\mathcal{M}_{\text{BEM}}$ single-stage procedure. The procedure is designed for use in multi-factor multinomial experiments with multiplicativity. In other words, it is used when there are multiple factors of interest that the experimenter wishes to test and all of the responses from a factor-level are completely independent of the responses from all the levels of the other factors. The procedure enables the user to study multiple factors with only slightly more than half the total observations than would be necessary to study the two factors independently.

An example of such an experiment is a survey where an advertising agency wishes to determine the favorite soft drink, the favorite brand of toilet paper, and the favorite long distance telephone service from a given set of options. Assuming that a person's response for their favorite soft drink has no relationship to their choices of toilet paper or long distance telephone service, and so on, then the multi-factor procedure can be used. Another example can be taken from a simulation experiment. Assume that we are interested in both the mean and variance of the daily average waiting times of customer's at a local bank. Given that mean and variance estimators are independent of each other, we could use the multi-factor procedure to study them simultaneously, rather than having to perform two independent experiments.

As mentioned, $\mathcal{M}_{\mathrm{BGJ}}$ is a single-stage procedure that is a multi-factor extension to $\mathcal{M}_{\mathrm{BEM}}$. The experimenter selects the number of observations to take from each factor level based on budget constraints. Alternatively, the user can specify a desired PCS and θ value and use a table of $\mathcal{M}_{\mathrm{BEM}}$ results to determine the maximum n value needed in order to obtain the desired PCS (see Bechhofer, Goldsman, and Jennison 1989 or Bechhofer, Santner, and Goldsman 1995 for extended $\mathcal{M}_{\mathrm{BEM}}$ tables). The procedure then takes all observations from all factor-levels, and selects the level with the most wins within each factor as the overall winner for that factor. The notation and formal specification of the procedure taken from Bechhofer et al. (1989) follow. Note that the procedure is given for a two-factor (bivariate) selection problem, but the extension to more factors is straightforward.

Suppose we are interested in two independent multinomial factors A and B each with a and b number of levels respectively. Let X_{ijr} $(1 \le i \le a, 1 \le j \le b)$, $r = 1, 2, \ldots, n$, denote independent matrix observations from a single bivariate multinomial population Π with unknown probability matrix p_{ij} $(1 \le i \le a, 1 \le j \le b)$. Let p_{ij} $(p_{ij} \ge 0, \sum_{i=1}^{a} \sum_{j=1}^{b} p_{ij} = 1)$ denote the probability that $X_{ijr} = 1$. Note that if $X_{ijr} = 1$ then all other values are zero for that matrix observation, i.e., $X_{i'j'r} = 0$ for all $i' \ne i$ or $j' \ne j$. Let $\alpha_i = \sum_{j=1}^{b} p_{ij}$ $(1 \le i \le a)$ and $\beta_j = \sum_{i=1}^{a} p_{ij}$ $(1 \le i \le b)$ denote the marginal success probabilities for the row and column factors, respectively. Then $\sum_{i=1}^{a} \alpha_i = \sum_{j=1}^{b} \beta_j = 1$. We assume that all factor-level responses are independent of those of other factors, and therefore $p_{ij} = \alpha_i \cdot \beta_j$ for all i and j. Due to the assumed independence, $\{\sum_{j=1}^{b} X_{ijr}$ $(1 \le i \le a)\}$ and $\{\sum_{i=1}^{a} X_{ijr}$ $(1 \le j \le b)\}$, $r = 1,\ 2,\ \ldots, n$, can be regarded as independent vector-observations from a single-factor a-level $(a \ge 2)$ and b-level $(b \ge 2)$ multinomial population with probability vector $\boldsymbol{\alpha} =$

$(\alpha_1, \ldots, \alpha_a)$ and $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_b)$ respectively. We will denote the ordered probabilities as $\alpha_{[1]} \leq \cdots \leq \alpha_{[a]}$ and $\beta_{[1]} \leq \cdots \leq \beta_{[b]}$. Therefore we want to select the factor-levels corresponding to $\alpha_{[a]}$ and $\beta_{[b]}$. The probabilities, their order, and their pairings with the alternatives are unknown.

The procedure uses an odds-ratio indifference zone, which is specified by the experimenter prior to experimentation and used to look up the appropriate number of observations. Therefore, we guarantee the following indifference-zone probability requirement:

$$\text{PCS} \geq \text{P}^* \text{ whenever } \alpha_{[a]} \geq \theta_\alpha \alpha_{[a-1]} \ \underline{\text{and}}\ \beta_{[b]} \geq \theta_\beta \beta_{[b-1]}, \tag{9}$$

where the θ_α, θ_β and P^* values are specified by the user prior to experimentation.

After n matrix-observations have been observed, let $y_{ijn} = \sum_{r=1}^{n} x_{ijr}$, $A_{in} = \sum_{j=1}^{b} y_{ijn}$ and $B_{jn} = \sum_{i=1}^{a} y_{ijn}$. The quantities $A_{[1]n} \leq \cdots \leq A_{[a]n}$ and $B_{[1]n} \leq \cdots \leq B_{[b]n}$ denote the ordered values of the A_{in} and B_{jn} respectively. If equal values of A_{in} or B_{jn} occur then the tied values should be arranged in any definite order. The procedure follows.

$\mathcal{M}_{\text{BGJ}}$ Procedure

Setup: For given a and b, select θ and P^*, and using the table, select n, or select n based on budget constraints.

Sampling: Take n independent matrix-observations, X_{ijr} $(1 \leq i \leq a, 1 \leq j \leq b)$, $r = 1, \ldots, n$, in one stage.

Terminal decision rule: Calculate $A_{[a]n}$ and $B_{[b]n}$. If exactly s $(1 \leq s \leq a)$ of the A_{in} are tied for largest, and exactly t $(1 \leq t \leq b)$ of the B_{jn} are tied for largest, then select one of these $s \cdot t$ event combinations at random, and assert that it is the event associated with $\alpha_{[a]}$ and $\beta_{[b]}$.

As mentioned above, this procedure allows the user to test two or more factors (f) simultaneously, while requiring fewer observations (exactly $1/f$ of the observations in the symmetric case) than if the tests were run independently. Detailed results can be found in Bechhofer, Goldsman, and Jennison (1989). The most savings in total observations arise in experiments which are symmetrical in nature, i.e., each factor has the same number of levels and same indifference-zone parameter θ.

Bechhofer, Goldsman, and Jennison (1989) showed that the overall PCS can be factored into the product of the individual PCS values. Let us denote the PCS of the multi-factor experiment by $\mathrm{P}^{(n)}_{(a,b)}\{\mathrm{CS}|\boldsymbol{\alpha},\boldsymbol{\beta}\}$ where a and b are the number of levels for factor A and B, respectively, and n is the number of observations to be taken from each factor-level. Then

$$\mathrm{P}^{(n)}_{(a,b)}\{\mathrm{CS}|\boldsymbol{\alpha},\boldsymbol{\beta}\} = \mathrm{P}^{(n)}_{(a)}\{\mathrm{CS}|\boldsymbol{\alpha}\} \times \mathrm{P}^{(n)}_{(b)}\{\mathrm{CS}|\boldsymbol{\beta}\}$$

for all (a,b,n) with $a,b \geq 2$, $n \geq 1$, under multiplicativity.

In selecting the value of n, we want to identify the smallest n that will satisfy Equation (9) under the Least Favorable Configuration (LFC) of the α_i and β_j. Thus the resulting value of n will be the smallest n from all possible configurations of the probabilities that satisfy Equation (9). The LFC is the configuration that will minimize the value of $\mathrm{P}^{(n)}_{(a,b)}\{\mathrm{CS}|\boldsymbol{\alpha},\boldsymbol{\beta}\}$ according to Equation (9). Kesten and Morse (1959) solved the LFC problem for single-factor experiments. Due to the factorization of the PCS into the individual single-factor PCS values, we can use their result in order to define the LFC (also know as the slippage configuration in this case) for our problem. For a two-factor experiment, the LFC is given by

$$\alpha_{[1]} = \cdots = \alpha_{[a-1]} = \frac{1}{\theta_\alpha + a - 1},\ \alpha_{[a]} = \frac{\theta_\alpha}{\theta_\alpha + a - 1}$$
$$\beta_{[1]} = \cdots = \beta_{[b-1]} = \frac{1}{\theta_\beta + b - 1},\ \beta_{[b]} = \frac{\theta_\beta}{\theta_\beta + b - 1}.$$

For symmetric factors (same θ values and number of levels), it was shown that the smallest n that will satisfy Equation (9) is chosen from a table of results from single-factor experiments, where

$$\mathrm{P}^{(n)}_{(a)}\{\mathrm{CS}|\theta,\mathrm{LFC}\} \geq \sqrt{\mathrm{P}^*}$$

where $a = b$, $\theta_a = \theta_b = \theta$ and P^* is the desired PCS for the multi-factor experiment. Therefore, under the symmetric case, if we have a table of results for the $\mathcal{M}_{\mathrm{BEM}}$ single-factor procedure that covers sufficient values of n, we can find our desired n value by looking up the value that corresponds to θ and $\sqrt{\mathrm{P}^*}$. For example, given $a = b = 2$, $\theta = 1.8$ and

$P^* = 0.90$ for a two-factor experiment, we look up the value of n in the single-factor table that corresponds to $\theta = 1.8$ and the smallest P^* value such that $P^* \geq \sqrt{0.90} = 0.9487$ which gives us $n = 31$. Bechhofer, Goldsman, and Jennison (1989) provide extended table results for the single-factor $\mathcal{M}_{\text{BEM}}$ procedure for use in selecting n.

For the nonsymmetric case, the optimal choice of n is much more complicated. In this case, we wish to select the smallest n such that $P_1 \cdot P_2 \geq P^*$ where $P_1 = P^{(n)}_{(a)}\{\text{CS}|\theta_\alpha, \text{LFC}\}$ and $P_2 = P^{(n)}_{(b)}\{\text{CS}|\theta_\beta, \text{LFC}\}$ for the two-factor experiment.

5.2 A Multi-Factor $\mathcal{M}_{\text{BK}}$ Procedure with Early Termination

The $\mathcal{M}_{\text{BGJ}}$ multi-factor multinomial procedure with multiplicativity is a single-stage procedure which does not allow for early termination when the winner is clear. We propose a procedure that is a generalization of the $\mathcal{M}_{\text{BK}}$ multinomial procedure which does allow for early termination. In the following subsections we will introduce the procedure, discuss the factorization of the PCS, show how to select n, and give our experimental setup and results.

5.2.1 The Multi-Factor $\mathcal{M}_{\text{MGK}_{\text{BK}}}$ Procedure

The notation for $\mathcal{M}_{\text{MGK}_{\text{BK}}}$ is the same as that used for the $\mathcal{M}_{\text{BGJ}}$ procedure as is much of the procedure itself. The experimenter selects P^* and θ_i for all factors being tested. Tables are used to determine n, the maximum number of observations that will be taken from each factor-level, or n is set due to budget constraints. We sample until we reach n or until all factors are able to stop sampling; this happens whenever one level is sufficiently ahead so that all other levels can at best catch up to the winner by the end of sampling. The detailed procedure is given below for a two-factor (bivariate) selection problem, but the extension to more factors is straightforward.

$\mathcal{M}_{\text{MGK}_{\text{BK}}}$ **Procedure**

Setup: For given a and b, select θ and P^*, and using the table, select n, or select n based on budget constraints.

Sampling: At the rth stage of sampling, take a random multinomial matrix-observation, X_{ijr} $(1 \leq i \leq a, 1 \leq j \leq b)$.

Stopping rule: Calculate the sample sums $A_{[i]r}$ and $B_{[j]r}$ for all i and j through stage r. Stop sampling at the first stage r where there exist two factor-levels A_i and B_j such that

$$A_{[a]r} \geq A_{[a-1]r} + n - r$$

and

$$B_{[b]r} \geq B_{[b-1]r} + n - r.$$

Remark: If the experimenter wants to run the experiment where individual factors can terminate sampling before all factors have made a selection, then the above stopping rule is altered to indicate that sampling stops from factor f whenever its individual stopping constraint is met and continues for the rest of the factors until they meet their individual stopping constraint.

Terminal decision rule: Let the random variable N represent the value of r when sampling terminates. If $N < n$, the procedure will terminate with a single category (made up of two factor-levels) — the factor-levels with the largest sample sums. Select this category as the winner. If $N = n$ and exactly s $(1 \leq s \leq a)$ of the A_{in} are tied for largest, and exactly t $(1 \leq t \leq b)$ of the B_{jn} are tied for largest, then select one of these $s \cdot t$ event combinations at random, and assert that it is the event associated with $\alpha_{[a]}$ and $\beta_{[b]}$.

5.2.2 Factorization of the PCS for the $\mathcal{M}_{\text{MGK}_{\text{BK}}}$ Procedure

The PCS for this multi-factor procedure factors into the PCS values of the individual single-factor experiments just as it does in the $\mathcal{M}_{\text{BGJ}}$ procedure. This is because for the single as well as multi-factor experiments, we stop only when the outcome, and thus the PCS values, cannot be changed. Ignoring the chance for a system to catch up and tie the best, the preceding statement is straightforward. However, even by employing strong curtailment, where we stop sampling when the second best system could still potentially tie the first, we still maintain the same PCS as in the single-stage selection procedure. The proof is analogous to the one given for the $\mathcal{B}_{\text{BK}}$ Bernoulli selection problem, which also employs strong curtailment. See Bechhofer and Kulkarni (1982) for more details. Therefore, our procedure achieves the same PCS as the $\mathcal{M}_{\text{BGJ}}$ procedure, just as the $\mathcal{M}_{\text{BK}}$ single-factor experiment achieves the same PCS as the $\mathcal{M}_{\text{BEM}}$ procedure.

As an example regarding the factorization of the PCS, we present the following exact PCS calculation for 2 systems each with 2 levels and $n = 3$. The matrices are the sums of all wins at the time of termination. For instance $\begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}$ denotes a point where sampling has stopped after A_2 and B_2 have each won twice. If we sum up the variance components in Table 13 from each potential stopping point, we get a PCS of

$$\alpha_2^2\beta_2^2(1 + 8\alpha_1\beta_1 + 2\alpha_1\beta_2 + 2\alpha_2\beta_1) = \alpha_2^2\beta_2^2(1 + 2\alpha_1)(1 + 2\beta_1)$$

after a little algebra. The results obtained for one factor, if we were to run each factor independently, are shown in Table 14.

Table 13: Exact PCS Calculation of the $\mathcal{M}_{\text{MGK}_{\text{BK}}}$ Procedure for Two Systems, Each with Two Levels and Max $n = 3$.

possible correct selection outcomes	probability of outcome
$\begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}$	$\alpha_2^2\beta_2^2$
$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$	$2\alpha_1\alpha_2^2\beta_1\beta_2^2$
$\begin{pmatrix} 0 & 0 \\ 1 & 2 \end{pmatrix}$	$2\alpha_1\alpha_2^2\beta_2^3$
$\begin{pmatrix} 0 & 1 \\ 0 & 2 \end{pmatrix}$	$2\alpha_2^3\beta_1\beta_2^2$
$\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$	$6\alpha_1\alpha_2^2\beta_1\beta_2^2$

Thus the PCS for the individual experiment is $\alpha_2^2 + 2\alpha_1\alpha_2^2$ and assuming independence, the overall PCS for both experiments run independently would be $\alpha_2^2\beta_2^2(1 + 2\alpha_1)(1 + 2\beta_1)$. It is clear that in this case the PCS results for the combined multi-factor experiment and two independently run experiments are equal.

Table 14: Exact PCS Calculation of the $\mathcal{M}_{\text{BK}}$ Procedure for One System with Two Levels and Max $n = 3$.

possible correct selection outcomes	probability of outcome
$\begin{pmatrix} 0 & 2 \end{pmatrix}$	α_2^2
$\begin{pmatrix} 1 & 2 \end{pmatrix}$	$2\alpha_1\alpha_2^2$

Now suppose that we allow the experimenter to stop sampling from one (or more) factor(s) once their individual results have been determined, regardless of whether the other factors must continue sampling. This could be especially useful if taking the extra observation had some additional time or cost associated with it. For instance, if a phone survey is being conducted, the extra time it takes to ask the second question could be saved if you no longer need to determine the result of that second factor. In this example, the primary cost of sampling comes from dialing the phone, making an introduction and convincing the person to answer the survey. The asking of the actual question is a small portion of the time involved, so asking additional questions does not add significant cost. However, some time is obviously added to the process when multiple questions are asked. Therefore, the elimination of one or more questions once their results have been conclusively determined could save some time in the overall process.

If we allow for early termination of sampling, the exact calculation of the PCS becomes more tedious. Table 15 lists all possible termination points (they are the same as those shown in the first table from this section). All possible paths to reach the given termination point are also enumerated and labelled for discussion purposes. For those paths where an early termination is possible, we continue to sample only from the system whose result is yet undetermined. In the situation where one factor is no longer sampled and where we are only taking additional observations from the one factor that remains, the samples are written as one-dimensional arrays.

Note that for the last three possible PCS outcomes there are only two possible paths for reaching the final result. This follows because if we obtain the first two sequential wins for A_2B_2, we will stop early and not take the final observation. Furthermore, by allowing

Table 15: Exact PCS Calculation of the $\mathcal{M}_{\mathrm{MGK_{BK}}}$ Procedure for Two Systems, Each with Two Levels and Max $n = 3$ Allowing for Early Termination.

possible CS outcomes	sampling path to reach outcome	probability of outcome
$\begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}$	(a.) $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	$\alpha_2^2\beta_2^2$
$\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$	(b.) $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	$\alpha_1\alpha_2^2\beta_1\beta_2^2$
	(c.) $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$\alpha_2^2\beta_1\beta_2^2$
	(d.) $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$\alpha_2^2\beta_1\beta_2^2$
	(e.) $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \end{pmatrix}$	$\alpha_1\alpha_2^2\beta_2^2$
	(f.) $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	$\alpha_1\alpha_2^2\beta_1\beta_2^2$
	(g.) $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \end{pmatrix}$	$\alpha_1\alpha_2^2\beta_2^2$
$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$	(h.) $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	$\alpha_1\alpha_2^2\beta_1\beta_2^2$
	(i.) $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	$\alpha_1\alpha_2^2\beta_1\beta_2^2$
$\begin{pmatrix} 0 & 0 \\ 1 & 2 \end{pmatrix}$	(j.) $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \end{pmatrix}$	already included in PCS
	(k.) $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \end{pmatrix}$	already included in PCS
$\begin{pmatrix} 0 & 1 \\ 0 & 2 \end{pmatrix}$	(l.) $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	already included in PCS
	(m.) $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	already included in PCS

the early termination of sampling for individual factors, we have eliminated some of the sampling paths that existed previously. Thus some of the termination points are produced by the same path as other termination points and therefore their contribution to the PCS is only included once. For instance, sampling paths (c.) and (l.) are the same, as are (g.) and (j.). Since they come to the same conclusion, only one is included in the PCS calculation.

When we sum up all of the PCS components we get

$$\mathrm{PCS} = \alpha_2^2\beta_2^2(1 + 2\alpha_1)(1 + 2\beta_1).$$

This is the same PCS as that obtained from multiplying the two individual PCS results. Therefore, the combined PCS can in fact be factored into the individual PCS values even if early termination is allowed. This result is intuitive, since if we can potentially stop one (or more) factor(s) early, the final outcome will not be affected if we stop sampling immediately or continue until the rest of the factors can terminate sampling.

Therefore, our procedure can be used either way: when all factor selections must be made before any factor can stop sampling, or when factors are allowed to terminate sampling as soon as their individual decision has been made. The PCS and $E(N)$ will be the same for each stopping rule. We suggest that the procedure be used in the manner that is easiest for the experimenter. If there is an added cost to taking the additional observations and it is easy stop sampling from one or more of the factors early, then it can be done with no loss in performance. However, if there is little or no cost to obtaining the additional information, then continue sampling from all factors until the terminal decision is made.

5.2.3 Selecting n for the $\mathcal{M}_{\mathrm{MGK_{BK}}}$ Procedure

Since the PCS values for $\mathcal{M}_{\mathrm{MGK_{BK}}}$ are identical to those obtained by the $\mathcal{B}_{\mathrm{BGJ}}$ procedure, and thus so is the factorization of the PCS, the minimum value of n needed to satisfy the constraints in Equation (9) for the symmetric case is the same as in Bechhofer, Goldsman, and Jennison (1989). Therefore, we can use a table of results for the single-stage procedure and determine n as the smallest value such that

$$\mathrm{P}_{(a)}^{(n)}\{\mathrm{CS}|\theta, \mathrm{LFC}\} \geq \sqrt{\mathrm{P}^*},$$

where $a = b$, $\theta_a = \theta_b = \theta$ and P^* is the desired PCS for the multi-factor experiment. The values of n corresponding to θ and P^* are identical for the $\mathcal{B}_{\mathrm{BEM}}$ and $\mathcal{B}_{\mathrm{BK}}$ single-stage tables. The only difference between the two is the $\mathcal{B}_{\mathrm{BK}}$ tables provide lower $E(N)$ values since it is sometimes not necessary to take all n observations in that procedure. So either table can be used to determine the value of n. However, we provide in this thesis extensive $\mathcal{B}_{\mathrm{BK}}$ tables which cover far more values of n and k than any previously available. Therefore, we recommend using the tables provided in Appendix D for choosing n. Furthermore, should the experimenter wish to test a scenario which is not covered in the tables, we have a

Monte Carlo simulation that can be used to efficiently generate results for any k, θ, and P^* combination.

As mentioned in Section 5.2.2, the optimal choice of n is more complicated for the nonsymmetric case. In this case, we wish to select the smallest n such that $\mathrm{P}_1 \cdot \mathrm{P}_2 \geq \mathrm{P}^*$ where $\mathrm{P}_1 = \mathrm{P}^{(n)}_{(a)}\{\mathrm{CS}|\theta_\alpha, \mathrm{LFC}\}$ and $\mathrm{P}_2 = \mathrm{P}^{(n)}_{(b)}\{\mathrm{CS}|\theta_\beta, \mathrm{LFC}\}$ for the two-factor experiment. More generally stated, for f factors, we wish to select the smallest n such that

$$\prod_{i=1}^{f} \mathrm{P}_i \geq \mathrm{P}^*.$$

Obviously, it can be tedious to determine n by hand since multiple combinations of the probabilities must be searched to find the combination which meets the PCS constraint and provides the smallest n value. We developed a computer program that will scan through the table values in order to make that determination automatically, so it is almost just as easy to run a nonsymmetric experiment as a symmetric one.

5.2.4 Experimental Setup and Results for $\mathcal{M}_{\mathrm{MGK_{BK}}}$

For simplicity, we will provide results for the symmetric case (each factor has the same number of levels and same IZ parameter θ) though a few examples of results for the non-symmetric case are also given. We test from 2 to 5 factors, each with 2 to 5 levels, and values of $\theta = 1.2$, 1.4, 1.6, 1.8, and 2.0. We select $\mathrm{P}^* = 0.90$ for all scenarios. The value of n is selected using our tables of Monte Carlo estimates for the $\mathcal{M}_{\mathrm{BK}}$ procedure (see the Appendix Tables 33–41). For those values of n not included in the table, our simulation was run until the desired estimates were obtained. The results for only one value of $\theta = 1.4$ are shown in Table 16. For the complete results see Table 42 in the Appendix. The tables provide results for $\widehat{\mathrm{PCS}}$ and $\widehat{E(N)}$, the Monte Carlo estimates of the PCS and $E(N)$ values, respectively. They also include individual $\widehat{E(N)}$ values which are estimates of the expected total number of observations required for a single factor run independently.

The results show that significant savings can be achieved if the experiments are run simultaneously rather than independently. For instance, for four factors, each with two levels and $\mathrm{P}^* = 0.90$ and $\theta = 1.4$, the combined experiment has an average of 123.53 observations. However, if we were to run each experiment separately we would need an

Table 16: Multi-Factor $\mathcal{M}_{\text{MGK}_{\text{BK}}}$ Monte Carlo Results for the Symmetric Case when $\theta = 1.4$ and $\text{P}^* = 0.9$.

factors	levels	max n	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	individual $\widehat{E(N)}$
2	2	95	0.901	85.81	81.89
2	3	185	0.902	173.78	168.93
2	4	275	0.900	263.04	257.79
2	5	370	0.902	357.62	352.02
3	2	117	0.901	107.62	100.82
3	3	220	0.902	208.76	200.70
3	4	320	0.902	308.25	299.67
3	5	430	0.902	417.72	408.61
4	2	133	0.901	123.53	114.63
4	3	245	0.900	233.80	223.37
4	4	355	0.898	343.29	332.19
4	5	470	0.897	457.97	446.29
5	2	146	0.896	135.59	124.94
5	3	265	0.903	253.77	241.43
5	4	385	0.903	373.14	360.02
5	5	500	0.897	488.17	474.55

average of 114.63 observations for each factor for a total of 458.52 in order to achieve the same simultaneous PCS. Regardless of the number of factors (f) tested, the combined experiment requires only a few more observations than one of the f independent single-factor experiments.

Since there are many more possible combinations of scenarios in the nonsymmetric case, only a few examples are provided here to illustrate the savings in $E(N)$ that can be expected. These results use the minimum value of n from the table of $\mathcal{M}_{\text{BK}}$ results such that $\prod_{i=1}^{f} \text{P}_i \geq \text{P}^*$. Table 17 provides an example of results for two factors and Table 18 provides results for three factors. In these tables, l_i is the number of levels for factor i and θ_i is the chosen IZ value for system i. The $\widehat{E(N)}_i$ values are the estimates of the expected number of total observations necessary to make a selection for factor i if it was run in a single-factor $\mathcal{M}_{\text{BK}}$ experiment. All results are for $\text{P}^* = 0.90$.

As observed in the symmetric case, the experimenter can gain significant savings in $E(N)$ in the nonsymmetric case as well if a single multi-factor experiment is run rather than separate independent experiments. For instance, with three factors each with 2, 3,

Table 17: Nonsymmetric Monte Carlo $\mathcal{M}_{\mathrm{MGK_{BK}}}$ Results for Two Factors.

l_1	l_2	θ_1	θ_2	n	$\widehat{E(N)}_1$	$\widehat{E(N)}_2$	$\widehat{\mathrm{PCS}}$	$\widehat{E(N)}$
2	3	1.6	1.6	73	59.98	64.45	0.901	65.80
2	3	1.6	1.8	54	43.58	46.26	0.900	47.67
2	4	1.6	1.6	102	82.76	93.31	0.900	93.89
2	4	1.6	1.8	71	58.34	63.40	0.902	64.48
3	4	1.6	1.6	116	102.09	106.06	0.901	107.94
3	4	1.6	1.8	89	78.53	79.26	0.901	81.93

Table 18: Nonsymmetric Monte Carlo $\mathcal{M}_{\mathrm{MGK_{BK}}}$ Results for Three Factors.

l_1	l_2	l_3	θ_1	θ_2	θ_3	n	$\widehat{E(N)}_1$	$\widehat{E(N)}_2$	$\widehat{E(N)}_3$	$\widehat{\mathrm{PCS}}$	$\widehat{E(N)}$
2	2	3	1.6	1.6	1.8	65	53.40	53.40	55.59	0.905	58.69
2	3	3	1.6	1.6	1.8	83	68.10	73.25	70.73	0.904	75.98
2	3	4	1.6	1.6	1.6	119	97.41	104.63	108.74	0.900	110.99

and 3 levels and $\theta = 1.6$, 1.6, and 1.8, respectively, $\mathcal{M}_{\mathrm{MGK_{BK}}}$ requires an average of 75.98 observations. If the experiments are run independently, it will take an average of 212.08 observations to obtain the same simultaneous PCS.

5.3 A Multi-Factor $\mathcal{M}_{\mathrm{BG}}$ Procedure with Early Termination

Bechhofer and Goldsman (1985, 1986) developed the $\mathcal{M}_{\mathrm{BG}}$ procedure which is based on the $\mathcal{M}_{\mathrm{BKS}}$ procedure introduced by Bechhofer, Kiefer, and Sobel (1968). Bechhofer and Goldsman (1985) studied $\mathcal{M}_{\mathrm{BKS}}$ and found that the PCS achieved in the LFC always substantially exceeds the lower bound of P^*. Thus $\mathcal{M}_{\mathrm{BKS}}$ always overprotects, resulting in more observations being taken than needed to satisfy the nominal lower bound of P^*. Therefore, Bechhofer and Goldsman add a truncation point n, the maximum number of observations to take from each system, to $\mathcal{M}_{\mathrm{BKS}}$ so that $E(N)$ is reduced while still maintaining the desired P^*. Due to the addition of n, strong curtailment can be employed. In addition to the original stopping constraint for $\mathcal{M}_{\mathrm{BKS}}$, $\mathcal{M}_{\mathrm{BG}}$ adds two additional ways to stop sampling: when n observations have been taken from each system, and whenever one system is sufficiently ahead so that the other systems can at best tie by the end of sampling. Bechhofer and Goldsman show that $\mathcal{M}_{\mathrm{BG}}$ is superior to $\mathcal{M}_{\mathrm{BKS}}$ in terms of $E(N)$ uniformly in $\boldsymbol{p}$.

Therefore, if truncation numbers exist for the given k, θ, and P^* the experimenter wishes to test, then $\mathcal{M}_{\mathrm{BG}}$ should always be used. The complete $\mathcal{M}_{\mathrm{BG}}$ procedure follows.

$\mathcal{M}_{\mathrm{BG}}$ **Procedure**

Setup: For given number of systems k, select θ and P^*, and select n using the table of truncation numbers for $\mathcal{M}_{\mathrm{BG}}$ (see Bechhofer, Santner, and Goldsman 1995).

Sampling: At the rth stage of sampling, take a random multinomial matrix-observation, X_{ir} $(1 \leq i \leq k)$.

Stopping rule: Calculate the sample sums $Y_{[i]r}$ for all i through stage r. Also calculate

$$z_r = \sum_{i=1}^{k-1} (1/\theta)^{(Y_{[k]r} - Y_{[i]r})}.$$

Stop sampling at the first stage when

$$z_r \leq \frac{1-\mathrm{P}^*}{\mathrm{P}^*} \quad \text{or} \quad r = n \quad \text{or} \quad Y_{[k]r} \geq Y_{[k-1]r} + n - r.$$

Terminal decision rule: Let the random variable N represent the value of r when sampling terminates. If $N < n$, the procedure will terminate with a single category — the one with the largest sample sum. Select this category as the winner, i.e., the one associated with $p_{[k]}$. If $N = n$ and there are multiple categories tied for the largest sample sum, then randomize to pick the winner.

We propose a procedure that is a generalization of the $\mathcal{M}_{\mathrm{BG}}$ multinomial procedure. $\mathcal{M}_{\mathrm{BG}}$ was designed to select the best level from a single-factor multinomial experiment. We extend $\mathcal{M}_{\mathrm{BG}}$ to allow for multiple factors to be studied simultaneously. In the following subsections we will introduce the procedure and will present our experimental setup and results.

5.3.1 The Multi-Factor $\mathcal{M}_{\mathrm{MGK_{BG}}}$ Procedure

The notation for $\mathcal{M}_{\mathrm{MGK_{BG}}}$ is the same as that used for the $\mathcal{M}_{\mathrm{MGK_{BK}}}$ procedure as is much of the procedure itself. The only change to the procedure is the inclusion of an additional stopping constraint from $\mathcal{M}_{\text{BG}}$. The experimenter selects P^* and θ_i for all factors being tested. Tables of truncation values for $\mathcal{M}_{\text{BG}}$ (found in Bechhofer, Santner, and Goldsman 1995) are used to determine n, the maximum number of observations that will be taken from each factor-level. We sample until we reach n or until all factors are able to stop sampling. Recall that a factor can stop sampling whenever one level is sufficiently ahead so that all other levels can at best catch up to the winner by the end of sampling. In this procedure, we allow for individual factors to stop sampling before all other factors have concluded sampling. The procedure can be used in two ways: when all systems must stop sampling at the same time or also when early termination of a factor is allowed, whichever setup is more easily applied to the experiment. However, more savings in terms of $E(N)$ can be realized when we allow for early termination of sampling for individual factors (more discussion on this topic will follow). Therefore, we describe the procedure in this manner, though it is still applicable to the other setup as well. The detailed procedure for two factors is given below with the extension to three or more factors being straightforward.

$\mathcal{M}_{\text{MGK}_{\text{BG}}}$ Procedure

Setup: For given a and b, select θ and P^*, and select n using the table of truncation numbers for $\mathcal{M}_{\text{BG}}$ (see Bechhofer, Santner, and Goldsman 1995).

Sampling: At the rth stage of sampling, take a random multinomial matrix-observation, X_{ijr} $(1 \leq i \leq a, 1 \leq j \leq b)$.

Stopping rule: Calculate the sample sums $A_{[i]r}$ and $B_{[j]r}$ for all i and j through stage r. Also calculate

$$z_{r_A} = \sum_{i=1}^{a-1} (1/\theta)^{(A_{[a]r} - A_{[i]r})}$$

and

$$z_{r_B} = \sum_{i=1}^{b-1} (1/\theta)^{(B_{[b]r} - B_{[i]r})}.$$

Stop sampling from factor A at the first stage when

$$z_{r_A} \leq \frac{1 - \text{P}^*}{\text{P}^*} \quad \text{or} \quad r = n \quad \text{or} \quad A_{[a]r} \geq A_{[a-1]r} + n - r.$$

Stop sampling from factor B at the first stage when

$$z_{r_B} \leq \frac{1 - \mathrm{P}^*}{\mathrm{P}^*} \quad \text{or} \quad r = n \quad \text{or} \quad B_{[b]r} \geq B_{[b-1]r} + n - r.$$

Remark: If the experimenter wants to run the experiment where all factors must terminate sampling together, then the above stopping rule is altered to indicate that sampling stops from both systems only if factors A and B both meet one of the stopping criteria.

Terminal decision rule: Let the random variable N represent the value of r when sampling terminates. If $N < n$, the procedure will terminate with a single category (made up of two factor-levels) consisting of the factor-levels with the largest sample sums. Select this category as the winner. If $N = n$ and exactly s $(1 \leq s \leq a)$ of the A_{in} are tied for largest, and exactly t $(1 \leq t \leq b)$ of the B_{jn} are tied for largest, then select one of these $s \cdot t$ event combinations at random, and assert that it is the event associated with $\alpha_{[a]}$ and $\beta_{[b]}$.

5.3.2 Experimental Setup and Results for $\mathcal{M}_{\mathrm{MGK_{BG}}}$

In the symmetric case, the number of levels and θ values are the same for each factor. Due to the assumed independence between factors, we select n in the same manner as in the $\mathcal{M}_{\mathrm{MGK_{BK}}}$ procedure. Therefore, if f is the number of factors, we find the minimum value of n from the truncation tables such that P^* for the single-factor $\mathcal{M}_{\mathrm{BG}}$ is $\geq (\mathrm{P}^*)^{(1/f)}$. However, the nature of the problem changes slightly in the nonsymmetric case and our future research will study the optimum choice of n and application of this procedure to the nonsymmetric case. For now, we will only consider this procedure for symmetric experiments.

As an example of performance of this procedure, results are provided for the two-factor (bivariate) problem. When $\mathrm{P}^* = 0.9$, the single-factor table value of P^* is approximately 0.95, which is available in the truncation table. We test the procedure under both stopping configurations: when individual factors can stop early, and when all factors must stop together. Values of $\theta = 1.2$, 1.4, 1.6, 1.8, and 2.0 are tested in the LFC for 2, 3, 4, and 5 levels. All results are for the symmetric case. Each Monte Carlo simulation is run for

100,000 replications and estimates of PCS and expected total observations ($\widehat{\text{PCS}}$ and $\widehat{E(N)}$) are calculated.

In order to present results for a case in which $k > 2$, we test the procedure for three factors, two levels for each factor, and $\text{P}^* = 0.857$. The odd value of P^* was chosen so that the truncation values corresponding to $\text{P}^* = 0.95$ for the $\mathcal{M}_{\text{BG}}$ single-factor problem could be used.

The results for the two-factor experiments where early stopping of individual factors is allowed are shown in Table 19. Note that values of $\widehat{E(N)}$ for the $\mathcal{M}_{\text{MGK}_{\text{BK}}}$ procedure presented earlier in this thesis are provided for comparison purposes. The results for the three-factor experiments with two levels where early stopping of individual factors is allowed are shown in Table 20. Similar results for the case when all systems must stop together are shown in Table 43 and Table 44 in the Appendix for the two-factor and three-factor experiments, respectively. All of these tables provide results for $\widehat{\text{PCS}}$ and $\widehat{E(N)}$, the Monte Carlo estimates of the PCS and $E(N)$ values, respectively. The tables also include individual $\widehat{E(N)}$ values which are estimates of the expected total number of observations required for a single factor run independently.

The results for both stopping rules and number of factors studied show that significant savings can be realized by performing the multi-factor $\mathcal{M}_{\text{MGK}_{\text{BG}}}$ versus running independent $\mathcal{M}_{\text{BG}}$ procedures for each factor. For instance, when we assume that early termination of sampling from individual systems is possible, for two systems, each with 5 levels and $\theta = 1.2$, $\widehat{E(N)} = 963$ when the systems are run in a combined multi-factor experiment. However, a single $\mathcal{M}_{\text{BG}}$ procedure has an $\widehat{E(N)} = 742$. So in order to study all five factors an $\widehat{E(N)} = 3710$ would be necessary. Thus $\mathcal{M}_{\text{MGK}_{\text{BG}}}$ would result in a savings of 2747 observations on average over running the experiments independently.

The results for both stopping rules also show lower $E(N)$ values than those obtained from the $\mathcal{M}_{\text{MGK}_{\text{BK}}}$ procedure with the same configuration. This result was expected since in the single-factor case $\mathcal{M}_{\text{BG}}$ is superior to $\mathcal{M}_{\text{BK}}$. Therefore, for multi-factor multinomial experiments where independence between factors can be assumed, the use of $\mathcal{M}_{\text{MGK}_{\text{BG}}}$ is recommended if truncation values are available. If the table results for truncation values

Table 19: $\mathcal{M}_{\mathrm{MGK_{BG}}}$ Monte Carlo Results for Two Factors in the Symmetric Case where $\mathrm{P}^* = 0.90$, Assuming Factors Can Stop Sampling Independently.

levels	θ	n	$\widehat{\mathrm{PCS}}$	$\widehat{E(N)}$	individual $\widehat{E(N)}$	$\mathcal{M}_{\mathrm{MGK_{BK}}}$ $\widehat{E(N)}$
2	1.2	455	0.902	224.55	166.54	306.36
2	1.4	151	0.902	65.40	48.31	85.81
2	1.6	59	0.902	34.91	26.56	42.87
2	1.8	35	0.904	23.01	18.03	26.52
2	2.0	27	0.908	16.85	13.09	19.30
3	1.2	960	0.901	455.65	346.42	618.66
3	1.4	266	0.904	130.70	98.88	173.78
3	1.6	125	0.901	66.13	50.32	85.28
3	1.8	71	0.903	42.37	32.63	52.96
3	2.0	52	0.905	29.96	23.03	36.17
4	1.2	1500	0.903	701.53	537.1	947.36
4	1.4	380	0.903	198.63	152.72	263.04
4	1.6	180	0.902	98.73	76.06	128.68
4	1.8	106	0.903	62.38	47.80	78.53
4	2.0	74	0.901	43.69	33.86	54.62
5	1.2	2000	0.904	963.28	741.70	1291.66
5	1.4	510	0.902	269.98	209.64	357.62
5	1.6	240	0.903	133.25	103.53	176.14
5	1.8	142	0.903	83.19	64.37	106.16
5	2.0	98	0.904	57.92	45.01	74.22

Table 20: $\mathcal{M}_{\mathrm{MGK}_{\mathrm{BG}}}$ Monte Carlo Results for Three Factors, Each with Two Levels in the Symmetric Case where $\mathrm{P}^* = 0.857$, Assuming Factors Can Stop Sampling Independently.

θ	n	$\widehat{\mathrm{PCS}}$	$\widehat{E(N)}$	individual $\widehat{E(N)}$
1.2	455	0.856	260.03	166.54
1.4	151	0.858	75.69	48.31
1.6	59	0.858	39.78	26.56
1.8	35	0.859	25.91	18.03
2.0	27	0.870	19.01	13.09

are unavailable, $\mathcal{M}_{\mathrm{MGK}_{\mathrm{BK}}}$ should be used until we can provide more detailed results for obtaining truncation points.

In Table 21 we compare the results obtained from $\mathcal{M}_{\mathrm{MGK}_{\mathrm{BG}}}$ for both types of stopping rules: when factors must stop together and when factors can terminate sampling individually. When all systems must stop at the same time, for the selected n the procedure is overly conservative. In other words, the PCS values obtained from the procedure are significantly higher than the specified nominal value P^* and therefore more samples are taken than needed to guarantee the PCS requirement. However, the PCS results where factors can stop sampling early are approximately equal to our desired P^* and correspondingly require fewer observations. The reason for the difference in PCS results is that for this procedure the outcome of the experiment could potentially change when factors that could have otherwise stopped sampling are forced to continue. In the $\mathcal{M}_{\mathrm{MGK}_{\mathrm{BK}}}$ procedure, factors can only stop sampling early when their selection decision has already been made. Therefore, if a factor A is forced to continue sampling because another factor has not yet reached a decision, the end result for factor A will remain unchanged (ignoring the possibility of ties) regardless of the additional samples taken. However, the added constraint in the $\mathcal{M}_{\mathrm{MGK}_{\mathrm{BG}}}$ procedure allows factors to stop sampling before the selected level has enough wins so that no other level could possibly catch up. If a factor is forced to continue sampling after it could have stopped, the level that is ultimately chosen for that factor could potentially change. The additional samples will increase the likelihood of a correct selection since they will give the best system more time to catch up if it is behind. That is why the PCS values are greater. In addition, by potentially allowing other systems to catch up to the best in the additional samples, it is possible that the stopping rule for a factor A is no longer met at the current observation, even though a selection could have been made previously. This is the cause for the increased $E(N)$ when all systems must stop together.

Table 21: Comparison of $\mathcal{M}_{\mathrm{MGK}_{\mathrm{BG}}}$ Monte Carlo Results for the Symmetric Cases Assuming Factors Can Stop Sampling Early, Versus the Case in Which They Must Stop Together.

					Stop Together		Can Stop Early	
factors	levels	θ	P*	n	$\widehat{\mathrm{PCS}}$	$\widehat{E(N)}$	$\widehat{\mathrm{PCS}}$	$\widehat{E(N)}$
2	2	1.2	0.9	455	0.923	236.76	0.902	224.55
2	2	1.4	0.9	151	0.928	69.19	0.902	65.40
2	2	1.6	0.9	59	0.918	36.11	0.902	34.91
2	2	1.8	0.9	35	0.913	23.62	0.904	23.01
2	2	2.0	0.9	27	0.921	17.26	0.908	16.85
2	3	1.2	0.9	960	0.930	477.83	0.901	455.65
2	3	1.4	0.9	266	0.928	136.90	0.904	130.70
2	3	1.6	0.9	125	0.926	69.13	0.901	66.13
2	3	1.8	0.9	71	0.922	44.01	0.903	42.37
2	3	2.0	0.9	52	0.923	31.23	0.905	29.96
2	4	1.2	0.9	1500	0.932	734.23	0.903	701.53
2	4	1.4	0.9	380	0.927	207.21	0.903	198.63
2	4	1.6	0.9	180	0.925	103.32	0.902	98.73
2	4	1.8	0.9	106	0.924	64.88	0.903	62.38
2	4	2.0	0.9	74	0.922	45.19	0.901	43.69
2	5	1.2	0.9	2000	0.932	1006.26	0.904	963.28
2	5	1.4	0.9	510	0.930	280.50	0.902	269.98
2	5	1.6	0.9	240	0.926	138.29	0.903	133.25
2	5	1.8	0.9	142	0.925	86.052	0.903	83.19
2	5	2.0	0.9	98	0.923	59.928	0.904	57.92
3	2	1.2	0.857	455	0.900	282.06	0.856	260.03
3	2	1.4	0.857	151	0.909	83.17	0.858	75.69
3	2	1.6	0.857	59	0.888	41.88	0.858	39.78
3	2	1.8	0.857	35	0.878	26.82	0.859	25.91
3	2	2.0	0.857	27	0.889	19.78	0.870	19.01

To keep the procedure from being overly conservative when systems must stop sampling together, new truncation values could be found for the multi-factor experiment. However, even though it is less efficient for the procedure not to allow individual factors early termination, the $E(N)$ values are still lower that those of all other existing multi-factor multinomial experiments. It is recommended that the procedure allow for early termination of individual sampling. However, if that is not feasible due to the nature of the experiment, there will still be savings in terms of the average number of total observations necessary to make a selection.

5.4 Conclusion for Multinomial Selection

As the results for $\mathcal{M}_{\mathrm{MGK_{BK}}}$ and $\mathcal{M}_{\mathrm{MGK_{BG}}}$ show, significant savings in observations can be realized by using a combined multi-factor experiment instead of performing multiple single-factor experiments independently. For $\mathcal{M}_{\mathrm{MGK_{BK}}}$, the total number of observations in the symmetric case is approximately $1/f$ of the total number of observations when running f single-factor $\mathcal{M}_{\mathrm{BK}}$ procedures. The savings resulting from the nonsymmetric case are not quite as large, but they are still significant. In addition, $\mathcal{M}_{\mathrm{MGK_{BK}}}$ provides the same PCS guarantee as does $\mathcal{M}_{\mathrm{BGJ}}$ but with fewer observations. For instance, for four factors, each with two levels, and $\mathrm{P}^* = 0.90$ and $\theta = 1.4$, $\mathcal{M}_{\mathrm{MGK_{BK}}}$ takes an average of 123.53 observations. The same experiment using $\mathcal{M}_{\mathrm{BGJ}}$ takes 133 observations. Thus using $\mathcal{M}_{\mathrm{MGK_{BK}}}$ provides an average savings of 9.47 observations for this experiment. Therefore, for testing multiple factors where independence can be assumed, the $\mathcal{M}_{\mathrm{MGK_{BK}}}$ procedure should be used instead of $\mathcal{M}_{\mathrm{BGJ}}$ as long as a sequential procedure is applicable to the factors being studied.

However, we also introduce $\mathcal{M}_{\mathrm{MGK_{BG}}}$, the multi-factor extension to the $\mathcal{M}_{\mathrm{BG}}$ procedure. $\mathcal{M}_{\mathrm{MGK_{BG}}}$ provides significant savings in $E(N)$ when multiple factors are tested simultaneously versus running independent $\mathcal{M}_{\mathrm{BG}}$ procedures for each factors. It also provides lower $E(N)$ results than $\mathcal{M}_{\mathrm{MGK_{BK}}}$ while maintaining the same level of confidence. For instance, for two factors, each with two levels, $\mathrm{P}^* = 0.90$ and $\theta = 1.2$, $\mathcal{M}_{\mathrm{MGK_{BG}}}$ takes an average of 224.55 observations whereas the same experiment using $\mathcal{M}_{\mathrm{MGK_{BK}}}$ takes an average of 306.36 observations; an average savings of 81.81 observations. Therefore, for multi-factor multinomial experiments where independence between factors can be assumed, the use of $\mathcal{M}_{\mathrm{MGK_{BG}}}$ is recommended over all currently existing multi-factor multinomial procedures. However, due to the limited tabulated results for obtaining truncation values, it may not be feasible to run $\mathcal{M}_{\mathrm{MGK_{BG}}}$ in many situations, in which case, the $\mathcal{M}_{\mathrm{MGK_{BK}}}$ should be used until more detailed results of truncation points for $\mathcal{M}_{\mathrm{MGK_{BG}}}$ are available.

CHAPTER VI

CONTRIBUTIONS OF THIS RESEARCH

This thesis contains contributions in two major areas of work, Bernoulli and multinomial ranking and selection. The contributions are a modification of the $\mathcal{KN}$++ procedure for use with Bernoulli data and extensions to several procedures in the general class of multinomial ranking and selection. These contributions are:

- A modification to the $\mathcal{KN}$++ designed for Bernoulli data that employs the use of common random numbers in order to reduce the expected total number of observations needed to make a selection in certain cases.
- A Monte Carlo simulation of the $\mathcal{M}_{\text{BK}}$ procedure that is used to provide more extensive results than were previously available. Exact results for $k = 2$ are also provided.
- A multi-factor extension to the $\mathcal{M}_{\text{BK}}$ procedure that allows multiple factors to be studied simultaneously while providing significant savings compared to studying the same factors independently.
- A multi-factor extension to the $\mathcal{M}_{\text{BG}}$ procedure that provides the best performance in terms of expected total observations compared to all existing multi-factor multinomial procedures.

A summary of each of these contributions follows.

One contribution of this research is in the area of Bernoulli selection. Refer to Sections 2.1 and 2.2 for examples of applications of the Bernoulli selection problem. A modification to the $\mathcal{KN}$++ procedure is introduced which employs the use of common random numbers in experimentation for data which is Bernoulli in nature. We are the first to provide such a procedure that is valid for more than two systems and makes no assumptions about the systems other than the capability of inducing some amount of positive correlation between them. This procedure is designed for use in simulation experiments, where each observation comes from a simulation run, and where correlation to reduce unwanted variability can often be induced by using common random numbers. In certain cases, this procedure has the potential for significant savings in total observations over existing procedures.

Another contribution is in the area of single-factor multinomial selection. Refer to Chapter 4 for applications of multinomial selection procedures. We introduce a Monte Carlo simulation of the $\mathcal{M}_{\mathrm{BK}}$ procedure for multinomial ranking and selection. Using this simulation we are able to provide much more extensive results than are currently available. These results are important because they are needed for certain uses of $\mathcal{M}_{\mathrm{BK}}$ in order to select an input parameter for the procedure and thus the configurations that could previously be tested were very limited. We also provide an exact calculation of results for the case when $k = 2$ in order to verify that our results are accurate to two decimal places.

The remaining two contributions are in the area of multi-factor multinomial selection. For examples of applications of this type of selection procedure, see Section 5.1. We introduce $\mathcal{M}_{\mathrm{MGK_{BK}}}$, a multi-factor extension to the $\mathcal{M}_{\mathrm{BK}}$ procedure. This procedure enables the experimenter to study multiple factors simultaneously and requires approximately $1/f$ of the total number of observations compared to running f single-factor $\mathcal{M}_{\mathrm{BK}}$ procedures. $\mathcal{M}_{\mathrm{MGK_{BK}}}$ provides the same PCS with fewer expected observations than $\mathcal{M}_{\mathrm{BGJ}}$, the previous best procedure for this problem, and therefore should be chosen over $\mathcal{M}_{\mathrm{BGJ}}$ for experiments where sequential sampling is possible. In addition, due to our extension of the tables for $\mathcal{M}_{\mathrm{BK}}$, table values that are needed to determine n (the maximum number of observations the procedure needs to take from each system) are readily available, making this procedure easy to implement.

We further introduce $\mathcal{M}_{\mathrm{MGK_{BG}}}$, a multi-factor extension of the $\mathcal{M}_{\mathrm{BG}}$ procedure. This procedure also enables multiple factors to be simultaneously studied with fewer observations than would be necessary to test each factor independently. Furthermore, this procedure requires fewer observations than $\mathcal{M}_{\mathrm{MGK_{BK}}}$ while providing the same probability requirement.

$\mathcal{M}_{\text{MGK}_{\text{BG}}}$ is superior to all existing multi-factor multinomial procedures where independence between factor-level responses can be assumed and should always be used whenever table values for the truncation value n are available.

APPENDIX A

$\mathcal{KN}+$ RESULTS FOR INDEPENDENT BERNOULLI

Table 22: $\mathcal{KN}+$ Independent Bernoulli Results when $c = 1$.

δ	k	$n_0 = 30$	$n_0 = 50$	$n_0 = 70$	$\mathcal{B}_{\mathrm{P}}$
0.06	2	0.951 268	0.953 260	0.953 262	0.963 250
	3	0.952 568	0.955 540	0.954 533	0.961 501
	4	0.951 871	0.955 828	0.956 811	0.959 731
	5	0.952 1179	0.956 1120	0.957 1098	0.956 935
0.11	2	0.958 94	0.963 112	0.969 144	0.970 85
	3	0.956 189	0.961 197	0.965 230	0.965 163
	4	0.957 286	0.961 288	0.964 322	0.958 223
	5	0.957 389	0.961 384	0.962 417	0.964 311

APPENDIX B

$\mathcal{KN}$++ RESULTS WITH CRN FOR BERNOULLI

Table 23: $\mathcal{KN}$++ Bernoulli EC and SC Results when $p_k = 0.85$.

k	δ	ρ	Config.	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	$\mathcal{B}_{\text{P}}$ $\widehat{E(N)}$
2	0.06	0.05	SC	0.944	225	250
2	0.06	0.10	SC	0.943	213	
2	0.06	0.15	SC	0.942	201	
2	0.06	0.20	SC	0.941	187	
2	0.06	0.25	SC	0.937	176	
2	0.06	0.50	SC	0.923	116	
2	0.11	0.05	SC	0.940	77	85
2	0.11	0.10	SC	0.938	73	
2	0.11	0.15	SC	0.936	70	
2	0.11	0.20	SC	0.937	66	
2	0.11	0.25	SC	0.938	63	
2	0.11	0.50	SC	0.944	48	
3	0.06	0.05	SC	0.949	475	501
3	0.06	0.10	SC	0.948	451	
3	0.06	0.15	SC	0.944	426	
3	0.06	0.20	SC	0.943	398	
3	0.06	0.25	SC	0.938	374	
3	0.06	0.50	SC	0.913	245	
3	0.06	0.05	EC	0.972	413	394
3	0.06	0.10	EC	0.972	390	

Table 23: (continued).

k	δ	ρ	Config.	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	$\mathcal{B}_{\text{P}}$ $\widehat{E(N)}$
3	0.06	0.15	EC	0.970	367	
3	0.06	0.20	EC	0.968	347	
3	0.06	0.25	EC	0.966	324	
3	0.06	0.50	EC	0.950	214	
3	0.11	0.05	SC	0.937	159	163
3	0.11	0.10	SC	0.935	151	
3	0.11	0.15	SC	0.933	143	
3	0.11	0.20	SC	0.930	135	
3	0.11	0.25	SC	0.929	128	
3	0.11	0.50	SC	0.929	92	
3	0.11	0.05	EC	0.965	141	130
3	0.11	0.10	EC	0.966	134	
3	0.11	0.15	EC	0.963	127	
3	0.11	0.20	EC	0.961	120	
3	0.11	0.25	EC	0.959	113	
3	0.11	0.50	EC	0.960	82	
4	0.06	0.05	SC	0.949	727	731
4	0.06	0.10	SC	0.948	690	
4	0.06	0.15	SC	0.947	651	
4	0.06	0.20	SC	0.943	613	
4	0.06	0.25	SC	0.939	571	
4	0.06	0.50	SC	0.907	373	
4	0.06	0.05	EC	0.981	593	518
4	0.06	0.10	EC	0.980	561	
4	0.06	0.15	EC	0.979	529	

Table 23: (continued).

k	δ	ρ	Config.	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	$\mathcal{B}_{\text{P}}$ $\widehat{E(N)}$
4	0.06	0.20	EC	0.978	498	
4	0.06	0.25	EC	0.976	465	
4	0.06	0.50	EC	0.961	308	
4	0.11	0.05	SC	0.941	243	223
4	0.11	0.10	SC	0.936	230	
4	0.11	0.15	SC	0.933	219	
4	0.11	0.20	SC	0.930	207	
4	0.11	0.25	SC	0.925	195	
4	0.11	0.50	SC	0.918	141	
4	0.11	0.05	EC	0.977	202	159
4	0.11	0.10	EC	0.976	192	
4	0.11	0.15	EC	0.975	183	
4	0.11	0.20	EC	0.972	173	
4	0.11	0.25	EC	0.971	164	
4	0.11	0.50	EC	0.965	120	
5	0.06	0.05	SC	0.950	986	935
5	0.06	0.10	SC	0.948	935	
5	0.06	0.15	SC	0.947	884	
5	0.06	0.20	SC	0.944	827	
5	0.06	0.25	SC	0.940	773	
5	0.06	0.50	SC	0.912	507	
5	0.06	0.05	EC	0.986	775	619
5	0.06	0.10	EC	0.985	735	
5	0.06	0.15	EC	0.984	692	
5	0.06	0.20	EC	0.984	653	

Table 23: (continued).

k	δ	ρ	Config.	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	$\mathcal{B}_{\text{P}}$ $\widehat{E(N)}$
5	0.06	0.25	EC	0.982	610	
5	0.06	0.50	EC	0.970	405	
5	0.11	0.05	SC	0.939	330	311
5	0.11	0.10	SC	0.935	313	
5	0.11	0.15	SC	0.931	297	
5	0.11	0.20	SC	0.926	280	
5	0.11	0.25	SC	0.922	266	
5	0.11	0.50	SC	0.906	190	
5	0.11	0.05	EC	0.983	267	211
5	0.11	0.10	EC	0.981	254	
5	0.11	0.15	EC	0.980	242	
5	0.11	0.20	EC	0.977	228	
5	0.11	0.25	EC	0.975	216	
5	0.11	0.50	EC	0.968	159	

Table 24: $\mathcal{KN}$++ Bernoulli Tests of n_0 Values when $\mathrm{p}_k = 0.85$ and $\delta = 0.06$ in the Slippage Configuration.

k	n_0	ρ	$\widehat{\mathrm{PCS}}$	$\widehat{E(N)}$	$\mathcal{B}_{\mathrm{P}}$ $\widehat{E(N)}$
2	10	0.05	0.944	225	250
2	10	0.10	0.943	213	
2	10	0.15	0.942	201	
2	10	0.20	0.941	187	
2	10	0.25	0.937	176	
2	10	0.50	0.923	116	
2	20	0.05	0.943	226	
2	20	0.10	0.944	213	
2	20	0.15	0.941	201	
2	20	0.20	0.939	188	
2	20	0.25	0.936	176	
2	20	0.50	0.922	116	
2	30	0.05	0.946	228	
2	30	0.10	0.944	216	
2	30	0.15	0.943	203	
2	30	0.20	0.942	191	
2	30	0.25	0.940	178	
2	30	0.50	0.926	119	
2	40	0.05	0.948	231	
2	40	0.10	0.947	218	
2	40	0.15	0.946	206	
2	40	0.20	0.946	195	
2	40	0.25	0.943	183	
2	40	0.50	0.936	128	
3	10	0.05	0.949	475	501

Table 24: (continued).

k	n_0	ρ	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	$\mathcal{B}_{\text{P}}$ $\widehat{E(N)}$
3	10	0.10	0.948	451	
3	10	0.15	0.944	426	
3	10	0.20	0.943	398	
3	10	0.25	0.938	374	
3	10	0.50	0.913	245	
3	20	0.05	0.948	476	
3	20	0.10	0.947	450	
3	20	0.15	0.943	425	
3	20	0.20	0.941	399	
3	20	0.25	0.939	373	
3	20	0.50	0.914	245	
3	30	0.05	0.950	478	
3	30	0.10	0.948	451	
3	30	0.15	0.947	427	
3	30	0.20	0.944	400	
3	30	0.25	0.942	374	
3	30	0.50	0.916	248	
3	40	0.05	0.951	478	
3	40	0.10	0.951	454	
3	40	0.15	0.947	427	
3	40	0.20	0.947	403	
3	40	0.25	0.944	377	
3	40	0.50	0.928	253	
4	10	0.05	0.949	727	731
4	10	0.10	0.948	690	
4	10	0.15	0.947	651	

Table 24: (continued).

k	n_0	ρ	$\widehat{\mathrm{PCS}}$	$\widehat{E(N)}$	$\mathcal{B}_{\mathrm{P}}$ $\widehat{E(N)}$
4	10	0.20	0.943	613	
4	10	0.25	0.939	571	
4	10	0.50	0.907	373	
4	20	0.05	0.949	729	
4	20	0.10	0.948	690	
4	20	0.15	0.946	652	
4	20	0.20	0.942	612	
4	20	0.25	0.940	570	
4	20	0.50	0.911	374	
4	30	0.05	0.951	730	
4	30	0.10	0.950	690	
4	30	0.15	0.949	652	
4	30	0.20	0.946	613	
4	30	0.25	0.942	574	
4	30	0.50	0.911	377	
4	40	0.05	0.953	732	
4	40	0.10	0.951	693	
4	40	0.15	0.950	654	
4	40	0.20	0.948	615	
4	40	0.25	0.945	577	
4	40	0.50	0.920	383	
5	10	0.05	0.950	986	935
5	10	0.10	0.948	935	
5	10	0.15	0.947	884	
5	10	0.20	0.944	827	
5	10	0.25	0.940	773	

Table 24: (continued).

k	n_0	ρ	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	$\mathcal{B}_{\text{P}}$ $\widehat{E(N)}$
5	10	0.50	0.912	507	
5	20	0.05	0.951	987	
5	20	0.10	0.949	934	
5	20	0.15	0.947	881	
5	20	0.20	0.944	827	
5	20	0.25	0.941	773	
5	20	0.50	0.911	507	
5	30	0.05	0.953	987	
5	30	0.10	0.951	935	
5	30	0.15	0.950	883	
5	30	0.20	0.948	828	
5	30	0.25	0.943	778	
5	30	0.50	0.913	511	
5	40	0.05	0.954	988	
5	40	0.10	0.953	936	
5	40	0.15	0.952	884	
5	40	0.20	0.948	831	
5	40	0.25	0.947	780	
5	40	0.50	0.918	517	

Table 25: $\mathcal{KN}$++ Bernoulli with CRN Results for $k = 2$.

k	δ	ρ	Config.	p_k	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	$\mathcal{B}_{\text{P}}$ $\widehat{\text{PCS}}$	$\mathcal{B}_{\text{P}}$ $\widehat{E(N)}$
2	0.03	0.00	SC	0.85	0.951	893	0.954	850
2	0.03	0.05	SC	0.85	0.951	856		
2	0.03	0.10	SC	0.85	0.951	804		
2	0.03	0.15	SC	0.85	0.950	761		
2	0.03	0.20	SC	0.85	0.949	713		
2	0.03	0.25	SC	0.85	0.947	669		
2	0.03	0.50	SC	0.85	0.937	430		
2	0.06	0.00	SC	0.85	0.946	238	0.963	250
2	0.06	0.05	SC	0.85	0.944	225		
2	0.06	0.10	SC	0.85	0.943	213		
2	0.06	0.15	SC	0.85	0.942	201		
2	0.06	0.20	SC	0.85	0.941	187		
2	0.06	0.25	SC	0.85	0.937	176		
2	0.06	0.50	SC	0.85	0.923	116		
2	0.09	0.00	SC	0.85	0.940	113	0.971	126
2	0.09	0.05	SC	0.85	0.939	107		
2	0.09	0.10	SC	0.85	0.936	101		
2	0.09	0.15	SC	0.85	0.934	96		
2	0.09	0.20	SC	0.85	0.933	91		
2	0.09	0.25	SC	0.85	0.931	84		
2	0.09	0.50	SC	0.85	0.931	60		
2	0.03	0.00	SC	0.35	0.951	1451	0.956	1403
2	0.03	0.05	SC	0.35	0.951	1374		
2	0.03	0.10	SC	0.35	0.951	1305		
2	0.03	0.15	SC	0.35	0.952	1234		
2	0.03	0.20	SC	0.35	0.951	1159		
2	0.03	0.25	SC	0.35	0.951	1087		
2	0.03	0.50	SC	0.35	0.949	722		
2	0.06	0.00	SC	0.35	0.951	356	0.955	333
2	0.06	0.05	SC	0.35	0.951	338		
2	0.06	0.10	SC	0.35	0.951	320		
2	0.06	0.15	SC	0.35	0.949	302		
2	0.06	0.20	SC	0.35	0.948	284		
2	0.06	0.25	SC	0.35	0.948	266		
2	0.06	0.50	SC	0.35	0.936	173		
2	0.09	0.00	SC	0.35	0.948	155	0.967	167
2	0.09	0.05	SC	0.35	0.947	147		
2	0.09	0.10	SC	0.35	0.944	139		
2	0.09	0.15	SC	0.35	0.944	131		
2	0.09	0.20	SC	0.35	0.944	123		
2	0.09	0.25	SC	0.35	0.941	115		
2	0.09	0.50	SC	0.35	0.931	77		

Table 26: $\mathcal{KN}$++ Bernoulli with CRN Results for $k = 3$ and $p_k = 0.85$.

k	δ	ρ	Config.	p_k	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	$\mathcal{B}_{\text{P}}$ $\widehat{\text{PCS}}$	$\mathcal{B}_{\text{P}}$ $\widehat{E(N)}$
3	0.03	0.00	SC	0.85	0.955	1874	0.953	1708
3	0.03	0.05	SC	0.85	0.955	1793		
3	0.03	0.10	SC	0.85	0.954	1689		
3	0.03	0.15	SC	0.85	0.955	1594		
3	0.03	0.20	SC	0.85	0.955	1504		
3	0.03	0.25	SC	0.85	0.951	1407		
3	0.03	0.50	SC	0.85	0.942	923		
3	0.03	0.00	MFC	0.85	0.975	1597	0.975	1349
3	0.03	0.05	MFC	0.85	0.976	1525		
3	0.03	0.10	MFC	0.85	0.975	1439		
3	0.03	0.15	MFC	0.85	0.975	1351		
3	0.03	0.20	MFC	0.85	0.974	1280		
3	0.03	0.25	MFC	0.85	0.973	1203		
3	0.03	0.50	MFC	0.85	0.966	790		
3	0.06	0.00	SC	0.85	0.951	503	0.961	501
3	0.06	0.05	SC	0.85	0.949	475		
3	0.06	0.10	SC	0.85	0.948	451		
3	0.06	0.15	SC	0.85	0.944	426		
3	0.06	0.20	SC	0.85	0.943	398		
3	0.06	0.25	SC	0.85	0.938	374		
3	0.06	0.50	SC	0.85	0.913	245		
3	0.06	0.00	MFC	0.85	0.973	433	0.981	394
3	0.06	0.05	MFC	0.85	0.972	413		
3	0.06	0.10	MFC	0.85	0.972	390		
3	0.06	0.15	MFC	0.85	0.970	367		
3	0.06	0.20	MFC	0.85	0.968	347		
3	0.06	0.25	MFC	0.85	0.966	324		
3	0.06	0.50	MFC	0.85	0.950	214		
3	0.09	0.00	SC	0.85	0.942	239	0.966	236
3	0.09	0.05	SC	0.85	0.942	225		
3	0.09	0.10	SC	0.85	0.940	214		
3	0.09	0.15	SC	0.85	0.934	203		
3	0.09	0.20	SC	0.85	0.931	192		
3	0.09	0.25	SC	0.85	0.929	180		
3	0.09	0.50	SC	0.85	0.912	125		
3	0.09	0.00	MFC	0.85	0.970	208	0.982	186
3	0.09	0.05	MFC	0.85	0.968	198		
3	0.09	0.10	MFC	0.85	0.969	188		
3	0.09	0.15	MFC	0.85	0.965	177		
3	0.09	0.20	MFC	0.85	0.964	168		
3	0.09	0.25	MFC	0.85	0.960	158		
3	0.09	0.50	MFC	0.85	0.952	111		

Table 27: $\mathcal{KN}$++ Bernoulli with CRN Results for $k = 3$ and $p_k = 0.35$.

k	δ	ρ	Config.	p_k	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	$\mathcal{B}_{\text{P}}$ $\widehat{\text{PCS}}$	$\mathcal{B}_{\text{P}}$ $\widehat{E(N)}$
3	0.03	0.00	SC	0.35	0.954	3052	0.955	2778
3	0.03	0.05	SC	0.35	0.955	2899		
3	0.03	0.10	SC	0.35	0.955	2743		
3	0.03	0.15	SC	0.35	0.957	2599		
3	0.03	0.20	SC	0.35	0.954	2450		
3	0.03	0.25	SC	0.35	0.955	2287		
3	0.03	0.50	SC	0.35	0.953	1529		
3	0.03	0.00	MFC	0.35	0.977	2517	0.977	2211
3	0.03	0.05	MFC	0.35	0.976	2407		
3	0.03	0.10	MFC	0.35	0.975	2271		
3	0.03	0.15	MFC	0.35	0.976	2150		
3	0.03	0.20	MFC	0.35	0.977	2023		
3	0.03	0.25	MFC	0.35	0.975	1896		
3	0.03	0.50	MFC	0.35	0.976	1267		
3	0.06	0.00	SC	0.35	0.955	755	0.959	697
3	0.06	0.05	SC	0.35	0.957	718		
3	0.06	0.10	SC	0.35	0.957	678		
3	0.06	0.15	SC	0.35	0.955	641		
3	0.06	0.20	SC	0.35	0.954	600		
3	0.06	0.25	SC	0.35	0.954	565		
3	0.06	0.50	SC	0.35	0.939	371		
3	0.06	0.00	MFC	0.35	0.976	619	0.979	554
3	0.06	0.05	MFC	0.35	0.975	587		
3	0.06	0.10	MFC	0.35	0.977	553		
3	0.06	0.15	MFC	0.35	0.974	523		
3	0.06	0.20	MFC	0.35	0.973	493		
3	0.06	0.25	MFC	0.35	0.973	462		
3	0.06	0.50	MFC	0.35	0.966	304		
3	0.09	0.00	SC	0.35	0.956	330	0.960	302
3	0.09	0.05	SC	0.35	0.954	314		
3	0.09	0.10	SC	0.35	0.954	297		
3	0.09	0.15	SC	0.35	0.952	280		
3	0.09	0.20	SC	0.35	0.948	263		
3	0.09	0.25	SC	0.35	0.947	246		
3	0.09	0.50	SC	0.35	0.929	164		
3	0.09	0.00	MFC	0.35	0.974	266	0.978	239
3	0.09	0.05	MFC	0.35	0.974	253		
3	0.09	0.10	MFC	0.35	0.973	238		
3	0.09	0.15	MFC	0.35	0.971	227		
3	0.09	0.20	MFC	0.35	0.970	212		
3	0.09	0.25	MFC	0.35	0.971	199		
3	0.09	0.50	MFC	0.35	0.958	135		

Table 28: $\mathcal{KN}$++ Bernoulli with CRN Results for $k = 4$ and $p_k = 0.85$.

k	δ	ρ	Config.	p_k	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	$\mathcal{B}_\text{P}$ $\widehat{\text{PCS}}$	$\mathcal{B}_\text{P}$ $\widehat{E(N)}$
4	0.03	0.00	SC	0.85	0.957	2871	0.956	2559
4	0.03	0.05	SC	0.85	0.958	2736		
4	0.03	0.10	SC	0.85	0.955	2584		
4	0.03	0.15	SC	0.85	0.957	2438		
4	0.03	0.20	SC	0.85	0.954	2297		
4	0.03	0.25	SC	0.85	0.953	2155		
4	0.03	0.50	SC	0.85	0.946	1408		
4	0.03	0.00	MFC	0.85	0.983	2277	0.983	1823
4	0.03	0.05	MFC	0.85	0.983	2169		
4	0.03	0.10	MFC	0.85	0.983	2047		
4	0.03	0.15	MFC	0.85	0.984	1934		
4	0.03	0.20	MFC	0.85	0.983	1821		
4	0.03	0.25	MFC	0.85	0.982	1711		
4	0.03	0.50	MFC	0.85	0.979	1127		
4	0.06	0.00	SC	0.85	0.953	767	0.959	731
4	0.06	0.05	SC	0.85	0.949	727		
4	0.06	0.10	SC	0.85	0.948	690		
4	0.06	0.15	SC	0.85	0.947	651		
4	0.06	0.20	SC	0.85	0.943	613		
4	0.06	0.25	SC	0.85	0.939	571		
4	0.06	0.50	SC	0.85	0.907	373		
4	0.06	0.00	MFC	0.85	0.982	623	0.986	518
4	0.06	0.05	MFC	0.85	0.981	593		
4	0.06	0.10	MFC	0.85	0.980	561		
4	0.06	0.15	MFC	0.85	0.979	529		
4	0.06	0.20	MFC	0.85	0.978	498		
4	0.06	0.25	MFC	0.85	0.976	465		
4	0.06	0.50	MFC	0.85	0.961	308		
4	0.09	0.00	SC	0.85	0.945	363	0.962	346
4	0.09	0.05	SC	0.85	0.943	345		
4	0.09	0.10	SC	0.85	0.942	327		
4	0.09	0.15	SC	0.85	0.936	310		
4	0.09	0.20	SC	0.85	0.934	292		
4	0.09	0.25	SC	0.85	0.928	274		
4	0.09	0.50	SC	0.85	0.914	187		
4	0.09	0.00	MFC	0.85	0.979	300	0.987	247
4	0.09	0.05	MFC	0.85	0.979	285		
4	0.09	0.10	MFC	0.85	0.976	271		
4	0.09	0.15	MFC	0.85	0.976	257		
4	0.09	0.20	MFC	0.85	0.974	243		
4	0.09	0.25	MFC	0.85	0.971	228		
4	0.09	0.50	MFC	0.85	0.966	158		

Table 29: $\mathcal{KN}$++ Bernoulli with CRN Results for $k = 4$ and $p_k = 0.35$.

k	δ	ρ	Config.	p_k	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	$\mathcal{B}_{\text{P}}$ $\widehat{\text{PCS}}$	$\mathcal{B}_{\text{P}}$ $\widehat{E(N)}$
4	0.03	0.00	SC	0.35	0.958	4673	0.954	4140
4	0.03	0.05	SC	0.35	0.958	4434		
4	0.03	0.10	SC	0.35	0.957	4199		
4	0.03	0.15	SC	0.35	0.958	3969		
4	0.03	0.20	SC	0.35	0.958	3741		
4	0.03	0.25	SC	0.35	0.957	3500		
4	0.03	0.50	SC	0.35	0.957	2335		
4	0.03	0.00	MFC	0.35	0.984	3547	0.985	2947
4	0.03	0.05	MFC	0.35	0.983	3368		
4	0.03	0.10	MFC	0.35	0.985	3184		
4	0.03	0.15	MFC	0.35	0.983	3011		
4	0.03	0.20	MFC	0.35	0.984	2839		
4	0.03	0.25	MFC	0.35	0.984	2663		
4	0.03	0.50	MFC	0.35	0.982	1779		
4	0.06	0.00	SC	0.35	0.958	1156	0.955	1010
4	0.06	0.05	SC	0.35	0.958	1098		
4	0.06	0.10	SC	0.35	0.960	1034		
4	0.06	0.15	SC	0.35	0.958	977		
4	0.06	0.20	SC	0.35	0.957	919		
4	0.06	0.25	SC	0.35	0.958	862		
4	0.06	0.50	SC	0.35	0.944	566		
4	0.06	0.00	MFC	0.35	0.984	862	0.984	720
4	0.06	0.05	MFC	0.35	0.984	819		
4	0.06	0.10	MFC	0.35	0.984	772		
4	0.06	0.15	MFC	0.35	0.983	730		
4	0.06	0.20	MFC	0.35	0.982	687		
4	0.06	0.25	MFC	0.35	0.983	644		
4	0.06	0.50	MFC	0.35	0.975	424		
4	0.09	0.00	SC	0.35	0.958	504	0.960	448
4	0.09	0.05	SC	0.35	0.958	479		
4	0.09	0.10	SC	0.35	0.956	452		
4	0.09	0.15	SC	0.35	0.955	427		
4	0.09	0.20	SC	0.35	0.954	400		
4	0.09	0.25	SC	0.35	0.950	374		
4	0.09	0.50	SC	0.35	0.928	247		
4	0.09	0.00	MFC	0.35	0.983	369	0.986	320
4	0.09	0.05	MFC	0.35	0.983	351		
4	0.09	0.10	MFC	0.35	0.983	331		
4	0.09	0.15	MFC	0.35	0.982	313		
4	0.09	0.20	MFC	0.35	0.981	294		
4	0.09	0.25	MFC	0.35	0.980	277		
4	0.09	0.50	MFC	0.35	0.970	188		

Table 30: $\mathcal{KN}$++ Bernoulli with CRN Results for $k = 5$ and $p_k = 0.85$.

k	δ	ρ	Config.	p_k	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	$\mathcal{B}_{\text{P}}$ $\widehat{\text{PCS}}$	$\mathcal{B}_{\text{P}}$ $\widehat{E(N)}$
5	0.03	0.00	SC	0.85	0.959	3871	0.956	3420
5	0.03	0.05	SC	0.85	0.957	3703		
5	0.03	0.10	SC	0.85	0.958	3500		
5	0.03	0.15	SC	0.85	0.956	3301		
5	0.03	0.20	SC	0.85	0.957	3109		
5	0.03	0.25	SC	0.85	0.956	2916		
5	0.03	0.50	SC	0.85	0.948	1921		
5	0.03	0.00	MFC	0.85	0.987	2957	0.988	2297
5	0.03	0.05	MFC	0.85	0.988	2815		
5	0.03	0.10	MFC	0.85	0.988	2664		
5	0.03	0.15	MFC	0.85	0.988	2516		
5	0.03	0.20	MFC	0.85	0.987	2366		
5	0.03	0.25	MFC	0.85	0.988	2222		
5	0.03	0.50	MFC	0.85	0.983	1477		
5	0.06	0.00	SC	0.85	0.954	1036	0.956	935
5	0.06	0.05	SC	0.85	0.950	986		
5	0.06	0.10	SC	0.85	0.948	935		
5	0.06	0.15	SC	0.85	0.947	884		
5	0.06	0.20	SC	0.85	0.944	827		
5	0.06	0.25	SC	0.85	0.940	773		
5	0.06	0.50	SC	0.85	0.912	507		
5	0.06	0.00	MFC	0.85	0.986	814	0.988	619
5	0.06	0.05	MFC	0.85	0.986	775		
5	0.06	0.10	MFC	0.85	0.985	735		
5	0.06	0.15	MFC	0.85	0.984	692		
5	0.06	0.20	MFC	0.85	0.984	653		
5	0.06	0.25	MFC	0.85	0.982	610		
5	0.06	0.50	MFC	0.85	0.970	405		
5	0.09	0.00	SC	0.85	0.944	492	0.963	446
5	0.09	0.05	SC	0.85	0.943	469		
5	0.09	0.10	SC	0.85	0.942	442		
5	0.09	0.15	SC	0.85	0.937	420		
5	0.09	0.20	SC	0.85	0.934	396		
5	0.09	0.25	SC	0.85	0.926	369		
5	0.09	0.50	SC	0.85	0.906	254		
5	0.09	0.00	MFC	0.85	0.985	393	0.989	298
5	0.09	0.05	MFC	0.85	0.983	374		
5	0.09	0.10	MFC	0.85	0.982	356		
5	0.09	0.15	MFC	0.85	0.982	338		
5	0.09	0.20	MFC	0.85	0.980	319		
5	0.09	0.25	MFC	0.85	0.978	300		
5	0.09	0.50	MFC	0.85	0.968	210		

Table 31: $\mathcal{KN}$++ Bernoulli with CRN Results for $k = 5$ and $p_k = 0.35$.

k	δ	ρ	Config.	p_k	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	$\mathcal{B}_{\text{P}}$ $\widehat{\text{PCS}}$	$\mathcal{B}_{\text{P}}$ $\widehat{E(N)}$
5	0.03	0.00	SC	0.35	0.959	6323	0.954	5456
5	0.03	0.05	SC	0.35	0.960	6000		
5	0.03	0.10	SC	0.35	0.962	5664		
5	0.03	0.15	SC	0.35	0.961	5362		
5	0.03	0.20	SC	0.35	0.960	5061		
5	0.03	0.25	SC	0.35	0.959	4737		
5	0.03	0.50	SC	0.35	0.959	3167		
5	0.03	0.00	MFC	0.35	0.988	4573	0.987	3672
5	0.03	0.05	MFC	0.35	0.988	4338		
5	0.03	0.10	MFC	0.35	0.987	4107		
5	0.03	0.15	MFC	0.35	0.988	3879		
5	0.03	0.20	MFC	0.35	0.988	3657		
5	0.03	0.25	MFC	0.35	0.988	3427		
5	0.03	0.50	MFC	0.35	0.987	2293		
5	0.06	0.00	SC	0.35	0.961	1560	0.956	1338
5	0.06	0.05	SC	0.35	0.961	1483		
5	0.06	0.10	SC	0.35	0.960	1398		
5	0.06	0.15	SC	0.35	0.961	1320		
5	0.06	0.20	SC	0.35	0.961	1246		
5	0.06	0.25	SC	0.35	0.960	1167		
5	0.06	0.50	SC	0.35	0.946	761		
5	0.06	0.00	MFC	0.35	0.988	1105	0.988	896
5	0.06	0.05	MFC	0.35	0.987	1049		
5	0.06	0.10	MFC	0.35	0.988	991		
5	0.06	0.15	MFC	0.35	0.987	937		
5	0.06	0.20	MFC	0.35	0.987	883		
5	0.06	0.25	MFC	0.35	0.986	827		
5	0.06	0.50	MFC	0.35	0.982	545		
5	0.09	0.00	SC	0.35	0.961	682	0.960	603
5	0.09	0.05	SC	0.35	0.961	648		
5	0.09	0.10	SC	0.35	0.958	613		
5	0.09	0.15	SC	0.35	0.957	576		
5	0.09	0.20	SC	0.35	0.956	541		
5	0.09	0.25	SC	0.35	0.953	506		
5	0.09	0.50	SC	0.35	0.931	333		
5	0.09	0.00	MFC	0.35	0.987	470	0.990	404
5	0.09	0.05	MFC	0.35	0.988	448		
5	0.09	0.10	MFC	0.35	0.987	423		
5	0.09	0.15	MFC	0.35	0.986	399		
5	0.09	0.20	MFC	0.35	0.986	376		
5	0.09	0.25	MFC	0.35	0.983	352		
5	0.09	0.50	MFC	0.35	0.976	242		

Table 32: Example of Performance of $\mathcal{B}_{\mathrm{P}}$ if CRN were Applied.

k	P^*	ρ	p_k	$\mathcal{B}_{\mathrm{P}}$ $\widehat{\mathrm{PCS}}$	$\mathcal{B}_{\mathrm{P}}$ $\widehat{E(N)}$
2	0.95	0.00	0.35	0.955	334.27
2	0.95	0.05	0.35	0.960	339.99
2	0.95	0.10	0.35	0.967	341.92
2	0.95	0.15	0.35	0.972	345.78
2	0.95	0.20	0.35	0.979	349.70
2	0.95	0.25	0.35	0.983	353.70
2	0.95	0.00	0.85	0.963	247.97
2	0.95	0.05	0.85	0.969	249.88
2	0.95	0.10	0.85	0.974	252.67
2	0.95	0.15	0.85	0.979	256.31
2	0.95	0.20	0.85	0.984	258.67
2	0.95	0.25	0.85	0.988	261.36
3	0.95	0.00	0.35	0.958	700.56
3	0.95	0.05	0.35	0.966	703.46
3	0.95	0.10	0.35	0.973	708.18
3	0.95	0.15	0.35	0.978	713.54
3	0.95	0.20	0.35	0.983	719.77
3	0.95	0.25	0.35	0.988	724.02
3	0.95	0.00	0.85	0.963	496.47
3	0.95	0.05	0.85	0.972	499.69
3	0.95	0.10	0.85	0.974	503.15
3	0.95	0.15	0.85	0.981	507.88
3	0.95	0.20	0.85	0.986	509.30
3	0.95	0.25	0.85	0.989	511.60

APPENDIX C

NORTA BACKGROUND INFORMATION

NORTA vectors are based on the transformation

$$X = \begin{pmatrix} F_{X_1}^{-1}[\Phi(Z_1)] \\ F_{X_1}^{-1}[\Phi(Z_2)] \\ . \\ . \\ . \\ F_{X_1}^{-1}[\Phi(Z_k)] \end{pmatrix}$$

where $(Z_1, Z_2, ..., Z_k)$ is a multivariate normal vector with the correlation matrix $\sum_Z$ and $F_{X_1}, F_{X_2}, ..., F_{X_k}$ are the marginal distributions that the user desires to generate.

We must adjust the correlation matrix $\sum_Z$ so that after the transformation we will obtain the desired correlation $\sum_X$ for the generated variables $X_1, ..., X_k$. The steps for the NORTA generation procedure as defined by Cario and Nelson are as follows:

1. 1. Set up: Determine a lower-triangular, nonsingular factorization $\mathbf{M}$ of $\sum_Z$ so that $\mathbf{MM}' = \sum_Z$.

2. 2. Generate $\mathbf{W} = (W_1, W_2, ..., W_k)'$ is a random vector made up of i.i.d. standard normal random variables.

3. 3. Set $\mathbf{Z} \leftarrow \mathbf{MW}$.

4. 4. Return $\mathbf{X}$ where $X_i \leftarrow F_{X_i}^{-1}[\Phi(Z_i)]$, $i = 1, 2, ..., k$.

5. 5. To generate more random variables, return to step 2 and repeat.

To find the correlation matrix $\sum_Z$ we use a bisectional search method. Then to find the lower-triangular matrix **M** a Cholesky decomposition can be used. The following C++ code can be used to determine **M**:

```
// Form A matrix (the covariance matrix of Z)

for (i = 0; i < Numsys; i++) {
    // Set all diagonal elements to 1
    A(i,i) = 1
}

for (i = 1; i < Numsys; i++) {
    for (j = 0; j < i; i++) {
        // Set all other elements to rhoZ(i,j) using transform
        A(i,j) = rhoZ((i*(i-1)/2)+j)
    }
}

// Find the factorization

for (int h = 0; h < Numsys-1; h++) {
    A(h,h) = double sqrt(A(h,h))
    for (i = h+1; i < Numsys; i++) {
        A(i,h) = A(i,h)/A(h,h)
    }
    for (j = h+1; j < Numsys; j++) {
        for (int p = j; p < Numsys; p++) {
            A(p,j) = A(p,j) - A(p,h)*A(j,h)
        }
    }
}

A(Numsys,Numsys) = sqrt(A(Numsys,Numsys))

// Generate the M matrix

for (i = 0; i < Numsys; i++) {
    for (j = 0; j < Numsys; j++) {
        // Initialize all values to zero
        M(i,j) = 0
    }
}

for (i = 1; i < Numsys; i++) {
    for (j = 0; j < i; j++) {
        // Fill in factored values in lower triangle (rest are 0)
        M(i,j) = A(i,j)
    }
}
```

APPENDIX D

$\mathcal{M}_{\mathrm{BK}}$ MULTINOMIAL PROCEDURE MONTE CARLO RESULTS

Table 33: Table of Monte Carlo Results for $\mathcal{M}_{\mathrm{BK}}$ when $k = 2$ and $\alpha = 0.05$.

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
1	0.500	0.524	0.545	0.565	0.583	0.600	0.615	0.643	0.667	0.688	0.722
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	0.500	0.524	0.545	0.565	0.583	0.600	0.615	0.643	0.667	0.688	0.722
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
3	0.500	0.536	0.568	0.597	0.624	0.648	0.670	0.709	0.741	0.768	0.812
	2.50	2.50	2.50	2.49	2.49	2.48	2.48	2.46	2.44	2.43	2.40
4	0.500	0.536	0.568	0.597	0.624	0.648	0.670	0.708	0.740	0.768	0.811
	2.50	2.50	2.50	2.49	2.48	2.48	2.48	2.46	2.45	2.43	2.40
5	0.500	0.545	0.585	0.621	0.654	0.683	0.709	0.754	0.790	0.820	0.865
	4.13	4.12	4.11	4.10	4.08	4.06	4.05	4.00	3.97	3.92	3.84
6	0.500	0.545	0.585	0.621	0.653	0.683	0.709	0.753	0.790	0.820	0.865
	4.13	4.12	4.11	4.10	4.09	4.07	4.05	4.01	3.96	3.92	3.84
7	0.500	0.552	0.599	0.640	0.677	0.710	0.739	0.788	0.826	0.857	0.901
	5.81	5.81	5.79	5.76	5.73	5.70	5.67	5.58	5.51	5.43	5.28
8	0.500	0.552	0.599	0.640	0.678	0.711	0.739	0.788	0.827	0.857	0.900
	5.81	5.80	5.78	5.76	5.73	5.69	5.66	5.58	5.50	5.43	5.30
9	0.500	0.558	0.611	0.657	0.698	0.733	0.765	0.816	0.855	0.885	0.926
	7.54	7.53	7.50	7.46	7.41	7.36	7.30	7.17	7.05	6.93	6.71
10	0.500	0.559	0.610	0.657	0.698	0.733	0.764	0.816	0.855	0.885	0.926
	7.54	7.52	7.51	7.45	7.41	7.35	7.30	7.18	7.05	6.93	6.72
11	0.500	0.564	0.621	0.673	0.716	0.753	0.786	0.839	0.877	0.907	0.944
	9.29	9.27	9.23	9.16	9.10	9.04	8.94	8.77	8.61	8.42	8.15
12	0.500	0.564	0.622	0.672	0.716	0.753	0.787	0.839	0.878	0.907	0.944
	9.29	9.27	9.23	9.17	9.10	9.03	8.93	8.77	8.59	8.44	8.15
13	0.500	0.569	0.631	0.685	0.731	0.771	0.805	0.859	0.896	0.924	0.958
	11.06	11.06	11.00	10.90	10.81	10.71	10.60	10.35	10.14	9.94	9.56
14	0.500	0.570	0.631	0.685	0.732	0.771	0.805	0.859	0.896	0.923	0.958
	11.08	11.04	11.00	10.91	10.80	10.71	10.60	10.35	10.15	9.94	9.56
15	0.500	0.574	0.641	0.697	0.746	0.786	0.822	0.875	0.913	0.938	0.968
	12.86	12.83	12.75	12.65	12.52	12.40	12.24	11.95	11.67	11.41	10.98
16	0.500	0.574	0.640	0.697	0.746	0.786	0.822	0.875	0.911	0.937	0.968
	12.84	12.83	12.76	12.66	12.53	12.40	12.24	11.96	11.70	11.42	10.98
17	0.500	0.579	0.649	0.709	0.759	0.802	0.836	0.889	0.925	0.948	0.975
	14.66	14.63	14.52	14.40	14.25	14.07	13.91	13.55	13.21	12.92	12.38
18	0.500	0.579	0.649	0.708	0.759	0.802	0.836	0.889	0.924	0.949	0.975
	14.67	14.63	14.53	14.41	14.25	14.06	13.91	13.55	13.22	12.91	12.39
19	0.500	0.584	0.655	0.718	0.771	0.815	0.850	0.902	0.934	0.957	0.981
	16.48	16.42	16.34	16.17	15.98	15.76	15.56	15.13	14.77	14.39	13.80
20	0.500	0.583	0.656	0.719	0.771	0.813	0.850	0.901	0.935	0.957	0.981
	16.46	16.43	16.34	16.15	15.98	15.78	15.56	15.15	14.74	14.39	13.79
21	0.500	0.587	0.664	0.728	0.783	0.827	0.862	0.912	0.944	0.964	0.985
	18.31	18.25	18.12	17.93	17.69	17.45	17.21	16.73	16.28	15.87	15.17
22	0.500	0.587	0.664	0.728	0.781	0.826	0.861	0.912	0.945	0.965	0.985
	18.31	18.25	18.11	17.92	17.74	17.46	17.22	16.73	16.26	15.85	15.18
23	0.500	0.591	0.670	0.737	0.792	0.837	0.872	0.921	0.952	0.971	0.988
	20.12	20.08	19.93	19.70	19.45	19.16	18.87	18.33	17.81	17.35	16.59
24	0.500	0.592	0.671	0.738	0.792	0.836	0.872	0.921	0.952	0.970	0.988
	20.11	20.06	19.92	19.69	19.44	19.17	18.87	18.32	17.81	17.34	16.58

Table 33: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
25	0.500	0.595	0.677	0.746	0.802	0.846	0.881	0.930	0.959	0.975	0.991
	21.98	21.90	21.72	21.47	21.16	20.86	20.55	19.91	19.31	18.81	17.97
26	0.500	0.595	0.677	0.745	0.802	0.847	0.880	0.930	0.958	0.976	0.991
	21.96	21.91	21.74	21.49	21.17	20.85	20.57	19.90	19.35	18.80	17.96
27	0.500	0.599	0.684	0.753	0.812	0.855	0.890	0.937	0.964	0.979	0.993
	23.81	23.74	23.53	23.26	22.89	22.57	22.20	21.48	20.83	20.29	19.36
28	0.500	0.599	0.684	0.753	0.811	0.856	0.890	0.937	0.963	0.979	0.993
	23.82	23.73	23.53	23.27	22.91	22.55	22.19	21.48	20.87	20.29	19.37
29	0.500	0.602	0.689	0.761	0.820	0.865	0.899	0.945	0.969	0.983	0.994
	25.65	25.59	25.35	25.04	24.62	24.23	23.84	23.03	22.37	21.75	20.74
30	0.500	0.602	0.690	0.762	0.819	0.864	0.898	0.943	0.969	0.983	0.995
	25.68	25.57	25.35	25.01	24.66	24.24	23.86	23.09	22.37	21.76	20.75
31	0.500	0.605	0.696	0.770	0.828	0.872	0.905	0.950	0.973	0.985	0.996
	27.51	27.42	27.16	26.79	26.36	25.93	25.50	24.63	23.88	23.24	22.14
32	0.500	0.606	0.696	0.769	0.828	0.871	0.905	0.949	0.973	0.986	0.996
	27.53	27.40	27.16	26.80	26.38	25.96	25.51	24.63	23.86	23.23	22.14
33	0.500	0.609	0.701	0.775	0.834	0.879	0.913	0.955	0.977	0.988	0.997
	29.38	29.25	29.00	28.61	28.15	27.65	27.14	26.21	25.36	24.66	23.52
34	0.500	0.608	0.700	0.777	0.835	0.879	0.912	0.955	0.976	0.988	0.997
	29.39	29.28	29.01	28.57	28.12	27.64	27.16	26.20	25.40	24.67	23.52
35	0.500	0.612	0.707	0.782	0.841	0.886	0.919	0.959	0.980	0.990	0.997
	31.24	31.12	30.80	30.38	29.88	29.33	28.81	27.81	26.89	26.16	24.91
36	0.500	0.612	0.706	0.783	0.842	0.886	0.918	0.959	0.980	0.990	0.998
	31.24	31.13	30.83	30.37	29.85	29.34	28.82	27.81	26.90	26.12	24.91
37	0.500	0.615	0.712	0.789	0.848	0.892	0.924	0.963	0.982	0.991	0.998
	33.11	32.99	32.61	32.16	31.61	31.04	30.44	29.37	28.43	27.61	26.34
38	0.500	0.615	0.711	0.788	0.848	0.892	0.924	0.964	0.982	0.992	0.998
	33.11	32.99	32.64	32.16	31.61	31.06	30.46	29.36	28.41	27.60	26.30
39	0.500	0.618	0.717	0.795	0.854	0.897	0.929	0.966	0.985	0.993	0.998
	34.99	34.84	34.45	33.95	33.36	32.75	32.12	30.95	29.90	29.05	27.69
40	0.500	0.618	0.716	0.795	0.854	0.899	0.930	0.967	0.985	0.993	0.998
	34.99	34.84	34.46	33.94	33.35	32.69	32.08	30.96	29.92	29.05	27.69
41	0.500	0.621	0.722	0.800	0.860	0.903	0.934	0.970	0.986	0.994	0.999
	36.84	36.70	36.27	35.76	35.08	34.45	33.76	32.50	31.45	30.53	29.10
42	0.500	0.621	0.722	0.801	0.860	0.904	0.934	0.970	0.986	0.994	0.999
	36.86	36.68	36.27	35.71	35.10	34.40	33.75	32.53	31.47	30.55	29.06
43	0.500	0.624	0.726	0.806	0.866	0.909	0.938	0.973	0.988	0.995	0.999
	38.72	38.55	38.12	37.51	36.80	36.10	35.41	34.08	32.94	31.98	30.46
44	0.500	0.623	0.726	0.806	0.867	0.909	0.938	0.972	0.988	0.995	0.999
	38.74	38.57	38.13	37.51	36.80	36.12	35.43	34.09	32.93	32.01	30.44
45	0.500	0.626	0.730	0.812	0.872	0.913	0.942	0.975	0.990	0.996	0.999
	40.62	40.44	39.98	39.29	38.54	37.82	37.05	35.64	34.47	33.44	31.85
46	0.500	0.626	0.731	0.812	0.872	0.914	0.943	0.976	0.990	0.996	0.999
	40.65	40.43	39.95	39.31	38.55	37.80	37.02	35.64	34.45	33.42	31.84
47	0.500	0.629	0.735	0.817	0.876	0.918	0.946	0.977	0.991	0.996	0.999
	42.48	42.29	41.79	41.09	40.31	39.50	38.70	37.22	35.97	34.93	33.25
48	0.500	0.629	0.735	0.817	0.876	0.918	0.946	0.978	0.991	0.996	0.999
	42.49	42.30	41.79	41.07	40.33	39.49	38.70	37.23	35.98	34.93	33.23

Table 33: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
49	0.500	0.632	0.740	0.821	0.882	0.922	0.950	0.980	0.992	0.997	1.000
	44.37	44.16	43.62	42.91	42.01	41.19	40.34	38.79	37.45	36.36	34.61
50	0.500	0.631	0.740	0.822	0.881	0.923	0.950	0.980	0.992	0.997	1.000
	44.38	44.18	43.61	42.87	42.04	41.16	40.33	38.80	37.47	36.38	34.61
51	0.500	0.634	0.744	0.827	0.886	0.927	0.954	0.982	0.993	0.997	1.000
	46.26	46.03	45.44	44.67	43.78	42.86	41.97	40.34	38.97	37.79	35.99
52	0.500	0.634	0.743	0.827	0.885	0.926	0.953	0.982	0.993	0.997	1.000
	46.23	46.01	45.47	44.65	43.81	42.87	41.98	40.30	38.96	37.80	35.98
53	0.500	0.636	0.748	0.831	0.890	0.931	0.956	0.983	0.994	0.998	1.000
	48.15	47.93	47.28	46.46	45.53	44.51	43.64	41.94	40.48	39.29	37.40
54	0.500	0.637	0.748	0.831	0.891	0.930	0.957	0.984	0.994	0.998	1.000
	48.17	47.90	47.29	46.47	45.48	44.56	43.58	41.90	40.44	39.26	37.38
55	0.500	0.638	0.751	0.835	0.895	0.934	0.959	0.985	0.995	0.998	1.000
	50.05	49.82	49.14	48.24	47.23	46.23	45.27	43.52	41.99	40.74	38.74
56	0.500	0.638	0.751	0.836	0.894	0.934	0.959	0.985	0.995	0.998	1.000
	50.06	49.83	49.14	48.23	47.27	46.24	45.27	43.47	41.98	40.71	38.77
57	0.500	0.641	0.755	0.840	0.898	0.937	0.962	0.987	0.996	0.998	1.000
	51.97	51.67	50.97	50.03	49.02	47.93	46.89	45.00	43.50	42.16	40.14
58	0.500	0.641	0.755	0.840	0.898	0.938	0.962	0.986	0.995	0.999	1.000
	51.96	51.68	50.96	50.04	48.99	47.91	46.90	45.05	43.43	42.18	40.17
59	0.500	0.643	0.758	0.845	0.903	0.941	0.964	0.988	0.996	0.999	1.000
	53.83	53.55	52.84	51.80	50.68	49.60	48.53	46.61	44.96	43.61	41.54
60	0.500	0.644	0.759	0.844	0.902	0.940	0.964	0.988	0.996	0.999	1.000
	53.84	53.53	52.80	51.79	50.73	49.60	48.54	46.65	44.99	43.67	41.54
61	0.500	0.646	0.763	0.848	0.906	0.943	0.967	0.989	0.997	0.999	1.000
	55.74	55.41	54.65	53.59	52.45	51.32	50.16	48.19	46.46	45.09	42.91
62	0.500	0.645	0.763	0.848	0.906	0.944	0.967	0.989	0.997	0.999	1.000
	55.74	55.45	54.65	53.59	52.47	51.27	50.17	48.13	46.50	45.09	42.92
63	0.500	0.648	0.766	0.852	0.910	0.947	0.969	0.990	0.997	0.999	1.000
	57.63	57.30	56.48	55.38	54.19	52.95	51.82	49.72	48.00	46.53	44.27
64	0.500	0.648	0.766	0.852	0.909	0.947	0.968	0.990	0.997	0.999	1.000
	57.63	57.31	56.52	55.40	54.21	52.95	51.84	49.72	47.98	46.54	44.33
65	0.500	0.650	0.770	0.855	0.913	0.949	0.971	0.991	0.997	0.999	1.000
	59.53	59.19	58.34	57.20	55.89	54.65	53.40	51.30	49.51	48.00	45.69
66	0.500	0.650	0.769	0.855	0.913	0.949	0.971	0.991	0.997	0.999	1.000
	59.55	59.20	58.36	57.21	55.90	54.63	53.45	51.33	49.49	48.01	45.73
67	0.500	0.652	0.772	0.858	0.916	0.952	0.972	0.992	0.998	0.999	1.000
	61.42	61.09	60.21	59.01	57.66	56.28	55.13	52.86	50.99	49.49	47.09
68	0.500	0.652	0.773	0.860	0.916	0.952	0.973	0.991	0.998	0.999	1.000
	61.44	61.11	60.15	58.93	57.65	56.32	55.02	52.83	50.98	49.48	47.06
69	0.500	0.654	0.776	0.862	0.919	0.954	0.974	0.992	0.998	1.000	1.000
	63.35	63.00	62.03	60.80	59.41	57.96	56.71	54.38	52.50	50.89	48.47
70	0.500	0.655	0.777	0.863	0.919	0.954	0.975	0.992	0.998	0.999	1.000
	63.33	62.95	61.99	60.74	59.40	58.04	56.70	54.40	52.47	50.94	48.46
71	0.500	0.657	0.779	0.866	0.922	0.956	0.976	0.993	0.998	0.999	1.000
	65.21	64.85	63.88	62.54	61.11	59.68	58.34	55.95	53.99	52.36	49.86
72	0.500	0.656	0.779	0.866	0.922	0.956	0.976	0.993	0.998	1.000	1.000
	65.27	64.88	63.86	62.54	61.14	59.71	58.30	55.96	53.96	52.32	49.84

Table 33: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
73	0.500	0.658	0.783	0.870	0.925	0.958	0.978	0.994	0.998	1.000	1.000
	67.14	66.77	65.69	64.32	62.82	61.42	59.98	57.51	55.51	53.83	51.24
74	0.500	0.658	0.782	0.870	0.925	0.959	0.978	0.994	0.998	1.000	1.000
	67.11	66.76	65.75	64.32	62.84	61.34	59.96	57.55	55.49	53.83	51.19
75	0.500	0.661	0.786	0.873	0.928	0.960	0.979	0.994	0.999	1.000	1.000
	69.06	68.63	67.55	66.12	64.58	63.09	61.63	59.10	56.99	55.23	52.64
76	0.500	0.661	0.786	0.872	0.927	0.960	0.979	0.994	0.999	1.000	1.000
	69.07	68.61	67.54	66.14	64.63	63.05	61.65	59.05	57.03	55.27	52.60
77	0.500	0.663	0.787	0.876	0.930	0.963	0.981	0.995	0.999	1.000	1.000
	71.01	70.52	69.45	67.91	66.33	64.68	63.22	60.61	58.49	56.71	54.00
78	0.500	0.663	0.789	0.875	0.930	0.962	0.980	0.995	0.999	1.000	1.000
	70.92	70.50	69.37	67.92	66.32	64.76	63.26	60.64	58.46	56.77	54.02
79	0.500	0.664	0.792	0.878	0.933	0.964	0.982	0.995	0.999	1.000	1.000
	72.90	72.43	71.22	69.71	68.03	66.39	64.88	62.20	60.00	58.19	55.37
80	0.500	0.665	0.792	0.879	0.933	0.964	0.981	0.995	0.999	1.000	1.000
	72.88	72.39	71.24	69.68	68.01	66.42	64.91	62.25	60.01	58.20	55.40
81	0.500	0.666	0.796	0.881	0.935	0.966	0.983	0.996	0.999	1.000	1.000
	74.79	74.33	73.03	71.51	69.77	68.10	66.54	63.76	61.53	59.65	56.78
82	0.500	0.667	0.796	0.882	0.935	0.965	0.982	0.996	0.999	1.000	1.000
	74.79	74.29	73.02	71.49	69.76	68.13	66.52	63.77	61.48	59.59	56.72
83	0.500	0.668	0.797	0.884	0.939	0.968	0.984	0.996	0.999	1.000	1.000
	76.71	76.21	74.91	73.28	71.44	69.74	68.10	65.27	62.94	61.06	58.16
84	0.500	0.668	0.798	0.884	0.938	0.968	0.983	0.996	0.999	1.000	1.000
	76.68	76.20	74.91	73.32	71.52	69.80	68.18	65.29	63.00	61.13	58.14
85	0.500	0.670	0.800	0.888	0.939	0.969	0.985	0.997	0.999	1.000	1.000
	78.64	78.11	76.78	75.01	73.26	71.42	69.74	66.89	64.49	62.55	59.55
86	0.500	0.670	0.801	0.887	0.940	0.969	0.984	0.997	0.999	1.000	1.000
	78.62	78.12	76.75	75.08	73.20	71.45	69.78	66.85	64.49	62.54	59.55
87	0.500	0.672	0.802	0.890	0.942	0.971	0.985	0.997	0.999	1.000	1.000
	80.51	79.99	78.65	76.86	74.93	73.12	71.45	68.40	65.98	64.02	60.87
88	0.500	0.672	0.804	0.889	0.941	0.971	0.986	0.997	0.999	1.000	1.000
	80.49	79.97	78.56	76.90	74.98	73.15	71.37	68.46	66.00	64.01	60.96
89	0.500	0.675	0.805	0.893	0.944	0.972	0.987	0.997	0.999	1.000	1.000
	82.47	81.84	80.48	78.60	76.69	74.83	73.03	69.96	67.50	65.42	62.31
90	0.500	0.673	0.805	0.892	0.943	0.972	0.986	0.997	0.999	1.000	1.000
	82.46	81.91	80.48	78.64	76.74	74.80	73.05	69.96	67.51	65.42	62.30
91	0.500	0.676	0.808	0.895	0.946	0.974	0.987	0.997	0.999	1.000	1.000
	84.41	83.77	82.31	80.44	78.42	76.40	74.65	71.53	69.03	66.95	63.69
92	0.500	0.676	0.808	0.894	0.946	0.973	0.988	0.997	0.999	1.000	1.000
	84.36	83.77	82.33	80.48	78.41	76.54	74.63	71.58	68.99	66.93	63.73
93	0.500	0.678	0.811	0.898	0.949	0.975	0.988	0.998	1.000	1.000	1.000
	86.28	85.63	84.16	82.11	80.09	78.13	76.30	73.09	70.46	68.39	65.10
94	0.500	0.678	0.811	0.897	0.947	0.974	0.989	0.997	1.000	1.000	1.000
	86.28	85.66	84.13	82.24	80.18	78.20	76.22	73.12	70.49	68.36	65.09
95	0.500	0.679	0.813	0.900	0.950	0.976	0.989	0.998	1.000	1.000	1.000
	88.21	87.58	86.03	84.00	81.89	79.82	77.92	74.65	72.04	69.83	66.46
96	0.500	0.679	0.814	0.901	0.949	0.976	0.989	0.998	1.000	1.000	1.000
	88.19	87.57	85.98	83.96	81.88	79.82	77.90	74.66	72.01	69.83	66.50

Table 33: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
97	0.500	0.682	0.817	0.902	0.952	0.977	0.990	0.998	1.000	1.000	1.000
	90.09	89.43	87.80	85.78	83.58	81.50	79.53	76.16	73.51	71.26	67.85
98	0.500	0.682	0.816	0.902	0.952	0.977	0.989	0.998	1.000	1.000	1.000
	90.14	89.41	87.83	85.76	83.60	81.48	79.59	76.23	73.49	71.26	67.90
99	0.500	0.683	0.818	0.906	0.953	0.978	0.990	0.998	1.000	1.000	1.000
	92.08	91.33	89.71	87.51	85.32	83.16	81.20	77.80	74.98	72.68	69.21
100	0.500	0.684	0.818	0.905	0.954	0.978	0.990	0.998	1.000	1.000	1.000
	92.01	91.31	89.70	87.56	85.27	83.17	81.17	77.79	75.02	72.73	69.21
101	0.500	0.685	0.821	0.906	0.954	0.979	0.991	0.998	1.000	1.000	1.000
	93.98	93.24	91.54	89.38	87.09	84.85	82.82	79.31	76.52	74.16	70.63
102	0.500	0.684	0.821	0.907	0.955	0.979	0.991	0.998	1.000	1.000	1.000
	94.00	93.28	91.54	89.37	86.99	84.83	82.76	79.31	76.50	74.17	70.54
103	0.500	0.686	0.823	0.908	0.956	0.980	0.991	0.998	1.000	1.000	1.000
	95.90	95.15	93.40	91.17	88.82	86.51	84.44	80.87	77.96	75.63	71.98
104	0.500	0.686	0.823	0.909	0.956	0.980	0.991	0.999	1.000	1.000	1.000
	95.89	95.16	93.37	91.12	88.76	86.49	84.44	80.91	77.96	75.62	71.99
105	0.500	0.688	0.825	0.912	0.958	0.981	0.992	0.999	1.000	1.000	1.000
	97.81	97.07	95.28	92.91	90.49	88.16	86.07	82.41	79.45	77.08	73.39
106	0.500	0.687	0.825	0.911	0.958	0.981	0.992	0.999	1.000	1.000	1.000
	97.80	97.09	95.25	92.94	90.49	88.15	86.07	82.48	79.47	77.09	73.38
107	0.500	0.690	0.827	0.913	0.960	0.981	0.992	0.999	1.000	1.000	1.000
	99.73	98.94	97.09	94.71	92.18	89.93	87.70	83.92	81.00	78.55	74.77
108	0.500	0.690	0.828	0.913	0.959	0.982	0.992	0.999	1.000	1.000	1.000
	99.78	98.93	97.03	94.68	92.20	89.83	87.69	84.05	80.95	78.57	74.82
109	0.500	0.690	0.830	0.915	0.960	0.983	0.993	0.999	1.000	1.000	1.000
	101.65	100.89	98.94	96.51	93.96	91.57	89.35	85.53	82.51	80.01	76.16
110	0.500	0.691	0.829	0.915	0.961	0.983	0.993	0.999	1.000	1.000	1.000
	101.62	100.86	98.93	96.50	93.93	91.51	89.32	85.55	82.52	80.05	76.14
111	0.500	0.693	0.831	0.917	0.962	0.983	0.993	0.999	1.000	1.000	1.000
	103.59	102.73	100.82	98.23	95.69	93.23	90.98	87.13	84.05	81.43	77.56
112	0.500	0.693	0.832	0.917	0.962	0.984	0.993	0.999	1.000	1.000	1.000
	103.59	102.72	100.77	98.25	95.61	93.16	90.98	87.04	84.06	81.44	77.55
113	0.500	0.694	0.835	0.919	0.963	0.984	0.994	0.999	1.000	1.000	1.000
	105.53	104.67	102.61	100.02	97.41	94.88	92.49	88.65	85.51	82.84	78.91
114	0.500	0.693	0.834	0.918	0.963	0.984	0.994	0.999	1.000	1.000	1.000
	105.48	104.70	102.63	100.10	97.43	94.88	92.62	88.67	85.49	82.92	78.95
115	0.500	0.696	0.836	0.920	0.964	0.985	0.994	0.999	1.000	1.000	1.000
	107.44	106.54	104.47	101.82	99.09	96.57	94.25	90.18	86.98	84.40	80.31
116	0.500	0.696	0.837	0.922	0.964	0.985	0.994	0.999	1.000	1.000	1.000
	107.43	106.55	104.42	101.77	99.16	96.53	94.20	90.19	86.95	84.36	80.32
117	0.500	0.697	0.840	0.923	0.966	0.986	0.994	0.999	1.000	1.000	1.000
	109.36	108.48	106.26	103.61	100.82	98.22	95.85	91.73	88.45	85.87	81.72
118	0.500	0.697	0.837	0.923	0.965	0.986	0.994	0.999	1.000	1.000	1.000
	109.27	108.51	106.38	103.60	100.90	98.19	95.84	91.73	88.49	85.80	81.70
119	0.500	0.698	0.841	0.924	0.966	0.986	0.995	0.999	1.000	1.000	1.000
	111.28	110.41	108.18	105.41	102.54	99.90	97.41	93.31	89.93	87.30	83.11
120	0.500	0.699	0.841	0.924	0.967	0.986	0.995	0.999	1.000	1.000	1.000
	111.27	110.38	108.14	105.39	102.56	99.91	97.49	93.32	89.97	87.28	83.09

Table 33: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
121	0.500	0.700	0.842	0.925	0.968	0.987	0.995	0.999	1.000	1.000	1.000
	113.25	112.27	110.03	107.21	104.27	101.49	99.09	94.86	91.54	88.71	84.44
122	0.500	0.700	0.842	0.925	0.968	0.987	0.995	0.999	1.000	1.000	1.000
	113.23	112.28	110.02	107.16	104.32	101.50	99.08	94.96	91.50	88.73	84.43
123	0.500	0.702	0.844	0.927	0.969	0.988	0.995	0.999	1.000	1.000	1.000
	115.12	114.18	111.89	109.02	106.01	103.21	100.70	96.42	92.97	90.17	85.88
124	0.500	0.701	0.843	0.927	0.969	0.988	0.995	0.999	1.000	1.000	1.000
	115.11	114.23	111.89	108.94	106.02	103.23	100.67	96.48	92.96	90.24	85.83
125	0.500	0.705	0.845	0.929	0.970	0.988	0.995	0.999	1.000	1.000	1.000
	117.07	116.00	113.76	110.74	107.73	104.91	102.35	98.01	94.46	91.64	87.25
126	0.500	0.703	0.846	0.928	0.970	0.988	0.996	0.999	1.000	1.000	1.000
	117.02	116.10	113.71	110.82	107.71	104.93	102.30	97.93	94.49	91.64	87.21
127	0.500	0.704	0.849	0.931	0.971	0.989	0.996	1.000	1.000	1.000	1.000
	118.99	118.04	115.53	112.54	109.45	106.58	103.98	99.50	96.00	93.06	88.60
128	0.500	0.705	0.849	0.931	0.971	0.988	0.996	1.000	1.000	1.000	1.000
	118.95	117.97	115.53	112.52	109.45	106.58	104.03	99.57	96.06	93.13	88.59
129	0.500	0.706	0.850	0.932	0.972	0.989	0.996	1.000	1.000	1.000	1.000
	120.90	119.88	117.39	114.28	111.12	108.26	105.57	101.13	97.53	94.58	90.03
130	0.500	0.706	0.851	0.933	0.972	0.989	0.996	1.000	1.000	1.000	1.000
	120.93	119.88	117.37	114.29	111.18	108.23	105.60	101.09	97.47	94.57	89.97
131	0.500	0.707	0.852	0.933	0.973	0.990	0.996	1.000	1.000	1.000	1.000
	122.79	121.83	119.26	116.13	112.84	109.94	107.20	102.64	99.00	95.99	91.40
132	0.500	0.707	0.853	0.934	0.973	0.990	0.996	1.000	1.000	1.000	1.000
	122.83	121.83	119.22	116.06	112.91	109.94	107.25	102.66	98.96	95.99	91.38
133	0.500	0.709	0.854	0.935	0.974	0.991	0.997	1.000	1.000	1.000	1.000
	124.82	123.71	121.08	117.93	114.63	111.57	108.85	104.19	100.56	97.46	92.77
134	0.500	0.709	0.853	0.935	0.975	0.990	0.997	1.000	1.000	1.000	1.000
	124.78	123.72	121.10	117.94	114.55	111.60	108.84	104.23	100.57	97.43	92.78
135	0.500	0.710	0.856	0.936	0.974	0.991	0.997	1.000	1.000	1.000	1.000
	126.74	125.61	122.92	119.72	116.37	113.23	110.48	105.77	102.00	98.86	94.20
136	0.500	0.711	0.855	0.936	0.975	0.991	0.997	1.000	1.000	1.000	1.000
	126.72	125.58	122.97	119.66	116.33	113.30	110.44	105.78	101.95	98.87	94.09
137	0.500	0.712	0.858	0.938	0.976	0.991	0.997	1.000	1.000	1.000	1.000
	128.65	127.50	124.80	121.42	118.02	114.95	112.10	107.31	103.44	100.35	95.50
138	0.500	0.711	0.857	0.938	0.975	0.991	0.997	1.000	1.000	1.000	1.000
	128.62	127.54	124.81	121.42	118.12	114.92	112.10	107.31	103.53	100.33	95.52
139	0.500	0.713	0.860	0.939	0.976	0.991	0.997	1.000	1.000	1.000	1.000
	130.57	129.41	126.58	123.26	119.78	116.67	113.64	108.90	105.01	101.78	96.93
140	0.500	0.713	0.859	0.940	0.976	0.991	0.997	1.000	1.000	1.000	1.000
	130.56	129.46	126.63	123.21	119.78	116.66	113.72	108.87	105.03	101.81	96.93
141	0.500	0.715	0.861	0.940	0.977	0.992	0.997	1.000	1.000	1.000	1.000
	132.48	131.27	128.47	125.01	121.54	118.34	115.33	110.46	106.48	103.23	98.29
142	0.500	0.715	0.860	0.940	0.976	0.992	0.997	1.000	1.000	1.000	1.000
	132.50	131.32	128.52	125.04	121.57	118.26	115.38	110.47	106.57	103.23	98.29
143	0.500	0.717	0.861	0.942	0.978	0.992	0.997	1.000	1.000	1.000	1.000
	134.43	133.18	130.44	126.80	123.26	119.95	116.96	111.93	107.98	104.75	99.65
144	0.500	0.716	0.863	0.941	0.977	0.992	0.997	1.000	1.000	1.000	1.000
	134.42	133.25	130.30	126.83	123.28	119.92	117.01	112.00	107.98	104.70	99.69

Table 33: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
145	0.500	0.717	0.864	0.943	0.978	0.993	0.998	1.000	1.000	1.000	1.000
	136.42	135.12	132.19	128.52	125.03	121.60	118.58	113.60	109.52	106.23	101.11
146	0.500	0.716	0.864	0.942	0.979	0.993	0.997	1.000	1.000	1.000	1.000
	136.35	135.19	132.18	128.64	124.94	121.55	118.66	113.51	109.45	106.14	101.05
147	0.500	0.718	0.865	0.943	0.980	0.993	0.998	1.000	1.000	1.000	1.000
	138.30	137.08	134.10	130.41	126.64	123.35	120.21	115.12	111.01	107.67	102.48
148	0.500	0.718	0.865	0.945	0.979	0.993	0.998	1.000	1.000	1.000	1.000
	138.40	137.06	134.07	130.26	126.69	123.32	120.28	115.12	111.03	107.58	102.51
149	0.500	0.720	0.868	0.945	0.980	0.993	0.998	1.000	1.000	1.000	1.000
	140.21	138.92	135.86	132.19	128.38	124.94	121.83	116.69	112.53	109.14	103.82
150	0.500	0.720	0.868	0.946	0.980	0.993	0.998	1.000	1.000	1.000	1.000
	140.22	138.95	135.83	132.14	128.39	124.98	121.84	116.68	112.57	109.05	103.84
155	0.500	0.723	0.871	0.950	0.982	0.994	0.998	1.000	1.000	1.000	1.000
	146.10	144.70	141.48	137.39	133.55	129.89	126.73	121.37	116.97	113.54	107.99
160	0.500	0.726	0.875	0.951	0.983	0.995	0.999	1.000	1.000	1.000	1.000
	149.93	148.53	145.10	141.07	136.98	133.36	129.96	124.48	120.05	116.36	110.77
165	0.500	0.731	0.879	0.953	0.984	0.995	0.999	1.000	1.000	1.000	1.000
	155.76	154.18	150.63	146.42	142.14	138.32	134.93	129.11	124.49	120.71	114.91
170	0.500	0.733	0.882	0.956	0.986	0.996	0.999	1.000	1.000	1.000	1.000
	159.61	157.99	154.33	149.94	145.56	141.61	138.10	132.23	127.50	123.69	117.72
175	0.500	0.737	0.886	0.958	0.987	0.996	0.999	1.000	1.000	1.000	1.000
	165.47	163.70	159.88	155.22	150.72	146.64	143.08	136.88	132.01	127.97	121.80
180	0.500	0.739	0.889	0.961	0.988	0.997	0.999	1.000	1.000	1.000	1.000
	169.31	167.57	163.57	158.73	154.20	149.99	146.31	140.04	134.99	130.86	124.65
185	0.500	0.743	0.892	0.963	0.989	0.997	0.999	1.000	1.000	1.000	1.000
	175.13	173.27	169.12	164.03	159.30	154.91	151.17	144.67	139.46	135.32	128.78
190	0.500	0.744	0.894	0.964	0.990	0.997	0.999	1.000	1.000	1.000	1.000
	179.00	177.11	172.87	167.71	162.64	158.39	154.36	147.72	142.47	138.17	131.54
195	0.500	0.747	0.898	0.967	0.990	0.998	0.999	1.000	1.000	1.000	1.000
	184.82	182.87	178.33	173.06	167.97	163.33	159.19	152.49	146.95	142.52	135.73
200	0.500	0.749	0.901	0.967	0.991	0.998	0.999	1.000	1.000	1.000	1.000
	188.76	186.73	181.98	176.57	171.40	166.63	162.48	155.58	150.00	145.50	138.45
210	0.500	0.755	0.906	0.971	0.992	0.998	1.000	1.000	1.000	1.000	1.000
	198.50	196.22	191.22	185.47	179.94	174.97	170.67	163.32	157.50	152.67	145.31
220	0.500	0.761	0.911	0.974	0.994	0.999	1.000	1.000	1.000	1.000	1.000
	208.17	205.73	200.46	194.30	188.50	183.22	178.80	171.11	165.03	160.00	152.24
230	0.500	0.766	0.916	0.977	0.994	0.999	1.000	1.000	1.000	1.000	1.000
	217.92	215.25	209.65	203.26	197.24	191.67	186.88	178.87	172.51	167.27	159.23
240	0.500	0.770	0.921	0.978	0.995	0.999	1.000	1.000	1.000	1.000	1.000
	227.67	224.90	218.92	212.14	205.68	200.00	194.97	186.60	180.06	174.47	166.15
250	0.500	0.776	0.924	0.981	0.996	0.999	1.000	1.000	1.000	1.000	1.000
	237.37	234.38	228.12	220.91	214.18	208.34	203.18	194.47	187.52	181.83	173.11

Table 34: Table of Monte Carlo Results for $\mathcal{M}_{\mathrm{BK}}$ when $k = 3$ and $\alpha = 0.05$.

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
1	0.333	0.355	0.375	0.394	0.412	0.429	0.444	0.474	0.500	0.524	0.565
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	0.333	0.355	0.375	0.394	0.412	0.429	0.444	0.474	0.500	0.524	0.565
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
3	0.333	0.362	0.389	0.416	0.441	0.463	0.486	0.527	0.563	0.594	0.651
	2.67	2.67	2.66	2.66	2.66	2.65	2.65	2.63	2.63	2.62	2.58
4	0.333	0.367	0.399	0.429	0.457	0.483	0.508	0.554	0.594	0.629	0.689
	2.89	2.89	2.89	2.88	2.87	2.87	2.85	2.83	2.81	2.79	2.74
5	0.333	0.370	0.404	0.437	0.467	0.497	0.523	0.573	0.618	0.655	0.719
	4.11	4.11	4.11	4.10	4.10	4.10	4.09	4.07	4.05	4.04	3.99
6	0.333	0.374	0.412	0.448	0.483	0.514	0.545	0.599	0.647	0.688	0.754
	4.93	4.92	4.92	4.90	4.88	4.87	4.83	4.78	4.72	4.66	4.54
7	0.333	0.377	0.419	0.459	0.495	0.530	0.563	0.621	0.670	0.714	0.781
	5.79	5.78	5.78	5.77	5.76	5.75	5.73	5.69	5.65	5.60	5.52
8	0.333	0.379	0.424	0.466	0.504	0.540	0.575	0.636	0.688	0.731	0.801
	6.72	6.72	6.70	6.68	6.66	6.63	6.60	6.53	6.45	6.37	6.20
9	0.333	0.383	0.430	0.475	0.517	0.555	0.591	0.656	0.710	0.755	0.823
	7.68	7.67	7.66	7.63	7.60	7.58	7.54	7.45	7.37	7.28	7.12
10	0.333	0.385	0.435	0.483	0.527	0.568	0.605	0.672	0.728	0.773	0.842
	8.53	8.52	8.50	8.47	8.44	8.41	8.37	8.27	8.15	8.04	7.81
11	0.333	0.388	0.440	0.488	0.534	0.576	0.616	0.685	0.742	0.788	0.857
	9.48	9.47	9.45	9.43	9.38	9.35	9.29	9.18	9.04	8.93	8.67
12	0.333	0.390	0.445	0.496	0.544	0.589	0.629	0.699	0.758	0.803	0.871
	10.43	10.42	10.40	10.36	10.32	10.26	10.20	10.06	9.91	9.76	9.45
13	0.333	0.393	0.450	0.503	0.554	0.599	0.641	0.714	0.770	0.818	0.884
	11.32	11.30	11.28	11.25	11.19	11.12	11.06	10.90	10.75	10.58	10.26
14	0.333	0.395	0.453	0.509	0.561	0.607	0.650	0.726	0.783	0.831	0.895
	12.28	12.26	12.23	12.18	12.12	12.06	11.98	11.77	11.60	11.40	11.04
15	0.333	0.397	0.458	0.516	0.570	0.618	0.663	0.737	0.796	0.842	0.905
	13.22	13.21	13.17	13.12	13.05	12.97	12.87	12.68	12.46	12.25	11.84
16	0.333	0.399	0.463	0.523	0.577	0.627	0.672	0.749	0.808	0.854	0.914
	14.12	14.12	14.07	14.01	13.94	13.86	13.76	13.53	13.29	13.06	12.63
17	0.333	0.401	0.466	0.527	0.583	0.635	0.682	0.757	0.817	0.863	0.922
	15.08	15.05	15.02	14.96	14.88	14.78	14.65	14.42	14.15	13.88	13.39
18	0.333	0.403	0.470	0.533	0.592	0.643	0.691	0.769	0.828	0.873	0.930
	16.04	16.01	15.97	15.90	15.79	15.69	15.56	15.29	15.00	14.71	14.17
19	0.333	0.405	0.474	0.539	0.599	0.653	0.700	0.778	0.837	0.882	0.937
	16.94	16.93	16.88	16.80	16.69	16.57	16.44	16.14	15.84	15.52	14.95
20	0.333	0.407	0.478	0.544	0.604	0.659	0.707	0.786	0.846	0.889	0.942
	17.90	17.89	17.83	17.74	17.64	17.50	17.35	17.03	16.69	16.36	15.71
21	0.333	0.409	0.481	0.549	0.611	0.667	0.715	0.797	0.854	0.896	0.948
	18.87	18.84	18.79	18.69	18.56	18.41	18.26	17.89	17.54	17.17	16.51
22	0.333	0.411	0.486	0.554	0.618	0.674	0.723	0.804	0.863	0.904	0.953
	19.78	19.77	19.69	19.60	19.46	19.31	19.16	18.76	18.37	17.98	17.26
23	0.333	0.412	0.488	0.559	0.624	0.680	0.731	0.812	0.869	0.909	0.957
	20.76	20.73	20.65	20.55	20.39	20.23	20.04	19.62	19.21	18.79	18.06
24	0.333	0.414	0.492	0.565	0.630	0.687	0.739	0.819	0.875	0.915	0.962
	21.70	21.68	21.61	21.47	21.33	21.15	20.94	20.51	20.07	19.62	18.79

Table 34: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
25	0.333	0.416	0.495	0.570	0.635	0.694	0.745	0.826	0.881	0.921	0.965
	22.65	22.59	22.53	22.38	22.25	22.04	21.83	21.37	20.91	20.42	19.59
26	0.333	0.417	0.498	0.574	0.642	0.700	0.752	0.832	0.889	0.926	0.968
	23.60	23.56	23.48	23.34	23.15	22.96	22.73	22.24	21.72	21.22	20.34
27	0.333	0.419	0.502	0.578	0.647	0.707	0.758	0.838	0.893	0.931	0.971
	24.58	24.53	24.44	24.29	24.10	23.87	23.63	23.11	22.59	22.07	21.12
28	0.333	0.421	0.504	0.583	0.652	0.713	0.765	0.846	0.899	0.935	0.974
	25.49	25.46	25.37	25.21	25.01	24.77	24.52	23.96	23.43	22.87	21.88
29	0.333	0.422	0.508	0.587	0.658	0.718	0.771	0.851	0.904	0.940	0.976
	26.44	26.42	26.32	26.15	25.93	25.69	25.41	24.83	24.25	23.66	22.64
30	0.333	0.424	0.510	0.591	0.663	0.724	0.776	0.856	0.910	0.942	0.978
	27.43	27.38	27.29	27.10	26.87	26.61	26.33	25.70	25.07	24.50	23.41
31	0.333	0.425	0.514	0.596	0.668	0.730	0.783	0.863	0.913	0.947	0.980
	28.36	28.33	28.21	28.01	27.79	27.52	27.20	26.55	25.92	25.30	24.19
32	0.333	0.427	0.516	0.599	0.672	0.735	0.787	0.867	0.919	0.950	0.982
	29.33	29.27	29.16	28.97	28.72	28.43	28.12	27.43	26.73	26.10	24.94
33	0.333	0.428	0.520	0.603	0.678	0.741	0.792	0.871	0.923	0.954	0.984
	30.28	30.24	30.11	29.91	29.64	29.33	29.02	28.31	27.55	26.88	25.70
34	0.333	0.430	0.523	0.607	0.682	0.744	0.798	0.877	0.927	0.957	0.984
	31.22	31.19	31.05	30.84	30.55	30.26	29.90	29.16	28.40	27.70	26.46
35	0.333	0.431	0.526	0.611	0.686	0.749	0.804	0.880	0.930	0.959	0.986
	32.21	32.14	31.99	31.79	31.49	31.16	30.78	30.03	29.24	28.50	27.23
36	0.333	0.433	0.528	0.614	0.691	0.757	0.808	0.886	0.934	0.962	0.987
	33.16	33.11	32.96	32.73	32.43	32.05	31.69	30.87	30.07	29.30	27.99
37	0.333	0.434	0.531	0.618	0.697	0.760	0.814	0.890	0.937	0.964	0.988
	34.08	34.06	33.89	33.67	33.31	32.97	32.56	31.72	30.89	30.12	28.75
38	0.333	0.435	0.534	0.622	0.700	0.763	0.818	0.894	0.940	0.967	0.990
	35.07	35.03	34.85	34.59	34.27	33.91	33.46	32.58	31.71	30.89	29.52
39	0.333	0.437	0.536	0.626	0.703	0.769	0.821	0.897	0.942	0.968	0.990
	36.04	35.98	35.80	35.55	35.22	34.80	34.38	33.47	32.57	31.70	30.27
40	0.333	0.438	0.539	0.630	0.708	0.774	0.827	0.901	0.945	0.971	0.991
	36.98	36.91	36.74	36.45	36.11	35.68	35.25	34.30	33.40	32.49	31.00
41	0.333	0.439	0.541	0.633	0.713	0.778	0.831	0.905	0.949	0.972	0.992
	37.96	37.88	37.71	37.41	37.03	36.59	36.14	35.16	34.20	33.30	31.79
42	0.333	0.441	0.544	0.636	0.715	0.783	0.837	0.910	0.950	0.974	0.993
	38.93	38.82	38.67	38.37	38.00	37.51	37.01	36.01	35.05	34.12	32.53
43	0.333	0.442	0.546	0.640	0.720	0.787	0.839	0.912	0.954	0.976	0.994
	39.88	39.80	39.62	39.28	38.89	38.40	37.93	36.88	35.83	34.93	33.28
44	0.333	0.444	0.549	0.643	0.725	0.791	0.843	0.914	0.956	0.977	0.994
	40.84	40.76	40.55	40.24	39.80	39.32	38.81	37.74	36.66	35.71	34.05
45	0.333	0.444	0.551	0.646	0.729	0.794	0.847	0.919	0.957	0.979	0.995
	41.79	41.74	41.53	41.19	40.72	40.25	39.71	38.55	37.52	36.52	34.80
46	0.333	0.446	0.553	0.651	0.731	0.799	0.850	0.922	0.961	0.980	0.995
	42.77	42.68	42.47	42.09	41.67	41.15	40.60	39.41	38.30	37.28	35.58
47	0.333	0.447	0.556	0.652	0.737	0.802	0.854	0.924	0.962	0.981	0.996
	43.74	43.64	43.40	43.07	42.57	42.04	41.49	40.29	39.13	38.10	36.30
48	0.333	0.448	0.558	0.656	0.740	0.806	0.858	0.927	0.964	0.983	0.996
	44.71	44.64	44.38	44.00	43.50	42.96	42.38	41.13	39.97	38.86	37.03

Table 34: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
49	0.333	0.450	0.560	0.659	0.743	0.809	0.862	0.930	0.966	0.983	0.996
	45.65	45.57	45.32	44.94	44.43	43.86	43.24	41.99	40.77	39.67	37.86
50	0.333	0.450	0.563	0.662	0.746	0.812	0.864	0.931	0.968	0.985	0.997
	46.62	46.56	46.28	45.88	45.36	44.78	44.16	42.85	41.58	40.45	38.59
51	0.333	0.452	0.565	0.666	0.749	0.816	0.867	0.934	0.969	0.986	0.997
	47.61	47.50	47.24	46.82	46.31	45.67	45.01	43.68	42.41	41.26	39.35
52	0.333	0.453	0.568	0.668	0.753	0.821	0.872	0.937	0.970	0.986	0.997
	48.54	48.46	48.19	47.76	47.20	46.57	45.89	44.55	43.24	42.03	40.08
53	0.333	0.454	0.569	0.671	0.755	0.824	0.874	0.939	0.971	0.987	0.997
	49.50	49.44	49.16	48.69	48.14	47.48	46.79	45.39	44.09	42.88	40.84
54	0.333	0.455	0.571	0.676	0.758	0.826	0.877	0.941	0.972	0.988	0.998
	50.49	50.41	50.12	49.61	49.08	48.39	47.68	46.26	44.90	43.65	41.61
55	0.333	0.456	0.574	0.678	0.763	0.829	0.879	0.944	0.974	0.989	0.998
	51.46	51.36	51.04	50.57	49.98	49.31	48.58	47.07	45.71	44.43	42.38
56	0.333	0.457	0.576	0.679	0.766	0.832	0.882	0.945	0.976	0.989	0.998
	52.43	52.33	52.02	51.54	50.89	50.20	49.47	47.96	46.51	45.25	43.14
57	0.333	0.459	0.579	0.682	0.769	0.837	0.885	0.947	0.977	0.990	0.998
	53.40	53.29	52.96	52.49	51.82	51.06	50.36	48.77	47.34	46.04	43.84
58	0.333	0.460	0.580	0.686	0.770	0.840	0.888	0.949	0.978	0.991	0.998
	54.36	54.23	53.93	53.39	52.77	51.99	51.21	49.68	48.16	46.82	44.61
59	0.333	0.461	0.582	0.688	0.774	0.842	0.891	0.950	0.978	0.991	0.999
	55.32	55.21	54.88	54.35	53.69	52.89	52.11	50.52	49.00	47.63	45.38
60	0.333	0.462	0.585	0.691	0.779	0.843	0.892	0.952	0.980	0.992	0.999
	56.28	56.19	55.84	55.28	54.57	53.85	53.04	51.36	49.79	48.43	46.11
61	0.333	0.463	0.587	0.692	0.780	0.846	0.897	0.954	0.981	0.992	0.999
	57.26	57.15	56.79	56.25	55.54	54.73	53.86	52.17	50.63	49.18	46.88
62	0.333	0.465	0.588	0.697	0.784	0.851	0.898	0.956	0.982	0.993	0.999
	58.20	58.09	57.74	57.16	56.43	55.61	54.75	53.03	51.41	50.02	47.57
63	0.333	0.465	0.590	0.700	0.787	0.852	0.900	0.957	0.983	0.993	0.999
	59.19	59.08	58.72	58.07	57.33	56.52	55.67	53.89	52.28	50.82	48.37
64	0.333	0.467	0.592	0.701	0.790	0.856	0.903	0.959	0.983	0.993	0.999
	60.19	60.02	59.66	59.06	58.26	57.40	56.51	54.75	53.09	51.62	49.11
65	0.333	0.468	0.595	0.706	0.793	0.857	0.906	0.960	0.984	0.994	0.999
	61.13	61.01	60.60	59.97	59.18	58.34	57.39	55.59	53.89	52.40	49.89
66	0.333	0.469	0.598	0.707	0.794	0.859	0.908	0.962	0.985	0.994	0.999
	62.14	61.97	61.53	60.93	60.12	59.26	58.28	56.41	54.68	53.18	50.63
67	0.333	0.470	0.598	0.709	0.796	0.863	0.909	0.963	0.986	0.995	0.999
	63.05	62.93	62.53	61.86	61.08	60.11	59.17	57.25	55.51	53.94	51.41
68	0.333	0.470	0.601	0.713	0.800	0.865	0.911	0.964	0.986	0.995	0.999
	64.04	63.93	63.46	62.79	61.97	61.03	60.04	58.11	56.31	54.75	52.16
69	0.333	0.472	0.601	0.713	0.803	0.867	0.914	0.965	0.987	0.995	0.999
	65.01	64.86	64.46	63.77	62.88	61.97	60.93	58.97	57.13	55.51	52.88
70	0.333	0.473	0.604	0.717	0.806	0.869	0.915	0.966	0.988	0.996	0.999
	65.98	65.83	65.40	64.68	63.80	62.85	61.84	59.84	57.96	56.32	53.66
71	0.333	0.473	0.606	0.718	0.808	0.872	0.917	0.968	0.988	0.996	0.999
	66.95	66.81	66.34	65.66	64.73	63.74	62.73	60.65	58.73	57.06	54.37
72	0.333	0.474	0.608	0.721	0.810	0.875	0.919	0.968	0.989	0.996	1.000
	67.94	67.79	67.30	66.59	65.66	64.63	63.59	61.52	59.57	57.88	55.13

Table 34: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
73	0.333	0.476	0.610	0.723	0.812	0.876	0.922	0.970	0.989	0.996	1.000
	68.86	68.73	68.27	67.51	66.57	65.58	64.45	62.34	60.41	58.65	55.92
74	0.333	0.477	0.613	0.727	0.814	0.878	0.923	0.971	0.990	0.997	1.000
	69.86	69.70	69.19	68.44	67.50	66.46	65.34	63.22	61.24	59.49	56.68
75	0.333	0.478	0.613	0.727	0.818	0.881	0.925	0.972	0.991	0.997	1.000
	70.82	70.67	70.19	69.41	68.39	67.36	66.22	64.00	62.01	60.24	57.42
76	0.333	0.479	0.616	0.730	0.818	0.884	0.926	0.973	0.991	0.997	1.000
	71.81	71.62	71.12	70.36	69.38	68.21	67.13	64.83	62.80	61.07	58.16
77	0.333	0.479	0.618	0.733	0.823	0.886	0.929	0.974	0.991	0.997	1.000
	72.79	72.63	72.06	71.26	70.21	69.10	67.95	65.72	63.69	61.83	58.91
78	0.333	0.481	0.619	0.734	0.824	0.887	0.929	0.974	0.992	0.997	1.000
	73.77	73.58	73.04	72.24	71.16	70.04	68.89	66.57	64.48	62.65	59.63
79	0.333	0.482	0.621	0.737	0.825	0.888	0.932	0.976	0.992	0.997	1.000
	74.70	74.53	74.00	73.16	72.13	70.97	69.73	67.38	65.25	63.43	60.46
80	0.333	0.482	0.623	0.738	0.828	0.891	0.933	0.977	0.992	0.998	1.000
	75.70	75.53	74.95	74.12	73.01	71.85	70.59	68.21	66.11	64.23	61.13
81	0.333	0.483	0.623	0.742	0.830	0.893	0.935	0.977	0.993	0.998	1.000
	76.67	76.52	75.94	75.03	73.94	72.72	71.47	69.09	66.90	64.96	61.92
82	0.333	0.484	0.625	0.743	0.832	0.894	0.935	0.978	0.993	0.998	1.000
	77.66	77.47	76.89	75.99	74.88	73.66	72.38	69.91	67.74	65.77	62.71
83	0.333	0.485	0.628	0.746	0.834	0.896	0.938	0.979	0.993	0.998	1.000
	78.61	78.43	77.83	76.92	75.79	74.54	73.25	70.73	68.54	66.58	63.43
84	0.333	0.486	0.629	0.749	0.836	0.897	0.939	0.980	0.994	0.998	1.000
	79.57	79.42	78.81	77.82	76.71	75.47	74.10	71.61	69.32	67.34	64.17
85	0.333	0.487	0.631	0.749	0.838	0.900	0.940	0.980	0.994	0.998	1.000
	80.56	80.36	79.74	78.81	77.60	76.34	75.00	72.41	70.15	68.20	64.90
86	0.333	0.488	0.633	0.751	0.840	0.902	0.942	0.981	0.994	0.998	1.000
	81.53	81.33	80.69	79.75	78.56	77.23	75.87	73.22	70.99	68.89	65.67
87	0.333	0.489	0.634	0.755	0.842	0.904	0.944	0.982	0.994	0.998	1.000
	82.51	82.32	81.69	80.64	79.46	78.11	76.71	74.15	71.81	69.76	66.40
88	0.333	0.490	0.635	0.756	0.842	0.905	0.945	0.982	0.995	0.998	1.000
	83.48	83.28	82.65	81.59	80.45	79.04	77.61	74.98	72.62	70.51	67.20
89	0.333	0.491	0.638	0.756	0.846	0.908	0.944	0.983	0.995	0.999	1.000
	84.50	84.25	83.55	82.59	81.32	79.88	78.53	75.79	73.39	71.35	67.91
90	0.333	0.491	0.640	0.759	0.849	0.908	0.947	0.984	0.996	0.999	1.000
	85.43	85.24	84.53	83.52	82.20	80.84	79.40	76.62	74.18	72.06	68.69
91	0.333	0.493	0.641	0.762	0.850	0.909	0.948	0.984	0.996	0.999	1.000
	86.40	86.17	85.50	84.43	83.11	81.74	80.27	77.48	75.05	72.88	69.43
92	0.333	0.494	0.643	0.763	0.851	0.911	0.949	0.985	0.996	0.999	1.000
	87.38	87.12	86.44	85.40	84.10	82.64	81.13	78.38	75.83	73.68	70.19
93	0.333	0.494	0.644	0.764	0.854	0.913	0.951	0.985	0.996	0.999	1.000
	88.36	88.15	87.41	86.32	84.97	83.49	81.98	79.16	76.66	74.44	70.92
94	0.333	0.496	0.646	0.767	0.856	0.914	0.952	0.986	0.996	0.999	1.000
	89.34	89.06	88.37	87.25	85.88	84.40	82.90	80.01	77.46	75.32	71.66
95	0.333	0.497	0.647	0.768	0.857	0.916	0.953	0.986	0.996	0.999	1.000
	90.33	90.05	89.33	88.20	86.82	85.27	83.76	80.82	78.23	76.06	72.45
96	0.333	0.497	0.649	0.771	0.859	0.916	0.953	0.986	0.996	0.999	1.000
	91.26	91.03	90.28	89.12	87.73	86.23	84.63	81.69	79.06	76.86	73.17

Table 34: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
97	0.333	0.498	0.651	0.772	0.860	0.918	0.955	0.987	0.997	0.999	1.000
	92.26	92.00	91.24	90.10	88.67	87.11	85.50	82.55	79.90	77.61	73.90
98	0.333	0.499	0.651	0.774	0.863	0.920	0.956	0.987	0.997	0.999	1.000
	93.25	92.96	92.23	91.02	89.56	87.98	86.37	83.37	80.75	78.43	74.72
99	0.333	0.500	0.654	0.777	0.864	0.921	0.956	0.988	0.997	0.999	1.000
	94.21	93.95	93.14	91.95	90.49	88.91	87.29	84.17	81.55	79.16	75.39
100	0.333	0.501	0.655	0.776	0.866	0.923	0.957	0.988	0.997	0.999	1.000
	95.14	94.91	94.13	92.95	91.38	89.75	88.14	85.04	82.32	79.94	76.19
101	0.333	0.501	0.655	0.780	0.868	0.924	0.958	0.989	0.997	0.999	1.000
	96.18	95.90	95.11	93.84	92.31	90.68	89.03	85.85	83.15	80.78	76.90
102	0.333	0.502	0.658	0.780	0.869	0.926	0.959	0.989	0.997	0.999	1.000
	97.13	96.88	96.04	94.82	93.22	91.51	89.87	86.72	83.94	81.54	77.68
103	0.333	0.504	0.660	0.783	0.870	0.926	0.960	0.989	0.998	0.999	1.000
	98.09	97.83	96.98	95.70	94.15	92.46	90.77	87.57	84.74	82.36	78.47
104	0.333	0.503	0.660	0.785	0.873	0.928	0.961	0.990	0.998	0.999	1.000
	99.11	98.83	97.98	96.65	95.05	93.35	91.58	88.36	85.54	83.12	79.16
105	0.333	0.505	0.663	0.786	0.873	0.928	0.962	0.990	0.998	0.999	1.000
	100.07	99.79	98.90	97.61	95.99	94.28	92.53	89.24	86.33	83.94	79.92
106	0.333	0.505	0.664	0.789	0.875	0.930	0.963	0.991	0.998	0.999	1.000
	101.05	100.76	99.87	98.52	96.90	95.17	93.41	90.10	87.12	84.77	80.64
107	0.333	0.506	0.664	0.790	0.877	0.931	0.964	0.991	0.998	0.999	1.000
	102.00	101.74	100.84	99.45	97.79	96.04	94.26	90.95	87.95	85.50	81.44
108	0.333	0.508	0.667	0.791	0.879	0.932	0.965	0.991	0.998	1.000	1.000
	102.97	102.67	101.77	100.41	98.68	96.94	95.11	91.79	88.85	86.27	82.21
109	0.333	0.508	0.668	0.793	0.879	0.933	0.965	0.992	0.998	1.000	1.000
	103.94	103.67	102.75	101.35	99.68	97.84	96.03	92.57	89.63	87.08	82.91
110	0.333	0.509	0.669	0.794	0.882	0.935	0.965	0.992	0.998	1.000	1.000
	104.94	104.62	103.72	102.29	100.54	98.74	96.90	93.42	90.45	87.91	83.65
111	0.333	0.509	0.670	0.797	0.882	0.936	0.967	0.992	0.998	1.000	1.000
	105.95	105.63	104.67	103.21	101.48	99.62	97.72	94.25	91.28	88.62	84.42
112	0.333	0.510	0.672	0.798	0.884	0.936	0.967	0.992	0.998	1.000	1.000
	106.87	106.61	105.64	104.18	102.37	100.48	98.61	95.11	92.05	89.44	85.22
113	0.333	0.511	0.674	0.799	0.885	0.938	0.969	0.992	0.999	1.000	1.000
	107.85	107.56	106.58	105.10	103.31	101.38	99.50	95.98	92.89	90.24	85.96
114	0.333	0.512	0.675	0.802	0.886	0.939	0.969	0.993	0.998	1.000	1.000
	108.86	108.55	107.54	106.01	104.23	102.31	100.35	96.77	93.65	91.05	86.68
115	0.333	0.513	0.676	0.803	0.888	0.939	0.970	0.993	0.999	1.000	1.000
	109.84	109.52	108.49	106.97	105.15	103.19	101.20	97.57	94.45	91.81	87.42
116	0.333	0.514	0.679	0.803	0.889	0.941	0.970	0.993	0.999	1.000	1.000
	110.79	110.45	109.43	107.94	106.05	104.06	102.09	98.46	95.29	92.61	88.22
117	0.333	0.514	0.679	0.806	0.890	0.942	0.971	0.993	0.999	1.000	1.000
	111.75	111.47	110.40	108.83	106.96	104.92	102.95	99.31	96.18	93.36	88.97
118	0.333	0.515	0.680	0.808	0.892	0.943	0.972	0.994	0.999	1.000	1.000
	112.78	112.43	111.39	109.74	107.91	105.86	103.85	100.13	96.91	94.16	89.70
119	0.333	0.516	0.682	0.808	0.894	0.943	0.973	0.994	0.999	1.000	1.000
	113.72	113.39	112.32	110.74	108.80	106.79	104.63	100.99	97.72	94.96	90.44
120	0.333	0.517	0.683	0.810	0.893	0.944	0.973	0.994	0.999	1.000	1.000
	114.72	114.34	113.28	111.67	109.73	107.65	105.58	101.77	98.56	95.72	91.23

Table 34: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
121	0.333	0.517	0.685	0.811	0.896	0.946	0.974	0.994	0.999	1.000	1.000
	115.69	115.36	114.24	112.60	110.65	108.54	106.44	102.66	99.32	96.54	91.97
122	0.333	0.518	0.686	0.814	0.896	0.947	0.973	0.994	0.999	1.000	1.000
	116.68	116.34	115.20	113.51	111.54	109.43	107.37	103.43	100.13	97.27	92.69
123	0.333	0.519	0.689	0.814	0.898	0.947	0.974	0.995	0.999	1.000	1.000
	117.66	117.31	116.10	114.46	112.46	110.31	108.19	104.31	100.99	98.07	93.46
124	0.333	0.520	0.687	0.816	0.900	0.949	0.975	0.995	0.999	1.000	1.000
	118.61	118.23	117.17	115.41	113.35	111.21	109.08	105.16	101.74	98.83	94.14
125	0.333	0.521	0.690	0.817	0.901	0.949	0.976	0.995	0.999	1.000	1.000
	119.60	119.23	118.08	116.34	114.26	112.11	109.94	106.04	102.62	99.63	94.95
126	0.333	0.521	0.691	0.818	0.902	0.950	0.976	0.995	0.999	1.000	1.000
	120.59	120.21	119.05	117.31	115.20	113.00	110.81	106.83	103.33	100.46	95.67
127	0.333	0.523	0.692	0.820	0.903	0.950	0.976	0.995	0.999	1.000	1.000
	121.57	121.15	120.01	118.22	116.10	113.94	111.69	107.66	104.21	101.21	96.45
128	0.333	0.523	0.694	0.821	0.904	0.952	0.977	0.995	0.999	1.000	1.000
	122.57	122.16	120.95	119.17	116.99	114.75	112.59	108.54	104.94	101.98	97.18
129	0.333	0.524	0.696	0.823	0.905	0.952	0.978	0.995	0.999	1.000	1.000
	123.53	123.12	121.87	120.08	117.94	115.73	113.43	109.36	105.82	102.83	97.92
130	0.333	0.524	0.695	0.824	0.905	0.953	0.978	0.996	0.999	1.000	1.000
	124.52	124.12	122.91	121.05	118.87	116.54	114.28	110.16	106.62	103.57	98.67
131	0.333	0.525	0.697	0.826	0.908	0.954	0.979	0.996	0.999	1.000	1.000
	125.49	125.07	123.85	121.95	119.75	117.48	115.15	110.96	107.42	104.39	99.42
132	0.333	0.527	0.700	0.826	0.908	0.955	0.979	0.996	0.999	1.000	1.000
	126.47	126.01	124.75	122.92	120.72	118.39	116.05	111.89	108.25	105.16	100.18
133	0.333	0.527	0.700	0.827	0.910	0.956	0.980	0.996	0.999	1.000	1.000
	127.42	126.99	125.74	123.89	121.59	119.23	116.87	112.66	109.01	105.96	100.93
134	0.333	0.527	0.701	0.828	0.910	0.956	0.980	0.996	0.999	1.000	1.000
	128.44	128.00	126.71	124.80	122.51	120.14	117.81	113.54	109.86	106.74	101.68
135	0.333	0.528	0.702	0.831	0.912	0.957	0.980	0.996	0.999	1.000	1.000
	129.37	128.96	127.67	125.71	123.40	121.03	118.68	114.32	110.57	107.50	102.39
136	0.333	0.529	0.704	0.832	0.913	0.958	0.981	0.996	0.999	1.000	1.000
	130.37	129.94	128.64	126.63	124.32	121.85	119.55	115.20	111.49	108.29	103.16
137	0.333	0.530	0.705	0.833	0.914	0.958	0.981	0.997	1.000	1.000	1.000
	131.33	130.91	129.60	127.63	125.24	122.79	120.35	116.07	112.26	109.06	103.95
138	0.333	0.531	0.704	0.835	0.915	0.959	0.982	0.997	0.999	1.000	1.000
	132.35	131.87	130.60	128.52	126.14	123.67	121.23	116.84	113.04	109.87	104.64
139	0.333	0.532	0.707	0.836	0.916	0.960	0.982	0.997	1.000	1.000	1.000
	133.32	132.82	131.49	129.48	127.08	124.54	122.10	117.70	113.91	110.66	105.39
140	0.333	0.531	0.707	0.838	0.916	0.961	0.983	0.997	1.000	1.000	1.000
	134.27	133.85	132.49	130.35	128.00	125.44	123.00	118.53	114.66	111.47	106.14
141	0.333	0.532	0.710	0.839	0.918	0.961	0.983	0.997	1.000	1.000	1.000
	135.29	134.81	133.42	131.28	128.91	126.34	123.87	119.37	115.44	112.26	106.96
142	0.333	0.532	0.710	0.839	0.918	0.962	0.984	0.997	1.000	1.000	1.000
	136.28	135.82	134.40	132.30	129.81	127.15	124.69	120.17	116.29	113.02	107.67
143	0.333	0.534	0.713	0.839	0.920	0.963	0.984	0.997	1.000	1.000	1.000
	137.18	136.76	135.30	133.26	130.69	128.09	125.55	121.05	117.09	113.79	108.46
144	0.333	0.535	0.713	0.842	0.920	0.963	0.984	0.997	1.000	1.000	1.000
	138.23	137.74	136.30	134.16	131.68	129.02	126.47	121.89	117.94	114.56	109.15

Table 34: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
145	0.333	0.535	0.715	0.843	0.921	0.963	0.984	0.997	1.000	1.000	1.000
	139.19	138.72	137.23	135.07	132.60	129.90	127.34	122.72	118.74	115.38	109.93
146	0.333	0.536	0.715	0.844	0.922	0.964	0.984	0.998	1.000	1.000	1.000
	140.18	139.66	138.24	136.00	133.48	130.78	128.27	123.53	119.55	116.17	110.64
147	0.333	0.537	0.716	0.845	0.924	0.965	0.985	0.998	1.000	1.000	1.000
	141.15	140.66	139.18	136.95	134.35	131.66	129.04	124.34	120.34	116.92	111.48
148	0.333	0.538	0.717	0.846	0.924	0.966	0.985	0.998	1.000	1.000	1.000
	142.10	141.58	140.15	137.87	135.26	132.57	129.95	125.22	121.21	117.69	112.22
149	0.333	0.538	0.719	0.847	0.925	0.965	0.986	0.998	1.000	1.000	1.000
	143.13	142.61	141.09	138.84	136.14	133.49	130.82	126.12	122.00	118.55	112.88
150	0.333	0.538	0.721	0.849	0.925	0.967	0.986	0.998	1.000	1.000	1.000
	144.11	143.59	142.01	139.76	137.14	134.35	131.70	126.86	122.79	119.29	113.67
155	0.333	0.542	0.726	0.855	0.930	0.969	0.987	0.998	1.000	1.000	1.000
	148.95	148.46	146.83	144.39	141.67	138.79	136.03	131.08	126.84	123.25	117.39
160	0.333	0.546	0.730	0.860	0.934	0.972	0.989	0.998	1.000	1.000	1.000
	153.93	153.32	151.66	149.06	146.21	143.25	140.37	135.28	130.91	127.14	121.14
165	0.333	0.549	0.735	0.864	0.937	0.974	0.990	0.999	1.000	1.000	1.000
	158.79	158.23	156.45	153.82	150.81	147.73	144.74	139.35	134.85	131.02	124.87
170	0.333	0.551	0.741	0.868	0.941	0.976	0.991	0.999	1.000	1.000	1.000
	163.72	163.13	161.20	158.50	155.32	152.08	149.06	143.59	138.96	135.00	128.64
175	0.333	0.555	0.747	0.874	0.944	0.977	0.992	0.999	1.000	1.000	1.000
	168.58	167.96	165.96	163.11	159.89	156.62	153.32	147.70	142.98	138.94	132.36
180	0.333	0.558	0.752	0.879	0.946	0.980	0.993	0.999	1.000	1.000	1.000
	173.51	172.84	170.75	167.79	164.49	160.96	157.75	151.92	147.01	142.86	136.12
185	0.333	0.561	0.755	0.883	0.950	0.981	0.993	0.999	1.000	1.000	1.000
	178.37	177.73	175.61	172.49	168.93	165.40	162.05	156.08	151.06	146.75	139.84
190	0.333	0.565	0.761	0.886	0.953	0.983	0.994	0.999	1.000	1.000	1.000
	183.33	182.59	180.33	177.16	173.46	169.81	166.44	160.19	155.12	150.73	143.61
195	0.333	0.567	0.766	0.890	0.955	0.984	0.995	0.999	1.000	1.000	1.000
	188.22	187.46	185.13	181.83	178.07	174.27	170.74	164.40	159.13	154.67	147.33
200	0.333	0.570	0.769	0.895	0.958	0.985	0.995	1.000	1.000	1.000	1.000
	193.13	192.37	189.96	186.44	182.53	178.68	175.05	168.65	163.17	158.56	151.13
210	0.333	0.577	0.778	0.903	0.963	0.988	0.996	1.000	1.000	1.000	1.000
	203.01	202.08	199.55	195.80	191.65	187.46	183.77	176.93	171.22	166.38	158.53
220	0.333	0.583	0.786	0.908	0.967	0.990	0.997	1.000	1.000	1.000	1.000
	212.85	211.87	209.09	205.12	200.70	196.40	192.31	185.26	179.28	174.22	166.04
230	0.333	0.587	0.794	0.915	0.971	0.991	0.997	1.000	1.000	1.000	1.000
	222.68	221.66	218.66	214.39	209.71	205.27	201.02	193.66	187.41	182.10	173.48
240	0.333	0.593	0.800	0.920	0.973	0.992	0.998	1.000	1.000	1.000	1.000
	232.48	231.46	228.29	223.78	218.85	214.06	209.71	201.90	195.41	189.94	180.96
250	0.333	0.598	0.810	0.926	0.976	0.993	0.998	1.000	1.000	1.000	1.000
	242.36	241.22	237.74	233.02	227.86	222.94	218.30	210.32	203.56	197.72	188.48

Table 35: Table of Monte Carlo Results for $\mathcal{M}_{\text{BK}}$ when $k = 4$ and $\alpha = 0.05$.

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
1	0.250	0.268	0.286	0.302	0.318	0.333	0.348	0.375	0.400	0.423	0.464
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	0.250	0.268	0.286	0.302	0.318	0.333	0.348	0.375	0.400	0.423	0.464
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
3	0.250	0.273	0.296	0.317	0.338	0.358	0.377	0.414	0.449	0.480	0.535
	2.75	2.75	2.75	2.75	2.74	2.74	2.74	2.73	2.72	2.71	2.69
4	0.250	0.278	0.305	0.331	0.356	0.380	0.403	0.446	0.487	0.523	0.586
	3.12	3.12	3.12	3.11	3.11	3.10	3.09	3.08	3.05	3.04	2.99
5	0.250	0.280	0.310	0.339	0.366	0.393	0.418	0.466	0.509	0.549	0.618
	4.18	4.17	4.16	4.17	4.16	4.16	4.15	4.14	4.13	4.12	4.09
6	0.250	0.283	0.315	0.346	0.376	0.405	0.433	0.486	0.532	0.575	0.650
	5.08	5.08	5.07	5.07	5.06	5.05	5.04	5.00	4.97	4.93	4.84
7	0.250	0.286	0.321	0.355	0.387	0.420	0.450	0.507	0.557	0.605	0.681
	6.07	6.07	6.06	6.05	6.05	6.03	6.01	5.97	5.94	5.88	5.79
8	0.250	0.288	0.326	0.363	0.399	0.432	0.464	0.526	0.581	0.628	0.708
	6.91	6.91	6.90	6.89	6.87	6.86	6.84	6.79	6.74	6.68	6.56
9	0.250	0.291	0.330	0.369	0.407	0.443	0.478	0.542	0.599	0.649	0.730
	7.88	7.88	7.87	7.86	7.83	7.82	7.80	7.75	7.67	7.61	7.47
10	0.250	0.292	0.334	0.376	0.416	0.454	0.490	0.557	0.616	0.669	0.754
	8.82	8.82	8.81	8.78	8.76	8.74	8.71	8.64	8.56	8.47	8.28
11	0.250	0.295	0.339	0.383	0.424	0.465	0.503	0.573	0.634	0.687	0.773
	9.78	9.78	9.77	9.74	9.73	9.69	9.65	9.57	9.48	9.38	9.16
12	0.250	0.297	0.343	0.388	0.433	0.474	0.516	0.587	0.653	0.706	0.790
	10.68	10.69	10.67	10.65	10.62	10.59	10.53	10.45	10.33	10.22	9.99
13	0.250	0.299	0.347	0.394	0.441	0.484	0.526	0.602	0.665	0.722	0.807
	11.65	11.65	11.64	11.62	11.58	11.54	11.49	11.37	11.26	11.12	10.84
14	0.250	0.300	0.351	0.400	0.447	0.493	0.536	0.614	0.681	0.736	0.821
	12.60	12.60	12.57	12.54	12.52	12.46	12.41	12.28	12.13	11.99	11.67
15	0.250	0.302	0.355	0.406	0.456	0.503	0.546	0.626	0.694	0.751	0.835
	13.57	13.57	13.54	13.51	13.45	13.41	13.36	13.21	13.05	12.87	12.52
16	0.250	0.304	0.358	0.411	0.463	0.511	0.556	0.639	0.707	0.764	0.848
	14.49	14.49	14.45	14.43	14.37	14.32	14.26	14.09	13.92	13.73	13.35
17	0.250	0.306	0.361	0.416	0.469	0.519	0.566	0.651	0.719	0.776	0.860
	15.46	15.44	15.44	15.38	15.34	15.27	15.20	15.00	14.83	14.62	14.19
18	0.250	0.307	0.364	0.421	0.475	0.528	0.575	0.662	0.730	0.788	0.871
	16.41	16.39	16.38	16.33	16.28	16.19	16.11	15.91	15.71	15.47	14.99
19	0.250	0.309	0.368	0.427	0.482	0.535	0.585	0.672	0.742	0.799	0.879
	17.38	17.36	17.34	17.29	17.23	17.16	17.05	16.84	16.62	16.35	15.86
20	0.250	0.310	0.371	0.431	0.489	0.543	0.592	0.682	0.754	0.810	0.890
	18.33	18.31	18.28	18.23	18.15	18.07	17.98	17.73	17.47	17.22	16.66
21	0.250	0.312	0.375	0.436	0.494	0.551	0.601	0.691	0.763	0.821	0.898
	19.28	19.27	19.24	19.18	19.12	19.01	18.92	18.65	18.38	18.08	17.50
22	0.250	0.313	0.377	0.440	0.501	0.557	0.610	0.699	0.774	0.829	0.905
	20.25	20.23	20.19	20.14	20.05	19.95	19.83	19.56	19.25	18.94	18.30
23	0.250	0.315	0.380	0.445	0.507	0.563	0.619	0.710	0.781	0.838	0.913
	21.22	21.20	21.17	21.10	21.00	20.91	20.76	20.47	20.16	19.82	19.14
24	0.250	0.316	0.383	0.450	0.513	0.571	0.626	0.718	0.792	0.847	0.919
	22.17	22.15	22.11	22.03	21.94	21.82	21.69	21.38	21.01	20.66	19.95

Table 35: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
25	0.250	0.318	0.387	0.453	0.518	0.578	0.632	0.726	0.798	0.855	0.925
	23.14	23.13	23.07	23.00	22.90	22.76	22.62	22.29	21.92	21.53	20.77
26	0.250	0.319	0.389	0.458	0.524	0.584	0.642	0.734	0.807	0.862	0.931
	24.09	24.08	24.03	23.95	23.83	23.70	23.53	23.19	22.81	22.38	21.57
27	0.250	0.321	0.392	0.462	0.529	0.592	0.647	0.743	0.816	0.868	0.935
	25.06	25.04	25.00	24.92	24.80	24.64	24.48	24.08	23.68	23.25	22.42
28	0.250	0.322	0.395	0.466	0.534	0.597	0.654	0.750	0.823	0.877	0.940
	26.02	26.01	25.94	25.86	25.73	25.58	25.40	25.00	24.55	24.09	23.23
29	0.250	0.323	0.398	0.470	0.539	0.604	0.660	0.756	0.829	0.883	0.944
	27.00	26.98	26.90	26.81	26.69	26.51	26.35	25.92	25.43	24.95	24.06
30	0.250	0.324	0.400	0.475	0.545	0.608	0.667	0.765	0.837	0.889	0.948
	27.95	27.94	27.87	27.75	27.63	27.47	27.26	26.79	26.30	25.79	24.85
31	0.250	0.326	0.403	0.478	0.549	0.615	0.675	0.772	0.843	0.894	0.953
	28.92	28.89	28.83	28.73	28.59	28.40	28.18	27.71	27.20	26.67	25.66
32	0.250	0.327	0.405	0.481	0.554	0.621	0.681	0.777	0.849	0.899	0.955
	29.90	29.84	29.80	29.69	29.53	29.33	29.10	28.62	28.08	27.51	26.47
33	0.250	0.328	0.408	0.487	0.561	0.626	0.686	0.785	0.854	0.904	0.959
	30.86	30.85	30.75	30.63	30.46	30.28	30.04	29.51	28.96	28.37	27.27
34	0.250	0.329	0.411	0.491	0.565	0.632	0.692	0.788	0.860	0.909	0.963
	31.82	31.80	31.71	31.58	31.42	31.21	30.96	30.45	29.83	29.22	28.05
35	0.250	0.331	0.413	0.494	0.568	0.636	0.697	0.795	0.865	0.913	0.965
	32.79	32.77	32.68	32.55	32.38	32.17	31.93	31.35	30.71	30.10	28.89
36	0.250	0.332	0.416	0.496	0.574	0.644	0.703	0.803	0.870	0.917	0.968
	33.77	33.72	33.65	33.51	33.31	33.07	32.83	32.20	31.58	30.95	29.69
37	0.250	0.333	0.418	0.500	0.578	0.648	0.710	0.809	0.875	0.922	0.969
	34.74	34.69	34.62	34.48	34.27	34.04	33.75	33.10	32.46	31.78	30.52
38	0.250	0.334	0.420	0.504	0.582	0.653	0.715	0.812	0.882	0.926	0.972
	35.71	35.68	35.59	35.43	35.21	34.96	34.68	34.04	33.30	32.61	31.30
39	0.250	0.335	0.423	0.508	0.587	0.659	0.721	0.818	0.885	0.930	0.974
	36.65	36.64	36.55	36.39	36.18	35.90	35.60	34.92	34.19	33.45	32.09
40	0.250	0.337	0.425	0.512	0.592	0.663	0.726	0.825	0.891	0.933	0.976
	37.64	37.59	37.50	37.33	37.11	36.84	36.52	35.80	35.04	34.29	32.93
41	0.250	0.337	0.427	0.514	0.594	0.668	0.731	0.828	0.895	0.936	0.978
	38.61	38.58	38.48	38.30	38.08	37.77	37.45	36.72	35.91	35.15	33.71
42	0.250	0.339	0.429	0.517	0.599	0.674	0.736	0.833	0.897	0.940	0.979
	39.59	39.53	39.45	39.26	39.03	38.70	38.36	37.62	36.81	36.01	34.51
43	0.250	0.340	0.431	0.521	0.603	0.677	0.741	0.839	0.902	0.942	0.981
	40.56	40.53	40.42	40.21	39.97	39.66	39.30	38.49	37.68	36.83	35.30
44	0.250	0.341	0.434	0.524	0.608	0.683	0.746	0.841	0.906	0.945	0.982
	41.52	41.49	41.37	41.19	40.89	40.58	40.22	39.42	38.52	37.67	36.12
45	0.250	0.342	0.436	0.527	0.612	0.686	0.748	0.846	0.910	0.949	0.984
	42.50	42.45	42.35	42.15	41.86	41.54	41.18	40.30	39.39	38.51	36.92
46	0.250	0.343	0.439	0.530	0.615	0.691	0.754	0.851	0.913	0.951	0.984
	43.47	43.42	43.30	43.10	42.82	42.46	42.08	41.19	40.27	39.37	37.73
47	0.250	0.344	0.441	0.534	0.620	0.694	0.759	0.856	0.916	0.953	0.986
	44.45	44.41	44.27	44.06	43.76	43.42	42.99	42.08	41.14	40.24	38.51
48	0.250	0.345	0.443	0.538	0.624	0.700	0.764	0.860	0.920	0.955	0.987
	45.41	45.39	45.23	45.00	44.70	44.32	43.90	42.98	41.99	41.07	39.32

Table 35: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
49	0.250	0.346	0.444	0.541	0.627	0.703	0.767	0.863	0.923	0.958	0.988
	46.39	46.35	46.23	45.97	45.67	45.28	44.86	43.89	42.85	41.88	40.13
50	0.250	0.347	0.447	0.542	0.631	0.707	0.772	0.867	0.926	0.960	0.989
	47.36	47.32	47.18	46.96	46.61	46.22	45.75	44.78	43.73	42.71	40.90
51	0.250	0.348	0.448	0.545	0.634	0.711	0.777	0.870	0.929	0.962	0.989
	48.36	48.29	48.17	47.92	47.58	47.16	46.68	45.67	44.57	43.56	41.69
52	0.250	0.348	0.452	0.550	0.638	0.715	0.781	0.874	0.931	0.963	0.990
	49.30	49.29	49.10	48.84	48.52	48.09	47.60	46.56	45.47	44.38	42.48
53	0.250	0.350	0.453	0.552	0.642	0.719	0.783	0.878	0.934	0.965	0.991
	50.29	50.22	50.09	49.83	49.45	49.02	48.54	47.42	46.31	45.21	43.28
54	0.250	0.351	0.455	0.555	0.644	0.723	0.788	0.882	0.935	0.967	0.991
	51.26	51.22	51.04	50.77	50.42	49.95	49.45	48.30	47.23	46.05	44.09
55	0.250	0.352	0.457	0.557	0.647	0.726	0.792	0.885	0.938	0.968	0.992
	52.24	52.20	52.03	51.75	51.38	50.90	50.37	49.21	48.06	46.91	44.88
56	0.250	0.353	0.458	0.562	0.652	0.730	0.795	0.888	0.940	0.970	0.993
	53.20	53.14	53.00	52.69	52.30	51.83	51.29	50.09	48.93	47.75	45.72
57	0.250	0.354	0.461	0.564	0.656	0.734	0.797	0.891	0.942	0.972	0.993
	54.19	54.13	53.96	53.65	53.25	52.76	52.24	51.01	49.80	48.59	46.50
58	0.250	0.354	0.463	0.567	0.658	0.737	0.802	0.893	0.945	0.974	0.994
	55.17	55.13	54.92	54.61	54.20	53.71	53.15	51.88	50.62	49.40	47.30
59	0.250	0.356	0.465	0.569	0.663	0.742	0.807	0.897	0.947	0.974	0.994
	56.14	56.06	55.89	55.58	55.14	54.62	54.04	52.78	51.50	50.26	48.05
60	0.250	0.357	0.468	0.570	0.665	0.744	0.809	0.899	0.950	0.975	0.995
	57.11	57.04	56.85	56.56	56.11	55.58	54.99	53.67	52.34	51.10	48.88
61	0.250	0.357	0.469	0.574	0.669	0.749	0.813	0.901	0.951	0.977	0.995
	58.10	58.03	57.82	57.50	57.04	56.50	55.88	54.57	53.19	51.96	49.67
62	0.250	0.358	0.470	0.577	0.672	0.752	0.817	0.905	0.953	0.978	0.995
	59.07	59.01	58.80	58.46	57.98	57.44	56.79	55.42	54.09	52.77	50.45
63	0.250	0.359	0.472	0.580	0.675	0.755	0.819	0.908	0.955	0.979	0.996
	60.04	60.00	59.77	59.42	58.95	58.38	57.74	56.31	54.91	53.59	51.24
64	0.250	0.361	0.474	0.583	0.679	0.757	0.823	0.910	0.957	0.979	0.996
	61.02	60.94	60.75	60.39	59.88	59.32	58.65	57.20	55.77	54.43	52.06
65	0.250	0.361	0.477	0.585	0.682	0.762	0.826	0.912	0.959	0.981	0.996
	62.01	61.92	61.70	61.36	60.84	60.23	59.55	58.12	56.65	55.27	52.88
66	0.250	0.362	0.478	0.588	0.683	0.766	0.829	0.916	0.959	0.982	0.997
	62.97	62.91	62.68	62.30	61.82	61.15	60.49	58.98	57.48	56.13	53.68
67	0.250	0.363	0.481	0.591	0.686	0.768	0.832	0.916	0.962	0.983	0.997
	63.97	63.88	63.63	63.25	62.76	62.10	61.38	59.89	58.32	56.96	54.42
68	0.250	0.364	0.483	0.593	0.691	0.771	0.834	0.919	0.963	0.983	0.997
	64.92	64.86	64.60	64.21	63.68	63.05	62.32	60.75	59.17	57.76	55.23
69	0.250	0.365	0.483	0.596	0.693	0.775	0.837	0.922	0.965	0.985	0.997
	65.90	65.84	65.60	65.18	64.66	63.96	63.26	61.66	60.08	58.58	56.01
70	0.250	0.366	0.485	0.598	0.696	0.775	0.841	0.922	0.965	0.985	0.997
	66.88	66.80	66.55	66.14	65.59	64.93	64.14	62.55	60.92	59.42	56.78
71	0.250	0.366	0.487	0.601	0.699	0.779	0.844	0.926	0.967	0.986	0.998
	67.88	67.79	67.53	67.09	66.54	65.85	65.05	63.40	61.76	60.23	57.59
72	0.250	0.367	0.489	0.603	0.702	0.784	0.847	0.928	0.967	0.987	0.998
	68.83	68.75	68.50	68.06	67.50	66.76	65.96	64.29	62.65	61.09	58.41

Table 35: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
73	0.250	0.368	0.492	0.605	0.704	0.786	0.849	0.929	0.969	0.987	0.998
	69.83	69.75	69.45	69.02	68.45	67.70	66.88	65.21	63.49	61.89	59.17
74	0.250	0.370	0.492	0.608	0.708	0.788	0.851	0.932	0.970	0.988	0.998
	70.81	70.69	70.44	69.98	69.38	68.64	67.82	66.08	64.37	62.72	59.97
75	0.250	0.370	0.494	0.610	0.709	0.792	0.854	0.933	0.971	0.988	0.998
	71.79	71.69	71.41	70.95	70.34	69.56	68.74	66.95	65.24	63.58	60.80
76	0.250	0.371	0.496	0.611	0.711	0.795	0.856	0.936	0.972	0.989	0.998
	72.77	72.67	72.39	71.92	71.30	70.48	69.65	67.80	66.08	64.39	61.57
77	0.250	0.372	0.498	0.615	0.716	0.796	0.859	0.937	0.974	0.989	0.999
	73.74	73.65	73.36	72.88	72.22	71.45	70.55	68.71	66.89	65.25	62.38
78	0.250	0.373	0.500	0.617	0.717	0.799	0.861	0.938	0.974	0.990	0.999
	74.72	74.61	74.31	73.83	73.19	72.36	71.48	69.57	67.82	66.05	63.15
79	0.250	0.374	0.501	0.619	0.721	0.802	0.864	0.940	0.975	0.991	0.999
	75.70	75.61	75.28	74.79	74.11	73.30	72.39	70.50	68.62	66.90	63.97
80	0.250	0.374	0.503	0.622	0.724	0.806	0.866	0.943	0.976	0.991	0.999
	76.67	76.57	76.27	75.73	75.06	74.20	73.31	71.35	69.46	67.74	64.79
81	0.250	0.375	0.504	0.624	0.726	0.808	0.868	0.944	0.977	0.991	0.999
	77.64	77.56	77.24	76.70	76.01	75.13	74.23	72.23	70.30	68.54	65.54
82	0.250	0.376	0.504	0.626	0.730	0.811	0.871	0.944	0.978	0.992	0.999
	78.62	78.53	78.24	77.68	76.93	76.06	75.13	73.15	71.17	69.34	66.34
83	0.250	0.376	0.507	0.628	0.731	0.813	0.873	0.947	0.978	0.992	0.999
	79.61	79.52	79.19	78.64	77.89	76.99	76.03	74.00	72.08	70.18	67.11
84	0.250	0.378	0.508	0.630	0.734	0.813	0.875	0.948	0.980	0.993	0.999
	80.60	80.48	80.18	79.60	78.83	77.97	76.96	74.85	72.91	71.03	67.88
85	0.250	0.378	0.511	0.634	0.736	0.817	0.878	0.949	0.980	0.993	0.999
	81.57	81.47	81.12	80.54	79.77	78.88	77.85	75.73	73.75	71.86	68.70
86	0.250	0.379	0.511	0.635	0.738	0.819	0.880	0.951	0.981	0.993	0.999
	82.55	82.45	82.11	81.50	80.74	79.80	78.75	76.63	74.59	72.68	69.45
87	0.250	0.380	0.513	0.637	0.741	0.822	0.882	0.951	0.981	0.994	0.999
	83.54	83.41	83.09	82.49	81.67	80.74	79.68	77.56	75.45	73.51	70.26
88	0.250	0.381	0.516	0.640	0.743	0.824	0.883	0.953	0.983	0.994	0.999
	84.51	84.39	84.03	83.42	82.63	81.66	80.58	78.41	76.26	74.35	71.04
89	0.250	0.382	0.517	0.642	0.747	0.827	0.885	0.954	0.983	0.994	0.999
	85.50	85.37	85.03	84.41	83.54	82.58	81.52	79.26	77.10	75.21	71.86
90	0.250	0.383	0.519	0.644	0.748	0.829	0.886	0.955	0.984	0.994	0.999
	86.48	86.34	85.98	85.36	84.53	83.52	82.44	80.18	77.95	76.00	72.64
91	0.250	0.384	0.520	0.645	0.750	0.831	0.889	0.957	0.984	0.995	0.999
	87.45	87.31	86.96	86.33	85.46	84.45	83.32	81.02	78.88	76.84	73.44
92	0.250	0.384	0.522	0.649	0.752	0.833	0.892	0.958	0.985	0.995	1.000
	88.45	88.30	87.92	87.25	86.41	85.37	84.25	81.92	79.70	77.66	74.25
93	0.250	0.384	0.523	0.650	0.755	0.836	0.893	0.959	0.985	0.995	1.000
	89.42	89.32	88.90	88.25	87.33	86.28	85.14	82.84	80.54	78.45	75.03
94	0.250	0.385	0.524	0.652	0.758	0.837	0.895	0.961	0.986	0.995	1.000
	90.38	90.26	89.87	89.21	88.28	87.24	86.07	83.65	81.40	79.35	75.80
95	0.250	0.387	0.527	0.654	0.759	0.840	0.897	0.961	0.987	0.996	1.000
	91.38	91.23	90.84	90.16	89.26	88.14	86.99	84.55	82.20	80.17	76.54
96	0.250	0.387	0.527	0.656	0.763	0.841	0.898	0.962	0.987	0.996	1.000
	92.34	92.24	91.83	91.11	90.16	89.09	87.88	85.38	83.08	81.01	77.42

Table 35: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
97	0.250	0.388	0.529	0.659	0.763	0.843	0.901	0.963	0.988	0.996	1.000
	93.33	93.21	92.79	92.06	91.14	90.02	88.80	86.30	83.90	81.79	78.15
98	0.250	0.388	0.531	0.659	0.766	0.847	0.903	0.965	0.988	0.996	1.000
	94.32	94.19	93.76	93.04	92.09	90.92	89.66	87.18	84.79	82.65	78.97
99	0.250	0.388	0.532	0.662	0.769	0.848	0.904	0.965	0.988	0.996	1.000
	95.33	95.21	94.73	94.00	93.02	91.85	90.59	88.09	85.67	83.43	79.76
100	0.250	0.390	0.536	0.663	0.771	0.851	0.904	0.966	0.989	0.997	1.000
	96.30	96.15	95.67	94.97	93.93	92.78	91.59	88.96	86.55	84.30	80.57
101	0.250	0.390	0.535	0.666	0.772	0.851	0.907	0.967	0.989	0.997	1.000
	97.27	97.14	96.69	95.90	94.92	93.72	92.42	89.82	87.35	85.08	81.38
102	0.250	0.392	0.537	0.669	0.774	0.853	0.909	0.967	0.990	0.997	1.000
	98.26	98.11	97.66	96.86	95.84	94.64	93.31	90.70	88.19	85.94	82.12
103	0.250	0.392	0.538	0.671	0.776	0.856	0.910	0.969	0.990	0.997	1.000
	99.22	99.09	98.63	97.83	96.81	95.57	94.25	91.61	89.01	86.78	82.91
104	0.250	0.393	0.540	0.671	0.778	0.857	0.912	0.970	0.990	0.997	1.000
	100.23	100.06	99.57	98.81	97.75	96.53	95.14	92.46	89.91	87.62	83.71
105	0.250	0.394	0.542	0.674	0.781	0.859	0.914	0.970	0.991	0.997	1.000
	101.20	101.04	100.57	99.74	98.67	97.45	96.09	93.31	90.72	88.42	84.52
106	0.250	0.394	0.541	0.675	0.782	0.860	0.915	0.971	0.991	0.997	1.000
	102.16	102.05	101.57	100.72	99.63	98.38	96.98	94.20	91.54	89.31	85.34
107	0.250	0.395	0.544	0.677	0.783	0.862	0.917	0.972	0.991	0.998	1.000
	103.13	103.02	102.51	101.68	100.58	99.29	97.88	95.05	92.46	90.06	86.09
108	0.250	0.396	0.545	0.680	0.786	0.865	0.917	0.973	0.992	0.998	1.000
	104.16	103.98	103.50	102.63	101.51	100.16	98.81	95.93	93.30	90.89	86.93
109	0.250	0.397	0.548	0.682	0.787	0.866	0.919	0.974	0.992	0.998	1.000
	105.13	104.96	104.43	103.58	102.49	101.12	99.73	96.81	94.13	91.74	87.66
110	0.250	0.397	0.548	0.683	0.791	0.869	0.920	0.974	0.992	0.998	1.000
	106.08	105.97	105.43	104.55	103.38	102.03	100.63	97.68	94.96	92.58	88.45
111	0.250	0.398	0.549	0.685	0.793	0.869	0.922	0.975	0.992	0.998	1.000
	107.08	106.93	106.41	105.53	104.32	102.95	101.51	98.56	95.85	93.36	89.22
112	0.250	0.399	0.552	0.686	0.795	0.871	0.922	0.975	0.993	0.998	1.000
	108.06	107.88	107.35	106.49	105.27	103.93	102.46	99.48	96.65	94.23	90.03
113	0.250	0.400	0.553	0.688	0.796	0.872	0.925	0.976	0.993	0.998	1.000
	109.05	108.87	108.35	107.46	106.21	104.81	103.31	100.30	97.49	95.04	90.82
114	0.250	0.399	0.553	0.690	0.796	0.874	0.925	0.977	0.994	0.998	1.000
	110.03	109.90	109.33	108.40	107.23	105.76	104.24	101.16	98.36	95.87	91.64
115	0.250	0.401	0.555	0.692	0.799	0.877	0.928	0.978	0.994	0.998	1.000
	111.01	110.86	110.31	109.35	108.10	106.66	105.12	102.01	99.18	96.69	92.42
116	0.250	0.401	0.557	0.693	0.802	0.877	0.928	0.978	0.994	0.998	1.000
	112.00	111.86	111.25	110.34	109.04	107.62	106.06	102.97	100.08	97.48	93.22
117	0.250	0.402	0.557	0.695	0.804	0.880	0.930	0.979	0.994	0.998	1.000
	112.99	112.81	112.24	111.27	109.96	108.52	106.97	103.80	100.90	98.33	94.03
118	0.250	0.403	0.558	0.697	0.803	0.879	0.930	0.979	0.995	0.999	1.000
	113.96	113.78	113.24	112.23	110.96	109.46	107.89	104.70	101.80	99.16	94.73
119	0.250	0.403	0.560	0.699	0.805	0.881	0.931	0.979	0.995	0.999	1.000
	114.96	114.78	114.21	113.18	111.90	110.40	108.74	105.58	102.61	99.94	95.57
120	0.250	0.405	0.563	0.700	0.808	0.883	0.933	0.980	0.995	0.999	1.000
	115.94	115.74	115.14	114.15	112.83	111.35	109.67	106.45	103.47	100.81	96.42

Table 35: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
121	0.250	0.405	0.563	0.702	0.810	0.884	0.934	0.981	0.995	0.999	1.000
	116.92	116.72	116.14	115.10	113.76	112.24	110.61	107.28	104.33	101.60	97.13
122	0.250	0.405	0.563	0.704	0.810	0.886	0.935	0.981	0.995	0.999	1.000
	117.91	117.71	117.13	116.08	114.73	113.16	111.48	108.20	105.14	102.46	97.95
123	0.250	0.406	0.565	0.706	0.813	0.888	0.937	0.981	0.995	0.999	1.000
	118.86	118.71	118.10	117.00	115.67	114.04	112.41	109.08	106.05	103.22	98.76
124	0.250	0.407	0.567	0.707	0.814	0.890	0.937	0.982	0.996	0.999	1.000
	119.86	119.67	119.06	117.99	116.62	114.98	113.30	109.94	106.87	104.11	99.53
125	0.250	0.407	0.568	0.708	0.816	0.891	0.939	0.982	0.996	0.999	1.000
	120.85	120.67	120.03	118.98	117.54	115.90	114.21	110.81	107.69	104.91	100.32
126	0.250	0.408	0.570	0.710	0.818	0.892	0.939	0.983	0.996	0.999	1.000
	121.82	121.64	120.98	119.91	118.48	116.83	115.13	111.64	108.59	105.71	101.07
127	0.250	0.409	0.572	0.712	0.820	0.893	0.940	0.984	0.996	0.999	1.000
	122.81	122.63	121.96	120.85	119.42	117.75	116.01	112.53	109.35	106.58	101.86
128	0.250	0.409	0.573	0.713	0.822	0.895	0.941	0.984	0.996	0.999	1.000
	123.79	123.61	122.94	121.85	120.34	118.69	116.93	113.42	110.27	107.42	102.70
129	0.250	0.410	0.573	0.716	0.824	0.895	0.942	0.984	0.997	0.999	1.000
	124.79	124.60	123.93	122.76	121.30	119.62	117.82	114.30	111.07	108.24	103.43
130	0.250	0.411	0.577	0.717	0.825	0.898	0.943	0.985	0.996	0.999	1.000
	125.73	125.54	124.85	123.73	122.21	120.52	118.71	115.17	111.93	109.02	104.24
131	0.250	0.411	0.577	0.718	0.826	0.899	0.945	0.985	0.996	0.999	1.000
	126.76	126.56	125.84	124.71	123.18	121.44	119.66	116.12	112.78	109.88	104.97
132	0.250	0.412	0.577	0.720	0.828	0.902	0.945	0.986	0.997	0.999	1.000
	127.73	127.52	126.83	125.66	124.12	122.33	120.52	116.90	113.61	110.74	105.81
133	0.250	0.413	0.579	0.723	0.829	0.902	0.947	0.986	0.997	0.999	1.000
	128.71	128.49	127.81	126.60	125.05	123.30	121.45	117.81	114.44	111.52	106.56
134	0.250	0.413	0.579	0.723	0.830	0.903	0.947	0.986	0.997	0.999	1.000
	129.71	129.50	128.79	127.58	126.04	124.25	122.33	118.72	115.31	112.31	107.39
135	0.250	0.414	0.582	0.725	0.832	0.904	0.948	0.987	0.997	0.999	1.000
	130.70	130.45	129.73	128.53	126.96	125.11	123.22	119.56	116.20	113.22	108.17
136	0.250	0.415	0.583	0.727	0.833	0.905	0.948	0.987	0.997	0.999	1.000
	131.68	131.46	130.70	129.48	127.91	126.05	124.17	120.47	117.01	113.99	108.94
137	0.250	0.416	0.583	0.728	0.836	0.906	0.950	0.987	0.997	0.999	1.000
	132.67	132.43	131.71	130.43	128.79	127.00	125.06	121.29	117.85	114.83	109.78
138	0.250	0.417	0.585	0.730	0.836	0.908	0.951	0.987	0.997	0.999	1.000
	133.66	133.40	132.66	131.38	129.79	127.90	125.98	122.20	118.70	115.67	110.55
139	0.250	0.417	0.585	0.730	0.838	0.909	0.952	0.988	0.997	1.000	1.000
	134.61	134.39	133.67	132.39	130.69	128.81	126.85	122.99	119.48	116.51	111.32
140	0.250	0.417	0.587	0.731	0.840	0.910	0.953	0.988	0.997	1.000	1.000
	135.63	135.39	134.63	133.35	131.63	129.77	127.73	123.93	120.43	117.30	112.10
141	0.250	0.418	0.589	0.734	0.841	0.910	0.953	0.989	0.998	1.000	1.000
	136.61	136.38	135.58	134.28	132.58	130.69	128.65	124.77	121.25	118.13	112.92
142	0.250	0.419	0.589	0.737	0.842	0.911	0.955	0.989	0.998	1.000	1.000
	137.57	137.33	136.58	135.22	133.51	131.60	129.52	125.67	122.10	118.90	113.72
143	0.250	0.420	0.591	0.738	0.843	0.913	0.955	0.989	0.998	1.000	1.000
	138.56	138.30	137.55	136.17	134.47	132.51	130.43	126.49	122.88	119.80	114.46
144	0.250	0.420	0.590	0.738	0.845	0.914	0.955	0.990	0.998	1.000	1.000
	139.53	139.31	138.57	137.16	135.40	133.41	131.38	127.36	123.80	120.60	115.25

Table 35: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
145	0.250	0.420	0.593	0.739	0.845	0.916	0.956	0.990	0.998	1.000	1.000
	140.52	140.31	139.47	138.12	136.37	134.30	132.24	128.23	124.65	121.41	116.09
146	0.250	0.421	0.593	0.742	0.848	0.916	0.956	0.990	0.998	1.000	1.000
	141.50	141.25	140.50	139.07	137.24	135.30	133.20	129.13	125.42	122.25	116.83
147	0.250	0.422	0.597	0.742	0.849	0.918	0.958	0.990	0.998	1.000	1.000
	142.51	142.27	141.40	140.08	138.20	136.17	134.07	129.95	126.34	123.06	117.66
148	0.250	0.423	0.596	0.744	0.851	0.919	0.958	0.990	0.998	1.000	1.000
	143.48	143.21	142.43	141.00	139.18	137.07	135.00	130.90	127.14	123.92	118.43
149	0.250	0.423	0.598	0.745	0.852	0.919	0.959	0.991	0.998	1.000	1.000
	144.45	144.21	143.38	141.97	140.10	138.06	135.88	131.76	127.98	124.69	119.25
150	0.250	0.423	0.599	0.745	0.853	0.920	0.960	0.991	0.998	1.000	1.000
	145.45	145.20	144.34	142.94	141.05	138.98	136.75	132.60	128.94	125.55	119.99
155	0.250	0.427	0.605	0.753	0.860	0.926	0.963	0.992	0.999	1.000	1.000
	150.42	150.11	149.22	147.72	145.68	143.54	141.29	137.00	133.05	129.63	123.94
160	0.250	0.429	0.611	0.760	0.864	0.929	0.966	0.993	0.999	1.000	1.000
	155.28	155.03	154.07	152.49	150.45	148.17	145.86	141.34	137.29	133.78	127.86
165	0.250	0.434	0.616	0.767	0.871	0.933	0.969	0.994	0.999	1.000	1.000
	160.22	159.91	158.93	157.24	155.12	152.76	150.27	145.62	141.51	137.91	131.87
170	0.250	0.435	0.622	0.774	0.876	0.938	0.972	0.995	0.999	1.000	1.000
	165.15	164.88	163.79	162.00	159.82	157.34	154.77	150.07	145.76	141.99	135.72
175	0.250	0.438	0.628	0.779	0.881	0.941	0.974	0.995	0.999	1.000	1.000
	170.09	169.79	168.66	166.83	164.54	161.94	159.35	154.40	149.93	146.11	139.65
180	0.250	0.441	0.633	0.785	0.888	0.945	0.975	0.996	0.999	1.000	1.000
	175.01	174.68	173.49	171.60	169.14	166.51	163.79	158.75	154.14	150.27	143.66
185	0.250	0.445	0.637	0.791	0.892	0.949	0.978	0.996	0.999	1.000	1.000
	179.96	179.58	178.42	176.40	173.81	171.08	168.30	163.11	158.42	154.33	147.59
190	0.250	0.447	0.643	0.797	0.896	0.952	0.979	0.997	1.000	1.000	1.000
	184.90	184.53	183.26	181.15	178.57	175.67	172.83	167.37	162.67	158.46	151.46
195	0.250	0.450	0.649	0.803	0.900	0.954	0.981	0.997	1.000	1.000	1.000
	189.82	189.42	188.08	185.89	183.24	180.26	177.25	171.70	166.83	162.57	155.39
200	0.250	0.453	0.652	0.806	0.904	0.958	0.983	0.998	1.000	1.000	1.000
	194.79	194.35	193.03	190.76	187.93	184.82	181.76	176.12	171.06	166.64	159.35
210	0.250	0.458	0.661	0.818	0.912	0.962	0.985	0.998	1.000	1.000	1.000
	204.63	204.19	202.73	200.24	197.24	194.04	190.82	184.70	179.46	174.91	167.25
220	0.250	0.463	0.671	0.825	0.919	0.966	0.987	0.999	1.000	1.000	1.000
	214.53	214.03	212.45	209.84	206.58	203.14	199.79	193.45	187.94	183.17	175.11
230	0.250	0.468	0.681	0.836	0.927	0.970	0.990	0.999	1.000	1.000	1.000
	224.39	223.89	222.12	219.32	215.87	212.24	208.69	202.18	196.34	191.32	182.93
240	0.250	0.473	0.688	0.843	0.931	0.974	0.991	0.999	1.000	1.000	1.000
	234.29	233.71	231.91	228.94	225.28	221.35	217.66	210.79	204.81	199.58	190.81
250	0.250	0.478	0.697	0.851	0.938	0.977	0.993	0.999	1.000	1.000	1.000
	244.12	243.56	241.58	238.44	234.55	230.50	226.57	219.43	213.19	207.78	198.68

Table 36: Table of Monte Carlo Results for $\mathcal{M}_{\text{BK}}$ when $k = 5$ and $\alpha = 0.05$.

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
1	0.200	0.216	0.231	0.245	0.259	0.273	0.286	0.310	0.333	0.355	0.394
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	0.200	0.216	0.231	0.245	0.259	0.273	0.286	0.310	0.333	0.355	0.394
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
3	0.200	0.219	0.237	0.256	0.273	0.291	0.308	0.340	0.370	0.399	0.453
	2.80	2.80	2.80	2.80	2.80	2.79	2.79	2.79	2.78	2.77	2.75
4	0.200	0.223	0.246	0.268	0.289	0.310	0.331	0.370	0.408	0.441	0.504
	3.28	3.28	3.27	3.27	3.27	3.26	3.25	3.24	3.22	3.21	3.16
5	0.200	0.226	0.251	0.276	0.300	0.324	0.347	0.391	0.432	0.471	0.539
	4.25	4.25	4.25	4.24	4.24	4.24	4.23	4.23	4.21	4.20	4.17
6	0.200	0.227	0.255	0.282	0.308	0.334	0.359	0.406	0.453	0.494	0.567
	5.16	5.15	5.15	5.14	5.14	5.13	5.12	5.10	5.07	5.04	4.99
7	0.200	0.229	0.259	0.288	0.317	0.345	0.372	0.424	0.475	0.518	0.599
	6.18	6.18	6.17	6.17	6.16	6.15	6.14	6.11	6.08	6.05	5.97
8	0.200	0.232	0.264	0.295	0.326	0.356	0.386	0.444	0.496	0.543	0.626
	7.10	7.10	7.08	7.08	7.07	7.06	7.04	6.99	6.96	6.91	6.81
9	0.200	0.234	0.268	0.301	0.335	0.367	0.399	0.460	0.514	0.565	0.652
	8.02	8.01	8.01	8.00	7.98	7.97	7.95	7.91	7.87	7.82	7.70
10	0.200	0.236	0.271	0.307	0.342	0.377	0.410	0.474	0.531	0.585	0.674
	8.96	8.96	8.95	8.94	8.93	8.91	8.89	8.84	8.78	8.71	8.57
11	0.200	0.237	0.274	0.312	0.350	0.385	0.421	0.487	0.547	0.602	0.696
	9.92	9.92	9.92	9.90	9.88	9.87	9.84	9.78	9.72	9.63	9.46
12	0.200	0.239	0.278	0.317	0.356	0.396	0.431	0.501	0.565	0.622	0.716
	10.89	10.89	10.89	10.87	10.85	10.81	10.79	10.72	10.63	10.53	10.33
13	0.200	0.240	0.282	0.323	0.363	0.403	0.441	0.514	0.579	0.639	0.733
	11.85	11.84	11.82	11.81	11.79	11.76	11.73	11.64	11.55	11.43	11.21
14	0.200	0.242	0.285	0.327	0.370	0.412	0.451	0.528	0.595	0.655	0.751
	12.79	12.79	12.78	12.76	12.73	12.69	12.67	12.57	12.45	12.34	12.08
15	0.200	0.243	0.287	0.332	0.376	0.419	0.461	0.539	0.608	0.669	0.765
	13.75	13.75	13.74	13.72	13.69	13.65	13.60	13.50	13.39	13.24	12.96
16	0.200	0.245	0.290	0.336	0.383	0.427	0.470	0.551	0.621	0.683	0.780
	14.71	14.72	14.70	14.68	14.64	14.60	14.55	14.43	14.31	14.16	13.84
17	0.200	0.246	0.294	0.341	0.389	0.435	0.480	0.562	0.635	0.697	0.795
	15.68	15.68	15.66	15.63	15.60	15.56	15.50	15.37	15.21	15.05	14.69
18	0.200	0.248	0.297	0.346	0.394	0.442	0.488	0.572	0.648	0.710	0.808
	16.64	16.64	16.61	16.59	16.56	16.50	16.44	16.30	16.13	15.95	15.56
19	0.200	0.249	0.299	0.351	0.401	0.450	0.496	0.583	0.659	0.723	0.820
	17.61	17.60	17.58	17.54	17.50	17.44	17.38	17.24	17.05	16.85	16.42
20	0.200	0.250	0.302	0.355	0.406	0.456	0.505	0.595	0.670	0.736	0.831
	18.57	18.57	18.54	18.51	18.46	18.41	18.34	18.14	17.97	17.73	17.28
21	0.200	0.252	0.305	0.359	0.413	0.463	0.514	0.603	0.681	0.748	0.841
	19.53	19.52	19.51	19.47	19.41	19.36	19.27	19.09	18.87	18.63	18.15
22	0.200	0.253	0.308	0.363	0.419	0.470	0.523	0.612	0.691	0.756	0.850
	20.51	20.49	20.47	20.43	20.37	20.31	20.21	20.03	19.80	19.54	19.01
23	0.200	0.254	0.310	0.366	0.423	0.477	0.528	0.622	0.703	0.766	0.860
	21.47	21.47	21.45	21.41	21.33	21.26	21.17	20.96	20.69	20.43	19.86
24	0.200	0.255	0.312	0.371	0.428	0.483	0.536	0.631	0.711	0.778	0.870
	22.45	22.43	22.41	22.36	22.28	22.21	22.12	21.89	21.62	21.31	20.70

Table 36: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
25	0.200	0.256	0.315	0.374	0.433	0.489	0.542	0.639	0.720	0.786	0.876
	23.42	23.41	23.38	23.32	23.26	23.17	23.07	22.82	22.53	22.22	21.57
26	0.200	0.257	0.318	0.378	0.438	0.497	0.551	0.648	0.731	0.794	0.884
	24.37	24.36	24.34	24.29	24.22	24.11	24.01	23.76	23.44	23.12	22.42
27	0.200	0.258	0.320	0.382	0.444	0.502	0.557	0.656	0.737	0.803	0.892
	25.35	25.36	25.31	25.26	25.17	25.08	24.97	24.68	24.36	23.99	23.27
28	0.200	0.260	0.322	0.386	0.447	0.508	0.565	0.664	0.746	0.811	0.897
	26.32	26.31	26.27	26.21	26.14	26.04	25.90	25.61	25.27	24.89	24.15
29	0.200	0.261	0.324	0.389	0.452	0.514	0.571	0.674	0.754	0.820	0.904
	27.29	27.27	27.25	27.18	27.10	26.98	26.85	26.52	26.17	25.76	24.98
30	0.200	0.262	0.327	0.393	0.457	0.521	0.579	0.681	0.762	0.827	0.910
	28.27	28.26	28.21	28.14	28.05	27.92	27.78	27.45	27.08	26.66	25.81
31	0.200	0.263	0.330	0.396	0.462	0.525	0.585	0.688	0.771	0.833	0.916
	29.25	29.22	29.17	29.12	29.02	28.90	28.74	28.39	27.98	27.56	26.65
32	0.200	0.264	0.331	0.400	0.466	0.531	0.591	0.695	0.777	0.842	0.921
	30.23	30.20	30.15	30.09	29.99	29.84	29.68	29.32	28.89	28.40	27.50
33	0.200	0.265	0.333	0.402	0.472	0.536	0.597	0.702	0.784	0.846	0.926
	31.18	31.17	31.13	31.07	30.93	30.80	30.63	30.24	29.80	29.32	28.34
34	0.200	0.266	0.335	0.407	0.476	0.543	0.602	0.709	0.792	0.854	0.929
	32.16	32.15	32.09	32.00	31.89	31.74	31.59	31.17	30.68	30.19	29.21
35	0.200	0.267	0.338	0.410	0.480	0.548	0.607	0.715	0.797	0.859	0.935
	33.13	33.13	33.07	32.98	32.85	32.70	32.54	32.10	31.60	31.08	30.04
36	0.200	0.268	0.340	0.413	0.483	0.552	0.615	0.721	0.804	0.866	0.938
	34.10	34.08	34.03	33.95	33.83	33.66	33.46	33.02	32.51	31.95	30.87
37	0.200	0.269	0.342	0.416	0.489	0.557	0.620	0.728	0.810	0.870	0.942
	35.09	35.08	35.01	34.91	34.78	34.61	34.42	33.94	33.41	32.84	31.72
38	0.200	0.270	0.344	0.420	0.493	0.563	0.626	0.733	0.815	0.875	0.945
	36.05	36.04	35.98	35.87	35.75	35.56	35.36	34.88	34.33	33.73	32.53
39	0.200	0.271	0.346	0.422	0.496	0.566	0.631	0.740	0.822	0.881	0.949
	37.02	37.02	36.94	36.86	36.71	36.54	36.31	35.80	35.22	34.59	33.39
40	0.200	0.272	0.348	0.426	0.501	0.572	0.638	0.746	0.828	0.884	0.952
	38.01	37.99	37.93	37.82	37.67	37.47	37.24	36.71	36.11	35.49	34.23
41	0.200	0.272	0.350	0.429	0.505	0.578	0.642	0.752	0.832	0.891	0.955
	38.98	38.98	38.91	38.79	38.63	38.42	38.20	37.64	37.03	36.34	35.05
42	0.200	0.274	0.352	0.431	0.509	0.580	0.648	0.757	0.839	0.896	0.958
	39.96	39.94	39.88	39.77	39.59	39.40	39.13	38.57	37.89	37.20	35.90
43	0.200	0.275	0.354	0.435	0.513	0.587	0.652	0.763	0.842	0.899	0.960
	40.93	40.91	40.84	40.73	40.57	40.34	40.10	39.48	38.84	38.10	36.73
44	0.200	0.276	0.356	0.438	0.515	0.590	0.657	0.768	0.848	0.905	0.962
	41.92	41.90	41.83	41.69	41.53	41.30	41.03	40.40	39.71	38.96	37.58
45	0.200	0.276	0.358	0.440	0.521	0.596	0.661	0.772	0.853	0.908	0.965
	42.90	42.88	42.80	42.67	42.48	42.24	41.99	41.34	40.60	39.84	38.41
46	0.200	0.277	0.360	0.443	0.525	0.599	0.668	0.778	0.858	0.911	0.967
	43.86	43.85	43.77	43.63	43.44	43.21	42.92	42.25	41.48	40.71	39.24
47	0.200	0.278	0.362	0.446	0.529	0.605	0.674	0.783	0.861	0.915	0.970
	44.85	44.82	44.74	44.60	44.40	44.15	43.84	43.16	42.40	41.61	40.03
48	0.200	0.279	0.364	0.450	0.532	0.608	0.677	0.788	0.866	0.918	0.970
	45.83	45.81	45.71	45.57	45.37	45.12	44.81	44.09	43.29	42.47	40.91

Table 36: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
49	0.200	0.280	0.365	0.453	0.536	0.613	0.682	0.795	0.871	0.922	0.973
	46.80	46.79	46.69	46.53	46.32	46.06	45.75	44.98	44.18	43.34	41.70
50	0.200	0.281	0.367	0.455	0.540	0.617	0.686	0.800	0.875	0.925	0.975
	47.77	47.75	47.68	47.51	47.28	47.01	46.69	45.89	45.07	44.19	42.54
51	0.200	0.282	0.369	0.458	0.541	0.621	0.691	0.801	0.878	0.928	0.976
	48.76	48.72	48.65	48.48	48.27	47.97	47.63	46.86	45.98	45.06	43.39
52	0.200	0.282	0.371	0.460	0.546	0.625	0.694	0.806	0.882	0.930	0.977
	49.74	49.71	49.62	49.46	49.22	48.93	48.58	47.76	46.86	45.98	44.22
53	0.200	0.283	0.373	0.463	0.549	0.630	0.699	0.810	0.886	0.933	0.978
	50.72	50.69	50.58	50.41	50.19	49.87	49.50	48.68	47.76	46.82	45.07
54	0.200	0.284	0.375	0.466	0.553	0.632	0.704	0.814	0.889	0.936	0.980
	51.70	51.68	51.56	51.39	51.14	50.84	50.46	49.61	48.65	47.70	45.87
55	0.200	0.285	0.376	0.469	0.557	0.638	0.708	0.820	0.891	0.938	0.981
	52.67	52.65	52.55	52.37	52.10	51.77	51.40	50.49	49.55	48.56	46.69
56	0.200	0.286	0.378	0.471	0.560	0.643	0.712	0.821	0.897	0.941	0.983
	53.65	53.62	53.51	53.34	53.08	52.72	52.34	51.46	50.42	49.40	47.51
57	0.200	0.286	0.380	0.474	0.564	0.646	0.716	0.828	0.899	0.944	0.983
	54.63	54.63	54.49	54.30	54.03	53.67	53.28	52.34	51.33	50.28	48.35
58	0.200	0.287	0.381	0.477	0.568	0.649	0.720	0.832	0.902	0.946	0.985
	55.62	55.58	55.48	55.27	54.98	54.64	54.23	53.23	52.20	51.13	49.17
59	0.200	0.288	0.383	0.480	0.571	0.652	0.725	0.835	0.906	0.948	0.986
	56.59	56.56	56.46	56.24	55.94	55.60	55.14	54.17	53.08	52.01	50.01
60	0.200	0.289	0.385	0.482	0.573	0.655	0.728	0.839	0.909	0.950	0.986
	57.58	57.55	57.42	57.21	56.92	56.56	56.11	55.07	53.98	52.89	50.85
61	0.200	0.290	0.387	0.482	0.576	0.660	0.732	0.842	0.911	0.951	0.987
	58.56	58.53	58.39	58.21	57.89	57.50	57.04	55.98	54.91	53.79	51.67
62	0.200	0.290	0.388	0.487	0.580	0.665	0.737	0.845	0.913	0.954	0.988
	59.54	59.51	59.38	59.15	58.85	58.43	57.98	56.92	55.77	54.62	52.47
63	0.200	0.291	0.390	0.490	0.581	0.667	0.740	0.848	0.916	0.956	0.989
	60.52	60.47	60.36	60.13	59.83	59.40	58.92	57.82	56.66	55.46	53.29
64	0.200	0.292	0.392	0.491	0.586	0.670	0.743	0.851	0.919	0.958	0.989
	61.50	61.46	61.33	61.10	60.76	60.36	59.86	58.76	57.54	56.34	54.14
65	0.200	0.293	0.394	0.494	0.589	0.674	0.745	0.854	0.922	0.960	0.990
	62.47	62.45	62.30	62.07	61.73	61.31	60.81	59.68	58.41	57.20	54.96
66	0.200	0.293	0.395	0.497	0.594	0.676	0.750	0.857	0.923	0.961	0.990
	63.47	63.42	63.28	63.05	62.67	62.28	61.74	60.58	59.34	58.08	55.76
67	0.200	0.295	0.396	0.499	0.596	0.681	0.753	0.861	0.927	0.962	0.991
	64.44	64.39	64.27	64.02	63.66	63.20	62.68	61.48	60.18	58.94	56.61
68	0.200	0.295	0.397	0.502	0.599	0.684	0.758	0.865	0.929	0.963	0.992
	65.44	65.40	65.25	64.98	64.61	64.16	63.61	62.39	61.08	59.82	57.44
69	0.200	0.296	0.400	0.504	0.600	0.687	0.761	0.867	0.930	0.965	0.992
	66.42	66.36	66.21	65.95	65.59	65.13	64.56	63.33	62.01	60.68	58.28
70	0.200	0.297	0.402	0.505	0.604	0.691	0.765	0.872	0.933	0.967	0.993
	67.39	67.33	67.20	66.95	66.55	66.07	65.48	64.19	62.82	61.50	59.07
71	0.200	0.297	0.403	0.508	0.606	0.694	0.766	0.873	0.935	0.969	0.993
	68.38	68.32	68.17	67.90	67.52	67.02	66.45	65.13	63.74	62.37	59.89
72	0.200	0.298	0.406	0.511	0.611	0.697	0.772	0.876	0.936	0.969	0.993
	69.36	69.32	69.13	68.87	68.46	67.97	67.36	66.04	64.63	63.23	60.70

Table 36: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
73	0.200	0.299	0.407	0.512	0.614	0.701	0.773	0.879	0.939	0.971	0.994
	70.36	70.29	70.11	69.85	69.42	68.92	68.31	66.93	65.50	64.10	61.56
74	0.200	0.299	0.408	0.516	0.615	0.702	0.777	0.881	0.941	0.972	0.994
	71.34	71.27	71.10	70.80	70.40	69.88	69.24	67.85	66.39	64.95	62.38
75	0.200	0.300	0.408	0.517	0.619	0.707	0.781	0.884	0.942	0.973	0.995
	72.31	72.25	72.11	71.79	71.35	70.81	70.18	68.76	67.31	65.82	63.21
76	0.200	0.301	0.410	0.521	0.621	0.710	0.783	0.884	0.944	0.974	0.995
	73.29	73.25	73.07	72.75	72.32	71.76	71.13	69.72	68.18	66.68	64.04
77	0.200	0.301	0.412	0.522	0.624	0.711	0.787	0.889	0.945	0.975	0.995
	74.26	74.23	74.04	73.73	73.27	72.74	72.06	70.59	69.05	67.55	64.86
78	0.200	0.302	0.414	0.524	0.626	0.717	0.790	0.890	0.947	0.976	0.996
	75.25	75.20	75.01	74.71	74.25	73.65	72.97	71.48	69.95	68.44	65.64
79	0.200	0.303	0.415	0.527	0.632	0.718	0.792	0.892	0.948	0.977	0.996
	76.25	76.17	75.99	75.67	75.17	74.62	73.93	72.43	70.83	69.29	66.49
80	0.200	0.304	0.416	0.529	0.632	0.721	0.795	0.895	0.950	0.978	0.996
	77.22	77.15	76.99	76.64	76.16	75.58	74.85	73.35	71.70	70.11	67.30
81	0.200	0.305	0.419	0.531	0.636	0.724	0.797	0.897	0.952	0.979	0.996
	78.21	78.13	77.93	77.62	77.10	76.53	75.82	74.22	72.57	71.00	68.13
82	0.200	0.305	0.420	0.533	0.639	0.727	0.801	0.901	0.953	0.979	0.997
	79.19	79.13	78.93	78.58	78.06	77.45	76.72	75.12	73.50	71.86	68.93
83	0.200	0.306	0.422	0.536	0.641	0.731	0.804	0.902	0.955	0.981	0.997
	80.18	80.12	79.90	79.55	79.04	78.39	77.66	76.04	74.35	72.68	69.78
84	0.200	0.306	0.423	0.538	0.643	0.734	0.804	0.905	0.956	0.981	0.997
	81.16	81.10	80.87	80.52	80.00	79.36	78.63	76.92	75.24	73.58	70.60
85	0.200	0.307	0.424	0.541	0.646	0.736	0.810	0.906	0.958	0.982	0.997
	82.14	82.08	81.87	81.47	80.95	80.30	79.52	77.84	76.08	74.42	71.43
86	0.200	0.308	0.426	0.543	0.648	0.738	0.812	0.910	0.959	0.982	0.997
	83.12	83.07	82.82	82.45	81.93	81.27	80.46	78.71	76.99	75.28	72.22
87	0.200	0.308	0.426	0.544	0.650	0.740	0.813	0.911	0.960	0.984	0.997
	84.10	84.05	83.83	83.44	82.89	82.21	81.42	79.66	77.88	76.11	73.06
88	0.200	0.310	0.428	0.547	0.654	0.744	0.817	0.912	0.962	0.984	0.997
	85.09	85.01	84.81	84.40	83.84	83.15	82.34	80.57	78.71	76.97	73.87
89	0.200	0.310	0.430	0.548	0.657	0.746	0.818	0.915	0.962	0.985	0.998
	86.05	86.00	85.78	85.39	84.79	84.11	83.31	81.46	79.65	77.85	74.71
90	0.200	0.311	0.431	0.551	0.659	0.748	0.822	0.917	0.965	0.986	0.998
	87.05	86.97	86.76	86.34	85.74	85.05	84.18	82.38	80.48	78.68	75.51
91	0.200	0.311	0.432	0.553	0.662	0.753	0.823	0.918	0.965	0.986	0.998
	88.03	87.97	87.74	87.32	86.72	85.98	85.16	83.30	81.38	79.59	76.35
92	0.200	0.312	0.434	0.554	0.664	0.755	0.827	0.920	0.966	0.986	0.998
	89.04	88.96	88.72	88.29	87.68	86.94	86.08	84.16	82.27	80.44	77.17
93	0.200	0.312	0.435	0.557	0.664	0.758	0.828	0.922	0.967	0.987	0.998
	89.99	89.94	89.71	89.27	88.68	87.87	87.03	85.09	83.14	81.30	77.98
94	0.200	0.313	0.437	0.558	0.667	0.760	0.831	0.923	0.968	0.987	0.998
	91.00	90.93	90.67	90.23	89.62	88.81	87.94	85.98	84.05	82.11	78.79
95	0.200	0.314	0.438	0.559	0.670	0.761	0.834	0.925	0.969	0.988	0.998
	91.98	91.89	91.65	91.22	90.57	89.79	88.86	86.90	84.90	82.95	79.66
96	0.200	0.315	0.439	0.562	0.674	0.764	0.835	0.926	0.970	0.989	0.999
	92.97	92.87	92.64	92.19	91.50	90.73	89.84	87.84	85.79	83.85	80.45

Table 36: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
97	0.200	0.315	0.441	0.564	0.676	0.766	0.839	0.928	0.971	0.989	0.999
	93.93	93.88	93.62	93.15	92.48	91.68	90.72	88.72	86.65	84.65	81.28
98	0.200	0.316	0.442	0.566	0.677	0.770	0.840	0.930	0.972	0.990	0.998
	94.91	94.86	94.61	94.12	93.46	92.61	91.69	89.59	87.51	85.55	82.09
99	0.200	0.317	0.445	0.569	0.680	0.771	0.843	0.931	0.972	0.990	0.999
	95.91	95.83	95.56	95.08	94.40	93.58	92.58	90.54	88.43	86.40	82.89
100	0.200	0.317	0.445	0.571	0.682	0.775	0.846	0.933	0.974	0.990	0.999
	96.92	96.82	96.54	96.05	95.36	94.50	93.49	91.42	89.27	87.28	83.74
101	0.200	0.318	0.448	0.571	0.684	0.776	0.848	0.934	0.974	0.990	0.999
	97.86	97.81	97.51	97.06	96.35	95.47	94.45	92.32	90.17	88.15	84.47
102	0.200	0.318	0.449	0.574	0.687	0.778	0.850	0.935	0.975	0.991	0.999
	98.87	98.78	98.49	98.03	97.28	96.43	95.38	93.23	91.01	88.96	85.38
103	0.200	0.319	0.451	0.574	0.689	0.781	0.851	0.937	0.975	0.991	0.999
	99.85	99.80	99.46	99.01	98.24	97.34	96.33	94.10	91.94	89.83	86.18
104	0.200	0.319	0.451	0.579	0.691	0.784	0.854	0.938	0.976	0.992	0.999
	100.86	100.76	100.46	99.93	99.21	98.27	97.26	95.02	92.81	90.65	86.98
105	0.200	0.321	0.452	0.580	0.694	0.784	0.854	0.941	0.977	0.992	0.999
	101.83	101.72	101.45	100.91	100.15	99.26	98.19	95.92	93.66	91.54	87.83
106	0.200	0.321	0.453	0.582	0.694	0.787	0.856	0.941	0.978	0.992	0.999
	102.81	102.73	102.43	101.87	101.15	100.19	99.15	96.83	94.57	92.41	88.63
107	0.200	0.322	0.454	0.583	0.697	0.790	0.859	0.943	0.979	0.993	0.999
	103.78	103.71	103.42	102.87	102.10	101.12	100.08	97.71	95.42	93.28	89.42
108	0.200	0.322	0.456	0.586	0.698	0.792	0.860	0.943	0.979	0.993	0.999
	104.78	104.68	104.38	103.81	103.08	102.08	100.98	98.63	96.30	94.06	90.23
109	0.200	0.323	0.457	0.588	0.701	0.793	0.862	0.945	0.980	0.993	0.999
	105.75	105.66	105.38	104.78	104.03	103.05	101.93	99.54	97.19	94.96	91.12
110	0.200	0.323	0.458	0.590	0.703	0.794	0.866	0.947	0.981	0.993	0.999
	106.75	106.65	106.35	105.76	104.97	104.02	102.83	100.41	98.04	95.80	91.91
111	0.200	0.324	0.460	0.593	0.706	0.798	0.867	0.947	0.981	0.994	0.999
	107.73	107.63	107.33	106.72	105.92	104.93	103.77	101.33	98.91	96.64	92.76
112	0.200	0.325	0.461	0.594	0.710	0.800	0.868	0.948	0.982	0.994	0.999
	108.72	108.62	108.32	107.70	106.83	105.87	104.70	102.24	99.80	97.50	93.55
113	0.200	0.325	0.462	0.595	0.710	0.802	0.871	0.949	0.982	0.995	0.999
	109.72	109.62	109.29	108.67	107.85	106.81	105.60	103.14	100.70	98.38	94.38
114	0.200	0.326	0.464	0.597	0.712	0.803	0.872	0.951	0.984	0.994	0.999
	110.68	110.59	110.26	109.66	108.80	107.76	106.57	104.01	101.53	99.24	95.18
115	0.200	0.326	0.466	0.599	0.714	0.805	0.873	0.952	0.983	0.995	1.000
	111.69	111.59	111.22	110.63	109.75	108.74	107.50	104.93	102.41	100.10	96.02
116	0.200	0.327	0.466	0.600	0.717	0.808	0.875	0.953	0.984	0.995	1.000
	112.67	112.57	112.23	111.60	110.70	109.64	108.42	105.81	103.31	100.97	96.79
117	0.200	0.327	0.469	0.602	0.719	0.810	0.878	0.954	0.984	0.995	1.000
	113.67	113.57	113.18	112.56	111.67	110.56	109.33	106.73	104.15	101.79	97.66
118	0.200	0.328	0.469	0.603	0.720	0.812	0.879	0.955	0.985	0.995	1.000
	114.65	114.53	114.18	113.55	112.64	111.51	110.28	107.64	105.02	102.69	98.44
119	0.200	0.329	0.470	0.607	0.721	0.812	0.880	0.956	0.985	0.995	1.000
	115.63	115.52	115.16	114.50	113.61	112.48	111.17	108.50	105.93	103.52	99.31
120	0.200	0.329	0.473	0.607	0.724	0.815	0.881	0.956	0.986	0.996	1.000
	116.61	116.52	116.11	115.49	114.54	113.41	112.17	109.41	106.81	104.35	100.08

Table 36: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
121	0.200	0.330	0.471	0.609	0.726	0.817	0.883	0.957	0.986	0.996	1.000
	117.60	117.49	117.14	116.47	115.50	114.37	113.10	110.34	107.66	105.24	100.94
122	0.200	0.330	0.473	0.610	0.731	0.818	0.886	0.958	0.987	0.996	1.000
	118.58	118.48	118.12	117.43	116.41	115.32	113.96	111.26	108.52	106.09	101.71
123	0.200	0.331	0.474	0.611	0.729	0.822	0.887	0.960	0.987	0.996	1.000
	119.56	119.45	119.08	118.41	117.44	116.25	114.91	112.16	109.45	106.90	102.53
124	0.200	0.332	0.476	0.614	0.731	0.823	0.888	0.961	0.987	0.996	1.000
	120.59	120.43	120.05	119.36	118.40	117.19	115.86	113.01	110.30	107.78	103.33
125	0.200	0.332	0.477	0.617	0.734	0.823	0.890	0.961	0.987	0.996	1.000
	121.57	121.44	121.05	120.31	119.32	118.16	116.79	113.93	111.20	108.59	104.19
126	0.200	0.332	0.477	0.619	0.736	0.827	0.891	0.962	0.988	0.997	1.000
	122.54	122.45	122.05	121.28	120.31	119.09	117.71	114.84	112.04	109.49	105.03
127	0.200	0.333	0.480	0.620	0.738	0.828	0.893	0.962	0.989	0.997	1.000
	123.52	123.41	123.00	122.26	121.26	120.02	118.64	115.76	112.89	110.29	105.82
128	0.200	0.334	0.480	0.622	0.738	0.829	0.894	0.963	0.989	0.997	1.000
	124.49	124.39	123.98	123.23	122.25	120.97	119.55	116.63	113.78	111.19	106.64
129	0.200	0.334	0.482	0.622	0.741	0.833	0.896	0.964	0.989	0.997	1.000
	125.51	125.39	124.97	124.22	123.17	121.87	120.47	117.51	114.66	112.01	107.44
130	0.200	0.334	0.482	0.624	0.743	0.834	0.896	0.965	0.990	0.997	1.000
	126.48	126.39	125.98	125.19	124.13	122.83	121.45	118.39	115.52	112.89	108.29
131	0.200	0.336	0.484	0.626	0.745	0.835	0.899	0.966	0.990	0.997	1.000
	127.46	127.33	126.93	126.15	125.08	123.74	122.32	119.31	116.39	113.67	109.12
132	0.200	0.336	0.486	0.628	0.746	0.836	0.899	0.966	0.990	0.997	1.000
	128.44	128.32	127.90	127.13	126.06	124.73	123.31	120.23	117.26	114.59	109.91
133	0.200	0.337	0.487	0.628	0.748	0.838	0.900	0.968	0.990	0.998	1.000
	129.45	129.32	128.88	128.11	126.99	125.65	124.26	121.08	118.13	115.42	110.73
134	0.200	0.337	0.488	0.631	0.750	0.841	0.903	0.969	0.991	0.998	1.000
	130.44	130.32	129.86	129.07	127.94	126.57	125.08	121.96	119.04	116.23	111.58
135	0.200	0.338	0.489	0.631	0.752	0.841	0.904	0.968	0.991	0.998	1.000
	131.41	131.28	130.86	130.06	128.89	127.55	126.07	122.96	119.89	117.15	112.35
136	0.200	0.338	0.491	0.633	0.754	0.841	0.905	0.969	0.991	0.998	1.000
	132.40	132.27	131.82	131.01	129.85	128.54	126.98	123.81	120.80	118.01	113.17
137	0.200	0.339	0.492	0.635	0.755	0.845	0.906	0.970	0.992	0.998	1.000
	133.39	133.27	132.79	131.99	130.84	129.43	127.93	124.71	121.67	118.81	113.99
138	0.200	0.339	0.493	0.637	0.757	0.846	0.907	0.970	0.992	0.998	1.000
	134.39	134.25	133.78	132.94	131.79	130.39	128.83	125.59	122.52	119.70	114.84
139	0.200	0.340	0.494	0.639	0.759	0.848	0.909	0.971	0.992	0.998	1.000
	135.37	135.23	134.74	133.89	132.74	131.34	129.74	126.52	123.34	120.57	115.65
140	0.200	0.341	0.495	0.641	0.760	0.849	0.910	0.972	0.993	0.998	1.000
	136.34	136.21	135.74	134.88	133.70	132.26	130.68	127.39	124.26	121.36	116.44
141	0.200	0.341	0.497	0.642	0.762	0.850	0.912	0.972	0.993	0.998	1.000
	137.34	137.20	136.72	135.85	134.66	133.23	131.55	128.29	125.13	122.24	117.25
142	0.200	0.342	0.496	0.645	0.764	0.852	0.912	0.972	0.993	0.998	1.000
	138.31	138.20	137.72	136.80	135.60	134.13	132.56	129.22	125.96	123.13	118.11
143	0.200	0.342	0.499	0.646	0.765	0.853	0.914	0.974	0.993	0.998	1.000
	139.30	139.20	138.66	137.78	136.56	135.11	133.47	130.06	126.86	123.92	118.89
144	0.200	0.343	0.499	0.646	0.767	0.855	0.915	0.974	0.993	0.998	1.000
	140.28	140.15	139.66	138.76	137.52	136.01	134.35	130.99	127.70	124.85	119.76

Table 36: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
145	0.200	0.343	0.502	0.649	0.767	0.858	0.916	0.975	0.993	0.999	1.000
	141.31	141.16	140.62	139.71	138.50	136.94	135.31	131.87	128.58	125.65	120.54
146	0.200	0.343	0.502	0.651	0.769	0.859	0.918	0.975	0.994	0.999	1.000
	142.28	142.16	141.62	140.68	139.44	137.86	136.22	132.76	129.54	126.55	121.32
147	0.200	0.345	0.503	0.652	0.773	0.860	0.917	0.975	0.994	0.998	1.000
	143.27	143.11	142.61	141.66	140.35	138.82	137.21	133.68	130.38	127.31	122.19
148	0.200	0.345	0.504	0.652	0.773	0.860	0.920	0.976	0.994	0.999	1.000
	144.24	144.10	143.57	142.65	141.34	139.77	138.02	134.57	131.22	128.18	122.94
149	0.200	0.345	0.506	0.653	0.775	0.862	0.919	0.977	0.994	0.999	1.000
	145.23	145.10	144.56	143.63	142.30	140.79	139.04	135.49	132.10	129.07	123.85
150	0.200	0.346	0.507	0.657	0.777	0.863	0.922	0.977	0.994	0.999	1.000
	146.22	146.08	145.53	144.55	143.25	141.67	139.93	136.42	132.98	129.92	124.61
155	0.200	0.348	0.513	0.663	0.784	0.870	0.928	0.980	0.995	0.999	1.000
	151.15	151.02	150.44	149.43	148.00	146.37	144.52	140.83	137.28	134.11	128.70
160	0.200	0.352	0.518	0.671	0.792	0.877	0.931	0.982	0.996	0.999	1.000
	156.13	155.91	155.33	154.25	152.79	151.05	149.21	145.29	141.69	138.43	132.81
165	0.200	0.354	0.523	0.675	0.799	0.881	0.935	0.984	0.996	0.999	1.000
	161.03	160.86	160.24	159.16	157.58	155.80	153.85	149.75	146.07	142.66	136.84
170	0.200	0.356	0.528	0.684	0.804	0.888	0.940	0.986	0.997	0.999	1.000
	165.97	165.83	165.15	163.96	162.35	160.43	158.38	154.24	150.39	146.91	141.01
175	0.200	0.360	0.533	0.689	0.812	0.894	0.944	0.987	0.998	1.000	1.000
	170.91	170.73	170.07	168.84	167.11	165.13	163.02	158.73	154.71	151.15	145.05
180	0.200	0.362	0.537	0.697	0.819	0.898	0.948	0.988	0.998	1.000	1.000
	175.87	175.66	174.98	173.65	171.85	169.86	167.57	163.19	159.08	155.40	149.08
185	0.200	0.364	0.544	0.704	0.826	0.904	0.952	0.989	0.998	1.000	1.000
	180.82	180.61	179.85	178.47	176.62	174.48	172.18	167.69	163.42	159.67	153.17
190	0.200	0.366	0.549	0.709	0.831	0.909	0.954	0.991	0.998	1.000	1.000
	185.78	185.55	184.73	183.36	181.37	179.16	176.84	172.14	167.81	163.92	157.22
195	0.200	0.368	0.555	0.716	0.835	0.913	0.957	0.991	0.999	1.000	1.000
	190.73	190.51	189.61	188.18	186.20	183.85	181.48	176.69	172.12	168.16	161.39
200	0.200	0.371	0.557	0.724	0.842	0.918	0.960	0.992	0.999	1.000	1.000
	195.66	195.43	194.58	192.96	190.91	188.52	186.03	181.04	176.54	172.44	165.48
210	0.200	0.376	0.569	0.733	0.852	0.925	0.966	0.993	0.999	1.000	1.000
	205.56	205.29	204.33	202.71	200.46	197.90	195.22	189.95	185.22	180.94	173.56
220	0.200	0.381	0.577	0.744	0.863	0.932	0.969	0.995	0.999	1.000	1.000
	215.43	215.14	214.15	212.37	209.94	207.20	204.48	198.86	193.84	189.41	181.73
230	0.200	0.385	0.586	0.754	0.870	0.938	0.973	0.996	1.000	1.000	1.000
	225.32	225.06	223.96	222.06	219.49	216.53	213.58	207.84	202.55	197.84	189.79
240	0.200	0.390	0.594	0.764	0.879	0.944	0.977	0.997	1.000	1.000	1.000
	235.25	234.92	233.77	231.68	229.02	225.88	222.71	216.68	211.21	206.32	197.96
250	0.200	0.394	0.603	0.774	0.887	0.950	0.979	0.997	1.000	1.000	1.000
	245.16	244.82	243.56	241.36	238.41	235.20	231.95	225.60	219.99	214.89	206.18

Table 37: Table of Monte Carlo Results for $\mathcal{M}_{\text{BK}}$ when $k = 6$ and $\alpha = 0.05$.

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
1	0.167	0.180	0.194	0.206	0.219	0.231	0.242	0.265	0.286	0.306	0.342
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	0.167	0.180	0.194	0.206	0.219	0.231	0.242	0.265	0.286	0.306	0.342
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
3	0.167	0.183	0.199	0.214	0.229	0.245	0.259	0.288	0.315	0.341	0.390
	2.83	2.83	2.84	2.83	2.83	2.83	2.83	2.82	2.82	2.81	2.80
4	0.167	0.186	0.205	0.224	0.243	0.261	0.279	0.314	0.348	0.380	0.440
	3.39	3.39	3.38	3.38	3.37	3.38	3.37	3.36	3.34	3.32	3.29
5	0.167	0.188	0.210	0.232	0.253	0.274	0.295	0.335	0.373	0.410	0.476
	4.32	4.32	4.32	4.32	4.31	4.31	4.30	4.30	4.28	4.27	4.25
6	0.167	0.190	0.214	0.237	0.261	0.284	0.306	0.350	0.391	0.430	0.502
	5.20	5.20	5.20	5.20	5.19	5.18	5.18	5.16	5.15	5.13	5.08
7	0.167	0.192	0.217	0.242	0.268	0.292	0.317	0.364	0.411	0.452	0.529
	6.23	6.23	6.23	6.22	6.21	6.21	6.20	6.18	6.16	6.13	6.07
8	0.167	0.193	0.220	0.248	0.275	0.302	0.328	0.380	0.429	0.475	0.558
	7.21	7.20	7.20	7.20	7.19	7.18	7.16	7.13	7.10	7.05	6.97
9	0.167	0.195	0.224	0.254	0.283	0.311	0.340	0.395	0.448	0.497	0.584
	8.14	8.14	8.13	8.12	8.12	8.11	8.09	8.05	8.01	7.97	7.87
10	0.167	0.197	0.227	0.259	0.289	0.321	0.351	0.410	0.464	0.516	0.607
	9.07	9.07	9.06	9.05	9.05	9.03	9.01	8.97	8.93	8.88	8.75
11	0.167	0.198	0.231	0.263	0.296	0.327	0.360	0.422	0.481	0.535	0.628
	10.03	10.02	10.01	10.01	10.00	9.99	9.96	9.92	9.85	9.80	9.66
12	0.167	0.199	0.233	0.268	0.301	0.335	0.369	0.435	0.494	0.551	0.647
	10.99	11.01	10.99	10.98	10.96	10.95	10.93	10.87	10.81	10.73	10.57
13	0.167	0.201	0.236	0.272	0.308	0.343	0.378	0.446	0.509	0.567	0.666
	11.97	11.96	11.97	11.95	11.93	11.91	11.88	11.83	11.75	11.67	11.47
14	0.167	0.202	0.239	0.276	0.314	0.351	0.388	0.459	0.524	0.583	0.684
	12.94	12.93	12.92	12.91	12.89	12.87	12.84	12.76	12.67	12.58	12.37
15	0.167	0.203	0.241	0.280	0.319	0.359	0.397	0.470	0.539	0.598	0.702
	13.89	13.89	13.88	13.87	13.85	13.80	13.78	13.71	13.61	13.50	13.26
16	0.167	0.205	0.244	0.284	0.325	0.366	0.405	0.481	0.551	0.613	0.717
	14.85	14.84	14.84	14.82	14.80	14.77	14.73	14.65	14.54	14.42	14.16
17	0.167	0.206	0.246	0.288	0.331	0.372	0.413	0.491	0.563	0.628	0.732
	15.83	15.81	15.81	15.79	15.76	15.73	15.69	15.60	15.47	15.34	15.06
18	0.167	0.207	0.249	0.292	0.336	0.380	0.421	0.502	0.576	0.641	0.748
	16.80	16.78	16.77	16.76	16.72	16.69	16.65	16.54	16.40	16.26	15.93
19	0.167	0.208	0.251	0.296	0.341	0.385	0.429	0.512	0.588	0.654	0.762
	17.76	17.76	17.75	17.72	17.70	17.65	17.60	17.48	17.33	17.17	16.81
20	0.167	0.209	0.254	0.299	0.345	0.392	0.436	0.522	0.598	0.666	0.775
	18.73	18.73	18.71	18.70	18.66	18.61	18.57	18.43	18.28	18.10	17.70
21	0.167	0.210	0.256	0.303	0.351	0.399	0.444	0.532	0.610	0.677	0.784
	19.70	19.69	19.68	19.66	19.61	19.57	19.51	19.37	19.21	19.02	18.60
22	0.167	0.211	0.258	0.307	0.355	0.404	0.452	0.542	0.619	0.690	0.795
	20.68	20.67	20.65	20.62	20.59	20.54	20.47	20.31	20.14	19.93	19.49
23	0.167	0.212	0.261	0.310	0.361	0.410	0.459	0.551	0.630	0.698	0.806
	21.64	21.64	21.62	21.59	21.54	21.50	21.43	21.26	21.07	20.86	20.37
24	0.167	0.213	0.263	0.314	0.366	0.415	0.466	0.557	0.641	0.710	0.817
	22.63	22.62	22.60	22.56	22.52	22.46	22.39	22.22	21.99	21.76	21.25

Table 37: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
25	0.167	0.214	0.265	0.318	0.369	0.422	0.473	0.567	0.649	0.720	0.825
	23.59	23.59	23.57	23.52	23.49	23.42	23.34	23.15	22.94	22.68	22.13
26	0.167	0.215	0.267	0.320	0.374	0.428	0.480	0.577	0.660	0.730	0.835
	24.57	24.56	24.54	24.50	24.45	24.39	24.29	24.09	23.86	23.60	23.00
27	0.167	0.216	0.269	0.324	0.378	0.434	0.486	0.585	0.668	0.740	0.844
	25.54	25.54	25.51	25.47	25.42	25.34	25.26	25.04	24.79	24.50	23.88
28	0.167	0.217	0.271	0.327	0.382	0.438	0.492	0.592	0.678	0.748	0.852
	26.52	26.50	26.48	26.44	26.39	26.32	26.22	25.99	25.71	25.42	24.74
29	0.167	0.218	0.273	0.330	0.386	0.444	0.498	0.598	0.684	0.756	0.862
	27.50	27.48	27.46	27.41	27.37	27.27	27.19	26.95	26.66	26.34	25.61
30	0.167	0.219	0.275	0.333	0.392	0.450	0.506	0.607	0.693	0.766	0.868
	28.47	28.47	28.44	28.39	28.32	28.23	28.13	27.88	27.58	27.22	26.50
31	0.167	0.220	0.277	0.337	0.395	0.455	0.510	0.615	0.703	0.775	0.873
	29.45	29.44	29.42	29.36	29.30	29.20	29.10	28.81	28.50	28.12	27.38
32	0.167	0.221	0.279	0.339	0.401	0.460	0.518	0.621	0.711	0.782	0.880
	30.43	30.41	30.38	30.33	30.25	30.16	30.05	29.76	29.41	29.04	28.26
33	0.167	0.221	0.281	0.343	0.404	0.465	0.524	0.628	0.719	0.790	0.887
	31.39	31.40	31.36	31.29	31.23	31.12	31.00	30.72	30.34	29.94	29.12
34	0.167	0.222	0.282	0.345	0.408	0.470	0.529	0.634	0.724	0.796	0.892
	32.38	32.37	32.34	32.29	32.20	32.09	31.97	31.66	31.28	30.87	29.98
35	0.167	0.223	0.285	0.348	0.412	0.475	0.534	0.643	0.732	0.803	0.897
	33.35	33.35	33.31	33.26	33.16	33.05	32.94	32.58	32.20	31.77	30.87
36	0.167	0.224	0.286	0.351	0.415	0.479	0.541	0.650	0.740	0.810	0.903
	34.32	34.34	34.29	34.23	34.14	34.02	33.87	33.53	33.12	32.67	31.71
37	0.167	0.225	0.288	0.354	0.420	0.485	0.546	0.656	0.747	0.817	0.908
	35.31	35.30	35.26	35.20	35.10	34.98	34.84	34.48	34.04	33.58	32.60
38	0.167	0.226	0.290	0.357	0.424	0.489	0.552	0.662	0.753	0.823	0.913
	36.29	36.27	36.25	36.17	36.08	35.95	35.80	35.41	34.96	34.47	33.47
39	0.167	0.227	0.292	0.359	0.427	0.494	0.557	0.670	0.758	0.828	0.917
	37.27	37.26	37.22	37.15	37.04	36.92	36.74	36.36	35.90	35.39	34.33
40	0.167	0.228	0.293	0.362	0.431	0.498	0.562	0.675	0.766	0.835	0.922
	38.24	38.23	38.20	38.13	38.02	37.88	37.71	37.30	36.81	36.26	35.17
41	0.167	0.228	0.295	0.365	0.435	0.504	0.568	0.681	0.773	0.840	0.925
	39.22	39.22	39.18	39.10	38.99	38.84	38.67	38.23	37.71	37.19	36.05
42	0.167	0.229	0.297	0.367	0.440	0.507	0.574	0.688	0.778	0.846	0.930
	40.21	40.20	40.15	40.08	39.94	39.81	39.61	39.17	38.65	38.07	36.89
43	0.167	0.230	0.299	0.370	0.442	0.512	0.578	0.692	0.784	0.851	0.933
	41.19	41.17	41.12	41.06	40.93	40.77	40.57	40.13	39.57	38.98	37.76
44	0.167	0.231	0.300	0.374	0.445	0.515	0.582	0.699	0.790	0.856	0.937
	42.16	42.15	42.10	42.01	41.91	41.75	41.54	41.05	40.48	39.87	38.63
45	0.167	0.231	0.302	0.375	0.449	0.521	0.588	0.705	0.793	0.860	0.940
	43.14	43.13	43.08	43.01	42.88	42.70	42.49	41.98	41.42	40.78	39.48
46	0.167	0.232	0.304	0.379	0.454	0.526	0.594	0.710	0.800	0.866	0.943
	44.12	44.12	44.06	43.97	43.83	43.66	43.43	42.92	42.31	41.67	40.34
47	0.167	0.233	0.305	0.381	0.457	0.529	0.598	0.713	0.805	0.871	0.946
	45.11	45.09	45.04	44.95	44.80	44.63	44.39	43.88	43.24	42.56	41.20
48	0.167	0.233	0.307	0.384	0.460	0.534	0.602	0.721	0.811	0.875	0.948
	46.09	46.07	46.01	45.92	45.78	45.59	45.35	44.80	44.15	43.45	42.05

Table 37: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
49	0.167	0.234	0.309	0.387	0.464	0.538	0.608	0.726	0.813	0.880	0.951
	47.07	47.06	47.00	46.89	46.74	46.55	46.31	45.73	45.07	44.34	42.92
50	0.167	0.235	0.310	0.388	0.466	0.541	0.613	0.731	0.819	0.885	0.954
	48.06	48.03	47.99	47.89	47.73	47.52	47.26	46.66	46.00	45.21	43.77
51	0.167	0.235	0.312	0.391	0.470	0.545	0.617	0.734	0.823	0.886	0.956
	49.04	49.04	48.95	48.85	48.69	48.49	48.22	47.64	46.92	46.17	44.62
52	0.167	0.236	0.313	0.393	0.474	0.551	0.621	0.739	0.829	0.891	0.959
	50.02	50.01	49.94	49.82	49.66	49.42	49.19	48.56	47.83	47.04	45.48
53	0.167	0.237	0.314	0.397	0.477	0.553	0.626	0.746	0.833	0.896	0.961
	51.00	50.97	50.93	50.80	50.63	50.42	50.13	49.47	48.75	47.90	46.33
54	0.167	0.238	0.316	0.399	0.481	0.558	0.630	0.751	0.838	0.897	0.963
	51.99	51.96	51.90	51.78	51.59	51.36	51.09	50.40	49.63	48.81	47.19
55	0.167	0.239	0.318	0.401	0.484	0.561	0.635	0.755	0.843	0.903	0.964
	52.97	52.94	52.88	52.75	52.57	52.34	52.04	51.36	50.55	49.71	48.04
56	0.167	0.239	0.320	0.403	0.486	0.566	0.638	0.760	0.847	0.905	0.966
	53.96	53.92	53.85	53.72	53.53	53.29	52.99	52.27	51.46	50.62	48.90
57	0.167	0.240	0.321	0.405	0.490	0.570	0.642	0.763	0.850	0.909	0.969
	54.93	54.92	54.85	54.72	54.51	54.25	53.97	53.23	52.37	51.48	49.74
58	0.167	0.241	0.323	0.408	0.493	0.572	0.648	0.769	0.854	0.912	0.970
	55.92	55.89	55.81	55.68	55.49	55.24	54.89	54.14	53.31	52.37	50.61
59	0.167	0.241	0.324	0.411	0.496	0.576	0.650	0.771	0.857	0.914	0.971
	56.90	56.87	56.80	56.66	56.47	56.19	55.87	55.11	54.21	53.28	51.46
60	0.167	0.242	0.325	0.413	0.500	0.579	0.656	0.775	0.862	0.918	0.973
	57.88	57.86	57.79	57.62	57.42	57.17	56.80	56.04	55.11	54.14	52.28
61	0.167	0.242	0.327	0.416	0.502	0.584	0.659	0.779	0.865	0.920	0.974
	58.86	58.85	58.76	58.60	58.40	58.12	57.78	56.97	56.03	55.04	53.15
62	0.167	0.243	0.329	0.417	0.505	0.587	0.663	0.784	0.868	0.923	0.976
	59.86	59.83	59.74	59.60	59.37	59.10	58.72	57.90	56.94	55.92	54.01
63	0.167	0.244	0.330	0.420	0.507	0.593	0.667	0.789	0.872	0.925	0.977
	60.83	60.81	60.72	60.55	60.35	60.02	59.70	58.80	57.84	56.83	54.83
64	0.167	0.244	0.331	0.423	0.512	0.595	0.671	0.791	0.875	0.929	0.978
	61.83	61.79	61.71	61.53	61.30	60.99	60.62	59.76	58.76	57.71	55.71
65	0.167	0.245	0.333	0.425	0.515	0.599	0.676	0.796	0.878	0.930	0.979
	62.81	62.77	62.68	62.52	62.27	61.96	61.56	60.67	59.68	58.60	56.55
66	0.167	0.246	0.335	0.426	0.517	0.601	0.677	0.801	0.883	0.932	0.980
	63.78	63.76	63.65	63.50	63.25	62.93	62.55	61.59	60.55	59.50	57.41
67	0.167	0.247	0.335	0.429	0.520	0.605	0.682	0.803	0.884	0.935	0.981
	64.77	64.74	64.65	64.48	64.22	63.89	63.49	62.55	61.47	60.35	58.24
68	0.167	0.247	0.337	0.431	0.524	0.608	0.684	0.806	0.888	0.937	0.982
	65.75	65.72	65.64	65.45	65.18	64.85	64.46	63.48	62.41	61.26	59.09
69	0.167	0.248	0.338	0.432	0.527	0.612	0.689	0.810	0.890	0.940	0.983
	66.76	66.70	66.61	66.44	66.14	65.82	65.40	64.42	63.28	62.14	59.95
70	0.167	0.248	0.340	0.437	0.530	0.615	0.694	0.814	0.894	0.942	0.984
	67.71	67.70	67.59	67.39	67.12	66.78	66.34	65.34	64.17	63.03	60.75
71	0.167	0.249	0.342	0.438	0.531	0.619	0.697	0.816	0.896	0.943	0.985
	68.69	68.66	68.57	68.38	68.11	67.73	67.29	66.28	65.09	63.92	61.66
72	0.167	0.250	0.342	0.440	0.534	0.621	0.700	0.820	0.897	0.945	0.986
	69.69	69.66	69.56	69.36	69.08	68.72	68.24	67.20	66.02	64.81	62.46

Table 37: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
73	0.167	0.250	0.344	0.441	0.536	0.626	0.703	0.823	0.902	0.947	0.986
	70.68	70.64	70.55	70.34	70.05	69.65	69.21	68.12	66.91	65.69	63.35
74	0.167	0.251	0.345	0.444	0.539	0.629	0.706	0.828	0.904	0.949	0.987
	71.65	71.62	71.52	71.31	71.03	70.62	70.16	69.03	67.82	66.61	64.20
75	0.167	0.251	0.347	0.447	0.543	0.631	0.710	0.830	0.905	0.950	0.988
	72.65	72.61	72.51	72.28	71.98	71.59	71.12	69.98	68.76	67.47	65.02
76	0.167	0.252	0.349	0.449	0.545	0.636	0.713	0.833	0.909	0.953	0.989
	73.63	73.59	73.47	73.26	72.97	72.53	72.06	70.91	69.61	68.31	65.87
77	0.167	0.252	0.350	0.451	0.549	0.638	0.716	0.837	0.911	0.954	0.989
	74.62	74.59	74.46	74.24	73.91	73.50	73.03	71.81	70.54	69.24	66.71
78	0.167	0.253	0.351	0.453	0.552	0.641	0.719	0.839	0.913	0.956	0.990
	75.61	75.57	75.45	75.22	74.88	74.48	73.97	72.75	71.45	70.08	67.55
79	0.167	0.254	0.353	0.455	0.555	0.643	0.722	0.842	0.917	0.958	0.990
	76.59	76.55	76.42	76.20	75.84	75.44	74.92	73.69	72.32	70.96	68.43
80	0.167	0.255	0.354	0.457	0.556	0.647	0.727	0.843	0.918	0.958	0.990
	77.58	77.52	77.41	77.18	76.85	76.40	75.85	74.63	73.23	71.88	69.25
81	0.167	0.255	0.355	0.458	0.559	0.651	0.729	0.847	0.919	0.961	0.991
	78.54	78.53	78.40	78.16	77.80	77.34	76.82	75.53	74.17	72.72	70.08
82	0.167	0.256	0.356	0.461	0.561	0.655	0.731	0.850	0.921	0.962	0.992
	79.55	79.50	79.37	79.13	78.79	78.29	77.76	76.48	75.08	73.62	70.95
83	0.167	0.256	0.357	0.464	0.563	0.656	0.736	0.853	0.924	0.962	0.992
	80.54	80.48	80.38	80.09	79.75	79.28	78.70	77.42	75.96	74.52	71.78
84	0.167	0.257	0.359	0.464	0.566	0.660	0.739	0.857	0.926	0.964	0.992
	81.51	81.48	81.33	81.10	80.73	80.22	79.65	78.30	76.82	75.36	72.63
85	0.167	0.258	0.361	0.467	0.570	0.663	0.741	0.858	0.927	0.966	0.993
	82.48	82.45	82.31	82.07	81.69	81.18	80.61	79.24	77.78	76.25	73.47
86	0.167	0.258	0.362	0.469	0.572	0.665	0.744	0.860	0.929	0.966	0.993
	83.48	83.45	83.32	83.04	82.65	82.16	81.58	80.19	78.66	77.16	74.34
87	0.167	0.259	0.364	0.472	0.576	0.667	0.746	0.863	0.932	0.967	0.993
	84.45	84.43	84.26	84.00	83.60	83.14	82.52	81.10	79.57	77.99	75.13
88	0.167	0.259	0.364	0.473	0.577	0.671	0.749	0.866	0.934	0.968	0.994
	85.47	85.43	85.26	84.99	84.59	84.07	83.48	82.02	80.47	78.89	76.00
89	0.167	0.260	0.365	0.474	0.578	0.673	0.753	0.867	0.936	0.970	0.994
	86.44	86.39	86.26	85.99	85.58	85.03	84.40	82.97	81.35	79.76	76.86
90	0.167	0.260	0.367	0.476	0.582	0.675	0.757	0.870	0.937	0.970	0.995
	87.43	87.40	87.23	86.97	86.54	86.00	85.34	83.90	82.25	80.68	77.69
91	0.167	0.261	0.368	0.479	0.585	0.679	0.758	0.873	0.938	0.972	0.995
	88.41	88.38	88.22	87.94	87.50	86.96	86.31	84.80	83.17	81.54	78.55
92	0.167	0.261	0.371	0.482	0.588	0.682	0.761	0.875	0.938	0.973	0.995
	89.38	89.36	89.18	88.89	88.46	87.91	87.26	85.73	84.13	82.43	79.39
93	0.167	0.262	0.370	0.481	0.589	0.685	0.764	0.877	0.941	0.973	0.995
	90.39	90.36	90.19	89.91	89.45	88.85	88.20	86.66	84.97	83.29	80.25
94	0.167	0.262	0.372	0.483	0.591	0.687	0.767	0.880	0.943	0.974	0.995
	91.38	91.35	91.18	90.89	90.43	89.84	89.13	87.55	85.86	84.18	81.08
95	0.167	0.263	0.374	0.487	0.593	0.689	0.769	0.881	0.944	0.975	0.996
	92.36	92.34	92.13	91.84	91.39	90.81	90.09	88.50	86.77	85.04	81.89
96	0.167	0.263	0.374	0.488	0.597	0.689	0.772	0.885	0.945	0.977	0.996
	93.36	93.32	93.14	92.82	92.36	91.80	91.04	89.40	87.69	85.91	82.76

Table 37: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
97	0.167	0.264	0.375	0.489	0.599	0.694	0.775	0.886	0.948	0.977	0.996
	94.34	94.28	94.13	93.81	93.31	92.73	91.97	90.33	88.54	86.80	83.59
98	0.167	0.265	0.376	0.491	0.600	0.697	0.775	0.888	0.949	0.977	0.996
	95.31	95.28	95.12	94.79	94.31	93.66	92.99	91.26	89.45	87.69	84.45
99	0.167	0.265	0.377	0.494	0.603	0.700	0.779	0.891	0.950	0.979	0.997
	96.31	96.26	96.10	95.75	95.26	94.63	93.90	92.19	90.33	88.53	85.30
100	0.167	0.266	0.380	0.495	0.605	0.702	0.781	0.893	0.952	0.980	0.997
	97.30	97.24	97.06	96.75	96.23	95.59	94.84	93.08	91.21	89.40	86.10
101	0.167	0.266	0.381	0.498	0.608	0.705	0.784	0.894	0.952	0.980	0.997
	98.29	98.23	98.05	97.70	97.20	96.54	95.78	94.01	92.16	90.28	86.96
102	0.167	0.267	0.382	0.498	0.611	0.706	0.787	0.896	0.953	0.981	0.997
	99.28	99.23	99.02	98.70	98.15	97.54	96.72	94.95	93.04	91.20	87.83
103	0.167	0.268	0.383	0.502	0.612	0.709	0.788	0.898	0.955	0.981	0.997
	100.26	100.20	100.00	99.65	99.15	98.46	97.70	95.87	93.97	92.06	88.63
104	0.167	0.268	0.383	0.503	0.614	0.712	0.791	0.900	0.956	0.982	0.997
	101.24	101.18	101.02	100.65	100.12	99.44	98.65	96.76	94.88	92.91	89.53
105	0.167	0.268	0.386	0.503	0.616	0.712	0.795	0.902	0.957	0.982	0.998
	102.23	102.21	101.98	101.64	101.10	100.41	99.53	97.71	95.78	93.85	90.34
106	0.167	0.269	0.386	0.507	0.619	0.716	0.796	0.904	0.957	0.983	0.998
	103.21	103.14	102.97	102.60	102.05	101.36	100.51	98.59	96.64	94.68	91.16
107	0.167	0.269	0.387	0.509	0.624	0.719	0.798	0.905	0.959	0.983	0.998
	104.23	104.16	103.96	103.57	102.98	102.30	101.48	99.54	97.55	95.58	92.04
108	0.167	0.270	0.389	0.511	0.622	0.721	0.801	0.906	0.961	0.984	0.998
	105.19	105.13	104.93	104.53	104.00	103.27	102.39	100.51	98.40	96.47	92.86
109	0.167	0.271	0.390	0.511	0.624	0.724	0.800	0.907	0.961	0.985	0.998
	106.18	106.13	105.91	105.53	104.99	104.21	103.40	101.42	99.34	97.28	93.76
110	0.167	0.271	0.392	0.514	0.628	0.726	0.805	0.910	0.962	0.985	0.998
	107.18	107.12	106.89	106.50	105.91	105.18	104.30	102.31	100.26	98.20	94.54
111	0.167	0.272	0.392	0.514	0.629	0.729	0.808	0.912	0.962	0.986	0.998
	108.16	108.09	107.90	107.51	106.89	106.12	105.24	103.23	101.15	99.06	95.37
112	0.167	0.273	0.394	0.517	0.633	0.729	0.810	0.914	0.965	0.986	0.998
	109.15	109.06	108.85	108.47	107.85	107.11	106.18	104.18	102.02	99.94	96.22
113	0.167	0.272	0.394	0.518	0.634	0.730	0.813	0.914	0.964	0.987	0.999
	110.15	110.10	109.85	109.46	108.83	108.10	107.12	105.10	102.93	100.82	97.06
114	0.167	0.273	0.395	0.521	0.637	0.735	0.814	0.917	0.966	0.987	0.998
	111.14	111.06	110.85	110.41	109.79	109.01	108.08	105.95	103.80	101.73	97.93
115	0.167	0.273	0.396	0.523	0.640	0.737	0.816	0.918	0.968	0.988	0.999
	112.13	112.07	111.83	111.40	110.75	109.96	109.03	106.91	104.65	102.54	98.74
116	0.167	0.274	0.397	0.525	0.641	0.739	0.818	0.919	0.967	0.988	0.999
	113.10	113.03	112.83	112.36	111.72	110.91	109.99	107.86	105.62	103.43	99.58
117	0.167	0.275	0.400	0.526	0.643	0.743	0.820	0.921	0.968	0.989	0.999
	114.07	114.03	113.77	113.35	112.70	111.87	110.90	108.74	106.49	104.33	100.43
118	0.167	0.275	0.400	0.527	0.645	0.744	0.822	0.921	0.969	0.989	0.999
	115.09	115.01	114.76	114.34	113.67	112.82	111.84	109.72	107.41	105.23	101.25
119	0.167	0.276	0.402	0.529	0.646	0.745	0.824	0.923	0.971	0.989	0.999
	116.07	115.99	115.75	115.29	114.64	113.79	112.82	110.60	108.25	106.04	102.10
120	0.167	0.276	0.402	0.529	0.650	0.748	0.826	0.925	0.971	0.990	0.999
	117.05	116.98	116.75	116.32	115.58	114.74	113.74	111.47	109.16	106.95	102.98

Table 37: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
121	0.167	0.277	0.403	0.533	0.650	0.751	0.827	0.926	0.971	0.990	0.999
	118.05	117.97	117.73	117.27	116.58	115.69	114.70	112.42	110.06	107.81	103.80
122	0.167	0.278	0.405	0.533	0.652	0.753	0.829	0.929	0.972	0.990	0.999
	119.04	118.94	118.71	118.25	117.56	116.64	115.66	113.30	110.99	108.75	104.69
123	0.167	0.278	0.406	0.535	0.654	0.754	0.831	0.929	0.973	0.991	0.999
	120.02	119.96	119.69	119.23	118.54	117.62	116.60	114.27	111.91	109.54	105.47
124	0.167	0.279	0.407	0.538	0.656	0.756	0.833	0.931	0.974	0.991	0.999
	121.00	120.93	120.67	120.19	119.50	118.58	117.54	115.17	112.76	110.42	106.34
125	0.167	0.279	0.408	0.538	0.657	0.757	0.835	0.931	0.974	0.991	0.999
	122.00	121.93	121.65	121.18	120.48	119.56	118.48	116.08	113.66	111.32	107.16
126	0.167	0.279	0.410	0.540	0.661	0.759	0.838	0.932	0.975	0.992	0.999
	123.00	122.90	122.64	122.16	121.40	120.50	119.40	117.02	114.54	112.21	107.95
127	0.167	0.280	0.410	0.542	0.662	0.762	0.839	0.934	0.976	0.992	0.999
	123.97	123.92	123.64	123.14	122.39	121.45	120.33	117.94	115.45	113.08	108.86
128	0.167	0.281	0.411	0.543	0.664	0.763	0.842	0.935	0.976	0.992	0.999
	124.97	124.89	124.62	124.13	123.36	122.41	121.29	118.82	116.32	113.97	109.67
129	0.167	0.281	0.413	0.545	0.665	0.767	0.843	0.936	0.977	0.992	0.999
	125.95	125.87	125.59	125.08	124.33	123.32	122.24	119.77	117.28	114.84	110.56
130	0.167	0.281	0.414	0.548	0.667	0.767	0.846	0.937	0.977	0.993	0.999
	126.95	126.85	126.59	126.04	125.31	124.33	123.13	120.67	118.12	115.68	111.35
131	0.167	0.282	0.414	0.549	0.671	0.769	0.846	0.938	0.978	0.993	0.999
	127.92	127.86	127.58	127.03	126.23	125.29	124.10	121.60	119.05	116.57	112.21
132	0.167	0.283	0.416	0.551	0.671	0.771	0.849	0.939	0.979	0.993	0.999
	128.91	128.81	128.53	128.00	127.24	126.25	125.04	122.55	119.89	117.44	113.01
133	0.167	0.283	0.417	0.553	0.672	0.773	0.849	0.940	0.979	0.994	0.999
	129.91	129.82	129.55	128.97	128.21	127.19	125.99	123.44	120.78	118.29	113.87
134	0.167	0.283	0.419	0.554	0.674	0.776	0.851	0.942	0.980	0.994	0.999
	130.90	130.83	130.50	129.96	129.17	128.12	126.97	124.33	121.67	119.19	114.69
135	0.167	0.284	0.419	0.555	0.677	0.777	0.852	0.943	0.981	0.994	1.000
	131.89	131.79	131.52	130.95	130.13	129.06	127.91	125.24	122.54	120.01	115.57
136	0.167	0.284	0.420	0.556	0.680	0.780	0.855	0.944	0.981	0.994	0.999
	132.87	132.79	132.49	131.93	131.09	130.01	128.84	126.18	123.48	120.94	116.42
137	0.167	0.285	0.422	0.558	0.681	0.780	0.857	0.945	0.981	0.994	1.000
	133.85	133.76	133.46	132.90	132.06	130.99	129.74	127.09	124.36	121.84	117.24
138	0.167	0.285	0.423	0.559	0.684	0.784	0.858	0.947	0.982	0.995	1.000
	134.85	134.77	134.45	133.89	133.02	131.94	130.70	127.96	125.27	122.65	118.09
139	0.167	0.286	0.424	0.561	0.684	0.784	0.860	0.946	0.982	0.995	1.000
	135.83	135.76	135.42	134.85	134.00	132.92	131.62	128.90	126.15	123.52	118.98
140	0.167	0.286	0.424	0.564	0.687	0.786	0.861	0.949	0.983	0.995	1.000
	136.85	136.74	136.43	135.81	134.94	133.89	132.56	129.80	127.03	124.42	119.77
141	0.167	0.287	0.425	0.565	0.688	0.788	0.863	0.948	0.983	0.995	1.000
	137.83	137.72	137.43	136.81	135.93	134.80	133.52	130.78	127.93	125.26	120.59
142	0.167	0.287	0.426	0.566	0.691	0.790	0.865	0.950	0.983	0.995	1.000
	138.80	138.75	138.40	137.78	136.88	135.78	134.48	131.68	128.85	126.20	121.47
143	0.167	0.288	0.427	0.567	0.692	0.792	0.867	0.951	0.984	0.995	1.000
	139.81	139.69	139.40	138.76	137.87	136.72	135.38	132.54	129.73	127.04	122.29
144	0.167	0.288	0.429	0.569	0.693	0.793	0.868	0.952	0.984	0.995	1.000
	140.79	140.69	140.34	139.73	138.83	137.68	136.35	133.47	130.65	127.88	123.09

Table 37: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
145	0.167	0.289	0.430	0.570	0.695	0.795	0.869	0.953	0.985	0.996	1.000
	141.78	141.68	141.33	140.72	139.79	138.63	137.31	134.36	131.51	128.74	123.96
146	0.167	0.289	0.430	0.572	0.697	0.797	0.870	0.952	0.985	0.996	1.000
	142.77	142.66	142.33	141.70	140.76	139.59	138.22	135.33	132.39	129.64	124.83
147	0.167	0.290	0.432	0.573	0.700	0.798	0.872	0.955	0.986	0.996	1.000
	143.75	143.64	143.31	142.68	141.71	140.53	139.19	136.18	133.26	130.53	125.67
148	0.167	0.290	0.433	0.574	0.699	0.801	0.873	0.955	0.986	0.996	1.000
	144.76	144.67	144.29	143.66	142.71	141.46	140.12	137.11	134.18	131.39	126.46
149	0.167	0.291	0.434	0.577	0.700	0.803	0.874	0.956	0.986	0.996	1.000
	145.74	145.63	145.29	144.61	143.70	142.43	141.07	138.07	135.08	132.27	127.29
150	0.167	0.291	0.435	0.578	0.704	0.804	0.875	0.957	0.987	0.996	1.000
	146.72	146.63	146.29	145.60	144.61	143.39	142.01	138.95	135.93	133.14	128.17
155	0.167	0.293	0.440	0.585	0.711	0.811	0.883	0.960	0.988	0.997	1.000
	151.70	151.58	151.20	150.50	149.46	148.18	146.66	143.52	140.40	137.50	132.37
160	0.167	0.295	0.444	0.592	0.720	0.820	0.889	0.964	0.990	0.997	1.000
	156.62	156.53	156.13	155.38	154.30	152.88	151.41	148.11	144.86	141.90	136.57
165	0.167	0.298	0.450	0.601	0.728	0.826	0.896	0.967	0.991	0.998	1.000
	161.57	161.45	161.05	160.23	159.11	157.68	156.07	152.63	149.32	146.26	140.72
170	0.167	0.300	0.456	0.607	0.734	0.834	0.902	0.970	0.992	0.998	1.000
	166.50	166.40	165.94	165.13	163.97	162.40	160.77	157.22	153.73	150.60	144.97
175	0.167	0.302	0.459	0.612	0.741	0.840	0.906	0.973	0.993	0.998	1.000
	171.47	171.36	170.90	170.01	168.80	167.21	165.49	161.79	158.19	154.95	149.11
180	0.167	0.304	0.465	0.618	0.748	0.848	0.913	0.974	0.994	0.999	1.000
	176.43	176.31	175.80	174.92	173.63	171.96	170.13	166.38	162.65	159.30	153.34
185	0.167	0.305	0.469	0.627	0.756	0.852	0.916	0.977	0.995	0.999	1.000
	181.36	181.28	180.72	179.77	178.42	176.73	174.82	170.86	167.10	163.64	157.51
190	0.167	0.309	0.475	0.633	0.764	0.859	0.920	0.979	0.995	0.999	1.000
	186.32	186.18	185.65	184.66	183.24	181.43	179.56	175.44	171.52	168.00	161.70
195	0.167	0.310	0.479	0.638	0.769	0.864	0.925	0.981	0.996	0.999	1.000
	191.27	191.16	190.57	189.54	188.11	186.22	184.23	180.00	176.00	172.30	165.85
200	0.167	0.313	0.482	0.644	0.774	0.869	0.929	0.983	0.997	0.999	1.000
	196.22	196.08	195.53	194.43	192.92	190.98	188.88	184.59	180.45	176.65	170.12
210	0.167	0.317	0.492	0.656	0.789	0.880	0.937	0.985	0.997	1.000	1.000
	206.14	205.99	205.35	204.20	202.48	200.44	198.21	193.68	189.30	185.37	178.41
220	0.167	0.321	0.501	0.669	0.800	0.889	0.943	0.988	0.998	1.000	1.000
	216.03	215.90	215.17	213.92	212.11	209.98	207.57	202.72	198.24	194.05	186.81
230	0.167	0.325	0.509	0.679	0.810	0.898	0.949	0.990	0.998	1.000	1.000
	225.98	225.79	225.03	223.68	221.76	219.40	216.89	211.83	207.05	202.74	195.16
240	0.167	0.329	0.517	0.690	0.823	0.907	0.954	0.992	0.999	1.000	1.000
	235.88	235.68	234.91	233.44	231.30	228.85	226.30	220.96	215.87	211.38	203.58
250	0.167	0.333	0.525	0.700	0.830	0.913	0.960	0.993	0.999	1.000	1.000
	245.83	245.59	244.74	243.17	240.97	238.33	235.51	230.01	224.81	220.15	211.96

Table 38: Table of Monte Carlo Results for $\mathcal{M}_{\text{BK}}$ when $k = 7$ and $\alpha = 0.05$.

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
1	0.143	0.155	0.167	0.178	0.189	0.200	0.211	0.231	0.250	0.268	0.302
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	0.143	0.155	0.167	0.178	0.189	0.200	0.211	0.231	0.250	0.268	0.302
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
3	0.143	0.157	0.171	0.184	0.198	0.211	0.224	0.249	0.274	0.296	0.342
	2.86	2.86	2.86	2.86	2.85	2.85	2.85	2.85	2.84	2.84	2.83
4	0.143	0.159	0.176	0.192	0.209	0.225	0.241	0.273	0.303	0.331	0.387
	3.47	3.47	3.46	3.46	3.47	3.45	3.45	3.44	3.43	3.42	3.39
5	0.143	0.162	0.181	0.199	0.218	0.237	0.256	0.292	0.327	0.360	0.423
	4.38	4.38	4.38	4.38	4.38	4.38	4.37	4.36	4.35	4.35	4.32
6	0.143	0.163	0.184	0.205	0.226	0.246	0.266	0.306	0.345	0.382	0.451
	5.25	5.24	5.25	5.25	5.23	5.23	5.23	5.22	5.20	5.18	5.14
7	0.143	0.165	0.187	0.209	0.231	0.253	0.275	0.319	0.361	0.400	0.474
	6.25	6.25	6.25	6.25	6.25	6.24	6.24	6.22	6.19	6.18	6.14
8	0.143	0.166	0.189	0.213	0.237	0.262	0.285	0.332	0.378	0.419	0.500
	7.26	7.25	7.26	7.26	7.24	7.23	7.23	7.20	7.17	7.15	7.08
9	0.143	0.167	0.192	0.218	0.244	0.269	0.296	0.345	0.394	0.442	0.525
	8.23	8.23	8.22	8.21	8.21	8.20	8.19	8.17	8.13	8.08	8.01
10	0.143	0.169	0.195	0.223	0.250	0.277	0.305	0.359	0.410	0.460	0.548
	9.16	9.16	9.16	9.15	9.15	9.13	9.12	9.09	9.05	9.00	8.90
11	0.143	0.170	0.198	0.227	0.256	0.285	0.314	0.371	0.425	0.476	0.569
	10.10	10.10	10.10	10.09	10.08	10.07	10.06	10.02	9.98	9.93	9.82
12	0.143	0.171	0.201	0.231	0.262	0.292	0.323	0.382	0.438	0.493	0.590
	11.07	11.07	11.06	11.06	11.04	11.03	11.01	10.97	10.93	10.87	10.73
13	0.143	0.172	0.203	0.234	0.266	0.298	0.331	0.393	0.453	0.508	0.607
	12.05	12.05	12.04	12.03	12.02	12.00	11.98	11.93	11.88	11.81	11.66
14	0.143	0.173	0.205	0.238	0.272	0.306	0.338	0.404	0.466	0.524	0.627
	13.03	13.03	13.02	13.00	12.99	12.97	12.96	12.90	12.82	12.74	12.57
15	0.143	0.175	0.208	0.242	0.276	0.312	0.345	0.415	0.479	0.539	0.643
	14.00	13.99	13.99	13.98	13.97	13.93	13.91	13.85	13.77	13.68	13.48
16	0.143	0.176	0.210	0.245	0.282	0.318	0.354	0.424	0.492	0.554	0.661
	14.95	14.96	14.95	14.94	14.92	14.89	14.87	14.80	14.71	14.62	14.38
17	0.143	0.176	0.212	0.248	0.286	0.325	0.363	0.435	0.503	0.566	0.675
	15.92	15.92	15.91	15.91	15.88	15.85	15.82	15.75	15.66	15.55	15.30
18	0.143	0.178	0.214	0.252	0.291	0.330	0.369	0.445	0.515	0.581	0.690
	16.90	16.89	16.88	16.87	16.84	16.82	16.79	16.70	16.61	16.48	16.21
19	0.143	0.179	0.216	0.255	0.295	0.336	0.375	0.454	0.527	0.592	0.703
	17.87	17.86	17.86	17.84	17.82	17.79	17.76	17.66	17.54	17.42	17.13
20	0.143	0.179	0.218	0.259	0.300	0.342	0.383	0.462	0.538	0.604	0.719
	18.85	18.85	18.84	18.82	18.79	18.76	18.72	18.62	18.49	18.35	18.01
21	0.143	0.180	0.220	0.262	0.304	0.348	0.391	0.472	0.546	0.615	0.730
	19.83	19.82	19.81	19.79	19.77	19.72	19.67	19.58	19.45	19.29	18.94
22	0.143	0.181	0.222	0.265	0.309	0.351	0.397	0.481	0.559	0.627	0.742
	20.80	20.79	20.78	20.77	20.73	20.71	20.64	20.53	20.38	20.22	19.83
23	0.143	0.182	0.224	0.269	0.313	0.358	0.404	0.490	0.569	0.639	0.752
	21.76	21.77	21.75	21.72	21.70	21.67	21.60	21.48	21.32	21.14	20.75
24	0.143	0.183	0.226	0.271	0.317	0.364	0.411	0.498	0.578	0.652	0.766
	22.75	22.74	22.73	22.70	22.67	22.63	22.56	22.43	22.27	22.06	21.63

Table 38: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
25	0.143	0.184	0.228	0.274	0.321	0.369	0.417	0.505	0.589	0.660	0.776
	23.72	23.73	23.70	23.68	23.64	23.59	23.53	23.40	23.20	23.01	22.54
26	0.143	0.185	0.230	0.277	0.326	0.375	0.423	0.514	0.597	0.671	0.785
	24.70	24.69	24.67	24.65	24.61	24.57	24.50	24.35	24.16	23.93	23.44
27	0.143	0.185	0.231	0.280	0.329	0.380	0.427	0.522	0.606	0.681	0.795
	25.69	25.68	25.66	25.63	25.60	25.53	25.48	25.30	25.10	24.85	24.34
28	0.143	0.186	0.233	0.282	0.333	0.384	0.434	0.529	0.615	0.691	0.806
	26.67	26.66	26.64	26.60	26.57	26.50	26.44	26.26	26.05	25.78	25.21
29	0.143	0.187	0.235	0.286	0.338	0.388	0.441	0.538	0.624	0.698	0.813
	27.64	27.63	27.61	27.57	27.53	27.48	27.40	27.21	26.98	26.72	26.12
30	0.143	0.188	0.237	0.289	0.341	0.394	0.448	0.546	0.634	0.708	0.819
	28.61	28.61	28.59	28.55	28.51	28.45	28.35	28.16	27.91	27.64	27.03
31	0.143	0.189	0.239	0.291	0.345	0.397	0.451	0.551	0.641	0.716	0.830
	29.58	29.58	29.56	29.53	29.48	29.43	29.34	29.12	28.86	28.57	27.90
32	0.143	0.189	0.240	0.294	0.349	0.402	0.457	0.559	0.648	0.727	0.837
	30.57	30.56	30.55	30.50	30.45	30.39	30.30	30.08	29.81	29.48	28.79
33	0.143	0.190	0.242	0.296	0.352	0.408	0.463	0.566	0.656	0.734	0.844
	31.55	31.55	31.53	31.48	31.43	31.36	31.26	31.03	30.74	30.40	29.69
34	0.143	0.191	0.243	0.299	0.355	0.413	0.470	0.573	0.665	0.742	0.852
	32.54	32.53	32.51	32.46	32.40	32.32	32.21	31.98	31.68	31.32	30.57
35	0.143	0.192	0.245	0.302	0.359	0.418	0.474	0.579	0.671	0.748	0.859
	33.51	33.51	33.48	33.44	33.38	33.29	33.20	32.94	32.61	32.26	31.45
36	0.143	0.192	0.247	0.304	0.363	0.423	0.481	0.588	0.681	0.756	0.864
	34.49	34.50	34.46	34.41	34.35	34.26	34.14	33.87	33.54	33.18	32.35
37	0.143	0.193	0.248	0.306	0.367	0.426	0.484	0.594	0.685	0.763	0.870
	35.47	35.47	35.44	35.41	35.32	35.23	35.13	34.84	34.50	34.09	33.25
38	0.143	0.194	0.250	0.309	0.370	0.431	0.490	0.600	0.693	0.770	0.876
	36.45	36.44	36.42	36.37	36.29	36.20	36.09	35.79	35.43	35.00	34.13
39	0.143	0.194	0.251	0.312	0.374	0.435	0.497	0.604	0.701	0.778	0.881
	37.44	37.44	37.40	37.34	37.27	37.19	37.04	36.76	36.36	35.93	35.01
40	0.143	0.195	0.253	0.314	0.377	0.440	0.500	0.612	0.707	0.783	0.887
	38.43	38.40	38.39	38.33	38.25	38.14	38.03	37.69	37.30	36.86	35.89
41	0.143	0.196	0.254	0.317	0.381	0.444	0.505	0.617	0.713	0.791	0.891
	39.40	39.39	39.37	39.30	39.22	39.11	38.99	38.66	38.25	37.75	36.78
42	0.143	0.196	0.256	0.319	0.383	0.447	0.510	0.623	0.720	0.795	0.897
	40.39	40.38	40.34	40.28	40.20	40.10	39.94	39.60	39.17	38.69	37.65
43	0.143	0.197	0.257	0.322	0.388	0.454	0.515	0.630	0.728	0.802	0.901
	41.36	41.36	41.33	41.25	41.17	41.04	40.92	40.55	40.09	39.62	38.54
44	0.143	0.198	0.259	0.324	0.390	0.456	0.520	0.635	0.731	0.807	0.906
	42.36	42.34	42.30	42.24	42.14	42.03	41.87	41.50	41.05	40.53	39.42
45	0.143	0.198	0.260	0.326	0.394	0.460	0.524	0.640	0.739	0.813	0.910
	43.34	43.32	43.29	43.22	43.12	43.00	42.85	42.45	41.96	41.43	40.31
46	0.143	0.199	0.262	0.328	0.398	0.464	0.530	0.646	0.744	0.819	0.914
	44.33	44.30	44.27	44.21	44.09	43.97	43.81	43.40	42.90	42.34	41.19
47	0.143	0.200	0.263	0.330	0.400	0.469	0.535	0.654	0.751	0.824	0.918
	45.31	45.28	45.25	45.20	45.08	44.94	44.77	44.34	43.82	43.26	42.05
48	0.143	0.200	0.265	0.334	0.404	0.472	0.540	0.658	0.755	0.829	0.922
	46.29	46.26	46.23	46.15	46.04	45.91	45.73	45.30	44.77	44.16	42.95

Table 38: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
49	0.143	0.201	0.266	0.335	0.406	0.476	0.543	0.663	0.763	0.834	0.925
	47.26	47.25	47.22	47.14	47.03	46.88	46.70	46.25	45.68	45.09	43.84
50	0.143	0.201	0.268	0.337	0.411	0.479	0.549	0.670	0.766	0.839	0.929
	48.25	48.25	48.19	48.13	47.99	47.86	47.65	47.18	46.62	46.00	44.67
51	0.143	0.202	0.269	0.340	0.413	0.483	0.553	0.673	0.770	0.842	0.932
	49.23	49.23	49.17	49.10	48.98	48.83	48.62	48.16	47.56	46.93	45.56
52	0.143	0.203	0.271	0.342	0.415	0.489	0.555	0.679	0.776	0.849	0.935
	50.21	50.20	50.16	50.07	49.95	49.78	49.61	49.08	48.49	47.79	46.45
53	0.143	0.203	0.271	0.345	0.419	0.492	0.562	0.684	0.782	0.852	0.937
	51.22	51.19	51.15	51.05	50.93	50.76	50.55	50.05	49.41	48.75	47.31
54	0.143	0.204	0.273	0.347	0.422	0.496	0.565	0.690	0.784	0.856	0.941
	52.18	52.17	52.13	52.03	51.91	51.72	51.53	50.97	50.36	49.66	48.18
55	0.143	0.204	0.274	0.348	0.424	0.499	0.571	0.695	0.791	0.861	0.942
	53.18	53.17	53.10	53.02	52.89	52.70	52.48	51.92	51.27	50.55	49.08
56	0.143	0.205	0.276	0.351	0.427	0.503	0.574	0.698	0.793	0.864	0.946
	54.16	54.14	54.08	54.00	53.86	53.67	53.44	52.89	52.22	51.48	49.94
57	0.143	0.206	0.277	0.353	0.431	0.507	0.580	0.703	0.798	0.869	0.948
	55.14	55.12	55.08	54.99	54.83	54.64	54.40	53.83	53.14	52.39	50.79
58	0.143	0.206	0.278	0.356	0.433	0.509	0.584	0.707	0.805	0.873	0.950
	56.12	56.11	56.06	55.95	55.82	55.63	55.36	54.78	54.04	53.28	51.70
59	0.143	0.207	0.280	0.357	0.437	0.514	0.586	0.710	0.807	0.877	0.953
	57.11	57.09	57.04	56.95	56.78	56.58	56.34	55.75	55.00	54.19	52.56
60	0.143	0.207	0.281	0.359	0.439	0.516	0.589	0.716	0.813	0.879	0.956
	58.09	58.10	58.02	57.92	57.77	57.57	57.31	56.67	55.91	55.11	53.39
61	0.143	0.208	0.282	0.361	0.443	0.521	0.594	0.722	0.817	0.885	0.956
	59.08	59.08	59.01	58.90	58.73	58.52	58.27	57.60	56.82	55.98	54.32
62	0.143	0.209	0.284	0.363	0.445	0.525	0.600	0.724	0.820	0.887	0.959
	60.06	60.04	60.00	59.89	59.72	59.48	59.22	58.58	57.78	56.91	55.14
63	0.143	0.209	0.285	0.366	0.448	0.529	0.603	0.731	0.825	0.890	0.961
	61.05	61.05	60.98	60.86	60.68	60.45	60.18	59.48	58.69	57.84	56.01
64	0.143	0.210	0.286	0.368	0.451	0.531	0.607	0.734	0.827	0.894	0.962
	62.05	62.02	61.96	61.84	61.67	61.44	61.15	60.44	59.64	58.71	56.91
65	0.143	0.210	0.288	0.370	0.455	0.534	0.610	0.737	0.832	0.897	0.964
	63.03	63.02	62.94	62.82	62.63	62.42	62.12	61.41	60.55	59.62	57.78
66	0.143	0.211	0.289	0.372	0.456	0.540	0.614	0.744	0.836	0.900	0.966
	64.01	63.99	63.92	63.81	63.63	63.36	63.07	62.32	61.45	60.52	58.64
67	0.143	0.211	0.291	0.374	0.459	0.540	0.616	0.747	0.840	0.902	0.968
	65.00	64.98	64.90	64.78	64.61	64.37	64.05	63.26	62.37	61.42	59.47
68	0.143	0.212	0.291	0.377	0.462	0.543	0.621	0.749	0.841	0.905	0.969
	65.98	65.96	65.90	65.75	65.57	65.34	64.99	64.23	63.33	62.36	60.36
69	0.143	0.212	0.293	0.378	0.463	0.547	0.626	0.752	0.846	0.908	0.971
	66.96	66.96	66.88	66.75	66.57	66.29	65.96	65.18	64.22	63.26	61.22
70	0.143	0.213	0.293	0.380	0.468	0.553	0.629	0.757	0.849	0.910	0.971
	67.96	67.93	67.87	67.72	67.52	67.24	66.92	66.10	65.16	64.15	62.12
71	0.143	0.214	0.294	0.383	0.471	0.554	0.632	0.762	0.854	0.913	0.973
	68.94	68.92	68.86	68.69	68.50	68.23	67.89	67.05	66.09	65.04	62.95
72	0.143	0.214	0.297	0.385	0.472	0.557	0.636	0.764	0.857	0.916	0.974
	69.92	69.90	69.83	69.68	69.48	69.20	68.85	68.01	66.98	65.94	63.83

Table 38: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
73	0.143	0.215	0.298	0.386	0.474	0.561	0.640	0.769	0.861	0.919	0.975
	70.91	70.88	70.81	70.67	70.46	70.17	69.81	68.93	67.92	66.83	64.67
74	0.143	0.215	0.299	0.388	0.477	0.565	0.642	0.771	0.863	0.921	0.977
	71.90	71.88	71.80	71.65	71.43	71.11	70.77	69.89	68.84	67.74	65.59
75	0.143	0.216	0.300	0.390	0.480	0.568	0.645	0.777	0.866	0.923	0.977
	72.90	72.87	72.79	72.64	72.40	72.09	71.75	70.80	69.77	68.64	66.46
76	0.143	0.216	0.301	0.391	0.482	0.570	0.650	0.778	0.867	0.925	0.978
	73.88	73.85	73.77	73.63	73.39	73.08	72.70	71.77	70.73	69.58	67.28
77	0.143	0.217	0.302	0.394	0.486	0.573	0.652	0.781	0.869	0.927	0.979
	74.86	74.82	74.76	74.59	74.36	74.05	73.67	72.72	71.63	70.45	68.24
78	0.143	0.218	0.304	0.395	0.489	0.576	0.656	0.786	0.874	0.930	0.980
	75.85	75.81	75.73	75.58	75.32	75.01	74.62	73.65	72.54	71.36	69.05
79	0.143	0.218	0.304	0.397	0.490	0.578	0.660	0.788	0.879	0.931	0.981
	76.83	76.81	76.75	76.57	76.32	76.00	75.56	74.62	73.42	72.28	69.91
80	0.143	0.218	0.306	0.399	0.493	0.581	0.662	0.792	0.880	0.934	0.982
	77.83	77.80	77.70	77.54	77.29	76.96	76.55	75.55	74.37	73.16	70.77
81	0.143	0.219	0.307	0.402	0.495	0.584	0.667	0.795	0.882	0.936	0.983
	78.81	78.80	78.69	78.51	78.28	77.93	77.48	76.46	75.30	74.06	71.62
82	0.143	0.219	0.308	0.403	0.498	0.588	0.667	0.798	0.885	0.938	0.983
	79.79	79.76	79.68	79.52	79.24	78.88	78.48	77.43	76.21	74.95	72.50
83	0.143	0.220	0.310	0.406	0.501	0.591	0.673	0.802	0.889	0.939	0.985
	80.78	80.76	80.67	80.47	80.21	79.85	79.41	78.34	77.13	75.84	73.33
84	0.143	0.221	0.310	0.407	0.504	0.594	0.675	0.804	0.890	0.941	0.985
	81.77	81.74	81.65	81.47	81.18	80.83	80.38	79.30	78.05	76.75	74.20
85	0.143	0.221	0.312	0.408	0.505	0.595	0.678	0.807	0.893	0.943	0.986
	82.75	82.74	82.63	82.46	82.17	81.82	81.35	80.25	78.93	77.65	75.09
86	0.143	0.221	0.313	0.411	0.508	0.599	0.682	0.811	0.897	0.945	0.986
	83.74	83.74	83.63	83.42	83.15	82.77	82.29	81.16	79.87	78.52	75.93
87	0.143	0.222	0.314	0.411	0.510	0.603	0.685	0.814	0.896	0.946	0.987
	84.73	84.69	84.62	84.42	84.12	83.73	83.26	82.10	80.81	79.44	76.80
88	0.143	0.223	0.314	0.413	0.512	0.604	0.687	0.816	0.899	0.948	0.987
	85.72	85.68	85.61	85.41	85.11	84.72	84.22	83.07	81.70	80.32	77.68
89	0.143	0.223	0.316	0.416	0.515	0.609	0.690	0.818	0.902	0.949	0.988
	86.71	86.68	86.58	86.38	86.08	85.66	85.19	84.00	82.64	81.25	78.53
90	0.143	0.223	0.317	0.418	0.517	0.611	0.694	0.822	0.903	0.951	0.989
	87.69	87.67	87.57	87.36	87.05	86.63	86.12	84.91	83.55	82.11	79.42
91	0.143	0.224	0.317	0.421	0.519	0.615	0.697	0.824	0.906	0.953	0.989
	88.69	88.66	88.57	88.32	88.04	87.59	87.08	85.89	84.50	83.02	80.25
92	0.143	0.225	0.320	0.422	0.523	0.616	0.700	0.826	0.907	0.954	0.990
	89.67	89.65	89.53	89.31	88.98	88.57	88.06	86.81	85.39	83.93	81.12
93	0.143	0.225	0.321	0.423	0.524	0.617	0.701	0.832	0.909	0.955	0.990
	90.68	90.66	90.52	90.30	89.99	89.56	89.03	87.71	86.32	84.82	81.97
94	0.143	0.225	0.322	0.425	0.528	0.621	0.705	0.834	0.911	0.957	0.991
	91.66	91.61	91.51	91.28	90.94	90.51	89.99	88.65	87.24	85.70	82.85
95	0.143	0.226	0.322	0.426	0.529	0.625	0.707	0.836	0.914	0.958	0.991
	92.65	92.60	92.50	92.26	91.92	91.46	90.93	89.60	88.12	86.60	83.69
96	0.143	0.226	0.324	0.428	0.531	0.626	0.711	0.840	0.915	0.959	0.991
	93.63	93.60	93.48	93.26	92.90	92.44	91.88	90.52	89.06	87.49	84.60

Table 38: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
97	0.143	0.227	0.326	0.429	0.533	0.630	0.713	0.840	0.916	0.959	0.991
	94.60	94.57	94.46	94.26	93.89	93.40	92.85	91.47	90.00	88.42	85.46
98	0.143	0.227	0.327	0.433	0.535	0.631	0.717	0.842	0.919	0.961	0.992
	95.60	95.57	95.43	95.20	94.87	94.40	93.79	92.42	90.91	89.28	86.27
99	0.143	0.228	0.328	0.434	0.538	0.636	0.719	0.845	0.921	0.962	0.993
	96.59	96.56	96.42	96.20	95.82	95.34	94.77	93.35	91.82	90.18	87.14
100	0.143	0.228	0.329	0.436	0.540	0.637	0.721	0.847	0.924	0.963	0.993
	97.59	97.57	97.42	97.17	96.82	96.33	95.71	94.30	92.67	91.06	88.01
101	0.143	0.229	0.330	0.437	0.544	0.640	0.724	0.849	0.924	0.965	0.993
	98.55	98.53	98.40	98.15	97.77	97.28	96.70	95.23	93.62	91.98	88.87
102	0.143	0.229	0.330	0.439	0.546	0.642	0.727	0.852	0.928	0.965	0.993
	99.55	99.53	99.40	99.12	98.75	98.26	97.64	96.18	94.51	92.90	89.79
103	0.143	0.230	0.332	0.440	0.548	0.646	0.730	0.854	0.928	0.967	0.994
	100.55	100.51	100.38	100.13	99.72	99.20	98.59	97.10	95.44	93.75	90.61
104	0.143	0.230	0.333	0.441	0.551	0.649	0.732	0.856	0.929	0.969	0.994
	101.53	101.48	101.35	101.11	100.69	100.17	99.53	98.05	96.37	94.64	91.47
105	0.143	0.231	0.334	0.444	0.551	0.650	0.733	0.859	0.931	0.968	0.995
	102.54	102.48	102.35	102.08	101.69	101.15	100.52	98.99	97.28	95.57	92.34
106	0.143	0.231	0.335	0.446	0.554	0.651	0.737	0.860	0.932	0.969	0.995
	103.53	103.47	103.34	103.07	102.65	102.14	101.47	99.94	98.20	96.43	93.18
107	0.143	0.231	0.335	0.447	0.555	0.655	0.739	0.863	0.934	0.971	0.995
	104.50	104.49	104.35	104.06	103.65	103.08	102.44	100.84	99.07	97.34	94.04
108	0.143	0.232	0.338	0.448	0.558	0.657	0.743	0.865	0.936	0.971	0.995
	105.50	105.45	105.30	105.05	104.61	104.06	103.35	101.79	99.98	98.24	94.90
109	0.143	0.232	0.338	0.451	0.562	0.659	0.744	0.868	0.937	0.972	0.996
	106.47	106.45	106.30	106.01	105.56	105.02	104.32	102.70	100.91	99.14	95.77
110	0.143	0.233	0.340	0.452	0.562	0.661	0.747	0.869	0.938	0.972	0.995
	107.48	107.43	107.27	107.00	106.55	106.00	105.29	103.67	101.85	100.02	96.59
111	0.143	0.233	0.340	0.453	0.566	0.664	0.749	0.872	0.939	0.974	0.996
	108.46	108.43	108.28	107.99	107.51	106.96	106.25	104.56	102.78	100.90	97.51
112	0.143	0.234	0.341	0.456	0.568	0.667	0.750	0.875	0.942	0.974	0.996
	109.47	109.42	109.26	108.96	108.49	107.92	107.23	105.50	103.66	101.84	98.39
113	0.143	0.234	0.342	0.458	0.569	0.668	0.753	0.876	0.942	0.976	0.996
	110.46	110.41	110.24	109.93	109.48	108.90	108.17	106.43	104.60	102.64	99.21
114	0.143	0.235	0.343	0.458	0.570	0.670	0.756	0.878	0.944	0.976	0.996
	111.42	111.38	111.23	110.93	110.47	109.86	109.10	107.37	105.46	103.60	100.05
115	0.143	0.235	0.344	0.460	0.572	0.673	0.760	0.879	0.944	0.977	0.996
	112.44	112.36	112.21	111.91	111.46	110.83	110.06	108.30	106.45	104.49	100.93
116	0.143	0.235	0.345	0.461	0.575	0.675	0.760	0.882	0.947	0.978	0.997
	113.41	113.36	113.20	112.90	112.41	111.78	111.03	109.23	107.26	105.35	101.80
117	0.143	0.236	0.346	0.463	0.577	0.677	0.763	0.883	0.947	0.978	0.997
	114.40	114.36	114.19	113.87	113.38	112.76	111.98	110.17	108.23	106.27	102.66
118	0.143	0.236	0.347	0.464	0.577	0.680	0.766	0.886	0.948	0.979	0.997
	115.39	115.34	115.18	114.87	114.39	113.74	112.94	111.07	109.17	107.16	103.52
119	0.143	0.237	0.348	0.466	0.580	0.683	0.768	0.886	0.949	0.979	0.997
	116.38	116.33	116.18	115.84	115.35	114.68	113.90	112.05	110.07	108.02	104.36
120	0.143	0.237	0.350	0.468	0.584	0.684	0.769	0.888	0.950	0.980	0.997
	117.38	117.32	117.15	116.83	116.30	115.66	114.85	112.98	111.00	108.92	105.22

Table 38: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
121	0.143	0.237	0.350	0.471	0.583	0.688	0.771	0.890	0.952	0.980	0.997
	118.37	118.32	118.14	117.78	117.30	116.60	115.82	113.87	111.85	109.87	106.12
122	0.143	0.238	0.352	0.471	0.588	0.690	0.774	0.892	0.953	0.981	0.998
	119.34	119.30	119.11	118.78	118.24	117.57	116.78	114.83	112.76	110.71	106.90
123	0.143	0.238	0.352	0.472	0.588	0.691	0.776	0.893	0.954	0.981	0.998
	120.35	120.31	120.13	119.76	119.25	118.54	117.71	115.79	113.66	111.59	107.77
124	0.143	0.239	0.353	0.475	0.592	0.695	0.777	0.896	0.957	0.983	0.998
	121.34	121.27	121.12	120.74	120.20	119.48	118.68	116.69	114.53	112.50	108.68
125	0.143	0.239	0.355	0.477	0.594	0.694	0.780	0.896	0.955	0.983	0.998
	122.32	122.29	122.08	121.71	121.15	120.50	119.62	117.63	115.52	113.40	109.52
126	0.143	0.240	0.356	0.477	0.594	0.699	0.783	0.900	0.958	0.983	0.998
	123.31	123.27	123.06	122.72	122.17	121.43	120.57	118.53	116.39	114.30	110.38
127	0.143	0.240	0.357	0.480	0.595	0.699	0.784	0.899	0.958	0.984	0.998
	124.30	124.25	124.02	123.68	123.15	122.42	121.53	119.48	117.34	115.12	111.23
128	0.143	0.241	0.357	0.480	0.599	0.703	0.786	0.901	0.958	0.984	0.998
	125.30	125.22	125.05	124.68	124.11	123.34	122.50	120.45	118.26	116.06	112.06
129	0.143	0.241	0.359	0.482	0.601	0.703	0.789	0.903	0.959	0.985	0.998
	126.29	126.22	126.02	125.66	125.06	124.36	123.43	121.36	119.14	116.92	112.96
130	0.143	0.242	0.359	0.484	0.601	0.707	0.791	0.904	0.961	0.985	0.998
	127.26	127.22	127.02	126.63	126.07	125.28	124.39	122.31	120.05	117.83	113.85
131	0.143	0.242	0.360	0.485	0.604	0.709	0.793	0.906	0.962	0.985	0.998
	128.27	128.19	128.02	127.62	127.02	126.24	125.34	123.20	120.98	118.74	114.62
132	0.143	0.243	0.362	0.487	0.606	0.709	0.796	0.906	0.962	0.986	0.998
	129.24	129.20	128.98	128.59	128.01	127.24	126.31	124.17	121.89	119.66	115.53
133	0.143	0.243	0.362	0.490	0.606	0.712	0.797	0.908	0.963	0.986	0.998
	130.24	130.20	129.98	129.56	129.00	128.18	127.24	125.11	122.78	120.52	116.39
134	0.143	0.243	0.363	0.490	0.609	0.715	0.800	0.910	0.964	0.987	0.999
	131.24	131.19	130.97	130.55	129.97	129.14	128.18	126.02	123.71	121.40	117.20
135	0.143	0.243	0.364	0.492	0.612	0.716	0.799	0.910	0.965	0.987	0.999
	132.23	132.18	131.95	131.53	130.93	130.13	129.21	126.96	124.61	122.29	118.09
136	0.143	0.244	0.365	0.493	0.613	0.718	0.803	0.915	0.965	0.988	0.999
	133.21	133.15	132.94	132.53	131.92	131.08	130.10	127.83	125.49	123.13	118.96
137	0.143	0.245	0.366	0.496	0.615	0.720	0.803	0.914	0.966	0.988	0.999
	134.19	134.14	133.92	133.47	132.89	132.05	131.10	128.82	126.40	124.06	119.88
138	0.143	0.245	0.368	0.496	0.617	0.721	0.807	0.915	0.967	0.988	0.999
	135.18	135.12	134.90	134.50	133.85	133.03	131.98	129.73	127.29	124.95	120.69
139	0.143	0.245	0.368	0.497	0.618	0.723	0.807	0.917	0.968	0.989	0.999
	136.18	136.14	135.89	135.45	134.84	134.02	133.01	130.62	128.22	125.84	121.53
140	0.143	0.246	0.369	0.497	0.620	0.725	0.811	0.919	0.969	0.989	0.999
	137.17	137.11	136.88	136.49	135.81	134.95	133.89	131.57	129.12	126.74	122.40
141	0.143	0.246	0.370	0.499	0.622	0.728	0.811	0.919	0.969	0.989	0.999
	138.17	138.10	137.87	137.45	136.77	135.91	134.87	132.53	130.04	127.59	123.22
142	0.143	0.247	0.371	0.502	0.624	0.730	0.812	0.920	0.970	0.990	0.999
	139.14	139.08	138.88	138.41	137.75	136.86	135.83	133.47	130.91	128.52	124.04
143	0.143	0.247	0.372	0.503	0.626	0.732	0.816	0.921	0.970	0.990	0.999
	140.13	140.07	139.86	139.41	138.71	137.82	136.77	134.41	131.86	129.37	124.94
144	0.143	0.248	0.374	0.504	0.628	0.733	0.817	0.923	0.971	0.990	0.999
	141.15	141.08	140.83	140.38	139.70	138.80	137.73	135.31	132.76	130.32	125.82

Table 38: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
145	0.143	0.248	0.374	0.505	0.630	0.735	0.818	0.924	0.972	0.990	0.999
	142.13	142.07	141.83	141.38	140.66	139.76	138.71	136.23	133.64	131.19	126.68
146	0.143	0.249	0.375	0.508	0.631	0.739	0.821	0.925	0.973	0.991	0.999
	143.11	143.03	142.81	142.35	141.63	140.69	139.62	137.15	134.56	132.05	127.56
147	0.143	0.249	0.376	0.508	0.633	0.739	0.823	0.926	0.973	0.991	0.999
	144.09	144.02	143.81	143.34	142.62	141.69	140.59	138.11	135.48	132.94	128.41
148	0.143	0.249	0.377	0.511	0.635	0.740	0.822	0.928	0.973	0.991	0.999
	145.06	145.03	144.78	144.30	143.59	142.69	141.59	139.01	136.39	133.89	129.24
149	0.143	0.250	0.378	0.512	0.634	0.744	0.824	0.928	0.975	0.991	0.999
	146.06	146.02	145.78	145.27	144.59	143.59	142.53	139.96	137.23	134.74	130.10
150	0.143	0.250	0.378	0.514	0.639	0.744	0.827	0.930	0.975	0.992	0.999
	147.10	147.00	146.76	146.26	145.53	144.57	143.44	140.88	138.22	135.64	131.01
155	0.143	0.252	0.384	0.521	0.647	0.753	0.835	0.935	0.977	0.993	0.999
	152.04	151.95	151.67	151.16	150.40	149.39	148.21	145.49	142.74	140.07	135.24
160	0.143	0.253	0.388	0.527	0.655	0.761	0.844	0.940	0.980	0.994	1.000
	156.97	156.94	156.63	156.10	155.27	154.20	152.95	150.12	147.27	144.49	139.54
165	0.143	0.256	0.392	0.531	0.661	0.771	0.850	0.944	0.982	0.995	1.000
	161.95	161.87	161.59	161.04	160.15	158.97	157.69	154.79	151.76	148.90	143.77
170	0.143	0.258	0.396	0.540	0.671	0.776	0.857	0.949	0.984	0.996	1.000
	166.90	166.81	166.52	165.91	165.00	163.85	162.44	159.44	156.33	153.43	148.09
175	0.143	0.260	0.402	0.546	0.678	0.785	0.865	0.952	0.985	0.996	1.000
	171.86	171.78	171.42	170.83	169.85	168.63	167.16	164.04	160.86	157.83	152.37
180	0.143	0.262	0.405	0.553	0.687	0.791	0.869	0.955	0.987	0.997	1.000
	176.81	176.72	176.40	175.71	174.72	173.45	171.98	168.66	165.36	162.24	156.62
185	0.143	0.264	0.409	0.559	0.694	0.799	0.877	0.959	0.989	0.997	1.000
	181.76	181.67	181.33	180.65	179.55	178.26	176.70	173.26	169.82	166.68	160.88
190	0.143	0.266	0.414	0.566	0.700	0.806	0.882	0.962	0.990	0.997	1.000
	186.72	186.63	186.27	185.52	184.44	183.05	181.43	177.93	174.42	171.14	165.21
195	0.143	0.267	0.418	0.571	0.709	0.814	0.888	0.966	0.991	0.998	1.000
	191.69	191.60	191.21	190.44	189.27	187.83	186.16	182.50	178.87	175.52	169.52
200	0.143	0.269	0.423	0.579	0.713	0.819	0.893	0.968	0.992	0.998	1.000
	196.65	196.55	196.14	195.33	194.17	192.62	190.87	187.15	183.45	179.98	173.76
210	0.143	0.273	0.431	0.589	0.727	0.831	0.901	0.972	0.994	0.999	1.000
	206.58	206.46	206.01	205.16	203.87	202.24	200.40	196.41	192.44	188.78	182.29
220	0.143	0.277	0.439	0.601	0.740	0.843	0.910	0.977	0.995	0.999	1.000
	216.48	216.36	215.88	214.95	213.53	211.80	209.82	205.61	201.49	197.66	190.82
230	0.143	0.279	0.447	0.612	0.753	0.853	0.919	0.979	0.996	0.999	1.000
	226.43	226.29	225.75	224.77	223.23	221.38	219.29	214.82	210.51	206.50	199.44
240	0.143	0.283	0.454	0.623	0.763	0.862	0.927	0.982	0.997	0.999	1.000
	236.35	236.21	235.65	234.54	232.96	230.96	228.77	224.05	219.54	215.31	207.98
250	0.143	0.287	0.461	0.633	0.774	0.871	0.934	0.985	0.997	1.000	1.000
	246.27	246.11	245.55	244.35	242.63	240.54	238.16	233.22	228.50	224.20	216.42

Table 39: Table of Monte Carlo Results for $\mathcal{M}_{\text{BK}}$ when $k = 8$ and $\alpha = 0.05$.

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
1	0.125	0.136	0.146	0.157	0.167	0.176	0.186	0.205	0.222	0.239	0.271
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	0.125	0.136	0.146	0.157	0.167	0.176	0.186	0.205	0.222	0.239	0.271
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
3	0.125	0.137	0.149	0.161	0.173	0.185	0.196	0.219	0.242	0.263	0.304
	2.88	2.88	2.88	2.87	2.87	2.87	2.87	2.87	2.86	2.86	2.85
4	0.125	0.139	0.154	0.168	0.183	0.197	0.211	0.239	0.266	0.294	0.345
	3.53	3.53	3.53	3.53	3.52	3.52	3.52	3.51	3.50	3.48	3.46
5	0.125	0.141	0.158	0.175	0.192	0.208	0.225	0.258	0.290	0.321	0.380
	4.43	4.43	4.43	4.43	4.43	4.43	4.43	4.41	4.41	4.40	4.38
6	0.125	0.143	0.161	0.180	0.198	0.217	0.236	0.271	0.307	0.341	0.406
	5.29	5.29	5.29	5.28	5.28	5.28	5.27	5.27	5.25	5.24	5.21
7	0.125	0.144	0.164	0.184	0.204	0.224	0.244	0.283	0.322	0.358	0.429
	6.27	6.27	6.27	6.27	6.27	6.26	6.26	6.25	6.23	6.22	6.18
8	0.125	0.145	0.166	0.187	0.209	0.230	0.252	0.294	0.336	0.376	0.453
	7.29	7.29	7.29	7.28	7.28	7.27	7.27	7.25	7.23	7.20	7.14
9	0.125	0.146	0.169	0.191	0.214	0.237	0.260	0.306	0.350	0.395	0.476
	8.28	8.28	8.28	8.28	8.27	8.26	8.25	8.24	8.21	8.17	8.10
10	0.125	0.148	0.171	0.195	0.219	0.244	0.269	0.318	0.365	0.413	0.498
	9.25	9.24	9.23	9.23	9.22	9.22	9.20	9.18	9.15	9.10	9.01
11	0.125	0.149	0.174	0.199	0.225	0.251	0.277	0.330	0.381	0.429	0.519
	10.17	10.18	10.17	10.17	10.16	10.16	10.14	10.11	10.07	10.03	9.93
12	0.125	0.150	0.176	0.203	0.230	0.258	0.285	0.340	0.393	0.444	0.539
	11.13	11.13	11.13	11.12	11.11	11.10	11.09	11.06	11.02	10.97	10.85
13	0.125	0.151	0.178	0.206	0.235	0.264	0.293	0.350	0.406	0.459	0.557
	12.10	12.09	12.09	12.09	12.08	12.06	12.05	12.01	11.97	11.92	11.78
14	0.125	0.152	0.180	0.209	0.239	0.269	0.299	0.360	0.418	0.474	0.576
	13.08	13.09	13.08	13.07	13.06	13.04	13.03	12.98	12.93	12.86	12.70
15	0.125	0.153	0.182	0.212	0.243	0.275	0.306	0.369	0.430	0.487	0.593
	14.06	14.07	14.06	14.05	14.04	14.01	14.00	13.96	13.89	13.82	13.64
16	0.125	0.154	0.184	0.215	0.247	0.281	0.314	0.380	0.442	0.502	0.607
	15.04	15.04	15.03	15.02	15.02	14.99	14.97	14.91	14.85	14.76	14.57
17	0.125	0.155	0.186	0.218	0.252	0.286	0.320	0.389	0.452	0.514	0.623
	16.01	16.01	16.00	16.00	15.98	15.96	15.94	15.86	15.80	15.71	15.50
18	0.125	0.155	0.187	0.221	0.257	0.292	0.327	0.398	0.465	0.528	0.638
	16.98	16.98	16.98	16.97	16.94	16.92	16.89	16.82	16.74	16.65	16.42
19	0.125	0.156	0.190	0.224	0.260	0.296	0.334	0.405	0.475	0.541	0.652
	17.96	17.95	17.94	17.93	17.92	17.89	17.86	17.79	17.70	17.58	17.33
20	0.125	0.157	0.191	0.227	0.264	0.302	0.339	0.415	0.485	0.551	0.665
	18.93	18.92	18.92	18.91	18.89	18.86	18.83	18.74	18.65	18.54	18.26
21	0.125	0.158	0.193	0.230	0.268	0.307	0.345	0.424	0.497	0.563	0.678
	19.91	19.90	19.90	19.89	19.87	19.83	19.81	19.70	19.60	19.48	19.18
22	0.125	0.158	0.195	0.233	0.272	0.313	0.352	0.431	0.506	0.574	0.691
	20.90	20.89	20.88	20.86	20.84	20.80	20.77	20.68	20.56	20.42	20.10
23	0.125	0.159	0.196	0.235	0.275	0.316	0.358	0.438	0.516	0.585	0.701
	21.87	21.86	21.86	21.84	21.82	21.79	21.74	21.65	21.51	21.37	21.04
24	0.125	0.160	0.198	0.238	0.279	0.322	0.365	0.448	0.526	0.597	0.715
	22.84	22.84	22.84	22.81	22.79	22.75	22.71	22.60	22.47	22.30	21.92

Table 39: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
25	0.125	0.161	0.200	0.241	0.283	0.326	0.369	0.455	0.533	0.607	0.726
	23.82	23.83	23.80	23.79	23.76	23.73	23.68	23.56	23.42	23.24	22.85
26	0.125	0.162	0.201	0.244	0.286	0.331	0.375	0.464	0.544	0.617	0.738
	24.80	24.80	24.79	24.76	24.74	24.70	24.65	24.52	24.36	24.18	23.76
27	0.125	0.162	0.203	0.245	0.291	0.336	0.381	0.470	0.552	0.625	0.748
	25.79	25.78	25.77	25.76	25.71	25.67	25.62	25.49	25.32	25.13	24.68
28	0.125	0.163	0.204	0.248	0.294	0.340	0.387	0.478	0.562	0.637	0.758
	26.77	26.76	26.74	26.73	26.69	26.65	26.59	26.45	26.27	26.05	25.57
29	0.125	0.164	0.206	0.250	0.297	0.345	0.392	0.484	0.570	0.646	0.767
	27.75	27.74	27.73	27.71	27.67	27.62	27.56	27.42	27.22	26.99	26.49
30	0.125	0.164	0.207	0.253	0.301	0.350	0.398	0.492	0.578	0.655	0.775
	28.73	28.71	28.71	28.70	28.64	28.59	28.53	28.37	28.18	27.94	27.40
31	0.125	0.165	0.209	0.256	0.304	0.353	0.402	0.499	0.586	0.664	0.785
	29.71	29.70	29.69	29.66	29.63	29.57	29.51	29.34	29.12	28.88	28.32
32	0.125	0.165	0.210	0.259	0.307	0.358	0.408	0.507	0.594	0.671	0.795
	30.69	30.69	30.67	30.63	30.61	30.55	30.47	30.28	30.07	29.81	29.20
33	0.125	0.166	0.212	0.260	0.311	0.363	0.414	0.513	0.603	0.680	0.802
	31.68	31.66	31.65	31.62	31.57	31.52	31.44	31.25	31.01	30.75	30.12
34	0.125	0.167	0.213	0.263	0.315	0.368	0.419	0.520	0.610	0.690	0.808
	32.65	32.64	32.64	32.59	32.55	32.49	32.41	32.21	31.96	31.66	31.03
35	0.125	0.167	0.215	0.265	0.318	0.370	0.425	0.527	0.616	0.696	0.816
	33.63	33.63	33.61	33.57	33.53	33.47	33.37	33.17	32.92	32.62	31.93
36	0.125	0.168	0.216	0.267	0.321	0.375	0.429	0.532	0.625	0.704	0.824
	34.61	34.62	34.59	34.56	34.51	34.43	34.36	34.14	33.86	33.55	32.85
37	0.125	0.169	0.218	0.270	0.324	0.379	0.434	0.538	0.633	0.712	0.831
	35.60	35.59	35.58	35.54	35.48	35.41	35.33	35.10	34.80	34.48	33.74
38	0.125	0.169	0.219	0.271	0.327	0.383	0.438	0.545	0.639	0.718	0.837
	36.58	36.58	36.56	36.53	36.47	36.40	36.30	36.05	35.76	35.43	34.65
39	0.125	0.170	0.220	0.274	0.331	0.387	0.444	0.551	0.644	0.725	0.845
	37.57	37.56	37.54	37.50	37.44	37.36	37.27	37.02	36.72	36.36	35.54
40	0.125	0.170	0.221	0.276	0.334	0.391	0.448	0.558	0.653	0.734	0.849
	38.55	38.55	38.53	38.48	38.41	38.35	38.24	37.97	37.65	37.26	36.45
41	0.125	0.171	0.223	0.278	0.337	0.395	0.454	0.564	0.659	0.740	0.855
	39.54	39.53	39.50	39.45	39.39	39.32	39.20	38.93	38.61	38.22	37.34
42	0.125	0.172	0.224	0.280	0.340	0.398	0.458	0.570	0.665	0.746	0.860
	40.52	40.50	40.50	40.45	40.38	40.30	40.18	39.89	39.55	39.14	38.26
43	0.125	0.172	0.226	0.283	0.343	0.403	0.462	0.576	0.671	0.752	0.868
	41.49	41.50	41.47	41.41	41.35	41.26	41.16	40.85	40.50	40.08	39.13
44	0.125	0.173	0.227	0.285	0.345	0.408	0.467	0.581	0.680	0.760	0.872
	42.49	42.48	42.46	42.41	42.33	42.23	42.12	41.80	41.42	40.99	40.03
45	0.125	0.174	0.228	0.287	0.349	0.409	0.472	0.586	0.683	0.765	0.878
	43.47	43.45	43.44	43.39	43.32	43.23	43.09	42.77	42.38	41.93	40.92
46	0.125	0.174	0.230	0.290	0.352	0.415	0.476	0.592	0.690	0.772	0.883
	44.45	44.45	44.41	44.36	44.29	44.19	44.07	43.73	43.34	42.85	41.82
47	0.125	0.174	0.230	0.291	0.355	0.418	0.479	0.597	0.697	0.779	0.887
	45.44	45.43	45.41	45.35	45.27	45.16	45.05	44.68	44.26	43.78	42.72
48	0.125	0.175	0.232	0.294	0.357	0.420	0.485	0.603	0.704	0.784	0.892
	46.43	46.41	46.38	46.33	46.25	46.16	46.00	45.65	45.20	44.71	43.61

Table 39: (continued).

	$\theta=1$	$\theta=1.1$	$\theta=1.2$	$\theta=1.3$	$\theta=1.4$	$\theta=1.5$	$\theta=1.6$	$\theta=1.8$	$\theta=2$	$\theta=2.2$	$\theta=2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
49	0.125	0.176	0.233	0.295	0.360	0.426	0.490	0.607	0.708	0.787	0.895
	47.40	47.41	47.38	47.32	47.23	47.11	46.97	46.62	46.16	45.66	44.52
50	0.125	0.176	0.235	0.297	0.363	0.428	0.495	0.614	0.714	0.794	0.898
	48.40	48.36	48.35	48.30	48.21	48.11	47.95	47.56	47.10	46.56	45.42
51	0.125	0.177	0.236	0.299	0.366	0.433	0.498	0.618	0.721	0.799	0.904
	49.40	49.37	49.34	49.29	49.19	49.07	48.92	48.53	48.02	47.49	46.30
52	0.125	0.177	0.237	0.301	0.369	0.437	0.502	0.624	0.725	0.804	0.906
	50.37	50.36	50.32	50.27	50.16	50.04	49.89	49.48	48.97	48.41	47.20
53	0.125	0.178	0.238	0.304	0.371	0.439	0.506	0.629	0.730	0.810	0.910
	51.38	51.35	51.32	51.25	51.15	51.03	50.85	50.44	49.91	49.32	48.09
54	0.125	0.178	0.239	0.306	0.375	0.443	0.511	0.634	0.735	0.814	0.915
	52.35	52.33	52.30	52.23	52.12	52.00	51.82	51.39	50.86	50.26	48.96
55	0.125	0.179	0.241	0.308	0.377	0.446	0.515	0.638	0.740	0.819	0.916
	53.31	53.31	53.26	53.21	53.11	52.98	52.79	52.35	51.80	51.18	49.90
56	0.125	0.180	0.241	0.310	0.380	0.450	0.519	0.643	0.745	0.825	0.922
	54.31	54.29	54.27	54.19	54.09	53.95	53.76	53.30	52.73	52.09	50.73
57	0.125	0.180	0.243	0.311	0.382	0.454	0.522	0.646	0.751	0.828	0.923
	55.31	55.29	55.24	55.19	55.07	54.92	54.74	54.28	53.67	53.03	51.65
58	0.125	0.180	0.244	0.312	0.385	0.458	0.526	0.653	0.753	0.831	0.928
	56.30	56.28	56.23	56.18	56.05	55.89	55.71	55.22	54.65	53.96	52.52
59	0.125	0.181	0.246	0.315	0.388	0.461	0.530	0.657	0.759	0.835	0.930
	57.27	57.26	57.21	57.16	57.03	56.87	56.68	56.17	55.57	54.89	53.41
60	0.125	0.181	0.246	0.318	0.391	0.465	0.534	0.661	0.764	0.841	0.933
	58.25	58.24	58.21	58.11	58.00	57.83	57.65	57.14	56.50	55.81	54.30
61	0.125	0.182	0.248	0.319	0.393	0.466	0.539	0.666	0.768	0.845	0.935
	59.24	59.24	59.19	59.11	59.00	58.83	58.61	58.10	57.44	56.71	55.18
62	0.125	0.183	0.248	0.321	0.396	0.470	0.541	0.670	0.774	0.846	0.939
	60.23	60.21	60.18	60.09	59.96	59.80	59.60	59.05	58.37	57.66	56.06
63	0.125	0.183	0.250	0.322	0.399	0.473	0.546	0.675	0.777	0.852	0.941
	61.23	61.20	61.17	61.07	60.95	60.78	60.55	60.00	59.32	58.57	56.95
64	0.125	0.183	0.251	0.325	0.400	0.477	0.551	0.679	0.781	0.859	0.944
	62.20	62.20	62.15	62.06	61.94	61.74	61.52	60.95	60.26	59.44	57.84
65	0.125	0.184	0.252	0.326	0.403	0.480	0.553	0.683	0.787	0.861	0.945
	63.20	63.18	63.14	63.05	62.91	62.72	62.50	61.90	61.18	60.40	58.73
66	0.125	0.184	0.254	0.329	0.407	0.482	0.557	0.689	0.789	0.865	0.947
	64.18	64.17	64.11	64.01	63.88	63.71	63.46	62.85	62.14	61.31	59.62
67	0.125	0.185	0.255	0.330	0.408	0.487	0.562	0.692	0.794	0.868	0.950
	65.16	65.15	65.10	65.01	64.88	64.67	64.42	63.81	63.07	62.23	60.52
68	0.125	0.185	0.256	0.333	0.412	0.488	0.564	0.695	0.796	0.872	0.952
	66.15	66.15	66.08	65.98	65.84	65.68	65.40	64.78	64.02	63.14	61.38
69	0.125	0.186	0.256	0.334	0.414	0.493	0.566	0.701	0.801	0.874	0.953
	67.16	67.13	67.09	66.97	66.82	66.63	66.38	65.72	64.96	64.08	62.27
70	0.125	0.186	0.258	0.335	0.416	0.497	0.572	0.704	0.806	0.877	0.955
	68.13	68.13	68.06	67.97	67.81	67.59	67.34	66.67	65.87	64.98	63.16
71	0.125	0.187	0.259	0.337	0.419	0.498	0.575	0.708	0.809	0.881	0.957
	69.12	69.09	69.05	68.95	68.78	68.59	68.31	67.62	66.81	65.88	64.03
72	0.125	0.187	0.260	0.339	0.421	0.503	0.578	0.712	0.814	0.882	0.958
	70.09	70.10	70.02	69.93	69.77	69.55	69.28	68.58	67.72	66.85	64.93

Table 39: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
73	0.125	0.188	0.261	0.340	0.423	0.506	0.581	0.716	0.818	0.886	0.960
	71.09	71.09	71.02	70.93	70.76	70.52	70.25	69.53	68.64	67.75	65.83
74	0.125	0.188	0.262	0.342	0.426	0.508	0.587	0.720	0.819	0.889	0.962
	72.08	72.07	72.01	71.91	71.73	71.50	71.20	70.49	69.62	68.65	66.67
75	0.125	0.189	0.262	0.344	0.428	0.510	0.590	0.724	0.824	0.892	0.963
	73.08	73.05	73.01	72.89	72.70	72.48	72.16	71.42	70.53	69.56	67.56
76	0.125	0.189	0.264	0.347	0.429	0.514	0.593	0.726	0.827	0.895	0.965
	74.04	74.03	73.99	73.86	73.71	73.44	73.15	72.40	71.47	70.49	68.46
77	0.125	0.190	0.266	0.348	0.433	0.518	0.596	0.731	0.830	0.896	0.966
	75.05	75.02	74.95	74.85	74.67	74.42	74.11	73.33	72.40	71.42	69.32
78	0.125	0.190	0.267	0.350	0.436	0.520	0.600	0.737	0.832	0.900	0.968
	76.02	76.02	75.94	75.84	75.65	75.39	75.07	74.26	73.36	72.28	70.21
79	0.125	0.190	0.267	0.351	0.438	0.522	0.603	0.737	0.836	0.901	0.969
	77.04	77.01	76.95	76.83	76.63	76.38	76.05	75.25	74.29	73.26	71.09
80	0.125	0.191	0.268	0.353	0.441	0.526	0.607	0.741	0.841	0.905	0.970
	78.02	78.00	77.93	77.81	77.61	77.35	77.00	76.19	75.19	74.14	72.01
81	0.125	0.191	0.269	0.354	0.443	0.529	0.610	0.745	0.843	0.908	0.971
	79.02	78.98	78.93	78.79	78.59	78.32	77.97	77.14	76.12	75.09	72.88
82	0.125	0.192	0.270	0.356	0.445	0.532	0.612	0.747	0.845	0.910	0.972
	79.97	79.97	79.92	79.78	79.57	79.30	78.96	78.10	77.08	75.97	73.73
83	0.125	0.192	0.273	0.358	0.448	0.534	0.616	0.752	0.848	0.913	0.974
	80.99	80.94	80.87	80.76	80.55	80.28	79.92	79.03	78.00	76.88	74.62
84	0.125	0.193	0.272	0.359	0.451	0.538	0.620	0.755	0.851	0.915	0.974
	81.97	81.95	81.88	81.75	81.52	81.24	80.88	79.99	78.94	77.77	75.49
85	0.125	0.193	0.273	0.362	0.453	0.539	0.622	0.757	0.853	0.917	0.976
	82.97	82.94	82.88	82.72	82.50	82.24	81.86	80.95	79.89	78.70	76.35
86	0.125	0.194	0.274	0.364	0.455	0.543	0.626	0.762	0.857	0.919	0.976
	83.95	83.92	83.86	83.70	83.48	83.18	82.81	81.89	80.79	79.62	77.26
87	0.125	0.194	0.276	0.365	0.456	0.545	0.629	0.764	0.860	0.920	0.978
	84.92	84.92	84.84	84.70	84.48	84.16	83.78	82.84	81.73	80.54	78.11
88	0.125	0.194	0.277	0.367	0.458	0.547	0.631	0.766	0.861	0.923	0.979
	85.93	85.90	85.83	85.69	85.46	85.15	84.76	83.81	82.68	81.44	79.01
89	0.125	0.195	0.278	0.367	0.460	0.550	0.634	0.771	0.864	0.924	0.979
	86.91	86.89	86.82	86.68	86.44	86.12	85.72	84.74	83.61	82.35	79.91
90	0.125	0.195	0.279	0.370	0.464	0.555	0.637	0.773	0.869	0.926	0.981
	87.90	87.89	87.80	87.65	87.41	87.07	86.68	85.68	84.51	83.29	80.73
91	0.125	0.196	0.280	0.371	0.466	0.557	0.640	0.777	0.870	0.928	0.981
	88.88	88.86	88.77	88.64	88.39	88.06	87.66	86.64	85.46	84.18	81.63
92	0.125	0.196	0.281	0.373	0.468	0.558	0.643	0.779	0.872	0.931	0.982
	89.87	89.88	89.76	89.62	89.37	89.04	88.64	87.59	86.39	85.07	82.53
93	0.125	0.197	0.282	0.374	0.470	0.561	0.646	0.783	0.875	0.932	0.982
	90.86	90.84	90.76	90.61	90.36	90.02	89.57	88.54	87.32	86.00	83.38
94	0.125	0.197	0.283	0.376	0.472	0.564	0.648	0.786	0.879	0.934	0.983
	91.85	91.83	91.75	91.59	91.33	90.98	90.56	89.47	88.20	86.89	84.25
95	0.125	0.197	0.283	0.378	0.475	0.567	0.651	0.787	0.879	0.936	0.984
	92.85	92.83	92.75	92.57	92.30	91.96	91.53	90.44	89.18	87.82	85.15
96	0.125	0.198	0.285	0.379	0.476	0.570	0.654	0.791	0.882	0.938	0.984
	93.83	93.81	93.73	93.56	93.30	92.93	92.50	91.39	90.09	88.71	86.04

Table 39: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
97	0.125	0.198	0.285	0.381	0.479	0.571	0.656	0.793	0.885	0.939	0.985
	94.84	94.80	94.73	94.54	94.27	93.92	93.48	92.34	91.02	89.64	86.93
98	0.125	0.199	0.287	0.383	0.480	0.577	0.658	0.796	0.885	0.940	0.986
	95.82	95.80	95.70	95.52	95.26	94.86	94.45	93.29	91.96	90.55	87.77
99	0.125	0.199	0.288	0.385	0.484	0.577	0.661	0.799	0.890	0.941	0.986
	96.81	96.77	96.68	96.49	96.21	95.85	95.40	94.22	92.86	91.47	88.66
100	0.125	0.200	0.289	0.387	0.484	0.580	0.665	0.800	0.890	0.943	0.987
	97.79	97.76	97.68	97.48	97.23	96.82	96.36	95.20	93.81	92.36	89.52
101	0.125	0.200	0.290	0.388	0.487	0.582	0.668	0.805	0.893	0.945	0.987
	98.79	98.77	98.65	98.48	98.19	97.82	97.31	96.10	94.72	93.28	90.39
102	0.125	0.200	0.291	0.389	0.490	0.584	0.671	0.807	0.895	0.946	0.988
	99.78	99.75	99.64	99.46	99.16	98.78	98.28	97.07	95.65	94.20	91.30
103	0.125	0.201	0.292	0.391	0.491	0.587	0.673	0.811	0.896	0.948	0.989
	100.78	100.75	100.64	100.44	100.15	99.75	99.25	97.99	96.59	95.08	92.17
104	0.125	0.201	0.292	0.392	0.492	0.590	0.677	0.812	0.900	0.949	0.989
	101.73	101.73	101.64	101.44	101.13	100.72	100.22	98.95	97.49	95.98	93.01
105	0.125	0.201	0.294	0.394	0.496	0.590	0.678	0.816	0.902	0.950	0.989
	102.75	102.73	102.61	102.42	102.11	101.71	101.18	99.89	98.41	96.90	93.88
106	0.125	0.202	0.294	0.395	0.498	0.594	0.683	0.818	0.903	0.951	0.990
	103.73	103.70	103.61	103.41	103.10	102.68	102.13	100.83	99.36	97.83	94.76
107	0.125	0.202	0.295	0.396	0.501	0.597	0.686	0.818	0.904	0.953	0.990
	104.73	104.71	104.60	104.40	104.07	103.65	103.08	101.81	100.29	98.69	95.68
108	0.125	0.203	0.297	0.398	0.502	0.600	0.686	0.823	0.906	0.954	0.991
	105.72	105.70	105.58	105.38	105.06	104.60	104.07	102.73	101.23	99.63	96.53
109	0.125	0.204	0.298	0.401	0.504	0.602	0.691	0.824	0.909	0.955	0.991
	106.72	106.65	106.57	106.34	106.03	105.60	105.01	103.71	102.11	100.53	97.40
110	0.125	0.203	0.298	0.402	0.506	0.604	0.692	0.827	0.912	0.956	0.991
	107.70	107.67	107.56	107.35	107.02	106.57	105.99	104.62	103.03	101.41	98.30
111	0.125	0.204	0.299	0.403	0.508	0.605	0.693	0.827	0.913	0.958	0.992
	108.68	108.66	108.54	108.32	107.99	107.54	106.97	105.59	103.96	102.34	99.17
112	0.125	0.204	0.300	0.405	0.511	0.609	0.696	0.830	0.914	0.958	0.992
	109.68	109.66	109.52	109.31	108.96	108.50	107.93	106.54	104.91	103.24	100.06
113	0.125	0.205	0.301	0.407	0.511	0.612	0.701	0.833	0.916	0.960	0.993
	110.68	110.64	110.54	110.29	109.96	109.48	108.86	107.46	105.83	104.14	100.88
114	0.125	0.205	0.303	0.407	0.513	0.614	0.701	0.835	0.917	0.961	0.993
	111.67	111.62	111.49	111.29	110.94	110.44	109.85	108.42	106.75	105.04	101.74
115	0.125	0.205	0.303	0.409	0.517	0.617	0.706	0.837	0.918	0.962	0.993
	112.65	112.62	112.49	112.26	111.90	111.41	110.80	109.37	107.70	105.96	102.67
116	0.125	0.206	0.303	0.410	0.517	0.619	0.705	0.839	0.921	0.963	0.993
	113.63	113.61	113.51	113.25	112.90	112.38	111.81	110.32	108.60	106.86	103.55
117	0.125	0.206	0.305	0.413	0.519	0.620	0.710	0.843	0.921	0.963	0.994
	114.63	114.61	114.47	114.22	113.89	113.37	112.72	111.24	109.52	107.79	104.40
118	0.125	0.207	0.306	0.414	0.523	0.623	0.712	0.845	0.923	0.966	0.994
	115.62	115.58	115.46	115.21	114.83	114.33	113.69	112.14	110.48	108.64	105.32
119	0.125	0.207	0.306	0.415	0.524	0.626	0.715	0.846	0.925	0.966	0.994
	116.62	116.58	116.46	116.21	115.82	115.30	114.66	113.13	111.36	109.58	106.14
120	0.125	0.207	0.307	0.416	0.525	0.627	0.715	0.849	0.926	0.967	0.994
	117.61	117.57	117.44	117.20	116.82	116.28	115.65	114.08	112.30	110.45	106.97

Table 39: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
121	0.125	0.208	0.308	0.419	0.529	0.630	0.718	0.850	0.927	0.967	0.994
	118.59	118.57	118.45	118.16	117.76	117.24	116.60	115.00	113.23	111.41	107.90
122	0.125	0.208	0.309	0.420	0.529	0.630	0.722	0.854	0.929	0.968	0.995
	119.58	119.54	119.43	119.16	118.77	118.27	117.54	115.92	114.17	112.31	108.81
123	0.125	0.208	0.310	0.420	0.531	0.634	0.724	0.854	0.932	0.969	0.995
	120.57	120.55	120.41	120.15	119.75	119.20	118.53	116.89	115.06	113.20	109.63
124	0.125	0.209	0.311	0.422	0.534	0.635	0.725	0.856	0.932	0.970	0.995
	121.58	121.52	121.40	121.15	120.73	120.19	119.50	117.83	115.96	114.13	110.54
125	0.125	0.209	0.312	0.424	0.534	0.639	0.726	0.859	0.933	0.971	0.995
	122.56	122.51	122.39	122.12	121.72	121.14	120.46	118.78	116.93	115.01	111.39
126	0.125	0.210	0.313	0.426	0.537	0.640	0.730	0.860	0.934	0.972	0.995
	123.55	123.53	123.38	123.09	122.67	122.11	121.42	119.73	117.88	115.92	112.30
127	0.125	0.210	0.314	0.426	0.538	0.643	0.731	0.862	0.936	0.973	0.996
	124.55	124.51	124.36	124.09	123.67	123.08	122.39	120.65	118.75	116.79	113.12
128	0.125	0.210	0.315	0.429	0.542	0.643	0.736	0.864	0.937	0.973	0.996
	125.54	125.49	125.35	125.06	124.62	124.08	123.31	121.61	119.68	117.72	114.00
129	0.125	0.211	0.315	0.429	0.542	0.648	0.738	0.866	0.938	0.974	0.996
	126.50	126.48	126.35	126.06	125.63	125.02	124.29	122.55	120.58	118.62	114.90
130	0.125	0.211	0.317	0.431	0.545	0.648	0.739	0.868	0.939	0.974	0.996
	127.51	127.48	127.32	127.05	126.59	126.03	125.26	123.47	121.54	119.54	115.77
131	0.125	0.212	0.317	0.432	0.545	0.652	0.740	0.869	0.941	0.975	0.996
	128.50	128.47	128.33	128.03	127.61	126.95	126.22	124.44	122.44	120.43	116.67
132	0.125	0.212	0.318	0.435	0.547	0.654	0.744	0.872	0.942	0.976	0.996
	129.49	129.46	129.31	129.00	128.57	127.92	127.17	125.35	123.33	121.31	117.55
133	0.125	0.212	0.319	0.435	0.550	0.657	0.745	0.874	0.942	0.976	0.996
	130.48	130.46	130.29	129.99	129.54	128.89	128.14	126.29	124.30	122.26	118.42
134	0.125	0.213	0.319	0.436	0.552	0.657	0.745	0.873	0.944	0.977	0.997
	131.47	131.45	131.30	130.99	130.51	129.89	129.13	127.28	125.21	123.18	119.26
135	0.125	0.213	0.321	0.438	0.554	0.660	0.749	0.876	0.944	0.977	0.997
	132.48	132.43	132.27	131.97	131.50	130.83	130.08	128.20	126.15	124.05	120.14
136	0.125	0.214	0.322	0.440	0.558	0.661	0.752	0.878	0.945	0.978	0.997
	133.47	133.42	133.25	132.93	132.44	131.82	131.02	129.13	127.07	124.98	121.01
137	0.125	0.214	0.322	0.441	0.556	0.662	0.753	0.880	0.947	0.979	0.997
	134.45	134.42	134.25	133.94	133.46	132.80	132.01	130.06	128.01	125.83	121.88
138	0.125	0.214	0.323	0.442	0.559	0.666	0.752	0.880	0.948	0.979	0.997
	135.43	135.41	135.24	134.93	134.42	133.78	132.99	131.04	128.90	126.77	122.73
139	0.125	0.215	0.324	0.444	0.561	0.667	0.757	0.883	0.949	0.980	0.997
	136.43	136.38	136.23	135.91	135.42	134.73	133.92	131.94	129.81	127.61	123.61
140	0.125	0.215	0.325	0.445	0.562	0.669	0.760	0.883	0.950	0.979	0.997
	137.42	137.39	137.21	136.89	136.39	135.72	134.85	132.93	130.70	128.57	124.53
141	0.125	0.215	0.326	0.447	0.564	0.671	0.761	0.886	0.951	0.980	0.998
	138.43	138.37	138.21	137.87	137.39	136.68	135.83	133.83	131.69	129.43	125.37
142	0.125	0.216	0.326	0.447	0.566	0.674	0.763	0.888	0.952	0.981	0.998
	139.40	139.35	139.20	138.87	138.33	137.64	136.81	134.78	132.58	130.37	126.19
143	0.125	0.216	0.329	0.450	0.567	0.675	0.765	0.888	0.953	0.982	0.998
	140.40	140.35	140.16	139.84	139.34	138.62	137.76	135.76	133.49	131.26	127.07
144	0.125	0.216	0.329	0.451	0.568	0.677	0.767	0.890	0.954	0.982	0.998
	141.40	141.35	141.18	140.83	140.32	139.59	138.70	136.65	134.41	132.20	128.00

Table 39: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
145	0.125	0.217	0.329	0.452	0.572	0.679	0.770	0.891	0.955	0.983	0.998
	142.38	142.33	142.17	141.83	141.28	140.55	139.65	137.59	135.34	133.08	128.85
146	0.125	0.217	0.330	0.454	0.573	0.680	0.771	0.894	0.957	0.983	0.998
	143.37	143.33	143.16	142.80	142.26	141.54	140.65	138.53	136.22	134.01	129.73
147	0.125	0.218	0.331	0.455	0.575	0.683	0.772	0.895	0.957	0.984	0.998
	144.37	144.30	144.13	143.79	143.24	142.49	141.63	139.50	137.16	134.82	130.60
148	0.125	0.218	0.333	0.456	0.577	0.685	0.775	0.896	0.957	0.984	0.998
	145.36	145.32	145.11	144.77	144.21	143.46	142.55	140.42	138.11	135.78	131.43
149	0.125	0.218	0.332	0.458	0.580	0.689	0.776	0.897	0.958	0.984	0.998
	146.37	146.30	146.13	145.75	145.17	144.41	143.53	141.37	139.01	136.66	132.33
150	0.125	0.219	0.334	0.459	0.579	0.688	0.778	0.899	0.959	0.984	0.998
	147.34	147.29	147.11	146.74	146.19	145.43	144.47	142.30	139.90	137.60	133.22
155	0.125	0.220	0.337	0.466	0.588	0.698	0.786	0.905	0.962	0.987	0.999
	152.29	152.25	152.07	151.66	151.07	150.24	149.29	146.97	144.56	142.08	137.54
160	0.125	0.222	0.342	0.472	0.596	0.707	0.796	0.911	0.966	0.989	0.999
	157.25	157.20	157.00	156.58	155.96	155.10	154.05	151.70	149.13	146.58	141.93
165	0.125	0.224	0.346	0.478	0.604	0.715	0.805	0.917	0.970	0.990	0.999
	162.23	162.15	161.94	161.51	160.87	159.96	158.84	156.41	153.70	151.07	146.29
170	0.125	0.226	0.350	0.484	0.610	0.723	0.810	0.922	0.972	0.991	0.999
	167.18	167.11	166.90	166.45	165.77	164.81	163.70	161.06	158.30	155.57	150.63
175	0.125	0.227	0.355	0.490	0.621	0.731	0.820	0.928	0.975	0.992	0.999
	172.12	172.08	171.84	171.37	170.61	169.63	168.42	165.75	162.87	160.09	155.02
180	0.125	0.229	0.359	0.498	0.627	0.740	0.826	0.931	0.977	0.993	1.000
	177.10	177.04	176.78	176.26	175.51	174.47	173.24	170.44	167.46	164.58	159.37
185	0.125	0.230	0.362	0.502	0.635	0.747	0.833	0.938	0.979	0.994	1.000
	182.08	182.01	181.74	181.22	180.39	179.31	178.03	175.12	172.11	169.11	163.72
190	0.125	0.232	0.365	0.509	0.642	0.754	0.840	0.941	0.982	0.995	1.000
	187.02	186.96	186.70	186.14	185.28	184.17	182.84	179.76	176.61	173.60	168.10
195	0.125	0.234	0.371	0.514	0.649	0.762	0.846	0.944	0.983	0.995	1.000
	191.98	191.92	191.63	191.06	190.16	188.99	187.61	184.51	181.24	178.05	172.42
200	0.125	0.235	0.374	0.522	0.656	0.767	0.851	0.948	0.985	0.996	1.000
	196.95	196.90	196.58	195.95	195.06	193.86	192.39	189.12	185.79	182.57	176.74
210	0.125	0.238	0.382	0.533	0.668	0.782	0.864	0.955	0.988	0.997	1.000
	206.89	206.82	206.47	205.80	204.84	203.47	201.96	198.43	194.94	191.54	185.43
220	0.125	0.242	0.389	0.543	0.683	0.795	0.876	0.961	0.990	0.998	1.000
	216.83	216.72	216.36	215.66	214.56	213.12	211.46	207.78	204.03	200.52	194.11
230	0.125	0.245	0.395	0.554	0.695	0.806	0.884	0.965	0.992	0.998	1.000
	226.76	226.64	226.29	225.49	224.33	222.80	221.03	217.15	213.20	209.55	202.82
240	0.125	0.248	0.403	0.563	0.706	0.819	0.894	0.971	0.993	0.999	1.000
	236.68	236.59	236.18	235.36	234.12	232.40	230.54	226.43	222.33	218.47	211.50
250	0.125	0.251	0.410	0.574	0.719	0.828	0.903	0.974	0.994	0.999	1.000
	246.61	246.52	246.07	245.21	243.82	242.10	240.06	235.74	231.45	227.46	220.12

Table 40: Table of Monte Carlo Results for $\mathcal{M}_{\mathrm{BK}}$ when $k = 9$ and $\alpha = 0.05$.

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
1	0.111	0.121	0.130	0.140	0.149	0.158	0.167	0.184	0.200	0.216	0.245
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	0.111	0.121	0.130	0.140	0.149	0.158	0.167	0.184	0.200	0.216	0.245
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
3	0.111	0.122	0.133	0.144	0.154	0.165	0.175	0.196	0.216	0.235	0.273
	2.89	2.89	2.89	2.89	2.89	2.89	2.89	2.88	2.88	2.88	2.87
4	0.111	0.124	0.137	0.149	0.162	0.175	0.188	0.214	0.239	0.263	0.311
	3.57	3.58	3.58	3.58	3.57	3.57	3.57	3.56	3.55	3.54	3.51
5	0.111	0.126	0.141	0.155	0.170	0.185	0.200	0.230	0.259	0.288	0.343
	4.48	4.48	4.48	4.48	4.48	4.48	4.47	4.46	4.46	4.45	4.43
6	0.111	0.127	0.143	0.160	0.177	0.193	0.210	0.244	0.277	0.308	0.369
	5.33	5.32	5.33	5.32	5.32	5.32	5.31	5.31	5.29	5.29	5.25
7	0.111	0.128	0.146	0.164	0.182	0.200	0.218	0.254	0.290	0.324	0.391
	6.29	6.29	6.29	6.29	6.29	6.29	6.28	6.27	6.26	6.25	6.22
8	0.111	0.129	0.148	0.167	0.186	0.206	0.225	0.264	0.303	0.340	0.412
	7.31	7.30	7.30	7.31	7.30	7.29	7.29	7.27	7.26	7.23	7.19
9	0.111	0.130	0.150	0.170	0.191	0.212	0.232	0.274	0.316	0.357	0.433
	8.32	8.32	8.32	8.31	8.31	8.30	8.30	8.28	8.25	8.22	8.17
10	0.111	0.131	0.152	0.174	0.196	0.217	0.239	0.285	0.329	0.372	0.454
	9.29	9.30	9.30	9.29	9.28	9.27	9.27	9.24	9.22	9.19	9.11
11	0.111	0.132	0.154	0.177	0.200	0.224	0.247	0.296	0.343	0.387	0.474
	10.24	10.25	10.24	10.24	10.23	10.23	10.21	10.18	10.15	10.13	10.04
12	0.111	0.133	0.156	0.180	0.205	0.230	0.255	0.306	0.355	0.404	0.494
	11.19	11.19	11.19	11.19	11.18	11.16	11.15	11.12	11.09	11.05	10.95
13	0.111	0.134	0.158	0.183	0.209	0.235	0.262	0.314	0.367	0.418	0.511
	12.15	12.15	12.15	12.14	12.13	12.13	12.12	12.09	12.04	11.99	11.89
14	0.111	0.135	0.160	0.186	0.213	0.240	0.268	0.324	0.378	0.431	0.530
	13.13	13.13	13.13	13.12	13.11	13.10	13.08	13.05	13.01	12.95	12.82
15	0.111	0.136	0.162	0.188	0.217	0.246	0.274	0.332	0.389	0.445	0.545
	14.11	14.11	14.11	14.11	14.09	14.08	14.06	14.03	13.97	13.91	13.77
16	0.111	0.136	0.163	0.191	0.220	0.250	0.280	0.342	0.401	0.457	0.561
	15.10	15.10	15.10	15.09	15.08	15.07	15.05	14.99	14.93	14.88	14.71
17	0.111	0.137	0.165	0.194	0.224	0.255	0.286	0.351	0.410	0.471	0.574
	16.08	16.08	16.08	16.07	16.06	16.04	16.02	15.95	15.90	15.82	15.65
18	0.111	0.138	0.167	0.197	0.228	0.260	0.293	0.357	0.421	0.481	0.591
	17.06	17.06	17.05	17.03	17.02	17.01	16.98	16.93	16.86	16.78	16.58
19	0.111	0.139	0.168	0.200	0.232	0.265	0.300	0.367	0.432	0.494	0.605
	18.02	18.03	18.02	18.01	17.99	17.98	17.95	17.89	17.81	17.73	17.51
20	0.111	0.139	0.170	0.202	0.236	0.270	0.304	0.374	0.442	0.503	0.619
	18.99	19.00	18.99	18.99	18.96	18.94	18.92	18.86	18.76	18.68	18.44
21	0.111	0.140	0.171	0.205	0.239	0.274	0.310	0.380	0.453	0.518	0.633
	19.98	19.97	19.97	19.95	19.94	19.92	19.89	19.83	19.72	19.62	19.36
22	0.111	0.141	0.173	0.207	0.242	0.279	0.316	0.389	0.460	0.527	0.646
	20.96	20.95	20.94	20.93	20.92	20.90	20.87	20.79	20.69	20.58	20.29
23	0.111	0.141	0.174	0.209	0.246	0.283	0.321	0.396	0.470	0.539	0.658
	21.94	21.94	21.93	21.92	21.90	21.87	21.84	21.76	21.66	21.53	21.23
24	0.111	0.142	0.176	0.212	0.249	0.287	0.327	0.405	0.478	0.548	0.671
	22.93	22.92	22.92	22.90	22.88	22.86	22.81	22.72	22.62	22.48	22.15

Table 40: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
25	0.111	0.143	0.177	0.214	0.252	0.292	0.331	0.412	0.488	0.559	0.682
	23.91	23.90	23.90	23.88	23.86	23.82	23.80	23.69	23.57	23.43	23.07
26	0.111	0.143	0.179	0.216	0.257	0.297	0.337	0.418	0.496	0.570	0.692
	24.89	24.89	24.87	24.86	24.82	24.80	24.76	24.66	24.53	24.38	24.02
27	0.111	0.144	0.180	0.219	0.259	0.300	0.343	0.425	0.505	0.580	0.702
	25.87	25.86	25.85	25.83	25.81	25.78	25.73	25.63	25.50	25.32	24.94
28	0.111	0.145	0.181	0.221	0.263	0.305	0.347	0.433	0.514	0.588	0.712
	26.85	26.85	26.83	26.82	26.78	26.75	26.71	26.59	26.45	26.27	25.87
29	0.111	0.145	0.183	0.223	0.265	0.309	0.353	0.439	0.522	0.598	0.721
	27.83	27.82	27.82	27.80	27.77	27.73	27.68	27.56	27.41	27.22	26.79
30	0.111	0.146	0.184	0.226	0.269	0.312	0.358	0.447	0.532	0.606	0.732
	28.81	28.82	28.79	28.76	28.74	28.71	28.65	28.52	28.35	28.17	27.71
31	0.111	0.146	0.185	0.227	0.272	0.317	0.363	0.453	0.536	0.615	0.741
	29.80	29.79	29.78	29.76	29.72	29.69	29.63	29.50	29.33	29.12	28.63
32	0.111	0.147	0.187	0.230	0.275	0.321	0.367	0.459	0.546	0.625	0.751
	30.78	30.78	30.77	30.74	30.71	30.66	30.62	30.46	30.28	30.06	29.54
33	0.111	0.147	0.188	0.232	0.278	0.324	0.373	0.466	0.554	0.631	0.759
	31.76	31.76	31.74	31.72	31.69	31.64	31.58	31.44	31.24	31.00	30.47
34	0.111	0.148	0.189	0.234	0.281	0.329	0.378	0.473	0.561	0.641	0.769
	32.75	32.74	32.73	32.70	32.67	32.62	32.55	32.40	32.20	31.94	31.37
35	0.111	0.149	0.191	0.236	0.283	0.332	0.381	0.477	0.567	0.649	0.775
	33.72	33.72	33.71	33.69	33.65	33.60	33.54	33.38	33.15	32.89	32.31
36	0.111	0.149	0.192	0.238	0.286	0.336	0.386	0.485	0.575	0.657	0.784
	34.72	34.71	34.70	34.67	34.62	34.57	34.51	34.33	34.11	33.83	33.21
37	0.111	0.150	0.193	0.240	0.289	0.340	0.392	0.490	0.581	0.665	0.792
	35.70	35.69	35.68	35.66	35.60	35.55	35.48	35.31	35.07	34.77	34.12
38	0.111	0.150	0.194	0.242	0.293	0.344	0.396	0.499	0.589	0.672	0.800
	36.66	36.68	36.66	36.63	36.58	36.53	36.45	36.25	36.02	35.73	35.03
39	0.111	0.151	0.195	0.245	0.296	0.348	0.400	0.504	0.597	0.680	0.804
	37.67	37.66	37.66	37.61	37.56	37.50	37.44	37.21	36.97	36.66	35.98
40	0.111	0.151	0.197	0.246	0.298	0.352	0.407	0.509	0.604	0.686	0.813
	38.65	38.65	38.63	38.60	38.55	38.49	38.39	38.19	37.93	37.61	36.87
41	0.111	0.152	0.198	0.248	0.301	0.356	0.408	0.513	0.609	0.694	0.817
	39.63	39.64	39.61	39.58	39.53	39.46	39.39	39.17	38.89	38.55	37.80
42	0.111	0.152	0.199	0.250	0.304	0.359	0.413	0.520	0.617	0.699	0.825
	40.63	40.63	40.59	40.57	40.51	40.43	40.37	40.13	39.84	39.49	38.68
43	0.111	0.153	0.200	0.252	0.306	0.362	0.418	0.527	0.626	0.707	0.830
	41.61	41.60	41.59	41.54	41.50	41.42	41.33	41.08	40.76	40.42	39.62
44	0.111	0.153	0.202	0.254	0.309	0.366	0.422	0.530	0.630	0.714	0.836
	42.60	42.59	42.56	42.53	42.48	42.40	42.31	42.07	41.72	41.36	40.54
45	0.111	0.154	0.202	0.255	0.312	0.369	0.427	0.537	0.636	0.720	0.843
	43.57	43.57	43.56	43.52	43.46	43.38	43.28	43.02	42.69	42.30	41.43
46	0.111	0.154	0.203	0.258	0.314	0.374	0.429	0.543	0.643	0.724	0.847
	44.56	44.57	44.54	44.49	44.44	44.34	44.27	43.98	43.64	43.26	42.35
47	0.111	0.155	0.205	0.259	0.317	0.376	0.434	0.548	0.646	0.732	0.854
	45.56	45.54	45.52	45.49	45.43	45.33	45.23	44.95	44.61	44.19	43.25
48	0.111	0.155	0.206	0.261	0.320	0.380	0.438	0.553	0.653	0.738	0.859
	46.54	46.54	46.51	46.47	46.41	46.31	46.22	45.92	45.56	45.11	44.16

Table 40: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
49	0.111	0.156	0.207	0.263	0.323	0.383	0.443	0.559	0.661	0.743	0.864
	47.53	47.51	47.50	47.46	47.38	47.29	47.18	46.88	46.49	46.07	45.06
50	0.111	0.156	0.208	0.265	0.326	0.387	0.447	0.563	0.664	0.749	0.870
	48.51	48.51	48.48	48.44	48.36	48.27	48.16	47.85	47.46	47.00	45.96
51	0.111	0.157	0.209	0.266	0.327	0.389	0.451	0.569	0.673	0.755	0.873
	49.50	49.50	49.46	49.43	49.35	49.26	49.14	48.80	48.39	47.94	46.88
52	0.111	0.157	0.210	0.269	0.330	0.392	0.456	0.574	0.676	0.760	0.878
	50.48	50.47	50.44	50.41	50.34	50.24	50.11	49.77	49.36	48.88	47.80
53	0.111	0.158	0.211	0.270	0.334	0.396	0.459	0.577	0.681	0.767	0.881
	51.47	51.46	51.43	51.40	51.31	51.21	51.09	50.75	50.32	49.80	48.69
54	0.111	0.158	0.212	0.273	0.336	0.400	0.463	0.585	0.688	0.771	0.886
	52.47	52.46	52.44	52.36	52.29	52.18	52.06	51.69	51.25	50.74	49.60
55	0.111	0.159	0.214	0.274	0.338	0.404	0.467	0.588	0.691	0.777	0.889
	53.44	53.44	53.41	53.36	53.27	53.15	53.03	52.67	52.22	51.66	50.50
56	0.111	0.159	0.215	0.276	0.341	0.405	0.471	0.593	0.698	0.781	0.893
	54.44	54.42	54.39	54.34	54.26	54.14	54.00	53.62	53.15	52.61	51.42
57	0.111	0.160	0.215	0.277	0.343	0.409	0.475	0.599	0.704	0.788	0.897
	55.42	55.41	55.40	55.34	55.24	55.13	54.98	54.59	54.09	53.52	52.31
58	0.111	0.160	0.217	0.279	0.345	0.413	0.478	0.602	0.709	0.793	0.900
	56.41	56.40	56.37	56.31	56.22	56.09	55.96	55.56	55.04	54.47	53.22
59	0.111	0.160	0.218	0.281	0.347	0.416	0.482	0.608	0.713	0.795	0.905
	57.39	57.39	57.35	57.29	57.22	57.08	56.93	56.50	56.00	55.41	54.10
60	0.111	0.161	0.219	0.283	0.350	0.419	0.486	0.610	0.718	0.800	0.908
	58.38	58.38	58.35	58.28	58.20	58.06	57.91	57.49	56.95	56.34	55.00
61	0.111	0.161	0.220	0.285	0.352	0.422	0.491	0.618	0.721	0.804	0.911
	59.37	59.38	59.33	59.26	59.17	59.05	58.86	58.42	57.90	57.29	55.91
62	0.111	0.162	0.221	0.287	0.355	0.425	0.494	0.621	0.726	0.810	0.915
	60.36	60.35	60.33	60.25	60.16	60.02	59.84	59.40	58.84	58.20	56.79
63	0.111	0.162	0.222	0.288	0.359	0.428	0.497	0.623	0.732	0.813	0.917
	61.35	61.33	61.29	61.24	61.12	61.00	60.83	60.38	59.79	59.14	57.71
64	0.111	0.163	0.223	0.289	0.359	0.431	0.501	0.629	0.735	0.818	0.921
	62.34	62.32	62.29	62.23	62.13	61.98	61.80	61.34	60.75	60.07	58.59
65	0.111	0.163	0.224	0.292	0.363	0.434	0.505	0.636	0.740	0.823	0.923
	63.32	63.32	63.28	63.20	63.10	62.96	62.77	62.27	61.68	60.97	59.50
66	0.111	0.163	0.225	0.293	0.364	0.437	0.507	0.636	0.745	0.828	0.927
	64.31	64.32	64.27	64.19	64.09	63.95	63.75	63.25	62.63	61.90	60.38
67	0.111	0.164	0.226	0.295	0.368	0.440	0.512	0.640	0.750	0.830	0.928
	65.29	65.28	65.26	65.17	65.06	64.92	64.71	64.23	63.56	62.86	61.33
68	0.111	0.165	0.227	0.296	0.369	0.443	0.516	0.646	0.753	0.832	0.930
	66.30	66.27	66.24	66.17	66.06	65.90	65.69	65.18	64.52	63.81	62.19
69	0.111	0.165	0.228	0.298	0.371	0.446	0.517	0.652	0.757	0.837	0.933
	67.29	67.28	67.23	67.15	67.04	66.87	66.68	66.12	65.47	64.71	63.12
70	0.111	0.165	0.229	0.300	0.375	0.447	0.522	0.656	0.763	0.841	0.936
	68.25	68.26	68.21	68.13	68.01	67.87	67.64	67.08	66.40	65.65	63.99
71	0.111	0.166	0.230	0.301	0.376	0.451	0.524	0.657	0.765	0.845	0.938
	69.26	69.25	69.20	69.12	69.00	68.85	68.63	68.07	67.37	66.57	64.87
72	0.111	0.166	0.231	0.302	0.379	0.455	0.530	0.661	0.769	0.847	0.940
	70.24	70.23	70.19	70.12	69.98	69.81	69.59	69.03	68.31	67.52	65.79

Table 40: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
73	0.111	0.167	0.232	0.304	0.380	0.457	0.532	0.668	0.772	0.852	0.942
	71.23	71.21	71.18	71.09	70.98	70.80	70.57	69.96	69.25	68.44	66.70
74	0.111	0.167	0.233	0.306	0.383	0.461	0.536	0.671	0.776	0.857	0.944
	72.24	72.22	72.16	72.08	71.95	71.76	71.53	70.93	70.22	69.35	67.58
75	0.111	0.167	0.234	0.308	0.385	0.463	0.538	0.672	0.782	0.859	0.947
	73.21	73.21	73.14	73.06	72.95	72.76	72.52	71.91	71.12	70.28	68.50
76	0.111	0.168	0.235	0.310	0.388	0.465	0.543	0.676	0.784	0.863	0.949
	74.22	74.19	74.13	74.05	73.91	73.74	73.48	72.87	72.10	71.20	69.34
77	0.111	0.168	0.236	0.312	0.390	0.470	0.546	0.682	0.790	0.865	0.950
	75.19	75.19	75.14	75.03	74.89	74.70	74.45	73.80	73.00	72.12	70.27
78	0.111	0.168	0.236	0.313	0.392	0.471	0.548	0.686	0.790	0.868	0.951
	76.18	76.18	76.13	76.02	75.89	75.70	75.44	74.77	73.99	73.06	71.17
79	0.111	0.169	0.237	0.315	0.395	0.475	0.552	0.689	0.797	0.870	0.953
	77.17	77.17	77.12	77.01	76.86	76.66	76.39	75.73	74.89	73.99	72.05
80	0.111	0.170	0.239	0.315	0.396	0.477	0.555	0.694	0.799	0.873	0.955
	78.15	78.13	78.10	78.01	77.85	77.64	77.38	76.68	75.85	74.92	72.97
81	0.111	0.170	0.239	0.317	0.398	0.481	0.556	0.696	0.802	0.877	0.956
	79.15	79.13	79.09	78.98	78.83	78.60	78.38	77.65	76.78	75.85	73.85
82	0.111	0.170	0.241	0.318	0.402	0.482	0.562	0.699	0.806	0.880	0.958
	80.13	80.14	80.07	79.98	79.81	79.61	79.31	78.62	77.72	76.77	74.76
83	0.111	0.171	0.241	0.320	0.403	0.486	0.564	0.703	0.807	0.882	0.960
	81.14	81.12	81.06	80.97	80.80	80.57	80.30	79.57	78.70	77.69	75.62
84	0.111	0.171	0.242	0.322	0.405	0.487	0.569	0.708	0.811	0.885	0.961
	82.12	82.10	82.05	81.95	81.79	81.56	81.25	80.50	79.63	78.60	76.52
85	0.111	0.171	0.243	0.324	0.407	0.490	0.571	0.710	0.814	0.888	0.963
	83.11	83.09	83.03	82.91	82.77	82.54	82.24	81.48	80.58	79.54	77.42
86	0.111	0.172	0.245	0.325	0.410	0.493	0.573	0.712	0.818	0.890	0.963
	84.10	84.08	84.02	83.91	83.74	83.51	83.22	82.47	81.50	80.45	78.31
87	0.111	0.172	0.245	0.328	0.413	0.497	0.577	0.716	0.820	0.892	0.966
	85.08	85.08	85.02	84.89	84.70	84.48	84.18	83.40	82.44	81.38	79.15
88	0.111	0.173	0.246	0.327	0.414	0.499	0.580	0.719	0.825	0.895	0.965
	86.09	86.06	86.01	85.90	85.72	85.47	85.15	84.37	83.38	82.29	80.12
89	0.111	0.173	0.247	0.330	0.418	0.502	0.582	0.723	0.827	0.898	0.968
	87.06	87.05	87.00	86.88	86.68	86.45	86.13	85.32	84.33	83.21	80.97
90	0.111	0.173	0.248	0.331	0.417	0.505	0.587	0.726	0.829	0.900	0.969
	88.05	88.05	87.98	87.86	87.69	87.41	87.09	86.27	85.26	84.14	81.86
91	0.111	0.174	0.249	0.333	0.420	0.507	0.590	0.730	0.832	0.902	0.970
	89.04	89.04	88.98	88.84	88.66	88.40	88.07	87.23	86.22	85.09	82.76
92	0.111	0.174	0.249	0.333	0.423	0.509	0.590	0.732	0.835	0.905	0.972
	90.05	90.02	89.97	89.84	89.64	89.38	89.07	88.20	87.15	86.00	83.61
93	0.111	0.175	0.251	0.335	0.424	0.512	0.595	0.736	0.839	0.906	0.971
	91.03	91.00	90.93	90.83	90.62	90.36	90.02	89.14	88.08	86.91	84.55
94	0.111	0.175	0.252	0.337	0.425	0.514	0.598	0.737	0.842	0.908	0.973
	92.02	92.00	91.94	91.81	91.62	91.34	90.99	90.12	88.99	87.85	85.40
95	0.111	0.175	0.252	0.338	0.430	0.518	0.601	0.743	0.844	0.911	0.974
	93.02	93.01	92.93	92.80	92.57	92.30	91.96	91.03	89.96	88.75	86.30
96	0.111	0.175	0.253	0.340	0.430	0.519	0.604	0.746	0.845	0.914	0.974
	93.99	93.98	93.91	93.78	93.59	93.30	92.92	91.98	90.89	89.64	87.21

Table 40: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
97	0.111	0.176	0.254	0.341	0.433	0.525	0.605	0.748	0.850	0.914	0.976
	94.98	94.98	94.90	94.79	94.55	94.25	93.91	92.97	91.82	90.62	88.13
98	0.111	0.176	0.255	0.342	0.435	0.525	0.608	0.751	0.852	0.918	0.976
	95.98	95.96	95.89	95.78	95.53	95.24	94.88	93.90	92.78	91.50	89.00
99	0.111	0.177	0.256	0.345	0.437	0.528	0.612	0.755	0.855	0.919	0.978
	96.98	96.94	96.88	96.73	96.52	96.22	95.85	94.86	93.69	92.42	89.86
100	0.111	0.177	0.256	0.345	0.436	0.530	0.615	0.757	0.857	0.920	0.979
	97.96	97.94	97.89	97.75	97.53	97.20	96.81	95.81	94.65	93.37	90.78
101	0.111	0.177	0.258	0.348	0.441	0.531	0.618	0.759	0.860	0.922	0.979
	98.96	98.93	98.86	98.71	98.48	98.21	97.78	96.80	95.56	94.31	91.66
102	0.111	0.178	0.259	0.349	0.442	0.536	0.621	0.761	0.861	0.924	0.980
	99.94	99.92	99.85	99.71	99.47	99.13	98.76	97.75	96.54	95.19	92.53
103	0.111	0.178	0.260	0.350	0.444	0.538	0.623	0.765	0.865	0.926	0.981
	100.94	100.92	100.83	100.69	100.47	100.12	99.72	98.69	97.44	96.12	93.42
104	0.111	0.179	0.260	0.351	0.446	0.539	0.627	0.767	0.867	0.927	0.981
	101.92	101.89	101.84	101.69	101.44	101.12	100.69	99.65	98.39	97.05	94.35
105	0.111	0.179	0.260	0.353	0.447	0.541	0.628	0.770	0.868	0.930	0.983
	102.92	102.90	102.85	102.67	102.44	102.09	101.67	100.60	99.34	97.97	95.18
106	0.111	0.179	0.262	0.354	0.451	0.544	0.629	0.772	0.872	0.931	0.983
	103.91	103.89	103.80	103.66	103.40	103.07	102.67	101.56	100.26	98.90	96.07
107	0.111	0.179	0.263	0.356	0.452	0.546	0.635	0.777	0.873	0.932	0.983
	104.90	104.88	104.80	104.64	104.40	104.05	103.59	102.50	101.21	99.81	96.98
108	0.111	0.180	0.264	0.357	0.454	0.549	0.636	0.778	0.874	0.934	0.984
	105.89	105.87	105.78	105.63	105.38	105.02	104.58	103.46	102.16	100.74	97.89
109	0.111	0.180	0.265	0.357	0.457	0.551	0.637	0.781	0.877	0.936	0.984
	106.89	106.85	106.78	106.62	106.35	106.01	105.57	104.42	103.08	101.62	98.77
110	0.111	0.180	0.264	0.361	0.457	0.552	0.641	0.783	0.880	0.936	0.985
	107.88	107.86	107.78	107.59	107.34	107.00	106.52	105.39	103.99	102.57	99.63
111	0.111	0.181	0.266	0.361	0.460	0.555	0.643	0.786	0.882	0.938	0.985
	108.86	108.85	108.75	108.59	108.32	107.95	107.49	106.31	104.92	103.46	100.55
112	0.111	0.181	0.267	0.363	0.462	0.558	0.647	0.788	0.883	0.939	0.986
	109.85	109.84	109.74	109.56	109.30	108.92	108.45	107.28	105.89	104.39	101.43
113	0.111	0.182	0.268	0.364	0.463	0.560	0.649	0.792	0.885	0.941	0.986
	110.86	110.82	110.74	110.56	110.30	109.92	109.43	108.23	106.86	105.32	102.31
114	0.111	0.182	0.268	0.366	0.465	0.562	0.651	0.794	0.886	0.943	0.987
	111.84	111.81	111.74	111.55	111.28	110.89	110.40	109.17	107.76	106.19	103.18
115	0.111	0.183	0.270	0.367	0.468	0.563	0.653	0.796	0.890	0.943	0.987
	112.84	112.79	112.70	112.52	112.25	111.88	111.39	110.13	108.67	107.15	104.08
116	0.111	0.183	0.271	0.367	0.469	0.567	0.657	0.797	0.890	0.945	0.988
	113.82	113.79	113.69	113.54	113.25	112.83	112.33	111.11	109.66	108.06	104.97
117	0.111	0.183	0.271	0.369	0.472	0.571	0.659	0.801	0.893	0.946	0.989
	114.83	114.79	114.69	114.52	114.21	113.79	113.30	112.05	110.55	108.97	105.88
118	0.111	0.184	0.272	0.370	0.473	0.571	0.661	0.805	0.894	0.948	0.989
	115.79	115.76	115.69	115.51	115.21	114.79	114.27	112.96	111.49	109.90	106.75
119	0.111	0.183	0.273	0.372	0.476	0.575	0.662	0.805	0.897	0.949	0.989
	116.80	116.78	116.68	116.49	116.17	115.75	115.26	113.97	112.40	110.81	107.67
120	0.111	0.184	0.273	0.374	0.476	0.575	0.665	0.807	0.899	0.950	0.990
	117.77	117.75	117.68	117.47	117.17	116.76	116.24	114.92	113.34	111.71	108.49

Table 40: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
121	0.111	0.184	0.275	0.375	0.479	0.579	0.667	0.809	0.900	0.951	0.990
	118.78	118.75	118.64	118.46	118.14	117.71	117.21	115.88	114.29	112.64	109.41
122	0.111	0.184	0.275	0.376	0.481	0.581	0.671	0.812	0.901	0.953	0.990
	119.76	119.75	119.66	119.45	119.13	118.69	118.16	116.82	115.23	113.54	110.31
123	0.111	0.185	0.276	0.377	0.482	0.581	0.674	0.813	0.902	0.954	0.991
	120.75	120.73	120.63	120.45	120.13	119.69	119.12	117.76	116.18	114.44	111.17
124	0.111	0.185	0.276	0.378	0.485	0.584	0.676	0.817	0.905	0.953	0.991
	121.76	121.73	121.62	121.43	121.10	120.65	120.08	118.69	117.10	115.44	112.06
125	0.111	0.185	0.278	0.381	0.486	0.587	0.677	0.819	0.906	0.954	0.991
	122.74	122.74	122.60	122.41	122.08	121.63	121.06	119.64	118.02	116.31	112.95
126	0.111	0.186	0.279	0.382	0.488	0.588	0.681	0.821	0.909	0.957	0.992
	123.75	123.71	123.60	123.38	123.06	122.61	122.02	120.62	118.95	117.20	113.82
127	0.111	0.186	0.279	0.382	0.490	0.592	0.683	0.823	0.910	0.958	0.992
	124.72	124.69	124.61	124.40	124.04	123.58	122.99	121.56	119.90	118.11	114.75
128	0.111	0.187	0.280	0.385	0.491	0.592	0.684	0.823	0.910	0.959	0.992
	125.72	125.69	125.58	125.35	125.03	124.57	123.97	122.53	120.87	119.06	115.61
129	0.111	0.187	0.281	0.386	0.493	0.596	0.686	0.825	0.913	0.959	0.993
	126.72	126.68	126.58	126.36	126.02	125.53	124.93	123.49	121.76	119.95	116.47
130	0.111	0.188	0.282	0.386	0.496	0.596	0.690	0.829	0.915	0.961	0.993
	127.70	127.66	127.56	127.35	126.98	126.53	125.89	124.41	122.66	120.87	117.39
131	0.111	0.188	0.282	0.389	0.498	0.599	0.691	0.831	0.916	0.961	0.993
	128.70	128.66	128.55	128.32	127.96	127.49	126.87	125.37	123.61	121.82	118.30
132	0.111	0.188	0.283	0.389	0.499	0.602	0.694	0.833	0.919	0.961	0.993
	129.69	129.67	129.55	129.33	128.96	128.46	127.82	126.30	124.52	122.73	119.18
133	0.111	0.188	0.284	0.391	0.501	0.603	0.694	0.835	0.918	0.962	0.993
	130.69	130.65	130.54	130.30	129.92	129.45	128.83	127.26	125.49	123.66	120.04
134	0.111	0.188	0.284	0.392	0.501	0.607	0.697	0.837	0.920	0.964	0.994
	131.66	131.64	131.53	131.30	130.94	130.40	129.80	128.23	126.42	124.53	120.92
135	0.111	0.189	0.286	0.393	0.504	0.607	0.700	0.838	0.922	0.965	0.994
	132.65	132.63	132.51	132.29	131.89	131.39	130.74	129.18	127.32	125.43	121.79
136	0.111	0.189	0.287	0.395	0.505	0.609	0.702	0.840	0.924	0.966	0.994
	133.65	133.64	133.49	133.28	132.89	132.38	131.71	130.10	128.25	126.33	122.70
137	0.111	0.190	0.287	0.397	0.508	0.611	0.704	0.842	0.924	0.967	0.995
	134.64	134.62	134.50	134.24	133.86	133.34	132.68	131.07	129.19	127.26	123.61
138	0.111	0.190	0.288	0.398	0.508	0.615	0.707	0.844	0.925	0.967	0.995
	135.64	135.61	135.49	135.24	134.86	134.31	133.63	132.01	130.13	128.18	124.46
139	0.111	0.190	0.289	0.401	0.509	0.616	0.708	0.845	0.927	0.968	0.995
	136.64	136.60	136.46	136.19	135.86	135.28	134.62	132.97	131.07	129.10	125.31
140	0.111	0.191	0.289	0.399	0.513	0.618	0.710	0.849	0.928	0.968	0.995
	137.61	137.59	137.47	137.23	136.81	136.26	135.58	133.89	132.01	130.02	126.26
141	0.111	0.191	0.289	0.401	0.514	0.619	0.710	0.850	0.930	0.969	0.995
	138.62	138.59	138.48	138.20	137.81	137.26	136.58	134.85	132.91	130.97	127.16
142	0.111	0.191	0.292	0.403	0.515	0.621	0.715	0.851	0.930	0.969	0.995
	139.62	139.58	139.45	139.19	138.79	138.25	137.52	135.82	133.89	131.88	127.94
143	0.111	0.191	0.292	0.404	0.518	0.624	0.715	0.855	0.932	0.970	0.996
	140.58	140.58	140.44	140.19	139.77	139.18	138.52	136.73	134.77	132.77	128.87
144	0.111	0.192	0.292	0.405	0.518	0.625	0.718	0.856	0.932	0.971	0.996
	141.60	141.57	141.44	141.17	140.76	140.17	139.48	137.67	135.77	133.71	129.78

Table 40: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
145	0.111	0.192	0.293	0.406	0.522	0.629	0.720	0.856	0.933	0.971	0.996
	142.60	142.56	142.43	142.16	141.72	141.13	140.43	138.66	136.68	134.63	130.64
146	0.111	0.193	0.295	0.408	0.523	0.631	0.724	0.858	0.935	0.972	0.996
	143.57	143.54	143.40	143.14	142.70	142.11	141.38	139.60	137.61	135.51	131.56
147	0.111	0.193	0.295	0.408	0.524	0.631	0.723	0.860	0.937	0.974	0.996
	144.58	144.54	144.40	144.13	143.69	143.10	142.38	140.55	138.47	136.39	132.42
148	0.111	0.193	0.296	0.410	0.526	0.633	0.726	0.862	0.937	0.974	0.996
	145.56	145.53	145.37	145.11	144.68	144.06	143.33	141.50	139.40	137.32	133.28
149	0.111	0.193	0.296	0.413	0.527	0.633	0.729	0.863	0.939	0.975	0.996
	146.56	146.52	146.40	146.07	145.65	145.08	144.30	142.46	140.32	138.24	134.18
150	0.111	0.194	0.298	0.413	0.529	0.636	0.730	0.866	0.939	0.975	0.997
	147.54	147.50	147.36	147.08	146.64	146.03	145.25	143.38	141.31	139.14	135.09
155	0.111	0.195	0.301	0.421	0.538	0.647	0.740	0.873	0.945	0.978	0.997
	152.53	152.46	152.32	151.99	151.54	150.89	150.09	148.12	145.96	143.67	139.49
160	0.111	0.197	0.306	0.426	0.545	0.656	0.748	0.881	0.948	0.981	0.998
	157.47	157.42	157.26	156.95	156.46	155.77	154.93	152.87	150.62	148.26	143.90
165	0.111	0.198	0.310	0.431	0.553	0.663	0.758	0.887	0.953	0.982	0.998
	162.44	162.39	162.20	161.90	161.37	160.65	159.75	157.61	155.25	152.83	148.34
170	0.111	0.200	0.312	0.436	0.562	0.674	0.767	0.894	0.957	0.984	0.998
	167.41	167.37	167.19	166.84	166.26	165.49	164.56	162.33	159.86	157.39	152.71
175	0.111	0.201	0.315	0.443	0.569	0.682	0.776	0.900	0.961	0.986	0.999
	172.36	172.32	172.16	171.77	171.18	170.37	169.36	167.07	164.54	161.92	157.19
180	0.111	0.203	0.319	0.448	0.575	0.688	0.781	0.905	0.964	0.988	0.999
	177.31	177.28	177.09	176.71	176.10	175.28	174.24	171.78	169.13	166.47	161.59
185	0.111	0.204	0.324	0.454	0.582	0.697	0.790	0.911	0.967	0.989	0.999
	182.28	182.26	182.03	181.64	181.01	180.13	179.04	176.51	173.77	171.03	166.00
190	0.111	0.206	0.327	0.459	0.591	0.704	0.798	0.918	0.970	0.990	0.999
	187.28	187.20	187.01	186.60	185.89	184.98	183.85	181.20	178.38	175.56	170.42
195	0.111	0.207	0.330	0.466	0.598	0.713	0.805	0.922	0.973	0.992	0.999
	192.23	192.18	191.96	191.50	190.80	189.83	188.66	185.91	182.99	180.07	174.81
200	0.111	0.209	0.333	0.472	0.605	0.719	0.812	0.926	0.975	0.993	1.000
	197.20	197.14	196.92	196.43	195.71	194.71	193.51	190.63	187.61	184.71	179.21
210	0.111	0.211	0.340	0.481	0.617	0.735	0.823	0.934	0.979	0.994	1.000
	207.11	207.08	206.84	206.34	205.52	204.40	203.15	200.09	196.89	193.72	188.01
220	0.111	0.214	0.348	0.492	0.631	0.748	0.837	0.940	0.982	0.995	1.000
	217.07	217.01	216.73	216.19	215.30	214.12	212.74	209.51	206.10	202.85	196.83
230	0.111	0.216	0.354	0.503	0.642	0.761	0.849	0.948	0.985	0.996	1.000
	227.00	226.95	226.65	226.05	225.12	223.84	222.32	218.95	215.34	211.87	205.65
240	0.111	0.220	0.360	0.514	0.655	0.773	0.859	0.954	0.988	0.997	1.000
	236.91	236.85	236.57	235.90	234.93	233.53	231.93	228.32	224.55	221.00	214.42
250	0.111	0.222	0.366	0.523	0.667	0.783	0.868	0.959	0.990	0.998	1.000
	246.88	246.81	246.48	245.79	244.68	243.28	241.56	237.73	233.81	230.09	223.20

Table 41: Table of Monte Carlo Results for $\mathcal{M}_{\mathrm{BK}}$ when $k = 10$ and $\alpha = 0.05$.

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
1	0.100	0.109	0.118	0.126	0.135	0.143	0.151	0.167	0.182	0.196	0.224
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	0.100	0.109	0.118	0.126	0.135	0.143	0.151	0.167	0.182	0.196	0.224
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
3	0.100	0.110	0.120	0.129	0.139	0.148	0.158	0.177	0.195	0.213	0.248
	2.90	2.90	2.90	2.90	2.90	2.90	2.90	2.89	2.89	2.89	2.88
4	0.100	0.111	0.123	0.134	0.146	0.157	0.169	0.192	0.215	0.237	0.281
	3.62	3.62	3.62	3.62	3.62	3.62	3.61	3.60	3.60	3.59	3.56
5	0.100	0.113	0.126	0.140	0.153	0.167	0.180	0.208	0.234	0.260	0.313
	4.52	4.52	4.52	4.52	4.52	4.52	4.51	4.50	4.50	4.49	4.47
6	0.100	0.114	0.129	0.144	0.159	0.174	0.190	0.220	0.250	0.280	0.338
	5.36	5.37	5.36	5.36	5.35	5.35	5.35	5.35	5.34	5.33	5.30
7	0.100	0.115	0.131	0.148	0.164	0.180	0.197	0.230	0.264	0.296	0.359
	6.31	6.31	6.32	6.31	6.31	6.31	6.30	6.29	6.29	6.27	6.25
8	0.100	0.116	0.133	0.150	0.168	0.186	0.203	0.239	0.275	0.310	0.377
	7.32	7.32	7.31	7.31	7.31	7.31	7.31	7.29	7.28	7.26	7.23
9	0.100	0.117	0.135	0.153	0.172	0.191	0.210	0.248	0.286	0.324	0.396
	8.35	8.34	8.34	8.34	8.33	8.33	8.32	8.31	8.29	8.27	8.22
10	0.100	0.118	0.137	0.156	0.176	0.196	0.217	0.258	0.298	0.339	0.418
	9.33	9.34	9.33	9.33	9.32	9.32	9.31	9.29	9.27	9.24	9.17
11	0.100	0.119	0.138	0.159	0.180	0.202	0.223	0.268	0.311	0.355	0.438
	10.30	10.30	10.29	10.29	10.28	10.28	10.27	10.24	10.23	10.19	10.11
12	0.100	0.120	0.140	0.162	0.184	0.207	0.229	0.277	0.323	0.369	0.455
	11.25	11.25	11.25	11.25	11.23	11.22	11.22	11.19	11.17	11.13	11.04
13	0.100	0.121	0.142	0.165	0.188	0.212	0.236	0.286	0.335	0.381	0.474
	12.20	12.19	12.19	12.20	12.19	12.18	12.17	12.14	12.11	12.08	11.97
14	0.100	0.121	0.144	0.167	0.192	0.217	0.243	0.293	0.345	0.395	0.489
	13.17	13.17	13.16	13.16	13.16	13.15	13.13	13.11	13.07	13.02	12.92
15	0.100	0.122	0.145	0.170	0.195	0.221	0.248	0.302	0.355	0.406	0.505
	14.15	14.14	14.15	14.15	14.13	14.13	14.11	14.08	14.04	13.99	13.86
16	0.100	0.123	0.147	0.172	0.199	0.226	0.254	0.309	0.364	0.419	0.518
	15.14	15.14	15.13	15.13	15.12	15.11	15.09	15.06	15.01	14.95	14.82
17	0.100	0.123	0.148	0.175	0.202	0.230	0.260	0.317	0.375	0.431	0.535
	16.13	16.13	16.12	16.12	16.10	16.09	16.07	16.03	15.98	15.92	15.76
18	0.100	0.124	0.150	0.177	0.205	0.234	0.263	0.324	0.385	0.442	0.550
	17.12	17.11	17.10	17.10	17.09	17.08	17.06	17.01	16.95	16.88	16.69
19	0.100	0.125	0.151	0.179	0.208	0.239	0.270	0.332	0.392	0.453	0.564
	18.09	18.08	18.09	18.07	18.07	18.04	18.02	17.98	17.92	17.84	17.65
20	0.100	0.125	0.153	0.182	0.212	0.244	0.275	0.341	0.403	0.466	0.576
	19.06	19.06	19.05	19.05	19.03	19.02	18.99	18.93	18.87	18.78	18.59
21	0.100	0.126	0.154	0.184	0.215	0.248	0.281	0.346	0.413	0.476	0.590
	20.04	20.03	20.03	20.01	20.01	19.99	19.96	19.92	19.84	19.75	19.52
22	0.100	0.127	0.155	0.186	0.218	0.252	0.285	0.355	0.422	0.486	0.603
	21.01	21.01	21.01	21.00	20.99	20.97	20.95	20.87	20.79	20.69	20.46
23	0.100	0.127	0.157	0.188	0.221	0.256	0.290	0.362	0.431	0.497	0.614
	22.00	22.00	21.99	21.97	21.97	21.95	21.92	21.84	21.76	21.65	21.41
24	0.100	0.128	0.158	0.190	0.225	0.261	0.296	0.367	0.439	0.507	0.626
	22.98	22.98	22.97	22.97	22.95	22.91	22.89	22.83	22.73	22.61	22.33

Table 41: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
25	0.100	0.128	0.159	0.192	0.227	0.264	0.300	0.374	0.448	0.517	0.638
	23.97	23.96	23.96	23.95	23.93	23.90	23.88	23.80	23.69	23.57	23.28
26	0.100	0.129	0.161	0.194	0.231	0.268	0.306	0.383	0.456	0.526	0.649
	24.96	24.96	24.94	24.93	24.90	24.88	24.85	24.77	24.66	24.53	24.21
27	0.100	0.129	0.162	0.197	0.233	0.272	0.311	0.388	0.464	0.536	0.658
	25.93	25.94	25.92	25.91	25.89	25.86	25.82	25.74	25.62	25.48	25.16
28	0.100	0.130	0.163	0.199	0.237	0.275	0.315	0.395	0.472	0.546	0.671
	26.92	26.92	26.91	26.89	26.87	26.85	26.81	26.71	26.58	26.44	26.08
29	0.100	0.130	0.164	0.201	0.239	0.279	0.320	0.402	0.481	0.555	0.682
	27.90	27.90	27.89	27.87	27.85	27.82	27.78	27.67	27.54	27.38	27.00
30	0.100	0.131	0.165	0.203	0.242	0.283	0.324	0.408	0.489	0.563	0.691
	28.88	28.88	28.87	28.84	28.82	28.79	28.76	28.65	28.50	28.34	27.94
31	0.100	0.131	0.167	0.204	0.244	0.286	0.329	0.413	0.497	0.572	0.700
	29.86	29.86	29.85	29.84	29.81	29.78	29.74	29.62	29.47	29.29	28.88
32	0.100	0.132	0.168	0.207	0.248	0.290	0.333	0.420	0.502	0.578	0.708
	30.85	30.85	30.84	30.82	30.79	30.76	30.72	30.60	30.44	30.26	29.81
33	0.100	0.133	0.169	0.208	0.251	0.293	0.338	0.425	0.510	0.588	0.719
	31.84	31.83	31.82	31.81	31.77	31.75	31.68	31.57	31.40	31.21	30.74
34	0.100	0.133	0.170	0.210	0.253	0.298	0.342	0.433	0.518	0.596	0.728
	32.83	32.82	32.81	32.79	32.77	32.72	32.67	32.53	32.35	32.17	31.66
35	0.100	0.134	0.171	0.212	0.256	0.300	0.347	0.438	0.525	0.603	0.737
	33.80	33.80	33.79	33.77	33.74	33.70	33.65	33.51	33.33	33.11	32.59
36	0.100	0.134	0.172	0.214	0.259	0.303	0.353	0.444	0.533	0.612	0.744
	34.79	34.79	34.78	34.76	34.72	34.69	34.61	34.47	34.29	34.06	33.51
37	0.100	0.134	0.173	0.215	0.261	0.308	0.356	0.449	0.539	0.621	0.753
	35.77	35.77	35.77	35.74	35.70	35.66	35.60	35.46	35.25	35.00	34.45
38	0.100	0.135	0.175	0.218	0.264	0.311	0.359	0.456	0.547	0.627	0.760
	36.76	36.76	36.74	36.72	36.68	36.64	36.58	36.41	36.21	35.96	35.36
39	0.100	0.135	0.176	0.220	0.267	0.316	0.364	0.461	0.552	0.635	0.766
	37.75	37.74	37.73	37.70	37.67	37.61	37.56	37.40	37.18	36.91	36.29
40	0.100	0.136	0.177	0.221	0.269	0.318	0.368	0.467	0.559	0.642	0.773
	38.74	38.72	38.72	38.71	38.65	38.60	38.54	38.36	38.14	37.86	37.24
41	0.100	0.136	0.178	0.223	0.271	0.321	0.372	0.474	0.565	0.649	0.783
	39.72	39.71	39.69	39.67	39.64	39.59	39.51	39.32	39.10	38.81	38.14
42	0.100	0.137	0.179	0.225	0.274	0.325	0.376	0.477	0.572	0.657	0.789
	40.71	40.70	40.69	40.67	40.61	40.56	40.49	40.31	40.06	39.76	39.05
43	0.100	0.137	0.180	0.227	0.276	0.329	0.381	0.485	0.577	0.662	0.795
	41.69	41.69	41.68	41.65	41.60	41.54	41.46	41.26	41.03	40.71	40.00
44	0.100	0.138	0.181	0.228	0.279	0.331	0.384	0.488	0.585	0.671	0.802
	42.68	42.68	42.65	42.63	42.59	42.53	42.45	42.25	41.98	41.65	40.90
45	0.100	0.138	0.182	0.229	0.282	0.334	0.388	0.494	0.590	0.677	0.806
	43.67	43.67	43.65	43.63	43.56	43.51	43.43	43.21	42.95	42.60	41.85
46	0.100	0.139	0.183	0.232	0.284	0.338	0.393	0.500	0.599	0.681	0.813
	44.66	44.64	44.64	44.60	44.55	44.49	44.40	44.17	43.88	43.56	42.75
47	0.100	0.139	0.184	0.233	0.286	0.341	0.395	0.503	0.603	0.691	0.819
	45.64	45.62	45.62	45.59	45.54	45.47	45.39	45.16	44.86	44.49	43.67
48	0.100	0.140	0.185	0.236	0.288	0.345	0.399	0.509	0.609	0.696	0.826
	46.63	46.62	46.61	46.56	46.52	46.44	46.37	46.12	45.81	45.44	44.57

Table 41: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS $E(N)$	PCS $E(N)$	PCS $E(N)$	PCS $E(N)$	PCS $E(N)$	PCS $E(N)$	PCS $E(N)$	PCS $E(N)$	PCS $E(N)$	PCS $E(N)$	PCS $E(N)$
49	0.100	0.140	0.186	0.237	0.292	0.347	0.404	0.514	0.616	0.700	0.830
	47.61	47.62	47.58	47.56	47.49	47.43	47.34	47.10	46.77	46.40	45.51
50	0.100	0.140	0.187	0.238	0.293	0.351	0.407	0.517	0.620	0.708	0.836
	48.60	48.59	48.57	48.54	48.50	48.41	48.32	48.08	47.73	47.33	46.44
51	0.100	0.141	0.188	0.240	0.296	0.353	0.413	0.526	0.628	0.714	0.841
	49.59	49.59	49.57	49.54	49.48	49.40	49.29	49.02	48.67	48.28	47.34
52	0.100	0.141	0.189	0.242	0.298	0.357	0.416	0.529	0.632	0.721	0.847
	50.58	50.58	50.55	50.52	50.45	50.37	50.27	50.01	49.64	49.22	48.25
53	0.100	0.142	0.190	0.243	0.301	0.359	0.421	0.534	0.636	0.725	0.850
	51.58	51.57	51.54	51.51	51.43	51.36	51.23	50.97	50.61	50.16	49.18
54	0.100	0.142	0.191	0.245	0.302	0.363	0.423	0.539	0.642	0.729	0.856
	52.55	52.56	52.52	52.50	52.44	52.34	52.23	51.94	51.57	51.12	50.08
55	0.100	0.142	0.192	0.246	0.305	0.365	0.426	0.543	0.648	0.735	0.860
	53.54	53.54	53.52	53.48	53.41	53.32	53.21	52.91	52.51	52.05	51.02
56	0.100	0.143	0.193	0.248	0.308	0.369	0.431	0.547	0.653	0.741	0.864
	54.53	54.52	54.49	54.46	54.39	54.30	54.18	53.89	53.48	53.00	51.91
57	0.100	0.143	0.193	0.250	0.310	0.371	0.435	0.555	0.657	0.746	0.867
	55.52	55.52	55.51	55.45	55.38	55.28	55.16	54.83	54.45	53.94	52.87
58	0.100	0.144	0.195	0.251	0.312	0.375	0.438	0.557	0.662	0.750	0.875
	56.52	56.51	56.48	56.44	56.37	56.26	56.13	55.81	55.40	54.91	53.74
59	0.100	0.144	0.195	0.253	0.314	0.377	0.441	0.563	0.667	0.756	0.876
	57.51	57.50	57.47	57.42	57.36	57.25	57.12	56.77	56.36	55.84	54.68
60	0.100	0.144	0.197	0.254	0.316	0.381	0.444	0.567	0.674	0.761	0.881
	58.49	58.48	58.46	58.41	58.34	58.23	58.10	57.75	57.30	56.78	55.57
61	0.100	0.145	0.197	0.256	0.320	0.383	0.448	0.569	0.679	0.766	0.886
	59.48	59.47	59.45	59.40	59.30	59.21	59.07	58.74	58.25	57.71	56.49
62	0.100	0.145	0.198	0.257	0.321	0.387	0.451	0.575	0.683	0.769	0.889
	60.46	60.46	60.44	60.39	60.30	60.18	60.07	59.69	59.21	58.68	57.40
63	0.100	0.145	0.199	0.260	0.323	0.390	0.455	0.579	0.688	0.775	0.893
	61.45	61.46	61.42	61.35	61.28	61.17	61.03	60.66	60.16	59.59	58.31
64	0.100	0.146	0.200	0.261	0.326	0.393	0.458	0.585	0.691	0.781	0.895
	62.44	62.44	62.42	62.36	62.27	62.15	62.01	61.60	61.14	60.51	59.23
65	0.100	0.146	0.201	0.262	0.328	0.396	0.463	0.587	0.698	0.785	0.899
	63.44	63.43	63.40	63.35	63.26	63.13	62.97	62.59	62.06	61.44	60.12
66	0.100	0.147	0.202	0.264	0.330	0.398	0.464	0.592	0.702	0.789	0.903
	64.43	64.41	64.38	64.33	64.24	64.12	63.97	63.56	63.01	62.42	61.03
67	0.100	0.147	0.203	0.265	0.332	0.399	0.468	0.599	0.706	0.792	0.905
	65.41	65.39	65.37	65.32	65.23	65.12	64.95	64.50	63.98	63.36	61.95
68	0.100	0.148	0.204	0.267	0.334	0.403	0.472	0.602	0.711	0.797	0.907
	66.40	66.38	66.35	66.31	66.21	66.09	65.92	65.48	64.93	64.29	62.88
69	0.100	0.148	0.205	0.268	0.337	0.406	0.475	0.604	0.714	0.802	0.912
	67.38	67.38	67.35	67.29	67.19	67.07	66.90	66.46	65.88	65.22	63.77
70	0.100	0.148	0.206	0.270	0.338	0.410	0.479	0.610	0.718	0.805	0.914
	68.38	68.38	68.33	68.27	68.18	68.04	67.86	67.41	66.85	66.16	64.68
71	0.100	0.149	0.207	0.271	0.340	0.412	0.481	0.612	0.724	0.807	0.918
	69.37	69.36	69.32	69.27	69.16	69.03	68.86	68.39	67.78	67.12	65.58
72	0.100	0.149	0.207	0.273	0.342	0.415	0.485	0.616	0.727	0.812	0.919
	70.36	70.35	70.32	70.25	70.16	70.01	69.83	69.36	68.75	68.04	66.47

Table 41: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
73	0.100	0.149	0.208	0.274	0.345	0.418	0.489	0.620	0.731	0.815	0.922
	71.35	71.35	71.31	71.24	71.14	70.98	70.80	70.32	69.71	68.98	67.39
74	0.100	0.150	0.209	0.275	0.347	0.421	0.492	0.625	0.736	0.820	0.923
	72.34	72.32	72.29	72.24	72.13	71.97	71.79	71.29	70.64	69.92	68.34
75	0.100	0.150	0.210	0.277	0.350	0.424	0.495	0.629	0.740	0.824	0.928
	73.33	73.32	73.28	73.22	73.10	72.94	72.76	72.25	71.59	70.85	69.21
76	0.100	0.151	0.210	0.279	0.351	0.425	0.497	0.633	0.743	0.828	0.929
	74.32	74.30	74.28	74.20	74.10	73.93	73.74	73.20	72.54	71.79	70.11
77	0.100	0.151	0.212	0.280	0.352	0.428	0.502	0.636	0.745	0.831	0.932
	75.31	75.30	75.26	75.19	75.09	74.92	74.71	74.18	73.53	72.71	71.01
78	0.100	0.151	0.213	0.281	0.356	0.431	0.504	0.640	0.752	0.834	0.934
	76.30	76.29	76.25	76.20	76.06	75.90	75.69	75.14	74.44	73.67	71.94
79	0.100	0.152	0.213	0.283	0.357	0.433	0.507	0.643	0.753	0.839	0.936
	77.30	77.28	77.25	77.16	77.06	76.88	76.67	76.11	75.42	74.59	72.84
80	0.100	0.152	0.214	0.285	0.359	0.435	0.511	0.648	0.758	0.842	0.938
	78.28	78.28	78.22	78.15	78.03	77.86	77.64	77.06	76.35	75.52	73.74
81	0.100	0.152	0.215	0.286	0.362	0.437	0.516	0.650	0.762	0.844	0.939
	79.27	79.25	79.22	79.14	79.01	78.86	78.60	78.06	77.30	76.47	74.66
82	0.100	0.153	0.216	0.287	0.364	0.441	0.518	0.654	0.766	0.847	0.943
	80.28	80.25	80.22	80.13	80.00	79.82	79.59	79.02	78.25	77.40	75.52
83	0.100	0.153	0.217	0.289	0.366	0.443	0.520	0.658	0.770	0.850	0.944
	81.25	81.24	81.20	81.12	80.99	80.82	80.58	79.97	79.18	78.34	76.45
84	0.100	0.153	0.218	0.290	0.367	0.446	0.522	0.659	0.771	0.853	0.946
	82.23	82.24	82.20	82.11	81.99	81.79	81.57	80.96	80.16	79.27	77.36
85	0.100	0.154	0.218	0.292	0.367	0.449	0.526	0.666	0.774	0.855	0.948
	83.22	83.22	83.18	83.09	82.98	82.77	82.52	81.89	81.11	80.24	78.25
86	0.100	0.154	0.219	0.293	0.372	0.450	0.529	0.670	0.781	0.859	0.949
	84.22	84.20	84.17	84.09	83.94	83.77	83.50	82.84	82.03	81.15	79.17
87	0.100	0.154	0.220	0.294	0.373	0.453	0.533	0.672	0.783	0.862	0.951
	85.21	85.19	85.15	85.08	84.92	84.75	84.47	83.82	82.98	82.06	80.09
88	0.100	0.155	0.221	0.296	0.375	0.456	0.535	0.675	0.786	0.865	0.952
	86.21	86.18	86.15	86.06	85.91	85.72	85.45	84.79	83.94	82.98	80.98
89	0.100	0.155	0.222	0.297	0.377	0.459	0.537	0.681	0.790	0.867	0.953
	87.20	87.19	87.14	87.05	86.90	86.70	86.44	85.73	84.88	83.96	81.89
90	0.100	0.155	0.223	0.299	0.378	0.460	0.540	0.683	0.792	0.870	0.956
	88.19	88.17	88.13	88.03	87.89	87.69	87.41	86.70	85.85	84.86	82.78
91	0.100	0.156	0.223	0.301	0.381	0.464	0.544	0.683	0.795	0.873	0.957
	89.17	89.15	89.13	89.01	88.88	88.66	88.38	87.70	86.79	85.79	83.66
92	0.100	0.156	0.224	0.302	0.383	0.466	0.547	0.687	0.799	0.877	0.958
	90.17	90.14	90.13	90.00	89.87	89.64	89.35	88.64	87.73	86.71	84.60
93	0.100	0.156	0.225	0.302	0.384	0.469	0.547	0.693	0.801	0.878	0.959
	91.18	91.14	91.10	91.02	90.85	90.62	90.36	89.60	88.71	87.64	85.49
94	0.100	0.157	0.226	0.304	0.386	0.471	0.552	0.696	0.804	0.880	0.960
	92.16	92.14	92.08	91.99	91.84	91.60	91.32	90.56	89.64	88.59	86.42
95	0.100	0.157	0.227	0.306	0.390	0.473	0.555	0.699	0.807	0.883	0.962
	93.15	93.13	93.06	92.97	92.80	92.59	92.28	91.52	90.59	89.51	87.27
96	0.100	0.157	0.227	0.307	0.390	0.475	0.556	0.700	0.811	0.886	0.963
	94.14	94.13	94.06	93.96	93.80	93.57	93.29	92.51	91.53	90.44	88.20

Table 41: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
97	0.100	0.158	0.228	0.309	0.392	0.479	0.560	0.705	0.813	0.888	0.965
	95.13	95.11	95.06	94.95	94.79	94.54	94.23	93.45	92.48	91.36	89.07
98	0.100	0.158	0.229	0.309	0.395	0.482	0.563	0.708	0.819	0.890	0.966
	96.12	96.10	96.05	95.95	95.76	95.52	95.22	94.40	93.37	92.32	90.00
99	0.100	0.159	0.230	0.311	0.395	0.482	0.566	0.709	0.819	0.891	0.966
	97.10	97.09	97.03	96.92	96.76	96.52	96.19	95.38	94.36	93.27	90.91
100	0.100	0.159	0.231	0.312	0.399	0.485	0.567	0.713	0.821	0.895	0.968
	98.11	98.09	98.02	97.93	97.74	97.49	97.18	96.35	95.33	94.18	91.80
101	0.100	0.159	0.231	0.313	0.400	0.488	0.571	0.716	0.825	0.896	0.969
	99.10	99.07	99.02	98.91	98.73	98.46	98.14	97.31	96.26	95.11	92.67
102	0.100	0.159	0.232	0.315	0.403	0.490	0.574	0.718	0.825	0.899	0.969
	100.08	100.07	100.00	99.89	99.70	99.44	99.12	98.28	97.23	96.02	93.57
103	0.100	0.160	0.233	0.315	0.403	0.492	0.575	0.721	0.829	0.900	0.970
	101.08	101.06	101.00	100.89	100.70	100.43	100.10	99.23	98.16	96.99	94.50
104	0.100	0.160	0.234	0.317	0.407	0.495	0.578	0.726	0.834	0.904	0.972
	102.07	102.05	101.98	101.87	101.67	101.41	101.09	100.18	99.06	97.86	95.41
105	0.100	0.160	0.235	0.318	0.408	0.498	0.583	0.726	0.834	0.906	0.972
	103.06	103.04	102.97	102.86	102.66	102.38	102.03	101.17	100.05	98.80	96.30
106	0.100	0.161	0.235	0.321	0.410	0.498	0.584	0.732	0.836	0.907	0.973
	104.04	104.04	103.97	103.84	103.65	103.38	103.02	102.10	101.00	99.74	97.22
107	0.100	0.161	0.236	0.321	0.411	0.503	0.587	0.733	0.840	0.909	0.975
	105.04	105.04	104.97	104.82	104.64	104.34	104.00	103.07	101.92	100.67	98.09
108	0.100	0.161	0.237	0.323	0.413	0.504	0.590	0.737	0.842	0.910	0.976
	106.03	106.01	105.95	105.82	105.62	105.33	104.97	104.01	102.87	101.62	98.98
109	0.100	0.162	0.237	0.325	0.415	0.505	0.593	0.739	0.845	0.913	0.976
	107.03	107.00	106.94	106.80	106.60	106.32	105.95	104.98	103.82	102.53	99.89
110	0.100	0.162	0.238	0.325	0.416	0.507	0.596	0.743	0.849	0.915	0.976
	108.01	108.00	107.93	107.81	107.61	107.30	106.90	105.92	104.73	103.44	100.79
111	0.100	0.162	0.239	0.327	0.419	0.511	0.598	0.744	0.849	0.915	0.978
	109.01	108.98	108.93	108.78	108.58	108.28	107.89	106.90	105.71	104.41	101.68
112	0.100	0.162	0.240	0.329	0.421	0.512	0.600	0.747	0.851	0.917	0.978
	109.99	109.98	109.92	109.77	109.55	109.26	108.86	107.86	106.65	105.35	102.60
113	0.100	0.163	0.241	0.329	0.421	0.514	0.602	0.749	0.854	0.919	0.979
	111.00	110.98	110.90	110.76	110.56	110.25	109.84	108.83	107.58	106.24	103.47
114	0.100	0.163	0.242	0.331	0.424	0.517	0.606	0.751	0.856	0.921	0.979
	111.98	111.97	111.88	111.74	111.53	111.22	110.79	109.81	108.53	107.19	104.36
115	0.100	0.163	0.241	0.332	0.427	0.519	0.608	0.755	0.858	0.924	0.980
	112.98	112.97	112.91	112.75	112.50	112.21	111.78	110.75	109.48	108.10	105.30
116	0.100	0.164	0.243	0.333	0.428	0.522	0.610	0.755	0.860	0.924	0.980
	113.97	113.96	113.87	113.73	113.50	113.18	112.74	111.73	110.43	108.99	106.20
117	0.100	0.164	0.244	0.334	0.429	0.523	0.611	0.761	0.862	0.925	0.981
	114.96	114.93	114.86	114.72	114.50	114.17	113.75	112.65	111.35	109.98	107.07
118	0.100	0.164	0.244	0.335	0.432	0.526	0.614	0.764	0.863	0.928	0.983
	115.96	115.95	115.86	115.72	115.47	115.14	114.72	113.60	112.32	110.87	107.94
119	0.100	0.164	0.245	0.337	0.433	0.528	0.617	0.764	0.867	0.928	0.983
	116.93	116.93	116.84	116.70	116.46	116.11	115.69	114.59	113.25	111.82	108.86
120	0.100	0.165	0.246	0.337	0.435	0.530	0.619	0.769	0.867	0.930	0.983
	117.94	117.91	117.83	117.69	117.45	117.10	116.67	115.51	114.21	112.75	109.78

Table 41: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
121	0.100	0.165	0.246	0.339	0.437	0.532	0.622	0.768	0.870	0.932	0.984
	118.93	118.91	118.85	118.67	118.42	118.08	117.64	116.52	115.15	113.65	110.66
122	0.100	0.166	0.248	0.340	0.438	0.533	0.624	0.772	0.873	0.933	0.984
	119.94	119.89	119.81	119.67	119.41	119.09	118.62	117.46	116.06	114.57	111.57
123	0.100	0.166	0.248	0.341	0.439	0.537	0.627	0.775	0.874	0.935	0.985
	120.92	120.90	120.82	120.66	120.42	120.04	119.57	118.42	117.04	115.48	112.45
124	0.100	0.166	0.249	0.342	0.443	0.539	0.629	0.777	0.877	0.936	0.985
	121.91	121.89	121.79	121.66	121.37	121.01	120.56	119.38	117.96	116.44	113.36
125	0.100	0.167	0.249	0.345	0.442	0.539	0.630	0.778	0.878	0.937	0.985
	122.88	122.85	122.80	122.62	122.38	122.02	121.55	120.35	118.90	117.36	114.22
126	0.100	0.166	0.250	0.345	0.445	0.542	0.633	0.780	0.881	0.939	0.986
	123.89	123.88	123.78	123.62	123.35	122.98	122.52	121.32	119.85	118.29	115.14
127	0.100	0.167	0.251	0.347	0.447	0.544	0.637	0.784	0.883	0.939	0.986
	124.89	124.87	124.77	124.61	124.34	123.97	123.46	122.25	120.78	119.24	116.07
128	0.100	0.167	0.251	0.348	0.448	0.546	0.637	0.787	0.884	0.941	0.987
	125.88	125.85	125.78	125.59	125.33	124.94	124.46	123.19	121.72	120.11	116.95
129	0.100	0.168	0.253	0.350	0.450	0.551	0.640	0.788	0.886	0.942	0.988
	126.88	126.84	126.75	126.57	126.31	125.88	125.42	124.17	122.65	121.06	117.83
130	0.100	0.168	0.253	0.351	0.454	0.551	0.642	0.789	0.887	0.943	0.988
	127.88	127.84	127.74	127.56	127.27	126.90	126.41	125.14	123.61	122.00	118.74
131	0.100	0.169	0.255	0.351	0.453	0.554	0.645	0.791	0.887	0.945	0.988
	128.86	128.82	128.72	128.56	128.30	127.86	127.38	126.09	124.55	122.90	119.65
132	0.100	0.169	0.255	0.353	0.456	0.557	0.649	0.794	0.891	0.946	0.989
	129.83	129.82	129.73	129.54	129.24	128.84	128.33	127.06	125.47	123.84	120.53
133	0.100	0.169	0.255	0.354	0.457	0.556	0.651	0.795	0.892	0.947	0.989
	130.84	130.81	130.73	130.53	130.25	129.86	129.29	128.01	126.42	124.76	121.43
134	0.100	0.169	0.256	0.355	0.459	0.559	0.652	0.798	0.895	0.948	0.990
	131.83	131.83	131.73	131.53	131.22	130.83	130.29	128.96	127.34	125.69	122.34
135	0.100	0.169	0.257	0.356	0.461	0.561	0.653	0.801	0.895	0.949	0.989
	132.83	132.79	132.71	132.52	132.21	131.80	131.27	129.92	128.32	126.60	123.22
136	0.100	0.170	0.257	0.358	0.461	0.563	0.655	0.802	0.897	0.951	0.990
	133.81	133.80	133.71	133.50	133.22	132.79	132.24	130.87	129.24	127.52	124.12
137	0.100	0.170	0.259	0.358	0.464	0.566	0.657	0.804	0.900	0.951	0.990
	134.79	134.77	134.68	134.50	134.18	133.75	133.23	131.84	130.18	128.44	125.03
138	0.100	0.170	0.259	0.361	0.465	0.565	0.661	0.805	0.900	0.951	0.991
	135.79	135.77	135.66	135.47	135.17	134.77	134.17	132.81	131.12	129.37	125.92
139	0.100	0.171	0.260	0.361	0.467	0.569	0.661	0.809	0.902	0.954	0.991
	136.78	136.76	136.68	136.47	136.16	135.72	135.16	133.71	132.05	130.29	126.83
140	0.100	0.171	0.260	0.362	0.468	0.571	0.665	0.809	0.904	0.954	0.992
	137.78	137.76	137.67	137.47	137.15	136.70	136.13	134.73	132.99	131.20	127.67
141	0.100	0.171	0.261	0.364	0.470	0.572	0.666	0.813	0.905	0.955	0.992
	138.78	138.75	138.66	138.45	138.12	137.68	137.10	135.64	133.95	132.14	128.59
142	0.100	0.172	0.262	0.364	0.472	0.575	0.668	0.815	0.905	0.956	0.992
	139.77	139.73	139.65	139.45	139.10	138.66	138.08	136.62	134.92	133.03	129.48
143	0.100	0.172	0.262	0.366	0.474	0.577	0.672	0.816	0.907	0.957	0.992
	140.76	140.75	140.64	140.43	140.08	139.62	139.03	137.57	135.82	133.96	130.38
144	0.100	0.172	0.263	0.367	0.474	0.580	0.674	0.817	0.909	0.957	0.993
	141.77	141.73	141.62	141.41	141.09	140.60	140.00	138.55	136.78	134.92	131.27

Table 41: (continued).

	$\theta = 1$	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.5$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2$	$\theta = 2.2$	$\theta = 2.6$
n	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS	PCS
	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$	$E(N)$
145	0.100	0.172	0.264	0.369	0.475	0.582	0.673	0.821	0.910	0.958	0.993
	142.74	142.72	142.62	142.40	142.09	141.58	141.01	139.47	137.71	135.82	132.16
146	0.100	0.173	0.264	0.369	0.479	0.582	0.677	0.823	0.911	0.960	0.993
	143.74	143.70	143.61	143.40	143.04	142.57	141.97	140.42	138.67	136.70	133.09
147	0.100	0.173	0.265	0.371	0.481	0.585	0.678	0.824	0.913	0.960	0.993
	144.74	144.70	144.60	144.37	144.02	143.55	142.95	141.39	139.59	137.68	133.92
148	0.100	0.173	0.266	0.372	0.481	0.587	0.680	0.826	0.915	0.961	0.993
	145.73	145.70	145.59	145.37	145.02	144.52	143.92	142.34	140.48	138.62	134.85
149	0.100	0.173	0.267	0.372	0.482	0.589	0.683	0.828	0.916	0.963	0.994
	146.73	146.70	146.58	146.38	146.03	145.51	144.89	143.30	141.46	139.48	135.72
150	0.100	0.174	0.267	0.372	0.486	0.590	0.685	0.831	0.918	0.963	0.994
	147.72	147.70	147.59	147.38	146.98	146.48	145.84	144.22	142.36	140.41	136.64
155	0.100	0.175	0.270	0.381	0.492	0.599	0.695	0.837	0.923	0.966	0.995
	152.68	152.64	152.56	152.29	151.91	151.40	150.70	149.04	147.08	145.04	141.11
160	0.100	0.177	0.274	0.386	0.500	0.608	0.704	0.850	0.929	0.970	0.996
	157.64	157.62	157.49	157.23	156.84	156.28	155.56	153.74	151.75	149.66	145.54
165	0.100	0.178	0.277	0.391	0.507	0.618	0.713	0.855	0.935	0.972	0.997
	162.63	162.57	162.45	162.19	161.77	161.15	160.44	158.56	156.46	154.26	150.04
170	0.100	0.179	0.281	0.397	0.515	0.627	0.723	0.862	0.939	0.975	0.997
	167.56	167.54	167.40	167.14	166.69	166.03	165.26	163.34	161.10	158.83	154.50
175	0.100	0.181	0.284	0.401	0.521	0.634	0.732	0.870	0.944	0.978	0.997
	172.55	172.50	172.37	172.09	171.62	170.95	170.11	168.08	165.81	163.46	158.93
180	0.100	0.182	0.287	0.409	0.529	0.641	0.737	0.875	0.948	0.980	0.998
	177.49	177.48	177.34	177.00	176.52	175.85	175.01	172.87	170.45	168.04	163.47
185	0.100	0.183	0.291	0.412	0.537	0.651	0.747	0.883	0.951	0.983	0.998
	182.48	182.45	182.28	181.97	181.44	180.71	179.83	177.61	175.16	172.61	167.92
190	0.100	0.184	0.294	0.418	0.544	0.658	0.754	0.890	0.957	0.984	0.998
	187.45	187.42	187.25	186.93	186.37	185.62	184.67	182.34	179.78	177.25	172.35
195	0.100	0.186	0.297	0.423	0.551	0.668	0.765	0.895	0.959	0.986	0.999
	192.41	192.38	192.23	191.87	191.28	190.47	189.47	187.12	184.52	181.80	176.82
200	0.100	0.187	0.300	0.429	0.558	0.673	0.771	0.899	0.962	0.987	0.999
	197.39	197.34	197.16	196.80	196.19	195.36	194.35	191.92	189.15	186.38	181.27
210	0.100	0.190	0.307	0.439	0.571	0.689	0.784	0.910	0.966	0.989	0.999
	207.32	207.28	207.08	206.70	206.03	205.12	204.03	201.36	198.54	195.59	190.16
220	0.100	0.192	0.314	0.448	0.584	0.702	0.798	0.920	0.972	0.991	0.999
	217.29	217.23	217.00	216.59	215.85	214.89	213.70	210.87	207.79	204.80	199.04
230	0.100	0.194	0.319	0.459	0.597	0.716	0.809	0.927	0.977	0.993	1.000
	227.19	227.15	226.93	226.45	225.69	224.64	223.39	220.34	217.07	213.92	208.01
240	0.100	0.197	0.325	0.469	0.608	0.727	0.822	0.935	0.980	0.995	1.000
	237.15	237.09	236.85	236.35	235.54	234.41	233.04	229.81	226.42	223.04	216.93
250	0.100	0.199	0.331	0.478	0.620	0.741	0.833	0.942	0.982	0.996	1.000
	247.09	247.06	246.78	246.23	245.33	244.14	242.70	239.28	235.77	232.27	225.80

APPENDIX E

MULTI-FACTOR $\mathcal{M}_{\text{MGK}}$ RESULTS

Table 42: Multi-Factor $\mathcal{M}_{\text{MGK}}$ Monte Carlo Results for the Symmetric Case when P* = 0.90.

factors	levels	θ	max n	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	individual $\widehat{E(N)}$
2	2	1.2	325	0.900	306.36	298.06
2	2	1.4	95	0.901	85.81	81.89
2	2	1.6	49	0.903	42.87	40.34
2	2	1.8	31	0.901	26.52	24.63
2	2	2.0	23	0.906	19.30	17.81
2	3	1.2	640	0.901	618.66	609.07
2	3	1.4	185	0.902	173.78	168.93
2	3	1.6	93	0.902	85.28	81.98
2	3	1.8	59	0.904	52.96	50.52
2	3	2.0	41	0.899	36.17	34.20
2	4	1.2	970	0.903	947.36	937.37
2	4	1.4	275	0.900	263.04	257.79
2	4	1.6	137	0.902	128.68	125.06
2	4	1.8	85	0.903	78.53	75.73
2	4	2.0	60	0.899	54.62	52.34
2	5	1.2	1315	0.901	1291.66	1281.09
2	5	1.4	370	0.902	357.62	352.02
2	5	1.6	185	0.904	176.14	172.18
2	5	1.8	113	0.901	106.16	103.14
2	5	2.0	80	0.904	74.22	71.70

Table 42: (continued).

factors	levels	θ	max n	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	individual $\widehat{E(N)}$
3	2	1.2	400	0.902	380.07	366.08
3	2	1.4	117	0.901	107.62	100.82
3	2	1.6	61	0.902	54.64	50.16
3	2	1.8	39	0.903	34.25	30.95
3	2	2.0	29	0.909	25.04	22.37
3	3	1.2	750	0.900	728.98	713.36
3	3	1.4	220	0.902	208.76	200.70
3	3	1.6	108	0.896	100.50	95.11
3	3	1.8	69	0.899	63.08	58.97
3	3	2.0	49	0.900	44.12	40.77
3	4	1.2	1130	0.898	1107.88	1091.34
3	4	1.4	320	0.902	308.25	299.67
3	4	1.6	160	0.903	151.75	145.86
3	4	1.8	98	0.892	91.75	87.18
3	4	2.0	69	0.894	63.80	60.08
3	5	1.2	1520	0.900	1497.18	1480.16
3	5	1.4	430	0.902	417.72	408.61
3	5	1.6	210	0.900	201.49	195.22
3	5	1.8	130	0.898	123.33	118.39
3	5	2.0	90	0.895	84.51	80.48
4	2	1.2	455	0.899	435.93	417.61
4	2	1.4	133	0.901	123.53	114.63
4	2	1.6	69	0.902	62.62	56.71
4	2	1.8	45	0.904	40.09	35.64
4	2	2.0	33	0.910	28.95	25.36
4	3	1.2	840	0.900	818.94	798.80

Table 42: (continued).

factors	levels	θ	max n	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	individual $\widehat{E(N)}$
4	3	1.4	245	0.900	233.80	223.37
4	3	1.6	121	0.898	113.42	106.44
4	3	1.8	77	0.898	71.05	65.72
4	3	2.0	55	0.898	50.07	45.71
4	4	1.2	1245	0.899	1223.10	1201.79
4	4	1.4	355	0.898	343.29	332.19
4	4	1.6	175	0.899	166.90	159.35
4	4	1.8	109	0.898	102.72	96.81
4	4	2.0	77	0.900	71.73	66.89
4	5	1.2	1680	0.903	1657.05	1635.28
4	5	1.4	470	0.897	457.97	446.29
4	5	1.6	235	0.903	226.35	218.21
4	5	1.8	143	0.897	136.39	130.06
4	5	2.0	100	0.895	94.52	89.27
5	2	1.2	500	0.901	479.84	457.98
5	2	1.4	146	0.896	135.59	124.94
5	2	1.6	75	0.900	68.63	61.63
5	2	1.8	49	0.905	44.06	38.79
5	2	2.0	35	0.900	31.09	26.89
5	3	1.2	910	0.897	889.02	865.11
5	3	1.4	265	0.903	253.77	241.43
5	3	1.6	131	0.897	123.43	115.15
5	3	1.8	83	0.899	77.09	70.73
5	3	2.0	60	0.906	54.96	49.79
5	4	1.2	1345	0.898	1323.07	1297.83
5	4	1.4	385	0.903	373.14	360.02

Table 42: (continued).

factors	levels	θ	max n	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	individual $\widehat{E(N)}$
5	4	1.6	190	0.903	181.82	172.83
5	4	1.8	117	0.896	110.74	103.80
5	4	2.0	84	0.905	78.63	72.91
5	5	1.2	1785	0.900	1762.68	1736.88
5	5	1.4	500	0.897	488.17	474.55
5	5	1.6	250	0.903	241.45	231.95
5	5	1.8	155	0.902	148.30	140.83
5	5	2.0	107	0.897	101.52	95.42

Table 43: $\mathcal{M}_{\text{MGK}_{\text{BG}}}$ Monte Carlo Results for Two Factors in the Symmetric Case when $P^* = 0.90$, Assuming Both Factors Must Stop Together.

levels	θ	n	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	individual $\widehat{E(N)}$
2	1.2	455	0.923	236.76	166.54
2	1.4	151	0.928	69.19	48.31
2	1.6	59	0.918	36.11	26.56
2	1.8	35	0.913	23.62	18.03
2	2.0	27	0.921	17.26	13.09
3	1.2	960	0.930	477.83	346.42
3	1.4	266	0.928	136.90	98.88
3	1.6	125	0.926	69.13	50.32
3	1.8	71	0.922	44.01	32.63
3	2.0	52	0.923	31.23	23.03
4	1.2	1500	0.932	734.23	537.1
4	1.4	380	0.927	207.21	152.72
4	1.6	180	0.925	103.32	76.06
4	1.8	106	0.924	64.88	47.8
4	2.0	74	0.922	45.19	33.86
5	1.2	2000	0.932	1006.26	741.70
5	1.4	510	0.930	280.50	209.64
5	1.6	240	0.926	138.29	103.53
5	1.8	142	0.925	86.052	64.37
5	2.0	98	0.923	59.928	45.01

Table 44: $\mathcal{M}_{\text{MGK}_{\text{BG}}}$ Monte Carlo Results for Three Factors Each with Two Levels in the Symmetric Case when $P^* = 0.857$, Assuming Both Factors Must Stop Together.

θ	n	$\widehat{\text{PCS}}$	$\widehat{E(N)}$	individual $\widehat{E(N)}$
1.2	455	0.900	282.06	166.54
1.4	151	0.909	83.17	48.31
1.6	59	0.888	41.88	26.56
1.8	35	0.878	26.82	18.03
2.0	27	0.889	19.78	13.09

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