A Real Options Approach to Modeling Investments in Competitive, Dynamic Retail Markets

A thesis
Presented to
The Academic Faculty

by

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In Partial Fulfillment
of the Requirements for the degree of
Doctor of Philosophy in Industrial and Systems Engineering

School of Industrial and Systems Engineering
Georgia Institute of Technology
August 2008

A Real Options Approach to Modeling Investments in Competitive, Dynamic Retail Markets

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To my mother, Homa

and

my father, Cyrus

Acknowledgments

Throughout the course of writing this dissertation, there have been several individuals who helped facilitate its successful completion. First and foremost, I would like to thank my advisor, Bill Rouse, for his unbelievable support. I met Bill first in his decision-making class in spring '04 when I was still a masters student in the Building Construction program at Georgia Tech. His exceptional ability to explain complex concepts in clear and simple language and combine theoretical discussions with interesting case studies developed my interest in the field of decision analysis. Fortunately after the successful completion of his course Bill extended me an offer to continue my PhD in the Tennenbaum Institute, the newly established multidisciplinary center at Georgia Tech. His enthusiastic support gave me confidence to change my major and explore several research ideas before finally focusing on the research topic of this dissertation. Working with him is an invaluable life time experience for me.

I would like to thank my coadvisor, Nicoleta Serban, and my committee member, Doug Bodner, who spent several hours to read my works and provided me great comments and suggestions. I also thank various individuals, some anonymous, who reviewed my thesis and provided me invaluable feedback. Particularly, I would like to thank my reading committee members Leon McGinnis and Godfried Augenbroe for their comments.

I would like to thank my officemate and fellow Ph.D. student, Michael Pennock, who, as the sixth member of my committee, helped me in a number of occasions and provided me critical inputs throughout this process. Also, I would like to thank my

Last but not least, I would like to thank my mother, Homa, and my father, Cyrus. They have simply sacrificed everything they had to ensure that my brother and I enjoy the best education. I dedicate this dissertation to these two exceptional people.

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Summary

The retail industry is considered to be a very competitive industry in the United States since there are so many players in the almost saturated retail markets that provide similar products and services at similar price levels to customers. Market selection has been identified as an important strategy to differentiate a retailer in this competitive market. Therefore in this thesis, we describe a conceptual framework to evaluate retailers' investment opportunities in dynamic, competitive retail markets. The objective is to describe a conceptual investment analysis framework to address the strategic aspects of a retailer's investment opportunity as well as the dynamic uncertainty of a retail market in a single framework. This conceptual framework outlines a strategic view towards retail stores as flexible assets of a retail enterprise. This conceptual framework is general and can be adjusted and applied to investments options in other services.

In addition, we develop an integrated investment analysis approach based on dynamic programming to explore retailers' investment behaviors in dynamic markets. *The objective is to determine retailers' optimal investment thresholds in noncompetitive and competitive markets*. We consider two retailers to illustrate our approach and use a simple game theory treatment to address competition in retail markets. We use our integrated investment analysis model based on a real options methodology to evaluate the apparent tendency for the small discount retailer invests earlier in a new developing market due to the competition effect from the large discount retailer. This early entry gives the small retail a first-mover advantage and delays the big retailer's entry into the competitive market. In addition, we conduct sensitivity analysis to characterize how

significantly the values of our model parameters impact the retailers' investment decisions.

We also develop an integrated investment analysis approach based on contingent claims analysis to explore retailers' investment behaviors in dynamic markets. The objective is *to determine retailers' optimal investment thresholds in noncompetitive and competitive markets. The equivalent risk neutral evaluation approach is presented in this thesis as an extended version of the contingent claims analysis approach, which facilitates the market-oriented valuation of the retailer's investment option in dynamic markets*. Sensitivity analysis is conducted to study how retailers' optimal investment thresholds change as the values of parameters in this equivalent risk neutral evaluation approach change. The relationship between the dynamic programming and the equivalent risk neutral evaluation approach is also summarized in this thesis to identify the similarities and the differences between these two investment analysis approaches. One of the most important objectives of this comparison is *to determine in what market conditions the choice of investment analysis approach is critical and dramatically changes the retailer's optimal investment threshold*.

Finally, we empirically examine an important aspect of our theoretical work that the big retailer invests and opens a store relatively later in markets with a small retailer compared to markets without a small retailer. In addition, the big retailer opens a store at relatively higher retail market potential in markets with a small retailer compared to markets without a small retailer. In this thesis, we discuss some empirical evidence to support these theoretical results. We chose Wal-Mart and Dollar General as the big and small retailers, respectively, in our empirical study. While our empirical results do not

validate the theory, these results do, however, provide supporting evidence for our theoretical work.

Chapter 1

Introduction and Background

The retail industry is considered as an essential component in any industrialized economic system [1]. In 2005 in the United States, retail is the third-largest private industry in number of establishments and number of employees after 'educational services, health care, and social assistance' and 'professional and business services'. The retail trade accounts for approximately 12.4 percent of all business establishments in the United States and for about 11.6 percent of U.S. employment [2].

The retail industry is very competitive in the U.S. This requires that retailers constantly improve their operations and differentiate themselves from their competitors [3]. One of the most important business strategies that any retailer uses to differentiate itself from the other competitors is the selection of appropriate geographic markets to open stores. It is widely accepted that market selection is one of the most basic and significant elements in defining the retail firm's strategy in addition to reducing price and running promotions and loyalty programs [4].

Despite the importance of retail market selection, many retailers have been using qualitative approaches, which are mainly based on experts' opinions, for potential market evaluation [5-9]. Many retailers have widely expressed great interest in the development of appropriate methods and techniques that could help them evaluate systematically the potential markets for investment and development of stores [10, 11].

Therefore, market selection analysis has become an important subject in retail research. There is an extensive body of research in marketing that deals with retail market selection (for a comprehensive review of this literature see [4, 12]). This body of research primarily deals with the determination of the most significant factors that impact the store's performance in a particular market.

However, this body of research does not address retail market selection from the investment analysis point of view, which is an important aspect of decision-making in the retail industry. The retail firm's management recognizes the development of each store in the market as an investment opportunity whose success enhances the financial positions of the retail firm. Thus, it is critical to consider the market selection problem from the investment perspective since this is one of the most important strategies of a retail firm to improve its operational and investment cash flows.

Investment analysis is a classical topic in capital budgeting. Capital budgeting (or investment appraisal) is the planning process used to allocate resources among investment activities on a long-term basis in order to enhance the firm's financial position. Consider that the financial objective of a retail firm is to maximize its shareholders' (residual owners) wealth, which is identified by the price of the retail firm's stock.

This stock price depends on the retail firm's expected profits, which in turn depend on the firm's cash flow over time. The value, timing, and riskiness of this cash flow determine the market value of the firm's stock and subsequently its shareholders' wealth. Hence, the firm should use an appropriate capital budgeting approach that is consistent with the criterion of maximizing the market value of the firm's stock, which helps the

shareholders maximize their wealth and subsequently their utilities of consumptions over time.

Investment analysis in the traditional capital budgeting is usually conducted using the Net Present Value (NPV) approach. NPV is used to select from among multiple alternatives for investment. The retailer chooses the best combinations of alternative markets to open stores based on the projected values of stores' NPVs and its budgeting limitations. There is a similar approach in the traditional capital budgeting approach that is also widely used in practical investment analysis. The Internal Rate of Return (IRR) is a capital budgeting metric used by firms to decide whether they should make investments. It is an indicator of the efficiency of an investment, as opposed to net present value (NPV), which indicates value or magnitude.

However, the NPV calculation is based on some implicit assumptions that reflect the basic inadequacy of this approach. The limitations of the NPV are widely documented in the literature [13-15]. These limitations can be overcome by using a different perspective on investment under uncertainty, which is recognized as real options. It is indicated that the real options methodology is a promising candidate for addressing managerial flexibility and strategic behaviors of decision makers under dynamic uncertainty [13-15].

Options formulations were first appeared in the seminal works of the late Fisher Black, Myron Scholes, and Robert Merton [16, 17]. Their works led to the Black-Scholes formula that determines the foundation for options and derivatives pricing, expanding the scope of options by considering equity as an option on the firm. However, Stewart Myers indicates that the value of the firm itself does depend on its options to develop real assets, for which the term real options have been used [18].

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Real options provides an analytical framework to evaluate management flexibility in decision-making on whether or how to proceed with business investment while it considers the dynamic uncertainty involved in the future values of the underlying factors. There are a number of instances that show the real options theory has already been used on corporate finance and strategy practice and market performance [19-21].

The retailer's flexibility in the investment decision and the ability to revise the original operating strategy (such as the option to expand or contract the store operation) as uncertainty evolves over time, expands the value of an investment opportunity compared to this value under passive management as it is represented in the traditional NPV approach. The real options approach captures the value of the active retailer's strategy, which protects against the unexpected losses due to the bad retail market conditions by deferring or dropping the investment option. The retailer's active strategy also improves the true value of the investment option in the good retail market by expanding the store operation.

Note that the real options approach does not undermine the essential value of the store's expected NPV. In fact, it enhances the evaluation process of the retailer's investment opportunity by adding the value of active management and strategic interaction to the store's expected NPV, which is still the most important component in the value structure. In the next chapter, we show how a real options methodology is applicable to evaluate retailer's investment opportunities to open stores in retail markets.

Our search in the literature was not successful in locating any research that addresses the retailer's market evaluation from the investment analysis perspective. Therefore in this thesis, we describe an analytical framework to evaluate retailers' investment

opportunities in dynamic, competitive retail markets. This framework is based on the real options methodology and outlines a strategic view of retail stores as flexible assets of a retail enterprise. This thesis is structured, as follows.

A conceptual real options framework for retail store investment analysis is presented in Chapter 2. This conceptual investment analysis framework addresses the strategic aspects of retailers' investment opportunities as well as the dynamic uncertainty of retail markets in a single framework. This framework outlines a strategic view towards retail stores as flexible assets of a retail enterprise. In addition, this framework is general and can be adjusted and applied to investment options in other services.

An integrated investment analysis approach based on dynamic programming is developed in Chapter 3 to determine retailers' optimal investment thresholds in noncompetitive and competitive markets. We use this integrated investment analysis approach to evaluate the apparent tendency of the small discount retailer to invest earlier in a new developing market due to the competition effect from the large discount retailer. This early entry gives the small retail a first-mover advantage and delays the big retailer's entry into the competitive market.

In Chapter 4, we revisit the evaluation problem of the retailer's investment option in competitive and noncompetitive markets. An integrated investment analysis approach based on contingent claims analysis is developed in this chapter to determine retailers' optimal investment thresholds. The equivalent risk neutral evaluation approach is presented in this chapter as an extended version of the contingent claims analysis approach, which facilitates the market-oriented valuation of the retailer's investment option in dynamic markets. The relationship between the dynamic programming and the

equivalent risk neutral evaluation approach is also summarized in this chapter to identify the similarity and the difference between these two investment analysis approaches.

In Chapter 5, we empirically examine an important aspect of our theoretical discussions in Chapter 3 and Chapter 4, namely, that the big retailer invests and opens a store relatively later in markets, in which the small retailer has a store opened, compared to markets where the small retailer does not have a store opened. We choose Wal-Mart and Dollar General as the big and small retailers, respectively, in our empirical study. While our empirical results do not validate the theory, these results do, however, provide supporting evidence for our theoretical work. This thesis ends with conclusions and future work that are summarized in Chapter 6.

Chapter 2

A Conceptual Real Options Framework for the Evaluation of a Retailer's Investment Opportunity in a Competitive, Dynamic Market

2.1 Abstract

The retail industry is considered to be a very competitive industry in the United States since there are so many players in the almost saturated retail markets that provide similar products and services at similar price levels to customers. Market selection has been identified as an important strategy to differentiate a retailer in this competitive market. Therefore in this chapter, we describe a conceptual framework to evaluate retailers' investment opportunities in dynamic, competitive retail markets. The objective is to describe a conceptual investment analysis framework to address the strategic aspects of retailers' investment opportunities as well as the dynamic uncertainty of retail markets in a single framework. This conceptual framework outlines a strategic view towards retail stores as flexible assets of a retail enterprise. This conceptual framework is general and can be adjusted and applied to investments options in other services.

2.2 Introduction

The retail industry is considered as an essential component in any industrialized economic system [1]. In 2005 in the United States, retail is the third-largest private industry in number of establishments and number of employees after 'educational services, health care, and social assistance' and 'professional and business services'. The retail trade accounts for approximately 12.4 percent of all business establishments in the United States and for about 11.6 percent of U.S. employment [2]. Figure 2.1 shows the number of full-time equivalent employees in the retail industry for the years 1998 to 2005.
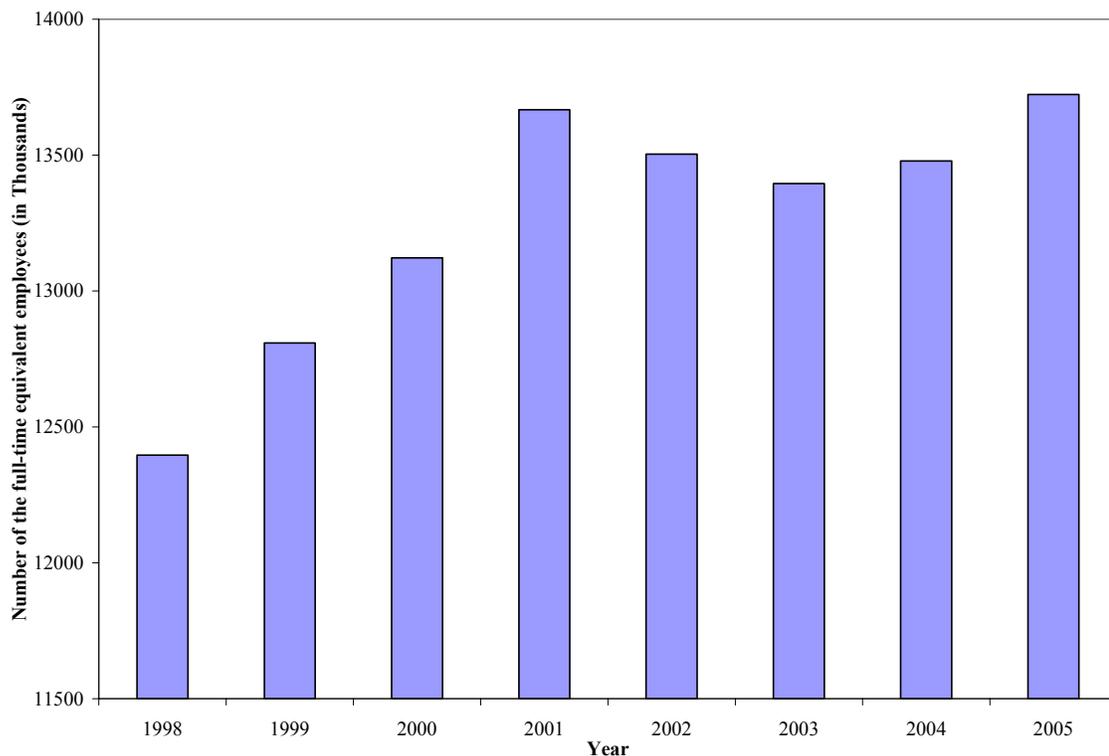
Figure 2.1. Number of the full-time equivalent employees in the retail industry (in Thousands) for the years 1998 to 2005 (Source: Bureau of Economic Analysis, U.S. Department of Commerce, available at http://www.bea.gov/industry/gdpbyind_data.htm)

The U.S. retail industry generates $3.8 trillion in retail sales annually ($4.2 trillion if food service sales are included) [2]. The retail industry is also one of the largest industries

in the United States in terms of the value added to the Gross Domestic Product (GDP). Figure 2.2 shows the retail industry's value added as a percentage of Gross Domestic Product (GDP) for the years 1998 to 2006. It is interesting to observe that the retail industry contribution to the U.S. GDP is decreasing slowly while the number of employees in the retail industry is slowly increasing.
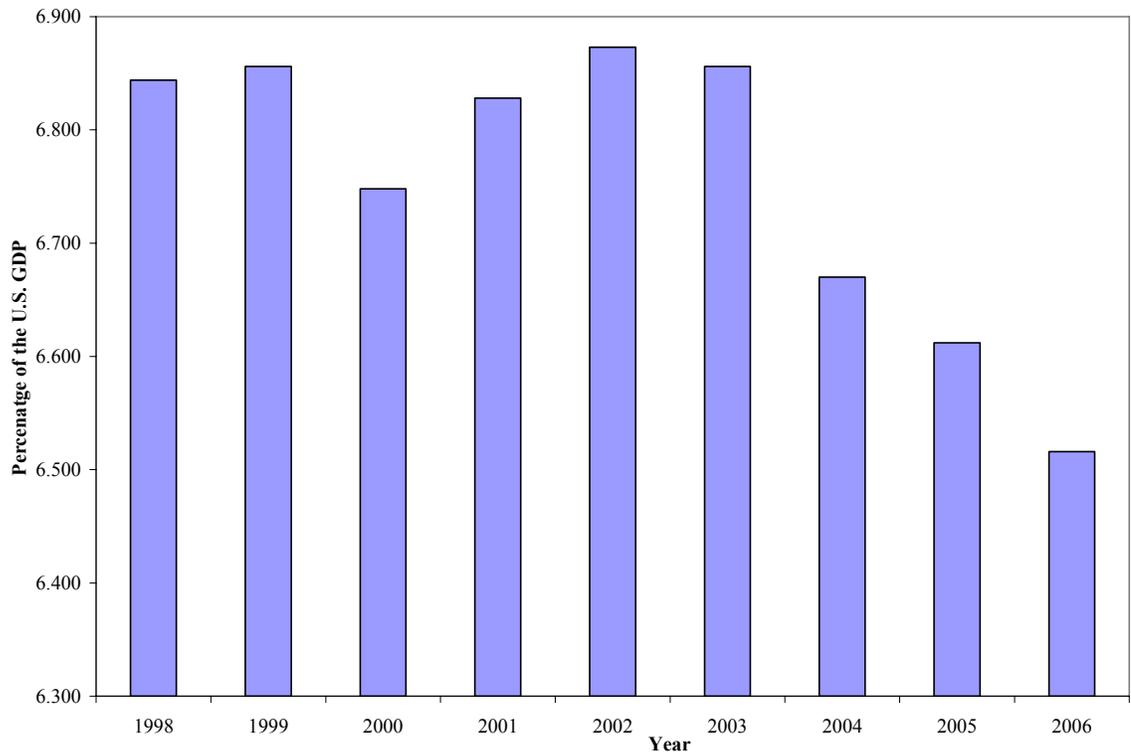
Figure 2.2. The retail industry's value added as a percentage of the U.S. GDP for the years 1998 to 2006 (Source: Bureau of Economic Analysis, U.S. Department of Commerce, available at http://www.bea.gov/industry/gdpbyind_data.htm)

The retail industry is very competitive in the U.S. since there are so many players in the almost saturated retail market that provide similar products and services at similar price levels to customers. Consequently, retailers must constantly improve their operations and differentiate themselves from the other competitors in order to survive in this highly competitive environment [3].

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One of the most important business strategies that any retailer uses to differentiate itself from the other competitors is the selection of appropriate geographic markets to open stores. It is widely accepted that market selection is one of the most basic and significant elements in defining the retail firm's strategy in addition to reducing price and running promotion and loyalty programs [4]. Retailers constantly look for attractive markets in which to invest and develop stores in order to improve their market positions by attracting and retaining more customers.

Despite the importance of retail market selection, many retailers have been using non-systematic approaches, which are mainly based on experts' opinions, for potential market evaluation [5-9]. Many retailers have widely expressed great interest in the development of appropriate methods and techniques that could help them evaluate systematically the potential markets for investment and development of stores [10, 11].

Therefore, market selection analysis has become an important subject in retail research. There is an extensive body of research in marketing that deals with retail market selection (for a comprehensive review of this literature see [4, 12]). This body of research primarily deals with the determination of the most significant factors that impact the store's performance in a particular market. Several standard methods in marketing research have been applied to market selection analysis as summarized below.

1. **The trial and error approach** [4]: this approach is used when there is no need for a systematic method for evaluating store markets (e.g., market growth is guaranteed for a stable market position). Nevertheless, intuitive judgment is important in the practical location decision-making.

2. **Checklist method** [22, 23]: the checklist method provides a systematic procedure for evaluating information about a potential market and facilitates the data collection procedure and to some extent the comparability of information among different potential markets. A typical checklist consists of many market-oriented factors such as socioeconomic and demographic variables, the level of competition, and consumer expenditure patterns.

3. **Analogue procedures** [24]: the first step in analogue procedures is to identify an existing market (or markets) similar to the one that is to be evaluated. Analogue procedures use customer surveys in these similar markets to determine the geographic pattern of trade areas. However, the results of analogue procedures are heavily dependent on the analyst's ability to make judicious selection of analogous markets. Also the method does not directly consider the competitive environment in evaluating the markets.

4. **Direct utility assessment** [25-28]: in this approach, consumer utility functions are estimated from simulated choice data using information integration, conjoint, or logit techniques. These methods build consumer utility functions through consumer evaluations of hypothetical store and market descriptions and not from the past choices. These hypothetical configurations should reflect the entire spectrum of possible values for a store or a market attribute and should be realistic to ensure meaningful responses.

5. **Location allocation models** [29, 30]: Location allocation models systematically evaluate a large number of possible market configurations, assign demand to these locations, and select the one which maximizes the retail firm's financial performance. The competition effect in the retail market is captured in these models by estimating the market share for a new store.

6. **Multiple linear regression models** [5, 31-34]: multiple linear regression analysis formally attempts to define the correlation between sales – or other store performance measures – and the variables influencing performance and develops a statistical relationship between store performance and trade area characteristics expected to influence sales. Several variables are included in regression analysis to explain different aspects of a store's trade area, such as socioeconomic and demographic variables, the level of competition, consumer expenditure patterns, governmental concerns, and store-specific characteristics. However, the definition of the store's trade area and the measurement of the competition effect can be problematic in this approach. These problems should be considered in addition to the general problems of regression analysis such as over-fitting and multi-colinearity when we use regression analysis in practice.

7. **Principal Components Analysis (PCA) and Classification And Regression Trees (CART)** [35]: the financial performance of a retail store is influenced by a large number of factors. Principal Components Analysis (PCA) is used for dimension reduction in the number of variables that are used to explain variations in stores' sales. These components are then used as new latent variables and input to regression models (i.e., new predictors of stores' sales). Classification And Regression Trees (CART) is used to develop a predictive model for stores' sales and clustering.

8. **Revealed preference approaches**: these approaches are based on the notion of consumer utility to explain and predict the retail store patronage among a number of retail stores. Several models are developed using this concept that are summarized below.

8.1. **The Huff model** [36, 37]: this model predicts the probability of visiting a store by a consumer. In its original formulation, this model only used one factor,

which is the store's size as the store attractiveness variable, to predict this probability. However, the Huff model has many extension forms that use more factors to predict the patronage of a particular retail store.

8.2. **Retail gravity models** [38]: in its original formulation, the retail gravity model was used to predict the point between two cities where trade between them would be divided. The form of the model has been transformed to predict the probability that a customer would patronize one of two or more markets, given the relative attraction of these markets and the distance of the customer to each.

8.3. **Spatial interaction models** [39]: Spatial interaction models describe and predict shopping behavior of consumers at the aggregated flow level of shopping trips between origin and destination locations.

8.4. **Multiplicative Competitive Interaction (MCI) models** [30, 40-42]: MCI models use a multiplicative function of several store's attributes as explanatory variables to explain and predict stores' market shares.

8.5. **Discrete choice models such as Multinomial Logit (MNL) models** [43, 44]: Discrete choice models assume that the consumer compares a limited set of discrete choice alternatives and chooses the alternative, which maximizes a utility value. This utility value is decomposed into a structural and a random component. Depending on different assumptions for the distribution of the error terms, one can derive different discrete choice models. One of the most widely used models is the Multinomial Logit (MNL) model that is derived from the assumption that the error terms are independently and identically double

exponential or Gumble distributed. The MNL model is used to specify the probability that the consumer selects to patronize a store (a choice alternative) from a given choice-set of the entire stores.

9. **Artificial Neural Network (ANN) models** [43]: ANN models are non-linear statistical data modeling tools that are used to model complex relationships between inputs and outputs or to find patterns in data. These models are used to predict stores' sales considering the complex interrelationships between store- and market-related variables.

However, this body of research does not address retail market selection from the investment analysis point of view, which is an important aspect of decision-making in the retail industry. When a retailer decides to open a store in a retail market he undertakes an investment activity whose success is revealed over time and depends on the uncertain market conditions. Therefore, the retail firm's management recognizes the development of each store in the market as an investment opportunity whose success enhances the financial position of the retail firm. Thus, it is critical to consider the market selection problem from an investment perspective since this is one of the most important strategies of a retail firm to improve its operational and investment cash flows.

Our search in the literature was not successful to locate any research that addresses the retailer's market evaluation from the investment analysis perspective. Therefore in this chapter, we describe a conceptual framework to evaluate retailers' investment opportunities in dynamic, competitive retail markets. The objective is to describe a conceptual investment analysis framework to address the strategic aspects of retailers' investment opportunities as well as the dynamic uncertainty of retail markets in a single

framework. This conceptual framework outlines a strategic view of retail stores as flexible assets of a retail enterprise. It is indicated that a real options methodology is a promising candidate for addressing managerial flexibility and strategic behaviors of decision makers under dynamic uncertainty [13-15].

Opening a store in a retail market by a firm is an example of an economic investment activity since it consists of an immediate cost (making an investment outlay to develop a store) in the expectation of future return (the store's revenue stream). The retailer's investment activity has the three important features of most investment decisions as outlined by Dixit and Pindyck [15], as follows.

1. **Irreversibility**: after the retailer decides to make an investment outlay and open a store in the retail market, this initial investment is partially sunk and he cannot recover all of this initial cost. We can think of several reasons for partial irreversibility of the investment in the retail market. First of all, most retailer-specific expenses to open the store are clearly sunk costs and cannot be recovered, e.g., the expenses on the outside and inside layout of a store and the other aspects of store operation are retailer-specific and cannot be recovered. The marketing and advertising expenses associated with the new store opening are also retailer-specific and cannot be recovered. On the other hand, if the retailer realizes the retail market is not as profitable as he originally thought he will not be able to sell the store at the price he purchased it or cancel the original lease on the store without paying the cancellation fee. The reason is that if an investment in a retail market becomes unsuccessful for a retailer due to the market condition it is also not an attractive investment for other similar retailers. Therefore, other retailers are not willing to compensate the retailer for its original investment cost in that retail market. In addition,

due to the general institutional agreements of the retail firm, the retailer cannot easily sell the store as an asset and reallocate the funds. The investment in the new workforce is also irreversible due to the substantial hiring, training, and firing. Therefore, the retailer's investment to open a store in the retail market is largely irreversible.

2. **Uncertainty**: the value of the future store's cash flow, which determines the value of the retailer's investment opportunity, is subject to uncertainty.

3. **The choice of timing**: the retailer has the ability to defer his investment option and postpone his decision to open a store as he gets more information about the future retail market. Note, however, that the future store's cash flow is still uncertain.

Several retailers' investment decisions have the above features to some extent. Therefore, it motivates us to use an appropriate investment analysis methodology that recognizes the significance and interaction between the above features in a systematic fashion. It is indicated that an investment analysis methodology, which is based on the financial-type real options is more appropriate than traditional capital budgeting approaches such as Net Present Value (NPV) and the other Discounted Cash Flow (DCF) methods to capture the uncertainty and management flexibility in investment decisions [13-15]. Therefore in this chapter, we develop a conceptual framework to categorize the retailers' investment opportunities in a real options framework. This chapter is structured, as follows.

The objectives of capital budgeting in a firm are discussed in section 2.3. Several methods that have been used in traditional capital budgeting are summarized in section 2.3.1. The major inadequacies of these techniques to evaluate the retailer's investment opportunities are also provided in section 2.3.2. A real options methodology is presented

in section 2.3.3 as an alternative investment analysis approach to overcome the limitations of the traditional capital budgeting approaches. The major differences between traditional capital budgeting approaches and the real options approach are also summarized in section 2.3.

The applicability of financial-type real options models to evaluate retailers' investment opportunities is discussed in section 2.4. The analogy between a real option on a retail store and a call option on a common stock, limitations of this analogy, categorization of investment options on the retail store, modeling the dynamic uncertainty, and evaluation approaches are the subjects of interest in sections 2.4.1, 2.4.2, 2.4.3, 2.4.4, and 2.4.5, respectively. Summary and future work are presented in section 2.5 at the end of this chapter.

2.3 The major differences between the traditional capital budgeting approaches and the real options approach

In this section, we describe the basic inadequacy of traditional capital budgeting approaches based on the NPV calculation in the evaluation of the retailer's investment opportunities.

2.3.1 Capital budgeting

Capital budgeting (or investment appraisal) is the planning process used to allocate resources among investment activities on a long-term basis. The tradeoff between consumption and investment is at the center of any choices that a firm makes regarding its investment opportunities. The financial objective of a firm is to help its shareholders (residual owners) to maximize their utilities of consumptions over time. In the finance

literature, it is commonly accepted that the firm can help shareholders achieve this goal simply by maximizing their shareholders' wealth. It is noted that the shareholders can adjust their income flows and investment portfolios in a perfect and complete capital market to satisfy their particular desirable level of consumption over time.

Therefore, it is a necessary condition that a firm maximizes the value of the portions of the shareholders' wealth that is related to the firm itself namely the market value of their stock. The market value of a stock depends on its expected profits, which depend on the firm's cash flow over time. The value, timing, and riskiness of this cash flow determine the market value of the firm's stock and subsequently its shareholders' wealth. Hence, the firm should use an appropriate capital budgeting approach that is consistent with the criterion of maximizing the market value of the firm's stock, which helps the shareholders maximize their wealth and subsequently their utilities of consumptions over time.

2.3.2 *Traditional capital budgeting*

In this section, we describe different approaches under the traditional capital budgeting that can be used to evaluate retailers' investment opportunities to open a store in a retail market.

Under certainty, the retailer can evaluate investment opportunities based on the NPV calculation, as follows. First, the retailer needs to specify the store's cash flow in terms of the store's expected costs and revenues over time. Then, the NPV of this cash flow will be calculated using an appropriate discount rate of return offered by comparable investment opportunities in the capital market (the opportunity cost of capital). If this NPV is greater than zero, the investment becomes attractive and the retailer could open

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store in the retail market. The investment evaluation under certainty follows this principle that as the store's NPV increases it becomes more attractive for investors.

NPV can also be used to select from among multiple alternatives for investment. The retailer chooses the best combinations of alternative markets to open stores based on the projected values of stores' NPVs and its budgeting limitations. There is a similar approach in the traditional capital budgeting approach that is also widely used in practical investment analysis. The Internal Rate of Return (IRR) is a capital budgeting metric used by firms to decide whether they should make investments. It is an indicator of the efficiency of an investment, as opposed to net present value (NPV), which indicates value or magnitude.

However, the NPV calculation under certainty is based on some implicit assumptions that reflect the basic inadequacy of this approach. The limitations of the NPV are widely documented in the literature [13-15]. Here we provide a brief summary of some of these limitations that are related to our problem of interest, which is to assess the retailer's investment opportunity in the retail market.

1. The NPV assumes that the retailer has to make the investment now or the investment will not be available in future. However, in the real world the retailer usually has the possibility to delay his investment option until new information about the retail market arrives and uncertainty about the future market is reduced. This possibility introduces the value component to the retailer's investment option that impacts the retailer's decision to invest and its timing. Note that *this option value is similar to the value of holding a financial call option on a common stock since the retailer has the right but not the obligation to exercise his investment option and open the store at his desired*

time. We will discuss this analogy in more detail in section 2.4.1. Therefore, when the retailer decides to exercise his investment option and makes an irreversible investment outlay to open a store he sacrifices the opportunity to receive more information about the retail market that may influence his decision to invest and its timing. This lost option value is an opportunity cost that should be included as part of the retailer's investment cost. Unfortunately, the traditional NPV analysis does not take into account this opportunity cost in the evaluation of the investment opportunities. Dixit and Pindyck [15] summarize many empirical studies that show how the value of this opportunity cost may be high in real-world investments and therefore, how the traditional capital budgeting approach can be wrong in explaining (or informing) the investors' behaviors. In addition, the findings from these empirical studies show that the value of the opportunity cost is sensitive to the volatility and uncertainty of the market and economic environment. Therefore, it is also shown that managers only invest in business activities whose values are several times higher than their investment outlays.

2. NPV has implicit assumptions regarding the expected store's cash flow and the retailer's commitment to a predetermined operating strategy. However, *in the real world the retailer revises his original plan for store operation as new information arrives about the retail market*. The retailer's flexibility in the store operation has important impact on the retailer's investment decision and its timing. Unfortunately, the NPV calculation is based on the assumptions that the retailer has to invest immediately, open the store in the retail market, and operate the store until the end of its predetermined life. Under these assumptions, the expected store's cash flow does not change regardless of how the retail market evolves and how the retailer may respond by changing his operating strategy in

the retail market. In the real world the retailer has management flexibility to change the original store operating strategy, e.g., the retailer can expand the store operation when the retail market is proven to be profitable. This retailer can temporarily shut down or contract the store operation when the retail market is proven not satisfactory.

3. *The existence of uncertainty in the future cash flow makes it difficult to apply the NPV approach in the evaluation of the investment options*. In the real world, the future store's cash flow is subject to uncertainty due to changes in the underlying factors such as the costs of labor and materials, the prices and the quantities of goods sold, the retailer's market share, competition, retail market growth, and government-related regulations. Consequently, the future store's cash flow should be characterized by a probability distribution instead of the single value as it would be specified under certainty. It is important to consider the uncertainty in the store's cash flow and the investor's attitude toward this risk since the primary objective of the retail firm is to open stores in retail markets that generate the most desirable cash flows that maximize the retail firm's profit and minimize its investment risk in order to enhance the retail firm's market value (i.e., increase the retail firm's stock values) and, subsequently, the shareholders' wealth.

4. The NPV calculation is based on the choice for the value of the discount rate that is assumed to be exogenous to the retailer's investment evaluation problem, i.e., it is assumed that the retailer can determine a correct discount rate for cash flow discounting for any period of time. *There is a significant need for a comprehensive model to determine the correct value of the discount rate at different periods as the store's cash flow changes over time*. This is probably one of the most important limitations of the NPV approach that will be discussed in more detail later in this section.

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5. *The NPV calculation does not reflect the strategic value of an investment opportunity in the competitive retail market*. Sometimes a retailer decides to open a store earlier in a market to take the first-mover advantage and delay the entry of his competitor to the market. The strategic value of this investment decision cannot be properly captured by the NPV calculation.

Many approaches have been developed in traditional capital budgeting to address the above limitations and enhance the evaluation process of investment options under uncertainty. Some of these approaches are specific to capital budgeting and some of them are general and applied into capital budgeting. We summarize three specific capital budgeting approaches and three general approaches in this chapter. Specific capital budgeting approaches are summarized below.

1. **Certainty equivalent approach to risk-adjustment**: in this approach, the uncertain value of the store's cash flow at each period is replaced by its certainty equivalent amount, which has the same present value as the expected value of the uncertain cash flow at that period. It is indicated that the certainty equivalent approach uses the risk-free rate to discount the certainty equivalent cash flow while using the opportunity cost of capital to discount the expected store's cash flow. Therefore, this approach accounts for both the time value of the money and the risk aversion of investors under uncertainty that is needed to compensate for the systematic risk associated with investments in retail markets. However, it is noted that *in the real world business environment it is difficult to determine the certainty equivalent by the traditional approach, particularly when the risk profile changes over time* [14].

2. **Risk-adjusted discount rate approach**: investment evaluation in this approach is similar to the NPV calculation under certainty except that the expected store's cash flow is discounted back using the risk-adjusted discount rate. This discount rate consists of two components: the risk-free rate that accounts for the time value of money and the discount risk premium rate that accounts for the rate that investors need to be compensated for the risk associated with the investment activities. The discount risk premium rate is determined by the retail firm and represents the retail firm's belief about the riskiness level of opening store in a retail market, i.e., the riskier the investment opportunity the higher the discount risk premium rate. This approach is easier to implement than the certainty equivalent approach since one needs not determine the certainty equivalent cash flow and since it uses the original expected value of the uncertain cash flow in the NPV calculation. However, it is noted that *this approach does not provide a systematic method to determine the discount risk premium rate and does not capture the variation of this rate along the lifetime of an investment in a retail market* [14].

3. **The Capital Asset Pricing Model (CAPM)**: this is a standard model in financial economics that is used to determine the theoretically appropriate required rate of return for a particular investment. As it was described earlier, the retail firm's financial objective is to select and open stores in the retail markets in order to maximize the shareholders' wealth. Therefore, the retail firm's investment strategy must be market-oriented with respect to the investment risk and compatible with the shareholders' expectations. There are two components in the total risk of an investment activity: market risk and firm-specific risk. Market risk (also known as systematic or nondiversifiable risk) represents the risk in the return of an investment that is related to the movements in

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the whole economy, i.e., recession, inflation, or budget deficit. The value of this market risk depends on the correlation between the investment's rate of return and the market's rate of return. Market risk is called nondiversifiable risk since the risk cannot be avoided or diversified away since it is related to the broad economic forces. However, the value of market risk is not the same for all investments since different economic sectors respond differently to economic movements and with differing correlation of losses between the return of an investment activity and the return of the total market. The second component of the total risk is firm-specific risk (also known as unique, unsystematic, idiosyncratic, or diversifiable risk). The firm-specific risk represents the risks that are associated with a particular firm or an industry, i.e., the success or failure of an R&D project. Since the shareholders can limit and diversify away this risk by simply holding a sufficiently large portfolio of many investment options in the competitively perfect capital market they do not ask the retail firm to compensate them with any risk premium. The CAPM model determines the expected rate of return that the shareholders require to be compensated for the systematic risk of an investment by the retailer in a retail market. This rate of return is the risk-adjusted discount rate or opportunity cost that should be used by the retail firm in the NPV calculation of an investment opportunity in a retail market as it was described in the risk-adjusted discount rate approach, i.e., the retail firm should only consider opening a store in a retail market that has the rate of return at least equal to the rate specified by the CAPM. To determine the true value of the expected rate of return for any period of time the retail firm needs to estimate the beta value of an investment, which is the ratio of the correlation between the expected rate of return on the investment and the market rate of return to the variance of the market rate of return. In addition to the statistical

measurement difficulties Myers and Turnbull [45] and Trigeorgis [14] summarize several important issues that make the correct estimation of the beta of an investment option difficult. *These difficulties are related to the accurate identification of the length and the growth rate of an investment opportunity, the pattern of an investment's cash flow over time, the process by which the management revises their original operating strategy, and the relationship between forecasted errors for the investment and the market rate of returns.* Since these issues create serious problems in the correct evaluation of investment opportunities, Trigeorgis suggests the use of option-based valuation to evaluate investment options. It is worth noting that the described difficulties are in common to all investment analysis approaches including the financial-type real options valuation approach. However, the real options approach provides a working framework to address and systematically treat these difficulties to provide more reliable results.

In addition to these three specific capital budgeting approaches, we summarize three general methodologies that can be applied in the traditional capital budgeting to evaluate the retailer's investment option under uncertainty, as follows.

1. **Sensitivity analysis**: the uncertainty over the future value of the store's cash flow lies at the uncertainty over the value of the underlying factors such as the value of retail market potential and the discount rate. Sensitivity analysis is an approach to determine extent to which the NPV of an investment is sensitive to the changes of the underlying investment factors and identify the most important variables that have the most significant impacts on the NPV of an investment. This approach determines the most crucial factors that could contribute the most to the variation in the NPV of an investment. These crucial factors become natural candidates for further exploration by the

retail firm to reduce uncertainty over their estimated values. Sensitivity analysis is usually done one factor at a time. It can also be carried out for two or many factors in combination. However, It is noted that the one- or multi-factor sensitivity analysis is unable to capture the interdependence among different factors that impact the NPV of their investment options. Therefore, Monte Carlo simulation is suggested to overcome this limitation.

2. **Monte Carlo simulation**: in this approach, the investment cash flow at each period is determined as a mathematical function of several underlying variables and their interdependencies that determine the value of the investment opportunity. Since some of these underlying variables are uncertain, a probability distribution should be identified for each variable either from historical data or subjectively by experts' opinions to describe the nature of uncertainty for each variable. Monte Carlo simulation can take advantage of sensitivity analysis since the probability distributions need be determined only for the crucial variables to save computational time and add more accuracy to the distribution specification. Monte Carlo simulation is carried out through random sampling from the distributions of the underlying crucial variables and the NPV calculation of the investment opportunity. This process repeats a substantially large number of times in order to determine the distribution of the NPV along with many statistical measures such as the expected value and the variance of the investment NPV. Trigeorgis summarize several limitations for Monte Carlo simulation approach, as follows.

a. The correct specification of the underlying probability distributions is difficult.

b. The simulation model building is complicated and should be done by experts. This makes it difficult for the management to understand the model, and correctly interpret

and commit to the results. However, this is more the limitation of the manager than the methodology and can be overcome by developing uncertainty analysis skills in the management.

c. The outcome of the Monte Carlo simulation is the risk profile of the investment NPV, which does not determine what discount rate has been used in the NPV calculation and does not provide any clear rule for the manager to base their investments on.

d. The risk profile of the investment NPV shows the total variability (total risk) of the investment opportunity, which is different from the investment systematic risk that the shareholders expect to be compensated for.

e. The risk profile of the investment NPV is usually symmetric due to the predetermined operating strategy. However, in the real world management has the flexibility to change the original operating strategy as more information becomes available about the business environment, which in turn makes the value of the investment opportunity asymmetric with negative skew. It is noted that simulation by itself is not appropriate to evaluate the investment value of the investment options that require determining the optimal investment times. Rather, simulation should be used as an aid to determine the risk neutral probability distributions in the real options approach, as will be discussed later.

3. **Decision tree analysis**: a decision-tree is a pictorial approach to structure the decision problem in a hierarchical fashion that accounts for both management flexibility to revise the original operating strategy and investment environment uncertainty as it evolves over time. Decision tree analysis forces the decision maker to be explicit about the sequence and the interdependency of their decisions over time. The investment

environment uncertainty is also shown at distinct, discrete points in time along with the sequence of management decisions. Simple dynamic programming (average out and fold back, starting from the very right end of the tree) is used to evaluate the value of the investment opportunity. The evaluation is based on the expanded expected NPV of the investment option, which consists of the static expected NPV and the total value of the management flexibility. Although decision tree analysis provides a single framework to assess the investment options considering both management flexibility and investment environment uncertainty, it has its own practical limitations. Trigeorgis [14] summarizes these limitations, as follows.

a. Practical investment decision problems are so complicated that makes it difficult to have a manageable decision tree, i.e., there are a large number of choices for management flexibility and their timings. In addition, there are a large number of states for the uncertainty over the investment environment that is usually resolved in a continuous fashion, not at discrete points of time.

b. Selection of the appropriate discount rate is assumed to be exogenous to decision tree analysis. It is an important issue since the discount rate captures the risk attitude of the shareholders over time.

Any of the above approaches has its own limitations as described above. The described limitations can be overcome by using a different perspective on investment under uncertainty, which is recognized as real options.

2.3.3 Real options investment analysis approach

Options formulations were first appeared in the seminal works of the late Fisher Black, Myron Scholes, and Robert Merton [16, 17]. Their works led to the Black-Scholes

formula that determines the foundation for options and derivatives pricing, expanding the scope of options by considering equity as an option on the firm. However, Stewart Myers indicates that the value of the firm itself does depend on its options to develop real assets, for which the term real options have been used [18].

Real options provides an analytical framework to evaluate management flexibility in decision-making concerning whether or how to proceed with business investment while it considers the dynamic uncertainty involved in the future values of the underlying factors. There are a number of instances that show the real options theory has already been used on corporate finance and strategy practice and market performance [19-21].

Trigeorgis [14] explains how management's flexibility to revise their original operating strategy according to the future conditions of the dynamic market represents an asymmetry or skewness in the probability distribution of the NPV. Note that in this thesis, we assume that retailers have flexibility to revise their original operating strategies and defer their investment options. However in practice, this assumption may not be true, for instance, a retailer may not be able to temporarily close or shut down a store when the market is not satisfactory since it may be against the overall corporate strategy or against the retail firm's agreements with its workers.

The retailer's flexibility of the investment timing to open a store in a dynamic market is a control strategy to limit the downside of the investment option since the retailer has the right to defer his investment opportunity until new information about the retail market arrives. If the retailer understands that the retail market is not profitable he can drop his investment opportunity. If the retailer's option is evaluated based on the traditional NPV without appreciating the flexibility value and the fact that the retailer has the right to

defer his investment option, the calculated NPV becomes negative when the market is not profitable. However, by using the real options approach – which explicitly appreciates the retailer's flexibility value – the revised NPV of the retailer's investment opportunity becomes zero when the retail market is not profitable since the retailer does not exercise his investment option and does not open a store in such a non-profitable retail market. This is due to the fact that the retailer is not assumed to be irrevocably committed to an untouchable operating strategy without any right to defer the investment option or revise the original operating strategy. The active management (or flexibility) value is incorporated into the conventional NPV to evaluate the retailer's investment opportunity.

The retailer's flexibility in the investment decision and the ability to revise the original operating strategy (such as the option to expand or contract the store operation) as uncertainty evolves over time expands the value of an investment opportunity compared to this value under passive management as it is represented in the traditional NPV approach. The real options approach captures the value of the active retailer's strategy, which protects against unexpected losses due to the bad retail market conditions by deferring or dropping the investment option. The retailer's active strategy also improves the true value of the investment option in the good retail market by expanding the store operation.

It is noted that the probability distribution of the NPV is reasonably symmetric in the absence of the retailer's active management to defer investment and revise the original operating strategy [14]. The static (passive) expected NPV (the mean of the symmetric distribution of static NPV) would coincide with its most likely estimate (mode). When the retailer's flexibility to defer the investment and revise the original store-operating

strategy is considered in the retailer's investment evaluation, the retailer limits the downside risk by reducing or removing negative cash flows. Therefore, the resulting distribution for the value of the retailer's investment opportunity becomes asymmetric with the positive skew. It is indicated that the expected value of this asymmetric distribution exceeds its mode, which is still the same as the mode of the NPV distribution under the passive management. The difference between the expected value and the mode of this asymmetric distribution is the retailer's flexibility value, which is denoted by Trigeorgis as the option premium [14].

This asymmetric distribution shows the value of the retailer's investment option that we refer to it in this thesis as the Net Option Value (NOV) of the retailer's investment opportunity. Trigeorgis denotes this NOV as the expanded (or strategic) NPV since it incorporates the managerial operating flexibility and strategic adaptability [14].

Hence, real options expand the static valuation of expected future cash flows by introducing the option premium to incorporate the value of flexibility and growth opportunities in an uncertain environment. The term static valuation is used in contrast to the dynamic valuation, which considers the uncertainty in retail markets that change over time. Trigeorgis defines this new expanded NPV using real options methodology (or NOV), as follows:

NOV of the retailer's investment option = "Expanded (strategic) NPV = Standard (static, passive or direct) NPV of the expected cash flows + option premium (value of operating and strategic options from active management and interaction effects of competition, synergy, and inter-project dependence)" [14].

Note that the real options approach does not undermine the essential value of the store's expected NPV. In fact, it enhances the evaluation process of the retailer's investment opportunity by adding the value of active management and strategic interaction to the store's expected NPV, which is still the most important component in the value structure. In the next section, we show how the real options methodology is applicable to evaluate retailer's investment opportunities to open stores in retail markets.

2.4 Applicability of financial-type real options models to evaluate retailers' investment opportunities in dynamic retail markets

In this section, we show how a financial-type real options methodology can be applied to evaluate retailers' investment opportunities in dynamic retail markets.

Consider a retailer that has the option (the right) to invest in a retail market and open a store for a predetermined period of time – note that it is only the right for this retailer and there is no obligation to open the store over this period. Whenever along the span of this investment opportunity the retailer can exercise his investment option, make investment expenditures, and open a store in the retail market.

It is also important to clarify how the retailer can acquire the investment opportunities in the first place. Sometimes, the retailer's investment opportunities are due to the ownership of a land or a property. But in general, the retailer is able to acquire the investment options considering his resources, reputation, market position, and possible scale, all of which have been developed over time [15]. In addition, the retailer may be able to pursue retail business in markets that other individuals or retailers cannot enter due to the substantially large entry expenses and lack of a close distribution center.

After the store becomes open, the retailer can start selling goods to the customers in this retail market. Therefore, this investment outlay generates a stream of revenue for the retailer. Of course, to operate the store the retailer has to spend money to provide necessary goods and services to his customers. The generated free cash flow defines the financial performance of the store and subsequently determines the financial value of this investment opportunity. Note that like any other business investments, the retailer's investment is not without risk since the values of the generated cash flow change partially randomly over time.

Thus, there are two important questions that are meaningful to be asked by this retailer. The first question is *how much it is worth to hold this investment option*. No matter whether and when the retailer decides to exercise his investment option there must be an inherent value to hold the right to open a store over a prespecified period of time since it provides an opportunity to make money through the generated revenue stream by the store. The second question is *about the optimal time to exercise the investment opportunity*. These two interrelated questions describe two significant aspects of a retailer's decision when he faces this investment opportunity. The retailer needs to know how much he should pay to acquire this investment option and decide when (if ever) to exercise his investment opportunity. Thus, one can conclude that the retailer's investment opportunity is analogous to a call option on a common stock.

2.4.1 *Analogy between a real option on a retail store and a call option on a common stock*

In this section, we show how the retailer's investment opportunity is analogous to a call option on a common stock. The retailer's investment option gives the retailer the right

(which the retailer needs not exercise) to make an investment outlay (the exercise price of the option) and open a store in a dynamic retail market. The state of this dynamic retail market determines the financial performance of the retailer's store that is determined by the store's free cash flow. In this thesis, we consider the value of retail market potential as the underlying factor in the retailer's investment option, i.e., the value of the retailer's investment option is derived from this market variable since the value of retail market potential determines the value of the store's free cash flow, which in turn defines the value of the retailer's investment option. In Chapter 3, we will show how the value of retail market potential can be used as an underlying factor in the formulation of store's cash flow in the dynamic retail market. The value of this retail market potential changes stochastically over time which is similar to the dynamic variation in the price of a common stock, which is the underlying asset in financial call options. The analogy between the real option on a retail store and the call option on a common stock is summarized in Table 2.1.

Table 2.1. The analogy between the real option on a retail store and the call option on a common stock.

Call Option on a Common Stock	Real Option on a Retail Store
Current stock price	Current value of retail market potential
Exercise price	Investment cost to develop the store
Time to expiration	Time before the retailer's investment opportunity disappears
Volatility of stock price	Volatility of the value of retail market potential
Risk-free interest rate	Risk-free interest rate
Dividend rate	Rate of opportunity cost

The current value of retail market potential is similar to the current stock price in this analogy since these two factors are the underlying factors whose variations impact the value of the retailer's investment option and the value of the call option, respectively. Note that the value of retail market potential can be retrieved from the retail market using actual demand for each product.

On the other hand, if the retailer decides to use his right and exercises his investment option he must spend some money to develop and open the store. This is similar to the exercise price of a call option on a common stock that a financial option holder has to pay to acquire the underlying stock. The retailer's investment opportunity becomes invalid after a certain period of time just like the call option on a common stock, which is only valid for a predetermined period of time. The value of retail market potential changes randomly over time. The value of the common stock also changes randomly over time. Volatility can be used for both situations to specify the standard deviation of the change in value of a common stock or the value of retail market potential with a specific time horizon. Therefore, volatility can be used to quantify the risk of these investments over that time period. The risk-free interest rate is the interest rate that it is assumed can be obtained by investing in financial instruments with no default risk. This interest rate is in common for both the retailer and the call option holder. Finally, similar to any investment options in the real world the retailer's investment option has a rate of opportunity cost that should be considered in the evaluation process. This rate is similar to the dividend rate on a common stock. We will discuss the significance of this rate of return shortfall in more details in section 4.3.3.

In this perspective, the store has strategic value for the retailer since it generates the revenue stream that determines the value of the retailer's investment option. However, this analogy between a real option on a retail store and a call option on a common stock is not exact. The limitations of the analogy between a real option on a retail store in a dynamic market and a call option on a common stock are discussed in the next section.

2.4.2 Limitations of the analogy between a real option on a retail store and a call option on a common stock

Although the analogy between a real option on a retail store and a call option on a common stock is useful at the conceptual level there are some essential differences between these two investment options that must be considered in the appropriate characterization and evaluation of the retailer's investment opportunities. These differences are documented in the real options literature [14, 15] and can be summarized in particular for the retailer's investment options, as follows.

1. **Nontradability of the real options**: there is a complete financial market for the call options on common stocks so that these options are traded with almost no constraint. However, the retailer's investment option on a store is an inside-firm investment opportunity that may not be transferred to another retailer. There is also no financial market in which the options on the value of retail market potential are traded. Despite this difference, we can still use capital budgeting approaches such as a real options methodology to evaluate the retailer's investment opportunities as if they would be traded in the complete market.

2. **Competitive interactions**: the holder of a call option on a common stock has the propriety right to decide whether and when to exercise his call option. This call option

holder needs not consider the competition effect in the evaluation of his call option since the competition does not affect the value of the underlying asset, which is the price of the common stock. Unlike call options, many of the investment opportunities in the retail markets are shared between several competitors, e.g., there are several retailers that can invest and open stores in a growing market. The entry of other competing retailers impacts the portion of retail market potential that the retailer acquires. Note that this portion of retail market potential is the underlying asset in the retailer's investment option from which the value of the retailer's investment opportunity is derived. Therefore, the existence of the competition effect must be addressed and formally treated in the valuation of the retailer's investment options. However, if the retailer does not expect the entry of the competitors to a particular market the retailer's investment option can be considered proprietary similar to call options on a common stock and therefore, can be evaluated without considering the competition effect.

3. **Interdependence between the real options**: it is noted that holding an option on a common stock is a simple investment option since its value only depends on the price of its underlying stock, for instance, consider the Balck-Scholes formula that is used for pricing European call options, which only depends on the volatility and expected growth of the underlying stock [16] . Many of the retailer's investment options are also simple since their values only depend on the value of the store's retail market potential. However, there are some other investment opportunities that are compound, e.g., strategic investments such as to open one store in a market with keeping an option to open another store in the nearby market if the first investment becomes successful. Strategic consideration should be allocated to the evaluation of these compound options since their

values are derived from their immediate retail market potential as well as the value of their subsequent investment decisions.

4. **The rate of return shortfall for the real options**: it is indicated that the real options on the nontraded assets such as the stores tend to have the rate of return below the equilibrium expected rate of return on the comparable traded assets of equivalent risk such as common stocks [14, 15]. Therefore, a dividend-like adjustment should be included in the evaluation of the retailer's investment options. This issue will be revisited in section 2.4.5.

The above differences appear in many practical retailers' investment options. In the next section, we summarize the retailer's practical investment opportunities in several categories with respect to their strategic characteristics.

2.4.3 Categorization of investment options on the retail store

In this section, we provide a conceptual framework that summarizes a retailer's investment opportunities into a framework that can be used for evaluation considering their similarities to a call-option on a common stock. The categorization helps us organize several aspects of management flexibility to defer an investment opportunity and revise the original operating strategy in the retail market.

In addition, strategic aspects of retailers' investment options are summarized in this categorization. Two strategic aspects of real options are of particular interest here, as follows.

1. **Effects of competition**: the retailer's investment opportunity in a retail market can be proprietary or shared. If the retailer has an exclusive right to manage his investment opportunity in a particular market without any significant effects from the other retailers'

initiatives the retailer's option is considered to be *proprietary* [14]. If the other retailers have entry barriers to a market due to its geographical location or their supply chain configurations, the retailer has the total flexibility to manage his investment and should evaluate his investment option without considering the existence of the expected entry of the other competitors. However, if the other retailers are already in the market or could easily enter the market the retailer must appreciate their existence or expected entries in his investment decisions. Therefore, the investment option is considered to be *shared*.

2. **Relationships between the investment opportunities**: the retailer's investment options can be classified as simple or compound depending on their significant impacts on other retailer's investment opportunities. Some investment opportunities are standalone investments whose values are derived from the value of retail market potential, e.g., an option to expand the store operation with no right to switch back to the original operation scale. These options are denoted as *simple options*. The value of some other investment opportunities does not only derive from their immediate retail market potential but also from their consequences to other investment options of the retailer, e.g., an option to open a store in a growing market with an option to expand it if the market turns out to be substantially large. These investment options are denoted as *compound options*.

Table 2 provides the categorization and the description of investment opportunities in retail markets. Consider that the retailer's investment option in each category can be proprietary or shared depending on the existence of other retailers in retail markets. In this table the major retailers' investment options are classified into six categories, as follows.

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1. **Defer option**: this is one of the most common options on a retail store. The retailer holds a right to open a store in an emerging market over a pre-specified period of time. Some retailers acquire this right by being involved in the development process of a growing market and owning or holding a lease on undeveloped land. The retailer decides whether and when to exercise his option based on his evaluation of the retail market whose uncertainty resolves over time. This is a simple option, which follows the described analogy to a call option on a common stock.

2. **Expansion (or contraction) option**: the retailer can revise his scale of operations after opening the store with respect to the state of retail market, i.e., the retail management holds a natural right to expand (contract) the scale of operations of the retail store after considering the necessary expenses when the market is realized to be growing (declining). This is a simple option, which follows the described analogy to a call option on a common stock.

3. **Temporarily suspension option**: the retail management also holds a natural right to temporarily suspend the store operation when the market appears to be unattractive unlike it was originally expected and resume the store operation when the market becomes attractive again. This is a compound option since the value of suspension option depends on the value of another option, which is the defer option to reopen the store (think of this option as an option on an option on a common stock). The necessary costs including the suspension, maintaining, and reactivation expenses should be considered in the evaluation of this compound option.

4. **Shut down option**: the retailer also has a right to totally close the store when the market is realized to be very unattractive. This is a simple option, which follows the described analogy to a call option on a common stock.

5. **Reconfiguration (renovation) option**: the retailer may consider renovating an old store with state-of-the-art technology to maintain his base customers and attract more customers depending on the market situation. This is a simple option, which follows the described analogy to a call option on a common stock.

6. **Growth option**: the retailer may consider adding new products and services to attract more customers in the emerging market. This is a simple option, which follows the described analogy to a call option on a common stock.

Table 2.2. Common categories of investment opportunities on retail store.

Category	Description
Defer option	Own or buy a lease on a commercial property or undeveloped land to develop a store
Expansion (or contraction) option	Switch the operating scale (expand or contract the store area) as the uncertainty about the retail market resolves
Temporary suspension option	Close the store when the retail market is realized to be unsatisfactory with an option to reopen in the future
Shut down option	Permanently close the store when the retail market is realized to be extremely unsatisfactory
Reconfiguration (renovation) option	Renovate an old store to attract more customers in a growing market

Growth option	Add (or remove) new business lines (products or services) to attract more customers in a growing market

All of the above options can be considered in two situations. First, is when the retailer can make his investment decisions without any significant competition effects from the other retailers (proprietary options). Second, it is when the retailer must consider the immediate competition from the other retailers in his investment decisions (shared options). This distinction is critical in the correct evaluation of retailers' investment options particularly for the shared options. The competition effect has been treated in two ways in the literature. The first treatment approach simply addresses competition exogenously and acknowledges its existence by adjusting the store's cash flows. The second treatment approach is more complicated since it addresses competition as an endogenous variable and searches for the equilibrium state of the retail market. Game theory is a methodology that is widely used in this endogenous treatment. In this thesis, we use this endogenous treatment of competition as it will be described in Chapter 3.

The similarity between the retailers' investment opportunities in the retail market and financial call options on a common stock motivates us to use a real options methodology for the evaluation of retailers' investment options. The analogy and categorization of retailers' investment opportunities in the real options framework is the first step toward options assessment. Next, we need to describe the nature of uncertainty in the retail market, which is the basis of any options evolution.

2.4.4 Modeling the dynamic uncertainty of the value of retail market

potential

To apply the financial options evaluation approach to the assessment of the retailer's investment opportunity, we need to identify an underlying asset on which the value of the investment option depends. In the case of the retailer's investment opportunity, *the value of retail market potential is the underlying asset from which the value of the retailer's investment option is derived*. This value changes randomly over time. We use a standard Geometric Brownian Motion (GBM) to model the stochastic behavior of the value of retail market potential. The choice of this GBM model is based on a noncontroversial assumption that the value of retail market potential grows at some rate plus random variation, i.e., the value of retail market potential grows exponentially with some random noise.

This stochastic model implies that the current value of retail market potential is known, but future values are lognormally distributed with the time horizon. Although the retailer observes the value of retail market potential as information arrives over time, the future value of retail market potential is always uncertain.

Note that the choice of GBM model to specify the dynamic uncertainty of the value of retail market potential is an abstraction from the real world store operation. Clearly, the choice of GBM model is not perfect but, as we will show, *it is useful*. The value of retail market potential changes dramatically over a very short period of time when a large group of people moves in or out of the market. This may occur when a subdivision or a building is built or demolished in the market. This sudden substantial change in the value of retail market over a short period of time cannot be appropriately captured by the

specifications of the GBM model. On the other hand, when a competitive retailer enters the retail market, the portion of retail market potential that the retailer acquires is not growing at the rate before. Therefore, the competing retailer's entry provides the upper limit for the value of retail market potential, which is again against the assumption of the GBM model. In addition, even we assume that the value of retail market potential grows exponentially the rate of this growth is not constant over time. We expect that a new retail market grows fast at the beginning but cools down after a certain time. This is not consistent with the constant expected growth rate assumption in the GBM process. In addition, the value of retail market potential does not grow infinitely as the GBM process assumes. Consider that the choice of the GBM model is appropriate in at least one regard. The value of retail market potential is strictly positive, which is consistent with the GBM model.

For the time being, we ignore the above issues regarding the use of the GBM model. In this thesis, we do not consider the market situations in which, the value of retail market potential experiences large swings in short periods of time, i.e., infrequent but discrete jumps. We assume that the incremental asset variability assumption holds true for most part in the evaluation of the retailer's investment option, i.e., the value of retail market potential grows at the noisy incremental rate as it can be described by the GBM process. To address dramatic changes of retail market potential in short periods of time one should look into the stochastic jump process that is a type of stochastic process that has large discrete movements (jumps), rather than small continuous movements. This is outside the scope of this thesis. Interested readers can refer to chapter three of [15] or [46] for further discussion.

Note that the GBM model is useful since it provides a simple, but powerful approach to treat the stochastic behavior of the retail market in the evaluation of the retailer's investment option. It is worth noting that many researchers including McDonald and Siegel [47] and Dixit and Pindyck [15] use the GBM model to specify the nature of uncertainty for the value of a project.

Our search in the literature could not locate any specific research that used the real options methodology to evaluate retailers' investment options. However, if we think of the physical retail store as a real estate property and believe that the value of the real estate asset is an indicator (or a proxy) of the value of retail market potential, then there exist several studies that used financial-type real options approach to evaluate the value of land and properties. Greden and Glicksman [48] provide a summary of studies that apply real options theory to several aspects of real estate development, e.g., use a financial option-pricing approach to explain the phenomenon of vacant urban land [49], use a perpetual call option to explore the effect of land-use choice [50], and use a perpetual American call option model to assess properties under construction and properties held for development [51]. What is in common among all these studies is that they all use a GBM process to model the value of the underlying assets of land and property.

In addition, by using GBM to model the stochastic behavior of retail market potential we can use an extensive body of research in finance to help us evaluate retailer's investment options. This is particularly valuable since we are interested in the evaluation of the retailer's investment opportunity to determine the optimal investment time to open

a store. Two techniques are described in the next section that can be used to assess retailers' investment opportunities.

2.4.5 Evaluation approaches of the retailers' investment options

One of the most important questions in real options analysis is to determine the optimal time, or the conditions under which, that the retailer should exercise his investment opportunity. This is similar to determining the optimal time that the holder of a financial call option should exercise his option on a common stock. Therefore, time plays a critical role in the evaluation of the retailers' investment opportunities since the value of the retailer's investment opportunity depends on the store's retail market potential that changes over time corresponding to the retailer's or his competitors' decisions and uncertainty of the retailer market. An appropriate investment analysis approach must be able to address these challenges. In this section, we summarize two general approaches that have been widely used in real options evaluation, as follows.

2.4.5.1 Dynamic programming

Dynamic programming is a standard approach in finance and economics to solve for the optimal solutions of problems involving sequential decisions under dynamic uncertainty. Here, we provide a brief overview of dynamic programming and how it can be used to evaluate retailer's investment options. A detailed description of dynamic programming can be found at [15, 52].

The dynamic programming approach is based on the decomposition of the whole problem into two basic components: the immediate decision and the value function that summarizes all the future subsequent decisions starting from the time of the immediate

decision. This idea is based on the Bellman's principle of optimality, which states that "an optimal policy has the property that, whatever the initial action, the remaining choices constitute an optimal policy with respect to the subproblem starting at the state that results from the initial actions" [15].

The Bellman equation can be used to determine the retailer's optimal exercise time for an investment option. At each time step the retailer has two alternatives: to defer the investment option or to carry out the investment. The fundamental equation of optimality indicates that the retailer should choose the alternative that maximizes the sum of these two components: the immediate profit and the continuation value (the expected value of the investment option at the next time step that is discounted back to the initial time step).

Dynamic programming can be easily applied in the evaluation of the retailer's investment opportunity using the idea that the underlying continuous stochastic variable can be modeled in a discrete fashion. Binomial and trinomial lattice models are developed to approximate the stochastic behavior of a stochastic variable that follows a GBM process [53, 54]. The lattice model determines the state of the uncertain variable (i.e., the value of retail market potential) over time in a discrete fashion and helps the retailer evaluate the investment option as a decision tree, as follows.

If the investment horizon is finite the retailer's investment decision at the last time step does not follow any subsequent decisions and therefore can be determined using the standard optimization methods. The decision is either to drop the investment option or to carry out the investment depending on the value of the underlying uncertain variable, which is the value of retail market potential. This solution serves as the valuation function for the time step before it on the tree. That, in turn, provides solutions for the

two time steps before the end of the tree, and so on. Therefore, this sequence of backward calculations can continue to the initial condition and determine the optimal policy for the retailer's investment behavior with respect to the value of retail market potential at any time step.

On the other hand, if the investment horizon is infinite each decision follows an exact similar problem to the original problem. The recursive nature of these decision problems simplifies the evaluation process since it facilitates numerical computation and sometimes makes it possible to obtain an analytical solution such as the Black-Scholes formula for the price of a European call option [16]. However, it is worth noting that the lattice approach with a substantially large number of small time steps can be used as a good approximate model to evaluate investment options with infinite horizons. The backward calculation procedure is the same as the finite-horizon investment options.

Finally, it is worth noting that the dynamic programming calculations are based on the assumption that the retail firm's cost of capital (interest rate to discount the future store's cash flow) can be specified exogenously. The contingent claims analysis approach is an alternative approach that does not require such an assumption regarding the value of the discount rate.

2.4.5.2 Contingent claims analysis

Contingent claims analysis is a standard valuation approach in financial economics to determine the market value of an investment opportunity in the complete financial market at equilibrium. Recall that the value of the retailer's investment option derives from the future value of retail market potential that changes randomly over time. It is also indicated that the objective of the retailer is to invest efficiently and open stores in

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appropriate retail markets in order to maximize the shareholders' wealth, i.e., the shareholders expect to be compensated for the systematic risk that is involved in the retailer's investment opportunity. Therefore, the retailer should invest and open stores in the retail markets that provide sufficient returns on their investments corresponding to the market values of their investment options. *Contingent claims analysis is a standard procedure to determine the true level of risk and the market value of an investment option by construction a replicating portfolio of traded assets in the complete financial market.*

In this market-oriented perspective, the retailer owns an asset (the store) or owns a leases on an asset (the store) that generates random cash flow over time. Although the retailer's asset (store) is not traded in the market, its true market value can be determined using the prices of the other existing traded assets that replicate the retailer's asset's return and risk characteristic. It is noted that all the retailer needs is to find a portfolio of traded assets in the market that exactly replicates the pattern of returns from this investment option, at every future date and in every future uncertain state [15]. The retailer is able to construct this replicating portfolio in the modern economy that presents a very rich menu of trading assets with different returns and risk characteristics.

The assumption of using the replicating portfolio for the evaluation of the retailer's investment options is consistent with the assumptions of the other techniques such as the NPV in the traditional capital budgeting. Recall that in the NPV calculation, we use the future cash flow of an investment opportunity to determine the correct discount rate based on the CAPM in the complete financial market as if the investment would be traded in the market. The correctness of using the replicating portfolio is similar to the correctness of using the NPV in the evaluation process and lies at the fundamental

assumption of market completeness, i.e. the retailer's decision should not expand the investor's opportunity set [14]. This assumption holds true for the retailer's investment opportunities since their investments are usually not unique. For the unique opportunities in the retail market both the traditional capital budgeting and the real options approaches return equally inadequate results. Hence, the replicating portfolio is an appropriate construct to assess the retailer's investment options in the retail market since the objective is to determine how much the retailer's investment option would be worth if it were traded in the market.

It is also indicated that the value of the retailer's investment option can be determined based on the equilibrium price of this replicating portfolio in the complete financial market. Note that at market equilibrium there are no arbitrage profit opportunities. Therefore, the no-arbitrage value of this replicating portfolio must be equal to the equilibrium value of the retailer's investment option. Otherwise, any difference between the price of this replicating portfolio and the retailer's investment option, which are exactly similar in their returns and risk patterns, provides a clear opportunity for those who seek arbitrage opportunities and this should not exist at the market equilibrium.

The calculation of the actual market value of an investment opportunity is the first important step toward the identification of the optimal retail investment policy regarding the optimal investment time and size. In contingent claims analysis, we also need to identify the stochastic behaviors of the store's retail market potential, which is the underlying asset in this investment option. For simplicity, we can assume that this underlying asset can be replicated by a traded portfolio that follows a GBM process (the justification and the limitations of this choice are discussed earlier). The remaining

calculations to determine the optimal investment time are very similar to the dynamic programming approach as described in section 2.4.5.1. The value function of dynamic programming and the value of the replicating portfolio in contingent claims analysis can be similarly used in the Bellman equation of optimality to determine the optimal investment times. The retailer exercises his investment option only when the expected future value of his replicating portfolio exceeds the value of holding an option on this replicating portfolio.

A simplified method of option valuation, which is denoted as the equivalent risk neutral valuation approach, is developed by Cox, Ross, and Rubinstein [53] to facilitate the process of contingent claims analysis. It is indicated that the risk neutral valuation of investment options is independent of the investor's risk attitudes and of consideration of capital market equilibrium [14]. In this approach, risk neutral probabilities are used instead of the actual probabilities to describe the stochastic behaviors of the underlying asset. These risk neutral probabilities can also be used in the lattice model that approximately determines the random variation of the underlying asset over time. Substituting the actual probabilities by their equivalent risk neutral probabilities enables us to evaluate an investment option in the risk-free world, i.e., the future cash flow is discounted back at the risk-free rate.

Apart from the similarity in the valuation process of investment options by dynamic programming and contingent claims analysis, there is an important difference between these two approaches. Contingent claims analysis offers a better treatment of the selection of the appropriate discount rate (as the measure of riskiness of an investment option) than the dynamic programming approach, i.e., contingent claims analysis

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evaluates an investment opportunity as if its risk would be traded in the complete financial market.

In the dynamic programming approach, an exogenous discount rate is assumed to adjust for the retail firm's cost of capital that is used to discount the future store's cash flow. Contingent claims analysis only takes the risk-free rate of return as an exogenous variable in the evaluation process. The value of this risk-free rate of return is determined by broad economic forces and can be easily determined from the financial market, for instance, the rate of return on the U.S. Treasury bill. Therefore, the total expected rate of return on an investment opportunity is determined from the CAPM, which is the fundamental model to specify the true level of compensation for the systematic risk in financial economics. However, this total expected rate of return needs to be adjusted since it is indicated that the non-traded assets such as the store may earn a return below the equilibrium expected rate of return of their replicating portfolio that has the same risk characteristics but is traded in the complete financial market [14, 15]. Therefore, to account for this rate of return shortfall, a dividend-like rate of return should be subtracted from the total expected rate of return to determine the expected rate of return for the replicating portfolio.

On the other hand, contingent claims analysis is based on a very demanding assumption regarding the construction of the replicating portfolio for the retailer's investment opportunity. The retailer requires studying a rich and large number of risky assets in the complete market to identify an appropriate combination of traded assets whose rate of returns exactly replicate the stochastic behavior of the value of retail market potential, i.e., the underlying stochastic components of the store's retail market

potential and its replicating portfolio must be perfectly correlated and follow an identical stochastic process. However, dynamic programming does not require the investment risk being traded in the financial market since it is based on the subjective evaluation of risk by the retailer, which is characterized by an exogenous discount rate. Thus, an appropriate combination of dynamic programming and contingent claims analysis can handle the valuation of the retailer's investment opportunities.

2.5 Conclusions

It is indicated that the retail industry is very competitive in the U.S. and hence, a retailer always strives to differentiate itself from other competitors to enhance its market position. Therefore, market selection is identified as an important strategy to differentiate a retailer in competitive retail markets. In this chapter, we show how the selection of appropriate markets to open stores can be considered as an investment decision by the retailer. Although market selection analysis is a well-established subject in retail and marketing research our search in the capital budgeting and investment analysis literature did not locate any study related to retail market analysis.

In addition, we summarize why the traditional capital budgeting approaches such as the NPV calculation are inadequate in addressing the strategic aspect of the retailer's investment decisions as well as its flexibility to change original, operational strategies as uncertainty about the dynamic retail market evolves over time. We also discuss why an investment analysis approach based on the real options methodology is a promising candidate to overcome some of these limitations.

Therefore, a conceptual framework is presented in this chapter to describe the analogy between the retailer's investment option in a dynamic market and the call option on a common stock. This analogy is useful since it outlines an investment analysis procedure based on the real options methodology to evaluate the retailer's investment option. As part of building this analogy, we discuss the value of retail market potential as an underlying asset for the valuation of the retailer's investment option, similar to the stock price, which is an underlying asset for the valuation of the call option. This conceptual framework is used in Chapter 3 and Chapter 4 to construct a computational framework for the valuation of the retailer's investment option, i.e., the value of retail market potential is used to derive the value of the retailer's optimal investment threshold in a competitive, dynamic market. However, we indicate that there are some essential differences between the retailer's investment option in a dynamic, competitive market and the call option on a common stock that must be considered in the appropriate characterization and evaluation of the retailer's investment opportunities.

Another conceptual framework is developed in this chapter to categorize different types of investment options that a retailer is faced in the real world. This categorization helps us organize several aspects of management flexibility to defer an investment opportunity and revise the original operating strategy in retail markets according to the competitive structure of a retail market.

It is concluded that the conceptual frameworks of this chapter can be used as a single framework to address the retailer's management flexibility as well as dynamic uncertainty and competition effect in retail markets. Therefore, these conceptual

frameworks provide a strategic view towards retail stores as flexible assets of a retail firm.

These conceptual frameworks are general and can be adjusted and applied to investments options in other services. Similar conceptual frameworks should be developed in other service industries and compared with our frameworks in the retail industry. Similarities, differences, challenges, and other related issues should be summarized in order to enhance our understanding regarding the application of the financial options methodology for the valuation of the real-world investment problems in different services.

Chapter 3

Evaluation of a Retailer's Investment Option in a Competitive, Dynamic Market: a Dynamic Programming Approach

3.1   Abstract

In this chapter, we develop an integrated investment analysis approach based on dynamic programming to explore retailers' investment behaviors in dynamic markets. *The objective is to determine retailers' optimal investment thresholds in noncompetitive and competitive markets*. We consider two retailers to illustrate our approach and use a simple game theory treatment to address competition in retail markets. We use our integrated investment analysis model based on a real options methodology to demonstrate the apparent tendency for the small discount retailer invests earlier in a new developing market due to the competition effect from the large discount retailer. This early entry gives the small retail a first-mover advantage and delays the big retailer's entry into the competitive market. In addition, we conduct sensitivity analysis to characterize how significantly the values of our model parameters impact the retailers' investment decisions.

3.2   Introduction

In this chapter, we develop an integrated investment analysis approach to explore retailers' investment behaviors in both competitive and noncompetitive, dynamic retail markets. This investment analysis approach incorporates a simple game theory treatment into the real options methodology framework to determine retailers' investment thresholds in dynamic markets. The objective of this chapter is to characterize how differences in retailers' cost parameters impact retailers' investment decisions in dynamic retail markets. In addition, we want to explore the effect of competition on retailers' entry decisions into growing markets.

This chapter is structured, as follows. A simple abstract demand model is presented in section 3.3 to characterize the demand side of a typical duopoly retail market. This demand function contains a variable that is time-dependent and can be used as a proxy variable to represent the dynamic changes in the value of retail market potential.

In section 3.4, we use a simple game theory treatment to address competition between two retailers in a market considering the demand model of section 3.3. The objective is to determine retailers' optimal quantities of the product and their respective profits in monopoly and duopoly markets.

In section 3.5, we present a particular type of continuous stochastic process called Geometric Brownian Motion (GBM) to model the dynamic uncertainty of a variable in the demand function that represents the value of retail market potential. It is important to mathematically formulate the dynamic uncertainty of this parameter since retailers' optimal decisions depend on the value of this parameter at the time of decision.

In section 3.6, we present an approximate model to describe this dynamic uncertainty in a discrete fashion similar to a decision tree. By using this lattice model, retailers' investment evaluation process simply reduces to a decision tree analysis problem.

In section 3.7, we describe a dynamic programming approach to determine the retailer's investment behavior in terms of the optimal time to enter and the quantity of the product to offer to the retail market. The objective of this section is to develop an investment analysis approach that determines when a retailer should exercise its investment option, enter a market, and open a store. We consider three general cases to illustrate our approach. We use this approach in section 3.8 and apply it on a simple notional example to illustrate how this approach can be used to determine two retailers' behaviors in noncompetitive versus competitive markets.

In section 3.9, we explore how differences in retailers' cost parameters impact their investment behaviors in terms of entry decisions to the growing markets. Next, we study how retailers' optimal investment thresholds change as the value of model parameters changes. Sensitivity analysis in section 3.10 shows us how significant the correct estimation of the value of a model parameter is to determine retailers' optimal investment thresholds. Summary and future works are provided in section 3.11 at the end.

3.3   Demand modeling

In this section, we determine an exogenous demand function that identifies the inverse relationship between the price of a product and the amount of it that customers purchase at this price in a retail market. This demand function contains a variable that shows a proxy for the value of retail market potential at any time step. This variable is modeled to

be time-dependent in order to capture the changes of the demand in this retail market over time.

We assume that retailers provide similar products at similar service levels to customers in this market. Customers are assumed to be only concerned about the price of a product when it is time to choose a store to shop for it. Therefore, the only product characteristic that determines its level of consumption is the price of the product. Customers decide to buy a product based on its price in the retail market and hence, the demand model needs describe the relationship between the price of the product and the amount of it (or its demand) that customers purchase at this specific product.

We use a notional product that both retailers provide, for modeling the demand side of the retail market. This notional product can be considered as an index for a basket of typical product that both retailers offer. The use of a single notional product is for modeling purpose to facilitate describing the demand side of the retail market in a reduced form. The price of this typical product is determined in the market based on the total amount of it in retailers' shelves. Therefore, we assume that the demand for retailing in this market by the demand for this notional product.

In this section, we determine a mathematical relationship between the demand for this notional product and its price in retail markets. We use a simple standard demand model to characterize this relationship at any time step in this retail market. Note that the price of a product determines the total amount of it that is demanded by customers in a market. We assume that there are only two retailers in this retail market that provide this product. However, this demand model can be applied for any other oligopoly markets with more than two retailers. The total amount of this product in these two retailers'

shelves determines the price of this product in the retail market. Consider that retailers can optimally select the amount of this product in their shelves in order to make satisfactory profits considering customer's attitude toward purchase of this product that is summarized in the demand function, as follows.

We use the following notations in our formulation in this thesis.

- (P(t)) is the price of this product at time step (t).

- (Q_1(t)) is the quantity of this product that is provided to the market by retailer 1 at time step (t).

- (Q_2(t)) is the quantity of this product that is provided to the market by retailer 2 at time step (t).

- (TQ(t)) is the total quantity of this product in the market at time step (t), i.e., (TQ(t) = Q_1(t) + Q_2(t)).

A simple linear function is used to present the inverse relationship between (P(t)) and (TQ(t)). Equation (3.1) summarizes this inverse relationship, as follows.

$$P(t) = -\gamma TQ(t) + X(t) = -\gamma(Q_1(t) + Q_2(t)) + X(t) \qquad (3.1)$$

where (γ) represents *the absolute constant slope of this price-quantity line* and (X(t)) represents *the intercept of this line that changes over time*. Figure 3.1 shows this linear demand function. Parameter (X(t)) in this figure represents the intercept of the demand line with the vertical price axis and identifies the maximum willingness to pay for this product in this retail market at time step (t). The units of the parameter (X(t)) are the units of the price of the product, which, in the United States, would be dollars. This parameter can be used as an indicator for the value of retail market potential (or the retail market

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size), i.e., the value of (X(t)) increases if the value of retail market potential increases. When the size of a retail market increases (or the value of retail market potential increases) it is more likely that consumers will pay a high price for a product and hence, the demand line shifts up and the value of its intercept (X(t)) increases. It is also assumed that both retailers have perfect knowledge regarding this demand model and the values of its parameters. The choice of this linear demand model is consistent with the literature in microeconomics (for instance, see [55, 56]).

Since the retail market is dynamic the value of the intercept parameter in this linear demand function (X(t)) is modeled to be a function of time. Therefore, we can capture the time varying aspect of this retail market in our simple demand model. We return back to this issue later in this chapter when we discuss the dynamic uncertainty of the retail market.
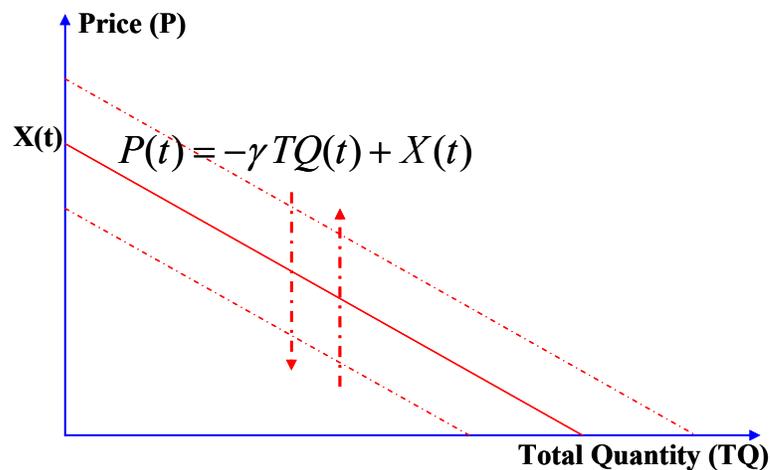
Figure 3.1. Inverse relationship between Price (P) and Total Quantity (TQ) of the notional product in the retail market and its dynamic variation.

In the next section, we will discuss how retailers can determine the quantity of this notional product at their shelves considering the described relationship between price and

total quantity. We use a game theory approach for this purpose to address the competition between two retailers.

3.4   A game theory approach to treat competition in the retail market

In this section, we use a simple game theory approach to address competition between two retailers in the retail market considering the inverse linear relationship between price and total quantity that is summarized in Equation (3.1). *The objective is to determine the optimal quantity of this product for these two retailers at time step (t) considering the market structure, the inverse price-quantity relationship, and retailers' cost structures.* But first we need to identify a profit function for these retailers since the retailer's objective is to maximize their profits by optimally selecting the quantity of the product that they should present in their shelves. In this thesis, we only consider a duopoly retail market but our discussion can be extended to other oligopoly markets with more than two retailers.

   Retailers are different in many aspects of their operations including store size, supply chain management, service levels, negotiations with suppliers, and etc. In this chapter, retailers' differences are summarized at higher level of abstraction in terms of their cost structures. Three cost parameters are used to characterize the important aspects of retailing business at the higher level of abstraction. These cost parameters summarize three important aspects of retailing expenditure, as follows.

- **The initial expenditure to establish, develop, and open a store in a market**: this is a one-time expenditure that a retailer must pay to develop a store in a market. We call this initial expenditure an investment cost and denote the retailer

i's *investment cost by (IC$_i$)* (i = 1, 2). The larger the size of the store the bigger the value of retailer's investment cost.

- **The operation expenditure to maintain a store in a market**: this operation expenditure includes the rental price, utility costs, and any other overhead or fixed expenses that are required to keep the store opened in a market regardless of its sales. We call this operation expenditure a fixed cost and denote the retailer i's *fixed cost by (FC$_i$)* (i = 1, 2). Fixed cost is an recurring cost that a retailer must spend at each time step to keep the store opened in the market. The larger the size of the store the bigger the value of retailer's fixed cost.

- **The marginal expenditure to acquire and provide one unit of product at the store shelves**: this marginal expenditure is the cost of acquiring and providing one unit of product at the market. We call this marginal expenditure a variable cost and denote retailer i's *variable cost by (VC$_i$)* (i = 1, 2). The larger the size of the retailer's supply chain structure and the more efficient the retailer's supply chain the less the value of retailer's variable cost.

The values of the above cost parameters are assumed to be exogenously determined for each retailer considering its store size, number of employees, supply chain configuration, and other related variables. It is also assumed that the values of retailers' cost parameters are constant and common knowledge between two retailers, i.e., retailer 1 knows the values of retailer 2's cost parameters in addition to the values of its cost parameters and vice versa. This assumption is particularly important when we note that a retailer simultaneously maximizes its profit by considering the best response of the other retailer that consists of the other retailer's cost parameters. In this section, we show how

these cost parameters can be included in the retailer's profit function to determine the optimal retailer's profit in a monopoly or a duopoly market.

It is indicated that the retailer's financial objective is to invest and open stores in the retail markets that have the most profitable cash flows. At time step (t), the retailer should determine the quantity of this product that he provides to the market such that its decision maximizes the store's cash flow at time step (t), i.e., retailer i should determine ($Q_i(t)$) in order to maximize its profit function that is summarized in Equation (3.2).

$$\Pi_i(t) = (P(t) - VC_i)Q_i(t) - FC_i \qquad i = 1, 2$$

(3.2)

where ($\Pi_i(t)$) is retailer i's profit at time step (t). The profit function has two major components. The first part is sales or revenue, which is equal to the quantity of product sold at the market at time step (t) multiplied by the price of the product. The second component is the retailer's cost part that consists of both variable and fixed costs. Variable cost equals the quantity of products sold multiplied by the retailer i's variable cost while the retailer i's fixed cost does not depend on the quantity of the products sold.

Note that there is no other retailer in this market that provides these two retailers' products and services to the market. Also assume that there is no other retailer that has the investment option to open a store in this market. Hence, customers buy their needed products only from these two retailers after they decide to open stores in this market. If only one retailer decided to exercise its investment option and opened a store in this market the market structure becomes monopoly and all customers in this market would patronize its store. If both retailers decided to open stores in this market the market

structure becomes duopoly and retailers would share this retail market and their market shares would be proportional to the quantities of the products they provide in this market.

In addition, since both retailers know the demand function in this retail market they are able to determine the quantities of the product on their shelves such that all quantities of the product would be sold in the market. Therefore, the retailers' quantities of the product are equal to the retailers' quantities of the product sold. In this section, we show how retailer 1 and 2 can optimally select their respective quantities of the product in monopoly and duopoly markets.

3.4.1 Monopoly retail markets

First, we consider a monopoly market in which there is only one retailer, e.g., retailer i (i may be 1 or 2). Substituting ($TQ(t) = Q_i(t)$) in Equation (3.1) we will have ($P(t) = -\gamma Q_i(t) + X(t)$). Substituting this price in Equation (3.2) we get Equation (3.3) that describes the profit function of retailer i in the monopoly market, as follows.

$$\Pi_i(t) = -\gamma(Q_i(t))^2 + (X(t) - VC_i)Q_i(t) - FC_i \qquad i = either\ 1\ or\ 2 \tag{3.3}$$

Retailer i's decision variable is the quantity of the product to provide in its shelves (Q_i(t)). Retailer i's profit function is a quadratic function of ($Q_i(t)$). In order to maximize this profit function, we apply the first order condition to derive the optimal value of ($Q_i(t)$), as follows.

$$\frac{\partial \Pi_i(t)}{\partial Q_i(t)} = -2\gamma Q_i(t) + X(t) - VC_i = 0 \quad i = either\ 1\ or\ 2 \tag{3.4}$$

We solve Equation (3.4) to find the optimal quantity of the product at the retailer i's

shelves at time step (t), which is denoted by (Q_i^M(t)). This optimal quantity is summarized

in Equation (3.5), as follows.

$$Q_i^M(t) = \frac{X(t) - VC_i}{2\gamma} \quad i = either\ 1or\ 2$$

(3.5)

Note that (Q_i^M(t)) must be positive since it represents the quantity of the product

offered at retailer i's shelves. Therefore, Equation (3.5) is only true if (X(t) $\geq$ VC$_i$),

otherwise, retailer i must offer zero quantity of the product at its shelves. Equation (3.6)

summarizes this discussion, as follows.

$$Q_i^M(t) = \begin{cases} \dfrac{X(t) - VC_i}{2\gamma} & When\,(X(t) \geq VC_i) \\ 0 & Otherwise \end{cases} \quad i = either\ 1or\ 2$$

(3.6)

We substitute the value of (Q_i^M(t)) from Equation (3.6) into the profit function of

Equation (3.3) in order to determine the optimal profit of retailer i in the monopoly

market. This optimal profit is denoted by (Πi^M(t)) and summarized in Equation (3.7), as

follows.

$$\Pi_i^M(t) = \begin{cases} \dfrac{(X(t))^2}{4\gamma} - \dfrac{(VC_i)(X(t))}{2\gamma} + \dfrac{VC_i^2}{4\gamma} - FC_i & When\,(X(t) \geq VC_i) \\ -FC_i & Otherwise \end{cases} \quad i = either\ 1or\ 2$$

(3.7)

Next, we consider two retailers in a duopoly retail market.

3.4.2 Duopoly retail markets

Secondly, we consider a duopoly market that both retailer 1 and 2 have stores opened in. Substituting (TQ(t) = $Q_1(t)+Q_2(t)$) from Equation (3.1) in Equation (3.2) results in the following profit functions for retailer 1 and 2 as summarized in Equation (3.8).

$$\begin{cases} \Pi_1(t) = -\gamma(Q_1(t))^2 + (X(t)-VC_1-\gamma Q_2(t))Q_1(t)-FC_1 \\ \Pi_2(t) = -\gamma(Q_2(t))^2 + (X(t)-VC_2-\gamma Q_1(t))Q_2(t)-FC_2 \end{cases} \tag{3.8}$$

Retailer 1 and 2's profit functions are quadratic functions with respective to their decision variables ($Q_1(t)$) and ($Q_2(t)$), respectively. However, retailer 1's profit function depends not only on its decision variable ($Q_1(t)$), but also on retailer 1's decision variable ($Q_2(t)$). The same thing is true for retailer 2's profit function, too. Therefore, each retailer should select the quantity of the product at its shelves considering the quantity of the product that is selected by the other retailer in order to optimize its own profit function. Since retailers simultaneously make decisions about their quantities of the product, we apply the first order condition to retailer 1's and 2's profit functions with respect to their decision variables ($Q_1(t)$) and ($Q_2(t)$), respectively. The first order condition equations are summarized in Equation (3.9), as follows.

$$\begin{cases} \dfrac{\partial \Pi_1(t)}{\partial Q_1(t)} = -2\gamma Q_1(t) + X(t)-VC_1-\gamma Q_2(t) = 0 \\ \dfrac{\partial \Pi_2(t)}{\partial Q_1(t)} = -2\gamma Q_2(t) + X(t)-VC_2-\gamma Q_1(t) = 0 \end{cases} \tag{3.9}$$

If we solve the above equations we get the best response function for retailer 1 and 2 that describes the value of retailer 1's and 2's quantity of the product as a function of

retailer 2's and 1's quantity, respectively. The retailer 1's best response function –

denoted by $R_1(Q_2(t))$ – determines the retailer 2's optimal quantity of the product when

retailer 2 provides $Q_2(t)$ at its shelves. Similarly, the retailer 2's best response function –

denoted by $R_2(Q_1(t))$ – determines the retailer 2's optimal quantity of the product when

retailer 1 provides ($Q_1(t)$) at its shelves. Retailers' best response functions are

summarized in Equation (3.10), as follows.

$$
\begin{cases}
Q_1(t) = R_1(Q_2(t)) = \dfrac{-\gamma Q_2(t) + X(t) - VC_1}{2\gamma} \\
Q_2(t) = R_2(Q_1(t)) = \dfrac{-\gamma Q_1(t) + X(t) - VC_2}{2\gamma}
\end{cases}
$$

(3.10)

Recall our assumption that retailers have perfect knowledge regarding the values of

their own cost parameters as well as the parameters of the retail market demand function.

Since retailers simultaneously make their decisions at time step (t) we can derive the

optimal values of retailers' quantities in the duopoly market by solving their best

response functions together. Retailer 1's and 2's optimal quantity of the product at the

duopoly market – denoted by ($Q_1^D(t)$) and ($Q_2^D(t)$), respectively – are summarized in

Equation (3.11).

$$
\begin{cases}
Q_1^D(t) = \dfrac{VC_2 - 2VC_1 + X(t)}{3\gamma} \\
Q_2^D(t) = \dfrac{VC_1 - 2VC_2 + X(t)}{3\gamma}
\end{cases}
$$

(3.11)

However, Equation (3.11) is valid only when both ($Q_1^D(t)$) and ($Q_2^D(t)$) are non-

negative. When ($Q_1^D(t)$) is non-negative and ($Q_2^D(t)$) is negative, retailer 1 has the

opportunity to be the only retailer in the market that provides non-zero quantity of the product since the retailer 2 must provide zero quantity of the product. We show in the appendix 1 that retailer 1 provides optimal monopoly quantity of the product to the market while retailer 2 provides zero quantity of the product.

Similarly, when $(Q_2^D(t))$ is non-negative and $(Q_1^D(t))$ is negative, retailer 2 provides optimal monopoly quantity of the product to the market while retailer 1 provides zero quantity of the product. When both $(Q_1^D(t))$ and $(Q_2^D(t))$ are negative both retailer provide zero quantity to the market. Equation (3.12) summarizes the optimal values of retailer 1's and 2's quantity of the product in the duopoly market considering the value of the other parameters.

$$
\begin{cases}
when\,((VC_2 - 2VC_1 + X(t)) \geq 0)\,\&\,((VC_1 - 2VC_2 + X(t)) \geq 0) \\
\quad \begin{cases}
Q_1^D(t) = \dfrac{VC_2 - 2VC_1 + X(t)}{3\gamma} \\
Q_2^D(t) = \dfrac{VC_1 - 2VC_2 + X(t)}{3\gamma}
\end{cases} \\
when\,((VC_2 - 2VC_1 + X(t)) \geq 0)\,\&\,((VC_1 - 2VC_2 + X(t)) < 0)\,\&\,(X(t) \geq VC_1) \\
\quad \begin{cases}
Q_1^D(t) = Q_1^M(t) = \dfrac{X(t) - VC_1}{2\gamma} \\
Q_2^D(t) = 0
\end{cases} \\
when\,((VC_2 - 2VC_1 + X(t)) < 0)\,\&\,((VC_1 - 2VC_2 + X(t)) \geq 0)\,\&\,(X(t) \geq VC_2) \\
\quad \begin{cases}
Q_1^D(t) = 0 \\
Q_2^D(t) = Q_2^M(t) = \dfrac{X(t) - VC_2}{2\gamma}
\end{cases} \\
Otherwise \\
\quad \begin{cases}
Q_1^D(t) = 0 \\
Q_2^D(t) = 0
\end{cases}
\end{cases}
\tag{3.12}
$$

We can calculate the optimal retailers' profits by substituting the above quantities in Equation (3.8). We get the following values for retailer 1 and 2's optimal profits in the duopoly market – denoted by (Π_1^D(t)) and (Π_2^D(t)), respectively – as summarized in Equation (3.13).

$$
\begin{cases}
when\,((VC_2 - 2VC_1 + X(t)) \geq 0)\,\&\,((VC_1 - 2VC_2 + X(t)) \geq 0) \\
\quad \begin{cases}
\Pi_1^D(t) = \dfrac{(X(t))^2}{9\gamma} + \dfrac{2(VC_2 - 2VC_1)(X(t))}{9\gamma} + \dfrac{(VC_2 - 2VC_1)^2}{9\gamma} - FC_1 \\[2ex]
\Pi_2^D(t) = \dfrac{(X(t))^2}{9\gamma} + \dfrac{2(VC_1 - 2VC_2)(X(t))}{9\gamma} + \dfrac{(VC_1 - 2VC_2)^2}{9\gamma} - FC_2
\end{cases} \\[3ex]
when\,((VC_2 - 2VC_1 + X(t)) \geq 0)\,\&\,((VC_1 - 2VC_2 + X(t)) < 0)\,\&\,(X(t) \geq VC_1) \\
\quad \begin{cases}
\Pi_1^D(t) = \Pi_1^M(t) = \dfrac{(X(t))^2}{4\gamma} - \dfrac{(VC_1)(X(t))}{2\gamma} + \dfrac{VC_1^2}{4\gamma} - FC_1 \\[2ex]
\Pi_2^D(t) = -FC_2
\end{cases} \\[3ex]
when\,((VC_2 - 2VC_1 + X(t)) < 0)\,\&\,((VC_1 - 2VC_2 + X(t)) \geq 0)\,\&\,(X(t) \geq VC_2) \\
\quad \begin{cases}
\Pi_1^D(t) = -FC_1 \\[2ex]
\Pi_2^D(t) = \Pi_2^M(t) = \dfrac{(X(t))^2}{4\gamma} - \dfrac{(VC_2)(X(t))}{2\gamma} + \dfrac{VC_2^2}{4\gamma} - FC_2
\end{cases} \\[3ex]
Otherwise \\
\quad \begin{cases}
\Pi_1^D(t) = -FC_1 \\[2ex]
\Pi_2^D(t) = -FC_2
\end{cases}
\end{cases}
$$

(3.13)

It can be seen from the above formulation that the optimal value of the retailer's quantity of the product in the monopoly or duopoly market depends on the retail market condition at time step (t), which is described by the parameter (X(t)) in the above formulas. Also consider that the retail market is uncertain and dynamic and therefore, the value of (X(t)) changes randomly over time. In addition, the optimal value of retailer 1's and 2's quantity of the product depends on the market structure that depends on the other

retailers' decision that is the optimal time to invest and open a store in the evolving market. On the other hand, the optimal time that a retailer decides to enter a market and opens a store also depends on the retail market condition that is characterized by the parameter (X(t)). Hence, *it is critical to determine a mathematical model for (X(t)) that captures the dynamic uncertainty of the value of retail market potential*. In the next section, a particular continuous stochastic process is presented to describe the dynamic uncertainty of (X(t)).

3.5 Modeling the dynamic uncertainty of the value of retail market potential

It is indicated that *parameter (X(t))* in the demand model of Equation (3.1) can be considered *as a proxy variable for the value of retail market potential* that is uncertain and changes over time. Since the retailer's optimal quantity of the product as well as the optimal time to enter the retail market depend on the value of (X(t)) we need to define a model to describe the stochastic variation of (X(t)) in a systematic fashion. Therefore in this section, we present a particular type of continuous stochastic process called *Geometric Brownian Motion (GBM) to model the dynamic uncertainty of (X(t))*.

To apply a financial options evaluation approach to the assessment of the retailer's investment opportunity, we need to identify an underlying asset that the value of the investment option depends on. We showed in the previous section that retailers' profit functions that determine the values of retailers' investment opportunities depend on the value of (X(t)). Therefore, (X(t)) can be considered as the underlying asset from which the value of the retailer's investment option is derived. Note that the value of (X(t)) changes over time as the value of retail market potential changes over time.

It is not controversial if we assume that the value of retail market potential grows exponentially at some positive rate plus some random variation due to the economic noise. Therefore, we use a standard GBM to model the stochastic behavior of (X(t)). The GBM model that describes the stochastic behavior of (X(t)) is summarized in Equation (3.14), as follows.

$$dX = \alpha X dt + \sigma X dz \qquad (3.14)$$

where (dz) is an increment of a Wiener process, ($\alpha > 0$) is the drift parameter, and ($\sigma > 0$) is the volatility parameter of this stochastic process (the interested reader can see [57] for detailed discussion on this particular continuous stochastic process). This continuous stochastic model implies that the current value of (X(t)) – denoted by (X_0) – is known, but the future values are lognormally distributed with the time horizon. Although the retailer observes the value of retail market potential as information arrives over time, the future value of retail market potential is always uncertain. Note that we assume that the retailer can observe the demand value for the notional product and hence, the demand for the retailing business in a retail market.

Note that the choice of GBM model to specify the dynamic uncertainty of the value of retail market potential is an abstraction from the real world. Clearly, the choice of a GBM model is not perfect but as we will show in this chapter it is useful. The value of retail market potential changes dramatically over a very short time increment when a large group of people moves in or out of the market. This may occur when a subdivision or a building is built or demolished in the market. This sudden substantial change in the value of retail market potential over a short time increment cannot be appropriately captured by the specifications of the GBM model. In addition, even we assume that the

value of retail market potential grows exponentially the rate of this growth is not constant over time. We expect that a new retail market grows fast at the beginning but cools down after a certain time. This is not consistent with the constant expected growth rate assumption in the GBM process. In addition, the value of retail market potential does not grow infinitely as the GBM process assumes.

For the time being, we ignore the above issue regarding the use of the GBM model. Note that the choice of the GBM model is appropriate in at least in one regard. The value of retail market potential is strictly positive, which is consistent with the GBM model. In addition, the GBM model is useful since it provides a simple, but powerful approach to treat the stochastic behavior of the value of retail market potential in the evaluation of the retailer's investment option.

Furthermore, by using the GBM to model the stochastic behavior of the value of retail market potential we can use an extensive body of research in finance to help us evaluate retailer's investment options. This is particularly valuable since we are interested in the evaluation of the retailer's investment opportunity to determine the optimal investment time to open a store.

Working with the continuous GBM model can be difficult. In addition, many options problems do not have any analytical closed-form solutions. Instead, researchers in finance have developed many numerical methods for the options valuation in simple discrete fashion. In the next section, we present an approximate discrete model for the GBM model to describe the dynamic uncertainty of $(X(t))$ in a discrete time fashion. This discrete approximation helps us evaluate retailers' investment options as simple as decision tree analysis.

3.6 A trinomial model to approximate continuous GBM in a discrete fashion

In this section, we present an approximate model to describe the dynamic uncertainty of (X(t)) in a discrete fashion similar to a decision tree. This discrete model is developed to be an appropriate approximation for the GBM, which is a continuous stochastic process. The motivation behind using this approximate method is to facilitate the valuation of the retailers' investment opportunities. We will show in the next section that the investment evaluation process simply reduces to a decision tree analysis problem by using this discrete model.

Numerical approximation procedures are widely used in financial evaluation and contingent claims analysis to find approximate solutions for situations in which the closed-form solutions are rare. The finite difference method is one of these numerical methods that is used by many researchers to solve the underlying partial differential equation (for instance, see [58-60]).

However in this chapter, we use another type of numerical method that is based on approximating the underlying stochastic processes of the state variables. For instance, Cox, Ross, and Rubinstein [53] develop a binomial lattice approach that explicitly establishes the arbitrage strategy that replicates the claim. More recently, Boyle equates the first two moments of the underlying lognormal distribution to those of the approximating distribution to develop a multi-period trinomial procedure to approximate a risk neutralized Geometric Wiener process [61]. Boyle extends his model to a five-jump model to approximate a joint bivariate lognormal process and uses it to value American style options that depend on prices of two state variables [54].

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However, Boyle's model is difficult to implement for the situations in which there are more than two state variables. Therefore, Boyle, Evnine, and Gibbs (BEG) [62] extends Boyle's model and makes it practical for (k) state variables. This approach is also based on equating the moment generating function of the approximating distribution to the true normal moment generating function in each time step [54].

In this chapter, we use a particular trinomial approximating approach that is developed by Kamrad and Ritchken [54] for valuing options on one state variable. Here our only state variable is (X(t)) that is described by a continuous GBM process. Using Kamrad and Ritchken's trinomial lattice formulation, we are able to approximate the continuous stochastic variations of (X(t)) via a discrete random walk process. It is noted that this discrete random walk process converges to the original continuous GBM process in the limit. Figure 3.2 shows how this approximation can be used to represent the dynamic changes of (X (t)) in a discrete random walk fashion.
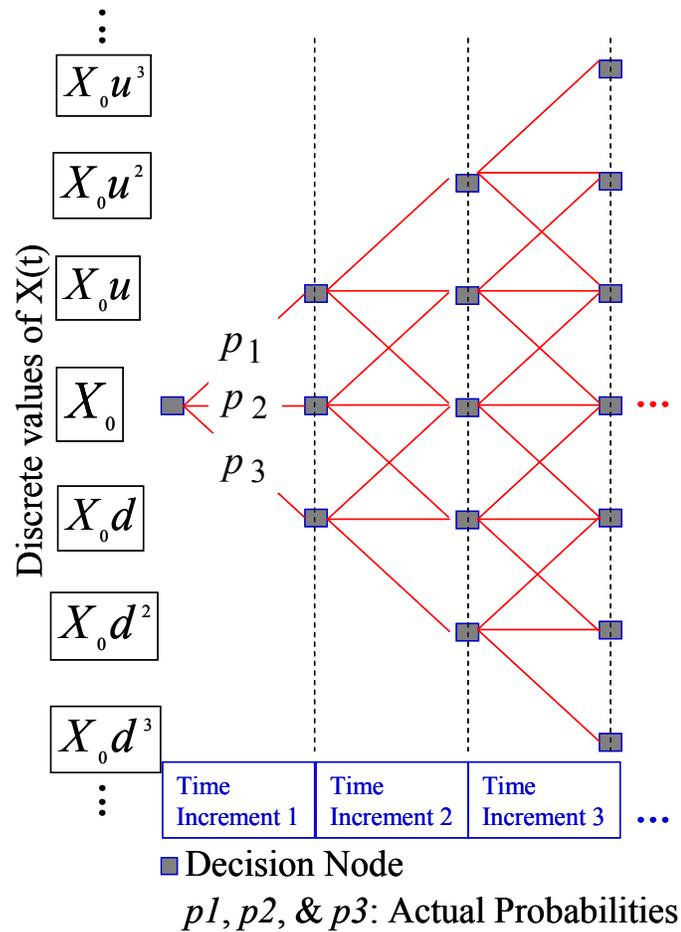
Figure 3.2. A trinomial lattice model to approximate the dynamic uncertainty of X(t) in a discrete fashion.

The discrete values of (X(t)) in this trinomial lattice are calculated, as follows. Assume the value of (X(t)) at the beginning of the first time step is (X_0). For the next time step, this value may increase by the ratio of (u>0), stay constant, or decrease by the ratio of (d=1/u) with actual probabilities of (p_1, p_2, and p_3), respectively. Assume the length of each time increment is (Δt). Therefore, the value of (X) at the next time step (X(t+Δt)) can be summarized, as follows.

$$X(t + \Delta t) = \begin{cases} X(t)u & \text{with probability } p_1 \\ X(t) & \text{with probability } p_2 \\ X(t)d & \text{with probability } p_3 \end{cases} \qquad (3.15)$$

This pattern continues for the subsequent time steps until it reaches the last time step. The total number of time steps should be selected large enough to cover several possible values for the stochastic state variable. It is noted that a trinomial lattice can be a fairly accurate representation of Geometric Brownian Motion if the time increment (Δt) used is small enough and the process occurs over a long enough time [54]. With the trinomial lattice, the probability distributions become discrete, and the investment option can be valued as a decision tree.

To completely define this approximate trinomial lattice, we need to determine the values of its parameters: the actual probabilities (p_1, p_2, and p_3) and the jump ratios (u,d). These values should be chosen in such a way that the true stochastic nature of (X(t)) is captured as faithfully as possible. Kamrad and Ritchken [54] shows that the following formulas (i.e., Equations (3.16a) and (3.16b)) present appropriate values for actual probabilities and jump ratios in their trinomial lattice model.

$$\begin{cases} u = e^{\lambda\sigma\sqrt{\Delta t}} \\ d = e^{-\lambda\sigma\sqrt{\Delta t}} \end{cases} \qquad (3.16a)$$

$$\begin{cases} p_1 = \dfrac{1}{2\lambda^2} + \dfrac{\mu\sqrt{\Delta t}}{2\lambda\sigma} \\ p_2 = 1 - \dfrac{1}{\lambda^2} \\ p_3 = \dfrac{1}{2\lambda^2} - \dfrac{\mu\sqrt{\Delta t}}{2\lambda\sigma} \end{cases} \qquad (3.16b)$$

where ($\mu = \alpha - (\sigma^2/2)$) and ($\lambda \geq 1$).

Recall that the motivation behind introducing the trinomial lattice model was to represent the stochastic variations of (X(t)) in a discrete random walk fashion to simplify the process of investment evaluation. Figure 3.2 also shows that this lattice representation can be used as a decision tree to determine the retailers' entry and quantity decisions in this competitive, dynamic market. Each node in this lattice represents a decision node for both retailers. Retailers decide to whether open a store or delay their investment opportunities with respect to the value of (X(t)) at that node. Decisions about their optimal quantity of the product follow this decision too.

In the next section, we explain how this lattice model can be used as a decision tree to explore the retailers' investment behaviors and determine their decision variables including investment timings and optimal quantities of the product. We will summarize the procedure for several investment situations.

3.7   A dynamic programming approach to evaluate retailers' investment options

Dynamic programming is a standard approach to solve for the optimal solutions of problems involving sequential decisions under dynamic uncertainty. In this section, we describe a dynamic programming approach to determine the retailer's investment behavior in terms of the optimal time to enter and the quantity of the product to offer to the retail market. The objective of this section is to develop an investment analysis approach that determines when a retailer should exercise its investment option, enter a market, and open a store. We consider three general cases to illustrate our approach.

However, the described procedure can be easily extended to the other investment cases with minor manipulation.

We consider three typical investment opportunities that commonly appear in retail markets to show how a dynamic programming approach can be used to evaluate retailers' decisions. These investment opportunities are summarized in terms of three retail market types, as follows.

- The market, in which only one retailer has an investment opportunity to enter and opens a store and the other retailer does not have the same opportunity.

- The market, in which one retailer has a store opened and the other retailer has an investment opportunity to enter and opens a store.

- The market, in which both retailers have investment opportunities and compete to enter and open stores.

It is assumed that once a retailer decides to open a store in any of the above markets the store will remain opened forever, i.e., the retailer does not have the option to temporarily suspend or shut down an operating store. It is also assumed that a retailer's investment option is free in our formulation[1]. Exercising an option is what costs money. This is modeled in our analysis approach as the retailer's investment cost to develop a store in a market. Therefore, first we need to define a procedure to determine the expected NPV of a store opened in the competitive dynamic retail market.

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[1] Retailers could, of course, buy options on real estate locations. However, this is not considered in this work.

3.7.1 The expected NPV of a store opened in a dynamic retail market

In this section, we summarize a procedure to determine the NPV of a store opened in a dynamic retail market. We use a trinomial lattice to approximate the dynamic changes of (X) in a discrete fashion and the expected NPV of a store opened using a decision tree. We determine this expected NPV for two market types: monopoly and duopoly.

3.7.1.1 The expected NPV of a store opened in a dynamic monopoly market

Consider a retailer (retailer i, i = either 1 or 2) that decided to open a store in a monopoly retail market. Assume the value of the underlying stochastic variable in this market (X) at the opening time of the store is (X_b). In addition, this store will remain opened forever. As the value of (X) changes over time retailer i revises the optimal quantity of the product in order to maximize its profit. Retailer i's optimal quantity of the product and the respective profit in the monopoly market are summarized in Equations (3.6) and (3.7), respectively, based on the value of (X) at time step (t). We use these equations to develop an approximate approach to calculate the expected NPV of retailer i's store opened in this market, as follows.

First, we construct a trinomial lattice for the future value of (X) in this retail market starting from the initial value if (X_b). We make this trinomial lattice long enough to accurately approximate the expected NPV of the store's future cash flow in this market. Note that the store's future cash flow should be discounted back to the first time step in this lattice to determine the expected NPV of the store. We use an appropriate discount rate, which is determined exogenously for this purpose. This discount rate is denoted by (ρ).

(ρ) is the retailer's cost of capital that is used as the appropriate discount rate to account for the time value of money as well as the riskiness of the investment option. The value of (ρ) is assumed to be determined by the retailer and therefore, it is exogenous to our investment analysis approach. In addition, we assume that both retail firms use the same discount rate in their investment valuation process. It is worth noting that this discussion is also consistent with the fundamental characteristics of a dynamic programming approach that uses an exogenous discount rate for investment evaluation. The dynamic programming calculations are based on the assumption that the retailer's cost of capital (interest rate to discount the store's future cash flow) can be specified exogenously.

Depending on the value of (ρ), we can determine how long the trinomial lattice of (X) should be to provide a good approximation for the expected NPV, for instance, if the value of (ρ) is 20% per year the store's cash flow in the thirty years after opening will be discounted back to the first time step at the rate of $((1/((1+0.2)^{30})) = 0.004)$, which represents an appropriate error level in the expected NPV approximation. However, one can continue the lattice for the longer time steps to get more a desirable level of error for approximation. In this chapter, we use thirty years as the base to approximate the expected NPV of a store that remains opened forever.

The following procedure shows how we can use the trinomial lattice of (X) to determine the expected NPV of a store opened in the monopoly market.

1. Construct a trinomial lattice that has 1,000 time steps (here months) for the state variable X using the given values of $(X_b, α, \text{ and } σ)$.

2. Set the time step under consideration as the first time step.

3. For every node in the time step under consideration:

   3.1. Calculate retailer i's optimal quantity of the product and the respective profit in the monopoly market with respect to the value of (X) at this node based on Equations (3.6) and (3.7), respectively. Use the value of (X) at this node as (X(t)) in these equations.

   3.2. Multiply retailer i's optimal profit by the length of each time increment (Δt = 1 month = 1/12 year) and store it as the store's cash flow for this node at this time step.

4. Move to the next time step. Set this time step as the time step under consideration. Repeat step 3 until reach the last time step.

5. For every node in the last time step:

   5.1. Calculate retailer i's optimal quantity of the product with respect to the value of (X) at this node.

   5.2. Calculate the final value of retailer i's investment in this monopoly market using equation (B4). This final value is the NPV of a store opened in this dynamic market if retailer i provides the optimal quantity of the product – as it was calculated in step 5.1 – to the monopoly market as long as the store is opened after the last time step. Use this final value as retailer i's cash flow for this node at the last time step.

6. Start from one time step before the last time step. Set this time step as the time step under consideration.

7. For every node in this time step:

7.1. Calculate the expected value of retailer i's store cash flow at the next time step and discount it back for one time increment.

7.2. Add retailer i's discounted expected store cash flow to the store's cash flow at this node for the current time step. Update the store's cash flow at this node.

8. Move to the previous time step. Set this time step as the time step under consideration. Repeat step 7 until the process reaches the first time step.

9. Return the store's cash flow at the only node of the first time step as the expected NPV of a store opened in the dynamic, monopoly market. Denote this expected NPV as (NPV_i^M), which represents the NPV of retailer i's store opened in this monopoly market.

Next, we show how we can use a similar approach to determine the expected NPVs of two retailers' stores opened in the dynamic market.

3.7.1.2 The expected NPVs of stores opened in a dynamic duopoly market

Consider two retailers (retailer 1 and 2) that decide to open stores in a duopoly retail market. Again assume that the value of the underlying stochastic variable in this market (X) at the opening time of the store is (X_b). In addition, retailers' stores will remain opened forever. As the value of (X) changes over time these retailers revise their optimal quantities of the product in order to maximize their own profits. The retailers' optimal quantities of the product and their respective profits in this duopoly market are summarized in Equations (3.12) and (3.13), respectively, based on the value of (X) at time step (t). We use these equations to develop an approximate approach to calculate the expected NPVs of these stores in this market, as follows.

First, we construct a trinomial lattice for the future value of (X) in this retail market given its initial value of (X_b). We also make this trinomial lattice long enough, consisting of 1,000 time steps (months) to approximate the expected NPVs of retailers' stores opened in this dynamic market. In addition, we use the exact same cost of capital (ρ) to discount both retailers' store cash flows.

The following procedure shows how we can use the trinomial lattice of (X) to determine the expected NPVs of retailers' stores opened in this duopoly market.

1. Construct a trinomial lattice that has 1,000 time steps for the state variable (X) using the given values of (X_b, α, σ).

2. Set the time step under consideration as the first time step.

3. For every node in the time step under consideration:

   3.1. Calculate retailer 1's and 2's optimal quantities of the product and their respective profits in the duopoly market considering the value of (X) at this node based on Equations (3.12) and (3.13), respectively. Use the value of (X) at this node as (X(t)) in these equations.

   3.2. Multiply the retailer 1's and 2's optimal profits by the length of each time increment ($\Delta t = 1/12$ year) and store them as retailer 1's and 2's store cash flows for this node at this time step.

4. Move to the next time step. Set this time step as the time step under consideration. Repeat step 3 until reach the last time step.

5. For every node in the last time step:

   5.1. Calculate retailer 1's and 2's optimal quantities of the product with respect to the value of (X) at this node.

5.2. Calculate the final value of retailer 1's and 2's investment in this duopoly market using Equation (C4). These final values are the NPVs of retailer 1's and 2's stores opened in this dynamic market if these retailers provide the optimal quantities of the product – as they were calculated in step 5.1 – to the duopoly market for as long as the stores are opened after the last time step. Use these final values as retailer 1's and 2's cash flows for this node at the last time step.

6. Start from one time step before the last time step (time step 999). Set this time step as the time step under consideration.

7. For every node in this time step:

7.1. Calculate the expected values of retailer 1's and 2's store cash flows at the next time step and discount them back for one time increment.

7.2. Add retailer 1's and 2's discounted expected store cash flows to retailer 1's and 2's store cash flows at this node, respectively. Update retailer 1's and 2's store cash flows at this node.

8. Move to the previous time step. Set this time step as the time step under consideration. Repeat step 7 until the process reaches the first time step.

9. Return retailer 1's and 2's store cash flows at the only node of the first time step as retailer 1's and 2's expected NPV of a store opened in this dynamic, duopoly market. Denote retailer 1's and 2's expected NPV of a store opened in this duopoly market by (NPV_1^D) and (NPV_2^D), respectively.

We use the expected NPV of the retailer's store opened as part of our dynamic programming approach to determine the retailer's investment behavior in dynamic retail

markets. We summarize our approach for three general investment cases in retail markets, as follows.

3.7.2 The market, in which only one retailer (retailer i, i = either 1 or 2) has an investment opportunity to enter and opens a store

Consider a retail (retailer i, i = either 1 or 2) that has a propriety investment option to open a store in a dynamic retail market. This investment opportunity is valid until time step (T) from now (time step (0)) and then becomes totally worthless. Also assume that the store opened in this market remains opened for ever. The objective of developing a dynamic programming approach is to study the investment behavior of retailer i in this dynamic market. Our particular interest is to explore when retailer i exercises its investment option and opens a store in this market. Retailer i's entry decision depends on the value of (X), which represents the dynamic state of the market at time step (t). In this section, we develop a dynamic programming approach to help us determine retailer i's investment threshold in terms of (X) and as function of time. This dynamic programming approach is based on the development of a lattice model to represent the dynamic uncertainty of (X) in a discrete fashion and then use it as a decision tree to determine the optimal value of (X) at time step (t), in which retailer i exercises its investment option.

The trinomial lattice for (X) can be constructed based on Equations (3.15) and (3.16), using the numerical values for (X_0), (α), and (σ). Note that we choose $(\Delta t = 1$ month = 1/12 year) to be the length of a time increment in the lattice and therefore, the constructed lattice consists of (N) time steps, in which N equals the ceiling value of $(T/\Delta t)$. The details of the backward calculation procedure that can be used to determine retailer i's investment threshold is summarized, as follows.

1. Construct a trinomial lattice that has (N) time steps for the state variable (X) using the given values of (X_0, α, σ, N).

2. Start from the nodes in the last time step. Set time step (N) as the time step under consideration.

3. For each node at the time step under consideration:

3.1. Calculate (NPV_i^M) corresponding to the value of (X) at this node. This is the value of retailer i's investment option at this node if retailer i decides to invest and opens a store in this market.

3.2. Calculate the value of waiting if retailer i decides to defer its investment option for one time step. This value is equal to the expected value of its investment option at the next time step, which is discounted back for one time increment. If the node is in the last time step the value of waiting becomes zero.

3.3. Determine whether retailer i invests at this node. Use the following decision rule. Retailer i invests in this market and opens a store if (NPV_i^M) exceeds the value of waiting at this node. Mark this node if retailer i decides to invest.

3.4. Store the maximum value between (NPV_i^M) and the value of waiting as the value of retailer i's investment option at this node.

4. For the time step under consideration, find the minimum value of (X) among the entire marked nodes. Return this value as retailer i's investment threshold at this time step.

5. Move to the previous time step. Set this time step as the time step under consideration. Repeat steps 3 and 4. Continue until reach the first time step of the lattice.

6. Return the value of retailer i's investment option at the only node of the first time step as the value of this investment option for retailer i.

7. Return retailer i's investment threshold in terms of the value of (X) at time step (t), in which retailer i exercises its investment option in this monopoly dynamic market.

Next, we consider another general market case to study the investment behaviors of a retailer in a competitive market.

3.7.3 The market, in which retailer 1 has a store opened and retailer 2 has an investment opportunity to enter and open a store

Consider retailer 2 has an investment option to open a store in a dynamic retail market. This investment opportunity is valid until time step (T) from now (time step (0)) and then becomes totally worthless. Also assume that the store opened in this market remains opened for ever. However, the difference between this investment case and what was described in the previous section is that there is another retailer (retailer 1) has already invested in and has a store opened in this market.

The objective of developing a dynamic programming approach is to study the investment behavior of retailer 2 in this dynamic market considering the existence of retailer 1. This dynamic programming approach is based on the development of a similar lattice model to represent the dynamic uncertainty of (X) in a discrete fashion and then use it as a decision tree to determine the optimal value of (X) at time step (t), in which retailer 2 exercises its investment option. The details of the backward calculation procedure that can be used to determine retailer 2's investment threshold are summarized, as follows.

1. Construct a trinomial lattice that has (N) time steps for the state variable (X) using the given values of (X_0, α, σ, N).

2. Start from the nodes in the last time step. Set time step (N) as the time step under consideration.

3. For each node at the time step under consideration:

3.1. Calculate retailer 2's (NPV_2^D) in this duopoly market corresponding to the value of (X) at this node. This is the value of retailer 2's investment option at this node if retailer 2 decides to invest and opens a store in this market.

3.2. Calculate the value of waiting if retailer 2 decides to defer its investment option for one time step. This value is equal to the expected value of its investment option at the next time step, which is discounted back for one time increment. If this node is in the last time step the value of waiting for retailer 2 is zero.

3.3. Determine whether retailer 2 invests at this node. Use the following decision rule. Retailer 2 invests in this market and opens a store if (NPV_2^D) exceeds the value of waiting at this node. Mark this node if retailer 2 decides to invest.

3.4. Store the maximum value between (NPV_2^D) and the value of waiting as the value of retailer 2's investment option at this node.

3.5. Calculate retailer 1's investment value. Use the following rule. The value of retailer 1's investment is equal to (NPV_1^D) if retailer 2 decides to invest. If retailer 2 decides to defer its investment option to the next time step retailer 1 will be the only firm in the market for one time increment. Therefore, retailer 1's investment value will have two components. The first component is retailer 1's optimal

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monopoly profit over a single time increment. The second component is the expected value of retailer 1's investment at the next time step, which is discounted back for one time increment. Adding these two components determines the value of retailer 1's investment. Note that if the node under consideration belongs to the last time step and retailer 2 decides not to invest, retailer 1's investment value is equal to (NPV_1^{M}) since retailer 1 has a monopoly market advantage for ever. Also consider that the investment cost should not be considered in the NPV calculation for retailer 2, neither at (NPV_1^{D}) nor at (NPV_1^{M}), since retailer 1 is assumed to be already in the market.

4. For the time step under consideration, find the minimum value of (X) among the entire marked nodes. Return this value as retailer 2's investment threshold at this time step.

5. Move to the previous time step. Set this time step as the time step under consideration. Repeat steps 3 and 4. Continue until the process reaches the first time step of the lattice.

6. Return the value of retailer 2's investment option at the only node of the first time step as the value of retailer 2's investment option for retailer 2.

7. Return retailer 2's investment threshold in terms of the value of (X) at time step (t), in which retailer 2 exercises its investment option in this dynamic market.

A very similar approach can be developed to evaluate retailer 1's investment behavior in a market, in which retailer 2 has a store opened and retailer 1 has an investment opportunity to enter and opens a store. Next, we consider a market that both retailer 1 and 2 have a shared investment option to open a store.

3.7.4 The market, in which both retailers have investment opportunities and compete to enter and open stores

Consider two retailers (retailer 1 and 2) that have a shared investment option to open stores in a dynamic retail market. Their investment opportunities are valid until time step (T) from now (time step (0)) and then become totally worthless. Also assume that the stores opened in this market remain opened for ever.

The objective of developing a dynamic programming approach is to study the investment behaviors of both retailers in this competitive, dynamic market. This dynamic programming approach is based on the development of a similar lattice model to represent the dynamic uncertainty of (X) in a discrete fashion and then use it as a decision tree to determine the optimal values of (X) at time step (t), in which retailer 1 and 2 exercise their investment options. We incorporate game theory in our evaluation procedure at the lattice nodes to explore the stable state of the market at each lattice node as a result of competition between retailers. The details of the backward calculation procedure that can be used to determine these thresholds are summarized, as follows.

1. Construct a trinomial lattice that has (N) time steps for the state variable (X) using the given values of (X_0, α, σ, N).

2. Start from the nodes in the last time step. Set time step (N) as the time step under consideration.

3. For each node at the time step under consideration:

3.1. Construct a table for this node that summarizes a 2X2 game between retailers. Retailers' investment values are stored in this table with respect to the following four market structures:

3.1.1. Both retailers exercised their investment options and open stores.

3.1.2. Both retailers defer their investment options for one time step (or drop their investment options if the time step under consideration is the last time step).

3.1.3. Retailer 1 already exercised its investment option and opened a store in this market and retailer 2 has an investment option in this market that he should decide to exercise or defer this option.

3.1.4. Retailer 2 already exercised its investment option and opened a store in this market and retailer 1 has an investment option in this market that he should decide to exercise or defer this option.

Note that we have already discussed the last two market structures in section 3.7.3.

3.2. Calculate retailer 1's (NPV_1^D) and retailer 2's (NPV_2^D) for the market structure, in which both retailers decide to exercise their investment options and open stores corresponding to the value of (X) at this node.

3.3. Retrieve retailer 1's investment value and retailer 2's investment option value from the corresponding node at this time step from the described lattice in section 3.7.3. These values represent retailers' investment values for the market, in which retailer 1 has a store and retailer 2 has an option to open a store. Note that we need to adjust retailer 1's investment value by subtracting retailer 1's investment cost from its investment value in section 3.7.3 since retailer 1 must pay this investment cost to acquire this investment opportunity.

3.4. Retrieve retailer 2's investment value and retailer 1's investment option value from the corresponding node at this time step from the described lattice in section 3.7.3. These values represent retailers' investment values for the market, in which retailer 2 has a store and retailer 1 has an option to open a store. Note that we need to adjust retailer 2's investment value by subtracting retailer 2's investment cost from its investment value in section 3.7.3 since retailer 2 must pay this investment cost to acquire this investment opportunity.

3.5. Calculate retailers' investment values for the market structure, in which both retailers decide to defer their investment options for one time step. These values are equal to the expected values of retailers' investment values at the stable state of the market in the next time step, which are discounted back for one time increment. Note that retailers' investment values for this market structure are zero if the time step under consideration is the last time step since retailers cannot defer their investment options further on. For any time step before the final time step, retailers' expected values in the next time step are the values of their investment options at the stable state of the market in the next time step. We will show how we can determine the stable state of the market in the next step.

3.6. Determine the pure- and mixed-strategy Nash equilibrium (equilibria) of the 2X2 game between these retailers at this node. Return whether any of the four market structures described in step (3.1.) is a pure-strategy Nash equilibrium for the 2X2 game between these retailers. Also return the values of retailers' investments in the stable state of the market at this node. In game theory, the Nash equilibrium is a solution concept of a game involving two or more players, in

which no player has anything to gain by changing only his or her own strategy unilaterally [63]. One or many of the described market structures might be the Nash equilibrium (equilibria) of the game between retailers at this node. Mark every market structure that represents a pure-strategy Nash equilibrium for retailers' investment game at this node. If there is only a single Nash equilibrium for this game, use the respective retailers' investment values corresponding to this unique Nash equilibrium market structure as retailers' investment values for this node at the stable state of the market. If there are more than one pure-strategy Nash equilibrium or if there is no pure-strategy Nash equilibrium for this game, calculate the mixed-strategy Nash equilibrium and use the retailers' investment values corresponding to this mixed-strategy as retailers' investment values for this node at the stable state of the market. In game theory a mixed strategy is a strategy, which chooses randomly between possible moves [63]. Mark mixed-strategy Nash equilibrium for this node if mixed-strategy is used to determine the retailers' investment values at the stable state of the market for this node.

4. For the time step under consideration, find the minimum value of (X) among the entire nodes that are marked for the market structure that retailer 1 was in and retailer 2 has an investment option to enter. We call this value (X_1). Also for the same time step, find the minimum value of (X) among the entire set of nodes that are marked for the market structure that retailer 2 was in and retailer 1 has an investment option to enter. We call this value (X_2). If the minimum value of (X) in the former case (X_1) is lower than the minimum value of (X) in the latter case (X_2) retailer 1 is the first retailer that invests and enters the market. Return (X_1) as retailer 1's investment

threshold at this time step. After retailer 1 opens a store, the market structure changes to the market in which retailer 1 has a store opened and retailer 2 has an option to open a store. Therefore, retailer 2's investment threshold at this time step becomes the retailer 2's investment threshold at the same time step in the general investment case described in section 3.7.3. In other words, this threshold is also equal to the minimum value of (X) for the market that both retailers decide to exercise their investment options and invest. Return this minimum value of (X) as retailer 2's investment threshold at this time step. Similarly, we can determine the retailers' investment thresholds at this time step when (X_2) is lower than (X_1). Return (X_2) as retailer 2's investment threshold at this time step. Also return the minimum value of (X) for the market that both retailers decide to open stores as retailer 1's investment threshold at this time step.

5. Move to the previous time step. Set this time step as time step under consideration. Repeat steps 3 and 4. Continue until the process reaches the first time step of the lattice.

6. Return the values of retailers' investments in the stable state of the market at the only node of the first time step as the retailers' values for this investment case.

7. Return retailer 1's and 2's investment thresholds in terms of the values of (X) at time step (t), in which retailer 1 and 2 exercise their investment options in this competitive, dynamic market, respectively.

In the next section, we use a simple numerical example to show how we can use the described investment analysis approaches in practice.

3.8   A numerical example

In this section, we use a simple example to illustrate how the game theory approach can be integrated in the lattice model to explore retailers' investment opportunities and determine their decisions as a decision tree problem. This example illustrates two retailers' behaviors in both noncompetitive and competitive, dynamic markets. The objective is to show how we can determine retailers' optimal investment thresholds and the values of their investment options using the described procedure in section 3.7.

Consider two retailers: retailer 1 and 2. Retailer 1's investment and fixed cost is higher than retailer 2's investment and fixed cost, respectively, i.e., (IC_1 = $400,000 > IC_2 = $200,000 and FC_1 = 200,000 $/Year > FC_2 = 100,000 $/Year), but retailer 2's marginal cost is higher than retailer 1's marginal cost (VC_1 = 80 $/Item Sold < VC_2 = 100 $/Item Sold). Also assume that retailers' investment options are valid for only one month, i.e., two-time step investment option. The values of the other model parameters are assumed to be: (ρ = 15%/Year), (α = 4%/Year), (σ = 0.1), (X_0 = 990), and (γ = 1).

We build a trinomial lattice for X using the following values: (α = 0.04, Δt = 1/12, σ = 0.1, N = 2, and X_0 = 990) and use it as a decision tree to determine retailers' optimal investment behaviors. This lattice is shown in Figure 3.3. The values of (X) at the end of the month are (X_1^+ = 1021.94, X_1^c = 990, and X_1^- = 959.05) with probabilities of (p_1 = 0.46, p_2 = 0.17, and p_3 = 0.37), respectively. These values are computed based on equations (3.15) and (3.16).

$$p_1 = 0.46 \rightarrow X_1^+ = 1021.94$$

$$X_0 = 990 \;\; \longleftarrow \;\; p_2 = 0.17 \rightarrow X_1^c = 990$$

$$p_3 = 0.37 \rightarrow X_1^- = 959.05$$
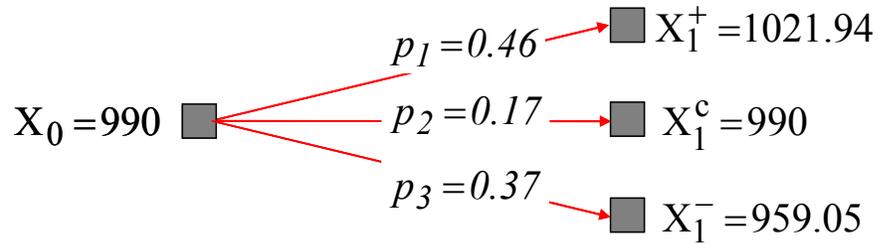
Figure 3.3. The trinomial lattice for the values of (X) and the actual probabilities in this dynamic market.

We use this lattice as a decision tree to analyze retailers' investment options. First, we study retailer 1's and 2's propriety options in this noncompetitive, dynamic market. We follow the procedure described in section 3.7.2 to determine the value of retailer 1's investment option in this dynamic market. This procedure is summarized in Figure 3.4. We start from the nodes in the last time step. Retailer 1 has a decision to make at each node: drop its investment option and receive zero or invest and open a store in this market and receives the NPV of the future cash flow of this store in this monopoly, dynamic market. It can be seen in Figure 3.4 that retailer 1 decides to invest in this market in all three nodes of the last time step since its investment value exceeds zero.

Then, we continue our evaluation process to the first phase. As it can be seen in Figure 3.4, retailer 1 decides to defer its investment option to the next time step at this node since the discounted, expected value of its investment at the next time step exceeds the NPV of its investment at this monopoly market respective to (X_0). Therefore, we can determine retailer 1's value of this investment option, which is denoted by (NOV_1) (i.e., retailer 1's Net Option Value) in Figure 3.4.

$$(NPV_1)^+ = \max(\text{Drop investment} = 0,$$
$$\textbf{Invest (Monopoly)} = \textbf{2,275,000})$$

$$(NOV_1) = \max(\textbf{Defer invetsment} = \textbf{2,021,000},$$
$$\text{Invest (Monopoly)} = 2,017,000)$$

$$(NPV_1)^c = \max(\text{Drop investment} = 0,$$
$$\textbf{Invest (Monopoly)} = \textbf{2,017,000})$$

$$(NPV_1)^- = \max(\text{Drop investment} = 0,$$
$$\textbf{Invest (Monopoly)} = \textbf{1,774,800})$$
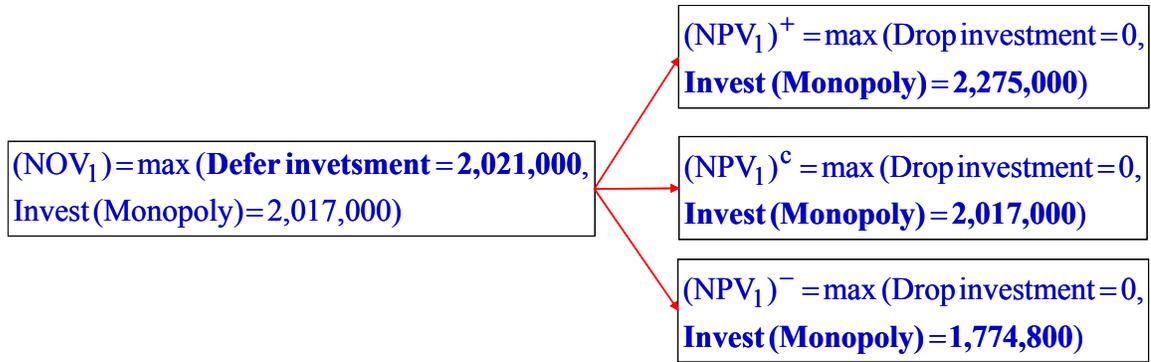
Figure 3.4. Retailer 1's NOV and optimal investment decisions.

Similarly, we can determine retailer 2's NOV and optimal investment decisions in this dynamic market. The results are summarized in Figure 3.5. Unlike retailer 1, retailer 2 exercises its investment opportunity at the first time step and does not wait for the second time step.

$$(NPV_2)^+ = \max(\text{Drop investment} = 0,$$
$$\textbf{Invest (Monopoly)} = \textbf{3,062,000})$$

$$(NOV_2) = \max(\text{Defer invetsment} = 2,801,000,$$
$$\textbf{Invest (Monopoly)} = \textbf{2,807,000})$$

$$(NPV_2)^c = \max(\text{Drop investment} = 0,$$
$$\textbf{Invest (Monopoly)} = \textbf{2,807,000})$$

$$(NPV_2)^- = \max(\text{Drop investment} = 0,$$
$$\textbf{Invest (Monopoly)} = \textbf{2,568,000})$$
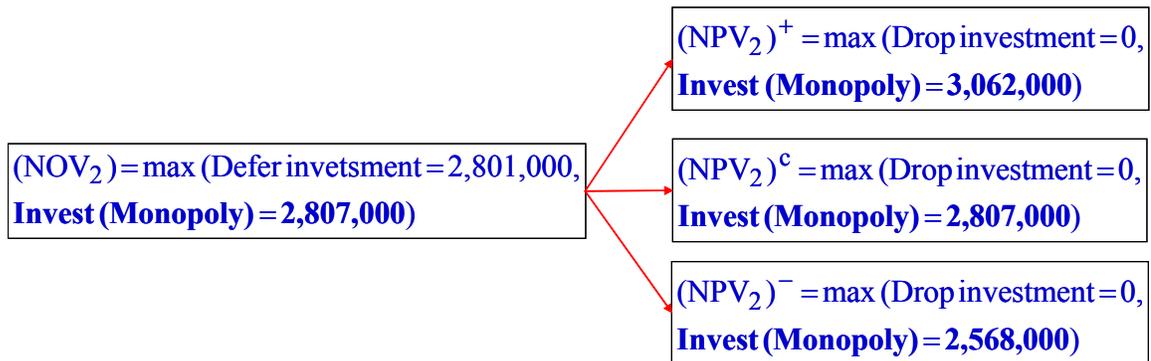
Figure 3.5. Retailer 2's NOV and optimal investment decisions.

Now, consider a market, in which retailer 2 has a store opened and retailer 1 has an investment option to open a store. We need to determine retailer 1's optimal decision at each node in order to determine retailer 2's value of investment at that node since retailer 1's decision determines the market structure and directly impacts the value of retailer 2's investment. We follow the procedure described in section 3.7.3 for this purpose.

Figure 3.6 summarizes retailer 1's NOV and its optimal investment decisions in this dynamic market. It can be seen that the only time retailer 1 exercises its investment option is in the second time step and the upper node corresponding to (X_1^+). Retailer 1's value of this investment option (denoted by NOV_1) is calculated based on discounting the expected investment value at the next time step.

$$(NPV_1)^+ = \max(Drop\,investment = 0,\ \textbf{Invest (Duopoly)} = \textbf{78,780})$$

$$(NOV_1) = \max(\textbf{Defer invetsment} = \textbf{35,725},\ Invest(Duopoly) = -37,380)$$

$$(NPV_1)^c = \max(\textbf{Drop investment} = \textbf{0},\ Invest(Duopoly) = -37,380)$$

$$(NPV_1)^- = \max(\textbf{Drop investment} = \textbf{0},\ Invest(Duopoly) = -146,280)$$
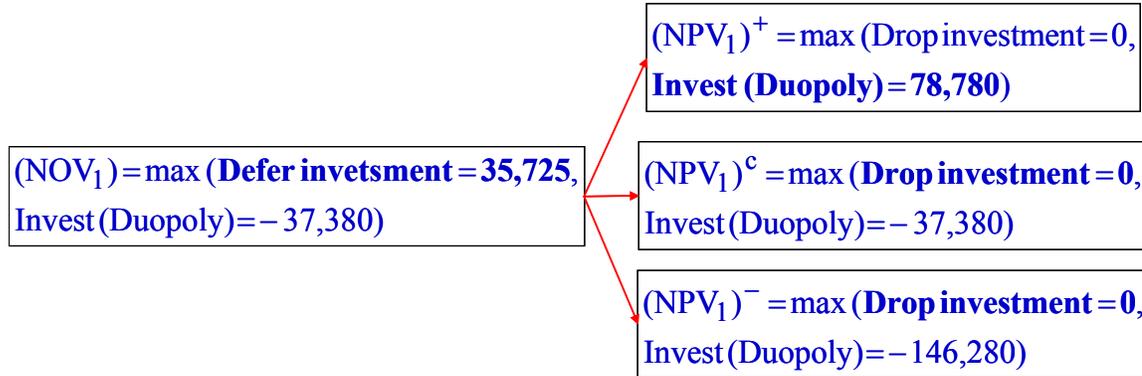
Figure 3.6. Retailer 1's NOV and optimal investment decisions in a dynamic market, in which retailer 2 has a store opened.

Corresponding to retailer 1's decisions, we can determine the value of retailer 2's investment (NPV_2), which represents the expected NPV of its store's cash flow that is opened in this dynamic market. These investment values are summarized in Figure 3.7. Retailer 2 receives NPV of a store opened in the duopoly market at the second time step when $(X(t) = X_1^+)$ since retailer 1 enters the market at this node. However, at the other two values of (X) at this time step (X_1^c) and (X_1^-) he receives NPV of a store opened in the monopoly market since retailer 1 decides to not enter the market. At the first time step, retailer 1's investment value has two components. The first component is the one time increment monopoly profit due to retailer 2's decision of deferring its investment option and the second component is the expected value of its investment at the second time step, which is discounted back for one time increment.

$$NPV_2^+ = 1,037,000 \, (\text{Duopoly})$$

$$NPV_2 = 1,998,000 \, (\text{Monopoly profit in the first period} + \text{Discounted expected value of the next stage})$$

$$NPV_2^c = 3,007,000 \, (\text{Monopoly})$$

$$NPV_2^- = 2,767,000 \, (\text{Monopoly})$$
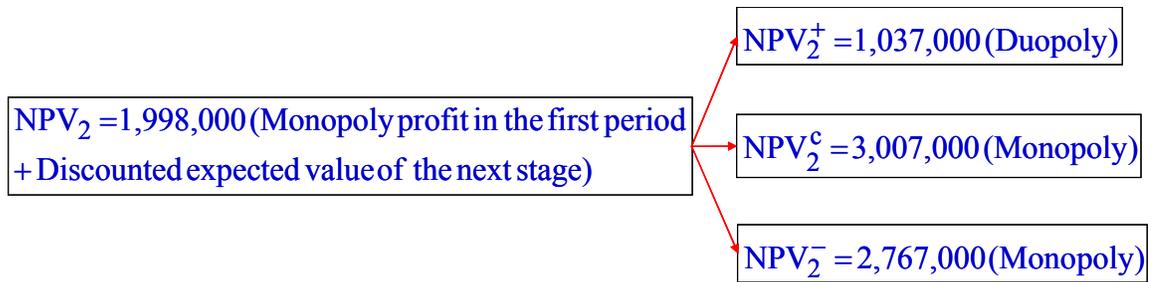
Figure 3.7. Retailer 2's NPV of a store opened in this market, in which retailer 1 has an investment option to open a store.

Next, we consider a similar market structure, in which retailer 1 has a store opened and retailer 2 has an investment option to enter. We can follow the same calculations based on the procedure described in section 3.7.3 to determine retailer 1's NPV and retailer 2's NOV. A summary of results are provided in Figure 3.8 and Figure 3.9, respectively.

$$(NPV_2)^+ = \max(\text{Drop investment} = 0, \mathbf{Invest\,(Duopoly)} = \mathbf{836,000})$$

$$(NOV_2) = \max(\mathbf{Defer\ invetsment} = \mathbf{728,000}, \text{Invest\,(Duopoly)} = 724,000)$$

$$(NPV_2)^c = \max(\text{Drop investment} = 0, \mathbf{Invest\,(Duopoly)} = \mathbf{724,000})$$

$$(NPV_2)^- = \max(\text{Drop investment} = 0, \mathbf{Invest\,(Duopoly)} = \mathbf{619,000})$$
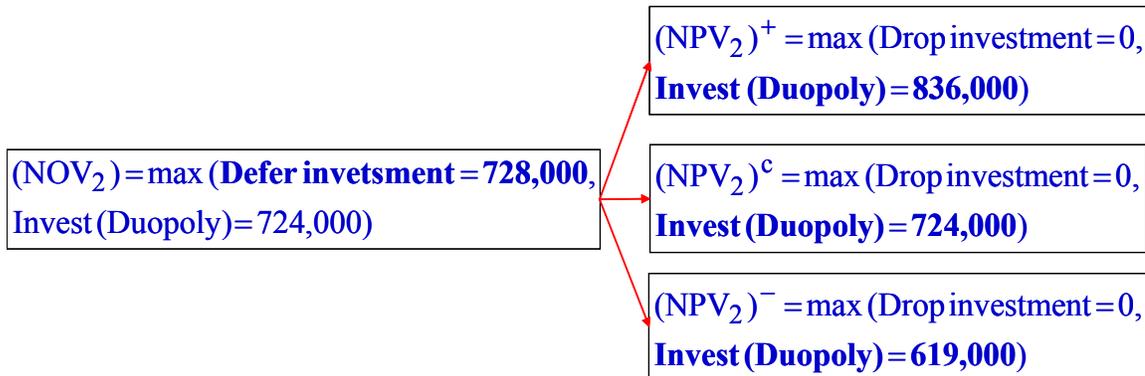
Figure 3.8. Retailer 2's NOV and optimal investment decisions in a dynamic market, in which retailer 1 has a store opened.

It can be seen in Figure 3.8 that retailer 2 defers its investment option to the second time step and invests at all three levels of (X) in that time step. As a result of retailer 2's optimal decisions, retailer 1's NPV of its store opened in this market follows the values shown in Figure 3.9.

$$NPV_1^+ = 479{,}000 \,(\text{Duopoly})$$

$$NPV_1 = 372{,}000 \,(\text{Monopoly profit in the first period} + \text{Discounted expected value of the next stage})$$

$$NPV_1^c = 363{,}000 \,(\text{Duopoly})$$

$$NPV_1^- = 254{,}000 \,(\text{Duopoly})$$
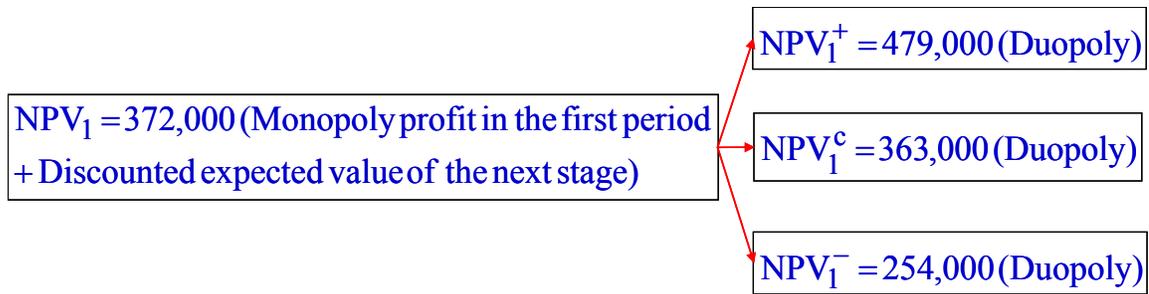
Figure 3.9. Retailer 1's NPV of a store opened in this market, in which retailer 2 has an investment option to open a store.

Next, we consider a more general case in which both retailers have investment options and compete to open stores in this dynamic market. We follow the procedure described in section 3.7.4 for this purpose. The results are shown in Figure 3.10 in the form of 2X2 tables that represent gaming between two retailers at each node. Each table shows the values of retailers' investment options under four possible market situations at the respective node. These possible market situations can be summarized, as follows.

- Both retailers exercise their investment options and open stores in this market. The respective retailers' investment values are summarized in the (I/I) cells of the 2X2 tables in Figure 3.10.

- Both retailers decide to defer their investment options at the first time step or drop their investment options at the second time step. The respective retailers' investment values are summarized in the (D/D) cells of the 2X2 tables in Figure 3.10.

- Retailer 1 is in the market and retailer 2 has an investment option to open a store in this market. The respective retailers' investment values are summarized in the (I/D) cells of the 2X2 tables in Figure 3.10.

- Retailer 2 is in the market and retailer 1 has an investment option to open a store in this market. The respective retailers' investment values are summarized in the (D/I) cells of the 2X2 tables in Figure 3.10.

At each node, we find the stable state of the market that determines retailers' optimal decisions. The Nash equilibria of games between retailers at each node are highlighted in this figure. It can be seen that the only time that both retailers are in the market is at the upper node of the second time step when $(X(t) = X_1^+)$. Retailer 2 is the first retailer that invests and opens a store in this market at the first time step due to its overall cost advantage. On the other hand, retailer 1 only invests and opens a store in this market at the upper node in the second time step when $(X(t) = X_1^+)$.
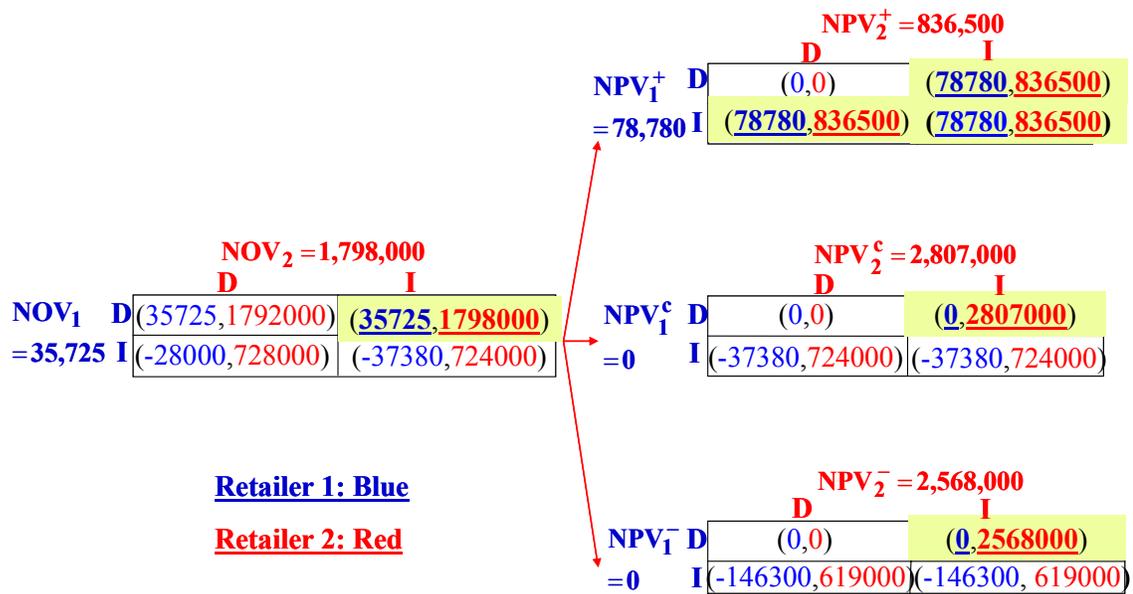
Figure 3.10. Retailer 1's and 2's investment values at the stable states of this dynamic market.

In the next section, we extend this simple example to the other retailers' investment options that are valid for a larger number of time steps. Through several numerical

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examples, we study how the values of retailers' cost parameters impact their optimal investment thresholds in noncompetitive and competitive dynamic markets.

3.9   Impact of retailers' cost structure on their optimal entry time decisions

In this section, we consider two retailers that have investment options to enter a dynamic market. The objective is to explore retailers' investment behaviors in terms of their entry decisions considering their difference in cost parameters and the effect of competition in dynamic markets.

We evaluate retailers' investment behaviors under two market situations. First, we consider a market situation, in which each retailer separately has a *proprietary investment option* to invest and opens a store. This market situation is not competitive. Secondly, we consider the same market situation, in which both retailers have *shared investment options* to invest and open stores. This market situation, on the other hand, is competitive. Therefore in this section, we study the effect of competition on retailers' investment behaviors.

In addition, we consider four general investment cases to study the impacts of retailers' cost parameters on their entry decisions. The described investment analysis approach in section 3.7 is used here to identify retailers' investment thresholds in terms of the values of X at time step (t) in the competitive, dynamic market. Four scenarios regarding the values of retailers' cost parameters are of particular interest, as summarized below.

- Retailer 1's fixed cost is higher than retailer 2's ficxed cost, i.e., $(FC_1 > FC_2)$, but the other cost parameters are equal $(IC_1 = IC_2$ and $VC_1 = VC_2)$

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- Retailer 1's investment cost is higher than retailer 2's investment cost, i.e., ($IC_1 >$ IC_2), but the other cost parameters are equal ($FC_1 = FC_2$ and $VC_1 = VC_2$)

- Retailer 1's marginal cost is higher than retailer 2's marginal cost, i.e., ($VC_1 >$ VC_2), but the other cost parameters are equal ($FC_1 = FC_2$ and $IC_1 = IC_2$)

- Retailer 1's investment and fixed cost is higher than retailer 2's investment and fixed cost, respectively, i.e., ($IC_1 > IC_2$ and $FC_1 > FC_2$), but retailer 2's marginal cost is higher than retailer 1's marginal cost ($VC_1 < VC_2$)

Retailers' investment behaviors are summarized in this section through a numerical example considering differences in the values of retailers' cost parameters in contrast to competitive versus noncompetitive markets.

3.9.1 *Two identical retailers with only the fixed cost difference*

Consider two retailers that have similar cost parameters and are only different in their fixed costs. The following numerical values are assumed for the cost parameters of these two retailers: ($IC_1 = IC_2 = \$400,000$, $VC_1 = VC_2 = 100$ \$/Item Sold and $FC_1 = 220,000$ \$/Year $> FC_2 = 200,000$ \$/Year). Also assume that retailers' investment options are valid for only two hundred months and then they become worthless. The number of months that retailers' investment options are valid is chosen arbitrarily. Our investment analysis approach can be extended to shorter or longer investment horizons. The values of the other model parameters are assumed to be: ($\rho = 15\%$/Year), ($\alpha = 4\%$/Year), ($\sigma = 0.1$), ($X_0 = 700$), and ($\gamma = 1$).

We use the described dynamic programming approach in section 3.7 for investment valuation. We build a trinomial lattice for X using the following values: ($\alpha = 0.04$, $\Delta t =$

1/12, $\sigma = 0.1$, N = 200, and $X_0 = 700$) and use it as a decision tree to determine retailers' optimal investment thresholds to enter the market.

Figure 3.11 shows retailers' optimal investment thresholds in two market situations as indicated by the values of X at time step (t), which is denoted by (X^*(t)) in the vertical axis. At time step (t) a retailer exercises its investment option and opens a store in a market when the value of retail market potential at this time step exceeds the optimal value of retail market potential at this time step, which is determined by our investment analysis approach as it is shown by the curve in Figure 3.11.

Since retailer 2 has the lower fixed cost than retailer 1 he has the advantage to enter the market first. Therefore, retailer 2's optimal investment threshold is lower than retailer 1's optimal investment threshold in both noncompetitive and competitive market structures.

The competition effect can be seen in Figure 3.11 too. Due to the competition, Retailer 2's investment threshold in the competitive market is lower than its investment threshold in the same market without any competition effect. Therefore, retailer 2 waits longer to exercise its investment option in the noncompetitive market. In the noncompetitive market retailer 2 can wait longer to receive more information regarding the future state of the market and then, exercises its investment option without having any fear regarding the entry of the other retailer.

On the other hand, the early investment by retailer 2 in the competitive, dynamic market has a disadvantage for retailer 1 since retailer 2's early entry makes retailer 1 wait longer before entering the competitive market. Given retailer 2 is already in the market retailer 1 must wait longer to exercise its shared investment option compared to the

market situation, in which the competition effect from retailer 1 does not exist. Retailer 1's optimal investment threshold in the competitive market is the threshold, in which the competitive market becomes large enough to support both retailers. Figure 3.11 shows that retailer 1 invests later when he holds a shared investment option. Thus, *our dynamic programming approach indicates the strategic aspect of early investment in competitive market through this numerical example*.

Also consider that the retailer's optimal investment threshold is lowered at the end because it is assumed that a retailer's investment opportunity disappears after the last time step. This lowered threshold at the end is simply an artifact of the end point that we choose in this numerical example. In the real world one can argue that the market opportunity does not really disappear and hence, the retailer's investment opportunity is perpetual and stays forever. However, it is indicated that the trinomial lattice formulation can be used as an accurate approximation for investment options that never expire [54]. Therefore, we can also use the investment evaluation procedure based on the trinomial lattice approximation that is described in this chapter for a retailer's investment opportunity that never expires. The only requirement is to construct a lattice with sufficiently large number of time steps in order to provide an appropriate approximation for the infinite time horizon of a retailer's investment option. Hence, the evaluation procedure of a retailer's investment option with an infinite time horizon is reduced to the evaluation procedure of a retailer's investment option with a very long, but finite time horizon as it was described in this chapter.

Figure 3.11. Impacts of competition and fixed cost differences on retailers' optimal investment thresholds.

3.9.2 Two identical retailers with only the marginal cost difference

Consider two retailers that have similar cost parameters and are only different in their marginal costs. The following numerical values are assumed for the cost parameters of these two retailers: ($IC_1 = IC_2 = \$400,000$, $FC_1 = FC_2 = 200,000$ \$/Year, and $VC_1 = 100$ \$/Item Sold $> VC_2 = 80$ \$/Item Sold). Also assume that retailers' investment options are valid for two hundred months. Again assume the same values for the other model parameters as section 3.9.1. We also use the same trinomial lattice built in section 3.9.1 to evaluate retailers' investment options in competitive and non-completive markets.

Figure 3.12 shows retailers' optimal investment thresholds in two market situations as indicated by ($X^*(t)$) in the vertical axis. At time step (t) a retailer exercises its investment option and opens a store in a market when the value of retail market potential

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at this time step exceeds the optimal value of retail market potential at this time step, which is determined by our investment analysis approach as it is shown by the curve in Figure 3.12.

Since retailer 2 has the lower marginal cost than retailer 1 he has the advantage to enter the market first. Therefore, retailer 2's optimal investment threshold is lower than retailer 1's optimal investment threshold in both noncompetitive and competitive market structures.

The competition effect can be seen in Figure 3.12 too. Due to the competition, retailer 2's investment threshold in the competitive market is lower than its investment threshold in the same market without any competition effect. Therefore, retailer 2 waits longer to exercise its investment option in the noncompetitive market. In the noncompetitive market retailer 2 can wait longer to receive more information regarding the future state of the market and then, exercises its investment option without having any fear regarding the entry of the other retailer.

On the other hand, early investment by retailer 2 in the competitive, dynamic market has a disadvantage for retailer 1 since retailer 2's early entry makes retailer 1 wait longer before entering the competitive market. Given retailer 2 is already in the market retailer 1 must wait longer to exercise its shared investment option compared to the market situation, in which the competition effect from retailer 1 does not exist. Retailer 1's optimal investment threshold in the competitive market is the threshold, in which the competitive market becomes large enough to support both retailers. Figure 3.12 shows that retailer 1 invests later when he holds a shared investment option. Thus, *our dynamic*

programming approach indicates the strategic aspect of early investment in competitive market through this numerical example.

Also consider that the retailer's optimal investment threshold is lowered at the end because it is assumed that a retailer's investment opportunity becomes disappeared after the last time step. This lowered threshold at the end is simply an artifact of the end point that we choose in this numerical example. In the real world one can argue that the market opportunity does not really disappear and hence, the retailer's investment opportunity is perpetual and stays forever. However, it is indicated that the trinomial lattice formulation can be used as an accurate approximation for investment options that never expire [54]. Therefore, we can also use the investment evaluation procedure based on the trinomial lattice approximation that is described in this chapter for a retailer's investment opportunity that never expires. The only requirement is to construct a lattice with sufficiently large number of time steps in order to provide an appropriate approximation for the infinite time horizon of a retailer's investment option. Hence, the evaluation procedure of a retailer's investment option with an infinite time horizon is reduced to the evaluation procedure of a retailer's investment option with a very long, but finite time horizon as it was described in this chapter.

Figure 3.12. Impacts of competition and marginal cost differences on retailers' optimal investment thresholds.

3.9.3 Two identical retailers with only the investment cost difference

Consider two retailers that have similar cost parameters and are only different in their investment costs. The following numerical values are assumed for the cost parameters of these two retailers: ($VC_1 = VC_2 = 100$ \$/Item Sold, $FC_1 = FC_2 = 200,000$ \$/Year, and $IC_1 = \$500,000 > IC_2 = \$400,000$). Also assume that retailers' investment options are valid for two hundred months. Again assume the same values for the other model parameters as section 3.9.1. We also use the same trinomial lattice built in section 3.9.1 to evaluate retailers' investment options in competitive and non-completive markets.

Figure 3.13 shows retailers' optimal investment thresholds in two market situations as indicated by ($X^*(t)$) in the vertical axis. At time step (t) a retailer exercises its investment option and opens a store in a market when the value of retail market potential

at this time step exceeds the optimal value of retail market potential at this time step, which is determined by our investment analysis approach as it is shown by the curve in Figure 3.13.

Since retailer 2 has the lower investment than retailer 1 he has the advantage to enter the market first. Therefore, retailer 2's optimal investment threshold is lower than retailer 1's optimal investment threshold in both noncompetitive and competitive market structures.

The competition effect can be seen in Figure 3.13 too. Due to the competition, retailer 2's investment threshold in the competitive market is lower than its investment threshold in the same market without any competition effect. Therefore, retailer 2 waits longer to exercise its investment option in the noncompetitive market. In the noncompetitive market retailer 2 can wait longer to receive more information regarding the future state of the market and then, exercises its investment option without having any fear regarding the entry of the other retailer.

On the other hand, early investment by retailer 2 in the competitive, dynamic market has a disadvantage for retailer 1 since retailer 2's early entry makes retailer 1 to wait longer before entering the competitive market. Given retailer 2 is already in the market retailer 1 must wait longer to exercise its shared investment option compared to the market situation, in which the competition effect from retailer 1 does not exist. Retailer 1's optimal investment threshold in the competitive market is the threshold, in which the competitive market becomes large enough to support both retailers. Figure 3.13 shows that retailer 1 invests later when he holds a shared investment option. Thus, *our dynamic*

programming approach indicates the strategic aspect of early investment in competitive market through this numerical example.

Also consider that the retailer's optimal investment threshold is lowered at the end because it is assumed that a retailer's investment opportunity becomes disappeared after the last time step. This lowered threshold at the end is simply an artifact of the end point that we choose in this numerical example. In the real world one can argue that the market opportunity does not really disappear and hence, the retailer's investment opportunity is perpetual and stays forever. However, it is indicated that the trinomial lattice formulation can be used as an accurate approximation for investment options that never expire [54]. Therefore, we can also use the investment evaluation procedure based on the trinomial lattice approximation that is described in this chapter for a retailer's investment opportunity that never expires. The only requirement is to construct a lattice with sufficiently large number of time steps in order to provide an appropriate approximation for the infinite time horizon of a retailer's investment option. Hence, the evaluation procedure of a retailer's investment option with an infinite time horizon is reduced to the evaluation procedure of a retailer's investment option with a very long, but finite time horizon as it was described in this chapter.

Figure 3.13. Impacts of competition and investment cost differences on retailers' optimal investment thresholds

3.9.4 Small and big retailers

In this section, we consider a more general case. Assume two retailers, retailer 1 and 2. Retailer 1's investment and fixed cost is higher than retailer 2's investment and fixed cost, respectively, i.e., (IC_1 = \$400,000 > IC_2 = \$200,000 and FC_1 = 200,000 \$/Year > FC_2 = 100,000 \$/Year), but retailer 2's marginal cost is higher than retailer 1's marginal cost (VC_1 = 80 \$/Item Sold < VC_2 = 100 \$/Item Sold).

Retailer 1 represents a large retailer that usually opens a large store in the market. Retailer 2 represents a small retailer that usually opens a small store in the market. Retailer 1's initial investment cost and the cost of running the store (fixed cost) is higher than retailer 2's investment and fixed cost, respectively, due to the scale of operation. However, retailer 1's marginal cost is lower than retailer 2's marginal cost since the large

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retailer has a better infrastructure and supply chain system that helps him provide the products with lower cost to the market compared to retailer 2.

Also assume that retailers' investment options are valid for two hundred months. Again assume the same values for the other model parameters as section 3.9.1. We also use the same trinomial lattice built in section 3.9.1 to evaluate retailers' investment options in competitive and non-completive markets.

Figure 3.14 shows retailers' optimal investment thresholds in two market situations as indicated by $(X^*(t))$ in the vertical axis. At time step (t) a retailer exercises its investment option and opens a store in a market when the value of retail market potential at this time step exceeds the optimal value of retail market potential at this time step, which is determined by our investment analysis approach as it is shown by the curve in Figure 3.14.

Since the small retailer has the overall cost advantage he enters the market before the big retailer. Therefore, retailer 2's optimal investment threshold is lower than retailer 1's optimal investment threshold in both noncompetitive and competitive market structures.

The competition effect can be seen in Figure 3.14 too. Due to the competition, retailer 2's investment threshold in the competitive market is slightly lower than its investment threshold in the same market without any competition effect. Therefore, retailer 2 waits a bit longer to exercise its investment option in the noncompetitive market. In the noncompetitive market retailer 2 can wait longer to receive more information regarding the future state of the market and then, exercises its investment option without having any fear regarding the entry of the other retailer.

On the other hand, early investment by retailer 2 in the competitive, dynamic market has a disadvantage for retailer 1 since retailer 2's early entry makes retailer 1 to wait longer before entering the competitive market. Given retailer 2 is already in the market retailer 1 must wait longer to exercise its shared investment option compared to the market situation, in which the competition effect from retailer 1 does not exist. Retailer 1's optimal investment threshold in the competitive market is the threshold, in which the competitive market becomes large enough to support both retailers. Figure 3.14 shows that retailer 1 invests later when he holds a shared investment option. Thus, *our dynamic programming approach indicates the strategic aspect of early investment by the small retailer in a competitive market through this numerical example*.

Also consider that the retailer's optimal investment threshold is lowered at the end because it is assumed that a retailer's investment opportunity becomes disappeared after the last time step. This lowered threshold at the end is simply an artifact of the end point that we choose in this numerical example. In the real world one can argue that the market opportunity does not really disappear and hence, the retailer's investment opportunity is perpetual and stays forever. However, it is indicated that the trinomial lattice formulation can be used as an accurate approximation for investment options that never expire [54]. Therefore, we can also use the investment evaluation procedure based on the trinomial lattice approximation that is described in this chapter for a retailer's investment opportunity that never expires. The only requirement is to construct a lattice with sufficiently large number of time steps in order to provide an appropriate approximation for the infinite time horizon of a retailer's investment option. Hence, the evaluation procedure of a retailer's investment option with an infinite time horizon is reduced to the

evaluation procedure of a retailer's investment option with a very long, but finite time horizon as it was described in this chapter.

Figure 3.14. Impacts of competition on small and big retailers' optimal investment thresholds.

It can be seen in Figure 3.14 that the small retailer's optimal investment thresholds are almost identical in the competitive versus noncompetitive markets. This happens since there are significant differences between the values of the small versus the big retailer's cost parameters. Therefore, the small retailer can wait longer and exercises its investment option at the higher level of $(X(t))$. On the other hand, when the values of retailers' cost parameters are similar the differences between the optimal investment thresholds of the first retailer in competitive versus noncompetitive markets are significant due to the higher level of competition in the market. This is shown in Figure 3.11 - Figure 3.13.

In the next section, we examine how retailers' investment thresholds are sensitive to the values of our model parameters.

3.10 Sensitivity analysis

In this section, we study how retailers' optimal investment thresholds change as the value of model parameters changes. We carry out one-factor sensitivity analysis for the following model parameters: (σ, ρ, and α). We use a numerical example to show how retailers' investment thresholds are sensitive to the variations in the values of the model parameters. Thus, sensitivity analysis shows us *how significant the correct estimation of the value of a model parameter is to determine retailers' optimal investment thresholds*.

In this section, *the objective of sensitivity analysis is to examine the characteristics of the optimal investment thresholds*. We use several numerical examples to help us identify the impacts of the variation in the model parameters on the optimal investment thresholds. We also carry out the first-order sensitivity analysis on the model parameters to determine how the investment thresholds change with respect to the variation in the model parameters. In addition, we use sensitivity analysis on the values of the retailer's cost parameters to show how the order in which, retailers enter the competitive market changes as a result of changes in costs parameter values.

3.10.1 Sensitivity analysis for the log-volatility of retail market potential (parameter (σ)) and its impact on retailers' optimal investment thresholds

In this section, we study how changes in the value of the log-volatility parameter (σ) impact retailers' optimal investment thresholds. We use the numerical example in section

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3.9.4 regarding small and big retailers' investment behaviors to illustrate this impact. Therefore, we consider two retailers with the same values of cost parameters in section 3.9.4 that have shared investment options to open stores in a competitive market. In addition, we assume that the values of the other model parameters are the same as their values in section 3.9.4. We consider three levels of (σ) for this sensitivity analysis: (σ = [0.15, 0.20, and 0.30]).

The value of (σ) is an indicator for the degree of dynamic uncertainty in the retail market since it represents the standard deviation of the growth rate of retail market potential as it was indicated in Equation (3.14). The value of (σ) increases when the dynamic uncertainty of the retail market increases. As the uncertainty in the value of retail market potential increases the value of the retailer's investment option increases since it becomes more likely that the retailer exercises its investment option at some time in the future although it may not be profitable to exercise its option now. Note that the upside potential future profit for the retailer is not limited while the downside loss is limited to zero when the retailer has an investment opportunity in the retail market. Therefore, as the value of (σ) increases the upside potential profit increases while the downside possible loss remains bounded at the constant zero level. Hence, the retailer's value of an investment option increases when the value of (σ) increases.

The retailer exercises its investment opportunity when its net investment value (the store's NPV minus the investment cost to open the store in the dynamic market) exceeds the value of its investment option, i.e., the retailer's optimal investment value is equal to the retailer's value of its investment option at the exercise point. Therefore, the optimal store's value that triggers the retailer to invest and open the store, increases when the

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value of (σ) increases. Figure 3.15 shows how the optimal thresholds of the retailers'

investment values increase as the value of (σ) increases from (0.15) to (0.30). Note that

changes in the value of (σ) do not impact the order, in which retailers enter the

competitive market. The small retailer invests earlier in the competitive market and

delays the big retailer's entry.
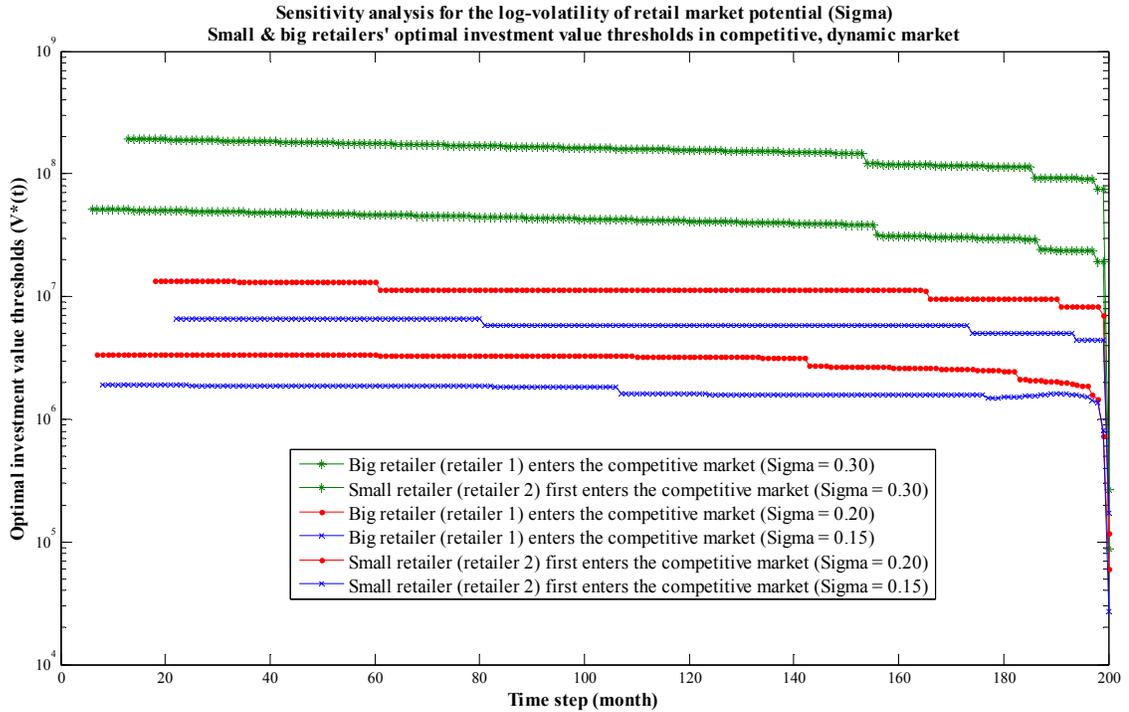
Figure 3.15. Sensitivity analysis on the value of (σ) and its impact on retailers' optimal thresholds for investment values (V^*(t)).

It is noted that the retailer requires the higher value of investment in order to exercise

its investment option when the value of (σ) increases. However, the impact of changes of

the value of (σ) on the value of (X) that triggers the retailer to exercise its investment

option is not trivial since it comes from a much more complicated process. The retailer's

investment value at the exercise time (V^*(t)) is calculated based on the described

procedure in section 3.7. A change in the value of (σ) changes the value of two sets of

parameters in the dynamic programming approach: the jump parameters (u and d) and the actual probabilities (p_1 and p_3) as described in Equation (3.16).

An increase in the value of (σ) on one hand, increases the value of the up jump parameter (u) while decreasing the value of the low jump parameter (d). Hence, the described lattice in section 3.7 contains the wider range of values for (X). This wider range of the underlying stochastic variable dramatically changes the retailer's investment value. An increase in the value of (σ) on the other hand, decreases the actual up probability (p_1) and increases the actual down probability (p_3). These changes in the values of actual probabilities decrease the retailer's investment value. The overall change in the retailer's investment value is the overall effect of these two interactive changes.

Since the retailer's investment value at the optimal exercise setting increases as the value of (σ) increases, we know that the overall change in the retailer's investment value is increasing. However, due to the sophisticated interaction between the above two effects, it is not easy to determine how the value of retail market potential should change to increase the retailer's optimal investment value when the value of (σ) increases. Note that the store's cash flows along the lattice nodes and the store's final values at the nodes in the end of the lattice are the quadratic functions of variable (X). This nonlinear relationship makes our calculation even more complicated. Therefore, we conduct several numerical examples to explore the direction of this change. We consider four levels of (σ) for this sensitivity analysis: (σ = [0.05, 0.10, 0.15, and 0.18]). The results are summarized in Figure 3.16. It can be seen that the retailer's investment threshold (denoted by ($X^*(t)$)) increases as the value of (σ) increases from (0.05) to (0.18). Therefore, an increase in the

optimal threshold value (X^*(t)) results in an increase in the optimal retailer's investment value (V^*(t)) when (σ) increases.
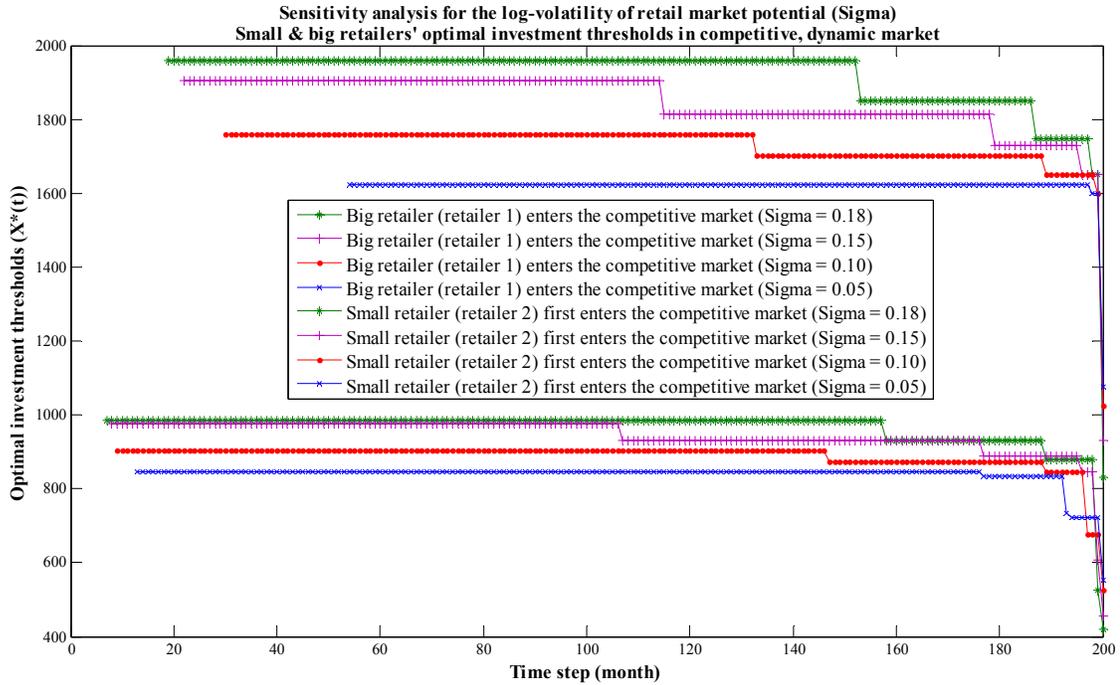
Figure 3.16. Sensitivity analysis for the log-volatility of retail market potential (parameter (σ)) and its impact on retailers' optimal investment thresholds.

It can be concluded that retailers' optimal investment thresholds and their optimal values for investment change as the value of (σ) changes. Hence, *retailers' investment behaviors are sensitive to the log-volatility of the retail market, irrespective of retailers' risk preferences*. This result is consistent with the characteristics of the optimal investment rule for an investment opportunity on a simple project that has been described by Dixit and Pindyck [15] (Ch. 5.4).

Dixit and Pindyck [15] also study the impact of changes in the value of (σ) on the overall retail firm's market value. Consider that the value of the retailer's investment opportunity increases as the value of (σ) increases. But for that very reason, the retailer becomes more reluctant to invest and open a store when the value of (σ) increases and

therefore, the amount of actual investment by the retail firm decreases. However, when the retail market becomes more uncertain the market value of the retail firm can go up even though the retail firm does less investment and perhaps opens fewer stores.

3.10.2 Sensitivity analysis for the expected growth rate (parameter (α)) and its impact on retailers' optimal investment thresholds

In this section, we study how changes in the value of the expected growth rate (α) impact retailers' optimal investment thresholds. Again we use the same numerical example in section 3.9.4 regarding small and big retailers' investment behaviors to illustrate this impact. Therefore, we consider two retailers with the same values of cost parameters in section 3.9.4 that have shared investment options to open stores in a competitive market. In addition, we assume that the values of the other model parameters are the same as their values in section 3.9.4. We consider three levels of (α) for sensitivity analysis: (α = [0.04,0.05, and 0.08]).

It is indicated in section 3.9.4 that retailer 1 invests first and enters the competitive market due to its overall cost advantage. However, retailers' optimal investment thresholds change as the value of (α) changes. The value of (α) represents the expected growth rate of the potential retail market, i.e., the value of (α) increases when the expected market potential increases. As the value of (α) increases the values of retailers' investment opportunities increase. This is also consistent with Dixit and Pindyck [15] (Ch. 5.1.B) that discuss the increase in the value an investment opportunity as a result of increase in the expected growth rate of return of the underlying stochastic asset.

Recall that a retailer exercises its investment option when the value of its investment exceeds the value of its investment option. Therefore, the optimal value of a retailer's

investment that triggers the option exercise increases as the expected growth rate of retail market potential (parameter (α)) increases. Figure 3.17 shows how the optimal thresholds of the retailers' investment values increase as the value of (α) increases from (0.04) to (0.08). Note that *changes in the value of (α) do not impact the order, in which retailers enter the competitive market*. The small retailer invests earlier in the competitive market and delays the big retailer's entry.
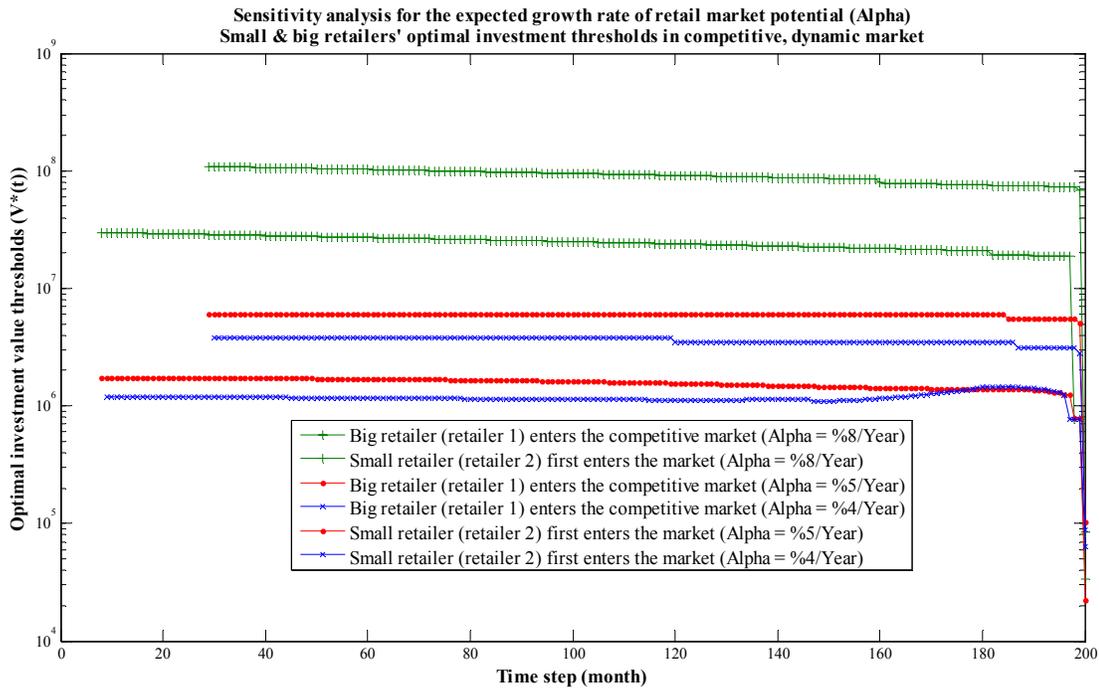
Figure 3.17. Sensitivity analysis for the expected growth rate (α) and its impact on retailers' optimal investment value thresholds (V^*(t)).

It is noted that the retailer requires the higher value of investment in order to exercise its investment option when the value of (α) increases. However, a change in the value of (α) changes the value of the actual probabilities (p_1 and p_3) as described in Equation (3.16b). An increase in the value of (α) increases the actual up probability (p_1) and decreases the actual down probability (p_3). These changes in the values of the actual probabilities increase the retailer's investment value. Therefore, the retailer's optimal

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investment threshold in terms of (X^*(t)) decreases as the expected growth rate of retail market potential (α) increases, i.e., it becomes optimal for a retailer to enter a market at lower level of retail market potential since the retail market grows at higher rate. We consider three levels of (α) for sensitivity analysis to illustrate this behavior: (α = [0.01,0.02, and 0.05]).

It can be seen in Figure 3.18 that *the retailers' optimal investment thresholds decrease as the value of (α) increases* from (0.01) to (0.05). Note that *changes in the value of (α) do not impact the order, in which retailers enter the competitive market.*
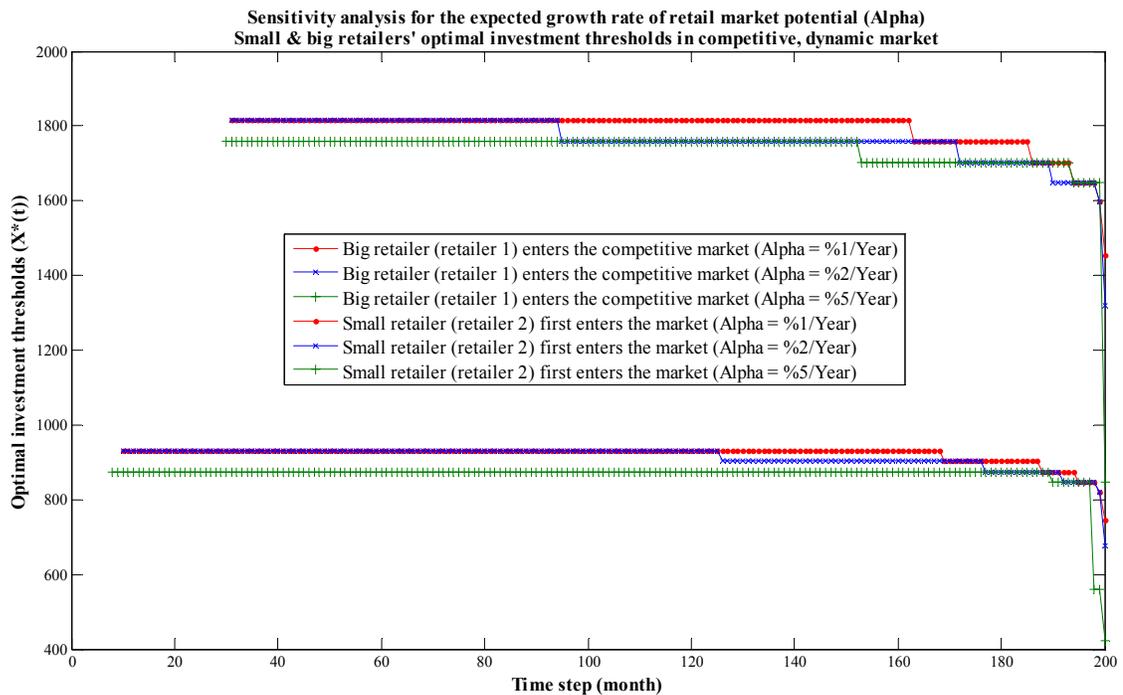
Figure 3.18. Sensitivity analysis for the expected growth rate (α) and its impact on retailers' optimal investment thresholds (X^*(t)).

3.10.3 Sensitivity analysis for the discount rate (parameter (ρ)) and its impact on retailers' optimal investment thresholds

In this section, we study how changes in the value of the discount rate (ρ) impact retailers' optimal investment thresholds. Again we use the same numerical example in section 3.9.4 regarding small and big retailers' investment behaviors to illustrate this impact. Therefore, we consider two retailers with the same values of cost parameters in section 3.9.4 that have shared investment options to open stores in a competitive market. In addition, we assume that the values of the other model parameters are the same as their values in section 3.9.4. However, we consider three levels of (ρ) for sensitivity analysis: ($\rho = [0.10, 0.12,$ and $0.15]$).

It is indicated in section 3.9.4 that retailer 1 invests first and enters the competitive market due to its overall cost advantage. However, retailers' optimal investment thresholds change as the value of (ρ) changes. The value of (ρ) increases when the riskiness of an investment option increases. The value of a retailer's investment option decreases when the value of (ρ) increases since the riskiness of this investment option increases. Therefore, the retailer's value of investment that triggers the option exercise decreases when the value of (ρ) increases. Figure 3.19 shows how the optimal retailer's investment value that triggers the option exercise ($V^*(t)$) decreases as the value of (ρ) increases from (0.1) to (0.15). Note that *changes in the value of (ρ) do not impact the order, in which retailers enter the competitive market.*
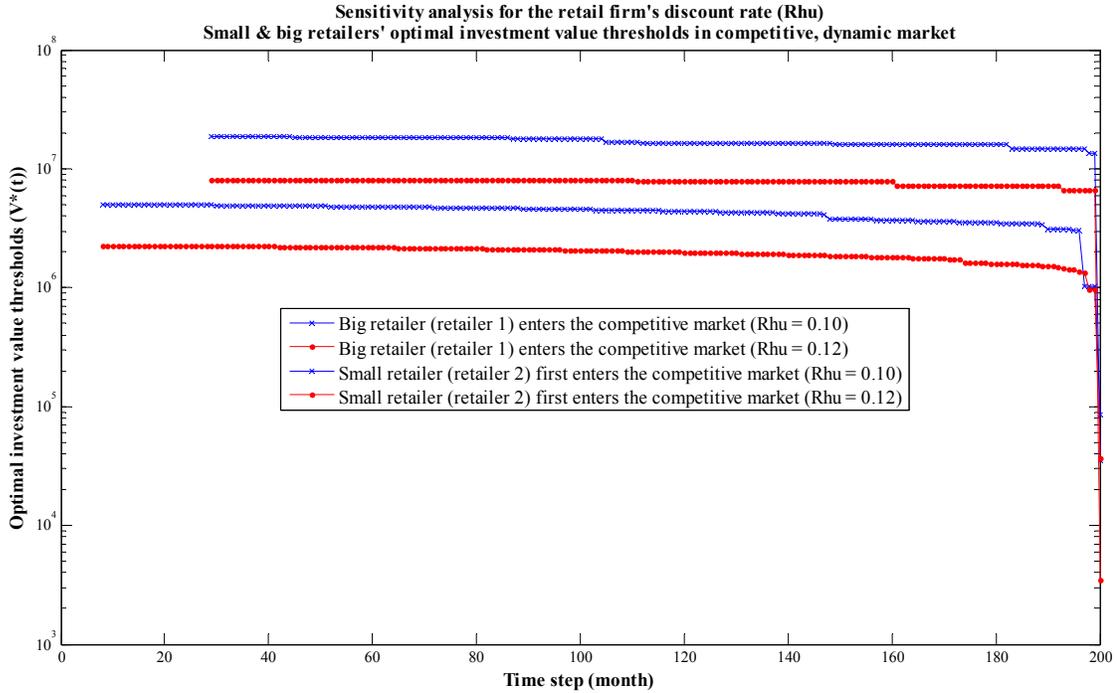
Figure 3.19. Sensitivity analysis for the discount rate (ρ) and its impact on retailers' optimal investment value thresholds (V*(t)).

It is noted that the retailer requires lower value of investment in order to exercise its investment option when the value of (ρ) increases. However, a decrease in the value of (ρ) decreases the value of a retailer's investment and hence, the retailer needs higher value of retail market potential to exercise its investment option. Therefore, the retailer's optimal investment threshold in terms of $(X^*(t))$ increases as the discount rate (ρ) increases, i.e., it becomes optimal for a retailer to enter a market at higher level of retail market potential since the riskiness of its investment option increases. We consider three levels of (ρ) for sensitivity analysis to illustrate this behavior: (ρ = [0.15, 0.20, and 0.30]).

It can be seen in Figure 3.20 that *the retailers' optimal investment thresholds increases as the value of (ρ) increases* from (0.15) to (0.30). Note that *changes in the value of (ρ) do not impact the order, in which retailers enter the competitive market.*

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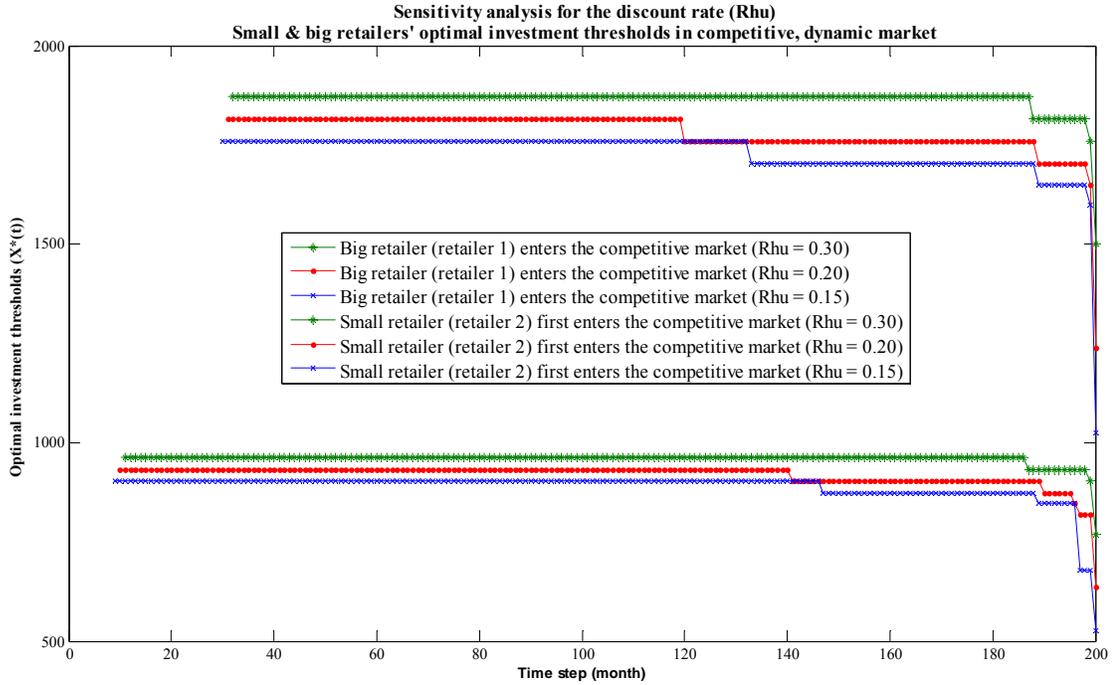
Figure 3.20. Sensitivity analysis for the discount rate (ρ) and its impact on retailers' optimal investment thresholds ($X^*(t)$).

3.10.4 Sensitivity analysis to assess the impacts of the changes in retailer 1's marginal cost on retailers' investment thresholds

In this section, we study how changes in the value of retailer's marginal cost (VC_1) impact retailers' optimal investment thresholds. Again we use the same numerical example in section 3.9.4 regarding small and big retailers' investment behaviors to illustrate this impact. Therefore, we consider two retailers with the same values of cost parameters in section 3.9.4 that have shared investment options to open stores in a competitive market. In addition, we assume that the values of the other model parameters are the same as their values in section 3.9.4. However, we consider three values for (VC_1) for sensitivity analysis: (VC_1 = [80, 40, and 20] \$/Item Sold). The objective is to determine how variations in the value of (VC_1) change the order in which retailers enter

the competitive market. We are also interested in exploring at what value of (VC_1) the big retailer (retailer 1) is able to enter the market first before the small retailer (retailer 2).

Figure 3.21 summarizes the results of this sensitivity analysis. As it was indicated in section 3.9.2 retailer 1's optimal investment threshold in the noncompetitive market decreases as (VC_1) decreases (consider the graphs in Figure 3.21 from left to the right). However, what is more interesting is that the decrease in the value of retailer 1's marginal cost (VC_1) also gives the big retailer (retailer 1) a strategic advantage over the small retailer (retailer 2) to enter the competitive market first. Retailer 2 is the first retailer that opens a store in the competitive market when $(VC_1 = 80$ \$/Item Sold) but retailer 1 is the first retailer that opens a store in the competitive market when $(VC_1 = [40$ and $20]$ \$/Item Sold). Figure 3.21 shows *the significance of the retailer's strategy for continuous improvement in supply chain systems in order to reduce the cost of providing the product*. This strategy helps the retailer invest early and enter the competitive market before the other retailers.

Sensitivity analysis for retailer 1's marginal cost (VC1)
Retailers' optimal investment thresholds in the dynamic market

Legend:
- Both retailers excercise their investment options and enter the competitive market
- Retailer 1 exercises his proprietary investment option and enters the non-competitive market
- Retailer 2 exercises his proprietary investment option and enters the non-competitive market
- Only one retailer excercises his option and enters the competitive market

Retailer 1's fixed cost (VC1 = 80 $/Item Sold)   Retailer 1's fixed cost (VC1 = 40 $/Item Sold)   Retailer 1's fixed cost (VC1 = 20 $/Item Sold)
Retailer 2 is fisrt in the competitive market   Retailer 1 is fisrt in the competitive market   Retailer 1 is fisrt in the competitive market

Figure 3.21. Sensitivity analysis for retailer 1's marginal cost (VC$_1$) and its impact on retailers' optimal investment thresholds.

3.10.5 *Sensitivity analysis to assess the impacts of the changes in retailer 1's fixed cost on retailers' investment thresholds*

In this section, we study how changes in the value of retailer's fixed cost (FC$_1$) impact retailers' optimal investment thresholds. Again we use the same numerical example in section 3.9.4 regarding small and big retailers' investment behaviors to illustrate this impact. Therefore, we consider two retailers with the same values of cost parameters in section 3.9.4 that have shared investment options to open stores in a competitive market. In addition, we assume that the values of the other model parameters are the same as their values in section 3.9.4. However, we consider three values for (FC$_1$) for sensitivity analysis: (FC$_1$ = [120,000, 180,000, and 220,000] $/Year). The objective of this sensitivity analysis is to determine how variations in the value of (FC$_1$) change the order

in which retailers enter the competitive market. We are also interested in exploring at what value of (FC_1) the big retailer (retailer 1) is able to enter the market first before the small retailer (retailer 2).

Figure 3.22 summarizes the results of this sensitivity analysis. As it was indicated in section 3.10.3 retailer 1's optimal investment threshold in the noncompetitive market decreases as (FC_1) decreases (consider the graphs in Figure 3.22 from left to the right). However, what is more interesting is that the decrease in the value of retailer 1's fixed cost (FC_1) also gives the big retailer (retailer 1) a strategic advantage over the small retailer (retailer 2) to enter the competitive market first. Retailer 1 is the first retailer that opens a store in the competitive market when (FC_1 = [120,000 and 180,000] \$/Year) but retailer 1 is the first retailer that opens a store in the competitive market when (FC_1 = 220,000 \$/Year). Figure 3.22 shows *the significance of the retailer's strategy for continuous improvement in store operation in order to reduce the fixed cost of providing the product*. This strategy helps the retailer invest early and enter the competitive market before the other retailers.

Retailers' optimal investment thresholds in the dynamic market

Legend:
- Both retailers excercise their investment options and enter the competitive market
- Retailer 2 exercises his proprietary investment option and enters the non-competitive market
- Retailer 1 exercises his proprietary investment option and enters the non-competitive market
- Only one retailer excercises his option and enters the competitive market

Retailer 1's fixed cost (FC1 = 120,000 \$/Year)
Retailer 1 is fisrt in the competitive market

Retailer 1's fixed cost (FC1 = 180,000 \$/Year)
Retailer 1 is fisrt in the competitive market

Retailer 1's fixed cost (FC1 = 220,000 \$/Year)
Retailer 2 is fisrt in the competitive market

Figure 3.22. Sensitivity analysis for retailer 1's fixed cost (FC_1) and its impact on retailers' optimal investment thresholds.

3.10.6 *Sensitivity analysis to assess the impacts of the changes in retailer 1's investment cost on retailers' investment thresholds*

In this section, we study how changes in the value of retailer's investment cost (IC_1) impact retailers' optimal investment thresholds. Again we use the same numerical example in section 3.9.4 regarding small and big retailers' investment behaviors to illustrate this impact. Therefore, we consider two retailers with the same values of cost parameters in section 3.9.4 that have shared investment options to open stores in a competitive market. In addition, we assume that the values of the other model parameters are the same as their values in section 3.9.4. However, we consider three values for (IC_1) for sensitivity analysis: (IC_1 = \$ [250,000, 380,000, and 450,000]). The objective of this sensitivity analysis is to determine how variations in the value of (IC_1) change the order

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in which retailers enter the competitive market. We are also interested in exploring at what value of (IC_1) the big retailer (retailer 1) is able to enter the market first before the small retailer (retailer 2).

Figure 3.23 summarizes the results of this sensitivity analysis. As it was indicated in section 3.9.4 retailer 1's optimal investment threshold in the noncompetitive market decreases as (IC_1) decreases (consider the graphs in Figure 3.23 from left to the right). However, what is more interesting is that the decrease in the value of retailer 1's investment cost (IC_1) also gives the big retailer (retailer 1) a strategic advantage over the small retailer (retailer 2) to enter the competitive market first. Retailer 2 is the first retailer that opens a store in the competitive market when (IC_1 = \$ [380,000 and 450,000]) but retailer 1 is the first retailer that opens a store in the competitive market when (IC_1 = \$250,000). Figure 3.23 shows *the significance of the retailer's strategy for selecting the right store size and shelf management in order to provide the same quantity of the product in the smaller area*. In addition, *the retailer should work with developers in new growing markets to get the better deals on commercial real estate properties for stores*. These strategy help the retailer invest early and enter the competitive market before the other retailers.
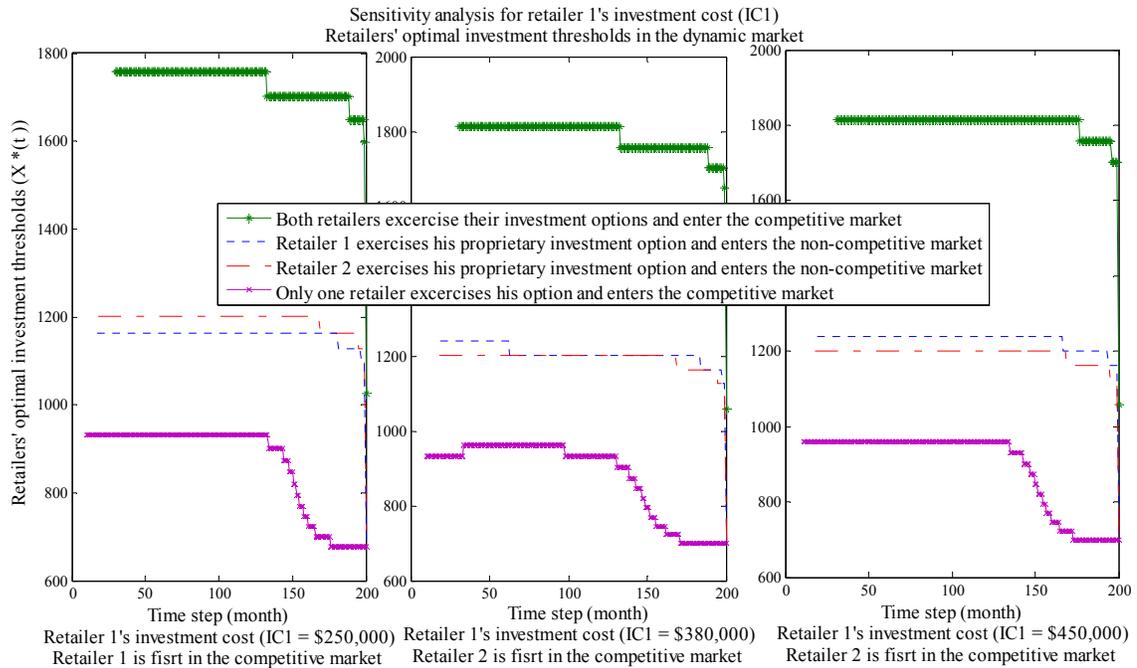
Figure 3.23. Sensitivity analysis for retailer 1's investment cost (IC_1) and its impact on retailers' optimal investment thresholds.

3.11 Comparison between the real options approach and the NPV calculation

In this section, we compare the real options approach for the evaluation of a retailer's investment opportunity that is described in this chapter to the traditional NPV approach. This comparison is based on the retailer's optimal investment threshold under each approach.

The major difference between the real options approach and the traditional NPV is in the way these two approaches deals with the management flexibility for deferring investment in a retail market. The NPV approach does not consider the management flexibility to defer an investment opportunity until the uncertainty about a retail market is evolved. Therefore, the decision rule in the NPV calculation is developed to determine

whether it is optimal for a retailer to invest in a retail market now or drop the investment opportunity forever. Hence, the NPV approach does not consider the possibility that a retailer can defer its investment opportunity until uncertainty about a retail market is evolved and then decide to open or not open a store in a market. A retailer can use this flexible strategy and defer its investment opportunity until the retail market proves to be profitable and hence, minimize the downside risk of its investment while it enhances the upside opportunity.

On the other hand, the real options approach addresses this possibility of deferring an investment opportunity in the evaluation of a retailer's investment opportunity. Based on what we described in this chapter a retailer's optimal investment threshold is determined by a decision tree analysis approach. This decision tree analysis approach is based on a lattice formulation that characterizes the volatility of a retail market in a random walk fashion. At any node in this tree a retailer is assumed to have flexibility to decide between two alternatives: exercise its investment option and open a store or defer its investment option for the next time step. This is the way that the real options approach extends the traditional NPV approach and incorporates the retail management flexibility in its investment evaluation.

The only nodes in the described decision tree of the real options approach that do not consider the flexibility of deferring an investment opportunity are the nodes in the last time step that the retailer's investment option is assumed to expire. In these nodes, the retailer's decision alternatives are either invest or drop the investment opportunity and the decision rule is that a retailer invests in this market if the expected NPV of its investment exceeds the value of waiting, which is zero. This investment rule is identical to the

decision rule in the NPV approach. Therefore, the retailer's investment threshold at the last time step in the decision tree of the real options approach is equal to the retailer's investment threshold based on the expected NPV approach, which does not change over the time that a retailer's investment option is valid.

Figure 3.24 shows the retailer's optimal investment thresholds based on the real options and the NPV investment analysis approaches. We use the same numerical example of section 3.9.4 for this purpose. The big and small retailer's optimal investment thresholds based on the NPV approach are lower than the big and small retailer's optimal investment thresholds based on the real options approach, respectively. It can be concluded that the NPV approach is too aggressive in the retailer's retailer investment evaluation since it does not consider the possibility of deferring an investment opportunity and hence, sets the retailer's optimal investment threshold lower than the corresponding real options approach.

The NPV approach is more aggressive than the real options approach in this investment case since it does not appreciate the value of waiting in the evaluation process of a retailer's investment option. The NPV approach evaluates a retailer's investment opportunity without considering a retailer's potential flexibility to defer the exercise of its investment opportunity. Since the NPV approach is based on now or never investment decision it results in more aggressive investment decisions compared to the real options approach, which considers the value of waiting as an important value component of a retailer's investment option.

Note that the retailers' optimal investment thresholds in Figure 3.24 are based on the values of (X) in time step (t). Recall that (X(t)) indicates the maximum price of the

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product in a retail market with respect to our demand model that was summarized in Equation (3.11). The difference between the NPV and the options approach is discussed in terms of this maximum price that triggers the retailer to invest and enter the market. Therefore, this difference does not imply that the NPV is larger or smaller than the NOV.

It is shown in Figure 3.24 that the NPV approach is more aggressive than the options approach since with the NPV approach, a lower maximum price $(X(t))$ triggers the decision to enter. In contrast, with the options approach, we need a higher maximum price $(X(t))$ to trigger this decision. Consider that the $(X(t))$ that triggers a decision is one where the NPV is greater than zero since uncertainty and management flexibility to defer investment in the market do not play into the NPV approach. The $(X(t))$ that triggers a decision takes into account the balance between waiting for higher $X(t)$, where the upside will inevitably happen, and foregoing revenue while the retailer is waiting.
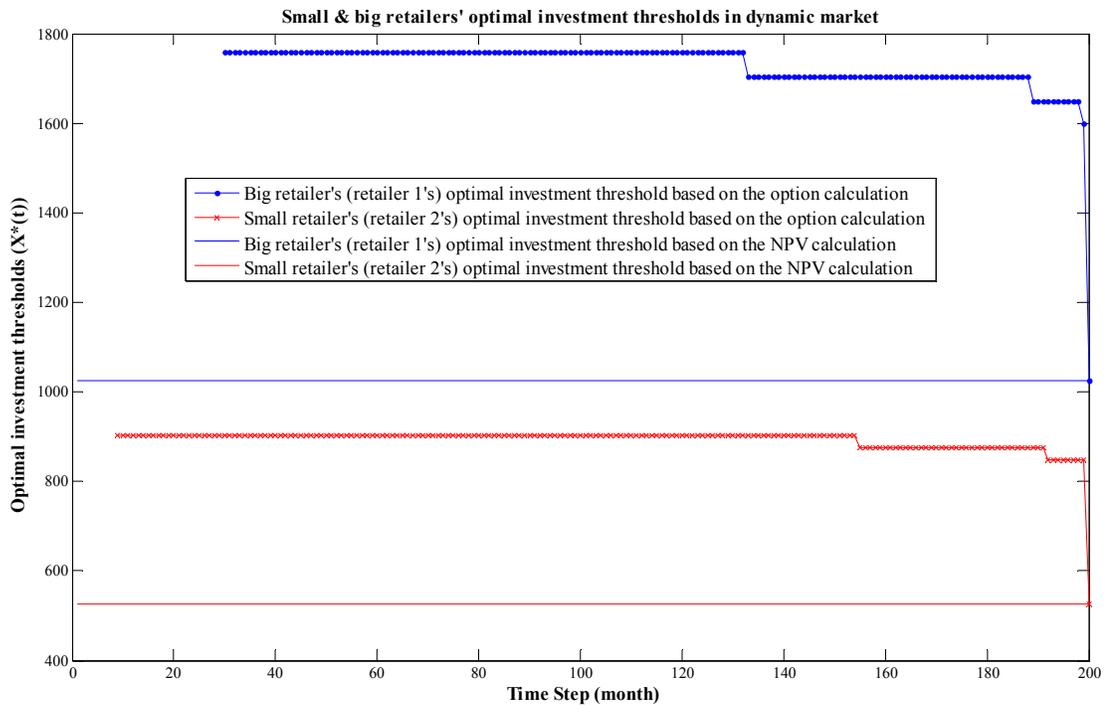
Figure 3.24. Comparison between the retailer's optimal investment thresholds based on the NPV and the options calculation.

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Note that the real options approach results in more aggressive decisions than the NPV approach in situation, in which a retailer has an option to terminate (the exit option) or flexibility to develop a store in multiple stages. In these cases, the real options approach considers the terminating value in the evaluation process, limits the downside loss of an investment risk, and hence, provides a more aggressive strategy for the investment evaluation.

Therefore, depending on what is flexible in the evaluation of a retailer's investment option either the NPV approach or the real options approach may be a more aggressive investment analysis approach. In this thesis, the NPV approach is more aggressive investment analysis approach since it does not appreciate the value of waiting as it is considered by the real options approach.

Hence, using the NPV approach in the retail decision-making and not considering the management flexibility to defer an investment opportunity results in aggressive decisions that do not correctly capture the hidden value of a retailer's investment opportunity. This result is consistent with the findings of other researchers such as Dixit and Pindyck [15] and Trigeorgis [14] that indicate that in the real world an investor requires a higher return than what is noted in the expected NPV approach for investing in a risky project.

However, there are some investment situations that the choice of the investment analysis approach does not change the retailer's investment thresholds. These situations can be summarized, as follows.

- *When a retailer does not have any flexibility to defer its investment option in a retail market*
- *When a retailer's investment option expires in a short time*

- *When the value of retail market potential is substantially larger than the retailer's investment optimal thresholds based on the NPV and the real options approach*

In other market situations, the optimal retailer's investment thresholds are very different based on these two approaches as shown in Figure 3.24 and hence, a retailer should use the real options approach for the correct investment evaluation.

3.12 How to use this real options approach in practical retail market analysis?

In this section, we identify how the described market analysis approach based on the real options methodology can be packaged to be used by retail store decision makers. The objective is to show how a retailer can apply this model in practice to determine the optimal investment threshold to exercise its investment option in a dynamic retail market. We discuss what inputs the need and what outputs are provided to them by this investment analysis approach.

A retailer needs to determine several inputs for our investment analysis approach, as summarized below. First, a retailer selects a retail market for possible investment. This market is a market that this retailer believes to hold an investment opportunity and can open a store at some point of time in future if he decides to. Note that the retailer's investment option is free in our formulation but exercising the options is what costs money since it requires an investment outlay to open a store. Also the retailer needs to identify when this investment option expires, i.e., how late a retailer can delay investment in this retail market. In addition, a retailer needs to identify how long a store remains opened in this market. This is important in calculating the terminating value of a retail

store in a market. Our formulation assumes that once a store is opened it remains opened for ever. However, it can be easily changed to any other time horizon. The retailer also specifies whether any of its competitors holds an investment option in this market. If there is a competition from the other retailer in this market the retailer's investment option must be analyzed as an investment option in a competitive market. Otherwise, it will be analyzed as an investment option in a noncompetitive market.

Next, this retailer needs to determine the values of expected growth rate and volatility of retail market potential (parameters (α) and (σ)) to describe the dynamic uncertainty of a retail market as summarized by variable ($X(t)$) in Equation (3.14). The retailer can use the data on demand of a typical product (or a basket of products) that it offers for this purpose. Suppose the demand for this typical product in a retail market over time are (D_0), (D_1), ..., (D_N). Note that the sub index shows the demand for different time steps starting from time step (0) to time step (N). Here we assume the time increment between two consecutive time steps is one year but the time increment can be any other time such as a quarter or a month. Luenberger [64] describes an estimation procedure to use these (N+1) time points of data to estimate the value of parameters (α) and (σ). The sample average and the sample standard deviation of the log-ratio of these (N+1) time points of demand data (denoted by ($\hat{\alpha}$) and ($\hat{\sigma}$) in Equation (3.17), respectively) are used as unbiased estimators of the expected growth rate and the volatility of ($X(t)$) whose dynamic uncertainty is formulated by the GBM process of Equation (3.14). Equation (3.17) shows the formula for this estimation process.

$$\begin{cases} \hat{\alpha} = \dfrac{1}{N} \sum_{k=0}^{N-1} \ln\left[\dfrac{D(k+1)}{D(k)}\right] = \dfrac{1}{N} \ln\left[\dfrac{D(N)}{D(0)}\right] \\[4mm] \hat{\sigma} = \sqrt{\dfrac{1}{N-1} \sum_{k=0}^{N-1}\left\{\ln\left[\dfrac{D(k+1)}{D(k)}\right] - \hat{\alpha}\right\}^2} \end{cases}$$

$$(3.17)$$

In addition to the above estimated values for the expected growth rate and the volatility for a retail market potential the retailer needs to specifies the current value of retail market potential (X_0) and the slope of the line (γ) in the demand function of Equation (3.1).

Then, the retailer needs to determine the values of its own cost parameters as well as the values of its competitor's cost parameters. It is assumed that the retailer exactly knows the values of its variable cost, fixed cost, and investment cost. The retailer can also estimate the values of these cost parameters for its competitor. However, when a retailer is not certain about the values of its competitor's cost parameters he can always conduct sensitivity analysis for the values of these parameters for its competitor in order to explore how the error in the correct estimation of the competitor's cost parameters change his optimal entry threshold to this market. Similar sensitivity analyses were conducted earlier in this chapter in sections 3.10.4 - 3.10.6. Note that a retailer only needs to estimate the values of its competitor's cost parameters when it is believed that the other retailer is expected to hold an investment option in this market.

Finally, the retailer needs to identify the value of the discount rate for the evaluation of its investment option in this market. The retail firm's cost of capital as it will be described in the next chapter in section 4.7.1 is a good estimate for the discount rate. However, sensitivity analysis should be conducted for different possible estimates for the

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discount rate based on the retailer's subjective assessment of the riskiness of an investment opportunity. Sensitivity analysis shows how this retailer's optimal investment threshold changes as the value of the discount rate changes. Similar study was conducted earlier in this chapter in section 3.10.3.

The first output of our investment analysis model is the optimal investment threshold in terms of the value of retail market potential that triggers the retailer to exercise its investment option in this retail market. This threshold changes over time as the retailer gets close to the time horizon that its investment option expires. The way this output can be interpreted by the retailer is that at any time step the retailer observes the value of retail market potential from the market and if this value exceeds the optimal investment threshold that is indicated by our model it is optimal for a retailer to exercise its investment option and opens a store in this market. Of courses, the retailer exercises its investment option the first time the value of retail market potential exceeds the optimal investment threshold.

In addition, our model provides the optimal investment threshold for the other competing retailer in this market if it exists. This output is important for a retailer in order to determine who will be the first retailer who opens a store in a new developing market and when the second retailer enters this competitive market. Our model provides the chance that a second retailer never enters the market over a certain period of time after the first retailer. The formulation for calculating this probability is summarized, as follows.

Suppose that the value of retail market potential (X) changes according to a GBM process with the known expected growth rate of (α) and the volatility of (σ). Hence,

variable ($W = \ln(X)$) follows a Brownian motion process with the expected growth rate of ($v = \alpha - (\sigma^2/2)$) and the volatility of (σ) as summarized in Equation (3.18).

$$dW = vdt + \sigma dz = (\alpha - (\sigma^2/2))dt + \sigma dz$$

(3.18)

Ross [65] describes a hitting time distribution for a Brownian motion process that can be used in our model to determine the probability that the second retailer enters the market after the first retailer in a specific amount of time. Let (M_t) denote the maximum value of (W) on the interval [0, t] when ($W(0) = 0$). The distribution of (M_t) follows Equation (3.19).

$$P\{M_t < y\} = \Phi\left(\frac{y - vt}{\sigma\sqrt{t}}\right) - e^{2vy/\sigma^2} \Phi\left(\frac{-y - vt}{\sigma\sqrt{t}}\right)$$

(3.19)

where (Φ) represents the cumulative distribution function of standard normal random variables. Equation (3.19) can be interpreted as the first time (t) at which the value of (W) hits (y), i.e., ($W(t) = y$). Therefore, the distribution in Equation (3.19) is called one-sided hitting or passage time distribution. Note that the results are consistent when ($W(0) = W_0$) by considering the Brownian motion process of ($W(t) - W_0$) instead of ($W(t)$). We use our example if section 3.9.4 to illustrate the process of calculating this probability.

Consider that based on our analysis in section 3.9.4, the small retailer enters the market first when the value of retail market potential hits the optimal value of ($X^* = 902.45$). The big retailer enters the market second when the value of retail market potential hits the optimal value of ($X^* = 1703.09$). We want to determine the probability that the big retailer enters the market within the five years after the small retailer enters

the market, i.e., the probability that the value of retail market potential (X) hits the optimal investment threshold of the big retailer ($X^* = 1703.09$) within the five years after it hits the optimal investment threshold of the small retailer ($X^* = 902.45$). Note that (X(t)) follows a GBM process with parameters ($\alpha = 0.04$ and $\sigma = 0.1$). Hence, ($W = \ln X$) follows a Brownian motion process with the expected growth rate of ($v = \alpha - (\sigma^2/2) = (0.04 - ((0.1)^2/2)) = 0.035$) and the volatility of ($\sigma = 0.1$). We can use Equation (3.19) to calculate the chance that over the five years the value of (W(t)) will never exceed the optimal investment threshold of the big retailer when its initial value is the optimal investment threshold of the small retailer. The following numerical values are used in Equation (3.19): ($v = 0.035$, $\sigma = 0.1$, $t = 5$, and $y = \ln((1703.09)/(902.45)) = 0.6351$) to calculate the probability of the big retailer never enters the market within the five years after the small retailer enters the market, as follows.

$$
\begin{aligned}
P\left\{\max_{0 \leq \tau \leq t} W_t < y\right\} &= P\{M_t < y\} = \Phi\left(\frac{y - vt}{\sigma\sqrt{t}}\right) - e^{2vy/\sigma^2}\,\Phi\left(\frac{-y - vt}{\sigma\sqrt{t}}\right) \\
&= \Phi\left(\frac{0.6351 - (0.035)(5)}{(0.1)\sqrt{5}}\right) - e^{2(0.035)(0.6351)/(0.1)^2}\,\Phi\left(\frac{-0.6351 - (0.035)(5)}{(0.1)\sqrt{5}}\right) \\
&= 0.9801 - (85.2595)(0.0001) = 0.96
\end{aligned}
\tag{3.20}
$$

It can be observed that it is very unlikely that the big retailer does not open a store in this market within the five years after the small retailer. Therefore, the calculation of the probability of the expected entry of the other retailer within a specific timeframe after a retailer opens a store in a market is an important output of our investment analysis approach.

3.13 Conclusions

In this chapter, we looked at retail market analysis from a theoretical investment perspective. We developed an integrated investment analysis approach to explore retailers' investment behaviors in competitive versus noncompetitive markets. It is concluded that retailers have different optimal investment thresholds in competitive versus noncompetitive markets. It is shown that the small retailer invests earlier in the market where it expects entry from the big retailer. Therefore, by use of this option-based model the observation that the small retailer should invest earlier in a new market is confirmed.

In addition, a complete sensitivity analysis was carried out to explore how retailers' investment thresholds are sensitive to the changes in the values of the model parameters. This sensitivity analysis was used to prioritize the variables that retailers should pay more attention to in their investment analysis process. It is concluded that retailers' optimal investment thresholds increase as the value of (σ) or (ρ) increases. However, retailers' optimal investment thresholds decrease as the value of (α) increases.

Retailers continuously improve their supply chain systems in order to reduce the marginal cost of providing products to the market. The low variable cost gives a retailer a first-mover advantage to enter a competitive market first and preempts the market from the entry of other competing retailers. Retailers also improve their store operations management strategies to reduce the overhead cost of providing products in store shelves. The low fixed cost gives a retailer a first-mover advantage to enter a competitive market first and preempts the market from the entry of other competing retailers. In addition, retailers work with developers closely in new growing markets to take advantage of

appropriate prices on commercial real estate properties for stores. In addition, they improve their shelf management strategies and select appropriate sizes for their prospective store in order to reduce the development cost. The low investment cost gives a retailer a first-mover advantage to enter a competitive market first and preempts the market from the entry of other competing retailers.

The real validity of the proposed model should be investigated in future research. This research should be extended to show how actual retailers could use this model in the investment evaluation process of competitive, dynamic retail markets. The major challenge that should be formally treated is to define an estimation process for the model parameters, particularly, (α, σ, and ρ). Thus, this option-based investment analysis approach can be useful for retailers that have long been known to take a qualitative approach to the evaluation of new markets for store development [6, 66-68].

On the other hand in this chapter, we use a simple demand model to characterize the retail market. This model is a linear demand function that relates the quantity of the product to its price at any time period. This model is based on several assumptions that are adopted for the ease of formulation. For instance, we assume that the retailer only decides on the quantity of the product at its shelves and not it price. In the real world, the retailer not only decides about the quantity of the product but also on the price of the product. This introduces another decision variable into the retailer's investment decision and makes our options formulation difficult since we now should consider the trade off between these two decision variables. Particularly, finding the equilibrium state of the market in the competition between two retailers is not an easy problem in game theory when we work with two decision variables for each player (i.e., retailer).

This discussion becomes more interesting and, of course, challenging when we consider that the retailer also decides about another decision variable, which describes the service (or quality) level of the retail store. It is indicated that the output of a retailer is not only the physical product but also the services that are associated with presenting this product to customers [69]. However, it is not easy to determine appropriate concepts for different aspects of service and it is even harder to measure these service aspects in the real world. Future research is needed to develop more appropriate demand models that describe the relationship between price, quantity, and service for the retail activity. Perhaps, works such as [70, 71] could provide a good start to understand the economics of retail firms.

In addition, in this chapter we do not use any explicit model to describe the supply side of retail market, i.e., we do not use any model that explicitly determines the relationship between the price of the notional product and the total quantity of this product that a retailer can provide to the market at this price. Instead, we use three cost parameters to characterize the important aspects of retailing businesses at the higher level of abstraction. In future works, an appropriate supply curve such as the Cobb-Douglas production function [72] should be developed and integrated into our investment analysis approach to describe the supply side of the retail market. The supply function is used to study how the retailer's investment behavior changes as the values of its cost parameters change.

Chapter 4

Evaluation of a Retailer's Investment Options in a Competitive, Dynamic Market: a Contingent Claims Analysis Approach

4.1 Abstract

In this chapter, we develop an integrated investment analysis approach based on contingent claims analysis to explore retailers' investment behaviors in dynamic markets. The objective is *to determine retailers' optimal investment thresholds in noncompetitive and competitive markets. The equivalent risk neutral evaluation approach is presented in this chapter as an extended version of the contingent claims analysis approach, which facilitates the market-oriented valuation of the retailer's investment option in dynamic markets*. Sensitivity analysis is conducted to study how retailers' optimal investment thresholds change as the values of parameters in this equivalent risk neutral evaluation approach change. The relationship between the dynamic programming and the equivalent risk neutral evaluation approach is also summarized in this chapter to identify the similarities and the differences between these two investment analysis approaches. One of the most important objectives of this comparison is *to determine in what market conditions the choice of investment analysis approach is critical and dramatically changes the retailer's optimal investment threshold*.

4.2   Introduction

In this chapter, we use an alternative approach to dynamic programming – that is summarized in Chapter 3 – to evaluate retailers' investment options in dynamic markets. This alternative approach is contingent claims analysis that is considered as a generalization of option pricing theory [73].

Contingent claims analysis is a standard approach in financial economics that is used to value investment opportunities by constructing a replicating portfolio of traded assets in the complete market. Contingent claims analysis uses economic theories that describe the decisions of investors, the market equilibria resulting from the aggregation of such decisions, and the equilibrium prices of assets [15]. Therefore, we will have *a rich menu of traded assets in the modern economies with a variety of returns and risk characteristics*. The basic idea of contingent claims analysis is to evaluate a new investment opportunity by replicating its return and risk characteristics using a portfolio of existing traded assets. Hence, the price of this replicating portfolio will be equal to the value of this new investment option since any difference represents an arbitrage opportunity to make sure profit, which is assumed that could not last in the complete market at equilibrium. This chapter is structured, as follows.

A brief overview of contingent claims analysis is presented in section 4.3. The fundamental assumption of contingent claims analysis is the existence of a replicating portfolio to span stochastic variations of the underlying asset with a combination of traded assets in the market. Fundamental issues related to the construction of a portfolio of traded assets *to replicate the value of retail market potential as the underlying state variable* in our investment analysis problem, are discussed in section 4.3.1. A GBM

process is presented in section 4.3.2 to characterize the dynamic uncertainty of the price of this replicating portfolio in the financial market. The role of opportunity cost or rate of return shortfall in the evaluation of the retailer's investment option is discussed in section 4.3.3. The equivalent risk neutral evaluation approach is presented in section 4.3.4 as an extended version of the contingent claims analysis approach, which facilitates the market-oriented valuation of the retailer's investment option in dynamic markets. An approximate trinomial lattice is presented in section 4.3.5 to simplify investment analysis based on the equivalent risk neutral approach.

The described trinomial lattice of section 2.5 is extended in section 4.4 to develop appropriate decision trees for the evaluation of retailers' investment options. We use this decision tree to study the impact of retailers' cost structures on their optimal entry time decisions in competitive, dynamic markets in section 4.5. Sensitivity analysis is conducted in section 4.6 to study how retailers' optimal investment thresholds change as the values of the parameters in this equivalent risk neutral evaluation approach change.

The relationship between the dynamic programming and the equivalent risk neutral evaluation approach is summarized in section 4.7 to identify the similarities and the differences between these two investment analysis approaches. In section 4.8, we compare the retailer's optimal investment thresholds based on these two investment analysis approaches and compare how they differ from each other corresponding to the market conditions. The objective is to determine in what market conditions the choice of investment analysis approach is critical and dramatically changes the retailer's optimal investment threshold that triggers the option exercise. This chapter concludes with conclusions and future works in section 4.9.

4.3   Contingent claims analysis

A contingent claim is an asset whose future payoff depends on or is contingent on the values of other underlying assets. The value of the contingent claim changes as the values of the underlying assets change over time. An investment option in the retail market is an example of a contingent claim, whose value derives from the store's free cash flow. As it was indicated in Chapter 3 the value of the store's free cash flow also derives from the value of retail market potential, which is summarized by $(X(t))$ in the abstract demand model of Equation (3.1) in Chapter 3. Therefore, if we consider an investment option in the retail market a contingent claim, its underlying asset will be $(X(t))$. Contingent claims analysis is a mathematical approach to analyze how the changes in the value of the underlying assets over time impact the value of the contingent claim. It is indicated that contingent claims analysis is the application of the replication methodology used in option pricing to the valuation of other assets [74].

Contingent claims analysis is a standard valuation approach in financial economics to determine the market value of an investment opportunity in the complete financial market at equilibrium. We use contingent claims analysis in this chapter as our investment analysis approach to evaluate the retailer's investment option in the dynamic market. As it was noted in Chapter 2 the retail firm's financial objective is to maximize its shareholders' wealth. The shareholders expect to be compensated for the systematic risk that is involved in the retailer's investment. The retailer achieves this financial objective by opening stores in the markets that generate satisfactory cash flows and increase the market value of the firm. Contingent claims analysis is a suitable investment

methodology to evaluate the retailer's investment options and select the most appropriate markets to invest in order to maximize the market value of the retail firm. Therefore, compared to the dynamic programming approach, contingent claims analysis is aimed at defining the correct market value of an investment option.

As it was discussed in Chapter 3, dynamic programming calculations are based on using an exogenous discount rare as the retailer's cost of capital. The problem with this approach is that it is not clear where this discount rate comes from or why it should be constant over time [15]. Contingent claims analysis uses a slightly modified approach to evaluate the retailer's investment options in order to maximize the market value of the retail firm. This approach is a standard procedure to determine the true level of risk and the market value of an investment option in the complete financial market at equilibrium. To apply contingent claims analysis for the investment evaluation of retailers' investment options, we need to address two issues. Firstly, we need to specify the underlying asset, from which the value of the retailer's investment option derives. In addition, we need to find a replicating portfolio of traded assets in market whose price is perfectly correlated to the value of this underlying portfolio. Secondly, we need to characterize how the value of this replicating changes randomly over time.

The underlying asset for the retailer's investment option (i.e., contingent claim) in the market is the value of retail market potential (or $(X(t))$) whose value changes randomly over time. This dynamic change in the value of the underlying asset impacts the value of the retailer's investment option. In order to use contingent claims analysis for the assessment of the retailer's investment option, we need to fully address an important assumption behind this investment analysis approach. It is noted that the stochastic

variations of the underlying asset must be spanned by a replicating portfolio of existing

assets that are traded in the complete capital market, for instance, in the financial market

such as stock markets. In the next section, we elaborate on the concept of replicating

portfolio and show how it is applicable for investment options in retail markets.

4.3.1 Contingent claims analysis

A replicating portfolio is a combination of traded assets in the capital market, that the

price of which is absolutely correlated with the value of the underlying asset (here the

value of retail market potential) at any time step. The price of such a replicating portfolio

at time (t) is equal to the value of the retailer's investment option at time (t). Consider

that this replicating portfolio is dynamic and therefore, the weight of each asset in the

portfolio is continuously adjusted to acknowledge the variations in the store's cash flow

and the asset prices. This requires a sufficiently large market that provides a menu of

assets with different rate of returns and risk characteristics. Therefore, in principle, one

can replicate stochastic variations of $(X(t))$ by a dynamic portfolio of traded assets. In

addition, the capital market must be complete such that the retailer's investment decisions

do not change the opportunity set available to its shareholders [15]. Interested readers can

refer to [75] for a list of necessary conditions needed for spanning.

The fundamental assumption of contingent claims analysis is the existence of a

replicating portfolio to span the stochastic variations of the underlying asset with a

combination of traded assets in the market. This assumption holds true for most

commodities such as oil that are typically traded on both spot and futures markets. It is

noted that the assumption of spanning also holds true for manufactured products whose

prices are to some extent correlated with the values of company's shares [15]. The only

place that the applicability of this assumption may be subject to question is for investment options that have some unique characteristics that make it very difficult and sometimes impossible to replicate their future values by a combination of existing assets. Investment options in R&D and new product development projects are good examples of such options in that their outcomes may be hard to predict and their values are unrelated to any existing assets.

The value of retail market potential, which is the underlying asset in the retailer's investment option, is not traded in any spot or futures markets. However, one can argue that the financial performance of a retail store depends on the value of retail market potential. On the other hand, the financial performance of a retail store is to some extent correlated with the retail firm's stock price or stock prices of similar retail firms in market. Therefore, dynamic variations of retail market potential can be spanned by constructing a replicating portfolio of retail firms' common stocks in the financial markets. Spanning holds for the retailer's investment option since there are a large number of retailers that have stores opened in a variety of markets. Hence, one can always find an appropriate combination of retailers' stocks to replicate a particular value of retail market potential.

There is also another way to look at the construction of a portfolio of assets to replicate the value of retail market potential for the evaluation of retailers' investment options. If we think of a physical retail store as a real estate property and believe that the value of this real estate asset is an indicator of its market attractiveness (i.e., retail market potential) then there exist several studies that used financial-type real options approach to evaluate the value of land and properties (for a summary of these studies see [48, 76]).

Therefore, one can replicate the value of retail market potential by the value of commercial retail properties in the surrounding market, i.e., the dynamic variation of the rental price in the commercial retail space is similar to the dynamic variation of the value of retail market potential in its neighborhood. Since there is a well-established market for commercial retail rental the price of the replicating portfolio can be observed from this market and used as the value of the retailer's option in the market. It is worth noting that *what is in common among the entire studies that use financial-type real options to evaluate properties and land is that all use a GBM to model the value of the underlying assets such as land and commercial properties*.

Therefore in this chapter, we assume spanning holds and in principle, we can replicate the uncertainty over the future value of retail market potential by a portfolio of existing assets. This portfolio replicates the return and risk characteristics of the original store's cash flow. This is not a controversial assumption due to the large number of retailers and a variety of retail markets. This also helps us evaluate the retailer's investment option using contingent claims analysis in a way to maximize the market value of the retail firm. However, even if spanning does not hold for a particular retailer's investment option we can still use dynamic programming to evaluate the retailer's investment option using an exogenous discount rate.

The price of the replicating portfolio at the equilibrium state in the complete market must be equal to the market value of the retailer's investment option. Any difference between these two values introduces a chance for sure profit or an arbitrage opportunity because one can buy whichever is cheaper, repackage it, and sell it at the higher price in

the market. This is based on the essential principle that the price discrepancies for equivalent assets or portfolio could not last in market equilibrium [15].

The price of this replicating portfolio changes randomly over time. In order to apply contingent claims analysis to evaluate the retailer's investment option, we need to describe the stochastic variation of this price in a systematic fashion. In the next section, we present a GBM to model dynamic changes of the price of this replicating portfolio.

4.3.2 GBM model for dynamic uncertainty of replicating portfolio

In Chapter 3, we modeled dynamic uncertainty of the future value of the store's free cash flow by incorporating a parameter in an abstract demand model (consider parameter $X(t)$ in Equation (3.1) of Chapter 3). Then, stochastic variations of this parameter in the dynamic retail market are modeled by a GBM model as it was noted in Equation (3.14) of Chapter 3. In this section, we assume that spanning holds, and therefore, one can perfectly replicate stochastic variations of the value of retail market potential by a dynamic portfolio of traded assets in the complete market. Traded assets could be a simple asset such as a stock or futures contract, or a dynamic portfolio of simple assets whose contents are adjusted continuously such that the overall portfolio's return and risk characteristics are perfectly correlated with the underlying asset under consideration.

We denote the price of this replicating portfolio of assets at time (t) by (Y(t)), which is perfectly correlated with the value of retail market potential at time (t). For simplicity and convenience of calculation, we assume (Y(t)) changes according to the following GBM, as summarized in Equation (4.1) below.

$$dY = \mu Y dt + \sigma Y dz \tag{4.1}$$

where (dz) is an increment of a Wiener process, ($\mu > 0$) is the drift parameter, and ($\sigma > 0$) is the volatility parameter of this stochastic process (interested reader can see [57] for detailed discussion on this particular continuous stochastic process). This continuous stochastic model implies that the current value of ($Y(t)$) – denoted by (Y_0) – is known, but the future values are lognormally distributed with the time horizon. Although the price of this replicating portfolio can be observed from the market, the future price of this portfolio is always uncertain.

The choice of GBM model is consistent with our modeling assumption in Chapter 3. In Chapter 3, we use GBM to describe the dynamic variation of ($X(t)$) in the retail market. Note that ($X(t)$) is a proxy variable for the value of retail market potential that we assume to grow at some positive rate plus some random variation due to economic noise. It was indicated in Chapter 3 that the choice of GBM model to specify the dynamic uncertainty of ($X(t)$) is not perfect and is an abstraction from the real world. However in Chapter 3, we ignored these modeling limitations and showed how useful this choice could be to evaluate the retailer's investment options.

In this section, we use the same justification and assume the price of this replicating portfolio grows at some positive rate plus some random variation due to the economic noise in the financial market. Recall that this is not a controversial assumption since the GBM is a fairly standard model to describe dynamic variations of common stock prices [16]. The price of this replicating portfolio is strictly positive, which is also consistent with the GBM model's assumption. However, the price of this replicating portfolio may change over a very short time increment due to the sudden changes in financial markets.

This sudden substantial change in the price of this replicating portfolio cannot be appropriately modeled by GBM model.

For the time being, we ignore the limitations of using GBM to model dynamic uncertainty of the replicating portfolio. Consider that the GBM model is useful since it provides a simple, but powerful approach to treat the stochastic behavior of the replicating portfolio. Furthermore, by using the GBM model we can use an extensive body of research in finance to help us in the evaluation process of retailers' investment options. This is particularly valuable since we are interested in determining the retailer's optimal investment time to exercise its investment option and opens a store.

The *parameter (μ)* in Equation (4.1) represents *the expected rate of return from holding the replicating portfolio of assets in the financial market*. An investor who holds this portfolio of assets in the complete market expects to be compensated for this portfolio's systematic (nondiversifiable) risk. We had a thorough discussion in Chapter 2 that this investor does not require to be compensated for the nonsystematic (diversifiable) risk because the entire market portfolio provides the maximum available diversification for the investors. Therefore, the risk premium of holding this replicating portfolio of assets should be determined based on the covariance of the rate of return on this portfolio with that on the whole market portfolio.

It is also assumed that this replicating portfolio does not pay any dividends and the entire return of holding this portfolio comes from the capital gain. Therefore, the expected rate of return from holding this replicating portfolio (μ) is identified using the Capital Asset Pricing Model (CAPM), which is a standard approach in economics that describes the relationship between risk and expected return and that is used in the pricing

of risky securities. Equation (4.2) summarizes the fundamental condition of equilibrium from the CAPM to determine the risk-adjusted expected rate of return that investors require to hold this replicating portfolio of assets.

$$\mu = r_f + \varphi \rho_{ym} \sigma$$

<div align="right">(4.2)</div>

where (r_f) is the risk-free interest rate, (φ) is the market price of risk, which is an aggregate market parameter, (ρ_{ym}) is the correlation of the price of the replicating portfolio (Y) with the entire market portfolio, and ($\sigma > 0$) is the volatility parameter of the GBM process that is used to describe stochastic variations of (Y(t)).

In contingent claims analysis we assume that the risk-free rate of return (r_f) is exogenously specified. This risk-free rate represents the interest an investor would expect from an absolutely risk-free investment over a specified time increment. In practice, however, the risk-free rate does not exist because even the safest investments carry a very small amount of risk, for instance, some risk because of inflation. Thus, the interest rate on a three-month U.S. Treasury bill is often used as the risk-free rate.

(φ) is the market price of risk, that is determined based on ($\varphi = (r_m - r_f)/\sigma_m$), where ($r_m$) is the expected return on the entire market portfolio and (σ_m) is the standard deviation of that return. Dixit and Pindyck [15] specify the following values for the above parameters corresponding to the New York Stock Exchange Index in 1990 as the entire market portfolio: ($r_m - r_f \approx 0.08$) and ($\sigma_m \approx 0.2$), so ($\varphi \approx 0.4$).

(ρ_{ym}) is the correlation of the price of the replicating portfolio (Y) with the entire market portfolio. Recall that the price of this replicating portfolio (Y) is perfectly correlated with the value of retail market potential (X) and therefore, they both have the

same correlation with the entire market portfolio, i.e., if (ρ_{xm}) is the correlation of the value of retail market potential and the entire market portfolio we will have ($\rho_{xm} = \rho_{ym}$).

Contingent claims analysis is based on an important assumption regarding the expected rate of return on the replicating portfolio (μ). Recall that in Chapter 3 the expected growth rate of retail market potential is denoted by parameter (α). It is assumed that this expected growth rate of retail market potential (α) is less than the expected rate of return of its replicating portfolio (μ) in the complete market. Dixit and Pindyck [15] show that the firm would be better off waiting and deferring its investment option for as long as the option is valid when the expected growth rate of retail market potential (α) is greater than the expected rate of return of its replicating portfolio (μ) in the complete market. The difference between (μ) and (α) is denoted by parameter (δ), i.e., ($\delta = \mu - \alpha$). Note that (δ) must be positive, i.e., ($\delta > 0$). The *parameter (δ)* represents *the opportunity cost of deferring the investment opportunity and instead keeping the investment option alive*. In the next section, we elaborate on the role of (δ) as an explicit or implicit dividend in investment evaluation through contingent claims analysis.

4.3.3 *The role of opportunity cost in contingent claims analysis*

In this section, we discuss the role of opportunity cost (i.e., parameter (δ)) in investment evaluation through contingent claims analysis. First, we start from investment evaluation of financial call options on common stocks and then use the analogy between the retailer's investment option and call options to move our discussion forward.

Consider financial call options on a common stock as contingent claims whose underlying asset is the common stock. The value of the financial call option derives from the price of a share of the common stock that changes randomly over time. Assume that

the expected rate of growth of this common stock price is (α), which represents the expected rate of capital gain, and every share of this stock pays dividend at the rate (δ) to the shareholder. Therefore, the total expected rate of return on a share of this common stock – denoted by (μ) – is equal to the summation of the expected rate of growth of this stock price and the dividend rate, i.e., (μ = α + δ).

It is indicated that the holder of an American call option never exercises his option until the maturity date when the dividend rate of the underlying stock is zero [15], i.e., (δ = 0). This is true since the entire return on the stock is captured in its price movement and there is no cost to keeping the option alive. When the dividend rate is positive (δ > 0) there is an opportunity cost to keeping the option alive since by not exercising the option, an option holder sacrifices the dividend stream. The value of this forgone dividend stream increases as the price of the stock increases. As some satisfactory high price level, the opportunity cost of this forgone opportunity cost becomes greater than the value of keeping the call option alive and therefore, the option holder exercises his option some time before its maturity date.

For the retailer's investment option, (μ) represents the risk-adjusted expected rate of return that retailer requires from owning the operating store in the retail market under consideration. When (δ > 0), the expected growth rate of retail market potential (α) is below this risk-adjusted expected rate of return. Therefore, the expected rate of capital gain on the store's free cash flow is lower than the risk-adjusted expected rate of return that the retailer requires from its store opened in this retail market. One can consider *(δ) as an opportunity cost of deferring opening of the store and keeping the investment option alive in the retail market*. Note that when (δ = 0) the retailer's behavior is similar

to the holder of a call option on the common stock. The retailer never exercises its investment option to open a store in the retail market until the time that the investment option expires. No matter how high the value of retail market potential and how high the value of the store's free cash flow the retailer keeps its investment option alive since there is no opportunity cost to hold the investment option.

In order to keep the retailer's investment problem interesting, we assume that the opportunity cost is positive (i.e., $\delta > 0$). Note that the value of the retailer's investment option decreases as the value of parameter (δ) increases. The reason is that the opportunity cost for keeping the investment option alive increases as (δ) increases. If (δ) becomes infinitely large ($\delta \to \infty$) the retailer's investment option is worthless and the retailer's decision becomes invest now or never depending on the initial expected value of the store's free cash flow.

The parameter (δ) can be interpreted in many different ways depending on the context of the investment options. In general, the discussion of (δ) is important for the investment option whose underlying asset earns an expected rate of return lower than necessary to induce investors to hold it [77].

The most intuitive case is the call option on a common stock. The parameter (δ) represents the dividend rate on a share of common stock. The interpretation of (δ) is simple in this case. In addition, the value of this parameter can be correctly determined from the financial market.

Dixit and Pindyck [15] summarize two types of dividends for investment options. The first dividend type is direct dividend that appears in investment options on the

product that naturally grows and generates profit for the owner of the asset, for instance, this product might be a tree that grows, yielding more wood.

The second type of dividend is the indirect dividend that appears in investment options on physical commodities. This implicit, indirect dividend is recognized as convenience yield in economics literature. Convenience yield is the benefit or premium associated with holding an underlying product or physical good, rather than the contract or derivative product [78, 79]. Sometimes, due to irregular market movements such as an inverted market, the holding of an underlying good or asset may become more profitable than owning the contract or investment option, due to its relative scarcity versus high demand. An example would be purchasing physical bales of wheat rather than future contracts or options. Whenever the demand for wheat rises due to the sudden drought the difference between the original purchase price of the wheat and its price after the market shock determines the convenience yield or implicit dividend for the investment option on wheat.

It is indicated that the holder of the option on natural resources such as oil or copper may be a firm that uses the natural resource as an important input in its operation process. Therefore, he may find it convenient to hold its own inventory of the natural resource rather than relying on the futures market to acquire his inventory needs [15]. This preference introduces convenience yield or implicit dividend for investment options on the natural resource assets. Therefore for the storable commodity, (δ) represents the net marginal convenience yield from storage that is the flow of benefits less storage cost that the marginal stored unit provides. Pindyck [80, 81] summarizes three benefits for owning

the natural resource commodity versus the investment option on the same commodity, as follows.

- Increase ability to smooth production

- Avoid stockouts

- Facilitate scheduling of production and sales

In the case of the retailer's investment option, *(δ)* can be interpreted as the parameter, which *describes the possible entry and capacity expansion of competitors in the retail market*. Therefore, *the value of parameter (δ) can be considered as the rate of return shortfall of the retailer's investment in the retail market*. The reason for the existence of the return shortfall in the retail market is that as the value of retail market potential increases the other retailers become interested in opening new stores or expanding their stores in this market. The existence of such competition threat in the retail market should be considered in the form of the opportunity cost of keeping store opening option alive and deferring the entry to the retail market. Parameter (δ) represents this opportunity cost or rate of return shortfall in the retail market.

Several researchers indicate that the value of opportunity cost (parameter δ) should be taken into account in order to correctly evaluate the investment options, for instance see [82-85]. Therefore in this chapter, we use an exogenously specified opportunity cost in the process of contingent claims analysis for the retailer's investment option. In addition, we assume that the value of parameter *(δ)* is *constant over the course of the retailer's investment option*. This is a simplified assumption because in reality the value of rate of return shortfall changes stochastically over time with response to market-wide pressure. We address this issue by conducting a sensitivity analysis on the value of

parameter (δ) to explore how the retailer's investment decisions are sensitive to the changes in the value of (δ).

Therefore in this chapter, we assume that (δ) is a basic parameter and independent of parameters (σ) and (r_f). Note that the risk-free rate of return (r_f) is exogenous to contingent claims analysis and assumed to be fixed since it summarizes the larger consideration of the whole capital market and is not influenced by what happens to any one asset or firm or even industry. Similarly, the aggregate market price of risk (φ) in Equation (4.2) is assumed to be fixed. Therefore, any changes in the volatility of retail market potential (σ) impact the value of the risk-adjusted rate of return (μ) as it is indicated in Equation (4.2). Consequently, either the value of the expected growth rate of market potential (parameter α) or the value of opportunity cost (parameter δ) must change to accommodate the changes in parameter (μ) due to the changes in parameter (σ). However in this chapter, we ignore this possibility and assume that the entire set of parameters (α, σ, and δ) represents the basic model parameters that can change independently from each other. Therefore, we conduct separate sensitivity analyses over the range of possible values for each parameter. In addition, we conduct sensitivity analyses over the range of possible values for a set of two possible ranges of values for two parameters all together to address the interdependence between the model parameters in reality. These sensitivity analyses help us understand how the retailer's investment thresholds change with respect to the changes in the values of model parameters.

On the other hand in this chapter, we use a particular approach to simplify the process of using contingent claims analysis in the retailer's investment valuation. This approach is equivalent risk neutral valuation, which is embedded in the relationship

between dynamic programming and contingent claims analysis valuation. This approach is discussed in the next section.

4.3.4 Equivalent risk neutral evaluation

Financial assets in the capital market have different expected rates of return depending on their particular levels of risk. When the variability in the price of an asset increases (or the volatility in the price of an asset increases) the asset becomes more risky. It is generally acceptable that the more risky assets have a greater expected rate of return than less risky assets.

It is indicated that it is possible to calculate the price of an asset assuming there was no risk. In this approach the future cash flow of a risky asset is discounted back using the risk-free rate of return. However, one still needs to take into account the riskiness of an asset and the volatility in its price not by changing the discount rate but by adjusting the actual probability measures for the price variation. The adjusted probability measures, that are used to price a risky asset as if it exists in the risk-free world, are called risk neutral measures or probabilities. A risk neutral measure is the probability measure that results when one assumes that the future expected value of all financial assets is equal to the future payoff of the asset discounted at the risk-free rate of return, i.e., when the asset prices are corrected so that there is no risk, the probabilities that result are those of the risk neutral measure.

It is worth noting that risk neutral probabilities are only conceptual measures and developed to facilitate the options valuation. Therefore, these probability measures cannot be interpreted as the actual probabilities. For a more detailed explanation of risk neutral probabilities and their existence interested reader can refer to [86, 87].

Equivalent risk neutral evaluation is a standard approach in financial economics that uses risk neutral probabilities to assess investment opportunities in the capital market. In this chapter, we use a particular method that is developed by Dixit and Pindyck [15] to construct an equivalent stochastic process for a GBM model.

Here we again mention Equation (3.14) in Chapter 3 that summarizes the stochastic behavior of retail market potential (parameter $(X(t))$) as a GBM model. Equation (4.3) shows this GBM model that specifies the expected growth rate of $(X(t))$ is (α) and its log-volatility is (σ).

$$dX = \alpha X dt + \sigma X dz$$
(4.3)

Recall that the future cash flow of the retailer's investment option is discounted back using an exogenous cost of capital denoted by (ρ). Dixit and Pindyck presents an equivalent risk neutral evaluation approach, which is based on replacing the exogenously specified discount rate (ρ) by the risk-free rate of return (r_f) and the expected growth rate of this GBM model (α) by $(r_f - \delta)$. Therefore, the future cash flow can be discounted back at the risk-free rate of return (r_f) if we assume the underlying asset grows according to an adjusted GBM process with the expected rate of $(r_f - \delta)$. This new artificial stochastic variable is denoted by $(X'(t))$ that starts at the same initial point $(X(t))$ at time (t), but thereafter follows the new GBM, which is summarized in Equation (4.4) below.

$$dX' = \alpha' X' dt + \sigma X' dz \equiv (r_f - \delta) X' dt + \sigma X' dz$$
(4.4)

We use this adjusted GBM process to evaluate the retailer's investment option. Therefore in this chapter, we use the equivalent risk neutral evaluation approach to

evaluate the retailer's investment opportunity. This approach is also consistent with the fundamental objective of contingent claims analysis to evaluate the retailer's investment option in a way that maximizes the market-value of the retail firm. Hence, the equivalent risk neutral evaluation approach extends the theory of contingent claims analysis and simplifies the process of investment valuation by adjusting the parameters of the underlying state variable.

In addition, the valuation process using the equivalent risk neutral evaluation approach is very similar to the dynamic programming approach since we can also use a discrete lattice model to evaluate retailer's investment option as a decision tree. The motivation behind using an approximate discrete approach to evaluate the retailer's investment option is the same as the dynamic programming approach. Working with the continuous GBM model is difficult and many options problems do not have any analytical closed-form solutions. In the next section, we present an approximate discrete model for the GBM model of Equation (4.4) to describe the dynamic uncertainty of $(X'(t))$ in a discrete time fashion. This discrete approximation helps us evaluate retailers' investment options as simple as decision tree analysis.

4.3.5 A trinomial lattice model to approximate the stochastic variation of $(X'(t))$ in a discrete fashion

In this section, we adjust the approximate model that was presented in Chapter 3 to describe the dynamic uncertainty of $(X(t))$ in a discrete fashion. Using the discrete approximation model helps us evaluate the retailer's investment option as a decision tree as we will show in the next section.

We again use the same approximating approach that is developed by Kamrad and Ritchken [54] – as it was shown in Chapter 3 – for valuing options on one state variable. Here our only state variable is (X'(t)) that is described by a continuous GBM process of Equation (4.4). Using Kamrad and Ritchken's trinomial lattice formulation, we are able to approximate the continuous stochastic variations of (X'(t)) via a discrete random walk process.

The discrete values of (X'(t)) in this trinomial lattice are calculated, as follows. Assume the value of (X'(t)) at the beginning of the first time step is X_0 (Recall that the initial value of X'(t) must be equal to the initial value of (X(t)) since both GBM processes are equivalent). For the next time step, this value may increase by the ratio of (u>0), stay constant, or decrease by the ratio of (d=1/u) with probabilities of (q_1, q_2, and q_3), respectively. Consider that these probabilities are the risk neutral probabilities that are specified according to the equivalent GBM process of (X'(t)) in equation (4.4). Assume the length of each time increment is (Δt). Therefore, the value of (X') at the next time step (X'(t+Δt)) can be summarized, as follows.

$$X'(t+\Delta t) = \begin{cases} X'(t)u & \text{with probability } q_1 \\ X'(t) & \text{with probability } q_2 \\ X'(t)d & \text{with probability } q_3 \end{cases}$$

(4.5)

This pattern continues for the subsequent time steps until it reaches the last time step. As indicated in Chapter 3 the total number of time steps should be selected large enough to cover several possible values for the stochastic state variable. To completely define this approximate trinomial lattice, we need to determine the values of its parameters: the risk neutral probabilities (q_1, q_2, and q_3) and the jump ratios (u, d). These values should

be chosen in such a way that the true stochastic nature of (X'(t)) is captured as faithfully as possible. We again use Kamrad and Ritchken's [54] formulation (i.e., Equations (4.6a) and (4.6b)) to determine values for risk neutral probabilities and jump ratios in the described trinomial lattice model, as follows.

$$
\begin{cases}
u = e^{\lambda\sigma\sqrt{\Delta t}} \\
d = e^{-\lambda\sigma\sqrt{\Delta t}}
\end{cases}
\tag{4.6a}
$$

$$
\begin{cases}
q_1 = \dfrac{1}{2\lambda^2} + \dfrac{\mu\sqrt{\Delta t}}{2\lambda\sigma} \\[2ex]
q_2 = 1 - \dfrac{1}{\lambda^2} \\[2ex]
q_3 = \dfrac{1}{2\lambda^2} - \dfrac{\mu\sqrt{\Delta t}}{2\lambda\sigma}
\end{cases}
\tag{4.6b}
$$

where ($\mu = r_f - \delta - (\sigma^2/2)$) and ($\lambda \geq 1$).

This trinomial lattice is very similar to the one that was constructed for the dynamic programming approach in Chapter 3. The only difference is that we adjust the expected growth rate of (X) (parameter α) by the expected growth rate of (X'), which is ($r_f - \delta$).

Recall our discussion in Chapter 3 that a trinomial lattice can be a fairly accurate representation of geometric Brownian motion if the time step (Δt) used is small enough and the process occurs over a long enough time [54]. In addition with the trinomial lattice, the probability distributions become discrete, and the investment option can be valued as a decision tree. Figure 3.2 in Chapter 3 can also be used as a framework that shows how decision tree analysis can be incorporated with the options framework to

evaluate the retailer's investment opportunity using the contingent claims analysis approach.

In the next section, we show how the lattice representation in this figure can be used as a decision tree to determine the retailers' entry and quantity decisions in this competitive, dynamic market. We will summarize the investment analysis procedure for the investment situations that are described in the dynamic programming approach.

4.4   An equivalent risk neutral evaluation approach to analyze retailers' investment options

In this section, we summarize an investment analysis approach based on the decision tree analysis approach and equivalent risk neutral evaluation to analyze retailers' investment options in several market structures. This options-based approach is considered as an "economically corrected" version of decision tree analysis [14] since it addresses the asymmetry in the retailer's profit and its risk characteristics in a single framework. This investment analysis approach is very similar to the dynamic programming approach that was described in Chapter 3 and therefore, we only highlight the differences in the evaluation procedure in this chapter.

The objective of this section is to develop an economically corrected decision tree using the risk neutral probabilities to determine the retailer's investment behavior in terms of the optimal time to enter and the quantity of the product to offer to the retail market. The major question of interest is to find out when a retailer should exercise its investment option, enter a market, and open a store. We consider the same three general cases that we discussed in Chapter 3 to illustrate our approach. However, the described procedure can be easily extended to the other investment cases with minor manipulation.

Here we again summarize those three retail market types to refresh our readers, as follows.

- The market, in which only one retailer has an investment opportunity to enter and opens a store while the other retailer does not have the same opportunity.

- The market, in which one retailer has a store opened and the other retailer has an investment opportunity to enter and opens a store.

- The market, in which both retailers have investment opportunities and compete to enter and open stores.

Again we assume that once a retailer decides to open a store in any of the above markets the store will remain opened forever. It is also assumed that a retailer's investment option is free in our formulation but exercising an option is what costs money that is modeled in our analysis approach as the retailer's investment cost to develop a store in a market.

Therefore first, we need to define a procedure to determine the expected NPV of a store opened in the competitive, dynamic retail market. This expected NPV is calculated in the exact similar fashion as described in sections 3.7.1 and 3.7.2. There are only two differences in this market-oriented expected NPV calculation. First we need to use the adjusted trinomial lattice of (X'(t)) instead of the original lattice of X(t). Consider that the values of the two underlying state variables ((X) and (X')) are identical at similar places in these two lattices since the same values of the jump ratios (u and d) are used to construct these two lattices according to Equation (4.6a). This is due to the fact that the values of (u and d) are only dependent on the value of log-volatility (σ), which is the same for both the original and the adjusted GBM process. However, the values of the

discrete probabilities are different in these two lattices. The actual probabilities are used in the original lattice using the expected growth rate of (α) while the risk neutral probabilities are used in the market-adjusted lattice using the adjusted growth rate of ($r_f -\delta$). This difference has an important impact on the expected calculation in decision tree analysis.

The second difference is related to the value of discount rate that is used to discount back the future cash flow along each lattice. While the original lattice of ($X(t)$) is discounted back by the exogenously defined cost of capital (ρ) this market-adjusted lattice of ($X'(t)$) should be discounted back at the risk-free rate of return (r_f).

The market-oriented expected NPV of retailer i's investment option in the monopoly, dynamic market is determined according to the procedure described in section 3.7.1 considering the above two differences. Note that retailer i's optimal quantity of the product and the respective profit in the monopoly market are the same as those that are summarized in Equations (B1) and (B2) in Appendix B, respectively. However, the market-oriented final value of retailer i's investment option in this monopoly, dynamic market (denoted by ($FV_i^M(t)$) corresponding to the value of ($X_F(t)$) at any node in the final time step) is slightly different from Equation (B4) in Appendix B since the continuous stream of retailer i's monopoly cash flow ($\Pi_{i,F}^M(t)$) should be discounted back to the final time step using the risk-free rate of return (r_f) in order to determine the final value ($FV_i^M(t)$). This market-oriented final value of retailer i's investment option in the monopoly, dynamic market ($FV_i^M(t)$) is summarized in Equation (4.7), as follows.

$$FV_i{}^M(t) = \int_0^\infty \Pi_{i,F}^M(\tau)e^{-rf\tau}d\tau$$

$$= \begin{cases} \dfrac{1}{r_f}\left(\dfrac{(X_F(t))^2}{4\gamma} - \dfrac{(VC_i)(X_F(t))}{2\gamma} + \dfrac{VC_i{}^2}{4\gamma} - FC_i\right) & When\,(X_F(t) \geq VC_i) \\[2ex] -\dfrac{FC_i}{r_f} & Otherwise \end{cases} \quad i = 1,2$$

<div align="right">(4.7)</div>

The market-oriented expected NPV of retailer 1's and 2's investment options in the duopoly, dynamic market is determined according to the procedure described in section 3.7.2 considering the above two differences. Note that retailer 1's and 2's optimal quantities of the product $((Q_{1,F}^D(t))$ and $(Q_{2,F}^D(t)))$ and respective optimal profits $((\Pi_{1,F}^D(t))$ and $(\Pi_{2,F}^D(t)))$ are the same as those that are summarized in Equations (C1) and (C2) of Appendix C, respectively. However, the market-oriented final values of retailer 1's and 2's investment options in this duopoly, dynamic market (denoted by $(FV_1^D(t))$ and $(FV_2^D(t))$ corresponding to the value of $(X_F(t))$ at any node in the final time step) are slightly different from Equation (C4) in Appendix C since the continuous stream of retailer 1's and 2's duopoly cash flows $((\Pi_{1,F}^D(t))$ and $(\Pi_{2,F}^D(t)))$ should be discounted back to the final time step using the risk-free rate of return (r_f) in order to determine the final value $((FV_1^D(t))$ and $(FV_2^D(t)))$. These market-oriented final values of retailer 1's and 2's investment options in the duopoly, dynamic market $((FV_1^D(t))$ and $(FV_2^D(t)))$ are summarized in Equation (4.8), as follows.

$$
\left\{
\begin{array}{l}
when\left((VC_2 - 2VC_1 + X_F(t)) \geq 0\right) \& \left((VC_1 - 2VC_2 + X_F(t)) \geq 0\right) \\[4pt]
\left\{
\begin{array}{l}
FV_1^D(t) = \dfrac{1}{r_f}\left(\dfrac{(X_F(t))^2}{9\gamma} + \dfrac{2(VC_2 - 2VC_1)(X_F(t))}{9\gamma} + \dfrac{(VC_2 - 2VC_1)^2}{9\gamma} - FC_1\right) \\[12pt]
FV_2^D(t) = \dfrac{1}{r_f}\left(\dfrac{(X_F(t))^2}{9\gamma} + \dfrac{2(VC_1 - 2VC_2)(X_F(t))}{9\gamma} + \dfrac{(VC_1 - 2VC_2)^2}{9\gamma} - FC_2\right)
\end{array}
\right. \\[16pt]
when\left((VC_2 - 2VC_1 + X_F(t)) \geq 0\right) \& \left((VC_1 - 2VC_2 + X_F(t)) < 0\right) \& \left(X_F(t) \geq VC_1\right) \\[4pt]
\left\{
\begin{array}{l}
FV_1^D(t) = FV_1^M(t) = \dfrac{1}{r_f}\left(\dfrac{(X_F(t))^2}{4\gamma} - \dfrac{(VC_1)(X_F(t))}{2\gamma} + \dfrac{VC_1^2}{4\gamma} - FC_1\right) \\[12pt]
FV_2^D(t) = -\dfrac{FC_2}{r_f}
\end{array}
\right. \\[16pt]
when\left((VC_2 - 2VC_1 + X_F(t)) < 0\right) \& \left((VC_1 - 2VC_2 + X_F(t)) \geq 0\right) \& \left(X_F(t) \geq VC_2\right) \\[4pt]
\left\{
\begin{array}{l}
FV_1^D(t) = -\dfrac{FC_1}{r_f} \\[12pt]
FV_2^D(t) = FV_2^M(t) = \dfrac{1}{r_f}\left(\dfrac{(X_F(t))^2}{4\gamma} - \dfrac{(VC_2)(X_F(t))}{2\gamma} + \dfrac{VC_2^2}{4\gamma} - FC_2\right)
\end{array}
\right. \\[16pt]
Otherwise \\[4pt]
\left\{
\begin{array}{l}
FV_1^D(t) = -\dfrac{FC_1}{r_f} \\[12pt]
FV_2^D(t) = -\dfrac{FC_2}{r_f}
\end{array}
\right.
\end{array}
\right.
\tag{4.8}
$$

We use these market-oriented expected NPVs to determine retailers' investment behaviors in terms of the optimal time to enter and the quantity of the product to offer to the retail market. The above three general market structures are selected to illustrate how equivalent risk neutral evaluation approach can be used to assess retailers' investment options. The assessment procedure is exactly similar to the procedures described in sections 3.7.2 - 3.7.4. We should only consider the major two differences that were described above. First, we should use the market-adjusted lattice of (X'(t)) that is built using the risk neutral probabilities, as the basis for our economically corrected decision

tree analysis. Second, we should use the risk-free rate of return (r_f) to discount back the retailer's cash flow along this market-oriented lattice.

Using these revised procedures, we are able to develop an equivalent risk neutral approach to find out when a retailer should exercise its investment option, enter a market, and open a store. In the next section, we use this economically-corrected decision tree analysis approach to explore how the values of retailers' cost parameters impact their optimal investment thresholds in noncompetitive and competitive dynamic markets. We conduct several numerical studies for this purpose.

4.5 Impact of retailers' cost structure on their optimal entry time decisions

In this section, we use the described procedures of section 4.4 for equivalent risk neutral evaluation to explore retailers' investment behaviors in terms of their entry decisions considering their difference in cost parameters and the effect of competition in dynamic markets. In section 3.9, we studied retailers' investment behaviors using the dynamic programming approach. In this section, we want to show that retailers' investment behaviors in terms of entry decisions remain the same using the market-oriented approach of contingent claims analysis. Again we consider the same four general investment cases of Chapter 3 summarized below.

- Retailer 1's fixed cost is higher than retailer 2's fixed cost, i.e., ($FC_1 > FC_2$), but the other cost parameters are equal ($IC_1 = IC_2$ and $VC_1 = VC_2$)

- Retailer 1's investment cost is higher than retailer 2's investment cost, i.e., ($IC_1 > IC_2$), but the other cost parameters are equal ($FC_1 = FC_2$ and $VC_1 = VC_2$)

- Retailer 1's marginal cost is higher than retailer 2's marginal cost, i.e., ($VC_1 >$ VC_2), but the other cost parameters are equal ($FC_1 = FC_2$ and $IC_1 = IC_2$)

- Retailer 1's investment and fixed cost is higher than retailer 2's investment and fixed cost, respectively, i.e., ($IC_1 > IC_2$ and $FC_1 > FC_2$), but retailer 2's marginal cost is higher than retailer 1's marginal cost ($VC_1 < VC_2$)

In the next sections, several numerical examples are conducted to explore retailers' entry decisions considering the differences in their cost parameters and their investment options in competitive versus noncompetitive markets.

4.5.1 Two identical retailers with only the fixed cost difference

Consider the same two retailers that we considered in section 3.9.1, which have similar cost parameters and are only different in their fixed costs. The values of their cost parameters are: ($IC_1 = IC_2 = \$400,000$, $VC_1 = VC_2 = 100$ \$/Item Sold and $FC_1 = 220,000$ \$/Year $> FC_2 = 200,000$ \$/Year). Also assume that retailers' investment options are valid for two hundred months. The values of the other parameters in the equivalent risk neutral evaluation approach are assumed to be: ($r_f = 5\%$/Year), ($\delta = 5\%$/Year), ($\sigma = 0.1$), ($X_0 = 700$), and ($\gamma = 1$).

Figure 4.1 shows these retailers' optimal investment thresholds in the competitive versus noncompetitive markets that are indicated by the values of (X) at time (t) (denoted as ($X^*(t)$) in the vertical axis). The results from the equivalent risk neutral evaluation approach are consistent with the results from the dynamic programming approach summarized in section 3.9.1. At time step (t) a retailer exercises its investment option and opens a store in a market when the value of retail market potential at this time step

exceeds the optimal value of retail market potential at this time step, which is determined by our investment analysis approach as it is shown by the curve in Figure 4.1.

Retailer 2 enters the market first due to its lower fixed cost advantage. It can be seen from Figure 4.1 that retailer 2's investment threshold in the competitive market is lower than its threshold in the same market without any competition effect. Therefore, retailer 2 enters the competitive market early enough in order to preempt the market from the entry of the other retailer (retailer 1). Retailer 2's first-mover advantage pushes up retailer 1's investment threshold and delays retailer 1's entry to the market. Thus, this numerical example based on the equivalent risk neutral evaluation approach shows the significance of the strategic aspect of early investment by the retailer with the lower fixed cost. This result is also consistent with the result from our numerical example based on the dynamic programming approach in section 3.9.1.

Also consider that the retailer's optimal investment threshold is lowered at the end because it is assumed that a retailer's investment opportunity becomes disappeared after the last time step. This lowered threshold at the end is simply an artifact of the end point that we choose in this numerical example. In the real world one can argue that the market opportunity does not really disappear and hence, the retailer's investment opportunity is perpetual and stays forever. However, it is indicated that the trinomial lattice formulation can be used as an accurate approximation for investment options that never expire [54]. Therefore, we can also use the investment evaluation procedure based on the trinomial lattice approximation that is described in this chapter for a retailer's investment opportunity that never expires. The only requirement is to construct a lattice with sufficiently large number of time steps in order to provide an appropriate approximation

for the infinite time horizon of a retailer's investment option. Hence, the evaluation procedure of a retailer's investment option with an infinite time horizon is reduced to the evaluation procedure of a retailer's investment option with a very long, but finite time horizon as it was described in this chapter.

However, consider that the retailer's optimal investment threshold based on the equivalent risk neutral evaluation approach falls off more quickly than the dynamic programming approach as the retailer reaches the end point. The reason is that unlike the dynamic programming approach, which uses a constant actual discount rate to evaluate a retailer's investment option, the equivalent risk neutral valuation approach does not use a constant actual discount rate to evaluate a retailer's investment option. The actual value of the discount rate in the risk neutral valuation approach changes in different time steps and different decision nodes in the approximate lattice model. Recall that the actual discount rate for a retailer's investment option is based on the actual probabilities of the change in retail market potential, which are different from the values of the risk neutral probabilities in the risk neutral evaluation approach. In addition, the values of these actual discount rates change more rapidly when a retailer reaches the end point since the retailer's investment opportunity becomes worthless after the last time step. Therefore, the values of the retailer's optimal investment threshold that depends on the values of these actual discount rates change more rapidly when a retailer reaches the end point. We do not observe this quick fall off behavior in the retailer's optimal investment threshold under the dynamic programming approach since the value of actual discount rate remains constant when a retailer reaches the end point.

Figure 4.1. Impacts of competition and fixed cost differences on retailers' optimal investment thresholds (the equivalent risk neutral evaluation approach).

4.5.2 Two identical retailers with only the marginal cost difference

Consider the same two retailers that were described in section 3.9.2, which have similar cost parameters and are only different in their marginal costs. The values of their cost parameters are: ($IC_1 = IC_2 = \$400,000$, $FC_1 = FC_2 = 200,000$ \$/Year, and $VC_1 = 100$ \$/Item Sold > $VC_2 = 80$ \$/Item Sold). The other model parameters are keeping at their same values as section 4.5.1.

Figure 4.2 shows these retailers' optimal investment thresholds in the competitive versus noncompetitive markets that are shown by ($X^*(t)$) in the vertical axis. The results from the equivalent risk neutral evaluation approach are consistent with the results from the dynamic programming approach summarized in section 3.9.2. At time step (t) a retailer exercises its investment option and opens a store in a market when the value of

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retail market potential at this time step exceeds the optimal value of retail market potential at this time step, which is determined by our investment analysis approach as it is shown by the curve in Figure 4.2.

Retailer 2 enters the market first due to its lower marginal cost advantage. It can be seen from Figure 4.2 that retailer 2's investment threshold in the competitive market is lower than its threshold in the same market without any competition effect. Therefore, retailer 2 enters the competitive market early enough in order to preempt the market entry of the other retailer (retailer 1). Retailer 2's first-mover advantage pushes up retailer 1's investment threshold and delays retailer 1's entry to the market. Thus, this numerical example based on the equivalent risk neutral evaluation approach shows the significance of the strategic aspect of early investment by the retailer with the lower marginal cost. This result is also consistent with the result from our numerical example based on the dynamic programming approach in section 3.9.2.

Also consider that the retailer's optimal investment threshold is lowered at the end because it is assumed that a retailer's investment opportunity becomes disappeared after the last time step. This lowered threshold at the end is simply an artifact of the end point that we choose in this numerical example. In the real world one can argue that the market opportunity does not really disappear and hence, the retailer's investment opportunity is perpetual and stays forever. However, it is indicated that the trinomial lattice formulation can be used as an accurate approximation for investment options that never expire [54]. Therefore, we can also use the investment evaluation procedure based on the trinomial lattice approximation that is described in this chapter for a retailer's investment opportunity that never expires. The only requirement is to construct a lattice with

sufficiently large number of time steps in order to provide an appropriate approximation for the infinite time horizon of a retailer's investment option. Hence, the evaluation procedure of a retailer's investment option with an infinite time horizon is reduced to the evaluation procedure of a retailer's investment option with a very long, but finite time horizon as it was described in this chapter.

However, consider that the retailer's optimal investment threshold based on the equivalent risk neutral evaluation approach falls off more quickly than the dynamic programming approach as the retailer reaches the end point. The reason is that unlike the dynamic programming approach, which uses a constant actual discount rate to evaluate a retailer's investment option, the equivalent risk neutral valuation approach does not use a constant actual discount rate to evaluate a retailer's investment option. The actual value of the discount rate in the risk neutral valuation approach changes in different time steps and different decision nodes in the approximate lattice model. Recall that the actual discount rate for a retailer's investment option is based on the actual probabilities of the change in retail market potential, which are different from the values of the risk neutral probabilities in the risk neutral evaluation approach. In addition, the values of these actual discount rates change more rapidly when a retailer reaches the end point since the retailer's investment opportunity becomes worthless after the last time step. Therefore, the values of the retailer's optimal investment threshold that depends on the values of these actual discount rates change more rapidly when a retailer reaches the end point. We do not observe this quick fall off behavior in the retailer's optimal investment threshold under the dynamic programming approach since the value of actual discount rate remains constant when a retailer reaches the end point.

Figure 4.2. Impacts of competition and marginal cost differences on retailers' optimal investment thresholds (the equivalent risk neutral evaluation approach).

4.5.3 Two identical retailers with only the investment cost difference

Consider the same two retailers that were described in section 3.9.3, which have similar cost parameters and are only different in their investment costs to develop stores. The values of their cost parameters are: ($VC_1 = VC_2 = 100$ \$/Item Sold, $FC_1 = FC_2 = 200,000$ \$/Year, and $IC_1 = \$500,000 > IC_2 = \$400,000$). The other model parameters keep at their same values as section 4.5.1.

Figure 4.3 shows these retailers' optimal investment thresholds in the competitive versus noncompetitive markets that are shown by ($X^*(t)$) in the vertical axis. The results from the equivalent risk neutral evaluation approach are consistent with the results from the dynamic programming approach summarized in section 3.9.3. At time step (t) a retailer exercises its investment option and opens a store in a market when the value of

retail market potential at this time step exceeds the optimal value of retail market potential at this time step, which is determined by our investment analysis approach as it is shown by the curve in Figure 4.3.

Retailer 2 enters the market first due to its lower investment cost advantage. It can be seen from Figure 4.3 that retailer 2's investment threshold in the competitive market is lower than its threshold in the same market without any competition effect. Therefore, retailer 2 enters the competitive market early enough in order to preempt the market from the entry of the other retailer (retailer 1). Retailer 2's first-mover advantage pushes up retailer 1's investment threshold and delays retailer 1's entry to the market. Thus, this numerical example based on the equivalent risk neutral evaluation approach shows the significance of the strategic aspect of early investment by the retailer with the lower investment cost. This result is also consistent with the result from our numerical example based on the dynamic programming approach in section 3.9.3.

Also consider that the retailer's optimal investment threshold is lowered at the end because it is assumed that a retailer's investment opportunity becomes disappeared after the last time step. This lowered threshold at the end is simply an artifact of the end point that we choose in this numerical example. In the real world one can argue that the market opportunity does not really disappear and hence, the retailer's investment opportunity is perpetual and stays forever. However, it is indicated that the trinomial lattice formulation can be used as an accurate approximation for investment options that never expire [54]. Therefore, we can also use the investment evaluation procedure based on the trinomial lattice approximation that is described in this chapter for a retailer's investment opportunity that never expires. The only requirement is to construct a lattice with

sufficiently large number of time steps in order to provide an appropriate approximation for the infinite time horizon of a retailer's investment option. Hence, the evaluation procedure of a retailer's investment option with an infinite time horizon is reduced to the evaluation procedure of a retailer's investment option with a very long, but finite time horizon as it was described in this chapter.

However, consider that the retailer's optimal investment threshold based on the equivalent risk neutral evaluation approach falls off more quickly than the dynamic programming approach as the retailer reaches the end point. The reason is that unlike the dynamic programming approach, which uses a constant actual discount rate to evaluate a retailer's investment option, the equivalent risk neutral valuation approach does not use a constant actual discount rate to evaluate a retailer's investment option. The actual value of the discount rate in the risk neutral valuation approach changes in different time steps and different decision nodes in the approximate lattice model. Recall that the actual discount rate for a retailer's investment option is based on the actual probabilities of the change in retail market potential, which are different from the values of the risk neutral probabilities in the risk neutral evaluation approach. In addition, the values of these actual discount rates change more rapidly when a retailer reaches the end point since the retailer's investment opportunity becomes worthless after the last time step. Therefore, the values of the retailer's optimal investment threshold that depends on the values of these actual discount rates change more rapidly when a retailer reaches the end point. We do not observe this quick fall off behavior in the retailer's optimal investment threshold under the dynamic programming approach since the value of actual discount rate remains constant when a retailer reaches the end point.

Figure 4.3. Impacts of competition and investment cost differences on retailers' optimal investment thresholds (the equivalent risk neutral evaluation approach).

4.5.4 Small and big retailers

In this section, we consider the same two retailers (retailer 1 and 2) in section 3.9.4. Retailer 1, with the higher investment and fixed costs and the lower marginal cost, represents a large retailer that usually opens large retail stores while the other retailer (retailer 2) represents a small retailer that usually opens small stores. The values of these retailers' cost parameters are   (IC_1 = $400,000 > IC_2 = $200,000, FC_1 = 200,000 $/Year > FC_2 = 100,000 $/Year, and VC_1 = 80 $/Item Sold < VC_2 = 100 $/Item Sold). The other model parameters keep at their same values as section 4.5.1.

Figure 4.4 shows these retailers' optimal investment thresholds in the competitive versus noncompetitive markets that are shown by (X^*(t)) in the vertical axis. The results from the equivalent risk neutral evaluation approach are consistent with the results from

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the dynamic programming approach summarized in section 3.9.4. At time step (t) a retailer exercises its investment option and opens a store in a market when the value of retail market potential at this time step exceeds the optimal value of retail market potential at this time step, which is determined by our investment analysis approach as it is shown by the curve in Figure 4.4.

Retailer 2 enters the market first due to its overall cost advantage. Another interesting observation from Figure 4.4 is that the small retailer's investment threshold in the competitive market is the same as its investment threshold in the noncompetitive market. Since the values of retailers' costs parameters are very different from each other the small retailer is confident that he opens the first store in the competitive market. Therefore, *the small retailer waits longer to fully take advantage of the value of waiting and invests in the satisfactory high value of retail market potential when uncertainty over the future state of retail market potential is reduced*. However, the small retailer's first-mover advantage pushes up big retailer's investment threshold and delays the big retailer's entry to the market.

Thus, this numerical example based on the equivalent risk neutral evaluation approach shows the significance of the strategic aspect of early investment by the retailer with the overall cost advantage. This result is also consistent with the result from our numerical example based on the dynamic programming approach in section 3.9.4.

Also consider that the retailer's optimal investment threshold is lowered at the end because it is assumed that a retailer's investment opportunity becomes disappeared after the last time step. This lowered threshold at the end is simply an artifact of the end point that we choose in this numerical example. In the real world one can argue that the market

opportunity does not really disappear and hence, the retailer's investment opportunity is perpetual and stays forever. However, it is indicated that the trinomial lattice formulation can be used as an accurate approximation for investment options that never expire [54]. Therefore, we can also use the investment evaluation procedure based on the trinomial lattice approximation that is described in this chapter for a retailer's investment opportunity that never expires. The only requirement is to construct a lattice with sufficiently large number of time steps in order to provide an appropriate approximation for the infinite time horizon of a retailer's investment option. Hence, the evaluation procedure of a retailer's investment option with an infinite time horizon is reduced to the evaluation procedure of a retailer's investment option with a very long, but finite time horizon as it was described in this chapter.

However, consider that the retailer's optimal investment threshold based on the equivalent risk neutral evaluation approach falls off more quickly than the dynamic programming approach as the retailer reaches the end point. The reason is that unlike the dynamic programming approach, which uses a constant actual discount rate to evaluate a retailer's investment option, the equivalent risk neutral valuation approach does not use a constant actual discount rate to evaluate a retailer's investment option. The actual value of the discount rate in the risk neutral valuation approach changes in different time steps and different decision nodes in the approximate lattice model. Recall that the actual discount rate for a retailer's investment option is based on the actual probabilities of the change in retail market potential, which are different from the values of the risk neutral probabilities in the risk neutral evaluation approach. In addition, the values of these actual discount rates change more rapidly when a retailer reaches the end point since the

retailer's investment opportunity becomes worthless after the last time step. Therefore, the values of the retailer's optimal investment threshold that depends on the values of these actual discount rates change more rapidly when a retailer reaches the end point. We do not observe this quick fall off behavior in the retailer's optimal investment threshold under the dynamic programming approach since the value of actual discount rate remains constant when a retailer reaches the end point.

Figure 4.4. Impacts of competition on small and big retailers' optimal investment thresholds (the equivalent risk neutral evaluation approach).

A close look at the results in section 4.5 leads us to this conclusion that *the investment threshold of the retailer with the cost advantage in the competitive market is substantially lower than its threshold in the same market without competition particularly when the values of the cost parameters of two competing retailers are similar*.

In the next section, we examine how retailers' investment thresholds are sensitive to the values of the model parameters in the equivalent risk neutral evaluation approach.

4.6   Sensitivity analysis

In this section, we study how retailers' optimal investment thresholds change as the values of the parameters in this equivalent risk neutral evaluation approach change. First, we conduct three one-factor sensitivity analyses to determine how retailers' investment thresholds and the values of their investment options at the exercise times change with respect to the change in the value of three essential factors in the equivalent risk neutral evaluation approach including (σ, r_f, and δ). Then, we conduct three two-factor sensitivity analyses on the values of these basic model parameters to examine how retailers' investment thresholds change regarding the variations in the values of two basic model parameters.

Finally, we examine how retailers' investment thresholds and their entry decisions to the competitive market are impacted by the changes in the values of the retailer's cost parameters. We use several numerical examples to illustrate our purpose.

4.6.1   Sensitivity analysis on the log-volatility of retail market potential (σ)

In this section, we study how changes in the value of the log-volatility of retail market potential (i.e., parameter (σ) in Equation (4.1)) impact retailers' optimal thresholds for investment values (i.e., $V^*(t)$) and their optimal investment thresholds (i.e., $X^*(t)$). Recall that we conducted a similar study on parameter (σ) of the dynamic programming approach in Chapter 3. We expect that the results from this study will be similar to the results in section 3.10.1 since parameter (σ) represents the same underlying factor in both dynamic programming and equivalent risk neutral evaluation approaches.

We use the same numerical example of section 4.5.4, regarding the competition between the small and big retailers, to conduct sensitivity analysis in this section. We consider four levels of (σ) for sensitivity analysis: (σ = [0.05, 0.10, 0.15, and 0.18]). Also assume that the values of the other model parameters including (r_f and δ) remain constant at their levels in section 4.5.4.

The value of (σ) is an indicator for the degree of dynamic uncertainty in the retail market since it represents the standard deviation of the growth rate of retail market potential as it was indicated in Equation (4.1). The value of (σ) increases when the dynamic uncertainty of the retail market increases. As the uncertainty in the retail market increases the value of the retailer's investment option increases since it becomes more likely that the retailer exercises its investment option at some time in the future although it may not be profitable to exercise its option now. Note that the upside potential future profit for the retailer is not limited while the downside loss is limited to zero when the retailer has an investment opportunity in the retail market. Therefore, as the value of (σ) increases the upside potential profit increases while the downside possible loss remains bounded at the constant zero level. Hence, the retailer's value of an investment option increases when the value of (σ) increases.

The retailer exercises its investment opportunity when its net investment value (the store's NPV minus the investment cost to open the store in the dynamic market) exceeds the value of its investment option, i.e., the retailer's optimal investment value is equal to the retailer's value of its investment option at the exercise point. Therefore, the optimal store's value that triggers the retailer to invest and open the store, increases when the value of (σ) increases. Figure 4.5 shows how the optimal thresholds of the retailers'

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investment values increase as the value of (σ) increases from (0.05) to (0.18). Note that *the changes in the value of (σ) do not impact the order, in which retailers enter the competitive market*. The small retailer invests earlier in the competitive market and delays the big retailer's entry.
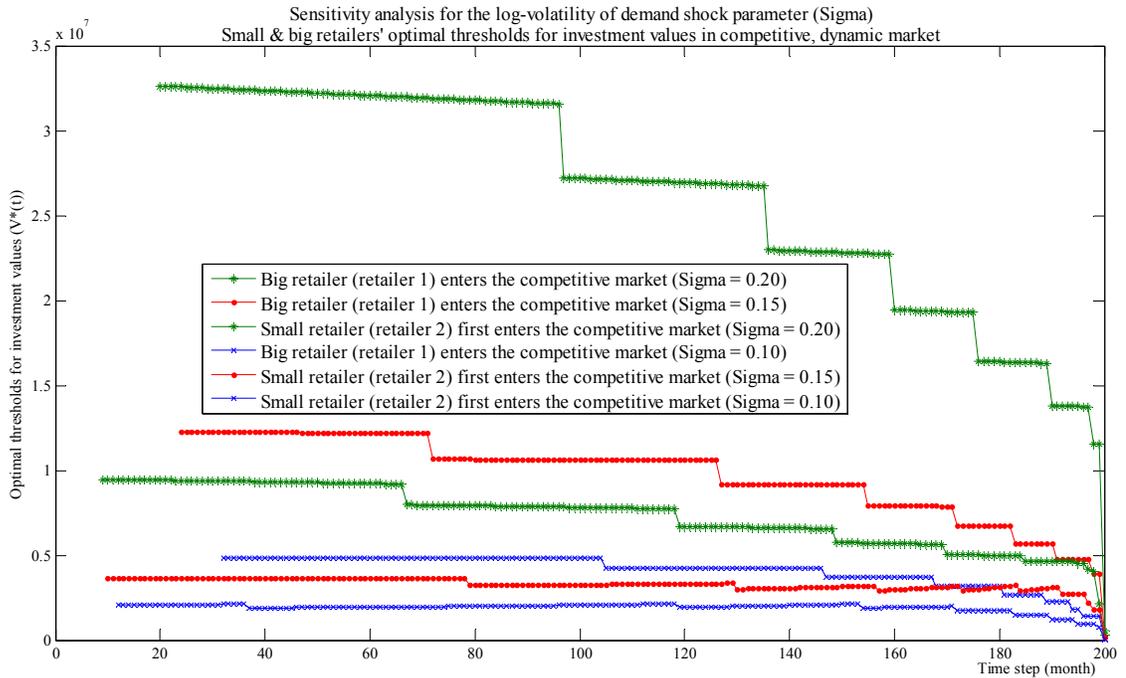
Figure 4.5. Sensitivity analysis on the value of (σ) and its impact on retailers' optimal thresholds for investment values (V*(t)) (the equivalent risk neutral evaluation approach).

It is noted that the retailer requires the higher value of investment in order to exercise its investment option when the value of (σ) increases. However, the impact of changes of the value of (σ) on the value of X that triggers the retailer to exercise its investment option is not trivial since it comes from a much more complicated process. The retailer's investment value at the exercise time (V*(t)) is calculated based on the described procedure in section 4.4. A change in the value of (σ) changes the value of two sets of parameters in the equivalent risk neutral evaluation approach: the jump parameters (u and d) and the risk neutral probabilities (q_1 and q_3) as described in Equation (4.6).

An increase in the value of (σ) on one hand, increases the value of the up jump parameter (u) while decreasing the value of the low jump parameter (d). Hence, the described lattice in section 4.4 contains the wider range of values for (X). This wider range of the underlying stochastic variable dramatically changes the retailer's investment value. An increase in the value of (σ) on the other hand, decreases the up risk neutral probability (q_1) and increases the down risk neutral probability (q_3). These changes in the values of risk neutral probabilities decrease the retailer's investment value. The overall change in the retailer's investment value is the overall effect of these two interactive changes.

Since the retailer's investment value at the optimal exercise setting increases as the value of (σ) increases, we know that the overall change in the retailer's investment value is increasing. However, due to the sophisticated interaction between the above two effects, it is not easy to determine how the value of retail market potential should change to increase the retailer's optimal investment value when the value of (σ) increases. Note that the store's cash flows along the lattice nodes and the store's final values at the nodes in the end of the lattice are the quadratic functions of variable X. This nonlinear relationship makes our calculation even more complicated. Therefore, we conduct several numerical examples to explore this direction of change. The results are summarized in Figure 4.6. It can be seen that the retailer's investment threshold (denoted by ($X^*(t)$)) increases as the value of (σ) increases. Therefore, an increase in the optimal threshold value ($X^*(t)$) results in an increase in the optimal retailer's investment value ($V^*(t)$) when (σ) increases.
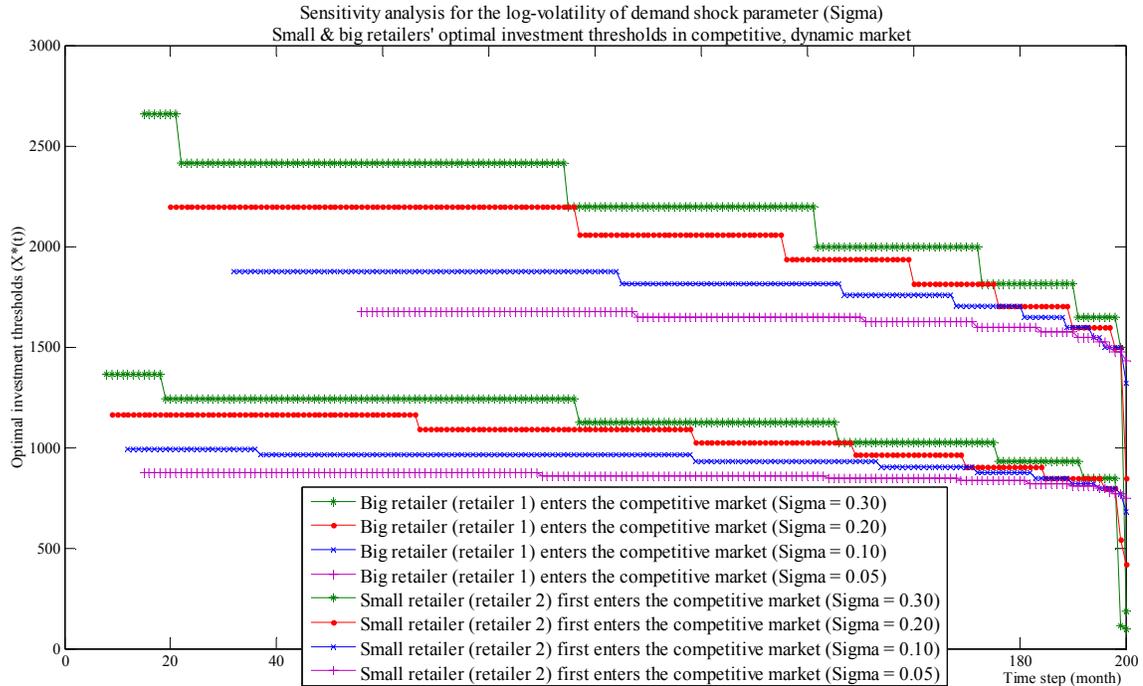
Figure 4.6. Sensitivity analysis on the value of (σ) and its impact on retailers' optimal investment thresholds (X*(t)) (the equivalent risk neutral evaluation approach).

It can be concluded that retailers' optimal investment thresholds and their optimal values for investment change as the value of (σ) changes. Hence, *retailers' investment behaviors are sensitive to the log-volatility of the retail market, irrespective of retailers' risk preferences*. This result is consistent with the characteristics of the optimal investment rule for an investment opportunity on a simple project that has been described by Dixit and Pindyck [15] (Ch. 5.4). Our results are also consistent with the results in section 3.10.1 that we conducted a similar sensitivity analysis on the value of (σ) in the dynamic programming approach. The direction of changes in retailers' investment thresholds with respect to the changes in the value of (σ) in that section are the same as our results in this section.

Dixit and Pindyck [15] also study the impact of changes in the value of (σ) on the overall retail firm's market value. Consider that the value of the retailer's investment

opportunity increases as the value of (σ) increases. But for that very reason, the retailer becomes more reluctant to invest and open a store when the value of (σ) increases and therefore, the amount of actual investment by the retail firm decreases. However, when the retail market becomes more uncertain the market value of the retail firm can go up even though the retail firm does less investment and perhaps opens fewer stores.

4.6.2 *Sensitivity analysis on the risk-free rate of return (r_f)*

In this section, we study how changes in the value of the risk-free rate of return (i.e., parameter (r_f) in the equivalent risk neutral evaluation approach) impact retailers' optimal thresholds for investment values (i.e., $V^*(t)$) and their optimal investment thresholds (i.e., $X^*(t)$). We use the same numerical example of section 4.5.4 regarding the competition between the small and big retailers to conduct sensitivity analysis in this section. We consider three levels of (r_f) for sensitivity analysis: (r_f = [0.03, 0.05, and 0.07]). Also we assume that the values of the other model parameters including (σ and δ) remain constant at their levels in section 4.5.4.

The low value of risk-free rate of return as the interest rate in the equivalent risk neutral evaluation approach makes the future relatively more important than the present and therefore, motivates the retailer to exercise its investment option earlier and opens a store to receive the store's future cash flow as soon as the retail market provides satisfactory return. Hence, the retailer's value of an investment option decreases when the value of (r_f) decreases.

On the other hand, the retailer exercises its investment option when the store's NPV (i.e., the value of the retailer's investment) exceeds the value of the retailer's investment option. Therefore, the critical store NPV that triggers the retailer to exercise its

investment option to open the store, decreases when the value of (r_f) decreases. The reason is that at the optimal time for exercising the investment option, the value of the retailer's investment option decreases when the value of (r_f) decreases. Hence, a decrease in the value of (r_f) shifts down the optimal value of the retailer's investment. Figure 4.7 shows how the optimal thresholds of retailers' investment values (denoted by ($V^*(t)$)) decrease as the value of (r_f) decreases from (0.07) to (0.03).

This decrease in the optimal retailer's investment value due to the decrease in the value of (r_f) can be explained in another fashion. Recall that the expected growth rate of retail market potential in our equivalent risk neutral evaluation approach is (r_f-δ). When the risk-free rate of return (r_f) decreases the expected growth rate of retail market potential falls, and hence, the value of the retailer's investment option as well as the investment value to exercise this investment option decreases. Therefore, the optimal threshold of retailer's investment value ($V^*(t)$) decreases as the value of (r_f) decreases.

Note that the *changes in the value of (r_f) do not impact the order, in which retailers enter the competitive market. The small retailer invests earlier in the competitive market and delays the big retailer's entry*.
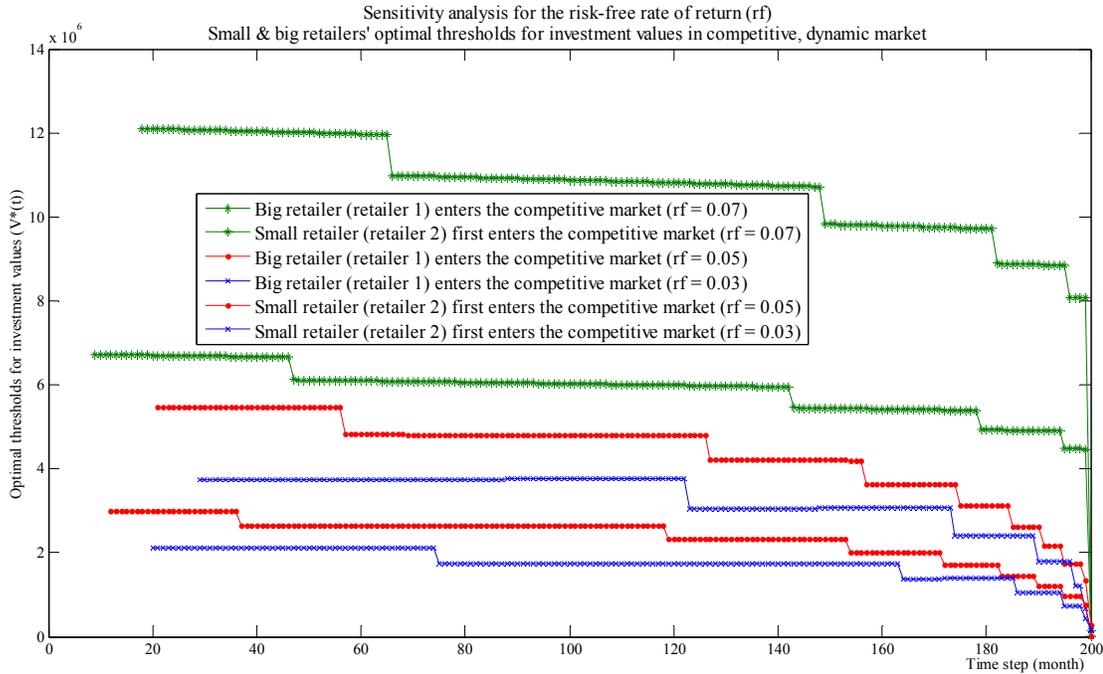
Figure 4.7. Sensitivity analysis on the value of (r_f) and its impact on retailers' optimal thresholds for investment values ($V^*(t)$) (the equivalent risk neutral evaluation approach).

It is noted that the retailer requires the higher value of investment in order to exercise its investment option when the value of (r_f) increases. However, the impact of changes of the value of (r_f) on the value of (X) that triggers the retailer to exercise its investment option is not trivial since it comes from a much more complicated nonlinear process. The retailer's investment value at the exercise time ($V^*(t)$) is calculated based on the procedure described in section 4.4. A change in the value of (r_f) changes the value of two sets of parameters in the equivalent risk neutral evaluation approach: the risk neutral probabilities (q_1 and q_3) as described in Equation (4.6) and the interest rate to discount the future store's cash flow.

An increase in the value of (r_f) on one hand, increases the up risk neutral probability (q_1) and decreases the down risk neutral probability (q_3). These changes in the values of risk neutral probabilities that are included in the expected value calculation, increase the

retailer's investment value. An increase in the value of (r_f) on the other hand, increases the value of interest rate that is used to discount back the store's future cash flow. This change decreases the retailer's investment value based on the procedure described in section 4.4. The overall change in the retailer's investment value is the overall effect of these two conflicting changes.

Since the retailer's investment value at the optimal exercise setting increases as the value of (r_f) increases, we know that the overall change in the retailer's investment value is increasing. However, due to the sophisticated interaction between the above two effects, it is not easy to determine how the value of retail market potential should change to increase the retailer's optimal investment value when the value of (r_f) increases. Note that the store's cash flows along the lattice nodes and the store's final values at the nodes in the last time step of the lattice are the quadratic functions of variable (X). This nonlinear relationship makes our calculation even more complicated. Therefore, we conduct several numerical examples to explore the direction of changes in (X) as a result of changes in (r_f). The results are summarized in Figure 4.8. It can be seen that the retailer's investment threshold (denoted by ($X^*(t)$)) decreases as the value of (r_f) increases. Therefore, a decrease in the optimal threshold value ($X^*(t)$) results in an increase in the optimal retailer's investment value ($V^*(t)$) when (r_f) increases.
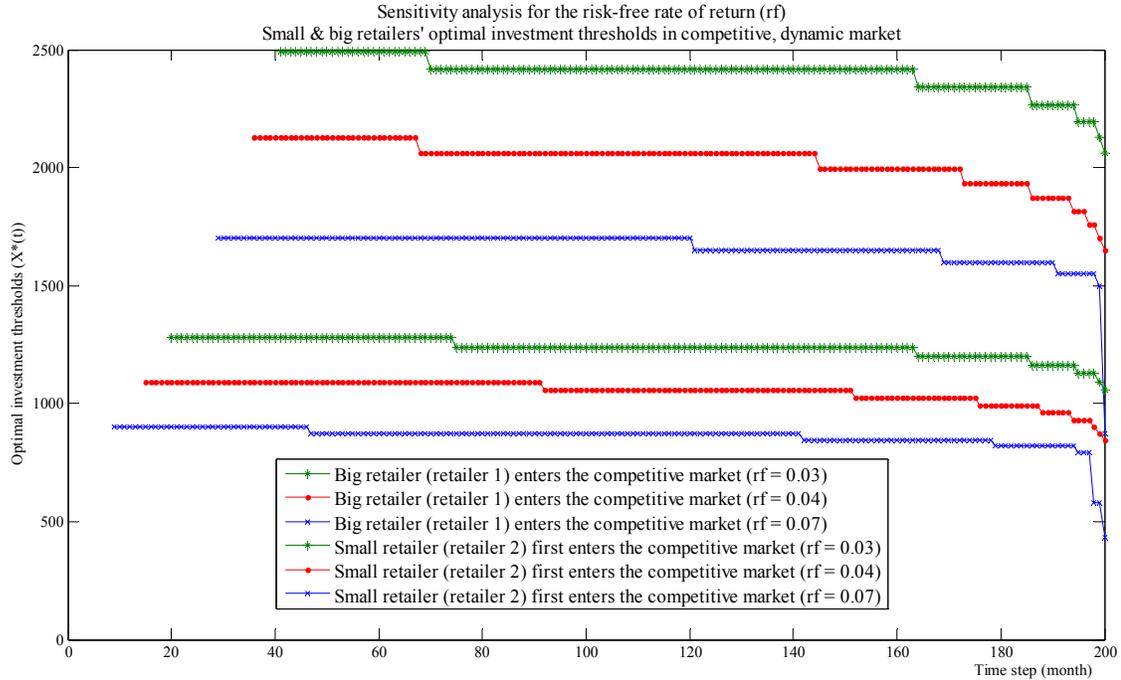
Figure 4.8. Sensitivity analysis on the value of (r_f) and its impact on retailers' optimal investment thresholds ($X^*(t)$) (the equivalent risk neutral evaluation approach).

Note that the results of our sensitivity analysis on the value of (r_f) are consistent with the characteristics of the optimal investment rule for an investment opportunity on a simple project that has been described by Dixit and Pindyck [15] (Ch. 5.4). In addition, Dixit and Pindyck [15] discuss the impact of changes in the value of (r_f) on the overall retail firm's market value. Consider that the value of each retailer's investment opportunity increases as the value of (r_f) increases. But for that very reason, the amount of actual investment by the retail firm decreases and the retailer only exercises fewer of these options. However, *when the value of (r_f) increases, the market value of the retail firm can go up even though the retail firm does less investment and perhaps opens fewer stores*.

4.6.3 Sensitivity analysis on the value of opportunity cost (parameter δ)

In this section, we study how changes in the value of opportunity cost (i.e., parameter (δ) in the equivalent risk neutral evaluation approach) impact retailers' optimal thresholds for investment values (i.e., $(V^*(t))$) and their optimal investment thresholds (i.e., $(X^*(t))$). We use the same numerical example of section 4.5.4 regarding the competition between the small and big retailers to conduct sensitivity analysis in this section. We consider three levels of (δ) for sensitivity analysis: (δ = [0.03, 0.05, and 0.10]). Also assume that the values of the other model parameters including (σ and r_f) remain constant at their levels in section 4.5.4.

As the value of opportunity cost (parameter (δ) or rate of return shortfall) increases the expected growth rate of retail market potential decreases (recall that the expected growth rate of retail market potential in our equivalent risk neutral evaluation approach is $(r_f-δ)$). Therefore, holding everything constant except the value of (δ), the expected growth rate of retail market potential falls when the value of (δ) increases (this is why we call parameter (δ) the rate of return shortfall). Then, the value of the retailer's investment option decreases as the value of (δ) increases since it becomes costlier for the retailer to wait rather than exercise its investment option. As a result the retailer's investment value (i.e., the store's NPV) that triggers the retailer to exercise its investment option decreases as the value (δ) increases.

In section 4.3.3, we indicated that parameter (δ) can be interpreted as the rate of return shortfall due to the possible entry and capacity expansion of competitors in the retail market. Therefore, as the value of (δ) increases, the possible threat from the other competing retailers in the market increases. This forces the retailer to invest earlier in the

market and consequently, the retailer's investment value to trigger the option exercise decreases. Hence, an increase in the value of (δ) shifts down the optimal value of the retailer's investment. Figure 4.9 shows how the optimal thresholds of retailers' investment values (denoted by ($V^*(t)$)) decrease as the value of (δ) increases from (0.03) to (0.10). Note that *changes in the value of (δ) do not impact the order, in which retailers enter the competitive market. The small retailer invests earlier in the competitive market and delays the big retailer's entry.*
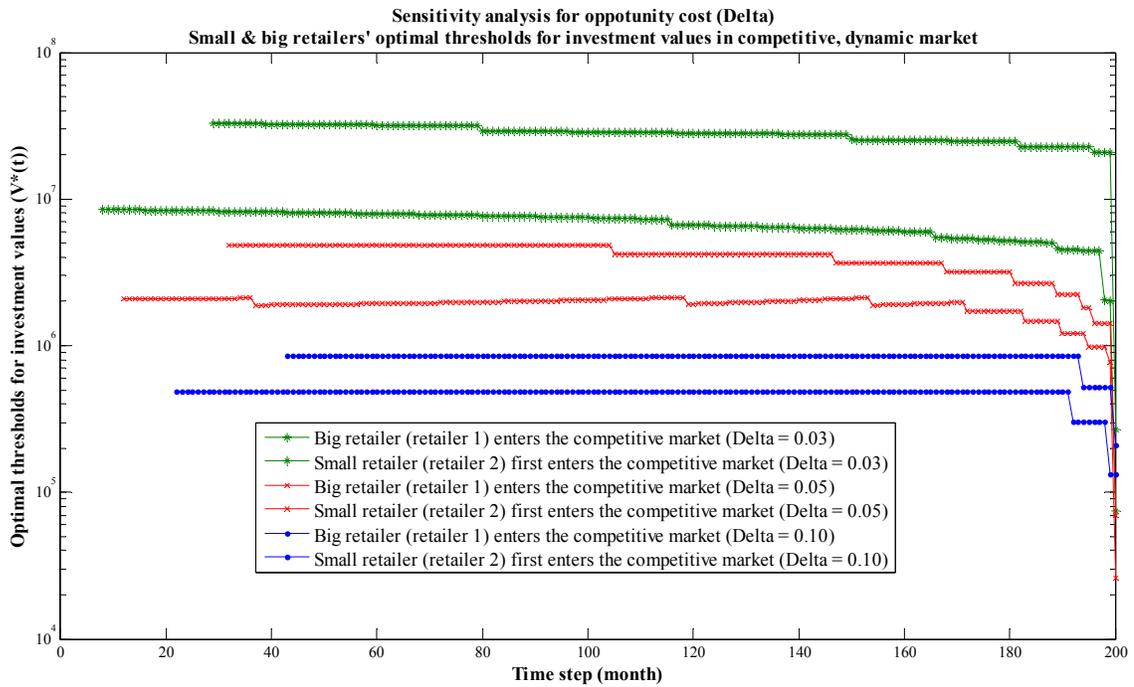

Figure 4.9. Sensitivity analysis on the value of (δ) and its impact on retailers' optimal thresholds for investment values ($V^*(t)$) (the equivalent risk neutral evaluation approach).

Now we study how changes in the value of (δ) impact the retailer's optimal investment threshold. Exploring the direction of changes is not as complicated as our discussions in the previous two sections on sensitivity analyses on the values of parameters (σ and r_f) since changes in the value of parameter (δ) only impact the values of risk neutral probabilities in the equivalent risk neutral evaluation approach.

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It can be seen in Equation (4.6) that an increase in the value of (δ) decreases the up risk neutral probability (q_1) and increases the down risk neutral probability (q_3). These changes in the values of risk neutral probabilities decrease the retailer's investment value. Therefore, the retailer needs the higher value for retail market potential (variable (X)) to compensate for the low expectation as a result of increase in the value of (δ). This result is shown in Figure 4.10, in which the retailer's optimal investment threshold (denoted by ($X^*(t)$)) increases as the value of (δ) increases from (0.02) to (0.10). Note that an increase in the optimal threshold value ($X^*(t)$) results in a decrease in the optimal retailer's investment value ($V^*(t)$) when (δ) increases.
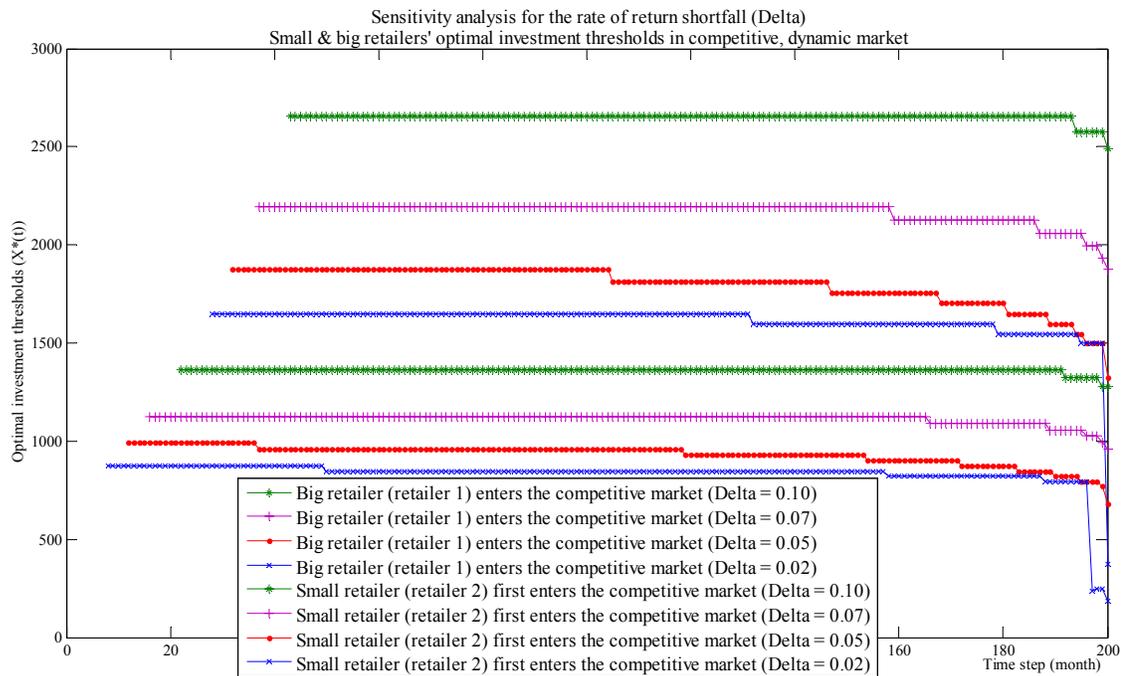
Figure 4.10. Sensitivity analysis on the value of (δ) and its impact on retailers' optimal investment thresholds ($X^*(t)$) (the equivalent risk neutral evaluation approach).

Note that the results of our sensitivity analysis on the value of (δ) are consistent with the characteristics of the optimal investment rule for an investment opportunity on a simple project that has been described by Dixit and Pindyck [15] (Ch. 5.4).

4.6.4 Sensitivity analysis on the values of log-volatility and opportunity cost

(parameters σ and δ)

In this section, we consider variations in the values of two basic parameters in the equivalent risk neutral evaluation approach (parameters (σ) and (δ)) and study how the retailer's optimal investment values that trigger the option exercise, change as the values of these two basic parameters change. Note that we still assume that these two parameters are independent from each other. However, we know from the CAPM, which is discussed in Equation (4.2), that (δ) should be adjusted according to variations in (σ) as Equation (4.9) describes the relationship between these two parameters.

$$\delta = \mu - \alpha = r_f + \varphi \rho_{ym} \sigma - \alpha \tag{4.9}$$

In this thesis, we conduct two-factor sensitivity analysis based on the assumption that parameters (σ) and (δ) are independent from each other and their values are determined from different sources. Again consider the same numerical example of section 4.5.4 regarding the competition between the small and big retailers for conducting sensitivity analysis in this section. We consider three levels of (σ) and three levels of (δ) for sensitivity analysis: (σ = [0.10, 0.15, and 0.2] and δ = [0.02, 0.05, and 0.07]). Also assume that the values of the other model parameters including (r_f) remain constant at their levels in section 4.5.4. We conduct sensitivity analyses on the retailer's optimal investment value that triggers the option exercise. Recall that retailers' investment options in this competitive market are valid for two hundred months. This retailer's

optimal investment value is the critical value of the retailer's investment at time (0) that triggers the retailer to exercise its investment option.

Figure 4.11 and Figure 4.12 show how retailer 1's and 2's optimal investment values at the option exercise change according to changes in the values of parameters (σ) and (δ). It can be seen that the retailer's decision to exercise its investment option is sensitive to variations in parameters (σ) and (δ). The retailer's optimal investment value rises sharply when the retail market uncertainty increases (i.e., the value of (σ) increases). In addition, when the opportunity cost (i.e., rate of return shortfall) decreases the retailer has less motivation to exercise its investment option and requires the higher value for its investment to exercise its option.
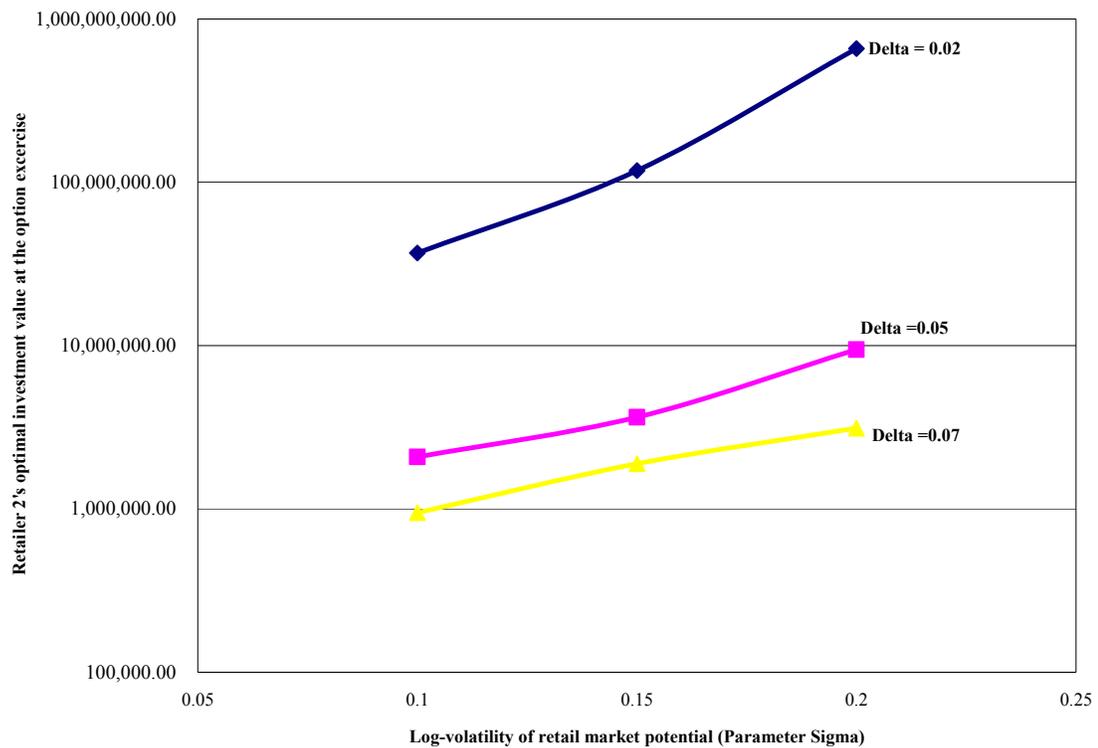
Figure 4.11. Sensitivity analysis on values of (σ and δ) and its impact on small retailer's (retailer 2's) optimal investment threshold (the equivalent risk neutral evaluation approach).
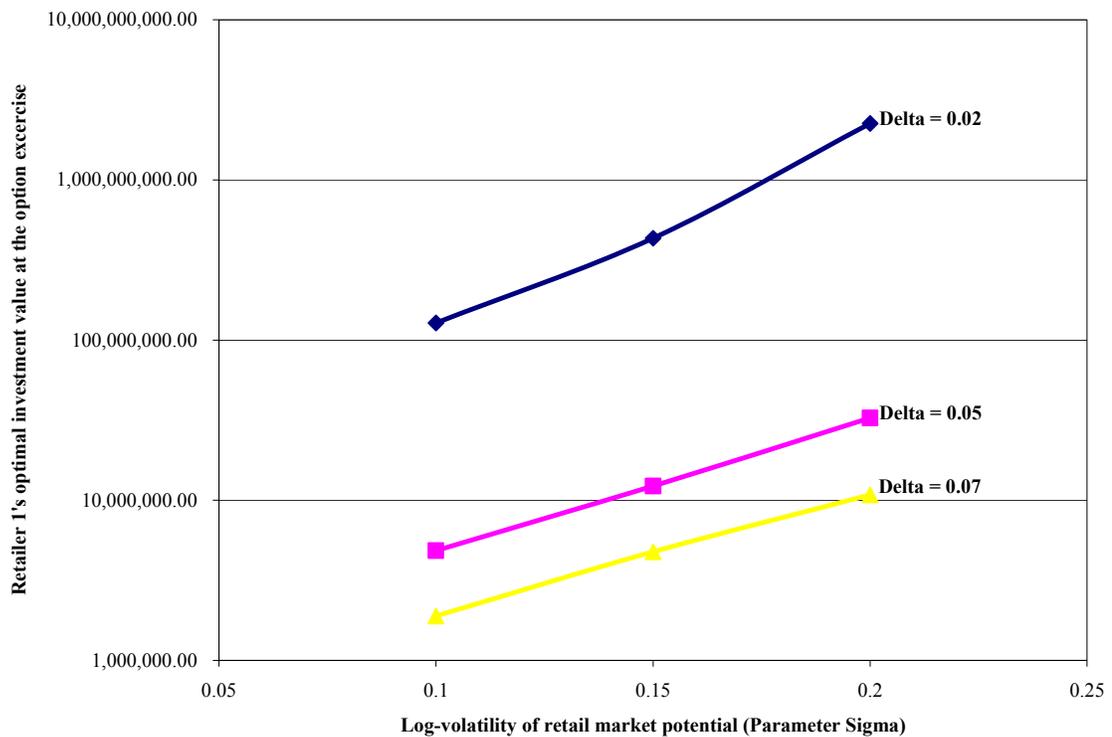
Figure 4.12. Sensitivity analysis on values of (σ and δ) and its impact on big retailer (retailer 1's) optimal investment threshold (the equivalent risk neutral evaluation approach).

4.6.5 *Sensitivity analysis on the values of risk-free rate of return and opportunity cost (parameters (r_f) and ($δ$))*

In this section, we consider variations in the values of two basic parameters in the equivalent risk neutral evaluation approach (parameters (r_f) and ($δ$)) and study how the retailer's optimal investment values that trigger the option exercise, change as the values of these two basic parameters change.

In this thesis, we conduct two-factor sensitivity analysis based on the assumption that parameters (r_f) and ($δ$) are independent from each other and their values are determined from different sources. Again consider the same numerical example of section 4.5.4 regarding the competition between the small and big retailers for conducting sensitivity

analysis in this section. We consider three levels of (r_f) and three levels of (δ) for sensitivity analysis: (r_f = [0.03, 0.05, and 0.07] and δ = [0.02, 0.05, and 0.07]). Also assume that the values of the other model parameters including (σ) remain constant at their levels in section 4.5.4. We conduct sensitivity analyses on the retailer's optimal investment value that triggers the option exercise. Recall that retailers' investment options in this competitive market are valid for two hundred months. This retailer's optimal investment value is the critical value of the retailer's investment at time (0) that triggers the retailer to exercise its investment option.

Figure 4.13 and Figure 4.14 show how retailer 1's and 2's optimal investment value at the option exercise change according to changes in the values of parameters (r_f) and (δ). It can be seen that the retailer's decision to exercise its investment option is sensitive to variations in parameters (r_f) and (δ). When the opportunity cost (i.e., rate of return shortfall) decreases the retailer has less motivation to exercise its investment option and requires the higher value for its investment to exercise its option. In addition, the retailer's optimal investment value decreases when the risk-free rate of return decreases since a lower discount rate discourages investment. This issue is indicated by Dixit and Pindyck [15] as a pure manifestation of the option idea that *a low interest rate makes the future relatively more important and increases the opportunity cost of the option exercise*.
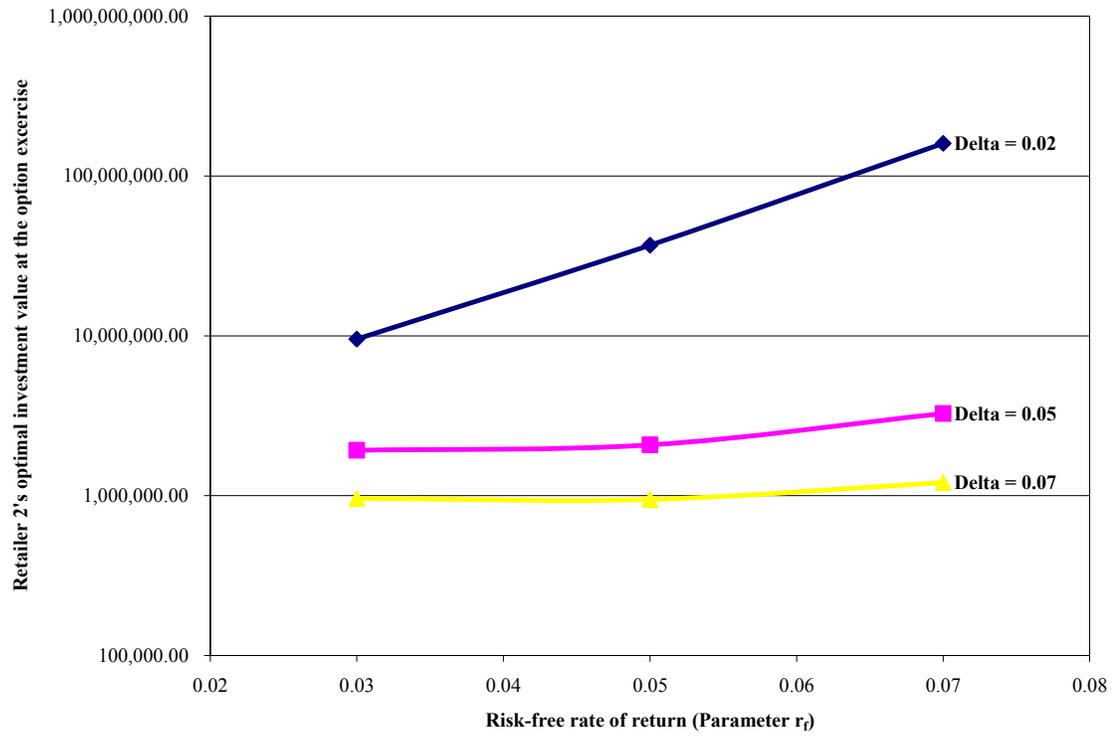
Figure 4.13. Sensitivity analysis on values of (r_f and δ) and its impact on small retailer's (retailer 2's) optimal investment threshold (the equivalent risk neutral evaluation approach).
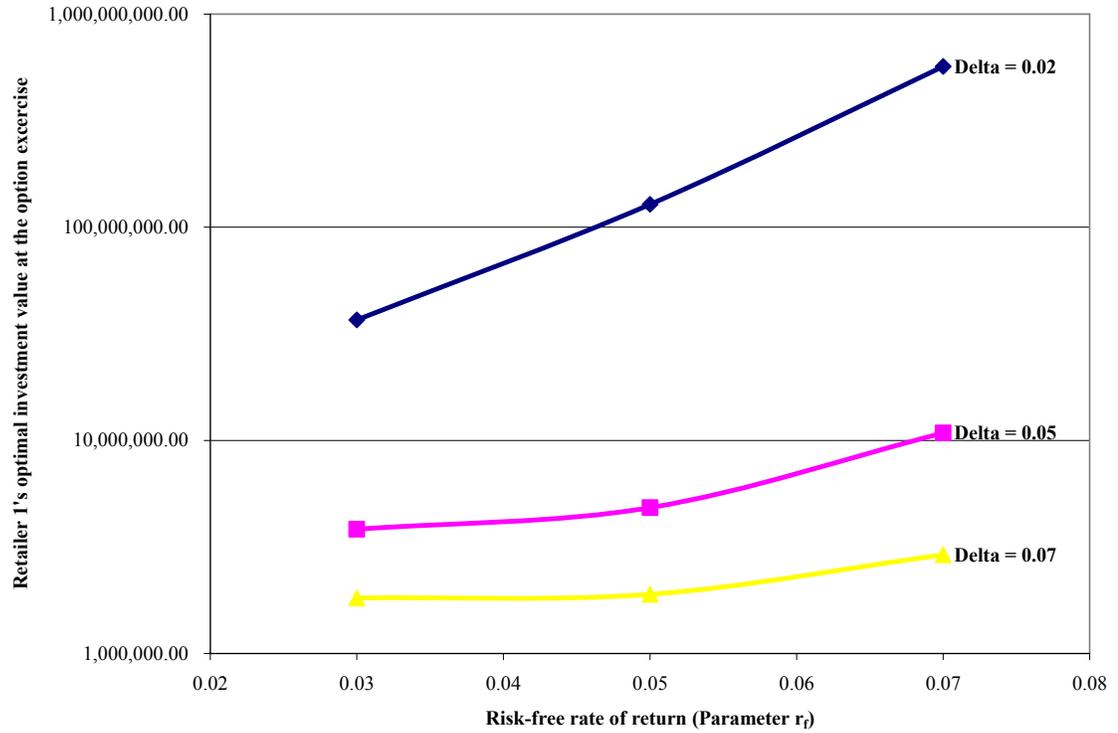
Figure 4.14. Sensitivity analysis on values of (r_f and δ) and its impact on big retailer's (retailer 1's) optimal investment threshold (the equivalent risk neutral evaluation approach).

We summarized how the retailer's investment decisions in the competitive, dynamic market are sensitive to the changes in the values of the parameters in the equivalent risk neutral evaluation approach. In the next section, we compare this approach to the dynamic programming approach, which is discussed in Chapter 3.

4.7 Relationship between the dynamic programming and the equivalent risk neutral evaluation approach

In this section, we discuss the similarities and the differences between the two investment analysis approaches that have been used in this thesis; the dynamic programming approach discussed in Chapter 3 and the equivalent risk neutral evaluation approach discussed in this chapter.

Consider the investment evaluation procedures that were described in this chapter and Chapter 3. It can be noticed that the dynamic programming and the equivalent risk neutral evaluation approaches are very similar in the assessment procedure. We can use the same numerical approximation methods (the trinomial lattice model in this thesis) to systematically treat dynamic uncertainty of retail markets, i.e., we use the trinomial lattice model to approximate dynamic uncertainty of retail market potential in a discrete fashion. The only difference between these two evaluation approaches with respect to this approximate trinomial lattice is related to the values of parameters that are used to construct the lattice. The dynamic programming approach uses the actual expected growth rate of retail market potential (i.e., parameter (α)) while the equivalent risk neutral evaluation approach uses the market-adjusted growth rate (i.e., ($r_f - \delta$)). However, both approaches use the same value for the log-volatility of retail market potential, which in turn, generates the same discrete values for the underlying state variable in the lattice formulation.

These two evaluation approaches are almost identical in the procedures that are used to determine retailers' optimal investment thresholds. These procedures are based on the same principle of dynamic optimization that is summarized in the Bellman equation of optimality [88], i.e., the retailer exercises its investment option when the value of its investment exceeds the value of keeping the investment option alive. In this thesis, we incorporate the optimality condition with the trinomial lattice approximation to develop appropriate decision trees for the retailer's investment evaluation. Note that both dynamic programming and equivalent risk neutral evaluation approaches use identical investment rules as part of their integrated decision tree analysis approaches.

Apart from the significant similarity between dynamic programming and equivalent risk neutral evaluation approaches, these two evaluation approaches are different in some aspects. The equivalent risk neutral evaluation approach is an investment analysis approach that evaluates the retailer's investment option in a way to maximize the market-value of the retail firm. Therefore this approach, which is an adjusted approach based on contingent claims analysis, represents an economically correct approach to evaluate the retailer's investment opportunities in theory. Contingent claims analysis provides a market-oriented approach for investment analysis and therefore, the market-value of an investment option will be determined using the price of its replicating portfolio in a complete market with no arbitrage opportunity. This approach is independent of the choice of the discount rate that may be problematic due to underestimation or overestimation of the true investment's risk. Therefore, using contingent claims analysis in investment decision-making provides a single platform to discuss the riskiness of an investment option since the true value of an investment opportunity is determined in the outside retail firm in the complete market with no arbitrage opportunity.

Thus, in theory, the retailer should use the contingent claims analysis approach to evaluate an investment opportunity in a retail market since this approach correctly determines the appropriate value of a risky investment risk in the complete market at no arbitrage. However in practice, this approach is very difficult to implement since it is based on a very demanding assumption.

It was noted earlier in this chapter that the contingent claims analysis approach requires the existence of sufficiently large and rich menu of risky assets in the capital market in order to replicate the stochastic behavior of the underlying state variable (the

value of retail market potential or variable (X) in this thesis). This necessary requirement states that the stochastic component (dz) in Equation (4.1) must be exactly replicated by the stochastic competent of the return on some traded assets (or a dynamic portfolio of traded assets). This assumption can be quite demanding since it requires not only these two stochastic components obey the same probability law but also that each and every path (or realization) of one process is perfectly correlated and replicated by the other process [15]. Even if the construction of this replicating portfolio was possible it requires a substantial amount of work to study a large number of traded assets and their risk and return characteristics to develop the replicating portfolio and adjusts it over time as the stochastic behavior of the underlying state variable (X) changes.

On the other hand, the equivalent risk neutral evaluation approach reduces the problem of finding a replicating portfolio of traded assets for the underlying state variable into the problem of estimating the rate of return shortfall (parameter (δ)) in the retailer's investment evaluation. It was indicated earlier in this chapter that this rate of return shortfall or the opportunity cost (parameter (δ)) represents the shortfall in the value of retail market potential due to the entry threat and competition effect by the other competing retailers in the market. Although the significance of considering this return shortfall in the investment valuation is documented in several places [82-85] there is not a practical procedure to estimate the correct value of parameter (δ) in the competitive, dynamic retail market.

In general, the estimation of the rate of return shortfall is difficult in any investment opportunity that is not directly traded in the capital market. In theory, parameter (δ) represents the difference between the expected rate of rerun of the replicating portfolio of

traded assets (i.e., parameter (μ) in Equation (4.1)) and the expected growth rate of the underlying asset of the investment option (i.e., parameter (α) in Equation (3.14)). Therefore, for that very reason that the construction of a replicating portfolio is difficult in the contingent claims analysis approach the estimation of the expected rate of return of this replicating portfolio (i.e., parameter (μ) in Equation (4.1)) is difficult in the equivalent risk neutral evaluation approach and consequently, the estimation of rate of return shortfall (parameter (δ)) is problematic.

Parameter (δ) can only be accurately estimated for the freely traded assets such as common stocks in the financial market since it is equal to the dividend rate on a share of a common stock. This parameter can be accurately estimated for the investment options on commodities that are traded in the futures markets. In this case, parameter (δ) represents the convenience yield that accounts for the benefit or premium associated with holding a physical asset, rather than the contract or derivative asset. However for the retailer's investment opportunity, the estimation of parameter (δ) is difficult and sometimes impossible due to the lack of data on the entry and capacity decisions of the other competing retailers in the market. In addition, the value of this parameter changes over the time as the retail market condition changes.

Since the correct estimation of the value of parameter (δ) is difficult in the retail market one can conclude that the problem of constructing a replicating portfolio for the underlying state variable (X) in the contingent claims analysis approach is reduced to the problem of finding the correct estimate for the values of parameters (μ) and (δ) in the equivalent risk neutral evaluation approach. Later in this chapter, we conduct a parametric study to explore how important the exact estimation of parameter (δ) is when

we compare these two investment analysis approaches for the evaluation of the retailer's investment option under different dynamic market conditions.

Although the difficulty in the estimation of parameter (δ) is a disadvantage in using the equivalent risk neutral evaluation approach, this approach is convenient for discounting purpose since it uses the risk-free rate of return as the discount rate to evaluate the retailer's investment option. We discussed this issue in section 4.3.4 by introducing the equivalent stochastic process that transfers the actual investment opportunity in the real world to its equivalent investment option in the risk-free world. The risk-free rate of return can be used as the discount rate for this equivalent investment option since the investment analysis calculation is carried out in the risk-free world.

Using the risk-free rate of return as the discount rate in the equivalent risk neutral evaluation approach can be considered as an advantage for this analysis method since the value of the risk-free rate of return is exogenous to the retailer's investment option and independent from the retail firm's capital structure. The appropriate value for the risk-free rate of return is derived from the much wider economy forces and can be determined from the capital market at any time step, for instance, from the rate of return on U.S. Treasury bills. Therefore, the equivalent risk neutral evaluation uses a market-oriented approach, which is independent of a particular retail firm's capital structure, for the evaluation of the retailer's investment option.

On the other hand, the dynamic programming approach does not require the demanding assumption of constructing the replicating portfolio for the retailer's investment option as it is asked in the equivalent risk neutral evaluation approach. The retailer's investment evaluation by the dynamic programming approach is not based on

the assumption that the risk in the retailer's investment option can be traded in the complete market in order to derive the market-oriented value of the retailer's investment option. Even if the retailer's investment option cannot be traded in the capital market the dynamic programming approach is still useful since it uses the retailer's subjective valuation of risk, which is summarized in the form of appropriate discount rate in investment analysis. The dynamic programming approach can be used in any circumstances to evaluate the retail firm's investment opportunity since it only requires estimation for the discount rate.

In the dynamic programming approach, an exogenous discount rate is assumed to adjust for the retailer's cost of capital that is used to discount the future store's cash flow. While the risk-free rate of return is the same for all retailers in the market the cost of capital is specific to each retail firm and reflects the retail firm's capital structure. Therefore, the equivalent risk neutral evaluation approach offers a more convenient and unified treatment for the discount rate than the dynamic programming that uses the firm-specific cost of capital as the exogenous discount rate. In the next section, we discuss how the value of the retail firm's cost of capital is different from the risk-free rate of return and how it can be determined for a specific firm.

4.7.1 The retail firm's cost of capital

The retail firm can raise capital in a variety of fashions. In general, we can divide these sources of capital in two broad categories: debt and equity. The retail firm should pay an appropriate price to capital owners that risk their money and invest in the firm. The retail firm usually pays different rates to raise these two sources of capital-- equity usually exhibits the higher price. The retail firm's capital structure depends on the relative

importance of debt and equity in the overall firm's capital and its cost of capital is a weighted sum of the cost of equity and the cost of debt.

Cost of debt is the return that creditors demand when they lend to the retail firm. It is computed by taking the rate on a non-defaulting bond whose duration matches the term structure of corporate debt, then adding a default premium that rises as the amount of debt increases. Since the firm's debt is a deductible expense, cost of debt (denoted by (k_d)) is the after-tax return on the firm's bond, which is determined by Equation (4.10), as follows.

$$k_d = (1-t)i \tag{4.10}$$

where (i) represents the interest rate in the market, in which the retail firm seeks a loan and (t) is the corporate tax rate. The values of these two parameters are exogenous to the retail firm.

Cost of equity (denoted by (k_e)) is a little more difficult to determine as equity does not pay a fixed return to its investors. There are two general approaches to calculate the retail firm's cost of equity. The first approach is the Gordon dividend growth model [89] (also known as dividend valuation approach) that is a simple shareholder valuation model as summarized in Equation (4.11).

$$k_e = \frac{D_0}{P_0} + g \tag{4.11}$$

Where (D_0) is the current annual dividend per share of stock that is paid to the regular retail firm's shareholder, (P_0) is the current price of a retail firm's share of common stock, and (g) is the annual growth rate of dividends of the retail firm's common

stock. The Gordon dividend model assumes that the price investors are willing to pay for a share of the retail firm's common stock as a function of the dividends that the retail firm will pay in the future and some rate of growth in the stock price itself (i.e., capital appreciation). Therefore, the retail firm can use this model to infer the discount rate applied by the financial market to investment in the retail firm as the retail firm's cost of equity (k_e).

The second approach to determine cost of equity is the Capital Asset Pricing Model (CAPM) that was summarized earlier in Equation (4.2) in this chapter. The CAPM is a standard model in financial economics to determine the theoretically appropriate rate of return of an asset that investors require in order to hold it in their portfolio. The CAPM determines the required rate of return of an asset considering its systematic or nondiversifiable risk. The asset under consideration in our context is the common stock of the retail firm. Here, we outline a slightly different version of the CAPM formula that is appropriate to determine the retail firm's cost of equity, as summarized in Equation (4.12).

$$k_e = r_f + \beta(E(r_m) - r_f) = r_f + \beta(RP)$$

(4.12)

where (r_f) is the risk-free rate of return, (β) is the retail firm's beta, $E(r_m)$ is the expected return of the entire market, and (RP) is the entire market risk premium.

The risk-free rate of return (r_f) is the interest rate that it is assumed to be obtained from investment instruments in the financial market such as U.S. Treasury bills with no default risk.

Parameter (β) represents the retail firm's beta, which is a statistical measure of the retail firm's market-related risk and shows how the return on the firm's common stock tends to co-vary with the entire market return. Equation (4.13) shows the formulation for parameter (β).

$$\beta = \frac{Cov(r_a, r_m)}{Var(r_m)}$$

(4.13)

where, (r_a) is the rate of return on the retail firm's common stock and (r_m) is the rate of return on the entire market, which usually consists of a large number of stocks in a well-diversified portfolio.

$E(r_m)$ is the expected return of the entire market that is influenced by the aggregate risk aversion of investors and the volatility of the entire market return. As the uncertainty over the market return increases (or the volatility of the market return increases) the value of $E(r_m)$ increases.

(RP) represents the entire market risk premium that represents the difference between the expected rate of return on the entire market that is required by investors $(E(r_m))$ and the risk-free rate of return (r_m). Different researchers indicate different values to estimate the entire market risk premium. These estimates range from (4%) to (8%) [90].

The retail firm's cost of debt (k_d) and the cost of equity (k_e) are combined to determine the overall retail firm's cost of capital (denoted by parameter (ρ) in this thesis) that is used as the interest rate for discounting future cash flows of firm's investments. The relative importance of debt and equity in the retail firm's capital structure defines the firm's cost of capital. The relative significance of debt and equity in the firm's capital

structure is summarized by an important financial ratio called the debt to equity ratio (D/E). The (D/E) determines the relative portion of equity and debt used to finance a company's assets. This ratio is often taken from the retail firm's balance sheet or statement of financial position. Using this ratio, we can determine the weighted average cost of capital of the retail firm, as summarized in Equation (4.14) below.

$$\rho = \text{Weighted Average Cost Of Capital} = w_1 k_d + (1 - w_1) k_e \qquad (4.14)$$

where (w_1) is the debt to capital (D/(D+E)) of the retail firm. Parameter (ρ) or the weighted average cost of capital represents the opportunity cost of the resources that the retail firm employs to generate benefits or value. This parameter is the rate of return that a retail firm would receive if it invested in a different vehicle with similar risk. Therefore, the weighted average cost of capital is the required rate of return by the retail firm in its capital budgeting process and the basis for selection of appropriate markets for investment and store opening.

The weighted average cost of capital (parameter (ρ)) is calculated according to the financial data of the retail firm and is used as an exogenous discount rate in the dynamic programming approach to evaluate the retailer's investment option. In the next section, we apply both dynamic programming and equivalent risk neutral evaluation approaches on the same investment analysis problem and compare the results from each approach for the retailer's optimal investment thresholds.

4.8 How critical is the choice of the investment analysis approach?

In section 4.7, we discussed the major differences between the dynamic programming and equivalent risk neutral evaluation approaches in the assessment of the retailer' investment option in dynamic markets. It was indicated that the equivalent risk neutral evaluation approach offers the better treatment for pricing the investment risk and determining the market-value of the retailer's investment option as the investment risk is traded freely in the financial market. It is also noted that in theory, the equivalent risk neutral evaluation approach is the correct approach to determine the market-value of a retailer's investment opportunity to compensate the retail firm's shareholders for the systematic risk of the investment.

However, the accurate estimation of the values of parameters (μ) and (δ) that are fundamental parameters in the equivalent risk neutral evaluation approach is so difficult that makes it challenging to apply this approach in practical investment analysis problems. Instead, the retail firm can use the dynamic programming approach, which is a more workable approach, for the investment evaluation. The dynamic programming approach does not require the demanding assumption of the replicating portfolio of traded assets and evaluates an investment option based on an appropriate estimate for the discount rate, which shows the retailer's subjective belief regarding the riskiness of the investment option.

Although the dynamic programming approach is a practical investment analysis approach and can be used for any investment evaluation it does not provide a correct calculation for the retailer's optimal investment threshold. In this section, we want to examine how the retailer's optimal investment threshold is different when the retail firm uses the dynamic programming approach instead of the equivalent risk neutral evaluation

approach considering the lack of data to estimate the necessary parameters. We conduct two separate studies to study the relationship between the dynamic programming and the equivalent risk neutral evaluation approach.

In the first study, we show how the retailer's optimal investment threshold changes under these two investment analysis approaches with respect to the values of the expected growth rate and the log-volatility of retail market potential that are noted by parameters (α) and (σ), respectively. In the second study, we show how the retailer's optimal investment threshold changes under these two investment analysis approaches with respect to the values of the expected rate of return of the replicating portfolio and the discount rate that are noted by parameters (μ) and (ρ), respectively.

In the first study, we use the retail firm's cost of equity as the expected rate of return on the portfolio of traded assets that replicates the growth of retail market potential. We assume that the stochastic change of the growth of retail market potential is identical with the stochastic change of the return on the retail firm's traded common stocks. This assumption is based on the idea that the retail firm's financial performance that is shown in the value of the retail firm's common stock (i.e., the retail firm's equity) is perfectly correlated with the value of retail market potential that the firm opens a store. This is not a perfect assumption since the price of retail firm's stock is the result of the aggregated performance of its entire stores in a variety of markets. However, for many retail markets the expected growth of retail market potential is at least partially correlated with the expected return on the retail firm's common stock since the financial performance of an individual store somehow implies the overall performance of the retail firm. Another advantage of this assumption is that the retail firm's cost of equity can be easily

determined using the retail firm's publicly available financial data. Our search in the literature did not locate any work that applied real options for the retailer's investment evaluation. However, other researchers use the volatility of stock price for the valuation of an investment option in R&D [91] and the pharmaceutical industry [21]. Therefore, we use the expected rate of return of the retail firm's stock as a proxy for the expected rate of return of a retailer's investment option.

Therefore, in the first study we assume that the value of retail firm's cost of equity (k_e) is equal to the value of parameter (μ) in Equation (4.1). Recall that Equation (4.1) describes a GBM process that is used to model the stochastic variation of the replicating portfolio for a retail firm's investment option.

The objective of our study is to compare the retailer's optimal investment threshold in different market types using these two analysis approaches and determine when the difference between these two approaches becomes significant. We investigate the difference between these two analysis approaches at various levels of parameters (α) and (σ). We use a simple notional example for this purpose to explore how the retailer's optimal investment threshold under these two investment analysis approaches changes as conditions in the retail market change. But first, we need to determine the retail firm's cost structure in order to calculate its cost of equity and its cost of capital that will be used in our analysis.

We use a study by Cho and Sayers [92] to determine appropriate estimates for the cost of equity and the weighted average cost of capital in our notional example. Cho and Sayers [92] use financial data of a national retail firm, Wal-Mart, and apply the formulas described in section 4.7.1 to calculate this national firm's cost of equity and weighted

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average cost of capital. We use their estimates for these parameters that are calculated in 2004 for our notional example in this section. Note that one can easily calculate cost of equity and weighted average cost of capital for any retail firm using its publicly available financial data at any point in time by applying formulas in Equations (4.11) and (4.12), respectively.

Hence, in this example we assume that the retail firms' cost of equity is (9%) and their weighted average cost of capital is (8%), i.e., ($k_e = 9\%$ and WACC = 8%). Note that the value of cost of equity (k_e) equals the value of expected rate of return on the replicating portfolio in the equivalent risk neutral approach and hence, we have ($\mu = k_e = 9\%$). On the other hand, the value of discount rate in the dynamic programming approach equals the value of the weighted average cost of capital and hence, we have (ρ = WACC = 8%). In addition, we assume that the rate of risk-free rate of return that is used in the equivalent risk neutral evaluation approach can be determined easily by looking at the rate of return on the thirty-year U.S. Treasury bill. We use (6%) as an appropriate estimate for this risk-free rate of return, i.e., ($r_f = 6\%$). Consider that the value of parameters (μ, r_f, and ρ) can be exogenously determined using the retail firm's financial data regarding its capital structure as well as some data regarding the entire capital market. Therefore for the first study, we assume the aforementioned values remain constant over the course of our comparative study between these two investment analysis approaches.

Again consider the same numerical example of section 4.5.4 regarding the competition between the small and big retailers for conducting a comparative study in this section. We consider investment options of these small and big retailers in different

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market conditions. The expected growth rate of retail market potential (parameter (α) in Equation (4.1)) and the log-volatility of this expected growth rate (parameter (σ) in equitation (1)) are the basic parameters that describe conditions of the retail market in which these retailers hold investment options to open stores. Therefore, we consider variations in the values of these two basic parameters (parameters (α) and (σ)) and study how the retailer's optimal investment values that trigger the option exercise, change as the values of these two basic parameters change. Note that parameters (α) and (σ) are independent from each other.

Therefore in this section, we conduct two-factor sensitivity analysis on the values of these two basic parameters, separately for each investment analysis approach and then, compare the retailer's optimal investment thresholds from each investment analysis approach to explore the value of the difference in results from these two approaches corresponding to different combinations of parameters (α) and (σ). We consider four levels of (α) and three levels of (σ) for sensitivity analysis: (α = [0.02, 0.04, 0.06, and 0.08] and σ = [0.10, 0.20, and 0.30]). Note that the corresponding values for parameter (δ) that is used in the equivalent risk neutral evaluation approach are (δ = [0.07, 0.05, 0.03, and 0.01]) since ($\delta = \mu - \alpha = 0.09 - \alpha$).

We conduct sensitivity analyses on the retailer's optimal investment value that triggers the option exercise, separately for each investment analysis approach. Recall that retailers' investment options in this competitive market are valid for two hundred months. The retailer's optimal investment value that is used in our study is the critical value of the retailer's investment at time (0) that triggers the retailer to exercise its investment option.

Figure 4.15 and Figure 4.16 show how small retailer's (retailer 2's) and big retailer's (retailer 1's) optimal investment values at the option exercise change according to changes in the values of parameters (α) and (σ) and the choice of the investment analysis approach. Note that in these two figures, retailer's critical investment values are labeled by DP if they are calculated base on the dynamic programming approach and are labeled as ERNE if they are calculated based on the equivalent risk neutral evaluation approach. These figures confirm some important results that we already derived from our studies in this chapter and Chapter 3.

First, observe that retailers' critical investment thresholds increase as the value of parameters (σ) increase. This trend is consistent between the two investment analysis approaches. We discuss the reason for this increasing trend in section 3.10.1 for the dynamic programming approach and in this chapter, sections 4.6.1 and 4.6.4, for the equivalent risk neutral evaluation approach. Secondly, observe that retailers' critical invest thresholds decrease as the value of parameter (α) increases. This trend is also consistent between the two investment analysis approaches. Recall that the value of parameter (δ) decreases as the value of parameter (α) increases considering (δ = μ − α) and the constant value of parameter (μ). We discuss the reason of this decreasing trend in section 3.10.2 for the dynamic programming approach and in sections 4.6.2, 4.6.4, and 4.6.5 for the equivalent risk neutral evaluation approach.

Now consider the retailers' critical investment values in Figure 4.15 and Figure 4.16. These values are calculated in two fashions for each combination of parameters (α) and (σ) in these figures; once they are calculated based on the dynamic programming approach and then, they are calculated based on the equivalent risk neutral evaluation

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approach. Observe that retailers' critical investment values are not exactly the same in these two approaches. Notice that in general the difference between these two approaches reduces as the value of parameter (α) increases or the value of parameter (σ) decreases. This observation can be explained, as follows.

When the value of retail market potential is expected to grow at a substantially high rate the retailer's investment is less subject to investment risk due to the high expected potential benefits of opening a store in this market. Therefore, *in the highly growing markets the accurate estimation of the opportunity cost (parameter (δ)) is not significant in the retailer's decision to exercise its investment option. In this situation, one can confidently apply the dynamic programming approach and uses the retail firm's cost of capital as the exogenous discount rate in the evaluation of the retailer's investment option*. The same argument can be made when the value of retail market potential is expected to have low volatility since there is low risk involved in the investment in this market. Observe that retailers' critical investment values are almost identical when the retail market is expected to have high growth rate and low volatility.

The further exploration to determine a more accurate estimation for parameter (δ) is only worth it when the value of retail market potential is expected to grow at the low rate or have the high volatility. In other market situations, the dynamic programming approach provides satisfactorily accurate results for retailers' critical investment thresholds. However, one should consider that the dynamic programming approach tends to return the critical thresholds that are usually below the critical investment thresholds of the equivalent risk neutral evaluation approach. In other words, the dynamic programming approach underestimates the waiting value of an investment option and

hence, tends to determine the retailer's investment threshold below its correct optimal threshold as it is identified by the equivalent risk neutral evaluation approach. This underestimation is more significant in markets with low expected growth rate of retail market potential than markets with high expected growth rate of retail market potential. Therefore, using the dynamic programming approach for the retailer's investment evaluation returns non-reliable results in markets with low expected growth rate.
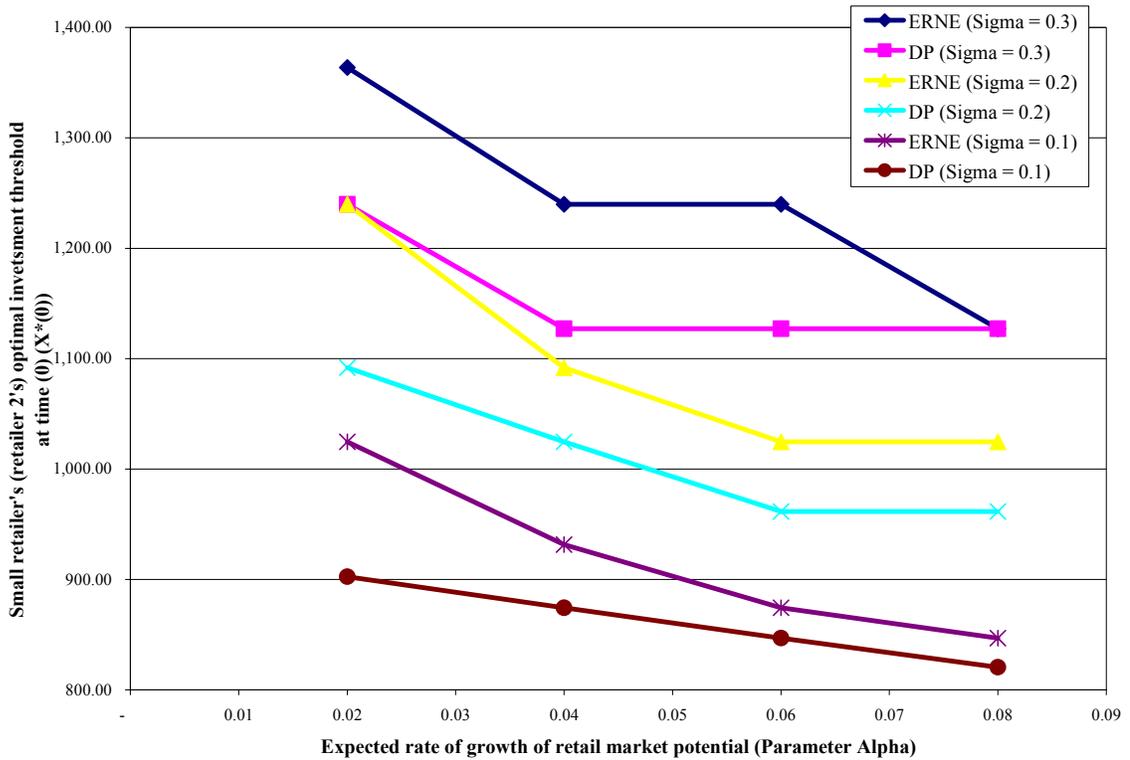
Figure 4.15. Comparison between the dynamic programming and the equivalent risk neutral evaluation approaches through small retailer's (retailer 2's) investment thresholds at time (0) ($X^*(0)$).
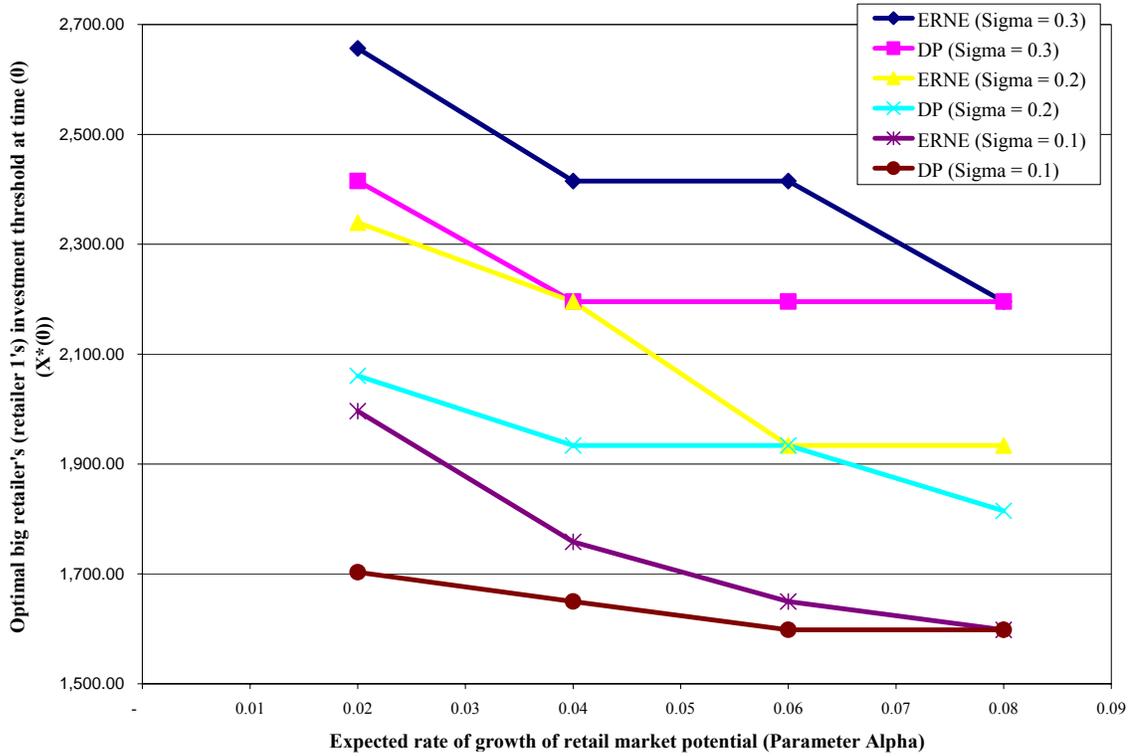
Figure 4.16. Comparison between the dynamic programming and the equivalent risk neutral evaluation approaches through big retailer's (retailer 1's) investment thresholds at time (0) ($X^*(0)$)

In the first study, we assume that the value of the expected rate of return of the replicating portfolio (parameter (μ)) depends on the retail firm's capital structure. Recall that the value of parameter (μ) is assumed to be equal to the value of the retail firm's cost of equity (k_e) that is a part of the retail firm's cost of capital (parameter (ρ)) as it is described in Equation (14.4). In the second study, we assume that these two parameters (parameters (μ) and (ρ)) change independently from each other.

Therefore in the second study, we investigate how the retailer's optimal investment threshold based on the dynamic programming is different from the retailer's optimal investment threshold based on the equivalent risk neutral evaluation approach at different combinations of parameters (μ) and (ρ). For this purpose, we use the same numerical example of small and big retailers that were described in section 4.5.4.

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We conduct two-factor sensitivity analysis on the values of parameters (μ) and (ρ), separately for each investment analysis approach and then, compare the retailer's optimal investment thresholds from each investment analysis approach to explore the value of difference in results from these two approaches corresponding to different combinations of parameters (μ) and (ρ). We consider three levels of (μ) and four levels of (ρ) for sensitivity analysis: (μ = [0.06, 0.08, and 0.10] and ρ = [0.08, 0.10, 0.12, and 0.15]). We also assume the following values for the other model parameters: (α = 0.04), (σ = 0.1), and (r_f = 0.06). Therefore, the respective values for parameter (δ) that is used in the equivalent risk neutral evaluation approach are (δ = [0.02, 0.04, and 0.06]) since ($\delta = \mu - \alpha = \mu - 0.04$).

We conduct sensitivity analyses on the retailer's optimal investment value that triggers the option exercise, separately for each investment analysis approach. Recall that retailers' investment options in this competitive market are valid for two hundred months. The retailer's optimal investment value that is used in our study is the critical value of the retailer's investment at time (0) that triggers the retailer to exercise its investment option.

Figure 4.17 and Figure 4.18 show how small retailer's (retailer 2's) and big retailer's (retailer 1's) optimal investment values at the option exercise change according to changes in the values of parameters (μ) and (ρ) and the choice of the investment analysis approach.

First, consider that these two figures confirm an important conclusion of our work in section 4.6.3, which indicates that the value of the retailer's optimal investment threshold ($X^*(0)$) increases as the value of parameter (μ) and in turn parameter (δ) increases. Also consider that the value of the retailer's optimal investment threshold ($X^*(0)$) increases as

the value of parameters (ρ) increases. This observation is also concluded in section 3.10.3. Note that the value of the retailer's optimal investment threshold does not change as the value of parameter (μ) changes since this change only incorporates into the equivalent risk neutral evaluation process and not in the dynamic programming approach.

The most important conclusion from our results in Figure 4.17 and Figure 4.18 is that the choice of the discount rate is significant in the true evaluation of an investment option in a retail market. The equivalent risk neutral evaluation approach shows the correct retailer's optimal investment threshold considering the true price of risk of this investment option in the complete market. It can be seen in Figure 4.17 and Figure 4.18 that the retailer's optimal investment threshold based on the dynamic programming approach deviates from the true market-oriented threshold at different levels of parameters (μ) and (ρ). This deviation can be summarized, as follows. *At the low value of the expected rate of return on the replicating portfolio (i.e., the low value of parameter (μ)) the low value of the discount rate provides a more accurate approximation for the true level of the retailer's optimal investment threshold. As the value of parameter (μ) increases the high value of the discount rate provides a more accurate approximation for the true level of the retailer's optimal investment threshold*.

Comparison between dynamic programming and equivalent risk neutral evaluation approaches The small retailer's (retailer 2's) optimal invetsment thresholds when the option exercises at time (0)

Legend:
- Dynamic Programming (Rhu = 8%)
- Dynamic Programming (Rhu = 10%)
- Dynamic Programming (Rhu = 12%)
- Dynamic Programming (Rhu = 15%)
- Equivalent Risk Neutral Evaluation

Y-axis: Small retailer's (retailer 2's) optimal investment threshold X2*(0)

X-axis: Expected rate of return on the replicating portfolio (Mu)

Figure 4.17. Comparison between the dynamic programming and the equivalent risk neutral evaluation approaches through small retailer's (retailer 2's) investment thresholds at time (0) (X2*(0)) with respect to the values of parameters (μ) and (ρ).

Comparison between dynamic programming and equivalent risk neutral evaluation approaches The big retailer's (retailer 1's) optimal invetsment thresholds when the option exercises at time (0)
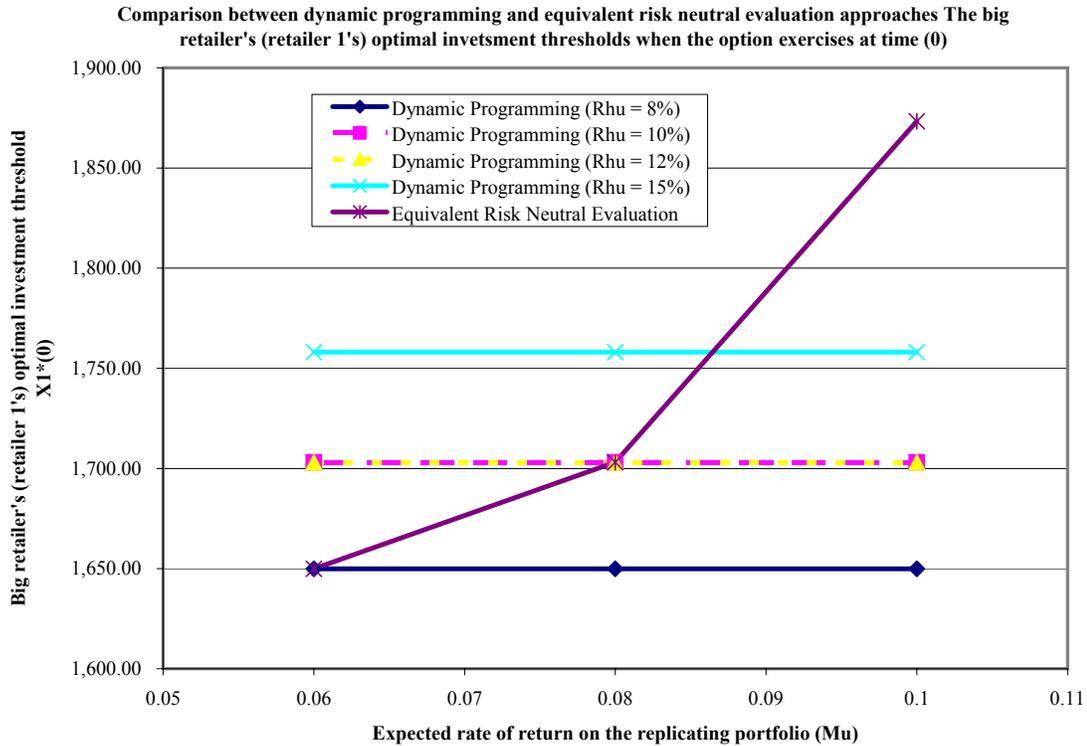
Figure 4.18. Comparison between the dynamic programming and the equivalent risk neutral evaluation approaches through big retailer's (retailer 1's) investment thresholds at time (0) (X1*(0)) with respect to the values of parameters (μ) and (ρ).

Therefore, the selection of the discount rate is absolutely critical in the success of the dynamic programming approach for the correct evaluation of the retailer's optimal investment threshold. Sensitivity analysis over the possible range of the values for this parameter is a must in the dynamic programming approach. Thus, an appropriate combination of dynamic programming and contingent claims analysis can handle the valuation of the retailer's investment opportunities in several market conditions.

4.9   Conclusions and future works

In this chapter, an integrated investment analysis approach based on the equivalent risk neutral evaluation approach is developed to explore retailers' investment behaviors

in competitive versus noncompetitive dynamic markets. This approach is based on a market-oriented perspective and determines the correct market-value of the retailer's investment opportunity in order to enhance the market-value of the retail firm.

It was shown that the retailer with the overall cost advantage enters the competitive market firsts and its optimal investment threshold in the competitive market is lower than its threshold in the same market without any competition effect, i.e., the retailer with the overall cost advantage enters the competitive market early enough to preempt the market from the entry of the other retailer. This first-mover advantage pushes up the other retailer's investment threshold and delays its entry to the market. Thus, the equivalent risk neutral evaluation approach shows the significance of the strategic aspect of early investment by the retailer with the overall cost advantage. This conclusion is also consistent with the other investment analysis approach based on dynamic programming as it was described in Chapter 3.

It is also concluded that the investment threshold of the retailer with the cost advantage in the competitive market is substantially lower than its threshold in the same market without competition particularly when the values of the cost parameters of two competing retailers are similar.

In addition, sensitivity analysis was conducted to study how retailers' optimal investment thresholds change as the values of parameters in this equivalent risk neutral evaluation approach change. It is concluded that the optimal store value that triggers the retailer to invest and open the store, increases when the value of (σ) or (r_f) increases. This optimal value decreases as the value of (δ) increases. However, the retailer's optimal investment threshold in terms of the state variable (X) increases as the value of (δ) or (σ)

increases. This optimal threshold decreases as the value of (r_f) increases. Note that changes in the value of any of these parameters do not impact the order, in which retailers enter the competitive market.

Comparative study between the equivalent risk neutral evaluation and the dynamic programming approaches show that the retailers' critical investment values are slightly different under each approach. However, it is concluded that one can confidently use the dynamic programming approach to evaluate the retailer's investment option in the dynamic market that is expected to grow at the high rate and has low volatility. Particular attention should be allocated to the retailer's investment option in the other market conditions specifically into the estimation of opportunity cost or the rate of return shortfall. An appropriate estimation procedure is required to be developed in order to determine the value of opportunity cost (parameter (δ)) for investments in competitive, dynamic retail markets.

Chapter 5

Empirical Evidence

5.1 Abstract

In this chapter, we empirically examine an important aspect of our theoretical work that was discussed in sections 3.9.4 and 4.5.4. It was noted that the big retailer invests and opens a store relatively later in markets with a small retailer compared to markets without a small retailer. In addition, the big retailer opens a store at relatively higher retail market potential in markets with a small retailer compared to markets without a small retailer. In this chapter, we discuss some empirical evidence to support these theoretical results. We choose Wal-Mart and Dollar General as the big and small retailers, respectively, in our empirical study. While our empirical results do not validate the theory, these results do, however, provide supporting evidence for our theoretical work.

5.2 Introduction

In this chapter, we conduct an empirical study to explore actual investment decisions of retailers in dynamic markets in terms of time to enter, and compare them with our theoretical results discussed in Chapter 3 and Chapter 4. We particularly focus on an important aspect of our theoretical work regarding the entry behavior of a big retailer into a market with a small retailer and compare it with the entry of the same big retailer into a market without the same small retailer. The small and big retailers in our empirical study

are considered to serve similar low-income communities and thus, they can provide a good case to examine the big retailer's investment behavior. In addition, we assume that the small and big retailers in our case study follow the same cost relationships that were previously considered in sections 3.9.4 and 4.5.4, i.e., the value of big retailer's fixed and investment costs are greater than the value of small retailer's fixed and investment costs, respectively, while the value of the big retailer's variable cost is less than the small retailer's variable cost. Consider that our theoretical results in sections 3.9.4 and 4.5.4 indicate that the big retailer invests relatively later in markets with a small retailer compared to markets without a small retailer when both retailers follow the stated cost relationships. In addition, the big retailer open a store at relatively higher market potential in markets in which a small retailer has a store opened compared to markets in which a small retailer does not have a store opened.

In this chapter, we empirically examine these theoretical results by looking at the investment behavior of a big discount retailer (Wal-Mart in our case study) in two types of retail markets. The first type of retail markets are markets in which a small discount retailer (Dollar General in our case study) has a store opened and the second type of retail markets are markets in which Dollar General does not have a store opened. In this chapter, we refer to the first type of retail markets as competitive retail markets and to the second type of retail markets as noncompetitive retail markets. Consider that the notion of competitiveness in this market classification is based on the Wal-Mart's perspective.

We empirically test the significance of the competition effect in the entry decision of Wal-Mart into these two market types. Therefore in this chapter, we study actual investment timings and market populations of Wal-Mart stores in competitive retail

markets and compare them with its investment timings and market populations in noncompetitive markets. *The objective is to determine whether any significant difference exists in Wal-Mart's investment timings and market populations in these two market types*. Our belief or expectation, which is based on our theoretical results in section 3.9.4 and 4.5.4, is that Wal-Mart enters competitive markets relatively later and at higher market potential than noncompetitive markets, i.e., in practice, Wal-Mart's preferred strategy is to invest relatively later and at the higher market potential in competitive markets than noncompetitive markets.

This chapter is structured, as follows. In section 5.3, we describe Wal-Mart and Dollar General as big and small retailers, respectively, in the context of our empirical study. We describe the process of data collection in section 5.4. Data analysis and empirical results are provided in section 5.5. Conclusions are presented at the end in section 5.6.

5.3   Wal-Mart and Dollar General

In this section, we present two discount retailers that we selected for the purpose of our empirical study. The first retailer is Wal-Mart, which is considered as the big retail firm in our case study. The second retailer is Dollar General, which is considered as the small retail firm in our study.

Wal-Mart Stores, Inc. (NYSE: WMT) is an American public corporation that runs a chain of large discount stores. This big retail firm is the world's largest public corporation by revenue according to the 2007 Fortune Global 500 [93]. Wal-Mart Stores Division U.S. is Wal-Mart's largest business subsidiary, accounting for 67.2% of net sales for

fiscal year 2006 [94]. Wal-Mart discount stores vary in size from 51,000 square feet to 224,000 square feet, with an average store covering about 102,000 square feet [94]. As of January 31, 2008, there were 971 Wal-Mart discount stores in the United States [95].

As for the small retail firm in our case study, we chose Dollar General, which is a chain of discount stores operating in approximately 8,205 stores (as of June 1, 2007) in 35 U.S. states [96].

We choose Wal-Mart and Dollar General for our case study since in geography and customer base, these two retail firms have much in common [97]. The average Wal-Mart customer's annual income is $35,000, which is below the national average [98, 99]. Dollar General also serves lower income communities whose average household annual income is below $35,000 [97]. Approximately 80 percent of Dollar General customers earn below $25,000 a year [100]. Therefore, both retail firms serve low income communities.

In addition, Wal-Mart faces serious competition from Dollar General since they both provide low price products to similar pools of customers [100, 101]. Dollar General stores are much smaller in size than Wal-Mart discount stores, which makes shopping at Dollar General stores more convenient than Wal-Mart discount stores, in part because they can be located in more convenient places than the much larger Wal-Mart stores. Although a big Wal-Mart discount store provides a magnet for one-stop, many-item shoppers hurried shoppers are attracted to small size Dollar General stores since they can save substantial time in shopping at small Dollar General stores, i.e., the vastness of Wal-Mart stores has its own disadvantages.

Dollar General stores are also conveniently located in the marketplace that provides easy access for many customers [102]. David A. Perdue, Dollar General's ex-chief executive, points to this issue when he says "We compete on price and convenience while Wal-Mart competes on price and assortment [101]." Wal-Mart is currently testing its own dollar stores to compete with Dollar General and other small discount dollar stores [101].

Moreover, most of Dollar General stores are conveniently located in small and medium towns of 25,000 population or less [97, 100]. Dollar General prefers to serve low income customers who must drive approximately twenty minutes to the nearest Wal-Mart [100]. Most of Wal-Mart stores, on the other hand, are strategically located between a few towns to serve a large number of customers in low income communities.

Hence, Wal-Mart and Dollar General were selected for our case study in this chapter since they have much in common in geography and customer base. Note that we do not consider the difference between Wal-Mart and Dollar General in terms of the convenience of shopping and easy access and its impacts on the customer's shopping behavior. These two retail firms consider similar growing retail markets for investment. However, Dollar General stores are smaller than Wal-Mart stores and therefore, they require relatively smaller market sizes for support, i.e., when we consider investment in a growing market, Dollar General can open a store relatively earlier than Wal-Mart since a Dollar General store requires a relatively smaller demand size or market potential to ensure its profitability. On the contrary, a Wal-Mart store requires a relatively high level of market potential to accommodate its substantial investment and operation costs.

This apparent tendency of early investment by Dollar General is consistent with our theoretical results in sections 3.9.4 and 4.5.4, which indicate that the small retailer opens

a store at the relatively lower level of retail market potential when it compares to the large retailer. In addition, we noted in sections 3.9.4 and 4.5.4 that the preferred strategy for a big retailer such as Wal-Mart is to open a store relatively later in markets with a small retailer such as Dollar General. In this chapter, we examine whether Wal-Mart in practice follows this strategy and enters competitive markets relatively later than noncompetitive markets. We describe the data that we use for our empirical study in the next section.

5.4   Data

In this section, we describe the data that we use in this case study. We chose Wal-Mart stores in the state of Georgia for our purpose. As of December 2006, there are 118 Wal-Mart stores opened in Georgia. The number of Wal-Mart stores in the state of Georgia is large enough to provide a meaningful sample for our case study. In addition, Wal-Mart opened its first store in Georgia in 1981 and then gradually expanded its operation across the state. Currently, Wal-Mart operates in many geographical locations across the state and therefore, our sample represents a variety of markets in this state.

We manually retrieved information regarding the Wal-Mart store number, store location (its physical address as well as its latitude/longitude information), and the year store opened from the Wal-Mart store locator [103]. We also manually retrieved the same data for Dollar General stores in the state of Georgia from the Dollar General store locator [104]. As of December 2006, there are 347 Dollar General stores serving several markets across the state of Georgia and therefore, provide a meaningful sample for our case study.

We use a standard approach based on the concept of market mile to define the retail market around a Wal-Mart or a Dollar General store. The retail market for a store is usually measured in terms of market mile around the store [105, 106]. This store's market mile specifies the distance of customers from the retail store, i.e., market mile determines the maximum distance that a customer must travel to reach the store.

The market mile definition to identify the store's market is also a good proxy for the traveling time that customers spend to reach the store. There is some evidence that a straight line is a good proxy for travel time. For instance, Phibbs and Luft [107] shows a correlation of 0.826 between straight line distance and travel times for distances below 15 miles.

Therefore in this case study, we use the concept of market mile around a retail store as a standard approach to define the retail market that is served by the store. We refer to a notional circle centered at the store's location with radius of store's market mile as the store's retail market. Note that this store's retail market is a notional circle on the surface of earth in the spherical dimension system. Since a retail store mainly serves its nearby market we refer to people who live inside this notional circle as store's customers.

We use this definition of store's market to identify average per capita income of customers who live in proxy to a Wal-Mart or Dollar General store. We want to show that both Wal-Mart and Dollar General usually open stores close to low income communities in Georgia as it was indicated in section 5.3. We retrieved per capita income data from the U.S. Census Bureau as our source of demographic data in this study [108]. Figure 5.1 and Figure 5.2 show histograms for per capita income of Wal-Mart and Dollar General customers in Georgia in year 2005, respectively. Note that the market mile for a

Wal-Mart store is considered to be 10 miles. However, the market mile for a Dollar General store is usually lower than 10 miles. In this research we use a constant five miles for market miles of Dollar General stores in Georgia.

Histogram of Per Capita Income of Wal-Mart customers in Georgia in 2005
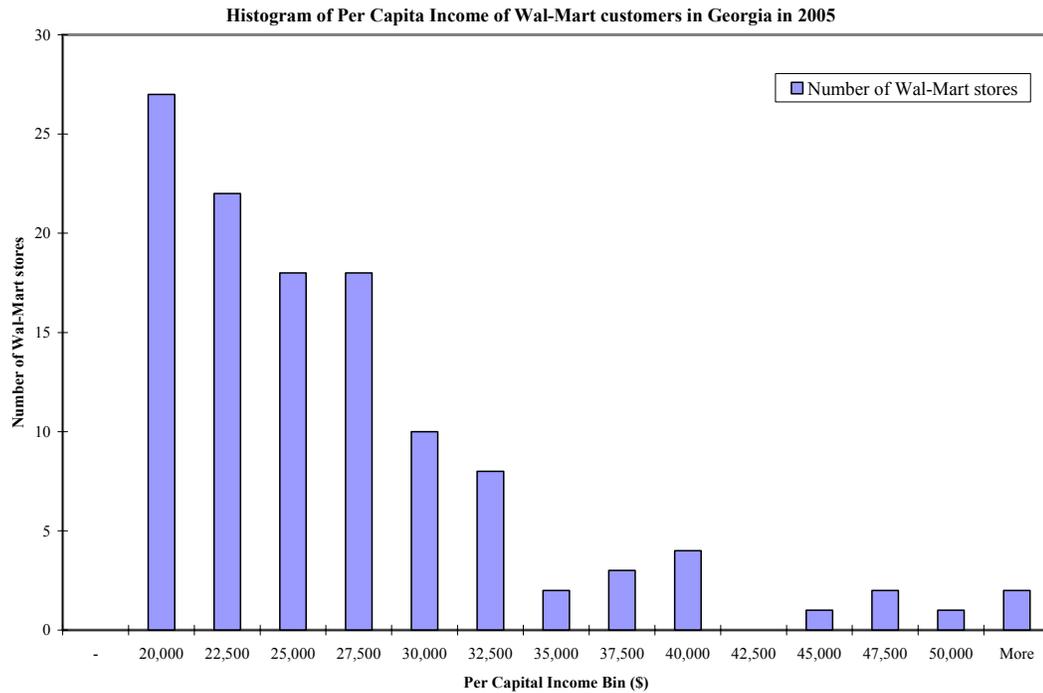
Figure 5.1. Histogram of per capita income of Wal-Mart customers in Georgia in year 2005.
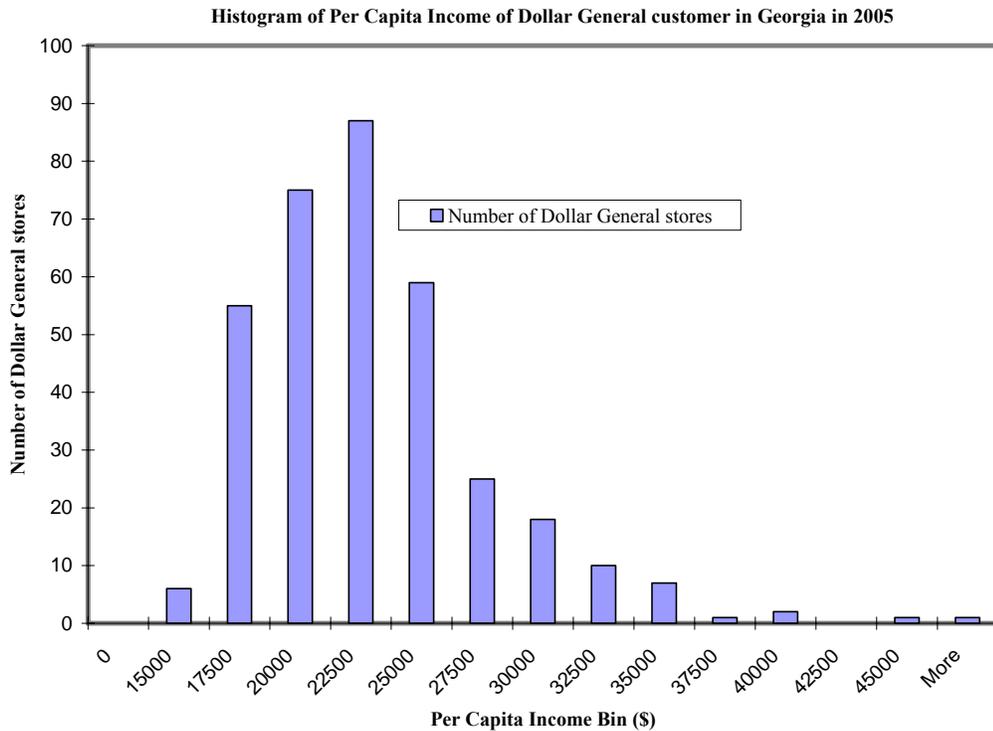
Figure 5.2. Histogram of per capita income of Dollar General customers in Georgia in year 2005.

Average per capita income of Wal-Mart customers in Georgia is approximately $25,000. The distribution of this per capita income is shown in histogram of Figure 5.1. On the other hand, average per capita income of Dollar General stores in Georgia is approximately $22,000. The distribution of this per capita income is also shown in Figure 5.2. It can be noticed that both Wal-Mart and Dollar General opened stores in Georgia in markets that are located closely to low income communities.

However, Wal-Mart stores serve larger markets with more population than Dollar General. Average population of Wal-Mart store's market in Georgia is approximately 200,000 in year 2005. The distribution of this population is shown in Figure 5.3. Note that we will use these Wal-Mart stores' market population data in an empirical study later in this chapter. The average population of a Dollar General store's market in Georgia is

approximately 44,000 in year 2005. The distribution of this population is also shown in Figure 5.4.

Histogram of market population of Wal-Mart stores in Georgia in 2005
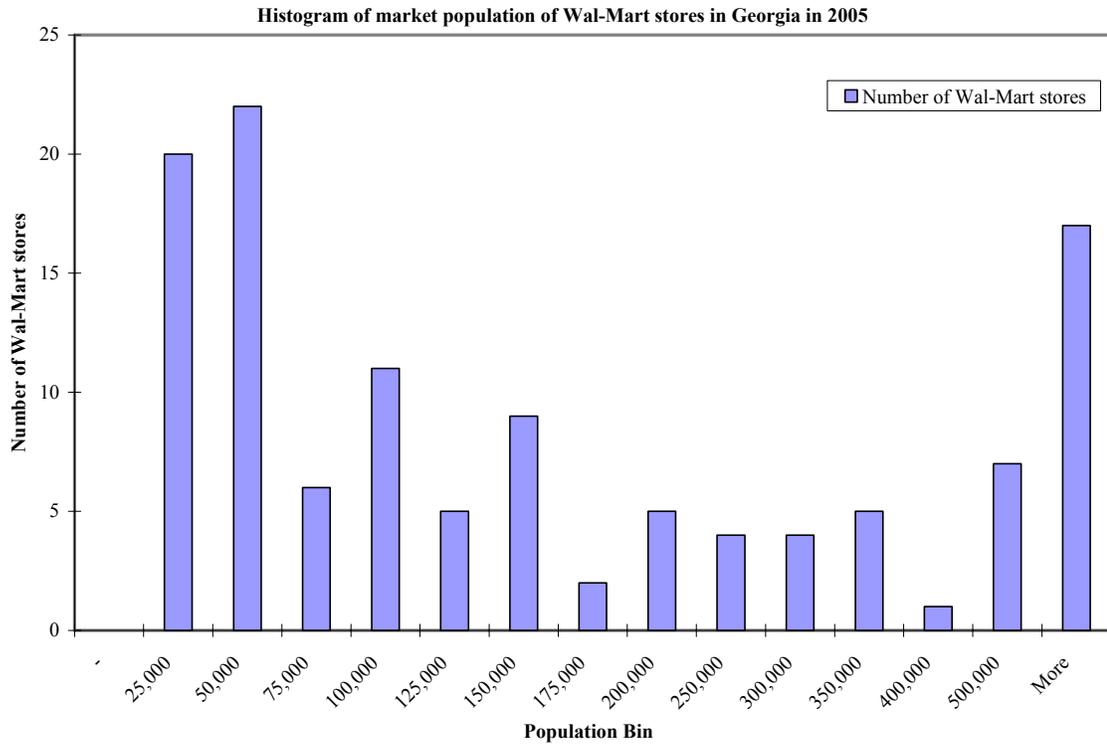
Figure 5.3. Histogram of market population of Wal-Mart store in Georgia in year 2005.
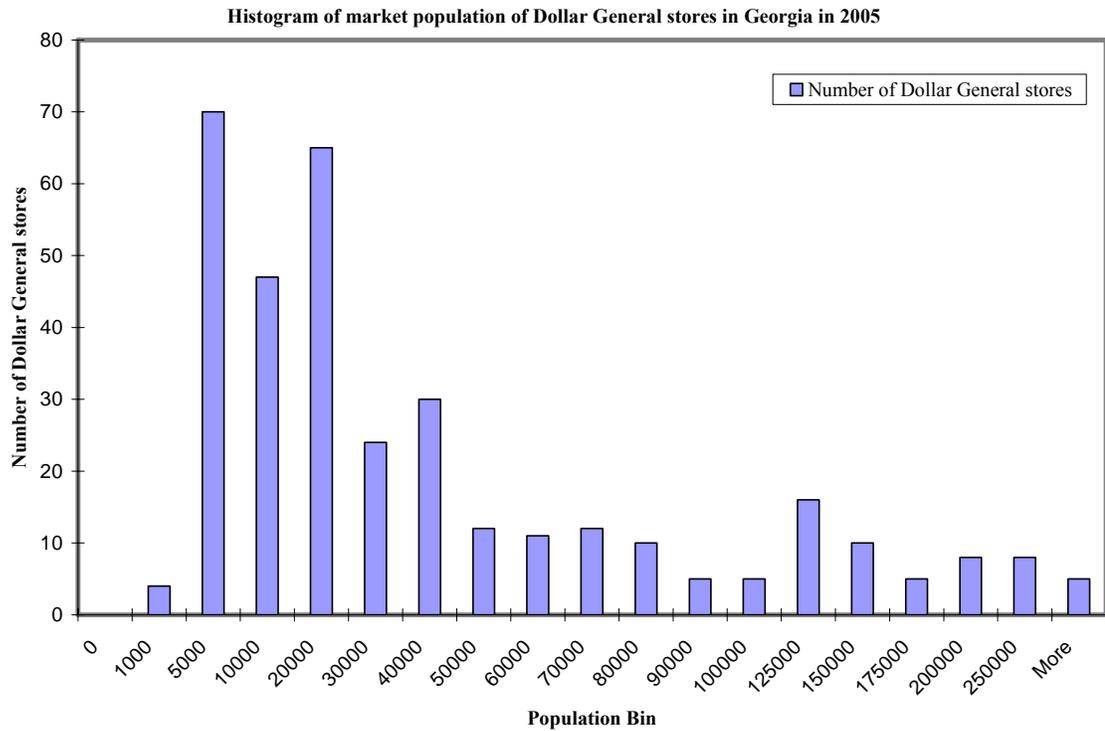
Figure 5.4. Histogram of market population of Dollar General stores in Georgia in year 2005.

Now we use latitude/longitude data of the location of Wal-Mart and Dollar General stores to determine whether there was a Dollar General in a retail market when Wal-Mart opened a store. Calculations are conducted by the Mapping Toolbox of Matlab®. This toolbox has the capability to use the latitude/longitude information of Wal-Mart stores, construct a notional circle as the Wal-Mart store's retail market, and check whether any Dollar General store is located inside this circle considering its latitude/longitude. We call a Wal-Mart store's market competitive, when there is a Dollar General store that is located in this market and opened before this Wal-Mart store. We call a Wal-Mart store's market noncompetitive, when there is no Dollar General store that is located in this market or there is a Dollar General store that is located in this market but opened after this Wal-Mart store. Figure 5.5 shows an overview of these competitive and

noncompetitive Wal-Mart markets in Georgia. Red circles show competitive Georgia markets in which Wal-Mart opened a store. Green dots in these red circles point to locations of Dollar General stores in these competitive markets. On the other hand, blue circles show noncompetitive Georgia market in which Wal-Mart opened stores. Wal-Mart stores' market miles are considered to be 10 miles in this figure.
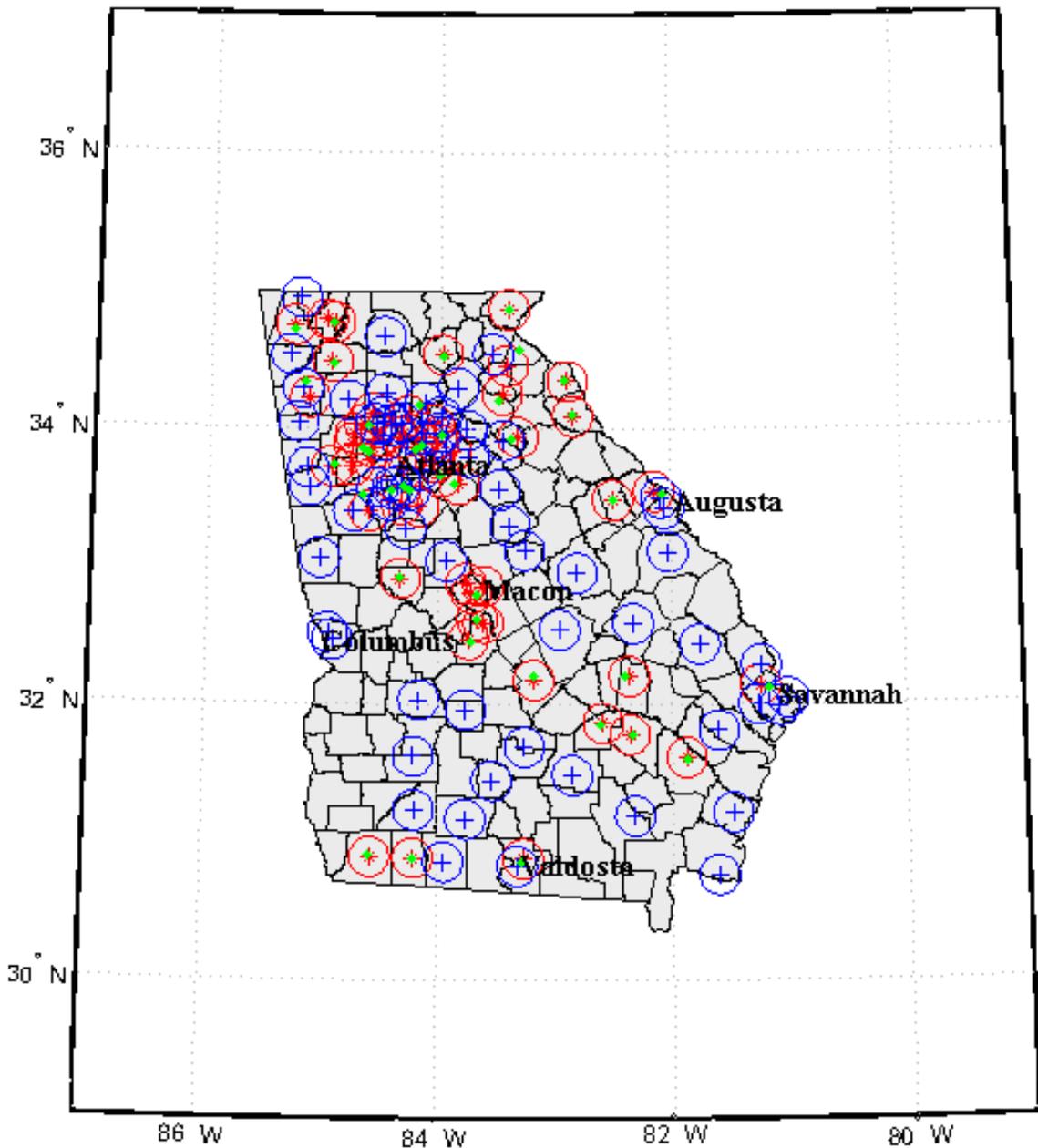
Figure 5.5. Competitive and noncompetitive Wal-Mart retail markets in Georgia.

Note that we consider a Wal-Mart store's retail market noncompetitive for the situation in which Dollar General opens a store after a Wal-Mart store. We call this Wal-Mart store's market noncompetitive for two reasons. First, from the Wal-Mart's stand point, this market is noncompetitive at the investment time since there was no Dollar General store in a nearby market. Secondly, Dollar General opens a store in this market much later than Wal-Mart. It is observed that on average, Dollar General opens a store in such markets on average more than eight years after Wal-Mart. Figure 5.6 shows a boxplot for the number of years that Dollar General opens a store after Wal-Mart in these markets. Therefore, it is not controversial to call these markets noncompetitive since Wal-Mart opened stores in these markets without considering the competition effect from Dollar General.

Boxplot of number of years Dollar General opens stores after Wal-Mart
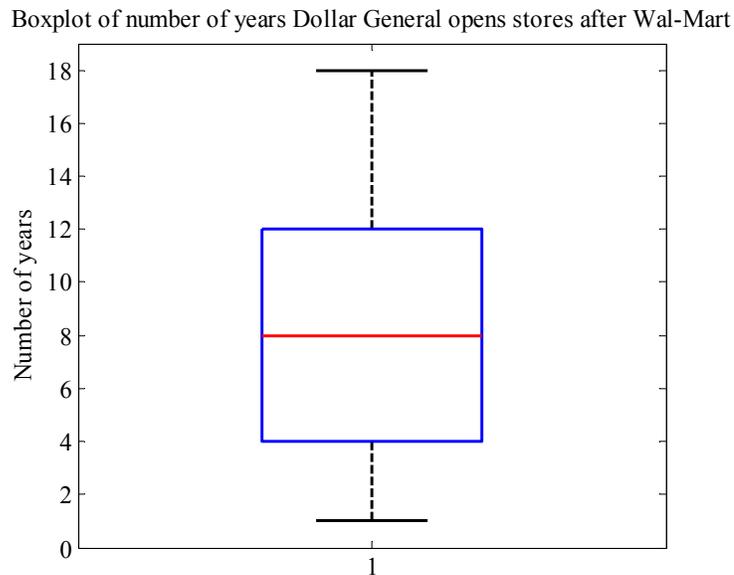
Figure 5.6. Boxplot of number of years Dollar General opens a store after Wal-Mart in Wal-Mart stores' noncompetitive markets.

We also choose two different values for market mile to see whether our results are sensitive to the choice of market mile. First, we divide Wal-Mart stores into two categories using 15 miles as market mile. Table D.1 and Table D.2 summarize a list of Wal-Mart stores in competitive and noncompetitive markets in the state of Georgia, respectively, when market mile equals 15 miles. Secondly, we divide Wal-Mart stores into two categories using 10 miles as market mile. Table D.3 and Table D.4 summarize a list of Wal-Mart stores in competitive and noncompetitive markets in the state of Georgia, respectively, when market mile is equal to 10 miles. The market population of every Wal-Mart store is also summarized in these tables with respect to the choice of the market mile.

In the next section, we use these four tables to examine whether Wal-Mart opens stores relatively later in competitive markets than noncompetitive markets.

5.5   Data analysis

In this section, we use a standard two-sample t-test to examine whether on average Wal-Mart opens stores in competitive markets relatively later than noncompetitive markets. We conduct two separate statistical tests for market miles of 15 and 10. For this purpose, we compare two columns related to the year Wal-Mart opens a store in Table D.1 and Table D.2 for which market miles are considered to be 15 miles. Separately, we compare similar two columns in Table D.3 and Table D.4 for which market miles are considered to be 10 miles.

We use a standard statistical hypothesis testing approach for our comparison purpose. Our null hypothesis in this test is that the mean value of the year Wal-Mart

opens stores in competitive markets is not different from the mean value of the year Wal-Mart opens stores in noncompetitive markets. The alternative hypothesis in our empirical study, which refers to our initial expectation, is that the mean value of the year Wal-Mart opens stores in competitive markets is greater than the mean value of the year Wal-Mart opens stores in noncompetitive markets. Our hypothesis test is summarized in Equation (5.1).

$$\begin{cases} H_0 : \mu_1 = \mu_2 \\ H_1 : \mu_1 > \mu_2 \end{cases}$$

(5.1)

where (μ_1) is the mean value of the year Wal-Mart opens stores in competitive markets and (μ_2) is the mean value of the year Wal-Mart opens stores in noncompetitive markets.

We carry out a standard two-sample t-test between two unequal samples with unequal, unknown variances as it is described in [109]. Consider that our two unequal samples are the years that Wal-Mart opens a store in competitive and noncompetitive markets, respectively.

We use t-test for our purpose since we compare the mean values of two normally distributed populations. We assume that Wal-Mart decides to open stores in competitive markets independent of noncompetitive markets and vice versa. Therefore, the sample of years Wal-Mart opens stores in competitive and noncompetitive markets are independent of each other. In addition, we assume that opening a store in a competitive market is independent from opening another store in another competitive market in Georgia. Therefore, we can assume that the years Wal-Mart opens stores in competitive markets are independently chosen from normal distribution and hence, these years are

independent and identically distributed (i.i.d.) from some normal distribution. A similar assumption can be made for the years Wal-Mart opens stores in noncompetitive markets. Thus, standard t-test can be used to test whether the mean values of two normally distributed populations (i.e., the mean values of the years Wal-Mart opens stores in competitive and noncompetitive markets) are equal.

We use a two-sample t-test with unequal sample sizes since numbers of stores in competitive and noncompetitive markets are unequal. There are 58 stores in competitive markets versus 60 stores in noncompetitive when market miles are 15, and 54 stores in competitive markets versus 64 stores in noncompetitive when market miles are 10. Since there is no knowledge regarding the true values of population variance for these two samples we use t-test with unequal and unknown variances.

We conduct this two-sample t-test in Minitab 15®, which is a standard statistical package. Figure 5.7 shows a summary of results for this two-sample t-test for the years Wal-Mart opens stores in competitive markets versus noncompetitive markets as described in Equation (5.1). It can be seen that we can strongly reject the null hypothesis in Equation (5.1) using the data related to two samples of the years Wal-Mart opens stores in competitive and noncompetitive markets that are summarized in Table D.1 and Table D.2, respectively. Therefore, we can conclude that (μ_1) is greater than (μ_2) at the (5%) significance level. Note that the p-value is almost zero, which emphasizes on the strength of rejecting the null hypothesis.

Two-Sample T-Test and CI: Competitive Markets, Noncompetitive Markets

```
Two-sample T for Competitive Markets vs Noncompetitive Markets (15 Miles)

                       N    Mean   StDev   SE Mean
Competitive Markets    58   1995.69   7.82     1.0
Noncompetitive Markets 60   1986.08   4.02     0.52


Difference = mu (Competitive Markets) - mu (Noncompetitive Markets)
Estimate for difference:  9.61

95% lower bound for difference:  7.69
T-Test of difference = 0 (vs >): T-Value = 8.35  P-Value = 0.000  DF = 84
```

Figure 5.7. Two-sample t-test to compare the year Wal-Mart opens a store in competitive markets versus noncompetitive markets (market mile radius is assumed to be 15 miles).

Thus, our data provide significant evidence that Wal-Mart opens a store in competitive markets relatively later than noncompetitive markets. We can observe the difference between these two groups of data using the boxplot as shown in Figure 5.8. It can be noticed that on average, Wal-Mart opens a store in competitive markets relatively later than noncompetitive markets. Note that Figure 5.7 and Figure 5.8 are based on the data that are for the case, in which market miles are considered to be 15 miles for Wal-Mart stores.
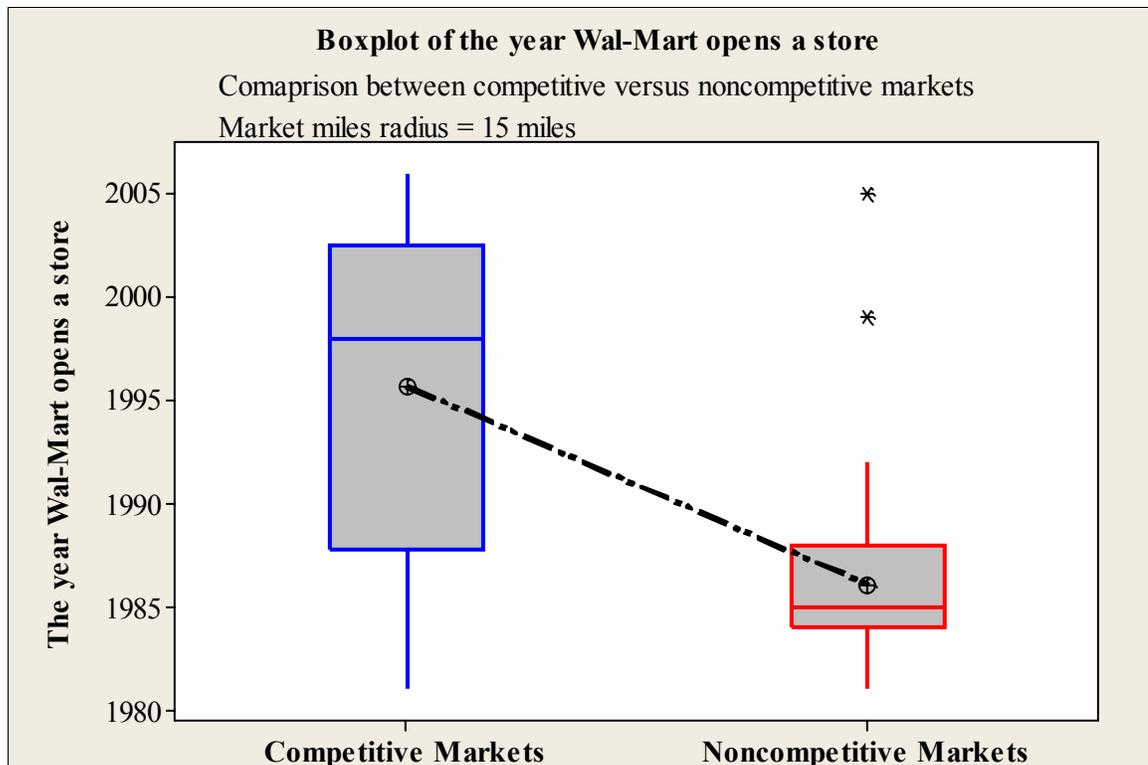
Figure 5.8. Boxplot of the year Wal-Mart opens a store in competitive versus noncompetitive markets (market mile radius is assumed to be 15 miles).

However, similar pattern can be observed when market miles are considered to be 10 miles. Figure 5.9 shows how the null hypothesis is again rejected using our data based on 10 miles as market miles. We can also observe the difference between these two groups of data using the boxplot as shown in Figure 5.10.

Figure 5.9. Two-sample t-test to compare the year Wal-Mart opens a store in competitive markets versus noncompetitive markets (market mile radius is assumed to be 10 miles).
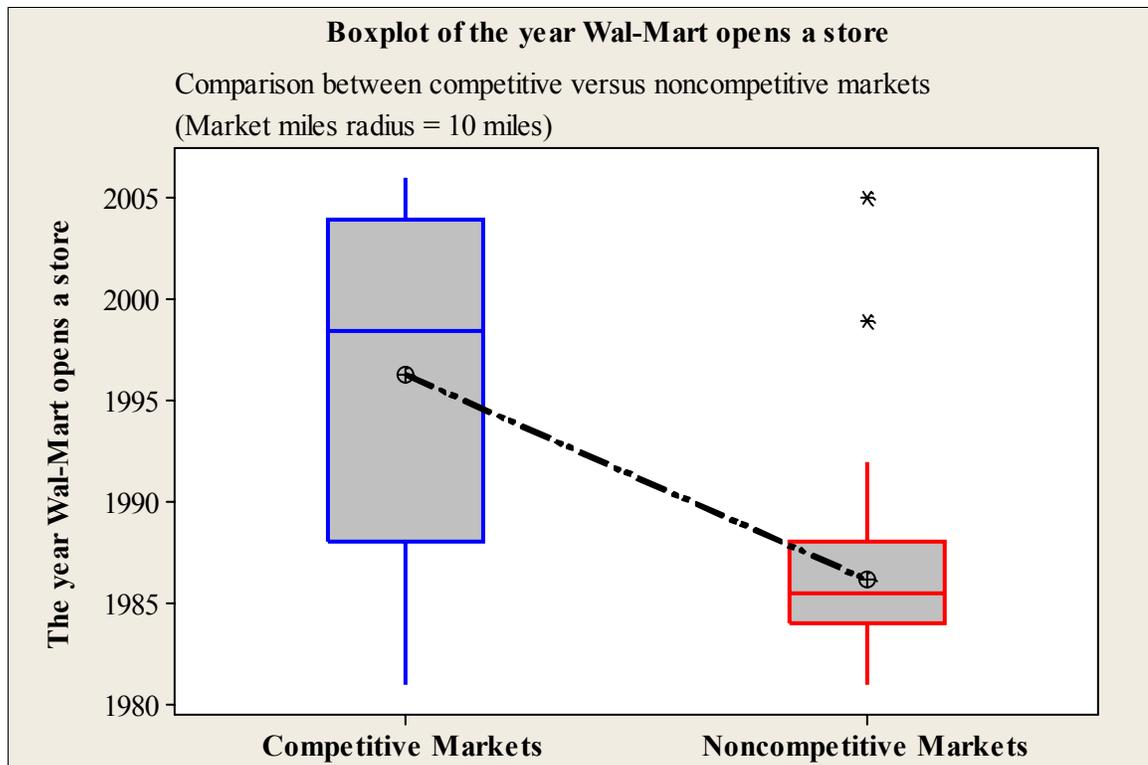
Figure 5.10. Boxplot of the year Wal-Mart opens a store in competitive versus noncompetitive markets (market mile radius is assumed to be 10 miles).

In addition to the comparison between the years that Wal-Mart opens a store in competitive and noncompetitive markets, we conduct another empirical study for these two types of markets. In Chapter 3 and Chapter 4, we describe retailers' optimal investment thresholds in terms of the value of retail market potential at the decision time. In this section, we assume that market population can be used as a proxy for the value of retail market potential, i.e., the retailer decides to open a store in a market whose population is large enough to provide satisfactory returns for the retail store.

Our theoretical results in sections 3.9.4 and 4.5.4 note that a big retailer such as Wal-Mart opens a store at the relatively higher level of retail market potential in markets with a small retailer such as Dollar General than markets without Dollar General, i.e., Wal-Mart opens a store at the relatively higher retail market potential in competitive market

than noncompetitive markets. If we use market population as a proxy for retail market potential then our theoretical results in sections 3.9.4 and 4.5.4 indicate that Wal-Mart opens a store at the relatively higher market population in competitive markets than noncompetitive markets. In this section, we want to empirically test this theoretical result.

Therefore, we again use a two-sample t-test with unequal sample sizes since numbers of stores in competitive and noncompetitive markets are unequal. Similar arguments as our discussions for opening year comparison can be made here to show how appropriate a two-sample t-test is to compare the mean values of two normally distributed market populations. There are 54 stores in competitive markets versus 64 stores in noncompetitive when market miles are 10. Since there is no knowledge regarding the true values of population variance for these two samples we use t-test with unequal and unknown variances.

Figure 5.11 shows a summary of results for this two-sample t-test for the market population of a Wal-Mart store in the opening year, in competitive markets versus noncompetitive markets as described in Equation (5.1). Note that (μ_1) in this test is the mean value of a Wal-Mart store's market population in a competitive market and (μ_2) is the mean value of a Wal-Mart store's market population in a noncompetitive market. It can be seen that we can strongly reject the null hypothesis in Equation (5.1) using Wal-Mart stores' market population data. Therefore, we can conclude that (μ_1) is greater than (μ_2) at the (5%) significance level. Note that the p-value is also very low (approximately 1%), which emphasizes on the strength of rejecting the null hypothesis.

```
Two-Sample T-Test and CI: Competitive markets, Noncompetitive markets
Market population at the year Wal-Mart opens a store (market miles = 10 miles)
Two-sample T for Competitive markets vs Noncompetitive markets

                         N    Mean    StDev   SE Mean
Competitive markets      54  195956   226347    30802
Noncompetitive markets   64  116423   135218    16902


Difference = mu (Competitive markets) - mu (Noncompetitive markets)
Estimate for difference:  79533
95% lower bound for difference:  21089

T-Test of difference = 0 (vs >): T-Value = 2.26  P-Value = 0.013  DF = 83
```

Figure 5.11. Two-sample t-test to compare the market population at the year Wal-Mart opens a store in competitive markets versus noncompetitive markets (market mile radius is assumed to be 10 miles).

Thus, our data provide significant evidence that Wal-Mart opens a store in competitive markets relatively at the higher market population than noncompetitive markets. We can observe the difference between these two groups of data using the boxplot as shown in Figure 5.12. It can be noticed that on average, Wal-Mart open stores in the relatively more populated market with Dollar General than markets without Dollar General.
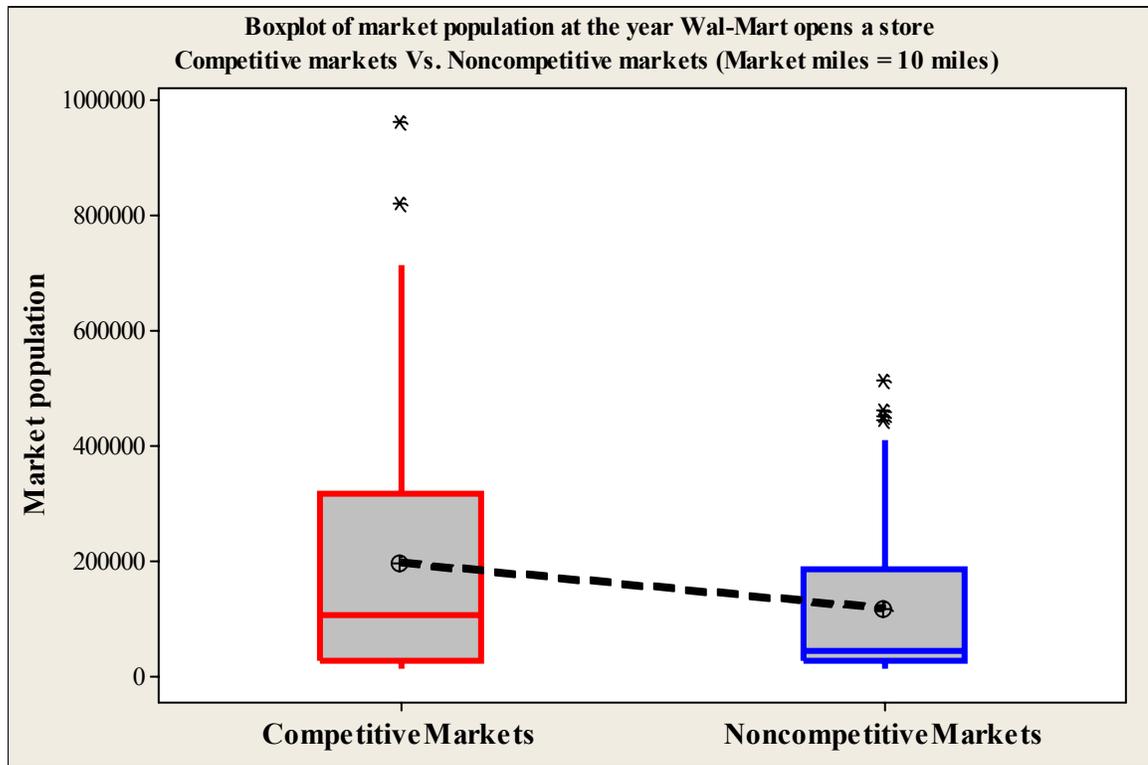
Figure 5.12. Boxplot of the Wal-Mart store's market population in the opening year, in competitive versus noncompetitive markets (market mile radius is assumed to be 10 miles).

However, similar pattern can be observed when market miles are considered to be 15 miles. Figure 5.13 shows how the null hypothesis is again rejected using our data based on 15 miles as market miles. We can also observe the difference between these two groups of data using the boxplot as shown in Figure 5.14.

```
Two-Sample T-Test and CI: Competitive markets, Noncompetitive markets
Market population at the year Wal-Mart opens a store (market miles = 15 miles)
Two-sample T for Competitive markets vs Noncompetitive markets

                         N    Mean    StDev    SE Mean
Competitive markets      58   405051  456160   59897
Noncompetitive markets   60   208374  272517   35182


Difference = mu (Competitive markets) - mu (Noncompetitive markets)
Estimate for difference:  196677
95% lower bound for difference:  81255
T-Test of difference = 0 (vs >): T-Value = 2.83  P-Value = 0.003  DF = 92
```

**Figure 5.13. Two-sample t-test to compare the market population at the year Wal-Mart opens a store in competitive markets versus noncompetitive markets (market mile radius is assumed to be 15 miles).**
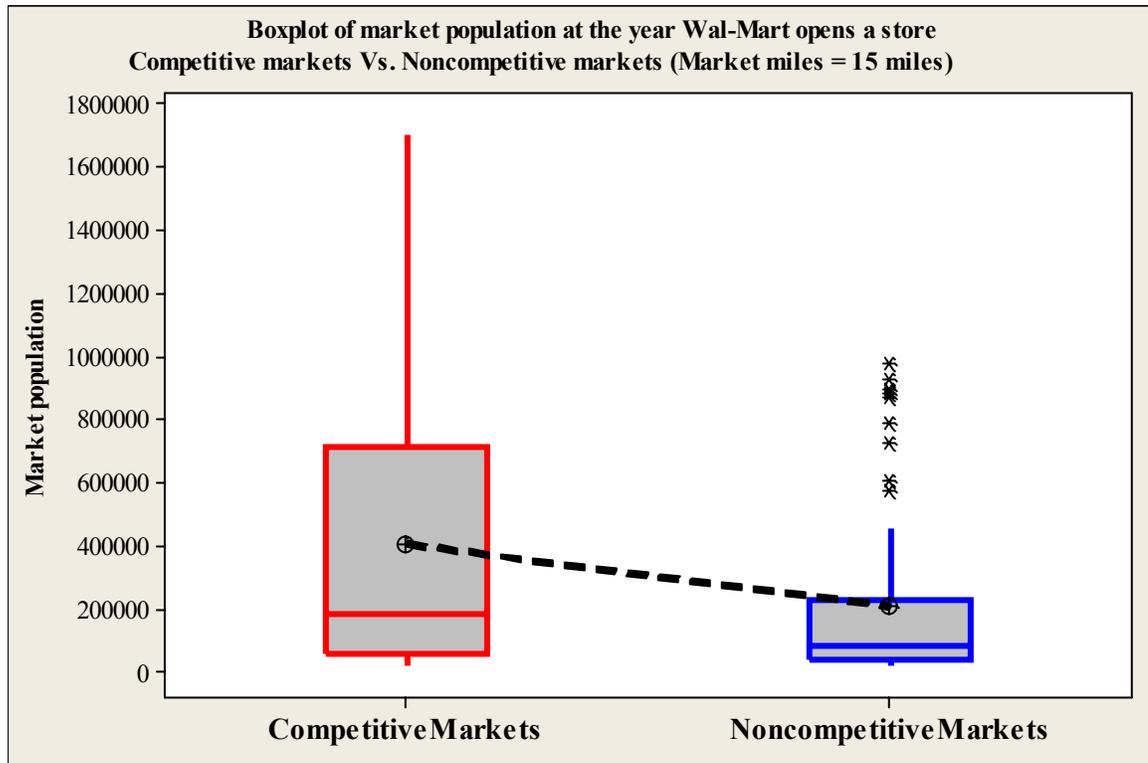

Figure 5.14. Boxplot of the Wal-Mart store's market population in the opening year, in competitive versus noncompetitive markets (market mile radius is assumed to be 15 miles).

However, conducting a two-sample t-test is based on an important assumption of the normality of the underlying data. We used t-test since our sample size is large enough (greater than 30 data), which is a rule of thumb to use t-test for comparison between two data sets. Here we explore the validity of this assumption by conducting the normality test on our data.

We use three standard normality tests in this chapter to determine whether our underlying data on the year Wal-Mart opens a store in a market or the market population when Wal-Mart opens a store are normally distributed. These three standard normality tests are summarized, as follow.

- **Anderson-Darling test** [110]: is one of the most powerful statistical tests for detecting departure from normality. This test compares the empirical cumulative distribution function (CDF) of our sample data with the expected normal CDF. The test rejects the null hypothesis of that sample data follow a normal distribution when the difference between these two CDFs is substantially large.

- **Ryan-Joiner normality test** [111]: This statistical test examines the normality of sample data by finding the correlation between the sample data and the normal scores of the sample data. If this correlation is below a certain critical value the null hypothesis, which indicates the population normality will be rejected. This test is similar to the Shapiro-Wilk normality test [112].

- **Kolmogorov-Smirnov normality test** [113]: this is a general statistical test to examine whether sample data follow a hypothetical distribution, for instance normal distribution. This test compares the empirical cumulative distribution function of the sample data with the data expected from a normal distribution. The null hypothesis of population normality will be rejected if this difference is substantially large.

We conduct all three normality test on our sample data. The results are consistent and the assumption of population normality is strongly rejected for our entire dataset of the year Wal-Mart opens a store and the market population when Wal-Mart opens a store. However, here we only report our results from Anderson-Darling normality test.

Figure 5.15 and Figure 5.16 show the normal probability plot for the year Wal-Mart opens a store in competitive and noncompetitive markets where market miles are

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considered to be 10 and 15 miles, respectively. It cane be observed that the null hypothesis of the population normality is rejected at 5% significance level based on the Anderson-Darling normality test since the sample data are not entirely located inside the region of 95% confidence level.
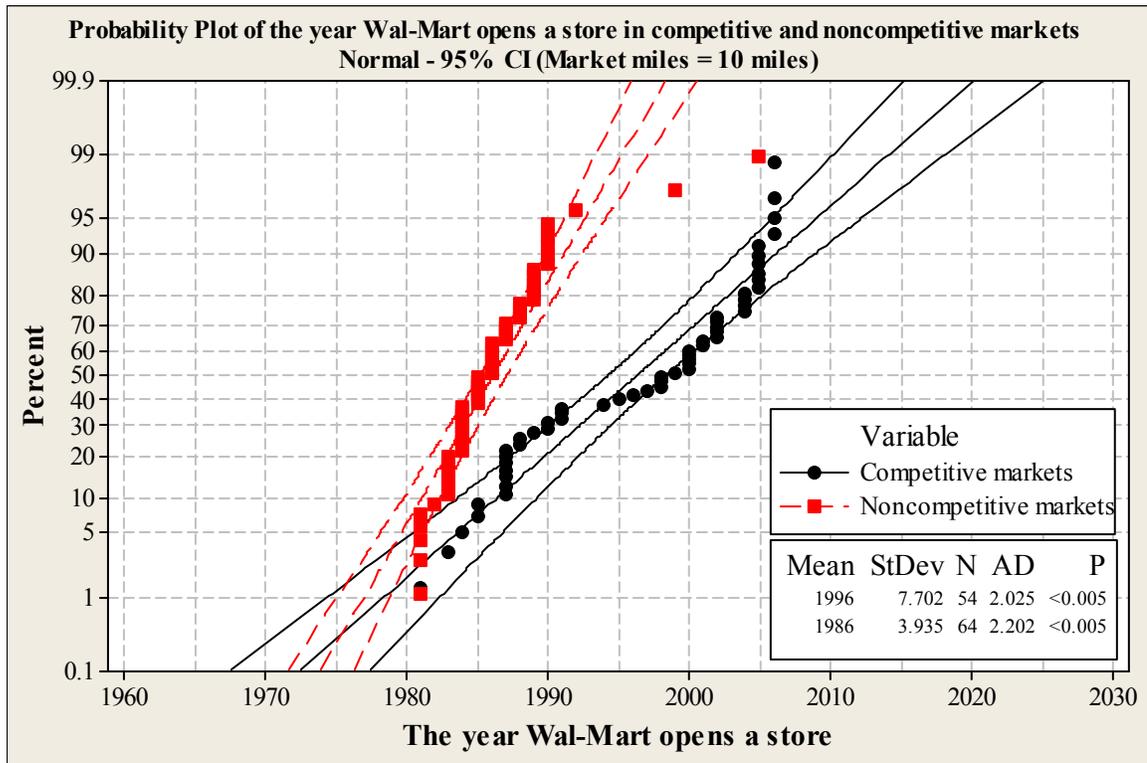
Figure 5.15. Normal probability plot of the year Wal-Mart opens a store in competitive and noncompetitive market (market miles are considered to be 10 miles).
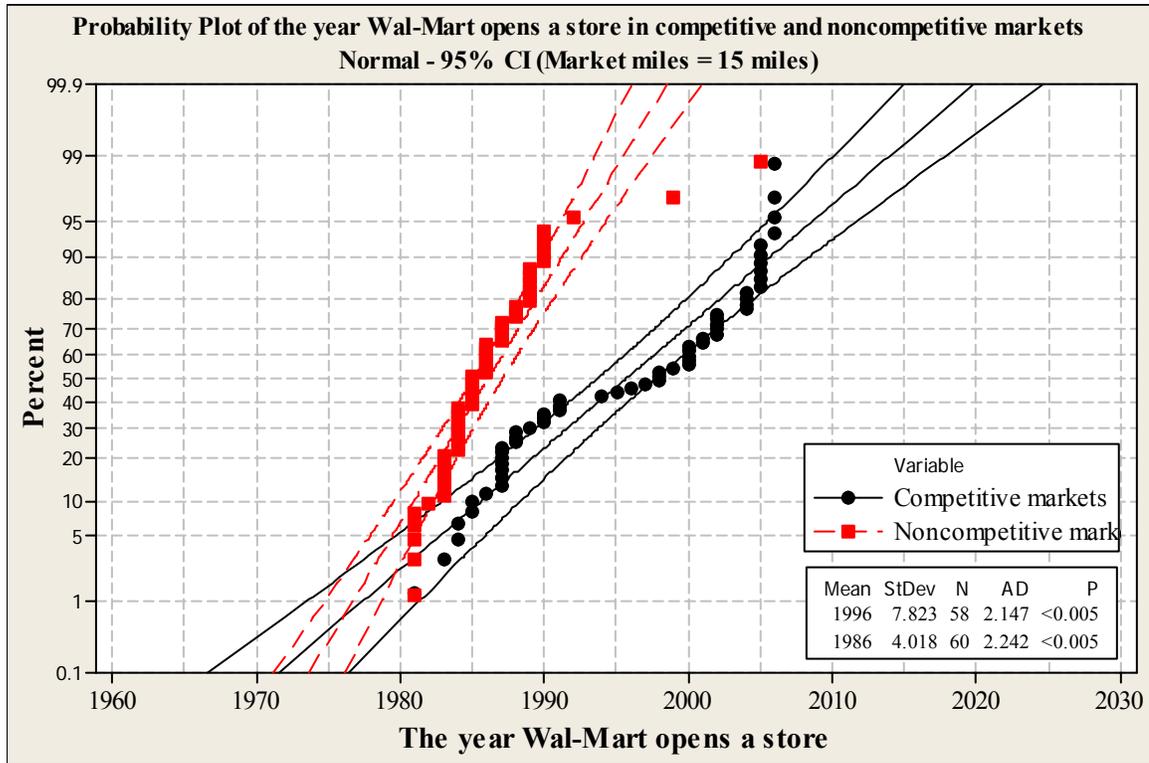
Figure 5.16. Normal probability plot of the year Wal-Mart opens a store in competitive and noncompetitive market (market miles are considered to be 15 miles).

Figure 5.17 and Figure 5.18 show the normal probability plot for the market population when Wal-Mart opens a store in competitive and noncompetitive markets where market miles are considered to be 10 and 15 miles, respectively. It cane be observed that the null hypothesis of the population normality is rejected at 5% significance level based on the Anderson-Darling normality test since the sample data are not entirely located inside the region of 95% confidence level.
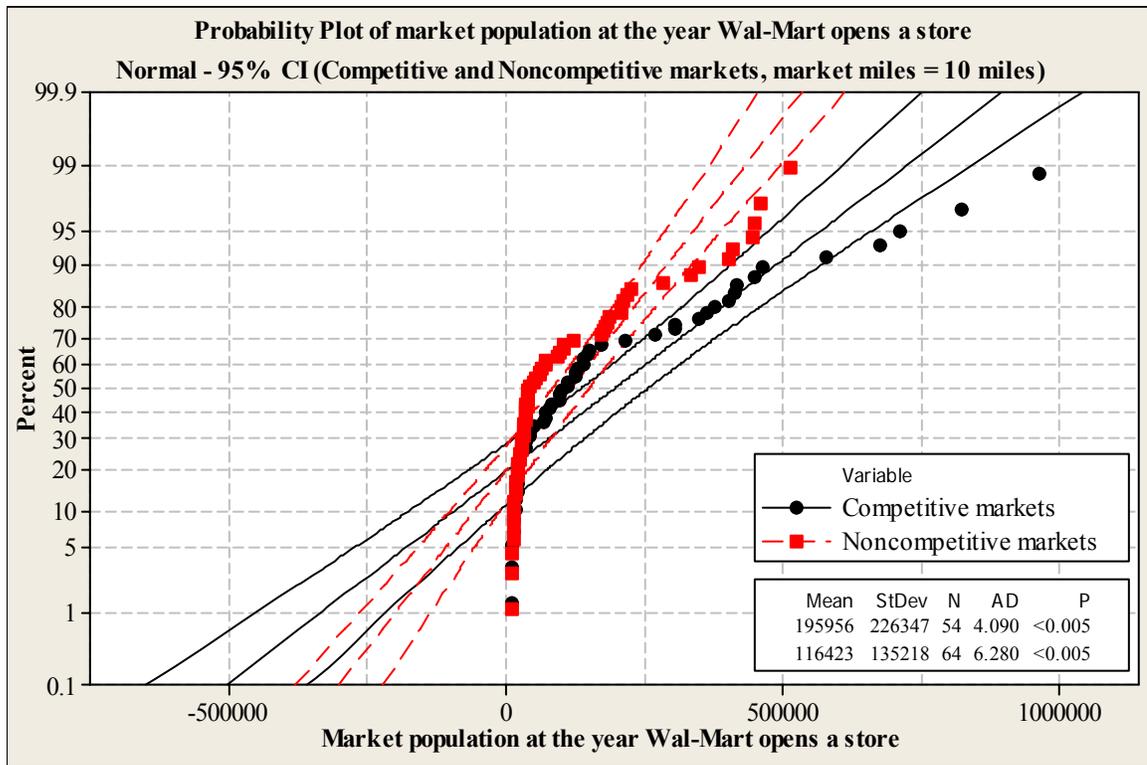
Figure 5.17. Normal probability plot of market population of a Wal-Mart store in competitive and noncompetitive market (market miles are considered to be 10 miles).
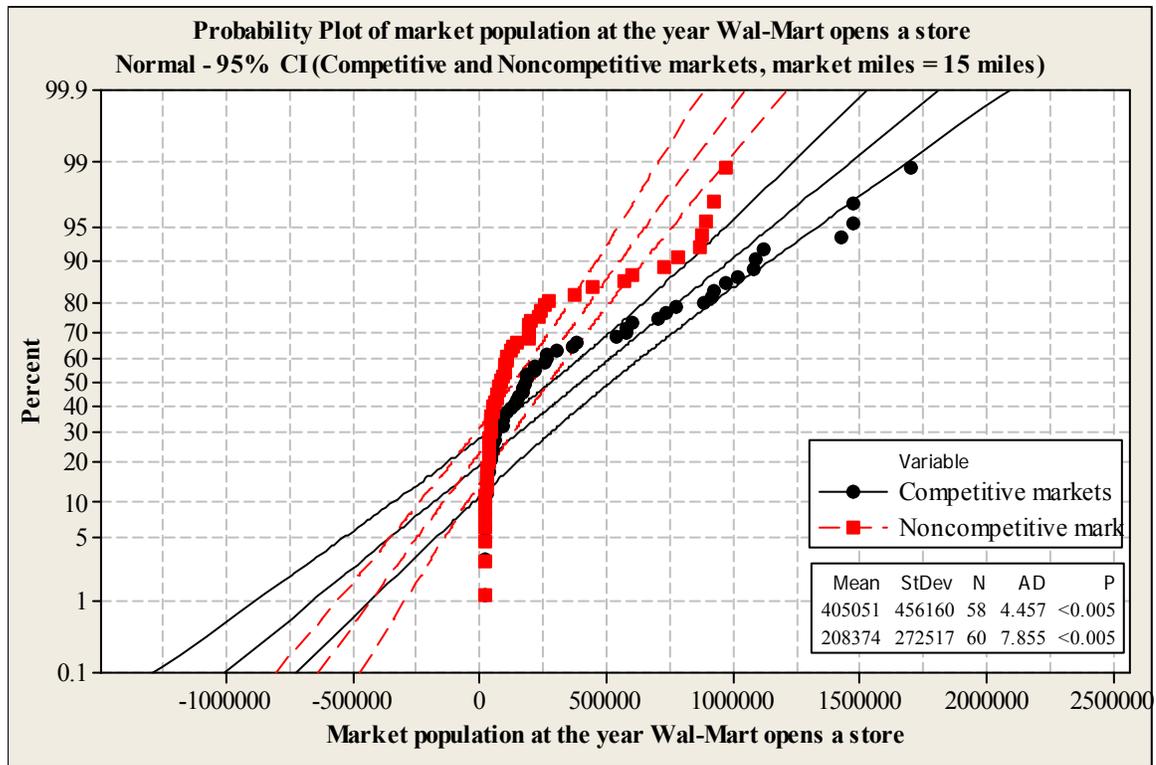
Figure 5.18. Normal probability plot of market population of a Wal-Mart store in competitive and noncompetitive market (market miles are considered to be 15 miles).

Since our sample data do not support the assumption of the population normality for the year Wal-Mart opens a store and the market population of a Wal-Mart store at the year it opens we may not be able to confidently use a t-test to examine the difference between the mean values of these variables in competitive and noncompetitive markets. Instead, in this chapter we use a nonparametric statistical test called Wilcoxon rank-sum test [114, 115] for comparing our two variables of interest (the year Wal-Mart opens a store and the market population when Wal-Mart opens a store) in competitive and noncompetitive markets.

The Wilcoxon rank-sum test (also known as Mann-Whitney U test, the Mann-Whitney-Wilcoxon (MWW) test, or Wilcoxon-Mann-Whitney test) is a nonparametric test to examine whether two sample data come from the same distribution. The null

hypothesis in this test is that two sets of observations are independently sampled from an identical distribution against the alternative hypothesis that these two sets of observations do not have equal medians. We use 'ranksum' function in Matlab® statistical package for the purpose of our calculation. The null hypothesis is rejected in the following cases.

- The null hypothesis of the year Wal-Mart opens a store in competitive markets and the year Wal-Mart opens a store in noncompetitive markets have the same median where market miles are 15 miles is strongly rejected at 5% significance level with the very low p-value of (5.5508e-11).

- The null hypothesis of the year Wal-Mart opens a store in competitive markets and the year Wal-Mart opens a store in noncompetitive markets have the same median where market miles are 10 miles is strongly rejected at 5% significance level with the very low p-value of (2.1212e-18).

- The null hypothesis of the market population when Wal-Mart opens a store in competitive markets and the market population when Wal-Mart opens a store in noncompetitive markets have the same median where market miles are 15 miles is strongly rejected at 5% significance level with the p-value of (0.0153).

- The null hypothesis of the market population when Wal-Mart opens a store in competitive markets and the market population when Wal-Mart opens a store in noncompetitive markets have the same median where market miles are 10 miles is not rejected at 5% significance level. However, this null hypothesis is rejected at 10% significance level with the p-value of (0.0716).

Therefore, these results indicate that the Wal-Mart's investment behavior is different in markets with Dollar General compared to markets without Dollar General. It can be also observed in the boxplots of Figure 5.8 and Figure 5.10 that the median of the year Wal-Mart opens a store in markets with Dollar General is greater than the median of the year Wal-Mart opens a store in markets without Dollar General. In addition, based on Figure 5.12 and Figure 5.14 we can conclude that the median of the market population of a Wal-Mart store in markets with Dollar General is greater than the median of the market population of a Wal-Mart store in markets without Dollar General.

Thus, the results from this nonparametric test shows that in Georgia Wal-Mart in practice follows the preferred strategy for a big retailer that is to open a store relatively later in markets in which the small retailer such as Dollar General has a store opened.

5.6   Conclusions

In this chapter, we presented an empirical study to examine Wal-Mart's investment behaviors in competitive versus noncompetitive markets. It is concluded that in Georgia Wal-Mart's practices are in accordance with the preferred strategy for a big retailer, that is to open a store relatively later in markets in which the small retailer such as Dollar General has a store opened. In addition, Wal-Mart in practice follows the preferred strategy for a big retailer that is to open a store in more populated markets in which the small retail such as Dollar General has a store opened. Therefore, our empirical results are consistent with our expectations from our theoretical discussions in sections 3.9.4 and 4.5.4. It should be noted that our empirical results do not validate our theory. However, these results do provide supporting evidence for our theoretical work.

However, our empirical results should be extended to other retail markets located in other states to examine whether the same pattern holds across the nation. In addition, similar case studies should be conducted for other discount retailers to examine their investment behavior under the competition effect. For instance, it is interesting to study the competition between retailers with similar cost structures such as Target and Wal-Mart or Dollar General and Family Dollar in order to test our theoretical results in Chapter 3 and Chapter 4.

In this chapter, we use market miles in terms of a notional circle around the store as our approach to determine the Wal-Mart store's market. However, we could use market miles in terms of actual traveling distance between Wal-Mart and Dollar General stores and conduct similar data analysis to ensure our results still hold.

In addition, we assume that customers have perfect knowledge regarding the existence and the location of Wal-Mart and Dollar General stores. We also assume that customers only choose between these two stores when it is time to shop for some products that they both provide. The other assumption in this empirical study is that Wal-Mart and Dollar General have similar abilities to open a store, for instance, site purchase or lease, preparation, and construction. It could take Wal-Mart longer to open a store because it is more difficult to find a large site than a smaller one. Wal-Mart also has more difficulty to acquire a permit for its store than Dollar General. However, these assumptions may or may not be true in the actual practice. Structured customer surveys should be conducted across these stores to validate these assumptions and evaluate our theoretical results.

Chapter 6

Conclusions and Future Works

In this chapter, we provide a summary of conclusions and future works for this thesis. It was indicated that the retail industry is very competitive in the U.S. and hence, a retailer always strives to differentiate itself from other competitors to enhance its market position. Therefore, market selection is identified as an important strategy to differentiate a retailer in competitive retail markets. In Chapter 2, we show how the selection of appropriate markets to open stores can be considered as an investment decision by a retailer. Although market selection analysis is a well-established subject in retail and marketing research, our search in the capital budgeting and investment analysis literature did not locate any study related to retail market analysis.

In addition in Chapter 2, we summarize why the traditional capital budgeting approaches such as the NPV calculation are inadequate in addressing the strategic aspects of a retailer's investment decision as well as its flexibility to change the original, operational strategies as uncertainty about the dynamic retail market evolves over time. We also discuss why an investment analysis approach based on the real options methodology is a promising candidate to overcome some of these limitations.

Therefore, a conceptual framework is presented in Chapter 2 to describe the similarities between a retailer's investment option in a dynamic market and a call option on a common stock. This analogy is useful since it outlines an investment analysis procedure based on the real options methodology to evaluate a retailer's investment

option. As part of building this analogy, we discuss the value of retail market potential as an underlying asset for the valuation of a retailer's investment option, similar to the stock price, which is an underlying asset for the valuation of a call option. This conceptual framework is used in Chapter 3 and Chapter 4 to construct a computational framework for the valuation of a retailer's investment option, i.e., the value of retail market potential is used to derive the value of the retailer's optimal investment threshold in a competitive, dynamic market. However, we indicate that there are some essential differences between a retailer's investment option in a dynamic, competitive market and a call option on a common stock that must be considered in the appropriate characterization and evaluation of the retailer's investment opportunities.

Another conceptual framework is developed in Chapter 2 to categorize different types of investment options that a retailer is faced in the real world. This categorization helps us organize several aspects of the management flexibility to defer an investment opportunity and revise the original operating strategy in retail markets according to the competitive structure of a retail market.

It is concluded that the conceptual frameworks of Chapter 2 can be used as a single framework to address the retailer's management flexibility as well as dynamic uncertainty and competition effect in retail markets. Therefore, these conceptual frameworks provide a strategic view towards retail stores as flexible assets of a retail firm.

An integrated investment analysis approach based on dynamic programming is developed in Chapter 3 to determine retailers' optimal investment thresholds in noncompetitive and competitive markets. It is concluded that retailers have different

optimal investment thresholds in competitive versus noncompetitive markets. My theoretical options-based investment analysis approach indicates that that the small retailer prefers to invest earlier in the market when it expects an entry from the big retailer. This early investment of the small retailer provides a first-mover advantage for the retailer and delays the entry of the big retailer to the market. Therefore, my theoretical model indicates that the big retailer ought to invest relatively later in markets with a small retailer compared to markets without a small retailer.

In addition, a complete sensitivity analysis is carried out in Chapter 3 to explore how the retailer's investment thresholds are sensitive to the changes in the values of the model parameters. This sensitivity analysis is used to prioritize the variables that retailers should pay more attention to in their investment analysis process. It is concluded that the retailer's optimal investment thresholds increase as the value of the log-volatility of retail market potential (parameter (σ)) or the retail firm's discount rate (parameter (ρ)) increases. However, retailers' optimal investment thresholds decrease as the expected value of retail market potential (parameter (α)) increases.

Retailers continuously improve their supply chain systems in order to reduce the marginal cost of providing products to the market. The low variable cost gives a retailer a first-mover advantage to enter a competitive market first and preempts the market from the entry of other competing retailers. Retailers also improve their store operations management strategies to reduce the fixed cost of providing products in store shelves. The low fixed cost gives a retailer a first-mover advantage to enter a competitive market first and preempts the market from the entry of other competing retailers. In addition, retailers work with developers work closely with developers in new growing markets to

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take advantage of appropriate prices on commercial real estate properties for stores. In addition, they improve their shelf management strategies and select appropriate sizes for their prospective store in order to reduce the development cost. The low investment cost gives a retailer a first-mover advantage to enter a competitive market first and preempts the market from the entry of other competing retailers.

The real validity of the proposed model in Chapter 3 should be investigated in future research. This research should be extended to show how actual retailers could use this model in the investment evaluation process of competitive, dynamic retail markets. The major challenge that should be formally treated is to define an estimation process for the model parameters, particularly, (α, σ, and ρ). The most important question is what (surrogate) measures of retail market potential and volatility provide are the best indicators for our model. Thus, this option-based investment analysis approach can be useful for retailers that have long been known to take a qualitative approach to the evaluation of new markets for store development [6, 66-68].

In Chapter 4, we revisit the evaluation problem of a retailer's investment option in competitive and noncompetitive markets. An integrated investment analysis approach based on the equivalent risk neutral evaluation approach is developed in Chapter 4 to explore retailers' investment behaviors in competitive versus noncompetitive dynamic markets. This approach has a market-oriented perspective and determines the correct market-value of a retailer's investment opportunity in order to enhance the market-value of the retail firm.

My theoretical results in Chapter 4 show that the retailer with the overall cost advantage enters the competitive market first and its optimal investment threshold in the

competitive market is lower than its threshold in the same market without any competition effect, i.e., the retailer with the overall cost advantage enters the competitive market early enough in order to preempt the market from the entry of the other retailer. This first-mover advantage pushes up the other retailer's investment threshold and delays its entry to the market. Thus, the equivalent risk neutral evaluation approach shows the significance of the strategic aspect of early investment by the retailer with the overall cost advantage. This theoretical result is also consistent with our result from the investment analysis approach based on dynamic programming as it was described in Chapter 3.

Our theoretical model in Chapter 4 shows that the investment threshold of the retailer with the cost advantage in the competitive market is substantially lower than its threshold in the same market without competition particularly when the values of the cost parameters of two competing retailers are similar. In addition, sensitivity analysis is conducted in this chapter to study how retailers' optimal investment thresholds change as the values of parameters in this equivalent risk neutral evaluation approach change. It is concluded that the optimal store's value that triggers the retailer to invest and opens the store, increases when the value of log-volatility of retail market potential (parameter (σ)) or the risk-free rate of return (parameter (r_f)) increases. This optimal value decreases as the value of opportunity cost (parameter (δ)) increases. However, the retailer's optimal investment threshold in terms of the state variable (X) increases as the value of (δ) or (σ) increases. This optimal threshold decreases as the value of (r_f) increases. Note that changes in the value of any of these parameters do not impact the order in which retailers enter the competitive market.

Comparative studies between the equivalent risk neutral evaluation and the dynamic programming approaches in Chapter 4 show that the retailers' critical investment values are different under each approach. However, it is concluded that one can confidently use the dynamic programming approach to evaluate the retailer's investment option in the dynamic market that is expected to grow at the high rate. Particular attention should be allocated to the retailer's investment option in other market conditions specifically into the estimation of opportunity cost. An appropriate estimation procedure is required to be developed in order to determine the value of opportunity cost (parameter (δ)) for investments in competitive, dynamic retail markets.

In Chapter 5, we empirically examine an important aspect of our theoretical discussion in Chapter 3 and Chapter 4, namely, that *the big retailer invests and opens a store relatively later and at higher market potential in markets in which the small retailer has a store opened compared to markets in which the small retailer does not have a store opened*. It is concluded that across the state of Georgia, Wal-Mart opens a store relatively later at higher market population (as proxy for market potential) in markets in which Dollar General has a store opened compared to markets in which Dollar General does not have a store opened. Therefore, these empirical observations are indicative of our theoretical results in sections 3.9.4 and 4.5.4. It should be noted that our empirical study does not validate our theoretical work. However, these results do provide supportive evidence for our investment analysis theory that is developed in this thesis.

Overall the contributions of this thesis can be summarized, as follows.

- **Developed a conceptual investment analysis framework based on the real options methodology for a retailer's investment option in a competitive,**

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dynamic market: this framework summarized the similarities and the differences between a retailer's investment option in a competitive, dynamic market and a call option on a common stock. In addition, this framework categorized different types of investment options in retail markets and provides a strategic view towards retail stores as flexible assets of a retail firm.

- **Developed an integrated investment analysis approach based on the dynamic programming and the equivalent risk neutral evaluation methodology to determine a retailer's optimal investment threshold in a competitive, dynamic market**: we developed two separate models based on the dynamic programming and the equivalent risk neutral evaluation approaches to evaluate a retailer's investment option in a competitive, dynamic market. These investment analysis models considered the retail management flexibility, the dynamic uncertainty of retail markets, and the competition effects in a single computational framework. In addition, my theoretical options-based models indicate that that the small retailer prefers to invest earlier in the market when it expects an entry from the big retailer. This early investment of the small retailer provides a first-mover advantage for the small retailer and delays the entry of the big retailer to the market. Therefore, my theoretical model indicates that the big retailer ought to invest relatively later and at higher market potential in markets with a small retailer compared to markets without a small retailer.

- **Provided empirical evidence for our theoretical result that the big retailer (Wal-Mart in our case study) invests relatively later and at higher market potential in markets with a small retailer (Dollar General in our case study)**

compared to markets without a small retailer: we conducted an empirical study in the state of Georgia and shows how in practice Wal-Mart follows the preferred strategy of a big retailer and opens stores later and at higher population in markets with Dollar General compared to markets without Dollar General

- **Conducted a comparative study between the dynamic programming and the equivalent risk neutral evaluation approaches**: it is concluded that although the equivalent risk neutral evaluation approach determines the correct market-value of a retailer's investment option the dynamic programming approach is a more practical investment analysis approach. It was shown that the dynamic programming approach only provides a reliable approximation for the retailer's optimal investment threshold in markets whose expected growth rate is high. In addition, our sensitivity analysis results show that the change in the value of the following pairs of model parameters impacts the optimal retailer's investment threshold in a similar fashion: the discount rate (ρ) in dynamic programming and the opportunity cost (δ) in equivalent risk neutral evaluation, the expected growth rate (α) in dynamic programming and risk-free rate of return (r_f) in equivalent risk neutral evaluation, and the log-volatility of retail market potential (σ) in dynamic programming and log-volatility of replicating portfolio (σ) in equivalent risk neutral evaluation.

However, our empirical study should be extended to other retail markets located in other states to examine whether the same pattern holds across the nation. In addition, similar case studies should be conducted for other discount retailers to examine their investment behaviors under the competition effect. For instance, it is interesting to study

the competition between retailers with similar cost structures such as Target and Wal-Mart or Dollar General and Family Dollar in order to test some of our theoretical results in Chapter 3 and Chapter 4.

In Chapter 5, we use market miles in terms of a notional circle around the store as our approach to determine the Wal-Mart store's market. However, we can use market miles in terms of actual traveling distance between Wal-Mart and Dollar General stores and conduct similar data analysis to ensure our results still hold.

All in all, this thesis is based on many assumptions that are used for simplifying the evaluation process. Relaxing any of these assumptions can provide opportunities for further research. For instance in this thesis, we use a simple demand function, which is exogenous to our investment analysis model to characterize the inverse relationship between price and quantity in a retail market. This model is based on an assumption that both retailers provide a product at the price in a retail market. We also assume that customers only consider the price of a product at their decision of choosing a retail store for shopping. Therefore, customers are indifferent between shopping at any of these two retail stores. These assumptions are made to facilitate the formulation and ease the options calculation.

However, in the real world retailers compete on price and hence, the price of a product may or may not be the same in two retailers' shelves. This discussion becomes more interesting and of course, challenging when we consider that a retailer also decides about another decision variable, which describes the service (or quality) level of its store. It is indicated that the output of a retailer is not only the physical product but also the services that are associated with presenting this product to customers [69]. However, it is

not easy to determine appropriate concepts for different aspects of service and it is even harder to measure these service aspects in the real world.

Therefore, appropriate demand models should be developed to consider the customer utility and address the way customers make tradeoffs between the price of a product and the service of a retail store when they make a decision to choose a store for shopping. One of the most important aspects of service in retailing is the convenience of shopping and accessibility of a retail store. These service attributes are critical in the competition between retailers with similar quality levels in store operation such as Wal-Mart and Dollar General in our case study. Thus, these two service factors should be considered along with other service attributes and the price of a product in describing the customer's utility for choosing a retail store for shopping.

However, these service attributes of retailing introduce several decision variables for a retailer in the evaluation process of a retailer's investment option and hence, make our options formulation difficult. We should consider that a retailer should determine an optimal combination of the price of a product and the service attributes of its store considering its own cost structure, its competitor's cost structure, and the customer utility in order to maximize its profit. This represents a highly complicate problem with many decision variables. Particularly, finding the equilibrium state of the market in the competition between two retailers is not an easy problem in game theory when we work with many decision variables for each player (i.e., retailer). Future research is needed to develop more appropriate demand models that describe the relationship between price, quantity, and service for the retail activity. Perhaps, works such as [70, 71] could provide a good start to understand the economics of retail firms.

In addition in this thesis, we do not use any explicit model to describe the supply side of retail market, i.e., we do not use any model that explicitly determines the relationship between the price of the notional product and the total quantity of this product that a retailer can provide to the market at this price. Instead, we use three cost parameters to characterize the important aspects of retailing business at higher abstract level. In future works, an appropriate supply curve such as the Cobb-Douglas production function [72] should be developed and integrated in our investment analysis approach to describe the supply side of the retail market. The supply function is used to study how the retailer's investment behavior changes as the values of its cost parameters change.

We also assume that customers have perfect knowledge regarding the existence and the location of Wal-Mart and Dollar General stores. We also assume that customers only choose between these two stores when it is time to shop for some products that they both provide. However, these assumptions may or may not be true in the actual practice. Structured customer surveys should be conducted across these stores to validate these assumptions and evaluate our theoretical results.

In addition, in this thesis we assume that the construction of a retail store is instantaneous, i.e., as soon a retailer decides to exercise its investment option he can immediately open a store. However, in the real world finding a place to open a store is not immediate and takes a substantial amount of time. This is particularly true for big retailers such as Wal-Mart that require a large site and extensive development. Future work should include consideration of construction time, which could be very different for large and small retailers. There are a few research papers that address the time to build issue in the options valuation and investment decisions, for instance [116].

Further, our investment analysis approach should be extended to consider the valuation of opening a series of stores in several retail markets. This investment opportunity requires a multi-stage options valuation technique that has been developed and used in different domains, for instance in investments in information technology [117, 118] and R&D value creation [119].

Last but not least, this thesis is the first step to understand the dynamics of investments in retail markets. The conceptual frameworks and the evaluation procedures are general and can be adjusted and applied to investments options in other services. Similar studies should be conducted in other service industries and compared with our analysis in the retail industry. Similarities, differences, challenges, and other related issues should be summarized in order to enhance our understanding regarding the application of the financial options methodology for the valuation of the real world investment problems in different services.

Appendix A

Proof of the Optimal Retailer's Quantity in the Monopoly Market

We know that $((X(t)+VC_2-2VC_1)\geq 0)$ and $((X(t)+VC_1-2VC_2)<0)$ since $(Q_1^D(t)\geq 0)$ and $(Q_2^D(t)<0)$. Therefore, $(((X(t)+VC_2-2VC_1)- (X(t)+VC_1-2VC_2))>0)$ or $(VC_2>VC_1)$. Under this circumstance in the retail market, retailer 1 can optimize its profit function by choosing the optimal monopoly quantity value as its decision variable. When retailer 1 chooses to provide $Q_1^M(t)$ as the value of its quantity of the product in the retail market the retailer 2's best response follows Equation (A1).

$$Q_2(t) = R_2(Q_1^M(t)) = \frac{-\gamma Q_1^M(t) + X(t) - VC_2}{2\gamma}$$

$$= \frac{-\gamma \dfrac{X(t)-VC_1}{2\gamma} + X(t) - VC_2}{2\gamma} = \frac{VC_1 - 2VC_2 + X(t)}{4\gamma} \tag{A1}$$

It can be seen that the value of retailer 2's best response function $Q_2(t)$ is negative since $((X(t)+VC_1-2VC_2)<0)$. Since retailer 2 cannot provide any positive quantity of the product the market structure becomes monopoly for the retailer 1. Hence, the optimal value of the retailer 2's decision variable will be equal to its optimal monopoly quantity – described in Equation (3.7) – and the retailer 2's optimal quantity is zero under this market situation.

Appendix B

Derivation of the Final Value in the Monopoly Market

In this appendix, we determine the final value of retailer i in the monopoly market that is used in the procedure to find retailer i's NPV in the monopoly, dynamic market in section 3.7.1.1. Assume the value of X at the final node under consideration is $X_F(t)$. Retailer i's optimal quantity of the product ($Q_{i,F}{}^M(t)$) and respective optimal profit ($\Pi_{i,F}{}^M(t)$) are calculated using Equations (3.6) and (3.7) and the results are summarized below in Equations (B1) and (B2), respectively. Note that X(t) is replaced by $X_F(t)$ in these formula.

$$Q_{i,F}^M(t) = \begin{cases} \dfrac{X_F(t) - VC_i}{2\gamma} & When\,(X_F(t) \geq VC_i) \\ 0 & Otherwise \end{cases} \quad i = 1,2 \tag{B1}$$

$$\Pi_{i,F}{}^M(t) = \begin{cases} \dfrac{(X_F(t))^2}{4\gamma} - \dfrac{(VC_i)(X_F(t))}{2\gamma} + \dfrac{VC_i{}^2}{4\gamma} - FC_i & When\,(X_F(t) \geq VC_i)_{\;i=1,2} \\ -FC_i & Otherwise \end{cases} \tag{B2}$$

To simplify the final value calculation, we assume that the value of X does not change randomly after the final time step. In fact, this value stays constant for ever any time step after the final time step. The calculation error associated with this assumption decreases as the number of time steps in the lattice increases since the final value is going to be discounted at the very low rate. Therefore, retailer i does not need to change its decision variable ever after the final time step and continues to provide the same optimal

monopoly quantity of the product $Q_{i,F}^M(t)$ to the market. Therefore, retailer i receives the cash flow stream of the value $(\Pi_{i,F}^M(t))$ from the last time step until infinity. The value of this perpetual cash flow of optimal monopoly profit at the last time step determines the final value of retailer i's investment in this monopoly market, which is denoted by $FV_i^M(t)$ corresponding to the value of $X_F(t)$ at any node in the final time step. The continuous stream of retailer i's monopoly cash flow $(\Pi_{i,F}^M(t))$ should be discounted to the final time step using the discount rate (ρ) in order to determine the final value $(FV_i^M(t))$. This is summarized in Equation (B3).

$$FV_i^M(t) = \int_0^\infty \Pi_{i,F}^M(\tau)e^{-\rho\tau}\,d\tau \tag{B3}$$

Substituting from Equation (B3) into the above integral we derive Equation (B4) for the final value of retailer i's investment at the last time step $(FV_i^M(t))$.

$$FV_i^M(t) = \begin{cases} \dfrac{1}{\rho}(\dfrac{(X_F(t))^2}{4\gamma} - \dfrac{(VC_i)(X_F(t))}{2\gamma} + \dfrac{VC_i^2}{4\gamma} - FC_i) & When\,(X_F(t) \geq VC_i) \\ -\dfrac{FC_i}{\rho} & Otherwise \end{cases} \quad i = 1,2 \tag{B4}$$

Derivation of the Final Value in the Duopoly Market

In this appendix, we determine the final value of retailer 1 and 2 in the duopoly market that are used in the procedure to find retailer 1's and 2's NPVs in the duopoly, dynamic market in section 3.7.1.2. Similar to Appendix 2, assume the value of X at the final node under consideration is $X_F(t)$. Retailer 1's and 2's optimal quantities of the product ($Q_{1,F}^D(t)$ and $Q_{2,F}^D(t)$) and respective optimal profits ($\Pi_{1,F}^D(t)$ and $\Pi_{2,F}^D(t)$) are calculated using Equations (3.11) and (3.12), respectively. These values are summarized in Equations (C1) and (C2), respectively. Note that X(t) is replaced by $X_F(t)$ in these formula.

$$
\begin{cases}
when\,((VC_2 - 2VC_1 + X_F(t)) \geq 0)\,\&\,((VC_1 - 2VC_2 + X_F(t)) \geq 0) \\
\quad \begin{cases}
Q_{1,F}^D(t) = \dfrac{VC_2 - 2VC_1 + X_F(t)}{3\gamma} \\[2mm]
Q_{2,F}^D(t) = \dfrac{VC_1 - 2VC_2 + X_F(t)}{3\gamma}
\end{cases} \\
when\,((VC_2 - 2VC_1 + X_F(t)) \geq 0)\,\&\,((VC_1 - 2VC_2 + X_F(t)) < 0)\,\&\,(X_F(t) \geq VC_1) \\
\quad \begin{cases}
Q_{1,F}^D(t) = Q_{1,F}^M(t) = \dfrac{X_F(t) - VC_1}{2\gamma} \\[2mm]
Q_{2,F}^D(t) = 0
\end{cases} \\
when\,((VC_2 - 2VC_1 + X_F(t)) < 0)\,\&\,((VC_1 - 2VC_2 + X_F(t)) \geq 0)\,\&\,(X_F(t) \geq VC_2) \\
\quad \begin{cases}
Q_{1,F}^D(t) = 0 \\[2mm]
Q_{2,F}^D(t) = Q_{2,F}^M(t) = \dfrac{X_F(t) - VC_2}{2\gamma}
\end{cases} \\
Otherwise \\
\quad \begin{cases}
Q_{1,F}^D(t) = 0 \\[2mm]
Q_{2,F}^D(t) = 0
\end{cases}
\end{cases}
$$

$$(C1)$$

$$
\begin{cases}
when\ ((VC_2 - 2VC_1 + X_F(t)) \geq 0)\ \&\ ((VC_1 - 2VC_2 + X_F(t)) \geq 0) \\
\quad \begin{cases}
\Pi_{1,F}^D(t) = \dfrac{(X_F(t))^2}{9\gamma} + \dfrac{2(VC_2 - 2VC_1)(X_F(t))}{9\gamma} + \dfrac{(VC_2 - 2VC_1)^2}{9\gamma} - FC_1 \\[4mm]
\Pi_{2,F}^D(t) = \dfrac{(X_F(t))^2}{9\gamma} + \dfrac{2(VC_1 - 2VC_2)(X_F(t))}{9\gamma} + \dfrac{(VC_1 - 2VC_2)^2}{9\gamma} - FC_2
\end{cases} \\
when\ ((VC_2 - 2VC_1 + X_F(t)) \geq 0)\ \&\ ((VC_1 - 2VC_2 + X_F(t)) < 0)\ \&\ (X_F(t) \geq VC_1) \\
\quad \begin{cases}
\Pi_{1,F}^D(t) = \Pi_{1,F}^M(t) = \dfrac{(X_F(t))^2}{4\gamma} - \dfrac{(VC_1)(X_F(t))}{2\gamma} + \dfrac{VC_1^{\,2}}{4\gamma} - FC_1 \\[4mm]
\Pi_{2,F}^D(t) = -FC_2
\end{cases} \\
when\ ((VC_2 - 2VC_1 + X_F(t)) < 0)\ \&\ ((VC_1 - 2VC_2 + X_F(t)) \geq 0)\ \&\ (X_F(t) \geq VC_2) \\
\quad \begin{cases}
\Pi_{1,F}^D(t) = -FC_1 \\[2mm]
\Pi_{2,F}^D(t) = \Pi_{2,F}^M(t) = \dfrac{(X_F(t))^2}{4\gamma} - \dfrac{(VC_2)(X_F(t))}{2\gamma} + \dfrac{VC_2^{\,2}}{4\gamma} - FC_2
\end{cases} \\
Otherwise \\
\quad \begin{cases}
\Pi_{1,F}^D(t) = -FC_1 \\[2mm]
\Pi_{2,F}^D(t) = -FC_2
\end{cases}
\end{cases}
\tag{C2}
$$

Similar to Appendix 2, we use a simplification assumption to calculate retailer 1's and 2's final values. We assume that the value of X does not change randomly after the final time step. In fact, this value stays constant for ever any time step after the final time step. Therefore, retailer 1 and 2 do not need to change their decision variables ever after the final time step and continue to provide the same optimal duopoly quantities of the product ($Q_{1,F}^D(t)$ and $Q_{2,F}^D(t)$) to the market. Therefore, retailer 1 and 2 receive the cash flow streams of the values ($\Pi_{1,F}^D(t)$ and $\Pi_{2,F}^D(t)$) from the last time step until infinity. The value of these perpetual cash flows of optimal duopoly profits at the last time step determine the final value of retailer 1's and 2's investments in this duopoly market, which are denoted by $FV_1^D(t)$ and $FV_2^D(t)$, respectively corresponding to the value of $X_F(t)$ at any node in the final time step. The continuous stream of retailer 1's and 2's

duopoly cash flows ($\Pi_{1,F}{}^D$(t) and $\Pi_{2,F}{}^D$(t)) should be discounted to the final time step using the discount rate (ρ) in order to determine the final values ($FV_1{}^D$(t) and $FV_2{}^D$(t)). This is summarized in Equation (C3).

$$FV_i^D(t) = \int_0^\infty \Pi_{i,F}^D(\tau)e^{-\rho\tau}\,d\tau \qquad i=1,2$$

$$(C3)$$

Substituting from Equation (C2) into the above integral we derive Equation (C4) for the final values of retailer 1's and 2's investments at the last time step ($FV_1{}^D$(t) and $FV_2{}^D$(t)).

$$
\begin{cases}
when\,((VC_2 - 2VC_1 + X_F(t)) \geq 0)\,\&\,((VC_1 - 2VC_2 + X_F(t)) \geq 0) \\
\quad\begin{cases}
FV_1^D(t) = \dfrac{1}{\rho}\left(\dfrac{(X_F(t))^2}{9\gamma} + \dfrac{2(VC_2 - 2VC_1)(X_F(t))}{9\gamma} + \dfrac{(VC_2 - 2VC_1)^2}{9\gamma} - FC_1\right) \\[2ex]
FV_2^D(t) = \dfrac{1}{\rho}\left(\dfrac{(X_F(t))^2}{9\gamma} + \dfrac{2(VC_1 - 2VC_2)(X_F(t))}{9\gamma} + \dfrac{(VC_1 - 2VC_2)^2}{9\gamma} - FC_2\right)
\end{cases} \\
when\,((VC_2 - 2VC_1 + X_F(t)) \geq 0)\,\&\,((VC_1 - 2VC_2 + X_F(t)) < 0)\,\&\,(X_F(t) \geq VC_1) \\
\quad\begin{cases}
FV_1^D(t) = FV_1^M(t) = \dfrac{1}{\rho}\left(\dfrac{(X_F(t))^2}{4\gamma} - \dfrac{(VC_1)(X_F(t))}{2\gamma} + \dfrac{VC_1^2}{4\gamma} - FC_1\right) \\[2ex]
FV_2^D(t) = -\dfrac{FC_2}{\rho}
\end{cases} \\
when\,((VC_2 - 2VC_1 + X_F(t)) < 0)\,\&\,((VC_1 - 2VC_2 + X_F(t)) \geq 0)\,\&\,(X_F(t) \geq VC_2) \\
\quad\begin{cases}
FV_1^D(t) = -\dfrac{FC_1}{\rho} \\[2ex]
FV_2^D(t) = FV_2^M(t) = \dfrac{1}{\rho}\left(\dfrac{(X_F(t))^2}{4\gamma} - \dfrac{(VC_2)(X_F(t))}{2\gamma} + \dfrac{VC_2^2}{4\gamma} - FC_2\right)
\end{cases} \\
Otherwise \\
\quad\begin{cases}
FV_1^D(t) = -\dfrac{FC_1}{\rho} \\[2ex]
FV_2^D(t) = -\dfrac{FC_2}{\rho}
\end{cases}
\end{cases}
$$

$$(C4)$$

Appendix D

A List of Wal-Mart Stores in Georgia Divided into Two Market Types: Competitive versus Noncompetitive Markets

In this Appendix, we provide four tables that summarize a list of Wal-Mart stores in the state of Georgia considering the competition effect from Dollar General. Table D.1 and Table D.2 summarize a list of Wal-Mart stores in competitive and noncompetitive markets in the state of Georgia, respectively, when market miles equal to 15 miles. Table D.1 provides a list for Wal-Mart store number, address, and the year store opened as well as Dollar General store number in its nearby competitive market. Table D.2 provides a list for Wal-Mart store number, address, and the year store opened in noncompetitive markets.

Table D.1. A list of Wal-Mart stores located in competitive markets in the state of Georgia along with the competitive Dollar General Store Number located in the nearby market (Market miles = 15 miles).

Index	Store Number	Year Opened	Market Population	Wal-Mart Store Address	Dollar General Store Number
1	3	1988	93,642	30983 Highway 441 S, Commerce, GA, 30529	997
2	459	1983	91,246	9218 Highway 278 NE, Covington, GA, 30014	85
3	618	1984	303,190	4166 Jimmy Lee Smith Pkwy, Hiram, GA, 30141	613
4	669	1981	45,157	2545 E Walnut Ave, Dalton, GA, 30721	46
5	**756**	**1984**	**42,597**	**13427 Highway 27, Trion, GA, 30753**	**101**
6	858	1985	32,949	361 8th Ave NE, Cairo, GA, 39828	849
7	864	1985	541,411	3109 E 1st St, Vidalia, GA, 30474	674
8	**937**	**1986**	**24,827**	**2795 Chastain Meadows Pkwy, Marietta, GA, 30066**	**613**
9	1018	1987	23,639	1099 Indian Dr, Eastman, GA, 31023	659
10	1061	1987	182,433	136 E Jarman St, Hazlehurst, GA, 31539	855
11	1076	1987	30,226	1401 Gray Hwy, Macon, GA, 31211	2168
12	1111	1987	51,641	1572 Anderson Hwy, Hartwell, GA, 30643	76
13	1112	1987	59,657	855 N Church St, Thomaston, GA, 30286	2048
14	1122	1987	216,777	3886 Hwy 17, Toccoa, GA, 30577	948
15	**1153**	**1987**	**46,361**	**6020 Harrison Rd, Macon, GA, 31216**	**2059**

16	1181	1988	1,124,138	1785 Cobb Pkwy S, Marietta, GA, 30060	613
17	1215	1988	58,838	450 W Belmont Dr, Calhoun, GA, 30701	24
18	1367	1989	175,064	2720 Watson Blvd, Warner Robins, GA, 31093	2059
19	1403	1990	62,812	2160 Hwy 441 N, Cornelia, GA, 30531	948
20	1488	1990	258,073	7001 Concourse Pkwy, Douglasville, GA, 30134	613
21	1586	1990	888,837	1133 E West Connector SW, Austell, GA, 30106	613
22	1658	1991	38,025	2205 Harrison Rd, Thomson, GA, 30824	631
23	1701	1991	27,301	955 Lower Hood St, Elberton, GA, 30635	2085
24	1720	1991	580,312	1550 Scenic Hwy, Snellville, GA, 30078	2564
25	2154	1994	925,860	2635 Pleasant Hill Rd, Duluth, GA, 30096	2564
26	2360	1995	1,475,714	4725 Ashford Dunwoody Rd, Atlanta, GA, 30338	2564
27	2475	1996	386,985	1436 Dogwood Dr, Conyers, GA, 30013	85
28	2513	1997	59,860	270 Wal Mart Way, Dahlonega, GA, 30533	571
29	2584	1998	1,473,316	4375 Lawrenceville Hwy, Tucker, GA, 30084	2564
30	2615	1998	91,218	3274 Inner Perimeter Rd, Valdosta, GA, 31602	2440
31	2630	1998	27,617	1100 N 1st St, Jesup, GA, 31545	301
32	2732	2000	138,265	600 St Hwy 61, Villa Rica, GA, 30180	630
33	2733	1999	188,668	1009 Saint Patricks Dr, Perry, GA, 31069	2059
34	2753	2000	20,624	1455 Highway 441 S, Clayton, GA, 30525	3251
35	2811	2000	146,217	4375 Lexington Rd, Athens, GA, 30605	3547
36	2860	2000	220,574	160 Pooler Pkwy, Pooler, GA, 31322	2956
37	2890	2004	107,583	5955 Zebulon Rd, Macon, GA, 31210	2168
38	2941	2002	368,095	5200 Windward Pkwy, Alpharetta, GA, 30004	1025
39	2988	2000	1,024,584	2625 N Hwy 27, La Fayette, GA, 30728	101
40	3201	2001	973,667	101 Willow Ln, McDonough, GA, 30253	1923
41	3205	2001	270,335	1100 Thornton Rd, Lithia Springs, GA, 30122	613
42	3388	2005	919,722	1400 Lawrenceville Hwy, Lawrenceville, GA, 30044	2564
43	3389	2006	777,867	3435 Centerville Hwy, Snellville, GA, 30039	85
44	3403	2006	267,972	3615 Marietta Hwy, Dallas, GA, 30157	613
45	3461	2002	187,624	2717 Highway 54, Peachtree City, GA, 30269	4778
46	3462	2002	122,191	3245 Lawrenceville Suwanee Rd, Suwanee, GA, 30024	2564
47	3471	2002	158,769	3105 Cobb Pkwy NW, Kennesaw, GA, 30152	613
48	3570	2002	583,157	4471 Washington Rd, Evans, GA, 30809	3877
49	3709	2006	1,089,398	2427 Gresham Rd SE, Atlanta, GA, 30316	2498
50	3750	2006	708,974	502 Booth Rd, Warner Robins, GA, 31088	2059
51	5151	2004	601,104	825 Cartersville Hwy SE, Rome, GA, 30161	2584
52	5173	2004	1,429,773	815 Shugart Rd, Dalton, GA, 30720	46
53	5252	2004	25,105	4221 Atlanta Hwy, Loganville, GA, 30052	2893
54	5275	2005	30,307	6435 Bells Ferry Rd, Woodstock, GA, 30189	1757
55	5363	2005	1,080,083	11499 Tara Blvd, Lovejoy, GA, 30250	2498
56	5390	2005	735,690	210 Cobb Pkwy S, Marietta, GA, 30060	613
57	5392	2005	1,702,086	980 W Parker St, Baxley, GA, 31513	10944
58	5422	2005	174,802	500 E Alice St, Bainbridge, GA, 39819	2349

Table D.2. A list of Wal-Mart stores located in noncompetitive markets in the state of Georgia (Market miles = 15 miles).

Index	Store Number	Year Opened	Market Population	Wal-Mart Store Address
1	494	1982	198,076.75	1025 Bullsboro Dr, Newnan, GA, 30265
2	510	1983	57,148.87	400 Shallowford Rd, Gainesville, GA, 30504
3	518	1983	90,722.84	1550 Riverstone Pkwy, Canton, GA, 30114
4	520	1984	66,836.80	440 Atlanta Hwy NW, Winder, GA, 30680
5	548	1983	71,686.52	630 Collins Hill Rd, Lawrenceville, GA, 30045

6	555	1983	71,874.01	2101 Veterans Blvd, Dublin, GA, 31021
7	556	1983	103,987.48	2425 Memorial Dr, Waycross, GA, 31503
8	575	1984	113,095.58	12182 Hwy 92, Woodstock, GA, 30188
9	588	1984	274,862.60	2825 Ledo Rd, Albany, GA, 31707
10	593	1984	35,681.63	1450 SE Bowens Mill Rd, Douglas, GA, 31533
11	594	1984	44,400.65	125 Pavillion Pkwy, Fayetteville, GA, 30214
12	605	1983	203,834.56	1955 E Montgomery Xrd, Savannah, GA, 31406
13	606	1983	197,060.43	4725 US Hwy 80 E, Savannah, GA, 31410
14	614	1984	106,374.34	803 New Franklin Rd, LaGrange, GA, 30240
15	615	1984	451,522.36	101 Market Place Blvd, Cartersville, GA, 30121
16	635	1981	122,164.79	6000 Ogeechee Rd, Savannah, GA, 31419
17	639	1981	29,457.24	150 Altama Connector Blvd, Brunswick, GA, 31525
18	658	1981	373,320.91	2510 Redmond Cir, Rome, GA, 30165
19	686	1985	58,057.90	120 Benjamin H Hill Dr W, Fitzgerald, GA, 31750
20	722	1981	86,149.62	1735 S Park St, Carrollton, GA, 30117
21	727	1981	603,053.87	109 Davis Rd, Cedartown, GA, 30125
22	745	1984	39,048.49	5600 N Henry Blvd, Stockbridge, GA, 30281
23	754	1984	34,638.25	730 Northside Dr E, Statesboro, GA, 30458
24	758	1984	26,438.53	1809 US Highway 280 E, Americus, GA, 31709
25	780	1985	86,106.86	2050 W Spring St, Monroe, GA, 30655
26	787	1985	727,866.44	7050 Hwy 85, Riverdale, GA, 30274
27	836	1985	20,271.59	6586 GA Highway 40 E, Saint Marys, GA, 31558
28	843	1985	24,417.54	131 N US Hwy 19 & Sylvester R, Camilla, GA, 31730
29	855	1986	76,676.41	4375 Jonesboro Rd, Union City, GA, 30291
30	856	1985	42,936.44	404 Highway 27 N Byp, Bremen, GA, 30110
31	862	1985	145,917.95	751 W Oglethorpe Hwy, Hinesville, GA, 31313
32	878	1985	575,090.49	1500 Market Place Blvd, Cumming, GA, 30041
33	889	1986	45,479.92	15328 S US Hwy 19, Thomasville, GA, 31757
34	899	1986	80,403.76	340 Norman Dr, Valdosta, GA, 31601
35	907	1986	36,989.83	120 N Lee St, Forsyth, GA, 31029
36	932	1986	101,518.04	1569 N Expressway, Griffin, GA, 30223
37	952	1986	40,643.66	641 East Bypass Southeast, Moultrie, GA, 31768
38	1006	1986	30,389.62	1215 E 16th Ave, Cordele, GA, 31015
39	1011	1986	44,292.59	434 S Columbia Ave, Rincon, GA, 31326
40	1024	1987	21,632.54	414 S Main St, Swainsboro, GA, 30401
41	1047	1987	882,808.02	6065 Jonesboro Rd, Morrow, GA, 30260
42	1070	1987	27,363.48	88 Highland Xing, Ellijay, GA, 30540
43	1072	1987	45,732.24	1830 US Highway 82 W, Tifton, GA, 31794
44	1121	1987	48,942.34	2592 N Columbia St, Milledgeville, GA, 31061
45	1143	1988	20,282.83	1308 S Harris St, Sandersville, GA, 31082
46	1184	1988	926,461.76	1825 Rockbridge Rd, Stone Mountain, GA, 30087
47	1227	1988	234,451.26	596 Bobby Jones Expwy, Augusta, GA, 30907
48	1293	1989	244,520.56	3209 Deans Bridge Rd, Augusta, GA, 30906
49	1311	1989	192,904.20	2801 B Airport Thwy, Columbus, GA, 31909
50	1314	1989	261,810.90	3795 Buford Dr, Buford, GA, 30519
51	1338	1989	196,773.13	4701 Buena Vista Rd, Columbus, GA, 31907
52	1340	1989	977,266.61	5401 Fairington Rd, Lithonia, GA, 30038
53	1363	1989	26,610.59	1681 Eatonton Rd, Madison, GA, 30650
54	1373	1990	870,318.43	4004 Lawrenceville Hwy NW, Lilburn, GA, 30047
55	1400	1990	135,839.72	1911 Epps Bridge Pkwy, Athens, GA, 30606
56	1458	1990	99,024.93	3040 Battlefield Pkwy, Fort Oglethorpe, GA, 30742
57	1578	1990	786,417.70	970 Mansell Rd, Roswell, GA, 30076
58	1766	1992	895,678.74	3100 Johnson Ferry Rd, Marietta, GA, 30062
59	2754	1999	29,007.72	1500 N Liberty St, Waynesboro, GA, 30830
60	5482	2005	40,100.18	201 Wal Mart Drive, Eatonton, GA, 31024

Table D.3 and

Table D.4 summarize a list of Wal-Mart stores in competitive and noncompetitive markets in the state of Georgia, respectively, when market miles equal to 10 miles. Table D.3 provides a list for Wal-Mart store number, address, and the year store opened as well as Dollar General store number in its nearby competitive market.

Table D.4 provides a list for Wal-Mart store number, address, and the year store opened in noncompetitive markets. It can be observed that when we reduce market miles from 15 miles to 10 miles only four Wal-Mart stores' markets (store numbers 756, 937, 1181, and 1403 as shown in bold red in Table D.1 and Table D.4) change from competitive to noncompetitive markets. In addition, the Dollar General store that competes with a particular Wal-Mart store changes in many cases when we decrease the market miles from 15 miles to 10 miles. These stores are highlighted in Table D.1 and Table D.3.

Table D.3. A list of Wal-Mart stores located in competitive markets in the state of Georgia along with the competitive Dollar General Store Number located in the nearby market (Market miles = 10 miles).

Index	Store Number	Year Opened	Market Population	Wal-Mart Store Address	Dollar General Store Number
1	3	1988	20,329	30983 Highway 441 S, Commerce, GA, 30529	997
2	459	1983	43,523	9218 Highway 278 NE, Covington, GA, 30014	85
3	618	1984	109,774	4166 Jimmy Lee Smith Pkwy, Hiram, GA, 30141	613
4	669	1981	68,592	2545 E Walnut Ave, Dalton, GA, 30721	46
5	858	1985	18,688	361 8th Ave NE, Cairo, GA, 39828	849
6	864	1985	22,463	3109 E 1st St, Vidalia, GA, 30474	674
7	1018	1987	12,167	1099 Indian Dr, Eastman, GA, 31023	659
8	1061	1987	12,546	136 E Jarman St, Hazlehurst, GA, 31539	855
9	1076	1987	147,762	1401 Gray Hwy, Macon, GA, 31211	2168
10	1111	1987	17,265	1572 Anderson Hwy, Hartwell, GA, 30643	76
11	1112	1987	34,804	855 N Church St, Thomaston, GA, 30286	2048
12	1122	1987	27,765	3886 Hwy 17, Toccoa, GA, 30577	948
13	**1153**	**1987**	**149,724**	**6020 Harrison Rd, Macon, GA, 31216**	**2168**
14	1215	1988	34,481	450 W Belmont Dr, Calhoun, GA, 30701	24
15	1367	1989	96,853	2720 Watson Blvd, Warner Robins, GA, 31093	2059
16	1488	1990	102,357	7001 Concourse Pkwy, Douglasville, GA, 30134	613
17	1586	1990	378,239	1133 E West Connector SW, Austell, GA, 30106	613

18	1658	1991	22,127	2205 Harrison Rd, Thomson, GA, 30824	631
19	1701	1991	15,255	955 Lower Hood St, Elberton, GA, 30635	2085
20	1720	1991	271,336	1550 Scenic Hwy, Snellville, GA, 30078	2564
21	2154	1994	464,079	2635 Pleasant Hill Rd, Duluth, GA, 30096	2564
22	**2360**	**1995**	**712,668**	**4725 Ashford Dunwoody Rd, Atlanta, GA, 30338**	**3164**
23	2475	1996	140,487	1436 Dogwood Dr, Conyers, GA, 30013	85
24	2513	1997	24,295	270 Wal Mart Way, Dahlonega, GA, 30533	571
25	2584	1998	821,875	4375 Lawrenceville Hwy, Tucker, GA, 30084	2564
26	2615	1998	71,508	3274 Inner Perimeter Rd, Valdosta, GA, 31602	2440
27	2630	1998	20,216	1100 N 1st St, Jesup, GA, 31545	301
28	**2732**	**2000**	**72,912**	**600 St Hwy 61, Villa Rica, GA, 30180**	**1749**
29	**2733**	**1999**	**51,844**	**1009 Saint Patricks Dr, Perry, GA, 31069**	**2061**
30	2753	2000	10,767	1455 Highway 441 S, Clayton, GA, 30525	3251
31	2811	2000	112,887	4375 Lexington Rd, Athens, GA, 30605	3547
32	2860	2000	78,064	160 Pooler Pkwy, Pooler, GA, 31322	2956
33	2890	2004	141,620	5955 Zebulon Rd, Macon, GA, 31210	2168
34	2941	2002	401,065	5200 Windward Pkwy, Alpharetta, GA, 30004	1025
35	2988	2000	42,896	2625 N Hwy 27, La Fayette, GA, 30728	101
36	**3201**	**2001**	**125,409**	**101 Willow Ln, McDonough, GA, 30253**	**4030**
37	3205	2001	413,792	1100 Thornton Rd, Lithia Springs, GA, 30122	613
38	3388	2005	577,110	1400 Lawrenceville Hwy, Lawrenceville, GA, 30044	2564
39	**3389**	**2006**	**130,996**	**3435 Centerville Hwy, Snellville, GA, 30039**	**2893**
40	3403	2006	449,675	3615 Marietta Hwy, Dallas, GA, 30157	613
41	**3461**	**2002**	**363,700**	**2717 Highway 54, Peachtree City, GA, 30269**	**1492**
42	**3462**	**2002**	**172,847**	**3245 Lawrenceville Suwanee Rd, Suwanee, GA, 30024**	**1566**
43	**3471**	**2002**	**81,803**	**3105 Cobb Pkwy NW, Kennesaw, GA, 30152**	**1757**
44	**3570**	**2002**	**97,838**	**4471 Washington Rd, Evans, GA, 30809**	**1094**
45	**3709**	**2006**	**216,343**	**2427 Gresham Rd SE, Atlanta, GA, 30316**	**3406**
46	3750	2006	349,031	502 Booth Rd, Warner Robins, GA, 31088	2059
47	5151	2004	306,725	825 Cartersville Hwy SE, Rome, GA, 30161	2584
48	5173	2004	676,583	815 Shugart Rd, Dalton, GA, 30720	46
49	**5252**	**2004**	**13,990**	**4221 Atlanta Hwy, Loganville, GA, 30052**	**1566**
50	5275	2005	21,352	6435 Bells Ferry Rd, Woodstock, GA, 30189	1757
51	5363	2005	416,130	11499 Tara Blvd, Lovejoy, GA, 30250	2498
52	**5390**	**2005**	**305,224**	**210 Cobb Pkwy S, Marietta, GA, 30060**	**2565**
53	5392	2005	963,405	980 W Parker St, Baxley, GA, 31513	10944
54	5422	2005	126,420	500 E Alice St, Bainbridge, GA, 39819	2349

Table D.4. A list of Wal-Mart stores located in noncompetitive markets in the state of Georgia (Market miles = 15 miles).

Index	Store Number	Year Opened	Market Population	Wal-Mart Store Address
1	494	1982	38,815	1025 Bullsboro Dr, Newnan, GA, 30265
2	510	1983	70,571	400 Shallowford Rd, Gainesville, GA, 30504
3	518	1983	36,704	1550 Riverstone Pkwy, Canton, GA, 30114
4	520	1984	38,644	440 Atlanta Hwy NW, Winder, GA, 30680
5	548	1983	121,610	630 Collins Hill Rd, Lawrenceville, GA, 30045
6	555	1983	25,871	2101 Veterans Blvd, Dublin, GA, 31021
7	556	1983	32,311	2425 Memorial Dr, Waycross, GA, 31503
8	575	1984	209,303	12182 Hwy 92, Woodstock, GA, 30188
9	588	1984	104,756	2825 Ledo Rd, Albany, GA, 31707
10	593	1984	21,335	1450 SE Bowens Mill Rd, Douglas, GA, 31533

11	594	1984	184,494	125 Pavillion Pkwy, Fayetteville, GA, 30214
12	605	1983	186,243	1955 E Montgomery Xrd, Savannah, GA, 31406
13	606	1983	171,987	4725 US Hwy 80 E, Savannah, GA, 31410
14	614	1984	44,373	803 New Franklin Rd, LaGrange, GA, 30240
15	615	1984	39,273	101 Market Place Blvd, Cartersville, GA, 30121
16	635	1981	97,573	6000 Ogeechee Rd, Savannah, GA, 31419
17	639	1981	50,091	150 Altama Connector Blvd, Brunswick, GA, 31525
18	658	1981	66,582	2510 Redmond Cir, Rome, GA, 30165
19	686	1985	18,905	120 Benjamin H Hill Dr W, Fitzgerald, GA, 31750
20	722	1981	41,027	1735 S Park St, Carrollton, GA, 30117
21	727	1981	30,717	109 Davis Rd, Cedartown, GA, 30125
22	745	1984	210,417	5600 N Henry Blvd, Stockbridge, GA, 30281
23	754	1984	29,424	730 Northside Dr E, Statesboro, GA, 30458
24	**756**	**1984**	**23,454**	**13427 Highway 27, Trion, GA, 30753**
25	758	1984	25,356	1809 US Highway 280 E, Americus, GA, 31709
26	780	1985	34,405	2050 W Spring St, Monroe, GA, 30655
27	787	1985	335,577	7050 Hwy 85, Riverdale, GA, 30274
28	836	1985	14,006	6586 GA Highway 40 E, Saint Marys, GA, 31558
29	843	1985	14,693	131 N US Hwy 19 & Sylvester R, Camilla, GA, 31730
30	855	1986	227,056	4375 Jonesboro Rd, Union City, GA, 30291
31	856	1985	37,669	404 Highway 27 N Byp, Bremen, GA, 30110
32	862	1985	32,012	751 W Oglethorpe Hwy, Hinesville, GA, 31313
33	878	1985	55,525	1500 Market Place Blvd, Cumming, GA, 30041
34	889	1986	29,864	15328 S US Hwy 19, Thomasville, GA, 31757
35	899	1986	60,628	340 Norman Dr, Valdosta, GA, 31601
36	907	1986	15,461	120 N Lee St, Forsyth, GA, 31029
37	932	1986	60,258	1569 N Expressway, Griffin, GA, 30223
38	**937**	**1986**	**285,067**	**2795 Chastain Meadows Pkwy, Marietta, GA, 30066**
39	952	1986	27,968	641 East Bypass Southeast, Moultrie, GA, 31768
40	1006	1986	20,233	1215 E 16th Ave, Cordele, GA, 31015
41	1011	1986	17,673	434 S Columbia Ave, Rincon, GA, 31326
42	1024	1987	13,280	414 S Main St, Swainsboro, GA, 30401
43	1047	1987	408,213	6065 Jonesboro Rd, Morrow, GA, 30260
44	1070	1987	11,003	88 Highland Xing, Ellijay, GA, 30540
45	1072	1987	35,367	1830 US Highway 82 W, Tifton, GA, 31794
46	1121	1987	37,245	2592 N Columbia St, Milledgeville, GA, 31061
47	1143	1988	13,705	1308 S Harris St, Sandersville, GA, 31082
48	**1181**	**1988**	**513,421**	**1785 Cobb Pkwy S, Marietta, GA, 30060**
49	1184	1988	460,310	1825 Rockbridge Rd, Stone Mountain, GA, 30087
50	1227	1988	210,005	596 Bobby Jones Expwy, Augusta, GA, 30907
51	1293	1989	218,156	3209 Deans Bridge Rd, Augusta, GA, 30906
52	1311	1989	175,535	2801 B Airport Thwy, Columbus, GA, 31909
53	1314	1989	93,018	3795 Buford Dr, Buford, GA, 30519
54	1338	1989	179,463	4701 Buena Vista Rd, Columbus, GA, 31907
55	1340	1989	402,416	5401 Fairington Rd, Lithonia, GA, 30038
56	1363	1989	12,104	1681 Eatonton Rd, Madison, GA, 30650
57	1373	1990	444,539	4004 Lawrenceville Hwy NW, Lilburn, GA, 30047
58	1400	1990	105,349	1911 Epps Bridge Pkwy, Athens, GA, 30606
59	**1403**	**1990**	**32,444**	**2160 Hwy 441 N, Cornelia, GA, 30531**
60	1458	1990	70,897	3040 Battlefield Pkwy, Fort Oglethorpe, GA, 30742
61	1578	1990	347,123	970 Mansell Rd, Roswell, GA, 30076
62	1766	1992	449,149	3100 Johnson Ferry Rd, Marietta, GA, 30062
63	2754	1999	12,270	1500 N Liberty St, Waynesboro, GA, 30830
64	5482	2005	18,110	201 Wal Mart Drive, Eatonton, GA, 31024

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