

# Venture Capital Financing with Staged Investment, Agency Conflicts and Asymmetric Beliefs

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# Venture Capital Financing with Staged Investment, Agency Conflicts and Asymmetric Beliefs

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## SUMMARY

We consider a risk averse entrepreneur who approaches a diversified venture capitalist (VC) for financing of a project with positive potential return. We develop several models that capture key features of the venture financing, including staged investment, VC oversight costs and agency conflicts. The contract between the VC and the EN includes risk-free and pay-performance sensitive compensation. Moral hazard arises because the EN must exert effort for the project to succeed. Our model is novel in that it also allows for asymmetric beliefs about project quality due to the EN's optimism even when the VC and EN face symmetric information.

We first analyze the VC-EN relationship when the VC has bargaining power. We characterize the equilibrium levels for the pay-performance sensitivities, investment and effort over time and show they can be either increasing or decreasing or initially increasing and then decreasing. We find that asymmetric beliefs and risk aversion have opposite effects on the VC-EN relationship. When the EN is moderately more optimistic than the VC, he accepts more risk and exerts more effort and the VC responds with more investment. In contrast, risk aversion reduces effort and investment. Our model predicts a performance-sensitive investment policy where critical milestones must be achieved for investment to continue. These milestones increase with the risk aversion and decrease with the asymmetry in beliefs. Consequently, project duration increases with asymmetric beliefs and decreases with risk aversion.

We calibrate this core model to empirical data and use numerical analysis to demonstrate that the technical and systematic risks have opposite effects. The VC's payoff and the project's value and duration increase with technical risk and decrease with systematic risk.

We analyze the relationship when the EN has bargaining power, and find that the equilibrium and the corresponding implications for venture financing do change. In this setting, the negative effects due to risk aversion are more pronounced. We also find that

if the EN's effort cannot be observed by the VC, then the pay-performance sensitivities, investment and effort all increase.

# CHAPTER I

## INTRODUCTION

Venture capital is the primary means through which innovative ideas are financed, nurtured and brought to fruition and therefore plays a crucial role in economic growth. Indeed, Gompers and Lerner (2001a) calculate that over the years “venture capitalists have created nearly one-third of the total market value of all public companies in the United States.” The “Venture Capitalist-Entrepreneur” (VC-EN) relationship exhibits several proven features, each of which is essential to the understanding of VC financing. First, the process of developing, testing and marketing an innovative idea possesses inherently high levels of technical and systematic risks. The VC and EN have different attitudes towards risk, since the VC is more diversified than the EN. Second, empirical evidence documents that the VC and EN often have divergent views (“asymmetric beliefs”) about the economic potential of the project.<sup>1</sup> As noted in *The Economist*: “Entrepreneurs tend to be wildly over-optimistic; if they were not, they would never get past their first crisis.”<sup>2</sup> Third, several studies document the prevalence of staged investment to mitigate the inefficiencies created by the agency conflicts that naturally arise between the VC and EN. In the presence of imperfect information, staged investment over time is a sensible means to avoid large capital investments before learning more about the project’s true quality.

In this thesis, we develop, to the best of our knowledge, the first theoretical framework of venture capital investment that incorporates the essential features of venture capital relationships in a dynamic setting—the different attitudes and components of risk, asymmetric beliefs, agency conflicts, imperfect information and dynamic learning, staged investment. We examine this framework under three different settings. The first, which we call the Basic

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<sup>1</sup>See, for example, Sahlman (1990), Gladstone and Gladstone (2002), and Landier and Thesmar (2005). Lerner (1998) argues that an entrepreneur’s strong sense of commitment to the firm he founded makes him loathe to admit failure and accept the true value of the firm. Gompers and Lerner (2001b) emphasize the prevalence of high levels of imperfect information about project qualities in venture capital financing.

<sup>2</sup>*The Economist*, April 16, 2005, p. 68.

Model, assumes the VC possesses the bargaining power. In the second setting, named the Shift of Power Model, we assume the EN holds the bargaining power. In the third model, the Unobservable Effort Model, we assume *information asymmetry* between the VC and the EN caused by the VC's inability to observe the EN's effort levels.

We demonstrate that the interactions between risk, asymmetric beliefs and the agency conflicts have a major impact on the key characteristics of venture capital relationships, namely, the economic value they generate, the structure of the long-term contracts between VCs and entrepreneurs, how VC investment is staged over time, and the duration of VC relationships. We examine and characterize the robustness of these results to the assumption of bargaining power and the observability of effort. Theoretical and numerical analysis of our framework suggests several novel testable implications for the financing, development, and economic value of new ventures. Chief among them are:

- i) VC's have significant incentives to "feed" entrepreneur optimism and exploit it to their advantage;
- ii) the equilibrium long-term contract for the EN features either increasing or decreasing pay-performance sensitivities; that is, the EN's compensation will either be always more or less sensitive to performance in earlier stages as compared with later stages;
- iii) the equilibrium staged VC investments over time (contingent on continuation) will either increase, decrease or initially increase and then subsequently decrease;
- iv) firm value and the VC's expected payoff are actually *enhanced* when there is *greater* noise in the perception of project quality, a striking *normative* implication;
- v) the relationship duration *decreases* with the project's systematic risk but *increases* with the project's technical risk or the degree of asymmetry of beliefs;
- vi) the pay-performance sensitivity and investment are lower when the EN has bargaining power as compared to when the VC enjoys bargaining power; and
- vii) the pay-performance sensitivity and investment increase when the VC cannot observe the EN's labor investment.

Our framework incorporates a dynamic principal-agent model where a cash-constrained, risk-averse entrepreneur (EN) with a project approaches a well-diversified, risk-neutral venture capitalist (VC) for financing at the initial date. The project generates potential value through physical capital investments by the VC and human capital (effort) investments by the EN. Both the VC and the EN have imperfect information about the project and may, in general, differ in their initial assessments of the project’s quality with the EN being more optimistic. The VC’s investment in the project may be *staged* over time. Future investment is contingent on intermediate observations of the project’s *termination value*, the fundamental state variable that represents the value of the project from the perspective of “outside” investors.<sup>3</sup> These observations serve as “signals” that enable the VC and the EN to update their assessments of its quality in a Bayesian manner. All payoffs occur when the relationship is terminated.<sup>4</sup> The EN is provided with inter-temporal incentives to invest human capital through a long-term *renegotiation-proof* contract that may depend on the *entire path* of the project’s termination value process. Either the VC or the EN may terminate the relationship at any intermediate date.

Under the assumption that the EN has CARA preferences, we derive and characterize the equilibrium long-term contract between the VC and the EN, which describes the VC’s investments over time, the EN’s path-dependent payoff upon termination, and the inter-temporal performance targets that must be met for the relationship to continue. Keep in mind that the duration of the relationship (or the number of stages of financing) is *endogenously* determined by the characteristics of the underlying project. Conditional on continuation, the VC’s staged investments, the sensitivities of the EN’s compensation to performance over each period (the *pay-performance* sensitivities), and the EN’s effort in each period, are all *deterministic* functions of time. The paths of investment, pay-performance sensitivities, and effort crucially depend on the relative magnitudes of the initial degree

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<sup>3</sup>We assume the VC and the EN possess specific skills and neither is permitted to supplying them to a third party. Hence, the termination value of the project at any date is lower than its *rational expectations* market value, namely, the value of the project under hypothetical full commitment by the VC and the EN, which incorporates the effect of their future physical and human capital investments. The termination value of the project is observable and verifiable, but the rational expectations market value is non-verifiable.

<sup>4</sup>Our analysis could be generalized to incorporate intermediate cash flows without qualitatively altering our main results.

of asymmetry in beliefs about project quality and the *cost of risk*, which increases with the EN's risk aversion and the project's total (systematic + technical) risk. If the initial degree of asymmetry in beliefs is below a threshold relative to the cost of risk, investments, pay-performance sensitivities, and effort increase monotonically over time. If the degree of asymmetry in beliefs is above this threshold, however, the pay-performance sensitivities and effort decrease monotonically over time, while the VC's investment schedule initially increases and subsequently decreases with time. Our theory therefore provides a potential explanation for the significant heterogeneity in contractual structures and investment schedules reported in earlier empirical studies (for example, Gompers, 1995).

The intuition for these results, described in greater detail in the thesis, hinges on the complex interplay among the value-enhancing effort by the EN that is positively affected by his optimism, the costs of risk-sharing due to the EN's risk aversion that is affected by the project's systematic and technical risk, and the effect of both the VC's physical capital investment and the EN's effort on output. The passage of time causes technical risk to be resolved thereby *lowering* the costs of risk-sharing. However, the passage of time also lowers the degree of asymmetry in beliefs of the VC and the EN, since successive project realizations cause the EN to revise his optimistic assessment of project quality. The decline in the degree of asymmetry in beliefs lowers the rents that the VC can extract by exploiting the EN's optimism. If the initial degree of asymmetry in beliefs is below a threshold, the beneficial effect of time on the costs of risk-sharing dominates so that the EN's pay-performance sensitivities and effort increase. As the EN's effort increases over time, the VC optimally *increases* her investment over time. An increase in the project's systematic or technical risk and/or a decrease in the degree of asymmetry in beliefs increases the costs of risk sharing compared with the economic rents that the VC can extract from the EN's optimism. If the degree of asymmetry in beliefs is above a threshold, the EN is willing to accept all the risk of the project so that his risky compensation and effort are initially *high*. The negative effect of the evolution of time on the degree of asymmetry in beliefs, however, dominates its positive effect on the costs of risk-sharing so that the EN's risky compensation effort declines over time. The VC's investments initially increase to "compensate" for the

decrease in effort of the EN. After a certain point in time, however, the decreasing effort of the EN makes it optimal for the VC to also lower her capital investments.

We derive the sensitivity of the equilibrium dynamics to the project's systematic and technical risk and the degree of asymmetry in beliefs. The EN's pay-performance sensitivities decline with risk and increase with the degree of asymmetry in beliefs. The effects of risk and the degree of asymmetry in beliefs on the VC's investment path, however, depend on their relative magnitudes. If the initial degree of asymmetry in beliefs is below a threshold relative to the cost of risk, the VC's investments decrease with systematic and technical risk and increase with the degree of asymmetry in beliefs. If the degree of asymmetry in beliefs is above a threshold relative to the cost of risk, however, the VC's investments actually *increase* with risk in early periods and *decrease* in later periods, whereas the VC's investments actually *decrease* with the degree of asymmetry in beliefs in early periods and *increase* in later periods.

With respect to the duration of the relationship, we demonstrate that it increases with the degree of asymmetry in beliefs and decreases with the EN's risk aversion. An increase in the degree of asymmetry in beliefs and/or a decrease in the EN's risk aversion raises the economic rents that the VC captures due to the EN's optimism relative to the costs of risk-sharing, thereby inducing her to prolong the relationship. The negative relation between duration and the degree of asymmetry in beliefs is consistent with the evidence in Kaplan and Stromberg (2003) that experienced entrepreneurs, who are likely to have more realistic beliefs, receive fewer rounds of financing.

We numerically implement and calibrate the parameters of our structural model to empirical evidence on venture capital financing. We demonstrate that our model does reasonably well in matching data on the durations of venture capital relationships and the distributions of returns from venture capital investment reported by Sahlman (1990) and Gompers (1995). We then analyze the calibrated model and numerically derive the effects of the degree of asymmetry of beliefs, the project's technical and systematic risk and the project's output elasticity of capital on the duration, firm value and VC's expected payoff.

Consistent with our earlier analytical results, EN optimism significantly enhances firm

value as well as the expected payoff to the VC.<sup>5</sup> The increase in the VC's expected payoff due to EN optimism is generally disproportionately greater than the increase in firm value, which reflects the substantial rents that the VC may extract by "feeding" EN optimism. The positive effects of EN optimism are consistent with the empirical evidence reported in Gelderen, Thurik and Bosma (2005). We also find that firm value is positively related to the duration of the relationship, which is also consistent with the evidence in Gompers (1995).

We demonstrate analytically for a two-period model and numerically for the general model that systematic and technical risk have dramatically *opposite* effects on duration, firm value, and the VC's expected payoff. All three output variables generally *increase* with the project's initial technical risk, but *decrease* with its systematic risk. The intuition for these results hinges on a subtle interplay between the effects of technical and systematic risk on the "speed of learning" about project quality, and the mean and variance of the assessments of project quality, which affect the VC's "option value" of continuing the relationship. An increase in the initial technical risk increases the variance of the distribution of project quality assessments, since assessments are more responsive to signals due to higher signal to noise ratios. Hence, the likelihood of "high" realizations of project quality assessments is increased. In the presence of limited liability, where the VC will terminate the relationship if it is no longer profitable for her to continue, the "option value" of continuing the relationship at any date increases, which leads to a higher expected duration, firm value, and expected payoff to the VC. On the other hand, an increase in the project's systematic risk lowers the signal to noise ratio, which generally leads to a decline in the variance of the distribution of project quality assessments, as they are less responsive to intermediate signals. Hence, the "option value" of continuing the relationship declines leading to a shorter expected duration, firm value, and expected payoff to the VC.

We show that duration, firm value, and the VC's expected payoff all decrease with the *physical* capital intensity of the underlying project and *increase* with its human capital

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<sup>5</sup>Firm value is the initial "rational expectations" market value of the firm from the perspective of the VC.

intensity. With a constant returns-to-scale production technology, an increase in the human capital intensity lowers the physical capital intensity and, therefore, increases the relative contribution of the EN's effort. As the EN's human capital is the key driver of value in our model, an increase in the marginal product of human capital increases firm value, duration, and the VC's expected payoff. These results also represent potentially testable implications of our theory.

In the later models of this thesis we check the robustness of our results to some of the assumptions made in the Basic Model. In the Shift of Power Model we assume a competitive VC market and assume the EN enjoys the bargaining power. As a result, the pay-performance sensitivities, capital investment and human effort decrease.

We introduce asymmetric information in the Unobservable Effort Model. We show that when effort is unobservable, the VC will invest more and the EN will receive more incentives to exert effort. The EN will indeed respond with higher effort levels. These results are similar to Gibbons and Murphy (1992). In our model, the increased investment provides additional incentive to the EN.

While the effects of agency conflicts and imperfect information have been studied in several contexts by prior studies, theoretical literature that incorporates asymmetric beliefs is relatively nascent. Landier and Thesmar (2005) develop a VC model with asymmetric beliefs, but focus solely on debt financing. They show that optimistic entrepreneurs tend to rely on short term debt rather than long term debt. Their model, however, does not allow for investment to be staged over time and limits contracts to debt alone. Cuny and Talmor (2005) analyze the effects of asymmetric beliefs in a VC finance model that compares the performance of firms funded by milestone staging to those funded by investment rounds. They find that when the EN is more optimistic than the VC, the advantages associated with round financing are increased. Their analysis of the effects of asymmetric beliefs is, however, of limited scope as they focus only on comparing the two types of finance mentioned above.

With respect to staging of investment, Neher (1999) shows that staging is essential to overcome the hold-up problem. As in Neher (1999), staging arises endogenously in our model with the number of stages also being determined endogenously. As Neher's (1999)

model is fully deterministic, however, his framework cannot be used to study the effects of risk, imperfect information, and asymmetric beliefs, which is a key focus of our study.<sup>6</sup>

Our framework shares features of dynamic principal-agent models that incorporate imperfect information (for example, Gibbons and Murphy, 1992, Holmstrom, 1999). Our study, however, differs significantly from these studies in that both the VC (the principal) and the EN (the agent) make investments (physical and human capital) over time, have asymmetric beliefs about project quality, and the relationship is terminated endogenously.<sup>7</sup> Our model could also be applied to study the financing of research and development. Berk, Green and Naik (2003) develop an R&D model in which staging is *exogenous*. Since their focus is on the valuation of R&D ventures, they do not incorporate agency conflicts or asymmetric beliefs.

The plan for the rest of the thesis is as follows. In Chapter 2, we provide a comprehensive literature review of recent research related to this thesis. In Chapter 3, we develop and analyze the Basic Model. Chapter 4 presents a risk analysis of a two-period version of the Basic Model. In Chapter 5, we describe the numerical implementation and calibration of the model and its findings. In Chapter 6, we develop and analyze the Shift of Power Model. In Chapter 7, we develop and analyze the Unobservable Effort Model. Chapter 8 provides concluding remarks and some suggestions for further research. Proofs are provided in the last section of each chapter. The code design of the numerical analysis Matlab code is provided in the Appendix.

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<sup>6</sup>Kockesen and Ozerturk (2004) argue that some sort of EN “lock-in” is essential for staged financing to occur. Egli, Ongena and Smith (2005) argue that staging can be used to build an EN’s credit rating.

<sup>7</sup>Admati and Pfleiderer (1994) and Fluck et. al. (2005) analyze two-period models of venture capital investment. We differ significantly from these studies in that we analyze the effects of asymmetric beliefs and agency conflicts on VC relationships in a dynamic framework where staging and project durations are endogenously determined.

## CHAPTER II

### LITERATURE REVIEW

Our model may be applied to a number of interrelated fields including venture finance, managerial incentives and project R&D. In his seminal work describing the venture capital industry, Sahlman (1990) describes three central motifs in the VC - EN relationship.

1. The inflow of capital is installed over time rather than provided upfront.
2. The contract between the VC and the EN is structured so that cashflow rights and control rights may be separated.
3. The VC continuously monitors and oversees the project and provides valuable advice to the EN.

Researchers employ a number of approaches to explain why these features developed (mainly 1 and 2), and how they affect the industry (mainly 3). One approach, which we do not employ, is the real options analysis (Cossin et. al. (2002), Berger et. al. (1996), Benaroch and Kauffman (1999) among others). Another approach, which we consider, is by means of the *principal-agent problem* also known as the *agency problem*. The agency problem rises from frictions and asymmetries between the VC and the EN. For example, if the VC is risk-neutral but the EN is risk-averse, the EN's objectives may be unaligned with the VC's and consequently the EN may employ investment strategies that are not optimal to the VC and inefficient society-wise. Similarly, if the EN is able to divert funds from the firm to his private consumption he may act in an efficient manner. Finally, agency conflicts from information or belief asymmetries may result in the VC's and EN's actions colliding. Kaplan and Stromberg (2001) provide a comprehensive review of empirical findings pertaining venture finance and the agency problem and Hart (2001) provides a review of theoretical models. Our review covers many of the papers in those review papers as well as other, more recent, papers. Section 2.1 describes empirical findings in the field of venture finance and Section 2.2

reviews literature related to venture finance and managerial incentives emphasizing agency conflicts.

## ***2.1 Empirical Evidence***

### **2.1.1 Staging Investment**

The single most important tool employed by the VC to guarantee his return is the staging of investment (Sahlman (1990)). The life of a project is divided into stages or investment rounds, starting from seed investments, whose sole purpose is to evaluate the project and its prospects for success, through development and expansion stages until liquidation stages or going to initial public offering (IPO). As the firm moves from one round to the next it usually requires ever-increasing investment, which may be provided by the same VC. The cost of capital to the firm, however, will decrease from round to round due to lower risks associated with better forecasts of project earnings (Plummer (1987)).

Using data from 794 venture-backed firms Gompers (1995) finds that staging investment enables the VC to acquire knowledge about the firm, monitor it and, if necessary, abandon it. They find that VCs concentrate their efforts in early stage firms where informational asymmetries between the VC and the EN are high and for which VC's monitoring and insight is of importance. They find that firms that are successful get more funding rounds and receive more total investment. Further, Gompers finds that unsuccessful firms are revealed (and discontinued) earlier and receive less funds than successful firms (success measured by going to IPO). Our model makes similar predictions. In our model, if a firm gets a positive signal in the first period its expected project duration (and consequently, its expected total funding) is larger. If the same firm received a negative signal, its expected project duration and its expected total funding decreases. We use data from Gompers (1995) to calibrate the parameters of our model for the numerical analysis.

In our model staging is allowed and the exact number of stages is endogenously derived. The capital investment in each period is set endogenously, and we are able to characterize when investment is increasing or decreasing or non-monotonic over time. Sahlman (1990) reports that there is typically up to eight different stages. In the numerical analysis, we

find that the probability for more than eight stages is negligible. Our model’s prediction that experienced entrepreneurs, who possess a more realistic belief about the firm’s quality (small asymmetry in beliefs), will be funded in fewer rounds is supported by Kaplan and Stromberg (2003).

### **2.1.2 Contract structure**

Sahlman (1990) reports that the VC-EN contract is a stock purchase agreement in which the VC guarantees capital at a certain schedule for which he receives some form of stock and other rights. Typically, the stock will be in a form of convertible preferred stock and the contract specifies the exact terms of the stock including conversion price, liquidation schemes and dividend terms. Other rights include (i) the “right of first refusal” in which the insider VC is given priority over outside investors in participation in new investments in the firm, (ii) information rights providing independent access to all information concerning the progress of the firm, and (iii) voting and control rights. The VC-EN contract also typically includes a number of restrictions on the EN such as a “no compete clause” that prevents the EN from working in the same industry for a period of time should he leave the firm. Finally, the contract specifies vesting schedules on the EN’s equity share, and the VC’s rights to buy-back those shares in case of the EN’s early resignation.

In their survey of 213 VC investments, Kaplan and Stromberg (2003) find the structure of the contracts is carefully designed to mitigate known problems such as the aforementioned principal-agent problem and the *hold-up problem*. The hold-up problem stems from the lack of the legal means to enforce EN commitment to the project. This problem is most severe when the entrepreneur is critical to firm success. With respect to the agency problem, they find that the VC-EN contract is designed to separate the allocation of cash flow rights, control rights and liquidation rights so that if the project performs poorly, the VC is able to independently increase his control and liquidation rights, whereas if the project’s performance is quite positive, the VC can reduce those rights while retaining his cash flow rights.

Gibbons and Murphy (1992) find empirical evidence that the EN's contingent compensation is increasing over time. In their theoretical model, which we discuss in length in Section 2.2.2, the EN can signal to the market about his ability. They show that this signaling is very strong in the early years of the EN's career (high career concerns) and is minimal in later years. Accordingly, the EN's contingent compensation is increasing over the years as his incentives shift from career concerns to immediate consumption concerns.

Our model predicts that when the EN is risk averse and the asymmetry in beliefs is sufficiently small, the contingent compensation is increasing over time. This result is robust to small changes in the initial degree of asymmetry in beliefs, who has the bargaining power (EN or VC) and whether effort is observable or not. Our assumption with regard to the EN's right to repudiation is supported by the fact that the VC must devise different schemes to ensure EN's long-term participation in the project.

### **2.1.3 VC Oversight**

VC monitoring and oversight is another central theme in VC finance, and is considered essential for firm success. The purpose of this oversight is multi-fold. In contrast to "arms-length funding", where the EN is not monitored by the financier, VC finance is a "relationship funding", and the EN not only receives the necessary capital but also critical advice, business ties and managerial support. Sahlman (1990) claims this is an essential advantage to the VC-EN relationship. Sahlman also reports that by monitoring firm performance the VC is able to avoid further investment if progress is not satisfactory. Indeed, Lerner (1995) finds that VC oversight increases during CEO change, a "sensitive" time in a project's life.

Oversight, however, does not come without cost, as reported by Sahlman (1990) and Kaplan and Stromberg (2004). While Sahlman does not give an estimate to the actual cost of this oversight, he reports that VC fund managers usually receive a managerial fee that is on average 2.5% of the capital invested by the fund, and that only few VC fund managers were paid according to the portfolio value. However, this does not represent the true costs of oversight because in addition to the mentioned VC management fee these VC fund managers receive at least 20% of profits (Gompers and Lerner (1999)). According to

Sahlman, the VC's way of financing the oversight costs is through the required expected rate of return, which is relatively high in comparison to other forms of funding. This, Sahlman explains, is due to the additional monitoring and oversight costs, and due to the "well-known bias in financial projections made by entrepreneurs" (p. 512). Lerner (1995) finds that companies physically nearer to the VC are more likely to be chosen for funding due to the reduced oversight costs.

Another consequence of the VC oversight is the choice of projects to be funded. In Hellmann and Puri (2000), candidate projects are labeled as either innovative or imitating. Innovative projects develop a new technology or non-existent service, while imitating projects continue already established products or services. They find that innovative projects are more likely to be financed by VC's than imitating projects, which they claim is due to the greater advantage oversight offers with innovative projects. They also find that for innovative projects VC financing is associated with a reduction of the time to bring the product to market. In another paper, Hellmann and Puri (2002) find yet another effect of the VC's oversight. They report that firms funded by VC's are more likely to hire marketing vice presidents, develop human resource policies and other professional measures than firms financed by other means.

We assume the VC considers oversight costs when investing in a firm. This corresponds to the empirical reports that find that these costs are substantial, and specifically to Lerner (1998), who reports evidence to strong VC consideration of oversight costs. In our model, we assume that the VC's cost of monitoring is exogenous and aggregate it with depreciation costs and losses to competition.

#### **2.1.4 Bargaining Power**

Baker and Gompers (2003) study 1,116 firms of which a third are backed by venture capitalists. They report that tenured CEO's have greater bargaining power and are able to increase the number of insiders sitting in the board. However, VC finance decreases the CEO's power, and they find that the influence of the EN (i.e. his bargaining power) is decreasing with the VC reputation. They explain this last result by assuming that a more

reputable VC gains bargaining power since he has better contacts to find suitable replacement for the EN. Consequently, they turn to check the rate of CEO turnover and find it is increasing with the VC's reputation.

Using valuation data for 4069 firms, Gompers and Lerner (2000) find that in periods with greater amounts of capital available in venture funds, the evaluations of venture firms increase. This implies that in times when less venture money is available the VC gets more for his money, effectively implying the VC's bargaining power is decreasing at times of abundant venture capital.

We test our model when the VC has bargaining power and when the EN has bargaining power. We find that when the EN has bargaining power he will have less contingent compensation.

### **2.1.5 Risk Analysis**

Kaplan and Stromberg (2004) conduct a study of 67 portfolio investments in which they classify investment risks and uncertainties into one of the following three categories:

1. Internal Risks — risks that are associated with asymmetries between the VC and the EN (agency conflict risks). These risks may include the EN's ability, his willingness to exert effort, insider's information about the project, etc.
2. External Risks — risks that are equally uncertain for the VC and the EN, such as market condition, competition, etc.
3. Complexity Risks — risks that are equally uncertain to the VC and the EN but that are partly under the control of the EN. Success of developing a product or executing management strategy are examples for complexity risk.

Kaplan and Stromberg find that internal risks are associated with more VC control, more contingent investment in a given round and more contingent compensation to the EN. External risks are associated with more VC liquidation rights, in contrast to the theoretical view of optimal risk sharing between the risk neutral VC and the risk averse EN. Complexity

risks are associated with more vesting of the EN's compensation, which corresponds to mitigating hold-up problems.

Gompers and Lerner (2001b) describe why little is known about the risk of early stage venture funded firms. VCs avoid pricing their firms until they go public and use the firm's book value as the firm's value prior to IPO. Thus, when many firms go public there is an upwards bias in the returns of venture firms, when in fact many of the gains reported in the IPO year were realized in the years preceding the IPO. They stress the importance of learning about the risk involved with venture capital due to the fact more and more public institutions allocate ever increasing fractions of their portfolios in this market.

Our model tackles issues related to the risk of venture firms. In a two-period model we demonstrate analytically many results with respect to risk effects on venture duration and VC's share. We are also able to numerically demonstrate different effects of risk in the multi-period model.

## ***2.2 Theoretical Models***

### **2.2.1 Staging Investment**

Neher (1999) provides a theoretical framework to show that staging is essential to overcome the hold-up problem. In this model, the investments made by the VC can be materialized into salvageable physical assets only upon completion of an investment period. If the EN decides to abandon the project during a period, then all the current period's investment is lost. Therefore, if the venture capitalist provides the whole required investment upfront, the entrepreneur can, prior to completion, force renegotiation on the VC. At this point the VC has already put in all the money required and therefore has no bargaining power. Thus, renegotiation will always result in the VC incurring losses, and therefore no VC will ever finance such a project in the first place. By staging investment, the VC can build collateral to his prior investments and give him bargaining power in case of renegotiation. In early investment periods, the VC's bargaining power stems from the fact that he has not yet invested much, whereas in later periods he has already built a physical collateral to preempt renegotiation. In this manner, the VC can assure his bargaining power at any

given time in the project's life, thus enabling him to get his required rate of return.

Our model is similar to Neher (1999) in the sense that we too see staging as a technique to overcome inefficiency due to the agency problem. The aspect of the agency problem that our staging overcomes is not the commitment problem that Neher addresses, but rather the inefficiency due to the EN's risk-aversion and the effort he must invest in the project (Neher's model is fully deterministic and so the issue of risk-aversion does not rise). The friction that rises as result of the EN's effort is commonly called a *moral hazard problem* and results in the EN considering not only the firm's value but also his effort level. Consequently, the EN's objectives are different from the VC's objectives that emphasize only firm's value. Another difference between our model and Neher's is in the compensation to the EN. Neher assumes that if the EN repudiates prior to project completion he receives no income, whereas in our model if the EN repudiates he receives his previously committed share of the project value. Both models, however, share the notion that a project accumulates value through investment even prior to its completion.

Another explanation for staging is provided by Kockesen and Ozerturk (2004), who find that some sort of EN lock-in is required for staged financing to occur. The reason is that following the VC's initial investment, the EN can opt out and seek finance from another VC. In this case, the first VC gets zero return for his investment and therefore no VC will want to make the initial investment. However, if the EN can be locked into the initial VC, VC finance may be feasible; even more so, it may be more attractive than "upfront finance" in which the entire investment is made at the beginning. A natural lock-in is an "information lock-in". This happens when a signal indicating the success of the project is received after some initial investment is made. This signal can be observed only by the EN and the initial VC, and therefore if the signal indicates success the EN will prefer staying with the original VC, because any alternative VC is unaware of the project's promise, and will therefore make a less appealing offer to the EN. This lock-in results in the EN having less bargaining power over the VC and hence, the VC can extract surplus when writing the second period contract. Another consequence is that the EN will overinvest in the initial period before information is revealed. This extra level of investment can be viewed as the cost to the

VC to be an insider in the project. The added value to the VC enables him to invest even when the project is rejected from an “upfront finance” point of view. In such cases, or when an upfront investor barely breaks even, the EN will prefer to share surplus with a more willing “relationship” financier. When informational lock-in is technically impossible, Kockesen and Ozerturk find that it is necessary for the EN to lock himself into the VC by adding a clause to the initial agreement that prevents him from seeking alternative sources of finance.

In our model we do not allow informational lock-in to arise. We assume the EN can effortlessly convey to any prospect investor the traits of the project and that due to competitiveness in the VC market, all VC’s will make similar lending offers. The “right of first refusal”, reported by Sahlman (1990), justifies our assumption that the same VC will continue in consecutive rounds. In addition, when our model assumes the VC market is competitive and the EN has bargaining power, the terms in which the VC will continue to invest must be identical to the terms in which any other VC would invest. Therefore, even without maintaining informational lock-in or a non-compete clause, we are able to explain why the same VC will invest in consecutive stages.

Egli, Ongena and Smith (2005) provide another advantage to staging investments. They describe a world where there are two types of ENs. The first chooses never to default on a loan and the second defaults whenever it is profitable for him to do so. In these circumstances, they show that the EN may prefer to have the investment staged over time so that the EN can build his “credit worthiness” reputation, and therefore increase his access to inexpensive capital. Their model also helps explain why it is common for the EN to seek capital from the same VC in consecutive rounds. Once the EN is able to build positive credit reputation with the initial VC he will prefer to stay with him since he receives better financing conditions. Their model can also explain why VC’s require a decreasing rate of return between rounds (Plummer (1980)). The VC is assuming less risk due to the increasing EN credit worthiness.

At the heart of Egli, Ongena and Smith (2005) lies the assumption that an EN may prefer to repay a loan even when he is permitted to default. In our model, we make a

similar assumption by assuming that the EN does not default on his monetary agreement. To make this assumption less objectionable, we point out that in our model we assume firm value grows positively in such a way that the probability the EN might find it beneficial to default is negligible. Reputation concerns similar to those raised by Egli et. al. may further serve to remove objection to this assumption.

#### *2.2.1.1 Contract structure*

The moral hazard problem, mentioned above, may be caused by a different reason than the EN's distaste with effort. For example, moral hazard may arise when the EN receives private benefits from the firm. These private benefits, pecuniary or not, may induce the EN to practice business policies that are not optimal to either the VC (whose objective is maximizing firm value) or society (whose objective is maximizing benefits to both parties). One approach to address this issue is the *incomplete contracts* approach (Hart and Moore (1988), Aghion and Bolton (1992) among others). This approach assumes that many actions the EN takes are unobservable or unverifiable and thus non contractible. Further, there may be many cases in which unforeseen events happen under which it is unclear what actions should be taken. Accordingly, an important purpose of the contract is to state who takes control of the firm rather than just what actions should be taken. Following this approach, Aghion and Bolton (1992) show that due to moral hazard, the contract written between the VC and EN will include not only monetary remunerations but also, independently, allocation of control rights. They find that when efficiency (i.e. maximizing social surplus) emphasizes maximizing firm value then control should be transferred to the VC. In contrast, when the private benefits are significant, then maximizing firm value results in losses in social surplus and therefore control should be shifted to the EN. Hart (2001) extends a simple version of Aghion and Bolton's model to explain shifts of control between different types of investors such as creditors and shareholders (VCs).

In contrast to Aghion and Bolton (1992) and Hart (2001), Kirilenko (2001) allows control to be divided continuously between the VC and the EN. In Kirilenko's model the VC faces an EN that enjoys nonpecuniary benefits from the firm whose value are known only to the

EN. Kirilenko shows that the VC requires disproportionately higher control rights than his equity size, and that the VC's control rights grow with the severity of the agency conflict. The reward to the EN for the loss of control is the ability to get better terms of financing, and to shift some of the risk to the VC. Without the possibility to separate control rights from equity holdings, investment will not take place in the first place.

Trester (1998) shows that the popularity of preferred equity contracts over debt contracts in venture finance projects is due to asymmetric information between the VC and the EN. His model predicts that if auditing were inexpensive and feasible then debt contracts would be optimal. His analysis is somewhat limited in comparison to our model because he assumes that the EN is risk-neutral and he does not consider contracts of a mixed nature. Indeed, Trester conjectures that if risk-averseness is introduced then mixed debt-equity contracts may be optimal. In our model, we find evidence that increased risk-averseness results in less contingent compensation.

Control is also in the core of Chan, Siegel and Thakor (1990), who consider a two-period model in which the EN is replaceable. The EN's skill is unknown upfront but both parties share the same beliefs about it. In the first period, the VC invests an initial amount. The output at the end of the first period depends on the amount of effort the EN exerts. At the end of the first period the EN's true skill is revealed to both parties and the VC can decide whether to take over the control of the firm or leave it by the EN. At the end of the second period a second and final cash flow is received, which is shared according to the division rule set at date zero. Chan et. al. consider renegotiation proof contracts, which specify the monetary compensation and the second period control decision as a function of the firm's output and the EN's skill. Chan et. al. explain why the EN is prohibited from seeking alternative sources of finance (in the second period). This result also corresponds to the prevalence of no-compete clauses. In addition, they find that the VC takes control of the firm if the EN's revealed skill is lower than a critical value. In this case, the VC will pay the EN a fixed amount. This result explains why a VC may retain the option to buy out the EN's shares. In contrast, when the EN stays in control his compensation is increasing with his skill.

Admati and Pfleiderer (1994) explain that absent an insider VC with precise knowledge of the firm overinvesting will occur. This happens because when uniformed outside investors provide the funding, the decision maker EN has incentive to continue that project even when it is optimal to abandon it. On the other hand, having an insider investor who is not the sole owner may lead to underinvesting for a number of reasons, mainly, due to the EN getting some of the surplus for which the VC has invested. Admati and Pfleiderer show that the optimal contract is a fixed-fraction contract. Under this contract the insider VC does not increase nor decrease his share in subsequent financing rounds. They show the fixed fraction contract is robust in the sense that it is optimal for any probability distribution of the firm's output. This type of contract explains why in later rounds investors other than the initial VC invest in the firm. It also suggests that the insider VC should be chosen to set the price of newly issued securities in future rounds. This is because the VC retains a fixed fraction of the total securities and consequently has no incentive to misprice them, whereas the EN will gain if they are overpriced and the new investors will tend to underprice.

Fluck, Garrison and Myers (2005) show that when EN effort is determined endogenously, the fixed fraction contract is not optimal. This happens because in the event the VC has a fixed fraction contract he still has the incentive to overprice the firm since by overpricing the EN share's value the EN is induced to work harder. Consequently, outsider investors cannot trust the insider VC's information and inefficiency occurs. They find that in some cases by increasing the insider VC's share the VC will lose the incentive to misprice newly issued securities. The Fluck et. al. model assumes two investment periods in which investment is set exogenously. The results of their analysis, which is purely numerical, indicate that underinvesting is a severe problem. However, their focus is mostly on corporate structure rather than optimal investment levels and they do not consider important aspects of the EN-VC relationship such as optimal investment levels, risk averseness and VC's monitoring costs.

### 2.2.2 Managerial Incentives

One of the most important tools the VC employs to overcome the agency problem is giving incentives to the EN to act according to the VC's objectives. The typical example is when the EN must exert effort to increase firm value, but this effort comes to the EN at some cost. The question of what are the optimal incentive scheme is addressed in the literature of managerial incentives (see Holmstrom (1999), Holmstrom and Ricart I Costa (1986)) and is very much related to the field of venture finance since the EN is usually in the managerial position and the VC is the firm's board of director.

Gibbons and Murphy (1992) develop a model for optimal managerial incentives. The purpose of managerial incentives is to persuade managers to behave according to what is optimal for the firm. These incentives are required because optimal behavior requires the manager to exert effort, something he may be reluctant to do. On the other hand, if the manager's concerns are to build his reputation to attract future employers to him, also named "career concerns", he will invest more than the optimal amount. Recall, it is typically assumed the effort by the EN is either unobservable or unverifiable and, therefore, non-contractible. Therefore, an ideal incentive scheme will link the manager's income directly with his abilities in a manner that results in the EN himself choosing the optimal effort level. However, since the EN's ability is also unknown the scheme ties the manager's compensation with other contractible measures that are good proxies to the EN's ability, such as the the firm's performance or output. When the VC cannot observe the effort by the EN, information asymmetry develops between the VC and the EN with regard to the EN's ability. Models such as Holmstorm (1979), Gibbons and Murphy (1992) and Bergemann and Hege (1998) employ this scheme of learning about the effort and ability indirectly by observing output. The model we develop is an extension of Gibbons and Murphy (1992). We therefore present this model in more detail.

Gibbons and Murphy consider a  $T$  period model where in each period  $t$  the firm's output is

$$y_t = \eta + a_t + \epsilon_t,$$

where  $a_t$  is the manager's effort,  $\epsilon_t$ , is a noise signal and  $\eta$  is the manager's ability. There is symmetric information about  $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ . Before the first period there is symmetric information about the manager's ability which is assumed by both parties  $\eta \sim N(m_0, \sigma_0^2)$ . However, since the manager's effort can not be observed by the firm owner, informational asymmetries develop in consequent periods. The manager has disutility from effort measured by  $g(a_t)$  and his utility from consumption and effort is given by:

$$U(w_1, \dots, w_T, a_1, \dots, a_T) := -\exp \left\{ -r \left[ \sum_{t=1}^T \delta^{t-1} (w_t - g(a_t)) \right] \right\}.$$

Following Holmstrom and Milgrom (1987), Gibbons and Murphy assume the contract between the firm owner and the manager is a short term linear contract given by

$$c_t + b_t y_t$$

for each period  $t$ . Due to competition between firm owners the firm owner gains zero expected return. In the first period, after the parties have chosen the optimal contract, the manager chooses his optimal effort level. After both parties observe the firm's output they each update their estimate of the manager's ability according to Bayesian updating. Since the firm owner cannot observe the manager's actual level effort he makes a conjecture about the effort level the manager exerted and updates his beliefs with regard to the manager's ability according to this conjecture,  $\hat{a}_1$ . Since the prior distribution of  $\eta$  is normal, so is the posterior distribution and the volatility is agreed upon by the firm owner and the manager at each period.

In what follows, we describe the equilibrium results of a simplified two-period model of Gibbons and Murphy (1992). In the second period, the manager's optimal effort decision  $a_2^*(b_2)$  satisfies  $g'(a_2) = b_2$ . Competition between firm owners dictates

$$c_2(b_2) = (1 - b_2)E[y_2|y_1].$$

Substituting this into the manager's utility results with an optimal slope,  $b_2^*$ , that satisfies

$$b_2 = \frac{1}{1 + r(\sigma_1^2 + \sigma_\epsilon^2)g''(a_2^*(b_2))}$$

where  $\sigma_1^2$  is the posterior variance of  $\eta$ . Since, by assumption,  $g''(a)$  is positive it follows that  $b_2^* < 1$ . In the first period, since the manager knows he can affect the firm owner's future estimate of  $\eta$  through his unobservable exerted effort, the manager's optimal effort does not depend on this period alone but also on optimal decisions in the last period. Consequently, Gibbons and Murphy show the optimal effort,  $a_1^*(b_1)$ , that satisfies

$$g'(a_1) = b_1 + \delta(1 - b_2^*) \frac{\sigma_0^2}{\sigma_0^2 + \sigma_\epsilon^2} := B_1.$$

The total incentive to the manager in the first period,  $B_1$ , is obtained from the contingent compensation in the first period contract,  $b_1$ , and from career concerns  $\delta(1 - b_2^*) \frac{\sigma_0^2}{\sigma_0^2 + \sigma_\epsilon^2}$ . The optimal slope,  $b_1^*$ , satisfies

$$b_1 = \frac{1}{1 + r(\sigma_0^2 + \sigma_\epsilon^2)g''(a_1^*(b_1))} - \delta(1 - b_2^*) \frac{\sigma_0^2}{\sigma_0^2 + \sigma_\epsilon^2} - \frac{r\delta b_2^* \sigma_0^2 g''(a_1^*(b_1))}{1 + r(\sigma_0^2 + \sigma_\epsilon^2)g''(a_1^*(b_1))}.$$

Since  $b_2^* < 1$  it follows that  $b_1^* < b_2^*$ , namely, the compensation slopes are increasing from the first period to the second. In fact, Gibbons and Murphy show that this statement is true in a general  $T$  period model, i.e., the contractual incentives,  $b_i^*$ , increase over time. The reason for this is that in early periods the manager's career concerns are greater than his concern for current wages, whereas towards the end of his career the manager is mostly interested in his wages and his leisure. By gradually increasing the part of the contract tied with performance, the firm can suppress the tendency to over-invest in earlier periods, and induce the manager to invest more than he would have otherwise in later periods of his career.

We adapt the model introduced by Gibbons and Murphy (1992) to address venture-financed projects. While we adopt the treatment of contracting, risk-averseness, effort and learning, we differ from their model in a number of meaningful ways by introducing three important features of venture capital. First, we introduce investment by the VC and allow it to be derived endogenously. Second, we allow for losses due to VC oversight and value depreciation. Third, we introduce asymmetric beliefs between the VC and the EN.

Baker and Hall (2004) examine the relationship between firm size and CEO incentives. There are two natural ways to measure the strength of CEO incentives: (i) the change in CEO wealth relative to a dollar change in the firm's value or (ii) the absolute value of the

CEO's wealth. They find that according to the first measure the compensation to CEO's in small firms is extremely large as compared to CEO's in large firms. However, according to the second measure the results are opposite. To reconcile these results they develop a new measure for the strength of CEO incentives. In their model they assume a marginal product of the manager's effort,  $\gamma$ . That is, they assume that every unit of manager effort contributes only  $\gamma < 1$  units to increase firm value. Baker and Hall's approach is novel in the fact that they assume the marginal productivity ( $\gamma$ ) depends on firm size, and consider the elasticity of the marginal productivity to firm size as a critical factor in determining the strength of the CEO's incentive. They measure the incentive's strength as the CEO's sensitivity pay to performance multiplied by his marginal productivity. Using data from 1749 firms they find that while the sensitivity pay to performance sharply declines with firm size, the incentive's strength measure is roughly constant with firm size. In the context of our model, however, they find that the manager's marginal productivity increases with firm size. This empirical evidence resonates well with the numerical analysis of our model in which we find that increasing the EN's output elasticity of effort increases the firm's value. Simply put, our model also predicts that firms with higher output elasticity of effort will be of larger size.

### **2.2.3 Asymmetric Information and Beliefs**

Many of the models discussed above contain some form of informational asymmetry between the EN and the VC to incorporate this feature observed in the VC industry. This asymmetry is either between the VC and the EN (Gibbons and Murphy (1992), Trester(1998)) or between insider and outsider VC's (Admati and Pfleiderer (1994) Fluck et. al. (2005), Kockesen and Ozerturk (2004)) or between them all (Egli et. al. 2005). In our model, we do not consider asymmetries between prospective investors but only between VC and the EN. Our contribution to the understanding of the effects of asymmetry is that we allow for asymmetry in beliefs. In the basic model we only consider asymmetry in belief between the EN and the VC. In the unobservable model, we also allow for asymmetries in information to develop because of the unobservability of effort.

Our motivation to introduce asymmetry in beliefs is the extensive empirical evidence to support it. Sahlman (1990) and Gladstone and Gladstone (2002) report that even in face of the same information, managers and owners have differences in opinion. The EN will almost always have a more positive view with regard to the venture's success. To quote Palich and Bagby (1995): "In other words entrepreneurs are more likely to see the business world through 'rose colored glasses' ". When extreme, this optimism leads the EN to irrational beliefs or to seem risk tolerant (Cave and Minty (2004)). Indeed, in our model, we assume the VC's beliefs are the "true beliefs", and if the EN is more optimistic than the VC, he is unrealistic. Lerner (1998) provide additional insight to understanding a cause for EN's unrealistic optimism. Strong EN commitment to the firm he founded and the lack to admit failure prevents him from conceding to the true (and low) value of the firm.

The importance of beliefs cannot be underestimated. Gelderen, Thurik and Bosma (2005) follow 517 nascent entrepreneurs (i.e. EN's with active manifested desire to start a business before actually starting it) over a period of three years and examine their success. They find that the perceived risk of the market is negatively tied with EN success. They also find a strong positive relationship between the EN's ambition to succeed. Assuming a strong correlation between ambition and optimism, Gelderen, Thurik and Bosma support our findings of the positive effects of EN's optimism.

Asymmetry in beliefs between the VC and the EN is comprehensively reported by Landier and Thesmar (2005). They start by explaining the source of such asymmetry and confirm that ENs tend to be more optimistic than the investors. Their model focuses on debt contracts to show that optimistic entrepreneurs tend to rely on short term debt rather than long term debt. The early maturity of debt enables the investor to take control of the project in case the project is unsuccessfully managed. They continue to back their results with empirical data from the French industry, which heavily relies on debt finance. The significance of Landier and Thesmar (2005) to our model lies more in the theoretical support to the existence of asymmetry in beliefs even when the VC and the EN face identical information. With regard to venture finance, however, their model limits contracts to debt alone and investment is not staged. Nevertheless, their results may be interpreted to

support the conclusions of our model. For example, the fact that optimistic ENs receive short term debt is interpreted as a means for the VC to allocate control when downside information on the project is revealed. Indeed, our model suggests that the contingent compensation of an optimistic EN is decreasing with time due to the arriving information. In our model, this decrease is the sum of two forces, one resulting from the decrease in risk and the second due to the decrease in the EN's optimism.

Cuny and Talmor (2005) analyze the effects of asymmetric beliefs in a VC finance model that compares the performance of firms funded by milestone staging to those funded by investment rounds. In milestone staging further investment is guaranteed to the EN if a milestone is reached, whereas in investment rounds finance, the firm is given no guarantee that further investments will be given. They find that when the EN is more optimistic than the VC, the advantages associated with round financing are increased. Their analysis of the effects of asymmetric beliefs is of limited scope as they focus only on comparing the two types of finance mentioned above.

Bigus (2003) develops a single period, three states of the world model that incorporates asymmetric beliefs between the VC and the EN with respect to the project's mean value and riskiness. The VC is more pessimistic than the EN and assumes the project's payoff has a lower mean and higher volatility. Bigus introduces a moral hazard problem in the form of an EN who is able to consume perks, an action that negatively affects the firm's return. He then examines the optimal contract under different cases: with or without perk consumption and for different cases of asymmetries in beliefs. His model predicts that VC's equity increases with the asymmetry in beliefs with regard to risk and that the debt level increases with the asymmetry in beliefs with regard to the mean of the project's return.

The focus of Bigus (2003) is on the structure of the optimal contracts in face of asymmetries in beliefs. He does not incorporate staged investment or even learning, two ingredients that seem only natural in the context of venture capital and asymmetries between the VC and the EN. Moreover, since his analysis is limited to a three state model with many restrictive assumptions, many questions are raised with regard to the robustness of the results to changes in these assumptions.

Surprisingly, there is very little venture finance literature, empirical or theoretical, dedicated to understanding the effects of asymmetry in beliefs. In this context the model we develop is a step towards a comprehensive approach towards understanding these effects.

#### **2.2.4 VC Oversight**

The moral hazard problem can be extended to a two-sided problem where both the EN and the VC need to exert effort for the project to succeed. This problem, also known as a *double sided moral hazard*, is used in Repullo and Suarez (2004) and Inderst and Muller (2004) to model the need of the VC to offer advice and the EN to exert effort. These models assume both the EN and the VC have disutility from the effort and advice they have to exert. In our modeling of the VC's problem, we choose a slightly different course: We acknowledge the importance of the VC's effort but we do not view it in the context of a moral hazard. Instead, we aggregate it as an exogenous loss cost.

Another paper that considers the significance of oversight is Allen and Gale (1999). They compare the efficiency of financial markets and financial intermediaries in face of asymmetry in beliefs of investors. In the financial markets each investor monitors the firm closely and is well informed of the details of the investment and has full control on the decision to invest. By using intermediaries, however, investors have limited access to information and rely on the manager of the investment fund to make the investment decision. Allen and Gale show the advantage of financial markets over financial intermediaries increases with the diversity of the investors. Since the focus of Allen and Gale is with regard to the optimal financial method they put little attention on the investment process itself and their model does not involve staged investment. In addition, they focus on asymmetry within investor groups and not between the investor and the entrepreneur.

The intermediaries in Allen and Gale (1999) are close in spirit to the 3-Tier modeling of venture capital, described as in Holmstrom and Tirole (1997) and Dessi (2005). This modeling separates the investors in the venture capital fund from the VC who manages this fund, and the EN. This three-tier hierarchy of venture finance (investor - fund manager - EN) is similar to the double sided moral hazard, considered above, since the investors in

the venture capital fund need both the VC (fund manager) and the EN to exert effort.

### **2.2.5 Bargaining Power**

Many of the theoretical models assume the VC industry is competitive, thus assuming the VC has no bargaining power. One exception is Inderst and Muller (2004), who develop a model that incorporates the possibility of bargaining power being shared between the VC and the EN and who predict that bargaining power affects the valuation of firms. Their model, however, does not consider staged investment and agents are risk neutral. Although we do not allow for bargaining power to be shared we examine the effects of shifting the bargaining power between the EN and the VC, and find the EN's contingent compensation decreases when the EN has bargaining power.

### **2.2.6 Risk Analysis**

The staged evolution of the project is an important feature common to venture finance models and R&D models. Indeed, many papers labeled as venture finance can readily be applied to the field of R&D; see, for example, the concluding remarks in Wang and Zhou (2004). Berk, Green and Naik (2003) develop an R&D model in which staging is exogenously given. In their model, which addresses project valuation and the cost of risk, a project must complete  $N$  successful stages in order to begin producing a cash flow. This cash flow behaves according to a standard geometric Brownian motion and is the sole source of systematic risk in their model. Their model has several sources of idiosyncratic risks including risk of obsolescence, the technical risk pertaining to the success of an investment round, and the duration and the total cost of the project. Investment is necessary but not a guarantee for the project to move to the next stage. In fact, the probability for success is updated from one investment round to the other through the history of the project. The investment made in each investment round is a linear function of the projected cash flow. As part of their risk analysis, Berk et. al. compare the risk premium required for a non-venture project (one with completed R&D) to the risk premium required for a venture project when both projects share the same cash flow projection. Since idiosyncratic risk can be diversified, they argue that traditional analysis should result in equal risk premium for both projects.

However, they demonstrate this is not the case and that “required risk premium for the R&D (project) is higher than it would be were the R&D (project) complete and the venture a traditional, cash producing project” (p. 2). This, they claim, is a result of the fact that while pure idiosyncratic risk can be diversified, the decision to continue an R&D project involves the resolution of both systematic and idiosyncratic risks. Thus, the project behaves similarly to a compound option on systematic uncertainty, which bears higher systematic risk than the underlying asset.

Since Berk, Green and Naik (2003) develops an R&D model they do not consider agency conflict but rather consider a single entity that manages and finances the project. This analysis is more suitable for large corporations who have their own R&D management but less so for the standard VC-EN venture projects. We share Berk, Green and Naik’s notion of separating between different sources of risk. They find that projects that perform poorly are abandoned early in their development stage. We reach a similar result, if in a slightly different context. In our model, all projects are terminated, rather than either completed or abandoned, and we too find that poor performing projects will have a shorter project duration.

An alternative approach to the handling of risk can be found in Guo and Yang (2005), who propose that the risk of a project is not exogenous but rather can be managed by the EN and the VC. In their model, the managers (EN) optimize their utility by controlling for the mean and the risk. When risk is determined exogenously they find a negative relationship between risk and the contingent compensation to the EN and that effort increases with the contingent compensation. However, when allowing risk to be determined endogenously, these standard results do not necessarily hold.

Guo and Yang (2005) presents a single period model and they do not consider investment. However, their model is very exciting in the sense that it considers managing the risk of firm’s output and not only its mean output, an approach that we may consider for future research.

## CHAPTER III

### THE BASIC MODEL

#### *3.1 The Model*

We consider an infinite time horizon with discrete dates  $0, 1, 2, 3, \dots$  that are assumed to be equally spaced for convenience. Period  $i, i \geq 1$  refers to the time interval  $[i-1, i)$ . At date 0, a cash-constrained entrepreneur (EN) with a project approaches a venture capitalist (VC) for funding. The project can potentially generate value through physical capital investments by the VC and human capital (effort) investments by the EN. Both the VC and the EN have imperfect information about the project and differ, in general, in their initial assessments of the project's quality.

If the VC agrees to invest in the project, she offers the EN a long-term contract that describes her investments in the project over time, and the EN's compensation. Investments by the VC (if they occur) are made at the beginning of each period. Either the VC or the EN could terminate the relationship at any date and could also initiate a renegotiation of their contract, that is, there is two-sided lack of commitment. In equilibrium, therefore, the contract between the VC and the EN is *renegotiation-proof*. We assume the VC possesses all the bargaining power in any negotiation with the EN. We show, however, that in order to provide appropriate inter-temporal incentives to the EN, the EN's *reservation payoff* at any date, that is, his promised payoff if the VC-EN relationship were terminated, varies over time. Since the VC possesses all the bargaining power in negotiations with the EN, in equilibrium termination occurs at the VC's behest.

The fundamental state variable is the market value  $V_i$  of the project if the VC-EN relationship is terminated at date  $i$ . This is the value of the claim to future earnings from the project *outside the VC-EN relationship*, that is, from the perspective of *outside investors* at date  $i$ . Therefore,  $V_i$  is the total payoff to the VC and the EN if their relationship is terminated at date  $i$ . We assume that both the VC and the EN possess *project-specific*

skills that are not transferrable. Neither the VC nor the EN can commit to supplying these skills to a third party. Hence, the amount outside investors would be willing to pay for the project is, in general, lower than the value if full commitment by the VC and the EN were hypothetically possible. The value under full commitment is the “rational expectations” value of the project, that is, the value after rationally incorporating the effects of future physical capital investments by the VC and human capital investments by the EN.

To simplify the analysis and exposition, we assume the project does not generate intermediate cash flows so that all payoffs occur upon termination (our analysis can be generalized to allow for intermediate cash flows without altering any of our main results). Hereafter, we refer to the variable  $V_i$  as the project’s *termination value* at date  $i$ . The termination value at any date is observable and verifiable and, therefore, contractible. The hypothetical value of the project under full commitment by the VC and the EN, the rational expectations value, is non-verifiable.

The VC has linear inter-temporal preferences whereas the EN is risk-averse with inter-temporal CARA preferences. The VC chooses her dynamic investment policy, the long-term renegotiation-proof contract for the EN, and the termination time (that is a random stopping time in general) to maximize her expected utility payoff upon termination. The EN, in turn, dynamically chooses his effort to maximize his expected utility payoff upon termination. The contract between the VC and the EN, the VC’s investment policy, the EN’s effort policy, and the termination time are derived endogenously in a subgame-perfect equilibrium of the dynamic game between the VC and the EN.

The *incremental termination value*, that is, the change in termination value over any period, depends on the level of investment by the VC, the amount of effort exerted by the EN, the *intrinsic quality* of the project, and market risk. The VC closely monitors the EN so that the EN’s effort is observable to the VC. However, it is non-verifiable by a third party and, therefore, not directly contractible. Both the VC and the EN have imperfect information about the intrinsic quality of the project, but have priors on it that may, in general, differ from each other. The VC and the EN update their assessments of the project’s intrinsic quality in a Bayesian manner based on their observations of the project’s

termination values, investments by the VC, and human capital inputs by the EN.

We begin by first describing how the VC’s physical capital investments and the EN’s human capital (effort) investments affect the project’s termination value over time.

### 3.1.1 The evolution of the termination value

The termination value of the project in any period is proportional to the initial termination value  $V_0$ , which we hereafter normalize to one. In each period  $i \geq 1$ , the project’s termination value evolves as follows

$$V_i - V_{i-1} = (c_i^\alpha \eta_i^\beta - l_i) + \Theta + S_i. \quad (1)$$

The change in termination value is derived from three sources— “net discretionary excess output”, “project quality” and “systematic risk”—each of which is described below.

*Net discretionary excess output.* Discretionary excess output in period  $i$  is a direct result of the VC’s capital investment  $c_i$  and the EN’s effort  $\eta_i$ , and is described by the Cobb-Douglas production function  $c_i^\alpha \eta_i^\beta$ ,  $\alpha, \beta > 0$ . Net discretionary output in period  $i$  is output less the “operating costs”, which we represent by an exogenous constant  $l_i$ . The operating costs could include wages to salaried employees, depreciation expenses, decline in revenues due to increased competition, fixed costs arising from increases in the scale of the project, etcetera. These costs are assumed to increase through time. For convenience and concreteness, we assume  $l_i = Li^2$ ,  $L > 0$ , which will ensure termination occurs in finite time almost surely. All our results remain qualitatively unaltered under alternative (deterministic) functional specifications of the operating costs as long as they are convex over time.

*Systematic risk.* The  $S_i$  represent the “systematic” component of the project’s risk. It is common knowledge that the  $S_i$  are independently and identically distributed with common distribution  $N(0, s^2)$ .

*Project quality.* The variable  $\Theta$  represents the per-period increase in the project’s termination value arising from the intrinsic quality of the project. The VC and the EN have *imperfect* information about  $\Theta$  and may also differ in their beliefs about its value. Their respective beliefs are, however, common knowledge. The *uncertainty* in the value of  $\Theta$  may

be viewed as the project's *technical risk*. The technical risk is resolved over time as the VC and the EN update their priors on  $\Theta$  in a Bayesian manner based on observations of the firm's performance.

Specifically, we assume that the VC's and EN's initial priors on  $\Theta$  are normally distributed with  $\Theta \sim N(\mu_0^{VC}, \sigma_0^2)$  and  $\Theta \sim N(\mu_0^{EN}, \sigma_0^2)$ , respectively. Define the random variable

$$Y_i := V_i - V_{i-1} - c_i^\alpha \eta_i^\beta + l_i = \Theta + S_i, \quad i = 1, 2, \dots, T-1. \quad (2)$$

Since the VC's capital investment  $c_i$ , and the EN's effort  $\eta_i$  are observable, it follows from well-known formulae (DeGroot 1970) that the posterior distribution on  $\Theta$  for each date  $i \geq 1$  is  $N(\mu_i^\ell, \sigma_i^2)$ , where

$$\sigma_i^2 = \frac{s^2 \sigma_{i-1}^2}{s^2 + \sigma_{i-1}^2} = \frac{s^2 \sigma_0^2}{s^2 + i \sigma_0^2}, \quad (3)$$

$$\mu_i^\ell = \frac{s^2 \mu_{i-1}^\ell + \sigma_{i-1}^2 Y_i}{s^2 + \sigma_{i-1}^2} = \frac{s^2 \mu_0^\ell + \sigma_0^2 (\sum_{t=1}^i Y_t)}{s^2 + i \sigma_0^2}, \quad \ell = VC, EN. \quad (4)$$

Note that  $E[\mu_i^\ell | \mu_{i-1}^\ell] = \mu_{i-1}^\ell$  since  $E[Y_i | \mu_{i-1}^\ell] = \mu_{i-1}^\ell$  and that the  $\sigma_i$  tend to zero. Let

$$\Delta_i := \mu_i^{EN} - \mu_i^{VC} = \frac{s^2 \Delta_0}{s^2 + i \sigma_0^2} = \frac{\sigma_i^2}{\sigma_0^2} \Delta_0, \quad i = 0, 1, 2, \dots \quad (5)$$

denote the *degree of asymmetry in beliefs* at date  $i$ . It follows from (5) that the degree of asymmetry in beliefs is resolved deterministically over time, and there is a linear relationship between the resolution of the asymmetry of beliefs and the resolution of the technical risk. Following Landier and Thesmar (2005), Sahlman (1990) and other researchers, we assume the EN is initially more confident of the success of his ideas, and so  $\Delta_0 \geq 0$ .

For future reference, we denote the information filtration of the probability space generated by the random variables  $\{V_i, i \geq 0\}$  by  $\{F_i\}$ . We let  $\{G_i\}$  denote the information filtration describing the history of termination values, effort choices by the EN, and capital investments by the VC, which is known to both the VC and EN. Clearly,  $F_i \subset G_i$ .

### 3.1.2 VC-EN interaction

Since the project does not generate intermediate cash-flows, the contract between the VC and the EN describes the payoffs to be received by both parties upon termination. Further,

since either the VC or the EN could choose to terminate the relationship at any date, the contract specifies the payoffs to be received by both parties as if the project were terminated at any date in the set  $\{0, 1, \dots\}$ .

More precisely, a feasible contract is described by the stochastic process  $P(\cdot)$ , where  $P(i)$  is the EN's payoff and  $V_i - P(i)$  is the VC's payoff if the relationship is terminated at date  $i \geq 0$ . Since the EN owns the project at the initial date,  $P(0)$  equals  $V_0$ . As the project's termination value is the only economic quantity that is contractible, the process  $P(\cdot)$  is  $\{F_i\}$ -measurable. If the project is terminated at date  $\tau$  (where  $\tau$  is a  $\{G_i\}$ -stopping time), the EN's expected utility at date zero is given by

$$-E \left[ \exp \left\{ -\lambda \left( P(\tau) - \sum_{i=1}^{\tau-1} k\eta_i^\gamma \right) \right\} \right]. \quad (6)$$

In (6), the parameter  $\lambda \geq 0$  characterizes the EN's risk aversion. The EN's disutility from effort in period  $i$  is given by  $k\eta_i^\gamma$  with  $k > 0, \gamma > 0$ .

By (6), the EN has multiplicative separable inter-temporal CARA preferences. We follow Gibbons and Murphy (1992) in therefore restricting consideration to *affine* contractual structures, that is, contracts where

$$P(i) - P(i-1) = a_i + b_i(V_i - V_{i-1}), \quad i = 1, 2, \dots \quad (7)$$

In (7), the contractual parameters  $a_i, b_i$  are  $\{F_{i-1}\}$ -measurable. It will follow from our subsequent analysis that contracts where  $b_i < 0$  for any  $i$  cannot arise in equilibrium. Note that the *change* in the EN's *promised* payoff over period  $i$  is an affine function of the *incremental termination value*  $V_i - V_{i-1}$ . It follows easily from (7) that the process  $P(\cdot)$  describing the EN's contract is given by

$$P(\tau) = P(0) + \sum_{i=1}^{\tau} a_i + b_i(V_i - V_{i-1}). \quad (8)$$

We show that, in equilibrium, the "fixed" component  $a_i$  of the EN's compensation in period  $i$  depends on the history of past "signals"  $\{V_0, V_1, \dots, V_{i-1}\}$  whereas the "proportional" component  $b_i$  in period  $i$  is *deterministic*. It follows from (6) and (8) that the EN's expected utility at date 0 is

$$-E \left[ \exp \left\{ -\lambda \left( V_0 + \sum_{i=1}^{\tau} [a_i + b_i(V_i - V_{i-1}) - k\eta_i^\gamma] \right) \right\} \right]. \quad (9)$$

At each date  $i$ , the EN can choose to terminate the relationship with the VC and receive his payoff  $P(i)$ , and therefore chooses to continue the relationship over the next period if and only if his expected utility from continuation exceeds his utility from termination. The *continuation utility ratio* of the EN at time  $i$ ,  $CUR(i)$ , is defined as the ratio of his expected utility from continuing the relationship to his utility from termination. The continuation utility ratio is given by

$$CUR(i) := E_i^{EN} \left[ \exp \left\{ -\lambda \left( \sum_{j=i+1}^{\tau} a_j + b_j(V_j - V_{j-1}) - k\eta_j^\gamma \right) \right\} \right] \quad (10)$$

where the notation  $E_i^{EN}$  denotes the EN's expectation conditioned on the information available at date  $i$ , that is, the  $\sigma$ -field  $G_i$ . Since the EN has a negative exponential utility function, the EN prefers a *smaller* continuation utility ratio. He, therefore, chooses to continue the relationship if and only if his continuation utility ratio is *at most* one.

Similarly, the VC chooses to continue the relationship if her expected utility from continuing exceeds her utility from termination. We define the VC's *continuation value* as the expected increase in the VC's utility if she continues the relationship, namely, the expected value of the VC's future compensations less her capital investments. The VC's continuation value at date  $i$ ,  $CV(i)$ , is given by

$$CV(i) := E_i^{VC} \left[ \sum_{j=i+1}^{\tau} \left( (1 - b_j)(V_j - V_{j-1}) - a_j - c_j \right) \right], \quad (11)$$

where  $E_i^{VC}$  denotes the VC's expectation conditioned on the information available at date  $i$ . The VC chooses to continue the relationship at date  $i$  if and only if her continuation value is *non-negative*. Since the VC possesses all the bargaining power in negotiations with the EN, it follows that, in equilibrium, the EN's continuation utility ratio is equal to one at every date and in all states of the world. Moreover, the EN is indifferent between continuation and termination in all states. The VC, on the other hand, terminates the relationship when her continuation value is negative.

### 3.2 *Equilibrium*

In order to simplify the subsequent analysis and notation, we assume there exists a maximum possible date  $T > 0$  such that project termination will occur when the VC has a negative

continuation value or at  $T$ , whichever comes earlier. We later show that this assumption is not restrictive by demonstrating that a sufficiently large  $T$  could be chosen so that the VC *voluntarily* terminates the project at a date earlier than  $T$  with probability arbitrarily close to one. Our subsequent analysis also shows that the VC-EN relationship over any time interval  $[0, t]$  does not depend on the choice of time horizon  $T > t$ , that is, the VC's investments, the EN's contract  $P(\cdot)$ , and the EN's effort over the time interval  $[0, t]$  do not depend on the time horizon  $T > t$ .

The following two conditions on the parameters of the model are sufficient to ensure that an equilibrium exists, and will be assumed throughout the remainder of the thesis:

**Assumption 1**  $\gamma > \beta$ .

**Assumption 2**  $(1 - \alpha)\frac{\gamma}{\beta} \geq 2$ .

The first condition implies that the EN faces decreasing returns to scale from the provision of effort. The second condition implies that the decreasing returns to scale from the EN's effort provision are sufficiently pronounced that the VC's "contract choice" problem has a solution, that is, an equilibrium exists. In the next Section we add a third condition, which guarantees that the equilibrium is *unique* and *stable*.

We use *backward induction* to characterize the equilibrium. First consider the last *possible* investment period  $i = T$ . Suppose that the project has not been terminated as of the date  $T-1$  (i.e. the beginning of period  $T$ ). Recall that the EN and VC priors on  $\Theta$  as of date  $T-1$  are  $N(\mu_{T-1}^j, \sigma_{T-1}^2)$  with  $\mu_{T-1}^j$  and  $\sigma_{T-1}^2$  given by (4) and (3), respectively with the index  $i$  set to  $T$ . For subsequent convenience in our inductive derivation of the equilibrium, it will be convenient to use the index  $i$  to denote the time period. The index  $i = T$  for now, but it will later denote an arbitrary time period when we establish the inductive step in our analysis.

### 3.2.1 The EN's Optimal Effort in Period $T$ for a Given Contract

Suppose that, in period  $i$  (recall that  $i = T$ ), the VC's investment is  $c$  and the EN's contractual parameters are  $(a, b)$  (see 7). If the EN exerts effort  $\eta$  in period  $i$ , his continuation

utility ratio (10) is given by

$$CUR(i-1) = E_{i-1}^{EN} \left[ \exp \left\{ -\lambda \left( a + b(V_i - V_{i-1}) - k\eta^\gamma \right) \right\} \right], \quad (12)$$

Using the fact that

$$E_{i-1}^{EN} [V_i - V_{i-1}] = E_{i-1}^{EN} [c^\alpha \eta^\beta - l_i + Y_i] = c^\alpha \eta^\beta - l_i + \mu_{i-1}^{EN},$$

the EN's continuation utility ratio equals<sup>1</sup>

$$\exp \left\{ -\lambda \left( a + b(c^\alpha \eta^\beta - l_i + \mu_{i-1}^{EN}) - k\eta^\gamma - \frac{\lambda}{2} b^2 (\sigma_{i-1}^2 + s^2) \right) \right\}. \quad (13)$$

Since the EN prefers a lower continuation utility ratio he will choose the effort level to minimize (13). The optimal effort level is, therefore, given by<sup>2</sup>

$$\eta(b, c) := \left( \frac{\beta c^\alpha b}{\gamma k} \right)^{\frac{\beta}{\gamma - \beta}}. \quad (14)$$

### 3.2.2 The VC's Choice of Contract in Period $T$

The VC rationally anticipates the EN's best response to his contract. She therefore chooses her investment  $c$  and the EN's contractual parameters  $(a, b)$  so that the EN's participation constraint is satisfied, that is, the EN's continuation utility ratio is at most one (recall that the EN has a negative exponential utility function). Since the VC has the bargaining power, it is optimal for her to choose  $(a, b)$  so that the EN's continuation utility ratio is *equal* to one, that is, his participation constraint is satisfied with equality. We can then show that the relation between the parameters  $a$ ,  $b$ , and  $c$  in in period  $i$  (recall that  $i = T$ ) is

$$a(b, c) := \frac{\lambda}{2} b^2 (\sigma_{i-1}^2 + s^2) + k\eta(b, c)^\gamma - b(c^\alpha \eta(b, c)^\beta - l_i + \mu_{i-1}^{EN}). \quad (15)$$

The above condition guarantees that the EN's continuation utility ratio is exactly one *regardless of the state at date  $i-1$* , that is,

$$CUR(i-1) \equiv 1. \quad (16)$$

<sup>1</sup>Recall that  $E[\exp\{-\lambda X\}] = \exp\{-\lambda(\hat{\mu} - \frac{\lambda}{2}\hat{\sigma}^2)\}$  if  $X \sim N(\hat{\mu}, \hat{\sigma}^2)$ .

<sup>2</sup>Note that Assumption 1 is a necessary and sufficient condition for the EN's problem to be well-defined.

Incorporating the EN's best (effort) response, the VC's continuation value (11) at date  $i-1$  is given by

$$E_{i-1}^{VC} [(1-b)(V_i - V_{i-1}) - a(b, c) - c]. \quad (17)$$

Substituting the EN's optimal effort (14) into (17) and using the fact that

$$E_{i-1}^{VC} [V_i - V_{i-1}] = E_{i-1}^{VC} [c^\alpha \eta^\beta - l_i + Y_i] = c^\alpha \eta^\beta - l_i + \mu_{i-1}^{VC},$$

the VC's continuation value simplifies to

$$\Lambda_i(b, c) := \Delta_{i-1} b - \frac{1}{2} p_{i-1} b^2 + \phi(b) c^{\alpha \frac{\gamma}{\gamma-\beta}} - c + \mu_{i-1}^{VC} - l_i. \quad (18)$$

In (18),

$$p_{i-1} := \lambda(\sigma_{i-1}^2 + s^2) \quad (19)$$

$$\phi(b) := \left(\frac{1}{k}\right)^{\frac{\beta}{\gamma-\beta}} \left( \left(\frac{\beta b}{\gamma}\right)^{\frac{\beta}{\gamma-\beta}} \left(1 - \frac{\beta b}{\gamma}\right) \right), \quad (20)$$

where  $\sigma_{i-1}^2$  and  $\Delta_{i-1}$  are given in (3) and (5), respectively.

It remains to determine the VC's optimal choices for the capital investment  $c_i$  and risky compensation  $b_i$  to the EN to maximize her continuation value (18). We begin with the optimal investment  $c(b)$  as a function of the risky compensation  $b$ . The optimal investment will be zero whenever  $b \geq \gamma/\beta$ . This extraordinary outcome would occur if the degree of asymmetry  $\Delta_{i-1}$  is sufficiently high. We shall impose an upper bound on  $\Delta_0$  that guarantees the investment is positive if the project continues. Fix then a value of  $b \in (0, \gamma/\beta)$ . Assumptions 1 and 2 guarantee the function  $\Lambda_i(b, \cdot)$  is strictly concave in the investment  $c$  (the exponent on  $c$  is guaranteed to be less than 1). As a consequence, setting the partial derivative of  $\Lambda_i(b, \cdot)$  with respect to  $c$  equal to zero yields

$$c(b) := \hat{K} \phi(b)^{\frac{\gamma-\beta}{(1-\alpha)\gamma-\beta}}, \quad (21)$$

from which the VC's continuation value as a function of the risky compensation,  $\Lambda_i(b, c(b))$ , is given by

$$\Lambda_i(b, c(b)) = \Delta_{i-1} b - \frac{1}{2} p_{i-1} b^2 + K c(b) + (\mu_{i-1}^{VC} - l_i). \quad (22)$$

The constants  $\hat{K}$  and  $K$  in (21) and (22) are positive and depend on  $\alpha, \beta$  and  $\gamma$ . Let

$$F_i(b) := \Delta_{i-1}b - \frac{1}{2}p_{i-1}b^2 + Kc(b) \quad (23)$$

denote the variable portion of the VC's continuation value at the beginning of period  $i$ . The VC clearly chooses the risky compensation in period  $i$  to solve

$$F_i^* := \max_{b \geq 0} F_i(b). \quad (24)$$

### 3.2.3 The Inductive Step

We now set  $i = T - 1$ , and suppose the project has not been terminated as of the beginning of this period (i.e. date  $T-2$ ). If in period  $i$  the VC's investment is  $c$ , the EN's contractual parameters are  $(a, b)$ , and he exerts effort  $\eta$ , his continuation utility ratio (10) is given by

$$\begin{aligned} CUR(i-1) &= E_{i-1}^{EN} \left[ \exp \left( -\lambda(a + b(V_i - V_{i-1}) - k\eta^\gamma) \right) CUR(i) \right] \\ &= E_{i-1}^{EN} \left[ \exp \left( -\lambda(a + b(V_i - V_{i-1}) - k\eta^\gamma) \right) \right], \end{aligned} \quad (25)$$

The first line above follows by the law of iterated expectations and the second line follows by (16). Since the expression (25) is identical to (12), we may use our previous arguments to show that the EN's optimal effort is  $\eta(b, c)$  given in (14) and the "fixed" component of the EN's compensation is  $a(b, c)$  given in (15).

It remains to determine the VC's optimal choices for the investment and risky compensation. Incorporating the EN's best (effort) response, the VC's continuation value at the beginning of period  $i$  (recall that the index  $i = T - 1$ ) is given by

$$CV(i-1) = E_{i-1}^{VC} \left[ (1-b)(V_i - V_{i-1}) - a(b, c) - c + \max\{CV(i), 0\} \right] \quad (26)$$

$$= \Lambda_i(b, c) + E_{i-1}^{VC} \left[ \max\{CV(i), 0\} \right]. \quad (27)$$

The above follows from the fact that the expression

$$E_{i-1}^{VC} \left[ (1-b)(V_i - V_{i-1}) - a(b, c) - c \right]$$

is identical to (17) and hence (18). As the right-hand side of (27) is *unaffected* by the actions taken by the VC and EN during period  $i$ , we may use our previous arguments to show that

the VC's continuation value in period  $i$  will be maximized when the optimal investment is given by (21) and the optimal risky compensation solves (24).

We can clearly extend the above arguments by induction to *any* period  $i$  and thereby derive the unique equilibrium, as characterized in the following theorem.

**Theorem 1 (Characterization of Equilibrium)**

*Under Assumptions 1 and 2, if the project has not been terminated as of date  $i-1$ ,  $1 \leq i \leq T$ , then the equilibrium contract offered by the VC and the EN's effort in the period is characterized, as follows:*

- *The risky compensation is  $b_i^*$ , the unique solution to (24);*
- *The investment is  $c_i^* := c(b_i^*)$  defined in (21);*
- *The fixed compensation is  $a_i^* := a(b_i^*, c_i^*)$  defined in (15);*
- *The optimal effort level is  $\eta_i^* := \eta(b_i^*, c_i^*)$  defined in (14).*
- *The VC's maximum continuation value at date  $i-1$  is given by*

$$CV(i-1) = \overbrace{F_i^* + \mu_{i-1}^{VC} - l_i}^{\text{within-period flow}} + \overbrace{E_{i-1}^{VC} [\max\{CV(i), 0\}]}^{\text{future option value}}. \quad (28)$$

**3.3 Properties of the Equilibrium in Each Period**

We now begin our analysis of the properties of the equilibrium that is characterized in Theorem 1. In this section, we focus on the VC's equilibrium investment  $c_i^*$  and the EN's contractual parameters  $a_i^*$  and  $b_i^*$  in a given period  $i$  conditional on the project not having been terminated. By our earlier discussion, the VC continues funding the project in period  $i$  if and only if her continuation value (28) is nonnegative. Since the degree of asymmetry in beliefs  $\Delta_{i-1}$  and variance  $\sigma_{i-1}^2$  are deterministic functions of time, an examination of (24) and (28) reveals that the equilibrium values for the risky compensation, investment and effort in each period are also *deterministic*. In addition, the *controllable* portion of the “within-period flow” in period  $i$ , namely  $F_i^*$ , is also deterministic. The only component of the contract that is stochastic and is adjusted based on realizations of the termination

value  $V_i$  of the project (the “signal” of project quality) is the *fixed* component  $a_i^*$  of the EN’s compensation.

The within-period flow depends on the VC’s current assessment of project quality,  $\mu_{i-1}^{VC}$  and the operating costs,  $l_i$ . If the within-period flow is positive, the VC continues the project. If it is negative, the VC continues the project only if the “future option value” of continuing is large enough to compensate for the current period’s expected loss. Keep in mind that the equilibrium values  $b_i^*$ ,  $c_i^*$ ,  $\eta_i^*$  and  $F_i^*$  only “exist” if the project continues into period  $i$ .

### 3.3.1 VC’s Objective Function

By (23) and (24), the nature of the equilibrium crucially depends on the *VC’s objective function*

$$F_i(b) = \Delta_{i-1}b - \frac{1}{2}p_{i-1}b^2 + Kc(b) \quad (29)$$

The objective function consists of three components:

- *Economic rent from the EN’s optimism.* The term,  $\Delta_{i-1}b$ , reflects the rents that the VC extracts from the EN by “exploiting” his “optimism” about the project’s intrinsic quality (we elaborate on this later).
- *Cost of risk.* The term,  $\frac{1}{2}p_{i-1}b^2$ , reflects the VC’s costs of risk-sharing with the risk-averse EN. We refer to the parameter,  $p_{i-1} = \lambda(\sigma_{i-1}^2 + s^2)$ , as the “*price of risk*” in period  $i$ .
- *Return on investment.* The “return on investment” term,  $Kc(b)$ , reflects the VC’s expected return as a result of his investment and the EN’s effort.

From (29), the EN’s risky compensation in equilibrium clearly depends on the *optimal investment function*  $c(\cdot)$ . The following proposition establishes properties of this function that play a key role in our subsequent analysis.

### Proposition 1

- (i) The optimal investment function  $c(\cdot)$  is strongly unimodal<sup>3</sup> on  $[0, \frac{\gamma}{\beta}]$  and achieves its maximum at  $b = 1$ .
- (ii) The optimal investment function  $c(\cdot)$  is strictly concave on  $[0, b_M]$  and strictly convex on  $[b_M, \frac{\gamma}{\beta}]$ , where  $b_M \in (1, \frac{\gamma}{\beta})$  is the unique minimum of the marginal optimal investment function  $c'(\cdot)$ .

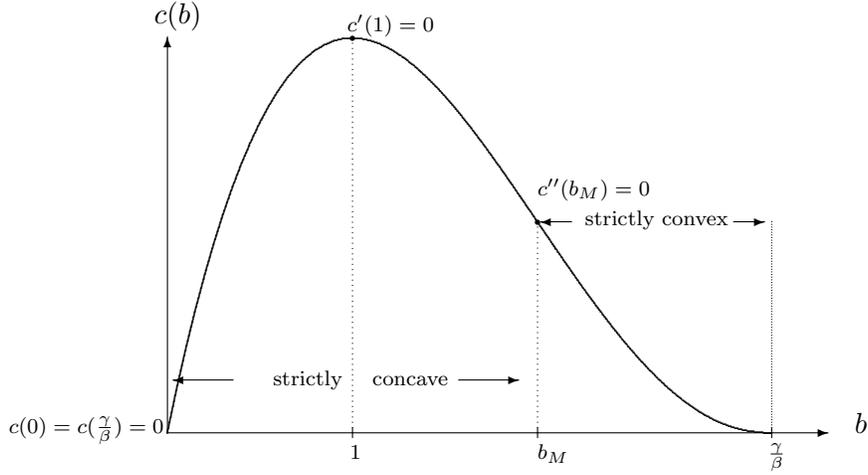
Figure 1 illustrates the structure of the optimal investment function.<sup>4</sup> The intuition for the *non-monotonicity* of the optimal investment function, which is important for understanding our subsequent results, is as follows. An increase in the risky compensation increases the EN's incentives to exert effort, but also increases the costs arising from the EN's higher disutility of effort (these costs are indirectly borne by the VC due to the EN's participation constraint) and the costs of risk-sharing with the risk-averse EN. For lower values of the risky compensation, the benefits of improved risk-sharing with the EN predominate so that the VC finds it beneficial to increase her investment. However, beyond a threshold level of risky compensation, the costs of risk-sharing outweigh the benefits so that the VC reduces her investment. In these regions, the VC induces the EN to exert high effort to generate value, but commits less money. In fact, our subsequent results establish that the equilibrium risky compensation for the EN exceeds 1 only if the degree of asymmetry in beliefs about the project's quality exceeds a threshold. In these scenarios, the VC "exploits" the EN's *exuberance* about the project's prospects.

The ratio of the initial degree of asymmetry of beliefs to the initial price of risk, namely,  $\Delta_0/p_0$ , provides an *a priori* bound on a solution to (24).

---

<sup>3</sup>Recall that a real-valued function of one variable  $f$  is strongly unimodal on the interval  $[a, b]$ ,  $a < b$ , if there exists an  $x^* \in (a, b)$  such that  $f$  is increasing on  $[a, x^*]$  and  $f$  is decreasing on  $[x^*, b]$ . Obviously, the value  $x^*$  maximizes  $f$  on  $[a, b]$ . This class of functions possesses a very simple but extremely useful property for analysis, which we repeatedly exploit: If  $f$  is strongly unimodal on  $[a, b]$  and also differentiable, then the sign of the derivative indicates the direction of the optimum solution, i.e., if  $f'(x) > 0$ , then  $x^* > x$ ; if  $f'(x) < 0$ , then  $x^* < x$ ; and if  $f'(x) = 0$ , then  $x^* = x$ . A simple and extremely fast (bisection search) algorithm will find the optimal solution to a strongly unimodal function.

<sup>4</sup>Unless otherwise stated, the parameters in this thesis are  $\alpha = 0.3875$ ,  $\frac{\gamma}{\beta} = 3.2653$ ,  $k = 0.1914$ ,  $\lambda = 1.0938$ ,  $s^2 = 0.5$  and  $\sigma_0^2 = 0.5$ ,  $\Delta_0 = 0.5$ . In Chapter 5 we explain why this particular choice of numbers.



**Figure 1:** Optimal investment function

**Proposition 2**

- (i) If  $\Delta_0/p_0 \leq 1$ , then an optimal solution to (24) is at most 1.
- (ii) If  $\Delta_0/p_0 > 1$ , then an optimal solution to (24) is less than  $\Delta_0/p_0$ .

In our subsequent analysis we assume that

**Assumption 3**  $\Delta_0/p_0 \leq b_M$ ,

where the parameter  $b_M$  is defined in Proposition 1. The above condition implies that the initial degree of asymmetry in beliefs  $\Delta_0$  of the EN and the VC is below a threshold relative to the price of risk  $p_0$ .

It follows from Proposition 2 and Assumption 3 that a solution to (24) must lie in the interval  $[0, b_M]$ . By Proposition 1, the VC's objective function is easily seen to be strictly concave and hence strongly unimodal. Consequently, there exists a unique solution  $b_i^*$  to (24), which we show must be positive.<sup>5</sup> We summarize these observations with the following proposition.

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<sup>5</sup>The marginal optimal investment evaluated at zero is infinite.

**Proposition 3**

Under Assumptions 1-3, each  $F_i(\cdot)$  is strictly concave and hence strongly unimodal on  $[0, b_M]$ , and the solution to (24) is positive and less than  $b_M$ .

**Remark 1**

The strong unimodality of the function  $F_i$  ensures the stability of the equilibrium described in the above theorem, that is, the EN's equilibrium risk-free and risky compensation and effort choices, and the VC's capital investments are continuous functions of the model parameters. Moreover, the equilibrium risky compensation can easily be numerically computed using an efficient bisection search—see footnote 3.

We now use the above results to further analyze the VC's objective function (29) and thereby determine the properties of the EN's equilibrium risky compensation. By the result of Proposition 3, the VC's objective function attains a unique, *interior* maximum so that its derivative must necessarily vanish at this maximum. The derivative of the VC's objective is

$$F'_i(b) = \Delta_{i-1} - p_{i-1}b + Kc'(b). \quad (30)$$

The EN's equilibrium risky compensation  $b_i^*$  in period  $i$  is therefore determined by the interplay among the *degree of asymmetry in beliefs*  $\Delta_{i-1}$ , the *price of risk*  $p_{i-1}$ , and the *marginal optimal investment* function  $c'(\cdot)$ . The following proposition precisely describes how the relation between the degree of asymmetry in beliefs and the price of risk affect the EN's equilibrium risky compensation in any period.

**Proposition 4**

Suppose that the VC-EN relationship is active in period  $i$ , that is, the project has not been terminated prior to date  $i$ . When the degree of asymmetry of beliefs  $\Delta_{i-1}$  at date  $i - 1$  is less than (equal, greater than) the price of risk  $p_{i-1}$ , the corresponding equilibrium risky compensation parameter  $b_i^*$  is less than (equal, greater than) 1.

The intuition for the results of the above proposition can be understood using the intuition for the non-monotonicity of the optimal investment function  $c(\cdot)$  described earlier. The VC's optimal choice of risky compensation for the EN reflects the tradeoff between

providing appropriate incentives for the EN to exert effort with the costs of risk-sharing with the risk-averse EN and the costs associated with the EN's disutility of effort. This tradeoff is significantly affected by the degree of asymmetry of beliefs about the project's quality as the VC could "exploit" the EN's optimism by inducing him to exert greater effort without incurring significant risk-sharing costs. When the degree of asymmetry of beliefs is lower than the price of risk (the EN is "reasonably optimistic"), the VC chooses a level of risky compensation less than one as the costs of risk sharing dominate the benefits of the EN's optimism. However, if the degree of asymmetry of beliefs exceeds the price of risk (the EN is "exuberant"), the VC "exploits" the EN by inducing her to accept a level of risky compensation that exceeds one. The effects of the EN's optimism and the costs of risk-sharing and effort are "perfectly balanced" when the degree of asymmetry in beliefs equals the cost of risk. In this scenario, the EN's risky compensation is exactly equal to one.

**Remark 2**

*As we subsequently demonstrate, when the EN is initially "reasonably optimistic", i.e.,  $\Delta_0 < p_0$ , he will remain so classified as time goes by. However, when the EN is initially "exuberant", i.e.,  $\Delta_0 > p_0$ , the resolution of the technical uncertainty about the project's intrinsic quality over time leads to a decline in the EN's level of optimism so that he eventually shifts from being "exuberant" to "reasonably optimistic".*

The following proposition describes the effect of the degree of asymmetry in beliefs  $\Delta_{i-1}$  and the price of risk  $p_{i-1} = \lambda(\sigma_{i-1}^2 + s^2)$  on the equilibrium levels of risky compensation and investment in period  $i$ .

**Proposition 5**

- (i) *The equilibrium risky compensation parameter  $b_i^*$  in period  $i$  is a decreasing function of the price of risk (and hence the individual parameters  $\lambda$ ,  $s^2$ , and  $\sigma_{i-1}^2$ ), and is an increasing function of the degree of asymmetry of beliefs.*
- (ii) *When the degree of asymmetry of beliefs is less (more) than the price of risk, the equilibrium level of investment  $c_i^*$  in period  $i$  is a decreasing (increasing) function of*

the price of risk and an increasing (decreasing) function of the degree of asymmetry of beliefs.

**Remark 3**

With regard to the equilibrium level of effort, it is clear from (14) that when the degree of asymmetry of beliefs is less than the price of risk, the equilibrium effort is a decreasing function of the price of risk and an increasing function of the degree of asymmetry of beliefs. However, when the degree of asymmetry exceeds the price of risk, the equilibrium effort might be non-monotonic.

An increase in the EN’s perception of the project’s intrinsic quality means he is willing to accept more risky compensation from the VC.

### 3.4 *Equilibrium Dynamics*

In the previous section, we described the “static” properties of the equilibrium, that is, the level of investment by the VC and the EN’s contractual parameters in a given period (conditional on continuation of the project). In this section, we investigate the *dynamics* of the equilibrium, that is, we describe the evolutions of the EN’s contract, his effort, and the VC’s investment. We show that the interplay between the *technical risk* of the project that represents the uncertainty about the project’s intrinsic quality, and the degree of asymmetry in beliefs about the project’s quality is the key determinant of the dynamics of the VC-EN relationship.

Before analyzing the general scenario where there is imperfect information as well as asymmetry in beliefs about the project’s quality, we briefly discuss two “benchmark” scenarios.

#### 3.4.1 **Symmetric Attitudes towards Risk and Symmetric Beliefs about Project Quality (Full Symmetry)**

In this scenario, the VC and the EN are both risk-neutral and have symmetric beliefs about the project’s quality. Therefore,  $\lambda = 0$  and  $\Delta_i = 0$  for all  $i$ .<sup>6</sup> It follows that the first and second components of the VC’s objective function (29) are zero. The third component, the

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<sup>6</sup>With CARA preferences, maximizing utility converges to maximizing the mean as  $\lambda \rightarrow 0$ .

return on investment, is always maximized at  $b = 1$  (Proposition 1). Therefore, the equilibrium levels of risky compensation, the VC's investment and the EN's effort are constant through time, and the VC's investment is at its highest possible level. These results follow from the fact that as the VC and the EN have symmetric attitudes towards risk and symmetric beliefs, they effectively function as a monolithic agent. Moreover, the risk-neutrality of the VC/EN implies that risk (systematic and technical) of the project does not affect the level of investment, the EN's contract, or his effort.

**Remark 4**

*While it is true that the investment levels are at their highest level in this scenario, it is important to emphasize that project value need not attain its maximum possible value. Optimism on the part of the EN can potentially be exploited by the VC by inducing greater effort from the EN thereby generating more value.*

**3.4.2 Perfect Information**

In the *perfect information* case the EN is risk averse ( $\lambda > 0$ ) but there is perfect information about the project's quality so that there is no technical risk ( $\sigma_0^2 = 0$ ) In this scenario, the VC's objective function

$$F_i(b) = F(b) := -\frac{\lambda}{2}s^2b^2 + Kc(b), \tag{31}$$

is independent of time. The time paths of risky compensation, investment and effort are all constant; we let  $b_p^*$ ,  $c_p^*$  and  $\eta_p^*$  denote the corresponding equilibrium values.

By Proposition 1 the optimal investment function achieves it maximum at  $b = 1$ , which implies that  $c'(1) = 0$ . Since

$$F'(b_p^*) = 0, \tag{32}$$

it follows from (31) that  $F'(1) < 0$ . The strong unimodality of  $F(b)$  now guarantees that  $b_p^* < 1$ , and therefore both  $c_p^*$  and  $\eta_p^*$  are less than the investment and effort levels in the “full symmetry” scenario where the VC and the EN are both risk-neutral and have symmetric beliefs about project quality.

### 3.4.3 Imperfect Information, Asymmetric Beliefs and Asymmetric Risk Attitudes - The Actual Scenario

We now analyze the scenario of interest where the attitudes towards risk as well as beliefs about project quality are asymmetric, that is,  $\sigma_0^2 > 0, \lambda > 0$ . Recall that  $b_i^*$ ,  $c_i^*$  and  $\eta_i^*$  denote, respectively, the equilibrium levels of risky compensation, investment and effort in period  $i$  (if the project has not been terminated).

The equilibrium *paths* of the EN's risky compensation, the  $(b_i^*)$ , the VC's investments, the  $(c_i^*)$ , and the EN's effort, the  $(\eta_i^*)$ , depend on the interplay among the three components of the VC's objective function (29)—the economic rent from the EN's optimism, the cost of risk and the return on investment. As the third component is obviously constant through time, the evolution of the optimal value  $F_i^*$  of the VC's objective function is determined by the evolutions of the first two components, which are deterministic functions of time. Both  $\Delta_{i-1}$  and  $p_{i-1}$  *decrease* with time so that the sum of the first two components is *not* necessarily monotonic with time. Consequently, the equilibrium values  $F_i^*$  (the maximum value of the VC's objective function 29), are not generally monotonic, either.

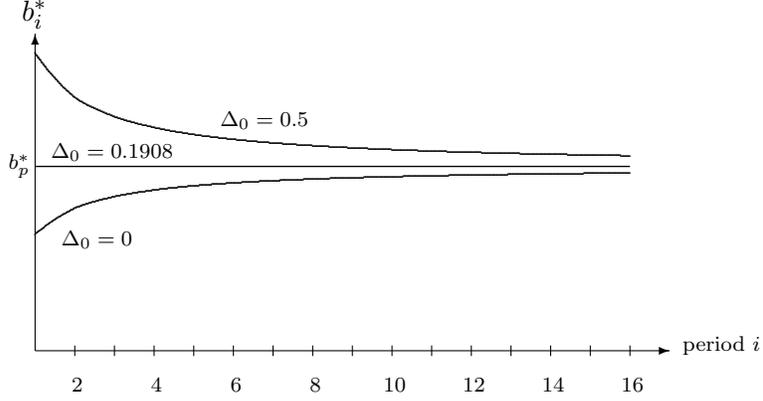
We now present a complete characterization of the equilibrium dynamics of the VC-EN relationship. The VC's objective function may be expressed as

$$F_i(b) = \left( \frac{\Delta_0}{\sigma_0^2} b - \frac{\lambda}{2} b^2 \right) \sigma_{i-1}^2 + F(b). \quad (33)$$

Since  $\sigma_i \rightarrow 0$ , it follows from the Envelope Theorem that  $b_i^* \rightarrow b_p^*$ , and thus  $(c_i^*, \eta_i^*) \rightarrow (c_p^*, \eta_p^*)$  by continuity. We now precisely describe the manner in which these economic variables converge to their asymptotic values. To simplify the subsequent exposition, we consider the index  $i$  as a continuous variable. Define

$$i^* := \max \left( \frac{\Delta_0 - \lambda s^2}{\lambda \sigma_0^2}, 0 \right). \quad (34)$$

Note that  $i^*$  is positive only when the EN is initially “exuberant”; otherwise, it is always zero. The following theorem describes the evolutions of the EN's risky compensation, his effort, and the VC's investment.



**Figure 2:** Optimal risky compensation path for different levels of initial asymmetry

**Theorem 2 (The Dynamics of the Equilibrium Contract)**

- (i) If  $\Delta_0 < \lambda\sigma_0^2 b_p^*$ , then the  $b_i^*$ ,  $c_i^*$  and  $\eta_i^*$  increase monotonically towards  $b_p^*$ ,  $c_p^*$  and  $\eta_p^*$ , respectively.
- (ii) If  $\Delta_0 = \lambda\sigma_0^2 b_p^*$ , then the  $b_i^*$ ,  $c_i^*$  and  $\eta_i^*$  are constant and equal  $b_p^*$ ,  $c_p^*$  and  $\eta_p^*$ , respectively.
- (iii) If  $\Delta_0 > \lambda\sigma_0^2 b_p^*$ , then the  $b_i^*$  decrease monotonically towards  $b_p^*$ , the  $c_i^*$  increase until  $i = i^*$  and then decrease monotonically towards  $c_p^*$ , and the  $\eta_i^*$  decrease monotonically towards  $\eta_p^*$  when  $i \geq i^*$ .

**Remark 5**

The value of  $i^*$  is precisely the point in time when the EN's risky compensation parameter is 1 and the investment is at its maximum. Prior to this point in time, the EN is “exuberant” and risky compensation exceeds one. After this point in time, the EN is “reasonably optimistic” and his risky compensation parameter is less than 1. The VC's equilibrium investment path is non-monotonic when the EN is initially “exuberant”. When the EN is initially “reasonably” optimistic, i.e.,  $i^* = 0$ , investments by the VC decrease over time.

The intuition for the results of Theorem 2 hinges on the complex interplay among the value-enhancing effort by the EN that is positively affected by his optimism, the costs of risk-sharing due to the EN's risk aversion that is affected by the project's systematic and technical risk, and the effect of both the VC's physical capital investment and the EN's effort on output.

The passage of time causes technical risk to be resolved thereby *lowering* the costs of risk-sharing. However, the passage of time also lowers the degree of asymmetry in beliefs of the VC and the EN as successive project realizations cause the EN to revise his optimistic assessment of project quality. The decline in the degree of asymmetry in beliefs lowers the rents that the VC can extract by exploiting the EN’s optimism.

If the initial degree of asymmetry in beliefs is below a threshold so that the EN is “reasonably optimistic”, the beneficial effect of time on the costs of risk-sharing dominate so that the EN’s risky compensation and effort both increase. As the EN’s effort increases over time, the VC optimally *lowers* her investment over time

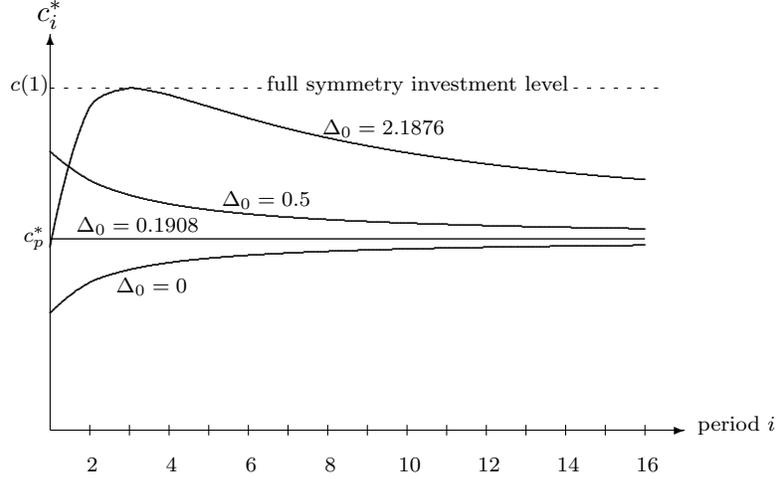
If the degree of asymmetry in beliefs is above a threshold so that the EN is “exuberant”, he is willing to accept all the risk of the project so that his risky compensation and effort are initially *high*. The negative effect of the evolution of time on the degree of asymmetry in beliefs, however, dominates its positive effect on the costs of risk-sharing so that the EN’s risky compensation effort declines over time. Due to the previously discussed non-monotonic relation between the VC’s investment and the EN’s risky compensation, the VC’s investment initially increases to “compensate” for the decrease in effort of the EN. After a certain point in time when the VC’s investment attains its maximum, the decreasing effort of the EN makes it optimal for the VC to also lower her capital investments.

There exists an initial degree of asymmetry of beliefs for which the positive effects of the resolution of technical risk on the costs of risk sharing and its negative effects on the EN’s incentives to exert effort due to his effort balance each other *exactly* so that risky compensation, investment, and effort are constant over time.

Figure 2 illustrates the results of Theorem 2 and the intuition underlying it. It describes three possible trajectories of risky compensation, whose outcomes depend on the initial degree of asymmetry in beliefs  $\Delta_0$ .<sup>7</sup> When the initial asymmetry is low compared to the initial technical risk and the EN’s risk-aversion, the positive effect of the resolution of technical risk dominates the negative effect of the resolution of the asymmetry. Consequently, risky compensation increases with time. When the initial degree of asymmetry in beliefs is high

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<sup>7</sup>The value  $0.1908 = \lambda\sigma_0^2 b_p^*$ .



**Figure 3:** Equilibrium investment paths

relative to the initial technical risk and degree of risk-aversion, the negative effect of the resolution of asymmetry now dominates the positive effect of the resolution of technical risk, and risky compensation now decreases with time.

Figure 3 depicts three possible trajectories of investment, whose outcomes depend on  $\Delta_0$ .<sup>8</sup> When  $\Delta_0 = 0.5$ , the economic rent component is not sufficiently high to place the initial risky compensation above 1, and so the risky compensation path lies below 1. Consequently, the value of  $i^*$  is zero, and so the equilibrium investment path is strictly decreasing.

### 3.5 Sensitivity of Equilibrium Dynamics

In this section, we investigate how the equilibrium dynamics are affected by changes in the degree of asymmetry in beliefs,  $\Delta_0$ , the EN’s risk aversion,  $\lambda$ , the project’s initial technical risk,  $\sigma_0^2$ , the EN’s cost of effort,  $k$ , and the systematic risk,  $s^2$ .

In light of Theorem 2, the subsequent analysis critically depends on the initial value of the degree of asymmetry in beliefs  $\Delta_0$ . The EN is termed “reasonably optimistic” if  $\Delta_0 \in [0, \lambda s^2 b_p^*)$  and “exuberant” if  $\Delta_0 \in (\lambda s^2 b_p^*, p_0 b_M)$ .<sup>9</sup>

<sup>8</sup>The value  $2.1876 = b_M p_0$ .

<sup>9</sup>Recall that Assumption 3 guarantees that  $\Delta_0 \leq p_0 b_M$ .

### 3.5.1 Risk

The following theorem characterizes the effects of the EN's risk aversion,  $\lambda$ , the initial technical risk,  $\sigma_0^2$ , and the systematic risk,  $s^2$  on the equilibrium dynamics. The statements below regarding the systematic risk,  $s^2$ , require the additional condition that

**Assumption 4**  $\Delta_0 < 2p_0$ .

This condition is automatically satisfied when  $b_M < 2$ . Recall that our prior assumptions ensure the  $b_i^*$  must lie below  $b_M$ ; it is quite reasonable to assume the problem parameters are such that the risky compensation offered to the EN by the VC would not exceed *twice* the change in termination value.

#### Theorem 3

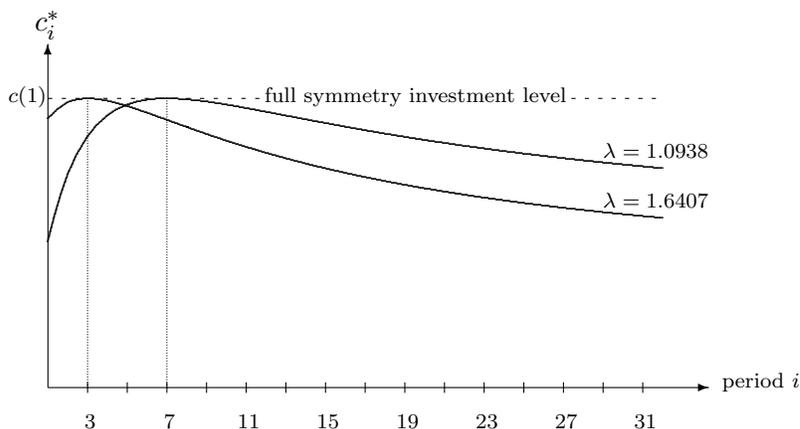
- (i) *The EN's equilibrium risky compensation in any period is a decreasing function of the price of risk and, therefore, the EN's risk aversion  $\lambda$  and the initial technical risk  $\sigma_0^2$ . This property also holds for the systematic risk  $s^2$  under Assumption 4.*
- (ii) *If the EN is initially reasonably optimistic, then the VC's equilibrium investment in any period is a decreasing function of the price of risk and, therefore, the EN'S risk aversion  $\lambda$  and the initial technical risk  $\sigma_0^2$ . This property also holds for the systematic risk  $s^2$  under Assumption 4.*
- (iii) *If the EN is initially exuberant, then the path of equilibrium investment by the VC changes as in Figure 4 as a result of a change in the price of risk and, therefore, a change in the EN's risk aversion  $\lambda$  and the initial technical risk  $\sigma_0^2$ .<sup>10</sup> (The time-path of investment moves "to the left" if the initial price of risk increases.) This property also holds for the systematic risk  $s^2$  under Assumption 4.*

#### Remark 6

*Figure 4 demonstrates that the path of equilibrium investment will converge to different limiting values depending on the EN's risk aversion.*

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<sup>10</sup>The parameters  $\sigma_0 = 0.1$  and  $\Delta_0 = 1.31256$ .



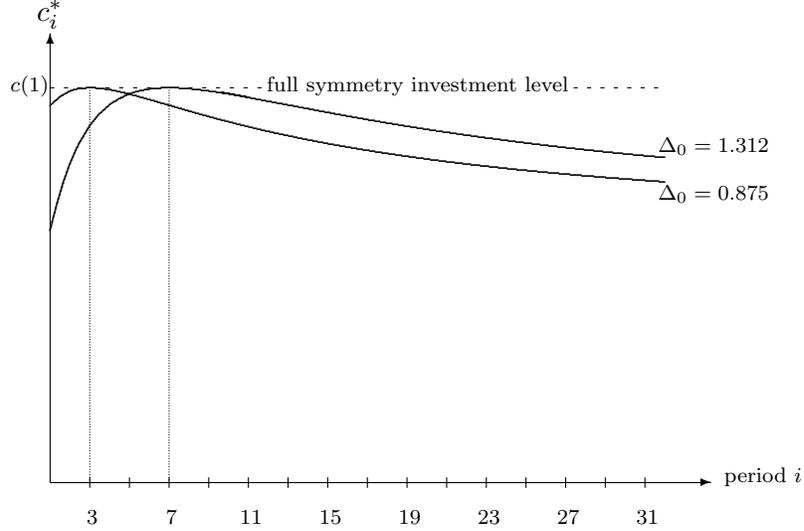
**Figure 4:** Sensitivity of equilibrium investment path to the price of risk.

The EN’s risky compensation parameter  $b_i^*$  in any period  $i$  declines with his risk aversion, the initial technical risk, and the systematic risk as an increase in any of these parameters increases the costs of risk-sharing between the VC and the EN.

The effects of risk aversion, systematic and technical risk on the VC’s investment path are, however, more subtle due to the presence of asymmetric beliefs. If the EN is initially reasonably optimistic, the costs of risk sharing outweigh the benefits of the EN’s optimism. The VC’s equilibrium investment path, therefore, declines *pointwise* with the EN’s risk aversion and the project’s market and technical risk in this region. If the EN is initially exuberant, an increase in risk increases the costs of risk sharing, thereby partially offsetting the VC’s rents from the EN’s exuberance. In early periods, it is beneficial for the VC to “compensate” for the resulting decline in the EN’s effort by increasing investment. As time passes, however, the EN’s degree of optimism declines thereby reducing the rents to the VC. The costs of risk-sharing, therefore, dominate in later periods so that an increase in risk results in a decline in the VC’s investment.

### 3.5.2 Asymmetry in beliefs

Not surprisingly, the effect of the initial degree of asymmetry in beliefs  $\Delta_0$  on the EN’s equilibrium risky compensation path and the VC’s equilibrium investment path is *opposite* to the effects of the cost of risk on these paths.



**Figure 5:** Sensitivity of equilibrium investment path to the initial asymmetry in beliefs

**Theorem 4**

- (i) *The EN's equilibrium risky compensation parameter in any period increases with the initial degree of asymmetry in beliefs  $\Delta_0$ .*
- (ii) *If the EN is initially reasonably optimistic, then the VC's equilibrium investment in any period is an increasing function of the initial degree of asymmetry in beliefs  $\Delta_0$ .*
- (iii) *If the EN is initially exuberant, then the path of equilibrium investment by the VC changes as in Figure 5 as a result of a change in the initial degree of asymmetry in beliefs  $\Delta_0$ .<sup>11</sup> (The time-path of investment moves “to the right” if the initial degree of asymmetry increases.)*

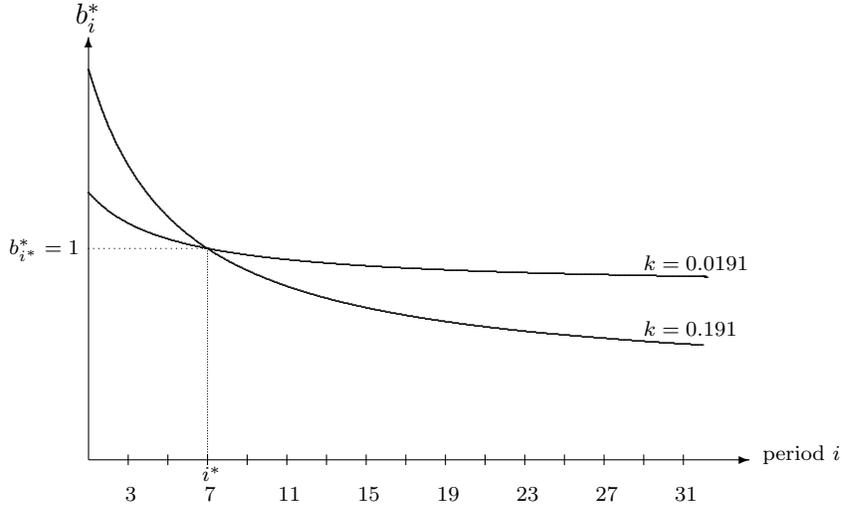
While the trajectory of  $b$  moves upward (downward) if  $\Delta_0$  increases (decreases), the influence of a perturbation of  $\Delta_0$  on the trajectory of  $c$  depends on whether  $\Delta_0$  lies in the increasing or decreasing region.

**3.5.3 Cost of effort**

The influence of a change in the cost of effort  $k$  on the EN's equilibrium risky compensation again depends on whether the EN is initially reasonably optimistic or exuberant.

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<sup>11</sup>The parameter  $\sigma_0 = 0.1$ .



**Figure 6:** Sensitivity of risky compensation to the cost of effort

**Theorem 5**

- (i) *If the EN is initially reasonably optimistic, then his equilibrium risky compensation parameter in any period is a decreasing function of the cost of effort  $k$ .*
- (ii) *If the EN is initially exuberant, then the change in his path of risky compensation as a result of a change in his cost of effort is as described in Figure 6.<sup>12</sup>*
- (iii) *The VC's equilibrium investment path is a pointwise decreasing function of the EN's cost of effort  $k$ .*

The intuition for the effect of a change in the cost of effort on the equilibrium paths of risky compensation and investment is as follows. When the degree of asymmetry of beliefs is “low” in comparison to the price of risk, the economic rents that the VC can potentially capture due to the EN’s exaggerated assessment of project quality are low compared with the costs of risk sharing and inducing effort from the EN. Therefore, as the EN’s cost of effort increases, the VC lowers the EN’s risky compensation as well as her own investment in the project in each period. On the other hand, if the EN is initially exuberant so that the degree of asymmetry of beliefs is “high” in comparison to the price of risk, the beneficial

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<sup>12</sup>The parameters  $\sigma_0 = 0.1$  and  $\Delta_0 = 1.31256$ .

effects of exploiting the EN's exuberance about the project's prospects dominate the costs of risk sharing and inducing effort in early periods. Therefore, the VC increases the EN's risky compensation in early periods, but lowers her own investment. As time evolves, project realizations cause the EN to revise his own assessment of project quality so that the costs of risk sharing eventually dominate the rents from exploitation. Hence, the EN's risky compensation and the VC's investment both decline in later periods.

### 3.6 *Project Duration*

In our analysis thus far, we have examined the dynamics of the VC's investments, the EN's compensation, and the EN's effort conditional on the project's continuation. As described earlier, the VC continues the project as long as her expected continuation value is positive. We now investigate the optimal termination decision of the VC, that is, the project's duration.

We first describe the optimal termination policy of the VC.

#### **Proposition 6**

*The optimal stopping policy for the VC is a trigger policy: there exist  $\mu_i^*$ ,  $0 \leq i \leq T - 1$ , such that the VC should terminate the project only if  $\mu_i^{VC} < \mu_i^*$ .*

The intuition for the above result is straightforward. At any date  $i$ , the VC's expected continuation value increases with her current assessment  $\mu_i^*$  of the project's quality. Since the VC continues the project if and only if her expected continuation value is nonnegative, at each date, there exists a trigger level such that she continues the project if and only if her current assessment of the project's quality exceeds the trigger.

This trigger policy can be also expressed to depend on the termination value instead of the quality assessment. By equation (1)

$$V_i - V_0 = \sum_{t=1}^i \Delta V_t = \sum_{t=1}^i (Y_t + c_t^{*\alpha} \eta_t^{*\beta} - l_t) \quad (35)$$

and therefore we can express

$$\sum_{t=1}^i Y_t = V_i - V_0 - \sum_{t=1}^i (c_t^{*\alpha} \eta_t^{*\beta} - l_t). \quad (36)$$

Substituting (36) in  $\mu_i$  given in (4) we have

$$\mu_i^{VC} = \frac{s^2 \mu_0^{VC} + \sigma_0^2 \left( V_i - V_0 - \sum_{t=1}^i (c_t^{*\alpha} \eta_t^{*\beta} - l_t) \right)}{s^2 + i\sigma_0^2} \quad (37)$$

and may conclude that

$$\mu_i \geq \mu_i^* \text{ if and only if } V_i \geq V_i^*,$$

where

$$V_i^* := V_0 + \sum_{t=1}^i (c_t^{*\alpha} \eta_t^{*\beta} - l_t) + \frac{(s^2 + i\sigma_0^2)\mu_i^* - s^2\mu_0}{\sigma_0^2}.$$

The sequence of the  $V_i^*$  may be thought of as the *performance targets* the firm must reach at each date or else it will terminate the project. Thus, either the  $\mu_i^*$  or the  $V_i^*$  may be used to define the trigger policy; the performance targets are more commonly used in practice.

The following result describes the effect of the EN's initial assessment of project quality, his risk aversion, and his cost of effort on the duration of the project.

**Proposition 7**

*The project duration  $\tau$  (i) increases with the EN's initial assessment of project quality,  $\Delta_0^{EN}$ , and (ii) decreases with his risk aversion,  $\lambda$ , and his cost of effort,  $k$ .*

As discussed earlier, an increase in the EN's initial degree of optimism about project quality increases the rents that the VC is able to extract by exploiting the EN's optimism thereby increasing her expected continuation value in *every* period. Hence, it is optimal for the VC to prolong the project's duration. An increase in the EN's risk aversion or cost of effort, however, increases the costs of risk-sharing for the VC, thereby lowering her continuation value in *every* period. Hence, the VC terminates the project earlier.

In the next section, we numerically analyze the effects of the project's market and technical risk on the duration of the VC-EN relationship. We demonstrate the striking result that market and technical risk generally have *opposing* effects on the project duration.

The following result establishes that the project is terminated in finite time almost surely.

### Proposition 8

For any  $\delta > 0$  there exists an  $N > 0$  such that, for any  $N' \geq N$ , in the scenario where the maximum possible number of periods is  $N'$ , the termination time is strictly less than  $N$  with probability greater than  $1 - \delta$ .

### 3.7 Proofs

To simplify the notation in the proofs to follow, we make a useful observation. The incremental change in termination value (1) depends on  $\eta$  only through the terms  $\eta^\beta, \eta^\gamma$ . There is no loss of generality if the unit of effort is redefined as  $z := \eta^\beta$ , the production function is taken as  $c^\alpha z$  and the disutility of effort is taken as  $z^{\gamma/\beta}$ . Note how the equilibrium, as characterized in Theorem 1, depends on the parameters  $\beta$  and  $\gamma$  only through their *ratio*  $\gamma/\beta$ . Accordingly, we shall hereafter normalize  $\beta$  to 1.

**Proof of Proposition 1.** The marginal (optimal) investment is given by

$$c'(b) \propto \left(\frac{1}{k}\right)^{\frac{1}{(1-\alpha)\gamma-1}} b^t (\gamma - b)^s (1 - b) \quad (38)$$

where

$$t := \frac{2 - (1 - \alpha)\gamma}{(1 - \alpha)\gamma - 1} \text{ and } s := \frac{\alpha\gamma}{(1 - \alpha)\gamma - 1}, \quad (39)$$

and where the symbol  $\propto$  means “equal up to a positive multiplicative constant”. Under Assumption 2, the parameter  $s$  is positive and the parameter  $t$  is negative. (Keep in mind that  $\beta$  is now 1.) Since  $\gamma > 1$  (Assumption 1), the strong unimodality of  $c(\cdot)$  easily follows from (38). Since  $c(0) = c(\gamma) = 0$  and  $c'(0) = +\infty$ , it also follows from (38) that  $c(\cdot)$  achieves its maximum at  $b = 1$ . Part (i) has been established.

As for part (ii), the second derivative is given by

$$c''(b) \propto b^{t-1} (\gamma - b)^{s-1} [t(\gamma - b)(1 - b) - sb(1 - b) - b(\gamma - b)]. \quad (40)$$

The expression inside the brackets is a strictly convex quadratic function whose value at 1 is negative, whose value at  $\gamma > 1$  is positive, and whose value at 0 is negative since  $t < 0$ . Consequently, there is exactly one root  $b_M$  of the quadratic in the interval  $(1, \gamma)$  such that  $c''(b_M) = 0$ . At  $b_M$  the marginal investment is at its *minimum*. Moreover, since  $c''(\cdot)$  is

negative on  $[0, b_M)$  and is positive on  $(b_M, \gamma)$ , the optimal investment function is therefore strictly concave on  $[0, b_M]$  and strictly convex on  $[b_M, \gamma]$ . This establishes part (ii). ■

We note the ratio of the asymmetry of beliefs  $\Delta_{i-1}$  to the price of risk  $p_{i-1} = \lambda(s^2 + \sigma_{i-1}^2)$  in period  $i$  may be expressed as

$$\frac{\Delta_{i-1}}{p_{i-1}} = \frac{\Delta_0}{\lambda(s^2 + i\sigma_0^2)}. \quad (41)$$

Using (41), we shall find it convenient to express the derivative of  $F_i$  (23) as

$$F'_i(b) = p_{i-1} \left[ \frac{\Delta_0}{\lambda(s^2 + i\sigma_0^2)} - b \right] + Kc'(b); \quad (42)$$

this functional form shall be repeatedly exploited in the proofs to follow.

**Proof of Proposition 2.** Obviously,  $p_{i-1} \left[ \frac{\Delta_0}{p_0} - b \right] \leq 0$  if  $\frac{\Delta_0}{p_0} \leq b$ . Parts (i) and (ii) now directly follow from (42) and Proposition 1. ■

**Proof of Proposition 3.** By Proposition 1, each  $F_i$  is the sum of a concave and strictly concave function on  $[0, b_M]$ , and so is strictly concave on this region, too. Since  $b_M > 1$ , it follows directly from Proposition 2 that an optimal solution to (24) must lie below  $b_M$ . Since  $F'_i(0) = +\infty$ , the optimal solution must be positive. ■

With a slight abuse of notation, for each parameter “ $\Pi$ ” we let  $b_i(\pi)$  and  $c_i(\pi)$  denote, respectively, the value of  $b$  and  $c$  at date  $i$  when the parameter  $\Pi$ 's value equals  $\pi$ , and we let  $b(\pi)$  and  $c(\pi)$  denote the entire time path of pay performance sensitivity and optimal investment when the parameter  $\Pi$ 's value equals  $\pi$ . We shall also write  $F'_i(b, \pi)$  to make explicit the functional dependence of the derivative of  $F_i$  on the parameter value  $\pi$ .

The following simple observation, embodied in the following Lemma, will be used repeatedly in the proofs to follow.

**Lemma 1**

*If  $F'_i(b, \pi)$  is an increasing (decreasing) function of  $\pi$ , then  $b_i(\pi)$  is an increasing (decreasing) function of  $\pi$ .*

**Proof.** Let  $\pi^1 < \pi^2$ . Suppose first that  $F'_i(b, \pi)$  is an increasing function of  $\pi$ . By definition,

$$0 = F'_i(b_i(\pi^2), \pi^2) = F'_i(b_i(\pi^1), \pi^1) < F'_i(b_i(\pi^1), \pi^2),$$

which immediately implies  $b(\pi^1) < b(\pi^2)$  by the strong unimodality of  $F_i$ . The proof in the decreasing case is analogous. ■

**Proof of Proposition 5.** Each part follows by a straightforward application of Lemma 1.

■

**Proof of Theorem 2.** We begin by proving the claim concerning the  $b_i^*$ . From (33) and (32),

$$F'_i(b_p^*) = \left( \frac{\Delta_0}{\sigma_0^2} - \lambda b_p^* \right) \sigma_{i-1}^2. \quad (43)$$

Therefore, the sign of  $F'_i(b_p^*)$  is identical to the sign of  $\Delta_0 - \lambda \sigma_0^2 b_p^*$ . The strong unimodality of each  $F_i(\cdot)$  now ensures that if this sign is negative (positive) then the  $b_i^*$  will lie strictly below (above)  $b_p^*$ . If the sign is zero then the  $b_i^*$  coincide with  $b_p^*$ . It remains to show the convergence is monotonic in the first and third cases. To this end suppose  $\Delta_0 < \lambda \sigma_0^2 b_p^*$ . Pick a period  $i$ . The optimal solution  $b_i^*$  satisfies

$$0 = F'_i(b_i^*) = \left( \frac{\Delta_0}{\sigma_0^2} - \lambda b_i^* \right) \sigma_{i-1}^2 + F'(b_i^*). \quad (44)$$

Since  $F(\cdot)$  is strongly unimodal, it follows from  $b_i^* < b_p^*$  and  $F'(b_p^*) = 0$  that  $F'(b_i^*) > 0$ . We may conclude from (44) that  $\Delta_0/\sigma_0^2 - \lambda b_i^* < 0$ . Since  $\sigma_i^2 < \sigma_{i-1}^2$ , it now easily follows that

$$F'_{i+1}(b_i^*) = \left( \frac{\Delta_0}{\sigma_0^2} - \lambda b_i^* \right) \sigma_i^2 + F'(b_i^*) > F'_i(b_i^*) = 0,$$

which implies  $b_{i+1}^* > b_i^*$  since  $F_{i+1}$  is strongly unimodal. Thus, the  $b_i^*$  increase monotonically towards  $b_p^*$ , as claimed. The argument when  $\Delta_0 > \lambda \sigma_0^2 b_p^*$  is analogous.

We now turn our attention to the  $c_i^*$ . Suppose first that  $\Delta_0 < \lambda \sigma_0^2 b_p^*$ . In this case the  $b_i^*$  increase monotonically towards  $b_p^*$ , which is less than one. Since  $c$  is strongly unimodal with a maximum at one, the  $c_i^*$  will increase monotonically towards  $c_p^*$ . The second case is obvious. As for the third case, the ratio  $\frac{\Delta_0}{\lambda(s^2 + i\sigma_0^2)}$  in (42) is greater than, equal or less than one depending on whether  $i$  is less than, equal or greater than  $i^*$ . Since  $c'$  is negative

on  $(1, \gamma)$  and positive on  $(0, 1)$ , it now follows easily from (42) that  $b_i^* > 1$  when  $i < i^*$ ;  $b_i^* = 1$  when  $i = i^*$ ; and  $b_i^* < 1$  when  $i > i^*$ . Since the  $b_i^*$  decrease monotonically towards  $b_p^*$ , initially the  $c_i^*$  will increase until  $i = i^*$ , and then will decrease monotonically towards  $c_p^*$ , as claimed.

The result for the  $\eta_i^*$  is the immediate consequence of the optimal effort function (14) and the results for pay performance sensitivity and investment. ■

**Proof of Theorem 3.** We start by establishing the claims for the parameters  $\pi = \lambda, \sigma_0^2$ . As for part (i), by substituting (3) and (5) in (42),

$$F_i'(b, \pi) = \frac{\Delta_0 s^2}{s^2 + (i-1)\sigma_0^2} - \lambda b s^2 \frac{s^2 + i\sigma_0^2}{s^2 + (i-1)\sigma_0^2} + K c'(b), \quad (45)$$

we see that  $F_i'(b, \pi)$  is clearly decreasing in  $\pi$ . The result now follows from Lemma 1.

As for part (ii), we first suppose  $\Delta_0$  lies in the increasing region. We know from Theorem 2 the  $b$  trajectory increases towards  $b_p^*$ , which is less than one. Since the trajectory of  $b$  is pointwise decreasing by part (i), and since  $c$  is an increasing function on  $[0, 1]$ , the first claim has been established.

Now suppose  $\Delta_0$  lies in the decreasing region. Suppose  $\pi^1 < \pi^2$ . Let  $i_j^*, j = 1, 2$ , denote the value of  $i^*$  (34) corresponding to  $\pi^j$ . Clearly,  $i_1^* > i_2^*$ . By Theorem 2, in the interval  $[0, i_2^*)$  both  $b(\pi^1)$  and  $b(\pi^2)$  lie above one; since  $b(\pi^1) > b(\pi^2)$ , it immediately follows that  $c(\pi^1) < c(\pi^2)$  in this interval. Analogously, by Theorem 2, in the interval  $(i_1^*, \infty)$  both  $b(\pi^1)$  and  $b(\pi^2)$  lie below one; since  $b(\pi^1) > b(\pi^2)$ , it immediately follows that  $c(\pi^1) > c(\pi^2)$  in this interval. By Theorem 2, we know  $c(\pi^1)$  is increasing in the interval  $[i_2^*, i_1^*]$  whereas  $c(\pi^2)$  is decreasing in this interval. Moreover, since  $c_{i_1^*}(\pi^1) = c(1) > c_{i_1^*}(\pi^2)$  and  $c_{i_2^*}(\pi^1) < c_{i_2^*}(\pi^2) = c(1)$ , the trajectories  $c(\pi^1)$  and  $c(\pi^2)$  cross exactly once in this interval.

We now turn our attention to  $s^2$ . By Proposition 1 it is sufficient to show that  $F_i'(b_i^*, s^2)$  is decreasing in  $s^2$ . By (45),  $F_1'(b_1^*, s^2)$  is clearly decreasing in  $s^2$ . Now suppose  $i \geq 2$ . The sign of the derivative of (45) with respect to  $s^2$  coincides with the sign of

$$-\left(\lambda b s^4 + \sigma_0^2(i-1)[b\lambda(2s^2 + i\sigma_0^2) - \Delta_0]\right), \quad (46)$$

and therefore the result will follow if we can establish that  $b_i^* \lambda(2s^2 + i\sigma_0^2) > \Delta_0$ . To this end let

$$\hat{b}_i := \frac{\Delta_0}{\lambda(2s^2 + i\sigma_0^2)}.$$

By assumption  $\Delta_0 < 2p_0$ , and so  $\hat{b}_i < 1$ , which implies  $c'(\hat{b}_i)$  is positive. Therefore, the derivative

$$F'(\hat{b}_i, s^2) = Kc'(\hat{b}_i) + \frac{\Delta_0 s^2}{s^2 + (i-1)\sigma_0^2} \left[ 1 - \frac{s^2 + i\sigma_0^2}{2s^2 + i\sigma_0^2} \right] \quad (47)$$

is positive, and we may conclude that  $b_i^* > \hat{b}_i$  since  $F_i$  is strongly unimodal. Thus,

$$b_i^* \lambda(2s^2 + i\sigma_0^2) > \hat{b}_i \lambda(2s^2 + i\sigma_0^2) = \Delta_0,$$

as required. ■

**Proof of Theorem 4.** Part (i) follows by a straightforward application of Lemma 1.

As for the proof of part (ii), we first suppose  $\Delta_0$  lies in the increasing region. We know from Theorem 2 the  $b$  trajectory increases towards  $b_p^*$ , which is less than one. Since  $c$  is an increasing function on  $[0, 1]$ , the first claim has been established.

Now suppose  $\Delta_0$  lies in the decreasing region. (Please refer to Figure 5.) Suppose  $\Delta_0^1 < \Delta_0^2$ . Let  $i_j^*$ ,  $j = 1, 2$ , denote the value of  $i^*$  (34) corresponding to  $\Delta_0^j$ . Clearly,  $i_1^* < i_2^*$ . By Theorem 2, in the interval  $[0, i_1^*)$  both  $b(\Delta_0^1)$  and  $b(\Delta_0^2)$  lie above one; since  $b(\Delta_0^1) < b(\Delta_0^2)$ , it immediately follows that  $c(\Delta_0^1) > c(\Delta_0^2)$  in this interval. Analogously, by Theorem 2, in the interval  $(i_2^*, \infty)$  both  $b(\Delta_0^1)$  and  $b(\Delta_0^2)$  lie below one; since  $b(\Delta_0^1) < b(\Delta_0^2)$ , it immediately follows that  $c(\Delta_0^1) < c(\Delta_0^2)$  in this interval. By Theorem 2, we know  $c(\Delta_0^1)$  is decreasing in the interval  $[i_1^*, i_2^*]$  whereas  $c(\Delta_0^2)$  is increasing in this interval. Moreover, since  $c_{i_1^*}(\Delta_0^1) = c(1) > c_{i_1^*}(\Delta_0^2)$  and  $c_{i_2^*}(\Delta_0^1) < c_{i_2^*}(\Delta_0^2) = c(1)$ , the trajectories  $c(\Delta_0^1)$  and  $c(\Delta_0^2)$  cross exactly once in this interval. ■

**Proof of Theorem 5.**

*Part (i).* Suppose first  $\Delta_0$  lies in the increasing region. We know from Theorem 2 the trajectory of  $b$  lies strictly below  $b_p^*$ , which is less than one. It follows the term  $Kc'(b)$  in (42) is always positive. Since  $c'(b)$  is a decreasing function of  $k$ , it follows that  $F_i'(b, k)$  is an increasing function of  $k$ , which establishes the claim by Lemma 1.

Now suppose  $\Delta_0$  lies in the decreasing region. The term  $Kc'(b)$  in (42) is negative when  $b > 1$  and positive when  $b < 1$ . Since  $c'(b)$  is a decreasing function of  $k$ , it follows that if  $k_1 < k_2$ , then  $F'_i(b, k_1) < F'_i(b, k_2)$  when  $b > 1$  and  $F'_i(b, k_1) > F'_i(b, k_2)$  when  $b < 1$ . For a fixed value of  $k$  the trajectory of  $b$  lies above 1 until time  $i = i^*$  at which point it lies below 1 thereafter, and the value of  $i^*$  is *independent* of  $k$ . The result now follows by Lemma 1.

*Part (ii).* When  $\Delta_0$  lies in the increasing region, the trajectory of  $b$  lies below one. The claim follows immediately from part (i).

Now suppose  $\Delta_0$  lies in the increasing region and let  $k_1 < k_2$ . By Part 1 of this theorem, (i)  $b(k_1) < b(k_2)$  in the interval  $[0, i^*)$ , which immediately implies  $c(k_1) > c(k_2)$  since both  $b(k_1)$  and  $b(k_2)$  lie above 1; and (ii)  $b(k_1) > b(k_2)$  in the interval  $(i^*, \infty)$ , which implies  $c(k_1) > c(k_2)$  since both  $b(k_1)$  and  $b(k_2)$  lie below 1. At period  $i^*$ ,  $b(k_1) = b(k_2) = 1$ . Since  $c'(b)$  is a decreasing function of  $k$ , it follows  $c_{i^*}(k_1) > c_{i^*}(k_2)$ . ■

We now make explicit the functional dependence of the VC's continuation value (28) on her current assessment of the project's intrinsic quality and write it as  $CV_i(\mu_i)$ . We drop the superscript on  $\mu_i$  since it shall always refer to the VC's assessment. Let  $Z$  denote the standard normal random variable. We note the continuation value may be expressed as

$$CV_i(\mu_i) = [F_i^* + \mu_i - l_{i+1}] + e_i(\mu_i), \quad (48)$$

where

$$e_i(\mu_i) := E_i \left[ \max\{CV_{i+1}(\mu_{i+1}), 0\} \right] \quad (49)$$

$$= E \left[ \max\{CV_{i+1}(\hat{\sigma}_i Z + \mu_i), 0\} \right] \quad (50)$$

and  $\hat{\sigma}_i^2 := \sigma_i^2 + s^2$ , and that there exists a uniform bound  $B$  on the  $F_i^*$ .<sup>13</sup>

**Lemma 2**

$CV_i(\mu_i)$  is nonnegative and bounded for each date  $i$ .

**Proof.** At date  $i$  the expected within-period value at any future date is bounded above by  $\mu_i + B$ , with a finite number of periods left  $(T - i)$ , and so the VC's continuation value at

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<sup>13</sup>For example, one may set  $B := \max_{b \geq 0} [\Delta_0 b - \lambda/2 s^2 b^2 + Kc(b)]$ .

date  $i$  is bounded above by the present value of this constant stream or zero, whichever is larger. ■

**Lemma 3**

The continuation value  $CV_i$  is a continuous, increasing function of  $\mu_i$  for each date  $i$ ,  $0 \leq i \leq T - 1$ .

**Proof.** The function

$$CV_{T-1}(\mu_{T-1}) = F_{T-1}^* + \mu_{T-1} - l_T$$

is obviously an increasing, continuous function of  $\mu_{T-1}$ , which implies from definition (50) that the function  $e_{T-2}$  and hence  $CV_{T-2}$  are each increasing functions of  $\mu_{T-2}$ , too. Given that  $e_{T-2}$  is an increasing function, it is obviously finite by Lemma 2. Since the function

$$\max\{CV_{T-1}(\hat{\sigma}_{T-2}Z + \mu_{T-2}), 0\}$$

is continuous in  $\mu_{T-2}$  (given the continuity of  $CV_{T-1}$ ), one may apply Lebesgue's bounded convergence theorem to establish that  $e_{T-2}$  is continuous, and hence  $CV_{T-2}$  is continuous, too. We have demonstrated that  $CV_{T-2}$  is an increasing, continuous function of  $\mu_{T-2}$ , as claimed. The process continues recursively by using the increasing and continuous properties of  $CV_{i+1}$  and Lemma 2 to establish the increasing, finite and then continuous properties of  $e_i$ . ■

**Proof of Proposition 6.** By Lemma 3 each function  $CV_i$  is continuous and increasing. The proof of Lemma 3 also shows that each  $e_i$  is increasing, which implies each  $CV_i$  is negative for sufficiently small  $\mu_i$ . Since each  $CV_i$  is obviously positive for sufficiently high  $\mu_i$ , there exists a unique value  $\mu_i^*$  for which  $CV_i(\mu_i^*) = 0$ . Clearly, the VC should terminate only if  $\mu_i < \mu_i^*$ . ■

**Proof of Proposition 7.** The objective function  $F_i$  (29) is an increasing function of  $\Delta_0^{EN}$ , which implies that  $F_i^*$  is also a increasing function of  $\Delta_0^{EN}$ . One may proceed exactly as in the proof of Lemma 3 to establish that each  $CV_i$  is a pointwise increasing function of  $\Delta_0^{EN}$ , too, and it should be clear from the proof of Proposition 6 that the trigger values will

decrease. Since a change in this parameter has no effect on the sample paths, the result (i) follows. The proof of (ii) is the same, except that each  $F_i$  is now a decreasing function of either  $\lambda$  or  $k$ , and thus the trigger values will increase. ■

**Proof of Proposition 8.** Pick  $\epsilon > 0$  and define  $\theta_0$  so that  $P(\Theta > \theta_0) = \epsilon$ . Now

$$P(\tau > i) = P\{\mu_t > \mu_t^* \text{ for all } t = 0, 1, \dots, i\} \quad (51)$$

$$\leq P\{\mu_i > \mu_i^*\} \quad (52)$$

$$\leq P\{\mu_i > \mu_i^* \mid \Theta \leq \theta_0\}P(\Theta \leq \theta_0) + P(\Theta > \theta_0) \quad (53)$$

$$\leq P\{\mu_i > \mu_i^* \mid \Theta = \theta_0\} + \epsilon. \quad (54)$$

By Proposition 6 and the assumed property of the  $l_i$ , the  $\mu_i^*$  eventually lie above a positive constant. Given this fact and the fact that the conditional distribution of  $\sum_{t=1}^i Y_t/i$  given  $\Theta = \theta_0$  is  $N(\theta_0, s^2/i)$ ,

$$P\{\mu_i \geq \mu_i^* \mid \Theta = \theta_0\} = P\left\{\frac{s^2\mu_0 + \sigma_0^2(\sum_{t=1}^i Y_t)}{s^2 + i\sigma_0^2} \geq \mu_i^* \mid \Theta = \theta_0\right\} \quad (55)$$

$$\leq P\left\{\frac{\sum_{t=1}^i Y_t}{i} \geq \mu_i^* - \frac{s^2\mu_0}{i\sigma_0^2} \mid \Theta = \theta_0\right\} \rightarrow 0 \text{ as } i \rightarrow \infty. \quad (56)$$

The result now follows from (54) and (56) since  $\epsilon$  was chosen arbitrarily.

## CHAPTER IV

### ANALYSIS OF RISK — A TWO-PERIOD MODEL

In Proposition 7 and its proof we find that continuation value and project duration are increasing in the initial asymmetry in beliefs,  $\Delta_0$ , and decreasing with the risk aversion,  $\lambda$ , and the cost of effort,  $k$ . In this chapter we continue this analysis by inquiring how continuation value and project duration depend on the systematic risk and the initial technical risk in a two-period model. These results illuminate the forces through which risk affects the firm's economics. We find clear distinction between the effects of the systematic risk and the effects of technical risk. We show that risk may have positive effects on the firm and demonstrate that risk effects need not be monotone.

#### 4.1 Risk and Continuation Value

We assume there are at most two investment periods. We examine how the VC's continuation value at date zero,  $CV_0$ , changes with initial technical risk and the systematic risk. Keep in mind, by (28),  $CV_0$  is the sum of the first period's within period flow and the second period's option value, namely,

$$CV_0 = F_1^* - l_1 + \mu_0^{VC} + E_0[\max\{CV_1, 0\}]. \quad (57)$$

where  $CV_1$ , the VC's continuation value at date 1, is given by

$$CV_1 = F_2^* - l_2 + \mu_1^{VC}. \quad (58)$$

For expositional purposes, in what follows, we remove the VC notation from the  $\mu_i$ . We also assume symmetric information ( $\Delta_0 = 0$ ) to simplify the analysis. (In our discussion we describe how asymmetry affects the results.) At date 0,  $CV_1$  is a random variable because the second period quality  $\mu_1$  is a random variable. In light of (4)

$$CV_1 \sim N\left(\underbrace{F_2^* - l_2 + \mu_0(\pi)}_a, \underbrace{\frac{\sigma_0^4}{s^2 + \sigma_0^2}}_{b(\pi)^2}\right). \quad (59)$$

where  $\pi \in \{\sigma_0^2, s^2\}$ . Notice that  $a(\cdot)$  is the expected second period's within period flow as viewed at date zero. The following two lemmas, whose proofs are in the last section of this chapter, are key to understanding the various effects of risk on the continuation value.

**Lemma 4**

*If  $\lambda > 0$  then  $F_i^*$  is decreasing in  $s^2$  and in  $\sigma_0^2$  for all  $i \geq 1$ . If  $\lambda = 0$  then  $F_i^*$  is independent of  $s^2$  and in  $\sigma_0^2$  for all  $i$ .*

**Lemma 5**

*The derivative of the second period option value with respect to  $s^2$  or  $\sigma_0^2$   $E_0[\max\{CV_1, 0\}]$  is of the form:*

$$a(\pi)'K_1 + b(\pi)'K_2, \quad \pi \in \{\sigma_0^2, s^2\}, \quad (60)$$

where  $a(\cdot)$  and  $b(\cdot)$  are defined in (59) and  $K_1$  and  $K_2$  are positive.

We now turn to characterize the relationship between  $CV_0$  and risk.

**Proposition 9**

*Assume symmetric information, i.e.,  $\Delta_0 = 0$ .*

- (i) *If the EN is risk averse ( $\lambda > 0$ ) then  $CV_0$  is decreasing in the systematic risk,  $s^2$ , but may be non-monotonic in the initial technical risk,  $\sigma_0^2$ .*
- (ii) *If the EN is risk neutral ( $\lambda = 0$ ) then  $CV_0$  is decreasing in the systematic risk,  $s^2$ , and increasing in the initial technical risk,  $\sigma_0^2$ .*

**Proof:** Observe that:

- (a) By Lemma 4 and the definition of  $a(\cdot)$ , if  $\lambda > 0$  then  $a(\cdot)$  is decreasing in  $s^2$  and in  $\sigma_0^2$ .
- (b) If  $\lambda = 0$ , then  $a(\cdot)$  is independent of  $s^2$  and in  $\sigma_0^2$ .
- (c)  $b(\cdot)$  is decreasing in  $s^2$  but increasing in  $\sigma_0^2$ .

First suppose that  $\lambda > 0$ . By observation (a) the derivatives of  $a(\cdot)$  and  $b(\cdot)$  with respect to  $s^2$  satisfy  $a'(\cdot) < 0$  and  $b'(\cdot) < 0$ . Thus,  $CV_0$  is decreasing with  $s^2$  by Lemma 5. By

observations (a) and (c) the derivatives of  $a(\cdot)$  and  $b(\cdot)$  with respect to  $\sigma_0^2$  satisfy  $a'(\cdot) < 0$  but  $b'(\cdot) > 0$ . Applying Lemma 5 reveals there are conflicting forces affecting the future option value and accordingly  $CV_0$  may be non-monotonic. Now suppose  $\lambda = 0$ . Observation (b) reveals that  $a(\cdot)$  is independent of risk. Since  $b(\cdot)$  is decreasing in  $s^2$ ,  $CV_0$  is also decreasing in the systematic risk by Lemma 5. Similarly, since  $b(\cdot)$  is increasing in  $\sigma_0^2$ ,  $CV_0$  is increasing in the initial technical risk by Lemma 5. ■

Proposition 9 illuminates two means by which risk can affect the continuation value. First, risk affects within period flow. As stated in Lemma 4, due to the added cost of risk both the technical risk and the systematic risk negatively affect the within period flow when the EN is risk averse. In addition, Lemma 5 reveals risk changes the future option value of the continuation value. A more volatile future is advantageous since in the presence of high volatility the VC enjoys the higher upside values without having to pay the price for the lower downside values. Interestingly, Lemma 5 shows that technical risk and systematic risk may have *opposite* effects on the option value. Increasing the systematic risk makes the learning more difficult, which results in a posterior assessment closer to the prior assessment. Thus, an increase in systematic risk is responsible for less volatility and less option value. Increasing the initial technical risk, on the other hand, increases the ratio between the technical and systematic risk and results in a more effective learning, which means that the parties are very sensitive to the signals and the posterior assessment is more volatile. Consequently, initial technical risk is positively tied with higher option value.

Proposition 9 shows that market risk is negatively tied with both components of the continuation value, and so the continuation value at date zero decreases with market risk. However, while technical risk increases the volatility, it simultaneously decreases the within period flow and the net affect on the future option value is unclear (as implied by Lemma 5). Therefore, as stated in Proposition 9, the net effect of technical risk on the continuation value is unclear and depends on the values of the model's parameters.

When the parties have asymmetric beliefs matters are more complicated since the within period flow,  $F_i^*$ , is not necessarily decreasing with the systematic risk. By (5) one can show

**Table 1:** Risk forces affecting continuation value

Type of Risk	Market Setting	$a'$		$b'$	Net Effect
		Due to Asymmetry	Due to Risk Aversion		
Systematic	No Agency	0	0	-	-
	Symmetric	0	-	-	-
	Asymmetric	+	-	-	$\pm$
Technical	No Agency	0	0	+	+
	Symmetric	0	-	+	$\pm$
	Asymmetric	-	-	+	$\pm$

that  $\Delta_i$  is decreasing in  $\sigma_0^2$  and increasing in  $s^2$ . This is because increasing the systematic risk diminishes the EN's learning ability and therefore he does not update (decrease)  $\Delta_i$  in as fast a pace. Increasing technical risk, however, has an opposite effect because it increases the technical to market risk ratio, making the market risk relatively smaller. The argument used in the proof of Lemma 4 may be used to show that  $F_i^*$  is not necessarily decreasing in systematic risk but is still (or even more so) decreasing in technical risk. Therefore, introducing asymmetric information on the one hand mitigates the positive effect of technical risk on the continuation value and on the other hand mitigates the negative effects of the systematic risk. We summarize how the “forces of risk” affect the continuation value in Table 1.

The effects of risk on continuation value are demonstrated in Figures 7 - 9, which displays lattices simulating the two-period model for different levels of initial technical risk. Recall,  $\sigma_0^\mu$ , the standard deviation of the VC's assessment of the firm's quality in the beginning of the second period, is given by (73). At the end of the first period the VC's assessment of the firm's quality moves up (u) by one standard deviation ( $\sigma_0^\mu$ ), stays at the same level (m) or moves down (d)  $\sigma_0^\mu$ . Due to this structure, the corresponding probabilities  $p^u, p^m, p^d$  are independent of the risk and are equal to (0.31,0.38,0.31), respectively. In each figure we display for each period the asymmetry level, the price of risk and  $F_i^* - l_i$ , the within period flow minus  $\mu_{i-1}$ . Except for  $\sigma_0^2$ , the value of the parameters is equal to the basic numbers

given in Table 6 and Table 8.

Figure 7 describes the evolution of the two-period model for a low level of technical risk ( $\sigma_0^2 = 0.1$ ) and for which  $CV_0 = 0.5$ . Figure 8 reveals that increasing  $\sigma_0^2$  to 0.5 decreases  $CV_0$  to 0.41. The reason for this is the steep decline of the deterministic component of the within period flow,  $F_i^* - l_i$ ,  $i = 1, 2$ . This loss of income cannot compensate for the gains due to the added volatility ( $CV_1^u$  for  $\sigma_0^2 = 0.1$  is higher than  $CV_1^u$  for  $\sigma_0^2 = 0.5$ ). Further increasing  $\sigma_0$  to 1 results in  $CV_0 = 0.43$  (Figure 9). The loss in  $F_i^* - l_i$ ,  $i = 1, 2$ , as result of the increase is less pronounced and therefore the increase in the volatility, manifested in the higher  $\mu_1^u$  and  $CV_1^u$ , is the dominant force.

The reason why continuation value is initially decreasing and then increasing in  $\sigma_0^2$  is explained by the magnitude of the within period flow when  $\sigma_0^2$  is low compared to when  $\sigma_0^2$  is high. In the first case, the within period flow is large, thus the potential for great losses. However, when  $\sigma_0^2$  is high the within period flow is small and it can never be negative. In this case the gains in the volatility are more dominant and hence the continuation value increases.

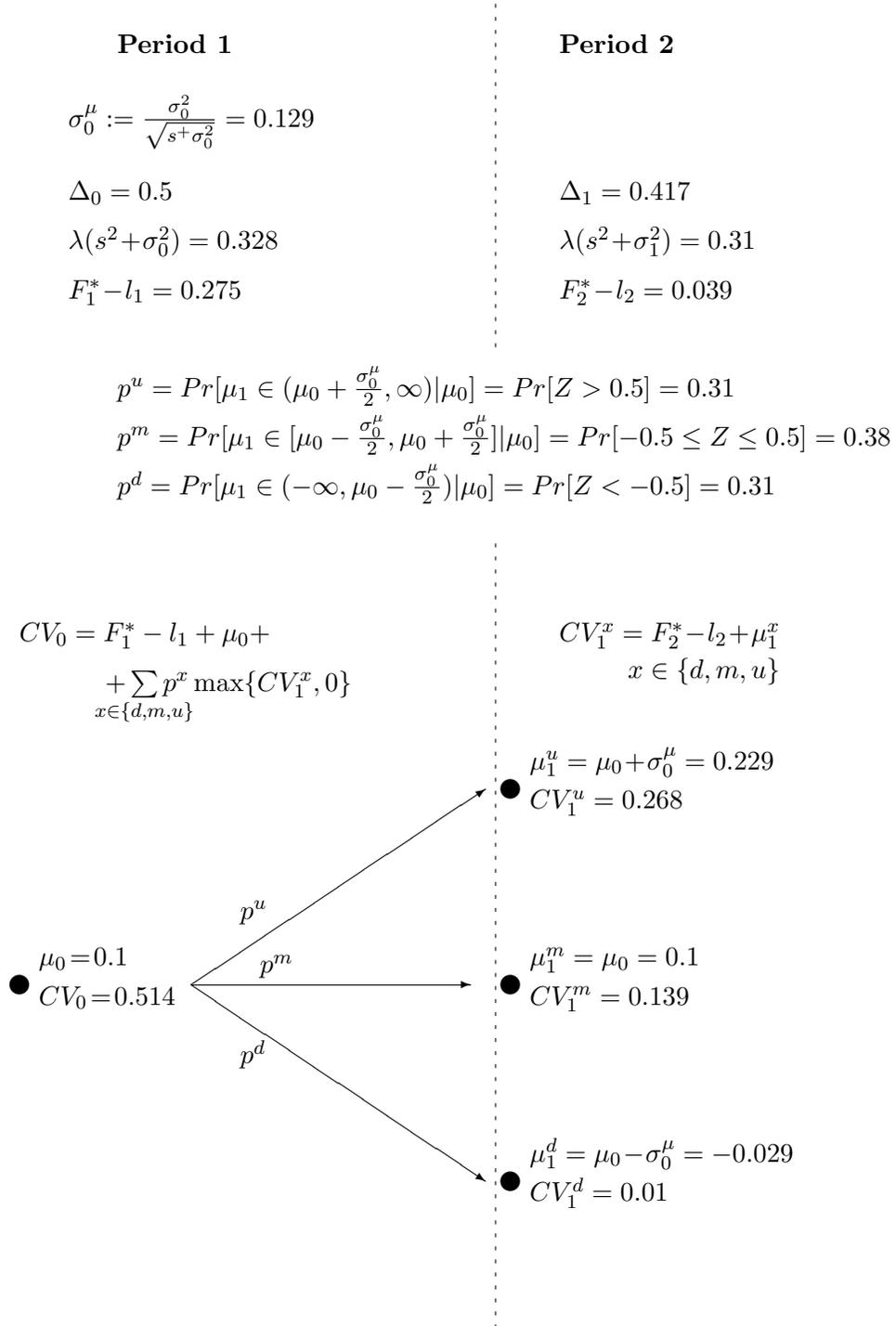
## 4.2 Risk and Project Duration

The expected timing of implementation,  $E[\tau]$ , is closely tied with continuation value since continued investment is conditioned on a positive continuation value. Assuming there is initial investment,  $E[\tau]$  depends only on the continuation value of the second period. As will be shown in the following proposition, we find it convenient to distinguish between the case where the expected second period flow,  $a(\cdot) := F_2^* - l_2 + \mu_0$ , is nonnegative and the case where it is negative. When the expected second period is nonnegative investment is guaranteed in the second period if the firm receives a neutral signal (i.e.  $\mu_1 = \mu_0$ ) and conversely if the expected second period flow is negative. The following proposition describes how the project's duration, assuming there is initial investment, changes with risk.

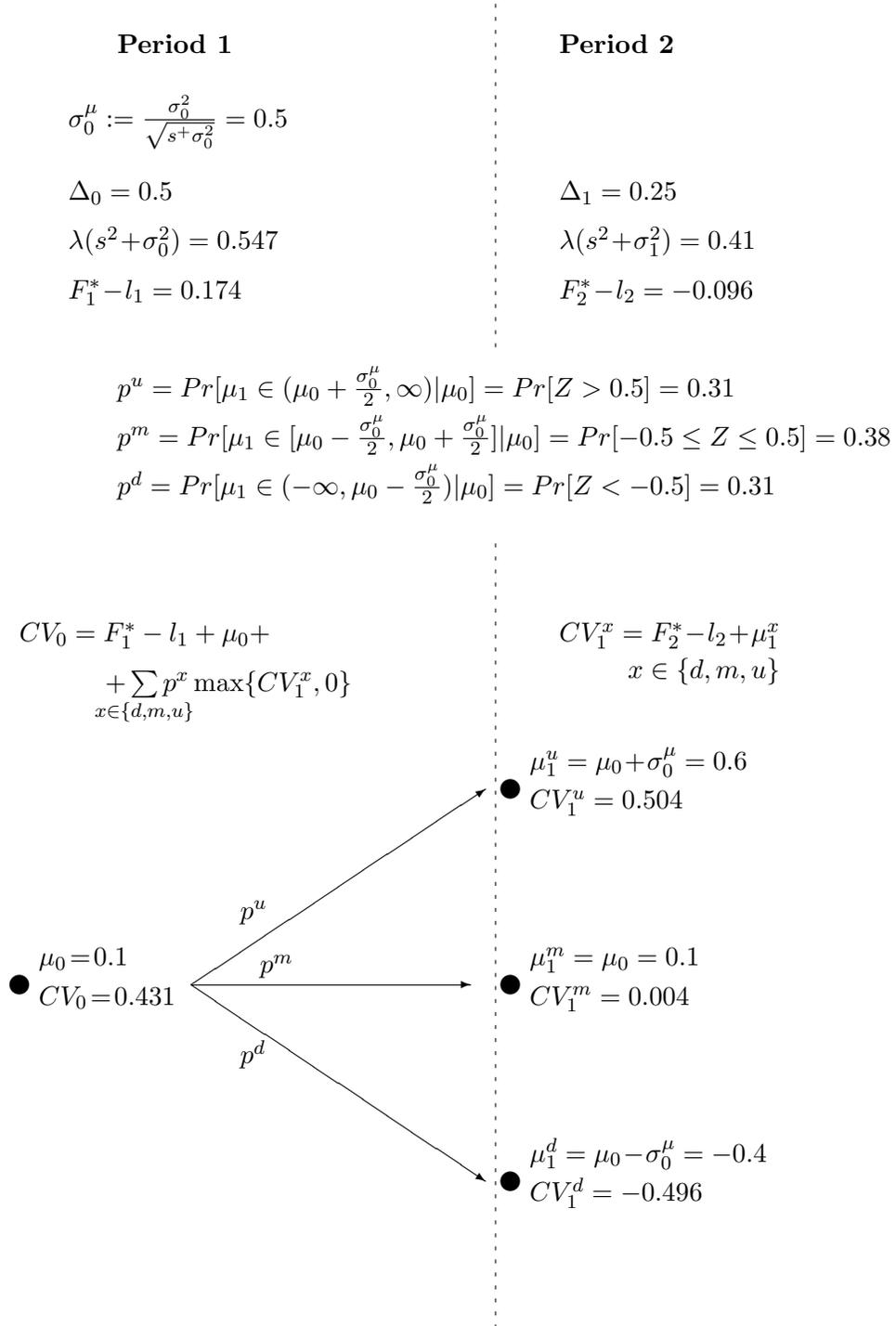
### Proposition 10

Assume symmetric information, i.e.,  $\Delta_0 = 0$ .

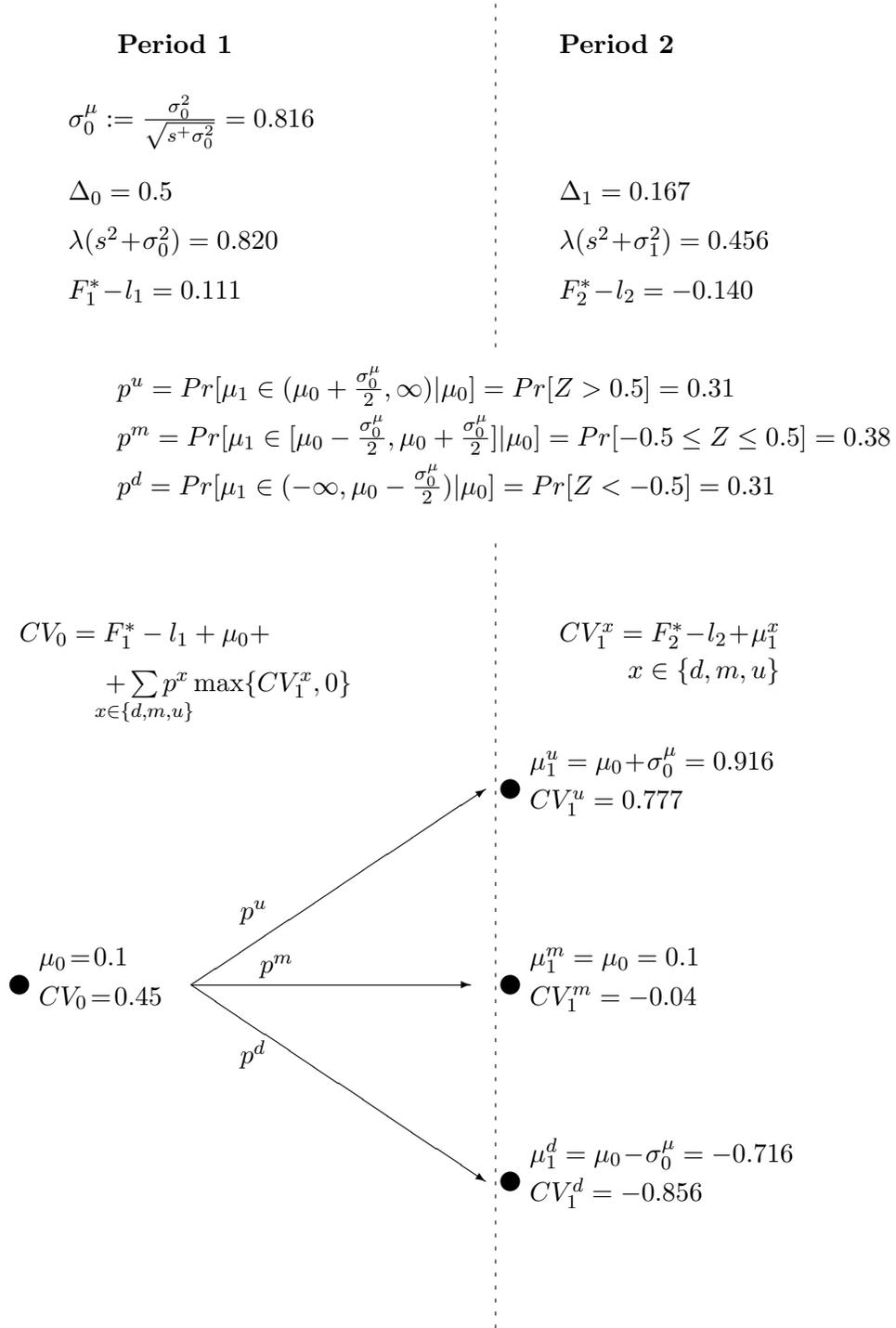
- (i) Suppose the expected second period flow is nonnegative. If the EN is risk averse



**Figure 7:** Two period lattice for base numbers and initial technical risk  $\sigma_0^2 = 0.1$



**Figure 8:** Two period lattice for base numbers and initial technical risk  $\sigma_0^2 = 0.5$



**Figure 9:** Two period lattice for base numbers and initial technical risk  $\sigma_0^2 = 1$

( $\lambda > 0$ ), then  $E[\tau]$  is decreasing in the initial technical risk,  $\sigma_0^2$ , but may be non-monotonic in the systematic risk,  $s^2$ . If the EN is risk neutral ( $\lambda = 0$ ), then  $E[\tau]$  is decreasing in the initial technical risk,  $\sigma_0^2$ , and increasing in the systematic risk,  $s^2$ .

(ii) Suppose the expected second period flow is negative. If the EN is risk averse ( $\lambda > 0$ ), then  $E[\tau]$  may be non-monotonic in the initial technical risk,  $\sigma_0^2$ , and decreasing in the systematic risk,  $s^2$ . If the EN is risk neutral ( $\lambda = 0$ ), then  $E[\tau]$  is increasing in the initial technical risk,  $\sigma_0^2$ , and decreasing in the systematic risk,  $s^2$ .

**Proof:** We continue the notation used in the proof of Proposition 9 and equation (59). We write the expected timing of implementation assuming initial investment takes place, ( $CV_0 > 0$ ), as

$$E[\tau] = 1 + Prob[CV_1 > 0] = 1 + \int_{-\frac{a}{b}}^{\infty} \underbrace{\frac{1}{\sqrt{2\pi}} \exp^{-\frac{x^2}{2}}}_{\phi(x)} dx. \quad (61)$$

The derivative of  $E[\tau]$  with respect to  $\pi \in \{s^2, \sigma_0^2\}$  is

$$-\phi\left(-\frac{a(\pi)}{b(\pi)}\right) \left(-\frac{a(\pi)}{b(\pi)}\right)', \quad (62)$$

which is proportional to

$$a'(\pi)b(\pi) - a(\pi)b'(\pi) \quad (63)$$

Keeping in mind that  $b(\pi) > 0$ , the rest of the proof follows immediately by considering whether  $a(\pi)$ , the expected second period flow, is nonnegative and observations (a) - (c) made in the proof to Proposition 9. ■

The effects of risk on the timing of implementation depend heavily on the sign of the expected second period flow (as viewed at date zero). In the case where it is positive, then the closer the second period assessment,  $\mu_1$ , is to its mean,  $\mu_0$ , the more likely investment will take place in the second period. Therefore, increasing the sensitivity to the market signals adversely affects the likelihood of investment in the second period. Accordingly, Proposition 10 states that increasing  $\sigma_0^2$  decreases the project duration. This happens because increasing  $\sigma_0^2$  not only increases the learning sensitivity but also decreases the deterministic part of

**Table 2:** Risk forces affecting project duration

Risk	$a > 0$	Market Setting	$a'$		$-ab'$	Net Effect
			Due to Asymmetry	Due to Risk Aversion		
Systematic	Yes	No Agency	0	0	+	+
		Symmetric	0	-	+	$\pm$
		Asymmetric	+	-	+	$\pm$
	No	No Agency	0	0	-	-
		Symmetric	0	-	-	-
		Asymmetric	+	-	-	$\pm$
Technical	Yes	No Agency	0	0	-	-
		Symmetric	0	-	-	-
		Asymmetric	-	-	-	-
	No	No Agency	0	0	+	+
		Symmetric	0	-	+	$\pm$
		Asymmetric	-	-	+	$\pm$

the second period flow,  $F_2^*$ , with both effects resulting in a higher probability for second period negative continuation value. Increasing the systematic risk results in two opposite effects. On the one hand, the parties ability to learn diminishes and therefore the second period quality assessment  $\mu_1$  is less volatile, which increases the probability for second period investment. On the other hand, increasing systematic risk results in a decrease in the second period deterministic return, which decreases the probability for second period investment. The net effect of these two forces depends on the specific values of the parameters.

When the expected second period flow is negative then the closer the second period quality is to its mean the higher the probability for no investment. Now volatility increases the probability for second period investment and therefore we get opposite effects.

Recall, when there is asymmetric information,  $F_i^*$  is still decreasing in the technical risk but is not necessarily increasing in systematic risk. Therefore, if  $\Delta_0 > 0$ ,  $a(\cdot)$  is still decreasing in technical risk but it is unclear how  $a(\cdot)$  changes with systematic risk, and accordingly the net effect on project duration may change. Table 2 summarizes the effects of risk on the timing of implementation for the different market settings (Asymmetric

Information, Symmetric Information and No Agency).

### 4.3 Proofs

**Proof of Lemma 4** Since  $\Delta_0 = 0$ , we have by (3) and (23)

$$F_i^* = \max_{b_i \geq 0} F_i(b_i) = Kc(b_i) - \lambda \left( s^2 + \frac{s^2 \sigma_0^2}{s^2 + i \sigma_0^2} \right) b_i^2$$

where  $Kc(b_i)$  is independent of either  $\sigma_0^2$  or  $s^2$ . Let  $r$  represent either the technical or systematic risk parameters,  $r \in \{s^2, \sigma_0^2\}$ ,  $F_i^*(r)$  denote  $F_i^*$  for a given level of  $r$ , let  $F_i(b_i, r)$  be similarly defined and let  $b_i^*(r)$  denote the optimal solution for a given level of  $r$ . Let  $r_1 < r_2$ . Since the expression  $s^2 + \frac{s^2 \sigma_0^2}{s^2 + i \sigma_0^2}$  is increasing in  $r$  we have that if  $\lambda > 0$ ,  $F_i(b_i, r_1) > F_i(b_i, r_2)$  for all  $b_i$ . Thus,

$$F_i^*(r_1) = F_i(b_i^*(r_1), r_1) > F_i(b_i^*(r_2), r_1) > F_i(b_i^*(r_2), r_2) = F_i^*(r_2)$$

where the first inequality follows from the uniqueness of the optimal solution and the second inequality follows from the observation that  $F_i(b_i, r_1) > F_i(b_i, r_2)$  for all  $b_i$ . When  $\lambda = 0$ ,  $F_i(b_i)$  is independent of either  $s^2$  or  $\sigma_0^2$ , hence  $F_i^*$  is independent of risk. ■

**Proof of Lemma 5** The future option value is

$$E_0[\max\{CV_1, 0\}] = \tag{64}$$

$$E_0[CV_1 \cdot 1_{\{CV_1 \geq 0\}}] = \tag{65}$$

$$E_0[CV_1 \cdot 1_{\{\frac{CV_1 - a}{b} \geq \frac{-a}{b}\}}] = \tag{66}$$

$$bE_0\left[\frac{CV_1 - a}{b} \cdot 1_{\{\frac{CV_1 - a}{b} \geq \frac{-a}{b}\}}\right] + aPr_0\left[\frac{CV_1 - a}{b} \geq \frac{-a}{b}\right] = \tag{67}$$

$$bE_0[Z \cdot 1_{\{Z \geq \frac{-a}{b}\}}] + aE_0[1_{\{Z \geq \frac{-a}{b}\}}]. \tag{68}$$

where  $Z$  denotes a standard normal random variable and where we use the notation

$$1_{\{X\}} = \begin{cases} 1 & \text{if } X, \\ 0 & \text{if not } X. \end{cases} \tag{69}$$

In integral form, (68) is equal to

$$a \int_{-\frac{a}{b}}^{\infty} \underbrace{\frac{1}{\sqrt{2\pi}} \exp^{-\frac{x^2}{2}}}_{\phi(x)} dx + b \int_{-\frac{a}{b}}^{\infty} x \underbrace{\frac{1}{\sqrt{2\pi}} \exp^{-\frac{x^2}{2}}}_{\phi(x)} dx \tag{70}$$

Applying Leibnitz's rule, the first derivative of (70) is

$$a' \int_{-\frac{a}{b}}^{\infty} \phi(x) dx + \underbrace{a \left[ -\phi\left(-\frac{a}{b}\right) \left(-\frac{a}{b}\right)' \right] + b \left[ -\left(-\frac{a}{b}\right) \phi\left(-\frac{a}{b}\right) \left(-\frac{a}{b}\right)' \right]}_{=0} + b' \int_{-\frac{a}{b}}^{\infty} x \phi(x) dx \quad (71)$$

$$a' \cdot \underbrace{\int_{-\frac{a}{b}}^{\infty} \phi(x) dx}_{\text{positive}} + b' \cdot \underbrace{\int_{-\frac{a}{b}}^{\infty} x \phi(x) dx}_{\text{positive}} \quad (72)$$

■

## CHAPTER V

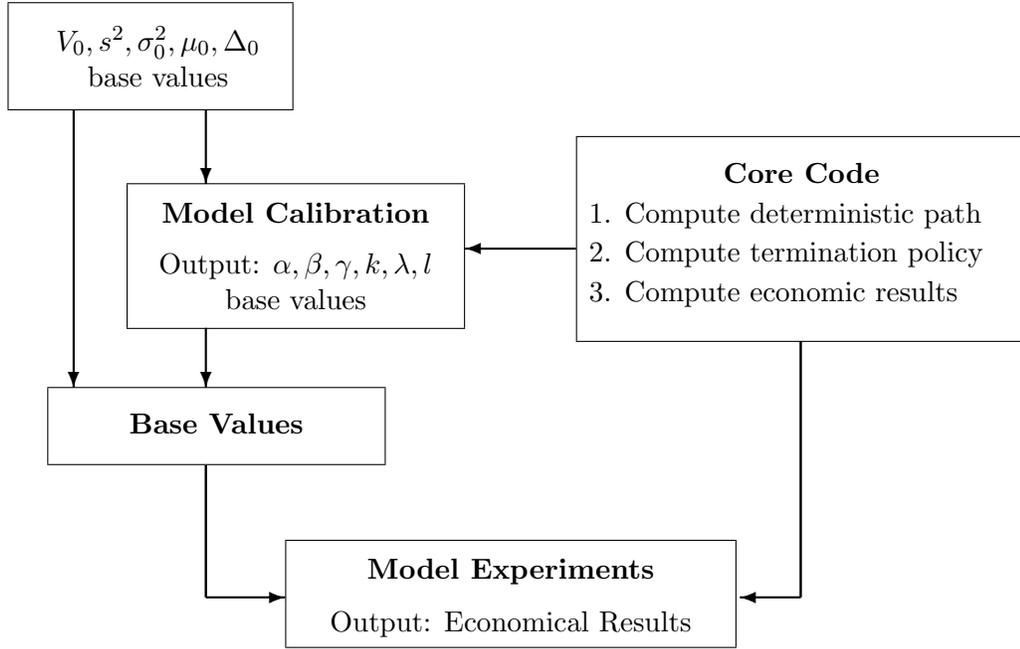
### NUMERICAL ANALYSIS AND RESULTS

#### 5.1 *Introduction*

Some of the questions we set out to investigate in this research cannot be answered analytically. Our analytical analysis is limited upfront by the lack of closed-form solution for the deterministic path (e.g. investment, EN's compensation). This problem is compounded when we attempt to find explicit solutions for the continuation value and the project duration. Consequently, we must use numerical methods to overcome the limitations posed by the analytical analysis and to this end we develop MATLAB code that simulates the model.

The greatest drawback of numerical analysis lies in the fact that the attained solutions are valid only for the specific values of the parameters used in the analysis. Consequently, any theoretical analysis based on these solutions is subject to the suspicion that it cannot be generalized to other parametric values. Therefore, if the parameter values are not a true reflection of empirical evidence and market behavior, the results of the numerical analysis are less meaningful and may not lead to a serious theoretical discussion. It is therefore imperative to set the parameter values so values given to the parameters be such that they properly reflect empirical evidence on the market. Unfortunately, for many of the parameters in our model there is no direct empirical findings that can help us set their values. In these cases, we set the value of the parameters indirectly to empirical evidence via a *calibration* process soon to be described. Using both the direct and indirect methods we calibrate the model's parameters and obtain a basic set of parametric values that closely reflect the venture capital industry. Thus, we can be comfortable in using the model to explain or predict other market phenomena.

The parameters we set directly to data are  $s^2$ , the systematic risk,  $\sigma_0^2$ , the initial technical risk,  $\mu_0^{VC}$ , the VC's belief on the initial firm quality, and  $\Delta_0$ , the initial asymmetry in belief. We normalize the initial firm termination value  $V_0$  to 1. We will refer to these parameters



**Figure 10:** Schematic overview of the numerical analysis

(including  $V_0$ ) as the *data driven parameters*. We set the *basic values* of these parameters directly according to empirical evidence.

The remaining parameters  $\lambda$ , the EN’s risk aversion,  $\gamma$  and  $k$ , the parameters associated with the EN’s disutility from effort,  $\alpha$  and  $\beta$ , the parameters associated with the firm’s production function, and  $L$ , the loss parameter, are calibrated to match a body of empirical evidence as closely as possible. These parameters will hereafter referred to as the *calibrated parameters*. Henceforward, we refer to these values as the *basic values* of the model.

The core code we develop to solve the model is therefore used in two different settings. First, it is used in the calibration phase in which we solve the model for many parametric values. Second, after we have set the basic values, we use the code to solve the model to obtain economical results and predictions. Figure 10 describes this relationship between the core code, the calibration and the numerical analysis processes.

In Section 5.2 we describe the core code’s components. In Section 5.3 we describe the calibration process and in Section 5.4 we describe the numerical analysis results.

## 5.2 *The Core Code*

The core code computes the solution to the model for a given value of the model’s parameters. It is executed in three steps: (1) compute the “deterministic path”; (2) compute the termination triggers; and (3) simulate the firm’s evolution (Monte Carlo simulation). We now proceed to describe in detail each of these steps.

### 5.2.1 Computing the deterministic path

In this step we compute the equilibrium time paths associated with the EN’s risky compensation, the VC’s investment and the EN’s exerted effort, namely, the  $b_i^*$ ,  $c_i^*$  and  $\eta_i^*$ . These results do not depend on the realization of the signals of project quality and are characterized in Theorem 1. Under our assumptions, the VC’s problem, (24), is strongly unimodal and therefore we may easily compute the EN’s risky compensation,  $b_i^*$ , via bisection search. The  $c_i^*$  and  $\eta_i^*$  are computed directly from  $b_i^*$ .

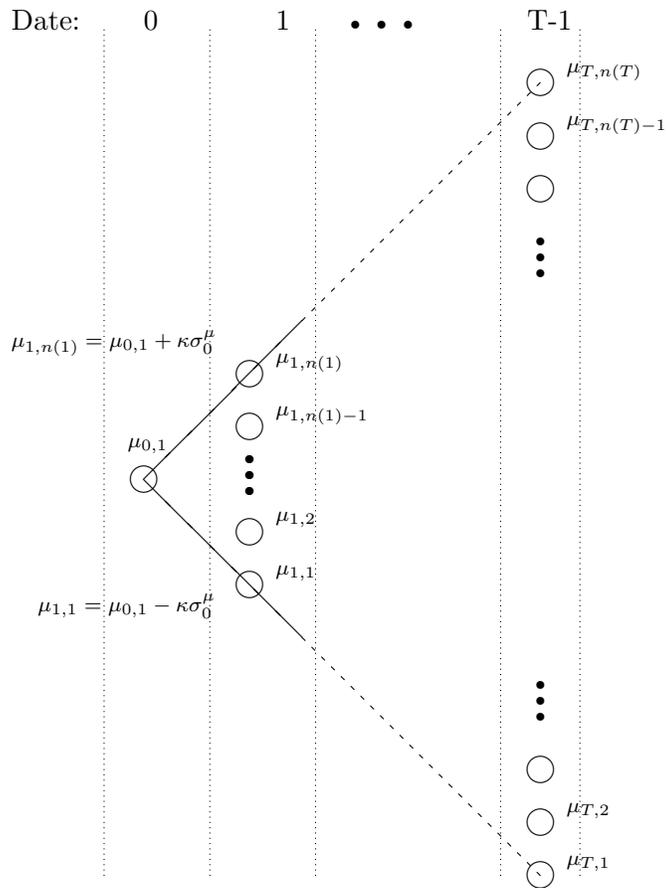
### 5.2.2 Computing the termination triggers

The continuation value  $CV_i(\cdot)$  at each date  $i = 0, 1, 2, \dots, T - 1$  is a continuous, increasing function of  $\mu_i$  (see Lemma 3). As a consequence (see Proposition 6), the optimal termination policy is a trigger policy: the VC continues investment if and only if the project quality at date  $i$ ,  $\mu_i$ , is greater than  $\mu_i^*$ , where  $CV_i(\mu_i^*) = 0$ . Given  $CV_i(\cdot)$  a simple bisection search will determine the trigger  $\mu_i^*$ .

It remains to compute the continuation value function. Here, we generate a lattice describing the evolution of the project’s quality over time. We now turn to describe in more detail the project quality lattice, the way the continuation value is computed and issues related with solution accuracy and computation time.

#### 5.2.2.1 Lattice design

The lattice, depicted in Figure 11, simulates the evolution of the project quality. Each column in the lattice represents a date in the life of the firm. At date 0 the project quality is given by  $\mu_0$ . At following dates the quality may be one of many different states, depending on the realization of the signal. Let



**Figure 11:** Schematic overview of the quality lattice

- $n(i)$  denote the number of states at date  $i$
- $\mu_{i,j}$  denote the firm's quality at the  $j$ th state at date  $i$ ,  $j = 1, \dots, n(i)$ .

The standard deviation of the project's quality at date  $i$ ,  $\sigma_i^\mu$ , may be derived from equation (4) and is given by:

$$\sigma_i^\mu = \frac{\sigma_i^2}{\sqrt{s^2 + \sigma_i^2}}. \quad (73)$$

We design the lattice so that the maximal state at date  $i$  is  $\kappa$  standard deviations above the maximal state at date  $i - 1$  for some  $\kappa > 0$ ; the minimum state is defined symmetrically. That is,

$$\mu_{i,n(i)} = \mu_{i-1,n(i-1)} + \kappa\sigma_{i-1}^\mu \quad (74)$$

$$\mu_{i,1} = \mu_{i-1,1} - \kappa\sigma_{i-1}^\mu. \quad (75)$$

The remaining  $n(i) - 2$  states are determined by setting their values to be equally spaced between the minimum and maximum states. That is,

$$\mu_{i,j+1} = \mu_{i,j} + \frac{\mu_{i,n(i)} - \mu_{i,1}}{n(i) - 1} \quad \text{for all } j = 1, \dots, n(i) - 1. \quad (76)$$

### 5.2.2.2 Continuation value

The last column in the lattice represents the possible project quality states at date  $T - 1$ , which is the last possible date for investment. At this point the continuation value is independent of the future and can be computed explicitly using the deterministic path values and the project quality. Thus, the terminal state  $\mu_{T-1,j}$  corresponds to a continuation value, denoted by  $CV_{T-1,j}$ , according to:

$$CV_{T-1,j} = \mu_{T-1,j} + F_{T-1}^* - l_{T-1}, \quad (77)$$

where  $F_i^*$  is the optimal solution to the variable portion of the VC's problem at date  $i$ , (24), and  $l_i$  is the loss at date  $i$ .

To compute the continuation value for each state at date  $i < T - 1$  we proceed as follows. Let  $D_{i,j}$  denote the set of all states that are "immediate descendants" of  $\mu_{i,j}$ — $\mu_{i+1,k} \in D_{i,j}$  if and only if  $\mu_{i+1,k}$  is within  $\pm\kappa\sigma_i^\mu$  from  $\mu_{i,j}$ . We shall say the firm transitions from state

$\mu_{i,j}$  to its descendant  $\mu_{i+1,k} \in D_{i,j}$  if the project quality changes from level  $\mu_{i,j}$  to a point in  $[\frac{1}{2}(\mu_{i+1,k} + \mu_{i+1,k-1}), \frac{1}{2}(\mu_{i+1,k} + \mu_{i+1,k+1})]$ . In case that  $\mu_{i+1,k}$  is a minimal (maximal) state we define  $\mu_{i+1,k-1} := -\infty$  ( $\mu_{i+1,k+1} := +\infty$ ). The transition probability  $p_{i,j,k}$  from state  $\mu_{i,j}$  to  $\mu_{i+1,k}$  is therefore given by

$$p_{i,j,k} = \Phi \left[ \left( \frac{1}{2}(\mu_{i+1,k} + \mu_{i+1,k+1}) - \mu_i \right) \frac{1}{\sigma_i^\mu} \right] - \Phi \left[ \left( \frac{1}{2}(\mu_{i+1,k} + \mu_{i+1,k-1}) - \mu_i \right) \frac{1}{\sigma_i^\mu} \right]. \quad (78)$$

In the discrete approximation, the continuation value at state  $\mu_{i,j}$  is given by:

$$CV_{i,j} = \mu_{i,j} + F_i^* - l_i + \sum_{\mu_{i+1,k} \in D_{i,j}} p_{i,j,k} \max(CV_{i+1,k}, 0). \quad (79)$$

Starting from (77) and working backwards through time in the familiar way, the continuation values for all states and dates are computed. Since the true continuation value function is continuous and increasing, we complete the approximation to  $CV_i(\cdot)$  via linear interpolation.

### 5.2.2.3 Computation time vs. solution accuracy

The accuracy of the triggers  $\mu_i^*$  computed in this step depends heavily on the number of states in each column of the lattice as well as on  $\kappa$ , the parameter that determines the range of firm quality represented in the lattice. In addition, the choice for the maximal number of periods,  $T$ , affects not only the accuracy of  $\mu_i^*$ , but also affects on the accuracy of the economic results of the model as compared to the infinite horizon solution. We let the number of states in the lattice increase linearly from period to period in the following manner:

$$n(i) = \begin{cases} 1, & \text{if } i = 0; \\ Mi, & \text{if } i > 0. \end{cases} \quad (80)$$

The value of  $M$  is set to 50 and the value of  $\kappa$  is set to  $\kappa = 2.5$ . In the many experiments we conducted, we found that an increase in  $M$  or  $\kappa$  or both did not change the value of  $\mu_i^*$  (to within a 3% tolerance). With regard to the choice of  $T$ , Sahlman (1990) provides empirical evidence there are at most 8 investment stages. We set  $T = 10$ . We note that in almost all of the experiments we conducted, the probability of the firm surviving to the 10th period was less than 0.1%.

### 5.2.3 Computing the model's results

In the last step of the core code we compute the economic statistics of the model by conducting a Monte Carlo simulation of  $N$  firms. In each experiment we simulate the change in the firm's quality over time. As the firm evolves we compute statistics such as the EN's payout, and upon termination we compute statistics such as number of periods, total investment and termination value. We set  $N = 100,000$ . In the many experiments we conducted, we found the results for expected net firm value, continuation value at date zero (expected net VC share) and expected project duration did not change by more than 1% when  $N$  was increased.

## 5.3 Calibration

We start the calibration process by setting values to the *data driven parameters*,  $s^2, \sigma_0^2, \mu_0$  and  $\Delta_0$ . These parameters are set relative to  $V_0$ , which has been normalized to 1. Next, we set the values of the *calibrated parameters*  $\alpha, \beta, \gamma, \lambda, k$  and  $L$ , through the calibration process to be described below.

### 5.3.1 Data driven parameters

The values for the volatilities,  $\sigma_0^2$  and  $s^2$ , and the parameters relating to the potential per-period gains if the parties took no action depend on the length of the period. Gompers (1995) provides data for the amount of time between funding for different investment stages. This data, provided in Table 3, reveals that the average time between investments for all stages is approximately 1 year (1.09). Hereafter, we set the investment period's length to one year.

**Parameter  $\mu_0$ :** The initial quality of the firm,  $\mu_0$ , is computed according to the CAPM model:

$$\frac{\mu_0}{V_0} = r_f + \text{beta} \cdot r_p, \quad (81)$$

where  $r$  is required rate of return,  $r_f$  is the risk-free rate,  $r_p$  is the risk premium. Following empirical findings by Kerins et. al. (2004) we set  $r_f = 0.04$ ,  $\text{beta} = 1$ , and  $r_p = 0.06$ . Accordingly,  $\mu_0 = 0.1V_0 = 0.1$ .

**Table 3:** Average time between funding for different investment stages

Industry	Time to Next Funding	Number of Investments
Seed	1.63	122
Startup	1.21	129
Early Stage	1.03	114
First Stage	1.08	288
Other Early	1.08	221
Expansion	1.26	377
Second Stage	1.01	351
Third Stage	0.86	181
Bridge	0.97	454

**Parameters  $\sigma_0^2$  and  $s^2$ :** The standard deviation in firm value is reported by Kerins et. al. (2004), Table 4, to be 102%. We round this figure and assume that the *initial* standard deviation is equal to the initial termination value,

$$\frac{\sqrt{s^2 + \sigma_0^2}}{V_0} = 1. \quad (82)$$

Further, we assume that the initial risk is equally divided between technical and systematic risk. Thus,  $s^2 = \sigma_0^2 = 0.5$ .

**Parameter  $\Delta_0$ :** We believe the EN’s experience is the most important factor in determining the asymmetry in beliefs parameter,  $\Delta_0$ , between the EN and the VC. A more experienced EN is expected to be more reasonable and realistic and hold opinions similar to the VC. We set  $\Delta_0 = 0.5V_0 = 0.5$ .

### 5.3.2 Calibrated parameters

The calibrated parameters are  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\lambda$ ,  $k$  and  $L$ . The value of these parameters is set so that the model’s output “best matches” empirical evidence described in Sahlman (1990) and Gompers (1995). An explicit definition of “best match” is given as part of our description of the calibration process. We start by explaining the difficulty associated with directly linking the parameters in this group to empirical data. Subsequently, we describe the empirical evidence, how we compute it in our model and the calibration process itself.

- **$\alpha$  and  $\beta$ :** The parameters  $\alpha$  and  $\beta$  represent the output elasticities of investment and effort, respectively. We limit the values of  $\alpha$  and  $\beta$  to  $\alpha + \beta = 1$  to model a constant returns-to-scale production function. Different values of  $\alpha, \beta$  describe different industries. Since the data we use to calibrate is cross-industry, the baseline values for  $\alpha$  and  $\beta$  must be calibrated.
- **$\gamma$  and  $k$ :** No data is available from which to estimate the values for  $\gamma$  and  $k$ , the parameters associated with the EN's disutility from effort. Recall Assumption 2,  $(1 - \alpha)\frac{\gamma}{\beta} \geq 2$ . Since  $\alpha + \beta = 1$  we have  $\gamma \geq 2$ . We set  $\gamma = 2$  and calibrate  $k$  to the data.
- **$\lambda$ :** Setting a value for risk-averseness directly from data is difficult if not impossible. Rabin and Thaler (2001), quoting Kandel and Stambaugh (1991), demonstrate that attempting to do so may result in absurdly high levels of risk-aversion. They warn researchers to be very careful when setting values to risk-aversion and conclude that “economists should use care in choosing the appropriate hypothetical examples when measuring risk aversion” (Rabin and Thaler (2001) p. 225).
- **$L$ :** The loss function is assumed to be increasing and convex. Specifically, we model the loss in period  $i$  to be

$$l_i = Li^2 . \tag{83}$$

As with  $\gamma$  and  $k$ , there is no empirical evidence from which to directly estimate the loss parameter,  $L$ , and therefore it must be calibrated.

### 5.3.2.1 Empirical evidence

The empirical evidence that we use consists of seven economical statistics about the venture capital industry reported by Sahlman (1990) and Gompers (1995). The average number of investment periods is computed indirectly from Gompers (1995), Table IV, and is summarized in Table 4. We conclude that:

1. The average number of rounds for all the firms is approximately 2.7.

**Table 4:** Average number of investment rounds by industry

Industry	Average Number of Rounds	Number of Firms
Communications	2.78	98
Computers	3.89	27
Computer Related	3.66	90
Computer Software	2.99	77
Electronic components	3.27	22
Other electronics	3.21	41
Biotechnology	3.69	29
Medical/health	2.98	90
Energy	1.91	22
Consumer products	2.14	103
Industrial products	2.09	89
Transportation	1.93	15
Other	1.60	96

Sahlman (1990, Figure 1) provides data about the distribution of the return from investment. Investments are divided into six groups. The first group contains all investments that ended with total loss, the second group contains investments that ended with partial loss, and the third, fourth, fifth and sixth groups comprise investments that returned a payoff of between  $[0, 2]$ ,  $[2, 5)$ ,  $[5, 10)$  and  $[10, \infty)$ , respectively. Table 5 summarizes his empirical findings. By merging adjacent groups we conclude that:

2. 34.5% of total investment resulted in a negative return.

**Table 5:** Investment categorized by return

Investments with return	Percentage of Total Amount Invested
Total Loss	11.5%
Partial Loss	23.0%
0 to 1.999	30.0%
2 to 4.999	19.8%
5 to 9.999	8.9%
$\geq 10$	6.8%

3. 49.8% of total investment resulted in a return between zero and five times the amount invested.
4. 15.7% of total investment resulted in a return greater than five times the amount invested.

Sahlman (1990, p. 485) also investigates the firm's rate of success and he reports that:

5. 32.4% of the companies (70 of 216) failed to yield the amount invested.
6. 67.6% of the companies yielded more than the amount invested.
7. 4.28 is the ratio between the total value of the firms and the total amount invested (\$1,049 million and \$245 million, respectively).

We summarize the empirical evidence (EE) in the following array:

$$\overline{EE} := (EE_1, \dots, EE_7) = (2.7, 0.345, 0.498, 0.157, 0.324, 0.676, 4.28) . \quad (84)$$

### 5.3.2.2 Code Outputs

In the simulation step of the core code we simulate a large number of firms. The statistics that are gathered in the simulation process are then used to compute economical results corresponding to the empirical evidence array,  $\overline{EE}$ . Let

- $N$  denote the number of simulated firms.
- $\tau_f$  denote the duration of firm  $f$ ,  $f = 1, \dots, N$ .
- $C_f$  denote the total amount of investment in firm  $f$ .
- $TV_f$  denote the value at termination of firm  $f$ .
- $Ret_f$  denote the return of firm  $f$ ,  $Ret_f := \frac{TV_f - V_0 - C_f}{C_f}$ .
- $Inv$  denote the total amount of investment in all the firms,  $Inv := \sum_{f=1}^N C_f$ .
- $I_1$  denote the sum of  $C_f$ 's such that  $Ret_f \in (-\infty, 0)$ .
- $I_2$  denote the sum of  $C_f$ 's such that  $Ret_f \in [0, 5)$ .

- $I_3$  denote the sum of  $C_f$ 's such that  $Ret_f \in [5, -\infty)$ .
- $N_1$  denote the number of firms such that  $Ret_f < 0$

The model's results are  $\overline{Res} := (Res_1, \dots, Res_7)$  where

1. the expected number of periods:  $Res_1 := \frac{1}{N} \sum_{f=1}^N \tau_f$ .
2. the percent of total investment resulting with a negative return:  $Res_2 := \frac{I_1}{Inv}$
3. the percent of total investment resulted with a return in  $[0, 5)$ :  $Res_3 := \frac{I_2}{Inv}$
4. the percent of total investment resulted with a return in  $[5, \infty)$ :  $Res_4 := \frac{I_3}{Inv}$
5. the percent of firms failing to return the investment:  $Res_5 := \frac{N_1}{N}$
6. the percent of firms succeeding to return the investment:  $Res_6 := 1 - Res_5$
7. the total return from total investment is  $Res_7 := \frac{1}{Inv} \sum_{f=1}^N (TV_f - V_0)$ .

In addition to the results computed for the calibration process,  $\overline{Res}$ , described above, the code computes other statistics to address a variety of research questions. Let:

- $NFV_f$  denote the net firm value of firm  $f$ ,  $NFV_f = TV_f - C_f$ .
- $ENP_f$  denote the EN's payout from firm  $f$ ,  $ENP_f = V_0 + \sum_{i=1}^{\tau_f} (a_i + b_i(V_i - V_{i-1}))$
- $NVCS_f$  denote the net VC share from firm  $f$ ,  $NVCS_f = TV_f - ENP_f - C_f$ .
- $N_{\tau=t}$  denote the number of firms with  $\tau_f = t$ ,  $t = 1, 2, \dots, T$ .

In addition to  $\overline{Res}$ , we compute:

1. Expected firm duration:  $E\tau := \frac{1}{N} \sum_{f=1}^N \tau_f$ .
2. The distribution of  $\tau$ :  $P[\tau = t] := \frac{1}{N} N_{\tau=t}$ .
3. The expected net firm value:  $ENFV := \frac{1}{N} \sum_{f=1}^N NFV_f$ .
4. The expected net VC share<sup>1</sup>:  $ENVCS := \frac{1}{N} \sum_{f=1}^N NVCS_f$ .

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<sup>1</sup>The expected VC share is the VC's continuation value

5. The expected total investment:  $EC := \frac{1}{N} \sum_{f=1}^N C_f$ .

### 5.3.2.3 The Calibration Process

The purpose of the calibration process is to determine the base values of the parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\lambda$ ,  $k$  and  $l$ . Keep in mind that we have already established the values of the other parameters of the model and the calibration process takes them as given. The process starts by setting a feasible range for each one of the parameters  $\alpha$ ,  $\gamma$ ,  $\lambda$ ,  $k$  and  $l$ . In the initial search we start by allowing each parameter to receive one of  $n$  equally spaced points in the parameter's feasible range. Since we are calibrating 5 parameters we will have to examine the economical results,  $\overline{Res}$ , of each of the  $n^5$  possible experiments. For each one of these  $n^5$  arrays we compute its sup-norm distance to the empirical evidence array  $\overline{EE}$ . The *best match* is the array  $\overline{Res}$  that minimizes the sup-norm distance to the empirical evidence array  $\overline{EE}$ . Formally, let

- the  $\ell$ th parameter,  $\ell = 1, \dots, 5$ , denotes the *calibrated parameters* in the following order  $(\alpha, \beta, \gamma, \lambda, k, l)$
- $[r_\ell^m, r_\ell^M]$  denotes the initial feasible range for parameter  $\ell$ .
- $step_\ell := \frac{r_\ell^M - r_\ell^m}{n-1}$  denotes the search resolution for parameter  $\ell$ .
- $(v_\ell^1, \dots, v_\ell^n)$  be  $n$  equally spaced points on the initial feasible range for parameter  $\ell$ , where

$$\begin{aligned} - v_\ell^1 &= r_\ell^m, \\ - v_\ell^{j+1} &= v_\ell^j + step_\ell \text{ for } j = 1, \dots, n-1, \\ - v_\ell^n &= r_\ell^M. \end{aligned}$$

- $V := (v_1^1, \dots, v_1^n) \times (v_2^1, \dots, v_2^n) \times \dots \times (v_5^1, \dots, v_5^n)$  denotes the search space, and notice the number of elements of  $V$  is  $n^5$ .
- $v \in V$  denotes a candidate value for the calibrated parameters,  $v = (v_1, \dots, v_5)$

- $\overline{Res(v)} := (Res(v)_1, \dots, Res(v)_7)$  denotes the model results corresponding to candidate  $v$ .

Then the initial best match values are:

$$\overline{bmv} := (bmv_1, \dots, bmv_5) = \arg \min_{v \in V} \left\{ \max_{1 \leq j \leq 7} \left\{ \frac{|Res(v)_j - EE_j|}{EE_j} \right\} \right\}. \quad (85)$$

At this point we start an iterative search process centered around the initial best match. In each iteration we define a grid around the current best match—in the first iteration the current best match values are  $\overline{bmv}$ —and search over the points of the grid. The distance between points on the grid is divided by two from one iteration to the next, and the search results of any iteration are set to be the new current best match. We repeat this process iteratively, until we reach the desired accuracy for the basic values. The last iteration's results are the *base values* of the model.

We now formally describe this process. Let

- the current best match values,  $\overline{cbmv} := \overline{bmv}$ ,

and repeat the following process 4 times:

1. the current best match values,  $\overline{cbmv} := \overline{bmv}$ .
2. the current step,  $cstep_\ell := step_\ell$  for each parameter  $\ell$ .
3.  $(v_\ell^1, v_\ell^2, v_\ell^3)$  be 3 equally spaced points around  $cbmv_\ell$  for parameter  $\ell$ , which
  - $v_\ell^1 = cbmv_\ell - cstep_\ell$ ,
  - $v_\ell^2 = cbmv_\ell$ ,
  - $v_\ell^3 = cbmv_\ell + cstep_\ell$ .
4. The search space,  $V := (v_1^1, v_1^2, v_1^3) \times (v_2^1, v_2^2, v_2^3) \times \dots \times (v_5^1, v_5^2, v_5^3)$ , and notice the number of elements of  $V$  is  $3^5$ .
5. Let  $v \in V$  denote a candidate value for the calibrated parameters,  $v = (v_1, \dots, v_5)$ .  $\overline{res(v)} := (res(v)_1, \dots, res(v)_7)$  be the model results corresponding to candidate  $v$ . Then the current best match values,  $\overline{cbmv} := (cbmv_1, \dots, cbmv_5)$  are given by (85).

**Table 6:** The initial feasible regions and base values for the calibrated parameters

Parameter	Region	Base Values
$\alpha$	[0.1 , 0.9]	0.3875
$\beta$	$1 - \alpha$	0.6125
$\gamma$	2	2
$\lambda$	[0.05 , 0.2]	1.0938
$k$	[0 , 0.1]	0.1914
$l$	[0.5 , 3]	0.0586

**Table 7:** Economic results of the calibration process

$\ell$	Economic Result	$Res_i$	$EE_i$
1	expected number of periods	1.929	2.7
2	percent of total investment resulting with a negative return	0.247	0.345
3	percent of total investment resulting with a return in $(0, 5)$	0.56	0.498
4	percent of total investment resulting with a return in $[5, \infty)$	0.194	0.157
5	percent of firms failing to return the investment	0.415	0.324
6	percent of firms succeeding to return the investment	0.585	0.676
7	total return from total investment	3.061	4.28

The *base values* for the calibrated parameters are contained in the vector  $\overline{cbmv}$  produced in the last iteration.

### 5.3.3 The Base Values

The initial feasible regions for the *calibrated parameters* are given in Table 6. The regions were chosen in a trial and error process so that the initial base numbers are interior points. Executing the calibration process on these regions with  $n$ , the number of points in the initial grid, set  $n = 9$  provided base values for the calibrated parameters as given in Table 6. The economic results,  $\overline{Res}$ , are given in Table 7, which compares our models results to the empirical evidence ( $\overline{EE}$ ). The distance of  $\overline{Res}$  from  $\overline{EE}$  is approximately 0.285, which means that the largest deviation was less than 29%. To complete the picture we summarize the base values for the *data driven parameters* in Table 8.

**Table 8:** The base values for the data driven parameters

Parameter	Base Value
$V_0$	100
$\mu_0$	0.1
$\sigma_0$	0.5
$s^2$	0.5
$\Delta_0$	0.5

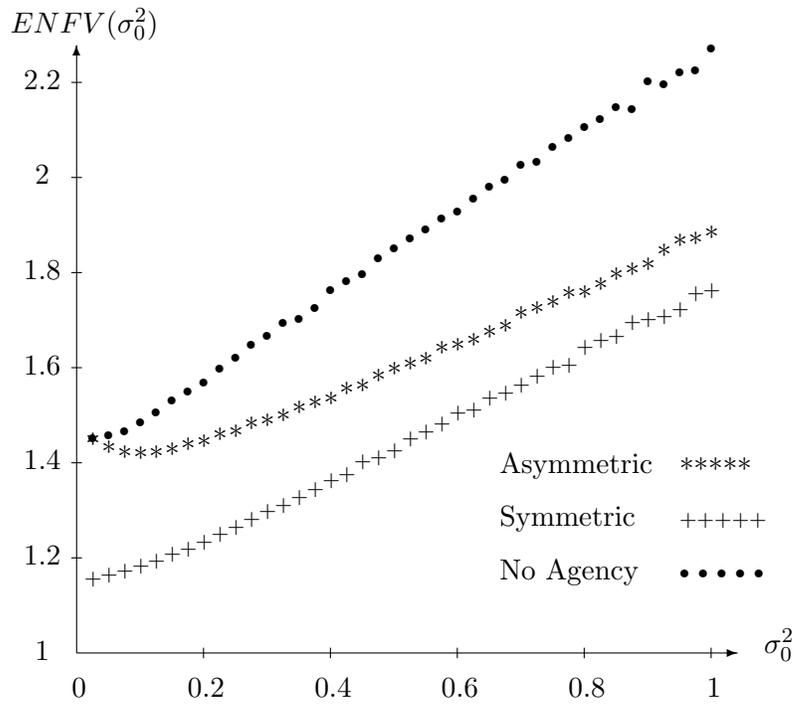
## 5.4 Numerical Results

We provide the multi-period model's numerical results given for the basic numbers. Our analysis focuses on finding how continuation value, project duration and net firm value behave. We examine this behavior for changes in the risky components,  $\sigma_0^2$  and  $s^2$  and for changes in the output elasticity to capital,  $\alpha$ . Similarly to our risk analysis in Chapter 4, we find clear distinction between the effects of the systematic risk and the effects of technical risk and demonstrate that these effects need not be monotone. In addition, we find that for firms with constant returns-to-scale increasing the output elasticity of labor, i.e. increasing  $\beta$  while maintaining  $\alpha + \beta = 1$ , increases continuation value, firm duration and expected net firm value.

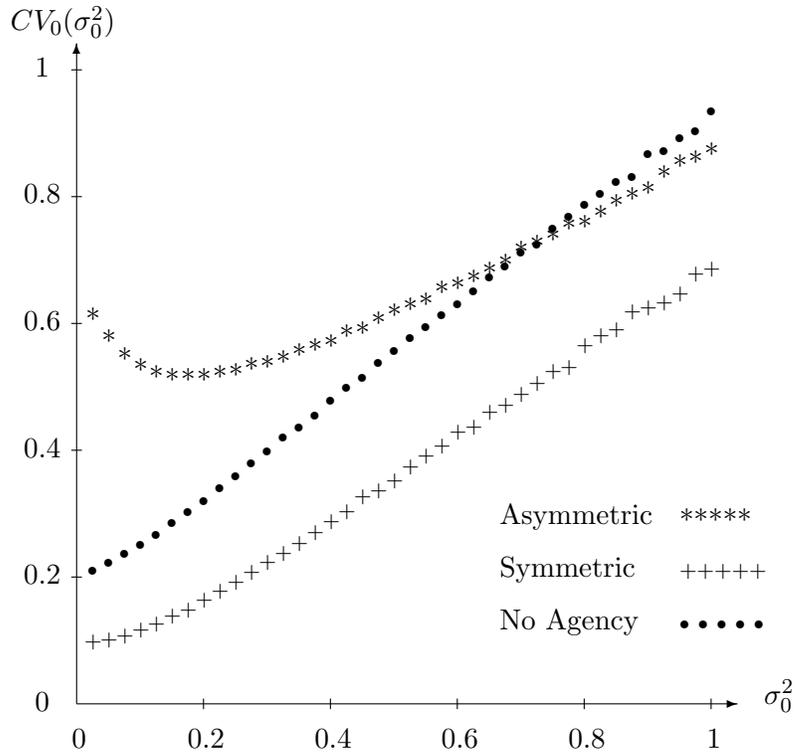
### 5.4.1 Risk Analysis

Experimenting with the base numbers for different values of  $\sigma_0^2$  provides similar results to those in the two period model. In the no agency and symmetric cases continuation value is increasing whereas under asymmetric information the relationship is non monotonic. These results are depicted in Figure 13, which describes the relationship between the continuation value and the initial asymmetric risk for the three market settings.

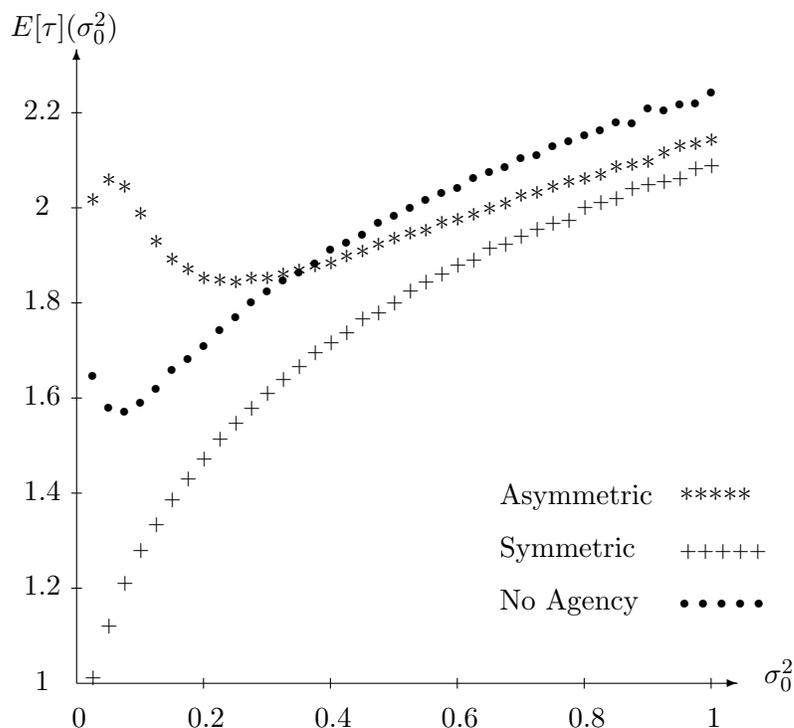
Figure 12 depicts the dependency of firm value on the initial systematic risk. For the symmetric and no agency market settings we have that firm value and the VC's share behave similarly. In both cases the dominant effect of the technical risk is to increase of the parties learning sensitivity. This effect is positive to the firm and prices it higher. When



**Figure 12:** Expected net firm value vs. initial technical risk



**Figure 13:** Date zero continuation value vs. initial technical risk

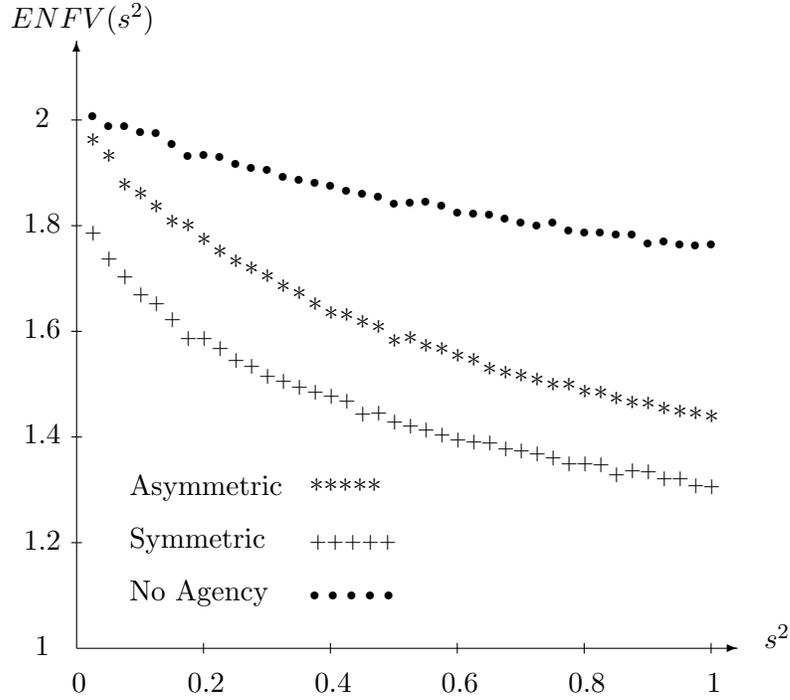


**Figure 14:** Expected firm duration vs. initial technical risk

$\Delta_0 > 0$ , the firm does not carry the losses to the VC's share due to increases in technical risk. Nevertheless, the firm is slightly affected by the VC's loss, due to the decrease in the expected number of investment periods. However, we find that for our base numbers this negative secondary effect is negligible compare to the positive effect of the enhanced sensitivity and consequently the asymmetric case is also increasing in Figure 12.

In contrast to technical risk, experimenting for different values of systematic risk on the basic numbers results in a consistent decreasing behavior. Figures 15 and 16 depict the net firm value and the expected VC share for the basic numbers when varying  $s^2$ . We find that even when  $\Delta_0 > 0$  the negative net effects due to the loss of learning and the increase in risk costs are dominant.

The main conclusion of the two-period model with regards to the effect of risk on the timing of implementation is that it depends on the sign of the expected within period flow of future periods. The loss function we use is quadratic form and, therefore, increases quickly which makes the deterministic part of the within period flow negative very quickly. We therefore expect, in the spirit of the second part of Proposition 10, that  $E[\tau]$  will decrease



**Figure 15:** Expected net firm value vs. market risk

with the systematic risk and increase with the initial technical risk. The complexity of the  $T$ -period model in comparison to the two-period model is manifested in Figure 14 where we see that  $E[\tau]$  is initially decreasing with  $\sigma_0^2$  in contrast to the 2-period model prediction. Notwithstanding, the  $T$ -period model results correspond very well to the two-period predictions. We now examine how  $E[\tau]$  behaves in the presence of conflicting forces such as when increasing  $s^2$  under symmetric or asymmetric information. In Figures 14 and 17 we present the expected project duration as a function of the initial technical risk and the systematic risk, respectively, for the three market settings. We find a strong relationship between project duration and the VC continuation value. Indeed, comparing Figure 14 with Figure 13 reveals that the presence of strong conflicting forces allows for non-monotonic behavior similar to the non-monotonic behavior measured for the continuation value.

#### 5.4.2 Labor and Capital Substitution

The parameters  $\alpha$  and  $\beta$  denote the returns to scale from capital and labor, respectively. Keep in mind our numerical analysis focuses on firms with constant returns-to-scale, i.e.

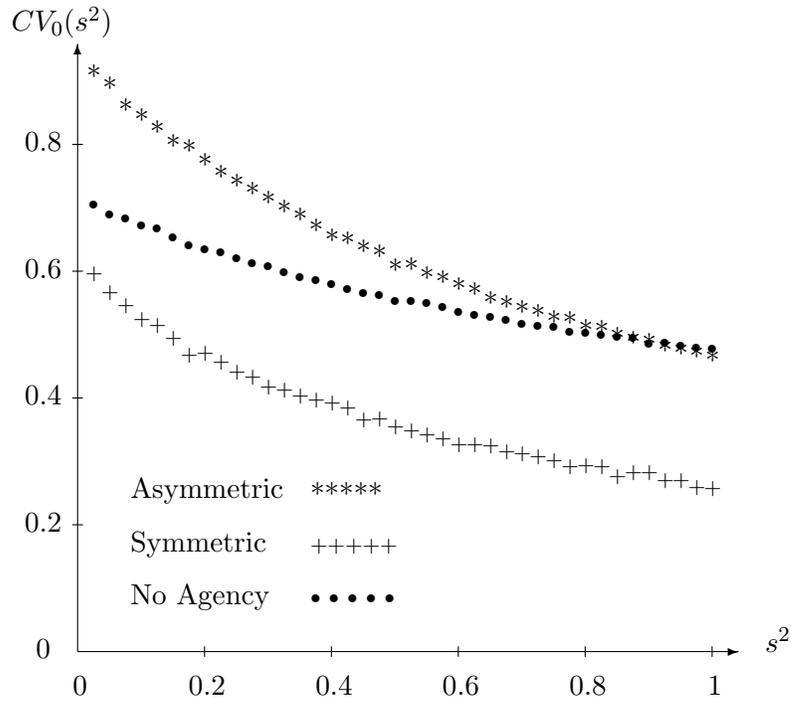


Figure 16: Date zero continuation value vs. market risk

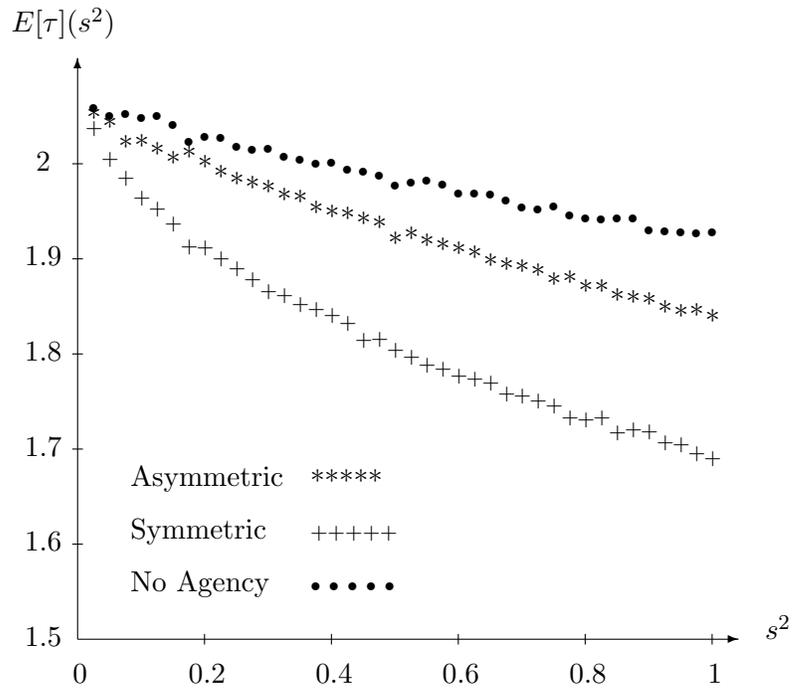


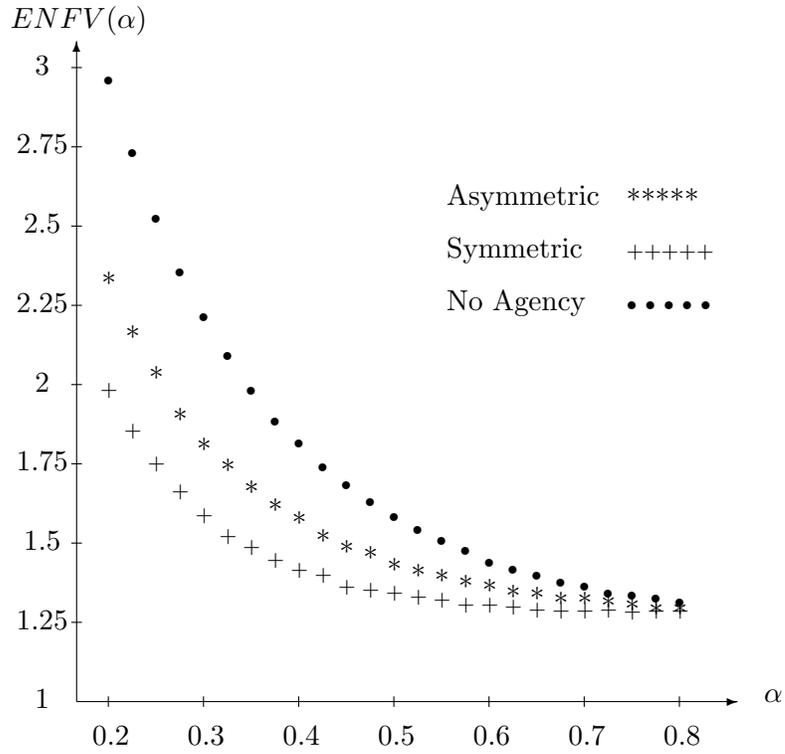
Figure 17: Expected firm duration vs. market risk

$\alpha + \beta = 1$ . Therefore, a firm with high  $\alpha$  will have a low  $\beta$  and therefore enjoys a high output elasticity to capital but a low output elasticity to effort. In this case we call the firm *capital elastic*. Conversely, a firm with low  $\alpha$  is *capital inelastic*. The numerical analysis examines how firms with different capital elasticity behave. We find that asymmetry and agency effects and VC's ability to exploit EN's optimism are more pronounced for capital inelastic industries.

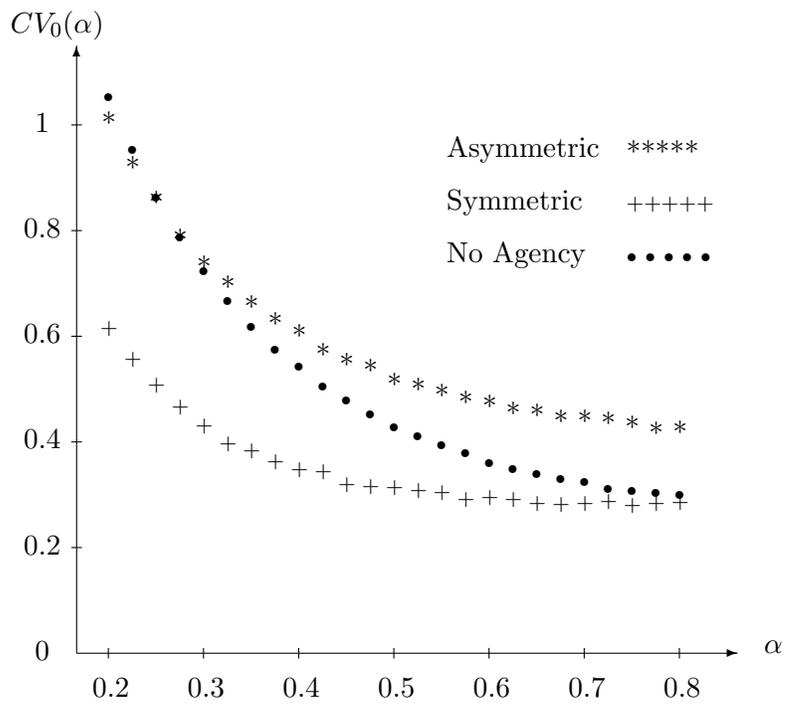
Figure 18 shows how the expected net firm value, changes with capital elasticity. The expected net firm value is described for asymmetric beliefs, symmetric beliefs and for the no agency case. In all market settings firm value declines with capital elasticity. Interestingly, we have that for sufficiently high capital elasticity firm value is almost equal for all three market settings. We explain these phenomena by noticing that the increase in termination value and the VC's within period low consist of an element that is contingent on effort and capital ( $c_i^\alpha \eta_i^\beta$  and  $F_i^*$ , respectively) and an independent contributor ( $\mu_{i-1} - l_i$ ). When the firm is capital inelastic, EN's effort comes "cheap" since the ratio between the cost of effort and the return from effort ( $\gamma/\beta$ ) is low and therefore the EN is willing to invest more effort. This extra effort has a significant positive effect on the firm's performance due to the high returns from labor. Increasing  $\alpha$  increases the relative cost of effort  $\gamma/\beta$  and discourages the EN and consequently the VC from activity in the firm. At some point, however, the change in the size of the contingent component is negligible compared to the independent contributor to the termination value and the within period flow. Since firm value and the continuation value are very much tied to the increase in termination value and the within period flow, respectively, we claim that the effects of elasticity are similar for firm value and continuation value, too. We also note the strong relationship between project duration and continuation value. Accordingly, for high capital elasticity firms, since firm's performance is almost unchanged by  $\alpha$ , we find similar results for its net value, the expected time to implementation and the initial continuation value. This result is illustrate in Figures 18 - 20. For high  $\alpha$  the graphs are almost constant.

The reason why firm value is almost identical for the asymmetric beliefs, symmetric beliefs and no agency cases for sufficiently high  $\alpha$  is explained by the relative high cost

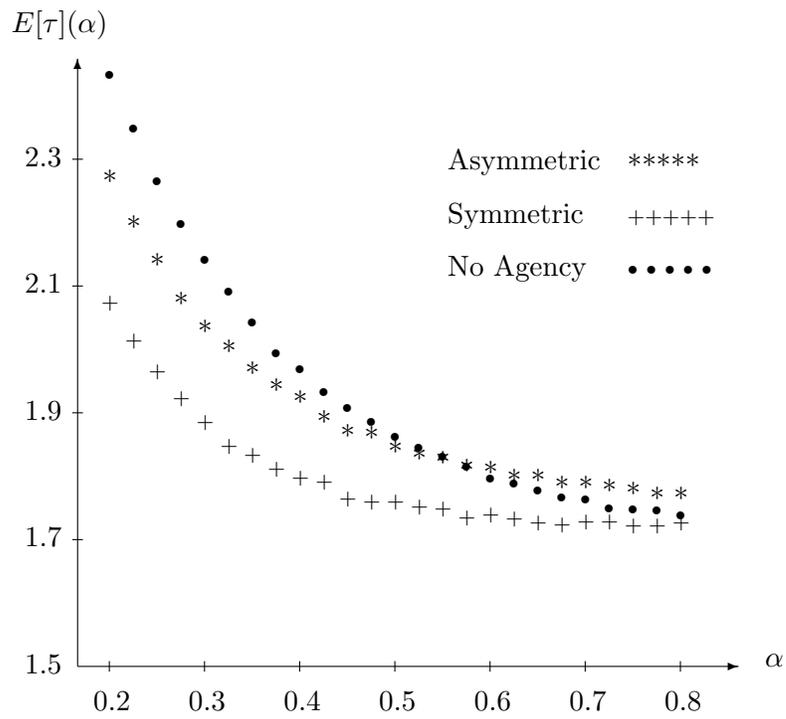
of effort. Agency and asymmetry costs are negligible compared to the high cost of EN effort. Nevertheless, as one can see from Figure 19, the VC will always be able to exploit the EN's optimism, because the EN is effectively willing to forgo some of the firm's gains and hand it over to the VC. Notice that when  $\alpha = 0.2$  the VC "harvests" almost all the asymmetry. With symmetric beliefs the VC's continuation value is slightly above 0.61, whereas with asymmetric beliefs ( $\Delta_0 = 0.5$ ) the VC's continuation value increases by 0.44, which is almost 90%  $\Delta_0$ . However, when  $\alpha = 0.8$  the VC gains less than 30% of  $\Delta_0$  as result of the asymmetric information. The reason for this is that for low capital elastic firms the EN does not need to share the risk with the VC, which is implied by the high equilibrium EN contingent compensation ( $b_i^*$ ) values. Therefore, the EN is highly exposed to losses due to asymmetric beliefs, hence the ability of the VC to fully exploit the asymmetry. However, when there is high capital elasticity, the EN requires more risk sharing, and is therefore more protected from losses due to the asymmetric beliefs. Finally, we note that in the asymmetry case project duration is longer than the other market settings for high capital elastic firms (see Figure 20). This phenomenon is explained by the higher level of continuation value.



**Figure 18:** Expected net firm value vs. capital elasticity



**Figure 19:** Date zero continuation value vs. capital elasticity



**Figure 20:** Expected firm duration vs. capital elasticity

## CHAPTER VI

### SHIFTING THE BARGAINING POWER

#### *6.1 Introduction*

In the previous chapters we assumed that the VC has the bargaining power in negotiations with the EN. We now examine the scenario where the EN has the bargaining power. For tractability, this model requires the additional assumption that the parties agree upfront on the number of investment periods,  $T$  and no early termination is possible. As the EN has all the bargaining power, the time horizon  $T$  is chosen such that the EN's expected utility is maximized. Through our analyses in the previous chapters and this one, we hope to shed more light on the effects of bargaining power on investments, labor supply and equilibrium contracts.

#### *6.2 The Model*

Our model is similar to the one described in Chapter 3. However, we now assume that the VC market is competitive and the EN possesses all the bargaining power in any negotiation with the VC. As in the basic model, the VC has linear inter-temporal preferences whereas the EN is risk-averse with CARA preferences. Since the EN possesses all the bargaining power, in contrast with the previous model, *the EN offers the VC a long-term renegotiation-proof contract* at date zero, which describes the VC's investments, the EN's compensation, and the termination time,  $T$ . The contract between the VC and the EN, the VC's investment policy, the EN's effort policy and the termination time are derived endogenously in a subgame-perfect equilibrium of the dynamic game between the VC and the EN.

As in the Basic Model, the project's termination value,  $V_i$ , evolves as follows:

$$V_i - V_{i-1} = (c_i^\alpha \eta_i^\beta - l_i) + \Theta + S_i.. \quad (86)$$

The EN's expected utility at date 0 is

$$-E \left[ \exp \left\{ -\lambda \left( V_0 + \sum_{i=1}^T [a_i + b_i(V_i - V_{i-1}) - k\eta_i^\gamma] \right) \right\} \right]. \quad (87)$$

The conditions that we assumed on the parameters in the Basic Model are also assumed to hold here. We now characterize the equilibrium.

### 6.3 *Equilibrium*

As before, we use backward induction to characterize the equilibrium. First consider the last investment period  $i = T$ . Recall that the EN and VC priors on  $\Theta$  as of date  $T-1$  are  $N(\mu_{T-1}^j, \sigma_{T-1}^2)$  with  $\mu_{T-1}^j$  and  $\sigma_{T-1}^2$  given by (4) and (3), respectively, with the index  $i$  set to  $T$ .

#### 6.3.1 Optimal Contractual Parameters in Period $T$

Suppose that at the beginning of period  $T$ , i.e. (date  $T-1$ ), the VC's investment is  $c$  and the EN's contractual parameters are  $(a, b)$ . If the EN exerts effort  $\eta$  in period  $T$ , his expected utility, (87), is given by

$$-E \left[ \exp \left\{ -\lambda \sum_{t=1}^{T-1} (a_t + b_t \Delta V_t - k\eta_t^\gamma) + (a + b \Delta V_T - k\eta^\gamma) \right\} \right] \quad (88)$$

At date  $T-1$ , that is, the beginning of period  $T$ ,

$$\Delta V_T = c^\alpha \eta^\beta - l_T + \Theta + S_T,$$

where, according to the EN,  $\Theta + S_T \sim N(\mu_{T-1}^{EN}, \sigma_{T-1}^2 + s^2)$ . Since past decisions and signal realization are observed by all, the EN's expected utility equals<sup>1</sup>

$$\max_{\eta} -\exp \left\{ -\lambda \sum_{t=0}^{T-1} (a_t + b_t \Delta V_t - k\eta_t^\gamma) \right\} \cdot \exp \left\{ -\lambda \left( a + b E[\Delta V_T] - k\eta^\gamma - \frac{1}{2} b^2 p_{T-1} \right) \right\}, \quad (89)$$

where, recall,  $p_{T-1} = \lambda(\sigma_{T-1}^2 + s^2)$ . Accordingly, the EN chooses his effort level to maximize:

$$bc^\alpha \eta^\beta - k\eta^\gamma \quad (90)$$

and the optimal effort level is

$$\eta(b, c) = \left( \frac{\beta c^\alpha b}{\gamma k} \right)^{\frac{\beta}{\gamma - \beta}}. \quad (91)$$

---

<sup>1</sup>Recall that  $E[\exp\{-\lambda X\}] = \exp\{-\lambda(\hat{\mu} - \frac{\lambda}{2}\hat{\sigma}^2)\}$  if  $X \sim N(\hat{\mu}, \hat{\sigma}^2)$ .

Competition between the VCs ensures that

$$a = -c + (1 - b)(c^\alpha \eta(b, c)^\beta - l_T + \mu_{T-1}^{VC}). \quad (92)$$

Since the EN has all the bargaining power, the contractual parameters maximize the EN's expected continuation utility expressed in (88). Substituting (92) in (89) and taking expectations, the optimal contractual parameters in period  $T$  solve

$$\max_{b,c} -\exp \left\{ -\lambda \left( -c + (c^\alpha \eta(b, c)^\beta - l_T + \mu_{T-1}^{VC}) + b \Delta_{T-1} \right) - k \eta(b, c)^\gamma - \frac{1}{2} b^2 p_{T-1} \right\}. \quad (93)$$

where recall  $\Delta_{T-1} := \mu_{T-1}^{EN} - \mu_{T-1}^{VC}$ . Substituting the optimal effort (91) into (93) and considering only the relevant expressions, the (93) simplifies to

$$\max_{b,c} \Delta_{T-1} b - \frac{1}{2} p_{T-1} b^2 + \phi(b) c^{\alpha \frac{\gamma}{\gamma-\beta}} - c, \quad (94)$$

where

$$\phi(b) := \left( \frac{1}{k} \right)^{\frac{\beta}{\gamma-\beta}} \left( \left( \frac{\beta b}{\gamma} \right)^{\frac{\beta}{\gamma-\beta}} \left( 1 - \frac{\beta b}{\gamma} \right) \right). \quad (95)$$

The problem (95) is, in fact, identical to problem (18). Since the assumptions on the parameters used for the Basic Model still apply, we can use the results from the Basic Model to deduce that the optimal investment as a function of the contingent compensation is given by

$$c(b) := \hat{K} \phi(b)^{\frac{\gamma-\beta}{(1-\alpha)\gamma-\beta}} \quad (96)$$

where

$$\hat{K} := \left( \frac{\alpha \gamma}{\gamma - \beta} \right)^{\frac{\gamma-\beta}{(1-\alpha)\gamma-\beta}} > 0.$$

Hence, (95) can be expressed as

$$\max_b G_T(b) := \Delta_{T-1} b - \frac{1}{2} p_{T-1} b^2 + K c(b) \quad (97)$$

where

$$K := \frac{(1 - \alpha)\gamma - \beta}{\alpha \gamma} > 0.$$

By the arguments used in the analysis of the Basic Model, Theorem 1 characterizes the optimal risky compensation,  $b_T^*$ , the optimal investment,  $c_T^*$ , and the optimal effort,  $\eta_T^*$ .

### 6.3.2 The inductive step

We note that  $b_T^*$ , the solution to (97), is independent of past decisions. This observation is critical to the analysis of the inductive step ( $i < T$ ). We will show going backwards that each period's decisions are independent of past decisions. Hence, in the inductive step we need not consider how the current decisions  $(b, c, \eta)$  affect future decisions,  $(b_t, c_t, \eta_t)$ ,  $t > i$ . Further, since at each future date competition between VC's ensures they receive zero return, they consider only the current period's returns. Now, assume we are at the beginning of period  $i < T$ . Suppose the VC investment is  $c$  and the EN's contractual parameters are  $(a, b)$ . If the EN exerts effort  $\eta$  in period  $i$ , the relevant part of his expected utility at the terminal date, (87), is given by

$$E \left[ \exp \left\{ -\lambda \left[ (a + b\Delta V_i - k\eta^\gamma) + \sum_{t=i+1}^T \left( a(b_t^*, c_t^*) + b_t^* \Delta V_t - k\eta_t^{*\gamma} \right) \right] \right\} \middle| \mathcal{G}_{i-1} \right] \quad (98)$$

as we need not consider past periods. Since  $\Delta V_i = c^\alpha \eta^\beta - l_i + \Theta + S_i$ , the EN's effort problem is identical to the  $T$  period problem and the optimal effort is given by (91). Competition between VC's ensure

$$a(b, c) = -c + (1 - b)(c^\alpha \eta(b, c)^\beta - l_i + \mu_{i-1}^{VC}), \quad (99)$$

which by the induction assumption is true for all  $i > t$ . Therefore, we rewrite (98) as

$$E \left[ \exp \left\{ -\lambda \left( -c + (1 - b)(c^\alpha \eta(b, c)^\beta - l_i + \mu_{i-1}^{VC}) + b\Delta V_i - k\eta(b, c)^\gamma + \sum_{t=i+1}^T \left( -c_t^* + (1 - b_t^*)(c_t^{*\alpha} \eta_t^{*\beta} - l_t + \mu_{t-1}^{VC}) + b_t^* \Delta V_t - k\eta_t^{*\gamma} \right) \right) \right\} \middle| \mathcal{G}_{i-1} \right]. \quad (100)$$

At date  $i - 1$ , the VC's future assessment of  $\Theta$ ,  $\mu_{t-1}^{VC}$ ,  $t > i$ , is a random variable. By (2) and (4) it may be expressed as

$$\mu_{t-1}^{VC} = \frac{s^2 \mu_{i-1}^{VC} + \sigma_{i-1}^2 \left( (t - i)\Theta + \sum_{j=i}^{t-1} S_j \right)}{s^2 + (t - i)\sigma_{i-1}^2}. \quad (101)$$

As the EN has the bargaining power and chooses the contract, the parameters  $b, c$  maximize the EN relevant utility. Keep in mind, by the induction assumption, future optimal decisions

$(b_t^*, c_t^*, \eta_t^*, t > i)$  are independent of  $b$  and  $c$ . Their only possible influence on the optimal choices of  $b, c$  may be through the risk. To avoid burdening the reader, in each of the following derivations we remove any expression that is irrelevant to the EN's maximization problem. Substituting  $\Delta V_i$ , (86), and  $\mu_{t-1}^{VC}$  in the relevant expected utility (100), the EN's objective is to maximize

$$E \left[ \exp \left\{ -\lambda \left( \phi(b) c^{\alpha \frac{\gamma}{\gamma-\beta}} - c - b \mu_{i-1}^{VC} + b(\Theta + S_i) + \sum_{t=i+1}^T \left( (1-b_t^*) \frac{\sigma_{i-1}^2 ((t-i)\Theta + \sum_{j=i}^{t-1} S_j)}{s^2 + (t-i)\sigma_{i-1}^2} + b_t^*(\Theta + S_t) \right) \right) \right\} \middle| \mathcal{G}_{i-1} \right]. \quad (102)$$

To compute the relevant expectation of (102) we point out that (102) is in the form of

$$E[\exp\{-\lambda(Z_i)\} | \mathcal{G}_{i-1}] \quad (103)$$

where  $Z_i$  is the sum of normal variables and hence, normally distributed. Recall, for normally distributed  $X$ ,  $E[\exp\{\lambda X\}] = \exp^{\lambda(E[X] - \frac{1}{2}Var[X])}$ , and therefore to proceed we need to find the relevant mean and the relevant variance of  $Z_i$ . The relevant mean of  $Z_i$  is given by

$$E[Z_i | \mathcal{G}_{i-1}] = E[\phi(b) c^{\alpha \frac{\gamma}{\gamma-\beta}} - c - b \mu_{i-1}^{VC} + b(\Theta + S_i) | \mathcal{G}_{i-1}] = \phi(b) c^{\alpha \frac{\gamma}{\gamma-\beta}} - c + b \Delta_{i-1} \quad (104)$$

because the VC is aware of the EN's beliefs and so  $E[\Theta | \mathcal{G}_{i-1}] = \mu_{i-1}^{EN}$ . Notice now that only  $\Theta$  and  $S_i$  are contained in risky expressions in  $Z_i$  that also contain  $b$  or  $c$ . Therefore, in computing the relevant variance of  $Z_i$  we need only consider the variance of the risky components  $\Theta$  and  $S_i$  (but not  $S_j, j > i$ ). The relevant risky component of  $Z_i$  is therefore given by

$$\Theta \left[ b + \sum_{t=i+1}^T \left( \frac{(1-b_t^*)\sigma_{i-1}^2(t-i)}{s^2 + (t-i)\sigma_{i-1}^2} + b_t^* \right) \right] + S_i \left[ b + \sum_{t=i+1}^T \frac{(1-b_t^*)\sigma_{i-1}^2}{s^2 + (t-i)\sigma_{i-1}^2} \right]. \quad (105)$$

Its variance is given by

$$\sigma_{i-1}^2 \left( b + \sum_{t=i+1}^T \left( \frac{(1-b_t^*)\sigma_{i-1}^2(t-i)}{s^2 + (t-i)\sigma_{i-1}^2} + b_t^* \right) \right)^2 + s^2 \left( b + \sum_{t=i+1}^T \frac{(1-b_t^*)\sigma_{i-1}^2}{s^2 + (t-i)\sigma_{i-1}^2} \right)^2, \quad (106)$$

which is equal to (after eliminating irrelevant expressions)

$$(\sigma_{i-1}^2 + s^2)b^2 + 2b \left( \sigma_{i-1}^2 \sum_{t=i+1}^T \left( \frac{(1-b_t^*)\sigma_{i-1}^2(t-i)}{s^2 + (t-i)\sigma_{i-1}^2} + b_t^* \right) + s^2 \sum_{t=i+1}^T \frac{(1-b_t^*)\sigma_{i-1}^2}{s^2 + (t-i)\sigma_{i-1}^2} \right) \quad (107)$$

$$= (\sigma_{i-1}^2 + s^2)b^2 + 2b\sigma_{i-1}^2 \sum_{t=i+1}^T \left( \frac{(1-b_t^*)\sigma_{i-1}^2(t-i) + (1-b_t^*)s^2}{s^2 + (t-i)\sigma_{i-1}^2} + b_t^* \right) \quad (108)$$

$$= (\sigma_{i-1}^2 + s^2)b^2 + 2b\sigma_{i-1}^2(T-i). \quad (109)$$

The relevant price of risk at date  $i-1$  is given by

$$\lambda \left( \frac{1}{2}p_{i-1}b^2 + b\sigma_{i-1}^2(T-i) \right) = \frac{\lambda}{2}p_{i-1}b^2 + \lambda\sigma_{i-1}^2(T-i)b \quad (110)$$

and includes the immediate period's cost of risk and the future cost that depends linearly on the number of periods left. Considering (102), the relevant price of risk and since, the EN's optimal contract problem may be expressed as

$$\max_{b,c} \left\{ -c + \phi(b)c^{\alpha \frac{\gamma}{\gamma-\beta}} + (\Delta_{i-1} - \lambda(T-i)\sigma_{i-1}^2)b - \frac{\lambda}{2}p_{i-1}b^2 \right\}. \quad (111)$$

The optimal investment is identical to the  $T$  period and is given by (96). Accordingly, the EN's maximization problem at the beginning of period  $i$  is

$$\max_b G_i(b) := (\Delta_{i-1} - \lambda\sigma_{i-1}^2(T-i))b - \frac{1}{2}p_{i-1}b^2 + Kc(b), \quad (112)$$

Recall,  $b_M \in (1, \frac{\gamma}{\beta})$  is the point where  $c''(b) = 0$  (Proposition 1). The following proposition ensures the solution to the EN's maximization problem exists and is unique.

**Proposition 11**

*Under Assumptions 1-3, each  $G_i(\cdot)$  is strictly concave and hence strongly unimodal on  $[0, b_M]$ , and the solution to (112) is positive and less than  $b_M$ .*

The proof of Proposition 11 is identical to Proposition 3, hence omitted. Since the EN's maximization problem (112) is independent of past decisions, we can extend the above arguments by backward induction to *any* period  $i$  and thereby derive the unique equilibrium, as characterized in the following theorem.

**Theorem 6 (Characterization of Equilibrium)**

*Under Assumptions 1 - 3 the equilibrium contract offered by the EN and his effort in period  $i$ ,  $1 \leq i \leq T$  is characterized, as follows:*

- The risky compensation is  $b_i^*$ , the unique solution to (112);
- The investment is  $c_i^* := c(b_i^*)$  defined in (96);
- The fixed compensation is  $a_i^* := a(b_i^*, c_i^*)$  defined in (99);
- The optimal effort level is  $\eta_i^* := \eta(b_i^*, c_i^*)$  defined in (91).

## 6.4 Analysis and Discussion

We now examine the effect of the shift in bargaining power on the EN's risky compensation parameters, the VC's investments, and the EN's effort  $(b_i^*, c_i^*, \eta_i^*)$ . When the EN is risk-neutral ( $\lambda = 0$ ) or when there is perfect information ( $\sigma_0^2 = 0, \Delta_0 = 0$ ), the EN's objective function,  $G(\cdot)$  is identical to  $F(\cdot)$ , the VC's objective function in the Basic Model. Hence, the optimal solution is independent of who has the bargaining power.

### Proposition 12

*The deterministic path trajectories when the EN is risk neutral or when there is perfect information are unaffected by whether the EN or the VC has the bargaining power.*

The following theorem shows that the allocation of bargaining power does matter when the EN is risk averse ( $\lambda > 0$ ) and there is imperfect information.

### Theorem 7 (Decreasing Contingent Compensation)

*Suppose the EN is risk averse.*

- (i) *Switching the bargaining power from the VC to the EN decreases the EN's risky compensation parameters  $(b_i^*)$  trajectory at all dates except the last for which the contingent compensation is equal for both models.*
- (ii) *If the EN is reasonably optimistic ( $\Delta_0 \leq p_0$ ) then switching the bargaining power from the VC to the EN decreases the optimal investment  $(c_i^*)$  and effort  $(\eta_i^*)$  trajectories at all dates except the last for which investment and effort are equal for both models.*

If the risk averse EN has the bargaining power, then the efficiency of risk-sharing is *reduced* so that the EN's risky compensation path, his effort, and the VC's investments are all lowered.

Recall  $b_p^*$  is the solution in the perfect information case and that  $b_p^* < 1$  (Section 3.4.2). For the Basic Model, Theorem 2 divides the value of  $\Delta_0$  into three distinct regions. In the first region, ( $\Delta_0 \in [0, \lambda\sigma_{i-1}^2 b_p^*]$ ), the EN's compensation path is increasing. In the second region, ( $\Delta_0 = \lambda\sigma_{i-1}^2 b_p^*$ ), the EN's compensation path is constant and in the third region, ( $\Delta_0 > \lambda\sigma_{i-1}^2 b_p^*$ ), the compensation path is decreasing. In the first and second region investment and effort behave analogously to the EN's compensation path. We now show that the region of  $\Delta_0$  for which the deterministic trajectories are increasing strictly contains the corresponding region when the EN has bargaining power:

**Theorem 8 (Increasing Trajectory Region)**

*If  $\Delta_0 \leq \lambda(1 + b_p)\sigma_0^2$  then the EN's contingent compensation trajectory is increasing. If, in addition,  $\Delta_0 \leq \lambda(T\sigma_0^2 + s^2)$  then the trajectories of investment and effort are also increasing.*

The region of  $\Delta_0$  for which investment, contingent compensation and effort are increasing is considerably larger when the EN has bargaining power than when the VC has bargaining power. This implies that the asymmetry in beliefs effects are much weaker when the EN has bargaining power. In the Basic Model when  $\Delta_0 = \lambda\sigma_{i-1}^2 b_p^*$  the simultaneous resolution of asymmetry and technical risk results in a constant risk sharing over time. This result is not replicated when the EN has bargaining power. Now, the effects of asymmetry are much weaker and consequently the resolution of risk is the dominant force. Consequently, even for relatively high levels of asymmetry we will see an increasing trajectory of contingent compensation as the EN is assuming more and more risk in response to the decrease in technical risk.

We now turn to consider parametric effects on the deterministic path:

**Theorem 9 (Comparative Statics)**

*If  $\Delta_0 \leq \lambda(T\sigma_0^2 + s^2)$  then in any period the EN's risky compensation, the EN's effort and the VC's investment are increasing in the initial asymmetry in beliefs,  $\Delta_0$ , and decreasing in the risk averseness,  $\lambda$ , the initial technical risk,  $\sigma_0^2$ , the systematic risk,  $s^2$ , and the effort parameter,  $k$ .*

By comparing Theorem 9 to the sensitivity of equilibrium theorems in the Basic Model (Section 3.5) we find that the sensitivity results that were limited to the “reasonably optimistic” region (i.e.  $\Delta_0 < \lambda(\sigma_0^2 + s^2)$ ) are now valid for a larger region. When the EN is in charge, the effects of asymmetric beliefs are strongly mitigated in the early periods because of the relatively high magnitude of the forces of the risk averseness. Consequently, for the EN to be considered overtly optimistic his asymmetry needs to surpass a higher threshold than in the Basic Model.

## 6.5 Proofs

**Proof of Theorem 7:** The derivative of the EN’s maximization problem, (112), is

$$G'_i(b) := (\Delta_{i-1} - \lambda\sigma_{i-1}^2(T - i)) - p_{i-1}b + Kc'(b), \quad (113)$$

Let  $b_i^X$  denote the contingent compensation at period  $i$  when  $X \in \{VC, EN\}$  has bargaining power. By definition,  $F'_i(b_i^{VC}) = 0$  and observe that  $G'_i(b) = F'_i(b) - (T-i)\lambda\sigma_{i-1}^2b$ . Hence,  $G'_i(b_i^{VC}) = -(T-i)\lambda\sigma_{i-1}^2b_i^{VC}$ . Unimodality of  $G(b)$  implies that  $b_i^{EN} < b_i^{VC}$  for any  $i < T$  and  $b_T^{EN} = b_T^{VC}$ , which establishes part (i). When  $\Delta_0 \leq p_0$ , by Proposition 2  $b_i^{EN} < b_i^{VC} \leq 1$  for any  $i < T$  and  $b_T^{EN} = b_T^{VC} \leq 1$ . Part (ii) now follows from the fact that  $c(b)$  is strictly increasing on  $[0, 1)$  and that  $\eta(b, c)$  is increasing. ■

**Proof of Theorem 8:** Let  $x := \frac{\Delta_0}{\lambda\sigma_0^2}$ . The derivative to the EN’s maximization problem, (113), can be rewritten

$$G'_i(b) := \sigma_{i-1}^2\lambda(x - (T - i) - b) \underbrace{-\lambda s^2 b + Kc'(b)}_{F'(b)}. \quad (114)$$

Notice that by Proposition 11  $G_i(b)$  and consequently,  $F(b)$  are strictly unimodal for all  $i$ . We start by proving the first statement of the theorem when  $\Delta_0 \leq \lambda b_p^* \sigma_0^2$ , i.e.  $0 \leq x \leq b_p^*$ . In Step 2 we prove for  $b_p^* < x \leq 1 + b_p^*$ . In Step 3 we complete the proof for the second statement.

**Step 1:** Suppose  $0 \leq x \leq b_p^*$ . Let  $i \in (2, \dots, T)$  and let  $b_i^*$  denote the optimal solution in date  $i$ . By Theorem 2, when  $x \leq b_p^*$  the optimal solution when VC has bargaining power lies below or equal to  $b_p^*$  for all  $i$  and as a consequence of Theorem 7 we have that

$b_i^* \leq b_p^*$ . Hence, by unimodality,  $F'(b_i^*) \geq 0$ . By definition,  $G'_i(b_i^*) = 0$  and therefore  $\sigma_{i-1}^2 \lambda(x - (T - i) - b_i^*) = -F'(b_i^*) \leq 0$ . Consider the EN's problem at period  $i - 1$ :

$$G'_{i-1}(b_i^*) = \sigma_{i-2}^2 \lambda(x - (T - i + 1) - b_i^*) + F'(b_i^*) \quad (115)$$

$$= \underbrace{(\sigma_{i-2}^2 - \sigma_{i-1}^2) \lambda(x - (T - i) - b_i^*)}_{\text{positive}} - \underbrace{\sigma_{i-2}^2 \lambda}_{\text{nonpositive}} < 0. \quad (116)$$

By unimodality,  $b_{i-1}^* < b_i^*$ .

**Step 2:** Suppose  $b_p^* < x \leq 1 + b_p^*$ . We start by establishing that the optimal solution at period  $T$  maintains  $x - b_T^* \leq 1$ . At period  $T$ ,  $G_T(\cdot) \equiv F_T(\cdot)$  and so by Theorem 2 we have  $b_T^* > b_p^*$  and by unimodality,  $F'(b_T^*) < 0$ . By definition,  $G'_T(b_T^*) = 0$  and therefore  $\sigma_{T-1}^2 \lambda(x - b_T^*) = -F'(b_T^*) > 0$  and consequently,  $x > b_T^*$ . Since  $x \leq 1 + b_p^*$  we have that  $x - b_T^* \leq 1$  since  $b_T^* > b_p^*$ . Let  $i \in (2, \dots, T)$  and let  $b_i^*$  denote the optimal solution in date  $i$ . Consider the EN's problem in period  $i - 1$ :

$$G'_{i-1}(b_i^*) = \sigma_{i-2}^2 \lambda(x - (T - i + 1) - b_i^*) - F'(b_i^*) \quad (117)$$

$$= \underbrace{(\sigma_{i-2}^2 - \sigma_{i-1}^2) \lambda(x - (T - i) - b_i^*)}_{\text{less than } \sigma_{i-2}^2} - \underbrace{\sigma_{i-2}^2 \lambda}_{\text{less than 1}} < 0. \quad (118)$$

By unimodality,  $b_{i-1}^* < b_i^*$ .

**Step 3:** By (5) and (3), (and algebraic manipulation),  $\Delta_0 \leq \lambda(T\sigma_0 + s^2)$  implies  $\Delta_{T-1} \leq \lambda(\sigma_{T-1}^2 + s^2)$ . Similarly to Proposition 2 we have that  $b_T^* \leq 1$ . Since,  $x < 1 + b_p^*$  we have from the previous steps of the proof that the optimal  $b$  are increasing over time and hence,  $b_i^* \leq 1$  for all  $i = 1 \dots T$ . The rest of the proof follows immediately from the fact that  $c(b)$  is strictly increasing on  $(0, 1)$  and  $\eta(b, c)$  is increasing. ■

**Proof of Theorem 9:** We begin by noting that for all  $i = 1, \dots, T$ , by Proposition 11  $G_i(\cdot)$  is strictly unimodal on  $[0, b_M]$  and therefore Lemma 1 and its proof apply to  $G_i(\cdot)$ . Since  $\Delta_0 < \lambda(T\sigma_0 + s^2)$ , similarly to Step 3 of the proof of Theorem 8 we have that  $b_i^* < 1$  and therefore  $c'(b_i^*) > 0$  for any period  $i$ . The rest of the proof is an immediate application of Lemma 1. ■

# CHAPTER VII

## NON-OBSERVABLE EFFORT

### 7.1 Introduction

In the previous chapters we assumed that the EN's effort choices are observable (but non-contractible). In this chapter we relax this assumption and examine the scenario where effort is unobservable. As in Chapter 6, we assume

- The VC market is competitive and the EN enjoys all the bargaining power.
- The timing of termination,  $T$ , is decided upfront at date zero.

The basic model is as described in Chapter 6, but we now assume that the EN's effort cannot be observed by the VC. The EN's assessment of project quality is still given by (3) and (4). However, the VC's learning now depends on his conjectures about the EN's effort in *past* periods. Specifically, if at the beginning of period  $i$  the VC conjectures the EN's past effort choices are  $(\hat{\eta}_1, \dots, \hat{\eta}_{i-1})$  then her posterior distribution on  $\Theta$  is  $N(\mu_i^{VC}, \sigma_i^2)$ , where

$$\mu_i^{VC} = \mu_i^{VC}(\hat{\eta}_1, \dots, \hat{\eta}_{i-1}) = \frac{s^2 \mu_0^{VC} + \sigma_0^2 \sum_{t=1}^{i-1} (\Delta V_t - c_t^\alpha \hat{\eta}_t^\beta + l_t)}{s^2 + (i-1)\sigma_0^2}. \quad (119)$$

The EN's information at any date is identical to the previous models. However, the VC's information set is now changed to exclude knowledge of the previous effort choices of the EN. We denote the EN and the VC's information set the beginning of date  $i$  by  $\mathcal{G}_{i-1}^{EN}$  and  $\mathcal{G}_{i-1}^{VC}$ , respectively. Clearly,  $\mathcal{G}_i^{VC} \subset \mathcal{G}_i^{EN}$ .

The dynamics of the equilibrium are similar to the Shift of Power Model with the exception that the EN and the VC have different information sets. Specifically, we assume that for any contract offer the EN responds in effort levels that take into account the fact the VC cannot observe the true effort but instead may conjecture them. Generally, the EN's effort may depend on the contract offer and the VC's conjectures. That is,  $\eta = \eta(a, b, c, \hat{\eta})$ .

The VC, on the other hand, must provide the EN with the most appealing contract. The VC, however, is aware of the EN's effort best response and therefore his conjecture with regard to the effort also depends on his choice of contract, i.e.  $\hat{\eta} = \hat{\eta}(a, b, c)$ . In equilibrium, the VC conjectures correctly and matches his conjecture to the EN's effort function (i.e.  $\hat{\eta}(a, b, c) = \eta(a, b, c, \hat{\eta}(a, b, c))$ )<sup>1</sup>. This equilibrium conjecture is used by the VC in order to maximize the EN's utility. To simplify the exposition of the equilibrium, we begin by developing the solution for a two-period model.

## 7.2 Two-Period Model

As usual, we use backward induction to derive the equilibrium. For given contractual parameters  $(b, c)$  the EN's utility for a choice of effort level  $\eta$  in the second period is

$$-\exp\{-\lambda(a_1 + b_1\Delta V_1 - k\eta_1^2)\} \cdot E\left[\exp\{-\lambda(a + b\Delta V_2 - k\eta^\gamma)\}|\mathcal{G}_1^{EN}\right], \quad (120)$$

where  $a_1, b_1, c_1$  and  $\eta_1$  are the first period decisions. Since  $\Delta V_2 = c^\alpha\eta^\beta - l_2 + \Theta + S_2$ , the EN's optimal choice of effort problem is

$$\max_{\eta} -E\left[\exp\{-\lambda(a + b(c^\alpha\eta^\beta - l_2 + \Theta + S_2) - k\eta^\gamma)\}|\mathcal{G}_1^{EN}\right]. \quad (121)$$

The EN's optimal choice of effort is

$$\eta(b, c) = \left(\frac{\beta c^\alpha b}{\gamma k}\right)^{\frac{\beta}{\gamma-\beta}}. \quad (122)$$

Competition among VCs ensures that the VC's offer guarantees her zero expected return.

$$-c + (1 - b)E[\Delta V_2|\mathcal{G}_1^{VC}] - a = 0, \quad (123)$$

which implies:

$$a = -c + (1 - b)(c^\alpha\eta(b, c)^\gamma + E[\Theta|\mathcal{G}_1^{VC}] - l_2). \quad (124)$$

At date 1, the VC's assessment of  $\Theta$ ,  $E[\Theta|\mathcal{G}_1^{VC}]$ , depends on his conjecture about the EN's first period's effort. By (119)

$$E[\Theta|\mathcal{G}_1^{VC}] = \mu_1^{VC}(\hat{\eta}_1) = \frac{s^2\mu_0^{VC} + \sigma_0^2(\Delta V_1 - c_1^\alpha\hat{\eta}_1 + l_1)}{s^2 + \sigma_0^2}. \quad (125)$$

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<sup>1</sup>In fact, we will show that  $\eta = \eta(a, b, c)$  and therefore the equilibrium condition about the conjecture is trivially set as  $\hat{\eta}(a, b, c) = \eta(a, b, c)$ . Since the EN's actions are independent of the conjectures, the conjectures need not be common knowledge.

Therefore,

$$a = a(b, c) := -c + (1 - b)(c^\alpha \eta(b, c)^\gamma - l_2 + \mu_1^{VC}(\hat{\eta}_1)) . \quad (126)$$

Substituting  $a(b, c)$  in the EN's utility, (121), where now we consider the VC's information set, we have the optimal contract must solve

$$\max_{b,c} -E \left[ \exp \left\{ -\lambda \left[ -c + (1-b)(c^\alpha \eta(b, c)^\beta - l_2 + \mu_1^{VC}(\hat{\eta}_1)) + b(c^\alpha \eta(b, c)^\beta - l_2 + \Theta + S_2) - k\eta(b, c)^\gamma \right] \right\} \middle| \mathcal{G}_1^{VC} \right] . \quad (127)$$

Since the VC's objective is to maximize the EN's utility he must consider the EN's beliefs about the project's quality. The VC's conjectures about past investment are consistent with (125) and so, in (127),  $E[\Theta | \mathcal{G}_1^{VC}]$  is given by

$$E[\Theta | \mathcal{G}_1^{VC}] = \mu_1^{EN}(\hat{\eta}_1) = \frac{s^2 \mu_0^{EN} + \sigma_0^2 (\Delta V_1 - c_1^\alpha \hat{\eta}_1 + l_1)}{s^2 + \sigma_0^2} . \quad (128)$$

Notice that the asymmetry in beliefs under the VC's information set  $\mathcal{G}_1^{VC}$  satisfies

$$\mu_1^{EN}(\hat{\eta}_1) - \mu_1^{VC}(\hat{\eta}_1) = \Delta_1 , \quad (129)$$

where  $\Delta_1$  is defined in (5). Taking the expectation of (127) and removing irrelevant expressions, the optimal contract must solve

$$\max_{b,c} - \exp \left\{ -\lambda \left[ -c + c^\alpha \eta(b, c)^\beta + b\Delta_1 - k\eta(b, c)^\gamma - \frac{\lambda}{2}(\sigma_1^2 + s^2)b^2 \right] \right\} , \quad (130)$$

where, in the above, we substitute  $E[\Theta | \mathcal{G}_1^{VC}] - \mu_1^{VC}(\hat{\eta}_1)$  according to (129). The rest of the derivation is identical to the Basic Model and, accordingly, the optimal investment is

$$c(b) := \hat{K} \phi(b)^{\frac{\gamma-\beta}{(1-\alpha)\gamma-\beta}} \quad (131)$$

and the optimal choices of contractual parameters solve

$$\max_b \Delta_1 b - \frac{1}{2} p_1 b^2 + K c(b) , \quad (132)$$

where  $K$  and  $\hat{K}$  are positive constants. We denote the optimal solution in the second period by  $b_2^*$  and the corresponding optimal risk free compensation, investment and effort

by  $a_2^* := a(b_2^*, c_2^*)$ ,  $c_2^* := c(b_2^*)$  and  $\eta_2^* := \eta(b_2^*, c_2^*)$ , respectively. We also note the second period optimal solution is independent of first period decisions.

In the first period, for contract  $(a, b)$ , investment  $c$  and effort level  $\eta$ , the EN's expected utility is given by

$$-E \left[ \exp \left\{ -\lambda \left( a + b\Delta V_1 - k\eta^2 + a_2^* + b_2^*\Delta V_2 - k\eta_2^{*\gamma} \right) \right\} \middle| \mathcal{G}_0^{EN} \right]. \quad (133)$$

Substituting  $a_2^*$  according to (126) and  $\Delta V_i$ ,  $i = 1, 2$ , according to (86), the EN's expected utility is given by

$$\begin{aligned} & -E \left[ \exp \left\{ -\lambda \left( (a + b(\Theta + c^\alpha \eta^\beta + S_1 - l_1) - k\eta^\gamma) \right. \right. \right. \\ & \left. \left. \left. - c_2^* + (1 - b_2^*)(c_2^{*\alpha} \eta_2^{*\beta} + \mu_1^{VC}(\hat{\eta}_1) - l_2) + b_2^*(\Theta + c_2^{*\alpha} \eta_2^{*\beta} + S_2 - l_2) - k\eta_2^{*\gamma} \right) \right\} \middle| \mathcal{G}_0^{EN} \right]. \end{aligned} \quad (134)$$

Substituting  $\mu_1^{VC}(\hat{\eta}_1)$  according to (125), respectively, we have the EN's choice of effort problem

$$\begin{aligned} \max_{\eta} \quad & -E \left[ \exp \left\{ -\lambda \left( a + b(\Theta + c^\alpha \eta^\beta + S_1 - l_1) - k\eta^\gamma \right. \right. \right. \\ & \left. \left. \left. - c_2^* + (1 - b_2^*)(c_2^{*\alpha} \eta_2^{*\beta} + \frac{s^2 \mu_0^{VC} + \sigma_0^2(\Theta + c^\alpha \eta^\beta + S_1 - c^\alpha \hat{\eta}_1^\beta)}{s^2 + \sigma_0^2} - l_2) \right. \right. \right. \\ & \left. \left. \left. + b_2^*(\Theta + c_2^{*\alpha} \eta_2^{*\beta} + S_2 - l_2) - k\eta_2^{*\gamma} \right) \right\} \middle| \mathcal{G}_0^{EN} \right]. \end{aligned} \quad (135)$$

Removing irrelevant terms, the EN's effort problem is

$$\max_{\eta} \quad bc^\alpha \eta^\beta - k\eta^\gamma + (1 - b_2^*) \frac{\sigma_0^2}{s^2 + \sigma_0^2} c^\alpha \eta^\beta \quad (136)$$

and the optimal effort

$$\eta(b, c) = \left( \frac{\beta c^\alpha B_1(b)}{\gamma k} \right)^{\frac{\beta}{\gamma - \beta}}, \quad (137)$$

where

$$B_1(b) := b + (1 - b_2^*) \frac{\sigma_0^2}{s^2 + \sigma_0^2}. \quad (138)$$

By the above analysis, *given the VC's conjecture of the EN's effort*  $\hat{\eta}_1$ ,  $\eta(b, c)$  is the EN's best response. In particular, the EN's best response is *independent* of the market's

conjecture. The VC's contract offer anticipates the EN's best response and therefore the VC's conjecture about the EN's effort depends on  $b$  and  $c$  as well. In equilibrium, the VC's conjecture *equals* the EN's best response. Since, however, the EN's best response is independent of the VC's conjecture we trivially set

$$\hat{\eta}_1 = \eta(b, c). \quad (139)$$

Competition among the VC's guarantees the contract satisfies

$$-c + (1 - b)E[\Delta V_1 | \mathcal{G}_0^{VC}] - a = 0 \quad (140)$$

and therefore

$$a(b, c) = -c + (1 - b)(c^\alpha \eta(b, c)^\beta + \mu_0^{VC} - l_1). \quad (141)$$

The contract maximizes the EN's expected utility (135). After substituting  $a(b, c)$  into (135), the EN's utility equals

$$\begin{aligned} -E \left[ \exp \left\{ -\lambda \left( -c + (1 - b)(c^\alpha \eta(b, c)^\beta + \mu_0^{VC} - l_1) + b(\Theta + c^\alpha \eta(b, c)^\beta + S_1 - l_1) - k\eta(b, c)^\gamma + \right. \right. \right. \\ \left. \left. - c_2^* + (1 - b_2^*)(c_2^{*\alpha} \eta_2^{*\beta} + \frac{s^2 \mu_0^{VC} + \sigma_0^2 (\Theta + c^\alpha \eta(b, c)^\beta + S_1 - c^\alpha \hat{\eta}_1^\beta)}{s^2 + \sigma_0^2} - l_2) \right. \right. \\ \left. \left. + b_2^*(\Theta + c_2^{*\alpha} \eta_2^{*\beta} + S_2 - l_2) - k\eta_2^{*\gamma} \right\} \mid \mathcal{G}_0^{VC} \right]. \quad (142) \end{aligned}$$

Applying (139) and removing (some) irrelevant terms the optimal contractual parameters solve

$$\begin{aligned} -E \left[ \exp \left\{ -\lambda \left( -c + c^\alpha \eta(b, c)^\beta + \mu_0^{VC} + b(\Theta + S_1 - \mu_0^{VC}) - k\eta(b, c)^\gamma + \right. \right. \right. \\ \left. \left. + (1 - b_2^*) \frac{\sigma_0^2 (\Theta + S_1)}{s^2 + \sigma_0^2} + b_2^*(\Theta + S_2) \right) \right\} \mid \mathcal{G}_0^{VC} \right]. \quad (143) \end{aligned}$$

We denote the exponent term in (143) by  $\lambda Z$  and note that since  $Z$  is the sum of normal variables it is normally distributed, too. To compute the relevant expectation of the VC's maximization problem we need to consider the relevant parts of  $E[Z | \mathcal{G}_0^{VC}]$  and  $Var[Z | \mathcal{G}_0^{VC}]$ . Since the VC is maximizing the EN's expectation,  $E[\Theta | \mathcal{G}_0^{VC}] = \mu_0^{EN}$  and the relevant components of  $E[Z | \mathcal{G}_0^{VC}]$  that affect the decision variables  $b$  and  $c$  are

$$\begin{aligned} & -c + c^\alpha \eta(b, c)^\beta + \mu_0^{VC} + b(E[\Theta | \mathcal{G}_0^{VC}] - \mu_0^{VC}) - k\eta(b, c)^\gamma \\ = & \phi(B_1(b))c^{\alpha \frac{\gamma}{\gamma - \beta}} - c + \Delta_0 b, \quad (144) \end{aligned}$$

where we obtain (144) by substituting optimal effort according to (137) similarly to the analysis in the Basic Model. The relevant risky components are those that multiply current period's decisions, in this case  $\Theta$  and  $S_1$ . Accordingly, the relevant variance of  $Z$  is

$$\underbrace{\left(b + (1 - b_2^*) \frac{\sigma_0^2}{s^2 + \sigma_0^2} + b_2^*\right)^2}_{B_1(b)} \cdot \underbrace{\text{Var}[\Theta | \mathcal{G}_0^{VC}]}_{\sigma_0^2} + \underbrace{\left(b + (1 - b_2^*) \frac{\sigma_0^2}{s^2 + \sigma_0^2}\right)^2}_{B_1(b)} \cdot \underbrace{\text{Var}[S_1 | \mathcal{G}_0^{VC}]}_{s^2}$$

and so the relevant cost of risk is given by

$$\frac{\lambda}{2} \left( (B_1(b) + b_2^*)^2 (s^2 + \sigma_0^2) - 2B_1(b)b_2^*s^2 \right). \quad (145)$$

Therefore, the contract problem is

$$\max_{b,c} \phi(B_1(b))c^{\alpha \frac{\gamma}{\gamma-\beta}} - c + \Delta_0 b - \frac{\lambda}{2} \left( (B_1(b) + b_2^*)^2 (s^2 + \sigma_0^2) - 2B_1(b)b_2^*s^2 \right). \quad (146)$$

The optimal investment is  $c(B_1(b))$  where  $c(\cdot)$  is given by (131). Since  $B_1(b)$  is of the form

$$B_1(b) = b + \text{Constant},$$

we can

- replace the term  $\Delta_0 b$  in (146) with  $\Delta_0 B_1$  and
- maximize over  $B_1 := B_1(b)$  instead of  $b$ .

We therefore rewrite the optimal contract problem as

$$\max_{B_1} Kc(B_1) + \Delta_0 B_1 - \frac{\lambda}{2} \left( (B_1 + b_2^*)^2 (s^2 + \sigma_0^2) - 2B_1 b_2^* s^2 \right). \quad (147)$$

where  $K$  is a positive constant. Rearranging, and removing irrelevant terms, the optimal contract problem is

$$\max_{B_1} Kc(B_1) + \Delta_0 B_1 - \frac{\lambda}{2} B_1^2 (s^2 + \sigma_0^2) + B_1 (\Delta_0 - \lambda b_2^* \sigma_0^2). \quad (148)$$

The optimal solution satisfies  $0 < B_1^* < b_2^*$  (we prove this for the  $T$ -period model). If we assume the EN is realistically optimistic ( $\Delta_0 < p_0$ ) then  $0 < b_2^* < 1$ .<sup>2</sup> Therefore, the optimal first period contingent compensation,  $b_1^* = B_1^* - (1 - b_2^*) \frac{\sigma_0^2}{s^2 + \sigma_0^2}$  satisfies  $b_1^* < B_1^* < b_2^*$  and may be negative.

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<sup>2</sup>Recall results from the Basic Model and, in particular, Proposition 2.

### 7.3 *T-period Model*

We use backward induction to characterize the equilibrium contract. As in the two-period model, we show the decisions in each period are independent of the past. In addition, competition in the VC market ensures that, in each period, the VC's expected return equals zero. We formalize the induction assumptions at the beginning of period  $i$ . Suppose the EN's contractual terms in period  $i$  are  $a, b, c$  and the EN's effort is  $\eta$ . Let  $(a_t^*, b_t^*, c_t^*, \eta_t^*)$  denote the optimal decisions in future periods,  $t, T \geq t > i$ .

1. Future decisions are independent of the current decision. That is,  $(b_t^*, c_t^*, \eta_t^*)$  are independent of  $(b, c, \eta)$ ,  $T \geq t > i$ .
2. The VC's expected return in each future period is zero, that is,

$$a_t^* := a_t(b_t^*, c_t^*) = -c_t^* + (1 - b_t^*)E[\Delta V_t | \mathcal{G}_{t-1}^{VC}], \quad T \geq t > i. \quad (149)$$

3. The optimal effort,  $\eta_t^* := \eta(B_t^*, c_t^*)$ , and the optimal investment,  $c_t^* := c(B_t^*)$ ,  $T \geq t > i$  where  $\eta(\cdot, \cdot)$  and  $c(\cdot)$  are given by (131) and (137), respectively, and where

$$B_t^* = b_t^* + \sum_{j=t+1}^T (1 - b_j^*) \frac{\sigma_0^2}{s^2 + (j-1)\sigma_0^2}. \quad (150)$$

We now describe the optimal contractual parameters in period  $i$ ,  $1 \leq i \leq T$ . Suppose that at the beginning of date  $i$  the EN's contractual parameters are  $(a, b)$  and the VC's investment is  $c$ . For an effort level,  $\eta$ , the relevant part of the EN's expected utility is

$$-E \left[ \exp \left\{ -\lambda \left[ (a + b\Delta V_i - k\eta^\gamma) + \sum_{t=i+1}^T (a_t(b_t^*, c_t^*) + b_t^* \Delta V_t - k\eta_t^{*\gamma}) \right] \right\} \middle| \mathcal{G}_{i-1}^{EN} \right]. \quad (151)$$

The fixed portion of the EN's compensation  $a_t(b_t^*, c_t^*)$ , depends on the VC's conjectures about the EN's *prior* effort choices. Specifically, suppose that the VC's conjectures about past EN effort are  $(\hat{\eta}_1, \dots, \hat{\eta}_{t-1})$ . Then

$$E[\Theta | \mathcal{G}_{t-1}^{VC}] = \mu_{t-1}^{VC}(\hat{\eta}_1, \dots, \hat{\eta}_{t-1}) := \frac{s^2 \mu_0^{VC} + \sigma_0^2 \sum_{j=1}^{t-1} (\Delta V_j - c_j^{*\alpha} \hat{\eta}_j^{*\beta} + l_j)}{s^2 + (t-1)\sigma_0^2}. \quad (152)$$

Substituting  $a_t(b_t^*, c_t^*)$ ,  $t > i$  and  $\Delta V_t$ ,  $t \geq i$ , according to (149) and (86), respectively, and considering (152), the EN's (relevant) expected utility (151) is

$$- E \left[ \exp \left\{ - \lambda \left[ a + b(\Theta + S_i + c^\alpha \eta^\beta - l_i) - k\eta^\gamma + \sum_{t=i+1}^T \left( -c_t^* + (1-b_t^*)(c_t^{*\alpha} \eta_t^{*\gamma} - l_t + \mu_{t-1}^{VC}(\hat{\eta}_1, \dots, \hat{\eta}_{t-1})) + b_t^*(c_t^{*\alpha} \eta_t^{*\gamma} - l_t + \Theta + S_t) - k\eta_t^{*\gamma} \right) \right] \right\} \middle| \mathcal{G}_{i-1}^{EN} \right]. \quad (153)$$

Keep in mind at the beginning of period  $i$ , for  $t > i$ , (152) can be rewritten

$$\mu_{t-1}^{VC}(\hat{\eta}_1, \dots, \hat{\eta}_{t-1}) = \frac{s^2 \mu_0^{VC} + \sigma_0^2 \left( \Theta + S_i + c^\alpha \eta^\beta - c^\alpha \hat{\eta}_i^\beta + \sum_{j=1, j \neq i}^{t-1} (\Delta V_j - c_j^{*\alpha} \hat{\eta}_j^\beta + l_j) \right)}{s^2 + (t-1)\sigma_0^2}. \quad (154)$$

and the only relevant component of  $\mu_{t-1}^{VC}(\hat{\eta}_1, \dots, \hat{\eta}_{t-1})$  to the EN's effort problem is the one containing the current period's effort,  $\eta$ . We disregard irrelevant terms and rewrite the EN's effort problem, (153)

$$\max_{\eta} \quad bc^\alpha \eta^\beta - k\eta^\gamma + \sum_{t=i+1}^T (1-b_t^*) \frac{\sigma_0^2 c^\alpha \eta^\beta}{s^2 + (t-1)\sigma_0^2}. \quad (155)$$

The optimal effort at date  $i$  is therefore given by

$$\eta(B_i, c) = \left( \frac{\beta c^\alpha B_i}{\gamma k} \right)^{\frac{\beta}{\gamma-\beta}}, \quad (156)$$

where

$$B_i = b + \sum_{t=i+1}^T (1-b_t^*) \frac{\sigma_0^2}{s^2 + (t-1)\sigma_0^2}. \quad (157)$$

Notice that the EN's optimal effort choice *does not depend* on the VC's conjectures of her *prior* effort choices. The VC is aware the EN's best effort response in each period  $t \geq i$  depends on the  $t$ th period contract offer  $(a_t, b_t, c_t)$ . Therefore, his *current* and *future* conjectures  $(\hat{\eta}_i, \dots, \hat{\eta}_{T-1})$  depend on the contract offer. Since, however, by (156) and the inductive assumption, the EN's effort does not depend on the VC's conjectures, in equilibrium, the VC's anticipation of the EN's effort in the current and future periods is correct and satisfies

$$\left( \hat{\eta}_i(b, c), \hat{\eta}_{i+1}(b_{i+1}, c_{i+1}), \dots, \hat{\eta}_{T-1}(b_{T-1}, c_{T-1}) \right) = \left( \eta(b, c), \eta_{i+1}^*, \dots, \eta_{T-1}^* \right). \quad (158)$$

Due to perfect competition among VCs, (149) holds true for the current period as well.

Hence,

$$a(b, c) = -c + (1 - b)(c^\alpha \eta(b, c)^\beta + \mu_{i-1}^{VC}(\hat{\eta}_1, \dots, \hat{\eta}_{i-1}) - l_i). \quad (159)$$

The optimal contract maximizes the EN's expected utility. We substitute  $a$  according to (159) in the EN's expected utility, (153), and remove irrelevant expressions. The contractual parameters,  $(b, c)$ , therefore, maximize

$$- E \left[ \exp \left\{ -\lambda \left[ -c + c^\alpha \eta(b, c)^\beta + b(\Theta + S_i - \mu_{i-1}^{VC}(\hat{\eta}_1, \dots, \hat{\eta}_{i-1})) - k\eta(b, c)^\gamma + \sum_{t=i+1}^T \left( (1 - b_t^*) \mu_{t-1}^{VC}(\hat{\eta}_1, \dots, \hat{\eta}_{t-1}) + b_t^*(\Theta + S_t) \right) \right] \right\} \middle| \mathcal{G}_{i-1}^{VC} \right]. \quad (160)$$

Substituting  $\eta(b, c)$  and  $\mu_{t-1}^{VC}(\hat{\eta}_1, \dots, \hat{\eta}_{t-1})$  according to (156) and (152), respectively, and considering the effort equilibrium, (158), the parameters  $(b, c)$  to maximize (again, removing irrelevant expressions)

$$- E \left[ \exp \left\{ -\lambda \left[ \phi(B_i) c^{\alpha \frac{\gamma}{\gamma-\beta}} - c + b(\Theta + S_i - \mu_{i-1}^{VC}(\hat{\eta}_1, \dots, \hat{\eta}_{i-1})) + \sum_{t=i+1}^T \left( (1 - b_t^*) \frac{\sigma_0^2 \left( \Theta + S_i + \sum_{j=i+1}^{t-1} (\Theta + S_j) \right)}{s^2 + (t-1)\sigma_0^2} + b_t^*(\Theta + S_t) \right) \right] \right\} \middle| \mathcal{G}_{i-1}^{VC} \right]. \quad (161)$$

In what follows, it is more convenient to express the optimal contract problem in terms of  $B_i$  rather than directly  $b$ . To compute the expectation of (161) we need to find the relevant mean and the relevant variance. Since the VC is maximizing the EN's utility according to the EN's beliefs,

$$E[\Theta | \mathcal{G}_{i-1}^{VC}] = \mu_{i-1}^{VC}(\hat{\eta}_1, \dots, \hat{\eta}_{i-1}) := \frac{s^2 \mu_0^{VC} + \sigma_0^2 \sum_{j=1}^{i-1} (\Delta V_j - c_j^* \alpha \hat{\eta}_j^{*\beta} + l_j)}{s^2 + (i-1)\sigma_0^2}.$$

Therefore, by (152),

$$E \left[ \Theta - \mu_{i-1}^{VC}(\hat{\eta}_1, \dots, \hat{\eta}_{i-1}) \middle| \mathcal{G}_{i-1}^{VC} \right] = \Delta_{i-1}. \quad (162)$$

The relevant risky components of (161) are the risky components that are in expressions containing  $b$  or  $c$ . Therefore, the only relevant risky components are  $\Theta$  and  $S_i$  (but not  $S_j$

for  $j > i$ ). The relevant risk is therefore

$$\Theta \left[ b + \sum_{t=i+1}^T \left( \frac{(1-b_t^*)(t-i)\sigma_0^2}{s^2+(t-1)\sigma_0^2} + b_t^* \right) \right] + S_i \left[ b + \sum_{t=i+1}^T \frac{(1-b_t^*)\sigma_0^2}{s^2+(t-1)\sigma_0^2} \right] \quad (163)$$

$$= \Theta \left( B_i + \sum_{t=i+1}^T B_t^* \right) + S_i B_i, \quad (164)$$

where  $B_t^*$  is given by (150) for the optimal level at date  $t$ . We show the derivation of (164) in the proofs. The corresponding variance is

$$\sigma_{i-1}^2 \left( B_i + \sum_{t=i+1}^T B_t^* \right)^2 + s^2 B_i^2, \quad (165)$$

equal to (eliminating irrelevant expressions)

$$(s^2 + \sigma_{i-1}^2) B_i^2 + 2\sigma_{i-1}^2 B_i \sum_{t=i+1}^T B_t^*. \quad (166)$$

Evaluating the expectation of (161), (that is, considering (166) and (162)), the optimal contract problem is

$$\max_{B_i, c} \phi(B_i) c^{\alpha \frac{\gamma}{\gamma-\beta}} - c + b \Delta_{i-1} - \frac{\lambda}{2} \left( (s^2 + \sigma_{i-1}^2) B_i^2 + 2\sigma_{i-1}^2 B_i \sum_{t=i+1}^T B_t^* \right). \quad (167)$$

The optimal investment,  $c(B_i)$ , is given by (131). Since  $B_i = b + \text{Constant}$  we can replace  $b \Delta_{i-1}$  with  $B_i \Delta_{i-1}$  in (167). Substituting the optimal investment the optimal contract problem is

$$\max_{B_i} H_i(B_i) := K c(B_i) - \frac{\lambda}{2} (s^2 + \sigma_{i-1}^2) B_i^2 + B_i \left( \Delta_{i-1} - \lambda \sigma_{i-1}^2 B_i \sum_{t=i+1}^T B_t^* \right). \quad (168)$$

We denote the optimal solution by  $B_i^*$ .

Recall,  $b_M \in (1, \frac{\gamma}{\beta})$  is the point where  $c''(b) = 0$  (Proposition 1). The following proposition ensures the solution to the EN's maximization problem exists and is unique.

**Proposition 13**

*Under Assumptions 1-3, each  $H_i(\cdot)$  is strictly concave and hence strongly unimodal on  $[0, b_M]$ , and the solution to (168),  $B_i^*$ , is positive and less than  $b_M$ .*

The proof of Proposition 13 is identical to Proposition 3, hence omitted. Since the current period's maximization problem and the other decision variables in period  $i$  satisfy

the induction assumptions we can extend the above arguments by backward induction to *any* period  $i$  and thereby derive the unique equilibrium, as characterized in the following theorem.

**Theorem 10 (Characterization of Equilibrium)**

Under Assumptions 1 - 3 the equilibrium contract offered by the VC and the EN's effort in period  $i$ ,  $1 \leq i \leq T$  is characterized, as follows:

- The risky compensation is  $b_i^* = B_i^* - \sum_{t=i+1}^T (1 - b_t^*) \frac{\sigma_0^2}{s^2 + (t-1)\sigma_0^2}$ ;
- The investment is  $c_i^* := c(B_i^*)$  defined in (131);
- The fixed compensation is  $a_i^* := a(b_i^*, c_i^*)$  defined in (159);
- The optimal effort level is  $\eta_i^* := \eta(B_i^*, c_i^*)$  defined in (156).

## 7.4 Analysis and Discussion

We first characterize the trajectory of the  $B_i$ 's.

**Proposition 14 ( $B_i^*$  Trajectory Region)**

Suppose the EN is reasonably optimistic ( $\Delta_0 < p_0$ )

- (i) The trajectory of  $B_i^*$  increasing.
- (ii)  $B_i^* \leq 1$  for all  $1 \leq i \leq T$ ;

Proposition 14 is significant since the optimal investment and effort behave according to  $B_i^*$ . When effort was observable, the EN's effort level was tied directly to his current period's contingent compensation. Now, however, effort depends on future contingent compensation as well.

We now turn to examine how optimal effort and investment levels compare to the Basic Model and the Shift of Power Model.

**Theorem 11**

Suppose the EN is realistically optimistic ( $\Delta_0 < p_0$ ). Prior to the last period, the effort level  $\eta_i^*$ , and the investment level,  $c_i^*$ , in each period are less than the effort and investment

levels in the Basic Model but are more than the effort and investment levels in the Shift of Power Model, respectively. In the last period investment and effort levels are identical for the three models.

Investment is monotonically increasing in the EN's contingent incentives (whether they are  $b_i^*$  as in the previous models or  $B_i^*$  as in our current model) as long as the EN is reasonably optimistic. Consequently, and assuming the EN is reasonable, increasing the contingent incentives to the EN increases the amount of investment. The EN's effort, which is induced by both the investment incentives and the contingent incentives, will behave similarly and increases with the contingent incentives.

## 7.5 Proofs

**Proof of Proposition 14:** The derivative of the EN's maximization problem, (168), is

$$H'_i(B) := \Delta_{i-1} - \lambda \sigma_{i-1}^2 \sum_{t=i+1}^T B_t^* - p_{i-1} B + K c'(B). \quad (169)$$

Let  $B_i^*$  be the optimal solution at period  $i$ . Therefore,  $H'_i(B_i^*) = 0$ . Notice that  $H'_{i-1}(B) = H'_i(B) - \lambda \sigma_{i-1} B_i^*$ . Since by Proposition 13  $B_i^* > 0$  we have  $H'_{i-1}(B_i^*) < 0$  and by unimodality  $B_{i-1}^* < B_i^*$ , which completes the proof to part (i). For period  $T$ ,  $H_T(\cdot) \equiv F_1(\cdot)$  and applying Proposition 2 we have  $B_T^* < 1$ . The rest of the proof now follows from part (i). ■

**Proof of Theorem 11:** Let  $b_i^X$  denote the contingent compensation at period  $i$  when  $X \in \{VC, EN\}$  has bargaining power (observable effort). We start by showing that for all  $i < T$ ,  $b_i^{EN} < B_i^* < b_i^{VC}$ . We note that:

$$H'_i(b) = F'_i(b) - \lambda \sigma_{i-1}^2 \sum_{t=i+1}^T B_t^* \quad (170)$$

$$H'_i(b) = G'_i(b) + \lambda \sigma_{i-1}^2 \sum_{t=i+1}^T (1 - B_t^*) \quad (171)$$

By definition,  $F'_i(b_i^{VC}) = 0$  and  $G'_i(b_i^{EN}) = 0$ . Since by Propositions 13 and 14  $0 < B_i^* < 1$ , we have  $H'_i(b_i^{VC}) < 0$  and  $H'_i(b_i^{EN}) > 0$  and by unimodality  $b_i^{EN} < B_i^* < b_i^{VC}$ . Since the EN is realistically optimistic, by Proposition 2  $b_i^{VC} < 1$ . Since investment and effort are identical functions for the three models and are increasing (since  $b < 1$ ) we have the

investment and effort ordered in an identical manner. The second statement immediately follows from the fact  $H_T(\cdot) = G_T(\cdot) = F_T(\cdot)$ . ■

**Derivation of Equation 164:** Let

$$x_t := (1 - b_t^*) \frac{\sigma_0^2}{s^2 + (t-1)\sigma_0^2}$$

and notice that

$$B_i^* = b_i^* + \sum_{t=i+1}^T x_t. \quad (172)$$

In addition, we note that

$$\begin{aligned} & \sum_{t=i+1}^T \sum_{k=t}^T x_k = \\ = & x_{i+1} + 2x_{i+2} + 3x_{i+3} + \dots + (T-i)x_T = \\ = & \sum_{t=i+1}^T (t-i)x_t \end{aligned} \quad (173)$$

By (157), the right hand side term of (164),  $S_i B_i$ , clearly equals

$$S_i \left[ b + \sum_{t=i+1}^T \frac{(1 - b_t^*)\sigma_0^2}{s^2 + (t-1)\sigma_0^2} \right],$$

the right hand side term of (163). It is therefore left to show the left hand side terms of (163) and (164) are equal. But,

$$\begin{aligned} & B_i + \sum_{t=i+1}^T B_t^* \\ = & b + \sum_{t=i+1}^T x_t + \sum_{t=i+1}^T \left( b_t^* + \sum_{k=t+1}^T x_k \right) \\ = & b + \sum_{t=i+1}^T b_t^* + \sum_{t=i+1}^T x_t + \sum_{t=i+1}^T \left( \sum_{k=t+1}^T x_k \right) \\ = & b + \sum_{t=i+1}^T b_t^* + \sum_{t=i+1}^T \left( x_t + \sum_{k=t+1}^T x_k \right) \\ = & b + \sum_{t=i+1}^T b_t^* + \sum_{t=i+1}^T \left( \sum_{k=t}^T x_k \right) \end{aligned} \quad (174)$$

and by (173) this equals to

$$\begin{aligned}
 & b + \sum_{t=i+1}^T b_t^* + \sum_{t=i+1}^T (t-i)x_t \tag{175} \\
 = & b + \sum_{t=i+1}^T \left( (t-i)x_t + b_t^* \right) \\
 = & b + \sum_{t=i+1}^T \left( \frac{(1-b_t^*)(t-i)\sigma_0^2}{s^2 + (t-1)\sigma_0^2} + b_t^* \right) . \blacksquare
 \end{aligned}$$

## CHAPTER VIII

### CONCLUSIONS

In this thesis we presented a model that incorporates key features of the venture financing process. Our model features staged investment, allows for contingent and risk-free compensation to the EN and considers costs of VC oversight. The VC's capital inflow commitment must be coupled with human capital investment by the EN. Another central feature of our model is the learning about the project's quality. We tested our model under different conflict scenarios between the VC and the EN including asymmetry in attitude towards risk and asymmetric beliefs about the project. In addition, we tested our model for different bargaining power assumptions and informational asymmetries.

Our research incorporates and explains empirical evidence about the VC-EN relationship. More importantly, we make predictions about the true meaning of empirical evidence. For example, Gibbons and Murphy (1992) find that pay-performance sensitivity of CEO's increase over time. We find similar findings. We predict, however, that this phenomena will happen only when the asymmetric beliefs between the VC and the EN are not too large. We also predict that it is more likely to happen when the EN has bargaining power.

We endogenously derive a milestone financing policy and find that the milestones are increasing with risk-averseness and decreasing with the asymmetry in beliefs. We also predict that the VC will benefit from an EN's optimism, which he can exploit.

Using a two-period version of our Basic Model, we conduct a full risk analysis and demonstrate the distinct effects of technical risk and market risk. Calibrating our model to empirical data enables us to conduct a more meaningful numerical analysis. The most significant conclusion from this analysis is that the two-period results are extended to the  $T$ -period model and that systematic risk and technical risk have opposite effects on the firm. Systematic risk is associated with decreasing payoffs to the VC, less firm value and shorter project duration. Conversely, technical risk is associated with future promise and

increases the VC payoff, firm value and project duration.

We predict that with a constant-return-to-scale production technology, increasing the labor output elasticity increases the firm's value, duration and appeal to the VC.

We examine the effects of asymmetric information by assuming the VC cannot observe the EN's effort. In this setting, we show that the EN receives more incentives to exert effort. More capital and human investment takes place.

There are a number of possible extensions to our research.

- VC oversight is a central feature in VC finance. There is empirical evidence about the positive effects of the VC's advice to the EN. In addition, researchers have shown that this advice requires considerable resources from the VC. In the current model we assume the VC's oversight is exogenous. An alternative model could consider a double-sided moral hazard formulation that allows the VC's monitoring costs and oversight to be determined endogenously. This approach may provide insight to explain how VC oversight behaves and address questions such as: When do we expect to see more VC oversight and when less? Will asymmetric beliefs allow VC to invest less resources in oversight? How does the EN's degree risk aversion affect oversight?
- Another way of introducing richness to the moral hazard problem in our model is by relaxing the assumption that the VC is a single entity. A more elaborate description of the VC will distinguish between the investors in the venture fund and the VC fund managers who interact directly with the EN. Incorporating this feature into our model will enable an improved understanding of the full scope of the VC finance process. Why do venture funds have a limited investing horizon? Why are VC funds structured as limited partnerships? How should the fund investors optimally compensate the VC fund manager? Do the answers to these questions depend on the VC-EN relationship?
- Our investigation of the effects of the bargaining power is limited to examining two extreme cases in which either the VC or the EN possesses *all* bargaining power. Under this assumption, we found that the deterministic path trajectories are lower when the EN has bargaining power. However, we do not know how the deterministic paths

behave when bargaining power is divided somewhere between the EN and the VC. Future research may incorporate a bargaining model that allows bargaining power to be shared between the parties. This will enable a more meaningful analysis of the effects of bargaining power.

- The stopping time in the Shift of Power and the Non Observable Effort models is determined upfront. Further research could allow one to relax this assumption and allow a dynamic investment policy with a random stopping time as in the Basic Model. This will enable one to examine how bargaining power and effort observability affect project duration.
- Our model may be generalized to a strategic management decision analysis model. As the firm's manager, the EN must allocate available resources between marketing and product improvement. The EN's allocation considerations may be influenced by project risks and by his ability to signal potential VC's about the project's potential.
- The contracts we assume in our model allow for per-performance sensitivities and risk free payments to the EN. By introducing explicit compensation securities it could be possible to better address specific empirical evidence about the EN's compensation schemes.
- In our model, we limit asymmetry in beliefs to  $\mu_0$ , the assessment of project quality. Introducing asymmetry in belief about the risk of the project will enable a further understanding of both the effects of asymmetric beliefs and the effects of risk.
- There is no quantitative empirical evidence about asymmetric beliefs and EN risk aversion. Most theoretical models assume the EN is risk averse. In contrast, anecdotal empirical evidence has led other researchers to consider EN's as risk-takers (Cave and Minty (2004)). Our model predicts a tradeoff between the effects of risk aversion and asymmetric beliefs that explains this contradiction. Many researchers conjecture (Sahlman (1990), Gladstone and Gladstone (2002) among others) the EN is very optimistic. Empirical research based on our model could attempt to quantify and

differentiate between these two features of the VC-EN relationship.

In closing, we believe our research contributes to the understanding of the multi-faceted phenomena of the VC-EN relationship.

# APPENDIX A

## CODE DESIGN

### *A.1 Module Overview*

The Matlab code contains a number of files, which we group into 6 modules, described below. In addition to these modules, there is a file containing all the parameters that control the experimentation or calibration processes. In this way, to conduct an experiment or run a calibration process one needs to change only the parameters gathered in this file.

- Module 1: Main Program. This module manages the numerical experiments. It produces an experiment scenario (i.e. assigned values to the parameters) and activates the other modules of the program.
- Module 2: Deterministic Path. This module produces the deterministic path results of the model.
- Module 3: Dynamic Evaluation. This module computes the continuation value for each state of the lattice and the trigger termination policy and the economic statistics.
- Module 4: Lattice Construction. This module generates the lattice representing the evolution of the state variable  $\mu_i^{VC}$ .
- Module 5: Result Presentation. Displays the deterministic and simulation results.
- Module 6: Calibration. This module manages the calibration of the parameters. It creates the experiment scenarios to be tested in the calibration process and activates the other modules of the program.

In the next section we describe the parameters of the code, given in the file. In Section A.3 we described the rest of the files of the code.

**Table 9:** Model parameters

alpha	$\alpha$ the capital elasticity
gamma	$\gamma$
gambet	$\frac{\gamma}{\beta}$
k_effort	$k$ , effort parameter
lambda	$\lambda$ , risk aversion
s_risk	$s^2$ , market risk
sigma_0	$\sigma_0$ , initial technical risk
Delta_0	$\Delta_0$ , Initial asymmetry
V_0	$V_0$ , Initial termination value
mu_o	$\mu_0^{VC}$ , initial VC assessment of project quality
loss1	parameter $L$ from loss formula $l_i = Li^x$
loss2	$x$ from loss formula

## A.2 Code Parameters - *Parameters.m*

In the file *Parameters.m*, we conveniently gather all the parameters that control the output of the code. The user of the code need only access these parameters when wanting to conduct an experiment or multiple experiments involving a change in one of the model's parameters or a calibration process. All variables on this file are global and unless stated otherwise, all variables are a single-cell. We group the parameters into four groups, model parameters, programming parameters, multiple experiment parameters and calibration parameters.

### A.2.1 Model Parameters:

The model parameters are described in Table 9. We point out that  $\beta$  does not appear because to solve the model we do not need both  $\beta$  and  $\gamma$  but rather their ratio,  $\frac{\gamma}{\beta}$ .

### Programming Parameters:

The programming parameters include all the parameters that control the lattice structure, the monte carlo simulation and the display of the results. These parameters are summarized in 10.

**Table 10:** Programming parameters

T	$T$ , Maximum number of periods
num_states	The number of nodes in date 1 (end of first period)
states_inc	Number of nodes added every date after date 1
std_inc	Number of standard deviations added to the extreme lattice values in each period
sim_num	Number of simulation runs to be executed in the Monte Carlo simulation
display_zero	Indicating whether ZERO DISPLAY or ALL DATES DISPLAY or both.
display_type	Indicating which of the lattice dynamic values be displayed.
val_num	Number of dynamic values stored in each node.
dynamic_names	Contains the labels of the lattice dynamic values.

There are two formats of the lattice display: ZERO DISPLAY and ALL DATES DISPLAY. ZERO DISPLAY will contain ALL lattice dynamic values but only for date zero, whereas ALL DATES DISPLAY displays results for all dates but the results can be limited through the parameters *display\_type*.

*display\_zero*: If equal 0 then only ZERO DISPLAY will be displayed. If equal 1 then both ZERO DISPLAY and ALL DATES DISPLAY. If equal 2 then only ALL DATES DISPLAY is displayed.

Keep in mind that date ZERO DISPLAY is a complete lattice display and will contain all dynamic values regardless of the values of *display\_type*. These parameters only affect ALL DATES DISPLAY in the following manner:

*dynamic\_names* and *display\_type* are arrays of the size  $[1 \times val\_num]$ . Each element in *display\_type* is either 1 or 0. An entry of 1 in the  $i$ th location indicates that the  $i$ th dynamic value is to be displayed in the lattice display. The dynamic values are given according to the following order:

1. State value ( $\mu_i$ );
2. Continuation value.

**Table 11:** Multiple experiment parameters

mult_flag	Indicating whether multi- or single-experiment scenario.
mult_param	Indicating the variable parameter - see below.
mult_values	An array consisting of the values of the var parameter.

**Table 12:** Calibrations parameters

calibration	Indicating whether calibration process or not.
num_values	The number of values we are searching on.
cal_limits	The initial feasible region for the calibrated parameters.
output_val	The empirical evidence data for the calibration.

**Remark:** To add dynamic values capabilities the following actions are needed: 1. Increase parameter *val\_num*. 2. Add an entry to the parameter *display\_type* 3. Update the functions *Values* to support the computation of the new dynamic value.

**Multiple Experiment Parameters:**

The parameters controlling the multiple experiment scenario are provided in Table 11

If *mult\_flag* =1 then the program executes a multi-experiment scenario, whereas zero indicates a single-experiment. In a multi-experiment scenario we allow one of the model parameters to change. The variable parameter is given by *mult\_param* according to the following rule: 1 = alpha, 2=gambet, 3=k\_effort, 4=lambda, 5=s\_risk, 6=sigma\_0, 7=Delta\_0, 8=V\_0, 9 = mu\_0, 10=loss1, 11=loss2, 12=d\_disc. The values the variable parameter receives are stored in *mult\_values*. The size of *mult\_values* depends on the number of experiments desired.

**Calibrations Parameters:**

The parameters of the calibration process are provided in Table 11

If *calibration* =1 then the program is running the calibration process instead of the regular single or multi experiment. This parameter affects the type of display. For example, in a calibration scenario we do not display the deterministic path, the lattice display nor

the trigger policy.

In the initial step of the calibration process the calibration grid is a five dimensional grid (each dimension representing one of the calibrated parameters -  $\alpha, \gamma, k, L$  and  $\lambda$ ). The number of points on each dimension is *num\_values* and these points are equally spaced between the lower limit and the upper limit of the initial feasible region for the dimension's parameter. The initial feasible regions for the calibrated parameters is stored in *cal\_limits* in the following order: alpha,gamma,k\_effort,loss2 and lambda. The size of *cal\_limits* is [2 X 5], where the first column is the lower limit and the second column is the upper limit.

$\overline{EE} = EE_1, \dots, EE_7$ ), the empirical evidence used in the calibration process, defined in Section 5.3.2.1, is stored in *output\_val*. The size of *output\_val* is [1 X 7]

### ***A.3 Module Design***

Unless stated otherwise, all data structures in each subroutine/function are private to the subroutine/function itself.

#### **A.3.1 Module 1 - Main Program**

##### **File List:**

1. *Program.m*
2. *UpdatePar.m* contains the function *UpdatePar*
3. *ExecProg.m* contains the function *ExecProg*

##### **Detailed Description:**

- *Program.m*: We allow for either a single-experiment execution or a multiple-experiment execution of the program. *Program* reads the parameters' values via *Parameters*, and then either executes a multi-experiment program or a single-experiment program. In a multi experiment scenario we allow (exactly) one of the model parameters to receive different values. The function *UpdatePar* sets the new value of the variable parameter. The number of experiments is equal to the size of *mult\_values*, which contains the different values the variable parameter receives.

- ***UpdatePar(i)***: In the multi-experiment setting, the variable parameter's value must be updated for each experiment. *UpdatePar.m* handles this. The function's parameter,  $i$ , is the experiment number.
- ***ExecProg(experiment)***: For experiment number  $experiment$ , *ExecProg* computes and displays the deterministic path and the dynamic values. Note that in a single-experiment setting  $experiment = 1$ .

### A.3.2 Module 2 - Deterministic Path

File name *DetPath.m* contains the following functions:

- ***DetPath***: The Main function in the file. This function takes no arguments and returns seven arrays that contain the deterministic path results. The arrays are *optb*, *optc*, *opteta*, *optf*, *loss*, *Delta* and *sigma* and they correspond to  $b_i^*$ ,  $c_i^*$ ,  $\eta_i^*$ ,  $F_i^*$ ,  $l_i$ ,  $\Delta_{i-1}$  and  $\sigma_{i-1}$ , respectively. These arrays are global and each is of the size  $[1 \times T]$ , where recall,  $T$  is the number of periods.

The following utility functions are used to generate the deterministic path results.

- ***BisectionSearch(sigmai,Deltai)***: Computes  $b_i^*$ . The arguments *sigmai* and *Deltai* are  $\sigma_{i-1}^2$  and  $\Delta_{i-1}$ , respectively.
- ***VCproblemDeriv(b,sigmai,Deltai)***: Computes  $F_i'(b)$ . The arguments *sigmai* and *Deltai* are  $\sigma_{i-1}^2$  and  $\Delta_{i-1}$ , respectively.
- ***VCproblemFunc(b,c,i,sigmai,Deltai)***: Computes  $F(b)$ . The arguments *sigmai* and *Deltai* are  $\sigma_{i-1}^2$  and  $\Delta_{i-1}$ , respectively. The argument  $c$  denotes  $c_i^*$
- ***LossFunc(i)***: Computes  $l_i$ .
- ***Investment(b)***: Computes  $c(b)$ .
- ***DerivInvestment(b)***: Computes  $c'(b)$ .
- ***Effort(b,c)***: Computes  $\eta(b,c)$

### A.3.3 Module 3 - Dynamic Evaluation

#### File List:

1. *DynamicVal.m* contains the function *DynamicVal*
2. *Values.m* contains the function *Values*
3. *MuStar.m* contains the function *MuStar*
4. *TestRes.m* contains the function *TestRes*

#### Detailed Description:

- *DynamicVal(i)*: It runs from the last period backwards until date  $i$  and computes the values associated with each state. It returns a matrix of size  $[val\_num*(T-i) \times CompSize(T-1)]$  where  $CompSize(T-1)$  is the number of states at the last date (see below in Module 4). Each  $val\_num$  rows correspond to a period (The first  $val\_num$  rows to period  $i+1$ , the next  $val\_num$  rows to period  $i+2$ , ..., the last  $val\_num$  rows to period  $T$ ). Each  $val\_num$  rows are ordered according to the order of the lattice dynamic values. In the rows corresponding to period  $i$ , the number of valid columns is  $CompSize(i)$ .
- *Values(c\_date,c\_list,n\_list,n\_value)*: Produces dynamic value arrays for all the states of date  $c\_date$ . To that end, it calls the *Descend* function (see Module 4 below), which computes the descendants and the probabilities of getting to the descendants. The other arguments of *Values* are:  $c\_list$ — the list of nodes in date  $c\_date$ .  $n\_list$ ,  $n\_value$ — the list of nodes in date  $c\_date+1$  and their dynamic values. Notice the size of  $n\_value$  is  $[val\_num \times \text{length of } n\_list]$ .

The function returns the value arrays packaged in a single matrix *answer* whose size is  $[val\_num \times \text{length of } n\_list]$ .

The computation of the values is as follows: for each state in  $c\_list$  we call *Descend* to receive its descendants and the probability to reach those descendants. The evaluation

of each dynamic value now follows from its recursive definition, which is described below.

Each row in *answer* stores a different value according to the order of the parameter *display\_type*.

Computation of the lattice dynamic values:

- The project quality ( $\mu^{VC}$ ) is given by the state's value (part of the Module 4).
- Continuation value at date  $i$  (CV):

$$CV = \begin{cases} \max(\mu_i^{VC} + F_i^* - l_{i+1}, 0), & \text{if } i = T - 1; \\ \max\left(0, \mu_i^{VC} + F_i^* - l_{i+1} + d \sum_{x \in desc} p_x CV_x\right), & \text{if } i \leq T - 2. \end{cases} \quad (176)$$

- ***MuStar(StVal)***: This function computes and returns the trigger policy. Its argument, *StVal*, contains the lattice dynamic values described above. The function returns an array size  $[1 \times T]$  containing  $\mu_1^*, \dots, \mu_T^*$ .
- ***TestRes(mu\_star)***: This function receives the trigger policy and runs a Monte Carlo simulation of the model. It computes the following economical results:  $Res_1, \dots, Res_7$ , expected VC share (i.e. the continuation value), expected net firm value and the expected total investment as defined in Section A.3.

#### A.3.4 Module 4 - Lattice Construction

##### File List:

1. ***States.m*** contains the function *States*.
2. ***Descend.m*** contains the functions *Descend* and *Probability*.
3. ***CompSize.m*** contains the function *CompSize*.

##### Detailed Description:

The design of the lattice is as follows. In date zero there is a single node whose state value is *mu\_0*. The number of states in the next date is *num\_states*, and thereafter in

each date *states\_inc* nodes are added. The value of a state at date *i* describes  $\mu_i$ . We allow for the state values to increase from date *i* to date *i* + 1 in the following manner. The highest value date will increase by *std\_inc* times  $\frac{\sqrt{s^2+\sigma_i^2}}{\sigma_i}$ . Similarly, the lowest value date will decrease by *std\_inc* times  $\frac{\sqrt{s^2+\sigma_i^2}}{\sigma_i}$ . (See derivations below in the discussion of the function *Probability*.) The other states' values will be equally spread between the two extreme states.

General Comment: All the state lists are given in arrays whose size is [1 X *CompSize(T-1)*] where *CompSize(T-1)* is the number of states at the last date (see below). This is to ensure that all lists are of the same length. However, the number of relevant cells in each array varies and depends on the number of states in the date.

- ***States(i)***: This function returns all the states of date *i*. An array containing all the possible values of  $\mu_i$  is returned to the calling command. The relevant cells in the returned array lie between the first cell and the *CompSize(i)*th cell.
- ***Descend(cdate, mu, NStates)***: A function that searches for the descendants of state *mu* from date *cdate*, where candidate descendants are given in *NStates*. *NStates* is the list of states in date following the date of *mu*, and *next\_size* is the *cdate*+1. The function returns pointers to the first and least descendant cells, *startpos* and *endpos*, respectively. (recall, the lattice structure is such that the state values are sorted increasing.) To be a descendant of *mu* a candidate state must be within *std\_inc* times *stdev* from *mu*.

In addition, the function returns *StateProb*, an array containing the probabilities to reach the descendant. Similarly to the state lists, the probability list is also of size [1 X *CompSize(T-1)*]. The relevant data of this array lies between cells *startpos* and *endpos*.

- ***Probability(mu,b\_range, t\_range)***: Computes the probability of going from state

$mu$ ) to the range  $[b\_range, t\_range]$ . Following the following formula

$$Prob[\mu_{i+1} < x] = \tag{177}$$

$$= Prob\left[\frac{s^2\mu_i + \sigma_i^2 Y_{i+1}}{s^2 + \sigma_i^2} < x\right] \tag{178}$$

$$= Prob\left[\mu_i + \frac{\sigma_i^2}{\sqrt{s^2 + \sigma_i^2}} Z < x\right] \tag{179}$$

$$= Prob\left[Z < (x - \mu_i) \frac{\sqrt{s^2 + \sigma_i^2}}{\sigma_i^2}\right] \tag{180}$$

where the third line is since  $Y_{i+1} \sim N(\mu_i, s^2 + \sigma_i^2)$ . Replacing  $x$  with  $b\_range$  and  $t\_range$  and taking the difference between the probabilities (top minus bottom) gives us the required probability.

- **Compsize(i)** Computes the number of nodes in date  $i$ .

### A.3.5 Module 5 - Result Presentation

File name **DispRes.m** contains the functions *DispRes*, *OpenDisplay*, *OpenDisCal*, *DisplayDetPath*, *DisplayZero*, *DisplayAll*, *DisplayCalExp*, *DispMuRes*, *DispCalBase* and *CloseDisplay*.

We describe the main function of this module, *DispRes*. The rest of the functions receive data from *DispRes* and display it.

**DisplayResults(phase,data,experiment)**: The argument *phase*'s value is from (1, 2, 3, 4, 5) with each value requiring the following actions:

1. Initializing the file. In this case *data* and *experiment* are disregarded.
2. Display the deterministic path. In this case *data* is the deterministic path matrix whereas *experiment* is disregarded. The deterministic path is displayed only in a single experiment scenario.
3. Display the lattice dynamic values. If single experiment scenario then displays either ZERO DISPLAY or ALL DATES DISPLAY or both (depending on the value of the parameter *display\_zero*). In multi experiment scenario only ZERO DISPLAY is displayed. In the calibration process there is no lattice dynamics value display.

4. Displays the trigger policy and the economic results of the model. If not a calibration scenario then displays all the economical results. If a calibration scenario then displays only the first seven results.
5. Closing the file. In this case *data* and *experiment* are disregarded.

### A.3.6 Module 6 - Calibration

#### File List:

1. *calibrate.m*
2. *CBaseNum.m* contains the functions *CBaseNum* and *CompDistance*.
3. *CCalVal3.m* contains the function *CCalVal3*.
4. *CFineTune.m* contains the function *CFineTune*.

#### Detailed Description:

- *calibrate.m*: This is the main procedure of the calibration process. Recall, in the initial step of the calibration process the calibration grid is a five dimensional grid with each dimension representing one of the calibrated parameters -  $\alpha, \gamma, k, L$  and  $\lambda$ ). The value of points on each dimension is stored in *cal\_val* (an array of size [5 X *num\_values*]). We also store the distance between the points (recall they are equally spaced so each dimension has one such distance to store) for each dimension/parameter in *step\_val* (an array of size [5 X 1]). We now call the function *CBaseNum* to receive the initial base numbers. In the next step we run a number of iteration to fine tune the grid search. The result of each iteration is another set of basic numbers. The basic numbers of the last iteration are chosen as the basic numbers of the calibrated parameters.
- *CBaseNum(num\_values, cal\_val)*: This function receives the number of points in each dimension of the grid (*num\_values*) and their values (*cal\_val*). For each permutation of these values it runs an experiment (computing the deterministic path, the

lattice dynamic values and the monte carlo simulation) using the other modules of the code. Given the experiment results, it computes the distance from the empirical evidence and returns the parameters' values of the experiment with the minimal distance. These values are the basic numbers.

- ***CompDistance(ExpResults)***: This function receives the current experiment's results and returns their distance from the empirical evidence.
- ***CFineTune(param\_val,cstep\_val,index)***: This function computes the new basic numbers in the second step of the calibration process. In this step we are fine tuning the grid iteratively. Given the current base numbers *param\_val* and the current distance between points on the grid *cstep\_val*, it invokes *CCalVal3* to receive the new grid parameters. Next it computes the new basic numbers from *CBaseNum*. The third argument of *CFineTune*, *index* denotes which iteration of basic numbers is now handled and is used for purpose of display. *CFineTune* returns the basic numbers (size [1 X 5]) and the new distance between points on the grid (size [5 X 1]).
- ***CCalval3(param\_val,cstep\_val)***: This function receives the current base numbers *param\_val* and the current distance between points on the grid *cstep\_val*. It returns the parameters for a finer grid search in the following manner. Each dimension of the new grid contains three points that are centered around the current base value of the parameter represented by that dimension. If the current base number is equal to the upper (lower) limit of the feasible region of that parameter then the three points are the current base number and below (above) it. The new distance between points is half (one third in the equal to limit cases) of the current distance between points. The function *CCalVal3* returns the new values of each dimension (size [5 X 3]) and the new distance between points (size [5 X 1]).

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