Topics in Fractional Airlines

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Yufeng Yao

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Topics in Fractional Airlines

Approved by:

Dr. Ellis Johnson, Advisor
Industrial and Systems Engineering
Georgia Institute of Technology

Dr. Ozlem Ergun
Industrial and Systems Engineering
Georgia Institute of Technology

Dr. Pinar Keskinocak
Industrial and Systems Engineering
Georgia Institute of Technology

Dr. Beril Toktay
Management
Georgia Institute of Technology

Dr. Irina Ioachim
Delta Airlines
Georgia Institute of Technology

Date Approved: April 9, 2007

To my husband, Wei Zhao

To my parents, Fuxin Yao and Xiufen Xu

To my lovely daughters, Danielle Zhao and Annie Zhao

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SUMMARY

Fractional aircraft ownership programs offer companies and individuals all the benefits of owning private jet, such as safety, consistency, and guaranteed availability, at a fraction of the cost of owning an aircraft. In the fractional ownership model, the partial owners of an aircraft are entitled to certain number of hours per year, and the management company is responsible for all the operational considerations and making sure an aircraft is available to the owners at the requested time and location.

This thesis research proposes advance optimization techniques to help the management company to optimally operate its available resources and provides tools for strategic decision making. The contributions of this thesis are:

(i) The development of optimization methodologies to assign and schedule aircraft and crews so that all flight requests are covered at the lowest possible cost. First, a simple model is developed to solve the crew pairing and aircraft routing problem with column generation assuming that a crew stays with one specific aircraft during its duty period. Secondly, this assumption is partially relaxed to improve resource utilization by revising the simple model to allow a crew to use another aircraft when its original aircraft goes under long maintenance. Thirdly, a new comprehensive model utilizing Bender's decomposition technique and a fleet-station time line is proposed to completely relax the assumption that crew stays with one specific aircraft. It combines the fleet assignment, aircraft routing, and crew pairing problems. In the proposed methodologies, real world details are taken into consideration, such as crew transportation and overtime costs, scheduled and unscheduled maintenance effects, crew rules, and the presence of non-crew-compatible fleets. Scheduling with time windows is also discussed.

(ii) The analysis of operational strategies to provide decision making support. Scenario analyses are performed to provide insights on improving business profitability and aircraft availability, such as impact of aircraft maintenance, crew swapping, effect of increasing demand by Jet-card and geographical business expansion, size of company owned aircraft, and strategies to deal with the stochastic feature of unscheduled maintenance and demand.

CHAPTER 1

INTRODUCTION

Fractional ownership is a growing option for business travel. Through this program, companies or individuals own a fractional share of an aircraft. The owners are entitled to a fixed number of flying hours, where they do not compete for time on a particular plane but are entitled to their time whenever they ask for it. The fact that the operational and maintenance issues are taken care of by the management company makes it a convenient option for the owners.

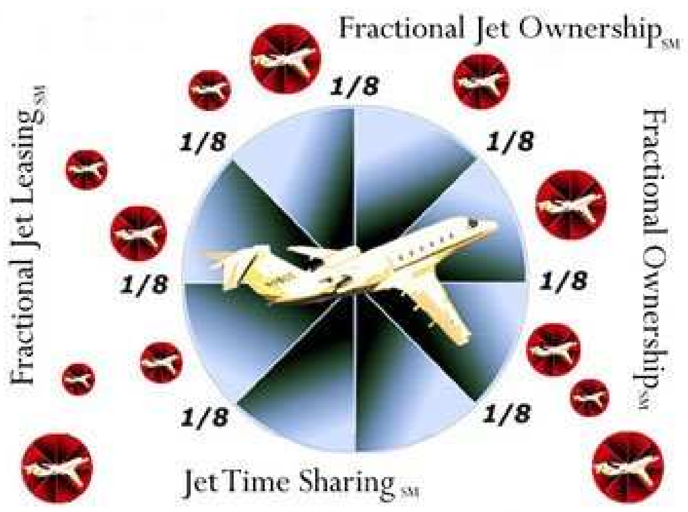


Figure 1.1 The concept of fractional ownership
(http://www.fractionaljetownership.com/index.html)

Fractional aircraft ownership is sometimes referred to as Fractional Jet Leasing or Jet Time Sharing. Figure 1.1 shows different names of the fractional ownership concept. A customer purchases an eighth interest in an aircraft, the plane in the blue center of the figure, and others purchase the remaining time portion of the same aircraft. If the customer's aircraft is not available, the fractional ownership management company will provide another aircraft, a plane in red circle of the figure, from its common fleet which is also fractionally owned or leased by others or a charter plane at no additional cost to the share owner.

More and more individuals and businesses prefer to become partial owners of an aircraft because this model offers relatively low cost (compared to whole aircraft ownership), flexibility, privacy, and guaranteed availability (with eight hours of advance notice), without aircraft and crew management bother. The fractional owner can fly directly anywhere among 5,500 airports (compared to 500 airports for commercial airlines) at any time with out check-in or security delays, or lost baggage, a significant benefit relative to commercial airline travel.

Although the fractional ownership program of private aircraft as a business model has been around since the 1960’s, it has become increasingly popular (Levere 1996; Michaels 2000) in the last twenty years. In 1986, there were three owners of fractionally held aircraft. By 1993, there were 110. From 2000 to 2004, the number of companies and individuals using fractional ownership grew by about 60 percent. Despite this rapid rate of expansion, many experts believe that only a small portion of the potential fractional business has been developed.

Unfortunately, the growth in the demand for fractional aircraft ownership has not translated into profitability for most management companies. According to Mcmillin (2006), fractional ownership companies operate almost 1000 business jets with a loss of about \$80 million in 2005, compared to a \$10 million profit in 2004. In fact, recently only one of the four largest management companies reported profits.

The primary drivers of the low profitability are: high repositioning cost, where empty aircraft have to be moved to pick up customers; expensive air charter cost, when peaks in demand can not be covered by available planes in the company. Rising fuel prices and lost time when aircraft are out of service for maintenance are also contributing factors. We believe that, by optimally arranging aircraft routes and crew schedules, significant improvement on the profitability of such businesses can be achieved with reduced operational costs and increased asset (crew and aircraft) utilization.

In this thesis, different models are developed to help the fractional management companies in assigning and scheduling aircraft and crews so that all flight requests are covered at the lowest possible cost. Scheduling problems have been extensively addressed in commercial airlines. However, the operation and planning processes in the fractional airlines are different from that in commercial airlines. The operation and planning problems arising in both types of airlines are briefly introduced in the following two sections.

1.1 Operations and Planning in Commercial Airlines

Usually, there are five phases in planning and scheduling processes in airline industry: flight scheduling, fleet assignment, aircraft routing, crew scheduling, and crew rostering. Yu (1998) contains a collection of articles in the field of commercial airlines. A variety of research and applications on airline operations research are addressed in Yu and Yang (1998), Barnhart et al. (2004), Ball (2004), and Clarke and Smith (2004). The first four phases related to this research are briefly described in the following sections.

1.1.1 Flight Schedule

The first phase of the airline planning process is to create a flight schedule. A *flight segment*, or *leg*, consists of the departure and arrival information, such as time and station. A *station* is an airport that an airline serves. According to the forecasted demand, the flight-scheduling phase determines all legs to be flown during a given period. Typically, the planner generates the basic schedule approximately 6 months in advance. In commercial airlines, most legs are flown every day of the week. The schedule is changed seasonally and small changes are made every month. Most domestic carriers have schedules that are the same every day with some changes for the weekends. Schedules are balanced (every arrival has a corresponding departure from the same station), and can be flown by the number of planes available.

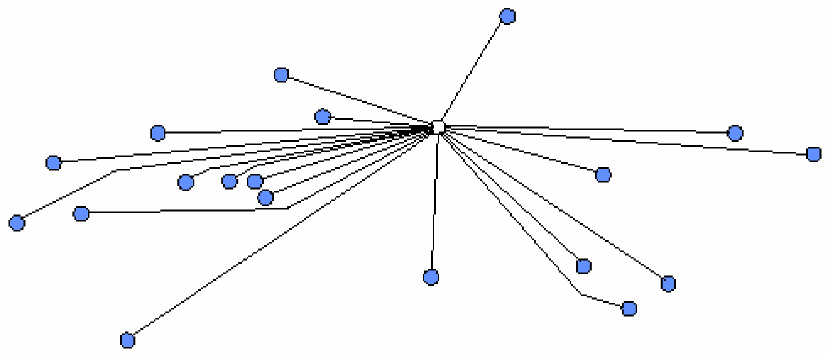


Figure 1.2 A hub-and-spoke network.

Most major commercial airlines use *hub-and-spoke* networks. Hubs are the airports with large number of daily flights, while the spokes are the airports with low activity. In this concept, spokes are connected through hubs, where customers can be combined together to form bigger passenger flow. Hubs have a large number of connecting flights to create many passenger itineraries. Hub-and-spoke networks provide a variety of departure-arrival pairs and are cheaper to operate than direct city-to-city flights because fewer aircraft are needed. Figure 1.2 demonstrates a sample network including twenty cities and one of them is the hub.

1.1.2 Fleet Assignment

The fleet assignment problem is addressed after preliminary flight schedules are completed. In general, the airline carriers have more than one type of aircraft. Each type of aircraft has different seating capacity, fuel consumption, and speed. A set of aircraft of the same type is defined as a *fleet*. Given the flight schedules and a list of available aircraft of different fleets, the planner assigns a specific fleet to fly each leg in order to maximize revenue (by matching seat capacity to passenger demand) and reduce costs (such as fuel, maintenance and airport gating). This model is called *fleet assignment Model (FAM)*.

The FAM is traditionally solved using multi-commodity flow models. The commodities are fleets and each leg must be assigned to exactly one fleet. Aircraft in a fleet departs from the same station where it lands. The total number of aircraft used in the FAM solution can not exceed the number of planes in a fleet. Abara (1989), Hane et al. (1995), Clarke et al. (1996), Barnhart et al. (1998), Rexing et al. (2000), Rosenberger et al. (2004) and Smith and Johnson (2006) describe the fleet assignment model in detail. Further discussion of the model is addressed in Chapter 3.

1.1.3 Aircraft Routing

Once the schedule and FAM are fixed, the planner determines the routing for each aircraft in the *aircraft routing* phase, or *aircraft rotation* phase. An aircraft route is a sequence of flights flown by an aircraft, identified by a unique tail number. In general, aircraft availability is determined with respect to scheduled and unscheduled maintenance events. After a certain number of flight hours, each aircraft goes under scheduled maintenance. All the planes that need to go under scheduled maintenance during a scheduling period together with the place and duration of the maintenance events are known beforehand. Hence the planner must assure that each aircraft scheduled to go under maintenance during the planning period arrives at the designated maintenance station at the designated time and is left on the ground during its maintenance period. Desaulniers, et al. (1997) and Clarke et al. (1997) discuss this problem in detail. Several modeling and solution approaches have been proposed to address the aircraft routing problem by Daskin and Panayotopoulos (1989) and Gopalan and Talluri (1998b). The aircraft routing problem in commercial airlines is always solved weeks in advance.

1.1.4 Crew Scheduling

Following aircraft routing phase, *crew scheduling* is considered. It is of particular importance because the crew costs are the second-largest operating expense faced by an airline, after fuel costs.

A *duty* contains a sequence of flights and related activities, such as briefing and debriefing, within a crew work day. The legality of crew composition and operations is defined by the Federal Aviation Administration (FAA) regulations. According to these regulations, only certain pairs of pilots are allowed to fly a certain type of aircraft given their current expertise and training status. The time that elapses between the beginning of a duty and the end of the duty is called *duty time*. It includes *briefing* and *debriefing* time before and after the trips. Furthermore, minimum *overnight rest* is required to take place between two consecutive duty periods. A *pairing* is sequence of duties, which can be legally flown by a single crew. Solving a crew-scheduling problem, also called a crew pairing problem, is equivalent to selecting a minimum cost set of crew pairings.

Crew bases are designated stations where crews must start their first duty and end their last duty. In commercial airlines, the crew schedule is made at least one week in advance. The pairing starts and ends at the same crew base, and it follows 8-in-24 planning rules, i.e. a crew must receive a rest if the crew flies more than 8 hours within a 24-hour period. Usually, a pairing contains at most 3 or 4 duties, or is determined by the upper bound of the *time away from base* (TAFB), which is the duration of a pairing. The time between two duties is defined as *overnight rest*, and the time between two flights is defined as *sit time* or *turn time*. The minimum turn time is 25 minutes.

1.2 Operations and Planning in Fractional Airlines

Fractional airlines have their own unique planning process due to the demand mechanism. The first four planning phases that are mentioned above in commercial airlines are different here.

1.2.1 Flight Schedule

Typically, a fractional management company requires that the owners request their flights at least eight hours before their desired departure time. Therefore in the flight scheduling phase, the legs are requested by the partial owners only days or hours ahead of time, instead of driven by the demand forecasting done months in advance. Owners call the scheduler in the management company to provide their departure location, departure time, and arrival location. Usually, the management company does not change a customer's request, except on *peak days*. Peak days are the days expected to have an unusual high amount of activities (such as the day before Thanksgiving). Some fractional management companies keep a contractual right for changing a customer's leg by shifting the departure time by at most $\pm\,\tau$ hours during these peak days.

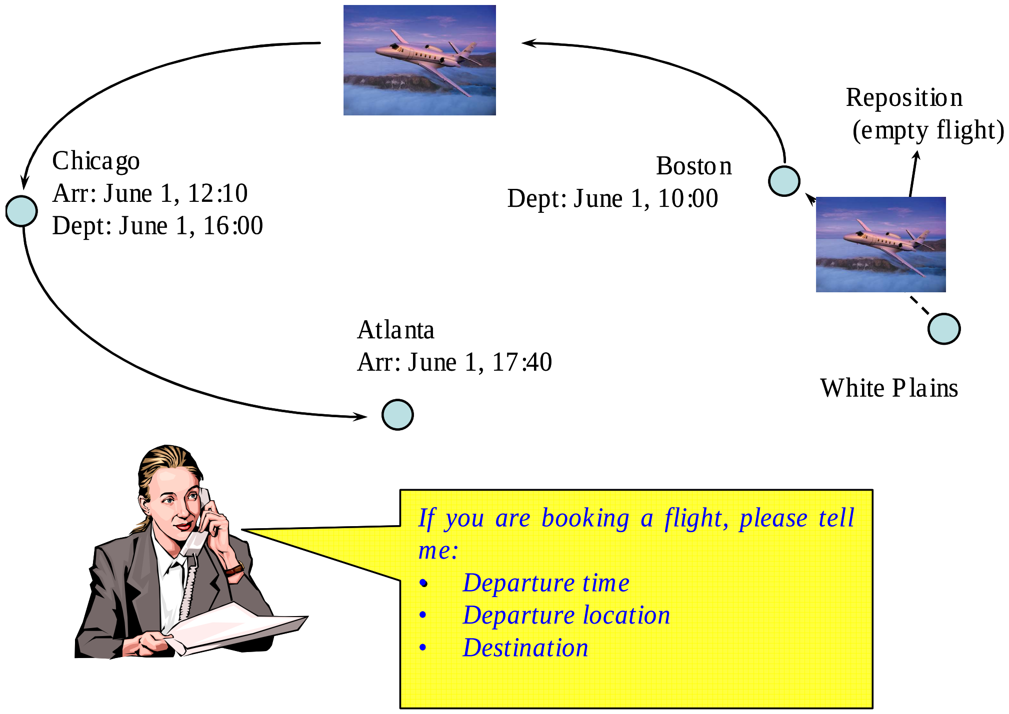


Figure 1.3 An example of the operation in fractional management company.

Figure 1.3 illustrates an example of the operation of fractional management company. An owner can call the management company requesting his desired flight with his departure and arrival location eight hours in advance. Therefore the flight schedule is normally neither fixed nor repeatable like the commercial airlines.

Another difference from most commercial airlines is that the legs are always direct flights from origination to destination, and hence a *point-to-point* network is used in fractional airlines (Figure 1.4).

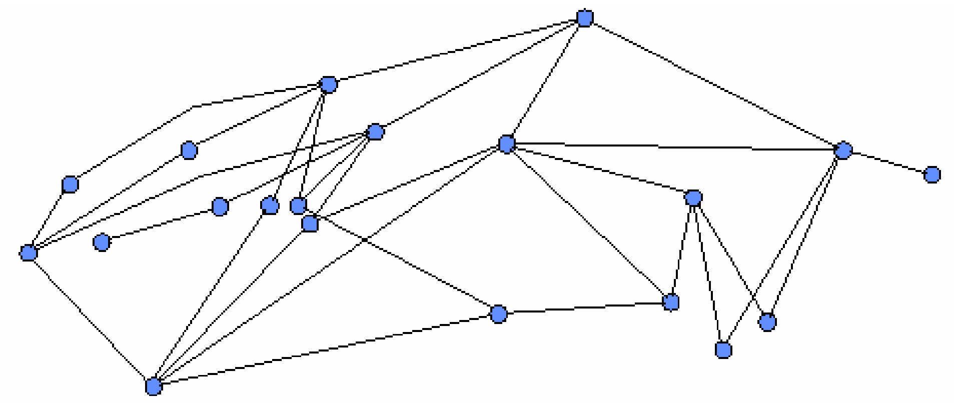


Figure 1.4 A point-to-point network.

1.2.2 Fleet Assignment

The fractional management company may operate a non-homogenous fleet with aircraft of different sizes. The objective of the FAM here is not to maximize profit based on demand for each flight. Usually the assigned aircraft type is the one that the customer owns. However, sometimes the planner may assign a different fleet type other than the customer owns. When an owner requests a flight the management company is obliged to serve this request with an aircraft that is at least as big as the owner's aircraft type. That is, the company may provide a larger aircraft without any additional expenses for the owner if it believes that the total operational costs can be decreased by this *complementary upgrade*, for instance, with a reduced reposition cost.

On the other hand, an owner can request an *upgrade* or a *downgrade* to a larger or a smaller aircraft, respectively. When such a request is received, the management company approves this request if the customer's contract includes guaranteed upgrade or downgrade hours. If a requested upgrade or downgrade is approved, then the flight hours to be deducted from the customer's account are adjusted with respect to the aircraft type. As a result, the remaining flight hours will be less when an upgrade is made or more if a downgrade is made. Moreover, the customer is required to pay the operating expenses resulting from this change.

1.2.3 Aircraft Routing

In aircraft routing phase, we refer to a customer requested flight as a leg. A crew of two pilots and an aircraft are assigned for flying each leg. If the assigned aircraft is not already at the departure station of the leg, the assigned crew flies the empty aircraft to this station. This empty flight is called a *reposition*. In Figure 1.3, an aircraft is flown to Boston from White Plains without passenger on board. A *trip* is either a reposition or a leg. Furthermore, a *taxi delay* is incurred for each take-off and landing and the length of this delay is determined with respect to the amount of traffic at a specific airport. Unlike commercial airlines, the routes here consist of not only revenue legs, flights with passengers on aboard, but also empty repositions. The repositions are major additional operational cost in fractional airlines, which are desired to be reduced as much as possible. Between any two trips, a minimum turn time of 45 minutes is required. The turn time is used for brief minor inspections and preparation of the aircraft for its next trip.

The operations of such direct service are similar to the pickup and delivery truckload problem in the trucking industry, where desired pickup times are given. It requires empty movements to drive the truck to the pickup location.

Moving empty resources between locations and minimizing the costs of empty moves is a primary challenge for the management company. These problems can be described as assignment models on time-space network and the objective is to minimize the total empty travel cost.

For this problem of minimizing empty repositioning moves, the routing planning quality can be measured by the *reposition ratio*, the ratio of total reposition miles to total trip miles. Note that the total trip miles here include both reposition and customer leg miles. A good routing has low reposition ratio.

1.2.4 Crew Scheduling

In fractional airlines, crew generally does not follow the more stringent rules required by commercial airlines. When selecting a legal crew pairing, the planner must meet the following requirements: a 14-hour maximum duty time, a 10-hour maximum flying time in a day, and 10-hour minimum overnight rest time between two duties. Unlike commercial airline, pilots can not travel as passengers on client flights. Therefore, when a pilot travels by a commercial airline from or to his crew base, a three-hour minimum connection time, due to the time needed to go through security, check in etc. at commercial airports, before the departure time of the commercial flight is assumed to be incurred. This connection time is also counted as a portion of the duty time. The connection time may include taxi time by automobile to a nearby airport that offers scheduled commercial flights.

In general, the pilots work on a schedule in which they stay on-duty for a specified number of days (e.g. one week) followed by an off-duty period (e.g. one week). We denote crews consisting of a captain and co-pilot who are starting their on-duty period as *coming-duty* crews. Coming-duty crews travel from their crew bases to the available aircraft locations. *Off-duty* crews are the crews that go back to their crew bases at the end of their on-duty period. Sometimes the management company may ask a coming-duty crew to fly to the station where an aircraft is located the day before the crew's duty-period starts to cover an early morning flight the next day. Also an off-duty crew may arrive to its home base a day after the end of its duty period due to flying a late flight the day before. In both of these cases the pilots are paid *overtime*. The exchange of a crew with another crew to fly an aircraft is called a "crew-swap" and the days of the week that the coming-duty crew starts its shift and the off-duty crew ends its shift are called "designated duty shift days".

For the management company aircraft availability is an important issue while scheduling flights. An aircraft is "idle" when it is ready to be assigned to a crew. Aircraft availability is determined with respect to scheduled and unscheduled maintenance events. According to FAA regulations, after a certain number of flight hours, each aircraft goes into scheduled maintenance. In our approach, we do not create the schedules for maintenance but use the maintenance information that is provided by the company. Therefore, we assume that all the planes that need to go into scheduled maintenance during a scheduling period together with the place and duration of the maintenance events are known beforehand. Hence the schedule must assure that each aircraft scheduled to go into maintenance during the planning period arrives at the designated maintenance station at the designated time and remains on the ground during its maintenance period. The crew assigned to fly a plane to its maintenance station either stays until the maintenance is completed or is reassigned to an idle aircraft depending on the duration of the maintenance event. All events that require a plane to be grounded for a period of time due to an unexpected problem are denoted as unscheduled maintenance events. When an unscheduled maintenance event occurs the plane stays at the destination station of its last flight until the problem is fixed, and the rest of the legs assigned to the aircraft are reassigned.

Due to the special feature of customer's demand in fractional ownership airline, the aircraft routing and crew scheduling is made only one or two days, or even hours ahead of departure. For instance, a crew is notified of any assignments (including any changes to an assigned leg due to owner requests or unscheduled maintenance) at least two hours prior to departure. The dynamic scheduling or operational policies to incorporating the dynamic nature are challenging topics. This thesis focus on crew scheduling that combines the aircraft routing and fleet assignment.

1.3 Previous Work on the Airline Planning Problems

For a summary of planning in the airline industry see Teodorovic (1988), Cook (1989), and Yu (1998). A good overview of the applications in the air transport industry can be found in Barnhart et al. (2003). We first review the related research for commercial airlines.

1.3.1 Previous Work on Commercial Airlines

The crew pairing problem in commercial airline industry has been addressed in numerous studies and various solution methods have been developed. Surveys of scheduling research in the airline industry can be found in Arabeyre, et al. (1969), Etschmaier and Mathaisel (1985), Richter (1989). Recent survey on the airline crew scheduling appears in Gopalan and Talluri (1998a) and Gopalakrishnan and Johnson (2005). The problem is generally formulated as a set-partitioning problem. The early work dates back to the 1960s (Steiger 1965; Niederer 1966). In the early 1970s, to avoid enumerating millions of potential pairings, American Airlines use a column-generation solution strategy, called TRIP, to heuristically select a solution for the daily domestic crew-scheduling problem (Gershkoff 1989; Anbil, et al. 1991a). Crainic and Rousseau (1987) and Lavoie, et al. (1988) formulate the problem as a set-covering problem and select a good set of pairings with a column generation algorithm. Klabjan and Schwan (1999) generate pairings with a parallel algorithm on a parallel machine. Klabjan, el al. (2001) use random pairing generation, combine with strong branching and a specialized branching rule while solving a large-scale airline crew pairing problem.

Regarding to the solution time, Lagrangian decomposition is exploited for early termination of column generation algorithm and speeds up the pricing algorithm (Wedelin 1995; Andersson, et al. 1998). Barahona and Anbil (1998) also present an extension to the sub-gradient algorithm, volume algorithm, to produce primal as well as dual solutions. This algorithm significantly improves computational time for solving crew pairing problem (Anbil, Ferrest and Pulleyblank 1998).

A great deal of attention has been given to the linear programming (LP) based branch-and-bound approach to solve crew-scheduling problems (Nemhauser and Wolsey, 1988). Anbil, Tanga and Johnson (1992) use SPRINT for solving LPs, where selected columns are added to the LP and the new LP is re-optimized. Chu, Gelman and Johnson (1997) improve the procedure for finding integer solutions. Hoffman and Padberg (1993) propose a brand-and-cut approach to solve a mixed integer program (MIP) for airline crew scheduling. After solving the LP, a violated valid inequality is created if the optimal solution is fractional.

Levine (1996) provides a hybrid genetic algorithm for airline crew scheduling problems and tests the algorithm on a set of 40 real-world problems. He also compares his algorithm with branch-and-cut and branch-and-bound algorithms. The branch-and-cut is determined to solve all the test problems to optimality within less time than the other two algorithms. The genetic algorithm can find feasible solutions for two larger problems when the branch-and-bound approach cannot.

The Branch-and-price approach, which combines column generation with branch-and-bound method to solve the LP relaxation at each node, is an exact algorithm. It dynamically generates columns throughout the branch-and-bound tree. For a survey of branch-and-price approaches see Barnhart et al. (1998a). One of the first branch-and-price methods to appear in the literature was the one presented in Desrochers and Soumis (1989) for the vehicle routing problem with time windows. Desrosiers et al. (1991) present the first application of branch-and-price to the airline crew scheduling problems. Vance et al. (1997a) provide a detailed description of column generation, branching, and search strategies for a branch-and-price algorithm. Barnhart et al. (1999) create a duty period network for crew scheduling problem, and exploit this algorithm with a duty-based formulation.

Hu and Johnson (1999) develop an algorithm based on primal-dual linear programming for the set-partitioning problem. Shaw (2003) extends this idea with a hybrid method, with which the column generation is delayed by enumerating sub-paths up front. Recently, Klabjan, Johnson, and Nemhauser (2000) present a parallel primal-dual algorithm and solve LP relaxations with this algorithm (Klabjan, et al., 2001).

Cordeau et al. (2001) apply Benders decomposition to simultaneously solve a single type of aircraft routing and crew scheduling problem. They solve the aircraft routing problem as a master problem and the crew pairing problem as a subproblem. A heuristic branch-and-bound method is used to obtain integer solutions. Barnhart et al. (1994) propose a long-haul crew assignment problem. They construct a long-haul network and generated columns by using a specialized shortest path search on the network. Part of this thesis extends their idea for the fractional airlines.

1.3.2 Previous Work on Fractional Ownership Airlines

For a fractional airline, the pairing problem poses a unique situation. Unlike commercial airlines, the flight legs in a fractional airline differ from day to day and week to week, and some are not known in advance. Repositioning requires flying an aircraft without any passengers on board, and repositioning may comprise 35% or more of the total flying. Keskinocak and Tayur (1998) study the fractional aircraft-scheduling problem for a single type of aircraft. They develop and test a zero-one IP for small- and medium-size problems (up to 20 planes and 50 trips) and provide a heuristic for solving larger instances. In their work, the multiple fleet types and crew duty restrictions are not considered. Ronen (2000) presents a decision-support system for scheduling charter aircraft. He develops a set-partitioning model that combines the fleet assignment and routing problems and incorporates maintenance activities and crew availability constraints. Larger scale problems (up to 48 aircraft and 92 trips) in one-day and two-day planning horizons are solved to minimize total cost of scheduling flights, subcontracting flights, and idling aircraft. They include subcontractor aircraft as a part of the owned aircraft but with different cost. Therefore, they consider selling off a sequence of flights.

Recently, Martin et al. (2002, 2003) extend the methods developed in Keskinocak and Tayur (1998) by including multiple types of aircraft and crew constraints. Their model considers multiple-day planning periods with 10-hour overnight rest between each day. Karaesmen et al. (2005) develop several mathematical models and heuristics that take into account the presence of multiple types of aircraft, scheduled maintenance, and crew constraints. They analyze the efficiency of these models through a computational study by solving daily scheduling problems. Hicks et al. (2005) develop an integrated optimization system for Bombardier Flexjet (www.flexjet.com), a large fractional aircraft management company. A column generation approach is applied to solve a large-scale mixed-integer nonlinear programming model, which is based on an integer multi-commodity network flow problem. A branch-and-bound approach is used to obtain integer solutions from selected columns, which represent the aircraft itineraries and crew schedules. Yang et al. (2006) extend the work in Karaesmen et al. (2003) to multi-day horizons. They first implement a network flow mode and create crew-feasible schedules. In their work, a branch and price method is proposed. Their experiments show the average utilization has increased to over 70% from 62%.

1.4 Dissertation Organization

The thesis is organized as follows: Chapter 2 introduces our algorithm for solving the crew pairing problem combined with the aircraft routing problem. The efficiency of our algorithm is evaluated with those of current methods on large-scale random data sets. Then maintenance issues and crew swap strategies are discussed. Chapter 3 describes a new model that allows full separation of crew and aircraft when simultaneously solving the fleet assignment, aircraft routing and crew scheduling problem. The model can be solved with Bender’s decomposition approach for large size instances. Chapter 4 investigates options for tactical and strategic planning. Chapter 5 concludes the research and discusses future works.

CHAPTER 2

INTEGRATING AIRCRAFT ROUTING AND CREW SCHEDULING

In this chapter, we assume that during its duty period a crew stays with one aircraft unless a long maintenance event occurs. Although this assumption provides schedules with low plane utilization, due to the high transportation costs and times incurred when the crews travel by commercial airlines and the increased operational complexity, most fractional management companies prefer to operate with such initial schedules and modify them in an ad-hoc manner if necessary. Hence the scheduling process is simplified to a two-stage assignment, which first assigns crews to aircraft in the beginning of a duty, and then assigns crews to a sequence of flight legs. In general this process is called crew pairing or crew scheduling. The crew pairing problem in commercial airline industry has been addressed in numerous studies and various solution methods have been developed. The problem is generally formulated as a set-partitioning problem (Marsten and Shepardson, 1981). One method that is commonly used to solve set partitioning problems is column generation. Column generation was initially introduced in Dantzig and Wolfe (1960) and there exist a number of papers where it was applied to solve airline crew scheduling problems (see for example Crainic and Rousseau (1987), Lavoie et al. (1988), and Barnhart et al. (1994)).

This chapter is organized as follows. Section 2.1 introduces our algorithm for solving the crew pairing problem combined with the aircraft routing problem. In Section 2.2, we compare the efficiency of our algorithm with those of current methods on large-scale random data sets. In Section 2.3, we present the results of different scenario analysis. Finally, we summarize the conclusions in Section 2.4.

2.1 Scheduling Approach

2.1.1 Basic Assumptions

We first assume that during its duty period a crew stays with one aircraft unless a long maintenance event occurs. Although this assumption provides schedules with low plane utilization, due to the high transportation costs and times incurred when the crews travel by commercial airlines and the increased operational complexity, most fractional management companies prefer to operate with such initial schedules and modify them in an ad-hoc manner if necessary. In our analysis, we will relax this assumption either when an aircraft goes under a long maintenance that lasts more than one day or when an extra crew is available and measure its effects on operational cost and plane utilization. Our modified model treats both the crew whose aircraft goes under long maintenance and the extra crew as special coming-duty crews.

The crew-swaps occur only during designated duty shift days or when an extra crew picks up an aircraft whose crew has already used up its maximum duty or fly time for the day. We also assume that the two pilots who form a crew pair do not split during an entire duty. These assumptions do not divert from the mode the fractional management companies operate in most of the time.

In terms of the cost, we assume that no additional cost or penalty is incurred if an aircraft or a crew idles on the ground. The charter rate is considered to be fixed in the planning horizon. Finally, we only incorporate complimentary upgrades since the fractional management company only incurs extra cost in these cases. The upgrade cost is the extra flying and reposition cost per hour when a leg is covered by an aircraft that is larger than requested.

We formulate the crew and aircraft scheduling problem as a set partitioning problem. We use a column generation method to solve a three-day scheduling problem where at each iteration all the known demand is incorporated. In our rolling horizon approach, after a three-day schedule is determined, the schedule for the first day is fixed and the problem is resolved using the next three-day demand data. We assume that this is a reasonable strategy given that the customer requests must arrive only eight hours prior to the departure time and in the industry on average 80-90% and 60-75% of the demand is known in advance for the second and third days respectively and this percentage drops significantly after the third day.

2.1.2 Formulation

In the three-day planning period, the crew pairing problem is formulated as a set-partitioning problem combined with aircraft and crew constraints. Given K legs, M planes, T fleet types, and R crews, let L be the set of legs in the three-day planning period; P be the set of available planes at the beginning of the planning period; W be the set of possible crew combinations; and CP be the set of all columns representing the possible pairings. Let x_j be a 0-1 variable indicating if column j, corresponding to a feasible pairing, is chosen in the solution or not and s_k be a slack variable indicating whether leg k is covered by a charter or not. Let c_j be the cost of column j and r_k be the charter cost for leg k. We assume that the set of legs, L, is ordered with respect to the departure time of the legs in the planning period.

We formulate the crew pairing problem as follows:

$$\text{(Q1)} \quad \text{Min} \quad \sum_{j\in CP} c_j x_j + \sum_{k\in L} r_k s_k$$

$$\text{s.t.} \quad \sum_{j\in CP} A_{kj} x_j + s_k = 1 \qquad \forall k \in L \qquad \pi_k \qquad (2.1.1)$$

$$\sum_{j\in CP} B_{kfj} x_j \le 0 \qquad \forall k \in L, f \in T \qquad \lambda_{kf} \qquad (2.1.2)$$

$$\sum_{j\in CP} E_{pj} x_j \le 1 \qquad \forall p \in P \qquad \delta_p \qquad (2.1.3)$$

$$\sum_{j\in CP} F_{wj} x_j \le 1 \qquad \forall w \in W \qquad \rho_w \qquad (2.1.4)$$

$$x_j \in \{0,1\} \qquad \forall j \in CP$$

$$s_k \in \{0,1\} \qquad \forall k \in L$$

where,

A_{kj} is 1 if leg k is included in column j, and 0 otherwise. B^t_{kj} is –1 in column j if a plane is left at the arrival station of the last leg $k \in L^t$ covered by an off-duty crew, 1 if a plane is picked up at the arrival station of leg $k \in L^t$ by a coming-duty crew operating type t planes, and 0 otherwise. These inequality constraints allow a plane left by an off-duty crew to either stay on ground or be picked up by a coming-duty crew. E_{pj} is 1 if plane p, an available aircraft in the beginning of the three-day planning period, is used in column j, and 0 otherwise. F_{wj} is 1 if crew w flies the sequence of legs in column j, and 0 otherwise.

Constraints (2.1.1) require that each customer leg be flown either by a company aircraft or a charter. π_k is the dual variable associated with the leg cover constraints. Constraints (2.1.2) insure that a coming-duty crew picks up an aircraft it can operate at the destination of the last leg the aircraft has flown only if an off-duty crew left it there. λ_{kf} is the dual variable associated with the aircraft connection constraints. Constraints (2.1.3) ensure that an aircraft is used only by one crew at any given time or is not used during the planning period. δ_p is the dual variable associated with the aircraft availability constraints. Constraints (2.1.4) ensure that a crew is assigned to only one pairing. ρ_w is the dual variable for the crew constraints.

Consider the sample constraint matrix in Table 2.1. The first column for Crew 1 represents a feasible duty where Crew 1 covers legs 1, 41, and 68 with aircraft 1. The last leg in the duty, which is also the last leg Crew 1 flies before going off duty on day 2, is leg 68, so a “-1” appears in the row of leg 68 in the aircraft connection constraints. The second column for Crew 2 represents a feasible pairing, where the aircraft that is left at the arrival station of leg 68 is picked up by Crew 2 and then is used for flying legs 70 and 111. In this case, a “1” appears in the intersection of row of leg 68 in the aircraft connection constraints and the second column for Crew 2 and no “1’s” appear in the aircraft constraints corresponding to this column. This ensures that Crew 2 only flies one aircraft. The third column for Crew 1, represents a scenario where Crew 1 does not fly any legs on day 2 and leaves the aircraft at the arrival station of the last leg it flies on day 1. Hence a “-1” appears in the row of leg 40 in this column in the aircraft connection constraints. In the third column for Crew 2, a “1” in the row of leg 40 in the aircraft connection constraints represents the fact that Crew 2 picks up the plane left at the arrival station of leg 40 by Crew 1. The first column for Crew 40, represents a scenario where Crew 40, which is neither coming on nor going off duty during the planning period, flies to cover legs on the first day and the third day and stays on the ground during the second day. We assume that in this example off-duty Crew 1 and coming-duty Crew 2 operate the same type of aircraft, namely Type 2. Therefore only fleet Type 2 connection constraints are shown in Table 2.1. In general, when there are *f* fleet types, the upper bound on the number of connection constraints is *f* times the number of legs in the planning horizon.

Table 2.1 A sample constraint matrix.

			Crew 1				Crew 2				…	Crew 40			Charter	RHS
			Go Off Duty At Day2				Come To Duty At Day3				...	On Duty For 3 Days				
		1	1	0	1	…					...	1	0	…		
		2	0	1	0	…					...	0	1	…		
	Day 1	…	…	…	…	…	0				...	…	…	…		
		40	0	1	1	…					...	1	1	…		
Leg		41	1	1	0	…					...	0	1	…		
Constraints		42	0	1	0	…					...	0	1	…	I	= 1
	Day 2	…	…	…	…	…	0				...	…	…	…		
		68	1	0	0	…					...	0	0	…		
		69					1	0	0	…	...	1	0	…		
		70					1	1	1	…	...	0	1	…		
	Day 3	…	0				…	…	…	…	...	…	…	…		
		111					0	1	1	…	...	1	1	…		
		1			0	…			0	…	...					
		2			0	…			0	…	...					
	Day 1	…	0		…	…	0		…	…	...	0				
Plane		40			-1	…			1	…	...					
Connection		41	0	0	0	…	0	0	0	…	...					
Constraints		42	0	-1	0	…	0	0	0	…	...					
	Day 2	…	0	0	…	…	0	0	…	…	...	0			0	≤ 0
Fleet Type 2		68	-1	0	0	…	0	1	0	…	...					
		69									...					
		70									...					
	Day 3	…	0				0				...	0				
		111									...					
	1		1	1	1	…	1	0	0	…	...	0	0	…		
	2		0	0	0	…	0	0	0	…	...	0	0	…		
Plane	3		0	0	0	…	0	0	0	…	...	0	0	…	0	≤ 1
Constraints	…		…	…	…	…	…	…	…	…	...	…	…	…		
	30		0	0	0	…	0	0	0	…	...	1	1	…		
	1		1	1	1	…	0	0	0	…	...	0	0	…		
	2		0	0	0	…	1	1	1	…	...	0	0	…		
Crew	3		0	0	0	…	0	0	0	…	...	0	0	…	0	≤ 1
Constraints	…		0	0	0	…	0	0	0	…	...	0	0	…		
	40		0	0	0	…	0	0	0	…	...	1	1	…		

2.1.3 Algorithm

Using column generation, we first solve the linear programming (LP) relaxation of the set partitioning problem formulated in the above section. Initially, we enumerate all feasible duties, each of which contains a bundle of legs that can be legally flown in a day, by using a depth first search algorithm. Then we create an auxiliary network for each available crew pair that is used to identify good pairings. Shortest paths on these auxiliary networks are used to create a set of initial columns that is fed into the initial LP. After solving the initial LP, we update the arc costs on the auxiliary networks using dual information provided by the LP, and solve a pricing problem by finding shortest paths on these networks with the new arc costs. The length of these shortest paths determines if the optimality of the solution found by the previous LP, and if not optimal what are the profitable columns to be added to the model. When we have an optimal solution for the LP relaxation, we feed all the columns present in the final LP into an integer programming solver. After the integer solution is obtained by solving the IP with all the existing columns in the final LP relaxation, the first day pairing for the three-day planning horizon is fixed and the procedure is repeated for the next three days using the first day information as initial conditions. The steps of the algorithm are shown as a flow chart in Figure 2.1. We give the details of the auxiliary networks and the related initial solution generation and pricing procedures in the rest of this section.

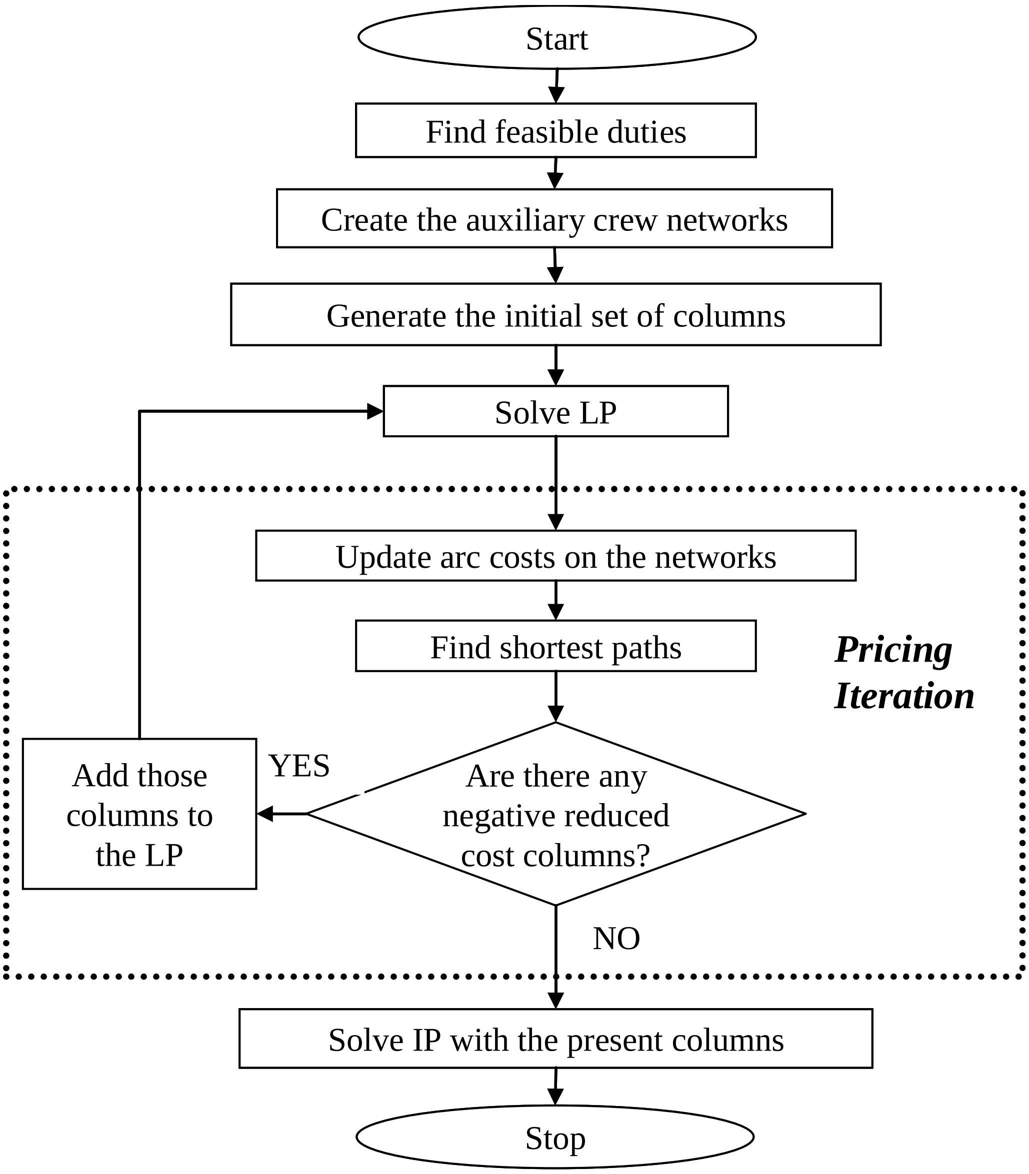


Figure 2.1 Flow chart of the algorithm.

2.1.4 Crew Network and the Pricing Out Step

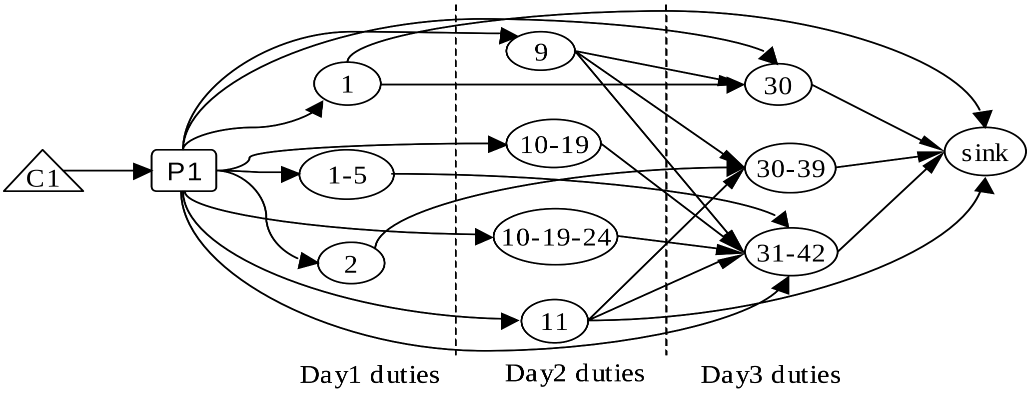
We show that the pricing iteration in the column generation process is equivalent to finding a set of shortest paths on appropriate crew networks. For each crew pair, we construct a network $G=(N, A)$, where the node set N in each network consists of a source node C representing the initial location of the crew, nodes P_i representing the available aircraft and their locations, a set of duty nodes representing feasible duties the crew can fly during the planning period, and a sink node.

The arcs emanating from a crew node depend on the crew's remaining on duty days. If the crew is already on duty and stays with a certain plane, the crew node has only one out-going arc that enters the node of this plane (Figure 2.2(a) and (c)). If the crew is a coming-duty crew, multiple out-going arcs connect the crew node to all the possible plane nodes and duty nodes where a plane is available from a previously flown leg. These arcs have costs consisting of the cost of the transportation between the crew's home base and the available plane's location and cost of overtime if it exists. For example, in Figure 2.2(b), Crew C2, coming on duty on day 3, can pick up the planes P2 or P3 that are idle at the beginning of the planning period or any other aircraft that is left at the arrival station of a leg completed before day 3. More specifically, in Figure 2.2(b) the dashed line between C2 and the duty node 1 and the solid lines between node 1 and nodes 30 and 30-39 correspond to the possibility that crew C2 may pick up the plane that flew leg 1 and then feasibly fly leg 30 or legs 30 and 39 on day 3.

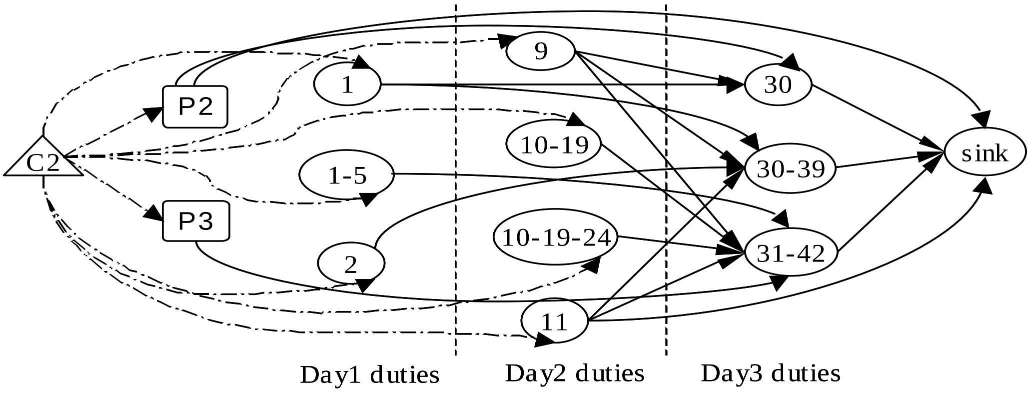
In the crew network, an arc between a plane node and the sink node with zero cost represents the fact that the plane stays idle on the ground during the planning period. For example, in Figure 2.2(b) the dashed line between node C2 and P2 together with the arc between node P2 and the sink implies that crew C2 comes on to duty on the third day and picks up plane P2 but stays on the ground without flying any duties during the course of the day. Furthermore, a plane node has an arc going into every duty node if the crew can transport to pick up the plane and then reposition and serve that duty legally.

A duty node connects to the duty nodes in the next day if the overnight rest legality constraint is satisfied. Also, a duty node in the first day can directly connect to any duty node in the third day implying that the crew will be on the ground during the second day between the first and third days' duties.

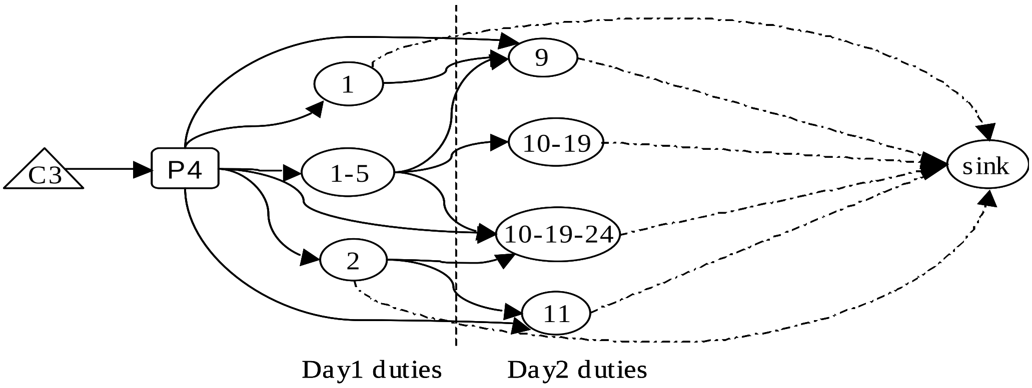
Finally, in all but the off-duty crew networks, all duty nodes connect to the sink node directly without cost. This incorporates the possibility that after flying the last leg of the duty, the crew may stay on the ground with its plane for the rest of the planning period. In the off-duty crew networks, the arcs between the duty nodes and the sink have costs consisting of the cost of the transportation between the arrival station of the last leg and the crew's home base and cost of overtime if it exists.



a. Crew on duty during the three-day planning period.



b. Crew comes on to duty on the third day.



c. Crew goes off duty at the end of the second day.

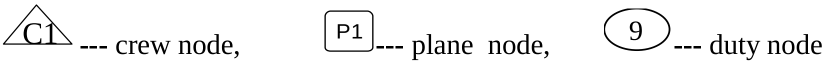


Figure 2.2 A partial crew network.

Note: Dashed lines represent crew travel.
Solid lines represent feasible connections for covering duties.

The crew networks are constructed once in the beginning of each planning period and then the arc costs are updated in the beginning of each pricing iteration of the column generation process. To generate our first restricted LP, we find a shortest path in each of the crew networks. These paths correspond to the best pairing for each crew with respect to the original costs. The initial restricted LP consists of columns corresponding to these shortest paths together with columns corresponding to charters. The initial restricted problem determines the initial dual variables that are passed on to the initial pricing problem.

At iteration i of the column generation process, first the linear programming relaxation of the set partitioning problem is solved using the subset of the columns corresponding to the profitable pairings identified in the previous iterations. Next, using the dual information obtained from this solution we solve the pricing problem to determine if the current solution is optimal, that is we determine if there exists any columns with negative reduced costs. Given the dual variables for (Q1) at iteration i the reduced cost for column j is calculated as follows:

$$\bar{c}_j^{\,i} = c_j - \sum_{k \in L} A_{kj} \pi_k^i - \sum_{k \in L,\, f \in T} B_{kfj} \lambda_{kf}^i - \sum_{p \in P} E_{pj} \delta_p^i - \sum_{w \in W} F_{wj} \rho_w^i \qquad \forall\, j \in CP. \qquad (2.1.5)$$

Determining if there exists profitable pairings reduces to solving a shortest path problem in each of the crew networks when the arc costs are updated using the dual information. Let c_{xy} and $\bar{c}_{xy}$ indicate the original and the updated costs of an arc between node x and node y, respectively. Then at the i^{th} iteration,

$$\bar{c}^{\,i}_{xy} = \begin{cases} c_{xy} - \delta^i_y, & \text{if node } x \text{ is a crew node and } y \text{ is a plane node,} \\ c_{xy} - \lambda^i_{yf}, & \text{if node } x \text{ is a crew node and } y \text{ is a duty node;} \\ & \text{or } x \text{ is a duty node and } y \text{ is a sink node,} \\ c^i_{xy} - \sum_{k \subseteq y} \pi^i_k, & \text{if } x \text{ is a plane or duty node and } y \text{ is a duty node.} \end{cases} \tag{2.1.6}$$

For example at iteration i in Figure 2.2(b), the cost on the arc between C2 and P2 becomes $c_{C2,P2} - \delta^i_{P2}$; the cost on the arc between C2 and duty node 1-5 becomes $c_{C2,1-5} - \lambda^i_{1-5,2}$ if C2 operates fleet type 2; and the cost on the arc between the duty nodes 1-5 in the second day and 31-42 in the third day becomes $c^i_{1-5,31-42} - \pi^i_{31} - \pi^i_{42}$.

Once the shortest paths are found for each crew pair it is easily checked if these correspond to columns with negative reduced costs by subtracting ρ^i_w from the cost of the shortest path for crew w. The column generation procedure terminates with an optimal solution for the LP relaxation of the set-partitioning problem when no columns with negative reduced costs exist.

2.2 Computational Efficiency of the Solution Methodology

In our column generation procedure described above, we solve the pricing problem by finding “the shortest path” in each crew network. Therefore, at each iteration at most one profitable column is identified and added to the LP for each crew. In our initial computational study, we noticed that adding at most one column per crew at each iteration resulted in a large number of pricing iterations we had to go through before an optimal solution was reached. To determine if adding a set of good columns per crew at each iteration increases the speed of our algorithm, we modified our price-out step so that instead of identifying the shortest path in each crew network, up to S shortest paths are determined. In the modified algorithm, first we determine the shortest paths to all of the duty nodes on the last day of the planning period. Then the subset of these paths corresponding to negative reduced cost columns is partially sorted and the best S of them is added to the previous LP. Although adding more than one column increases the time for solving the LP’s and requires the sorting of the paths at each iteration, in our computational experiments the overall running times were reduced.

The results of a computational study where one-day problems of different sizes are solved are presented in Table 2.2. The instances for this study are generated based on the demand and scheduled maintenance data obtained from a fractional management company. The first column gives the size of the instance as the number of planes and legs present. Given the data, the second column presents the number of duty nodes created. The next four columns present the size of the LP (in terms of number of columns in the final iteration) when we add 1, 5, 10, and 20 columns per crew at each pricing iteration. The LP run times and the number of pricing iterations the column generation process went through are listed in the next four columns. The last four columns display the solution times required for solving the MIP. In the last row of the table, we present the results for a 3-day planning problem with 100 planes and 200 legs. The running times are given in seconds. The number of pricing iterations is given in parenthesis in columns 7-10.

Table 2.2 Computational time comparisons

# Planes, # Legs	# Duty Nodes	LP Size				LP Time Sec. (Number of Pricing Iterations)				MIP Time sec.			
		1 Col	5 Cols	10 Cols	20 Cols	1 Col	5 Cols	10 Cols	20 Cols	1 Col	5 Cols	10 Cols	20 Cols
70, 210	26286	3728	5696	8377	12450	113.86 (51)	51.89 (16)	48.37 (12)	43.85 (9)	0.14	0.2	0.39	1.03
100, 200	22514	3361	10998	16932	23846	105.16 (32)	65.78 (22)	54.8 (17)	48.57 (13)	0.11	0.58	0.94	2.35
50, 150	8331	1650	3096	4565	6832	20.86 (31)	10.29 (12)	10.32 (9)	8.48 (7)	0.01	0.06	0.24	0.32
70, 140	6812	3038	4953	6968	7979	27.74 (42)	17.53 (14)	12.76 (10)	11.4 (6)	0.02	0.29	0.59	0.82
100, 100	2435	1869	4997	6893	9108	11.01 (18)	4.15 (10)	2.96 (7)	2.58 (5)	0.01	0.03	0.05	0.12
50, 100	2435	1532	3547	3924	4821	8.08 (29)	4.77 (14)	4.08 (8)	3.02 (5)	0.01	0.09	0.18	0.25
30, 90	2027	914	2418	3015	4177	4.61 (31)	1.532 (16)	1.603 (10)	1.42 (7)	0.01	0.03	0.05	0.09
50, 50	136	398	1264	1982	2895	1.05 (8)	0.24 (5)	0.24 (4)	0.34 (3)	0.01	0.02	0.03	0.03
3-day	3932	8331	15494	17599	21927	275.61 (81)	137.41 (32)	72.40 (18)	48.36 (12)	2.57	2.77	2.7	2.97

Analyzing the results presented in this table we conclude that adding more than one column has a significant effect in the total run time of the algorithm. Although the effect of adding more columns decreases as S is increased, the best run times, especially for the larger instances, are obtained when up to 20 columns are added per crew. In the rest of our computational study, we add up to twenty columns per crew in each iteration.

We perform a set of experiments to determine the computational efficiency and effectiveness of our solution approach with real data obtained from a fractional management company including the demand and scheduled maintenance. In Table 2.3, we present the performance of our scheduling tool on different planning horizons with different instance sizes. The sizes of the instances are given as the number of planes and legs. In the 2-day and 3-day instances, we run the algorithm once over 2 and 3 days, respectively, with all the demand and scheduled maintenance data. In these multi day instances, the initial conditions for crew and aircraft are assumed to be the same as that of the 1-day problem. The solution times given in the third column are the total times required for solving the LP and the IP. We use the value obtained from solving the LP as a lower bound on the optimal value of the IP. The last column in Table 1 presents the gaps between the value of the integer solutions we obtained and the LP lower bounds, and hence provides upper bounds on the optimality gaps for the solutions we obtain. We observe that as the size of the instances grows and the planning horizon gets longer, the solution time increases from less than one minute to around ten minutes. However, the run time stays within acceptable limits even for operational decision making and the optimality gap never goes above 0.06%.

Table 2.3 Computational time comparisons.

Planning Horizon	# Planes, # Legs	Solution Time (Sec)	Optimality Gap (%)
	35, 42	0.23	0
1-day	61, 101	0.79	0.02
	75, 125	1.11	0
	35, 83	0.72	0
2-day	61, 187	10.46	0.02
	75, 231	36.74	0
	35, 124	4.6	0
3-day	61, 261	70.4	0.04
	75, 341	435.2	0.06

2.3 Maintenance Issues and the Refinement of the Model

In this section, we first consider the impact of scheduled and unscheduled maintenance events on profitability through scenario analysis. We use data based on the real operational data provided to us by CitationShares, a fractional ownership management company. The company focuses on light and mid-size aircraft. Then we propose a refinement of the model to improve the utilization of plane and crew by re-assigning crew whose original assigned aircraft undergo long maintenance.

2.3.1 Incorporating Scheduled and Unscheduled Maintenance

To incorporate scheduled maintenance, we treat a maintenance event as a special mandatory leg that a specific aircraft has to fly. The arrival/departure location, S_m, of the maintenance leg is assigned to be the airport where the maintenance is scheduled to take place and the duration of the leg, from time t_{sm} to time t_{em}, is equal to the duration of the maintenance visit. For an aircraft with a scheduled maintenance event due during the current planning horizon we make sure that the aircraft flies this leg by having it arrive at S_m before t_{sm}. That is, if the aircraft is already assigned to a crew then we modify the crew network for this crew so that any path in the network includes the maintenance leg, and if the aircraft is not assigned to a crew we force the assignment of the aircraft to the nearest unassigned crew and make the necessary changes to this crew's network. We note that, our goal with this methodology is not to provide a maintenance plan but to make sure to incorporate the already scheduled maintenance into a plane's itinerary. Assuming that a plane mainly goes under unscheduled maintenance at the place it breaks down, we determine the start time and location of an unscheduled maintenance leg according to our solution and the maintenance record provided by the company. If a plane needs to go under unscheduled maintenance at t_{sm_un} according to the maintenance record and the plane is flying a trip during this time in our solution, we set the start time of the unscheduled maintenance event as the arrival time of the trip and change the end time accordingly. The start and end station of the maintenance are then set to the arrival station of the trip. To evaluate the effect of unscheduled maintenance, we use the real demand and the maintenance data for a month, in which 937 legs (1,441.1 hours) are covered with 35 available aircraft. During this month 197 scheduled, 104 overnight unscheduled, and 49 mid-day unscheduled maintenance events occur. We solve the scheduling problem with two scenarios: incorporating scheduled and overnight unscheduled maintenance; and incorporating scheduled and all of unscheduled maintenance events that occurred during this month. The designated crew swaps occur twice a week (on Tuesday and Thursday) and crew stays with plane during its duty period.

In Scenario 1, for each three-day planning period, the scheduled and overnight unscheduled maintenance events are added to the demand legs as special legs. In Scenario 2, after an integer solution is obtained for the problem in Scenario 1, we check if any mid-day unscheduled maintenance events exist for the current day. If so, we first fix the pairings for the trips finished before the unscheduled maintenance event occurs, then we add the maintenance leg and re-solve the scheduling problem. The characteristics of the schedule and a break down of operational costs are presented in Table 2.4. The results indicate that the mid-day unscheduled events, 14% of total number of maintenance requests in the month, increase the overall operational cost by up to 12.5%. This finding suggests that implementing a preventive maintenance program and/or forecasting unscheduled maintenance events based on aircraft maintenance history and incorporating this data into the scheduling algorithm are worthwhile directions to consider further.

Table 2.4 Schedule and cost characteristics with maintenance considerations

Scenario	Reposition Hours	Reposition Ratio	Reposition Cost	Upgrade Cost	Transportation & Overtime Cost	Charter Cost	Total Cost
1	732.06	0.337	634,708	29,869	104,951	27,283	796,811
2	786.98	0.353	670,251	36,658	120,538	69,293	896,740

The break-down costs of two scenarios are listed in Table 2.4. The results indicate that mid-day unscheduled events increase the overall operational cost by up to 12.5%. The steep increase in the cost is due to the increased repositioning hours flown and the fact that charter flights are required to cover some of the legs. It suggests that implementing a preventive maintenance program and/or incorporating unscheduled maintenance events based on aircraft maintenance history into the scheduling algorithm are worthwhile directions to consider further.

2.3.2 Crew Swap Strategies

In this section, we investigate the effect of adopting flexible crew swapping strategies on the operational costs. First, we analyze if designating four days a week for crew swaps instead of two is profitable. In other words, we compare dividing the crew into two pairs (four groups) versus four pairs (eight groups) for the seven-day off/on duty period. Next, we evaluate the effect of frequent crew swapping during a duty period by reassigning free crew whose aircraft goes under a long maintenance instead of swapping only on the designated duty shift days.

2.3.2.1 Increasing the number of designated crew swap days

The results presented in Table 2.3 for Scenarios 1 and 2, assumed crew swaps occurring twice a week. Using the same demand data and aircraft maintenance information as in Scenario 2, in Scenario 3, we allow the crew pairs to swap four times a week, by letting sets of four crew pairs to swap on Monday through Thursday. The total cost varies from week to week for both scenarios. However, neither is much better than the other.

Table 2.5 Schedule and cost characteristics with crew swap strategies

Scenario	Reposition Hours	Reposition Ratio	Reposition Cost	Upgrade Cost	Transportation & Overtime Cost	Charter Cost	Total Cost
2	786.98	0.353	670,251	36,658	120,538	69,293	896,740
3	788.54	0.354	691,065	40,473	136,600	44,563	912,701

The cumulative results of this computational study for a month are presented in Table 2.5. We note that the overall cost increases by 1.8%. Hence, we conclude that changing the number of designated crew swaps during a week to 4 times from 2 does not improve the overall operational performance. Furthermore, from the management point of view, it is more practical to operate if the number of designated crew swap days during a week is two. The rest of the computational study assumes that the designated crew swaps occur twice a week.

2.3.2.2 Separating crew from aircraft

In all scenarios above, we assume that the number of on-duty crew is the same as that of aircraft and a crew waits at the maintenance station or is sent to its crew base when its aircraft goes under maintenance. Hence there is a one to one correspondence between the number of on-duty crews and available aircraft, which results in the assumption that an available aircraft is assigned to a single crew during the crew's duty period. This assumption simulates the actual model of operations for most fractional management companies. When the crew is not separated from an airplane during its duty period the cost and the long elapsed time incurred when the crew is transported by commercial airlines is avoided. However under this assumption the utilization of crew and aircraft decreases. Under some circumstances, transporting a crew to fly another aircraft can improve the utilization of crew and aircraft. For instance, FAA regulations require that a crew cannot fly beyond its fly and duty time; its aircraft, however, is idle to be flown by another crew. Therefore, most fractional management companies create their initial schedules with this assumption and modify the schedules later on in an ad hoc manner if assigning a new crew to an aircraft seems to be profitable.

To increase plane utilization by separating crew from the airplane in the middle of a crew's duty period, we analyze two possibilities: when a plane goes under long maintenance its crew becomes free to pick up another plane or the management company maintains extra crew. First, we consider crew-swapping opportunities created when an aircraft needs a long maintenance that lasts more than one day hence freeing its crew to be reassigned to another aircraft. Under such a circumstance, a swap may occur between the free crew and a crew who has used up its allowable fly and/or duty hours for the current day. In Figure 2.3a, the free crew C*f* first covers leg 1 and takes P1 to the maintenance station. Then it is reassigned to fly another available plane P5 at the arrival station of leg 5 after the crew C5 finish the duty node 3-5 (Figure 2.3b), or an unassigned plane P3 to cover a duty node 8. As a result, when one or more aircraft go under long maintenance the number of on-duty crew becomes greater than that of available aircraft and different crews are assigned to operate the same aircraft within a day.

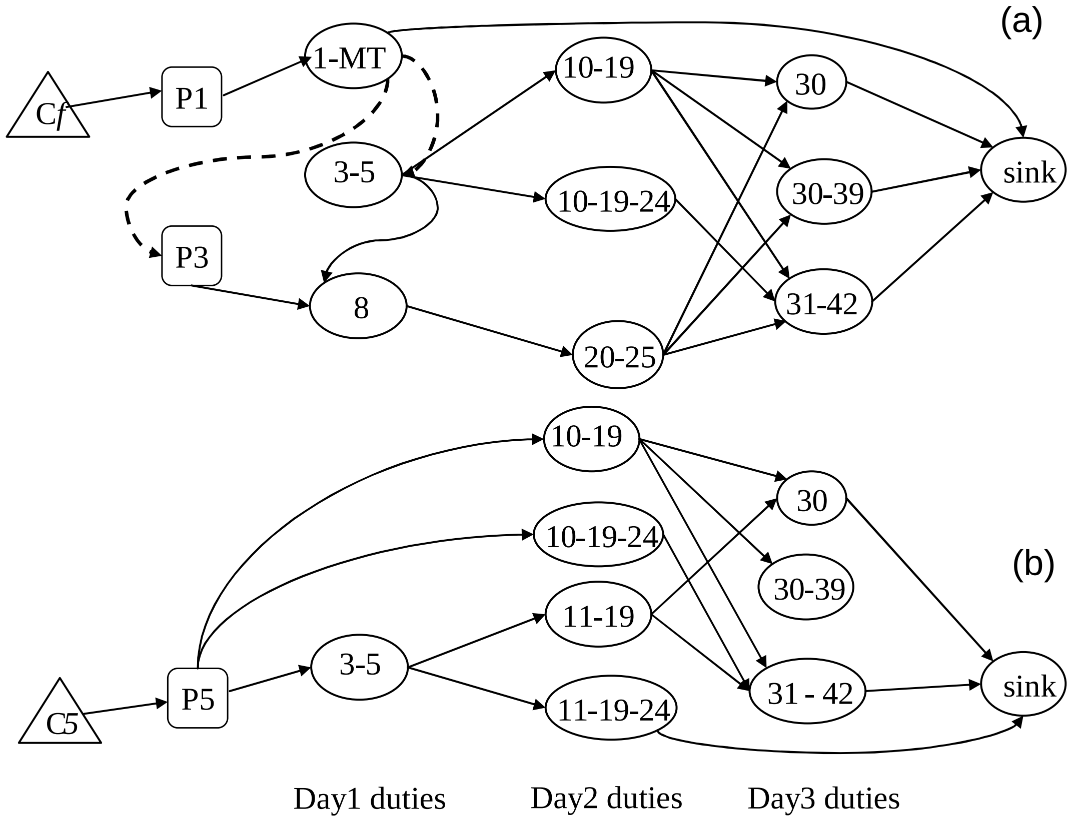


Figure 2.3 Reassignment of the free crew.

In this setting, the free crew is treated as a special coming-duty crew, whose available days are the number of days remaining in its duty period instead of seven days. This special crew can pick up an idle aircraft and cover some of the legs on the same day the swap occurs. Here, we identify an idle aircraft as an aircraft that has just finished its maintenance or an aircraft whose original crew has finished its duty for the current day. In either case, the free crew transportation time and cost are taken into account. After the swap, the free crew becomes an on-duty crew who can either fly uncovered legs during the current day or the next day's early legs that cannot be legally flown by the original on-duty crew. In the meanwhile, the original on-duty crew becomes a new free crew that waits for reassignment.

We demonstrate the effects of this crew swapping strategy with a computational study based on the same monthly data used in Scenario 2. The operational characteristics and costs of the monthly schedule obtained with this model (Scenario 4) are presented in the second row of Table 2.6. We conclude that the total cost was reduced by 11.7% with the new crew swapping strategy. We particularly note that, this strategy decreases the charter cost by 77% and the transportation and overtime costs by 30%. Furthermore, the number of leg hours flown over the month by a company aircraft was increased by 0.6% and the sum over the month of the number of aircraft that actually flew at least one leg during a day was decreased by 3%. When we analyze the daily plane utilization rates (that is the ratio of leg hours flown by company aircraft to the number of aircraft that covered at least one leg during the day) we observe improvements of up to 15%.

Table 2.6 Schedule and operational cost characteristics with crew and aircraft separation

	Reposition Hours	Reposition Ratio	Reposition Cost	Upgrade Cost	Transportation & Overtime Cost	Charter Cost	Total Cost
Scenario 2	786.98	0.353	670,251	36,658	120,538	69,293	896,740
Scenario 4	766.79	0.347	652,422	38,872	84,704	15,893	791,891
Scenario 5	761.84	0.346	645,385	39,014	83,427	15,893	783,719

We next evaluate if even further operational efficiencies can be obtained when the management company keeps extra crew during a planning period. This way more than one crew can be assigned to one aircraft during a day even when long maintenance events do not take place. The extra crew is treated in the same way as the free crew and the model is modified similarly. Row 3 of Table 2.6 (Scenario 5) presents the results of a computational study on the previous monthly data when only one extra crew is kept and swapping of crew whose aircraft goes under long maintenance is allowed. In this scenario, we assume that the home base for the extra crew is HPN, the most active station for CitationShares. The results indicate that including the extra crew provides a 1% cost improvement over Scenario 4. Hence, we conclude that it will only be profitable to keep an extra crew, if on average the monthly cost for hiring a crew (a pair of pilots) is no more than 1% of the monthly operational cost. In further computational testing, we saw that if more than one extra crew is kept they remain idle.

CHAPTER 3

A NEW MODEL AND BENDERS' DECOMPOSITION

In the previous section, we discussed approaches to increase crew and aircraft utilization by reassigning a free crew to an available aircraft. In this section, a more flexible model that allows feasible crew swap at any time, instead of only the case when a plane goes into long maintenance, is developed. Intuitively, a plane can fly all day long, but a crew has to follow FAA regulations. Besides reassigning a free crew, two crews can swap when they locate at the same station. For example, a crew, called early crew, finishes its duty earlier than another crew at the same station, but the first plane needs to be kept on the ground for overnight maintenance or other reasons; then the early crew can start to fly another plane on the ground immediately after its overnight rest to increase the utilization.

To consider the crew swap at the same station, we create a duty-based fleet-station time line, which is a unique extension from the original fleet-station time line applied in commercial airlines for solving FAM, to record the plane activities. With such a fleet-station time line structure, all crew swap possibilities are considered so crew can swap when it is feasible. A new model that integrates the fleet assignment, aircraft routing, and crew scheduling is proposed in this chapter.

Instead of sequentially solving the fleet assignment, aircraft routing, and crew scheduling, integrating them into one model is attractive because the three phases are correlated. Some recent research has integrated two consecutive phases of the above three. Some interesting contributions regarding to the integration of the fleet assignment and aircraft routing problem are presented by Desaulniers et al. (1997) and Barnhart et al. (1998b). Desaulniers et al. (1997) address daily aircraft routing and scheduling for a heterogeneous aircraft fleet to maximize the anticipated profits. They introduce two approaches: a set-partitioning model and a time constrained multi-commodity network flow model. Barnhart et al. (1998b) propose a flight string model to solve fleeting and routing problems.

Cordeau et al.(2001), Klabjan et al. (2002) and Cohn and Barnhart (2003) have shown that integrating the aircraft routing and crew scheduling problems can obtain significant better solutions than solving the problems sequentially. Klabjan et al. (2002) propose a solution approach to integrate aircraft and crew pairing by considering time window and plant count constraints in the crew pairing problem. Cordeau et al. (2001) and Mercier et al. (2003) introduce a model to integrate aircraft routing and crew scheduling with Bender’s decomposition approach. Cohn and Barnhart (2003) incorporate aircraft maintenance routine to solve crew-scheduling problem.

However, there are only few papers on integrating all the three phases. Research related to the introduction of maintenance and crew considerations in the fleet assignment problem is discussed in Clarke et al. (1997), Rushmeier and Kontogiorgis (1997) and Barnhart et al. (1998c). An integrated approximation model from fleet assignment and crew pairing model in Barnhart, et al. (1998c) is based on a former formulation on crew pairing problem, which is called duty-based pairing problem (DPP) by Vance et al. (1997). The model combines the basic fleet assignment model and DPP, and an advanced sequential solution approach is developed, where an integrated approximation model is solved to provide fleeting decision, then one crew pairing problem for each fleet is solved. Recently, Sandhu and Klabjan (2004) develop two solution methodologies to solve an integrated model, combining the three phases together, for a major commercial airline. One approach is a combination of Lagrangian relaxation with column generation and another is the Bender's decomposition approach. However, there is no literature available on the investigation of the integrated approach to apply on non-commercial airline operations planning, whose operational characteristics are very different than commercial airline.

We put the fleet assignment, aircraft routing and crew pairing together in one model to support the planning in fractional ownership airline operations. Considering the three phases in a holistic manner can better reflect the interdependency between them. In this model, the crew constraints in crew pairing are considered in the modified FAM, aircraft routing and aircraft maintenance are combined in crew pairing. The model increases the flexibility of the planning by separating crews from the aircraft. The advantage of the proposed model is that the crew and the aircraft are no longer required to stay together all the time, so that an aircraft can be used by any available crew.

This chapter is organized as follows. Section 3.1 reviews the basic FAM model. Section 3.2 introduces the integrated model based on the unique attributes in fractional airline. Section 3.3 presents some computational results.

3.1 Basic FAM

Sherali et al. (2005) provides an overview for the fleet assignment problem in airline. The paper discussed the basic FAM under the concept called the same-every-day fleet assignment, which means the same fleeting decision is used again and again for all the days. Two principal network structures are stated for formulating the fleet assignment problem: connection network, addressed in Abara (1989) and Rushmerier and Kontogiorgis (1997), and time-space network, adopted in Berge and Hopperstad (1993) and Hane et al. (1995). The idea of time-space network for the basic FAM is adopted in this thesis.

In the basic FAM, the solution should satisfy the aircraft balance constraints, which are controlling the activities at each station with a time line for each fleet. The activities can be described in a fleet-station time line. It records the departures and arrivals at the station for each fleet (Figure 3.1) to preserve aircraft flow conservation. Hane et al. (1995) originally creates it for commercial airline planning. In the time-line network, each node represents a departure time from a station or, more precisely, a *ready time* at a station. The ready time is the time when an aircraft is ready to takeoff after it arrives at the station. The balance is maintained by the flow conservation on a time-expanded multi-commodity network. Hence, the circle in the time line ensures the circulation through the network so that an aircraft arriving at the station must depart from the same station. There are two types of arcs: *flight arcs* and *ground arcs*. A flight arc represents a flight in the schedule starting from its departure node, or ending at its arrival node in the network. In Figure 3.1, arcs from 1 to 6 are flight arcs. A ground arc connects two successive nodes at one station in the network. It counts the number of planes in the fleet on the ground between the two nodes.

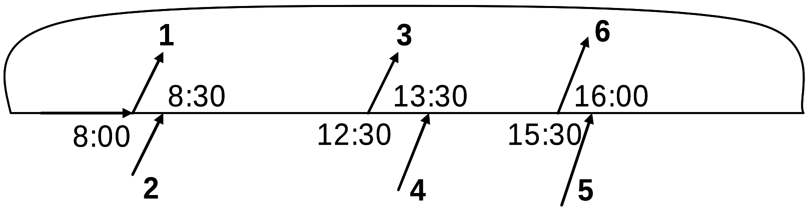


Figure 3.1 Station-fleet time line in commercial airlines

Barnhart, et al. (1998c) gives the following basic FAM model. Let *L* be the set of legs and *T* be the set of fleet type as in Chapter 2. N^f is the set of nodes in fleet *f*'s fleet station-time network, and G^f is the set of ground arcs in fleet *f*'s network. The objective of FAM is to minimize the total cost of assigning fleet type *f* to leg *k*. The binary decision variable y_{kf} equals 1 if fleet type *f* is assigned to leg *k*, and 0 otherwise. Another decision variable z_{gf} denotes the number of aircraft on the ground arc *g* in fleet type *f*. *V(f)* is the number of aircraft in fleet *f*. c_{kf} is the cost of assigning leg *k* to fleet *f*.

Basic FAM model:

$$Min \quad \sum_{f\in T}\sum_{k\in L} c_{kf}\, y_{kf}$$

$$s.t. \quad \sum_{f\in T} y_{kf} = 1 \qquad \forall\, k \in L \qquad (3.1.1)$$

$$\sum_{k\in L} b1_{knf}\, y_{kf} + \sum_{g\in G^f} b2_{gnf}\, z_{gf} = 0 \qquad \forall\, n \in N^f, \forall\, f \in T \qquad (3.1.2)$$

$$\sum_{k\in L} e1_{kf}\, y_{kf} + \sum_{g\in G^f} e2_{gf}\, z_{gf} \le V(f) \qquad \forall\, f \in T \qquad (3.1.3)$$

$$y_{kf} \in \{0,1\} \qquad \forall k \in L, \forall f \in T \qquad (3.1.4)$$

$$z_{gf} \geq 0 \qquad \forall g \in G^f, \forall f \in T \qquad (3.1.5)$$

where,

$$b1_{knf} = \begin{cases} -1, & \text{if flight arc (leg) } k \text{ leaves node } n \text{ in fleet } f\text{'s network;} \\ 1, & \text{if flight arc } k \text{ enters node } n \text{ in fleet } f\text{'s network;} \\ 0, & \text{otherwise.} \end{cases}$$

$$b2_{gnf} = \begin{cases} -1, & \text{if ground arc } g \text{ enters node } n \text{ in fleet } f\text{'s network;} \\ 1, & \text{if ground arc } g \text{ leaves node } n \text{ in fleet } f\text{'s network;} \\ 0, & \text{otherwise.} \end{cases}$$

$$e1_{kf} = \begin{cases} 1, & \text{if flight arc } k \text{ cross the ending time in fleet } f\text{'s network;} \\ 0, & \text{otherwise.} \end{cases}$$

$$e2_{gf} = \begin{cases} 1, & \text{if ground arc } g \text{ cross the count time in fleet } f\text{'s network;} \\ 0, & \text{otherwise.} \end{cases}$$

Constraints (3.1.1) ensure each leg will be assigned to exactly one fleet type *f*. Constraints (3.1.2) force the balance of aircraft flow in the network. Constraints (3.1.3) make sure that the total number of aircraft on the ground and in use does not exceed the available number of aircraft in the fleet at the count time.

The operations of fractional airlines are different from that of commercial airlines because of the reposition and changing demand. Thus the FAM corresponding to commercial airlines is not applicable. We propose an integrated model for fractional airlines operations in the following sections and start with an amended fleet-station time line.

3.2 The Fleet-Station Time Line in Fractional Airlines

Flights in commercial airlines operate mostly the same every day, while the flights in fractional airlines differ from day to day. Therefore, the fleet-station time line does not contain a repeating cycle as modeled in commercial airlines. The value of the ground arcs is constrained to be greater than or equal to zero to represent the number of planes on the ground. A departure can only be feasible when there is at least an aircraft on the ground and ready to takeoff.

Moreover, in the time line network for fractional airlines, the repositions must also be included as the flight arcs besides the customer requested flights to record the departures and arrivals at the station. However, if all possible repositions were indicated in the time line, the number of rows for constraint (3.1.2) would be overwhelming. Therefore, instead of using flight arcs and ground arcs, we present a duty-based fleet-station time line, created based on the crew duty network, with ground arcs and crew's duty arcs. A ground arc connects two consecutive nodes at one station in the time line. A duty arc indicates a crew's duty, containing a sequence of flights. In the duty-based fleet-station time line, a node represents the departure time of a duty or the ready time for the next take off.

Table 3.1 Flight schedules

Leg ID	Departure Time	Arrival Time	Departure Station	Arrival Station
1	8:00	11:00	S1	S4
2	6:00	8:30	S2	S1
3	12:30	15:00	S1	S3
4	10:00	13:30	S3	S1
5	13:00	16:00	S4	S1
6	15:30	18:00	S1	S2

In previous section, Figure 3.1 shows the activities incurred at station S1. Assume Table 3.1 gives the schedules for the flights that drawn in Figure 3.1. Let two crews available at stations S2 and S3 respectively. Assume that the feasible duties arcs for the crew at S2 are: d(2), d(2-3), d(3), and d(6). Numbers in these duties represent the leg ID. Another crew can cover one of the two duties d(4) and d(4-6). Therefore, four duty arcs leave S2 and two duty arcs leave S3. The take off time of a duty is the departure time of the first leg in the duty, if the crew is available at the first departure station. Otherwise, the crew has to reposition to the first departure station in a duty. We assume the reposition occurs at its latest possible time. This means the departure time of a reposition is the latest time that the crew has to takeoff so that the crew can fly the demand leg on time. The minimum turn time, assumed to be 45 minutes, between two trips should be taken into account. For instance (Figure 3.2), the crew who locates at S2 has to reposition to S1 to cover duty d(3). The latest take off time of the duty is 9:15. The ready time of a duty is the arrival time of the last leg in a duty plus the minimum turn time. So the ready time for duty d(3) is 15:45, the same as duty d(2-3).

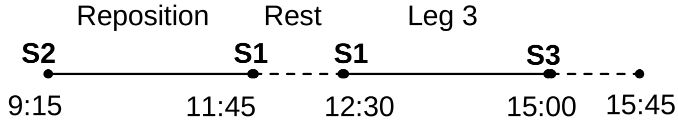


Figure 3.2 An example of how to decide when a duty starts and when it ends.

With the configuration descried above, a duty-based fleet-station time line in fractional airline for this example can be presented in Figure 3.3. All duty arcs start from station S2 or S3 where the two crews located. Because no duty starts from or ends at station S4, there is no time line records the duty activity for S4 in this fleet for this example. Leg 1 and leg 5 are not covered by these two crews but they can be covered by other crew who fly a plane in the same fleet or a larger plane.

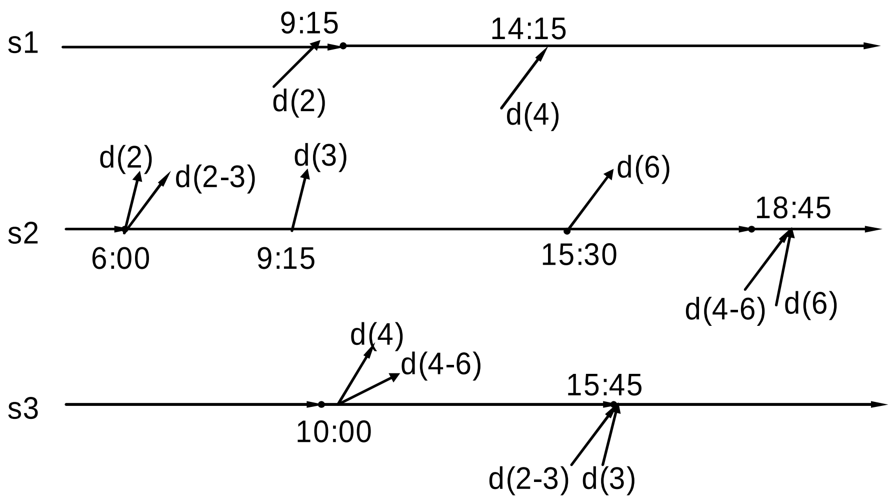


Figure 3.3 Duty-based fleet-station time line for one fleet

The discussion can be taken a step further about the crew's duty arcs in the duty-based fleet-station time line when maintenance is considered. As stated in Chapter 2, when an aircraft needs to go under maintenance for a long time, the crew is then called *free crew*, and is free to be reassigned to another available aircraft. The reassignment allows the crew to reach two duty nodes in one day. One is before the reassignment and the other one is after the reassignment. The whole duty for this crew should be indicated into two segments: an early duty whose last leg is the maintenance, and a later duty that the crew flies other flights with another aircraft. We give a simple example for one-day operations in a fleet in a time-space duty network (Figure 3.4).

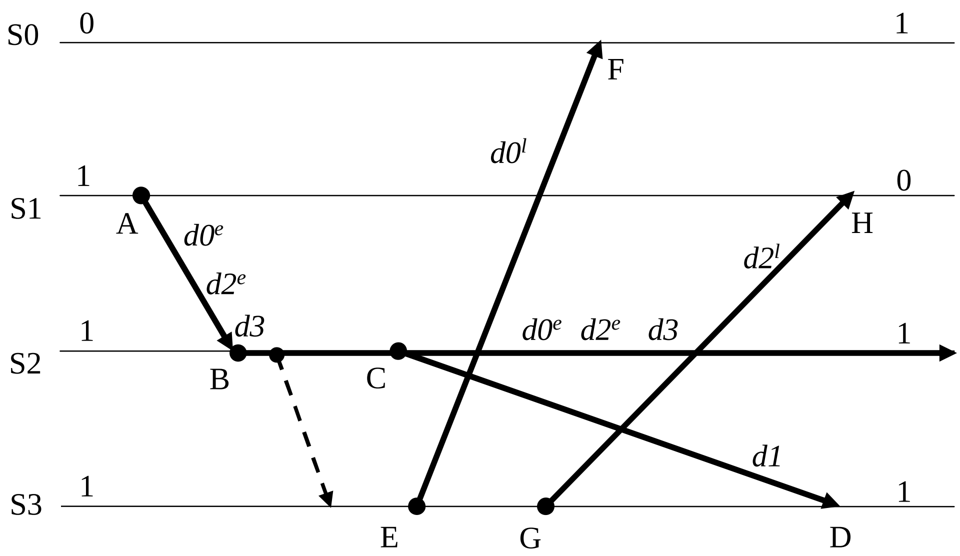


Figure 3.4 An example of time-space duty network for one fleet.

Assume crews C1 and C2 are available at stations S1 and S2 respectively and three aircraft are available at stations S1, S2, and S3. The aircraft at S1 needs go under maintenance at S2. It is possible that C1 takes the aircraft to S2 for its maintenance service that starts after time B (early duty AB-MT). Here, MT represents maintenance leg. Note that MT only means the crew has to fly the aircraft to its maintenance station before the maintenance starts, not fly MT itself. Then C1 travels to S3 and finishes the later duty EF or GH with the idle aircraft. The dashed line indicates that the crew travels from S2 to S3 via commercial airline. In this case, we have to divide both duties *d0* (AB-

MT-EF) and *d2* (AB-MT-GH) into two segments to keep information about changing aircraft. If it is an AF duty or an AH duty directly, the solution will be infeasible since the segment EF or GH will have to be covered with an aircraft which is actually under maintenance. Another feasible duty for this C1 is *d3* (AB-MT) and then stays with the aircraft at the maintenance station S2. Crew C2 covers duty *d1* (CD). However, leg EF and GH will not be covered, and need charter.

The fleet-station time line for this example is shown in Figure 3.5. Again, the nodes are the first take off time of each duty and the ready time for the next take off. Therefore, the duties $d0^e$, $d2^e$, and $d3$ leave at point A and are ready at point BB when MT leg finishes its maintenance service. Duties $d0^l$, $d2^l$ and $d1$ leave at point E, G, and C and are ready at point F', H' and D' respectively. points F', H', and D' are shifted from F, H, and D by the minimum turn time. Arcs Z_0 to Z_{11} are the ground arcs.

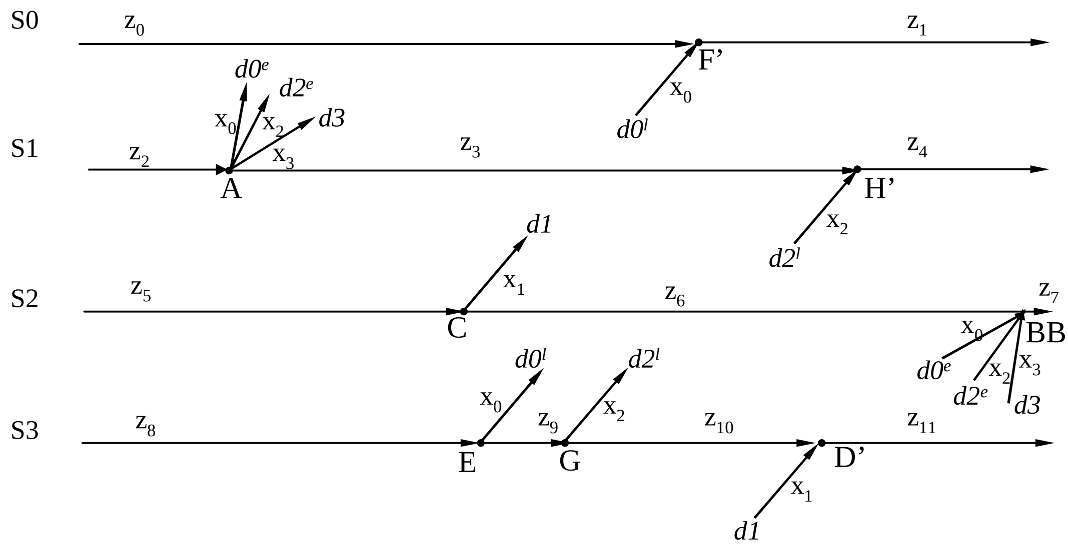


Figure 3.5 Duty-based fleet-station time lines for one fleet with a maintenance event

If the initial number of aircraft on the ground at each station is expressed with $\{Z_0, Z_2, Z_5, Z_8\}$ and their value equal to {0, 1, 1, 1}, then the summation of them should match the summation of the final number of aircraft on the ground at each station, which is $\{Z_1, Z_4, Z_7, Z_{11}\}$ would be {1, 0, 1, 1} in the solution (see example in Section 3.3).

3.3 The Formulation for the New Model

A new model is presented to include the duty-based fleet-station time line with the objective to minimize the total cost, which consists of reposition costs, upgrade costs, travel costs and charter costs. We define the following parameters:

L set of customer flights of fleet f in the planning period,

W set of crews,

N^f set of nodes in the fleet-station time line network of fleet f,

T set of fleet types,

G^f set of ground arcs in the network of fleet f,

$G^f_{Initial}$ set of ground arcs before the first node in each station time line in fleet f,

CP^f set of all columns representing the possible pairings,

$V(f)$ number of aircraft on the ground in fleet f in the beginning of a planning period (constant),

c_j cost of column j, which includes reposition cost, upgrade cost and travel cost. A column is a feasible pairing for a fleet, since the crew for this pairing can only fly one specific fleet type.

r_k chartering cost for flight k

A_{kj} 1 if flight k is included in column j, and 0 otherwise.

F_{wj} 1 if crew w flies the sequence of flights in column j, and 0 otherwise.

C_{nij} 1 if pairing j has duty i and it enters node n, –1 if pairing j has duty i and it leaves node n in the network of fleet f, and 0 otherwise.

D_{ngf} 1 if ground arc g leaves node n, –1 if ground arc g enters node n in the network of fleet f, and 0 otherwise.

The decision variables are:

x_j 1 if the solution picks a pairing at column j , and 0 otherwise.

s_k (slack variable) 1 if flight k is covered by a charter, and 0 otherwise.

z_{gf} the number of aircraft in fleet f on the ground arc g.

Given the initial value of z_{gf} for each $g \in G^f_{Initial}$ in fleet f with $V(f)$ in the beginning of the planning period, The new model (Q2) that uses fleet-station time line then is formulated as follows:

$$
\begin{array}{llll}
(Q2) \;\; Min & \sum_{f \in T} \sum_{j \in CP^f} c_j x_j + \sum_{k \in L} r_k S_k & & \\
s.t. & \sum_{f \in T} \sum_{j \in CP^f} A_{kj} x_j + s_k = 1 & \forall\, k \in L & (3.3.1) \\
 & \sum_{f \in T} \sum_{j \in CP^f} F_{wj} x_j \le 1 & \forall\, w \in W & (3.3.2) \\
 & \sum_{i: j \in CP^f} C_{nij} x_j + \sum_{g \in G^f} D_{ngf} z_{gf} = 0 & \forall\, n \in N^f, \forall\, f \in T & (3.3.3) \\
 & z_{gf} = V(f) & \forall\, g \in G^f_{initial},\; \forall\, f \in T & (3.3.4) \\
 & x_j \in \{0,1\} & \forall\, j \in CP^f, \forall\, f \in T & \\
 & s_k \in \{0,1\} & \forall\, k \in L & \\
 & z_{gf} \ge 0 & \forall\, g \in G^f, \forall\, f \in T &
\end{array}
$$

Constraints (3.3.1) are the leg covering constraints, which require that every leg k in L to be covered either by a company's aircraft or a charter aircraft. Constraints (3.3.2) are the crew constraints, which ensure that a crew is assigned to only one pairing. The aircraft balancing constraints (3.3.3) make sure the aircraft flow conservation. Constraints (3.3.4) initialize the number of planes available at the very beginning of each fleet-station time line.

In the simple example that given in Figure 3.4 and 3.5, let variables x_0 to x_3 represent pairings for the one-day period; s_0 to s_4 represent slack variables that customer's legs AB, EF, CD, GH, and the maintenance leg MT have to be chartered. Then the problem will be formulated as follows with assumed costs.

Minimize

obj: $3000\, x_0 + 1000\, x_1 + 2900\, x_2 + 1000\, x_3 + 4000\, s_0 + 6000\, s_1 + 4000\, s_2 + 5600\, s_3 + 100000\, s_4$

Subject To

$x_0 + x_2 + x_3 + s_0 = 1$ $x_0 + s_1 = 1$ $x_1 + s_2 = 1$ $x_2 + s_3 = 1$ $x_0 + x_2 + x_3 + s_4 = 1$	Leg coverage
$x_0 + x_2 + x_3 <= 1$ $x_1 <= 1$	Crew constraint
$- x_0 - z_0 + z_1 = 0$ $x_0 + x_2 + x_3 - z_2 + z_3 = 0$ $- x_2 - z_3 + z_4 = 0$ $- z_5 + x_1 + z_6 = 0$ $- z_6 - x_0 - x_2 - x_3 + z_7 = 0$ $x_0 - z_8 + z_9 = 0$ $x_2 - z_9 + z_{10} = 0$ $- x_1 - z_{10} + z_{11} = 0$	Plane flow conversation
$z_2 = 1$ $z_5 = 1$ $z_8 = 1$	Initial plane location

The solution for this problem gives: x={1, 1, 0}, s={0, 0, 0, 1}, z={0, 1, 1, 0, 0, 1, 0, 1, 1, 0, 0, 1}. The objective value is 9600 and leg GH is a charter. In the solution, the ground arc z_7=1 gives the aircraft that go under maintenance is ready to be flown at point BB.

We use the same methodology, i.e., column generation presented in Chapter 2, to solve the model (Q2). However, even creating the fleet-station time line with duty arcs instead of flight arcs, the large number of nodes and arcs on the time lines for different fleets and hundreds stations prohibits efficient use of the solution approach proposed in Chapter 2. The situation gets worse for a larger fleet type, where all the possibilities of the upgrades from smaller fleet types are included. Therefore, before applying the pricing out procedure in column generation, a preprocessing on constructing the fleet-station time line to reduce the network size is critical.

The preprocessing procedure we used is called *node aggregation*, which is described in Berge and Hopperstad (1993), Hane et al. (1995), and Sherali et al. (2005). The nodes on the duty-based fleet-station time line in this thesis represent the take off time (i.e., start time) of a duty and ready time for the next take off. When recording the flow on each station, as long as the time line represents the correct connections that departure after arrival, the exact time (either start time or ready time) pertaining to each node's event does not matter since the primary use of the fleet-station time line is to preserve aircraft conservation. Therefore, consecutive readies and the following consecutive departures can share one node so that each ready at the aggregated node can be feasibly connected to any departure at this node. The node aggregation example is presented at station S5 in Figure 3.6. The first time line lists all activities at station S5 with 7 nodes, and the second one shows its node aggregation with one 3 nodes. After preprocessing, the number of rows (nodes) and columns (ground arcs) decrease significantly.

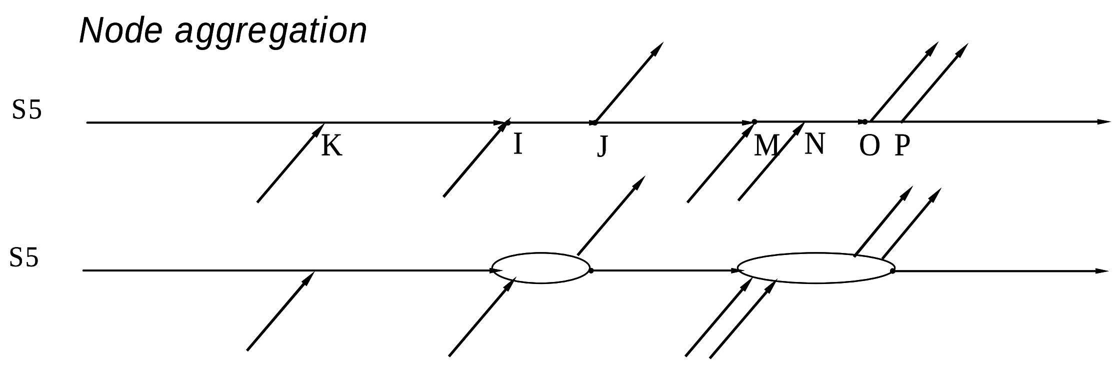


Figure 3.6 Node aggregation.

However, even after preprocessing, when the instance size increases, the solution approach described in Chapter 2 is not efficient to solve the new model (3.3.1)-(3.3.7). The number of nodes on the duty-based fleet-station time line becomes prohibitive for large size problems. For an instance that includes 3 fleets, 61 planes and 100 legs per day, the number of nodes are 745,766. The time for creating the networks grows exponentially. For a larger size, such as 5 fleets with 150 planes, we experienced out of memory computational difficulty. Hence, a new algorithm is needed to effectively reduce the number of nodes in each network.

3.4 Benders' Approach

In order to avoid the overwhelming number of nodes on the time line, we propose the use of Bender's decomposition approach. It is a partitioning method, introduced by Benders (1962), enables a divide-and-conquer strategy for solving large-scale mathematical programming problems. Benders' decomposition method has been applied to solve transportation problems (Cordeau et al., 2000, 2001; Sandhu and Klabjan 2004). In airline planning, Richardson (1976) applied Benders' algorithm in optimizing aircraft routing problem. Mercier and Soumis (2007) extends model in Cordeau et al. (2001) by including time windows and using simple Benders cuts to solve an integrated model.

In this Section, we first decide the fleet assignment for each leg and solve crew scheduling problem based on the assignment solution. The approach iteratively solves FAM as a master problem and crew scheduling and aircraft routing as a subproblem. The fleet assignment constraints along with the Benders cuts together form the restricted master problem (RMP). In each iteration, the dual of the subproblem provides cuts to the RMP. For a given assignment solution, we only solve the resulting subproblems and iteratively adjust the assignment variables until optimality is reached or a good solution with a given small gap is found. To apply the Benders decomposition, we will represent the model in the next section.

3.4.1 Benders Formulation

To be convenient, the integrated model (Q2) in Section 3.3 is re-stated as below for easy comparison.

$$
\begin{aligned}
(Q2)\quad & Min \quad \sum_{f\in T}\sum_{j\in CP^f} c_j x_j + \sum_{k\in L} r_k S_k & & \\
& s.t. \quad \sum_{f\in T}\sum_{j\in CP^f} A_{kj} x_j + s_k = 1 & \forall k\in L & \quad (3.4.1)\\
& \sum_{f\in T}\sum_{j\in CP^f} F_{wj} x_j \le 1 & \forall w\in W & \quad (3.4.2)\\
& \sum_{i:j\in CP^f} C_{nij} x_j + \sum_{g\in G^f} D_{ngf} z_{gf} = 0 & \forall n\in N^f, \forall f\in T & \quad (3.4.3)\\
& z_{gf} = V(f) & \forall g\in G^f_{initial},\ \forall f\in T & \quad (3.4.4)\\
& x_j \in \{0,1\} & \forall j\in CP^f, \forall f\in T & \\
& s_k \in \{0,1\} & \forall k\in L & \\
& z_{gf} \ge 0 & \forall g\in G^f, \forall f\in T &
\end{aligned}
$$

Instead of including all the possible fleet assignments (i.e. upgrades) as in one model (Q2), we explicitly introduce the fleet assignment variables in the new model. Let c_{kf} be the cost of assigning leg k to fleet f, which represents the upgrade cost if the customer of leg k requests a different fleet. Thus the new model (Q2) is reformulated as follows:

$$
\begin{aligned}
(Q3)\quad & Min \quad \sum_{f\in T}\sum_{j\in CP^f} c_j x_j + \sum_{k\in L} r_k s_k + \sum_{f\in T}\sum_{k\in L} c_{kf} y_{kf} & & \\
s.t.\quad & \sum_{f\in T} y_{kf} = 1 & \forall k\in L & \quad (3.4.5)\\
& \sum_{j\in CP^f} A_{kj} x_j + s_k = y_{kf} & \forall k\in L,\ \forall f\in T & \quad (3.4.6)\\
& \sum_{j\in CP^f} F_{wj} x_j \le 1 & \forall w\in W, \forall f\in T & \quad (3.4.7)\\
& \sum_{i:j\in CP^f} C_{nij} x_j + \sum_{g\in G^f} D_{ngf} z_{gf} = 0 & \forall n\in N^f, \forall f\in T & \quad (3.4.8)\\
& z_{gf} = V(f) & \forall g\in G^f_{initial},\ \forall f\in T & \quad (3.4.9)\\
& x_j \in \{0,1\} & \forall j\in CP^f, \forall f\in T & \\
& s_k \in \{0,1\} & \forall k\in L^f,\ \forall f\in T & \\
& z_{gf} \ge 0 & \forall g\in G^f, \forall f\in T & \\
& y_{kf} \in \{0,1\} & \forall k\in L^f, \forall f\in T &
\end{aligned}
$$

The objective is to minimize the operational cost, which consist of the pairing cost (i.e. the reposition cost, the crew transportation, and its overtime cost if any incurs), the charter cost, and the total upgrade cost. Fleet assignment constraints (3.4.5) make sure each leg k is only assigned to one fleet f. Constraints (3.4.6-3.4.9) is similar to the model in Q2, but decomposed by fleet without upgrade.

In the Benders approach, the fleet assignment is initially fixed to the customer requested fleet type. Hence, for some fixed fleet assignment $\bar{y} \in \{0,1\}$, the linear relaxed subproblem (SP) reads:

$$v(\bar{y}) = Min \quad \sum_{f \in T} \sum_{j \in CP^f} c_j x_j + \sum_{k \in L} r_k s_k$$

$$
\begin{aligned}
s.t. \quad & \sum_{j \in CP^f} A_{kj} x_j + s_k = \bar{y}_{kf} && \forall\, k \in L, \forall\, f \in T && (3.4.10)\\
& \sum_{j \in CP^f} F_{wj} x_j \leq 1 && \forall\, w \in W, \forall\, f \in T && (3.4.11)\\
& \sum_{i:j \in CP^f} C_{nij} x_j + \sum_{g \in G^f} D_{ngf} z_{gf} = 0 && \forall\, n \in N^f, \forall\, f \in T && (3.4.12)\\
& z_{gf} = V(f) && \forall\, g \in G^f_{initial}, \ \forall\, f \in T && (3.4.13)\\
& x_j \geq 0 && \forall\, j \in CP^f, \forall\, f \in T &&\\
& s_k \in \{0,1\} && \forall\, k \in L^f, \forall\, f \in T &&\\
& z_{gf} \geq 0 && \forall\, g \in G^f, \forall\, f \in T &&
\end{aligned}
$$

Let $\boldsymbol{\pi}=(\pi_k|k\in L)$, $\boldsymbol{\rho}=(\rho_w|w\in W)$, $\boldsymbol{\delta}=(\delta_n|n\in N^f,\ f\in T)$, and $\varsigma=(\varsigma_g|g\in G^f_{initial},\ f\in T)$ be the dual variables associated with constraints (3.4.10), (3.4.11), (3.4.12), and (3.4.13), respectively. Constraints (3.4.13) are initial plane count. The dual of above SP is the following dual subproblem (SPD):

$$
\begin{aligned}
Min \quad & \sum_{f\in T}\sum_{k\in L}\pi_k \bar{y}_{kf} + \sum_{w\in W}\rho_w \\
s.t. \quad & \sum_{k\in L} A_{kj}\pi_k + \sum_{w\in W} F_{wj}\rho_w + \sum_{i\in D^f} C_{ni}\delta_n \le c_j && \forall\ j\in CP^f, \forall\ f\in T && (3.4.14)\\
& \pi_k \le r_k && \forall\ k\in L && (3.4.15)\\
& \sum_{g\in G^f} D_{ng}\delta_n + \varsigma_{g\in G^f_{initial}} \le 0 && \forall\ n\in N^f, \forall\ f\in T && (3.4.16)\\
& \rho_w \ge 0 && \forall\ w\in W
\end{aligned}
$$

The linear relaxation SP is solved with the column generation process that described in Chapter 2. Since the SP always gives a feasible solution, the MP only consists of the Benders optimality cuts, the fleet assignment constraints (3.4.5). Let β be a free variable, the RMP then reads:

$$
\begin{aligned}
Min \quad & \beta + \sum_{f\in T}\sum_{k\in L} c_{kf}\, y_{kf} \\
s.t. \quad & \sum_{f\in T} y_{kf} = 1 && \forall\ k\in L && (3.4.17)\\
& \beta - \sum_{f\in T}\sum_{k\in L}\pi_k y_{kf} \ge \sum_{w\in W}\rho_w && && (3.4.18)\\
& y_{kf} \in \{0, 1\} && \forall\ k\in L, \forall\ f\in T && (3.4.19)
\end{aligned}
$$

3.4.2 Basic Algorithm

Let τ be the iteration counter. Let U be the set of extreme points defined by SPD (3.4.14)-(3.4.16). The basic Benders algorithm is summarized as follows.

1. Initialization: set $\beta^1 = -\infty$, $\tau=1$, $U^1=\emptyset$. Also chose $\bar{y}^1$=customer requested fleet.

2. Solving the master problem RMP, let $\bar{y}^\tau$ be an optimal solution of the RMP and gives a lower bound.

3. Solving the subproblem SP: taking $\bar{y}^{\tau}$ as an input. Because the SP is always feasible and finite, let $\mathbf{x}^{\tau}$ be an optimal solution of the SP and $(\boldsymbol{\pi}, \boldsymbol{\rho}, \boldsymbol{\delta})^{\tau}$ be a dual optimal solution of SPD given as an optimal extreme point.

    a. If $v(\bar{y}^{\tau})=\beta^{\tau}$, $(\mathbf{x}^{\tau}, \bar{y}^{\tau})$ is optimal for the linear relaxed SP and the RMP, stop the Benders procedure.

    b. Otherwise, set $U^{\tau+1}= U^{\tau} \cup (\boldsymbol{\pi}, \boldsymbol{\rho}, \boldsymbol{\delta})^{\tau}$ to generate an optimality cut.

    c. Set $\tau=\tau+1$, and go to step 2.

At each iteration, the subproblem is solved with column generation with the input of fleet assignment $\bar{y}^{\tau}$, meanwhile, exactly one constraint, an optimality cut, is added to the RMP. The Benders process terminates at step 3a with an optimal solution $(\mathbf{x}^{\tau}, \bar{y}^{\tau})$. With the optimal fleet assignment solution $\bar{y}^{\tau}$ yielded at step 3a, the integrality constraints are added back to the crew pairing subproblem. Then we solve this integer programming sublproblem. This subproblem is solved only once and the integer solutions for the original model Q2 are obtained.

3.4.3 Refinements of Benders Approach

With the decomposition, the problem size is reduced. In the meanwhile, we have observed that the solution time is increased from minutes to hours because the algorithm converges very slowly. Now we need techniques to improve the process. Magnanti and Wong (1981) introduce methodologies to accelerate Benders decomposition. For our application, we improve the Benders convergence with the following refinements.

3.4.3.1 Solving Individual Subproblems for Each Fleet

It is possible that all extreme points in set U have to be reached to find the optimal solution, which results in the slow convergence. For this worst case, we can decompose the subproblem to reduce the number of extreme points that defined by the dual of the subproblem. The crew pairing problem is separable for each fleet due to the presence of non-crew-compatible fleets. Therefore, the subproblem (SP) can be decomposed into $|t|$ (the number of fleets) subproblems, one for each fleet $f \in T$. We consider the SP (3.4.10) - (3.4.13) for an individual fleet type and add corresponding cuts according to the individual dual information of the $|t|$ subproblems. Let β_f be the free variable for the subproblem (SP^f), then each dual solution of SP^f should provides an optimality cut:

$$\beta_f - \sum_{k \in L^f} \pi_k y_{kf} \geq \sum_{w \in W^f} \rho_w$$

Thus, at each iteration of the algorithm, $|t|$ potential optimality cuts are generated after solving the subproblems. If the cut has been obtained previously, it is already satisfied and therefore should not be added to the RMP. This modification is very efficient because the subproblems are solved individually for each fleet.

3.4.3.2 Relaxing the Integrality Constraints in the RMP

Another major computational bottleneck is the master problem, which is an integer program needs to be solved repeatedly. McDaniel and Devine (1977) suggested relax the integrality constraints on the variables of the master problem. The application in Cordeau et al. (2000, 2001) for solving locomotive and car assignment problems has enormous reduced the solution time by solving linear relaxation of master problem. Hence, we first relax the integrality of the fleet assignment variables $y_{kf} \in \{0,1\}$ in the RMP.

With the relaxation, the algorithm thus may not be stopped at the criteria $v(\bar{y}^{\tau})=\beta^{\tau}$. However, the linear relaxation programming of the RMP provides a valid lower bound (LB) on value β_{LP}^{τ}. The subproblem provides an upper bound (UB) on value $v(\bar{y}^{\tau})$. The algorithm then can be stopped when the difference between UB and LB less than a chosen gap $\epsilon>0$, i.e., $v(\bar{y}^{\tau})-\beta_{LP}^{\tau}<\epsilon$.

Hence, the approach in 3.2 is now modified into two phases. In Phase I all integrality constraints are relaxed and the linear relaxation of the BIM is solved to optimality with the basic Benders algorithm. Retaining all optimality cuts generated in Phase I, Phase II adds the integrality constraints back to the RMP and use the basic algorithm to solve the integer program problem with the new optimality cuts.

3.4.3.3 Adding Initial Cuts

Some valid initial cuts may help accelerating the Benders convergence. First we consider limiting the number of upgrades. For example, the number of fleet-1 legs $k \in L^{f_1}$ upgrades to fleet-2 legs cannot be more than a chosen number m_1, such that

$$\sum_{k \in L^{f_1}} y_{kf_2} \leq m_1$$

We also limit the number of fleet-1 legs $k \in L^{f_1}$ and fleet-2 legs $k \in L^{f_2}$ upgrades to fleet-3 legs to a chosen number m_2, such that

$$\sum_{k \in L^{f_1}, L^{f_2}} y_{kf_3} \leq m_2 .$$

3.4.3.4 Adjusting the RMP Solution with Line Search

In order to control the objective value of the subproblem updated in an improving direction, the input of the SP $\bar{y}^{\tau}$ needs to be adjusted at some iterations. At each iteration of the basic algorithm, we keep tracking the value of the subproblem. Let the previous and current RMP solution be $\bar{y}^{\tau}$ and $\bar{y}^{\tau+1}$, respectively. The line search is done with the following logic:

1. If $v(\bar{y}^{\tau+1}) < v(\bar{y}^{\tau})$, the value of the SP is improving, then the new input for the next iteration is $\bar{y}^{\tau+1}$, yielded by solving the RMP that contains all cuts generated before τ-th iteration.
2. Otherwise, keeping the value of the SP from moving far away from the improving direction, we chose a solution between $\bar{y}^{\tau}$ and $\bar{y}^{\tau+1}$ by comparing the fleet assignment for each leg k as follows:

    a. If leg k is upgraded, i.e., $\bar{y}(k)^{\tau+1} > \bar{y}(k)^{\tau}$, but its dual price π_k is less than a chosen value, then do not upgrade and keep the assignment as in the previous solution $\bar{y}^{\tau}$. We only upgrade a leg when its dual price π_k is higher than the chosen value and use the assignment as in the current solution $\bar{y}^{\tau+1}$.

    b. Otherwise, keep all other assignments as in solution $\bar{y}^{\tau+1}$.

With this adjustment, the value of the subproblem will not move far from an improving direction. As a result, this refinement accelerates the convergence of the Benders algorithm.

3.5 Computational Efficiency

Three approaches have been proposed: The column generation for a simple model (Q1) and a modified model that allows crew reassignment (Q1') in Chapter 2; the column generation for a new model with the duty-based fleet-station time line (Q2); and Bender's decomposition for the new model (Q3) in this Chapter. Giving different data sets over one week, a comparison on the total cost is shown in Table 3.2. It shows that the longer the planning horizon is the more cost savings are. Model Q2 generates better solutions than model Q1'. The improvement is because there are more crew swap opportunities exist in the model Q2, where a crew is separated from its original assigned plane. The difference between Q3 and Q2 is less than 0.5%, which is within the range of the chosen gap $\epsilon>0$.

Table 3.2: The cost effectiveness comparison on the three models

		Total Cost			Improvement	
Instance Size (Fleet, Plane, Leg)	Planning Horizon (day)	Column Generation (Q1')	Time Line & Column Generation (Q2)	Benders' (Q3)	Q2 vs. Q1'	Q3 vs. Q2
3, 35, 257	1	392,563	390,375	391,483	0.60%	-0.30%
	2	386,451	384,940	383,152	0.40%	0.50%
	3	381,216	377,242	375,659	1.00%	0.40%
3, 61, 557	1	813,246	809,057	807,745	0.50%	0.20%
	2	805,184	801,368	801,642	0.50%	0.00%
	3	789,962	782,651	783,579	0.90%	-0.10%
5, 75, 888	1	1,988,441	1,970,475	1,969,783	0.90%	0.00%
	2	1,974,563	1,958,374	1,954,665	0.80%	0.20%
	3	1,942,374	1,921,650	1,919,384	1.10%	0.10%

However, the solution time increases at large instance size and long planning horizon. Solving model Q3 with Benders decomposition is much slower than other two approaches, especially when the data set includes more fleet types. One reason could be too much iterations on adding cuts due to the large amount of possible fleet assignments. For one of the runs during the week in an instance that contains 5 fleets and 75 planes and a 2-day planning horizon, solving the subproblem only takes 6.2 seconds on average, but it takes 79 iterations to solve the linear relaxation model of the RMP.

Table 3.3: The computational efficiency comparison on the three models

		Avg. Solution Time Per Run (s)		
Instance Size (fleet, Plane, Leg)	Planning Horizon (day)	Column Generation (Q1')	Time Line & Column Generation (Q2)	Bender's (Q3)
3, 35, 257	1	0.31	0.88	7.22
	2	1.06	3.12	27.75
	3	5.02	38.76	155.82
3, 61, 557	1	0.85	3.65	26.47
	2	12.54	74.36	155.82
	3	68.35	120.11	374.68
5, 75, 888	1	1.59	12.06	99.45
	2	40.54	106.02	622.78
	3	464.02	697.34	1563.88

In summary, a new model Q2 is proposed in this chapter to integrate duty-based fleet-station time line with the aircraft routing problem and the crew scheduling problem. The duty-based fleet-station time line records the activities on each station in the fleet as time goes during the planning horizon. The model can be solved with the column generation technique that in described in Chapter 2. However, for large scale problems, the column generation posts a computational barrier since it creates too many nodes in the crew network. The memory required in the model prohibits effective calculation with current computation capacity. Solving an instance (5 fleets, 150 planes, and 645 legs) with model (Q2) gives an out of memory error. Benders decomposition approach reduces the problem size and provides a solution in almost three hours. Although it is very slow, Benders decomposition combined with column generation approach shows a way to ease the burden on resource requirement.

CHAPTER 4

STRATEGIC PLANNING

The methodologies proposed in previous chapters provide means to quickly evaluate business ideas, and offer valuable insights to various options. In this chapter, scenario analyses are performed to support decision making on several tactical and operational issues in fractional management companies, and the effect of these analyses on the total operational cost are discussed. First, the question "What is the right demand size for a given fleet?" is examined. Next, different marketing strategies for expanding demand are investigated, and their impacts on profitability are compared. Furthermore, the options on company-owned core planes are studied. Finally, strategies are discussed to take stochastic events into account when evaluating operational strategies during the planning period.

4.1 Effect of Demand Size

The effects of increased demand on profitability are first analyzed. In Scenarios 1, 2 and 3, the same monthly data set as in the scenario 2 of chapter 2 is used, and new legs are then added to this base demand data amounting to a 5%, 10% and 15% increase in leg hours for the month. The new legs are selected randomly with replacement from the demand data provided by CitationShares for a different month. Note that, in this analysis we assume that the fleet size stays constant.

The details of the monthly operational cost, revenue, and profit under increased demand are presented in Table 4.1. In this analysis we take into account the extra revenue (the hourly flight rate the management company charges) generated by the new demand. The extra profit earned is calculated by subtracting the extra cost from the extra revenue generated. Note that the reposition ratio does not change significantly. However, charter costs as well as operational costs, including reposition, upgrade, transportation and overtime costs, start increasing immediately with increased demand density.

Table 4.1 Schedule and operational cost characteristics with increased demand.

Scenario		# of Legs	Leg Hours	Rep Hours	Rep Ratio	Extra Cost	Extra Revenue	Extra Profit
	Base	937	1441	787	0.353			
1	**5%**	970	1514	835.9	0.356	99,734	104,993	5,259
2	**10%**	1043	1585	873.3	0.355	198,320	193,093	-5,227
3	**15%**	1071	1657	904.8	0.353	298,966	289,255	-9,711

The revenue increases as demand increases, however, profit may not follow the same trend. To analyze the profitability of demand expansion we calculate the revenue according to the hourly flight rates the customers pay to the fractional management company. Different fleet types have different hourly rates and the revenue generated is the product of the total flight hours for each fleet type with the corresponding hourly flight rate. For example, the hourly rate assumed for a CJ1 is $1,200 and the new demand contains 17.8 CJ1 flight hours, hence the additional revenue generated by the CJ1 fleet is $21,360.

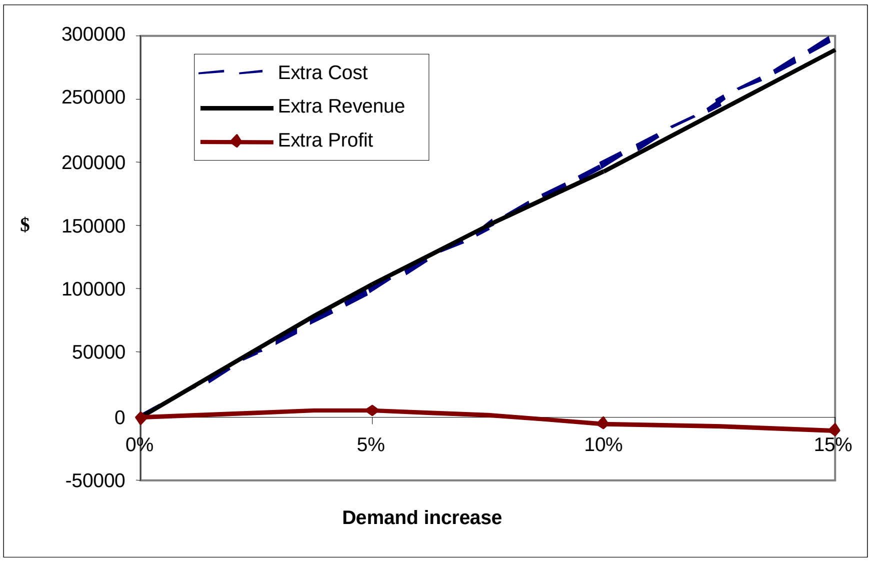


Figure 4.1 Change in profitability under increased demand with fixed capacity

When the demand is increased by 5%, the current capacity seems barely able to handle it, and the revenue increasing ratio is lower than the cost increasing ratio. Once the revenue increase is not enough to compensate cost increase, the profitability starts to drop. As demonstrated in Figure 4.1, adding new demand higher than 5% is not as profitable as one might think. It is noted that the net change in profitability under any of the three scenarios is less than 1%.

4.2 Strategies to Increase Profitability

In the following three subsections we analyze three strategies to increase profitability. From the previous analysis one concludes that creating more demand might potentially increase profit. In the first subsection, we consider increasing demand by introducing a new product, “jet-card”. Jet-cards are pre-paid flying cards, which give customers the opportunity to take business aviation advantage. Jet cards give the fractional management company more flexibility in satisfying customer demand. Hence, while generating revenue, the costs incurred per flight hour by the management company are kept lower.

Next, we study the effect of increasing demand by expanding the operations geographically. The data we use so far comes from a fractional management company that has its customer base in the east coast of United States. Hence, one possible strategy for increasing demand is to acquire new customers across the continental United States. We analyze this possibility, and demonstrate how adding new flight hours as legs which include a west coast origin and/or destination might change profitability.

4.2.1 Jet-card

A new product that has been recently introduced by most fractional management companies is a prepaid “jet-card.” Similar to a prepaid phone card, a jet-card allows customers use the business jet service in the future without purchasing the aircraft. Although it requires customer to pay up front, it gives the card owner freedom at the price lower than using charter aircraft. This concept increases the business aviation affordability and combines the safety, consistency, and guaranteed availability of fractional ownership with the simplicity and flexibility of charter (Kemp 2006).

Jet-cards target the customers who are not ready to make the contractual or monetary commitment for partially owning a jet. Many of these customers require less than 25 hours of flight time annually, however, some buy multiple cards. These programs enable individuals and companies to pre-pay for 25 hours of private flight time on the company-operated aircraft for each card purchased. In this way, the fractional management companies expand their private jet ownership business to include private jet “usership.” Jet-card holders receive almost the same benefits as fractional owners, including the safe, reliable, flexible and customer-focused service. It’s assumed that the jet-card holders act as 1/32-share owners and request two-hour trips on average.

Better flight arrangement can be achieved by creating a limited flexibility in satisfying demand, which means the ability to shift the departure time by a narrow time interval. When all the feasible duties are generated initially, the allowed flexibility in the departure times gives planner more room to route aircraft, which results in a larger number of possible duties. The duty generator is revised to accommodate the shift mechanism with a basic rule. The shift on the departure time is considered only if it enables a crew to fly an extra leg immediately before or after a leg.

Figure 4.2 explains how the shifting procedure works. In Figure 4.2, the solid lines represent the time interval of the customer legs, the dashed lines are the required reposition and turn times, and the bold arrows display the shifting departure times. The letters represent departure and arrival stations. In case (1), the duty generator does not shift the departure times because the time between the legs A-B and C-D is greater than the required reposition and turn times. On the other hand, in case (2), to operate both legs E-F and G-H with the same crew, the departure time of E-F or G-H or both must be shifted. Assuming the shift on E-F does not create an illegality for the crew, two duties are created to allow the departure time shift, either moves E-F earlier or G-H later, in a minimal way to limit the effect to the customer leg. In case (3), the departure time of the second leg K-L is earlier than the arrival time of the first leg I-J. In this case, if leg K-L can not be covered when either leg I-J or when K-L is moved given the allowable time window width, both of the departure time of I-J and K-L have to be shifted within the range of the customer agreements. Thus I-J starts early than requested, and K-L starts later than requested.

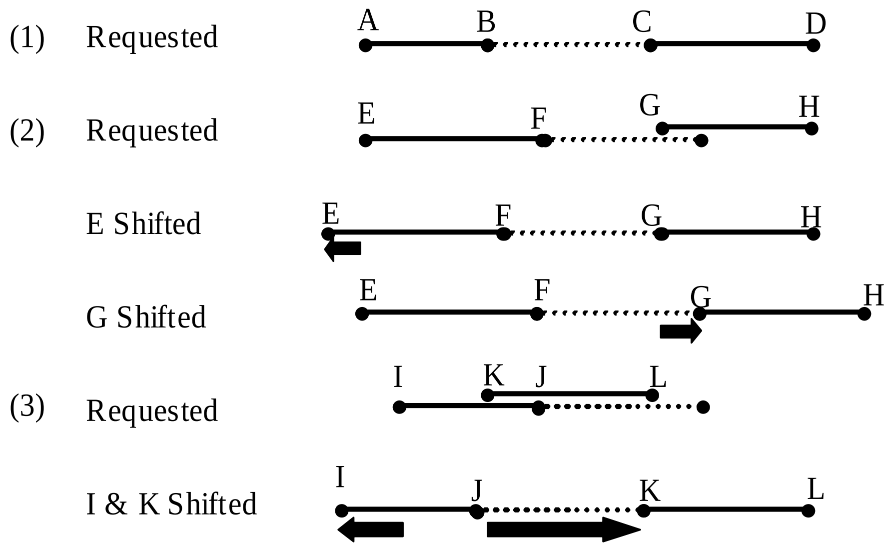


Figure 4.2 An example showing how shifting of leg departure times are executed

In scenarios, the same base data set is used and the same legs are added to increase the demand size by 5%, 10%, 15%, and 20%. The third column in Table 4.2 presents the extra profit obtained with no time window. The observation is that increasing demand only by 5% is profitable. Next, it is assumed that these card legs allow for one-hour time-window flexibility. Although allowing for leg departure time-windows is not popular across the industry yet, the analysis shows that this added flexibility has significant impact on increasing profitability. Table 4.2 shows that adding up to 15% demand as jet-card legs becomes profitable with departure time-window flexibility. However, for the same data set, only up to 5% new demand can be handled efficiently without departure time-windows. Furthermore, the average plane utilization in Scenario 4, 5, and 6 increases by 4.8%, 9.2%, and 13.6% with time-window policy.

Table 4.2 Extra profits made when card demand increased

Scenario		**Extra profit Without TW**	**Extra profit with TW**
	Base	0	0
4	5%	5,499	10,263
5	10%	-5,227	13,258
6	15%	-9,935	8,974
7	20%	-15,857	-6,453

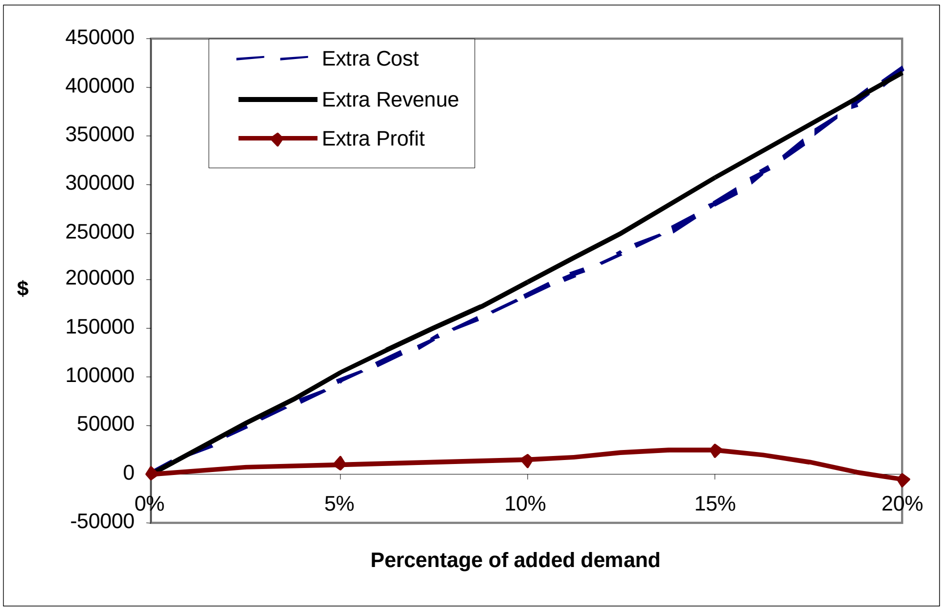


Figure 4.3 Changes in profitability on increasing demand with time windows

To examine the result sensitivity on the size of demand, four other data sets are selected: a real data in July 2005, two similar data generated from demand in April 2006, and a large data set that is doubled on the size of data in April 2006. Requests from jet-card holders are mixed with those from regular fractional owners. The analyses show at least 5% cost improvement with the flexibility of the departure time on the jet-card demand in Table 4.3. The owner hours are the flight hours that requested by owners and jet-card hours are the flight hours that requested by jet-card holders. The leg ratio in the fifth column is the ratio of the number of jet-card legs to the total legs, and the hour ratio in the sixth column is the ratio of the flight hours of jet-card to the total flight hours.

Table 4.3 Cost savings on the jet-card with one hour flexible departure time

Data Size Fleet, Plane, Leg	Num of Card legs	Owner Hours	Jet-card Hours	Leg Ratio	Hour Ratio	Save ($)	Save %
3, 61, 2164	162	3101.05	261.42	7.53%	7.63%	273,708	5.39%
5, 75, 2847	132	4685.45	260.68	4.64%	5.27%	534,663	6.39%
5, 75, 2790	306	4340.05	573.75	10.97%	11.68%	559,762	6.80%
5, 150, 5639	636	8684.45	1177.42	11.28%	11.94%	787,521	6.30%

4.2.2 Expanding Operations Geographically

In this subsection, the analysis is focused on the effects of increasing demand by expanding the customer base geographically. The data used in the previous sections reflects the operations of a fractional management company with customer and crew bases in the East Coast of the US. One growth direction for the company is to acquire new customers and crew based in the West. The particular interest is to study the impact of geographical expanding strategy, which implies increasing demand by including some longer flights.

Three new scenarios are created by adding 5%, 10%, and 15% total fight hours from the same base demand data set as before, in addition, new flights are required to depart and/or arrive in the West Coast or Rocky Mountains. To create a new flight, a leg is randomly selected from another demand data set. To incorporate stations in the western region into the leg for the analyses, the following approach is used. First, the departure time for the selected leg is retained. Then, a departure and/or arrival station in the western region is randomly drawn from the 51 western region airports given by the company. Once the departure and arrival locations of the new leg are determined, the arrival time is calculated based on the distance and departure time. New legs are created repeatedly until a total of 5%, 10%, or 15% flight hours of new demand are added to the base demand.

Note that although the new leg hours are the same as before, the number of new legs is less due to the longer flight time of these legs.

The computational results for this analysis are given in Table 4.4 and Figure 4.4. Comparing the results in Table 4.4 to those in Table 4.1, the observation is that the revenue and profitability will improve when demand is increased by up to around 10% with the expansion. However, a 15% increase in demand causes a significant jump in the operational costs. These results are probably due to those longer but fewer flights for the same total flight time. Furthermore, the geographic expansion results in increased reposition ratios and operational costs due to longer distance between stations.

Table 4.4 Schedule and operational cost characteristics with operations expansion

Scenario		# of Legs	Leg Hour	Rep Hour	Rep Ratio	Extra Cost	Extra Revenue	Extra Profit	% cost incr	% Rev. incr	% Profit incr
	Base	937	1441	787	0.353	0	0	0			
8	**5%**	966	1514	856.8	0.361	95,610	104,025	8,415	10.66%	4.89%	0.68%
9	**10%**	1,005	1585	895.9	0.361	203,423	203,985	562	22.68%	9.59%	0.05%
10	**15%**	1,043	1657	982.1	0.372	337,335	303,533	-33,802	37.62%	14.27%	-2.75%

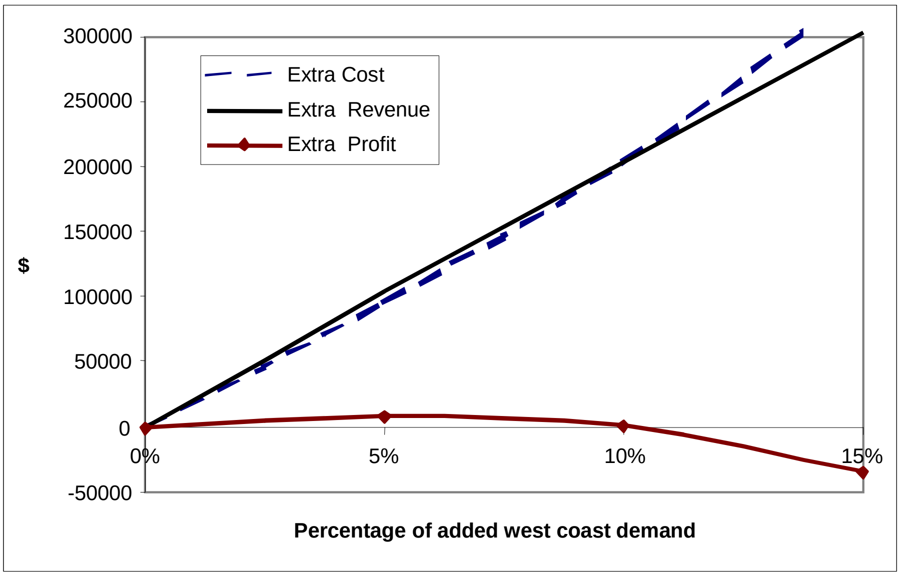


Figure 4.4 Change in profitability when operations are expanded geographically

4.3 The Right Size of Core Plane Fleet

One important factor in determining the profitability of a fractional management company is the ratio of the number of fractional owners to the number of aircraft in its fleet. In general, the management company has two categories of aircraft, customer-owned aircraft and company-owned core aircraft. Core planes supplement the customer planes to provide extra capacity when demand is high. Owning a core fleet drives down charter costs and increases customer satisfaction, since the customers prefer traveling with a company plane and crew. Frequent use of charters is not only expensive but also costs the company in terms of customers' good will. On the other hand, owning and maintaining a large core fleet with a low utilization rate hurts profitability. Hence, as the size of its business grows a fractional management company is faced with the crucial questions: how many and what type of planes should be kept in its core fleet?

To analyze how the size of the core fleet affects profitability, we use the same base demand data set from the previous computations. Within the month, the company used a fleet of 35 planes, 9 of which were in the core fleet. The remaining 26 fractionally owned planes are made up of 7 CJ1s, 9 Bravos and 10 Excels. First, the 9 core planes are removed from the available fleet. The following steps are used to add the core plane back to core fleet. One plane is added from one of the above plane types and a schedule is created for the whole month with the base demand data. Since there are three plane types, three options are created. Comparing among the three options, the plane giving the lowest operating cost to the core fleet is kept, and a new iteration starts until all of the 9 planes are added back to the fleet.

The fixed cost associated with owning and maintaining core aircraft is defined as the "core cost." It captures pilot salaries, core lease expense, insurance, and so on. Figure 4.5 presents how core cost changes versus charter and operation costs, as the core fleet size is increased. In general, the operation cost is stable. However, the charter cost decreases more rapidly than the increase in the core cost until 4 planes are added to the core. It concludes for the month that having 4 Bravos in the core provides the least cost schedule.

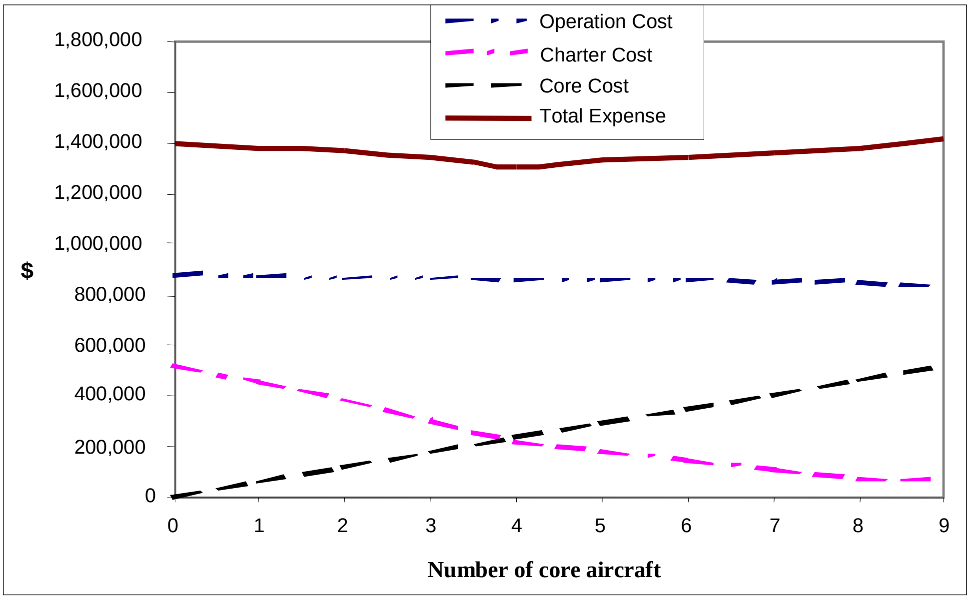


Figure 4.5 Effect of number of core planes on profitability

The same analyses are repeated when demand is increased by 5%, 10% and 15%, as in Scenarios 4, 5, and 6. Figure 4.6 presents the results of these analyses. Note that the optimal number of core planes increases as the demand increases. When the demand is increased by 5% keeping 4 or 5 Bravos gives the least cost schedule. When the demand is increased by 10% and 15% keeping 4 Bravos and 1 Excel, and 4 Bravos and 2 Excels are more profitable, respectively.

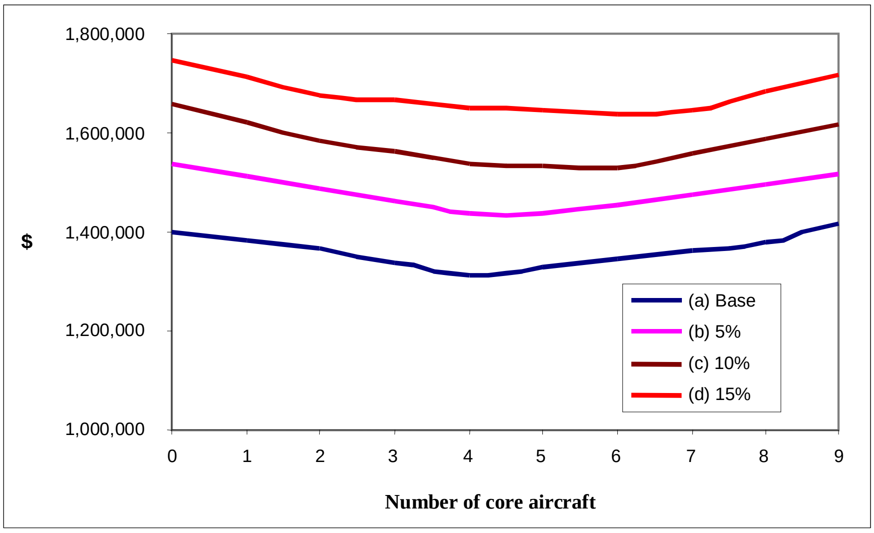


Figure 4.6 Number of core aircraft for four cases

Furthermore, different data sets are examined to obtain a picture on how many core planes should be kept when the company grows. The data sets are real data from the fractional management company, with the exception of data in large set, which are generated by folding a two-month demand data into one month and double the number of aircraft to mimic the business growth. The results in Table 4.5 show that around 10% to 15% of planes can be reserved as core planes. Note that the highest core ratio at the third column with 42 planes is corresponding to the highest average customer leg hours per leg. But it does not show an exact correlation because the lowest average leg hours per leg do not match the lowest core ratio.

Table 4.5 Number of Core Planes for Different Data Size

	June, 2003	Jan, 2004	July, 2005	April, 2006	Large Set
Data Size (Fleet, Plane, Leg)	3, 35, 937	3, 42, 1057	3, 61, 2164	5, 75, 2847	5, 150, 5639
Num of Core	4	6	8	10	19
Core Ratio	11.43%	14.29%	13.11%	13.33%	12.67%
Avg Leg Hours per Leg	1.61	1.95	1.5	1.74	1.75
Avg Leg Hours per Plane	41.73	48.92	53.19	65.95	65.74

4.4 Selecting Special Routes for Unreliable Aircraft

The analysis in Chapter 2 indicates the unscheduled maintenance is one of the primary causes of the operation expenses, the focus on the unscheduled events to limit the time lose due to aircraft out of service and improve the utilization. Some research has been reported on flight status monitoring and using prognostic maintenance to actively prevent unscheduled event from happening (Skormin 2002 and Ong 2004).

This problem can also be treated from the operational point of view, which tries to isolate the unscheduled maintenance, and minimize the propagation of the disruption. First data analysis is performed to identify unreliable aircraft, which are prone to unscheduled maintenance, and then treat them specially in scheduling so that the possible impact of unscheduled maintenance is reduced.

Two criteria, frequency and duration, are used to identify the unreliable aircraft. First, check the total number of requests for unscheduled maintenance of each individual aircraft identified by the tail number (Figure 4.7a). During the one-month testing period, there are 42 aircraft available for dispatch. 90% of aircraft went to unscheduled maintenance occasionally. Some aircraft requested more than 8 times, average once every 4 days. Figure 4.7b shows the total duration of these visits. There are four aircraft under unscheduled maintenance for more than 180 hours, which means that they are out of service in ¼ of the total time. Meanwhile, these four aircraft requested unscheduled maintenance for more than 9 times. We therefore identify them as unreliable aircraft during that month. Note that, to get an accurate estimate, one could apply more sophisticated analysis on longer period of historical data. We are providing a simple example to illustrate the methodology.

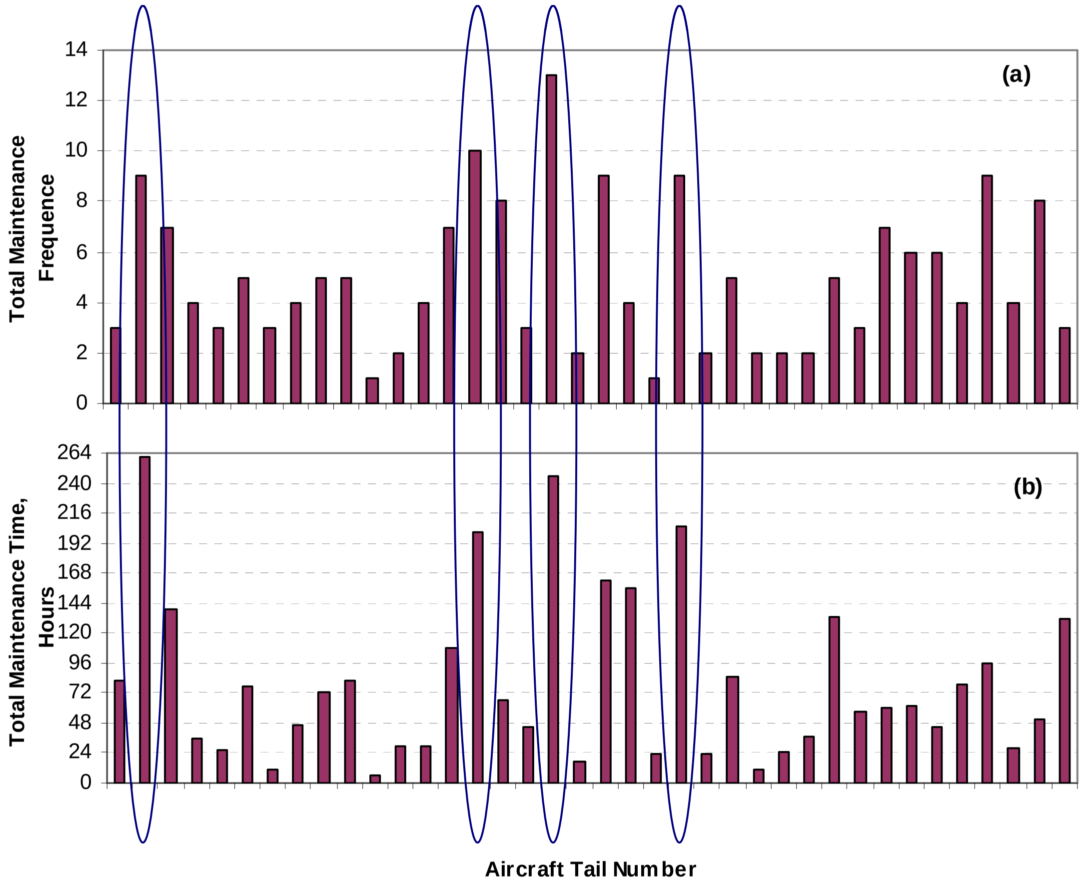


Figure 4.7 Unscheduled maintenance characteristics for each aircraft. (a) The requested times of each aircraft; (b) The total maintenance duration of each aircraft.

For the four unreliable aircraft selected above, an operation strategy is proposed: force them to fly close to a maintenance station. In this way, the problem can be fixed quickly and the aircraft can go back to service earlier. There is also a safety benefit through this strategy since the unstable aircraft will fly shorter distance to get repaired. If the fly time is less than one hour, a station is considered as close to a maintenance station. If it is not a major problem, the aircraft can make empty movement to the maintenance station. However, the trade off is that other aircraft have to fly longer trips. The computational result can indicate if it is worthy to restrict the flying distance of those unreliable aircraft.

The results of making special route for the unreliable aircraft are compared to those without special strategy in Table 4.6. Notice that even the special treatment for unreliable aircraft only save unscheduled maintenance duration by 10%, the total cost is reduced by 8.6%, which brings significant savings. It could save $111,672 in that month if some unreliable aircraft are flying near the maintenance stations.

Table 4.6 Results comparison for special route strategy on unreliable aircraft

	Reposition Cost	**Upgrade Cost**	**Charter Cost**	**Num of Charters**	**Total Cost**
Without Special Routes	889,405	34,196	376,552	24	1,300,153
With Special Routes	725,850	36,751	425,880	25	1,188,481

Furthermore, it is worthy to analyze the probability of a fleet may fail in each time period since the end of the last unscheduled maintenance (in Figure 4.8). Although the probability of failure is not as high as we thought, but it indicates an interesting trend that all fleet types are more likely to go back to unscheduled maintenance again in the first few hours after it is released from maintenance. Three examined fleet types, A, B, and C, are displayed in the similar trend.

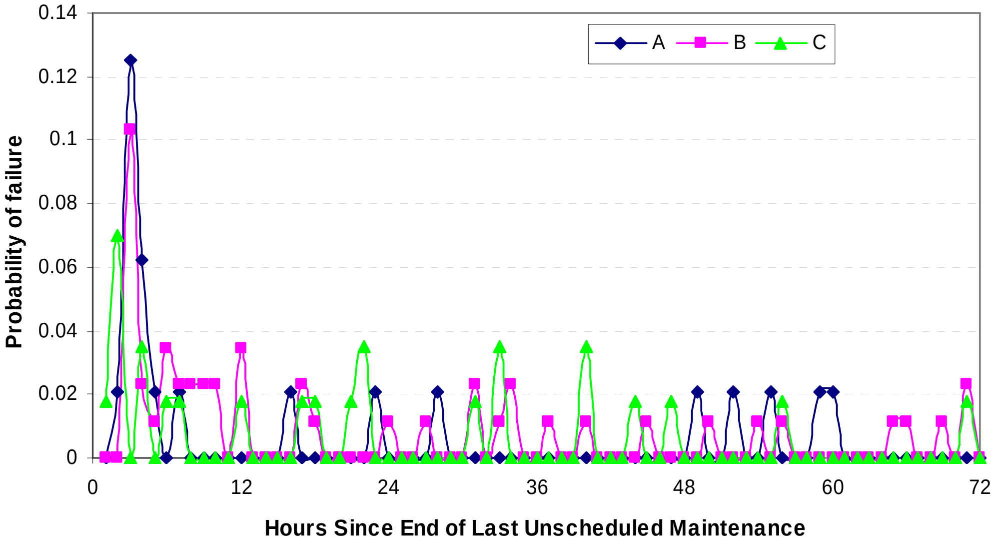


Figure 4.8 Probability of failure after the last unscheduled maintenance

Hence, we can make a further strategy for the unreliable aircraft that the special routing is only made for the day that it finishes maintenance. After the time period in which the aircraft has respectively higher risk of unscheduled maintenance, it can be dispatched to any station. Especially when the company faces the shortage on its own planes to meet high customer demands during the day.

4.5 Stochastic Demand

The scenario analyses in previous chapters assume that complete demand information is available. In reality, as mentioned before in a fractional airline, some of the legs are requested just eight hours before the departure. Hence, not all flights are known when the schedules are made, and effectively dealing with this dynamic nature is critical to the success of the planning. In this section, different strategies are evaluated to capture this dynamic nature and increase profitability: (i) repositioning crew(s) to the nearest hub with legality constraint; (ii) putting spare planes at hubs; (iii) incorporating demand forecast; (iv) allowing the decision to reject new demand in the peak days.

To relax the assumption of complete demand information, only a portion of the legs in the planning horizon is used in initial planning. First, it is assumed that all demand in the first day are known, and partial demand in the second and the third days are unknown, and the demand uncertainty in the third day is higher than that in the second day. This assumption is used for evaluating the strategy of legally repositioning crew to the nearest hub after it finishes its duty. In this situation, certain percentage of demand in the second and the third day is randomly removed from the real demand information, assuming those removed legs have not been requested when the schedule is made for the three-day planning period. A simple example illustrates how the demand data is created. For the planning period of days 1-2-3, 10% of the legs in day 2 and 20% of the legs in day 3 are removed from the real data based on historical trend. Then for the next planning period of days 2-3-4, all the legs in day 2 are known. Therefore all the day 2 legs, which were removed back in the previous planning period, are added. Half of the removed legs in day 3 are also added back. Similarly, 20% of the demand in day 4 are removed.

The next step is to simulate the stochastic demand showing in the first day of the three-day planning horizon. The process is similar to the one described above. Some demand in the first day is first hidden, and then add back as new demand. With this approach, the strategy of reserving spare planes at hubs is evaluated.

4.5.1 Repositioning Crew to the Nearest Hub

It is reasonable to assume that the total cost of the crew pairing solution is higher when the information is incomplete. One idea to potentially reduce cost is to let crew move the aircraft to a nearest hub, which is a station that has high flight activities. Thus the crew will have a higher chance to take a leg with less reposition in the next day. The cost savings in this strategy is from the potential coverage increase of the moved aircraft at a hub, having the benefit created is more than the reposition cost to a hub in the first day. Therefore, after the first day duty, a crew will be suggested to fly to a hub with a short empty reposition at a low reposition cost. It is defined as the nearest hub if the closest hub is within one hour flight distance.

The data set includes 42 crews and 276 legs in one week. In the scenario analyses, the stochastic demand is simulated in the same way described before. Assume 85% and 70% demands are known in the second and third day respectively. Three scenarios are compared: the first one uses model Q1 with no crew swap; the second scenario uses Q1' allowing crew swap when the crew's assigned aircraft goes into long unscheduled maintenance, but no reposition in advance; the third one applies Q1' and also moves crew to its nearest hub in advance after its current duty is finished. The results are listed in Table 4.7. A 5.3% saving on the total cost of the second scenario, comparing to the first scenario, again indicates reassigning crew when its plane goes under maintenance can improve the operation. From the costs listed in the last row, even with addition cost ($470) to fly plane to a closed hub, the total cost is reduced by 7.3% comparing to the first scenario. Note that the total cost is higher with stochastic demand than that of all the demands are known in advance which is $620,607.

Table 4.7 Cost comparison on repositioning crew to the nearest hub

	Reposition Cost	Upgrade Cost	Transport & Overtime Cost	Num of Overtime	Charter Cost	Num of Charters	Swap Cost	Move Cost	Total Cost
Q1	237,125	8,203	47,750	46	333,667	17			626,745
Q1’	233,828	8,678	40,332	38	309,453	15	1,197		593,488
Q1’+Hub	227,523	7,815	36,266	36	307,427	15	1,197	470	580,698

Investigations are performed on how the strategy impacts the cost at different uncertainty level on the demand information. Table 4.8 presents the comparison on the cost between the option that crews do not move to a hub (NoHubCost) versus move to a hub (HubCost) with different percentages of known demand in the second day and third day. HubCost includes the reposition cost that a crew moves its plane to the hub. The comparison is run on a smaller data set which includes 35 planes. The result indicates that higher uncertainty makes the repositioning crew strategy more cost efficient.

Table 4.8 Comparison of moving aircraft to hub or not when demand is uncertain

June	NoHubCost	HubCost	Cost Saving
Complete	174,899		
90%, 80%	194,473	193,546	0.5%
80%, 75%	235,121	233,376	0.7%
70%, 55%	270,032	265,132	1.8%

4.5.2 Putting Spare Planes at Hubs

The management company notices that requesting charter is mostly the result of the unexpected events, such as new demands and unscheduled maintenance. Moreover, some new demands coming in the current day makes it difficult to adjust from the original plan. Besides the strategies discussed, one alternative is to consider reserving spare plane(s) at hub to respond to the new events which may occur in the first day of planning.

When evaluating this strategy, one needs to select the type of spare plane and the hub to cover a new event. To answer these questions, an assignment model is suggested. The objective of the model is not to calculate and minimize the real reposition cost, but find the best hub to recover the unexpected event. The term, recover, means the demand can not be covered originally, and has to use the reserved spare plane.

Let $g' \in G^f$ be a ground arc, representing a hub where a spare plane in the fleet f could be repositioned to in advance.

The decision variables are:

$z_{g'f}$ the number of spare planes in fleet f on the ground arc g'.

$u_{kg'f}$ 1 if leg k is recovered by a spare plane in the fleet f at ground arc g', and 0 otherwise.

v_k a slack variable, it is 1 if leg k is not recovered by any spare plane, and 0 otherwise. In other words, it is 1 if leg k is covered by a regular plane, 0 if recovered by a spare plane.

The following parameters are also defined for this model:

$E_{kg'f}$ assignment cost: proportional to the cost of repositioning the spare plane in the fleet *f* at ground *g'* to recover leg *k*. The distance from ground *g'* to the departure station of leg *k* is used in this model with an adjustment factor.

b_k upper bound of reposition, which is the reposition cost from the departure location to the furthest selected hub

The objective of the model is to minimize the total assignment cost so that each leg can be recovered either by a spare plane at a hub or by repositioning a plane to the departure station of the leg. The formulation for the assignment problem is given as follows:

$$Min \quad \sum_{k\in L}\sum_{g'\in G^f}\sum_{f\in T} E_{kg'f}u_{kg'f} + \sum_{k\in L} b_k v_k$$

$$s.t. \quad u_{kg'f} \le z_{g'f} \qquad \forall\ k \in L \qquad (4.3.1)$$

$$\sum_{g'\in G^f}\sum_{f\in T} u_{kg'f} + v_k = 1 \qquad \forall\ k \in L \qquad (4.3.2)$$

$$\sum_{k\in L} u_{kg'f} \le M z_{g'f} \qquad \forall\ f \in T, \forall\ g' \in G^f \qquad (4.3.3)$$

Constraints (4.3.1) ensure that leg *k* can be recovered only if there is a spare plane in the fleet *f* on ground arc *g'*. Constraints (4.3.2) require that each leg must be covered by either one spare plane or a regular plane. Constraints (4.3.3) restrict that a spare plane in the fleet *f* on the ground arc *g'* can only recover at most *M* legs.

Based on the history customer demand, 31 specific hubs are selected nation wide to find out which set of legs will be recovered by a fleet type at a hub. Hence, there are 31 ground arcs *g'*, and the number of $z_{g'f}$ variables is *g'*f*. The scenario (A) without spare plane is compared with the scenario (B) using spare planes. The procedures for the simulation are listed below:

1. Randomly hide some demands, where the departure times are later than 8am. The hidden demands will be added as new legs. In practice, the optimizer is run whenever new demand comes. In this thesis, three runs are performed at 8am, 12pm, and 3pm. Therefore, hidden demands are selected based on assumption that they are requested in the time intervals (midnight, 8:30am], (8:30am, 12:30pm], and (12:30pm, 3:30pm].
2. Run the optimizer at midnight with the current known demand information for both scenarios. For scenario B, additionally run assignment model with partial demand. Among the idle planes obtained from the optimizer, spare planes are selected and moved to hubs based on the output of assignment model. They are freed as other regular planes when rerun optimizer at the next day morning.
3. The optimizer then runs three times with new demands we know so far. At the time of rerunning optimizer, flight schedules are fixed for next 3 hours, since there may be planes repositioning or ready to fly customer legs.
4. Rerun the optimizer whenever an unscheduled maintenance occurs.

The difference for the two scenarios is that spare planes are moved to hubs in advance for Scenario (B) in midnight. Then procedures 3 and 4 are the same in both scenarios. Several tests are performed based on different percentage of unknown demand. The instances have 3 fleets, 61 planes, and 872 legs in a ten-day experiment. Table 4.9 indicates that the spare plane strategy would be beneficial to the business. The numbers in the first column are the percentage of unknown demands in the three days. In addition, we notice that the improvement by putting spare plane(s) at hubs diminishes when there is less uncertainty on the demand information.

Table 4.9 Spare plane comparison with different level of uncertainty

% known	Without Spare	With Spare	Improvement
90/80/60	1,253,803	1,182,746	5.67%
92/85/70	1,199,267	1,157,531	3.48%
95/90/80	1,139,906	1,116,648	2.04%

The results demonstrate that having planes ready at some stations based on information provided by the assignment model to absorb the unexpected events could result about 2 to 6% cost reduction. The difficulty is to predict where and when the unexpected events occur and decide where to put a spare plane. Because of the possible benefit of the option, it is worthwhile to consider combining forecast information with the reserving plane strategy to absorb the impact of new event or new demand.

4.5.3 Incorporating Demand Forecasting

Forecasted information can be used to reduce the impact of new demand. Since it is not practical to directly predict a specific customer flight with the departure/arrival location and time, forecasting on the number of total required planes in a fleet at a time period would be helpful to determine how many planes to be reserved. For example, the forecast may predict that m planes in fleet CJ1 are needed at time t, and there are n planes scheduled for the known demand, then $r=m-n$ planes should be reserved to cover the difference between forecasted demand and known demand at time t.

Based on the departure and arrival time of the known demand, it is easy to estimate the number planes scheduled (called in-service planes) in each time period. For the estimation, a three-hour reposition including turn time is assumed. At time t, if there are two legs with departure time earlier than t+3 hours and arrival time later than t, then two planes are required in service at time t. If only one of them is requested by a CJ1 owner, the number of in-service planes at time t in fleet CJ1 is one, n =1.

If r=m-n>0, it means new demands come and require additional planes at time t. While r represents the difference between forecasted demands and known demands at time t, R is the number of planes in a fleet to be reserved in a day. Thus, R is max(r) rounded to the nearest integer. For instance, if max(r)=2.1, two planes will be reserved (R=2) when r reaches 0.5 and 1.5, respectively, in Figure 4.9.

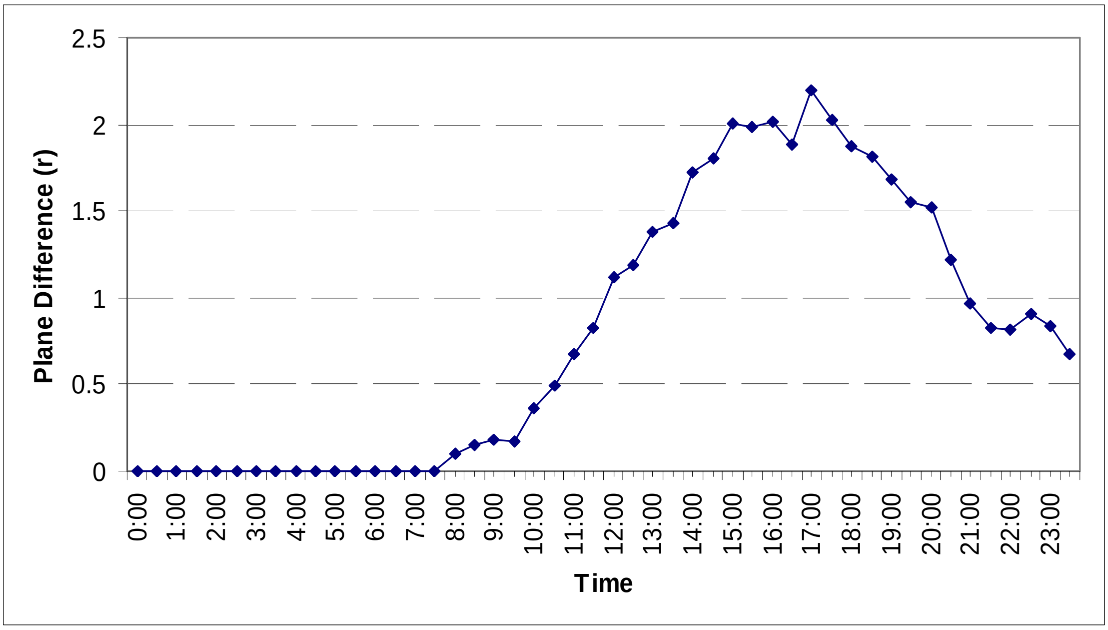


Figure 4.9 Example for reserving planes based on forecasting information

Once the number of reserved planes (*R*) is determined, the next step is to select the reserve locations. To reduce the risk of reserving a plane at a hub without using it, only busiest airports (hubs) are pre-selected as location candidates. The *R* hubs with the highest count of the nearby known demands (the departure location of the demands is located within certain radius of the hub) are selected.

To incorporate this strategy into planning, reserving planes at hub(s) can be represented by dummy legs with the same departure and arrival location. In Figure 4.9, the start time and end time of the dummy legs are [11:00, end of the day] (r=0.5) and [13:30, 20:00] (r=1.5). As time moves along, the reserved planes are activated to cover the rest of the legs and new demands. In another word, the dummy legs will be replaced by real new demand.

A computational experiment is made to investigate how this strategy would perform relative to the benchmark that solves the scheduling problem, whenever new demand comes in, without anticipating the additional demand. With twelve-day forecast information, Table 4.10 shows an average 2.4% improvement when reserving planes at some hub according to the demand forecast.

Table 4.10 Reserving planes with demand forecasting for one day planning horizon

Day	No Forecast	Forecast/Reserving	Improvement
1	140,845	133,486	5.2%
2	237,858	236,722	0.5%
3	183,368	170,826	6.8%
4	252,805	246,407	2.5%
5	176,211	177,317	-0.6%
6	300,458	280,305	6.7%
7	103,106	105,393	-2.2%
8	128,587	131,058	-1.9%

9	271,964	267,694	1.6%
10	313,772	334,445	-6.6%
11	282,418	268,473	4.9%
12	137,798	122,097	11.4%
Avg	**210,766 (c11)**	**206,185 (c12)**	**2.4%**

Note that some numbers in the last column are negative due to inaccuracy of the forecast. Low accuracy could result in moving aircraft to hub but not covering legs. Another reason could be the effect of unscheduled maintenance. The simulation in the 10th day shows the largest negative improvement when reserving planes in advance based on forecast information. It is mostly because of a charter due to an unscheduled maintenance, even though a plane is reserved at a hub.

Table 4.11 Reserving planes with demand forecasting for three day planning horizon

Day	No Forecast	Forecast/Reserving	Improvement
1	143,762	135,025	6.08%
2	228,643	225,642	1.31%
3	180,375	172,361	4.44%
4	250,574	240,745	3.92%
5	175,528	176,964	-0.82%
6	303,857	289,462	4.74%
7	101,746	100,473	1.25%
8	124,587	116,653	6.37%
9	266,477	269,496	-1.13%
10	306,722	301,565	1.68%
11	273,674	265,346	3.04%
12	129,662	120,421	7.13%
Avg	**207,134 (c21)**	**201,179 (c22)**	**3.2%**

In addition, investigation is deployed to find out whether the forecast information in the second and third day along with the strategy of reserving planes would further improve operations. The results display a positive answer in Table 4.11. The overall average improvement is increased to 3.2%. Therefore, longer demand information and forecast saves on the cost. Compared the costs in Table 4.10 and Table 4.11, having three-day demand information saves 1.7% more, which is (c21-c11)/c11, for “no forecast” scenario; and 2.4% more, which is (c22-c12)/c12, for “forecast/reserving” scenario over having only one-day demand information.

4.5.3 Rejecting or Different Pricing for the ‘Last Minute’ Demand in Peak Day

There are peak days each year that demand is very high, such as around Christmas and Thanksgiving. The company always has to face more charters in peak days, and charters are always to be avoided. First, requiring charter aircraft at the ‘last minute’ from the third parties is much more expensive than asking in advance. Also there may not be charter aircraft available for the new demand at its desired departure time. For this case, some options are proposed the customer, who requests his flight at “last minute” during the peak day, to change his flight to the following day or later; otherwise he has to pay a higher price for the trip. It may not be suitable for the fractional ownership program, since the owners are protected by their ownership that they can request flights any time they like with eight hours advance notice. However, it could be a valuable cost saving alternative in other on-demand air transportation mode, for instance jet-card holders. The analysis of this option is demonstrated on card holders.

Considering that most customers are based on East coast, it is reasonable to assume that most resources are located around East coast, and may be able to absorb the “last minute” demand in east region. Therefore, to illustrate the effect of late request, 8% new legs as the ‘last minute’ card demand are randomly generated in a peak day mainly in west and central region.

The desired departure time of those demands are mostly in the late afternoon varying from 19:00 to 23:15. Before 23:15 in that peak day, it is observed that there is no enough aircraft to cover the new demands (Figure 4.10). For example, the demand that desired to depart at 19:00, it request fleet B, while fleet B only has 17 aircraft. At time 19:00, the company already over capacity that it has 20 planes in service for other demands. Note that although it has 3 more planes in use than the number of planes in the fleet, it does not always mean need 3 charters. There could be some larger planes used for upgraded flight.

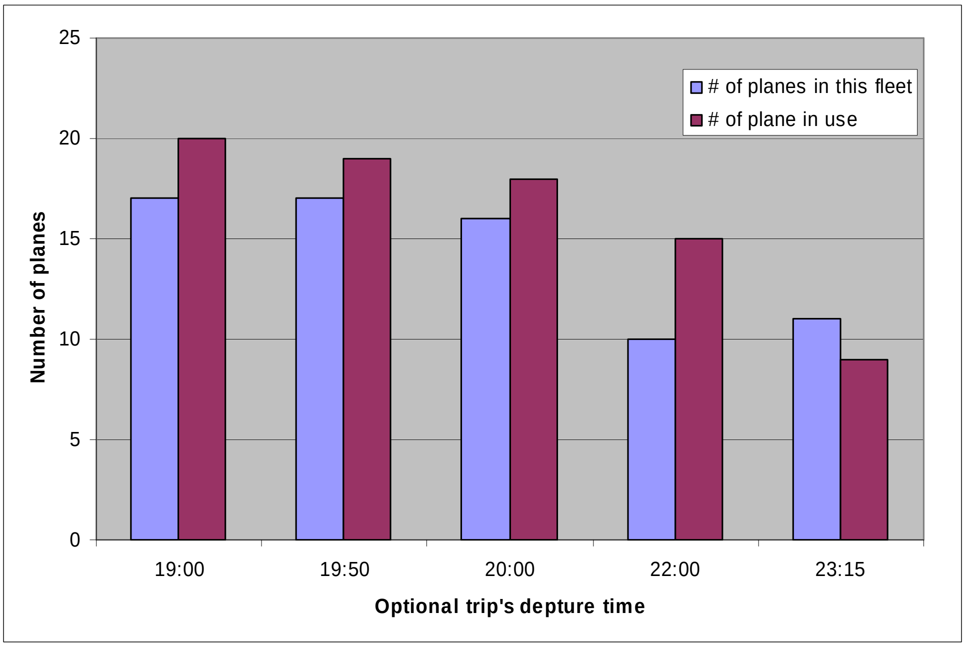


Figure 4.10 The number of planes in use.

When accepting a demand, there are two consequences comparing to reject it. First operational cost is higher with more demands. Second, the revenue is also higher. To decide whether accepting the new card demand, the net gain is examined, which is the difference between the revenue increase and cost increase in Table 4.12. Whenever a new card demand appears, the incremental cost is compared with the extra revenue generated if the new demand is honored. If the net gain by accepting the demand is positive, the demand should be accepted. The last column in Table 4.12 displays the decision if a demand should be accepted or not.

Table 4.12 Decision on accepting a new card demand

Demand	Card DeptTime	FlyTime	Cost	Revenue	Net gain	Accept
ASE-SPW	19	105	3,904	2,319	-1,585	No
AMA-MEV	19:50	201	827	4,439	3,612	Yes
UVA-GTU-FTW	20, 21:15	38, 37	2,108	2,650	542	Yes
TWF-FSD	22	129	6,330	3,710	-2,620	No
LAS-HHR	23:15	56	1,892	1,200	-692	No

Keeping a customer service level is important for a company, so the above analysis may not necessarily means rejecting a demand, but as a reference to support decision making. For instance, the company can ask card holders not to request the "last minute" flight in the peak days, or they have to pay the possible premium, such as $1,585 for the flight request in the first row of Table 4.12.

4.6 Conclusions

In this Chapter, several tactical and operational issues faced by the fractional management company are analyzed, and the impacts of these issues on profitability are demonstrated.

First, the question “What is the right demand size for a given fleet?” is examined. Starting with a base set of monthly data, demand size is increased by 5%, 10%, and 15%. It is concluded that if the new demand is similar to the current demand base, an increased demand up to 5% is profitable.

Different strategies to generate new demand are tested, and the right configurations are suggested based on profitability. We first consider increasing demand by introducing a new product, 25-hour prepaid jet-card. We assume that these new demand legs allow for one-hour time window flexibility on departures. Our analysis shows that this added flexibility has a significant effect in profitability.

We considered increasing demand by expanding operations geographically. The customer and the crew bases for the base data we use are concentrated in the eastern United States. Hence, we increase demand by adding new legs with a West Coast origin and/or destination. The computational experiments show that a geographic expansion results in an increased reposition ratio and operational costs but lower charter costs on average. We conclude that this may be due to having longer but fewer flights for the same total flight time and having more flights requested by larger fleet type owners. We demonstrate that under this scenario, profitability is increased when demand is increased by up to 10%.

Furthermore, we study the effect of number and type of core planes owned by the management company on profitability. We determine the breakeven points for the number of core planes for the base demand and when the demand is increased by 5%, 10%, and 15%. We remark that although the profit remains relatively flat in the indicated ranges, the mixture of costs is quite different as increased cost of the core planes eventually outweighs decreases in charter cost. Operational costs tend to drift downward but not dramatically, perhaps because there are more legs being flown as charters decrease. The investigation on different size of data set, which indicates the growth of the company, demonstrates that around 10% to 15% of planes can be kept as core planes.

This chapter also addressed strategies on stochastic events, unscheduled maintenance and new demand. Those stochastic events could disrupt the prior plan that made based on statistic information. Unscheduled maintenance, especially the one(s) occur during the middle of the day, could result in charter cost. The scenario analyses show that making special routes for most unreliable planes will reduce the operational cost, under the assumption that they can quickly go back to service if they fly close to maintenance stations.

Another dynamic situation is that new demands come during the first day of the planning horizon, when the initial routes and assignment have been made. Considering the potential benefit that the crew will have a higher chance to fly a leg with less reposition in the next day, crew(s) can reposition to the nearest hub after they finish their first day duty. The computational results show that with about 0.1% addition reposition cost that move plane to a nearest hub could provide about 2% reduction on the total cost.

Putting spare planes at hub is discussed to reduce the impact of mid-day unscheduled maintenance.

In addition, allowing the decision to reject some of ‘last minute’ card demands in the peak days would be beneficial compare to present method of operation in the fractional company. Alternatively, a customer can pay the possible premium when his flight causes a negative net gain.

CHAPTER 5

CONCLUSIONS AND FUTURE RESEARCH

Optimization methodologies are developed to help fractional management companies in efficiently managing their aircraft and crew so that all flight requests are covered at the lowest possible cost. The proposed models take into account: crew transportation cost and overtime cost, scheduled and unscheduled maintenance effects, crew constraints, and the presence of a non-homogeneous fleet. Using the proposed scheduling approaches, various scenario analyses on real operational data are carried out to assist the fractional management company in making strategic and tactical planning decisions.

The contributions of this thesis research are listed as follows:

1. Developed multiple methodologies to optimize the operations for the fractional ownership airline.
    a. A simple model is proposed and implemented to solve crew pairing problems with a combination of crew scheduling and aircraft routing problems. The crew is assumed to stay with an aircraft in its duty period, which is common initial scheduling rule in practice of most fractional management companies. The simple model is solved with column generation technique.
    b. A reformation of the simple model is implemented. This reformation allows crew be reassigned after its aircraft goes into long maintenance.

This approach improves the utilization of crew and plane. Maintaining an extra crew also could provide more opportunity of crew swap.

   c. To further improve the utilization, we proposed an integrated model which simultaneously solves the fleet assignment, aircraft routing and crew scheduling problem. A duty-based fleet-station time line is introduced to record the plane activities at each station. This model fully separate crew and plane which increase the utilization. Bender's decomposition is employed to overcome the hurdle of enormous memory requirement on big instance sizes. Some techniques are discussed to improve the computational efficiency on Bender's decomposition.

2. Provided valuable strategic decision-making supports based on scenario analysis with complete demand information. Te following issues are investigated:
   a. The effect of scheduled and unscheduled maintenance on operational costs. The mid-day unscheduled maintenance results in significant extra cost. Special rules to keep unreliable planes flying close to maintenance stations are proposed to reduce the impact.
   b. The effect of different crew-swap strategies on operational costs. The total cost of using two or four designed shifting days per week does not make significant difference on cost. However, crew swap during the duty period improves the resource utilization and also reduces operational cost.
   c. The effects of increased demand. Without increasing fleet size, the company has capacity to effectively handle extra demands to some degree. Net profit may drop if demand keeps growing without adding extra

resources. Moreover, two scenarios to expanding demand are evaluated. (i) Introduction of a new product, “jet-card”, where customers buy flight hours without becoming fractional owners. The analysis indicates that if the company allows an up to one-hour time window on departure time for jet-card flights, current capacity can be used to manage more demand increase from jet-card with profit. (ii) Operational expansion to different geographic areas. The investigation concludes expanding to west coast would be profitable.

d. The effect of number of core planes, which owned exclusively by the management company. Different scenario analyses on sets of data show that around one eighth of planes could be saved as core planes.

e. Strategies to tackle the stochastic nature of demand are investigated, which include moving a crew to the nearest hub after it finishes its current day duty, putting spare plane at hub in advance, reserving plane at hub based on forecast, and rejecting new card demand in peak days based on profitability.

The investigated strategies are valuable for the management company. The impact of these analyses may be very significant given that, the top 4 management companies share about 90% of the market and collectively operate a growing fleet currently numbering over 1000 aircraft strong. It’s estimated that even a 1% reduction in operating costs across this fleet would result in annual savings of over $20 million, at least $10 million of it in fuel costs. Currently, the fractional ownership management company we worked with is the only one sells its jet-card with one-hour flexible departure time window, which is the only product making profit.

This thesis focused on methodologies design and strategic investigations based on scenario analyses for the fractional airlines, which has unique and dynamic feature. Mixed integer programming is used to solve deterministic models and simulate the stochastic character in special way. The analyses provide several encouraging and operational suggestions.

There are several interesting topics along with the direction of this research could be beneficial:

To capture the dynamic nature more closely, further research for solving the scheduling problems with stochastic approaches, such as stochastic programming will be valuable.

The convergence of Benders decomposition could be further improved for computational efficiency.

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