

SMALL PARTS HIGH VOLUME ORDER PICKING SYSTEMS

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SMALL PARTS HIGH VOLUME ORDER PICKING SYSTEMS

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*To my father, Razmik (Alik) Khachatryan,
my mother, Karine Gevorgyan,
and my husband, Vardges Ter-Hovhannisyan*

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LIST OF SYMBOLS

D – the length of the picking area;

H – the height of the picking area;

s_h – speed of the picker in the horizontal direction;

s_v – speed of the picker in the vertical direction;

t_h – travel time it takes to move horizontally from the lower left-hand corner along entire pick area, $t_h = D/s_h$;

t_v – travel time it takes to move vertically from the lower left-hand corner across entire pick area, $t_v = H/s_v$;

s_c – speed of a conveyor;

t_c – time it takes for an order container to travel the segment of a conveyor along the pick zone (i.e. length D), $t_c = D/s_c$;

N_j – number of items in an order j , random number with known distribution;

M – constant batch size of orders;

X – size of a batch of entities, i.e. for item-buffers $X = \sum_{j=1}^M N_j$ and order buffers $X=M$;

B – number of buffers in the second station;

K – number of buffers in the second station, including space in station, $K=B+1$;

W – the time a container is released into a zone;

$\lambda, E[\tau], c_a^2$ – arrival rate, expected interarrival time and squared coefficient of variation of interarrival time, respectively, $E[\tau] = 1/\lambda$;

$\mu_i, E[S_i], c_{S_i}^2$ – service rate, expected service time and squared coefficient of variation of

service time, respectively, of serving at station $i, i = 1, 2; E[S_i] = 1/\mu_i$.

ρ_i – utilization of server i ;

T – cycle time of an entity;

L – number of entities in the pick station;

S^O – travel time of an order;

S^I – travel time of an item.

SUMMARY

This research investigates analytical models that might serve to support decisions in the early stages of designing high volume small parts order picking systems. Because the development of analytical closed-forms is challenging, a common approach is to use simulation models for detailed design performance assessment. However, simulation is not suitable for early stage design purposes; because simulation models are time-consuming (thus expensive) to construct and execute, especially when the number of alternatives to evaluate is large. If available, analytical models are computationally cheaper. They provide faster and more flexible solutions and though usually less detailed, may be adequate to support early stages of design. The challenge is to develop generic analytic models providing useful results for a class of problems.

This research focuses on a class of problems in high volume small parts order picking systems in which customer orders are progressively consolidated (or assembled) from pick buffers into order containers passing from one picking zone to another via conveyor, so that no sortation equipment is required downstream prior to packing and shipping and picking can precede order assembly. This is a new technology, and not yet in widespread use. The novelty in the modeling approach is the distinct separation of item-picking and order assembly operations which permits the development of performance models for both throughput and service level.

Essentially the system is modeled as a tandem queue, and the two detailed models for the picking and assembly subsystems are developed based on detailed description of

the operations. Solving the model provides estimates for performance measures, such as order cycle time and system throughput, which are essential in design. The approximation method requires estimating the squared coefficient of interdeparture times from the classical $G^X/G/1$ queuing model, and a suitable approximation is derived in this thesis. Computational tests show the model to provide reasonably accurate estimates of system performance, with minimal computational overhead.

To support the proposed queuing model, new models are developed for estimating mean and squared coefficient of variation for pick and assembly operation times. These models include the variability of order contents and the picking process, along with the physical layout. Results of the estimation compare very well with that of simulation.

CHAPTER 1

INTRODUCTION

1.1 Purpose and Scope of Research

Today's global supply chains require properly designed warehouses for efficient, cost effective performance. Designing warehouses is difficult because of conflicting trends, goals, and requirements. Warehousing involves a significant amount of product handling, which is time consuming and expensive, especially if done manually. Increasing costs and shrinking product lifecycles are driving inventory reduction. Because less inventory will be in warehouses, expectations are growing for more frequent low volume deliveries, with shorter and more reliable response times through the supply chain. In addition, factors such as marketing add even more pressure on the supply chain by requiring delivery of a wider variety of products with shorter lifecycles and lower total sales. As a result, warehouses operate in an environment where inventories are expected to be small and turn quickly while customer service is expected to improve by reducing transaction time and improving accuracy.

Order picking is the process of retrieving an appropriate amount of one or more products from specified storage locations to fill specific customer orders. A *line item* is referred to as a single detailed record (each line that reflects an item and a quantity) of a customer order. *Customer orders* usually consist of a list of line items that identifies types and amounts of products required by the customer. Often, to improve picking efficiency several customer orders are combined into one *warehouse order* which, in

addition to product type and quantity, identifies pick locations in the warehouse. This type of order picking is called *batch picking* (Sharp 2001). For batch picking two pick strategies are distinguished: *sort-while-pick* and *pick-and-sort*. Under the sort-while-pick strategy, the picker is provided with several containers to keep orders in a batch separately. Under the pick-and-sort strategy, the orders are picked simultaneously and sorted afterwards. This research focuses on sort-while-pick strategy, and specifically addresses system design.

While order picking represents only a part of the material handling operations, it is the most expensive operational area in warehouses, according to (Tompkins 1996). Design and operation of order picking systems is a very difficult task. In many situations designers are overwhelmed with issues such as uncertainty of material flow through the system, multiple and often conflicting objectives, large amounts of information to process, and large numbers of design alternatives to consider.

Often, key objectives in designing order picking systems include increasing efficiency or production rate, reducing cycle time, and increasing customer service level. The productivity of the order picking process depends on the storage systems and their layout, and the control mechanisms applied to labor and automation. Productivity issues are particularly critical when order picking systems must process a high volume (more than 100 orders per hour) of small orders (ten or fewer items per order), which arrive continually and require one hour or less response time - period between releasing and fulfilling an order, (Choe 1991). Typical examples of high-volume order picking systems include mail order companies, compact disks, contact lenses, pharmaceutical, and book distributors.

The scope of this research is limited to order picking systems that handle high volumes of small orders and that have a randomized storage policy within a pick zone. This policy results in high space utilization, although it may increase travel time to pick an item (Francis et al. 1992). The equipment types for storage and retrieval of small parts picking systems are classified into three categories:

- *goods-to-picker*: goods are transported from storage to pick area for picking operation to occur (e.g. horizontal or vertical carousels, or miniloads);
- *picker-to-goods*: order picker (either human or robotic) travels to a storage location for the picking operation to occur (e.g. bin shelving, gravity flow racks, pick-to-light systems, modular drawers);
- *automatic*: goods are dispensed from a storage unit to a conveyor or container on a conveyor (e.g., an A-Frame).

The fundamental issues to consider in designing order picking systems include:

- Selecting storage equipment and material handling technology.
- Specifying configuration and the layout of selected systems.
- Selecting appropriate operational strategies.

The storage equipment and material handling technology selection refers to determining equipment and degree of automation for storing, order picking, and material handling. For example, typical questions like: “should a conveyor system be used?”, and “should order picking be manual or automated?” are posed. In layout design a specific configuration of a system is provided, with details such as configuration of storage area and aisle structures. Based on the equipment selected and the layout, material flow in the order picking system is determined. Operational strategies include those decisions that

influence the overall design, for example, storage rules or order picking methods. The design decisions have tremendous impact on the system performance, and therefore are analyzed within the framework of system performance. Furthermore, it is advantageous to include operational decisions in the early design stage, because, once investment is made and equipment is installed it is very expensive and sometimes impossible to change design decisions.

The purpose of this research is to develop analytical models to serve as a decision aids in the early stages of designing and analyzing high volume order picking systems. Due to the complexity of order picking systems the development of analytical closed-forms is challenging. Therefore, a common approach today is to use simulation models for detailed design performance assessment. However, simulation is not always suitable for design purposes, especially when the number of alternatives is large. Moreover, analytical models are typically computationally cheaper than simulation. They provide faster and more flexible solutions than simulations, and though usually less detailed, still may be adequate to support early stages of design.

In designing an order picking system it is important to be able to assess the performance of the designed system. Often, the assessment is provided through the prism of metrics like throughput, utilization, and service level. A quick and accurate assessment with respect to these performance metrics allows designers to consider more alternatives and to make early decisions on which specific design alternatives should be further developed or eliminated at the early stage of design.

In the following sections of this chapter the description of industrial examples for high volume order-picking systems are presented, followed by further discussion, generalization and analysis.

1.2 Examples of High Volume Order Picking Systems

In this section three examples of high volume order picking systems are described: (1) the robotic order picking system adapted from (Kim et al. 2002); (2) a pick-to-light system observed in a facility of a distribution center located in Graz, Austria; (McGinnis 2004) and (3) a manual pick-to-belt order picking system for a mail order company described in detail by (de Koster 1994).

1.2.1. A Robotic Order Picking System

The automatic order picking system described in (Kim et al. 2002) is shown in Figure 1.1. The pick zones (or workstations) are arranged in a serial order and there is a common conveyor transporting order containers among them. Order containers are formed into trains (or order batches), which are released into the system in waves. This operational strategy employs a periodic fixed time interval release of a batch or train of orders into the zone. Each order container of a train contains one customer's order and can accommodate multiple line items.

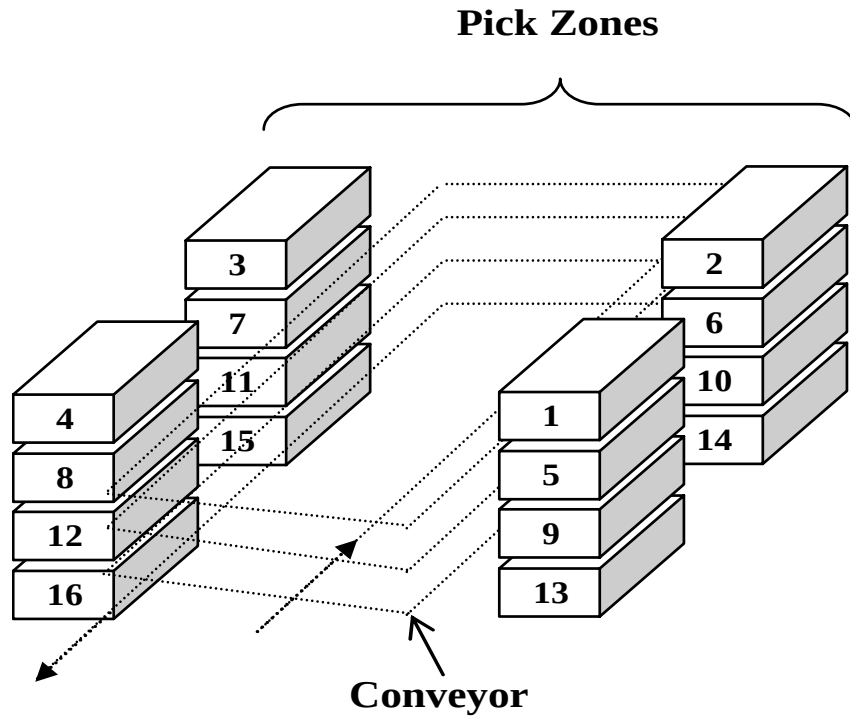


Figure 1.1 Automatic order picking system,
(modified from (Kim et al. 2002))

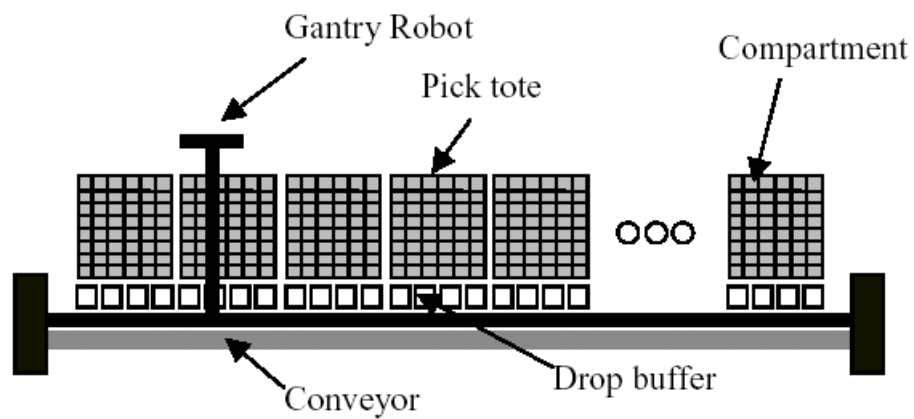


Figure 1.2 Zone Layout (reproduced from (Kim et al. 2002)).

Each order container passes sequentially from the first pick zone to the last. A layout of a single zone is depicted in Figure 1.2. Each pick zone has one picking gantry robot. A pick zone contains pick buffers where products picked by the gantry robot from the pick area are placed and wait for their order containers. Each pick buffer can accommodate only *one product item* at a given time. In this system, pick buffers are spread evenly along pick area.

The gantry robot must pick the items in its zone for an order, and place them in pick buffers prior to the order container passing the buffers. The conveyor moves at a constant speed. Appropriate sensing and control systems enable the pick buffer to release its content into the corresponding order container as it passes underneath. If the item is deposited in a pick buffer after the container designated to receive it has passed that buffer, a pick error occurs.

1.2.2. A Novel Pick-To-Light Order Picking System

Pick-to-light systems are semiautomatic paperless picking systems. Pick-to-light order picking systems, Figure 1.3, are rather common in small item picking systems with high volume deliveries. An interesting investigation by (Sharp 1996) discusses features of pick-to-light systems and presents results of a survey of ten pick-to-light systems with respect to productivity and quality analysis.

The pick zones (or workstations) are arranged in a serial order and there is a common conveyor transporting order containers among them. Every zone contains a pick-to-light system, and items are picked manually. Each item location has an

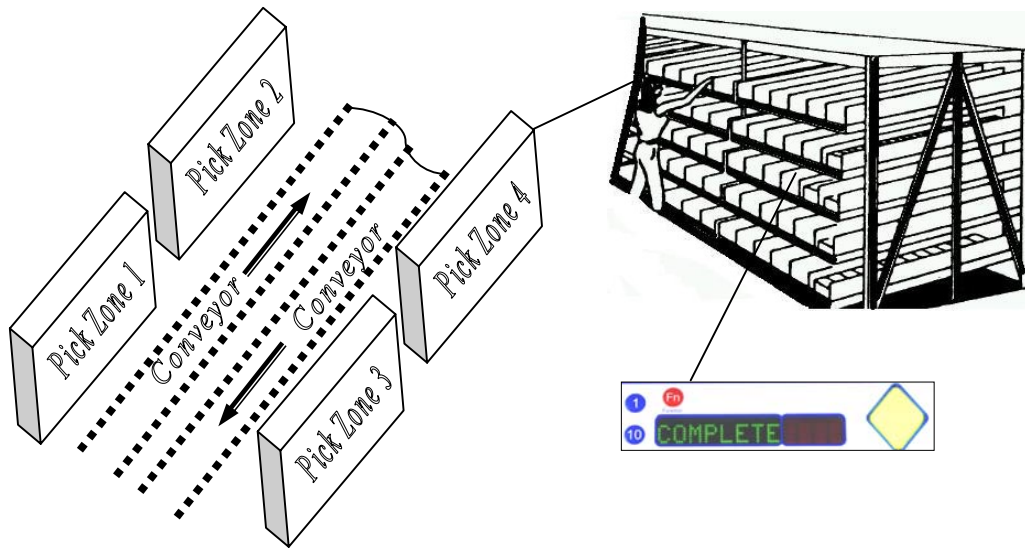


Figure 1.3 Pick-to-Light System (partial images from www.news.thomasnet.com).

individual digital display indicating quantity to pick and an acknowledgement button. After completing a picking operation the picker presses the acknowledgement buttons to confirm pick.

Unlike the conventional pick-to-light systems where a picker places the picked item(s) directly into an order container, here, a picker places item(s) into a temporary buffer designated for the corresponding customer order.

Order containers arrive on a conveyor to the picking stations. Each order container is designated for a single customer's order and can accommodate multiple line items. Each pick buffer is assigned to one *customer order*. In this system, pick buffers are located above conveyor and centrally located in the middle of the pick zone. Similar to the robotic order pick system described earlier, an appropriate sensing and control system enables the pick buffer to release its content into the corresponding order

containers as it passes underneath. A picker must pick all the items for an order in its zone, and place them in a pick buffer prior to the order container passing the buffer. If the item is deposited in a pick buffer after the container designated to receive it has passed that buffer, a pick error occurs.

1.2.3. Conventional Manual Pick-to-Belt System

Consider a conventional manual pick-to-belt OPS for a mail order company illustrated in Figure 1.4. Order containers arrive on a conveyor. Every container is assigned to one customer order. A printed order list is attached to a container, containing all line items that have to be picked. The conveyor system transports order containers sequentially to all workstations (or zones), where a product has to be picked from pick areas into the container.

If a product has to be picked from a specific pick station then the transportation system pushes out the corresponding container into the corresponding picking station, so that the main flow of containers is not blocked by containers waiting for picking. Each pick station is equipped with pick table where order containers reside during picking operation. When a pick table is full and cannot accommodate more order containers, the main flow of containers on a conveyor is disrupted by a “jam” created on the conveyor, i.e. order containers accumulate on the conveyor belt blocking a flow of other order containers. At the picking station the picker reads the order list for the particular zone, walks to the bin shelf area, picks the line items of the order and deposits them into the container. Having picked all the ordered items in the zone the picker pushes the

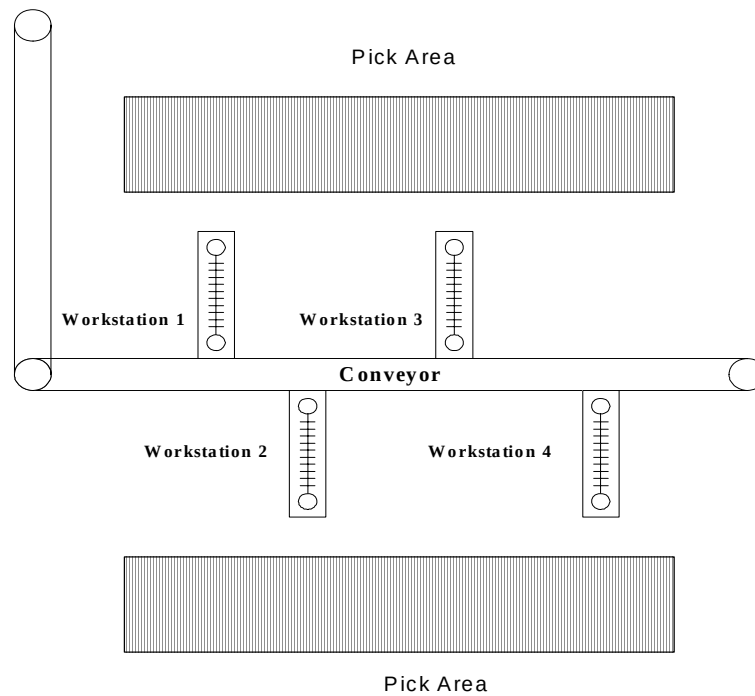


Figure 1.4 Manual pick-to-belt order picking system.

containers. At the picking station the picker reads the order list for the particular zone, walks to the bin shelf area, picks the line items of the order and deposits them into the container. Having picked all the ordered items in the zone, the picker pushes the container back to the conveyor system. The conveyor transports the container to the next picking station or to the packing area. Pick-to-belt systems are commonly used in the high-volume small parts picking operations (de Koster 1997).

1.3 High Volume Small Parts Order Picking Systems

This research focuses on high volume small parts order picking systems in which customer order is being progressively consolidated (or assembled) into order containers passing from one picking zone to another via conveyor, so that no sortation equipment is

required downstream prior to packing and shipping (Figure 1.5). Progressive assembly systems are categorized as zone picking strategies. In zone picking as defined by (Frazelle and Apple 1994), the storage area is partitioned into pick zones, order pickers are assigned a specific zone, and pick items only within that zone. Orders are moved from one zone to the next (usually on conveyor systems) as they are picked (also known as "pick-and-pass").

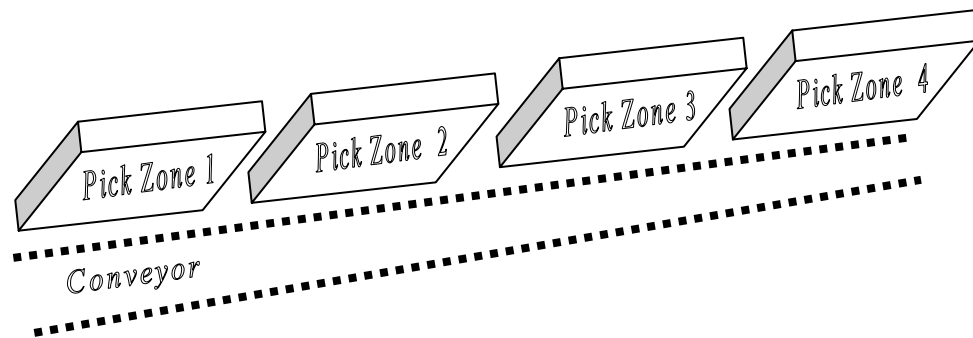


Figure 1.5 Typical Order Picking System
with Zones and Connecting Conveyor.

It is important to distinguish two separate operations in the systems without downstream sortation: order *picking* and order *assembly*. These operations are regarded as separate activities when items for a customer order are picked and set aside to wait temporarily in a pick buffer. Later, an assembly operation takes place when an appropriate container for a customer order arrives on a conveyor and items are transferred from buffers to the order container. However, if items are picked and placed directly into order container then both picking and assembly operations occur simultaneously.

Within the scope of this research, pick zones of the studied systems can be classified into two categories: *pick-to-buffer* zone picking workstations and *pick-to-container* zone picking workstations. These types of order picking systems may be used for items that differ in shapes. However, the items are restricted in size and weight, since they have to be small enough to fit into order containers, pick buffers and of course shelves.

1.3.1. Pick-to-Buffer Zone Picking Workstations

Pick-to-buffer zone picking workstations consist of a pick area, where items are stored for a pick, and pick buffers, where a picker deposits picked items, (Figure 1.6). In this system, two operations are required: item transfer from a pick area to buffer and item transfer from a buffer to order container arriving on a conveyor. Hence, in pick-to-buffer systems order picking and order assembly operations occur separately. An important observation here is that pick-to-buffer workstations implement sort-while-pick strategy since the content of a pick buffer (or group of pick buffers) is allocated to a single order and is deposited in the appropriate order container automatically, although at some time later than the pick itself. This way, items are separated by orders at the picking activity.

Order containers are placed on a conveyor, either one by one or in trains (or batches) and are transported sequentially to all pick zones. If order containers are released into the conveyor in trains then a warehouse order corresponds to the batch of customer orders included in that train.

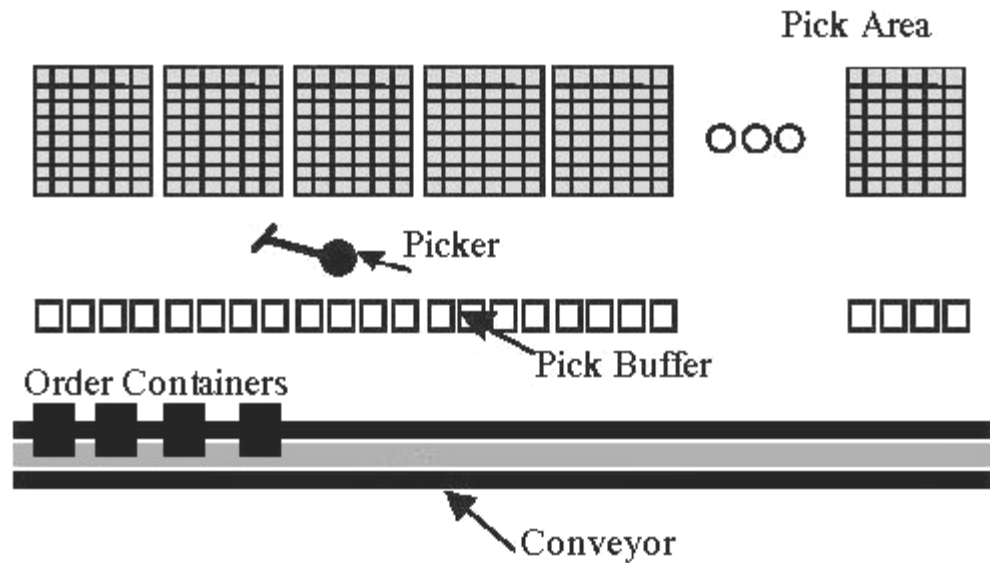


Figure 1.6 A typical *pick-to-buffer* pick zone with pick buffers.

Order pickers receive information about a warehouse order before order containers arrival to their workstation. Upon receiving information, order pickers begin picking items into the pick buffers. For a given warehouse order, pick operations may be performed simultaneously at several or all workstations. Moreover, a picker in the first workstation must respond to picking request much faster than a picker in the subsequent workstations, since the latter ones have more time to prepare items for delivery into order containers. Within a particular zone, when items of a warehouse order are picked the picker may start working on the next available warehouse order. However, pick activity is limited by the number of pick buffers in a zone; when all buffers are full, the picker must stop. It is assumed that enough pick buffers are provided in a single workstation to accommodate the largest warehouse order. One of the concerns of the designer is to determine the appropriate number of pick buffers in each workstation. The fundamental

issue in designing workstations with buffers is synchronization of the flow from the various picking zones, i.e. in order to perform a transfer of items to order containers they must be present in pick buffers when the order container arrives.

Pick zones may differ by the capacity and layout of pick buffers. With regard to capacity, pick buffers may be further classified into two categories: *item buffers* and *order buffers*. The former have their capacity limited to one item, while in the latter case the buffers are big enough to hold all the items for one order. In terms of layout, pick buffers may be evenly *spread* along pick area, (Figure 1.6), or be *centralized* in the middle of pick zone. Obviously, if pick buffers are centralized in the middle of a pick zone, then an order container passing underneath the pick buffers collects all the items for that order in a very short period of time.

To further facilitate our discussion, order pickers are categorized into two types: *manual* and *robotic*. Two major factors must be considered: *travel metrics* imposed by the order picker type, and the *capacity* of an order picker in terms of number of items that simultaneously can be handled by an order picker.

There are three types of travel metrics worth mentioning in this analysis: Euclidian, Rectilinear, and Chebyshev. The Euclidean distance is the straight line distance between two points. In a plane with one point at (x_1, y_1) and the other at (x_2, y_2) , the Euclidean distance is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. The Rectilinear distance is the distance between two points measured along axes at right angles. In a plane for one point at (x_1, y_1) and the other at (x_2, y_2) , the rectilinear distance between them is $|x_1 - x_2| + |y_1 - y_2|$. These two metric systems are more appropriate for measuring a distance traveled by a manual order picker. However, since in high volume setting for the studied systems it

makes more sense for a pick zone to have a single aisle structure where order picker travels only between pick area and buffers, the Euclidean and Rectilinear distances may produce similar result. Therefore, only rectilinear metric system, which is much easier to analyze, is considered. The third travel metric, the Chebyshev distance, is the maximum distance in either coordinate direction. In a plane with one point at (x_1, y_1) and the other at (x_2, y_2) , the Chebyshev distance is $\max(|x_1 - x_2|, |y_1 - y_2|)$. The Chebyshev distance applies only to robotic order pickers, since robots have two independent motors to travel simultaneously in the horizontal and vertical directions. The Chebyshev distance depends on a maximum travel of the horizontal and vertical movements.

In terms of order picker capacity, a manual picker may perform either single-item-pick (SIP) or multiple-items-pick (MIP) in one trip. To facilitate MIP, order pickers are usually provided with a pick vehicle with some sorting capability for placing items during a picking trip. For sorting line items, the sort-while-pick system can use a push cart that is divided into compartments to separate items that belong to other orders. Usually, physical limitations of robotic pickers do not allow them to hold multiple items simultaneously. Therefore, we will assume that robotic pickers pick only one item at a time, i.e. perform SIP. Table 1.1 summarizes the above discussion.

Table 1.1 Factors Affecting Order Pickers Performance.

Order Picker Type	Metric System	Capacity
<i>Manual</i>	Rectilinear	SIP/MIP
<i>Robotic</i>	Chebyshev	SIP

It is important to mention that customer orders must be preprocessed before being released for picking. Batching into warehouse orders, resource assignment for each line item, and work load balancing among pick zones all will have an impact on warehouse productivity.

1.3.2. Conventional Pick-to-Container Zone Picking

Pick-to-container zone picking workstations consist of a pick area, where items are stored and from which items are picked, and a pick station (usually a pick table), (Figure 1.7). Order containers are placed on a conveyor and are transported sequentially to all pick zones. If an order has an item to be picked at a certain zone then the order container is diverted from the conveyor to the corresponding workstation. Upon arrival, order containers wait in the pick station until order picker retrieves all the items required from that zone. At the station, a picker walks along the pick area, picks items and deposits them into the order container. When an order is completely picked the container is pushed back onto the conveyor and then transported either to shipping or to another pick station.

In systems with pick-to-container workstations picking and assembly operations occur simultaneously. A picker does not have advance notification about the content of a particular order whose container is yet to arrive. In order to perform a pick operation the order picker requires the order container to be present in the pick station. This type of picking is appropriate when no pick buffer is present in the system.

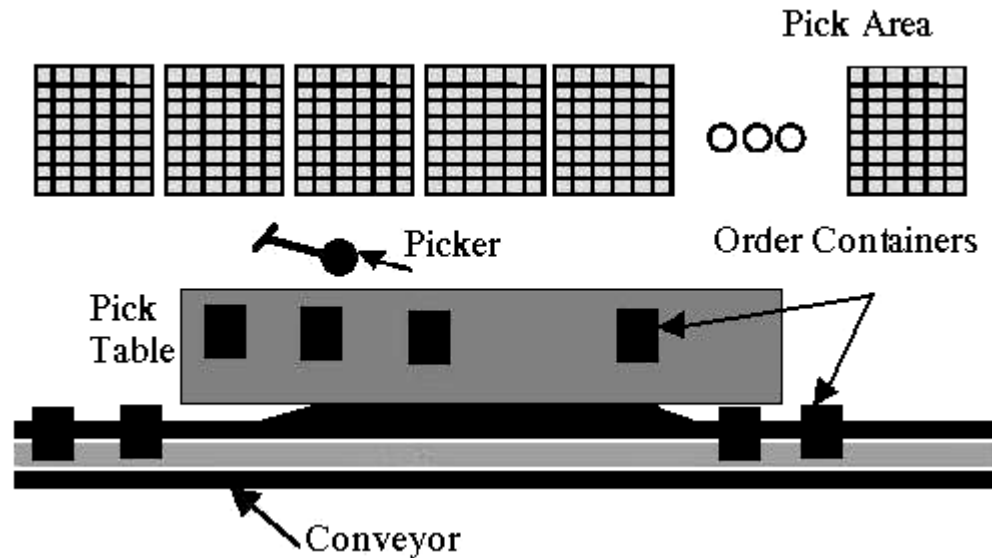


Figure 1.7 A typical *pick-to-container* pick zone without pick buffers.

Two types of order picking strategies can be implemented in systems with pick-to-container workstations: *single* order picking and *batch* picking. In single order picking each order is picked one by one in a single pick tour, for example, in first come first served order of container arrival. In batch picking two or more customer orders are batched and picked together in a single pick tour. At the same time, to pick an order, two *pick trip* strategies can be implemented, either by performing a single picking trip for the entire order or by making separate trips for every line item within the order. The pick trip strategies can be combined with order batching strategies.

If order batching is implemented with the single item pick trip strategy, then the following scenario must take place. If upon completion of picking a line item, a new order container has arrived to the pick table then two orders are combined and a single trip must be performed for every line item in the combined order. This scenario requires

smart control system that will indicate to the picker a new quantity of an item that must be picked in a single tour. The control system must also be able to show a picker the right quantity to be dropped into each order container.

If order batching is implemented together with order pick trip strategy then a picker might need to wait for order containers to arrive to the pick station to form a batch. Upon forming a batch the picker walks to shelf area and picks all items within that batch. The picker performs sort-while-pick operation by using push carts with separate compartments for each order. In this picking policy, mean pick time per order may be reduced due to batch picking. A number of policies for order batch picking may be considered:

- Batch picking of fixed number of orders, say b . The picker waits till b order containers arrive to a pick station and starts picking as soon as the number of containers reaches this size. If, on completion of a batch, a picker finds more than b orders waiting, the picker takes a batch of size b (e.g. in order of arrival), while others, in excess of b orders, wait on the pick table.
- There is a minimum number of orders in a batch, say a , that is less than or equal to capacity of the table, say b . The picker adopts the following policy: upon completion of picking a batch, the picker finds q orders waiting and if
 - i) $0 \leq q < a$, then a picker waits till the number of order containers in a station grows to a ;
 - ii) $a \leq q < b$, then a picker takes a batch of size q orders for service;

- iii) $q \geq b$, then a picker takes a batch of size b for service (e.g. in order of arrival), while those in excess of b orders, wait (typically creating a “jam” on a conveyor).

The order batching and pick trip strategies discussed above are presented for the completeness of the pick-to-container systems description. However, in this thesis these scenarios are not analyzed and are left for the future research.

1.4 The Physical System

Factors defining a physical system can be subdivided into *design* decisions and *operating policy* decisions. The design decisions specify the physical configuration of the system. Design decisions considered in this research include:

- number of pick zones;
- number of buffers for each zone in pick-to-buffer order picking systems;
- number of order container slots in the pick table for pick-to-container order picking systems;
- speed of the conveyor;

Operating policy decisions are used to control the actions of the system for efficient and effective performance. The operating policy decisions considered in this research include:

- number of orders in a batch for pick-to-buffer order picking systems;
- maximum number of containers allowed in a zone for pick-to-container order picking systems;

- time between two warehouse order releases to the system (or interarrival time of a warehouse order);
- time between two order trains (container) releases to a conveyor (or headway);

1.5 System Performance

Performance measures that are of interest to a designer include:

- throughput (number of orders processed in a unit time by the system or by a single pick zone);
- cycle time of an order in the system/zone (time elapsed between order release and order completion);
- average number of orders/items in a workstation or a system;
- average number of orders/items in the system waiting to be served;
- average time spent waiting by an order;
- service level, defined as the probability of order completely filled, an indication of system component (zone) synchronization;
- picker utilization; the fraction of time a picker is busy;
- conveyor utilization, the fraction of time a conveyor is busy;
- buffer utilization, the fraction of time a buffer is occupied.

1.6 Material Flow Analysis

Analyses of the material flow in warehousing systems can be conducted from three perspectives: *goods flow*, *order containers flow* and *picking operations flow*.

Goods Flow: The flow of goods through the picking system is shown schematically in Figure 1.8. After being replenished to the pick area, depending on order picking system type, goods flow to the order containers either directly or through intermediate pick buffers or avoiding them.

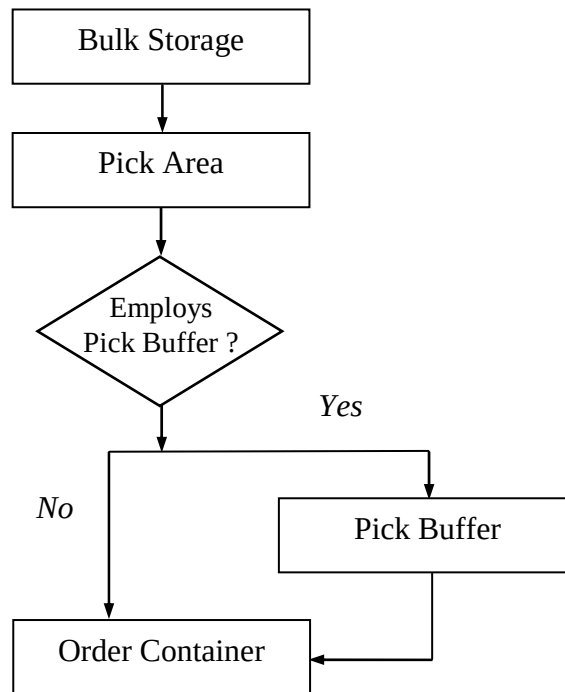


Figure 1.8 Flow of goods through the system.

Order Containers Flow: After entering the zone, order containers either flow on a conveyor to the next zone (or to the packing/shipping area) or remain in a workstation for a picking operation. In the first case, synchronization of the order container with pick operation is required. In the latter case, asynchronous flow of order containers is observed.

Picking Operations Flow: Picking operation for the system with buffers is straightforward. A picker transfers goods from the pick area to buffers. If buffer does not dispense goods to order containers automatically, then a picker must perform this transfer.

1.7 Outline of the Thesis

The rest of the thesis chapters are organized as follows. *Chapter 2* presents a discussion of the relevant research in the area of small parts order picking systems and their design. *Chapter 3* lays out the approach and directions for study of pick-to-buffer systems. *Chapter 4* describes simulation models constructed for the pick-to-buffer order picking systems. In *Chapters 5* and *6* the models for approximating performance measures of the pick-to-buffer systems and pick times are developed. *Chapter 7* and *8* present the approach in modeling assembly times of picking operation and service level of pick-to-buffer systems, respectively. *Chapter 9* demonstrates comparison of results of simulations with that of analytical models. Finally, *Chapter 10* discusses the future directions and remaining challenges that this study presents.

CHAPTER 2

LITERATURE REVIEW

During the last several decades, research on order picking system design and analysis has attracted a great deal of interest by practitioners and the academic community. The literature that directly or indirectly relates to this research is reviewed in this chapter.

2.1 Procedures to Support Order Picking Systems Design

Models to predict system behavior prior to actual construction have not been extensively and completely formulated in the existing scientific publications. As (Rouwenhorst et al. 2000) reports, an overwhelming majority of the published research is devoted to isolated sub-problems that are typically of analytical nature. It has been underlined that there is a need for pursuing design oriented studies that integrate various models and methods to develop a theoretical basis for an order picking system design methodology.

There have been many attempts to conceptualize and systematize the warehouse design problem. In a comprehensive review of warehouse design (Rouwenhorst et al. 2000) presents a structural approach to defining decision levels by classifying them into a hierarchy of three decision making categories depending upon the economic impact of each level.

- *Strategic* – decisions that have long term impact and are often the most costly of the three types of decisions (e.g. design of the process flow ,systems selection, design of processes,);
- *Tactical* – decisions that have medium term impact and are typically based on the outcome of the strategic decisions (e.g. resource dimensioning, equipment selection, layout);
- *Operational* – decisions that have short term impact and the lowest financial influence on the warehouse (e.g. batching, order sequencing, routing policies).

Design problems on higher levels of hierarchy are more interdependent than on lower ones, therefore it makes sense to group them for the integrated analysis. In the recent publication (Heragu et al. 2005) the authors develop a model that jointly determines the functional area sizes and the product allocation in a way that minimizes the total material handling cost. Design problems on lower levels of the Rouwenhorst hierarchy, on the other hand, can be considered independently.

(Gray et al. 1992) illustrate a method for a design process involving sequential and interactive steps through the hierarchy of decision levels. A combination of analytical and simulation models was used to narrow down the design space, then a detailed performance evaluation was done using simulation in order to select system configurations, equipment, and appropriate operational policies. Similar approach that combines simulation and analytical models were suggested by (Ashayeri et al. 1985). The authors pointed out that neither a purely analytical nor solely simulation approach will yield a practical design and a combination of the two may be needed to achieve a good design. A design procedure presented by (Rosenblatt and Roll 1984) and

(Rosenblatt et al. 1993) includes both simulation and analytical methods to find a global optimal solution in determining size and layout of a warehouse. In this formulation three types of costs are considered: initial investment, shortage costs, and costs associated with the storage policy.

Another attempt to conceptualize a design process is presented in the general framework of order picking system described in (Yoon and Sharp 1992), (Yoon and Sharp 1995), (Yoon and Sharp 1996). In this general framework a conceptual design procedure is established by decomposition and modification of the general structure of an order picking system which is a combination of functional areas and material flows, see also (Choe 1991). The importance of considering interdependencies between different design factors was also emphasized in (Vaughan and Petersen 1999), (Malmborg 2000), and (Sharp et al. 1990).

2.2 *Equipment*

The research concerning design problems on a strategic level appears to be very rare in spite of its importance in the entire design process. Here, design problems include decisions on the level of automation of the order picking system. Competing alternatives include fully automatic, semi-automated or manual order picking systems.

A description of small parts order picking systems equipment alternatives is provided by (Frazelle 1988) and (Houmas 1986). A common way to compare alternative systems is to assess the economic or operational performance of an order picking system. In a parametric analysis based on total cost factors (Sharp et al. 1994) compare several competing small part storage and retrieval equipment types that are commonly used for

item picking. The equipment types compared are shelving, storage drawers, gravity flow racks, horizontal carousels, and miniload storage and retrieval systems.

Using a cost-productivity analysis technique called hierarchy of productivity ratios, (Cox 1986) developed a methodology to evaluate different levels of automation. Based on approximations of system performance (Oser 1996) provides an analysis of an automated transfer car storage and retrieval system for small parts storage and high performance by comparing their system to conventional alternatives, such as a miniload, a horizontal carousel.

2.3 Parametric Modeling

Decisions at the tactical level of design influence the operating performance of an order picking system. Here, the main concerns include determination of order picking system layout, or physical arrangement. The decisions for a layout problem of concern involve determining number of zones and the storage space required per zone. Specifically, given a layout, operational policies, and storage assignment policy, it is necessary to divide the entire pick area into zones such that a certain objective is achieved. Examples of objective include maximizing the system throughput and minimizing workload imbalance between zones. The literature in this area is very scarce.

In the previously-mentioned hierarchical framework developed by (Gray et al. 1992), one of the warehouse design decisions includes zoning, where decisions such as number of zones and zone sizes are made. In the publication by (Petersen 2002) the authors use simulation to examine the configuration or shape of pick zones. The authors conclude that size and storage capacity of the zone, batch size, and storage policy all have

a significant effect on the best zone configuration. In a study of pick-and-pass order picking systems, (Le-Duc and de Koster 2005) develop an optimization model to determine number of zones by assigning items to pick routes in each zone. In his thesis, (Roodbergen 2001) develops a non-linear optimization model to determine the layout that minimizes average travel distances in a system with random storage. The decision variables are number of aisles, length of aisles, and the depot location.

Numerous related studies address AS/RS systems. Problems of interest in AS/RS include configuration problems such as determining the number of cranes, height and length of pick area, etc. The design of AS/RS end-of-aisle order picking system is addressed by (Bozer and White 1990). The authors develop a mathematical model to analyze the performance of AS/RS end-of-aisle order picking system (Bozer and White 1984) and present a design algorithm to determine the minimum number of aisles required to meet a given throughput requirement. The AS/RS design problem is also discussed in (Karasawa et al. 1980), (Ashayeri et al. 1985), (Malmberg 2001), and (Zollinger 1996).

2.4 Performance Evaluation

The performance evaluation problem is to assess specific performance measures for a given order picking system, and often is solved using analytical models. Thus, performance models are essential tools for designing order picking systems. Examples of performance measures include travel time, storage capacity, and operational cost. Travel time models deal with the estimation of expected travel time to perform a picking operation or tour. Most of the related literature focuses on travel time models. Travel

time models can be classified according to equipment type utilized, such as automated storage/retrieval systems (AS/RS), carousels, or multi-aisle shelf storage systems. In order to model travel time, certain assumptions must be made, e.g, regarding pick area layout, storage location assignment policies, or batching and routing policies.

Numerous studies address performance models of AS/RS. (Bozer and White 1990) address end-of-aisle order picking systems focusing on miniload AS/RS. The authors develop the performance model for a randomized storage system that consists of multiple aisles and propose a design algorithm to minimize number of storage aisles. In (Bozer and White 1996), they further generalize their algorithm to other configurations of the order picking system. (Foley and Frazelle 1991) develop a distribution of a travel time for AS/RS with randomized storage, while (Park et al. 2003) elaborate it for dedicated storage.

A common technique used to analyze the performance of systems is based on queuing theory. (Choe 1991) develops queuing models with a two-moment approximation to estimate the response time of an order under single-order-pick, sort-while-pick policy with two variations of batching: fixed time windows and fixed number of orders, and pick-and-sort policy that contains two subsystems: the picker system and accumulation/sorting system. A stochastic model for performance analysis of a carousel system is presented in (Rouwenhorst et al. 1996). The carousel is treated as a server in a simple queuing model, permitting the derivation of various queuing characteristics. (de Koster 1994) proposes a modeling and approximate analysis method for pick-to-belt order picking systems that can be used in a design phase to quickly evaluate alternatives.

Design alternatives include the length and layout of the pick zone and the system, the number of picking zones, and the conveyor speed.

Results for queuing models of travel time in conventional multi-aisle order picking systems have been presented in numerous studies. For example, (Pandit and Palekar 1993) present a queuing theoretic model to study the effect on response time of layout in a conventional multi-aisle warehouse, where response time includes both waiting and processing times. (Chew and Tang 1999) develop a distribution for length of a single order picking tour in a multi-aisle order picking system. This result is used in a queuing analysis of order batching and storage allocation problems.

Performance models of AS/RS and mini-load systems were also formulated using queuing theory. (Hur et al. 2004) propose a stochastic approach for the performance estimation of a unit-load AS/RS by using an M/G/1 queuing model with a single server and two queues. Their proposed approach gives satisfactory results with very high accuracy.

(Lee 1997) suggests a single server queuing model that has two queues to estimate the throughput of a mini-load system. However, his model limits travel times to exponentially distributed random variables which does not always reflect reality. (Park et al. 1999) model a miniload system AS/RS system as a two stage cyclic queuing network with limited capacity and both general and exponential servers. They obtain closed form expressions for the stationary probability and the throughput of the system. (Koh et al. 2005) study order picking systems based on a miniload AS/RS. The authors assume the order picking time is exponentially distributed. They model the system as a closed-queuing network with limited queue length and analyzed the model with the

supplementary variable technique presented in (Ohlson 1981) to find the long-run behavior of the system.

2.5 Summary

The existing literature mainly focuses on designing a rectangular warehouse with aisle layout. Here, each pick tour begins and ends at the centralized depot passing through one or several storage aisles. This layout is not very efficient for high volume order picking systems when a conveyor is present as a transporter. In this research it is assumed that in every zone a pick tour occurs between a storage location and a pick buffer/table, which is located within the same aisle. Thus, it is not surprising to conclude that there is room for research in this area of order picking design.

The performance characteristics of some equipment types such as AS/RS or carousels, have been addressed in many studies. However, the throughput of novel high volume pick technologies, such as automatic item picker described in the Chapter I, has not been fully examined. Queuing models are popular tools in assessing system behavior. Most of the prior studies assumed exponential distributions of random variables, which usually does not reflect the reality. Another area that offers prospect for the further research is the design of pick zones. Proper design of pick zones can potentially improve productivity.

The research effort of this thesis aims at studying design issues that are mainly tactical in nature. Indeed, the main focus of the study is to investigate trade-offs in decisions at this design level by comparing different alternatives. At the same time this research focuses on strategic level of design decision in terms of order picking system

selection. The research concerning design problems on a strategic level appears to be very rare in spite of its importance in the entire design process. In the proposed research, strategic decisions on the level of automation of the order picking system are to be made. Competing alternatives include fully automatic, semi-automated or manual order picking systems.

Small parts order picking is complicated. It captures all the unique features of order picking. Few analytical results are available for small parts order picking. This study attempts to fill the gap.

CHAPTER 3

THE APPROACH

3.1 *Introduction*

In this chapter a modeling framework for supporting high volume small parts order picking systems design is presented. The framework is based on a generic approach for system design decomposition that can be applied to the specific high volume order picking systems addressed in this thesis. In the early stages of design, aggregated representation of system elements and behavior must be explored to establish an initial design. Later, this design will serve as a starting point for further detailed exploration by performing “fine tuning” of design parameters.

3.2 *Design Model*

Factors affecting design of order picking systems include: system requirements (e.g. throughput, service level), space constraints, system alternatives, budgetary restrictions, etc. All of the above contribute to the complexity of order picking system design.

Although we are unable to estimate accurately the cost of a specific target order picking system, it can be argued that the major contributors to the overall cost in the automated system are gantry robots and in manual systems are fast pick storage technologies (e.g. pick-to-light systems). Thus, from a design perspective, it seems natural to minimize the number of zones in the systems. By no means is the cost of storage technologies in automatic systems disregarded, rather it is considered as a part of a whole package of

automatic picking systems, since special types of storage systems might be required for highly automated robots.

A design is technically feasible if it meets system requirements for throughput and service level. *Throughput* of the systems is defined as number of orders processed per unit time, while *Service Level* of the system is defined as a probability that a batch of orders is entirely picked before order container enters the zone. For the design to be optimal, it has to be feasible and require minimum number of zones. The design problem thus may be posed conceptually as an optimization model and written as:

Objective: Min Z

Subject to:

System Throughput (Z) $\geq$ TH

System Service Level (Z) $\geq$ SL

Z is a Positive Integer

where, Z is the number of zones, TH is the total throughput required and SL is the service level required.

The above model suggests that system throughput and/or service level must be functions of zone configurations. In this thesis we attempt to model system requirements by studying zones in isolation and later incorporating output parameters into decisions required for designing the entire multi-zone system.

In order to support system requirements, proper design of zones is required, so the factors affecting performance of the zone must be explored. Since order containers are placed on a conveyor in batches (or possibly of size one) with certain headway between batches and the speed of a conveyor is constant through the entire line there will never be

blocking between zones. Therefore, we assume the space in between zones to be unlimited. Thus, the overall throughput of the system is determined by the bottleneck zone, i.e. the zone that supports the lowest throughput.

Focusing on the design of a zone, one piece of equipment that also contributes to the overall performance is the pick buffer. Therefore, the design process must take into consideration the impact of pick buffers on system requirements and performance. As such, throughput and service level models must incorporate certain system design parameters, specifically number of zones, storage space dimensions, picker and conveyor speeds, number and layout of buffers within each zone, time between release of orders into the system (order release triggers pickers to start picking), and time between releases of order containers into system.

3.3 Performance Models

In order to assess the throughput of the system, the performance of each zone must be evaluated. In this thesis an approximation model is developed to estimate the performance of a zone. The performance model is applicable to the pick-to-buffer technologies described in *Chapter 1*.

The performance model takes the form of a tandem queuing system of two servers with a buffer area of a limited capacity between them. An entity that is tracked through the queuing system may be either an order, if an order buffer system is studied or an item, if item buffer system is considered. The performance model decomposes the activities occurring in the zone into two key operations: *picking* of entities into buffers, and *assembly* of entities

from buffers into order containers. In the queuing model each operation is represented as a separate server.

The picking and assembly operations occur in sequence and therefore affect one another due to the limited buffer between them. These two operations are modeled and studied separately and later the two models are integrated into a single integrated performance model. Figure 3.1 shows elements of the performance model and their relationship within the model. Details of queuing model and solution methodology are described in *Chapter 5*.

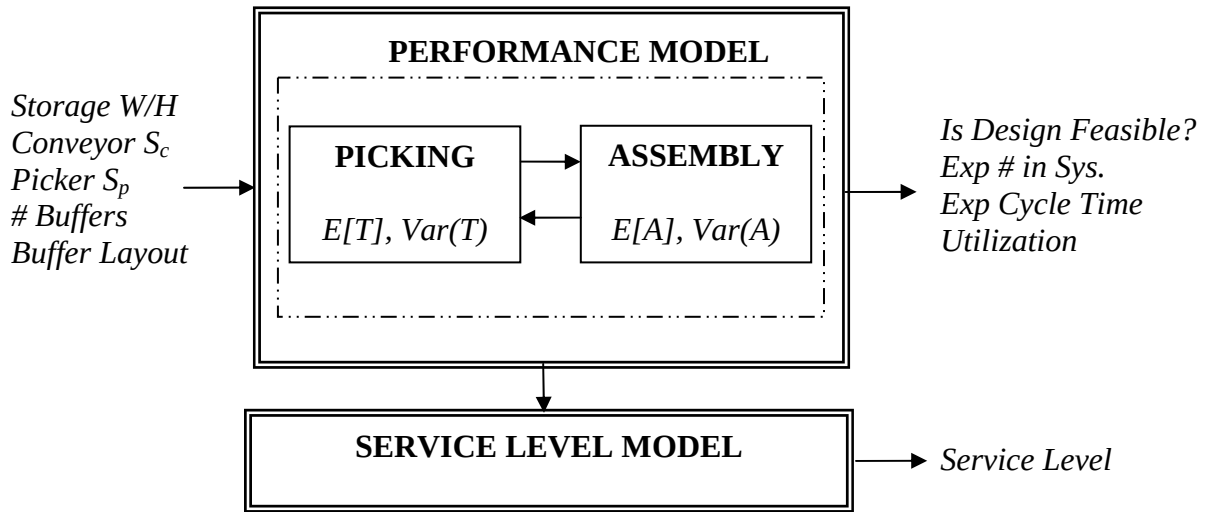


Figure 3.1 Modeling Approach

The performance model is used to determine if the design of the system is technically feasible, i.e. if the system in steady state meets throughput and service requirements, for given storage and space dimensions, picker and conveyor speeds, layout and number of buffers within each zone.

3.3.1. Picking Operation

The picking of entities into buffers is the first operational component of the performance model, i.e. the first server in the tandem queuing model. The picking operation starts at the instant when a picker begins to travel to pick an item of an order and ends when the item is deposited into a pick buffer. To estimate the picker's travel time approximation models are developed for different pick-to-buffer technologies. The travel time model approximates the expected time and the variance of picking activity given storage dimensions, picker speed and order profile. Indeed, the variability of order contents and picking process, and the physical layout make pick time approximation models very complex. Two travel time approximation models are developed corresponding to item buffers and order buffers.

For an *item buffer* technology with the robotic order picker type and spread layout the item flow is tracked through the system. The travel time model consists of two parts: approximation of horizontal movement and approximation of vertical movement. This model is also implemented for the manual order picker type in an item buffer system with spread layout by ignoring vertical travel as it is assumed that the pick area is ergonomically convenient for human picker to reach the highest pick location.

The travel time model for an *order buffer* technology with centralized layout tracks the flow of customer orders through the system. It approximates time for a single round trip that a picker must perform to pick an order and bring back to the central buffer area where orders are deposited into order buffers. Models for picking operation are developed and presented in *Chapter 6*.

3.3.2. Assembly Operation

The assembly of entities from buffers into order containers is the second component of the performance model. An assembly operation starts from the instant when the entity is deposited into a buffer and ends when it is collected (or assembled) into an order container. An entity is collected when an appropriate order container arrives on a conveyor to the buffer where the items of that order reside. Thus, entities must be held in buffers until container arrives. The entities that are deposited into a buffer after the appropriate order container passes underneath that buffer are considered late. To collect late entities a special “error container” is released into a system that cleans up buffers with late entities.

The assembly time model approximates the expected time and the variance of assembling operation given number of pick buffers, speed of a conveyor, and order release rate into the system. The model for assembly operation is developed in *Chapter 7*.

3.4 Service Level Model

Service level is a major consideration in the system requirements; therefore a model is developed to estimate service level. Service level depends on picker’s utilization, assembly operation. The service level is defined as the probability that a batch of orders is picked into pick buffers before corresponding order containers of that batch reach the pick zone. In *Chapter 8*, where the model for service level is developed, it is also shown that the time to pick a batch of orders have a normal distribution.

The results of a performance model can be used to evaluate the time to pick a batch of orders. Once an approximation of the pick time is available the value of service level can be

found by looking at the probability that the batch is picked before order containers reach the pick zone.

3.5 Conclusions

This thesis studies a problem in the design of pick-to-buffer high volume small parts order picking systems that has not been addressed before in literature. The novelty in the modeling approach is the distinct separation of order-picking operations which permits the development of performance model for both throughput and service level. Computationally fast solution methods are provided to compute results using the performance and service level models. *Chapter 9* presents details of model validations.

CHAPTER 4

SIMULATION MODEL

4.1 Introduction

This chapter describes the simulation models of pick-to-buffer systems, which are used later to validate analytic models. The simulation models are implemented using Brooks Automation's AutoMod TM simulation software.

The simulation method used in Automod is based on entities (or in AutoMod terminology *Loads*) that move through resources using material handling transports and storage systems. AutoMod provides built-in modules for modeling material handling transports. A substantial portion of the underlying model logic is automatically generated when using these modules. AutoMod material handling transports include: Conveyors, Path-Based Vehicle Movement, AS/RS, and Bridge Crane.

In this chapter, first we present a brief overview of AutoMod elements. Next, description of pick-to-buffer simulation models is given with further details on collected statistics and important parameters used in the models.

4.2 Essentials of AutoMod

Material handling transport systems can be created in an AutoMod model along with the process systems that connect the transport systems to the logical flow of loads (Brooks Automation Inc. 2003). In a process system, loads move between processes (or locations) and compete for resources (equipment, operators, and queues).

The process system is the backbone of an AutoMod model; it creates, processes, and removes loads in the simulation. Processes in AutoMod are the places where actions are performed or decisions are made. Each model has a process system and can optionally have one or more movement systems to transport loads between processing locations. The process system defines the logic that controls what activities a load sees during a simulation.

Loads are the moving entities in AutoMod and can be generated deterministically or probabilistically during the simulation. Loads are given types, such as “Item” or “Container,” and can have attributes such as “time in the system” or “time of arrival”. Types and attributes can be read and modified using AutoMod’s commands. Loads have three dimensional shapes that can be defined or set during the simulation. Loads are the driving force of a simulation; they trigger logic execution by moving from the process to process. Events correspond to the start or the end of a process or transport.

Resources in AutoMod are used to represent machines, operators, fixtures, containers, and any other finite capacity objects required to process or transport loads. A resource status indicates whether it is busy or idle while the availability specifies if the resource is up or down. A resource has a capacity limit, i.e. the number of loads that can use it simultaneously. Statistics are automatically collected for every resource in a model. Loads use resources for specified processing or transport times.

Queues in AutoMod represent physical space where loads can wait. They can have user defined capacity. Loads claim queues and compete with each other to occupy queue space. If a queue has reached its capacity the next load trying to occupy that queue is delayed in the previous location or queue until space becomes available.

Two types of material handling transports are used in the simulation of pick-to-buffer systems. These are conveyor and AS/RS. AutoMod provides built-in modules to model conveyors and AS/RS.

Conveyor modules define sections of a modeled conveyor. Each section may have different speed and orientation. Loads enter a conveyor at a point called a “station”, are transported from station to station and leave the conveyor at a station.

An *AS/RS* module defines a model of one or multiple aisles with storage racks where storage/retrieval machines pick up and retrieve loads. Each storage/retrieval machine serves a single aisle. The storage/retrieval machines move according to a Chebyshev travel metric, i.e. move simultaneously in horizontal and vertical dimensions. Each storage rack contains bins, or pick locations. Each row of bins is referred to as a tier; each column of bins is referred to as a bay. The storage/retrieval machine transports loads between bins and pickup and delivery stands or AS/RS zones. A pickup and delivery stand is a single bin. AS/RS zones are groups of bins. Processes can be connected to AS/RS Zones and to pickup and delivery stands.

The *AutoStat* program is a part of the AutoMod software. It is the statistical analysis tool that supports experimenting with a model, determining the warmup period and calculating confidence intervals for model statistics.

AutoMod automatically generates statistical reports specified by the user. This report provides information such as resource utilization, inventory levels and the total time loads are in a system.

4.3 Description of Pick-to-Buffer Simulation Models

A description of a single zone for two different pick-to-buffer systems is presented in this section: specifically, a robotic system with item buffers spread layout and a manual system with order buffer centralized layout. The models are parameterized to support a range of configurations. The simulation model consists of the process system and two material handling modules: Conveyors and AS/RS.

4.3.1. Item Buffers System with Spread Layout

A snapshot from a Simulation Run for the robotic system with item buffers spread layout is depicted in Figure 4.1. The area of the storage racks is $100 \times 20 \text{ m}$ with 100 bays and 20 tiers which yields 2,000 bins each bin having an area equal to one square meter. The vertical speed of the storage/retrieval machine is set to 0.9 m/sec while the horizontal speed to 3.15 m/sec . The values of acceleration/deceleration are set to a large number so that storage/retrieval machine reaches its speed instantaneously.

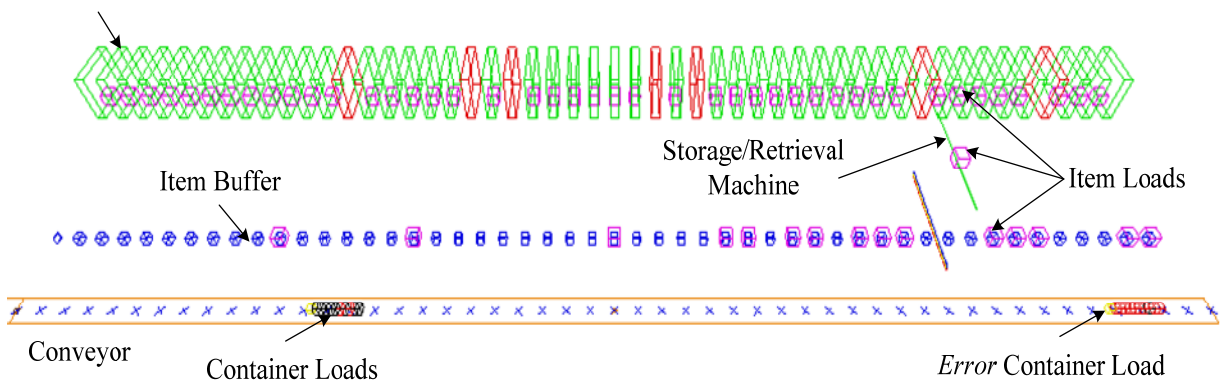


Figure 4.1 Snapshot of an Item Buffer System Simulation Run.

AS/RS zones and pickup and delivery stands are created in order to control the location and movement of pick items. The number of zone pickup and delivery stands is set to the same value as the number of item buffers in the system. In the example in Figure 4.1 fifty item buffers and hence fifty AS/RS zones and fifty pickup and delivery stands were created and numbered from one to fifty. Every AS/RS zone covers area of two bays and twenty tiers. Each pickup and delivery stand is located at the lower left hand corner of the AS/RS zone. For the robotic item buffer system the capacity of a storage/retrieval machine is set to one.

The conveyor module is defined so that it extends throughout the entire aisle and therefore has a physical length of *100 m*. All the way through its length the conveyor consists of one section since the speed is assumed to be constant. To control flowing *Container* loads the length of a conveyor is divided into fifty segments corresponding to fifty item buffers. Each segment of a conveyor has an input and an output station. An output station of a previous segment coincides with an input station of a next segment producing total of fifty one stations numbered from zero at the beginning of a section to fifty at the end of a section. The speed of the conveyor is set to the value of *0.6 m/sec* so that a container load travels the length of a conveyor within one zone within *166.667 seconds* (or *2.778 minutes*). The standard speed of conveyors varies from *0.3-0.6 m/sec* (*60 - 120 feet/min*), however high speed conveyors may attain up to *2.0 m/sec* (*400 feet/min*).

The process system is responsible for controlling of the flow of loads. Three main types of loads are created: *Batch*, *Container*, and *Item*. A *Batch* load is generated every $1/\lambda$ seconds. Upon arrival to the initial process a *Batch* load is then cloned into *M Container* loads, where *M* is the batch size, i.e. number of orders in batch. Now every *Container* load

physically represents an order container that must flow on a conveyor and assemble items from the item buffers. For each *Container* load that corresponds to a single order, a random numbers of items are generated according to a certain distribution (e.g. positive binomial). At this point in the simulation *Item* loads are generated for each order.

Once *Container* loads are created they are delayed for a certain time in order to allow the picker (i.e. storage/retrieval machine) to pick items into buffers. The delay is accomplished through a single resource with infinite capacity created beforehand in the simulation model. Upon completion of the delay, the *Container* load enters the conveyor section into the first station and flows from station to station until it reaches the last station of the conveyor where it leaves the system.

Immediately upon creation of *Item* loads, all the items of a batch begin the following process. For each item a pick location along the storage rack is randomly generated according to discrete uniform distribution in both vertical and horizontal directions. Next, they are placed into *Rack Queues* indicating their storage locations and wait until a picker picks them and places into buffers.

Fifty *Buffer Queues* with a capacity of one item are used to represent pick buffers. The *Buffer Queues* are also numbered from one to fifty so that each buffer corresponds to an AS/RS zone and pick/delivery stand. Once pick locations of all *Item* loads are determined the storage/retrieval machine starts picking the items in the batch one by one in a sequence starting from the leftmost location for the first batch and later by alternating a travel direction. The initial location of a storage/retrieval machine is at the beginning of a zone. When an *Item* load is picked, the process system determines the location of a closest empty buffer where the *Item* load must be deposited. The process system checks the capacity of the

Buffer Queue that corresponds to AS/RS zone of a pick location. If the capacity is reached the process system continues the logic and searches for the next closest available *Buffer Queue*. When an empty buffer is located, a storage/retrieval machine transports the *Item* load to pick/delivery stand location corresponding to the *Buffer Queue* number, and moves the *Item* load from the AS/RS into the *Buffer Queue*. In the *Buffer Queue* the *Item* load is delayed until a *Container* load moving on the conveyor reaches the station that has same number as a *Buffer Queue* number.

A *Container* load meanwhile flows on a conveyor and checks at every station if an *Item* load that belongs to its order resides in a corresponding *Buffer Queue*. If so, the *Container* load empties the buffer and *Item* load logically leaves the system. The *Container* load in its turn marks its attribute as “item picked” and continues to flow on the conveyor.

For cases when a pick error occurs, i.e. an *Item* load is moved into a *Buffer Queue* after a *Container* load passes it, a new *Error Container* load that is attached to the last container of every batch assembles it later. At the pick error event an *Item* load again empties a *Buffer Queue* and logically leaves the system.

4.3.2. Order Buffers System with Centralized Layout

Figure 4.2 displays a snapshot from a Simulation Run for the manual system with order buffers centralized layout. Many details of this system are not different the same as the robotic item buffer model. The main components of the system are the material handling transports, and loads; loads types are constructed as for the robotic system. The storage rack is again $100 \times 20 \text{ m}$ with 100 bays and 20 tiers which yields 2,000 bins each bin having an area equal to one square meter. For this system also the AS/RS module is used since it is

possible to convert the Chebyshev travel metric into a horizontal rectilinear travel, as is assumed for a manual picker. To accomplish that the vertical speed of storage/retrieval machine is set to a large number while the horizontal speed is set to 2 *m/sec*. The values of acceleration/deceleration are again set to a large number so that storage/retrieval machine reaches its speed instantaneously.

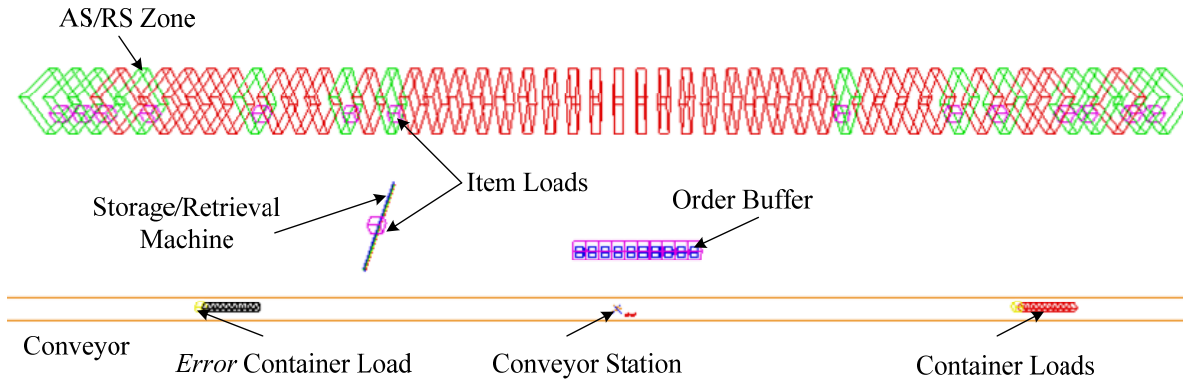


Figure 4.2 Snapshot of an Order Buffer System Simulation Run.

AS/RS zones and pickup and delivery stands are as described before. The rack here is divided into fifty zones to control the location and movement of pick items. In this example, each AS/RS zone covers two bays and twenty tiers. Each pickup and delivery stand is located at the lower left hand corner of the AS/RS zone. For the manual order buffer system the capacity of a storage/retrieval machine is set to infinity so that a picker can pick multiple items simultaneously as assumed in Section 1.2.

The conveyor module, as in the item buffer case, is defined so that it extends throughout the entire aisle and therefore has a physical length of 100 *m*. All the way through its length the conveyor consists of one section since the speed is assumed to be constant. To control flowing *Container* loads, the length of a conveyor is divided into two segments with

three stations: input, central buffer area and output stations. The speed of the conveyor is set to the value of 0.6 m/sec so that a container load travels the length of a conveyor within one zone within 166.667 seconds (or 2.778 minutes). Conveyor moves from left-to-right transporting *Container* loads from the input station to the output station.

Three main types of loads: *Batch*, *Container*, and *Item*, that are controlled by the process system are created similar to item buffer model.

Ten *Buffer Queues* with an infinite capacity (so that they can accommodate multiple items comprising an order) are used to represent pick buffers. Once pick location of *Item* loads are determined, the storage/retrieval machine starts picking all items of a batch. The initial location of a storage/retrieval machine is at the buffer area, i.e. the center of a pick area. Then, the storage/retrieval machine travels to the rightmost item location picking all items of a batch along the way. Once reaching the rightmost item location the picker travels back to the buffer area, i.e. center of a pick zone, deposits items into *Buffer Queues* sorting out each order into a separate buffer. Next, the storage/retrieval machine travels to the leftmost item location picking all the items along the way and again brings them back to deposit into *Buffer Queues*. This process repeats for each batch of orders.

The process system checks the availability of the *Buffer Queues*. If all buffers are occupied by other orders, the process system stops the storage/retrieval machine travel until one *Buffer Queue* becomes available and then a picker is given an opportunity to unload. When a *Buffer Queue* is available for a new order, a storage/retrieval machine moves all the *Item* loads from the AS/RS into the *Buffer Queue*. In the *Buffer Queues* the *Item* loads are delayed until a *Container* load moving on the conveyor reaches the second station that is located in the center of a conveyor line.

A *Container* load meanwhile flows on a conveyor. Upon reaching the second station it checks for *Item* loads that reside in *Buffer Queues*. If any *Buffer Queue* contains items of its order the *Container* load empties the buffer and all *Item* loads logically leave the system. The *Container* load marks its attribute as “items picked” and continues to flow on the conveyor. *Item* loads that do not belong to the passing *Container* load remain in *Buffer Queues* until the appropriate *Container* load passes underneath.

For cases when a pick error occurs, i.e. *Item* loads are moved into a *Buffer Queue* after the corresponding *Container* load passes it, a new *Error Container* load that is attached to the last container of every batch assembles it later on. At the pick error event *Item* loads again empties a *Buffer Queue* and logically leave the system.

4.4 Collecting Statistics

Statistical data are collected during simulation runs. Most are automatically generated by AutoMod while some must be collected using custom code. Table 4.1 shows a list of statistics collected along with the description of collecting method.

Table 4.1 Description of Collected Statistics from the AutoMod Simulation Run.

Collected Statistics	Collecting Method
Picker Utilization	AutoMod generated; Utilization of storage/retrieval machine
<i>Item</i> load Cycle Time	Coding; Time between generating an <i>Item</i> load and assembly by <i>Container</i> load from <i>Buffer Queues</i>
<i>Order</i> Cycle Time	Coding; Time between generating an order (which is same as a time to generating a <i>Container</i> load) and assembly of items of an order by a <i>Container</i> load from <i>Buffer Queues</i>

Table 4.1 (Continued).

<i>Item</i> load Pick Time	Coding; Time between the instant when storage/retrieval machine moves to pick an <i>Item</i> load and placing into a <i>Buffer Queue</i>
<i>Order</i> Pick Cycle Time	Coding; Time between the instant when storage/retrieval machine moves to pick the first <i>Item</i> load of an order and placing the last <i>Item</i> load of an order into a <i>Buffer Queue</i>
<i>Item</i> load Delay Time in the Buffer	Coding; Time between placing an <i>Item</i> load into a <i>Buffer Queue</i> and assembly by <i>Container</i> load
<i>Order</i> Delay Time in the Buffer	Coding; Time between placing the last <i>Item</i> load of an order into a <i>Buffer Queue</i> and assembly by <i>Container</i> load
<i>Item</i> Service Level	Coding; proportion of <i>Item</i> loads placed into a <i>Buffer Queue</i> during a period when <i>Container</i> load is delayed in a resource prior to it entering a conveyor section
Order Service Level	Coding; proportion of <i>Orders</i> placed into a <i>Buffer Queue</i> during a period when <i>Container</i> load is delayed in a resource prior to it entering a conveyor section

In order to determine the long-run or steady state behavior of a system it is necessary to specify a warm-up period for the simulation, that is a point in simulation when collection of the statistical data start and the initial data are deleted. The value of warm-up period is suggested by simulation software based on pilot runs. It is also possible to set the warm-up periods manually. AutoMod allows graphical plotting of output values using Welch's

method (Law and Kelton 2000). This method determines a warm-up period by analyzing the output curve. The time when the curve “flattens out” is considered the end of the warm-up period. By experimentation it was determined that 10 days for warm-up period is sufficient to reach steady state behavior of the system. The length of the simulation run is then determined to be four times more than the warm-up period, so that the total simulation runs for the systems totals to 40 days.

4.5 Model Parameters

In order to generate different configurations of the systems, the parameters of the models must be changed. Some parameters are possible to modify through coding in the process system, while others must be changed when the simulation model is constructed. Table 4.2 lists model parameters that can be changed to generate different configurations.

Table 4.2 Parameters of the Models that require modification to generate different system configurations.

Module	Parameter	Modified at
AS/RS	Number of Bays, Bay Width, Bay Height, Number of Tiers, Tier Height, Number of Pick and Delivery Stands, Pick and Delivery Stands Locations, Number of Zones, Location of Zones	Model construction
	Storage/Retrieval Machine Capacity, Horizontal and Vertical Acceleration, Deceleration, and Velocity	Model construction and Coding

Table 4.2 (Continued).

Conveyor	Length of Conveyor, Number of Sections, Location of Sections, Number of Stations, Location of Stations	Model construction
	Section Acceleration, Deceleration, and Velocity	Model construction and Coding
Process System	Number of Queues, Location of Queues, Dimensions of Vector Variables	Model construction
	Batch Arrival Distribution and Rate, Order Distribution and Distribution Parameters, Batch Size, Time Between Conveyor Releases into a Zone	Coding

4.6 Conclusions

This chapter describes a detailed simulation model for two variants of pick-to-buffer systems. The model is parameterized to allow experimentation with design parameters, such as length, height, opening size, travel speeds, and number of buffers.

CHAPTER 5

QUEUEING SYSTEMS

5.1 Introduction

An essential step in designing order picking systems is determining performance parameters such as throughput, order cycle time in the system, and utilization. It is also very important to establish a relationship between system performance measures and other design variables such as layout, dimensions, etc.

This chapter is concerned with the relationship between system performance measures and design. More specifically, the focus of the study is to determine expected throughput and cycle time of an order in the pick zone as a function of design.

A popular methodology for modeling nondeterministic manufacturing and material handling systems is queueing networks. Queueing networks are used in designing or redesigning a facility to evaluate design and configuration alternatives. A queueing network can be described as a group of interconnected components, called nodes, where each node represents a service station and the buffer in front of it. According to Kendall's notation (Kendall 1953) nodes can be classified symbolically. In the representation of the form *A/B/S/C* the interpretation is:

A - distribution of intervals between arrivals;

B - distribution of service times;

S - the number of servers;

C - the system capacity, including the customer in service. Omitted for unlimited queues;

The following symbols are used for arrival and service distributions:

M - the Poisson (Markovian) process with exponential distribution;

$_mE$ - the m -Erlang distribution;

H_2 - the balanced Hyperexponential-2 distribution;

D - the Deterministic times;

G - the General (any) distribution.

The arrival and service distributions might also have superscripts indicating batch arrival and/or process, respectively. For example, $D^X/M/1/10$ is deterministic arrivals of random batch sizes, one exponential server with maximum of 10 customers in the system.

Pick zones of pick-to-buffer zone picking systems can be modeled as an open queuing network with two stations in tandem and finite buffer between them. Picking and assembly operations are represented by serving stations and the pick buffer area is represented by buffers of finite capacity in front of the second, or assembly server, as illustrated in Figure 5.1. The arrival process into the queuing system is a stream of requests to pick items from pick area to buffers.

Due to the fact that arrival to the system is simply information flow to a picker, the buffer capacity in front of the first server is unlimited. The first server, representing a picking process, receives information about a batch of customer orders that contains multiple

items to pick. Then the picker picks line items from pick locations to buffers. There is a limited capacity for buffered items. The second server is responsible for dropping items from the buffer into the order container passing on a conveyor belt.

Two types of systems are distinguished: systems with *item-buffers* and systems with *order-buffers*. In one system a request is received by a picker to pick a batch of items, depositing each item into an individual item-buffer, while in the other system an operator receives a request to pick a batch of orders, depositing items for an order into an individual order-buffer. In the former case *items* and in the latter case *orders* are the relevant entities that must be tracked in the queuing system. From now on they are called *entities* of the system and the meaning (item or order) is clear from the type of buffer (item buffer or order buffer). Thus, in the system with item-buffers (order-buffer) every buffer can accommodate one entity, i.e. item (order).

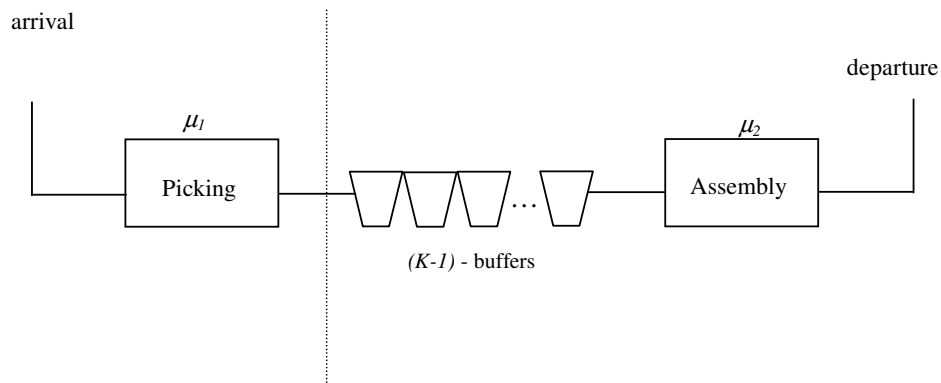


Figure 5.1 Queuing System for a Pick Zone.

5.2 Assumptions

The underlying assumptions required to analyze a queuing system of the type shown in Figure 5.1 are summarized as follows:

- (A1) The queue operates in a steady-state condition;
- (A2) Distribution of number of items in an order is known;
- (A3) Orders are grouped together in a batch and released into the system. The batch size is constant;
- (A4) All random variables are independent of each other;
- (A5) No entity is scrapped or otherwise lost;
- (A6) All items flow according to first-in-first-out discipline;
- (A7) Entity batch size is not larger than number of buffers in a station.

The following notation will be used throughout this chapter.

Parameters:

N_j – number of items in an order j , random number with known distribution;

M – constant batch size of orders;

X - size of a batch of entities, i.e. for item-buffers $X = \sum_{j=1}^M N_j$ and order buffers $X=M$;

K – number of buffers in the second station, including space in station;

$\lambda, E[\tau], c_a^2$ – arrival rate, expected interarrival time and squared coefficient of variation of

interarrival time, respectively, $E[\tau] = 1/\lambda$;

$\mu_i, E[S_i], c_{s_i}^2$ – service rate, expected service time and squared coefficient of variation of

service time, respectively, of serving at station i , for $i = 1, 2$; $E[S_i] = 1/\mu$.

Results of Decisions:

T – cycle time of an entity;

L – number of entities in the pick station;

Further notation will be developed as necessary.

5.3 Approximation Method

The queuing network described above is an open finite queuing network. The blocking caused by the finite buffer greatly increases complexity of the analysis. The performance measures for open queuing networks with general processing times cannot be evaluated exactly (Papadopoulos et al. 1993). Therefore, an approximation method for evaluating performance measures is needed.

The well-known approximation methodologies for queues with finite buffers are based on one or more of three techniques: isolation, decomposition and expansion. The isolation technique partitions the flow line into isolated sub-queues and studies them separately, (Labetoulle and Pujolle. 1980). The decomposition technique first partitions the flow line into isolated sub-queues, then considers the effect of an upstream and downstream station on the performance of the overall network (Altiok and Perros 1987; Bitran and

Tirupati 1988; Dallery and Frein 1993; Gershwin 1994). In the expansion technique, an additional virtual waiting space is added in front of every finite buffer for the items that are blocked by the full buffer. The jobs stay at this space until a buffer is freed (Jain and Smith 1994; Kerbache and Smith 1987; Singh and Smith 1997; Smith and Daskalaki 1988). To analyze the queuing model presented in Figure 5.1, a decomposition technique is proposed.

5.3.1. Decomposition Method

The queuing system presented in the Figure 5.1 is decomposed into two single-station sub-queues. Since the picker receives information about a customer order that contains multiple items (orders) to pick, the external arrival to the queuing system is modeled as a batch arrival. A picker picks an entity and drops it into the buffer. Thus, the first sub-queue can be presented as $G^X/G/1$ queue in Kendall's notation. The second single-server sub-queue is modeled as $G/G/1/K$ *lost* arrival system (or loss system), where K is the buffer capacity including the entity in service. In the stand-alone lost arrival system, arrivals are lost if queue capacity has reached its limit K (i.e. the queue is saturated).

In the real system, when a picker picks an entity and is not able to drop into a buffer because no buffer is available for a drop operation then the entity waits to be dropped and the first station is blocked. The picker holds the entity until at least one buffer is freed. Total service time of a picker then consists of two components: the actual serving time and the holding time.

In the model, a method for revising the upstream service time to include the additional holding time is implemented. The departure process from sub-queue I, which is at the same time an arrival process to sub-queue II carries information about saturation of the

sub-queue II. A saturation of sub-queue II leads to a blocking of the first server. In this system the blocked entity waits in the first server until a second sub-queue becomes unsaturated.

The decomposition method proposed here is based on an approximation model suggested by (Yannopoulos and Alfa 1994). In their approximation algorithm (Yannopoulos and Alfa 1994) start from the last node of the tandem line and revise the service time and variance of the previous nodes. The approximation proposed in this thesis is a modified version of the above approach. Here we begin with the first sub-queue of the network (i.e. $G^X/G/I$ system). The $G^X/G/I$ model is analyzed to find mean cycle time of an entity in the system, utilization of the server and the squared coefficient of variation of interdeparture times from the first sub-queue (which is the squared coefficient of variation of interarrival times into the second sub-queue) as if no blocking occurs. (Yannopoulos and Alfa 1994) assume squared coefficient of variation of interdeparture times from the first sub-queue is the same as squared coefficient of variation of interarrival times into the second sub-queue, which causes inaccurate approximation. Then the decomposition approach proceeds to analyze the second sub-queue, i.e. $G/G/I/K$ system. In this analysis both mean cycle time and blocking probability are computed. Using the value of blocking probability, the next step is to iterate back to sub-queue I for a revision of the service processes, i.e. mean and squared coefficient of variation of the first server, in the same manner as described in (Yannopoulos and Alfa 1994). In this way we are able to account for possible delays for the entity in the first sub-queue that is in the blocked server. Then the performance parameters such as mean cycle time of an order and average number of orders in the first sub-queue are again derived.

5.4 Analytical Model

Consider a tandem network as shown in Figure 5.1. External arrival of batches occurs only at the first node. Upon arrival at the first server they join a queue of unlimited size. Suppose the arrivals of batches follow a renewal process with rate λ , mean $E[\tau]=1/\lambda$, and squared coefficient of variation c_a^2 .

Depending on pick buffer capacity (item or order) it is necessary to define the arriving batch size X .

Item-Buffer System. Each order consists of a batch of items of a random size N with mean of $E[N]$, squared coefficient of variation c_N^2 , and probability mass function $P(N=n)=a_n$, $n=0, 1, 2, \dots$. M orders are batched together for processing. X is the total number of items in that batch. If flow of items through a queue is considered then

$$X = \sum_{j=1}^M N_j \quad (5.1)$$

Proposition 1: The mean and the squared coefficient of variation of batch size X are then

$$E[X] = E[N]M \quad (5.2)$$

$$c_X^2 = \frac{c_N^2}{M} \quad (5.3)$$

Proof: (Appendix A)

Order-Buffer System. If flow of orders through a queue is considered then

$$E[X] = M, \text{ and } c_X^2 = 0 \quad (5.4)$$

Thus, each arrival consist of a batch of entities of random size X with mean size of $E[X]$, squared coefficient of variation c_X^2 defined appropriately for each studied system. Service times at stations have general distributions with service rate μ_i entities/time unit and squared coefficient of variation $c_{s_i}^2$, $i = 1, 2$. The capacity of the second server is K entities (including the one in service). Departures from the network occur only from the second server. An entity that completed its service at server I joins a queue of the second server. If a sub-queue II is saturated then an entity remains at the first server until space becomes available at the buffer of the second server. During the time when server I is blocked it cannot process any other entity.

5.4.1. Computation of Performance Parameters of $G^X/G/I$ Model

Consider $G^X/G/I$ sub-queue. Entities are served by a single server with a general service time distribution. The service time of a server is arbitrarily distributed with rate μ_1 , mean $E[S_1] = 1/\mu_1$ and squared coefficient of variation $c_{s_1}^2$. The traffic intensity is computed by (5.5).

$$\rho_1 = \frac{\lambda E[X]}{\mu_1} \quad (5.5)$$

and it is assumed that the queue is stationary, i.e. $\rho_1 < 1$. Entities in each batch are numbered randomly. The service discipline is First In First Out (FIFO) within batches.

Proposition 2: The expected cycle time of an entity in a $G^X/G/I$ system is computed as

$$E[T_1] = \left\{ \frac{\rho_1}{(1-\rho_1)} \right\} \left\{ \frac{(c_a^2 + \rho_1^2 c_X^2 + \rho_1 \lambda E[S_1] c_{s_1}^2)}{(1 + \rho_1^2 c_X^2 + \rho_1 \lambda E[S_1] c_{s_1}^2)} \right\} \left\{ \frac{(\rho_1 + \rho_1 c_X^2 + \lambda E[S_1] c_{s_1}^2)}{2\lambda} \right\} + \frac{E[S_1](E[X]c_X^2 + E[X] + 1)}{2} \quad (5.6)$$

Proof: Let $S_{k,n}$ be the service time of the k th entity in the n th batch, assume the $S_{k,n}$ are independently and identically distributed random variables and for convenience, define $S_{0,n} \equiv 0$. Let

$$S_n^* = \sum_{k=1}^X S_{k,n}, \quad n = 1, 2, \dots \quad (5.7)$$

be the total service time of all arrivals of the n th batch. Assume the interarrival time, the service time and batch size are mutually independent.

As shown in (Buzacott and Shanthikumar 1993), the $G^X/G/1$ queue can be analyzed by using approximations of the $G/G/1$ queue. Each job of the $G/G/1$ queue approximation is the aggregate of the corresponding tasks associated with each job in the $G^X/G/1$ queue, and has an arrival process $\{A_n, n=1, 2, \dots\}$ and service time S_n^* .

Proposition 3: The mean and the squared coefficient of variation of service times S^* are then

$$E[S^*] = E[X]E[S] \quad (5.8)$$

$$C_{S^*}^2 = C_X^2 + \frac{1}{E[X]} C_S^2 \quad (5.9)$$

Proof: (see Appendix A)

The average number of entities, $E[L_1]$, in the system for $G^X/G/1$ queue as given in (Buzacott and Shanthikumar 1993) is the following

$$E[L_1] = \lambda E[X] w_o + \frac{\lambda (E[X^2] + E[X]) E[S_1]}{2} \quad (5.10)$$

where w_o – the waiting time of an item in $G/G/1$ system is given by

$$w_o = \left\{ \frac{\rho_1^2 (1 + C_{S_1}^2)}{1 + \rho_1^2 C_{S_1}^2} \right\} \left\{ \frac{C_a^2 + \rho_1^2 C_{S_1}^2}{2\lambda(1 - \rho_1)} \right\} \quad (5.11)$$

Using *Proposition 3* and (5.11), the waiting time of an entity in a $G^X/G/I$ system would be

$$w_o = \left\{ \frac{\rho_1}{(1-\rho_1)} \right\} \left\{ \frac{(C_a^2 + \rho_1^2 C_X^2 + \rho_1 \lambda E[S_1] C_{S_1}^2)}{(1 + \rho_1^2 C_X^2 + \rho_1 \lambda E[S_1] C_{S_1}^2)} \right\} \left\{ \frac{(\rho_1 + \rho_1 C_X^2 + \lambda E[S_1] C_{S_1}^2)}{2\lambda} \right\} \quad (5.12)$$

Substituting (5.12) into (5.10), the average number of entities in a $G^X/G/I$ system can be written as the following

$$E[L_1] = \left\{ \frac{\rho_1 E[X]}{(1-\rho_1)} \right\} \left\{ \frac{(c_a^2 + \rho_1^2 c_X^2 + \rho_1 \lambda E[S_1] c_{S_1}^2)}{(1 + \rho_1^2 c_X^2 + \rho_1 \lambda E[S_1] c_{S_1}^2)} \right\} \left\{ \frac{(\rho_1 + \rho_1 c_X^2 + \lambda E[S_1] c_{S_1}^2)}{2} \right\} + \frac{\rho_1 (E[X] c_X^2 + E[X] + 1)}{2} \quad (5.13)$$

For a batch arrival queue, Little's law has the form

$$E[T_1] = \frac{E[L_1]}{\lambda E[X]} \quad (5.14)$$

The expected cycle time of an entity in the first sub-queue shown in (5.6) can be computed using Little's law and the average number of entities in a $G^X/G/I$ system.

5.4.2. Coefficient of Variation of the Interdeparture Times

The proposed decomposition method analyzes the first sub-queue of the network to find the squared coefficient of interdeparture as if no blocking occurs. Let ΔD be interdeparture times from $G^X/G/I$ queue.

Proposition 4: Squared coefficient of variation of the interdeparture times, c_d^2 , of an entity from $G^X/G/I$ is obtained as follows:

$$c_d^2 = \rho_1^2 E[X] (c_X^2 - 2) + 2E[T_1] \rho_1 \mu_1 (\rho_1 - 1) + 2E[X] \rho_1 + c_a^2 E[X] + 2c_{S_1}^2 \rho_1^2 \quad (5.15)$$

Proof: By definition, the squared coefficient of variation of the interdeparture times is expressed through variance and expected interdeparture times.

$$c_d^2 = \frac{\text{var}(\Delta D)}{(E[\Delta D])^2} \quad (5.16)$$

Let A_n be the n th batch arrival time and let $D_{k,n}$ be the departure time of the k th job of the n th batch (Figure 5.2). Then $D_{1,n}$ is the departure time of the first job served in the n th batch and $D_{last,n-1}$ is the departure time of the last job served in the $(n-1)$ st batch. Therefore $\{D_{1,n}\}$ satisfies the following relation (5.17)

$$D_{1,n} = \max [D_{last, n-1} ; A_n] + S_{1,n} , \quad n=1,2,\dots \quad (5.17)$$

Let $[A]^+ = \max [A, 0]$. Flow time of the batch $T_n = D_{last,n} - A_n$ (see Figure 5.2) is given by (5.18)

$$\begin{aligned} T_n &= [D_{last, n-1} - A_n]^+ + S_n^* = \\ &= [(D_{last, n-1} - A_{n-1}) - (A_n - A_{n-1})]^+ + S_n^* = \\ &= [T_{n-1} - \tau_n]^+ + S_n^* \end{aligned} \quad (5.18)$$

where $\tau_n = A_n - A_{n-1}$ is the interarrival time between the $(n-1)$ st and n th batch arrival times.

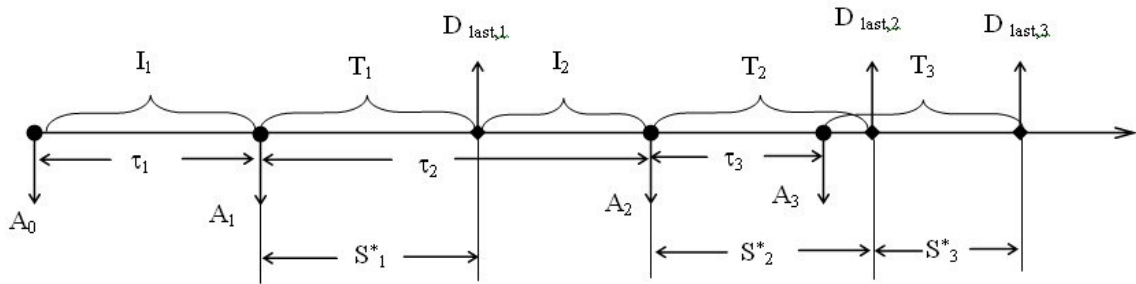


Figure 5.2 Sample System Behavior.

If $D_{last, n-1} < A_n$ or equivalently $T_{n-1} < \tau_n$ the server will be idling during the period of $(D_{last, n-1} ; A_n)$ for a duration of $I_n = (A_n - D_{last, n-1})$. Otherwise, the server will be processing

the first job of the (n) th batch immediately after departure of the last job of the $(n-1)$ st batch.

In this case we set $I_n = 0$. Therefore,

$$I_n = [\tau_n - T_{n-1}]^+, \quad n=1,2,\dots \quad (5.19)$$

Subtracting (5.19) from (5.18) we get

$$T_n - I_n = [T_{n-1} - \tau_n]^+ + S_n^* - [\tau_n - T_{n-1}]^+ = T_{n-1} - \tau_n + S_n^*, \quad n=1,2,\dots \quad (5.20)$$

We assume $E[T_{n-1}] \rightarrow E[T] < \infty$ and $E[S_{1,n}] \rightarrow E[S_1] < \infty$ as $n \rightarrow \infty$. Then taking expectations of both sides and taking the limit as $n \rightarrow \infty$ we get

$$E[I] = E[\tau] - E[S^*] \quad (5.21)$$

Therefore the interdeparture time, $\Delta D_{1,n}$, between the last job of the $(n-1)$ th batch and the first job of the n th batch is:

$$\Delta D_{1,n} = D_{1,n} - D_{last,n-1} = [A_n - D_{last,n-1}]^+ + S_{1,n} = [\tau_n - T_{n-1}]^+ + S_{1,n} = I_n + S_{1,n} \quad (5.22)$$

Now, by taking expectations of both sides and the limit as $n \rightarrow \infty$ we get the expected interdeparture time of the first customer in an unspecified batch as given by (5.23).

$$E[\Delta D_{1,\cdot}] = E[I] + E[S_1] = E[\tau] - E[S^*] + E[S_1] \quad (5.23)$$

On the other hand the interdeparture time, $\Delta D_{\cdot,n}$, within a batch is simply a service time of a job. The expected interdeparture time of jobs within a batch in an unspecified batch is

$$E[\Delta D_{\cdot,\cdot}] = E[S_1] \quad (5.24)$$

Thus, the expected interdeparture time of any unspecified job is obtained by

$$\begin{aligned} E[\Delta D] &= \frac{E[\Delta D_{1,\cdot}] + E[\Delta D_{\cdot,\cdot}](E[X] - 1)}{E[X]} = \\ &= \frac{E[\tau] - E[S^*] + E[S_1] + E[S_1](E[X] - 1)}{E[X]} \\ &= \frac{E[\tau]}{E[X]} \end{aligned} \quad (5.25)$$

Using (5.22) and the fact that I_n , and $S_{l,n}$ are independent, the variance of the interdeparture times between the first job of n th batch and the last job of the $(n-1)$ st batch, $var[\Delta D_{l,\cdot}]$, can be written as

$$var(\Delta D_{l,\cdot}) = var(I) + var(S) \quad (5.26)$$

Rewriting (5.20) as

$$T_n - S_n^* - I_n = T_{n-1} - \tau_n \quad (5.27)$$

and observing that if $T_n - S_n^* > 0 \Leftrightarrow I_n = 0$, and $T_n - S_n^* = 0 \Leftrightarrow I_n > 0$, we have:

$$(T_n - S_n^*) I_n = 0 \quad (5.28)$$

Taking the variance of both sides in (5.27) one sees

$$var(T_n - S_n^* - I_n) = var(T_{n-1}) + var(\tau_n) \quad (5.29)$$

Observe that in general,

$$var(T_n - S_n^* - I_n) = var(T_n - S_n^*) + var(I_n) - 2cov((T_n - S_n^*) I_n) \quad (5.30)$$

Also,

$$var(T_n - S_n^*) = var(T_n) + var(S_n^*) - 2cov(T_n, S_n^*) \quad (5.31)$$

A useful result given in (Buzacott and Shanthikumar 1993) shows that

$$E[T_n S_n^*] = E[T_n]E[S_n^*] + var(S_n^*) \quad (5.32)$$

Equation (5.32) can be used in the computation of $cov(T_n, S_n^*)$

$$\begin{aligned} cov(T_n, S_n^*) &= E[T_n S_n^*] - E[T_n]E[S_n^*] \\ &= E[T_n]E[S_n^*] + var(S_n^*) - E[T_n]E[S_n^*] = var(S_n^*) \end{aligned} \quad (5.33)$$

Substituting (5.33) into (5.31) yields

$$var(T_n - S_n^*) = var(T_n) + var(S_n^*) - 2var(S_n^*) = var(T_n) - var(S_n^*) \quad (5.34)$$

By definition of covariance,

$$\begin{aligned} cov((T_n - S_n^*) I_n) &= E((T_n - S_n^*) I_n) - E[T_n - S_n^*] E[I_n] = \\ &= -E[T_n - S_n^*] E[I_n] = -(E[T_n] - E[S_n^*]) E[I_n] \end{aligned} \quad (5.35)$$

Using (5.34) and (5.35) the equation (5.31) can be written as

$$var(T_n - S_n^* - I_n) = var(T_n) - var(S_n^*) + var(I_n) + 2(E[T_n] - E[S_n^*]) E[I_n] \quad (5.36)$$

Now, using (5.36) the equation (5.29) can be rewritten as follows

$$var(T_n) - var(S_n^*) + var(I_n) + 2(E[T_n] - E[S_n^*]) E[I_n] = var(T_{n-1}) + var(\tau_n) \quad (5.37)$$

so that,

$$var(I_n) = 2(E[S_n^*] - E[T_n])(E[\tau] - E[S^*]) + var(\tau_n) + var(S_n^*) \quad (5.38)$$

Finally, (5.26) becomes,

$$var(\Delta D_{1,\cdot}) = 2(E[S^*] - E[T])(E[\tau] - E[S^*]) + var(\tau) + var(S^*) + var(S_1) \quad (5.39)$$

It remains to derive the variance of interdeparture times for all jobs other than the first one in a batch. Note that it equals the variance of the service time.

$$var(\Delta D_{i,\cdot}) = var(S_i) \quad i \in [2, X] \quad (5.40)$$

Since the service times are iid RV, the interdeparture times are also iid RV. Thus, the variance of the interdeparture time of the unspecified job can be computed as

$$var(\Delta D) = \frac{var\left(\sum_{i=1}^X \Delta D_i\right)}{E[X]} = \frac{E\left[\left(\sum_{i=1}^X \Delta D_i\right)^2\right] - \left(E\left[\sum_{i=1}^X \Delta D_i\right]\right)^2}{E[X]} \quad (5.41)$$

Expanding (5.41) one sees that

$$\begin{aligned} var\left(\sum_{i=1}^X \Delta D_i\right) &= var\left(D_1 + \sum_{i=2}^X \Delta D_i\right) = var(D_1) + var\left(\sum_{i=2}^X \Delta D_i\right) = \\ &= var(D_1) + var(D_i) E[X-1] + (E[D_i])^2 var(X-1) \end{aligned} \quad (5.42)$$

which after simplifications is used in (5.41)

$$\text{var}(\Delta D) = \frac{2(E[S_1]E[X] - E[T])(E[\tau] - E[S_1]E[X]) + \text{var}(\tau) + 2\text{var}(S_1)E[X] + 2(E[S_1])^2 \text{var}(X)}{E[X]} \quad (5.43)$$

Finally, further simplifying (5.43) the squared coefficient of variation of the interdeparture times of an entity from $G^X/G/I$ becomes (5.15).

5.4.3. Computing Performance Parameters of G/G/1/K Model

Consider now the $G/G/I/K$ sub-queue. Entities arrive to the sub-queue II one by one with the same interarrival time as the interdeparture time from sub-queue I which is given by (5.25). Next, they are served by a single server with a general service time distribution with rate μ_2 , mean service time $E[S_2] = 1/\mu_2$ and squared coefficient of variation $C_{S_2}^2$. Then, the traffic intensity is computed by (5.44).

$$\rho_2 = \frac{\lambda E[X]}{\mu_2} \quad (5.44)$$

A single-server approximation method for a loss system is used to compute performance parameters of $G/G/I/K$ system. A comprehensive review of several competing approximation methods for $G/G/I/K$ system is given in (Springer and Makens 1992). Among five approximations presented in the review, the one with the best approximation of the probability of system being full, p_k , is chosen as this parameter will be used in the revision of the service processes of the first sub-queue.

(Gelenbe 1975) proposes approximating a discrete queuing process as a continuous diffusion process. Consider a stochastic process $\{N(t) \mid N(t) = 0, 1, \dots, N\}$, where $N(t)$ is the number of entities in a node (including the one in the service). The discrete stochastic

process $\{N(t)\}$ can be approximated by a continuous diffusion process $\{X(t) | X(t) \in [0, N]\}$ (Yao and Buzacott 1985). Preliminary studies on diffusion processes can be found in (Cox and Miller 1980).

A diffusion process is characterized by its drift, variance, and its boundaries. The drift β is the difference of the arrival and service rates:

$$\beta = \lambda_2 - \mu_2 \quad (5.45)$$

The variance α of a diffusion process is the function of the first two moments of the inter-arrival and the service time distributions.

$$\alpha = \lambda_2 c_d^2 + \mu_2 C_{s_2}^2 \quad (5.46)$$

The diffusion process $\{X(t)\}$ has two boundaries, 0 and N. Whenever $\{X(t)\}$ hits either boundary, it remains there for a random time then returns to the interval $(0, N)$ (Yao and Buzacott 1985). Now, the queuing process can be approximated as a continuous diffusion process to find the performance measures of interest.

A standard way to combine the drift and variance parameters into one term is by the following.

$$\gamma = 2\beta / \alpha \quad (5.47)$$

The performance measures of interest for $G/G/1/K$ queue are the expected number of entities in the system, $E[L_2]$, the expected cycle time of entities, $E[T_2]$; probability of the system being full, p_K ; and of it being empty, p_0 . The approximations for p_0 is given by (Gaver and Shedler 1973) as

$$p_0 = \begin{cases} \frac{1-e^{\gamma K} - \rho_2(1-e^{\gamma(K+1)})}{(1+\rho_2)(1-e^{\gamma K}) - \rho_2(1-e^{\gamma(K-1)})} & , \text{ if } \rho_2 \neq 1 \\ \frac{1}{K+1} & , \text{ if } \rho_2 = 1 \end{cases} \quad (5.48)$$

(Gaver and Shedler 1973) do not provide approximation for probability of the system being full and for the expected number of entities in the system. (Kimura 1986) extends their work and provides the following approximations for probability of the system being full,

$$p_K = \begin{cases} \frac{(1-p_0)(1-e^{\gamma})e^{\gamma(K-1)}}{1-e^{\gamma K}} & , \text{ if } \rho_2 \neq 1 \\ \frac{1-p_0}{K} & , \text{ if } \rho_2 = 1 \end{cases} \quad (5.49)$$

and expected number of entities in the system

$$E[L_2] = \begin{cases} \frac{1-p_0}{1-e^{\gamma}} - \frac{(1-p_0)Ke^{\gamma K}}{1-e^{\gamma K}} & , \text{ if } \rho_2 \neq 1 \\ \frac{(1-p_0)(K+1)}{2} & , \text{ if } \rho_2 = 1 \end{cases} \quad (5.50)$$

Finally, using Little's Law one obtains the expected cycle time of entities in the sub-queue II

$$E[T_2] = \frac{E[L_2]}{\lambda E[X]} \quad (5.51)$$

5.5 Reverse Step for Service Processes Revision

After completing approximation of parameters of interest of the second sub-queue the reverse step is performed, by returning to the analysis of sub-queue I for service processes revisions. Figure 5.3 demonstrates how the two sub-queues are coupled through parameters computed for each sub-queue.

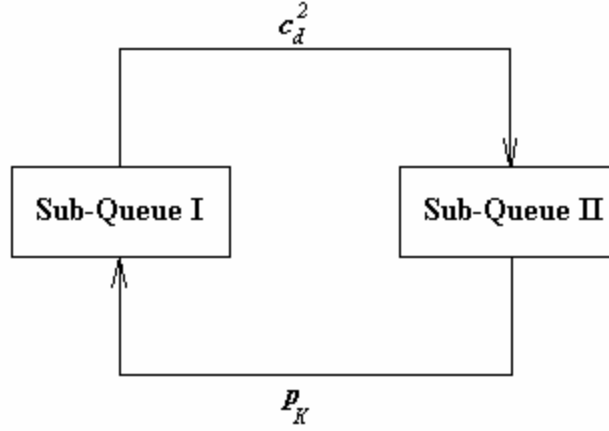


Figure 5.3 Forward and Reverse Coupling of Two Sub-Queues.

Thus, upon obtaining probability p_k by equation (5.49), this parameter is used to adjust the service time of the first server to account for the blocking delay. The first server blocking time is equivalent to the residual service time of the second server. The n th moment of the residual time of the second server is given in (Kleinrock 1975) as

$$E[R_2^n] = \frac{E[S_2^{n+1}]}{(n+1)E[S_2]} \quad (5.52)$$

where $E[S_2^{n+1}]$ is the n th moment of the service time of the second server. The variance of the residual time of the second server is given by

$$Var[R_2] = E[R_2^2] - (E[R_2])^2 \quad (5.53)$$

Then, the revised mean processing time, $E[S_1']$, and the variance for the first server,

$Var[S_1']$, are the following, (Yannopoulos and Alfa 1994)

$$E[S_1'] = E[S_1] + p_K E[R_2] \quad (5.54)$$

and

$$Var(S_1') = Var[S_1] + p_K^2 Var[R_2] \quad (5.55)$$

In order to obtain the third moment of the service time used in (5.55) the service distribution was approximated by phase-type Coxian-2 distribution (Yao and Buzacott 1985). According to Coxian laws with two phases, an entity undergoes service in phase 1, then leaves the system with probability p or continues its service in phase 2 with probability $(1-p)$, Figure 5.4. The service times in both phases are exponential with rate μ_1^C and μ_2^C respectively.

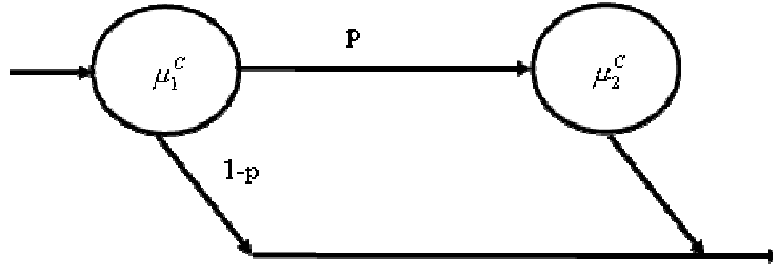


Figure 5.4 Coxian Distribution.

The first three moments of Coxian-2 distribution are the following

$$m_1 = \frac{1}{\mu_1^C} + \frac{p}{\mu_2^C} \quad (5.56)$$

$$m_2 = \frac{2}{(\mu_1^C)^2} + 2p \frac{\mu_1^C + \mu_2^C}{\mu_1^C (\mu_2^C)^2} \quad (5.57)$$

$$m_3 = \frac{6}{(\mu_1^C)^3} + 6p \frac{(\mu_1^C)^2 + \mu_1^C \mu_2^C + (\mu_2^C)^2}{(\mu_1^C)^2 (\mu_2^C)^3} \quad (5.58)$$

For a general distribution when first two moments of service distribution are given the parameters of Coxian-2 distribution can be found as suggested by (Marie 1980)

$$\mu_1^C = 2\mu_2 \quad (5.59)$$

$$\mu_2^C = \frac{\mu_2}{C_{s_2}^2} \quad (5.60)$$

$$p = \frac{1}{2C_{s_2}^2} \quad (5.61)$$

Having revised processing time and the variance for the first server, the decomposition algorithm suggests applying the $G^X/G/1$ model again in order to obtain final performance measure of the first sub-queue.

Thus, the total expected cycle time and expected number of entities in the system are obtained by

$$E[T] = E[T_1] + E[T_2] \quad (5.62)$$

and

$$E[L] = E[T]\lambda E[X] \quad (5.63)$$

5.6 Computational Results

To evaluate the approximation model a tandem queuing system is simulated using the AutoMod simulating package. Queuing systems with various distributions of arrival and service times, different utilizations of servers, number of buffers as well as batch sizes have been tested in order to gain better understanding of strengths and weaknesses of the approximation model.

Three types of distributions have been considered for interarrival and service times: Erlang-2, Exponential, and HyperExponential-2. These distributions cover situations where the coefficients of variation are low (0.5), moderate (1) and high (2), respectively. Four different batch sizes of arriving entities were included in the tests: batch of one (case of when

orders must be picked into order-buffer), batch of 5, 10 and 25 (case item-buffer systems). In order to make the study complete, server utilizations were set to vary from 0.2 to 0.8 by 0.2 increments. Finally, depending on batch size, different numbers of buffers in front of the second server have been simulated so that certain blocking phenomenon can be created. For all the tests, order size was assumed to have zero truncated binomial distribution, $ZTBin(p, k)$ with parameters $p \in (0, 1)$ and a positive integer k . The value of k was always set to the number of buffers in the system. In all cases the arrival rate, λ , is set to 0.05 batches/sec.

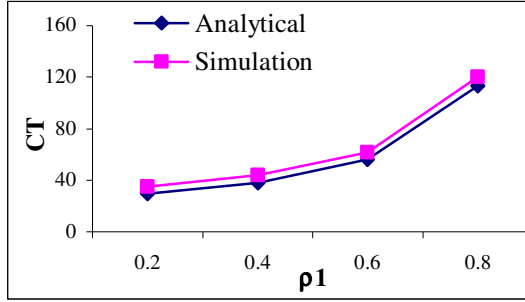
Table 5.1 shows results comparing analytical approximation with simulation for the adjusted utilization of the first server. The relative error indicates that the approximation works well in general. On average amongst all the comparisons the relative error is about 14.5%. The highest error is observed when the utilization of the first sub-queue is much less than utilization of the second sub-queue, a situation very unlikely to occur in the target application. This pattern is noted in all systems studied. One reason for this is due to inaccuracies in approximations of the second sub-queue. The approximation provided by (Gelenbe 1975) is not very accurate when the squared coefficient of the interarrival into the second sub-queue (which is same as the squared coefficient of interdeparture from the first sub-queue) is high. At the low utilizations of the first sub-queue the squared coefficient of interdeparture into the second sub-queue is higher than at higher utilizations, which causes the inaccuracy. For example for the system $M^{10}/_2Er/M/25$ when $\rho_1 = 0.2$ and $\rho_2 = 0.6$ the adjusted utilization of the first sub-queue produces 17.67% relative error, while when the utilization of the first sub-queue is increased to 0.8 the relative error reduces to (-0.353) %.

Table 5.1 Adjusted utilization of the first server.

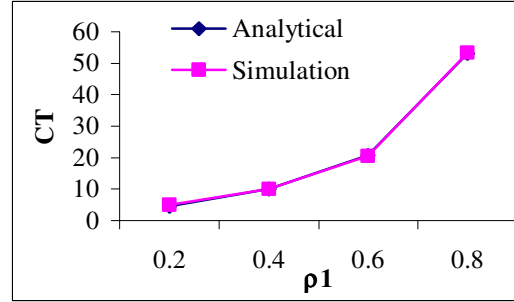
Queuing System (Arrival/Server1/Server 2/Number of Buffers)	ρ_1	ρ_2	Adjusted ρ_1 Analytical	Adjusted ρ_1 Simulation	Relative Error %
$M/M/M/5$	0.4	0.2	0.4002	0.4002	0.011
	0.6	0.2	0.6002	0.5994	-0.126
	0.8	0.6	0.8205	0.8234	0.355
	0.2	0.4	0.2031	0.204	0.428
	0.6	0.6	0.6205	0.628	1.211
	0.4	0.6	0.4205	0.4326	2.881
	0.8	0.8	0.8714	0.9064	4.015
	0.2	0.6	0.2205	0.2374	7.671
	0.6	0.8	0.6714	0.7332	9.203
	0.4	0.8	0.4714	0.5632	19.471
$M^5/M/M/10$	0.4	0.2	0.4	0.3998	-0.057
	0.6	0.2	0.6	0.6006	0.097
	0.6	0.6	0.621	0.6158	-0.833
	0.8	0.8	0.8571	0.8488	-0.966
	0.8	0.6	0.8113	0.8028	-1.05
	0.6	0.8	0.6811	0.7106	4.324
	0.4	0.6	0.4268	0.447	4.74
	0.2	0.4	0.2032	0.2158	6.219
	0.4	0.8	0.4935	0.6034	22.267
	0.2	0.6	0.2282	0.2942	28.91
${}_2Er^{10}/{}_2Er/{}_2Er/25$	0.4	0.2	0.4	0.4002	0.05
	0.8	0.6	0.8006	0.7994	-0.146
	0.6	0.2	0.6	0.601	0.167
	0.6	0.6	0.6037	0.6002	-0.575
	0.8	0.8	0.8137	0.804	-1.186
	0.2	0.4	0.2002	0.2026	1.2
	0.4	0.6	0.4065	0.4134	1.685
	0.6	0.8	0.6316	0.6558	3.825
	0.2	0.6	0.2073	0.245	18.161
	0.4	0.8	0.4417	0.5386	21.936
$M^{10}/{}_2Er/M/25$	0.6	0.2	0.6	0.6008	0.133
	0.4	0.2	0.4	0.4008	0.2
	0.8	0.6	0.8014	0.7986	-0.353
	0.6	0.6	0.6065	0.5998	-1.1
	0.8	0.8	0.8226	0.8074	-1.845
	0.2	0.4	0.2005	0.2044	1.967
	0.4	0.6	0.4106	0.4188	1.986
	0.6	0.8	0.6467	0.6644	2.743
	0.2	0.6	0.2118	0.2492	17.677
	0.4	0.8	0.4599	0.5426	17.988

Table 5.1 (continued).

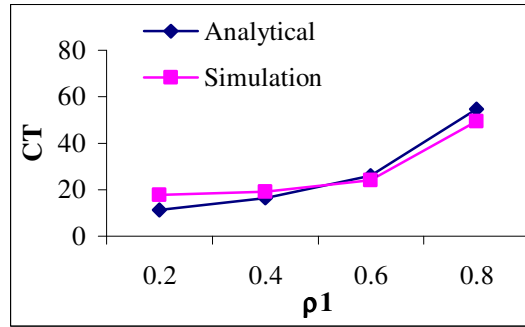
Queuing System (Arrival/Server1/Server 2/Number of Buffers)	ρ_1	ρ_2	Adjusted ρ_2 Analytical	Adjusted ρ_2 Simulation	Relative Error %
$2Er^{25}/M/2Er/50$	0.6	0.2	0.6	0.6006	0.1
	0.4	0.2	0.4	0.4006	0.15
	0.8	0.6	0.8001	0.7988	-0.168
	0.4	0.6	0.4025	0.4036	0.284
	0.2	0.4	0.2	0.2006	0.293
	0.6	0.6	0.6014	0.5994	-0.332
	0.8	0.8	0.8081	0.7996	-1.049
	0.6	0.8	0.6209	0.629	1.309
	0.2	0.6	0.2022	0.2138	5.76
	0.4	0.8	0.4268	0.4778	11.939
$M^{10}/H_2/M/25$	0.8	0.6	0.8025	0.803	0.061
	0.6	0.2	0.6	0.6004	0.067
	0.4	0.2	0.4	0.4006	0.15
	0.6	0.6	0.6076	0.6094	0.298
	0.8	0.8	0.8295	0.8366	0.857
	0.2	0.4	0.2005	0.2058	2.66
	0.4	0.6	0.4112	0.4242	3.157
	0.6	0.8	0.6505	0.6852	5.336
	0.2	0.6	0.2119	0.252	18.917
	0.4	0.8	0.4615	0.5576	20.825
$M^5/H_2/H_2/10$	0.6	0.2	0.6004	0.6012	0.139
	0.4	0.2	0.4004	0.4016	0.289
	0.8	0.6	0.8413	0.8696	3.362
	0.6	0.6	0.6534	0.69	5.599
	0.8	0.8	0.9332	0.9934	6.446
	0.6	0.8	0.7576	0.8336	10.035
	0.4	0.6	0.4597	0.5132	11.628
	0.2	0.4	0.211	0.241	14.23
	0.4	0.8	0.5695	0.697	22.383
	0.2	0.6	0.2606	0.3542	35.933



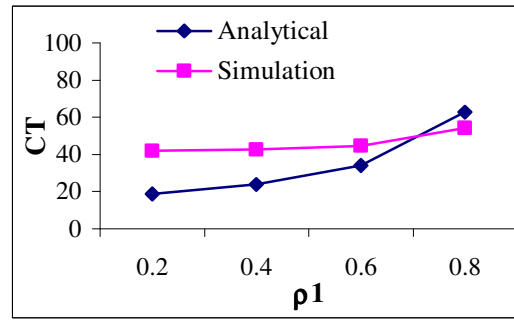
a) $EX=1, K=5, \rho_2=0.6$



b) $EX=5, K=10, \rho_2=0.2$

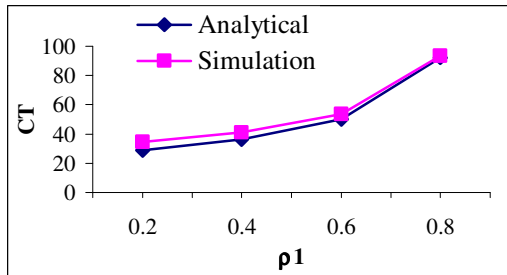


c) $EX=10, K=15, \rho_2=0.6$

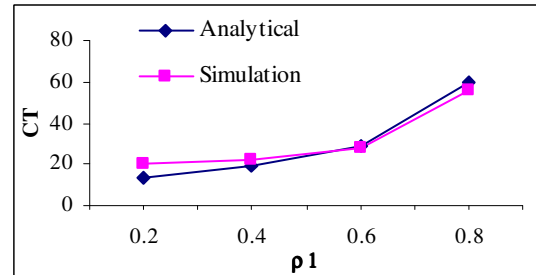


d) $EX=25, K=50, \rho_2=0.8$

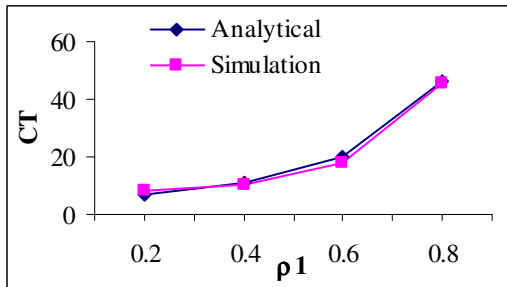
Figure 5.5 Total cycle time in $M^X/M/1$ and $G/M/1/K$ tandem queues.



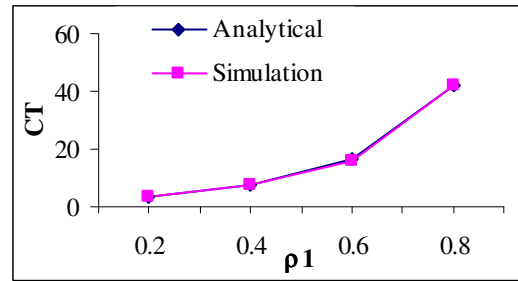
a) $EX=1, K=5, \rho_2=0.6$



b) $EX=5, K=10, \rho_2=0.6$

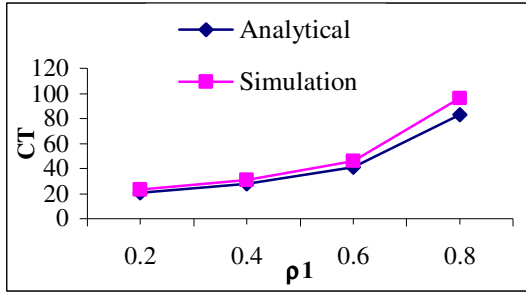


c) $EX=10, K=15, \rho_2=0.4$

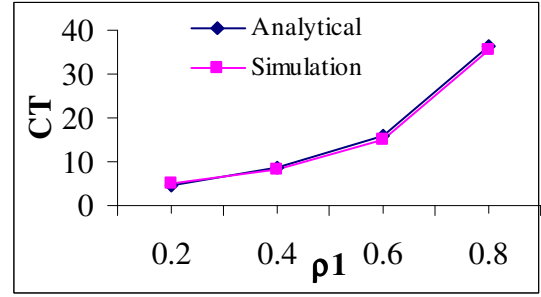


d) $EX=25, K=35, \rho_2=0.2$

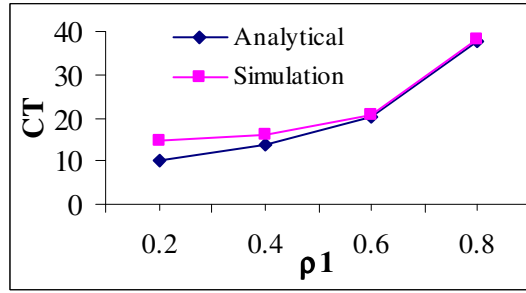
Figure 5.6 Total cycle time in $M^X/2Er/1$ and $G/M/1/K$ tandem queues.



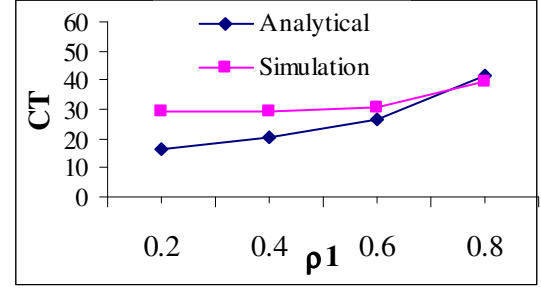
a) $EX=1, K=10, \rho_2=0.4$



b) $EX=5, K=10, \rho_2=0.2$

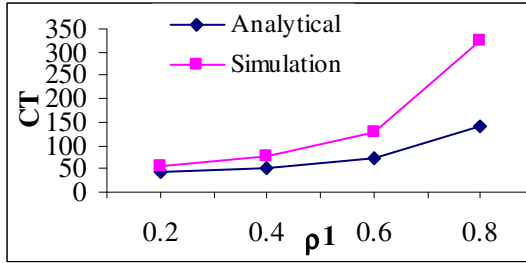


c) $EX=10, K=15, \rho_2=0.6$

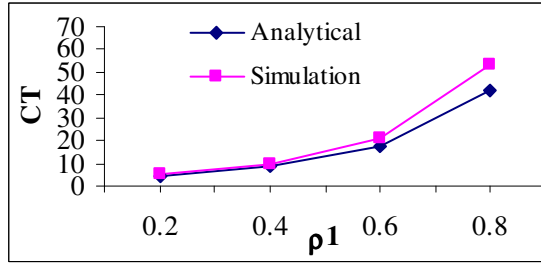


d) $EX=25, K=50, \rho_2=0.8$

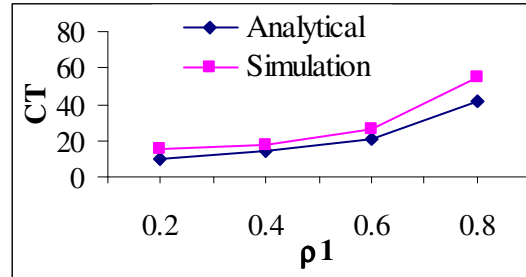
Figure 5.7 Total cycle time in ${}_2Er^X/M/1$ and $G/H_2/1/K$ tandem queues.



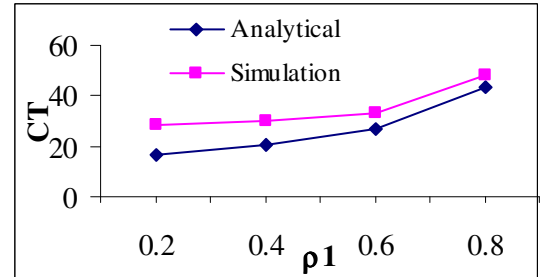
a) $EX=1, K=10, \rho_2=0.6$



b) $EX=5, K=10, \rho_2=0.2$



c) $EX=10, K=15, \rho_2=0.6$



d) $EX=25, K=50, \rho_2=0.8$

Figure 5.8 Total cycle time in ${}_2Er^X/H_2/1$ and $G/H_2/1/K$ tandem queues.

Another performance measure of interest is the total time the entity spends in the system, which is shown in Figures 5.5 – 5.8. As expected, as the utilization of the first sub-queue increases the total cycle time also increases. In general the plots of the approximation model fit the pattern of the plots from simulation results. As before, when the difference of service rates, hence utilizations, in two sub-queues is large the analytical model does not produce a good approximation. When the squared coefficient of variation of service times is high (case of Hyperexponential-2 distribution of service times, Figure 5.8) the deviation of an approximation from the simulation results is even larger. Once again this happens because the $G^X/G/1/K$ approximation for the second queue does not produce accurate result at high squared coefficient of variations.

5.7 Conclusions

In this chapter a performance model for pick-to-buffer order picking systems has been developed. The pick-to-buffer system is modeled as a tandem queue of two servers representing picking and assembly activities and buffers of limited capacity, representing pick-buffers. The queuing model then is solved by a decomposition method using iteration technique.

The solution of the queuing model provides such performance parameters as cycle time in the system, number of entities in the system, utilization. These parameters are crucial knowledge for designers.

To our knowledge there has not been prior work which included batch arrival into the queuing system. The solution to the queuing model is novel in the sense that first the squared coefficient of variation of the interdeparture times from the first sub-queue that has batch

arrivals is computed followed with approximations of performance parameters of the second sub-queue. Along with performance parameters, the approximation of the second sub-queue produces important parameters for our solution method, namely the blocking probabilities which affect performance parameters of the first sub-queue. With the iteration technique using those blocking probabilities, the adjusted performance parameters of the first sub-queue are then approximated.

To evaluate the performance of the queuing approximation, analytical results were compared to those from simulation of a two server tandem queue. Different distributions of interarrival and service times have been included in the study. These distributions were picked to cover situations with low, moderate and high variability. The results of comparing approximations of performance parameters with that of simulations indicate that the approximation produces fairly good estimates when utilization of the first sub queue is comparably close to the utilization of the second sub-queue.

CHAPTER 6

TRAVEL TIME APPROXIMATIONS

6.1 *Introduction*

An analytical expression for the travel time of the order picker is a key factor in designing small parts high volume order picking systems. Useful performance measures that can be derived from travel time models are the expected travel time and its variance.

In this chapter expected travel times and its variances are derived for particular pick methods and pick system physical characteristics assuming a pick-to-buffer system. In fact, approximation methods depend on system layout: centralized buffer and spread buffer layouts. Travel time is measured as the time it takes for an order picker to make a single pick trip between a pick buffer and item storage location to pick an order or batch orders.

This chapter is organized as follows. Assumptions of travel time models are first laid out followed by approximation methods for two technologies: robotic and manual systems. And finally, results of approximation method are compared to that of simulation for each technology considered.

6.2 *Assumptions*

The small parts high volume order picking systems considered have an item storage area in a single aisle layout, using conventional storage systems such as bin shelving or automated pick-to-light. We will not make any particular assumption on item storage equipment technology and only regard it as a *pick area* with openings where items are stored.

In this analysis the following assumptions are made:

- (A1) The number of item buffers (order buffers) is never less than the number of items (orders) in a batch of orders. Denote as B the number of item (order) buffers in the pick area;
- (A2) The length and height of a pick area, the length of a buffer area and order picker speed are known and constant;
- (A3) The pick area is modeled as a continuous rectangular area;
- (A4) Any point within a picking area is equally likely to be selected for a particular pick operation, i.e. pick locations are distributed uniformly along pick area;
- (A5) Item picked is placed in the closest and available pick buffer;
- (A6) Time to retrieve an item from a storage location, and time to place an item into a pick buffer are constant and small, and omitted from the analysis;
- (A7) Inventory shortage never happens; pick area inventory replenishment is performed in a way that does not interfere with order picking;
- (A8) Each line item in an order is composed of a single item unit;
- (A9) The number of items in a customer's order is modeled as random variable. Let N be the pick quantity with probability mass function $P(N=n)=a_n$, $n=0, 1, 2...u$.
Note, $u \leq B$;
- (A10) Customer orders are independent; the random variable, N , representing order size is independently and identically distributed.

Assumption (A1) on number of buffers is practical; if the number of item buffers (order buffers) is less than the number of items (order) in a batch of orders, then the batch can be split to satisfy this assumption.

Continuous representation of pick area, (A3), is appropriate since, in many cases, problems involve a large number of storage locations.

Assumption (A4) is not valid for most order picking systems. However, if class-based storage is used, then within a class this assumption is acceptable.

Many studies incorporate the retrieval time into the analysis of the travel time models. However, in the studied systems, the retrieval times usually have constant and small values relatively to walk time. Thus, retrieval times can be omitted from our study making assumption (A6) valid.

Assumption (A7) is itself one of the most important issues in designing of order picking systems. Therefore, replenishment systems are not incorporated into the study for simplicity.

Finally, assumption (A9) requires that order sizes have general bounded distribution which is imposed by the assumption (A1) and that the order has to have at least one line item requested. In this analysis, for illustration purposes, N is assumed to have zero truncated binomial distributions. *Appendix B(a)* contains a summary of this distribution and parameter estimators. *Appendix B(b)* illustrates how to derive a distribution of orders within a zone.

Parameters:

D – the length of the picking area;

H – the height of the picking area;

B – the number of buffers in the pick area;

s_h – speed of the picker in the horizontal direction;

s_v – speed of the picker in the vertical direction;

t_h –travel time it takes to move horizontally from the lower left-hand corner along entire pick

area, $t_h = D/s_h$;

t_v – travel time it takes to move vertically from the lower left-hand corner across entire pick

area, $t_v = H/s_v$;

N –number of items in an order, random number with known distribution.

Results of Decisions:

S^O – travel time of an order;

S^I – travel time of an item.

6.3 Robotic Picking

Recalling the discussion in *Chapter 1* regarding technological limitations of a robotic picker, it is assumed that only a single item is handled by the robot from the pick area into the buffer area. Therefore, only spread buffer layout (see description in *Section 1.3.1*) is considered for the robotic system. Robot movement is characterized by the Chebyshev metric, i.e. the total travel time between two locations depends on a maximum time of the horizontal and vertical moving times. The velocities are assumed to be constant for vertical and horizontal travels and acceleration is ignored. To ensure order accuracy, the picker is required to pick one line item at a time.

To minimize the gantry robot movement, the items in a batch are sorted according to their location along the picking area. The following *forward-reverse* pick strategy is implemented. Pick locations of the very first order are visited in the increasing x-coordinate direction (forward). This logic allows the gantry robot to pick an entire order batch in one

pass through the zone with minimum backtracking. The next order is picked by visiting pick locations in the reverse order (i.e. in the decreasing x-coordinate direction).

Following an approach used in (Bozer and White 1984) the pick area is modeled as a continuous area and normalized in the following way.

Let,

$$T = \max\{t_h, t_v\} \quad (6.1)$$

and

$$b = \min\{t_h/T, t_v/T\} \quad (6.2)$$

This implies that $0 \leq b \leq 1$; b is called a shape factor. If pick area dimensions and velocities are such that $t_h = t_v$ then we have $b = 1$ and the pick area is said to be *square-in-time*.

For any point in a picking area $X=(x,y)$, measured in travel time, the normalization implies $0 \leq x \leq 1$ and $0 \leq y \leq b$. So, the pick area is normalized to b time units in the vertical direction and *one* time unit in the horizontal direction. The pick buffer area is also modeled as a continuous area and is normalized in the same manner as the pick area. Thus, for any point in a pick buffer area $X^b=(x^b,y^b)$ in time we have $0 \leq x^b \leq 1$ and $y^b=0$. So, the buffer area is normalized to one time unit in the horizontal direction.

6.3.1. Spread Buffer Layout

Consider pick-to-buffer system where pick buffers are evenly spread the pick area. Assume pick buffers have capacity of one item, i.e. spread buffer layout with item buffers. For a system with item buffers, it makes sense to focus on the performance characteristics of item travel time.

Consider horizontal robot movement. Assume that points in the picking and buffer areas are visited in the increasing x-coordinate direction (Figure 6.1). This is accomplished by sorting pick points to be visited sequentially by their x-coordinate values. Similarly, the pick buffers to be visited are sorted according to their x-coordinate values. Thus, $X_{(1)} \leq X_{(2)} \leq \dots \leq X_{(n)}$ are the order statistics for independently and identically distributed random pick locations $X_1, X_2, \dots, X_n$ with a common probability distribution $f(x)$ and cumulative distribution $F(x)$. Similarly, $X_0^b \leq X_1^b \leq X_2^b \leq \dots \leq X_n^b$, where X_0^b is the location of the last buffer of the previous batch, are the ordered buffer locations $X_{(1)}^b, X_{(2)}^b, \dots, X_{(n+1)}^b$, such that from $X_{(k)}^b$ buffer location robot travels to k th ordered item and picks into $X_{(k+1)}^b$ buffer, $1 \leq k \leq n$. Note, that X_0^b buffer location is equivalent to ordered $X_{(1)}^b$ and represents the location of the last buffer of the previous batch. For modeling purposes X_0^b is assumed to be always located to the left of item locations of a new batch.

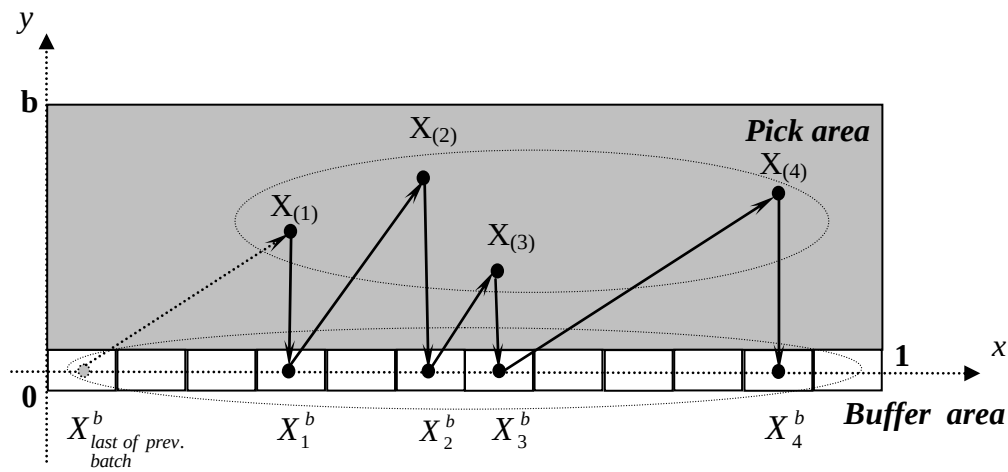


Figure 6.1 Order statistics for pick locations and pick buffers.

A pick/drop operation involves the two locations of an item and buffer. Prior to picking, the robot must perform an *empty* move from a previous buffer location to the pick location, and due to Chebyshev movement, $S_{E_k} = \max(|X_{(k)}^b - X_{(k)}|, Y_k)$. After picking, the robot performs a *loaded* move from the pick location to the corresponding buffer location, and again due to Chebyshev movement, $S_{L_k} = \max(|X_{(k)} - X_{(k+1)}^b|, Y_k)$. Thus, time to pick an item k is given by the following.

$$S_k^I = S_{L_k} + S_{E_k} \quad (6.3)$$

Proposition 6.1: The first and second moments of the travel time of the loaded travel from a pick location to a buffer location for a single item are given in (6.4) and (6.5), respectively.

$$E[S_L] = \frac{b}{2} \quad (6.4)$$

and

$$E[S_L^2] = \frac{b^2}{3} \quad (6.5)$$

The first and second moments of the travel time of the empty travel for a single item from a buffer location to a pick location when total number of picks is n are given in (6.6) and (6.7), respectively.

$$E[S_{En}] = \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{2(-1)^{n+k} n^2}{(n+k)} \left(\frac{b^{2n+1}}{4n(n+1)(2n+1)} - \frac{b^{n-k+1}}{(n-k)(n-k+1)(n-k+2)} - \frac{(n+k)}{(n-k+1)(2n+1)} \right) \quad (6.6)$$

and

$$E[S_{En}^2] = \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{(-1)^{n+k} n^2}{(n+k)} \left(\frac{b^{2n+2}}{n(n+1)(2n+3)} - \frac{4b^{n-k+2}}{(n-k)(n-k+2)(n-k+3)} - \frac{(n+k)}{(n-k+2)(n+1)} \right) \quad (6.7)$$

Proof: Details of the derivations of distributions of loaded and empty travel of robotic order pickers are given in *Appendix C(a)*.

Since the number of items in an order is a random variable generated from a discrete distribution that is bounded from above by the value u , first and second moments of the travel time of the empty travel from a buffer location to a pick location must be computed by the following

$$E[S_E] = \sum_{n=1}^u E[S_N | N = n] P\{N = n\} \quad (6.8)$$

and

$$E[S_E^2] = \sum_{n=1}^u E[S_N^2 | N = n] P\{N = n\} \quad (6.9)$$

where, N is the number of items in the customer orders that is independently and identically distributed, with *p.m.f.* $P\{N = n\}$ and $E[S_N | N = n]$ and $E[S_N^2 | N = n]$ are computed by (6.6) and (6.7), respectively. The equations (6.8) and (6.9) do not have nice closed forms for an arbitrary distribution of N . However, they are not difficult to evaluate numerically.

i) Expected Travel time

The time to pick an item is given by (6.3). Thus, expected time to pick an item is the following

$$E[S^I] = E[S_L + S_E] = E[S_L] + E[S_E] \quad (6.10)$$

In the following chapters it will be necessary to compute the expected time to pick an order. Thus, based on the well known Wald's equation (Ross 2000), time to pick an order is given by the following

$$E[S^O] = E\left[\sum_{i=1}^N (S_L + S_E)\right] = E[N](E[S_L] + E[S_E]) \quad (6.11)$$

where $E[N]$ is the expected number of items in the order.

ii) Variance of the Travel time

The time of an empty travel to a pick location and loaded travel to a buffer location are correlated. Therefore,

$$\text{Var}(S^I) = \text{Var}(S_L + S_E) = \text{Var}(S_L) + \text{Var}(S_E) + 2\text{Cov}(S_L, S_E) \quad (6.12)$$

where

$$\text{Cov}(S_L, S_E) = E[S_L S_E] - E[S_L]E[S_E] \quad (6.13)$$

Using the Cauchy-Schwartz Inequality, (4.14), to estimated the first term in (6.13) yields an upper bound on the covariance, given by (4.15):

$$E[S_L S_E] \leq \sqrt{E[(S_L)^2]E[(S_E)^2]} \quad (6.14)$$

$$\text{Var}(S^I) \leq \text{Var}(S_L) + \text{Var}(S_E) + 2\left(\sqrt{E[(S_L)^2]E[(S_E)^2]} - E[S_L]E[S_E]\right) \quad (6.15)$$

The variance of the time to pick an entire order follows from the relation given (6.16) in (Ross 1996).

$$\text{Var}(S^O) = \text{Var}\left(\sum_{i=1}^N (S^I)\right) = \text{Var}(S^I)E[N] + (E[S^I])^2 \text{Var}(N) \quad (6.16)$$

iii) Numerical Example and Analysis

To evaluate the robotic travel time model consider a small parts high volume robotic order picking system with the following parameters:

- Length of the picking area, $D = 100$ m;

- Height of the picking area, $H = 20 \text{ m}$;
- Speed of a robot in horizontal direction, $s_h = 3.15 \text{ m/sec}$;
- Speed of the robot in vertical direction, $s_v = 0.9 \text{ m/sec}$;
- Number of pick buffers in the zone, $B = 75$.

These parameters produce a shape factor, $b = 0.7$ and normalizing factor, $T = 31.75$.

In this example number of items per order, N , is assumed to be distributed as a zero truncated binomial distribution with parameters $p \in (0,1)$ and a positive integer k , $ZTBin(p,k)$ where $p = 0.2$ and $k=75$, i.e. $E[N] = 15$ items per order. Note, k must be less than equal to the number of buffers in the system. In all further examples we assume $k=B$, so that maximum number of items in an order is no more than number of buffers. From (6.10) and (6.15) an item requires

- average travel time, $E[S^I] = 22.379 \text{ sec}$;
- variance of travel time, $Var(S^I) = 159.774 \text{ sec}^2$,
- squared coefficient of variation, $scv(S^I) = 0.319$.

A Monte Carlo simulation model is developed to evaluate the analytical model. A pick area is assumed to have 100 columns and 20 levels yielding 2000 openings, such that width and height of each bin is 1 m. 75 buffers are evenly spread along pick area. First, orders are generated using the generator for $ZTBin$ variates described in *Appendix B*. Then, item pick locations along X and Y coordinates are generated independently for an order according to discrete uniform $DU(1,D)$, $DU(1,H)$ distributions. Next, pick locations are arranged in the increasing x-coordinates relative to the leftmost location.

The very first pick is initiated from the furthestmost left location of the pick area, point $(0,0)$. Once the picker reaches the first pick location it transports an item to the nearest

unused pick buffer to deposit. Then the picker travels to the next item in sequence, picks it, and again deposits it to the nearest unused buffer. After the current order is completed the picker starts on the next order but this time travels in the reverse direction, i.e. first picks rightmost located item, then nearest one on its left, etc. Travel time between the pick bin in the pick area and the pick buffer is computed from the center of the pick bin to the center of the buffer. The simulation length is set to 60,000 replications. Every replication represents an item travel time. Finally, the average travel time and the variance of travel time of every item is computed in the simulation.

For the above example, the Monte Carlo simulation model estimates an item average travel time of *22.358 sec* with variance *158.771 sec²* and squared coefficient of variation *0.318*. For a 90% 2-sided confidence interval, the upper confidence limit (UCL) and lower confidence limit (LCL) for average travel time are *22.337 sec* and *22.380 sec*, respectively, and the UCL and LCL for variance of the travel time are *157.790* and *159.89* respectively. In this example the approximation is fairly accurate; the relative gaps are less than 1% (i.e. the ratio of absolute gap to the corresponding result of the simulation).

To ensure that the model is accurate, results of an analytical model are compared to that of a simulation model with respect to various factors. Zero truncated binomial, *ZTBin(p, k)* is assumed for customer order size where the maximum number of items per order is varied from one to the number of buffers in the system. Simulation runs are performed for different numbers of buffers in the system which is set to 25, 50, 75, and 100.

Tables 6.1 and 6.2 display the expected value and variance of travel times for different average order sizes, $E[N]$, and numbers of buffers. The general pattern of expected value and the variance of travel times is illustrated in Figures 6.2 and 6.3, respectively.

It can be observed that the continuous approximation gives a very good performance. The average absolute value of the relative error for 250 runs is 0.20 % and 1.86% for the expected time and the variance, correspondingly. The relative gap trends for expected travel times and variance for travel times are depicted in Figures 6.4 and 6.5, respectively. The deviation decreases with the increase of buffers and expected number of picks in a batch. In general, the expected value decreases with the increase of expected number of items in the order since pick locations in the pick area become closer to each other. However, as the expected number of items in the batch approaches the number of buffers the expected travel

Table 6.1 Expected Item Travel times, $E[S^I]$, for the Robotic System.

E[N]	Analytical	Simulation			
		<i>25 buffers</i>	<i>50 buffers</i>	<i>75 buffers</i>	<i>100 buffers</i>
5	23.1220	23.3170	23.3346	23.32036	23.2990
10	22.5346	22.5663	22.5729	22.55139	22.5730
25	22.2843	22.5553	22.2885	22.2915	22.2814
50	22.2390	-	22.3797	22.2299	22.2284
75	22.2299	-	-	22.3153	22.2309
100	22.2266	-	-	-	22.2952

Table 6.2 Variance of Item Travel times, $Var[S^I]$, for the Robotic System.

E[N]	Analytical	Simulation			
		<i>25 buffers</i>	<i>50 buffers</i>	<i>75 buffers</i>	<i>100 buffers</i>
5	149.92	143.43	143.82	143.95	143.67
10	156.45	153.69	154.13	154.41	154.08
25	162.39	153.54	162.39	162.00	161.87
50	163.94	-	158.64	163.59	163.58
75	164.29	-	-	160.37	163.87
100	164.43	-	-	-	161.43

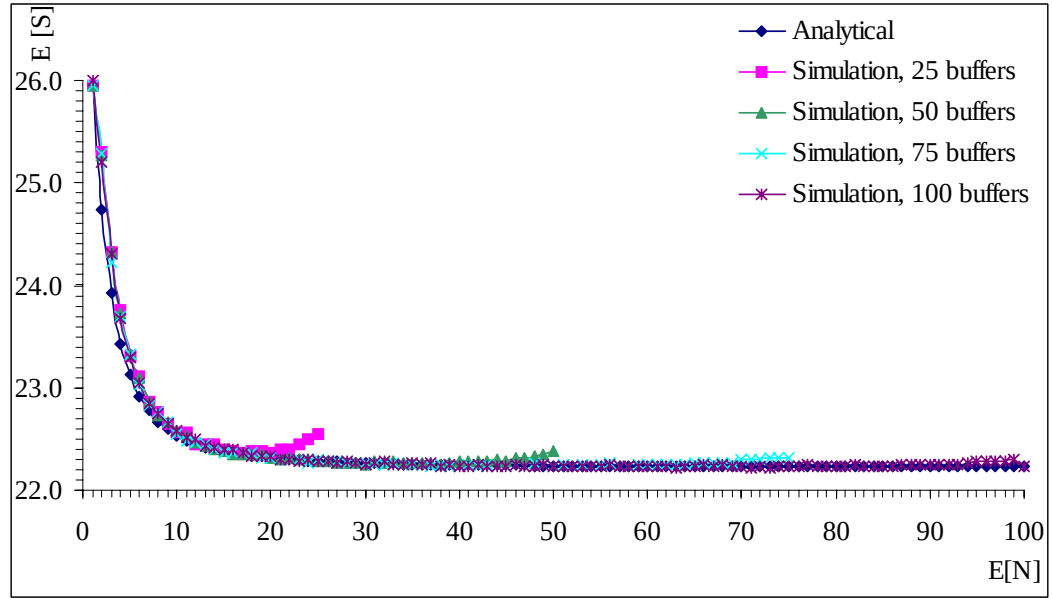


Figure 6.2 Expected Travel times, $E[S^I]$, of an Item for the Robotic System.

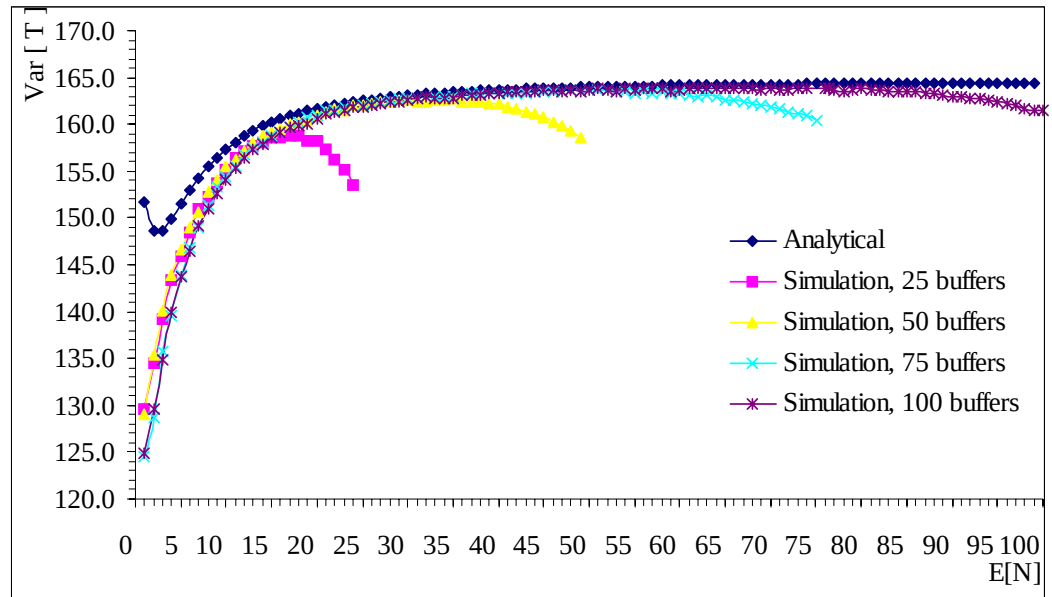


Figure 6.3 Variance of Travel times, $Var(S^I)$, of an Item for the Robotic System.

time increases (the elevated segments in the simulation plots in Figures 6.2 and 6.3). This can be attributed to the fact that amount of robot travel in search of the next closest available buffer increases. However, the relative gap for these cases is very small, e.g. in case of 25 buffers with expected batch size of 25 items in an order the relative gap is 1.2%.

There is a larger gap for the variance when expected number of items in the batch is small. The analytical model in these cases tend to overestimate since to approximate the covariance between empty and loaded travel times, the analytical model uses Cauchy-Schwartz Inequality which is an upper bound on the covariance. As the number of items in a batch increases the covariance approaches the upper bound of Cauchy-Schwartz Inequality. Another observation worth mentioning is that with the increase of expected number of items in a batch the variance increases, Figure 6.3. This can be explained by looking at the nature of Chebyshev's travel. Recall that Chebyshev travel time assumes the maximum time of the horizontal and vertical moving times. When number of items in a batch increases the sorted locations along x -coordinate become closer to each other, while locations along y -coordinate remain randomly distributed along rack's height. According to Chebyshev travel for an individual item the travel time might be determined by either very small value of travel time along x -coordinate, or relatively large value of travel time along y -coordinate. This combination of small and large values of travel times impacts the overall increasing trend of the variance, Figure 6.3.

From Figures 6.2 and 6.3, one can see that the analytic estimates of $E[S^I]$ and $Var(S^I)$ appear to converge to a constant value as the number of pick locations increases. Indeed, with the increase of expected number of items in the order, the expected travel time to pick an item is bounded by the height (in time) of the pick zone. To pick an item, a full height trip

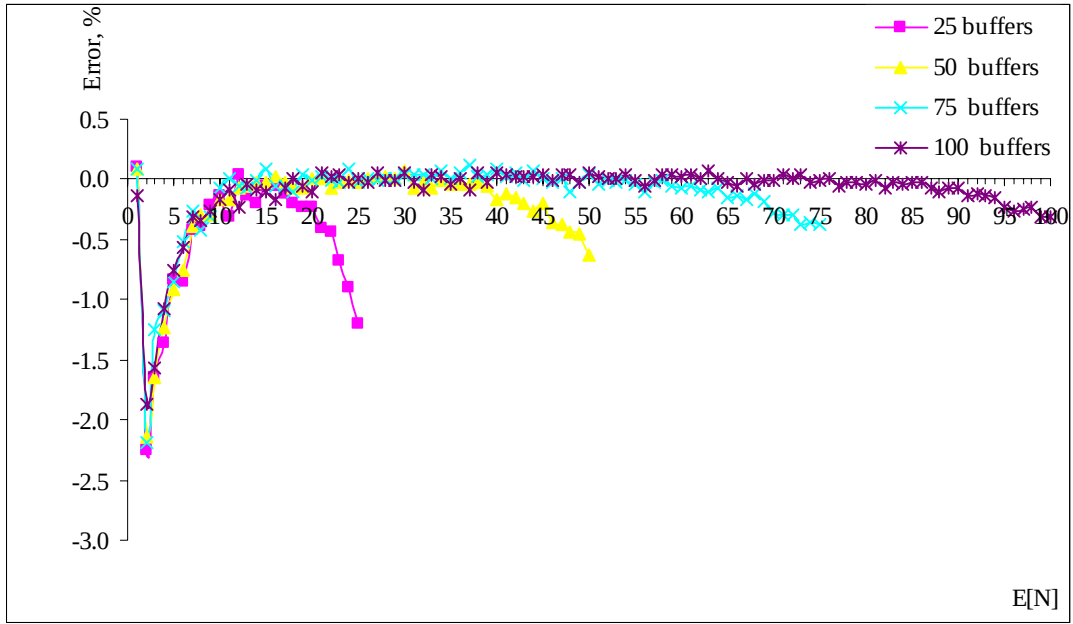


Figure 6.4 Relative Gap Trend, $E[S^I]$, of an Item for the Robotic System, when Shape Factor $b = 0.7$.

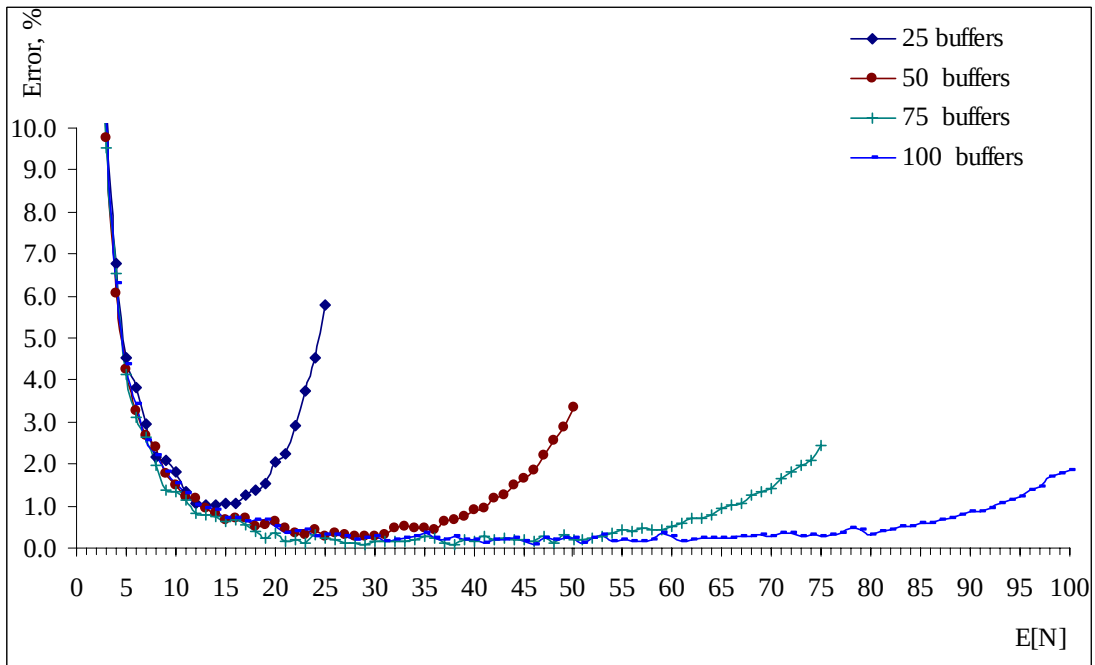


Figure 6.5 Relative Gap Trend, $Var(S^I)$, of an Item for the Robotic System, when Shape Factor $b = 0.7$.

is performed twice to make empty and loaded travels to the item location. Then the expected travel time is the bounded by the value of b . Similarity, the variance of the full length travel is $b^2/12$, passing the full length twice produces variance bound of $b^2/3$. As in the example considered above the height of the pick zone in time is $b=0.7$ sec., after de-normalization expected travel time is bounded by $(Tb)=22.22$ and the variance is bounded by $(T^2 b^2/3)=164.61$.

Lastly, the approximation model is very sensitive to the shape factor parameter. As the shape factor of the pick area increases the approximation becomes more accurate. Figures 6.6 and 6.7 illustrate relative gaps for a case when a shape factor is increased to the

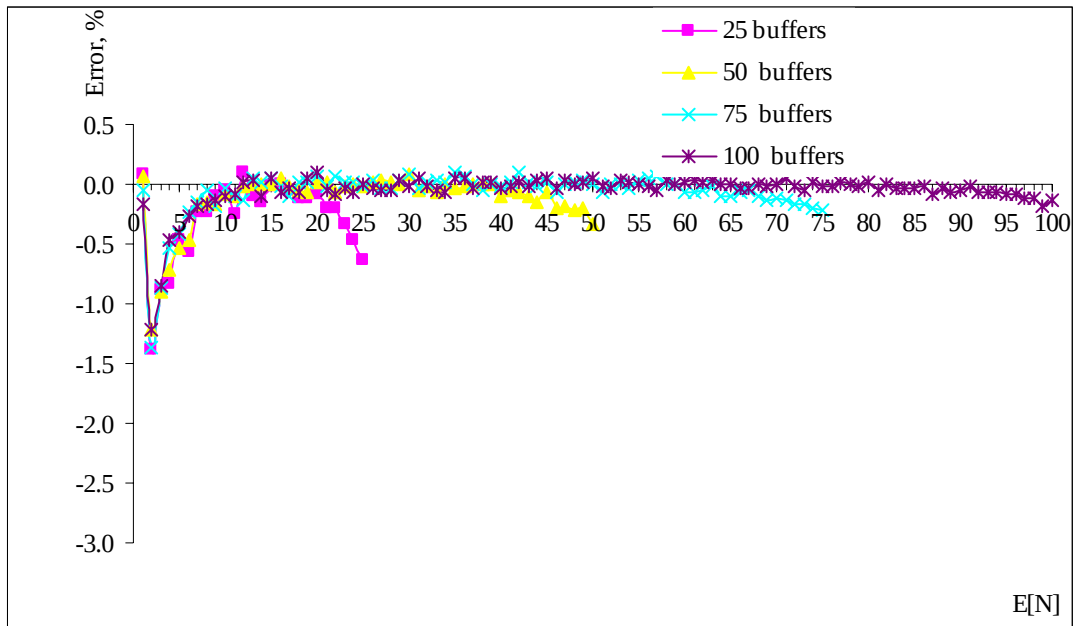


Figure 6.6 Relative Gap Trend, $E[S^I]$, of an Item for the Robotic System, when Shape Factor $b = 1$.

value one. This phenomenon can be explained again by looking at the nature of the Chebyshev travel. When the shape factor is large, travel time is more affected by the location of the item along y -coordinate. Thus, at the large shape factors the overall travel

time becomes less affected by backtracking and approximation model works much better. As we see, the relative gaps are smaller than in the case of shape factor being equal to 0.7.

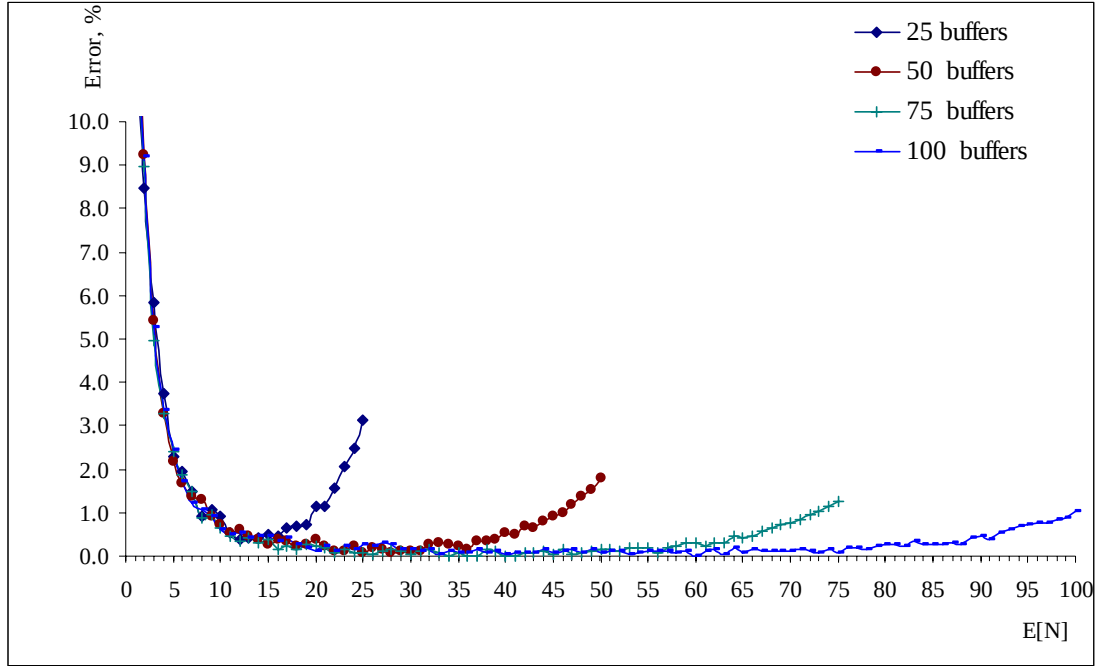


Figure 6.7 Relative Gap Trend,, $Var(S^I)$, of an Item for the Robotic System, when Shape Factor $b = 1$.

6.4 Manual Picking

As indicated in *Chapter 1*, travel time for a manual order picker is computed using rectilinear distance, also, multiple item picking is the most appropriate picking policy for a manual picker. All the assumptions and notations made previously in *Section 6.2* remain valid. Manual picking system for both centralized and spread buffer layouts are considered. To pick an item an operator walks between buffer and item storage locations. It is assumed that a travel distance across the aisle is insignificant. Also the height of the pick area is assumed to be ergonomically convenient for the order picker to reach the highest pick

locations. Therefore, only horizontal travel time of a picker along aisle impacts the travel time.

6.4.1. Centralized Buffer Layout

Consider a pick-to-buffer system where pick buffers are located in the middle of the pick area. As noted in Table 1.1 in *Chapter 1* the most appropriate picking policy for this layout is multiple item picking into order buffers. Since this case involves study of order buffers, it makes sense to focus on the performance measures of order travel time. The following picking strategy is implemented, and illustrated in Figure 6.8. An order picker starts a pick tour from buffer location, visits all the pick locations on one side of the pick area, walks back to buffer locations to deposit picked items into buffers and continues to visit pick locations to the other side of the pick area.

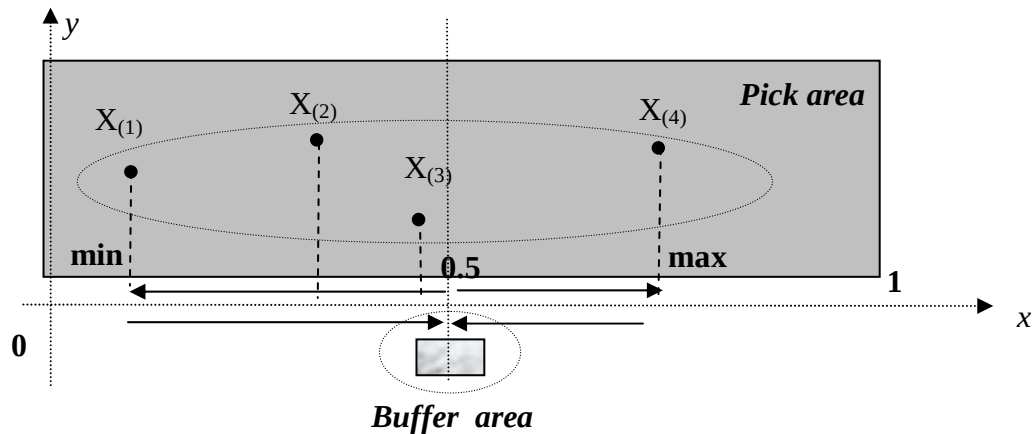


Figure 6.8 Manual Pick Tour into Centralized Buffers.

Pick buffers are located in the center of the pick area along horizontal axis, i.e. we assume a pick buffer area where no time is required to travel between buffers. Assume that deposit time of the picked items is not significant. In case of multiple order picking, a sort-while-pick policy must be implemented to separate items in different orders.

As before the pick area is modeled as a continuous area and is normalized in one direction only. Thus, for any point in a picking area x , measured in time we have $0 \leq x \leq 1$. So, the pick area is normalized to *one* time unit in the horizontal direction. Pick buffers are centered at the point $(x^b)=(1/2)$. The normalization factor $T = t_h$.

To pick items, an operator walks from the buffer location to the furthest right item location of an order along the aisle, denote it X_{max} . Once reaching X_{max} , the order picker turns back and walks to the buffer area, deposits items into buffers, walks to the furthest left item location of an order along the aisle, denote it X_{min} , and returns to the buffer area, see Figure 6.8.

Proposition 6.2: The first and second moments of the total travel time of an order when total number of picks is n are given in (6.17) and (6.18), respectively.

$$E[S_n] = \frac{2(1-2^{-n})(n-1+2^{1-n})}{n+1} \quad (6.17)$$

$$E[S_n^2] = \frac{2^{1-3n}(2^n-2)(2^n n - 2^n + 2)^2}{(n+1)^2} + \frac{2(1-2^{-n})(n^2-n+2)}{(n+1)(n+2)} \quad (6.18)$$

Proof: Details of the derivations are given in *Appendix C(b)*.

Since number of picks in an order is a random variable generated from a discrete distribution that is bounded from above by the value u , the first and second moments of time to pick an entire order can be computed by the following

$$E[S^O] = \sum_{n=1}^u E[S_N | N = n] P\{N = n\} \quad (6.19)$$

and

$$E[(S^O)^2] = \sum_{n=1}^u E[S_N^2 | N = n] P\{N = n\} \quad (6.20)$$

where, N is the number of items in the customer orders that is independently and identically distributed random variables with *p.m.f.* $P\{N = n\}$ and $E[S_N | N = n]$ and $E[S_N^2 | N = n]$ are computed by (6.17) and (6.18), respectively. Again, the equations (6.19) and (6.20) do not have nice closed forms for an arbitrary distribution of N . However, they are not difficult to evaluate numerically.

i) **Numerical Example and Analysis**

To evaluate the model consider a prototype of small parts high volume manual order picking system with the parameters as in *Section 6.2.1(iii)*. As before, let number of items per order, N , be a random variable having zero truncated binomial distribution with parameters $p \in (0,1)$ and a positive integer k , $ZTBin(p, k)$ where $p = 0.2$ and $k=75$, i.e. $E[N] = 15$ items per order. From (6.19) and (6.20) an order requires

- average travel time, $E[S^O] = 55.554 \text{ sec}$;
- variance of travel time, $Var(S^O) = 27.822 \text{ sec}^2$,
- squared coefficient of variation, $scv(S^O) = 0.0097$.

A Monte Carlo simulation model is developed to evaluate the analytical model. A pick area with length, D , of 100 columns is created. Width of each column is set to 1 m. The center of the buffer area is fixed at the fiftieth column (middle of the pick area). First, orders are generated using the generator for *ZTBin* variates described in *Appendix B*. Then, item

pick locations along x coordinate are generated for an order according to discrete uniform $DU(1,D)$ distribution. Next, pick locations are arranged in the increasing x -coordinate relative to the leftmost location. The very first pick is initiated from the buffer area. Then picker walks to the first pick location on the right and continues picking items until he/she reaches the furthest pick location. Once all items are picked on the right side relative to buffer area, picker travels back to buffer area, deposits items into order buffers and continues picking the rest of the items of the batch on the left side relative to buffer area. Travel time is computed from the center of the pick location to the center of the buffer area. The simulation length is set to 60,000 replications. Every replication corresponds to one order. And finally, the average travel time and the variance of travel time of an order is computed in the simulation.

For the above example, the Monte Carlo simulation model estimates an order average travel time of 55.13 sec with variance 29.531 sec^2 and squared coefficient of variation 0.0090 . For a 90% 2-sided confidence interval, the upper confidence limit (UCL) and lower confidence limit (LCL) for average travel time are 55.099 sec and 55.177 sec , respectively, and the UCL and LCL for variance of the travel time are 29.283 and 29.289 , respectively. In this example the approximation is fairly acceptable, producing 0.76% relative gap for expected travel time and 5.79% for the variance of travel time.

To investigate the model accuracy, results of an analytical model are compared to those of a simulation model with respect to various factors. Zero truncated binomial, $ZTBin(p, k)$ is assumed for customer orders where the maximum number of items per order is set to 50.

The general pattern of expected value and the variance of travel times are illustrated in Figures 6.9 and 6.10, respectively. One can see from figures that the expected value increases with the increase of expected number of items in the order since picker requires walking further in the pick area. On the other hand the variance of travel time decreases with the increase of expected number of items in the order since more pick locations are necessary to visit with fewer “outliers” in pick area. With the increase of expected number of items in the order the expected travel time is bounded by the twice total length (in time) of the pick area. As in the example considered above the length of the pick area in time is $D/s_h = 31.74$ sec, which is necessary to travel twice, giving a maximum trip time of 63.49 sec.

The relative gap trend for the expected value and the variance of travel times are depicted in Figure 6.11. It can be observed that the continuous model displays a very good performance. High relative gap in both parameters is detected when expected number of picks is low. This is affected by approximation on the picker travel when both the minimum and maximum pick locations are both on one side from the pick buffer.

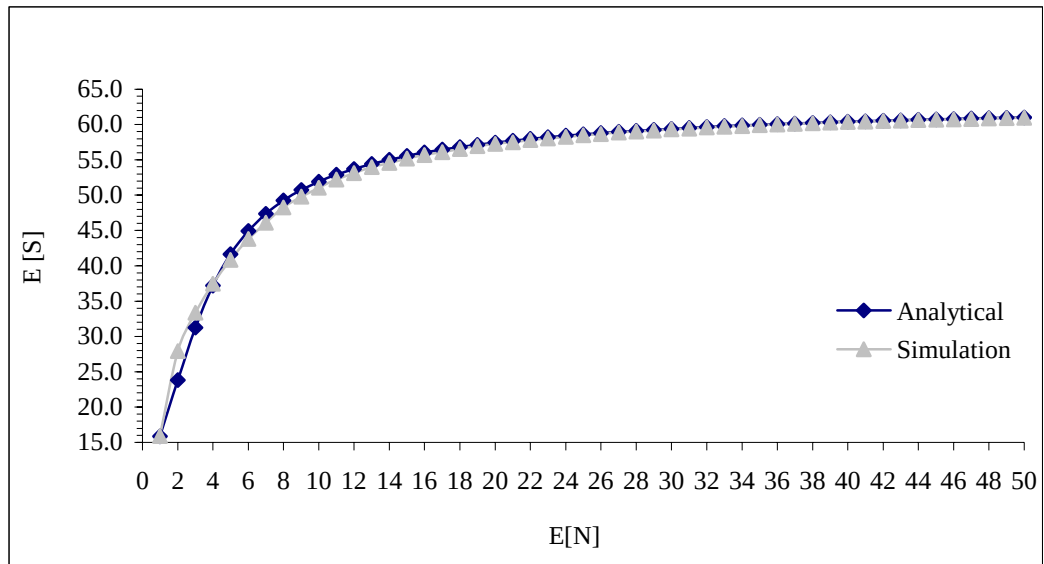


Figure 6.9. Expected Travel times, $E[S^O]$, for the Manual System with Centralized Buffers.

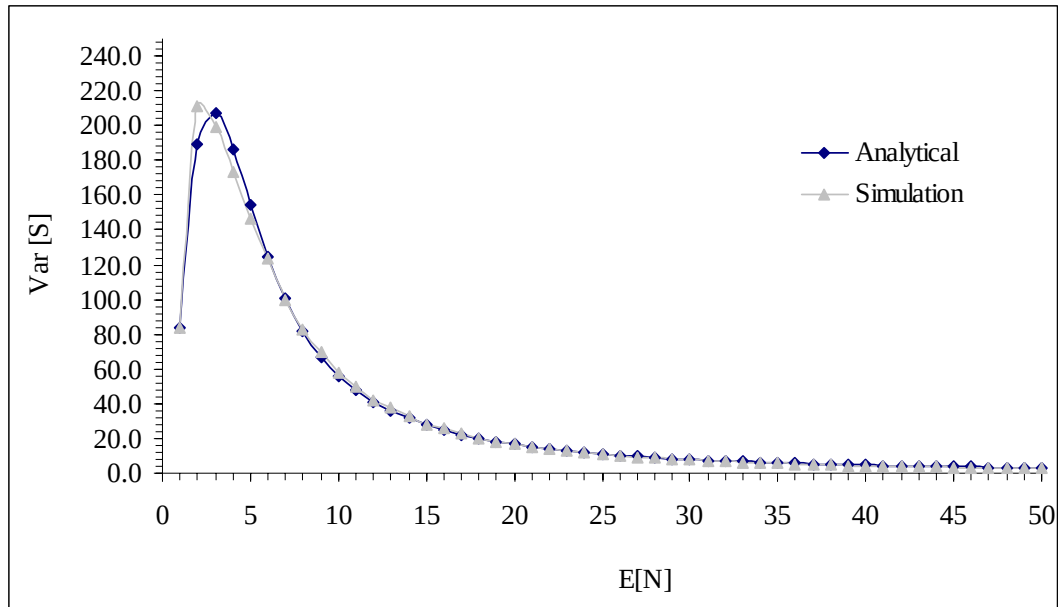


Figure 6.10 Variance of Travel times, $Var[S^O]$, for the Manual System with Centralized Buffers.

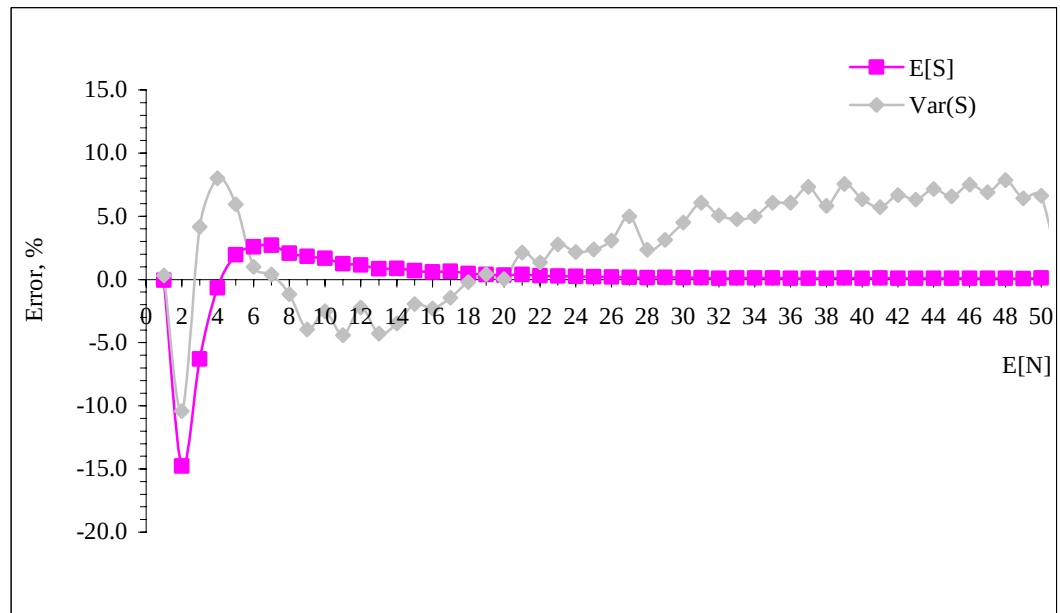


Figure 6.11 Relative Gap Trend, $E[S_o]$ and $Var(S_o)$, for the Manual System with Centralized Buffers.

6.4.2. Spread Buffer Layout

Consider pick-to-buffer scenario, when a picker collects items into item buffers with spread buffer layout (Figure 6.12). The following picking strategy is implemented: an order the picker travels down the aisle, pushing a cart, picking items into the cart as pick locations are encountered, and dropping items from the cart into pick buffers as empty pick buffers are encountered. If, at the end of the aisle, the picker has items remaining in the cart, direction is reversed until the first available pick buffer is encountered. This way a picker minimizes backtracking. Since the system studied employs item buffers then it makes sense to make an analysis of an item travel time.

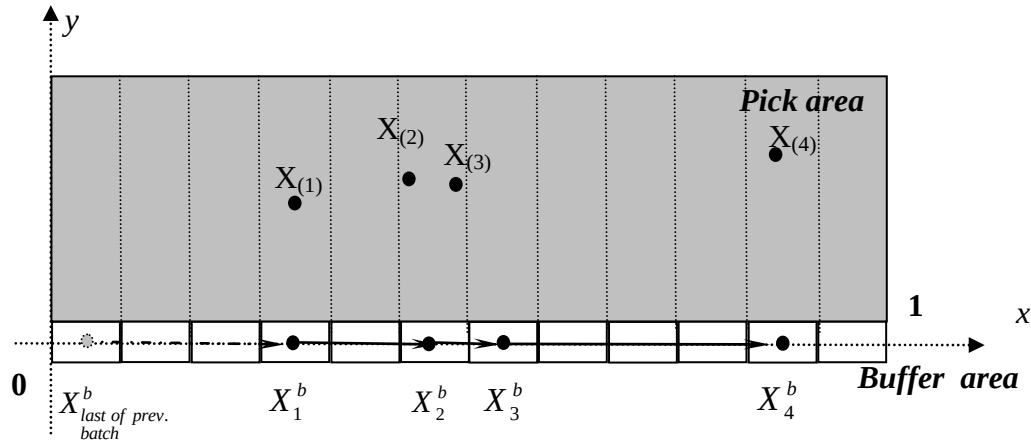


Figure 6.12 Order statistics for pick locations and pick buffers.

As before, the pick area is modeled as a continuous area and is normalized in one direction only. Thus, for any point in a picking area x in time we have $0 \leq x \leq 1$. So, the pick area is normalized to *one* time unit in the horizontal direction. This scenario is very similar to robotic pick system with spread item buffer layout. The main difference is that the height of the pick area does not impact on the travel time. This suggests that the approximation

made for the robotic picker is applicable to the manual picker case with the assumption that the height of the pick area is disregarded, i.e. shape factor $b = 0$.

Thus, the first and second moments of the travel time of the empty travel from a buffer location to a pick location when total number of picks is n can be simplified from (6.6) and (6.7) by setting b to zero, the results are given in (6.21) and (6.22), respectively.

$$E[S_{En}] = \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{2(-1)^{n+k} n^2}{(k-n+1)(2n+1)} \quad (6.21)$$

and

$$E[S_{En}^2] = \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{(-1)^{n+k} n^2}{(k-n+2)(n+1)} \quad (6.22)$$

Note, that the equations for the expected value and the variance of travel time of an item given by (6.10) and (6.15), respectively, are still valid for the manual pick system studied in this section.

i) **Numerical Example and Analysis**

To evaluate the model consider a prototype of small parts high volume manual order picking system with the parameters as in *Section 6.2.1(iii)*. As in the previous cases the number of items per order, N , is assumed having zero truncated binomial, $ZTBin(p, k)$ distribution with parameters $p \in (0,1)$ and a positive integer k , $ZTBin(p, k)$ where $p = 0.23$ and $k=80$, i.e. $E[N] = 15$ items per order. Note, k must be less than equal to the number of buffers in the system. From (6.21) and (6.22) an item requires

- average travel time, $E[S^I] = 2.014 \text{ sec}$;
- variance of travel time, $Var(S^I) = 3.931 \text{ sec}^2$,
- squared coefficient of variation, $scv(S^I) = 0.968$.

A Monte Carlo simulation model developed to evaluate the analytical model. A pick area is assumed to have 100 columns, with column width set to 1 m. 75 buffers are evenly spread along pick area. Items are dropped into the center of the buffer. Orders are generated using the generator for *ZTBin* variates described in *Appendix B*. Item pick locations along *X* coordinate are generated for an order according to discrete uniform $DU(1,D)$ distribution. and arranged in the increasing x-coordinate relative to the leftmost location.

For the first pick an operator starts at the point (0,0) and walks along the pick aisle pushing a cart. An operator picks items into the cart as pick locations are encountered, and drops items from the cart into pick buffers as empty pick buffers are encountered. If, at the end of the aisle, the picker has items remaining in the cart, direction is reversed until the first available pick buffer is encountered. The average travel time and the variance of travel time of an item is computed in the simulation. The simulation length is set to 60,000 replications. Every replication represents an item travel time.

For the above example, the simulation model estimates an item average travel time of 1.988 sec with variance 3.563 sec^2 and squared coefficient of variation 0.90. For a 90% 2-sided confidence interval, the upper confidence limit (UCL) and lower confidence limit (LCL) for average travel time are 1.984 sec and 1.993sec, respectively, and the UCL and LCL for variance of the travel time are 3.495 and 3.631, respectively. In this example the approximation is fairly accurate for the expected time, producing 1.29 % relative gap, while the variance has about 10.3% relative error.

The results of the analytical model are compared to those of a simulation model with respect to various factors. Zero truncated binomial, $ZTBin(p, k)$ is assumed for customer orders where the maximum number of items per order is varied from one to the number of

buffers in the system. Simulation runs are performed for different numbers of buffers in the system, which is set to 25, 50, 80, and 100. The general pattern of expected value and the variance of travel times are shown in Figures 6.13 and 6.14, respectively. As can be seen from figures, the expected value and variance to pick an item decreases with the increase of expected number of items in the order. For the order sizes that are much smaller than number of buffers the analytical results produce better approximations to that of simulation. As order size increases and approaches the number of buffers, the backtracking of a picker becomes more frequent due to unavailable item buffers. This situation can be seen by raising tails in simulation runs figures. Figures 6.15 and 6.16 show gaps for the expected value and the variance, respectively, of the travel time relative to simulation outcomes. As can be observed, the performance of the analytical model is acceptable for the expected travel time when number of items is within 75% of number of buffer. However, the variance does not perform very well. Yet when number of items in an order is not large, cases when we are more interested in, the relative gap is within 10 %.

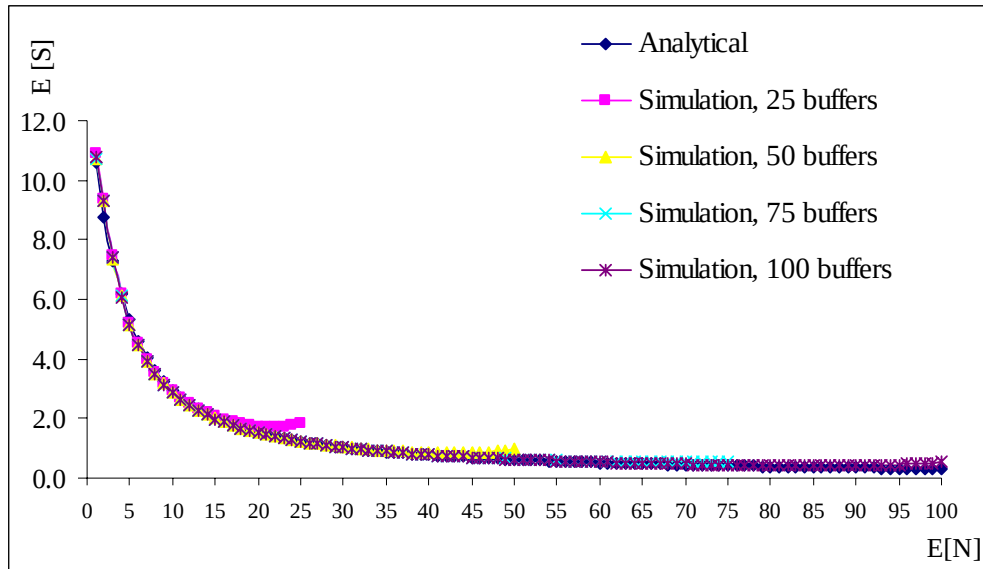


Figure 6.13 Expected Item Travel times, $E[S^I]$, for the Manual System with Spread Item Buffers.

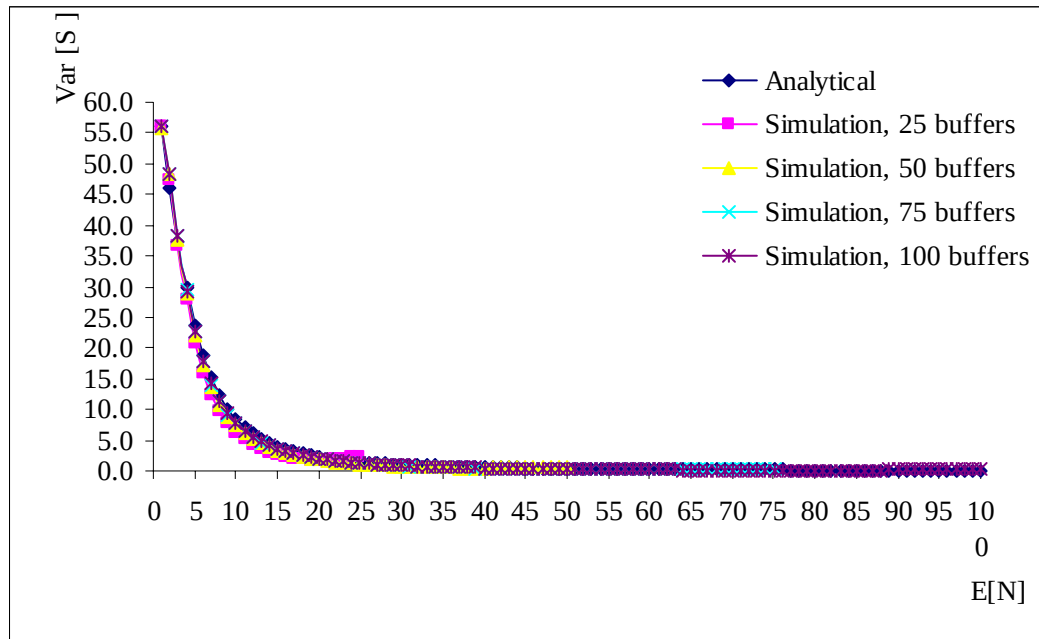


Figure 6.14 Variance of Item Travel times, $Var(S^I)$, for the Manual System with Spread Item Buffers.

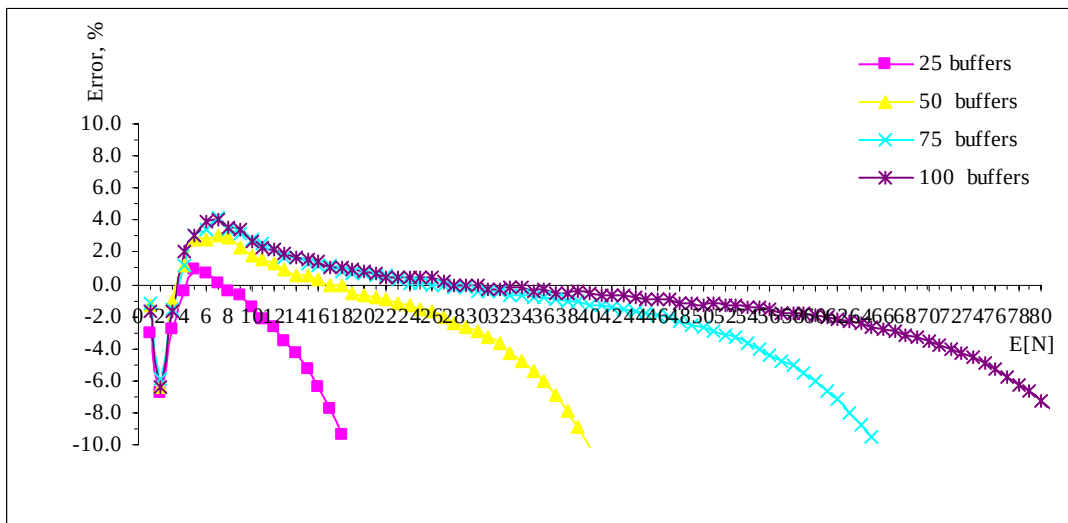


Figure 6.15 Absolute Relative Gap Trend for $E[S^I]$ in the Manual System with Spread Item Buffers.

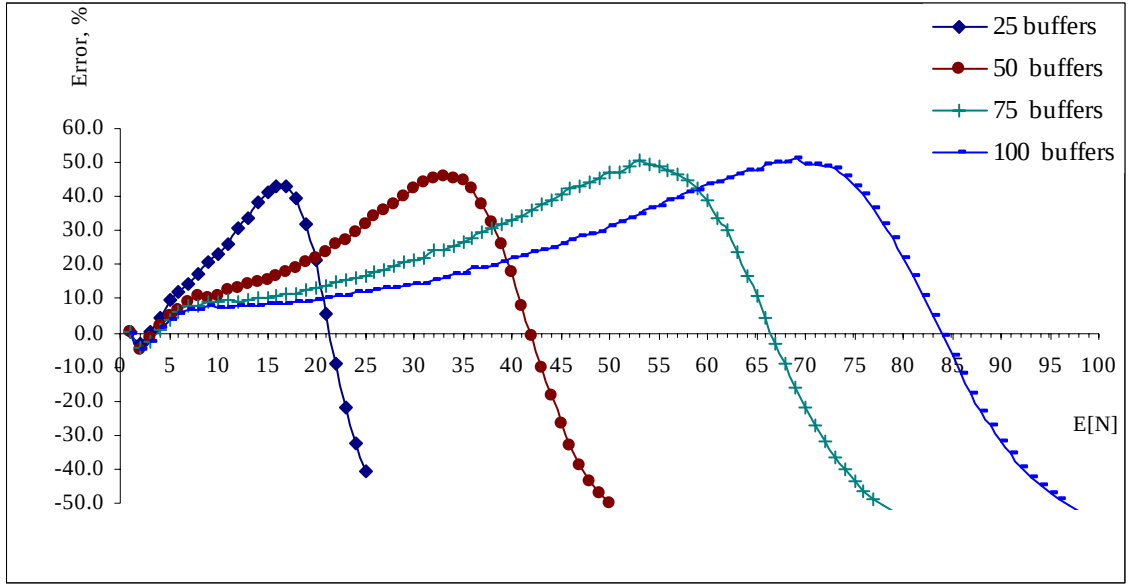


Figure 6.16 Absolute Relative Gap Trend for $Var(S^I)$ in the Manual System with Spread Item Buffers.

6.5 Conclusions

Analytical models of travel times of a picker are developed in the small parts order picking systems for robotic and manual pick-to-buffer technologies. Different configurations of buffer layouts and capacities are studied to develop closed form expressions.

The main assumptions of the analytical model are the following:

- pick area is represented as a continuous rectangular area;
- item locations of an order are independently and uniformly distributed over the pick area;
- buffers have no capacity limit and are available at all times.

The major performance measures as expected travel time and the variance of travel time are obtained through analytical models. To our knowledge, there has not been any prior work for approximating these performance measures for the described technologies.

The numerical examples, along with simulation, show that the analytical models are accurate enough for most configurations. These models are particularly useful for design problems that are yet at the early stage of development.

CHAPTER 7

ASSEMBLY MODEL

7.1 Introduction

In this chapter expected entity assembly time and its variance are derived for particular pick methods and pick system physical characteristics, assuming a pick-to-buffer order picking system. These parameters are necessary for evaluating the performance model described in *Chapter 5*.

The *assembly time* is defined as the time the order container spends traveling to a buffer to collect an entity after it has been picked. Figure 7.1 displays schematically the assembly times, S_1 , S_2 , and S_3 for items X_1 , X_2 and X_3 , respectively in an item buffer system. At time t_0 a picker starts picking the item X_1 . At time t_1 item X_1 is placed into buffer X_1^b . Next, the picker travels to items X_2 and X_3 , retrieves items and places them into buffers X_2^b and X_3^b at times t_2 and t_3 , respectively. In the meantime, an order container at time t_1 is S_1 time units away from reaching buffer X_1^b in order to collect (or assemble) item X_1 . Hence, S_1 is the assembly time of the item X_1 . On the other hand at time t_2 , when an item X_2 is placed into a buffer X_2^b an item X_1 is still being assembled and once an order container reaches buffer X_1^b an assembly process for item X_2 starts. Meanwhile item X_2 is waiting for E_2 time units to undergo an assembly process. At time t_3 an order container is S_3 time units away from reaching buffer X_3^b in order to collect (or assemble) item X_3 . Therefore, S_3 is the assembly time of the item X_3 .

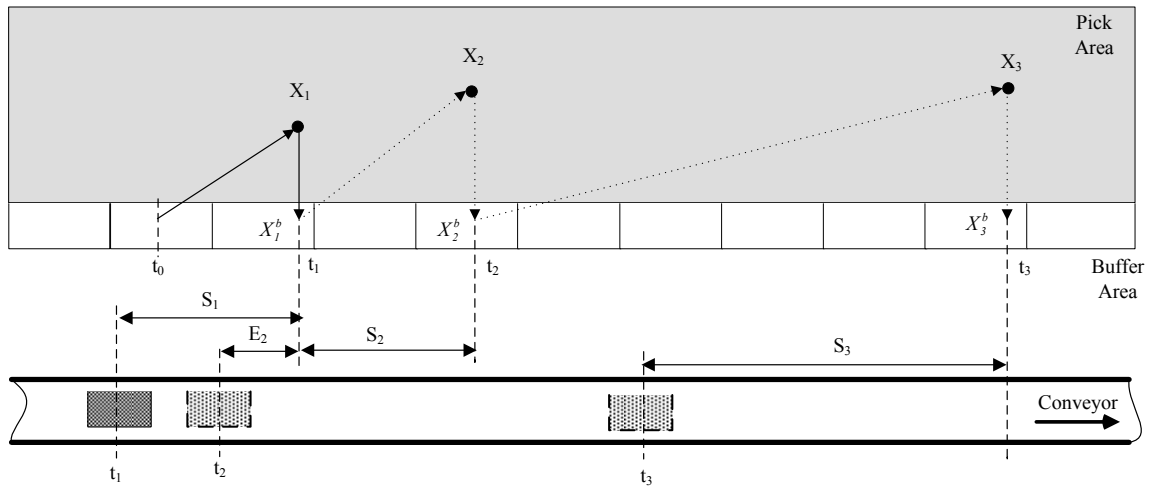


Figure 7.1 Assembly Process for Item Buffer Systems.

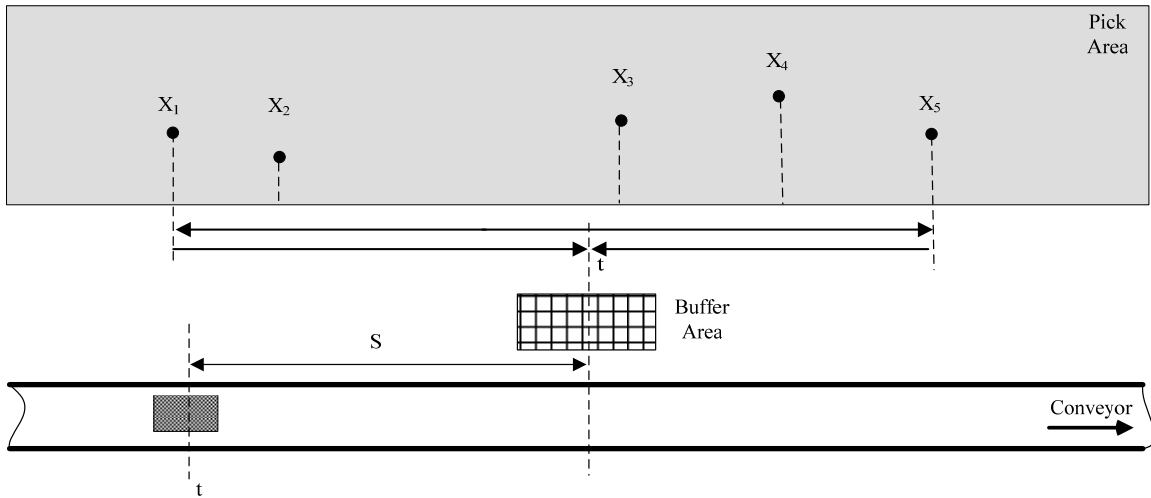


Figure 7.2 Assembly Process for Order Buffer Systems.

Figure 7.2 displays the assembly time, S for an order in an order buffer system. When an order is completely picked and all items are placed in the buffer, the order container is S time units away from buffer area. As in the item buffer case, before one order container assembles its items (S time units passed) another order container might be queued to start an assembly process, i.e. all items of the second order are picked and placed in the buffer. In this case, the second order needs to wait until the first order container completes its assembly process in order for it to initiate assembly service.

Let t_i be the time when an entity i is placed into a pick buffer and $\tilde{t}_i$ be the time when an entity i is collected from the pick buffer into its order container. Figure 7.3 displays an assembly process timeline. The time that an item i is delayed in the buffer is $\tilde{t}_i - t_i = E_i + S_i$, where S_i is the assembly time for an entity i and E_i is the wait to assemble time defined as

$$E_i = \begin{cases} \tilde{t}_{i-1} - t_i & \text{if } t_i < \tilde{t}_{i-1} \\ 0 & \text{if otherwise} \end{cases}. \text{ Therefore the assembly time for an entity } i \text{ can be written}$$

as

$$S_i = \begin{cases} \tilde{t}_i - \tilde{t}_{i-1} & \text{if } t_i < \tilde{t}_{i-1} \\ \tilde{t}_i - t_i & \text{if otherwise} \end{cases} \quad (7.1)$$

In this chapter the expected mean assembly time and its variance is determined.

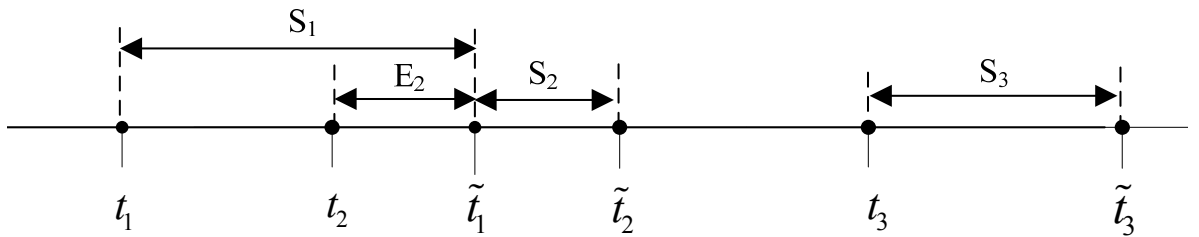


Figure 7.3 Assembly Process Timeline.

In both item buffer and order buffer systems order containers must be tracked through the system in order to determine their wait and assembly times. In addition, each item in each order must be analyzed. This stochastic behavior complicates the analysis of the assembly process. Therefore, to estimate the assembly time parameters, a search method is developed and described in this chapter.

The assembly of entities from buffers into order containers is one of two main components of the queuing model described in *Chapter 5*, consisting of two tandem servers with limited capacity buffers between them. The assembly operation represents the second server, parameters of which (expected time and the squared coefficient of variation of assembly) we attempt to estimate in this section.

As previously mentioned, because of the stochastic behavior of picker travel it is not straightforward to derive parameters of the assembly operation. These parameters are important in determining the impact of assembly time on picking time, i.e. when buffers are full and therefore a picker must idle what would be the additional delay of a picker due to blocking. If all the parameters of the model are known, the additional delay of a picker is computed using the analysis presented in *Chapter 5*. The challenge is to estimate the performance of the second subqueue, the assembly operation.

Suppose the squared coefficient of variation of assembly time was known, and further that it was possible to estimate the mean cycle time of an entity flowing through the system just by studying the flow of containers through the system (an approach to do exactly this is described later in this chapter). In that case, it would be possible to use the approximation results from *Chapter 5* to search for the expected value of assembly time. While the value of the squared coefficient of variation of assembly time is not known, an upper bound can be

computed by studying the relationship between assembly and picking operation; derivation of the upper bound is presented later in this chapter.

Thus, the approach for analyzing the order picking system is the following (see Figure 7.4). First, we obtain the total cycle time of an entity in the system by observing flow of containers. Next, the squared coefficient of the assembly time is computed. We then follow with the iterative procedure using Bisection method by searching the mean assembly time so that the total cycle time approximated by implementing the queuing model approaches the total cycle time obtained by analyzing the flow of containers.

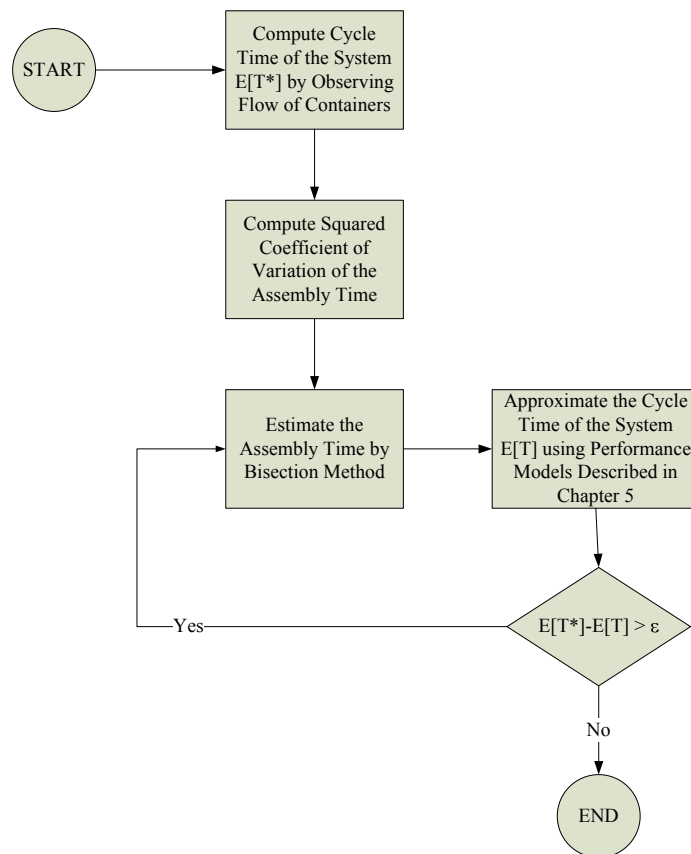


Figure 7.4 Flow Chart of Estimating Parameters of Assembly Time.

7.2 Assumptions

Let an *active item* be an item that has been scheduled to be picked (or released for picking) and not yet assembled into an order container. The time an item is in this active state is called the *item cycle time*. Similarly, an *order cycle time* is defined as the elapsed time between order release for picking and the completion of order assembly into the order container.

In this analysis the following assumptions are made:

- (A1) No two entities are assembled at the same time;
- (A2) All items of an order and the corresponding order container are released into the system at the same instant;
- (A3) The length of the buffer area is known;
- (A4) The speed of the conveyor is known and constant.

Parameters:

D – the length of the picking area;

W – the time from order release until the container enters the zone;

B – the number of buffers in a zone;

T – cycle time of an entity;

L – number of entities in the pick station;

s_c – speed of a conveyor;

t_c – time it takes for an order container to travel the segment of a conveyor along the pick zone (i.e. length D), $t_c = D/s_c$;

N – number of items in an order, random number with known distribution.

Results of Decisions:

S^O —assembly time of an order;

S^I — assembly time of an item

7.3 Cycle Time

The cycle time of an entity depends on the system configuration. Two cycle time models are presented in this section: one for item buffer technology, and the other for order buffer technology.

7.3.1. Item Buffer Systems

In order to approximate item pick times (see *Chapter 6*) it was assumed that pick locations are uniformly and independently distributed along horizontal and vertical directions. Recall that in the robotic item picking system, a picker performs a loaded move from item pick location to the closest buffer location. Thus we may assume that selected pick buffer locations, i.e. X_i^b , $i=1..N$, in Figure 7.1, also are uniformly distributed in horizontal direction assuming that the pick buffer which is the closest to the pick location is used.

The first item of an order is assembled when the order container passes underneath the buffer that has the first item of that order, i.e. X_1^b in Figure 7.1, the next item of the order is then assembled when the order container passes underneath buffer with the second item of that order, i.e. X_2^b , and so on.

Let C_i be the travel time of an order container on a conveyor after it enters a zone until it reaches the buffer X_i^b , $i = 1 \dots M$ for assembly. Then to reach buffer X_i^b , for $i = 2, \dots, M$, it takes $C_i = t_c X_i^b / B$ time units. On the other hand, the time it takes for an order container from the order release instant to reach buffer X_i^b is $T_i = W + C_i$, where W is the time from order release until the container enters the zone. Note, T_1 is the cycle time of the first item. Similarly, time it takes for an order container to reach X_2^b is $T_2 = W + C_2$ and in general, time it takes for an order container to reach X_i^b is the following

$$T_i = W + C_i = W + \frac{t_c}{B} X_i^b \quad (7.2)$$

The expected cycle time (7.2) can be re-written in the following form

$$E[T] = W + \frac{t_c}{B} E[X_i^b] \quad (7.3)$$

Since we assume $X_i^b \sim DU(1, B)$ (more on discrete uniform distribution see *Appendix B(a)*).

$$E[X_i^b] = \frac{1+B}{2} \quad (7.4)$$

Then expected cycle time of an item (7.3) becomes

$$E[T] = W + \frac{t_c (1+B)}{2B} \quad (7.5)$$

The expected cycle time of an item in item buffer system computed by (7.5) is an overestimate of the real mean cycle time. Consider a system with high utilization of buffers. In this system the buffers located in the beginning of a zone have lower utilization, since once all buffers are full these buffers are the first ones that free up. Once the buffers in the beginning of the zone free up, a picker transports new picked item into those buffers. As more items are picked into buffers closest to the beginning of a zone, shorter assembly cycle

times will be observed for items as they will be assembled as soon as the order containers enter the zone.

7.3.2. Order Buffer Systems

The cycle time of an order in the centralized order buffer picking system is trivial to compute. It is just time until it takes an order container to reach buffer area, which is located in the middle of zone.

$$E[T] = W + \frac{t_c}{2} \quad (7.6)$$

In contrast to spread item buffer system the cycle time of an order given by (7.6) is exact, and no over- or underestimations take place in compare to the real system.

7.4 Squared Coefficient of Variation of Assembly Time

One of the main parameters of assembly time is a squared coefficient of variation. In this section a value for squared coefficient of variation is found by comparing the travel time that a picker must perform to pick an entity and the time an order container travels on a conveyor to collect an entity.

In pick-to-buffer systems, while the picker travels to pick an item, the corresponding order container travels on the conveyor an amount of time equal to the picker's travel time. The pick buffer location always must be further in the zone in comparison to the container location on the conveyor, otherwise pick errors will be observed. In Figure 7.1 $S_i + E_i$ is the amount of the time by which pick precedes assembly for item i .

First, consider item buffer systems. Suppose the picker has to pick an item that requires short travel, e.g. item X_2 in Figure 7.1. To pick an item X_2 the picker spends $(t_2 - t_1)$

time units. Meanwhile the container travels on the conveyor a distance in time also equal to $(t_2 - t_1)$. To reach buffer location X_2^b the container still needs to travel $(E_2 + S_2)$ time units. Now suppose the picker travels a longer time to pick an item, e.g. item X_3 in Figure 7.1, both picker and container travel a distance in time equal to $(t_3 - t_2)$. To reach buffer location X_3^b the container needs to travel S_3 time units which is significantly larger than the assembly time of the item at X_2 . These observations suggest a positive correlation between picker travel and assembly times. On the presumption of a strong correlation, we assume a correlation coefficient of one for purposes of approximating the squared coefficient of variation of assembly time.

Next, consider an order buffer system and recall that items and containers are released at the same time. Since the buffer area is located in the middle of the pick zone, the longer it takes for a picker complete picking, the closer the order container will be to the buffer and the less time will be required for the assembly operation. This suggests a negative correlation between picker travel and assembly times. Again, on the presumption of a strong correlation, we assume a correlation coefficient to be a negative one.

The value for the squared coefficient of variation for assembly time is computed in the following manner. Let, P be travel time to pick an item/order. Then the following is true:

$$Corr(P, S) = \frac{Cov(P, S)}{\sqrt{Var(P)Var(S)}} \quad (7.7)$$

where S is the assembly time. Rewriting (7.7) and using assumption $Corr(P, S) = \pm 1$, one gets

$$Cov(P, S) = \sqrt{Var(P)Var(S)} \quad (7.8)$$

On the other hand

$$Cov(P, S) = E[P S] - E[P]E[S] \quad (7.9)$$

Using Cauchy-Schwartz Inequality, the equation (7.9) can be rewritten as

$$Cov(P, S) \leq \sqrt{(Var(P) + (E[P])^2)(Var(S) + (E[S])^2)} - E[P]E[S] \quad (7.10)$$

Based on equations (7.8) and (7.10) one can write

$$\sqrt{Var(P)Var(S)} \leq \sqrt{(Var(P) + (E[P])^2)(Var(S) + (E[S])^2)} - E[P]E[S] \quad (7.11)$$

By simplifying (7.11) and using the definition of squared coefficient of variation one can show by simple algebraic manipulations that squared coefficient of variation of assembly time is:

$$c_s^2 = \frac{Var(S)}{(E[S])^2} \leq \frac{Var(P)}{(E[P])^2} = c_p^2 \quad (7.12)$$

We will use c_p^2 as an upper bound for c_s^2 .

7.5 Expected Assembly Time

In this section an algorithmic computation of the expected assembly time is presented using a search method. The solution procedure for the model described in *Chapter 5* is implemented and used to compute expected assembly time. Recall that the performance model is a queuing system with two tandem servers with a limited capacity buffer in between them. The queuing system is decomposed into two sub-queues, one representing the picking operation, and one representing the assembly server with the buffers in front of it.

The parameters of the first server are estimated by the travel time model presented in *Chapter 6*. The squared coefficient of variation of assembly time is determined by equation (7.12). Then the cycle time of an entity is computed as described in Section 7.3. We call this

value a desired cycle time. The search method developed in this section is then implemented to determine the value of the expected assembly time at which the solution procedure from *Chapter 5* produces yields the desired cycle time. The notations used in this section are the same as in *Chapter 5*.

The search procedure is the well known Bisection Method (Burden and Faires 2001). The method starts with two initial values of assembly time, A and B forming an interval where we suspect the solution lies (i.e. the expected assembly time that will result a desired cycle time). The procedure then proceeds by iteratively evaluating the cycle time at the midpoint of the interval $C = (A + B) / 2$ and testing to see in which of the subintervals $[A, C]$ or $[C, B]$ the solution lies. The new interval becomes $[A, B]$ and the procedure is repeated with the new interval, iteration continues until $B - A \leq \varepsilon$, a specified accuracy.

An important observation is that the larger the value of service time (in this case assembly time), the larger the cycle time. Therefore, if the value of item cycle time in the midpoint is larger than the desired value then searching in the lower sub interval will result a new solution which is located closer to the desired value. Repeating in the same manner the convergence eventually will be achieved. A well known property of a Bisection Method is its linear convergence.

7.5.1. Initial Interval

The minimum time that an order container must travel to assemble is when at the time of placing an item into the buffer an order container is underneath that buffer and assembles at the same instant, i.e. the assembly time is zero.

The longest assembly time is when an order container must travel the segment of a conveyor before entering the zone, i.e. W , and the entire pick area in order to assemble an item. Therefore the initial interval can be set to the following values,

$$A_1 = 0 \text{ and } B_1 = W + t_c \quad (7.13)$$

7.5.2. The Algorithm

The algorithm for computing the mean assembly time is presented in Figure 7.5. During implementation of the algorithm it is important to check several conditions. If the predicted mean assembly time is very high then it would cause high value for probability of blocking, which in its turn would result in longer delays in picking activity (first server blocking). As the blocking increases, the utilization of the picker increases too, as it takes more time to process (pick) an entity. The approximation model implemented for solving first sub-queue assumes steady state behavior of the system and if the utilization of the first server is greater than one the approximation model does not work properly and therefore produces negative values for cycle times. As a result, during the algorithm it is necessary to check for the value of utilization, ρ_l . If it is greater than one the lower interval in the algorithm must be selected, i.e. set $B_{i+1}=C_i$. At the same time, the stopping condition for the algorithm must be implemented in case if after many iterations the steady state condition for the first server is not achieved. This would mean that the system with studied parameters is not feasible.

Step 1. INITIALIZE:

Compute A_1 and B_1 by (7.13);

Compute a desired cycle time $E[\bar{T}]$ given by (7.5) for item buffer systems or (7.6) for order buffer systems;

Choose tolerance level ε ;

Set $i = 1$

Step 2.

Compute $C_i = (A_i + B_i) / 2$

Set $E[S_2]$ to C_i

Implement the solution procedure of a queuing system described in *Chapter 5*

Step 3.

IF $|E[\bar{T}] - E[T]| < \varepsilon$ THEN

GOTO STEP 5

ELSE IF $E[\bar{T}] \leq E[T]$ THEN

SET $A_{i+1} = A_i$ AND $B_{i+1} = C_i$

ELSE

SET $A_{i+1} = C_i$ AND $B_{i+1} = B_i$

Step 4.

Set $i = i + 1$

GOTO STEP 2

Step 5. STOP.

Figure 7.5 Algorithm for Computing Assembly Time.

7.6 Numerical Example

To demonstrate the implementation of the above algorithm and the results obtained a numerical example for the robotic item buffer system with spread buffer is presented in this section. The input parameters are set to the following values:

- Length of the picking area, $D = 100 \text{ m}$;
- Height of the picking area, $H = 20 \text{ m}$;
- Speed of the picker in the horizontal direction, $s_h = 3.15 \text{ m/sec}$;
- Speed of the picker in the vertical direction, $s_v = 0.9 \text{ m/sec}$;

- Speed of a conveyor, $s_c = 0.6 \text{ m/sec}$;
- Number of buffers, $B = 50$;
- Number of items in an order has zero-truncated binomial distribution $N \sim \text{ZTBin}(0.0568, 50)$ such that $E[N]=3$ items and $\text{Var}(N)=2.23$;
- Batch size of orders, $M = 10$ orders;
- The time from order release until the container enters the zone, $W = 800 \text{ sec}$;
- Arrival rate of a batch of orders, expected interarrival time and squared coefficient of variation of interarrival time, $\lambda^b=0.015$ batches/sec; $E[\tau^b]=1/\lambda^b = 66.667 \text{ sec/batch}$; $c_{a^b}^2=0$, respectively, i.e. $\lambda=0.0005 \text{ items/sec}$; $E[\tau]=1/\lambda=2000 \text{ sec/item}$; $c_a^2=0$.

The parameter of interest is the utilization of the picker, ρ_l , i.e. whether a picker can handle the throughput with the above zone configuration and set parameters. The size of a batch of entities has zero-truncated binomial distribution $X \sim \text{ZTBin}(0.6, 50)$, such that $E[X]=30$ items and $\text{Var}(X)=12$.

The performance model described in *Chapter 5* has the following parameters. The batch of size X arrives at rate $\lambda^b=0.015$ batches/sec or $\lambda=0.0005 \text{ items/sec}$. The first server representing the picker has expected service time S_l and squared coefficient of variation $c_{S_1}^2$.

The expected travel time of a picker and the squared coefficient of variation are estimated using equations (4.8) and (4.9), yielding $E[S_l]=22.268 \text{ sec}$ and $c_{S_1}^2=0.3286$. The algorithm in Figure 7.3 is implemented to compute the parameters of the second server, i.e. $E[S_2]=61.4916 \text{ sec}$ and $c_{S_2}^2=0.3286$ which affects the first server such that the expected

travel time of a picker and the squared coefficient of variation become $E[S_1^{new}] = 22.6437$ sec and $c_{S_1^{new}}^2 = 0.3180$, respectively, with the utilization $\rho_I = 0.3397$.

7.7 Analysis

To ensure that the assembly model is accurate, results of the analytical model are compared to that of simulation model described in *Chapter 4*. The comparison is carried out for the time that an item is delayed in the buffer, i.e. the values of $E_2 + S_2$. Here, both spread item buffer and centralized order buffer systems are studied.

The arrival process into the system is assumed to be deterministic, as it reflects more realistic situation when warehouses send order requests to order picking departments in batches at fixed time intervals. For all the tests, order size was assumed to have zero truncated binomial distribution, $ZTBin(p, k)$ with parameters $p \in (0, 1)$ and a positive integer k , $ZTBin(p, k)$. The value of k is always set to the number of buffers in the system. Different batch sizes of arriving entities were included in the tests with values varying from 4 to 20 orders per batch depending on the system under consideration. The number of buffers is chosen so that picker's utilization varies within a range $[0.2, 0.95]$. Conveyor speed is set to 0.6 m/sec.

Figures 7.6-7.15 show results of comparing analytical approximation of the time that an entity is delayed in the buffer, i.e. $E_2 + S_2$ with the simulation output “Item load Delay Time in the Buffer”. The values for $E_2 + S_2$ are displayed on the left axes. The figures also include plots for picker's utilization, ρ_I , where $0 \leq \rho_I < 1$. The values for utilization should be read from the right axes. In every figure parts (a) and (b) are actual plots of simulation and

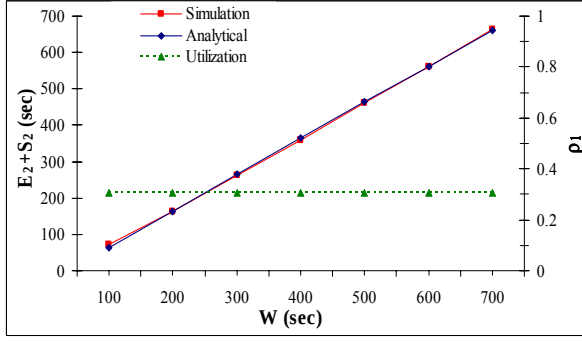
analytical results when picker utilization is *low to average* and *average to high*, respectively. Part (c) of every figure depicts the percent of a relative error gap for both plots (a) and (b).

7.7.1. Spread Item Buffer Systems

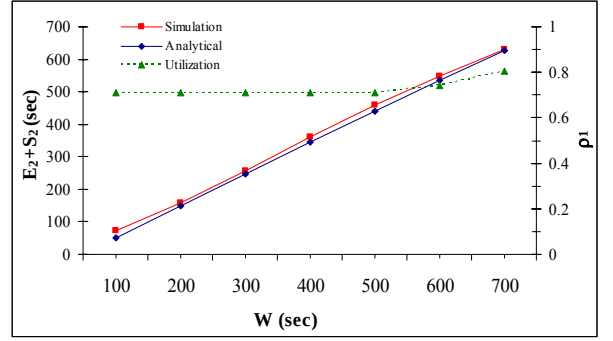
In the spread item buffer system, the length of the picking area is set to *100 m*, and the height to *20 m*. Speeds of the picker in the horizontal and vertical directions are set to *3.15 m/sec* and *0.9 m/sec*, respectively. These pick area dimensions and speeds of a picker result in a shape factor of 0.7 (more on shape factor see *Chapter 6*). The number of buffers is set to 25, 50, or 100. The expected number of items in an order is set to values of 2, 3, or 4. Depending on number of buffers, different batch size of orders has been tested so that picker's utilization can vary. The time from order release until the container enters the zone, W , is set to vary from *100* to *700 sec* by *100 sec* increment. The value of W consists of two parts: time that an order container is held until it is placed on a conveyor and the time order container is transported into the zone.

Figures 7.6-7.11 illustrate the comparisons for spread item buffer systems. For all buffers sizes with low W the relative gap is very high. This can be explained due to the fact that the time until order container reaches the buffer to assemble an item is very low with low W s.

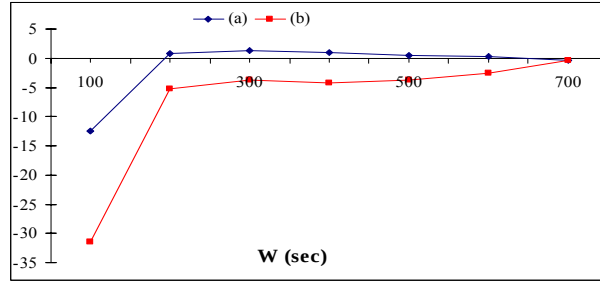
As the W increases the cycle time of an item in the system increases and the time that an item is delayed in the buffer also increases and the error gap decreases.



(a) Low to Average Utilization;

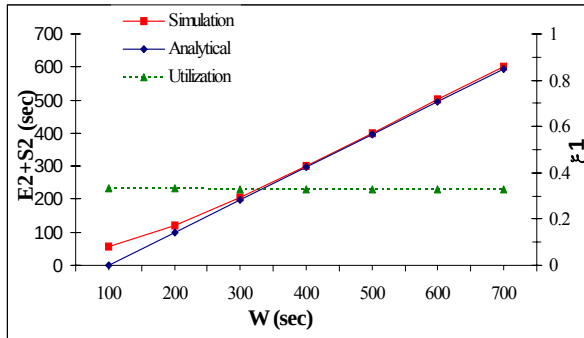


(b) Average to High Utilization;

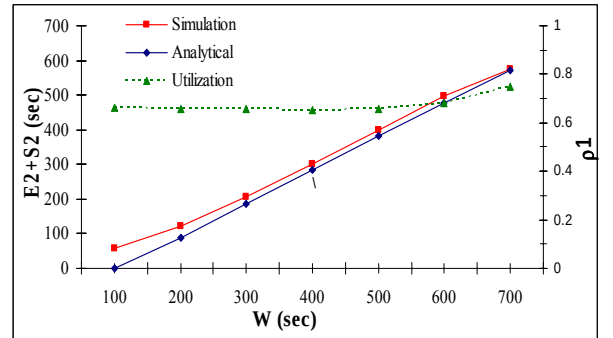


(c) Relative Error Gap, %;

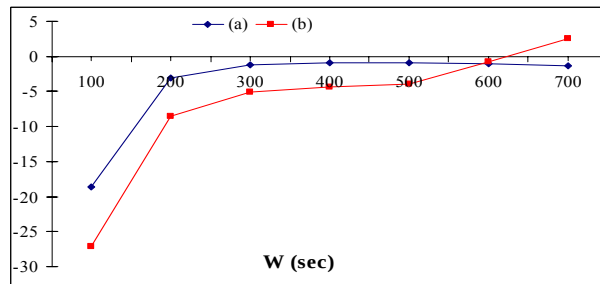
Figure 7.6 System with $B=25$, $M=3$, and $EN=3$.



(a) Low to Average Utilization;

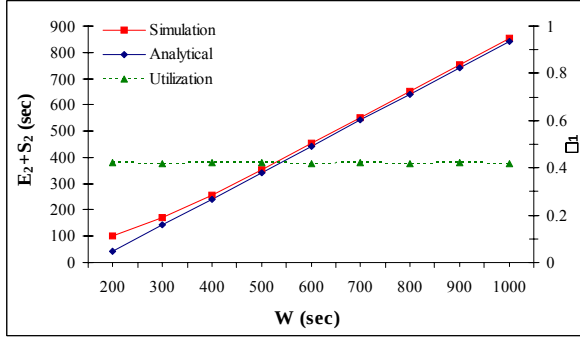


(b) Average to High Utilization;

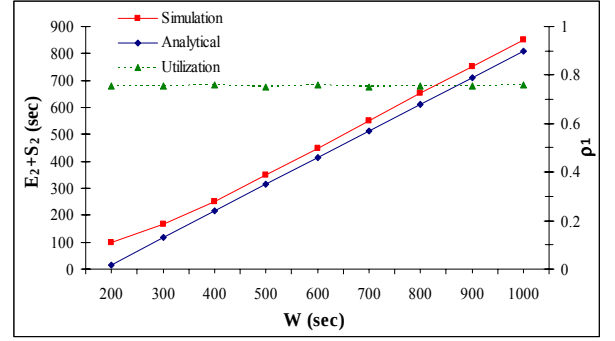


(c) Relative Error Gap, %;

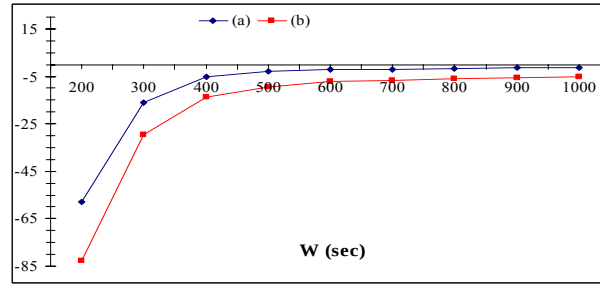
Figure 7.7 System with $B=25$, $M=5$, and $EN=3$.



(a) Low to Average Utilization;

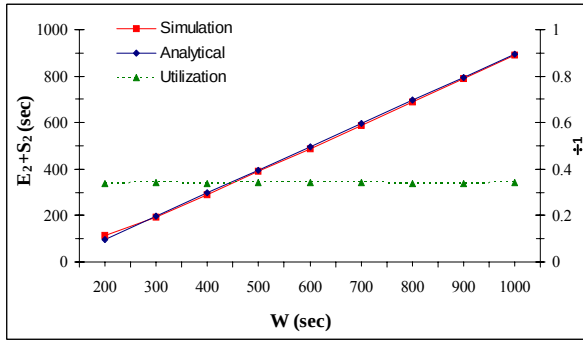


(b) Average to High Utilization;

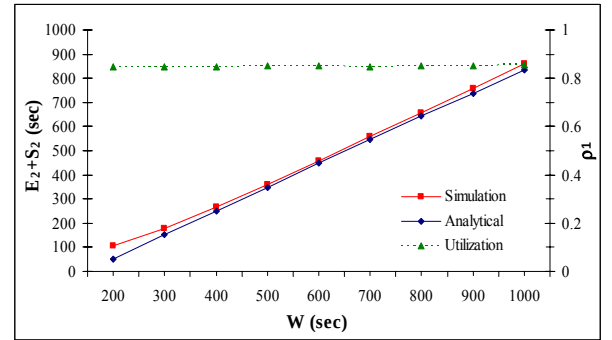


(c) Relative Error Gap, %;

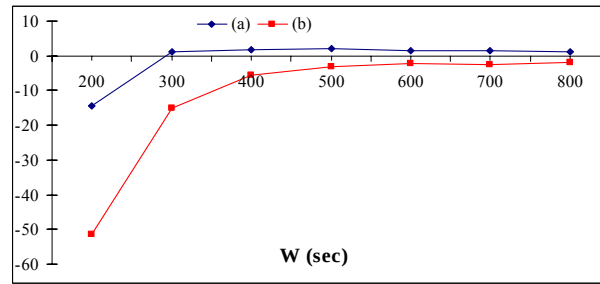
Figure 7.8 System with $B=50$, $M=10$, and $EN=2$.



(a) Low to Average Utilization;

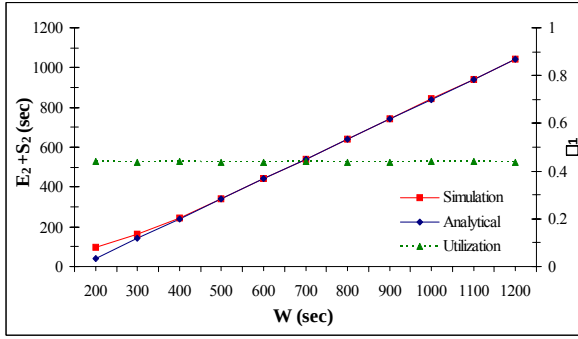


(b) Average to High Utilization;

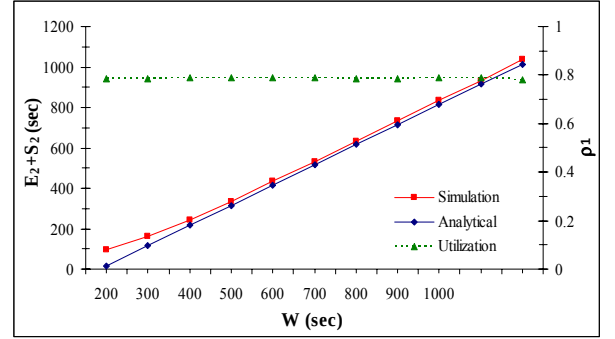


(c) Relative Error Gap, %;

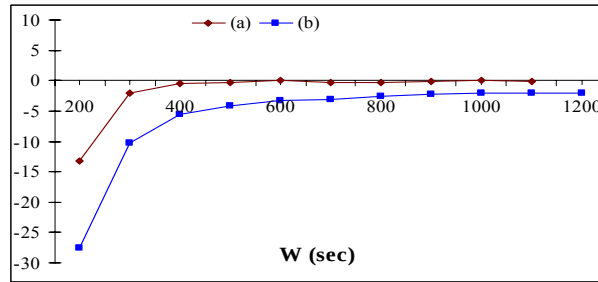
Figure 7.9 System with $B=50$, $M=5$, and $EN=3$.



(a) Low to Average Utilization;

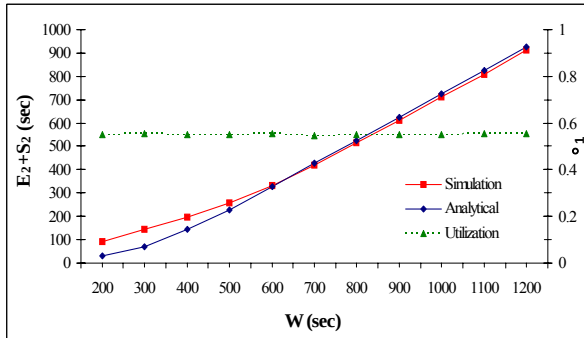


(b) Average to High Utilization;

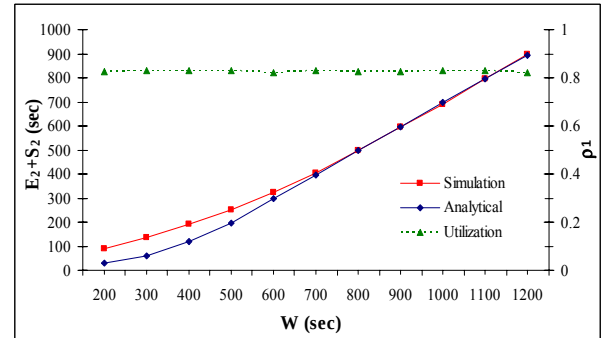


(c) Relative Error Gap, %;

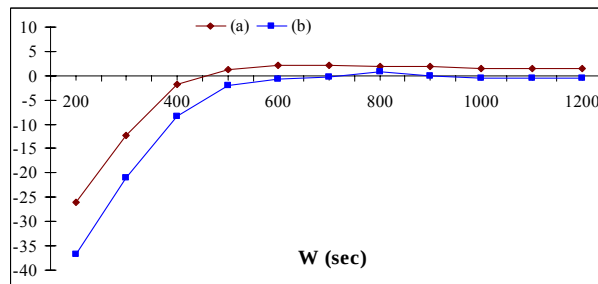
Figure 7.10 System with $B=100$, $M=10$, and $EN=2$.



(a) Low to Average Utilization;



(b) Average to High Utilization;



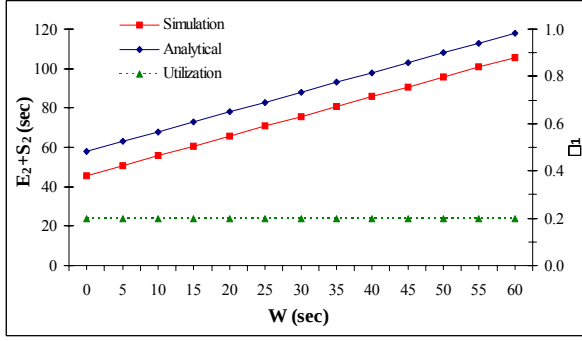
(c) Relative Error Gap, %;

Figure 7.11 System with $B=100$, $M=10$ and $EN=3$.

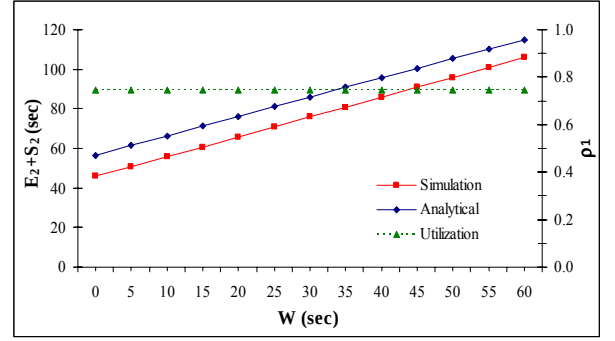
7.7.2. Centralized Order Buffer Systems

In the centralized order buffer system the length of the picking area is set to 100 m , and the height is assumed ergonomically convenient to reach the highest point for a manual picker. Speeds of the picker in the horizontal directions are set to 3.15 m/sec and very high value in the vertical direction to ensure that no additional time is spent on a vertical travel. Two different numbers of buffers are included in the test: 10 and 15. The expected number of items in an order is set to values 2, 3, and 7. Depending on number of buffers, different batch size of orders has been tested so that picker's utilization can vary. The time from order release until the container enters the zone, W , is set to vary from 0 to 60 sec by 5 sec increment. The value of W again consists of two parts: time that an order container is held until it is placed on a conveyor and the time order container is transported into the zone.

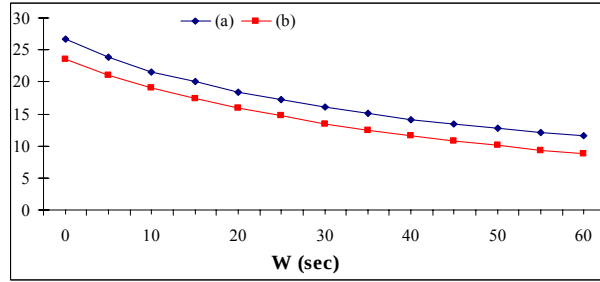
Figures 7.12-7.15 illustrate the comparisons for the centralized order buffer systems. For all buffers sizes with low W , the relative gap is high and it decreases with the increase of the value of W . As in the case of item buffers no matter what the picker utilization is, the relative gap has same pattern, decreasing with the increase of W . It is not surprising to note that the higher the batch size the worse is the approximation as more blocking is created. In fact the number of items in a batch has no impact on performance as it does not affect the blocking (see Figure 7.15, with Batch size $M=2$ and order size $EN=7$). The order size affects the travel time of the picker, however once items of an order are picked they occupy only one buffer in order buffer system thus leaving more buffer space for other orders without blocking phenomenon.



(a) Low to Average Utilization;

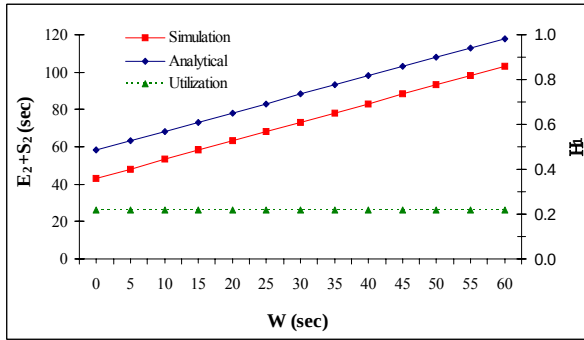


(b) Average to High Utilization;

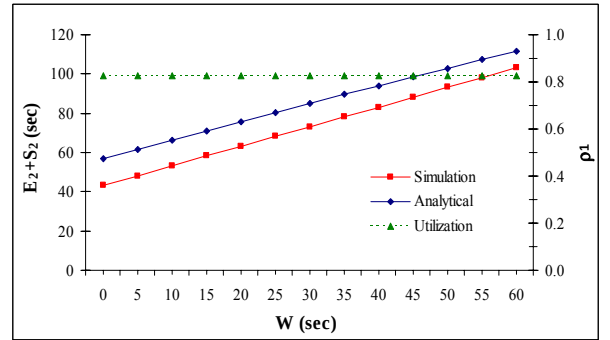


(c) Relative Error Gap, %;

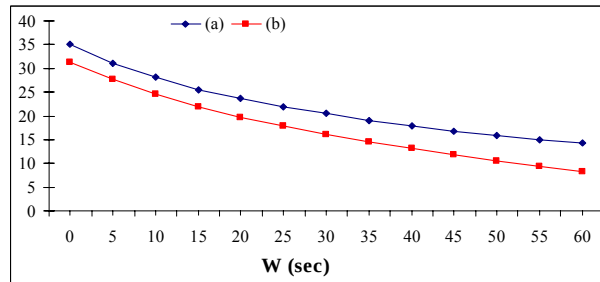
Figure 7.12 System with $B=10$, $M=3$, and $EN=3$.



(a) Low to Average Utilization;

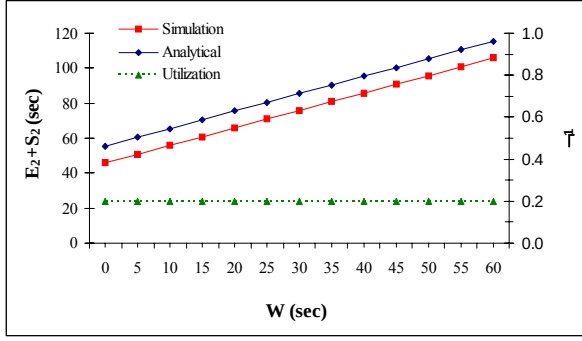


(b) Average to High Utilization;

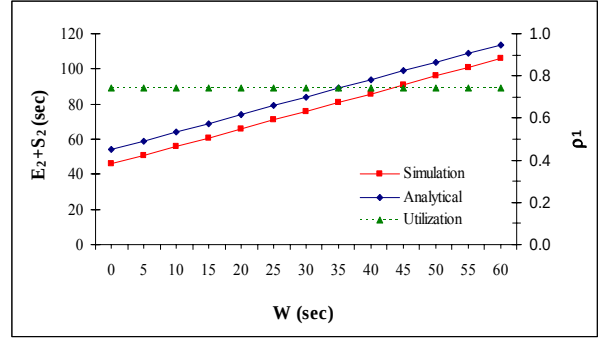


(c) Relative Error Gap, %;

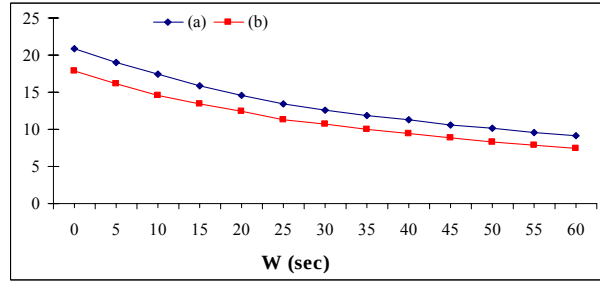
Figure 7.13 System with $B=10$, $M=5$ and $EN=3$.



(a) Low to Average Utilization;

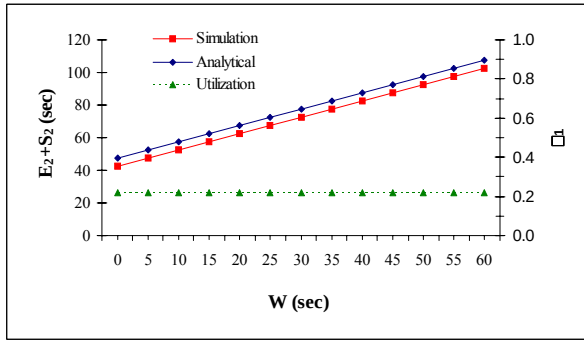


(b) Average to High Utilization;

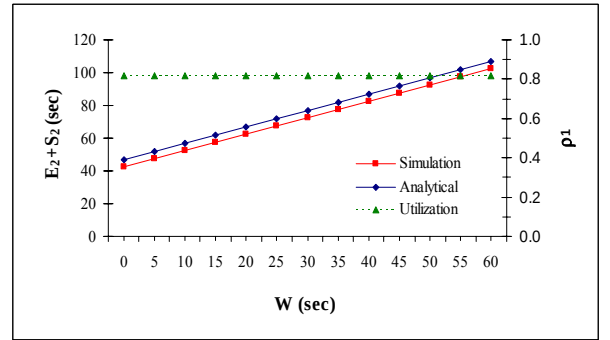


(c) Relative Error Gap, %;

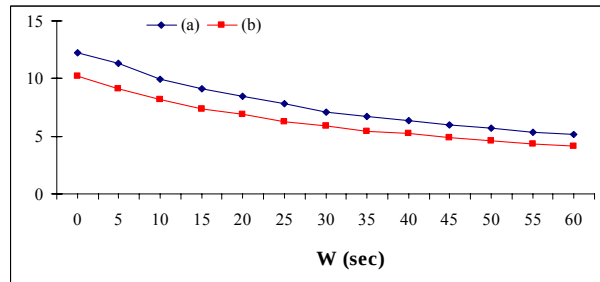
Figure 7.14 System with $B=15$, $M=3$, and $EN=3$.



(a) Low to Average Utilization;



(b) Average to High Utilization;



(c) Relative Error Gap, %;

Figure 7.15 System with $B=15$, $M=2$ and $EN=7$.

7.8 Conclusions

The primary focus of this chapter is to estimate the assembly time and the variance of the entity for particular pick methods and pick system physical characteristics, assuming a pick-to-buffer order picking system. The assembly operation of entities from buffers into order containers is one of two main components of a queuing model describing the underlying systems. In order to estimate the assembly times a search procedure based on Bisection method is implemented.

The cycle time of an entity flowing through the queuing system is used in the search procedure. Once the cycle time of an entity through system is known, the results of queuing model can be implemented in the search procedure to find parameters of assembly time and consequently utilization and travel time of a picker. On the other hand the cycle time of an entity can be calculated by studying a flow of order containers through the system. The search algorithm for finding of parameters of assembly time is described in this chapter.

To evaluate the accuracy of the assembly model, results of an analytical model are compared to that of simulation. The comparison is carried out for the time that an item is delayed in the buffer, i.e. the values of $E_2 + S_2$. Here, both spread item buffer and centralized order buffer systems are studied. The results show an adequate approximation, especially when the value of W increases.

CHAPTER 8

SERVICE LEVEL MODEL

8.1 Introduction

In this chapter the service level model is developed. The service level is defined as the probability that a batch of entities is entirely picked into buffers during the time a container is released into a zone, W . The service level is an important measure to the designers, it is a system requirement and introduced as a constraint in the design problem formulation.

The approach in measuring service level is to approximate the cycle time to pick a batch using a *Normal* distribution and then evaluate the probability that the normal curve covers length of W , i.e. if $S^B \sim N(\mu, \sigma)$, then the following probability

$$P\{S^B \leq W\} \tag{8.1}$$

is the service level of the process represented by the random variable S^B , where W is the value for the time between container release and entry into the zone. In fact, S^B is defined as a time to pick batch of entities.

In order to be able to validate the assumption of normality the statistical hypotheses of goodness-of-fit testing of a probability model is performed using SPSS statistical package. The value of expected mean and standard deviation of the time to pick a batch of entities, S^B , that is going to be used as parameters of Normal distribution is computed using previously estimated parameters for the travel time to pick a single item S^I or an order S^O depending on configuration of studied system.

Parameters:

S^O – travel time of an order;

S^I – travel time of an item.

S^B – travel time of a batch.

N – number of items in an order, random number with known distribution;

M – constant batch size of orders;

X – size of a batch of entities, i.e. for item-buffers $X = \sum_{j=1}^M N_j$ and order buffers $X=M$;

Results of Decisions:

SL – the service level

8.2 Service Level

The service level, SL , is defined as a probability that batch of entities is entirely picked into buffers during time between container release and entry into the zone, W . Thus, by definition

$$SL = P\{S^B \leq W\} = \Phi\left(\frac{W - E[S^B]}{\sqrt{Var(S^B)}}\right) \quad (8.2)$$

where $\Phi(x)$ is the cumulative distribution function of the standard normal distribution.

The values of $E[S^B]$ and $Var(S^B)$ are travel time parameters of a picker to pick batch of entities into buffers taking into account the blocking times. Thus, for item buffer systems the value of $E[S^B]$ can be computed by the following

$$E[S^B] = E[S^I]E[X] \quad (8.3)$$

The equation (III.8) can be rewritten to obtain $Var(S^B)$ in (8.4).

$$Var(S^B) = \left(Var(S^I)E[N] + (E[S^I])^2 Var(N) \right) M \quad (8.4)$$

$E[S^I]$ and $Var(S^I)$ are given by (4.54) and (4.55) respectively.

For order buffer systems the travel time parameters of a picker to pick a batch of orders into buffers taking into account the blocking times are given by

$$E[S^B] = E[S^O]E[X] \quad (8.5)$$

The equation (III.8) of *Appendix III* can be rewritten to obtain $Var(S^B)$ in (8.6).

$$Var(S^B) = Var(S^O)M \quad (8.6)$$

The validation of service level model is presented in the Chapter 9.

8.3 Normality Tests

The null hypothesis for a goodness-of-fit test for values of times to pick i th batch of entities, S_i^B , is the following:

H_0 : The S_i^B s are IID random variables with distribution function F .

One of the most powerful tests, the Kolmogorov-Smirnov test is used to determine whether an underlying probability distribution differs from a hypothesized distribution

($N(\mu, \sigma)$ in this case) based on finite samples. The samples are collected during the simulation of the studied systems. The one-sample Kolmogorov-Smirnov test compares the empirical distribution function with the cumulative distribution function specified by the null hypothesis (Hayter 1996). For normality testing, minor improvements to Kolmogorov-Smirnov test is made by Lilliefors (Law and Kelton 2000; Lilliefors 1967).

To demonstrate the normality of a travel time to pick a batch the statistical procedure of Kolmogorov-Smirnov test with Lilliefors significance correction is performed using a statistical package SPSS. The statistical data is collected using the simulation model described in *Chapter 4*. Figures 8.1, 8.2 and 8.3 display histograms for S_i^B when $X=15, 20, 30$, respectively. Results of the test of normality for these cases are shown in Tables 8.1, 8.2 and 8.3. Results of Kolmogorov-Smirnov tests with Lilliefors significance correction indicate that S_i^B fit the normal curve well, i.e. tests are not significant. In fact the higher the number of orders in the batch, the better is the fit due to the Central Limit Theorem which states that if the sum of the variables has a finite variance, then it will be approximately normally distributed the sum.

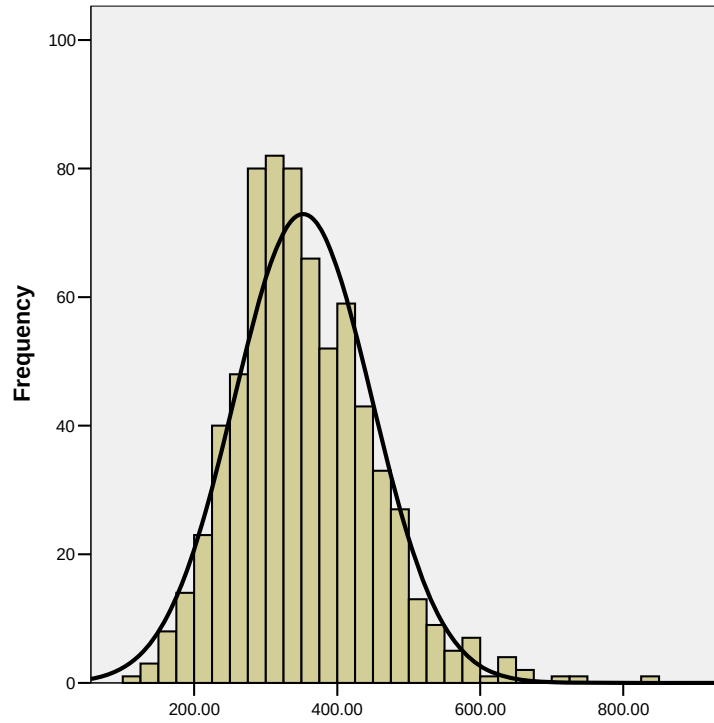


Figure 8.1 Histogram for S_i^B , when $X=15$.

Table 8.1 SPSS output ,when $X=15$.

Kolmogorov-Smirnov(a)		
Statistic	df	Sig. p-value
0.080	200	0.118

a Lilliefors Significance Correction

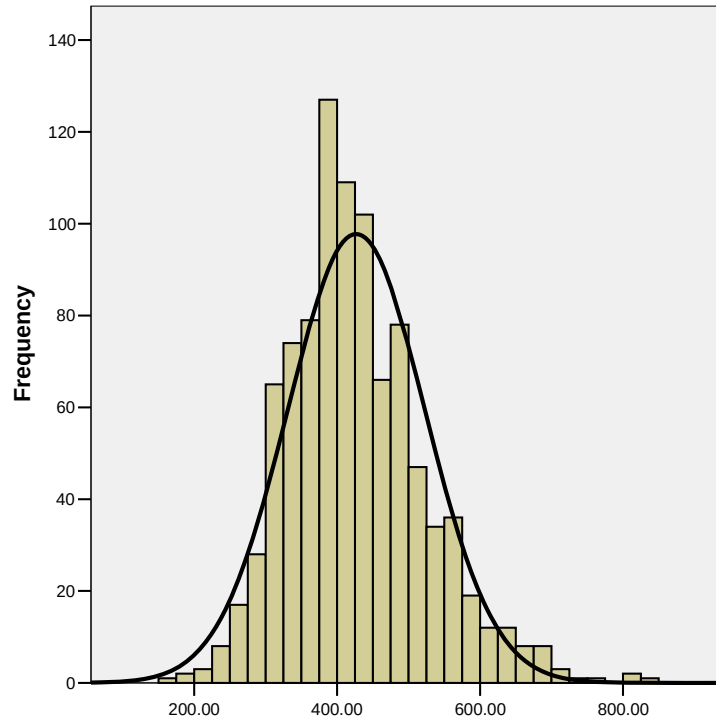


Figure 8.2 Histogram for S_i^B , when $X=20$.

Table 8.2 SPSS output ,when $X=20$.

Kolmogorov-Smirnov (a)		
Statistic	df	Sig. p-value
0.043	200	0.200(*)

* This is a lower bound of the true significance

a Lilliefors Significance Correction

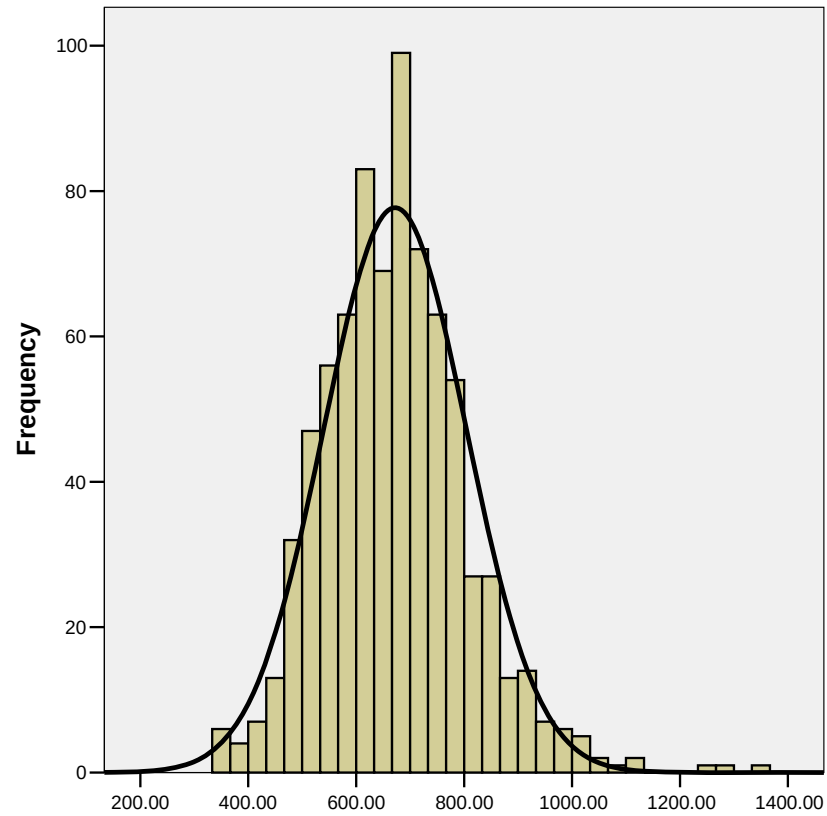


Figure 8.3 Histogram for S_i^B , when $X=30$.

Table 8.3 SPSS output ,when $X=30$.

Kolmogorov-Smirnov (a)		
Statistic	df	Sig. p-value
0.068	200	0.200(*)

* This is a lower bound of the true significance

a Lilliefors Significance Correction

8.4 Conclusions

This chapter presents the analytical model for approximating the service level of the pick-to-buffer systems. The service level is defined as the probability that a batch of entities is entirely picked into buffers during the time a container is released into a zone, W .

The approach in measuring service level is to approximate the cycle time to pick a batch through *Normal* distribution and evaluate the probability that the normal curve covers length of W .

In order to be able to validate the assumption of normality the statistical hypotheses of goodness-of-fit testing of a probability model is performed using SPSS statistical package. The values of expected mean and standard deviation of a time to pick a batch of entities that is going to be used as parameters of Normal distribution are computed using previously estimated parameters for a travel time to pick a single item S^I or an order S^O depending on configuration of studied system. The Kolmogorov-Smirnov shows that the hypothesized Normal distribution is a good fit for the cycle time to pick a batch. In the *Chapter 9* the validation of the service level model will be demonstrated.

CHAPTER 9

VALIDATIONS

9.1 Introduction

In this chapter the results of analytical model are compared to that of simulation. To evaluate the analytical model it is desirable to obtain the data set from an actual physical system. However, it is difficult to get actual data set, and therefore we use a simulation described in *Chapter 4* as a benchmark to our analytical model.

The analytical model is evaluated at a wide range of operating scenarios. The performance metrics that are compared are the picker utilization and the service level of the system.

9.2 Comparison

The comparison between analytical and simulation models has been conducted for systems with both spread item buffers and centralized order buffers. For both systems the arrival process into the system is assumed to be deterministic, as it reflects more realistic situation when warehouses send order requests to order picking departments in batches at fixed time intervals. For all the tests, order size was assumed to have zero truncated binomial distribution, $ZTBin(p, k)$ with parameters $p \in (0, 1)$ and a positive integer k , $ZTBin(p, k)$. The value of k is set to the number of buffers in the system. Different batch sizes of arriving entities were included in the tests with values varying from 2 to 20 orders per batch depending on the system under consideration. The number of buffers is chosen so that

picker's utilization varies within the range $[0.2, 0.9]$. The speed of the conveyor is set to 0.6 m/sec .

9.2.1. Spread Item Buffer Systems

In the spread item buffer system the length of the picking area is set to 100 m , and the height to 20 m . Speeds of the picker in the horizontal and vertical directions are set to 3.15 m/sec and 0.9 m/sec , respectively. Three different numbers of buffers are included in the test: 25, 50, and 100. The expected number of items in an order is set to values 2, 3, and 4. Depending on number of buffers, different batch size of orders has been tested so that picker's utilization can vary. The time from order release until the container enters the zone is set to vary from 100 to 700 sec by 100 sec increment.

Figures 9.5-9.10 show results of comparing analytical approximation of the picker utilization and service level with the simulation output "*Picker Utilization*" and "*Item Service Level*", respectively. Both parameters have values between zero and one, therefore it is very convenient to display them on the same graph. For every figure plots (a) and (b) display the dependency of the values of utilization and service level, respectively, on W , the time from order release until the container enters the zone. Plots (a) and (b) display cases with low to average utilization ($\rho_l < 0.5$) and average to high utilization ($\rho_l \geq 0.5$), respectively. Plots (c) and (d) of every figure display the relative error gap for service level and utilization respectively for cases illustrated in (a) and (b).

As we can see the approximations work very well. The error gap for service level is the highest when the value of W is the smallest. In fact with small W the values of service level are very low, and the absolute error between simulation and analytical results are also

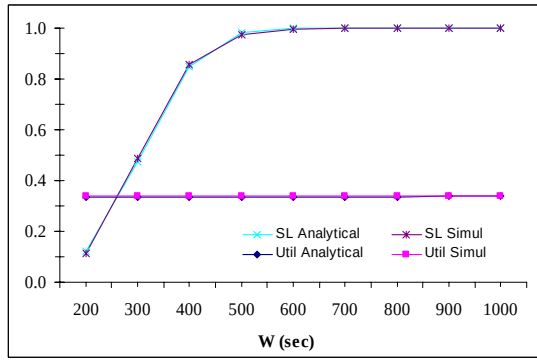
low, while the relative error is high. In this situation high relative gaps are acceptable. The highest relative error gap observed for the service level is 20%. Another observation is that for every case the relative error gap for service level almost always has the same pattern for both low to average and average to high utilizations.

The analytical values of utilization produce very good results with the observed highest relative error gap of about 10%.

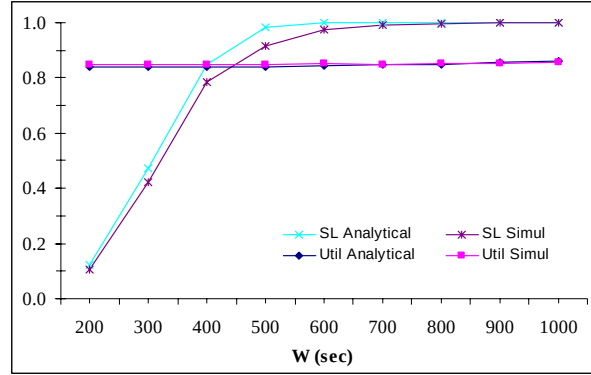
The above analysis becomes even more critical when the study of zones that are located at the end of the order picking system is carried out. The time from order release until the container enters the zone is longer in comparison to the first zone. During these times a picker receives information about arriving order contents and starts picking them into the buffers. Since time between order release and the container entering the zone are long the buffers will get filled up long before order containers arrive to the zone. Once buffers are filled to capacity, the picker waits for an order container arrival to free up a buffer. As soon as one buffer opens up the picker travels to that buffer to place an item.

With this strategy, the pick time of a picker increases as a result of two factors: a) more chaotic (not following a predefined pick strategy) behavior of placing items into buffers is observed; b) picker blocking time is higher due to increased waiting for buffers to free up. Here buffers and pickers have higher utilization than those of zones in the beginning of the order picking system.

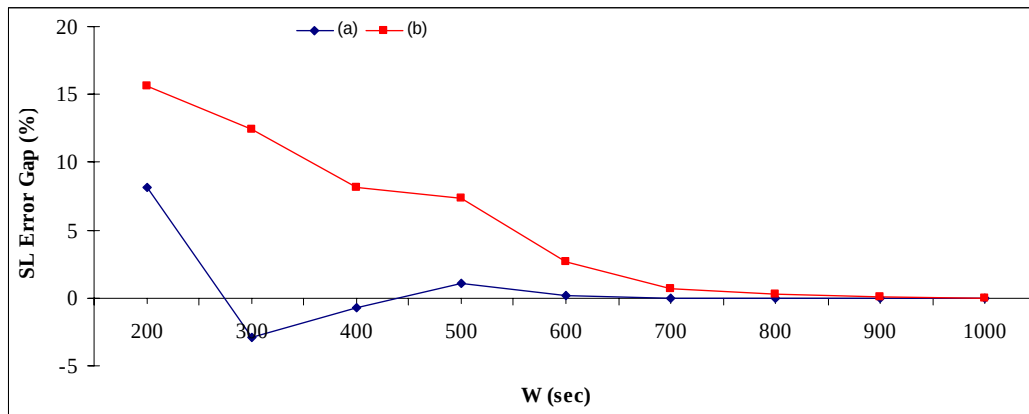
Figure 9.7 illustrates a situation when the value of W is high and the utilization of a picker drastically increases. Starting from about $W=800$ sec the utilization of a picker at low to high values produced by simulation is much higher than that of resulted from the analytical model. This can be explained by the increase of utilization due to chaotic behavior



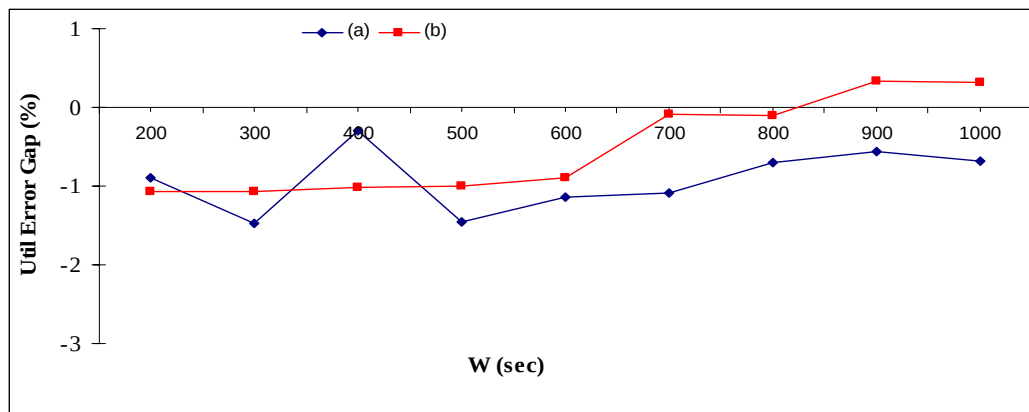
(a) Low to Average Utilization;



(b) Average to High Utilization;

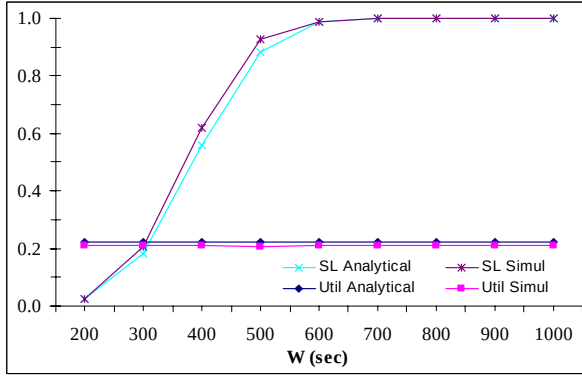


(c) SL Relative Error Gap, %;

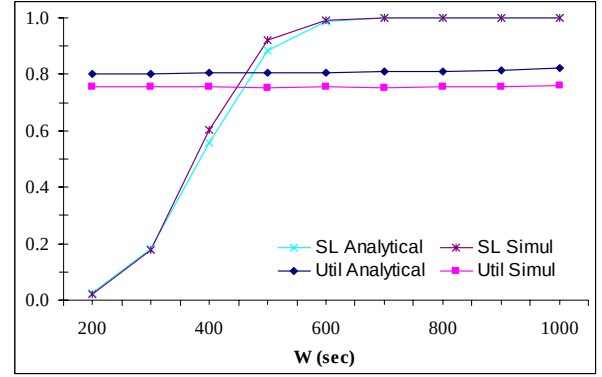


(d) Utilization Relative Error Gap, %;

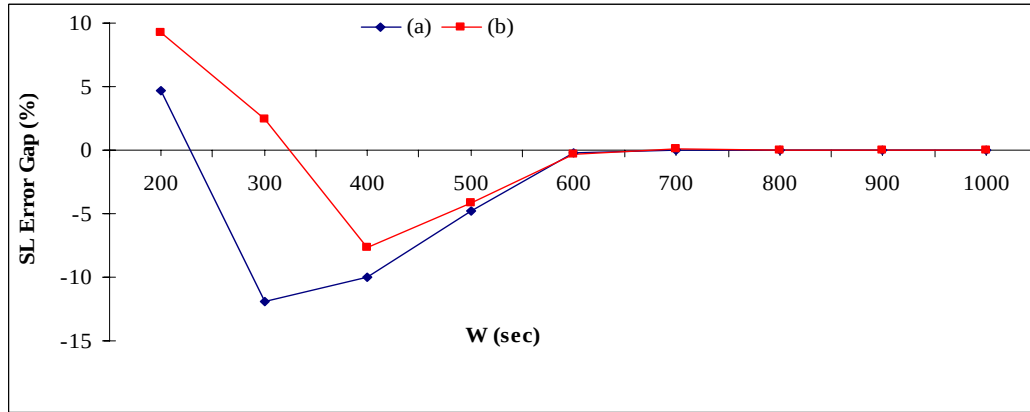
Figure 9.1 System with $B=50$, $M=5$, and $EN=3$.



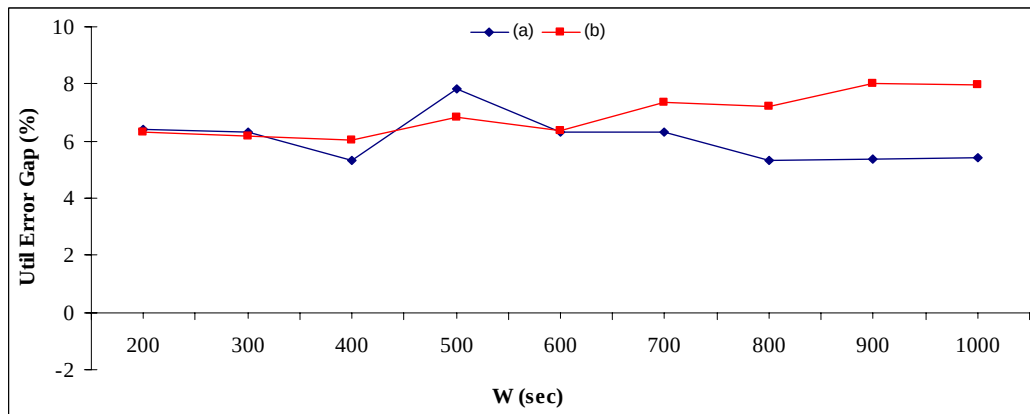
(a) Low to Average Utilization;



(a) Average to High Utilization;

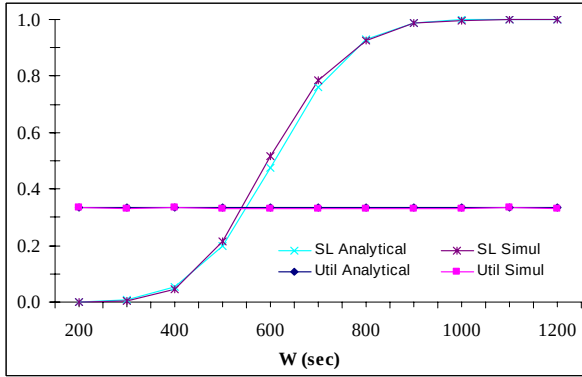


(b) SL Relative Error Gap, %;

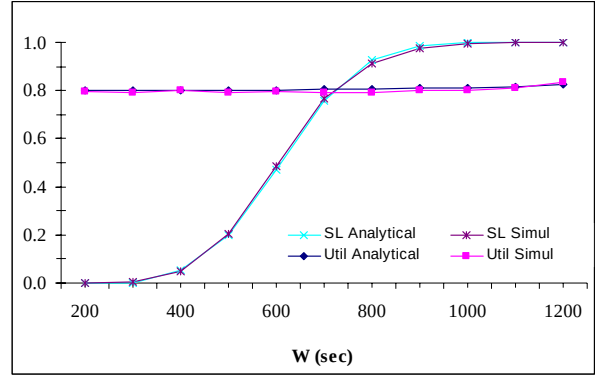


(d) Utilization Relative Error Gap, %;

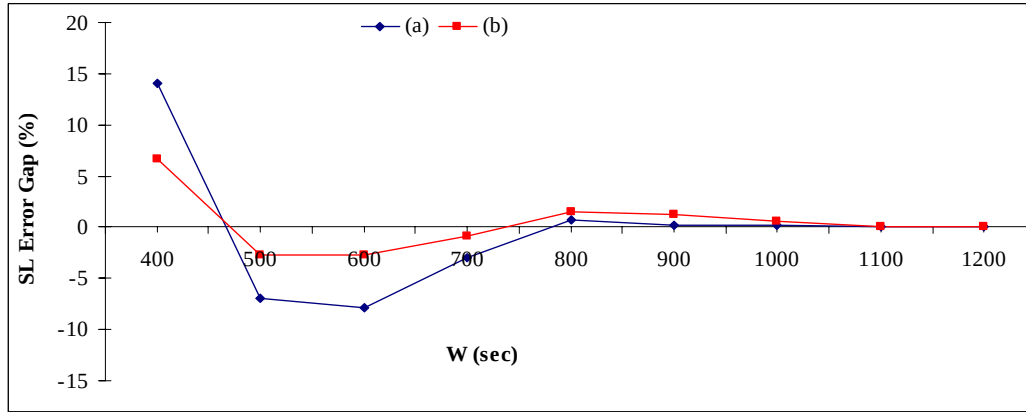
Figure 9.2 System with $B=50$, $M=10$, and $EN=2$.



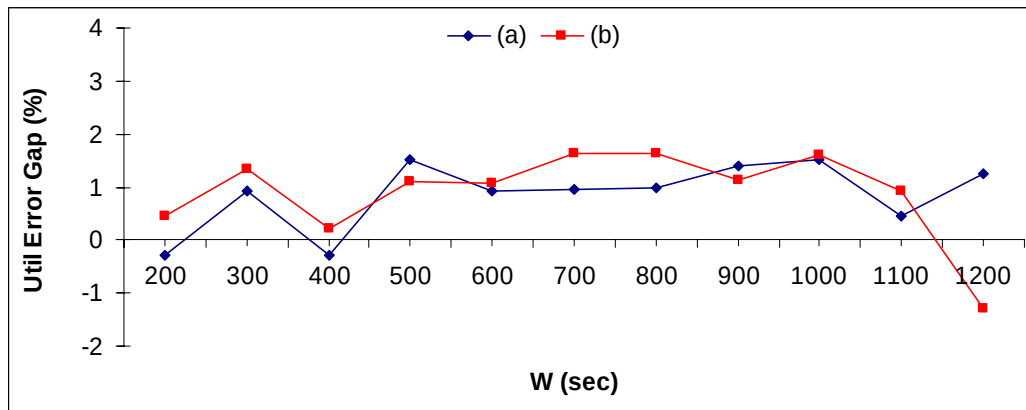
(a) Low to Average Utilization;



(c) Average to High Utilization;

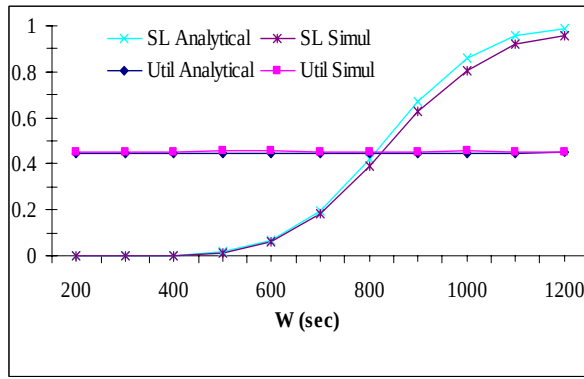


(c) SL Relative Error Gap, %;

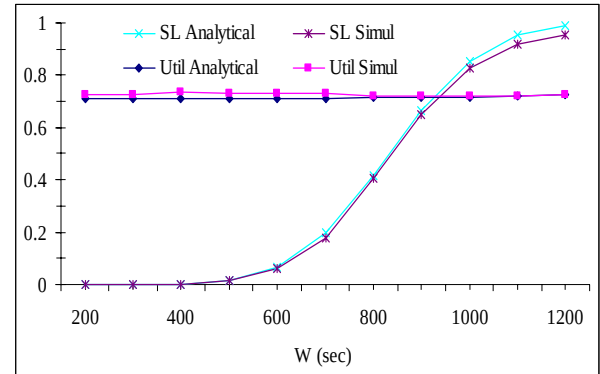


(d) Utilization Relative Error Gap, %;

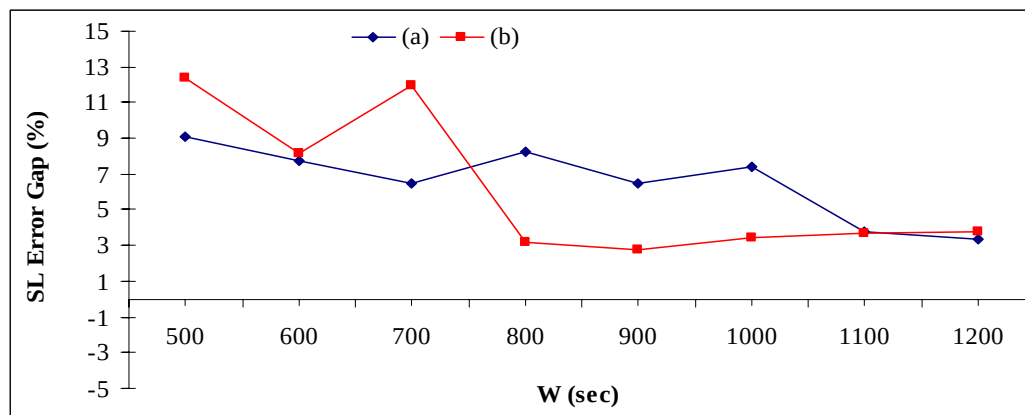
Figure 9.3 System with $B=50$, $M=10$, and $EN=3$.



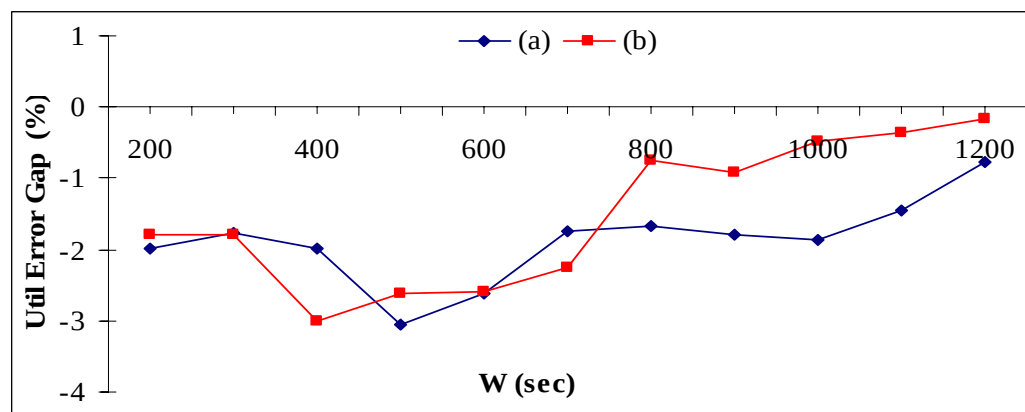
(a) Low to Average Utilization;



(b) Average to High Utilization;

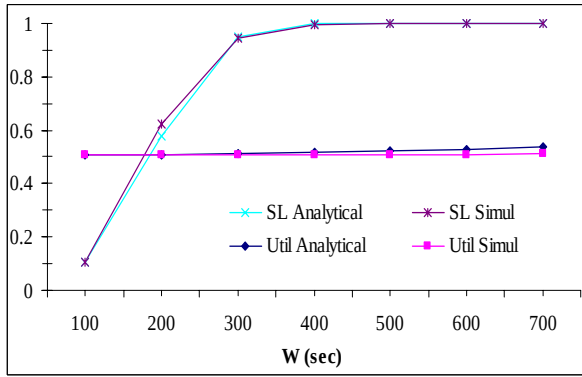


(c) SL Relative Error Gap, %;

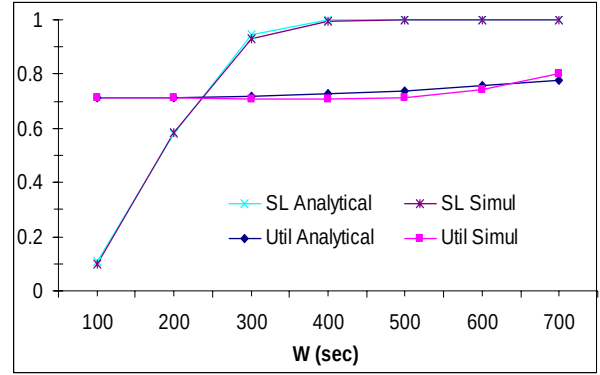


(d) Utilization Relative Error Gap, %;

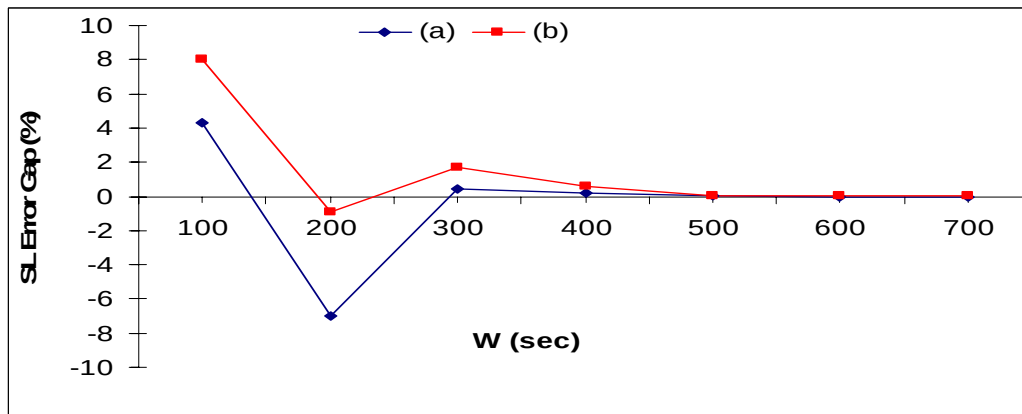
Figure 9.4 System with $B=50$, $M=10$, and $EN=4$.



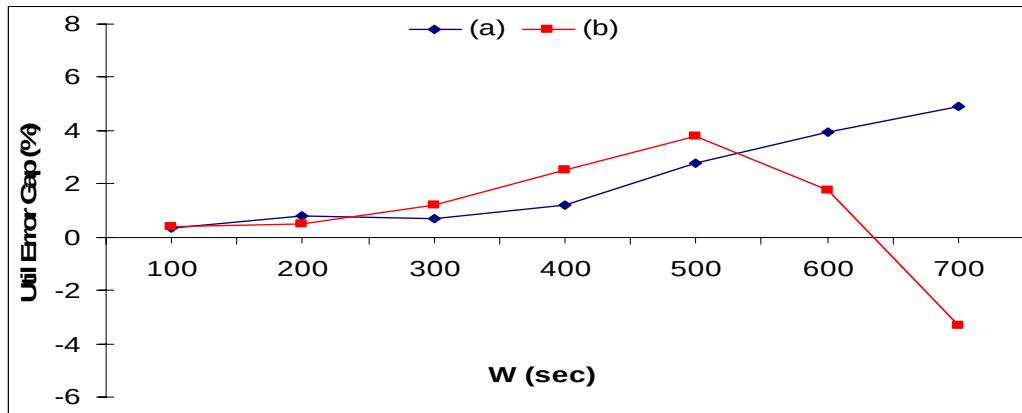
(a) Low to Average Utilization;



(b) Average to High Utilization;

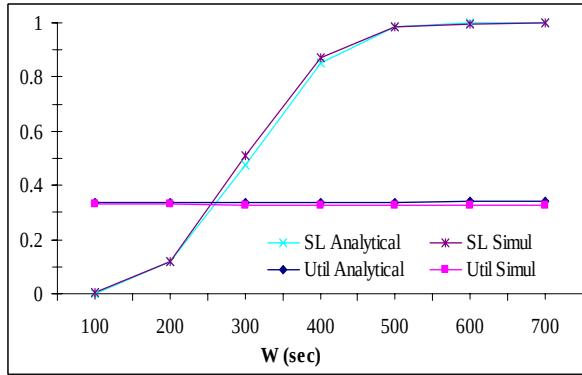


(c) SL Relative Error Gap, %;

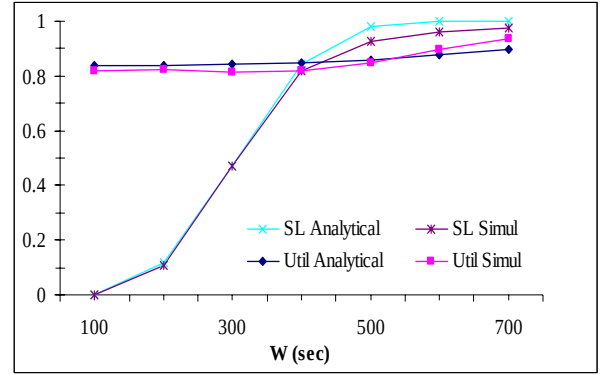


(d) Utilization Relative Error Gap, %;

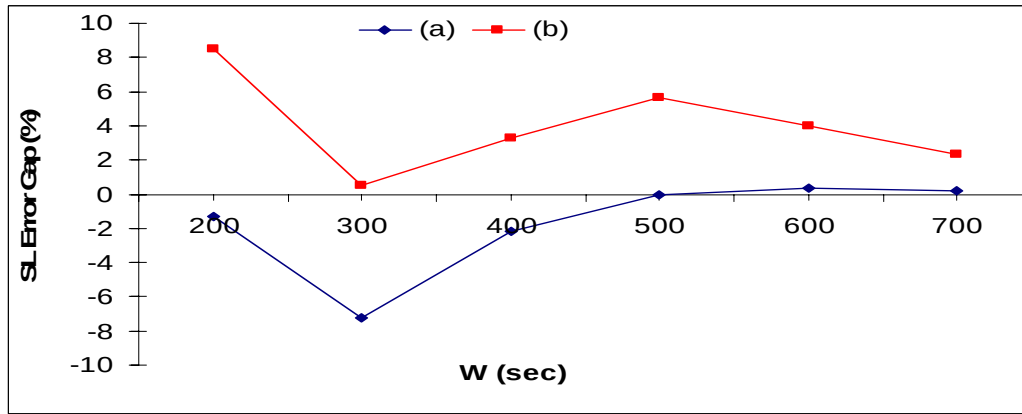
Figure 9.5 System with $B=25$, $M=3$, and $EN=3$.



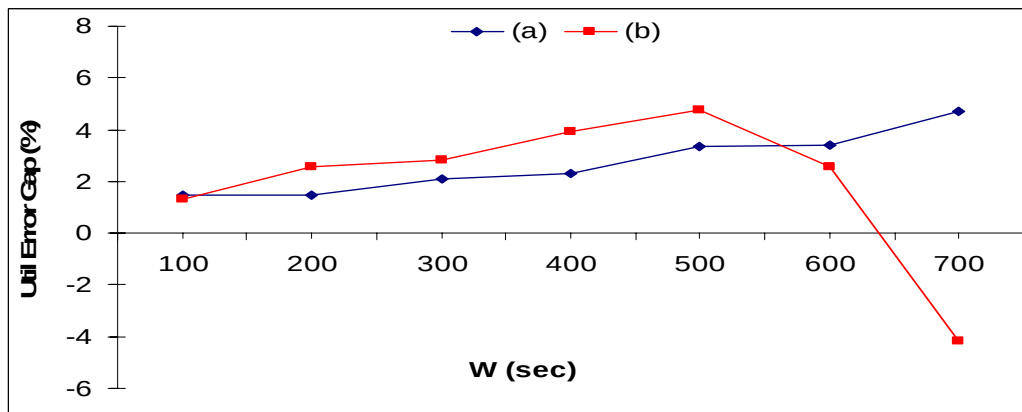
(a) Low to Average Utilization;



(b) Average to High Utilization;

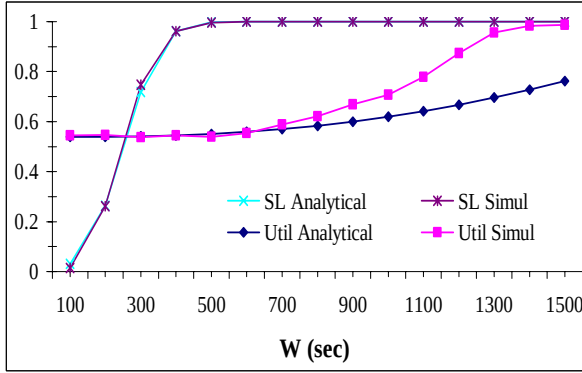


(c) SL Relative Error Gap, %;

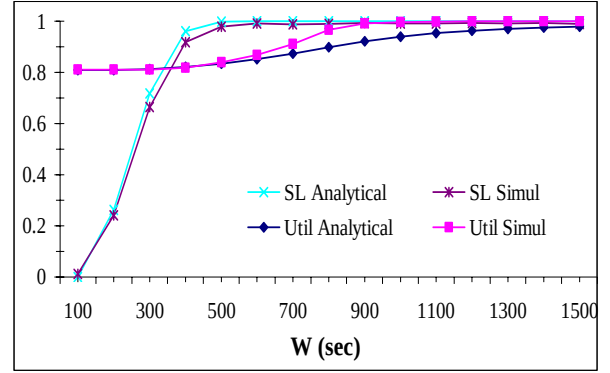


(d) Utilization Relative Error Gap, %;

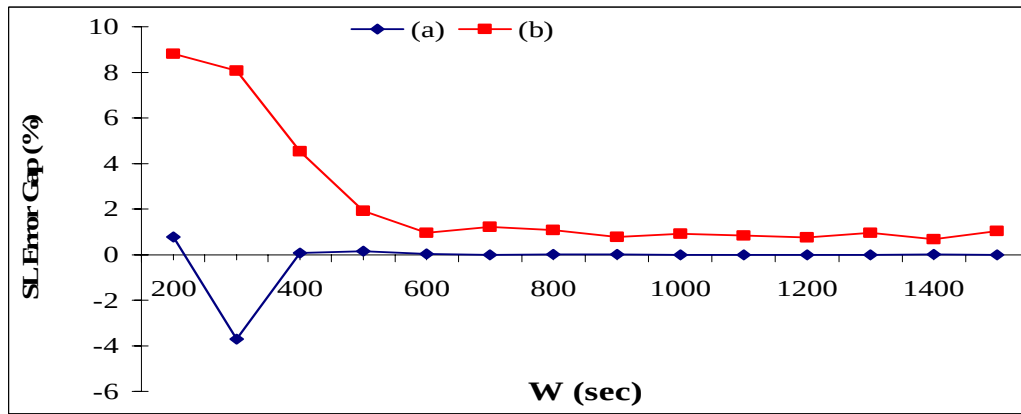
Figure 9.6 System with $B=25$, $M=5$, and $EN=3$.



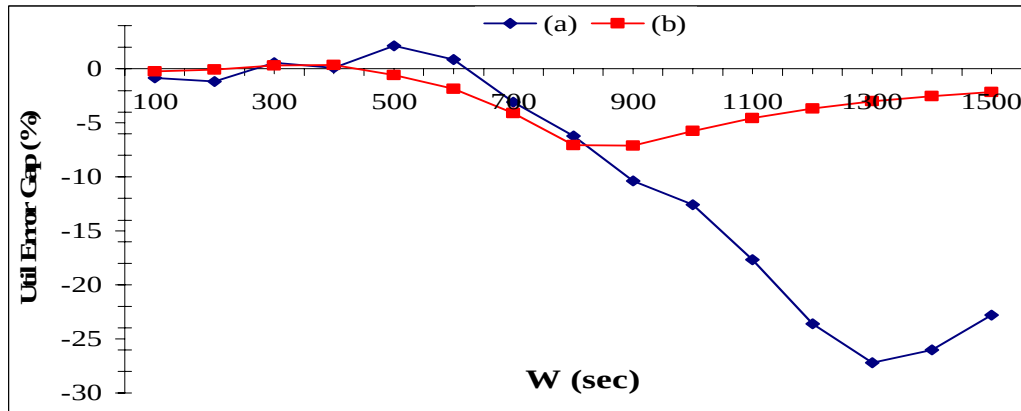
(a) Low to Average Utilization;



(b) Average to High Utilization;

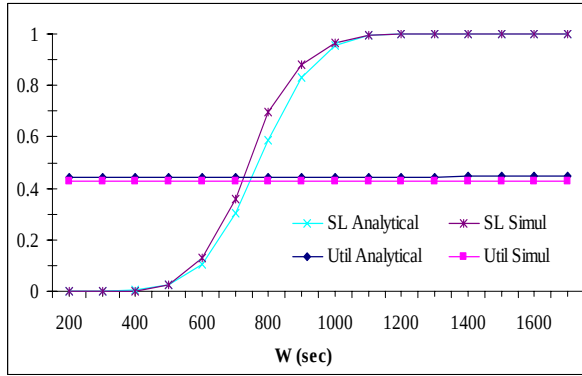


(c) SL Relative Error Gap, %;

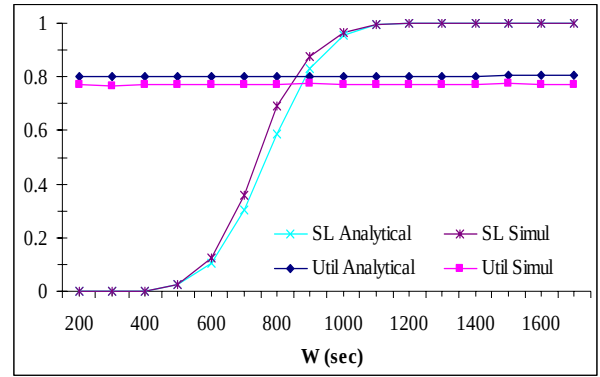


(d) Utilization Relative Error Gap, %;

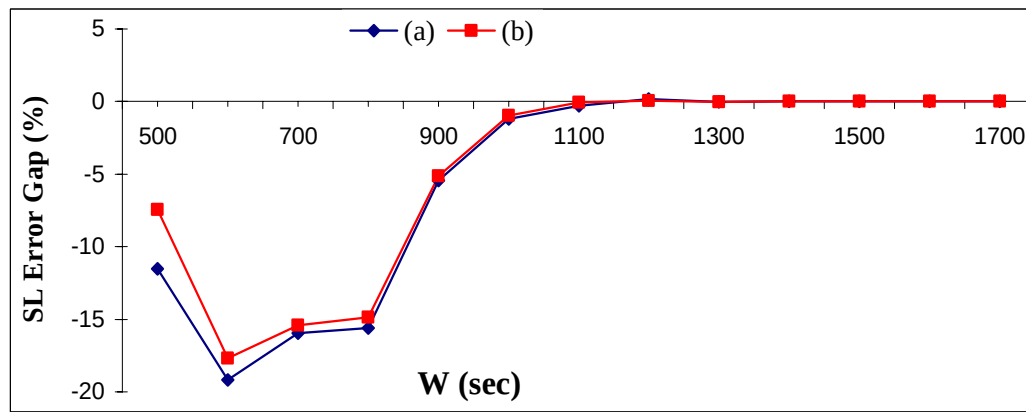
Figure 9.7 System with $B=25$, $M=3$, and $EN=4$.



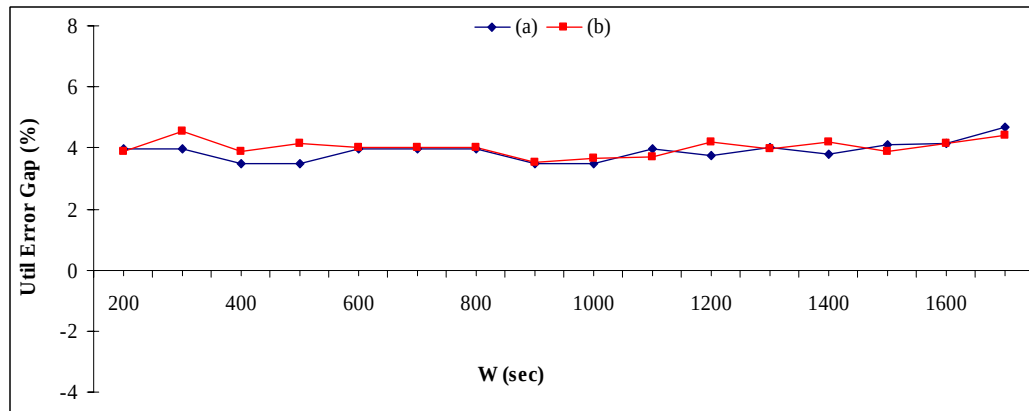
(a) Low to Average Utilization;



(b) Average to High Utilization;

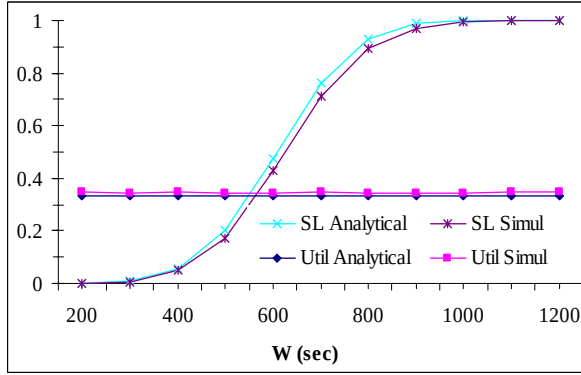


(c) SL Relative Error Gap, %;

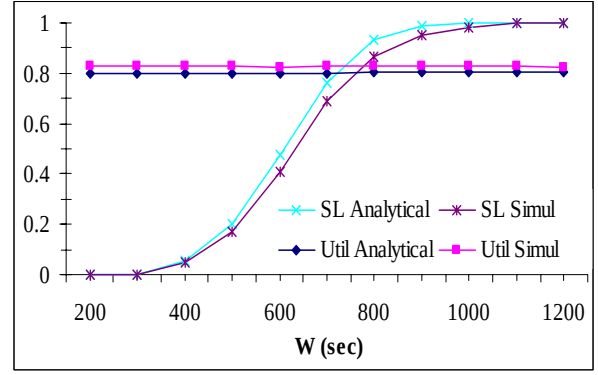


(d) Utilization Relative Error Gap, %;

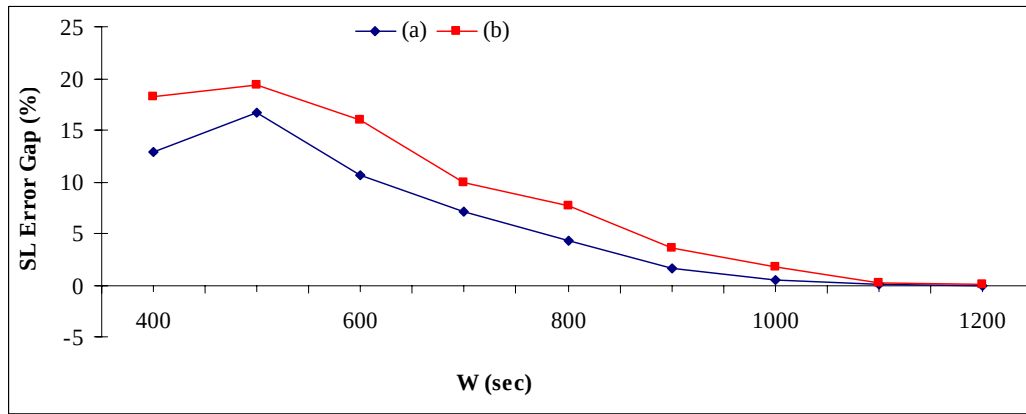
Figure 9.8 System with $B=100$, $M=20$, and $EN=2$.



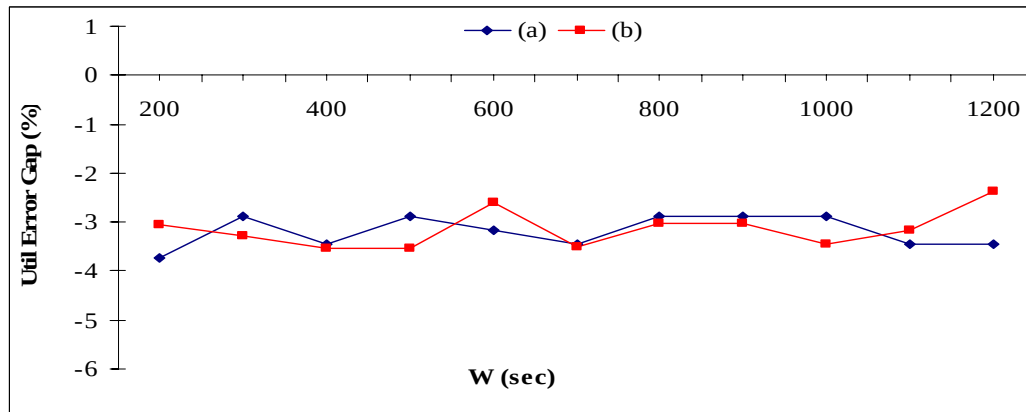
(a) Low to Average Utilization;



(b) Average to High Utilization;

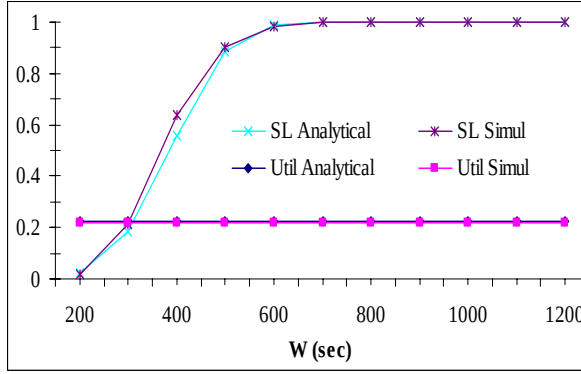


(c) SL Relative Error Gap, %;

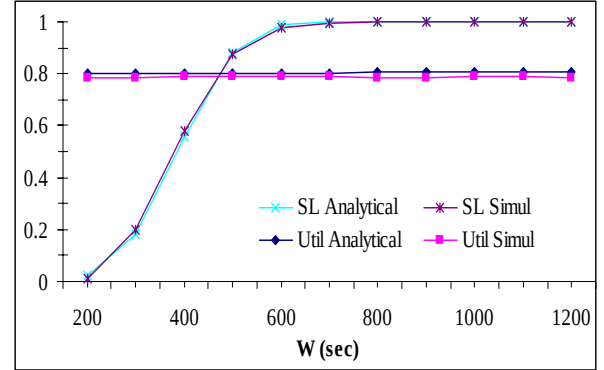


(d) Utilization Relative Error Gap, %;

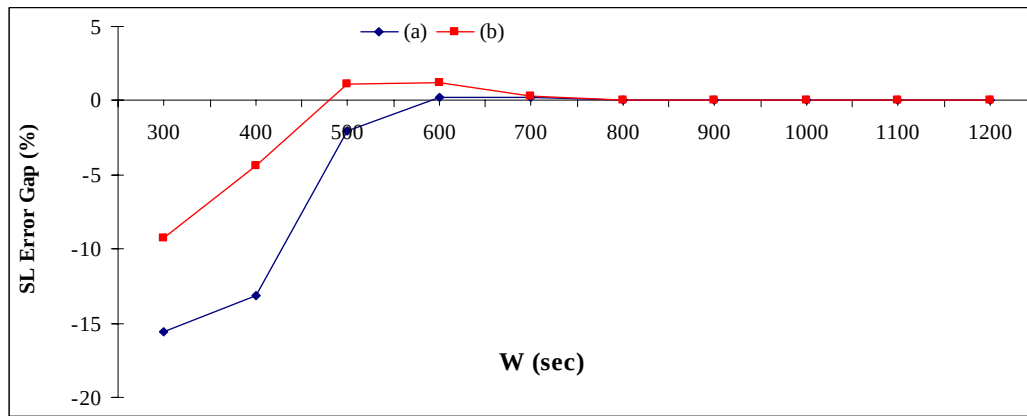
Figure 9.9 System with $B=100$, $M=10$, and $EN=3$.



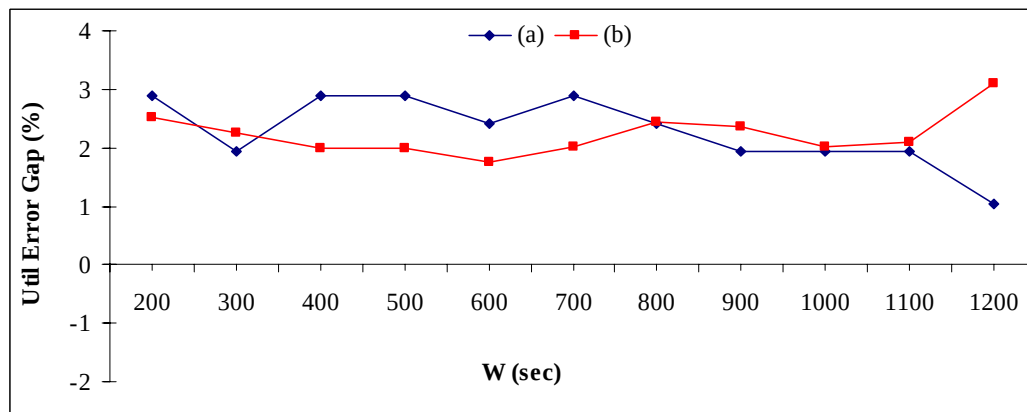
(a) Low to Average Utilization;



(b) Average to High Utilization;



(c) SL Relative Error Gap, %;



(d) Utilization Relative Error Gap, %;

Figure 9.10 System with $B=100$, $M=10$, and $EN=2$.

of placing items into buffers, which is not taken into account in an analytical model. The cases when $W \geq 800 \text{ sec}$ might be considered the time until an order container reaches a zone other than the first one.

One way to reduce the travel time and utilization of a picker is to supply information about orders to be picked relatively late and design each zone as if is the first zone of the order picking system.

9.2.2. Order Buffer Systems

In the centralized order buffer system, the length of the picking area is set to 100 m , and the height is assumed ergonomically convenient to reach the highest point for a manual picker. Speeds of the picker in the horizontal directions are set to 3.15 m/sec and very high number and vertical direction to ensure that no additional time is spent on a vertical travel. Two different numbers of buffers are included in the test: 10 and 15. Depending on number of buffers the expected number of orders in a batch is set to values 2, 3, 5, 7 so that picker's utilization can vary. The time from order release until the container enters the zone, W , is set to vary from 0 to 60 sec by 5 sec increment. The value of W again consists of two parts: time that an order container is held until it is placed on a conveyor and the time order container is transported into the zone.

Figures 9.11-9.17 illustrate the comparisons analytical approximation of the picker utilization and service level with the simulation output "*Picker Utilization*" and "*Order Service Level*", respectively. Similarly to the case of spread item buffers, figure plots (a) and (b) display the dependency of the values of utilization and service level on W . Plots (a) and (b) display cases with low to average utilization ($\rho_l < 0.5$) and average to high utilization

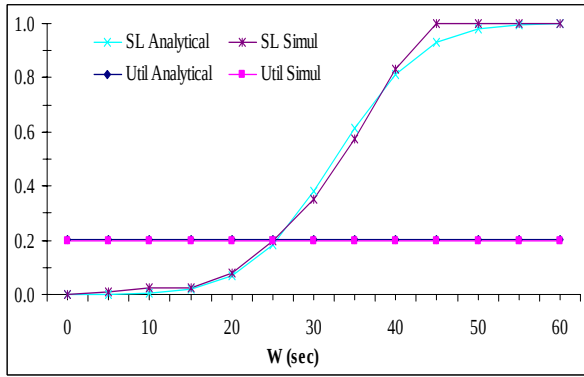
($\rho_l \geq 0.5$), respectively. Plots (c) and (d) of every figure display the relative error gap for service level and utilization respectively for cases illustrated in (a) and (b).

As we can see the approximations work fairly well. The error gap for service level is the highest when the value of W is the smallest. In fact with small W the values of service level are very low, and the absolute error between simulation and analytical results are also low. In this situation high relative gaps are acceptable. The highest relative error gap observed for the service level is 37% (this is when the utilization is about 85%).

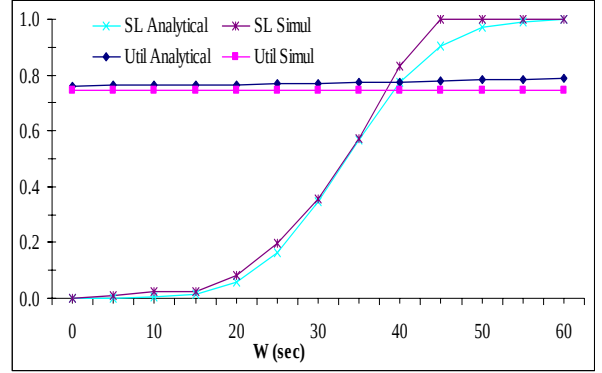
The relative error gap for service level is also high at the values of SL which is in between 0.2 and 0.7. However in these cases the error gap is not more than 15 %. The big deviation is because the time to pick a batch of orders does not have a perfect shape of a *Normal* distribution. However we can see that the assumption of normality is still acceptable for studied system as the relative gap for service level is within acceptable range. Again, as in the case of spread item buffer, for every case the relative error gap for service level almost always has the same pattern for both low to average and average to high utilizations.

The analytical values of utilization produce very good results. In rare cases the observed relative error gap for utilization reaches about 15%.

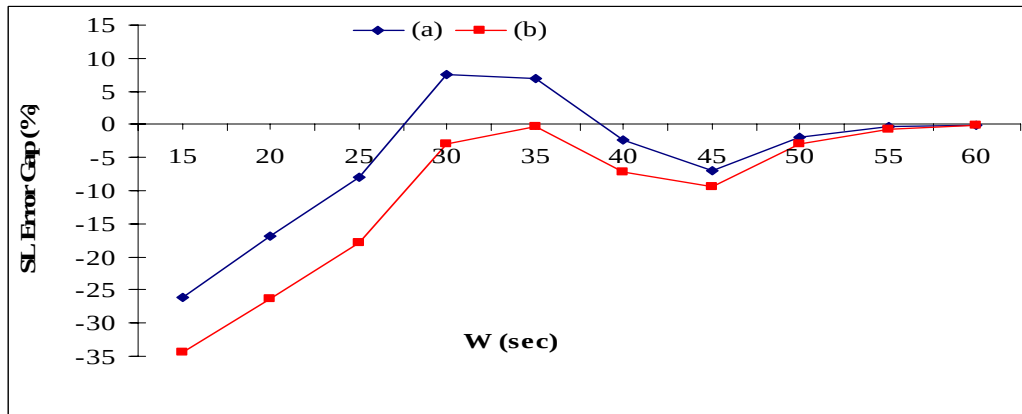
It is not surprising to note that the number of items in a batch has no impact on performance as it does not affect the blocking (see Figure 9.17, with batch size $M=2$ and order size $EN=7$). The order size affects the travel time of the picker, however once items of an order are picked they occupy only one buffer in order buffer system thus leaving more buffer space for other orders without blocking phenomenon.



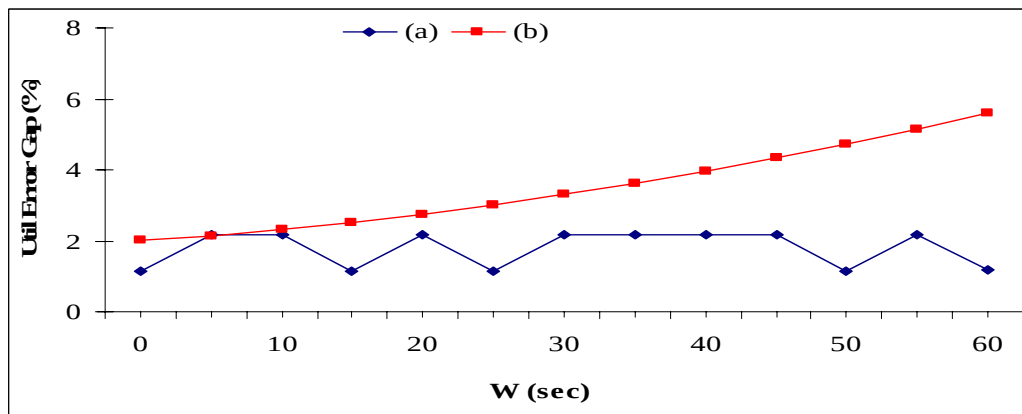
(a) Low to Average Utilization;



(b) Average to High Utilization;

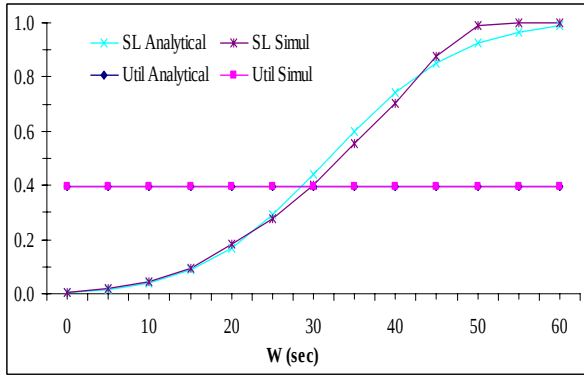


(c) SL Relative Error Gap, %;

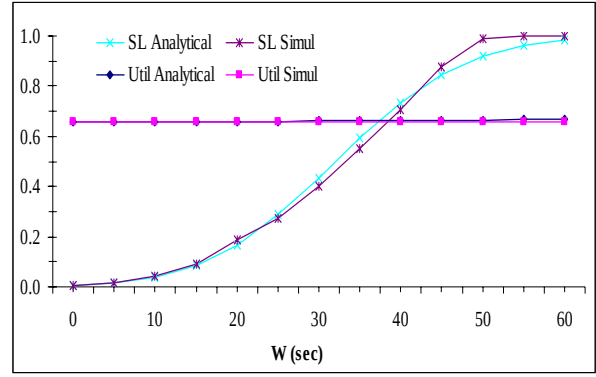


(d) Utilization Relative Error Gap, %;

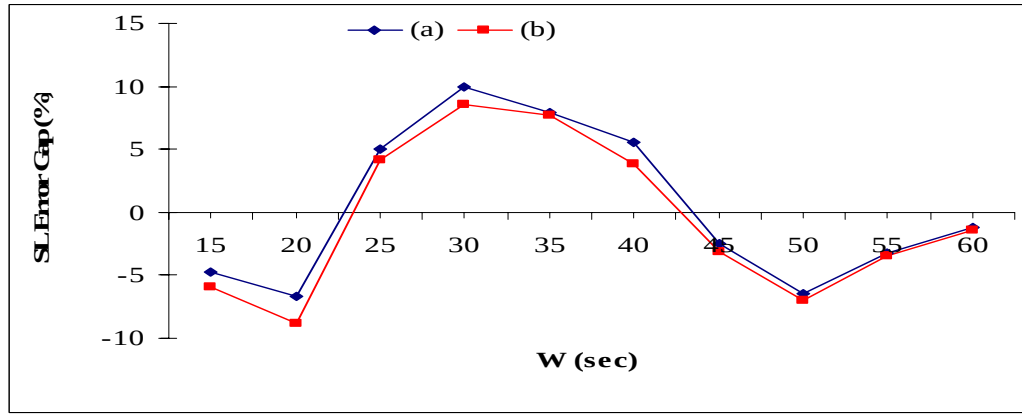
Figure 9.11 System with $B=10$, $M=3$, and $EN=3$.



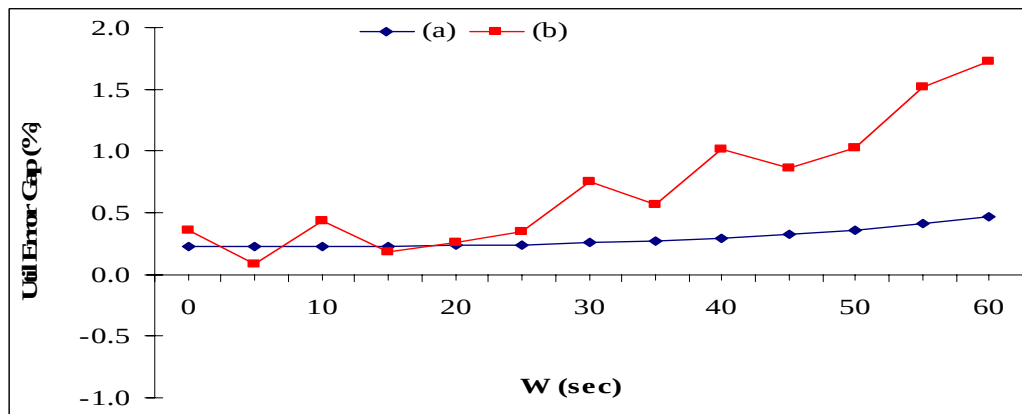
(a) Low to Average Utilization;



(b) Average to High Utilization;

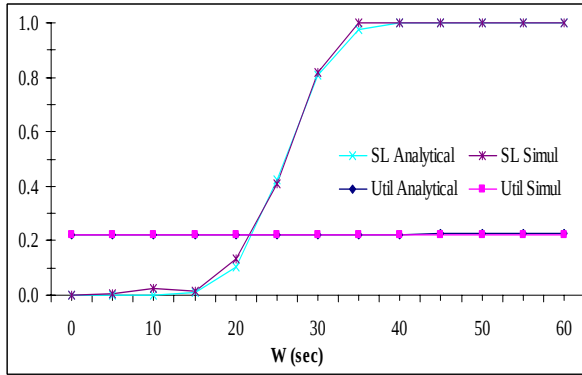


(c) SL Relative Error Gap, %;

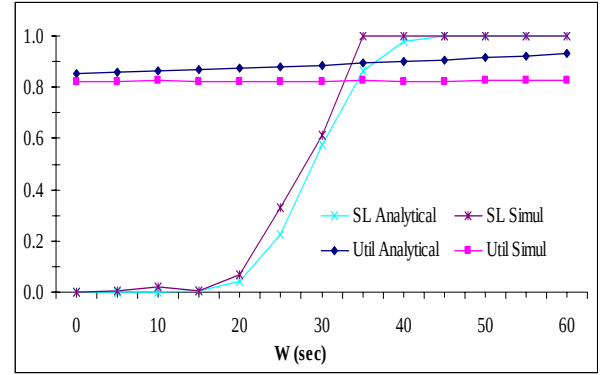


(e) Utilization Relative Error Gap, %;

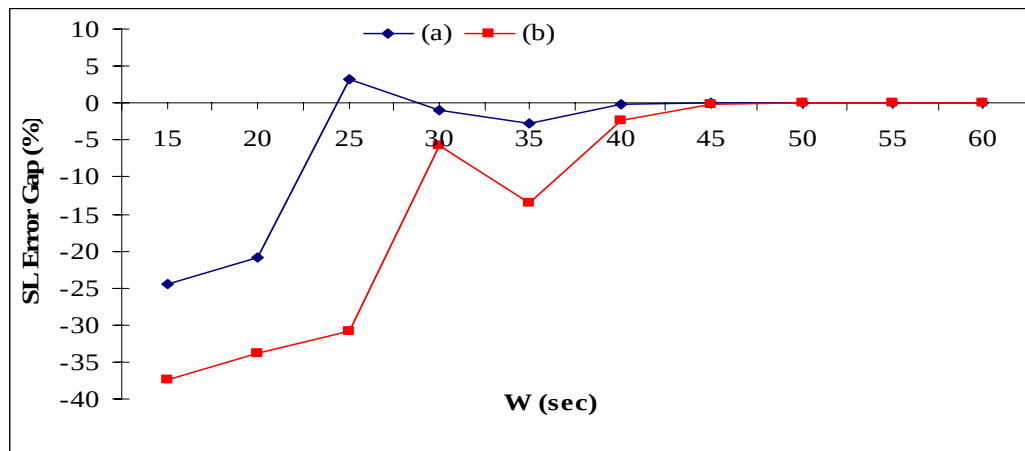
Figure 9.12 System with $B=10$, $M=2$, and $EN=3$.



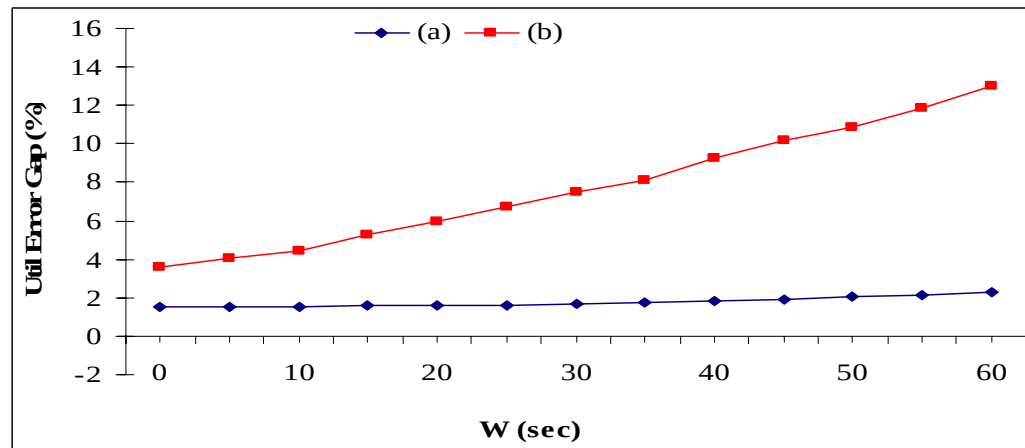
(a) Low to Average Utilization;



(b) Average to High Utilization;

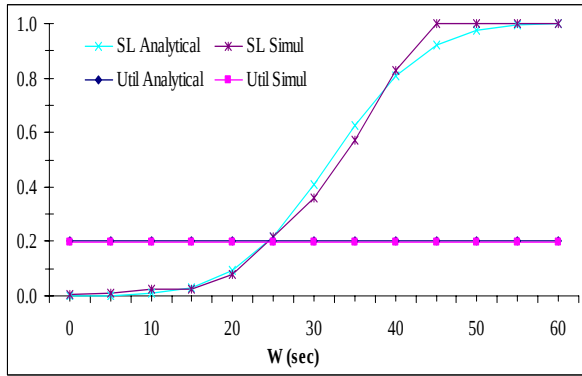


(c) SL Relative Error Gap, %;

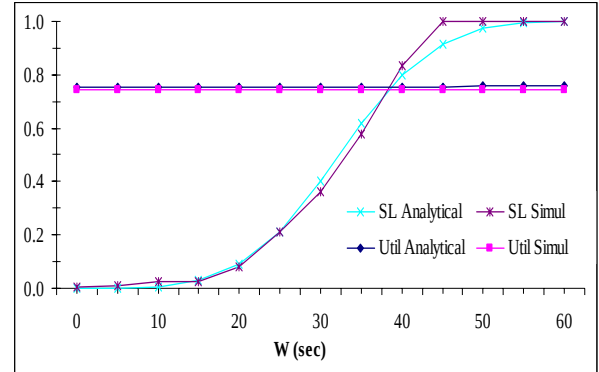


(d) Utilization Relative Error Gap, %;

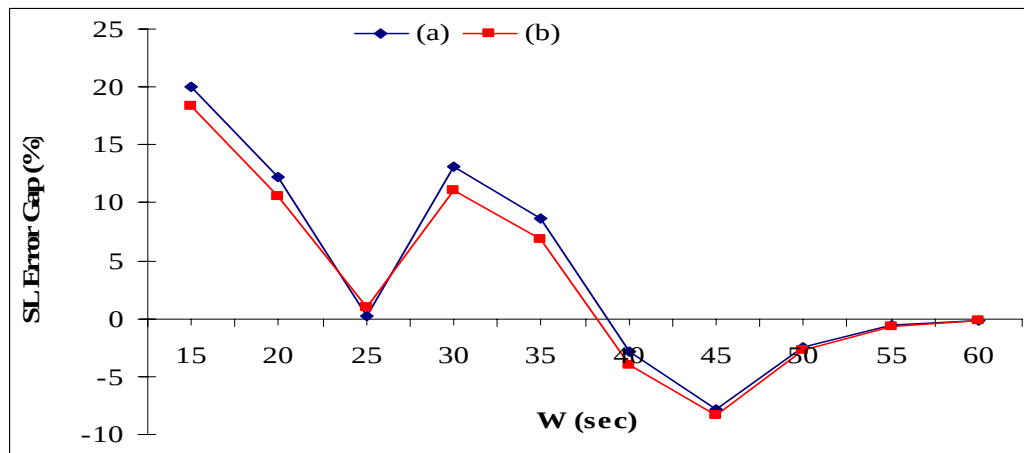
Figure 9.13 System with $B=10$, $M=5$, and $EN=3$.



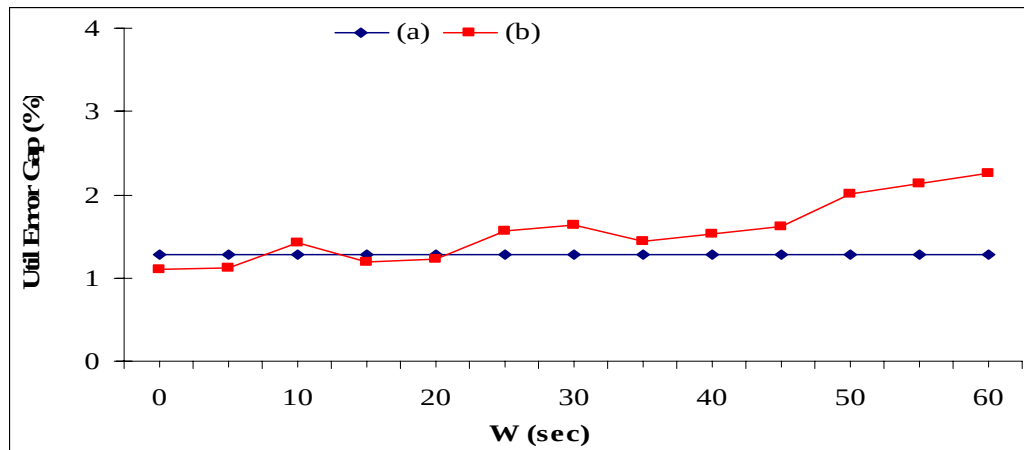
(a) Low to Average Utilization;



(b) Average to High Utilization;

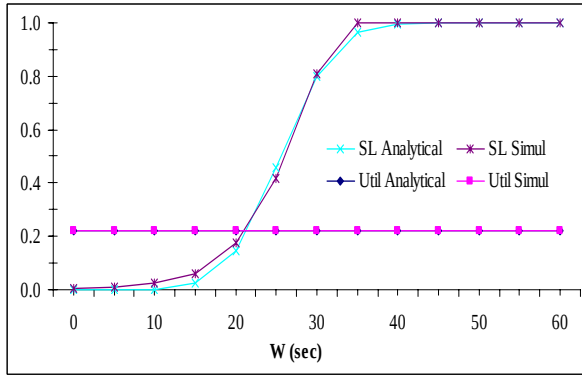


(c) SL Relative Error Gap, %;

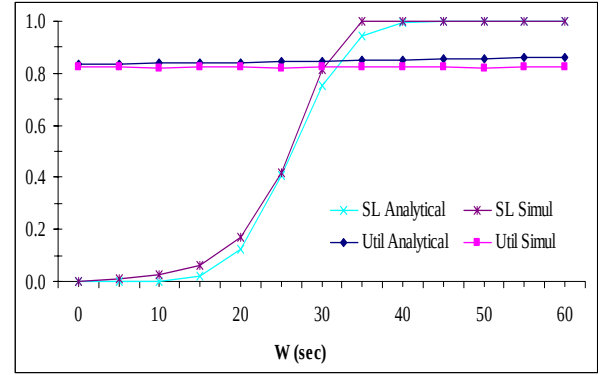


(d) Utilization Relative Error Gap, %;

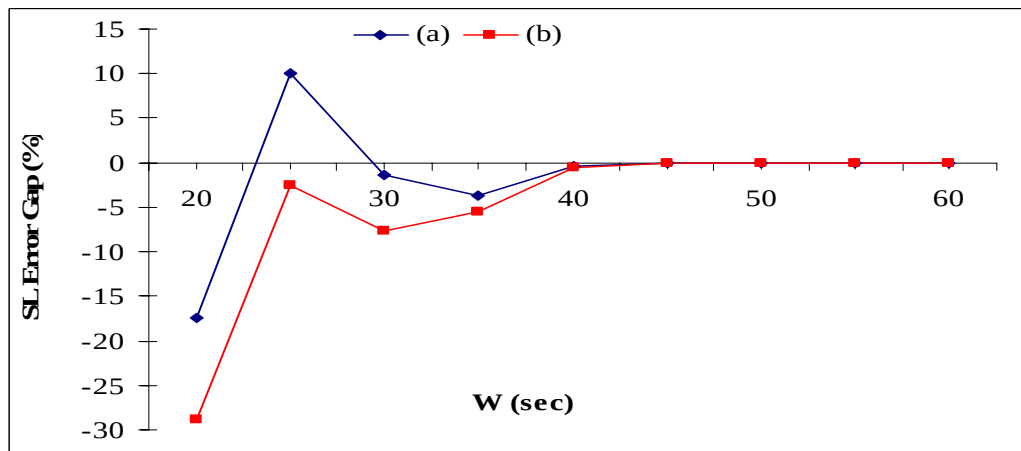
Figure 9.14 System with $B=15$, $M=3$, and $EN=3$.



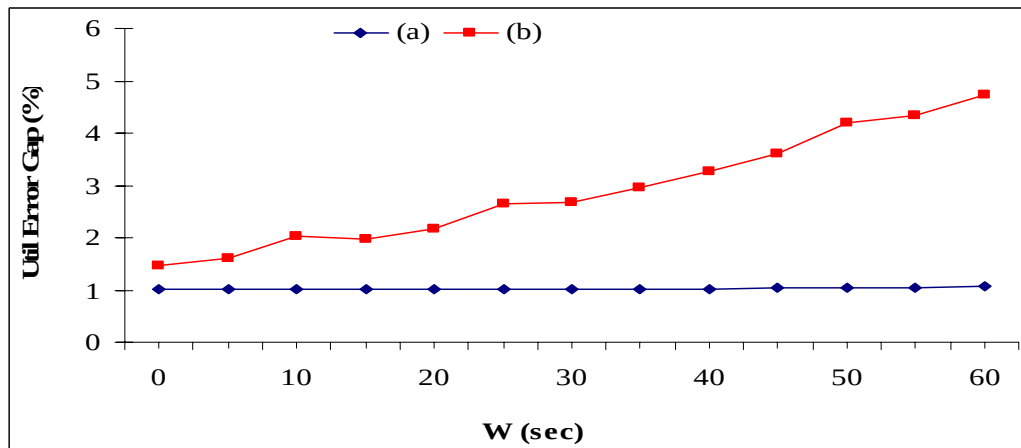
(a) Low to Average Utilization:



(b) Average to High Utilization;

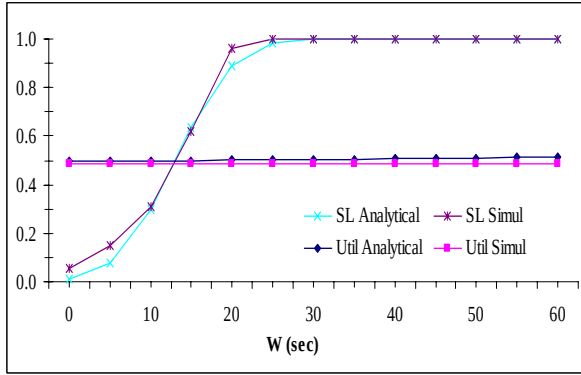


(c) SL Relative Error Gap, %;

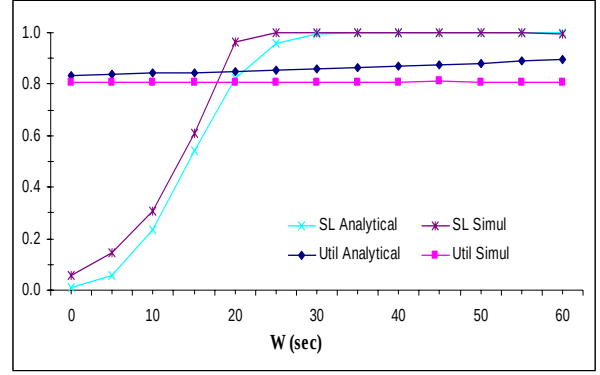


(d) Utilization Relative Error Gap, %;

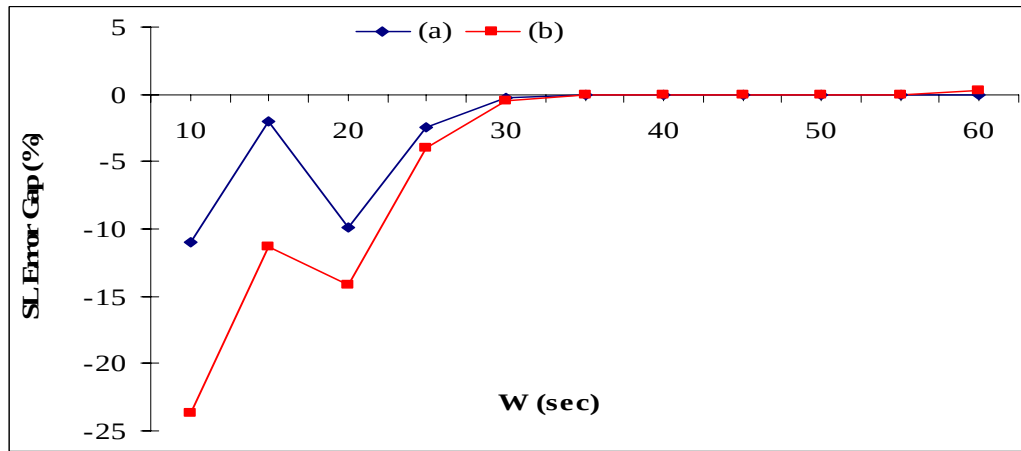
Figure 9.15 System with $B=15$, $M=5$, and $EN=3$.



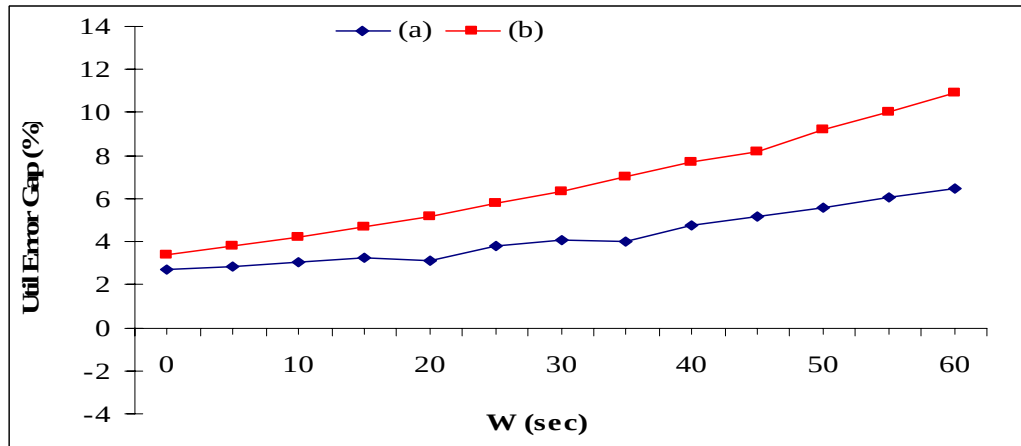
(a) Low to Average Utilization;



(b) Average to High Utilization;

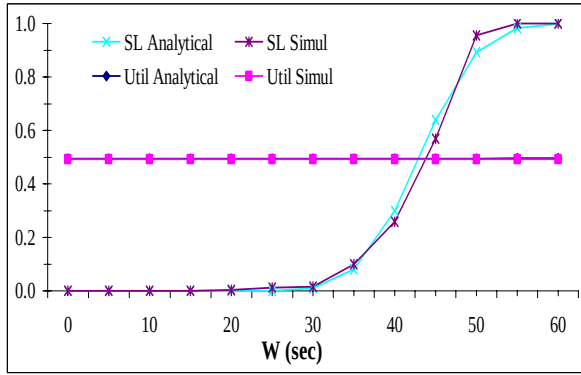


(c) SL Relative Error Gap, %;

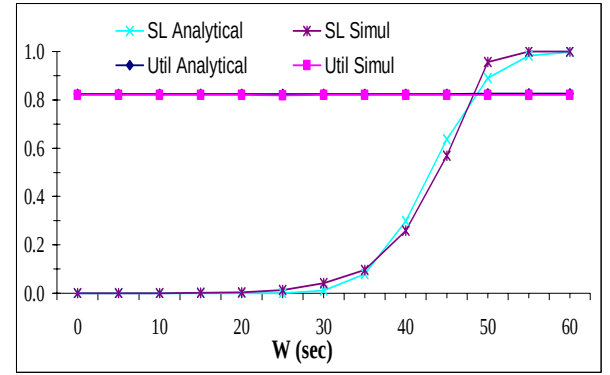


(d) Utilization Relative Error Gap, %;

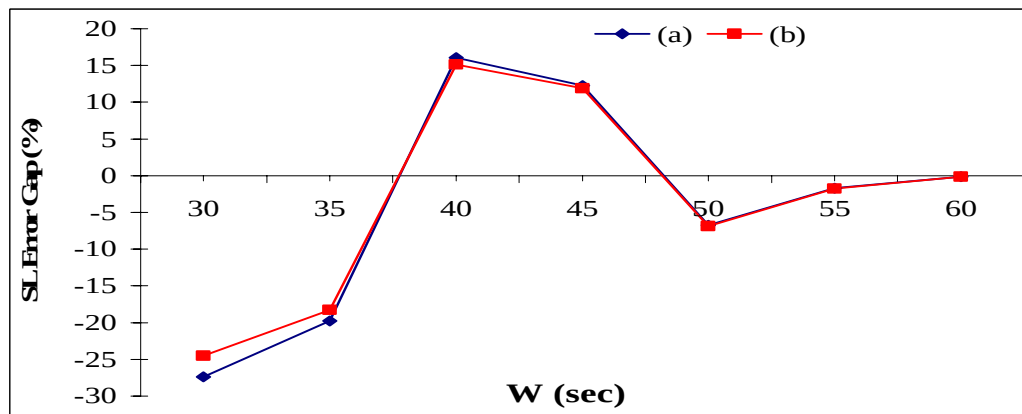
Figure 9.16 System with $B=15$, $M=7$, and $EN=2$.



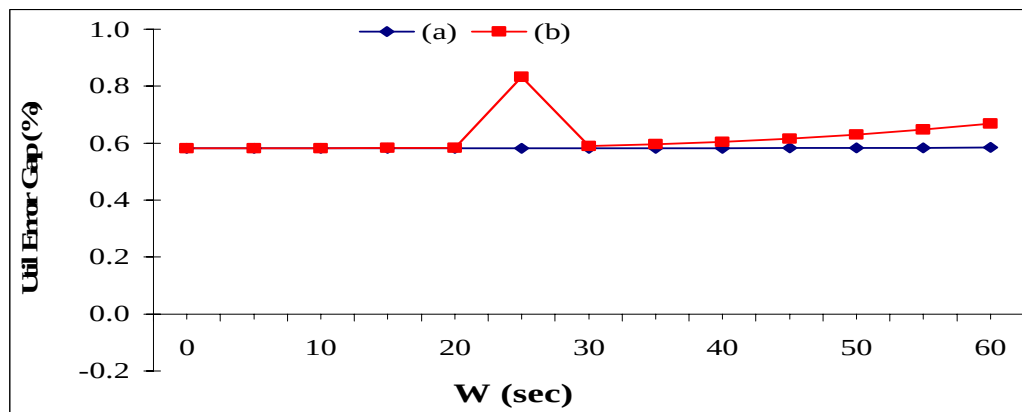
(a) Low to Average Utilization;



(b) Average to High Utilization;



(c) SL Relative Error Gap, %;



(d) Utilization Relative Error Gap, %;

Figure 9.17 System with $B=15$, $M=2$, and $EN=7$.

9.3 Conclusion

The analytical models for utilization and service level were tested on a data set obtained from a simulation models. The validation of the model was conducted through a comprehensive set of experiments over a wide range of values for the different pick-to-buffer order picking systems that might influence the accuracy of the analytical results.

The comparison results indicated that the analytical model performs very well for estimating the picker's utilization and is reasonably accurate for estimating the service level. Thus we can make a statement that the analytical model is a very valuable tool for designers in respect to many factors. First, the analytical model produces very satisfactory approximation results. Second, the designers save time and money on developing and executing simulation models to determine best design possible.

CHAPTER 10

CONCLUSIONS AND FUTURE WORK

10.1 Current Research

This research is concentrated on the initial design of innovative high volume small parts order picking systems using pick buffers and organized in zones which are connected by a conveyor. Customer order containers travel from zone to zone on the conveyor, and items are deposited automatically from the pick buffers. In this thesis, pick zones are studied in isolation. Within every zone, two types of operations are distinguished: item picking and order assembly. A tandem queuing model incorporating both operations is developed.

The solution methodology proposed for the particular queuing model developed to represent pick-to-buffer order picking systems uses a decomposition strategy. To analyze the two sub-queues, existing approximation methods from the research literature have been used. In order to account for interactions between the two sub-queues, a forward and reverse coupling methodology has been implemented. In that methodology, the forward coupling parameters of influence are mean and squared coefficient of variation of interdeparture times from the upstream sub-queue. The reverse coupling step is necessary since variability and limited capacity of the second sub-queue causes blocking in the first sub-queue. The reverse coupling is accomplished by adjusting service processes of the first sub-queue based on the analysis of the second sub-queue, specifically, the blocking probability. The results of analytical approximation of the given tandem queuing system are evaluated using simulation. The solution methodology provides satisfactory approximation.

Experimental comparisons of the values computed for utilization and service level using both the analytical models and a detailed simulation. were A comprehensive set of experiments varied the parameters which might influence the accuracy of the analytical models. The analytic results were consistent with the simulation results with reasonable error gaps.

The main contributions of the thesis work are the following:

- To our knowledge, this is the first analysis work on design and analysis of small parts high volume order picking systems with a pick-to-buffer configuration. The novelty in the modeling approach is the distinct separation of order-picking operations (picking and assembly) which permits the development of performance model for both throughput and service level.
- A novel stochastic model has been developed to estimate pick time of two specific types of pick-to-buffer systems. The model incorporates randomness in several dimensions: order size, item location and buffer location. Results of the estimation compare very well with that of simulation.
- To assess the performance parameters of pick and assembly operations of the system a tandem queuing model is constructed. To approximate performance parameters an adaptation of existing approximation methodologies is implemented.
- The approximation of a tandem queuing model requires estimation the squared coefficient of interdeparture times from the $G^X/G/1$ queuing model which is presented in this thesis. To our knowledge this is the first model to estimate squared coefficient of interdeparture times from the $G^X/G/1$ queuing system.

This research demonstrates that good analytical models can be developed for use in the design of pick-to-buffer systems. The designers must evaluate trade-offs in the system. With the increase of time until the order container enters the zone, W , the service level increases. However, in this situation the buffers become full faster and blocking increases which in turn affects the picker utilization. The utilization of a picker increases due to extra blocking time occurring in the system.

The above analysis becomes even more critical when the study of zones that are located at the end of the order picking system is carried out. The time from order release until the container enters the zone is longer in comparison to the first zone. During these times a picker receives information about arriving order contents and starts picking them into the buffers. Since time between order release and the container entering the zone are long the buffers will get filled up long before order containers arrive to the zone. Once buffers are filled to capacity, the picker waits for an order container arrival to free up a buffer. As soon as one buffer opens up the picker travels to that buffer to place an item. With this strategy, the pick time of a picker increases as a result of two factors: a) more chaotic (not following a predefined pick strategy) behavior of placing items into buffers is observed; b) picker blocking time is higher due to increased waiting for buffers to free up. Here buffers and pickers have higher utilization than those of zones in the beginning of the order picking system. This can be explained by the increase of utilization due to chaotic behavior. One way to reduce the travel time and utilization of a picker is to supply information about orders to be picked relatively late and design each zone as if it is the first zone of the order picking system.

Another design issue to consider is the dependency of service level and utilization on the ratio of expected entities to pick in a batch to the number of buffers in the system. When the number of buffers is fixed and the number of entities to pick in the batch increases, the service level decreases because more entities must be picked during time W . Also, in this case the utilization of a picker increases because of increasing blocking of buffers, i.e. more entities occupy buffers during time W .

10.2 Future Research

There are a number of issues to be pursued in future research.

An important problem for future research is to use the analytical models described in this thesis to develop a design model which can be efficiently solved to provide near optimal solutions. The challenge in developing a design model is that it must be modeled from the overall system perspective, and combine performance of every zone into one model.

One of the assumptions of the travel time models in this dissertation is that the items are spread uniformly along the pick area. The analysis on different assumptions on items layout along the pick area can be also carried out, e.g. allocation of items within the zone according to ABC classification. The assumption on different item allocations might make very hard to derive a travel time model. There are many mathematical manipulations, including multiple integrations and derivations that might not produce closed forms for expected values and variances of travel times. Then efficient approximations for travel models must be implemented.

At the same time different operational policies for pickers travel can be also explored. This is very important since in the early design stage there is a great uncertainty about how

the order picking systems are going to be operated. This can also be a subject of future research. Similar to the case of item allocations, different operational policies might make impossible to derive travel time models with closed form expressions.

Another problem worth future investigation is to consider different definitions of service level and develop new model for the problem. A service level in this thesis is defined as a probability of a batch of orders being completely picked before appropriate order containers enter the zone. An improved definition to the service level can be the probability an order is picked when the appropriate order containers leave the zone. There is a great deal of stochastic uncertainty in the systems studied, which complicates tracking of entities through the system. The synchronization issue between a picker and a flowing on a conveyor container must be taken into consideration in order to develop other models for service level.

For the future work the investigation on the optimal shape factor of the rack (applicable to robotic picking) can be carried out. The optimal travel time can be found by minimizing a total travel time to pick an item, i.e. sum of expected loaded and empty travel times. To do so a derivative of the expected travel time with respect to shape factor can be taken, later by setting the result equal to zero, and solving for the value of a shape factor.

APPENDIX A

MEAN AND SCV OF SUMS

Let

$$T = \sum_{k=1}^X S_k, \quad \text{A.1}$$

where both X and S are random variables with known mean and squared coefficient of variation.

Proposition: The mean and the squared coefficient of variation of service times S^* are then

$$E[T] = E[X]E[S] \quad \text{A.2}$$

and

$$C_T^2 = C_X^2 + \frac{1}{E[X]} C_S^2 \quad \text{A.3}$$

Proof: By definition

$$C_T^2 = \frac{\text{Var}(T)}{(E[T])^2} \quad \text{A.4}$$

Where from A.1 $E[T]$ can be written as

$$E[T] = E\left[\sum_{k=1}^X S_k\right] = E[X]E[S] \quad \text{A.5}$$

On the other hand again using A.2 we can write,

$$\text{Var}(T) = \text{Var}\left(\sum_{k=1}^X S_k\right) = E\left[\left(\sum_{k=1}^X S_k\right)^2\right] - \left(E\left[\sum_{k=1}^X S_k\right]\right)^2 \quad \text{A.6}$$

where

$$\begin{aligned}
E\left[\left(\sum_{k=1}^X S_k\right)^2\right] &= E\left[E\left[\left(\sum_{k=1}^X S_k\right)^2 \middle| X = j\right]\right] = \sum_{j=0}^{\infty} E\left[\left(\sum_{k=1}^X S_k\right)^2 \middle| X = j\right] P\{X = j\} = \\
&= \sum_{j=0}^{\infty} \left(\text{Var}\left(\sum_{k=1}^j S_k\right) + \left(E\left[\sum_{k=1}^j S_k\right]\right)^2 \right) P\{X = j\} = \sum_{j=0}^{\infty} \left(j \text{Var}(S) + j^2 (E[S])^2 \right) P\{X = j\} = \quad \text{A.7} \\
&= \text{Var}(S) \sum_{j=0}^{\infty} j P\{X = j\} + (E[S])^2 \sum_{j=0}^{\infty} j^2 P\{X = j\} = \text{Var}(S) E[X] + (E[S])^2 E[X^2]
\end{aligned}$$

Substituting A.7 and A.5 into A.6 we get

$$\begin{aligned}
\text{Var}(T) &= \text{Var}\left(\sum_{k=1}^X S_k\right) = E\left[\left(\sum_{k=1}^X S_k\right)^2\right] - \left(E\left[\sum_{k=1}^X S_k\right]\right)^2 = \\
&= \text{Var}(S) E[X] + (E[S])^2 E[X^2] - (E[X])^2 (E[S])^2 = \quad \text{A.8} \\
&= \text{Var}(S) E[X] + (E[S])^2 (E[X^2] - (E[X])^2) = \\
&= \text{Var}(S) E[X] + (E[S])^2 \text{Var}(X)
\end{aligned}$$

Then using A.8 the equation A.3 becomes

$$\begin{aligned}
C_T^2 &= \frac{\text{Var}(T)}{(E[T])^2} = \frac{\text{Var}(S) E[X] + (E[S])^2 \text{Var}(X)}{(E[X])^2 (E[S])^2} = \\
&= \frac{\text{Var}(S) E[X]}{(E[X])^2 (E[S])^2} + \frac{(E[S])^2 \text{Var}(X)}{(E[X])^2 (E[S])^2} = \quad \text{A.9} \\
&= \frac{\text{Var}(S)}{E[X] (E[S])^2} + \frac{\text{Var}(X)}{(E[X])^2} = \frac{C_S^2}{E[X]} + C_X^2
\end{aligned}$$

APPENDIX B

RANDOM DISTRIBUTION

(a) Bounded Distributions: Discrete Uniform and Zero Truncated Binomial

Zero Truncated Binomial Distribution

The probability mass function of a zero truncated binomial random variable having parameters (n,p) is given by

$$p(x) = \begin{cases} \binom{n}{x} \frac{p^x (1-p)^{n-x}}{1-(1-p)^n} & \text{for } x=1,2,\dots,n \\ 0 & \text{otherwise} \end{cases} \quad (\text{B.1})$$

The mean and variance of a zero truncated binomial distribution are given in

$$E(X) = \frac{np}{1-(1-p)^n} \quad (\text{B.2})$$

and

$$\text{Var}(X) = \frac{np(1-p-(1-p)^n(1-p+np))}{(1-(1-p)^n)^2} \quad (\text{B.3})$$

The values $E[1/X]$ and $E[1/X^2]$, which are given in (B.4) and (B.5), respectively do not reduce to a simple form, however they are not very hard in computations.

$$E[1/X] = \frac{(1-p)^{n-1} np \sum_{k=0}^{n-1} \frac{p^k}{(k+1)! (k+1)^2 (1-p)^k} \prod_{s=1}^k (s-n)}{1-(1-p)^n} \quad (\text{B.4})$$

and

$$E[1/X^2] = \frac{(1-p)^{n-1} np \sum_{k=0}^{n-1} \frac{p^k}{(k+1)!(k+1)^2 (1-p)^k} \prod_{s=1}^k (s-n)}{1-(1-p)^n} \quad (\text{B.5})$$

For the analysis purposes it is required to estimate the probability parameter p from (B.2). Since p cannot be written in a closed form, a numerical method is required to obtain the value. A simple method suggested by Newton-Raphson method (Johnson and Kotz 1972) yields a good estimator, $\bar{p}$, which is computationally not intensive.

Newton-Raphson Method for $\bar{p}$

Step 1. Set $i=1$

$$p_i = \frac{E[X]}{n}$$

Step 2. Set $i = i+1$

$$p_{i+1} = p_i - \frac{np_i - E[X] + E[X](1-p_i)^n}{n - nE[X](1-p_i)^{n-1}}$$

Step 3. IF $p_{i+1} = \left(\frac{np_i}{1-(1-p_i)^n} - E[X] \right) / E[X] < \varepsilon$ **THEN STOP**

ELSE GOTO STEP 2

It has been proved that Newton-Raphson Method monotonically converges to p .

However, for a distribution where $E[X] > 2.5$ the following approximation can be used instead of a numerical method.

$$\bar{p} = p^0 \frac{1-(1-p^0)^n}{1-E[X](1-p^0)^{n-1}} \quad (\text{B.6})$$

where p^0 is the probability parameter for the untruncated binomial distribution

$$p^0 = \frac{E[X]}{n} \quad (\text{B.7})$$

A generation form of the zero truncated binomial distribution can be obtained by the rejection method of the lower bound of a binomial distribution, (Law and Kelton 2000). A generation form is the following:

Step 1. Generate binomial random variate X_i ;

Step 2. IF $X_i > 0$ THEN Accept

ELSE Reject

Discrete Uniform Distribution

The random variable is said to be discretely uniformly distributed over the interval [a,b] if its probability mass function is given by

$$p(x) = \begin{cases} \frac{1}{b-a+1} & \text{for } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases} \quad (\text{B.8})$$

The mean and variance of a discrete uniform distribution are given in

$$E[X] = \frac{a+b}{2} \quad (\text{B.9})$$

and

$$\text{Var}[X] = \frac{(b-a+1)^2 - 1}{12} \quad (\text{B.10})$$

Throughout this thesis, in some approximations it will be required to use the values of $E[1/X]$ and $E[1/X^2]$ which are given in (B.11) and (B.12), respectively.

$$E\left[\frac{1}{X}\right] = \frac{\sum_{k=1}^b \frac{1}{k} - \sum_{k=1}^{a-1} \frac{1}{k}}{b-a+1} \quad (\text{B.11})$$

$$E\left[\frac{1}{X^2}\right] = \frac{\psi'(a) - \psi'(b) - \frac{1}{b^2}}{b-a+1} \quad (\text{B.12})$$

where $\psi'(z)$ is a Trigamma function $\psi'(z) = \sum_{k=0}^{\infty} \frac{1}{(k+z)^2}$

An asymptotic expansion in of a Ttrigamma function has a form

$$\psi'(z) \cong \frac{1}{z} + \frac{1}{2z^2} + \frac{1}{6z^3} - \frac{1}{30z^5} + \frac{1}{42z^7} - \frac{1}{30z^9} + \dots \quad (\text{B.13})$$

Note, $\psi'(1) = \frac{\pi^2}{6}$

(b) Order Size

In this section we determine the distribution of orders within a zone. Let N be the pick quantity for a single order with probability mass function $P(N=n) = a_n$, $n=0, 1, 2, \dots, u$. In one customer order each line item can be allocated in different zones, as such we are interested in determining the distribution of a customer order within a single zone.

For a given quantity of items to be picked within an order, we assume that each item has a probability p_i that is found in the i th zone. Then the number of units of items to be picked at each zone given N units of items to be picked follows multinomial distribution, (B.14). We can use a random variable N_i to indicate the number of times outcome number i was observed over the N trials, i.e. number of items to be picked in zone i out of total of N picks.

$$P\{N_1 = n_1, N_2 = n_2, \dots, N_Z = n_Z | N\} = \binom{N}{n_1, n_2, \dots, n_Z} p_1^{N_1} p_2^{N_2} \dots p_Z^{N_Z} \quad (\text{B.14})$$

Note, $\sum_{i=1}^Z p_i = 1$, $\sum_{i=1}^Z n_i = N$, $E[N_i] = Np_i$ and $\text{Var}(N_i) = Np_i(1-p_i)$, for $i=1 \dots Z$. N_i is distributed binomially with parameters $N_i \sim \text{Binomial}(n_i, p_i)$, for $i=1 \dots Z$. At the uniform demand rates at each zone $p_i = 1/Z$.

Since number of picks in an order, N , is a random variable generated from a discrete distribution that is bounded from above by the value u , then

$$P\{N_1 = n_1, N_2 = n_2, \dots, N_Z = n_Z\} = \sum_{n=1}^u P\{N_1 = n_1, N_2 = n_2, \dots, N_Z = n_Z | N\} P\{N = n\} \quad (\text{B.15})$$

Note, $E[N_i] = E[N]p_i$ and $\text{Var}(N_i) = E[N]p_i(1-p_i) + \text{Var}(N)(p_i)^2$, for $i=1 \dots Z$.

Incoming orders are batched. Batching is done by grouping a fixed number, M , of orders. We use well-known property of the binomial distribution: assume $N_j \sim B(n_j, p)$, $j=0 \dots M$, are independent binomial variables, then $\sum_{j=1}^M N_j$ is again a binomial variable; its

distribution is $\sum_{j=1}^M N_j \sim B\left(\sum_{j=1}^M n_j, p\right)$.

APPENDIX C

DERIVATIONS OF PICK TIMES

(a) Distributions of Loaded and Empty Travels of Robotic Order Pickers

Let t_L and t_E be the travel times for a single pick of a loaded and an empty moves, respectively. The distributions of t_L and t_E are of our interest. Then,

$$F^E(z) = P\{t_E \leq z\} = P\{\max(|X_i - X_i^b|, |Y_i - Y_i^b|) \leq z\} \quad \text{C.1}$$

$$F^L(z) = P\{t_L \leq z\} = P\{\max(|X_{i+1}^b - X_i|, |Y_{i+1}^b - Y_i|) \leq z\} \quad \text{C.2}$$

where (X_i, Y_i) is a random point in a picking area and (X_i^b, Y_i^b) is a random point in a buffer area (see Figure 4.1). Assuming X and Y coordinates are independently generated, $Y_i^b = 0$ and $X_{i+1}^b = X_i$, the cumulative distribution functions C.1 and C.2 can be rewritten in the following forms,

$$F^E(z) = P\{|X_i - X_i^b| \leq z\} P\{|Y_i - Y_i^b| \leq z\} = F_{|X_i - X_i^b|}^E(z) F_y(z) \quad \text{C.3}$$

$$F^L(z) = P\{Y_i^b \leq z\} = F_y(z) \quad \text{C.4}$$

In general, a distribution function of the absolute value can be written as,

$$F_{|Z|}(z) = P\{|Z| \leq z\} = P\{-z \leq Z \leq z\} = P\{Z \leq z\} - P\{Z \leq -z\} = F_Z(z) - F_Z(-z) \quad \text{C.5}$$

Then, equations C.3 becomes

$$F^E(z) = \left(F_{X_{i+1}^b - X_i}^E(z) - F_{X_{i+1}^b - X_i}^E(-z) \right) F_y(z) \quad \text{C.6}$$

Recall that $Y_i \sim \text{Uniform}(0, b)$, hence

$$F_y(y) = \begin{cases} 0 & \text{for } y < 0 \\ \frac{y}{b} & \text{for } 0 \leq y \leq b \\ 1 & \text{for } y > b \end{cases} \quad \text{C.7}$$

Thus, we are interested in finding distribution of Z_1 and Z_2 shown in C.8 and C.9, i.e. the difference of the i -th order statistics for $X_{(i)}$ and $X_{(i)}^b$ for an empty travel and Y_i for a loaded travel.

$$Z_1 = X_{(i)} - X_{(i)}^b \quad \text{C.8}$$

$$Z_2 = Y_i \quad \text{C.9}$$

In general, if random variables R and S are independent then the distribution function of their difference is the convolution of their distributions.

$$f_{R-S}(z) = \int_{-\infty}^{\infty} f_R(z+x) f_S(x) dx \quad \text{C.10}$$

Also, the density function, $f_{X_{(i)}}(x)$, of the i -th order statistics for $X_{(i)}$, when total of k points are ordered, is given by C.11 .

$$f_{X_{(i)}}(x) = \frac{k!}{(k-i)!(i-1)!} [F(x)]^{i-1} [1-F(x)]^{k-i} f(x) \quad \text{C.11}$$

Recall that $X_i, X_i^b \sim \text{Uniform}(0, 1)$, hence

$$f(x) = f_b(x) = \begin{cases} 1 & \text{for } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{C.12}$$

and

$$F(x) = F_b(x) = \begin{cases} 0 & \text{for } x < 0 \\ x & \text{for } 0 \leq x \leq 1 \\ 1 & \text{for } x > 1 \end{cases} \quad \text{C.13}$$

Since all points are uniformly distributed, the results will hold for any i value. Hence, without loss of generality, set $i=n$, i.e. consider the last pick. Note, that there are totally n item locations and $(n+1)$ buffer locations (n buffers to pick items into and one buffer, $X_{(1)}^b$, representing the location of the last buffer of the previous batch). Taking into account this assumption and using equations C.12 and C.13, density functions of the n -th order statistics in C.11 can be written as:

$$f_{X_{(n)}}(x) = f_{X_{(n)}^b}(x) = \begin{cases} nx^{n-1} & \text{for } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{C.14}$$

Plugging in C.14 into C.10 we get the density functions of Z_I presented in C.15

$$f_{Z_1}^E(z_1) = \int_0^1 f_{X_{(n)}^b}(z_1 + x) f_{X_{(n)}}(x) dx = \int_0^1 n(z_1 + x)^{n-1} nx^{n-1} dx \quad \text{C.15}$$

Distribution Function of an Empty Travel

Since $-1 \leq z \leq 1$ and $0 \leq z + x \leq 1$ (i.e. $-z \leq x \leq 1-z$), we consider different cases for z , the equation C.15 becomes:

$$f_Z^E(z) = \begin{cases} \int_{-z}^1 n(z+x)^{n-1} nx^{n-1} dx & z \in [-1, 0] \\ \int_0^{1-z} n(z+x)^{n-1} nx^{n-1} dx & z \in [0, 1] \\ 0 & \text{otherwise} \end{cases} \quad \text{C.16}$$

which after integration and simplifications becomes

$$f_z^E(z) = \begin{cases} \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{n^2}{n+k} z^{n-k-1} \left(1 - (-1)^{k+n} z^{k+n}\right) & z \in [-1, 0] \\ \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{n^2}{n+k} \sum_{m=0}^{n+k} \binom{n+k}{m} (-1)^m z^{n+m-k-1} & z \in [0, 1] \\ 0 & \text{otherwise} \end{cases} \quad \text{C.17}$$

Integrating C.17 we get a distribution function of Z_1

$$F_Z^E(z) = \begin{cases} 0 & z \in (-\infty, 0) \\ \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{n^2}{n+k} \left(\frac{z^{n-k} - (-1)^{n+k}}{n-k} + \frac{(-1)^{n+k} - (-1)^{n+k} z^{2n}}{2n} \right) & z \in [-1, 0] \\ \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{n^2}{n+k} \left(\sum_{m=0}^{n+k} \binom{n+k}{m} (-1)^m \frac{z^{n+m-k}}{n+m-k} \right) + \frac{(-1)^{n+k}}{2n} - \frac{(-1)^{k+n}}{(n-k)} & z \in [0, 1] \\ 1 & z \in (1, \infty) \end{cases} \quad \text{C.18}$$

Then a distribution function of the absolute value of two x-coordinates given in C.5 is

$$F_{|X-X^b|}^E(z) = \begin{cases} 0 & z \in (-\infty, 0) \\ \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{n^2}{n+k} \left(\sum_{m=0}^{n+k} \binom{n+k}{m} (-1)^m \frac{z^{n+m-k}}{n+m-k} \right) + \frac{(-1)^{k+n} z^{2n}}{2n} - \frac{(-1)^{n+k} z^{n-k}}{n-k} & z \in [0, 1] \\ 1 & z \in (1, \infty) \end{cases} \quad \text{C.19}$$

Finally, using C.3 a distribution function of a loaded travel is given by,

$$F^E(z) = \begin{cases} 0 & z \in (-\infty, 0) \\ \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{n^2}{b(n+k)} \left(\sum_{m=0}^{n+k} \binom{n+k}{m} (-1)^m \frac{z^{n+m-k+1}}{n+m-k} \right) + \frac{(-1)^{k+n} z^{2n+1}}{2n} - \frac{(-1)^{n+k} z^{n-k+1}}{n-k} & z \in [0, b] \\ \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{n^2}{n+k} \left(\sum_{m=0}^{n+k} \binom{n+k}{m} (-1)^m \frac{z^{n+m-k}}{n+m-k} \right) + \frac{(-1)^{k+n} z^{2n}}{2n} - \frac{(-1)^{n+k} z^{n-k}}{n-k} & z \in [b, 1] \\ 1 & z \in (1, \infty) \end{cases} \quad \text{C.20}$$

Differentiating C.20 and simplifying probability density function $f^L(z)$ can be written as

$$f^E(z) = \begin{cases} \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{n^2}{b(n+k)} \left(\sum_{m=0}^{n+k} \binom{n+k}{m} (-1)^m \frac{(n+m-k+1) z^{n+m-k}}{n+m-k} \right) + \frac{(-1)^{k+n} (2n+1) z^{2n}}{2n} - \frac{(-1)^{n+k} (n-k+1) z^{n-k}}{n-k} & z \in [0, b] \\ \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{n^2}{n+k} \left(\sum_{m=0}^{n+k} \binom{n+k}{m} (-1)^m \frac{z^{n+m-k-1}}{n+m-k-1} \right) + (-1)^{k+n} z^{2n-1} - (-1)^{n+k} z^{n-k-1} & z \in [b, 1] \\ 0 & \text{otherwise} \end{cases} \quad \text{C.21}$$

Then using C.22, the first moment of the pick time of the loaded move from a pick location to a buffer location when total number of picks is n , becomes C.23,

$$E(S) = \int_{-1}^1 z f(z) dz \quad \text{C.22}$$

or

$$E(S_{En}) = \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{2(-1)^{n+k} n^2}{(n+k)} \left(\frac{b^{2n+1}}{4n(n+1)(2n+1)} - \frac{b^{n-k+1}}{(n-k)(n-k+1)(n-k+2)} - \frac{(n+k)}{(n-k+1)(2n+1)} \right) \quad \text{C.23}$$

Finally, for computing the second moment of the pick time of the loaded move from a pick location to a buffer location when total number of picks is n is given by C.24.

$$E(S_{En}^2) = \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{(-1)^{n+k} n^2}{(n+k)} \left(\frac{b^{2n+2}}{2n(n+1)(2n+3)} - \frac{2b^{n-k+2}}{(n-k)(n-k+2)(n-k+3)} - \frac{(n+k)}{(n-k+2)(n+1)} \right) \quad \text{C.24}$$

Distribution Function of a Loaded Travel

A loaded travel is performed in y-coordinate direction only. By assumption, location of items are distributed Uniformly within the height of the pick area. Thus, $Y_i \sim U(0, b)$. Note, cumulative distribution function of uniform distribution is given by C.7.

(b) Travel Time of Manual Order Pickers For Centralized Buffers

Let T_{min} and T_{max} be the pick time to the leftmost and rightmost locations from a buffer area, respectively. The distributions of T_{min} and T_{max} are of our interest. Then,

$$F^{\max}(z) = P\{T_{\max} \leq z\} = P\{|X_{\max} - 1/2| \leq z\} \quad C.25$$

$$F^{\min}(z) = P\{T_{\min} \leq z\} = P\{|X_{\min} - 1/2| \leq z\} \quad C.26$$

where $X_{\max} = \max(X_1, X_2, \dots, X_n)$ and $X_{\min} = \min(X_1, X_2, \dots, X_n)$.

In general distributions of minimum and maximum points are given in C.27 and C.28, respectively.

$$F_{X_{\max}}(x) = \prod_{i=1}^n F_{X_i}(x) \quad C.27$$

$$F_{X_{\min}}(x) = 1 - \prod_{i=1}^n (1 - F_{X_i}(x)) \quad C.28$$

Since X_i 's are uniformly distributed over the interval (0,1) then distributions of the minimum and maximum points are the following.

$$F_{X_{\max}}(x) = \begin{cases} 0 & x \leq 0 \\ (x)^n & 0 < x \leq 1 \\ 1 & x > 1 \end{cases} \quad C.29$$

$$F_{X_{\min}}(x) = \begin{cases} 0 & x \leq 0 \\ 1 - (1-x)^n & 0 < x \leq 1 \\ 1 & x > 1 \end{cases} \quad C.30$$

Then the cumulative distribution and density functions of T_{min} and T_{max} are given in C.31 and C.32, respectively.

$$F^{\max}(x) = F^{\min}(x) = \begin{cases} 0 & x \leq 0 \\ (x + 1/2)^n - (1/2 - x)^n & 0 \leq x \leq 1/2 \\ 1 & \end{cases} \quad \text{C.31}$$

and

$$f^{\max}(x) = f^{\min}(x) = \begin{cases} 0 & \text{otherwise} \\ n(1/2 + x)^{n-1} + n(1/2 - x)^{n-1} & 0 \leq x \leq 1/2 \end{cases} \quad \text{C.32}$$

Then using C.32, the first and the second moments of the pick time from a buffer location to a maximum and minimum pick locations when total number of picks is n , becomes C.33 and C.34, respectively.

$$E[T_{\max}] = E[T_{\min}] = \frac{2^{1-n} + n - 1}{2(n+1)} \quad \text{C.33}$$

$$E[T_{\max}^2] = E[T_{\min}^2] = \frac{n^2 - n + 2}{4(n+1)(n+2)} \quad \text{C.34}$$

In order to find total pick time three cases must be considered.

- Minimum and maximum pick locations are in different sides from the pick buffer, i.e.

$$T_{\max} \geq 1/2, T_{\min} \leq 1/2;$$

- Minimum and maximum pick locations are on the right hand side from the pick buffer, i.e. $T_{\min} \geq 1/2$;

- Minimum and maximum pick locations are on the left hand side from the pick buffer, i.e. $T_{\max} \leq 1/2$.

When total number of picks is n the total pick time is as shown in C.35.

$$S_n = \begin{cases} 2T_{\max} + 2T_{\min} & \text{if } T_{\max} \geq 1/2 \text{ and } T_{\min} \leq 1/2 \\ 2T_{\max} & \text{if } T_{\min} \geq 1/2 \\ 2T_{\min} & \text{if } T_{\max} \leq 1/2 \end{cases} \quad \text{C.35}$$

Note, from C.29 and C.30

$$P\{T_{\max} \geq 1/2, T_{\min} \leq 1/2\} = 1 - (1/2)^{n-1} \quad \text{C.36}$$

$$P\{T_{\min} \geq 1/2\} = P\{T_{\max} \leq 1/2\} = (1/2)^n$$

Then the first and the second moments of the total pick time of an order when total number of picks is n , becomes C.37 and C.38, respectively.

$$E[S_n] = 4E[T_{\max}] \left(1 - (1/2)^n\right) \quad \text{C.37}$$

and

$$E[S_n^2] = 8E[T_{\max}^2] \left(1 - (1/2)^n\right) + 8(E[T_{\max}])^2 \left(1 - 2(1/2)^n\right) \quad \text{C.38}$$

After simplifications C.37 and C.38 become C.39 and C.40, respectively.

$$E[S_n] = \frac{2(1 - 2^{-n})(n - 1 + 2^{1-n})}{n + 1} \quad \text{C.39}$$

and

$$E[S_n^2] = \frac{2^{1-3n}(2^n - 2)(2^n n - 2^n + 2)^2}{(n + 1)^2} + \frac{2(1 - 2^{-n})(n^2 - n + 2)}{(n + 1)(n + 2)} \quad \text{C.40}$$

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