OPTIMAL COOPERATIVE AND NON-COOPERATIVE PEER-TO-PEER MANEUVERS FOR REFUELING SATELLITES IN CIRCULAR CONSTELLATIONS

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OPTIMAL COOPERATIVE AND NON-COOPERATIVE PEER-TO-PEER MANEUVERS FOR REFUELING SATELLITES IN CIRCULAR CONSTELLATIONS

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Dedicated to Dadabhai and Jyatha,

who would have been very happy

to see my dissertation.

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SUMMARY

On-orbit servicing (OOS) of space systems provides immense benefits by extending their lifetime, by reducing overall cost of space operations, and by adding flexibility to space missions. Refueling is an important aspect of OOS operations. The problem of determining the optimal strategy of refueling multiple satellites in a constellation, by expending minimum fuel during the orbital transfers, is challenging, and requires the solution of a large-scale optimization problem. The conventional notion about a refueling mission is to have a service vehicle visit all fuel-deficient satellites one by one and deliver fuel to them. A recently emerged concept, known as the peer-to-peer (P2P) strategy, is a distributed method of replenishing satellites with fuel. P2P strategy is an integral part of a mixed refueling strategy, in which a service vehicle delivers fuel to part (perhaps half) of the satellites in the constellation, and these satellites, in turn, engage in P2P maneuvers with the remaining satellites. During a P2P maneuver between a fuel-sufficient and a fuel-deficient satellite, one of them performs an orbital transfer to rendezvous with the other, exchanges fuel, and then returns back to its original orbital position. In terms of fuel expended during the refueling process, the mixed strategy outperforms the single service vehicle strategy, particularly with increasing number of satellites in the constellation. This dissertation looks at the problem of P2P refueling problem and proposes new extensions like the Cooperative P2P and Egalitarian P2P strategies. It presents an overview of the methodologies developed to determine the optimal set of orbital transfers required for cooperative and non-cooperative P2P refueling strategies. Results demonstrate that the proposed strategies help in reducing fuel expenditure during the refueling process.

CHAPTER I

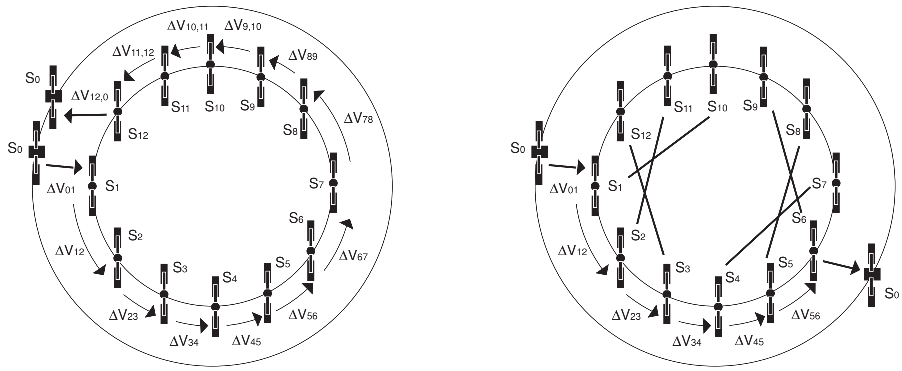
INTRODUCTION

The traditional practice in the space industry has been the development of large and complex monolithic spacecraft, resulting in high costs of overall space operations. In recent times, the need for several small satellites performing the equivalent job of a larger spacecraft has been recognized. Formation flying cluster of satellites provide means of cost reduction and addition of flexibility to space-based programs. Naturally, the areas of formation flight and satellite clusters,[35,68,88] or the more recently proposed fractionated spacecraft architecture[10] have been receiving significant attention. Typically, a spacecraft requires a regular fuel budget for station-keeping and orbital maneuvers. Hence, fuel on-board a spacecraft is one of the important factors in determining the design life-time of the spacecraft. A spacecraft may also encounter different kinds of failure that may degrade the performance of the spacecraft, or even make the spacecraft non-functional. Traditionally, the industry has focussed on replacing a spacecraft at the end of its life-time. However, there has been a growing interest in the new paradigm of on-orbit servicing, and refueling in particular. Capabilities to repair, upgrade, and replenish a spacecraft have immense potential to decrease the cost of overall space operations, and impart flexibility to space-based missions, apart from extending the design lifetime of the satellites. Although there have been several studies on the economic and technological feasibility of servicing missions, there are not enough studies on determining the best way of planning a servicing mission. In this dissertation, we look at this problem. We consider a simple system of a circular constellation of multiple satellites, and determine the best possible way of servicing these satellites. In particular, we address the refueling operation of servicing missions. We will show that even for this simple system, the problem of determination of the best way of refueling the satellites is challenging, as it requires the solution of a large-scale optimization problem. In this introductory chapter, we present a brief overview of our problem of study, literature survey and acquaint the reader with some basic notations.

1.1 Problem Overview

We will consider a system of multiple satellites moving in a circular orbit. These satellites are required to have a minimum amount of fuel. We also have a service-vehicle that can deliver fuel to all these satellites in the constellation. All satellites are required to satisfy the minimum fuel requirement at the end of the refueling process. A refueling mission comprises of several orbital transfers necessary to deliver fuel to the satellites in a constellation. We assume that the service-vehicle and the satellites employ chemical propulsion. We also assume the following information is available for each satellite: mass, initial position, initial fuel content, minimum fuel requirement, and specific thrust of the engine. The radius of the orbit, and the maximum time for the refueling mission, are also specified. Given all this information, we would like to answer the following question: What is the "best possible way" of refueling the system of satellites? By "best possible way," we mean that the fuel expended during all the orbital transfers taking place during refueling has to be a minimum.

The conventional notion of a refueling mission is to have a service vehicle visit the satellites in an optimal sequence and replenish them with fuel. This strategy of refueling satellites is referred to as a single service vehicle (SSV) refueling strategy. Fig 1(a) depicts the SSV refueling strategy, in which the service-vehicle delivers fuel to six satellites in the constellation. The Peer-to-Peer (P2P) refueling strategy provides an alternative way of distributing fuel among the satellites in the constellation, in the absence of a service vehicle. The fundamental concept behind P2P refueling is that



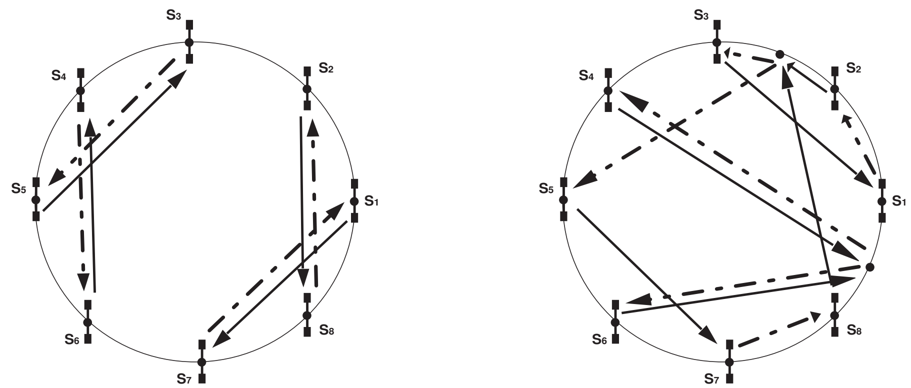
(a) Single Service Vehicle Strategy

(b) Mixed Strategy

Figure 1: Refueling Strategies in a Constellation.

if some of the satellites have more fuel than the remaining ones, then they can share their fuel with those deficient of it by engaging in P2P maneuvers. During a P2P maneuver between a fuel-sufficient and fuel-deficient satellite, one of the satellites performs an orbital transfer to rendezvous with the other, exchanges fuel, and then return to its original orbital position. The satellite which performs the orbital transfer is said to be active, while the satellite which remains in its orbital slot throughout the transfer is said to be a passive. Fig 2(a) depicts the P2P refueling strategy, in which three fuel-sufficient satellites engage in P2P maneuvers with three fuel-deficient satellites in order to exchange fuel. The forward trips are marked by solid arrows, while the return trips are marked by dotted arrows. P2P refueling is an integral part of a mixed refueling strategy, in which the service vehicle refuels part (potentially half) of the satellites in a constellation, and these satellites engage in P2P maneuvers with the remaining satellites in order to distribute the fuel in the constellation. The mixed refueling strategy is depicted in Figure 1(b).

The primary focus of this dissertation is the P2P refueling of satellite constellations. Of particular interest are two extensions of the P2P refueling problem. One of them is the Egalitarian P2P refueling, in which an active satellite is not constrained



(a) P2P Refueling Strategy (b) P2P Maneuvers

Figure 2: P2P Refueling Strategy.

to return to its original orbital position, and can return to any orbital slot originally occupied by a different satellite. The other case is the Cooperative P2P (C-P2P) refueling, in which both satellites, involving in a refueling transaction, are active and engage in a cooperative rendezvous. After the fuel exchange takes place, both satellites return to their original slots. Furthermore, the ideas of E-P2P and C-P2P refueling can be combined into one single Cooperative Egalitarian P2P (CE-P2P) refueling strategy. During a CE-P2P maneuver, both satellites participating in a refueling transaction are active and return to any available orbital position during their return trips. Fig. 2(b) depicts instances of C-P2P, E-P2P, and CE-P2P maneuvers. As before, the forward trips are marked by solid lines, while the return trips are marked by dotted lines. Satellites s_4 and s_6 engage in a C-P2P maneuver, that is, they rendezvous in the slot different from their original locations, exchange fuel, and return to their original locations. Satellites s_5 and s_7 engage in an E-P2P maneuver, in which satellite s_5 rendezvous with the satellite s_7 that stays in its original location, and after the fuel exchange is over, satellite s_7 returns to the slot originally occupied by satellite s_8. Similarly, satellites s_1 and s_3 engage in an E-P2P maneuver, in which the active satellite s_3 returns to its the slot initially occupied by the active satellite s_2. Satellites s_2 and s_8 engage in a CE-P2P maneuver during which the satellites engage in a cooperative rendezvous, and return to orbital slots initially occupied by the active satellites s_3 and s_5.

The primary goal of this dissertation is to develop methodologies to determine the optimal set of orbital transfers that yield the minimum fuel expenditure during P2P, E-P2P, C-P2P, and CE-P2P refueling strategies.

1.2 Literature Survey: On Orbit Servicing

Waltz[84] defines OOS as work done in space by man or machine or by a blend of both, in order to increase the operational life and capabilities of the space assets. OOS operations primarily include on-orbit assembly and maintenance of a space asset, as also replenishment of consumables. In recent years, there have been a growing interest in the OOS paradigm, and several studies have been performed on OOS operations including refueling. Most of these studies have focussed on reviewing the servicing missions that have taken place till date, identifying the benefits offered by servicing, capturing the cost-effectiveness of servicing operations, analyzing different servicing architectures, identifying design modifications necessary for satellites to be considered serviceable, and reviewing key technologies required for servicing.

1.2.1 Motivation: Servicing Missions

Although the current practice in the space industry is to replace spacecrafts after their design lifetime, there have been several instances when on-orbit servicing has proven to be beneficial. The first on-orbit servicing mission can perhaps be traced to a manned mission to SkyLab in 1973, when a substitute heat shield was deployed in the space station, after the original was damaged during launch.[36] Solar Maximum Repair Mission (SMM) in 1984 provides another instance of servicing mission that became necessary after the failure of the Coronagraph Polarimeter and the fuse failures of the

Attitude Control System.[50] Servicing missions were also undertaken for the Russian space station in order to deliver fuel, expendables, and other cargo.[6] The most prominent instance of OOS operations is perhaps the repair of Hubble Space Telescope (HST) in 1993, when a unit was installed in order to compensate for a manufacturing defect in the primary mirror. There have been several servicing missions for the HST after that, leading to an increase in productivity of the HST by 1 to 2 orders of magnitude, and an increase in data output by 2 to 3 times.[44] These missions were all manned missions. Recent efforts of the Defence Advanced Research Projects Agency (DARPA) led to the first demonstration of autonomous servicing in space. DARPA's Orbital Express program demonstrated several key on-orbit servicing technologies like automated rendezvous, transfer of fluid (hydrazine), and robotic arm transfers of Orbital Replacement Unit (ORU) components.[12]

1.2.2 Benefits of OOS operations

The servicing missions for SkyLab, SMM, HST, and the Russian Space Station clearly identifies the benefits of life-time extension and anomaly resolution offered by OOS operations. The study on Spacecraft Modular Architecture Design[65] (SMAD) identified six potential benefits of on-orbit servicing: (1) reduced life-cycle costs, (2) increased payload sensor availability achieved by replacing failed sensors, (3) extended spacecraft orbital lifetime achieved by replenishing consumables like propellant, (4) enhanced spacecraft capabilities achieved by insertion of new technologies, (5) enhanced mission flexibility and operational readiness because refueling capability allows for maneuvers which would otherwise shorten the spacecraft lifetime by high fuel consumption, (6) pre-launch spacecraft integration flexibility offered by a modular architecture of a serviceable spacecraft. A survey of spacecraft failures that could have been corrected by allowing for on-orbit servicing can be found in Ref. 78. Apart from providing the primary benefits of lifetime extension, upgrade, and anomaly resolution, OOS may also be used as a surveillance tool for customers willing to perform an inspection of their assets.[17] Ideas about OOS operations also include the use of space tugs to boost or relocate satellites to desired orbits.[46] The primary objectives of OOS operations can therefore be summarized as: life extension, repair, upgrade, relocate, and inspection.[39,46,71]

Refueling is one of the vital OOS operations, primarily because most satellites have a mission life driven by propellant usage.[17] Provision of refueling capabilities would allow for satellites to be launched with less fuel. This may either mean reduced launch costs, or additional revenue generation by dedicating the volume and mass, previously occupied by excess fuel, to additional payload.[51,83] The designers of Space Based Laser (SBL) have also identified refueling operations to be essential for replenishing an operational chemical laser system.[42,54] Furthermore, refueling capability enables new missions like extremely low-altitude high-drag orbits for Earth observation satellites.[48]

1.2.3 The Economic Perspective

In spite of OOS operations being highly successful in the case of SkyLab, SMM, and HST missions, and the potential benefits that OOS can offer, there are several concerns in the space industry regarding adopting the servicing paradigm. In order for servicing to be practical, both a serviceable spacecraft and a servicer are required. This brings up the primary issue referred to as the 'chicken vs. egg' dilemma.[17] On one hand, why would a manufacturer develop a serviceable spacecraft when there does not exist a servicing infrastructure? On the other hand, why would anyone develop a servicing infrastructure when there does not exist a customer base? Also, the costs associated with the development of a serviceable spacecraft and a servicing infrastructure have been considered too high to justify the acceptance of the OOS paradigm. Because of the prevalent competition among the satellite manufacturers, the focus has been on cutting down development costs, thereby hindering the development of a serviceable satellite.[46] In fact, commercial companies are not likely to adopt the OOS paradigm unless the cost to benefit ratio is substantially decreased.[64]

Several studies looked at the problem of servicing from the perspective of OOS providers and OOS customers.[32,33,46,51,71] An OOS provider would be concerned about the minimum price to charge a client for servicing. In order to determine this price, the OOS provider needs to consider the cost of the servicing mission and the cost of infrastructure development required for servicing operations. On the other hand, an OOS customer would be concerned about the maximum price to pay for a servicing mission. Hence, the customer needs to consider the savings in life-cycle cost offered by the servicing mission, and the value of flexibility offered by a servicing mission. Flexibility refers to the availability of a set of options, from which the customer can chose the one that best answers the uncertainties like changing market requirements.[71] The viewpoints of the provider and client are discussed in Ref. 51, with respect to servicing operations that aid in orbit-raising and station-keeping. The client and provider perspectives for the case of refueling are also discussed in Ref. 46. Different viewpoints of the provider and the client are presented for the case of satellite upgrade in Ref. 32, 33.

The OOS provider would be concerned about the servicing architecture. There is the cost of developing the infrastructure to service satellites. Also, the cost of a servicing operation needs to be estimated, based on the fuel expenditure for all maneuvers (orbital transfers) required for the mission, and the fuel and other requirements of the serviceable client satellites. Typically, in order to avail of OOS operations, client satellites need to be designed for servicing. However, refueling is an operation that presents an unique opportunity to OOS providers. Owing to the high volatility and toxicity of satellites' fuel, the current design and integration practices of satellites allows fuel to be loaded into the satellite just before launch. In other words, the fueling process is not an integral part of satellite's integration process. This implies that it is possible to refuel the currently operating satellites.[48]

From the perspective of the OOS client, servicing operations would allow the satellite operators to address various risks: (1) system failure may be encountered during long life-time of satellites, (2) technology on-board a satellite may become obsolete long before the end-of-life of satellites, (3) the market, that an operational system caters to, may itself become obsolete, leading to loss of revenue-earning capabilities of the system, (4) customer desires may evolve with time leading to change in mission requirements.[32] Regular upgrades sent to satellites via OOS operations can help the operators deal with the technology evolution and changing market requirements. Also, degraded performance of an operational system can be corrected by replacement of the defective module. Although upgrading helps to mitigate the risks mentioned above, there is a risk of the servicing operations itself. This is a key factor in determining the acceptance of the OOS paradigm by the space operators. There are also other factors that concern the operators: how new the upgrades are, and the delay to service. This would depend on whether the modules are kept in on-orbit depots, or launched on demand.[32] Furthermore, from the perspective of an OOS client, refueling capability allows for a satellite to be launched with less amount of fuel, so that the client can chose to launch the satellite with less mass, thereby incurring lesser launch costs. Otherwise, the client can chose to fill up the mass and volume, previously occupied by fuel, with additional revenue-generating transponders.[46] Refueling a satellite at end-of-life helps the satellite operator avoid the risk of loss of future revenue in the event servicing was to fail. Note that from the OOS client perspective, it does not matter what the form of servicing architecture is as long as the price, that the OOS provider charges, is acceptable. The operator may take a decision based on existing market condition: If the market is up and refueling would generate a profit, then refueling is the option. On the other hand, if the market is down, and refueling would result in loss, then the operator would decommission the satellite.[46]

1.2.4 Servicing Architectures and Cost-Effectiveness of OOS Operations

In one of the original studies on OOS, Reynerson[65] introduced a notion of cost in defining a serviceable spacecraft: "Any spacecraft for which the benefits of OOS outweigh the associated cost". The study also provides an overview of the Spacecraft Modular Architecture Design (SMAD) and discusses a low-cost servicing architecture design for servicing. It emphasizes the functional replacement strategy in OOS operations in order to minimize the cost and complexity of servicing missions. In Ref. 41, the authors outline different types of servicing missions and different orbits for servicing, analyze different servicing methods, including the transportation for OOS operations. There have also been studies on the analysis and design of OOS architecture with the Global Positioning System (GPS) constellation as a case study.[42] The structural modifications, necessary for satellites in GPS constellation so that they can be serviced, have also been identified.[29] A detailed discussion on different means of making a spacecraft cost-effective by frequent non-intrusive servicing can be found in Ref. 83. Non-intrusive servicing refers to operations such as propellant re-supply, power transfer, and visual inspection, and leaves out operations like equipment change-out and repair. A model can also be developed to compare servicing and non-servicing architectures in both mass and cost over the life of the targeted constellation and cost.[64] For GEO and MEO constellations, Ref. 13 discusses the OOS system architecture, lists various servicing tasks, the servicer vehicle mission scenario, and impacts on the satellite design.

Typically, the above-mentioned studies have focussed on identifying logistical support for a given space mission, analyzing different servicing architectures to achieve the servicing goals, designing the service vehicle, modifying the design of a satellite to make it serviceable, and finally determining the most cost-effective servicing architecture.[71] This is the traditional provider's perspective of looking at the servicing problem. A new customer-centric approach is introduced in Ref. 71, in order to capture the value of flexibility that is important from the point of view of the OOS customers. Ref. 51 provides a framework that breaks the OOS valuation analysis into two distinct parts: the client's value and the provider's value. With a case-study of GEO communication satellites, the study points out a viable GEO servicing market. It identifies cases that look promising both from the viewpoint of the OOS provider and the customer. Ref. 32 analyzes the value of flexibility offered by satellite upgrade, and in particular looked at the instance of upgrade of solar panels for GEO communication satellites. Based on the HST example, the model of a scientific serviceable mission is developed and the promise of on-orbit upgrade operations is emphasized in Ref. 33. Ref. 46 examined the end-of-life refueling case for two real-world communication satellites, and justifies the existence of a servicing market, along with benefit for potential customers. Studies have also been done to identify feasible non-commercial OOS markets like upgrade of government weather satellites in geosynchronous orbit.[47] A new value proposition that considers rapid response to technological or market change and design of less redundant satellites, is addressed in Ref. 48.

Although most studies have considered monolithic servicing architecture, an alternative fractionated servicing architecture have also been investigated. For instance, the Heterogeneous Expert Robots for On-Orbit Servicing (HEROS) architecture consists of a fleet of small, agile robots cooperatively performing servicing missions.[80,81]

1.2.5 Enabling Technologies for OOS Operations

One of the key technologies required for autonomous OOS operations is the Autonomous Rendezvous and Capture (ARC). Ref. 57 reviews some of the technology efforts related to ARC. Ref. 76 discusses the development of an autonomous servicing spacecraft simulator to validate autonomous control algorithms and different hardware required for ARC, and OOS operations in general. Tests have been performed at the Naval Research Laboratory in order to validate autonomous guidance, navigation, and control algorithms with a relative navigation sensor in the loop for satellite servicing and inspection.[9] For the ARC operations during the successful tests of the OE program, an Advanced Video Guidance Sensor was used to provide relative position and attitude between the two vehicles.[30,56] Ref. 87 describes a high performance image processing unit and its role in the ARC operations during servicing of a spacecraft.

One of the most important aspect of servicing missions is refueling which involves fluid exchange. The development of critical technologies related to on-orbit refueling is discussed in Ref. 27. It provides an overview of the necessary fluid coupling, and systems required to engage and disengage the fluid couplings. Ref. 18 describes the fluid transfer and propulsion system required for on-orbit propellant replenishment, for the Orbital Express program. The HERMES OOS provides an architecture with a minimalistic approach for enabling refueling services on-orbit. Ref. 38 discusses a small HERMES Quick Disconnect (QD) accessory that allows any satellite carrying the accessory perform the functions of a tanker spacecraft.

Studies have also been done to address servicing objectives other than refueling. Ref. 43 discusses the design of a small spacecraft to perform on-orbit servicing tasks. The MORPHbots presents a lightweight modular system, comprised of standardized actuator, sensor, and computational modules, that is capable of performing assembly and servicing works typically done by astronauts.[4] An Orbital Recovery System (ORS) is being developed with the goals of maintaining a telecommunications satellite in Geostationary (GEO) orbit for 10 or more additional years beyond its normal propellant end-of-life.[34] Ref. 49 focusses on simplifying the attitude control system of space cargo for reusable orbital logistics supply servicing systems, with potential application in logistics supply missions to International Space Station, or propellant supply mission for the Low Earth Orbit satellites. Front-end Robotics Enabling Near-Term Demonstration (FREND), which is a technology demonstration program at Naval Research Laboratory, is aimed at designing and building a robotic payload capable of grappling and repositioning existing satellites.[79] Ref. 3 discusses a dextrous robotic servicing system, based largely on the requirements for robotic servicing of HST. For the fractionated servicing architecture HEROS, Ref. 80 discusses the development of path-planning algorithms.

A detailed account of technical and economic feasibility of on-orbit satellite servicing can be found in Ref. 77. With the Orbital Express program, we can say that OOS operations has attained technological maturity. The major obstacle are therefore the high costs of servicing and the lack of a serviceable satellite market. The demonstration of one or more basic OOS operations with a low capital starting point may pave the way for the development of a serviceable market. One possible relatively low capital starting point of servicing missions could begin with the refueling of existing satellites.[46]

1.3 Literature Survey: Optimal Time-Fixed Impulsive Rendezvous

A servicing mission would involve several orbital transfers performed by a service vehicle and/or satellites. These transfers require the solution of time-fixed rendezvous problems. Assuming a chemical propulsion system for the service vehicle and the satellites, the orbital maneuvers would be impulsive in nature.

1.3.1 Non-Cooperative Rendezvous

Optimal fixed-time multi-impulse transfer trajectories can be determined by methods based on Lawden's primer vector theory.[40] Lion and Handelsman[45] applied the calculus of variations to the primer vector theory in order to obtain the first order conditions for optimal addition of an impulse along the trajectory of the transfer vehicle, or for the inclusion of initial and final coasting. Primer vector theory has been applied to determine multiple impulse fixed time solutions to rendezvous between two vehicles in circular orbits.[62] C-W equations[60] have also been used in the literature to obtain minimum fuel multiple-impulse orbital trajectories. In particular, the primer vector theory has also been applied to the C-W equations.[31] In our study, we are interested in minimum fuel two-impulse orbital transfer between coplanar circular orbits. This is essentially the well-known Lambert's problem.[60] The multiple revolution solutions to Lambert's problems, in which the transfer vehicle can complete several revolutions in the transfer orbit, have been studied. it has been shown that if the number of maximum possible revolutions is $N_{\max}$, then the optimal solution is determined by exhaustively investigating a set of $(2N_{\max} + 1)$ candidate minima.[61] However, it has been established that the optimal solution can be obtained by investigating at most two of the $(2N_{\max} + 1)$ candidate minima.[74]

1.3.2 Cooperative Rendezvous

Although most of the studies in the literature focus on active-passive (non-cooperative) rendezvous, there also exits works which consider the active-active (cooperative) case of rendezvous. The earliest works on cooperative rendezvous considered rendezvous between general linear or non-linear systems with various performance indices.[24,52] The idea of using differential games to study cooperative rendezvous problems has also been discussed in the literature.[86] A study has been done on the optimal terminal maneuver of the active satellites engaged in a cooperative impulsive rendezvous.[59] Determination of optimal terminal maneuvers involves the optimization of the common velocity vector after the rendezvous. Methods have also been developed for determining optimal time-fixed impulsive cooperative rendezvous using primer vector theory.[53] These accommodate cases of fuel-constraints on satellites and enables the addition of mid-course impulse(s) to a vehicle's trajectory. For the case of fixed-time impulsive maneuvers, cooperative rendezvous is advantageous when the time allotted for the maneuver is relatively short. Examples show that the non-cooperative solution becomes cheaper once the time allotted for the rendezvous is large enough for Hohmann transfers to be feasible. The minimum fuel rendezvous of two power-limited spacecrafts has also been studied[14,15] using non-linear analysis as well as C-W equations. For such spacecraft engaging in a rendezvous maneuver, cooperative rendezvous is always found to be cheaper than non-cooperative solution. Constrained and unconstrained circular terminal orbits have also been analyzed, and it has been found that the cooperative solution still remains the cheaper option to rendezvous. Analytical solutions using the C-W equations can be used to predict the nature of the terminal orbit of rendezvous.[15] For instance, in the case of a cooperative rendezvous between two satellites in a circular orbit, the two meet up at an orbital slot mid-way between the original slots, each satellite essentially removing half the phase angle.

1.4 Literature Survey: Optimal Scheduling for Refueling

One aspect of the study of servicing missions, or refueling missions in particular, is the determination of the optimal scheduling, which involves the solution of a large-scale optimization problem. The conventional notion of refueling fuel-deficient satellites in a constellation is to have a refueling spacecraft visit the latter one by one and impart fuel to them.[73] By an optimal schedule for single-service vehicle refueling strategy, we mean an optimal sequence in which the service vehicle would visit the fuel-deficient satellites in a constellation in order to replenish them with fuel; the optimal sequence corresponding to minimum total fuel expenditure of the service vehicle. Typically, for a service vehicle imparting fuel to satellites in a circular constellation, the optimal schedule is sequential visit of the satellites by the service vehicle in a clockwise or counter-clockwise fashion.[73] For the problem of servicing satellites in a constellation when plane changes are required, it has been shown, with the instance of servicing satellites in Geosynchronous orbit (GEO), that the optimal schedule may not be sequential.[5]

Recently, an alternative scenario for distributing fuel amongst a large number of satellites has been proposed.[74,75] In this scenario, no single spacecraft is in charge of the whole refueling process. Instead, all satellites share the responsibility of refueling each other on an equal footing. This is referred to as the peer-to-peer (P2P) refueling strategy.[75] This is achieved by having satellites with excess fuel sharing their resources (propellant) with those depleted of it. Although a stand-alone P2P strategy might seem unconventional at first notice, P2P comes as a natural choice in distributing fuel in the constellation in a mixed refueling strategy.[19,82] In such a scenario, an external refueling spacecraft, either launched from Earth or coming from a different orbit, replenishes half of the satellites in a constellation and returns back to its original orbit. The satellites which receive fuel from the external refueling spacecraft distribute the fuel amongst other satellites in the constellation via P2P refueling. Numerical studies have shown that the mixed refueling strategy is a competitive alternative to the single-service vehicle refueling strategy and, in fact, outperforms the latter, as the number of satellites[82] in the constellation increases and/or the time to refuel decreases. Furthermore, the incorporation of cost-reducing strategies such as the coasting time allocation strategy and asynchronous P2P maneuvers[19] provides further improvement by reducing the fuel expenditure of the P2P phase of the mixed refueling strategy.

In all of the above-mentioned studies, P2P refueling was perceived as a means to equalize fuel in the constellation. In order to achieve fuel equalization in the constellation, an optimization problem was formulated, such that the deviation of each satellite's fuel from the initial average fuel in the constellation is penalized. Under such a formulation, the problem of establishing optimal pairings of satellites reduces to a problem of finding the maximum weighted matching in the so-called constellation graph. This maximum matching problem can be solved using standard methods.[28] A decentralized approach that uses auctions has also been reported in Ref. 70. An alternative formulation for the P2P refueling problem is to impose a minimum fuel requirement for each satellite in the constellation in order to remain operational. Satellites having the required amount of fuel are fuel-sufficient, while those which do not have the required amount of fuel are fuel-deficient. We therefore seek to find the optimal satellite pairings so that all satellites end up being fuel-sufficient at the end of the refueling process. This is to be achieved by using as little fuel as possible in the process.[23,69]

Furthermore, the (baseline) P2P refueling strategy can be extended to cases like the Egalitarian P2P (E-P2P), the Cooperative P2P (C-P2P), and Cooperative Egalitarian P2P (CE-P2P) refueling strategies. Although the baseline P2P and the C-P2P refueling strategies can be formulated as a bipartite matching problem, or a two-index assignment problem,[22,69] the E-P2P and the CE-P2P problem would require the solution of a higher dimensional matching problem, or a multi-index assignment problem. In these cases, the problem becomes hard to solve. In particular, the E-P2P refueling strategy, can be formulated as a three-index assignment problem.[20] It is well known that the three-index assignment problem is NP-complete.[26] The general multi-index assignment problem was first stated by Pierskalla[55] as an extension of the two-index assignment problem. The three-dimensional assignment problem, which is a special case of the multi-index assignment problem, can be viewed as a matching problem on a complete tripartite graph. Several sub-optimal algorithms have been proposed for this problem. A branch-and-bound algorithm was proposed to solve the three-index assignment problem by Balas and Saltzman.[7] Approximation algorithms for three-index assignment problems with triangle inequalities were addressed by Crama and

Spieksma.[16] For multi-index assignment problems in k-partite graphs with decomposable costs*, Bandelt, Crama and Spieksma[8] introduced two approximate algorithms, each of which solves a sequence of two-index assignment problems. Another class of algorithms that has been developed for solving the three-index assignment problem includes the Greedy Random Adaptive Search Procedure (GRASP).[2,25,66] Feo and Resende[25] discussed GRASP as a means for solving general combinatorial optimization problems. Robertson[66] introduced four GRASP implementations for the multi-index assignment problem, which are combinations of two constructive methods (i.e., randomized reduced cost greedy, and randomized maximum regret) and two local search methods (i.e., two-assignment exchange, and variable depth exchange). Aiex et al.[2] proposed the use of GRASP with path relinking. This method was able to improve the quality of the heuristic solutions proposed in Refs. 7 and 16. Moreover, the GRASP method is shown to benefit from parallelization.

The GRASP method can be used to to solve the E-P2P refueling problem.[20] Alternatively, the problem can be formulated using network flows.[1,21,23] Similarly, the CE-P2P can also be formulated using network flows. The E-P2P, C-P2P, and the CE-P2P are the subjects of discussion in this dissertation, and will be discussed in great detail in the following chapters.

1.5 Preliminary Notations

In this section, we will introduce some introductory notations in order to facilitate the discussion in the forthcoming chapters.

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*By decomposable costs, we mean that the cost of a clique in the k-partite graph is a function of the cost of the edges induced by the clique. Note that a clique is a subgraph in which all vertices are pairwise adjacent. For a k-partite graph, a clique comprises of exactly one node from each partition of the k-partite graph.

1.5.1 Constellation Details

We consider the circular constellation to consist of n satellites, distributed over n orbital slots in a circular orbit of radius R. Let the set of n satellites be given by $\mathcal{S} = \{s_i : i = 0, 1, 2, \ldots, n\}$, where s_0 represents a fictitious satellite. Let the set of n orbital slots be given by $\Phi = \{\phi_i \in [0, 2\pi) : i = 1, 2, \ldots, n,\ \phi_i \neq \phi_j\}$. We introduce a mapping $\sigma_t : \Phi \mapsto \mathcal{S}$ that, at time $t \geq 0$, assigns to each orbital slot a satellite from $\mathcal{S}$. In particular, $\sigma_t(\phi_j) = s_i$ implies that the satellite s_i occupies the orbital slot ϕ_j at time t. If the slot ϕ_j is empty at time t, we write $\sigma_t(\phi_j) = s_0$. Also, let the fuel content of satellite s_i at time t be denoted by $f_{i,t}$. In particular, let the initial fuel content of satellite s_i be denoted by f_i^- and the final fuel content be denoted by f_i^+; that is, $f_i^- = f_{i,0}$ and $f_i^+ = f_{i,T}$, where T is the time allotted for refueling.

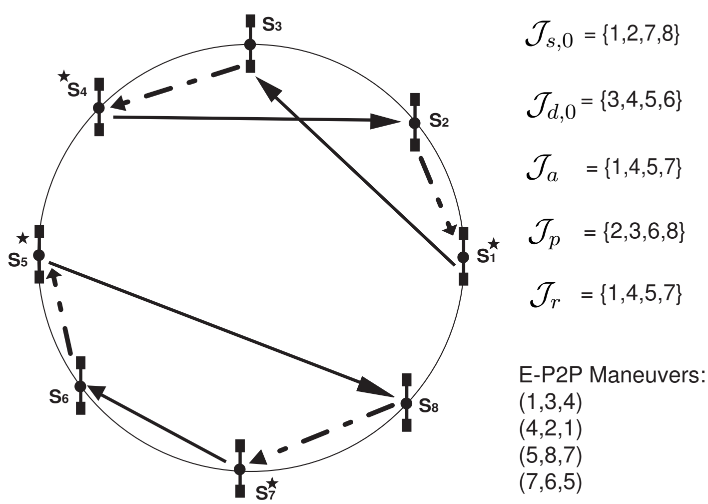


Figure 3: Notations explanation for refueling.

1.5.2 Satellite Roles During Refueling

It will be convenient to keep track of the indices of the satellites participating in the refueling process under different roles. To this end, let $\mathcal{I} = \{1, 2, \ldots, n\}$. We will refer to satellites as *fuel-sufficient* if they have excess fuel and thereby capable of sharing this amount of fuel to other satellites in the constellation. Satellites which are depleted of fuel are referred to as *fuel-deficient* satellites. Let $\mathcal{I}_{s,0}$ denote the set comprised of indices of the fuel-sufficient satellites, while let $\mathcal{I}_{d,0}$ denote the set comprised of indices of the fuel-deficient ones. Clearly, $\mathcal{I}_{s,0} \cup \mathcal{I}_{d,0} = \mathcal{I}$. During a P2P refueling transaction between a fuel-sufficient and a fuel-deficient satellite, one of them (henceforth referred to as the *active* satellite) performs an orbital transfer to rendezvous with the other satellite (henceforth referred to as the *passive* satellite). After the fuel exchange takes place between the two, the active satellite returns to its original orbital slot. We will denote the index set of active satellites by $\mathcal{I}_a \subseteq \mathcal{I}$ and the index set of passive satellites by $\mathcal{I}_p \subset \mathcal{I}$. For convenience, let $\mathcal{J}_{s,t} = \{j : \sigma_t(\phi_j) = s_i, i \in \mathcal{I}_{s,t}\}$ denote the index set of orbital slots occupied by fuel-sufficient satellites at time t, and let $\mathcal{J}_{d,t} = \{j : \sigma_t(\phi_j) = s_i, i \in \mathcal{I}_{d,t}\}$ denote the index set of orbital slots occupied by fuel-deficient satellites at time t. Also, let $\mathcal{J}_a = \{j : \sigma_0(\phi_j) = s_i, i \in \mathcal{I}_a\}$ denote the index set of orbital slots occupied by the active satellites before any orbital maneuver commences, let $\mathcal{J}_p = \{j : \sigma_0(\phi_j) = s_i, i \in \mathcal{I}_a\}$ denote the index set of orbital slots occupied by the passive satellites before any orbital maneuver commences. Fig. 3 shows some of the notations for a case of E-P2P refueling, which is an extension of P2P refueling. In this case, we consider $\sigma_0(\phi_i) = s_i$. Also, satellites s_1, s_2, s_7 and s_8 are the fuel-sufficient satellites and the remaining are the fuel-deficient satellites. The active satellites are marked with '$\star$', the forward trips are marked by solid arrow, while the return trips are marked by dashed arrow.

1.5.3 Satellite Properties

Furthermore, for each satellite s_i, we denote the mass of its permanent structure by m_{spi} and the specific thrust of its engine by I_{spi}. Also, we denote the gravitational acceleration on the surface of the earth by g_0. For each satellite s_i, we therefore define the characteristic constant $c_{0i} = g_0 I_{\text{spi}}$. Finally, we will denote the optimal rendezvous cost required for an orbital transfer from slot ϕ_i to ϕ_j by ΔV_{ij} and the fuel expended by a satellite s_μ to perform the orbital transfer from slot ϕ_i to slot ϕ_j is denoted by p_{ij}^{μ}.

1.6 Organization of the Dissertation

Let us now outline the organization of the dissertation. In Chapter 2, we discuss the problem of time-fixed impulsive rendezvous. We discuss the case of both cooperative and non-cooperative rendezvous, and assume that each orbital transfer comprises of two impulses. In Chapter 3, we discuss the different refueling strategies: the single service vehicle refueling strategy, the baseline peer-to-peer refueling strategy, and the mixed refueling strategy. In Chapter 4, we look into details the problem of non-cooperative and Cooperative P2P refueling. We also consider the case when the satellites are in two different circular orbits. In Chapter 5, the problem of Egalitarian P2P refueling is discussed. In Chapter 6, we discuss the problem of a Cooperative Egalitarian P2P refueling. Finally, in Chapter 7, we present the conclusions, the primary contributions of the dissertation, and also outline the potential future research areas.

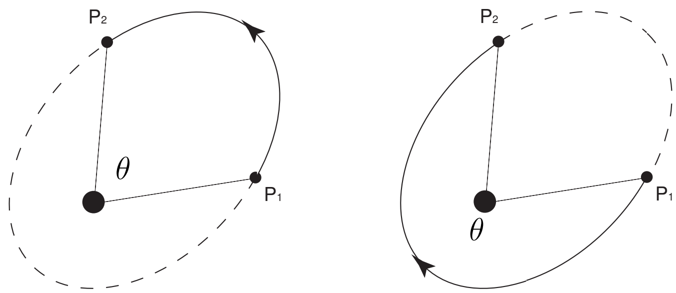
CHAPTER II

OPTIMAL TIME-FIXED RENDEZVOUS

A refueling mission comprises of several orbital transfers (rendezvous) made by the service vehicle and/or the satellites. Before discussing a complete refueling mission, let us look into one single rendezvous problem. Furthermore, let us consider that the satellites and the service vehicle employ a chemical propulsion system, so that the maneuvers are impulsive in nature. In particular, we look at the problem of two-impulse time-fixed rendezvous in this chapter. We discuss both cases of rendezvous between two satellites: (1) non-cooperative rendezvous, in which only one of the satellites performs the orbital transfer, (2) cooperative rendezvous, in which both satellites perform orbital transfers to complete the rendezvous. We motivate our discussion by taking a look at the well-known classical problem of Lambert.

2.1 Lambert's Problem

Lambert's problem can be stated as follows: Given two points P_1 and P_2 in space, the time of flight t_f, and a direction of flight, determine the transfer orbit that takes a spacecraft from P_1 to P_2 in the given time t_f. By direction of flight, we mean whether the spacecraft moves along the short way (transfer angle $\theta \leq \pi$), or it moves along the long way (transfer angle $\theta > \pi$), as illustrated in Figure 4. Given the two points P_1 and P_2 in space, there are two conjugate elliptical orbits for a given value of semi-major axis a of the transfer orbit. For the case shown in the Figure 5, F is the primary focus occupied by the primary gravitational body (Earth), and F_s and F_ℓ are the vacant foci corresponding to the two conjugate ellipses with the same value of a. The distance of P_1 and P_2 from F are given by r_1 and r_2, while d is the distance between the points P_1 and P_2. The two conjugate ellipses differ in their



(a) Short Way (b) Long Way

Figure 4: Lambert's Problem.

eccentricities. The ellipse with vacant focus F_s has the smaller eccentricity, while the one with vacant focus F_ℓ has the larger eccentricity. Lambert's theorem states that the time of flight from P_1 to P_2 is a function of the semi-major axis a of the transfer orbit, the distance d between the points, and the sum of the radii r_1 and r_2.

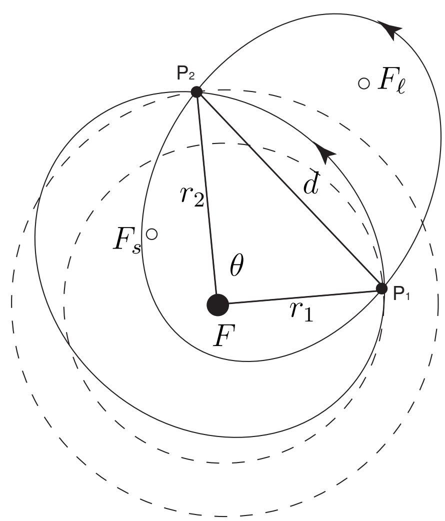


Figure 5: Transfer Orbit Geometry in Lambert's Problem.

Note that if the transfer time t_f permits, the spacecraft can complete several revolutions N in its transfer orbit. In general, the transfer time is given by

$$\sqrt{\mu}t_f = a^{3/2}\left[2N\pi + \alpha - \beta - (\sin\alpha - \sin\beta)\right], \tag{1}$$

where μ is the gravitational parameter, and α and β are parameters defined by

$$\sin\frac{\alpha}{2} = \left(\frac{s}{2a}\right)^{1/2}, \quad \sin\frac{\beta}{2} = \left(\frac{s-d}{2a}\right)^{1/2}, \tag{2}$$

where $s = (r_1 + r_2 + d)/2$. Figure 6 shows a typical variation of t_f with a for different

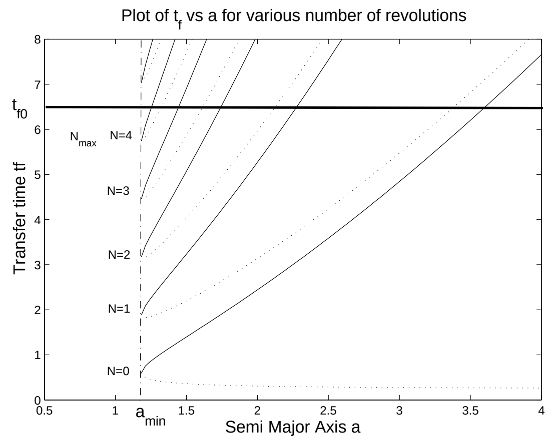


Figure 6: Example plot of transfer-time vs. semi-major axis.

values of N. Clearly, there is a minimum value of a, denoted by $a_{\min}$, for which a transfer is possible. Also, for a given value of N, there are two branches (marked by solid and lines in the plot) connected at $a_{\min}$. They correspond to the two different conjugate elliptical path mentioned before. Suppose, we are given a time of flight t_{f0} as shown in Figure 6. This time determines the maximum number of revolutions $N_{\max}$ that would be possible. A horizontal line through $t_f = t_{f0}$ intersects the branches corresponding to different number of revolutions. We accordingly have two intersections for each revolution $N = 1, 2, \ldots, N_{\max}$. For $N = 0$, there can be only one intersection as the lower branch is monotonically decreasing. Therefore, there are $2N_{\max} + 1$ intersections. In other words, $2N_{\max} + 1$ choices of a for the transfer orbit are possible for a given value of transfer time t_f.

2.2 Non-Cooperative Rendezvous

Let us consider that a satellite s_1 in a circular orbit of radius r_1 has to rendezvous with another satellite s_2 in a circular orbit of radius r_2. Without loss of generality, we can consider that the rendezvous starts at the time $t = 0$. At this instant of time, s_1 occupies the point P_1, as shown in Figure 7, and s_2 occupies the position P_2' with an angle of separation θ_0 with respect to P_1. At the instant of time $t = t_f$, the satellite s_2 occupies the position P_2. Hence, we are required to find the transfer orbit that takes the satellite s_1 from P_1 to P_2 in the given time t_f. In our discussion of the Lambert's problem, we found that there are $2N_{\max} + 1$ transfer orbits that would be possible. We need to select the best of these candidate solutions.

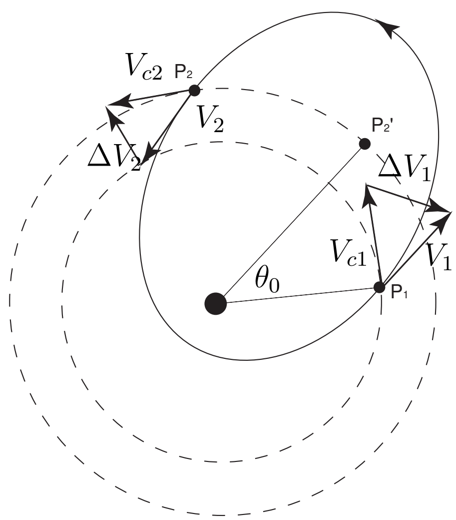
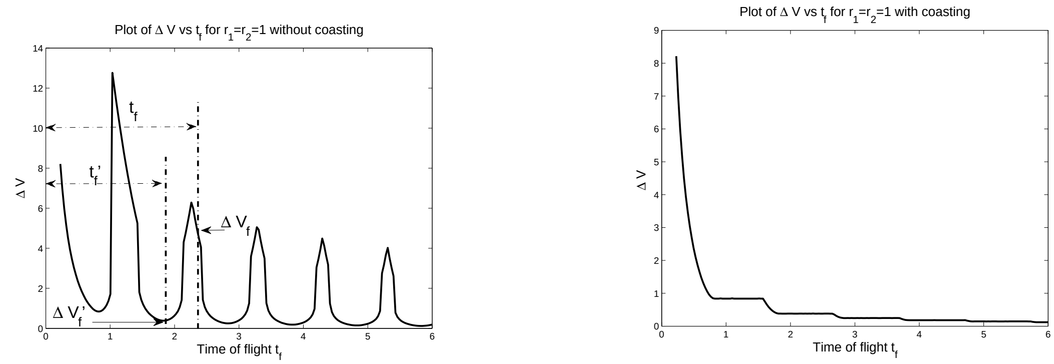


Figure 7: Two-Impulse Transfer.

In order to move from P_1 to P_2, the satellite s_1 uses two impulses. The first impulse results in a velocity change ΔV_1 at P_1 and places s_1 in the transfer orbit, while the second impulse causes another velocity change ΔV_2 at P_2 that places s_1 in the target orbit. Figure 7 shows these velocity changes incurred at points P_1 and P_2. The total velocity change incurred is then given by

$$\Delta V = \Delta V_1 + \Delta V_2 \tag{3}$$

Among all candidate solutions, the transfer orbit we chose for s_1 is the one that incurs the minimum possible ΔV. Note that this optimal transfer orbit is for a given initial angle θ_0 and a given time of flight t_f. A detailed variation of ΔV, with respect to changes in θ_0 and t_f, can be found in the contour plots in Ref. 73.

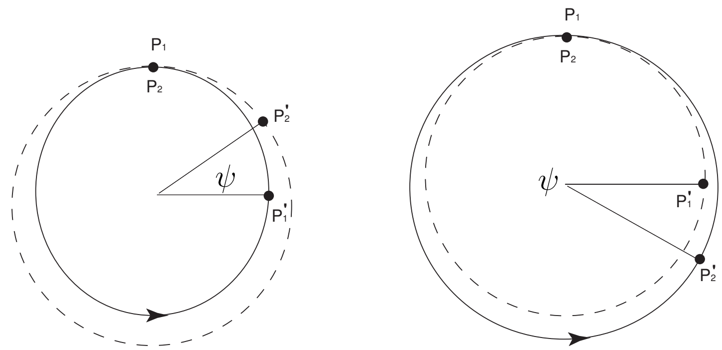


(a) Without Coasting

(b) With Coasting

Figure 8: Variation of ΔV with time ($r_1 = r_2$).

In particular, let us look at the case $r_1 = r_2$. In such a case, the points P_1 and P_2 are on the same circular orbit. Given an initial lead angle θ_0 of satellite s_2 with respect to satellite s_1, a typical variation of ΔV with time is shown in Figure 8(a). The plot shows alternating local maxima and minima. Now, suppose a time t_f is given for the rendezvous to complete. The corresponding $\Delta V = \Delta V_f$ is shown in the plot. Clearly, if we utilize a time t'_f lesser than the time t_f, it is possible to reduce the ΔV to $\Delta V'_f$, as shown in the figure. Thus, the actual transfer time t'_f is less than given time t_f, so that the satellite s_1 can coast for the remaining time. In other words, given a time t_f, we allow a coasting time $t_f - t'_f$ with $t'_f \leq t_f$, such that the nearest local minimum is attained. Each local minimum corresponds to a so-called Phasing Maneuver, for which points P_1 and P_2 coincide, that is, the impulses are provided at the same position on the transfer orbit. With the allowance of coasting, the ΔV becomes a non-increasing function of time, as shown in Figure 8(b). The plot has alternating cost-reducing and cost-invariant intervals. For the case of $r_1 \neq r_2$, the optimal rendezvous is a Hohmann transfer. It has been shown in Ref.[58] that the globally optimal two-impulse transfer that minimizes total ΔV is the Hohmann transfer. Note that, in order to have a Hohmann transfer, the time needs to be sufficient. We would look at both the Hohmannn transfer and the Phasing maneuver



(a) Sub-Synchronous (b) Super-Synchronous

Figure 9: Phasing Maneuver.

in the remainder of this section.

2.2.1 Phasing Maneuvers

In a phasing maneuver, the satellite s_1 transfers from one point to a different point in a circular orbit, such that the velocity changes occur at the same point in the transfer orbit. In other words, the points P_1 and P_2 coincide in this case. Depending on whether the transfer orbit has a lesser semi-major axis or not, the the Phasing maneuver is termed Subsynchronous or Supersynchronous. Figure 9 depicts both phasing maneuvers, with the transfer orbit shown in solid line, and the original orbit in dotted line. In the case of a sub-synchronous maneuver (Figure 9(a)), the satellite s_1 transfers to an orbit with a smaller time period in order to catch up with satellite s_2 that initially leads s_1 by an angle ψ. Similarly, in the case of a super-synchronous maneuver (Figure 9(b)), the satellite s_1 transfers to an orbit with a larger time period in order to catch up with satellite s_2 that initially lags s_1 by an angle ψ. Figure 9 shows the initial positions P_1' and P_2' of the satellites s_1 and s_2, as well as the phasing angle ψ of both phasing maneuvers. The velocity change required for a phasing maneuver can be calculated analytically.

Let us consider a phasing maneuver by the satellite located on the orbit of radius

$r_\star$. Also, let $T_\star$ denote the time period for the orbit of radius $r_\star$. Let us denote the phasing angle by ψ, where $-\pi \le \psi \le \pi$. We consider the cases of $\psi < 0$ and $\psi > 0$ separately. For each of these cases, we have one of the two maneuvers (supersynchronous or subsynchronous). We therefore have four cases to consider:[39]

i) Supersynchonous and $\psi > 0$: The velocity change required for this transfer is given by

$$\Delta V^p = 2\sqrt{\frac{\mu}{r_\star}}\left[\sqrt{2-\left(\frac{\ell-1}{\ell-\psi/2\pi}\right)^{2/3}}-1\right], \tag{4}$$

where

$$\ell = \lfloor\, T/T_\star + \psi/2\pi \,\rfloor. \tag{5}$$

ii) Supersynchonous and $\psi < 0$: The velocity change required for this transfer is given by

$$\Delta V^p = 2\sqrt{\frac{\mu}{r_\star}}\left[\sqrt{2-\left(\frac{\ell}{\ell-\psi/2\pi}\right)^{2/3}}-1\right]. \tag{6}$$

iii) Subsynchonous and $\psi > 0$: The velocity change required for this transfer is given by

$$\Delta V^p = 2\sqrt{\frac{\mu}{r_\star}}\left[1-\sqrt{2-\left(\frac{\ell}{\ell-\psi/2\pi}\right)^{2/3}}\right]. \tag{7}$$

iv) Subsynchonous and $\psi < 0$: The velocity change required for this transfer is given by

$$\Delta V^p = 2\sqrt{\frac{\mu}{r_\star}}\left[1-\sqrt{2-\left(\frac{\ell+1}{\ell-\psi/2\pi}\right)^{2/3}}\right]. \tag{8}$$

2.2.2 Hohmann Transfers

For a Hohmann transfer from an orbit of radius r_1 to an orbit of radius r_2 (where, for simplicity, we may assume that $r_1 < r_2$), the semi-major axis of the transfer orbit is given by $a = (r_1 + r_2)/2$, so that the velocity change corresponding to the first impulse is given by

$$\Delta v_1 = \sqrt{2\mu\left(\frac{1}{r_1} - \frac{1}{r_1 + r_2}\right)} - \sqrt{\frac{\mu}{r_1}}, \tag{9}$$

and the velocity change corresponding to the second impulse is given by

$$\Delta v_2 = \sqrt{\frac{\mu}{r_2}} - \sqrt{2\mu\left(\frac{1}{r_2} - \frac{1}{r_1 + r_2}\right)}. \tag{10}$$

Using the above expressions, we have the total ΔV requirement for the Hohmann transfer to be

$$\Delta V^H = \left(\sqrt{\frac{\mu}{r_2}} - \sqrt{\frac{\mu}{r_1}}\right) + \sqrt{2\mu}\left(\sqrt{\frac{1}{r_1} - \frac{1}{r_1 + r_2}} - \sqrt{\frac{1}{r_2} - \frac{1}{r_1 + r_2}}\right). \tag{11}$$

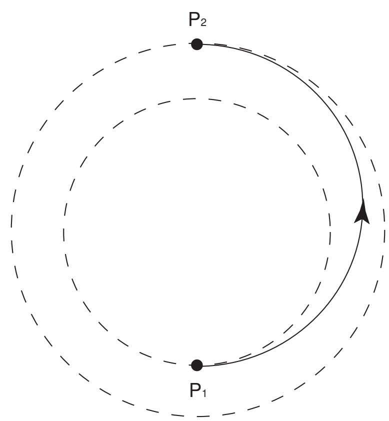


Figure 10: Hohmann Transfer.

The angle of separation required for a Hohmann transfer to be feasible is given by[11]

$$\theta_H = \pi\left[1 - \left(\frac{1 + r_1/r_2}{2}\right)^{3/2}\right]. \tag{12}$$

Unless this angle of separation is achieved, the satellite performing the transfer will need to coast for a time τ_H given by[11]

$$\tau_H = \frac{\theta_0 - \theta_H}{2\pi\left(1/T_1 - 1/T_2\right)}, \tag{13}$$

where θ_0 is the initial separation angle, and where $T_s = \left(1/T_1 - 1/T_2\right)^{-1}$ is the synodic period for the orbits concerned, with $T_i = 2\pi\sqrt{r_i^3/\mu}$, for $i = 1, 2$ is the orbital period.

Since we are concerned with fixed-time transfers, the maximum time allowed for coasting is given by

$$t_c \leq T - \pi\sqrt{\frac{(r_1 + r_2)^3}{8\mu}}. \tag{14}$$

Therefore, the separation angle required for a Hohmann transfer should lie between θ_H and $\theta_H + \Delta\theta$, where

$$\Delta\theta = \begin{cases} \dfrac{2\pi}{T_s}\left(T - \pi\sqrt{\dfrac{(r_1 + r_2)^3}{8\mu}}\right), & \text{if } r_1 < r_2, \\ -\dfrac{2\pi}{T_s}\left(T - \pi\sqrt{\dfrac{(r_1 + r_2)^3}{8\mu}}\right), & \text{if } r_1 > r_2. \end{cases} \tag{15}$$

A Hohmann transfer is therefore feasible for all separation angles $\theta_0 \in [\theta_H, \theta_H + \Delta\theta_H]$ if $r_1 < r_2$ and $\theta_0 \in [\theta_H + \Delta\theta_H, \theta_H]$ if $r_1 > r_2$. Therefore, all slots on r_i and r_o that satisfy the above condition on the separation angle will allow for a Hohmann transfer to take place.

2.3 Cooperative Rendezvous

In our discussion so far, we have looked at the problem of non-cooperative rendezvous, that is, only one of the satellites perform the orbital transfer necessary to complete the rendezvous. However, this need not be the case, and both satellites might be active and each performs an orbital transfer necessary for the rendezvous. We assume that during a cooperative rendezvous, each satellite performs a time-fixed two-impulse transfer. Furthermore, we assume that the terminal rendezvous orbit is circular.

In the remainder of this chapter, we look at the problem of cooperative rendezvous. To this end, let us consider two satellites s_μ and s_ν occupying the orbital slots ϕ_i and ϕ_j in the circular orbits of radius r_i and r_o respectively. Let the initial separation angle between these satellites be θ_0. Now, we consider various orbit for cooperative rendezvous, and discretize each orbit of radius r into a set of orbital slots Φ_r equally spaced along the orbit. Let $\mathcal{I}$ denote the set of indices for these slots. Now consider an orbital slot $\phi_{k_r} \in \Phi_r$ on the orbit of radius r where a cooperative rendezvous takes place, where $k_r \in \mathcal{I}$. The situation is depicted in Fig. 11.

Let the time allotted for a cooperative rendezvous between the two satellites be given by T, and let also the velocity change required for an orbital transfer from slot ϕ_i to slot ϕ_{k_r} be denoted by ΔV_{ik_r}, and the velocity change required for an orbital transfer from slot ϕ_j to slot ϕ_{k_r} be denoted by ΔV_{jk_r}. The total velocity change required for a cooperative rendezvous in which the satellites meet at slot $\phi_{k_r} \in \Phi_r$ is denoted by

$$\Delta V_{ij}^c|_{k_r} = \Delta V_{ik_r} + \Delta V_{jk_r}. \tag{16}$$

This is the total velocity change required for a cooperative rendezvous between the two satellites. Had the satellites been involved in a non-cooperative rendezvous, then the

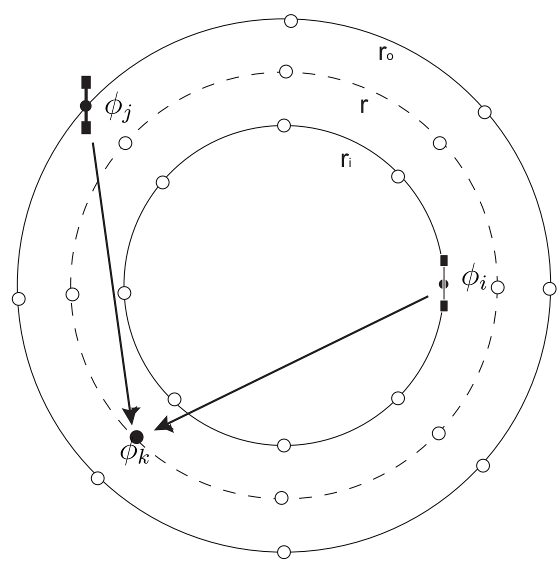


Figure 11: Cooperative rendezvous for the case $r_i \leq r \leq r_o$.

total velocity change required to complete the rendezvous would be ΔV_{ij} if satellite s_μ were active and ΔV_{ji} if satellite s_ν were active. Hence, the cases $\phi_{k_{r_i}} = \phi_i$ and $\phi_{k_{r_j}} = \phi_j$ correspond to the two cases of non-cooperative rendezvous.

One of the slots in Φ_r results in the cheapest cooperative maneuver between any two satellites meeting on the orbit of radius r. Let us denote this slot by $\phi_c(r)$ and the corresponding total velocity change by $\Delta V_c(r)$. We therefore have

$$\Delta V_c(r) \triangleq \min_{\phi_{k_r} \in \Phi_r} \Delta V_{ij}^c|_{k_r}, \tag{17}$$

and

$$\phi_c(r) \triangleq \arg \min_{\phi_{k_r} \in \Phi_r} \Delta V_{ij}^c|_{k_r}. \tag{18}$$

Assume now that the optimal cooperative rendezvous involving satellites s_μ and s_ν takes place in the orbit of radius $r_{\min}$ and at the orbital slot $\phi_{c,\min}$. This corresponds to the lowest ΔV over all possible orbits and all possible slots. We also let the corresponding optimal velocity change be $\Delta V_{c,\min}$. We therefore have,

$$\Delta V_{c,\min} \triangleq \min_r \Delta V_c(r) \tag{19}$$

and

$$\phi_{c,\min} \triangleq \phi_c(r_{\min}), \quad \text{where} \quad r_{\min} = \arg \min_r \Delta V_c(r). \tag{20}$$

Recall that optimal time-fixed two-impulse rendezvous is a Hohmann transfer (for different orbits) or a Phasing maneuver (for same orbit). Let us now investigate now two cooperative maneuvers that comprise of these optimal maneuvers. The first case of cooperative rendezvous that we study is a Hohmann-Hohmann Cooperative Maneuver, which comprises of two Hohmann transfers. The second case of cooperative rendezvous that we study is a Hohmann-Phasing Cooperative Maneuver, which comprises of a Hohmann transfer and a Phasing maneuver.

2.4 Hohmann-Hohmann Cooperative Maneuvers (HHCM)

Let satellites s_μ and s_ν engage in a HHCM rendezvous. We assume that for all orbits of radius r where a cooperative rendezvous can take place, there exists at least one slot $\phi_{k_r} \in \Phi_r$ at which both satellites can perform a Hohmann transfer. Using the expression for the cost of a Hohmann transfer in (11), the total velocity change required for the HHCM rendezvous, when $r_i \leq r \leq r_o$, is given by

$$\begin{aligned}\Delta V_c^H(r) &= \left(\sqrt{\frac{\mu}{r_o}} - \sqrt{\frac{\mu}{r_i}}\right) + \\ &\sqrt{2\mu}\left(\sqrt{\frac{1}{r_i} - \frac{1}{r_i + r}} - \sqrt{\frac{1}{r} - \frac{1}{r_i + r}} + \sqrt{\frac{1}{r} - \frac{1}{r_o + r}} - \sqrt{\frac{1}{r_o} - \frac{1}{r_o + r}}\right). \quad (21)\end{aligned}$$

Taking the derivative of the previous expression with respect to r, we have,

$$\begin{aligned}\sqrt{\frac{2}{\mu}}\frac{\mathrm{d}}{\mathrm{d}r}\left(\Delta V_c^H\right) &= \left(\frac{1}{r}\right)^2\left[\frac{1}{\sqrt{1/r - 1/(r_i + r)}} - \frac{1}{\sqrt{1/r - 1/(r_o + r)}}\right] \\ &+ \left(\frac{1}{r_i + r}\right)^2\left[\frac{1}{\sqrt{1/r_i - 1/(r_i + r)}} - \frac{1}{\sqrt{1/r - 1/(r_i + r)}}\right] \\ &+ \left(\frac{1}{r_o + r}\right)^2\left[\frac{1}{\sqrt{1/r - 1/(r_o + r)}} - \frac{1}{\sqrt{1/r_o - 1/(r_o + r)}}\right] \quad (22)\end{aligned}$$

By defining the following two parameters

$$\beta_1 = 2\left(r_o + r_i\right)^3, \qquad \beta_2 = r_o\left(r_o + 3r_i\right)^2, \tag{23}$$

and by substituting $r = r_i$ in (22), we have

$$\sqrt{\frac{2}{\mu}}\left[\frac{\mathrm{d}}{\mathrm{d}r}\left(\Delta V_c^H\right)\right]_{r=r_i^+} = \frac{\sqrt{\beta_1} - \sqrt{\beta_2}}{r_i^{3/2}\left(r_i + r_o\right)^{3/2}}. \tag{24}$$

Note that $\beta_1 - \beta_2 = \left(r_o - r_i\right)\left[r_o\left(r_o + r_i\right) - 2r_i^2\right] > 0$ since $r_o > r_i$. It follows that $0 < \sqrt{\beta_2} < \sqrt{\beta_1}$. We therefore have that

$$\left[\frac{\mathrm{d}}{\mathrm{d}r}\left(\Delta V_c^H\right)\right]_{r=r_i^+} > 0. \tag{25}$$

Substituting $r = r_o$ in (22), and by performing similar calculations, we obtain

$$\left[\frac{\mathrm{d}}{\mathrm{d}r}\left(\Delta V_c^H\right)\right]_{r=r_o^-} < 0. \tag{26}$$

Similarly, we consider the cost of a HHCM rendezvous for the cases $r < r_i < r_o$ and $r_i < r_o < r$. These two cases yield

$$\left[\frac{\mathrm{d}}{\mathrm{d}r}\left(\Delta V_c^H\right)\right]_{r=r_i^-} < 0, \tag{27}$$

and

$$\left[\frac{\mathrm{d}}{\mathrm{d}r}\left(\Delta V_c^H\right)\right]_{r=r_o^+} > 0. \tag{28}$$

The results can be summarized as

$$\left[\frac{\mathrm{d}}{\mathrm{d}r}\left(\Delta V_c^H\right)\right]_{r=r_i} \begin{cases} < 0, & \text{if } r < r_i, \\ > 0, & \text{if } r > r_i, \end{cases} \tag{29}$$

and

$$\left[\frac{\mathrm{d}}{\mathrm{d}r}\left(\Delta V\right)\right]_{r=r_o} \begin{cases} < 0, & \text{if } r < r_o, \\ > 0, & \text{if } r > r_o. \end{cases} \tag{30}$$

Therefore, we conclude that the ΔV cost for a HHCM rendezvous attains a local minimum when either $r = r_i$ or $r = r_o$. Note that a HHCM rendezvous for $r = r_i$ or $r = r_o$ is actually a non-cooperative Hohmann transfer. It follows that if either Hohmann transfer is possible, then the non-cooperative maneuvers are local minimizers. Let us denote the cost of a non-cooperative Hohmann transfer by ΔV_{nc}^H. Figure 12 shows how the cooperative rendezvous cost varies with r for different values of r_o (r_i is fixed at 1). For a cooperative rendezvous at an outer orbit, the cost of the maneuver increases rapidly. As r_o approaches r_i, the cooperative cost for any intermediate orbit r approaches the non-cooperative Hohmann transfer cost and the concave region flattens out. In the limiting case when $r_o \to r_i$, the minimum is obtained at $r = r_i = r_o$ with the total cost of transfer being zero.

For convenience, let the difference of the HHCM and the non-cooperative Hohmann maneuver costs be denoted by the function $\eta(r)$, given by

$$\eta(r) \triangleq \frac{\Delta V_c^H(r) - \Delta V_{nc}^H}{\sqrt{2\mu}}.$$

Clearly, $\eta(r_i) = 0$ and $\eta(r_o) = 0$. The function $\eta(r)$ can be calculated analytically, and its variation over r (Fig. 13(a)) shows that $\eta(r)$ is marginally sub-optimal for all $r \in (r_i, r_o)$ compared to $r = r_i$ or $r = r_o$. If enough time is available so that Hohmann

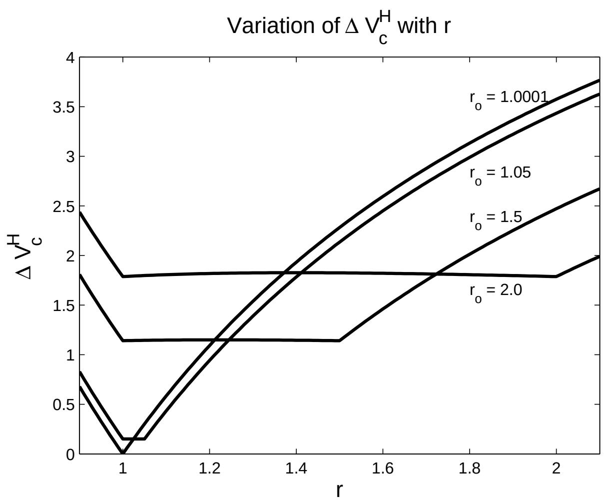
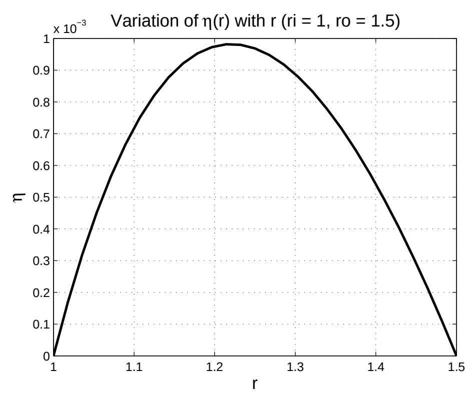
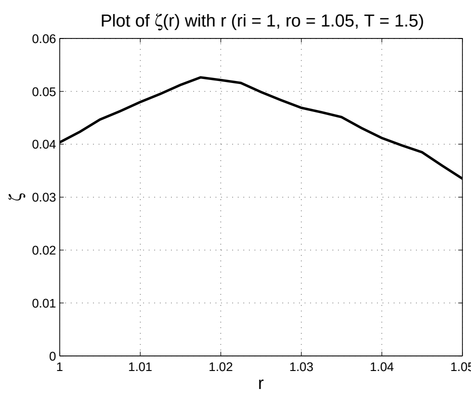


Figure 12: Variation of HHCM cost with r.

transfers are possible for the given separation of the satellites, the optimal rendezvous is non-cooperative.



(a) Variation of $\eta(r)$



(b) Variation of $\zeta(r)$

Figure 13: Variation of auxiliary functions $\eta(r)$ and $\zeta(r)$ with r.

If the optimal cooperative rendezvous is comprised of two Hohmann transfers, we have $\Delta V_c(r) = \Delta V_c^H(r)$. However, both Hohmann transfers may not be possible. In this case $\Delta V_c(r) \neq \Delta V_c^H(r)$. Let us define the following function:

$$\zeta(r) \triangleq \frac{\Delta V_c(r) - \Delta V_c^H(r)}{\sqrt{2\mu}}.$$

Since a Hohmann transfer is the optimal two-impulse transfer between all coplanar circular orbits, a HHCM rendezvous is the optimal cooperative rendezvous at a radius $r \notin \{r_i, r_o\}$. Hence, $\zeta(r)$ measures the sub-optimality of the cooperative rendezvous solution when a HHCM rendezvous is not feasible at any slot on the orbit. The function $\zeta(r)$ is shown in Fig. 13(b). Note that if a HHCM rendezvous is feasible at any slot on the orbit, we have $\zeta(r) = 0$.

2.5 Hohmann-Phasing Cooperative Maneuvers (HPCM)

In our case, either satellite s_μ or satellite s_ν performs a Phasing maneuver, that is, $r_\star \in \{r_i, r_o\}$. The satellite can transfer from its original slot (ϕ_i for s_μ and ϕ_j for s_ν) to another orbital slot $\phi_{k_{r_\star}}$ in the same orbit, by performing one of the following two maneuvers: (1) A *supersynchronous* maneuver, in which the transfer orbit has a higher apoapsis than $r_\star$, (2) A *subsynchronous* maneuver, in which the transfer orbit has a lower periapsis than $r_\star$.

Recall that a HPCM rendezvous comprises of a Hohmann transfer and a Phasing maneuver. Note that during a HPCM rendezvous, the Phasing maneuver can occur either on the orbit r_i or on the orbit r_o. We therefore have, $r_\star \in \{r_i, r_o\}$. The satellite can transfer from its original slot (ϕ_i for s_μ and ϕ_j for s_ν) to another orbital slot $\phi_{k_{r_\star}}$ in the same orbit. In the case $r_\star = r_i$, the satellite s_ν performs a Hohmann transfer from r_o to r_i and the satellite s_μ performs a Phasing maneuver. In the other case $r_\star = r_o$, the satellite s_μ performs a Hohmann transfer from r_i to r_o and the satellite s_ν performs a Phasing maneuver. This case is depicted in Fig. 14, in which ψ represents the phasing angle and θ_0 is the initial separation angle between satellites s_μ and s_ν. Denoting by $\overline{\Delta V}_c(r_\star)$ the cooperative cost, we therefore have

$$\overline{\Delta V}_c(r_\star) = \Delta V^p + \Delta V^H_{nc}, \qquad \star = i, o. \tag{31}$$

The total ΔV for a HPCM rendezvous depends on the phasing angle ψ. The phasing angle ψ determines the location of the cooperative rendezvous on the orbit

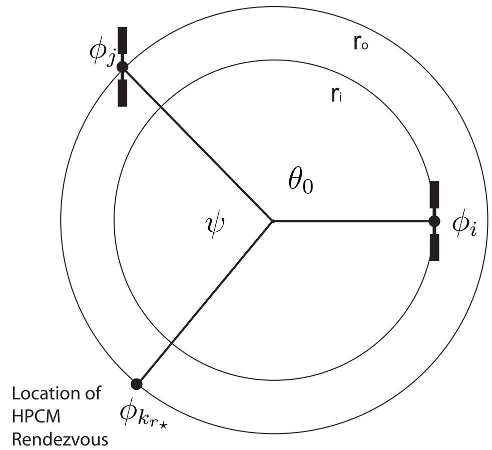


Figure 14: Hohmann-Phasing Cooperative Maneuver ($r_\star = r_o$).

$r_\star$, where $r_\star = r_i$ or $r_\star = r_o$. In this section, we consider the four cases of Phasing maneuvers and find the locations on $r_\star$ for which the corresponding HPCM rendezvous is cheaper (in terms of ΔV) than a cooperative maneuver on an intermediate orbit. According to the previous discussion, these are exactly the locations for which a HPCM rendezvous is feasible.

First, note the following expressions:

$$\lfloor T/T_\star - 1 \rfloor \leq \ell \leq \lfloor T/T_\star \rfloor \quad \text{if} \quad \psi \leq 0, \tag{32}$$

and

$$\lfloor T/T_\star \rfloor \leq \ell \leq \lfloor T/T_\star + 1 \rfloor \quad \text{if} \quad \psi \geq 0. \tag{33}$$

We therefore have,

$$\begin{aligned}
\frac{\Delta V_c(r) - \overline{\Delta V}_c(r_\star)}{\sqrt{2\mu}} &= \frac{\Delta V_c(r) - \Delta V_c^H(r)}{\sqrt{2\mu}} + \frac{\Delta V_c^H(r) - \Delta V_{nc}^H}{\sqrt{2\mu}} + \frac{\Delta V_{nc}^H - \overline{\Delta V}_c(r_\star)}{\sqrt{2\mu}} \\
&= \eta(r) + \zeta(r) - \frac{\overline{\Delta V}_c(r_\star) - \Delta V_{nc}^H}{\sqrt{2\mu}} \\
&= \eta(r) + \zeta(r) - \frac{\Delta V_p}{\sqrt{2\mu}} \qquad (34)
\end{aligned}$$

We are interested in finding the phasing angle such that $\Delta V_c(r) \geq \overline{\Delta V}_c(r_\star)$. It follows that

$$\eta(r) + \zeta(r) \geq \sqrt{\frac{2}{r_\star}} \left[\sqrt{2 - \left(\frac{\ell - 1}{\ell - \psi/2\pi} \right)^{2/3}} - 1 \right], \tag{35}$$

which gives

$$\sqrt{\frac{r_\star}{2}}\left(\eta\left(r\right)+\zeta\left(r\right)\right)\geq\sqrt{2-\left(\frac{\ell-1}{\ell-\psi/2\pi}\right)^{2/3}}-1. \tag{36}$$

Simple calculations lead to

$$\left(\frac{\ell-1}{\ell-\psi/2\pi}\right)\geq\left[2-\left(\sqrt{r_\star/2}\left(\eta(r)+\zeta(r)\right)+1\right)^2\right]^{3/2}. \tag{37}$$

This inequality yields

$$\frac{\psi}{2\pi}\geq\left(1-\frac{1}{\left[2-\left(\sqrt{r_\star/2}\left(\eta(r)+\zeta(r)\right)+1\right)^2\right]^{3/2}}\right)\ell+\frac{1}{\left[2-\left(\sqrt{r_\star/2}\left(\eta(r)+\zeta(r)\right)+1\right)^2\right]^{3/2}}. \tag{38}$$

Finally, using (5), the above inequality yields

$$\frac{\psi}{2\pi}\geq\lfloor T/T_\star\rfloor-\frac{\lfloor T/T_\star\rfloor-1}{\left[2-\left(\sqrt{r_\star/2}\left(\eta(r)+\zeta(r)\right)+1\right)^2\right]^{3/2}}. \tag{39}$$

Inequality (39) provides a lower bound $\psi_\ell^1(r)$ for the supersynchronous phasing angle. This lower bound is given by

$$\psi_\ell^1(r)=2\pi\left[1+\lfloor T/T_\star\rfloor\left(1-\frac{1}{\left[2-\left(\sqrt{r_\star/2}\left(\eta(r)+\zeta(r)\right)+1\right)^2\right]^{3/2}}\right)\right], \tag{40}$$

and determines the minimum value of the supersynchronous phasing angle that defines locations on $r_\star$ for which a HPCM rendezvous is feasible. Since for this case we have by definition $0\leq\psi\leq\pi$, the lower bound on the phasing angle is given by $\max\{0,\psi_\ell^1(r)\}$. Naturally, π represents an upper bound for the phasing angle. Similarly, for the case of a supersynchronous Phasing maneuver with $\psi<0$, we can show that

$$\frac{\psi}{2\pi}\geq\left(\lfloor T/T_\star\rfloor-1\right)\left(1-\frac{1}{\left[2-\left(\sqrt{r_\star/2}\left(\eta(r)+\zeta(r)\right)+1\right)^2\right]^{3/2}}\right) \tag{41}$$

ensures that a HPCM is cheaper than the optimal cooperative rendezvous on any orbit of radius $r \neq r_\star$. The above inequality imposes a lower bound on the supersynchronous phasing angle given by

$$\psi_\ell^2(r) \triangleq 2\pi\left(\lfloor T/T_\star \rfloor - 1\right)\left(1 - \frac{1}{\left[2 - \left(\sqrt{r_\star/2}\,(\eta(r) + \zeta(r)) + 1\right)^2\right]^{3/2}}\right). \tag{42}$$

For this case, we have $-\pi \leq \psi \leq 0$ by definition. The lower bound on the phasing angle is therefore given by $\max\{-\pi, \psi_\ell^2(r)\}$. Naturally, it follows by definition that the upper bound on the phasing angle is 0. In summary, for the case of a supersynchronous maneuver, the lower bound on the phasing angle is given by

$$\psi_\ell(r) = \begin{cases} \max\{0, \psi_\ell^1(r)\}, & \text{if } 0 \leq \psi \leq \pi, \\ \max\{-\pi, \psi_\ell^2(r)\}, & \text{if } -\pi \leq \psi \leq 0. \end{cases} \tag{43}$$

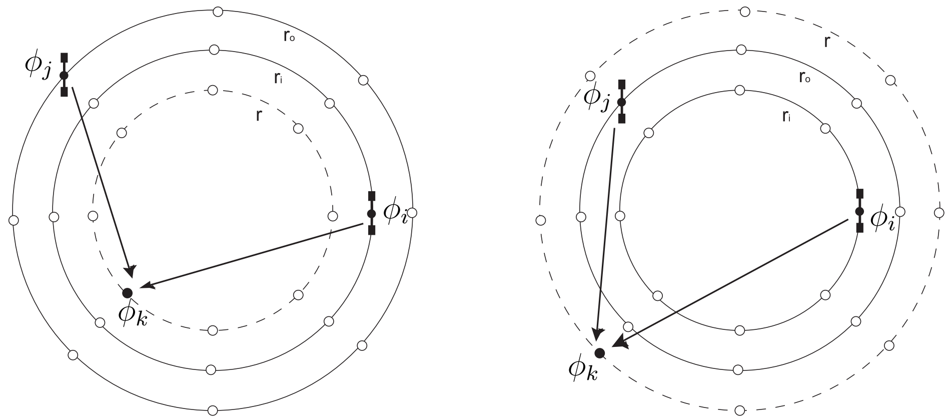
Note that the maximum value of $\psi_\ell(r)$ represents a lower bound on the phasing angle that defines the location on $r_\star$ for which a HPCM is optimal. Note also that the maximum for both $\psi_\ell^1(r)$ and $\psi_\ell^2(r)$ occurs when the quantity $\left(2 - \left(1 + \sqrt{r_\star/2}\,(\eta(r) + \zeta(r))\right)^2\right)$ is maximum, equivalently, when $\left(1 + \sqrt{r_\star/2}\,(\eta(r) + \zeta(r))\right)$ is minimum, which occurs when $\eta(r) + \zeta(r)$ is minimum.

For the case of a sub-synchronous maneuver with $\psi > 0$, we have

$$\frac{\psi}{2\pi} \leq \lfloor T/T_\star \rfloor - \frac{\left(\lfloor T/T_\star \rfloor + 1\right)}{\left[2 - \left(1 - \sqrt{r_\star/2}\,(\eta(r) + \zeta(r))\right)^2\right]^{3/2}} \tag{44}$$

as the corresponding condition that makes HPCM cheaper. The above inequality imposes an upper bound $\psi_u^3(r)$ on the subsynchronous phasing angle in a HPCM

$$\psi_u^3(r) \triangleq 2\pi\left(\lfloor T/T_\star \rfloor - \frac{\left(\lfloor T/T_\star \rfloor + 1\right)}{\left[2 - \left(1 - \sqrt{r_\star/2}\,(\eta(r) + \zeta(r))\right)^2\right]^{3/2}}\right), \tag{45}$$



(a) $r < r_i < r_o$ (b) $r_i < r_o < r$

Figure 15: Cooperative rendezvous.

such that a HPCM rendezvous is feasible. However, for this case, we have by definition, $0 \leq \psi \leq \pi$. Therefore, $\min\{\psi_u^3(r), \pi\}$ denotes the upper bound on the phasing angle, while the lower bound is zero. Finally, for the case of a sub-synchronous maneuver with $\psi < 0$, we can show that

$$\frac{\psi}{2\pi} \leq 1 + (\lfloor T/T_\star \rfloor - 1) \left(1 - \frac{1}{\left[2 - \left(1 - \sqrt{r_\star/2}\,(\eta(r) + \zeta(r))\right)^2\right]^{3/2}} \right) \tag{46}$$

is required to have a HPCM maneuver to be optimal. This inequality imposes an upper bound $\psi_u^4(r)$ on the subsynchronous phasing angle

$$\psi_u^4(r) \triangleq 2\pi \left[1 + (\lfloor T/T_\star \rfloor - 1) \left(1 - \frac{1}{\left[2 - \left(1 - \sqrt{r_\star/2}\,(\eta(r) + \zeta(r))\right)^2\right]^{3/2}} \right) \right], \tag{47}$$

which is a bound on the location on $r_\star$ for which a HPCM rendezvous is feasible. Since by definition, $-\pi \leq \psi \leq 0$, the upper bound on the phasing angle is given by $\min\{0, \psi_u^4(r)\}$, and the lower bound is given by $-\pi$. Therefore, by combining the two cases of subsynchronous maneuvers, we have the following expression

$$\psi_u(r) = \begin{cases} \min\{\pi, \psi_u^3(r)\}, & \text{if } 0 \leq \psi \leq \pi, \\ \min\{0, \psi_u^4(r)\}, & \text{if } -\pi \leq \psi \leq 0. \end{cases} \tag{48}$$

Note that the minimum of $\psi_u(r)$ over r represents an upper bound on the phasing angle that gives the position on $r_\star$ for which HPCM is feasible, hence also optimal. Note that the minimum of both $\psi_u^3(r)$ and $\psi_u^4(r)$ occurs when $\left(2 - \left(1 - \sqrt{r_\star/2}\,(\eta(r) + \zeta(r))\right)^2\right)$ is minimum, that is, when $\left(1 - \sqrt{r_\star/2}\,(\eta(r) + \zeta(r))\right)$ is maximum, which occurs when $\eta(r) + \zeta(r)$ is minimum.

The above analysis gives the location of the cooperative rendezvous of HPCM to be the cheapest rendezvous option between the two satellites.

2.6 Short Time to Rendezvous

In the previous sections it has been assumed that a phase-free Hohmann transfer is always possible between the orbits r_i and r_o. However, if the time allowed for rendezvous is sufficiently small, a Hohmann transfer between orbits r_i and r_o and vice versa becomes infeasible. In this section, we consider the case of short-time rendezvous between satellites s_μ and s_ν. We therefore assume that

$$T < \pi\sqrt{\frac{(r_i + r_o)^3}{8\mu}}. \tag{49}$$

Clearly, HPCM maneuvers are not possible in this case. However, HHCM maneuvers may be possible for some orbit $r \notin \{r_i, r_o\}$. Let us determine the orbits r for which HHCM maneuvers can occur for short-time rendezvous. To this end, let us investigate if the time T is sufficient for both satellites s_μ and s_ν to perform Hohmann transfers to an orbit of radius r. It follows from inequality (16) that

$$T < \pi\sqrt{\frac{(r + r_o)^3}{8\mu}}, \text{ for all } r \geq r_i. \tag{50}$$

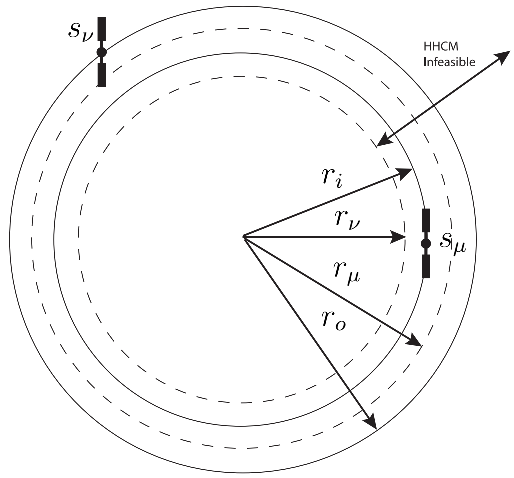


Figure 16: Short time of rendezvous: Feasibility of HHCM.

Therefore, satellite s_ν cannot perform a Hohmann transfer to an orbit of radius r if $r \geq r_i$. For the given time T, the satellite s_ν can nonetheless perform a Hohmann transfer from orbit r_o to an orbit r, provided $r \leq r_\nu$, where r_ν is defined as

$$r_\nu = \left(\frac{8\mu T^2}{\pi^2}\right)^{1/3} - r_o < r_i. \tag{51}$$

Similarly, for the given time T satellite s_μ can perform a Hohmann transfer from orbit r_i to an orbit of radius r, provided that $r \leq r_\mu$, where r_μ is defined as

$$r_\mu = \left(\frac{8\mu T^2}{\pi^2}\right)^{1/3} - r_i < r_o. \tag{52}$$

Note that $r_\nu < r_\mu < r_o$ and $r_\nu < r_i < r_o$. Hence, a HHCM rendezvous is feasible only for $r \leq r_\nu$. Consequently, if the optimal solution is a HHCM rendezvous, the location of rendezvous is at an orbit of radius $r \leq r_\nu$. Otherwise, the optimal rendezvous takes place on an orbit of radius $r > r_\nu$. Figure 16 shows the two satellites s_μ and s_ν in the orbits r_i and r_o respectively, along with the orbits r_μ and r_ν.

The results of the previous analysis are summarized in Table 1. In the table, T^H_{nc} denotes the time required for a non-cooperative Hohmann transfer between the satellites (which is a function of the initial separation angle θ_0) and T^H_{pf} denotes the time required for a phase-free Hohmann transfer.

Table 1: Summary of results.

Time of Rendezvous	Optimal Solution	Optimal Rendezvous Location
$T \geq T_{nc}^{H}(\theta_0)$	Non-Cooperative Hohmann Transfer	$r_\star = r_i$ or $r_\star = r_o$
$T_{pf}^{H} \leq T < T_{nc}^{H}(\theta_0)$	HPCM	$r_\star = r_i$ or $r_\star = r_o$
$T < T_{pf}^{H}$	Cooperative Rendezvous	$r_\star \leq r_\nu$ if HHCM

2.7 Fuel Expenditure During Cooperative Rendezvous

We have discussed so far only the minimization of total velocity change required for a cooperative rendezvous. In this section, we consider the minimization of the true objective, which is the fuel expenditure during the cooperative rendezvous between the satellites s_μ and s_ν. Let $m_{s\mu}$ and $m_{s\nu}$ denote the mass of the permanent structure of the satellites s_μ and s_ν respectively, while f_μ^- and f_ν^- denote the initial fuel content of satellites s_μ and s_ν, respectively. For the transfer of s_μ from ϕ_i to ϕ_{k_r}, let ΔV_{ik_r} denote the required velocity change. The fuel expenditure during the transfer is given by

$$p_{ik_r}^{\mu} = \left(m_{s\mu} + f_\mu^-\right)\left(1 - e^{-\frac{\Delta V_{ik_r}}{c_{0\mu}}}\right). \tag{53}$$

For the transfer of s_ν from ϕ_j to ϕ_ℓ, let $\Delta V_{j\ell}$ denote the required velocity change. The fuel expenditure during this transfer is given by

$$p_{jk_r}^{\nu} = \left(m_{s\nu} + f_\nu^-\right)\left(1 - e^{-\frac{\Delta V_{jk_r}}{c_{0\nu}}}\right). \tag{54}$$

The total fuel expenditure during the cooperative rendezvous between satellites s_μ and s_ν is therefore given by

$$p_{ik_r}^{\mu} + p_{jk_r}^{\nu} = \left(m_{s\mu} + f_\mu^-\right)\left(1 - e^{-\frac{\Delta V_{ik_r}}{c_{0\mu}}}\right) + \left(m_{s\nu} + f_\nu^-\right)\left(1 - e^{-\frac{\Delta V_{jk_r}}{c_{0\nu}}}\right), \tag{55}$$

and is a function of the location (slot ϕ_{k_r} of orbit of radius r) of the cooperative rendezvous. Now, let us assume that the minimum fuel expenditure occurs at the slot $\phi^\star$ of orbit of radius $r^\star$. We will denote all quantities associated with the optimal fuel expenditure by the subscript '$\star$'. In other words, we have

$$p^{\mu}_{ik_{r_\star}} + p^{\nu}_{jk_{r_\star}} \leq p^{\mu}_{ik_r} + p^{\nu}_{jk_r} \tag{56}$$

for all possible r and ϕ_{k_r}. Using (55), we have from (56),

$$\left(m_{s\mu} + f^-_\mu\right)\left(e^{-\frac{\Delta V_{ik_r}}{c_{0\mu}}} - e^{-\frac{\Delta V_{ik_{r_\star}}}{c_{0\mu}}}\right) + \left(m_{s\nu} + f^-_\nu\right)\left(e^{-\frac{\Delta V_{jk_r}}{c_{0\nu}}} - e^{-\frac{\Delta V_{jk_{r_\star}}}{c_{0\nu}}}\right) \leq 0 \tag{57}$$

Expanding the exponential term, and neglecting higher powers of $\Delta V/c_0 \ll 1^*$, we have

$$\left(m_{s\mu} + f^-_\mu\right)\left(\frac{\Delta V_{ik_{r_\star}}}{c_{0\mu}} - \frac{\Delta V_{ik_r}}{c_{0\mu}}\right) + \left(m_{s\nu} + f^-_\nu\right)\left(\frac{\Delta V_{jk_{r_\star}}}{c_{0\nu}} - \frac{\Delta V_{jk_r}}{c_{0\nu}}\right) \leq 0, \tag{58}$$

which reduces to

$$\frac{\left(m_{s\mu} + f^-_\mu\right)}{c_{0\mu}}\Delta V_{ik_{r_\star}} + \frac{\left(m_{s\nu} + f^-_\nu\right)}{c_{0\nu}}\Delta V_{jk_{r_\star}} \leq \frac{\left(m_{s\mu} + f^-_\mu\right)}{c_{0\mu}}\Delta V_{ik_r} + \frac{\left(m_{s\nu} + f^-_\nu\right)}{c_{0\nu}}\Delta V_{jk_r} \tag{59}$$

Note that the right-hand side of the above inequality is a function of r and ϕ_{k_r}. The inequality holds for all r and ϕ_{k_r}. Hence, we have

$$\min_{r,\phi_{k_r}}\left[\frac{\left(m_{s\mu} + f^-_\mu\right)}{c_{0\mu}}\Delta V_{ik_r} + \frac{\left(m_{s\nu} + f^-_\nu\right)}{c_{0\nu}}\Delta V_{jk_r}\right] = \frac{\left(m_{s\mu} + f^-_\mu\right)}{c_{0\mu}}\Delta V_{ik_{r_\star}} + \frac{\left(m_{s\nu} + f^-_\nu\right)}{c_{0\nu}}\Delta V_{jk_{r_\star}} \tag{60}$$

We therefore conclude that the total fuel expenditure is minimized at the location where a weighted sum of ΔV is minimized, the weights being a ratio of mass and specific impulse for each satellite. If this ratio is the same for the two satellites, that is, $m_{s\mu}/c_{0\mu} = m_{s\mu}/c_{0\mu}$, then the minimum fuel expenditure during cooperative rendezvous is equivalent to minimizing the total ΔV. Furthermore, if the satellites

*This assumption is justified because a typical value of $c_0 = 2943$ m/s and the ΔV requirement for the transfers would be much smaller (of the order of 10 m/s).

have the same engine characteristics and nearly the same mass, minimizing fuel is the same as minimizing total ΔV.

Note that for a cooperative rendezvous on an orbit of radius r, both satellites s_μ and s_ν must have enough fuel to complete the rendezvous at an orbital slot ϕ_{k_r} on the orbit r. Next, we determine the necessary conditions for the feasibility of a cooperative rendezvous at a slot ϕ_{k_r} on the orbit r.

For satellite s_μ to be able to complete the rendezvous, we must have

$$p_{ik_r} \leq f_\mu^-, \tag{61}$$

which, under the assumption $\Delta V/c_0 \ll 1$, implies

$$\frac{\left(m_{s\mu} + f_\mu^-\right)}{c_{0\mu}} \Delta V_{ik_r} \leq f_\mu^-. \tag{62}$$

Similarly, for the satellite s_ν to be able to complete the rendezvous, we must have

$$p_{jk_r} \leq f_\nu^-, \tag{63}$$

which, under the assumption $\Delta V/c_0 \ll 1$, yields

$$\frac{(m_{s\nu} + f_\nu^-)}{c_{0\nu}} \Delta V_{jk_r} \leq f_\nu^-. \tag{64}$$

Equations (62) and (64) imply that the radius r of the orbit for the cooperative rendezvous to take place is bounded above and below by $r_\ell \leq r \leq r_u$. Hence, minimizing the total fuel is equivalent to minimizing the weighted sum of ΔV over all locations of all orbits of radius r such that $r_\ell \leq r \leq r_u$.

Assuming the orbits r_i and r_o are close enough, that is $r_o - r_i \ll r_i$, we can derive explicit expressions for r_ℓ and r_u. To this end, let us consider the transfer of s_μ from the orbit r_i to some orbit $r > r_i$. For a given amount of fuel, the highest orbit the satellite s_μ can transfer to is the one given by a Hohmann transfer. The velocity change for a Hohmann transfer from orbit r_i to r is given by

$$\Delta V_{ik_r} = \Delta r_u \sqrt{\frac{\mu}{r_i^3}}, \tag{65}$$

where $\Delta r_u = r - r_i$ and where we have assumed that $\Delta r_u / r_i \ll 1$. Using (65), we obtain from (62),

$$\Delta r_u \leq \frac{f_\mu^-}{\left(m_{s\mu} + f_\mu^-\right)} c_{0\mu} \sqrt{\frac{r_i^3}{\mu}}. \tag{66}$$

This expression yields the upper bound r_u as follows

$$r_u = r_i + \frac{f_\mu^-}{\left(m_{s\mu} + f_\mu^-\right)} c_{0\mu} \sqrt{\frac{r_i^3}{\mu}}. \tag{67}$$

Let us now consider the transfer of s_ν from the orbit of radius r_o to the orbit $r < r_o$. For a given amount of fuel, the lowest orbit the satellite s_ν can transfer to is a Hohmann transfer from r_o and r. Letting $\Delta r_\ell = r - r_o$, and assuming again that $\Delta r_\ell / r_o \ll 1$, we have,

$$\Delta V_{jk_r} = \frac{1}{2} \Delta r_\ell \sqrt{\frac{\mu}{r_o^3}}. \tag{68}$$

Using the above expression, we obtain from (64),

$$\Delta r_\ell \leq 2 \frac{f_\nu^-}{\left(m_{s\nu} + f_\nu^-\right)} c_{0\nu} \sqrt{\frac{r_o^3}{\mu}}, \tag{69}$$

which yields the following expression for the lower bound r_ℓ as follows

$$r_\ell = r_o - 2 \frac{f_\nu^-}{\left(m_{s\nu} + f_\nu^-\right)} c_{0\nu} \sqrt{\frac{r_o^3}{\mu}}. \tag{70}$$

In summary, a cooperative rendezvous is feasible at an orbit of radius r if and only if $r_\ell \leq r \leq r_u$. If $r_\ell > r_i$ and $r_u > r_o$, none of the non-cooperative rendezvous are feasible and the rendezvous has to be cooperative.

From the above analysis (recall also (60), we find that the fuel expenditure is minimized when the weighted sum

$$\frac{\left(m_{s\mu} + f_\mu^-\right)}{c_{0\mu}} \Delta V_{ik_r} + \frac{\left(m_{s\nu} + f_\nu^-\right)}{c_{0\nu}} \Delta V_{jk_r} \tag{71}$$

is minimized for all $r_\ell \leq r \leq r_u$. Assume now that the satellites s_μ and s_ν perform a HHCM rendezvous at an orbit of radius r. For $r_i \leq r \leq r_o$, the total ΔV required for the HHCM rendezvous remains roughly constant, say, ΔV_0. For similar satellites, that is, $m_{s\mu} = m_{s\nu} = m_s$ and $c_{0\mu} = c_{0\nu}$, we have for the expression in (71)

$$\frac{m_s + f_\nu^-}{c_0}\Delta V_0 + \frac{f_\mu^- - f_\nu^-}{c_0}\Delta V_{ik_r} = \frac{m_s + f_\mu^-}{c_0}\Delta V_0 + \frac{f_\nu^- - f_\mu^-}{c_0}\Delta V_{jk_r}. \tag{72}$$

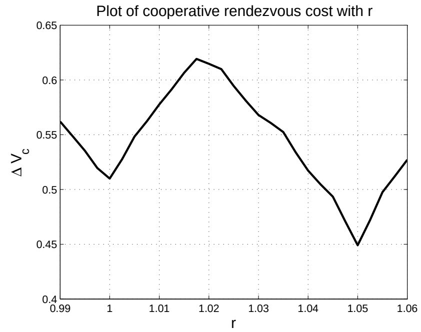
If $f_\mu^- < f_\nu^-$, the above expression is minimized when ΔV_{ik_r} is maximized, which occurs at $r = r_\mu$. Similarly, if $f_\nu^- < f_\mu^-$, the above expression is minimized when ΔV_{jk_r} is maximized, which occurs at $r = r_\nu$. In either case, the fuel-deficient satellite moves as close to the fuel-sufficient satellite as possible. This is a particularly important case for the refueling problem because refueling typically takes place towards the end of fuel life-time of the satellite. Hence, it is likely that the fuel-deficient satellites would be almost depleted of fuel. In such a case, even if enough time is permitted for Hohmann transfers to take place, the optimal rendezvous has to be cooperative, at an orbit of radius $r = r_\nu$ or $r = r_\mu$.

2.8 Numerical Example

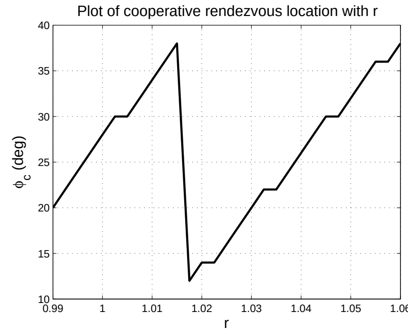
In this section, we first consider an example of a cooperative rendezvous between two satellites in two different circular orbits. According to the previous developments, the terminal orbit of the satellites at the end of the cooperative maneuver is assumed to be circular as well. With the help of this example, we illustrate that the optimal rendezvous that minimizes the total ΔV, is either a non-cooperative Hohmann transfer or a cooperative maneuver that is comprised of a Hohmann transfer and a Phasing maneuver, provided there is sufficient time to perform a phase-free Hohmann transfer between the orbits r_i and r_o.

Example 1. *Cooperative rendezvous between two satellites in different circular orbits.*

Let $r_i = 1$, $r_o = 1.05$ and $\theta_0 = 60\,\text{deg}$. First we determine the optimal cooperative rendezvous for a time-of-flight less than the one necessary for a Hohmann



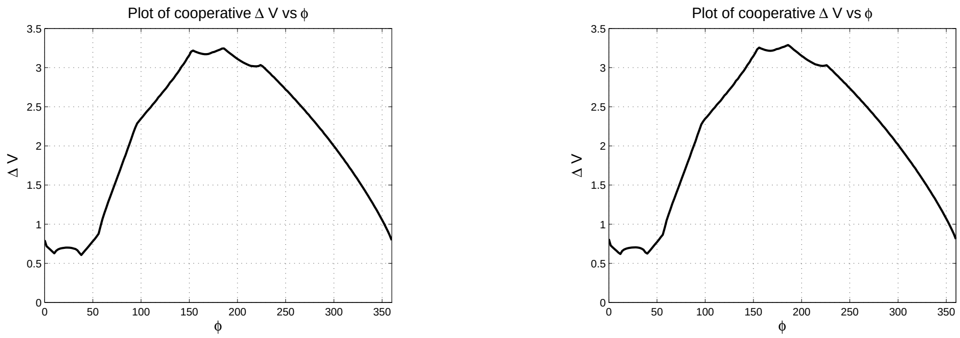
(a) Cooperative rendezvous cost



(b) Optimal location for rendezvous

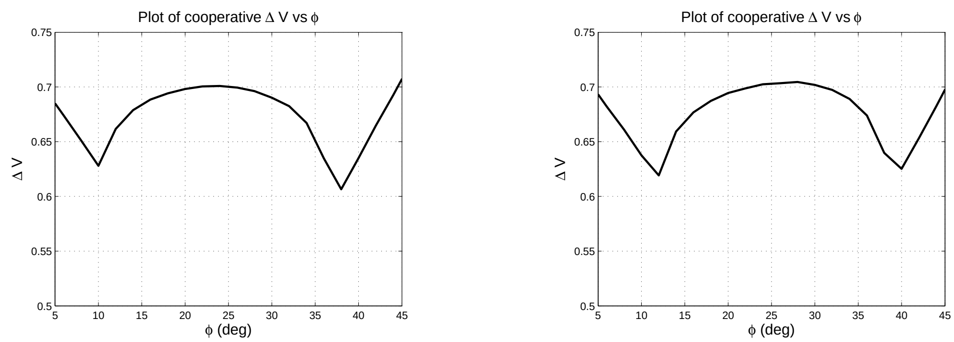
Figure 17: Case study ($r_i = 1, r_o = 1.05, \theta = 60\,\mathrm{deg}, T = 1.5$).

transfer for either one of the non-cooperative maneuvers. For this example, a non-cooperative Hohmann transfer for which s_μ is the active satellite becomes possible when $t = 2.6290$. The other non-cooperative Hohmann transfer, in which s_ν is the active satellite, becomes possible at $t = 3.1479$. In other words, if $t < 2.6290$, non-cooperative Hohmann transfers are not feasible between the two satellites. We therefore consider the time for rendezvous to be $t = 1.50$. We determine the total cost (ΔV) of a cooperative rendezvous for all possible slots and compute the minimum. We consider cooperative rendezvous to occur in orbits of radius r, where $0.98 \leq r \leq 1.07$. This allows us to consider all three cases of cooperative rendezvous, namely (i) $r < r_i < r_o$, (ii) $r_i < r < r_o$ and (iii) $r_i < r_o < r$. Figure 17(a) shows the variation of cooperative ΔV with the radius r of the orbit. On each orbit of radius r where the cooperative rendezvous takes place, there is an optimal location that yields the minimum ΔV for that particular orbit. Figure 17(b) depicts the variation of the optimal position of cooperative rendezvous $\phi_c(r)$ with r. The plot shows a discontinuity in the optimal rendezvous position as the value of r changes from $r = 1.0150$ to $r = 1.0175$. To investigate the reason for this discontinuity, let us consider the variation of $\Delta V_{ij}^c|_{k_r}$ over various slots Φ_r of a given intermediate orbit. Figure 18(a) shows such a variation for the orbit with radius $r = 1.0150$, while Fig. 18(b) shows



(a) $r = 1.0150$ (b) $r = 1.0175$

Figure 18: Explaining the discontinuity: Competing local minima.

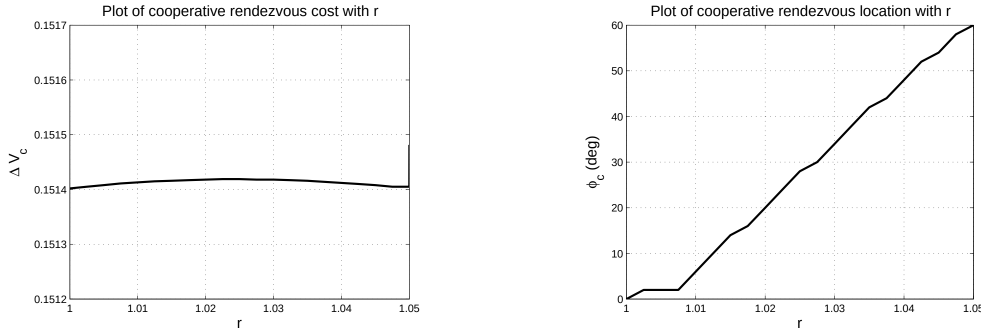


(a) $r = 1.0150$ (b) $r = 1.0175$

Figure 19: Explaining the discontinuity: Detail of the two competing local minima.

the same for the orbit with radius $r = 1.0175$. A detailed view of the two competing local minima is shown in Fig. 19.

Both these plots show two local minima that compete with each other for the cheapest solution for cooperative rendezvous on that particular orbit r. Each of these local minima corresponds to a cooperative maneuver in which one of the orbital transfers is a Hohmann transfer. As r changes from $r = 1.0150$ to $r = 1.0175$, there is a change of optimal cooperative rendezvous from one local minimum to the other. This shift of the optimal position appears as a discontinuity in the plot of ϕ. Naturally, there is no discontinuity in the variation of ΔV.

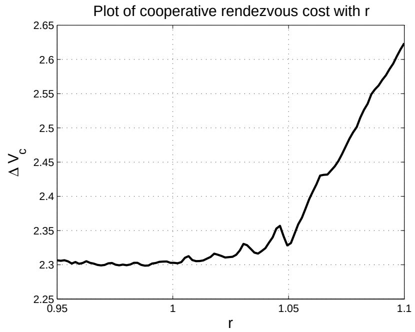


(a) Cooperative rendezvous cost (b) Optimal location for rendezvous

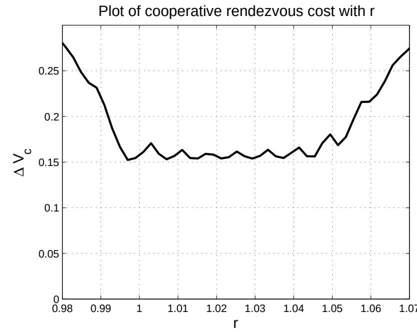
Figure 20: Case study ($r_i = 1, r_o = 1.05, \theta = 60\,\text{deg}, T = 3.0$).

Referring back to Fig. 17(a), we see that the minimum ΔV for cooperative rendezvous occurs at one of the orbits $r = r_i$ or $r = r_o$. The optimal cooperative rendezvous on orbit r_i occurs at the slot $\phi = 28\,\text{deg}$, while the optimal cooperative rendezvous on orbit r_o occurs at the slot $\phi = 32\,\text{deg}$. Calculations of the feasible slots for Hohmann transfers indicate that Hohmann transfers are possible for slots $\phi = 28.28\,\text{deg}$ to $\phi = 53.21\,\text{deg}$ on orbit r_i, while Hohmann transfers are possible for slots $\phi = 6.39\,\text{deg}$ to $\phi = 31.32\,\text{deg}$ on orbit r_o. These are obtained by calculating the lead angles necessary for a Hohmann transfer, as given by equation (15). Because of the discretization used for our calculation of slots at intervals of $2\,\text{deg}$, we obtain the optimal rendezvous locations at $\phi = 28\,\text{deg}$ (instead of $\phi = 28.28\,\text{deg}$) on orbit r_i and at $\phi = 32\,\text{deg}$ (instead of $\phi = 31.32\,\text{deg}$) on orbit r_o. The results indicate that the optimal rendezvous locations on orbits r_i and r_o occur near the slots where Hohmann transfers are possible, indicating that the optimal cooperative rendezvous is indeed a Hohmann-Phasing cooperative maneuver. Hence, when the time of rendezvous does not allow for a Hohmann non-cooperative transfer, the best possible cooperative maneuver is found to be comprised of a Hohmann transfer and a Phasing maneuver.

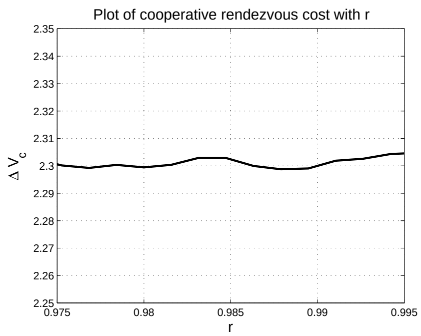
Next, we investigate the optimal cooperative rendezvous between two satellites



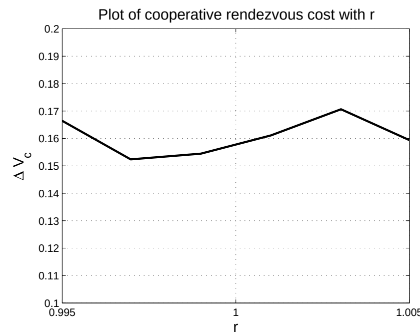
(a) $\theta = 60\,\text{deg}$, $T = 0.50$



(b) $\theta = 7\,\text{deg}$, $T = 0.518$



(c) $\theta = 60\,\text{deg}$, $T = 0.50$ (detail)

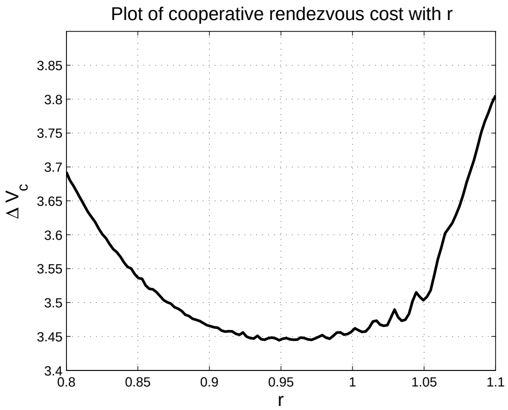


(d) $\theta = 7\,\text{deg}$, $T = 0.518$ (detail)

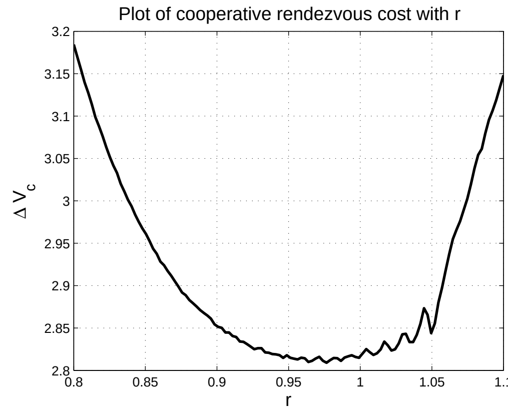
Figure 21: Case study for small time of flight ($r_i = 1, r_o = 1.05$).

for a time of maneuver $T = 3.0$ that allows for non-cooperative rendezvous using Hohmann transfers. Figure 20(a) shows the variation of cooperative ΔV with the radius r of the orbit where the cooperative rendezvous takes place. Figure 20(b) depicts the variation of the optimal position of the cooperative rendezvous $\phi_c(r)$ with r. It is found that the non-cooperative Hohmann transfers provide the best ΔV for the rendezvous of the two satellites. In summary, this numerical example shows that the optimal rendezvous between two satellites in different orbits is either non-cooperative Hohmann or it is a cooperative maneuver comprised of a Hohmann and a Phasing maneuver.

Let us now consider a time for the rendezvous T, so that a phase-free Hohmann transfer between orbits r_i and r_o is not possible. For our example, if $T < 0.519$ a



(a) $r_i = 1.00$, $r_o = 1.05$, $\theta_0 = 60\,\text{deg}$, $T = 0.40$



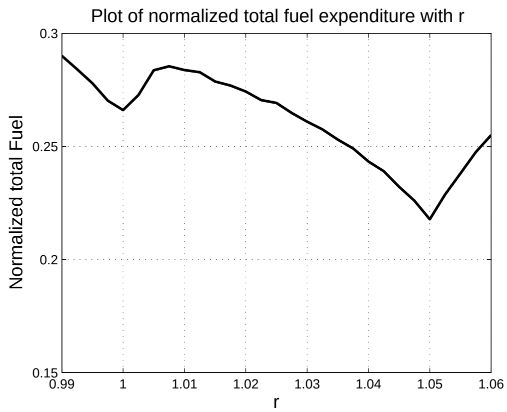
(b) $r_i = 1.00$, $r_o = 1.05$, $\theta_0 = 60\,\text{deg}$, $T = 0.45$

Figure 22: Optimal cooperative (but non-HHCM) rendezvous for short time of flight.

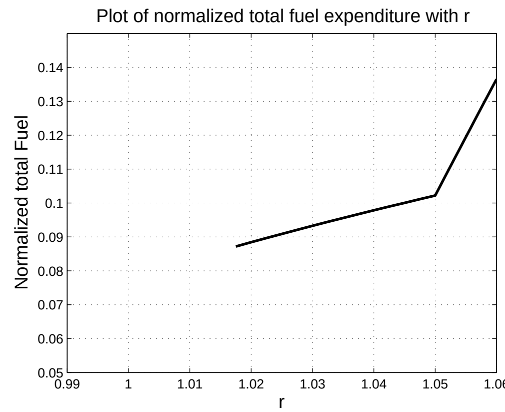
Hohmann transfer between r_i and r_o is not possible, so we let $T = 0.50$. In this case $r_\nu = 0.95$, so that HHCM maneuvers are not possible at any orbit of radius $r > 0.95$. The optimal rendezvous is cooperative and occurs at the orbit of radius $r = 0.9879$ (not a HHCM rendezvous). However, there are cases when the optimal solution is a HHCM rendezvous. For instance, when $r_i = 1$, $r_o = 1.05$, $\theta_0 = 7\,\text{deg}$ and $T = 0.518$, we have $r_\nu = 0.9975$ and the optimal maneuver is a HHCM rendezvous. The optimum occurs at the orbit of radius $r = 0.9975$, when the HHCM maneuver just becomes feasible. Figures 21(a) and 21(b) show the variation of ΔV for both of these cases. The optimal rendezvous in either case occurs at an orbit other than r_i or r_o. Figures 21(c) and 21(d) show the detail of the region where the minimum occurs. Note that the function is relatively flat in this region. We may use this result to compute (analytically) HHCM maneuvers that are only slightly suboptimal.

In order to confirm the occurrence of cooperative (but not HHCM) rendezvous when the time to rendezvous is very short, we repeated the analysis for the cases $T = 0.40$ and $T = 0.45$. The results are shown in Fig. 22. In both cases the HHCM maneuver is sub-optimal. In the first case the optimum occurs at $r = 0.9487$ whereas

$r_\nu = 0.6735$. In the second case the optimum occurs at $r = 0.9765$ whereas $r_\nu = 0.8143$. Note that the optimal cooperative maneuver in either case is substantially cheaper than the corresponding HHCM solution.



(a) Sufficient Fuel Case



(b) Insufficient Fuel Case

Figure 23: Variation of fuel with r.

Thus far we only considered the minimization of ΔV. Let us now consider the fuel expenditure during the cooperative rendezvous between the satellites in orbits $r_i = 1$ and $r_o = 1.05$ and with angle of separation $\theta_0 = 60$ deg. Fig. 23 shows the variation of fuel expenditure with r. The fuel expenditure has been normalized by dividing the total fuel expenditure by the maximum of the fuel capacities of the satellite. In the first case, the satellites have 25 and 5 units of fuel and the time for rendezvous is $T = 1.5$. The fuel-deficient satellite has enough fuel to complete the non-cooperative rendezvous. The plot shows that there are indeed two local minimum at $r = 1$ and $r = 1.05$, that is, at the end orbits. In this case, the fuel is minimized at the same location as the total ΔV and the optimal rendezvous is a HPCM. In the second case, the time of rendezvous is $T = 3.0$ and the fuel content of the satellites are 25 and 1.3 units respectively. Had the fuel-deficient satellite enough fuel to complete a non-cooperative Hohmann transfer, the optimal rendezvous would have taken place at $r = 1.00$. However, the 1.3 units of fuel is not sufficient for the fuel-deficient satellite to complete a non-cooperative Hohmann transfer. Consequently, the optimal rendezvous takes place at $r = 1.0175$. The fuel-deficient satellite uses all of its fuel in order to transfer to an orbit that is as close as possible to the fuel-sufficient satellite. The optimal rendezvous is HHCM.

2.9 Summary

In this chapter, we discussed the problem of determining optimal time-fixed, impulsive rendezvous. First, we looked at the case of two-impulse non-cooperative rendezvous. Next, we studied the problem of cooperative rendezvous between two satellites in circular orbits. We assume that the terminal orbit for each rendezvous maneuver is circular. We have specifically looked at cooperative maneuvers that are comprised of two Hohmann transfers (HHCM), or a Hohmann transfer and a Phasing maneuver (HPCM). If the time of maneuver allows for a non-cooperative Hohmann transfer, the optimal solution is non-cooperative. When the time to rendezvous is not sufficient for a non-cooperative Hohmann transfer between the satellites, the optimal rendezvous that yields the minimum ΔV is the Hohmann-Phasing cooperative maneuver. However, in both these cases, we assume that the time of transfer is sufficient for a (phase-free) Hohmann transfer to take place between the orbits of the satellites. If the time to complete the rendezvous is too short, then a cooperative rendezvous at a lower orbit is the optimal candidate, and it may or may not be a HHCM rendezvous. Recognizing that the real objective to minimize in an orbital transfer problem is fuel, we also discuss the problem of minimizing fuel expenditure during cooperative rendezvous.

CHAPTER III

ON-ORBIT REFUELING STRATEGIES

In the previous chapter, we discussed the problem of two-impulse time-fixed orbital transfers. During a refueling mission, several such orbital maneuvers would be necessary to deliver fuel to the fuel-deficient satellites in a constellation. Fuel is expended during these orbital maneuvers. Hence, we would like to minimize the total fuel spent during all maneuvers. In this chapter, we discuss the problem of determining the optimal set of orbital transfers that incur the minimum fuel expenditure during a complete refueling mission. To this end, we will look at both the Single Service Vehicle (SSV) and the Peer-to-Peer (P2P) refueling strategies, and illustrate how the optimal maneuvers required for refueling can be determined in either case. Finally, we discuss the mixed refueling strategy, which combines the ideas of SSV and P2P refueling.

3.1 Single Service Vehicle (SSV) Refueling Strategy

During a SSV strategy, a service vehicle visits the satellites one by one to deliver fuel to them. Let a service vehicle S_0 visits n satellites in the order $s_{\ell_1}, s_{\ell_2}, \ldots, s_{\ell_n}$, where $\ell_i \in \mathcal{I} = \{1, 2, \ldots, n\}$. For instance, Figure 24 shows a sequence of orbital transfers by the service vehicle that visits 6 satellites in a constellation in order to deliver fuel. For simplicity, let us consider that the service vehicle S_0 is already in rendezvous with the first satellite s_{ℓ_1} in the sequence. Let the total time given for S_0 to visit the remaining satellites in the sequence be T. Also, let the transfer time required by S_0 to move from s_{ℓ_i} to $s_{\ell_{i+1}}$ be given by $t_{i,i+1}$. Clearly, we must have

$$\sum_{i=1}^{n-1} t_{i,i+1} \leq T. \tag{73}$$

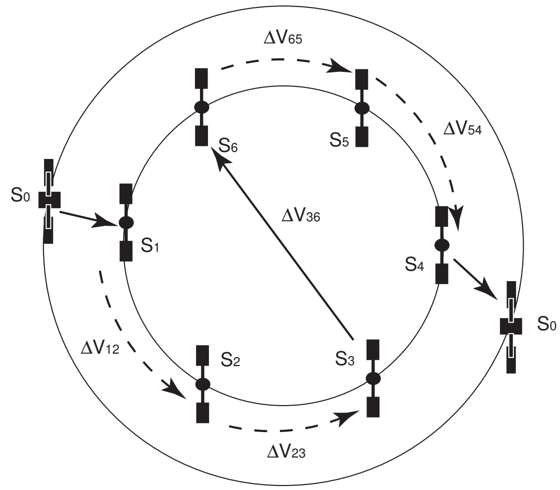


Figure 24: Single Service Vehicle Refueling Strategy.

Now, let $\Delta V_{\ell_i,\ell_{i+1}}$ denotes the velocity change required by S_0 to transfer from s_{ℓ_i} to $s_{\ell_{i+1}}$. Hence the total velocity change that is incurred during the orbital transfers is given by

$$\sum_{i=1}^{n-1} \Delta V_{\ell_i,\ell_{i+1}}. \tag{74}$$

We would like to minimize this objective function subject to the constraint given in (73).

Recall that, for an individual transfer, the velocity change is a non-increasing function of time with the allowance of coasting. Figure 25 shows the variation of $\Delta V_{\ell_i,\ell_{i+1}}$ with time. Inspection of the plot reveals that the total time can be divided into alternating cost-decreasing and cost-invariant intervals. Let $j_{i,\max}$ denotes the number of cost-decreasing intervals that is possible for the maximum time T available for the transfer. Let A_{ij-1} denotes the time at which j^{th} cost-reducing interval starts, while β_{ij} denotes the time at which the j^{th} cost-invariant interval starts. Clearly, $j = 1, 2, \ldots, j_{i,\max}$ and

$$j_{i,\max} = \max\{j : A_{ij-1} \leq T\} \tag{75}$$

Typically, for any j, $\beta_{ij} - A_{ij-1}$ is small compared to $A_{ij} - A_{ij-1}$, and also the difference increases with increasing j. In fact, as the transfer time increases, $\beta_{ij} - A_{ij-1} \to 0$.

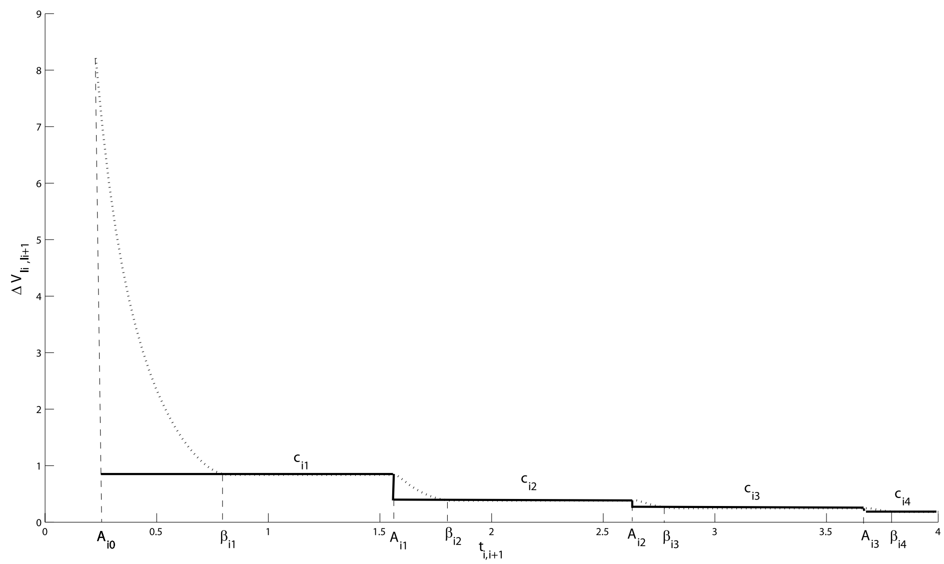


Figure 25: Step Function Approximation.

We may therefore approximate the $\Delta V_{\ell_i,\ell_{i+1}}$ with a series of step functions as shown in Figure 25. Let us now introduce binary variables as follows:

$$x_{ij} = \begin{cases} 1 & \text{if } t_{i,i+1} \in [A_{ij-1} A_{ij}]\,,\ j = 1, 2, \ldots, j_{i,\max}\ , \\ 0 & \text{otherwise.} \end{cases} \tag{76}$$

That is, if the time for the transfer from satellite s_{ℓ_i} to $s_{\ell_{i+1}}$ belongs to the interval $[A_{ij-1}A_{ij}]$, then the corresponding decision variable x_{ij} is 1. Otherwise, the decision variable is 0.

Let us assign a cost c_{ij} to each decision variable x_{ij}. The cost equals the value of the step function corresponding to the interval $[A_{ij-1}, A_{ij}]$. We can therefore determine the optimal time distribution for the trips made by S_0 by solving the integer program:

$$\min \sum_{i=1}^{n-1} \sum_{j=1}^{j_{i,\max}} c_{ij} x_{ij}, \tag{77}$$

subject to the following constraints

$$\sum_{j=1}^{j_{i,\max}} x_{ij} = 1, \text{ for all } i = 1, 2, \ldots, n-1, \tag{78}$$

$$\sum_{i=1}^{n-1} \sum_{j=1}^{j_{i,\max}} A_{ij} x_{ij} \geq T, \tag{79}$$

$$\sum_{i=1}^{n-1} \sum_{j=0}^{j_{i,\max-1}} A_{ij} x_{ij} \leq T. \tag{80}$$

Constraint (78) means that a transfer time has to be assigned to each of the $n-1$ transfers, while Constraints (79) and (80) signifies that all transfers must be completed within the given time T. The solution of this integer program yields the optimal time distribution for the SSV strategy for a particular sequence. Extensive numerical studies, performed in Ref. 74, indicates that of all possible sequences in which the service S_0 visits the satellites, the optimal sequence is sequential or bi-sequential.

3.2 Peer-to-Peer (P2P) Refueling Strategy

A P2P maneuver refers to two satellites, one fuel-sufficient and the other fuel-deficient, engaging in a fuel transaction. Currently, we consider that the fuel exchange takes place at the orbital slots of one of these satellites, that is, only one of them (active) performs the orbital transfer necessary for a rendezvous with the other (passive). The fuel exchange takes place at the orbital slot of the passive satellite and after it is over, the active satellite returns to its original orbital slot. We assume that a satellite can engage itself in at most one P2P maneuver. Given a constellation with satellites having different amounts of fuel, a set of such P2P maneuvers can be utilized to attain fuel equalization among the satellites. Under the notion of fuel equalization, we consider that the satellites involved in a P2P maneuver exchange an amount of fuel such that at the end of the maneuver, both satellites end up with equal amount of fuel. In this chapter, we briefly discuss the formulation of the problem of P2P refueling based on this notion of fuel equalization.[19,72,75]

3.2.1 Formulation of the P2P refueling problem

Under the notion of fuel equalization, the principal objective of refueling is to keep the fuel content of the satellites as close to the mean fuel in the constellation. This can be achieved by minimizing the deviation of the fuel among all satellites in the constellation. Ideally, this deviation should be calculated with respect to f_{av}^{+} which is the mean fuel in the constellation after all the P2P maneuvers take place. However, we would like to spend as little fuel as possible in order to achieve fuel equalization. Minimizing the deviation of fuel content of satellites from f_{av}^{+} does not ensure that the fuel expenditures during the ensuing orbital maneuvers is small. Hence, we would measure the deviation with respect to f_{av}^{-} which is the initial mean fuel in the constellation. Keeping all the fuel content of satellites close to f_{av}^{-} ensures that fuel expenses during the refueling process is kept small. Equivalently, the following cost function can be maximized[72,75]

$$\mathcal{C}_a = -\sum_{\mu \in \mathcal{I}} |f_{\mu}^{+} - f_{\text{av}}^{-}|. \tag{81}$$

Maximization of the objective function given in (81) needs to be achieved via a set of P2P maneuvers that redistributes the fuel in the constellation. Under this formulation of P2P refueling problem, we consider those satellites to be fuel-sufficient which have greater than the average fuel in the constellation, that is, $\mathcal{I}_{s,0} = \{i : f_i^{-} \geq f_{\text{av}}^{-}\}$. The remaining satellites are fuel-deficient, that is, $\mathcal{I}_{d,0} = \{i : f_i^{-} < f_{\text{av}}^{-}\}$.

The optimization problem involved in P2P refueling is typically formulated using a constellation graph (Figure 26) in which the vertices represent the orbital slots and the edges represent a P2P refueling transaction. To this end, let us define the undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ whose nodes are given by the set $\mathcal{V} = \mathcal{J}$ that essentially corresponds to the index set of all orbital slots in the constellation. An (undirected) edge between a pair of orbital slots represents a P2P maneuver between the satellites initially occupying the respective slots. For instance, $\langle i, j \rangle \in \mathcal{E}$ will denote that

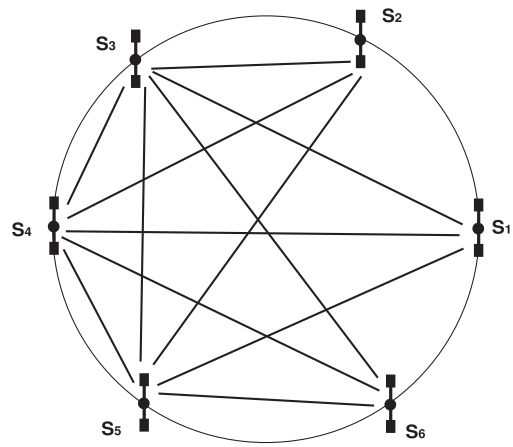


Figure 26: Constellation Graph.

the satellite $s_\mu = \sigma_0(\phi_i)$ initially occupying the orbital slot ϕ_i undergoes a refueling transaction with the satellite $s_\nu = \sigma_0(\phi_j)$ initially occupying the orbital slot ϕ_j. Each edge $\langle i, j\rangle \in \mathcal{E}$ has an associated cost which equals the fuel expenses incurred during the refueling process. Now, in an ensuing P2P maneuver between s_μ and s_ν, either of the two satellites can be active provided they both have enough fuel to carry out the orbital transfers. Let us denote the fuel expenditure incurred by satellite s_μ during an orbital transfer from ϕ_i to ϕ_j be denoted by p^μ_{ij}. Then, the fuel expenditure incurred during a P2P maneuver in which satellite s_μ transfers to the orbital slot of s_ν and comes back to its original position is given by

$$c^\mu_{ij} = p^\mu_{ij} + p^\mu_{ji}. \tag{82}$$

The amount of fuel spent by s_μ to rendezvous with s_ν is given by[72]

$$p^\mu_{ij} = (m_{\mathrm{s}\mu} + f^-_\mu)(1 - e^{-\Delta V_{ij}/c_{0\mu}}), \tag{83}$$

where $m_{\mathrm{s}\mu}$ is the mass of the permanent structure of satellite s_μ, ΔV_{ij} is the velocity change required the orbital transfer, and the parameter $c_{0\mu}$ is defined by $c_{0\mu} = g_0 I_{\mathrm{sp}\mu}$, g_0 being the acceleration due to gravity at the Earth's surface, and $I_{\mathrm{sp}\mu}$ is the specific thrust of satellite s_μ. The amount of fuel consumed by satellite s_μ to return back to its original position after a fuel exchange has taken place* is given by

$$p_{ji}^{\mu} = (2m_{s\mu} + f_{\mu}^{-} + f_{\nu}^{-} - p_{ij}^{\mu})\frac{(1 - e^{-\Delta V_{ji}/c_{0\mu}})}{(1 + e^{-\Delta V_{ji}/c_{0\mu}})}, \tag{84}$$

where ΔV_{ji} is the optimum rendezvous cost for the return journey. Similarly, the fuel expenditure incurred during a P2P maneuver in which satellite s_{ν} transfers to the orbital slot of s_{μ} and returns to its original position is given by

$$c_{ij}^{\nu} = p_{ji}^{\nu} + p_{ij}^{\nu}, \tag{85}$$

where the calculation of p_{ij}^{ν} and p_{ji}^{ν} are similar to the calculation of p_{ji}^{μ} and p_{ij}^{μ}.

In case when both satellites can be active, we will consider the refueling transaction that is cheaper. In other words, we assign to every edge $\langle i, j \rangle \in \mathcal{E}$ a unique cost c_{ij} given by

$$c_{ij} = \begin{cases} c_{ij}^{\mu}, & \text{if } s_{\mu} \text{ can be active, but } s_{\nu} \text{ cannot,} \\ c_{ij}^{\nu}, & \text{if } s_{\nu} \text{ can be active, but } s_{\mu} \text{ cannot,} \\ \min\{c_{ij}^{\mu}, c_{ij}^{\nu}\}, & \text{if either } s_{\mu} \text{ or } s_{\nu} \text{ can be active,} \\ \infty, & \text{if neither } s_{\mu} \text{ nor } s_{\nu} \text{ can be active.} \end{cases} \tag{86}$$

We are only concerned with edges that have finite cost, that is, edges that correspond to a feasible P2P fuel transaction. We therefore consider $\mathcal{E}$ to be consisting of edges that have finite cost. Also, for convenience, we define the neighbor $\mathcal{N}(i)$ of a node $i \in \mathcal{V}$ as the set of nodes that has an edge with i, that is, $\mathcal{N}(i) = \{j : \langle i, j \rangle \in \mathcal{E}\}$.

We are interested in a set $\mathcal{M} \subseteq \mathcal{E}$ of P2P maneuvers that would maximize the objective function $\mathcal{C}_a$. To this end, let us define a binary variable x_{ij} corresponding to each edge $\langle i, j \rangle \in \mathcal{E}$ as follows:

$$x_{ij} = \begin{cases} 1 & \text{if } \langle i, j \rangle \in \mathcal{M}, \\ 0 & \text{otherwise.} \end{cases} \tag{87}$$

*It is assumed that during the exchange of fuel the fuel-sufficient satellite gives enough fuel to the fuel-deficient satellite so that both have the same amount of fuel at the end of the fuel transaction.[75]

We will refer to any node $i \in \mathcal{V}$ as *matched* if there exists an edge $\langle i, j \rangle \in \mathcal{M}$. Otherwise, we will refer the node i as *unmatched*. The contribution of all matched vertices of $\mathcal{G}$ to $\mathcal{C}_a$ in equation (81) is easily computed as

$$- \sum_{\substack{\mu \in \mathcal{I} \\ s_\mu = \sigma_0(\phi_i)}} \sum_{\langle i,j \rangle \in \mathcal{N}(i)} |f_\mu^+ - f_{\mathrm{av}}^-| x_{ij}. \tag{88}$$

On the other hand, if satellite s_μ is not involved in a fuel transaction, then $f_\mu^+ = f_\mu^-$. The corresponding node in $\mathcal{G}$ does not have an edge that is part of the set $\mathcal{M}$. As a result, $x_{ij} = 0$ for all $\langle i, j \rangle \in \mathcal{N}(i)$. In fact, we have $x_{ij} = 0$ for all $\langle i, j \rangle \in \mathcal{E} \backslash \mathcal{M}$. The contribution to $\mathcal{C}_a$ from all unmatched vertices is

$$- \sum_{\substack{\mu \in \mathcal{I} \\ s_\mu = \sigma_0(\phi_i)}} \Big(1 - \sum_{\langle i,j \rangle \in \mathcal{N}(i)} x_{ij}\Big) |f_\mu^- - f_{\mathrm{av}}^-| = - \sum_{\mu \in \mathcal{I}} |f_\mu^- - \bar{f}^-| + \sum_{\substack{\mu \in \mathcal{I} \\ s_\mu = \sigma_0(\phi_i)}} \sum_{\langle i,j \rangle \in \mathcal{N}(i)} |f_\mu^- - f_{\mathrm{av}}^-| x_{ij}. \tag{89}$$

The term $\sum_{\mu \in \mathcal{I}} |f_\mu^- - f_{\mathrm{av}}^-|$ in the previous expression is constant, and thus it has no effect on the optimization process and hence can be neglected. From Equations (88) and (89), and summing up the contributions from all satellites, we finally have

$$\mathcal{C}_a' = \sum_{\substack{\mu \in \mathcal{I} \\ s_\mu = \sigma_0(\phi_i)}} \sum_{\langle i,j \rangle \in \mathcal{N}(i)} \big(|f_\mu^- - f_{\mathrm{av}}^-| - |f_\mu^+ - f_{\mathrm{av}}^-|\big) x_{ij}. \tag{90}$$

Recalling that each edge $\langle i, j \rangle \in \mathcal{E}$ has contributions from two vertices $i, j \in \mathcal{V}$ of the graph, and rewriting the summation in equation (90) as a summation over all edges in the constellation graph, the objective function to be maximized is given by

$$\mathcal{C}_a' = \sum_{\substack{\langle i,j \rangle \in \mathcal{E} \\ s_\mu = \sigma_0(\phi_i) \\ s_\nu = \sigma_0(\phi_j)}} \big(|f_\mu^- - f_{\mathrm{av}}^-| - |f_\nu^+ - f_{\mathrm{av}}^-| + |f_\nu^- - f_{\mathrm{av}}^-| - |f_\mu^+ - f_{\mathrm{av}}^-|\big) x_{ij} \tag{91}$$

Letting π_{ij} denote the coefficient of x_{ij} in the previous sum in (91), the problem becomes one of maximizing

$$\mathcal{C}_a' = \sum_{\langle i,j \rangle \in \mathcal{E}} \pi_{ij} x_{ij}. \tag{92}$$

subject to (87) and the following constraint

$$\sum_{\langle i,j\rangle\in\mathcal{N}(i)} x_{ij} \leq 1, \qquad i \in \mathcal{I}. \tag{93}$$

The constraint (93) ensures that each satellite is involved in at most one fuel transaction with another satellite. Since the objective of the refueling process is to equalize the fuel among all satellites in the constellation, we impose the constraint that after each fuel transaction between any pair of satellites, the two satellites end up with the same amount of fuel. In other words, we impose the condition that $f_\mu^+ = f_\nu^+$ for all satellite pairs s_μ and s_ν involved in a P2P refueling transaction. Noting that the difference between the total fuel in the satellites before and after refueling can be related to the total fuel burnt during the rendezvous,[75] one obtains

$$f_\mu^+ = f_\nu^+ = \frac{1}{2}(f_\mu^- + f_\nu^- - c_{ij}). \tag{94}$$

Using equation (94), the weight of each edge in the constellation graph becomes

$$\pi_{ij} = |f_\mu^- - f_{\mathrm{av}}^-| + |f_\nu^- - f_{\mathrm{av}}^-| - |f_\mu^- + f_\nu^- - 2f_{\mathrm{av}}^- - c_{ij}|, \tag{95}$$

where $s_\mu = \sigma_0(\phi_i)$ and $s_\nu = \sigma_0(\phi_j)$. Note that we can leave out all edges from the constellation graph which has negative cost. Hence, we can only consider a reduced constellation graph $\mathcal{G}_{\mathrm{r}} = (\mathcal{V}, \mathcal{E}_r)$ where $\mathcal{E}_{\mathrm{r}} = \{\langle i,j\rangle \in \mathcal{E} : \pi_{ij} > 0\}$. Given these weights on the edges of the reduced constellation graph $\mathcal{G}_{\mathrm{r}}$, we seek a matching $\mathcal{M}$ that will maximize the sum of the weights of all edges in $\mathcal{M}$. This is a standard maximum weight matching problem in graph theory.[28,85] The solution to this problem provides the pairs of satellites involved in the optimal distribution of fuel using a P2P refueling scheme. A similar optimization can also be done by using the square of the deviation of the fuel content of the satellites from the average,[19] instead of using the absolute value of the deviation.

Example 2. *A P2P refueling strategy for a constellation of* 14 *satellites.*[72]

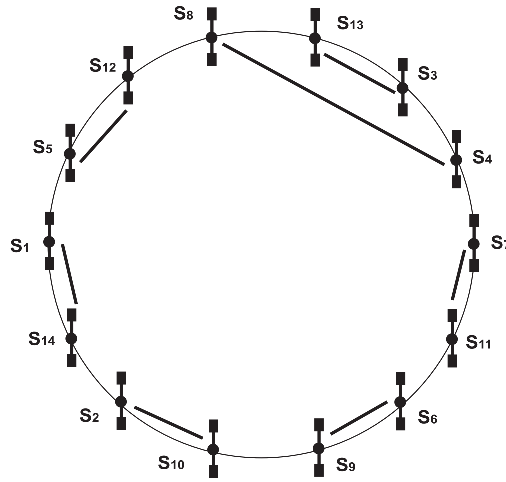


Figure 27: P2P refueling strategy in a constellation.

Consider a case of P2P refueling example in a constellation of 14 satellites with fuel contents 38.8, 36, 35.2, 32.8, 29.6, 27.6, 26.8, 17.6, 14, 8, 6.8, 6.4, 5.6 and 0.4 units. For each satellite, the mass of permanent structure for each of these satellites is 60 units, the specific thrust of each is 300 sec. The satellites are distributed evenly in the constellation as shown in Fig. 27. The altitude of the constellation is 500 Km. The allowed total time of refueling is $T = 12$ orbital periods. The average fuel in the constellation is 20.4 units, implying that satellites s_1, s_2, s_3, s_4, s_5,s_6 and s_7. The remaining satellites are fuel-deficient. The optimal P2P assignments for refueling satellites in this constellation are: $s_1 - s_{14}$, $s_2 - s_{10}$, $s_3 - s_{13}$, $s_4 - s_8$, $s_5 - s_{12}$, $s_6 - s_9$ and $s_7 - s_{11}$. The total fuel expenditure incurred in refueling is 30.1 units, and the final fuel content of the satellites are 17.7, 20.4, 18.8, 19.9, 16.4, 19.1, 15.3, 19.9, 19.2, 20.4, 15.3, 16.4, 18.8 and 17.7 units respectively.

3.3 A generalized cost function approach

The P2P refueling strategy, under the notion of fuel equalization, essentially has two objectives: (i) minimization of the fuel deviation among all satellites in the constellation, and (ii) minimization of the fuel expenditure during the orbital rendezvous transfers. Moreover, these two objectives are conflicting in nature. For instance, we can fulfil only the first objective by performing continuous orbital transfers until all satellites have the same amount of fuel (perhaps even null). On the other hand, we can satisfy only the second objective by not performing any orbital transfers at all. The cost function in equation (81) was introduced rather heuristically so that it *implicitly* takes into account both of these objectives. In this section we show that this rationale is valid. We do this by introducing an optimization criterion $\mathcal{C}_b$, that incorporates *explicitly* the previous two conflicting objectives, and by unraveling the relationship of the cost $\mathcal{C}_b$ with the cost $\mathcal{C}_a$ in equation (81).

Since we seek to minimize the fuel deviation among all satellites in the constellation at the end of the refueling process, we introduce the following cost function to be maximized

$$\mathcal{C}_1 = -\sum_{\mu \in \mathcal{I}} |f_\mu^+ - f_{\rm av}^+|^2. \tag{96}$$

Since we also want to minimize the cost incurred during the orbital maneuvers required for the fuel transfers, we also introduce the following cost to be maximized

$$\mathcal{C}_2 = -\sum_{\langle u,v\rangle \in \mathcal{M}} c_{uv}^2. \tag{97}$$

Given $\mathcal{C}_1$ and $\mathcal{C}_2$, we assign a relative weight between these two costs, and we combine them into a single cost function to be maximized, as follows

$$\mathcal{C}_b = \alpha \mathcal{C}_1 + (1-\alpha)\,\mathcal{C}_2, \tag{98}$$

where $0 \le \alpha \le 1$ takes care of the relative importance assigned to the two objectives.

The contribution to $\mathcal{C}_1$ from the satellites participating in fuel transactions is

$$-\sum_{\substack{\mu \in \mathcal{I} \\ s_\mu = \sigma_0(\phi_i)}} \sum_{\langle i,j\rangle \in \mathcal{N}(i)} |f_\mu^+ - f_{\rm av}^+|^2 x_{ij}. \tag{99}$$

The contribution to $\mathcal{C}_1$ from the satellites not participating in fuel transactions is

$$-\sum_{\substack{\mu \in \mathcal{I} \\ s_\mu = \sigma_0(\phi_i)}} \Big(1 - \sum_{\langle i,j\rangle \in \mathcal{N}(i)} x_{ij}\Big) |f_\mu^- - f_{\rm av}^+|^2. \tag{100}$$

Combining the contributions from the participating (matched) and nonparticipating (unmatched) satellites into (96), one obtains

$$\mathcal{C}_1 = -\sum_{\substack{\mu\in\mathcal{I}\\ s_\mu=\sigma_0(\phi_i)}}\sum_{\langle i,j\rangle\in\mathcal{N}(i)}|f_\mu^+ - f_{\text{av}}^+|^2 x_{ij} - \sum_{\substack{\mu\in\mathcal{I}\\ s_\mu=\sigma_0(\phi_i)}}|f_\mu^- - f_{\text{av}}^+|^2 + \sum_{\substack{\mu\in\mathcal{I}\\ s_\mu=\sigma_0(\phi_i)}}\sum_{\langle i,j\rangle\in\mathcal{N}(i)}|f_\mu^- - f_{\text{av}}^+|^2 x_{ij}. \tag{101}$$

The average fuel available in the constellation before and after refueling are related by

$$f_{\text{av}}^+ = f_{\text{av}}^- - \frac{1}{n}\sum_{\langle u,v\rangle\in\mathcal{M}} c_{uv}. \tag{102}$$

Using equation (102), we may rewrite equation (101) as

$$\mathcal{C}_1 = \sum_{\substack{\mu\in\mathcal{I}\\ s_\mu=\sigma_0(\phi_i)}}\sum_{\langle i,j\rangle\in\mathcal{N}(i)}\big(|f_\mu^- - f_{\text{av}}^-|^2 - |f_\mu^+ - f_{\text{av}}^-|^2 + \frac{2}{n}(f_\mu^- - f_\mu^+)\sum_{\langle u,v\rangle\in\mathcal{M}} c_{uv}\big)x_{ij} - \sum_{\mu\in\mathcal{I}}|f_\mu^- - f_{\text{av}}^+|^2. \tag{103}$$

A simple calculation yields

$$\begin{aligned}\sum_{\substack{\mu\in\mathcal{I}\\ s_\mu=\sigma_0(\phi_i)}}|f_\mu^- - f_{\text{av}}^+|^2 &= \sum_{\mu\in\mathcal{I}}\Big(|f_\mu^- - f_{\text{av}}^-|^2 + \frac{2}{n}(f_\mu^- - f_{\text{av}}^-)\sum_{\langle u,v\rangle\in\mathcal{M}} c_{uv}\Big)\\ &\quad + \frac{1}{n}\Big(\sum_{\langle u,v\rangle\in\mathcal{M}} c_{uv}^2 + \sum_{\langle u,v\rangle\in\mathcal{M}} c_{\nu\mu}\sum_{\langle m,k\rangle\in\mathcal{M}\setminus\langle u,v\rangle} c_{mk}\Big)\end{aligned}$$

Note that

$$\sum_{\substack{\mu\in\mathcal{I}\\ s_\mu=\sigma_0(\phi_i)}}(f_\mu^- - f_{\text{av}}^-) = 0.$$

Moreover, the term $\sum_{\mu\in\mathcal{I}}|f_i^- - \bar{f}^-|^2$ is constant for a given constellation, and plays no role in the optimization process. Excluding this constant term, we have

$$\sum_{\substack{\mu\in\mathcal{I}\\ s_\mu=\sigma_0(\phi_i)}}|f_\mu^- - f_{\text{av}}^+|^2 = \frac{1}{n}\Big(\sum_{\langle u,v\rangle\in\mathcal{M}} c_{uv}^2 + \sum_{\langle u,v\rangle\in\mathcal{M}} c_{\nu\mu}\sum_{\langle m,k\rangle\in\mathcal{M}\setminus\langle u,v\rangle} p_{mk}\Big).$$

Hence the cost function to be maximized can be written as

$$\begin{aligned}\mathcal{C}_b' &= \alpha\sum_{\substack{\mu\in\mathcal{I}\\ s_\mu=\sigma_0(\phi_i)}}\sum_{\langle i,j\rangle\in\mathcal{N}(i)}\Big(|f_\mu^- - f_{\text{av}}^-|^2 - |f_\mu^+ - f_{\text{av}}^-|^2 + \frac{2}{n}(f_\mu^- - f_\mu^+)\sum_{\langle u,v\rangle\in\mathcal{M}} c_{uv}\Big)x_{ij}\\ &\quad -\frac{\alpha}{n}\Big(\sum_{\langle u,v\rangle\in\mathcal{M}} c_{uv}^2 + \sum_{\langle u,v\rangle\in\mathcal{M}} c_{uv}\sum_{\langle m,k\rangle\in\mathcal{M}\setminus\langle u,v\rangle} c_{mk}\Big) - (1-\alpha)\sum_{\langle u,v\rangle\in\mathcal{M}} c_{uv}^2.\end{aligned} \tag{104}$$

Writing the above as a summation over the edges and using equation (102), it follows that the criterion to be maximized takes the form

$$
\begin{aligned}
\mathcal{C}'_b &= \alpha \sum_{\langle i,j\rangle\in\mathcal{E}} \Big(|f^-_\mu - f^-_{\text{av}}|^2 + |f^-_\nu - f^-_{\text{av}}|^2 - \frac{1}{2}|f^-_\mu + f^-_\nu - c_{ij} - 2f^-_{\text{av}}|^2\Big) x_{ij} \\
&\quad + \frac{\alpha}{n} \sum_{\langle i,j\rangle\in\mathcal{E}} c_{ij} \sum_{\langle m,k\rangle\in\mathcal{E}\setminus\langle i,j\rangle} c_{mk} x_{mk} x_{ij} - (1-\alpha-\frac{\alpha}{n}) \sum_{\langle u,v\rangle\in\mathcal{E}} c^2_{uv} x_{uv},
\end{aligned}
\tag{105}
$$

where $s_\mu = \sigma_0(\phi_i)$ and $s_\nu = \sigma_0(\phi_j)$. This expression consists of both linear and quadratic terms in the decision variables x_{ij}. This makes the problem a quadratic binary programming problem. One way to solve this problem is by introducing new variables in lieu of the quadratic terms. This also introduces new constraints involving the new and old variables. Formulating these as linear constraints, the problem can be converted to a linear binary programming problem for which efficient algorithms exist.

To this end, consider the quadratic term $x_{ij}x_{mk}$ where x_{ij} and x_{mk} are binary variables. Note that two edges that are part of the matching cannot share the same vertex, that is, if $i,\ j,\ m \in \mathcal{I}$, and $x_{im} = 1$, then $x_{ij} = 0$ for all $\langle i,j\rangle \in \mathcal{E},\ j \neq m$. Thus, we may only consider quadratic terms of the form $x_{ij}x_{mk}$, $\langle i,j\rangle, \langle m,k\rangle \in \mathcal{E}$ and $i,\ j,\ k,\ m \in \mathcal{I}$, all distinct. Let now $\mathcal{I}'$ be a set of indices (of cardinality $|\mathcal{E}|$) generated as follows

$$
q = n \times i + j, \quad \text{for all } \langle i,j\rangle \in \mathcal{E},\ i,j \in \mathcal{I}. \tag{106}
$$

Conversely, given $q \in \mathcal{I}'$ the corresponding indices i and j are obtained via integer division by n using (106). We can therefore establish a one-to-one correspondence between elements of $\mathcal{I}'$ and $\mathcal{E}$, and we write $q \sim \langle i,j\rangle$ to denote this correspondence.

Considering now distinct indices $i,j,m,k \in \mathcal{I}$, and $p,q \in \mathcal{I}'$ such that $p \sim \langle i,j\rangle$ and $q \sim \langle m,k\rangle$, we introduce the new variable defined by

$$
x_{pq} = x_{ij}x_{mk}, \tag{107}
$$

These new variables are also binary since

$$x_{pq} = \begin{cases} 1, & \text{when } x_{ij} = 1 \text{ and } x_{mk} = 1, \\ \\ 0, & \text{otherwise.} \end{cases} \tag{108}$$

The restrictions in equation (108) can be imposed on the new variable by introducing the following three linear constraints

$$x_{pq} \leq x_{ij}, \tag{109}$$

$$x_{pq} \leq x_{mk}, \tag{110}$$

$$-x_{pq} + x_{ij} + x_{mk} \leq 1. \tag{111}$$

The first two constraints ensure that whenever $x_{ij} = 0$ or $x_{mk} = 0$, we have $x_{pq} = 0$. The last of the previous three constraints ensures that $x_{pq} = 1$ when $x_{ij} = 1$ and $x_{mk} = 1$. Hence, the problem of minimizing the dual objectives absorbed in equation (98) is equivalent to the following *linear* binary integer programming problem

$$\begin{aligned} \mathcal{C}_b' &= \sum_{\langle i,j\rangle\in\mathcal{E}} \alpha \left(|f_\mu^- - f_{\text{av}}^-|^2 + |f_\nu^- - f_{\text{av}}^-|^2 - \frac{1}{2}|f_\mu^- + f_\nu^- - 2f_{\text{av}}^- - c_{ij}|^2 \right) x_{ij} \\ &\quad -(1 - \alpha - \frac{\alpha}{n}) \sum_{\langle i,j\rangle\in\mathcal{E}} c_{ij}^2 x_{ij} + \frac{2\alpha}{n} \sum_{\substack{\langle i,j\rangle\in\mathcal{E} \\ \langle m,k\rangle\in\mathcal{E}\setminus\langle i,j\rangle}} c_{ij} c_{mk} x_{pq}, \end{aligned} \tag{112}$$

(where $s_\mu = \sigma_0(\phi_i)$ and $s_\nu = \sigma_0(\phi_j)$) subject to the constraints given by equations (109), (110), (111), and equations (87)-(93).

The parameter α in equation (98) weighs the relative importance for the fulfilment of the two performance objectives we have set for a P2P refueling scenario. If $\alpha = 0$, no fuel equalization is desirable ($\mathcal{C}_b = \mathcal{C}_2$), and we only minimize the rendezvous costs. Obviously, in such a case the optimal solution involves no satellite pairings: all satellites remain at their initial orbital slots and the matching set $\mathcal{M}$ is empty. Equivalently, $|\mathcal{M}| = 0$. As we increase the value of α, fuel equalization becomes increasingly important and after a certain value of $\alpha = \bar{\alpha} > 0$ at least one pair of satellites performs a fuel transaction. The matching set $\mathcal{M}$ is non empty, and consequently $|\mathcal{M}| > 0$. For $\alpha = 1$ fuel equalization is the only optimization objective ($\mathcal{C}_b = \mathcal{C}_1$), which is achieved with a (perhaps) unacceptably large number of fuel transactions. A compromise between the performance objectives $\mathcal{C}_1$ and $\mathcal{C}_2$ is achieved via an intermediate value of α. We will now illustrate with an example the effect of α on the optimal satellite pairings. Also, with the help of this example, we now investigate numerically the relationship between the solutions obtained via the two costs given in (81) and (98).

Example 3. *Optimizing* $\mathcal{C}_b$ *for a constellation of* 10 *satellites.*

Let us consider the constellation a constellation of 10 satellites evenly distributed in a circular orbit. The initial fuel content of the satellites are: 25, 20, 8, 8, 2, 1, 22, 1, 2 and 6 units. For each of the satellites, the mass of the permanent structure is 70 units and specific thrust of engine is 300 sec. The total time of refueling is 12 orbital periods. For this constellation, a P2P refueling strategy as obtained by minimizing objective $\mathcal{C}_a$ yields a total fuel expenditure of 14.93 units. We can compute the values of $\mathcal{C}_1$ and $\mathcal{C}_2$ for this solution and they turn out to be $\mathcal{C}_1 = 58.57$ square units and $\mathcal{C}_2 = 88.34$ square units respectively. Now, we consider the minimization of the objective $\mathcal{C}_b$ for various values of α. Fig. 29 shows the variation of number of edges in the constellation graph for various values of α. When $\alpha = 0$, our sole objective is to minimize the fuel expenditure during the refueling process; naturally, this is achieved by having no P2P maneuver at all. Consequently, there are no edges in the constellation graph at this value of α. As we increase the value of α, that is, reduce the weightage on $\mathcal{C}_2$, more and more edges are included in the constellation graph. Fig. 28 shows the variation of the values of the two objective functions $\mathcal{C}_1$ and $\mathcal{C}_2$ for different values of α. Each point on the curve in these plots is optimal for different choice of values of α. The range of values of α for which the same pairings of satellites occur as with the optimization of $\mathcal{J}_a$ is also shown on these plots. Note that for this range of α the pairings of satellites are the same, hence the values of $\mathcal{C}_1$ and $\mathcal{C}_2$ are also the same. The plot depicts that solutions obtained via (81) correspond to solutions obtained via (98) for a range of values of α that achieve a balanced compromise between the original conflicting optimization objectives $\mathcal{C}_1$ and $\mathcal{C}_2$. For $\alpha = 0.26 - 0.33$ (highlighted in the plot by a solid dot because it corresponds

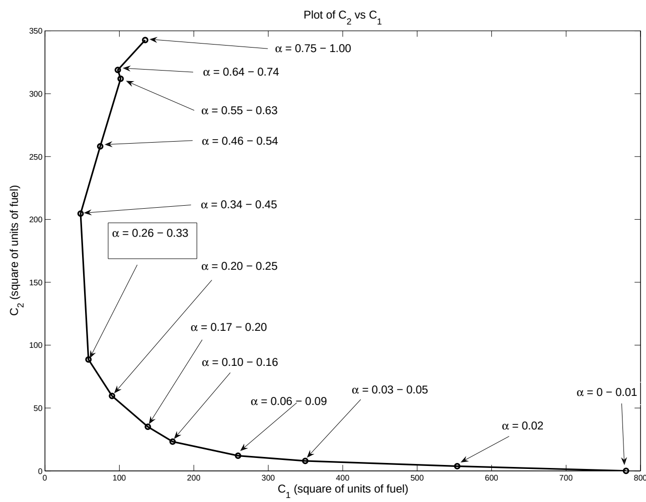


Figure 28: Variation of $\mathcal{C}_2$ and $\mathcal{C}_1$ with α.

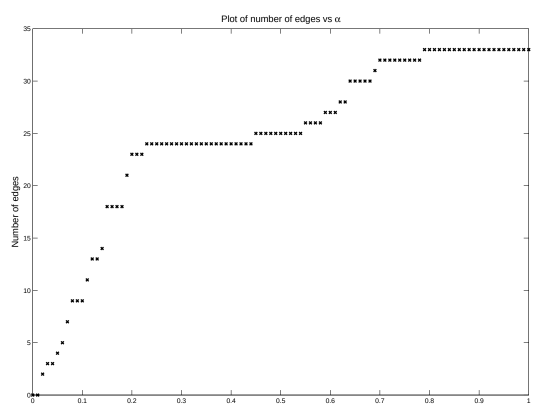
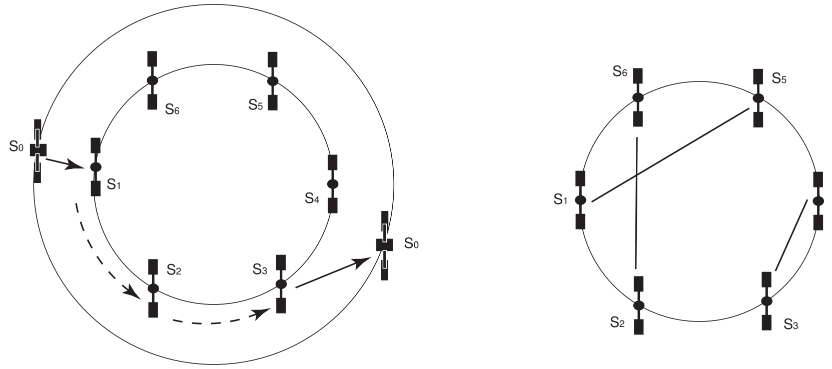


Figure 29: Variation of number of edges with α.

to the solution obtained by minimization of 81), we have a reasonable compromise between the two performance specifications $\mathcal{C}_1$ and $\mathcal{C}_2$. It can be seen that the use of



(a) First Step (b) Second Step

Figure 30: Mixed refueling Strategy.

the simpler cost $\mathcal{C}_a$ in lieu of $\mathcal{C}_b$ is justified, as the former results in solutions which are identical to those obtained via $\mathcal{C}_b$ for values of α that provide a balance between the objectives $\mathcal{C}_1$ and $\mathcal{C}_2$. The case for using $\mathcal{C}_a$ instead of $\mathcal{C}_b$ is made stronger in light of the fact that the calculation of the optimal matching using the cost $\mathcal{C}_b$ is computationally more intensive than using the cost $\mathcal{C}_a$ owing to the larger number of decision variables and the associated constraints; see (107)-(111). As a result, in practice one can bypass the optimization of $\mathcal{C}_b$ and deal only with the optimization of $\mathcal{C}_a$ when computing the optimal satellite pairings in a P2P scenario.

3.4 A Mixed Refueling Strategy

The mixed refueling strategy combines the ideas of SSV and P2P refueling strategies. In a mixed strategy, a service vehicle delivers fuel to part (perhaps, half) of the satellites in the constellation. The satellites which recieve fuel from the service vehicle distributes the same among all the remaining satellites by engaging in P2P maneuvers. Clearly, a mixed refueling strategy has two steps: (1) SSV phase, and (2) P2P phase. In this section, we will discuss the two steps of the mixed refueling strategy. To this end, let there be an even ($2n$) number of satellites in the constellation, and let T be the total time of the refueling mission. Figure 30 depicts the two phases of the mixed refueling strategy.

3.4.1 SSV Phase

During the SSV phase, the service vehicle S_0 delivers fuel sequentially to n of the satellites. Let $\mathcal{I}_1$ denote the index set of the satellites that receive fuel from S_0. Without loss of generality, we can consider that $\mathcal{I}_1 = \{1, 2, \ldots, n\}$. Also, let $T^{(1)}$ denotes the time allotted for the SSV phase. Let $t^{(1)}_{i,i+1}$ denotes the time taken by S_0 to transfer from satellite s_i to satellite s_{i+1}. Then, we have,

$$T^{(1)} = \sum_{i=1}^{n-1} t^{(1)}_{i,i+1}. \tag{113}$$

The individual transfers times $t^{(1)}_{12}, t^{(1)}_{23}, \ldots, t^{(1)}_{n-1,n}$ can be obtained by solving the integer program given by (77) -(80).

3.4.2 P2P Phase

During the P2P phase, the satellites $s_1, s_2, \ldots, s_n$ engage in P2P maneuvers with the remaining satellites whose indices are given by $\mathcal{I}_2 = \{n+1, n+2, \ldots, 2n\}$. Let $T^{(2)}$ denotes the time for the P2P phase. Hence, we have

$$T^{(2)} = T - T^{(1)}. \tag{114}$$

The optimal P2P maneuvers can be determined by solving a P2P problem with fuel-sufficient satellites given by indices $\mathcal{I}_1$ and fuel-deficient satellites given by indices $\mathcal{I}_2$.

It has been shown in Ref. 82 that, in terms of fuel expended during the refueling mission, the mixed strategy becomes better with increasing number of satellites in the constellation. This result is demonstrated by Figure 31, which compares the total amount of fuel expended during the two strategies as the number of satellites in a constellation changes. However, as we will show now, the mixed refueling strategy can be improved further by the inclusion of a couple of cost-reducing strategies.

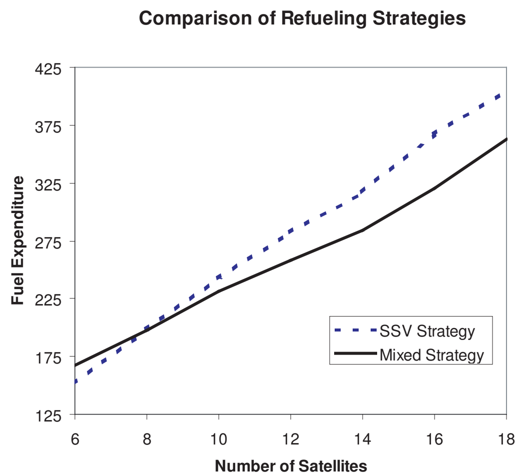


Figure 31: Refueling Strategies: SSV vs. Mixed

3.5 Cost-Reducing Strategies for Mixed Refueling

In this section, we discuss two means of that helps in reducing the fuel expenditure during mixed strategy.

3.5.1 Coasting Time Allocation (CTA) Strategy

The idea of allowing coasting intervals is utilized in this section to propose a strategy for reducing the overall P2P rendezvous cost. Already mentioned is that the optimal cost, when coasting is included, is a non-increasing function of time. That is, the inequality

$$\Delta V(t_{f1}) \leq \Delta V(t_{f2}), \quad \text{for} \;\; t_{f1} \geq t_{f2} \tag{115}$$

holds for any two transfer times t_{f1} and t_{f2}. Note that this monotonicity of ΔV versus the transfer time does not hold if there are no coasting intervals.

In the previous chapter, while discussing the P2P refueling strategy, we assumed that the time for forward and return trips are equal. In particular, we show that by allowing unequal transfer times between the forward and return journeys for each fuel transaction, one can reduce the transfer cost. To this end, let us consider a P2P maneuver between two satellites s_μ and s_ν occupying the orbital slots ϕ_i and ϕ_j

respectively. Let us denote by t_{ij} the total time allowed to complete both legs of the fuel transaction between satellites s_μ and s_ν. Let us consider s_μ to be active and s_ν to be passive. Moreover, let t^f_{ij} denote the time for the forward journey and t^r_{ij} denote the time for the return journey, so that

$$t_{ij} = t^f_{ij} + t^r_{ij}. \tag{116}$$

In case of an equal partition of the total time between the forward and return transfers, we have $t^f_{ij} = t^r_{ij} = t_{ij}/2$. Let us use the superscript I, to denote quantities associated with such an equal time partition transfer. We will use the superscript II to denote the quantities associated with a transfer with unequal time partition of t_{ij} such that the forward and return legs are completed within the time intervals $t^f_{ij} = t_{ij}/2 - t'_{ij}$ and $t^r_{ij} = t_{ij}/2 + t'_{ij}$, where t'_{ij} denotes the optimal final coasting time for the forward leg. Similarly, we will use the superscript III to denote the quantities associated with a transfer with unequal time partition of t_{ij} such that the forward and return legs are completed within the time intervals $t^f_{ij} = t_{ij}/2 + t''_{ij}$ and $t^r_{ij} = t_{ij}/2 - t''_{ij}$, where t''_{ij} denotes the optimal coasting time for the return leg. Let us concentrate on the case where coasting is part of the forward leg.

Note that since coasting periods do not have any effect on the cost,

$$\Delta V^{\mathrm{I}}_{ij} = \Delta V^{\mathrm{II}}_{ij}$$

which implies, according to (83) that

$$p^{\mu\mathrm{I}}_{ij} = p^{\mu\mathrm{II}}_{ij}. \tag{117}$$

For the return flight, and since $t_{ij}/2 + t'_{ij} \geq t_{ij}/2$ we have, via (84), that

$$\Delta V^{\mathrm{I}}_{ji} \geq \Delta V^{\mathrm{II}}_{ji},$$

which implies that $e^{-\Delta V^{\mathrm{I}}_{ji}/c_{0i}} \leq e^{-\Delta V^{\mathrm{II}}_{ji}/c_{0i}}$. Using this inequality, it follows that $1 - e^{-\Delta V^{\mathrm{I}}_{ji}/c_{0i}} \geq 1 - e^{-\Delta V^{\mathrm{II}}_{ji}/c_{0i}}$, and also $1 + e^{-\Delta V^{\mathrm{I}}_{ji}/c_{0i}} \leq 1 + e^{-\Delta V^{\mathrm{II}}_{ji}/c_{0i}}$. These two inequalities together yield

$$\frac{1 - e^{-\Delta V_{ji}^{\mathrm{I}}/c_{0i}}}{1 + e^{-\Delta V_{ji}^{\mathrm{I}}/c_{0i}}} \geq \frac{1 - e^{-\Delta V_{ji}^{\mathrm{II}}/c_{0i}}}{1 + e^{-\Delta V_{ji}^{\mathrm{II}}/c_{0i}}} \tag{118}$$

which, via (115), yields

$$p_{ji}^{\mu\mathrm{I}} \geq p_{ji}^{\mu\mathrm{II}}. \tag{119}$$

From equation (117) and inequality (119), the identity (82) yields

$$c_{ij}^{\mu\mathrm{I}} \geq c_{ij}^{\mu\mathrm{II}}. \tag{120}$$

A similar analysis holds when a coasting period of length t''_{ij} is part of the return leg. We have therefore shown the following proposition.

Proposition 1. *Given the total time for a fuel transaction to take place between two satellites in the same circular orbit, an equal time allocation between the forward and return legs of the two associated rendezvous transfers is always suboptimal.*

We will next utilize this idea to devise a coast time allocation (CTA) algorithm for reducing the fuel coast during each fuel transaction.

3.5.2 CTA Algorithm

The main idea behind the formulation of a fuel-reducing strategy is to allow for unequal time distribution between the forward and the return legs for each fuel transaction. To this end, we consider the following three cases:

- Case-I: $t_{ij}^{f} = t_{ij}^{r} = t_{ij}/2$
- Case-II: $t_{ij}^{f} = t_{ij}/2 - t'_{ij}$ and $t_{ij}^{r} = t_{ij}/2 + t'_{ij}$
- Case-III: $t_{ij}^{f} = t_{ij}/2 + t''_{ij}$ and $t_{ij}^{r} = t_{ij}/2 - t''_{ij}$

Assume a fuel transaction between satellites $s_\mu = \sigma_0(\phi_i)$ and $s_\nu = \sigma_0(\phi_j)$, where $i \in \mathcal{J}_a$ and $j \in \mathcal{J}_p$. Let $c_{ij}^{\mu\mathrm{I}}$, $c_{ij}^{\mu\mathrm{II}}$ and $c_{ij}^{\mu\mathrm{III}}$ denote the fuel spent for satellite s_μ to rendezvous with s_ν and return back to its original position, for each of the previous three cases, respectively. The optimal time sharing is the one that satisfies

$$c_{ij}^{\mu *} = \min\{c_{ij}^{\mu \mathrm{I}}, c_{ij}^{\mu \mathrm{II}}, c_{ij}^{\mu \mathrm{III}}\}. \tag{121}$$

The corresponding time allocation is then given by

$$(t_{ij}^f, t_{ij}^r) = \begin{cases} (t_{ij}/2, t_{ij}/2), & \text{if } c_{ij}^{\mu *} = c_{ij}^{\mu \mathrm{I}}, \\ (t_{ij}/2 - t'_{ij}, t_{ij}/2 + t'_{ij}), & \text{if } c_{ij}^{\mu *} = c_{ij}^{\mu \mathrm{II}}, \\ (t_{ij}/2 + t''_{ij}, t_{ij}/2 - t''_{ij}), & \text{if } c_{ij}^{\mu *} = c_{ij}^{\mu \mathrm{III}}. \end{cases}$$

We can similarly compute the cost of a single fuel transaction for the case $i \in \mathcal{J}_p$ and $j \in \mathcal{J}_a$. Finally, the optimum fuel consumption in a P2P maneuver between any two satellites s_μ and s_ν is given by

$$c_{ij}^{*} = \begin{cases} c_{ij}^{\mu *}, & \text{if } s_\mu \text{ can be active, but } s_\nu \text{ cannot be active,} \\ c_{ij}^{\nu *}, & \text{if } s_\nu \text{ can be active, but } s_\mu \text{ cannot be active,} \\ \min\{c_{ij}^{\mu *}, c_{ij}^{\nu *}\}, & \text{if either } s_\mu \text{ or } s_\nu \text{ can be active,} \\ \infty, & \text{if neither } s_\mu \text{ nor } s_\nu \text{ can be active.} \end{cases}$$

Ref. 19 shows the benefit of such a strategy in reducing the fuel expenditure of P2P refueling. In most cases, the CTA gives a reduced fuel consumption, but do not affect the optimal P2P assignments. However, there may be cases when the application of CTA changes the P2P assignments.[19] We will use this algorithm to compute the optimal fuel expenditure in P2P phase of the mixed refueling strategy which we focus on in the rest of the chapter.

3.5.3 Asynchronous P2P Refueling

The mixed refueling strategy discussed before considers the SSV and P2P phases to be distinct. In other words, the P2P maneuvers all take place simultaneously, at the end of SSV phase, during the time $T^{(2)}$ allotted for the P2P phase. However, note that this need not be the case. A satellite that receives fuel from the SSV need not wait for SSV phase to complete in order to start the P2P maneuvers. For instance, it can initiate an orbital transfer immediately after it receives fuel from S_0. In this case, the different P2P maneuvers will have different time-lengths. Hence, we refer to these as Asynchronous P2P (A-P2P) maneuvers. Note, however that the time $T^{(2)}$ is binding only for satellite s_n (the last satellite to be visited by s_0 during the first step of a mixed strategy). All other satellites s_i ($i = 1, \ldots, n-1$) have available $T^{(2)} + \sum_{k=i}^{n-1} t^{(1)}_{k,k+1}$ time units to perform their fuel transactions. Thus, the time available for s_i to complete the P2P maneuver with its matching pair s_j is given by

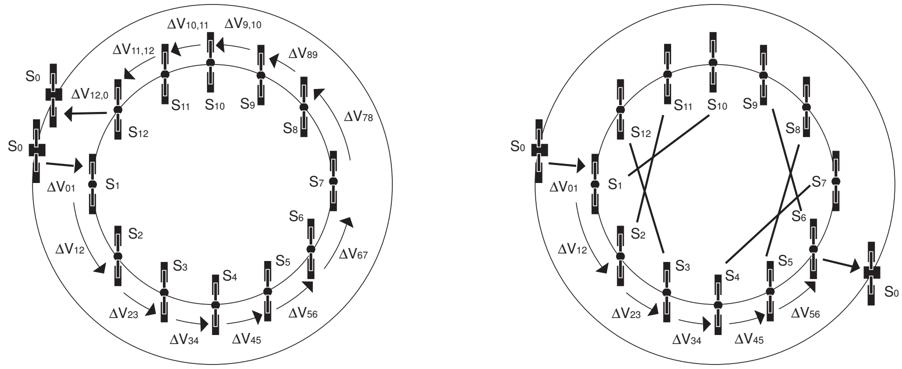
$$t^{(2)}_{ij} = \begin{cases} T^{(2)} + \sum_{k=i}^{n-1} t^{(1)}_{k,k+1}, & \text{if } i \in \mathcal{I}_1 \backslash \{n\}, \\ T^{(2)}, & \text{if } i = n. \end{cases} \tag{122}$$

Since $t^{(2)}_{ij} \geq T^{(2)}$ for all satellite pairs, referring again to equation (115) makes it clear that each rendezvous between two satellites will require less fuel than a synchronous implementation. Consequently, the overall fuel consumption for the whole constellation will also be reduced by using an asynchronous P2P implementation.

3.6 Comparison of Refueling Strategies

Let us consider a constellation in a circular orbit with an even number of satellites, say, $2n$. For the sake of simplicity, we may assume that all satellites are initially depleted of fuel, that is, $i \in \mathcal{I}_{d,0}$ for all $i \in \mathcal{I}$. Given a maximum refueling period, say T, we wish to refuel all of the satellites from a service vehicle s_0, such that after time T, they all end up with approximately the same amount of fuel. In the process, we also want to minimize the total fuel expenditure during the ensuing orbital maneuvers. Equivalently, we want to maximize the total amount of fuel that can be delivered to the constellation. We have two alternatives for solving this problem.

The first alternative is for s_0 to refuel (perhaps sequentially[73]) all satellites in the constellation. This scenario is shown in Figure 32(a).



(a) Single-spacecraft refueling scenario

(b) Mixed refueling scenario

Figure 32: Refueling Strategies in a Circular Constellation.

Example 4. *Mixed refueling strategy for a constellation of* 12 *satellites.*

We now apply the CTA algorithm along with an asynchronous (mixed) P2P refueling strategy to a constellation comprising 12 satellites. Through this example, we show how these improvements for a mixed refueling strategy make the latter a competitive alternative to a refueling strategy using a single service vehicle or to mixed synchronous P2P strategies.

Table 2: Optimal Fuel Consumption the Refueling with a Single Service Vehicle.

Segment	t_{ij}	ΔV_{ij}	Fuel Expense
$i = 1,\ j = 2$	1.9084	0.1821	35.9746
$i = 2,\ j = 3$	1.9084	0.1821	32.1287
$i = 3,\ j = 4$	1.9084	0.1821	28.5604
$i = 4,\ j = 5$	1.9084	0.1821	25.2497
$i = 5,\ j = 6$	1.9084	0.1821	22.1779
$i = 6,\ j = 7$	1.9084	0.1821	19.3278
$i = 7,\ j = 8$	1.9084	0.1821	16.6834
$i = 8,\ j = 9$	1.9084	0.1821	14.2299
$i = 9,\ j = 10$	1.9084	0.1821	11.9535
$i = 10,\ j = 11$	1.9084	0.1821	9.8414
$i = 11,\ j = 12$	0.9163	0.3805	15.8334

To this end, we assume a circular orbit constellation with an even number of satellites. The service spacecraft, denoted by s_0, starts with an initial amount of fuel

$f_0(0^-) = 500$ units. We assume that s_0 is initially at a higher circular orbit than the constellation orbit. It is required that s_0 returns to the same orbit after completing the refueling process with $f_0(T^+) = 10$ units of fuel, where $T = 20$ is the maximum allowed time for completing the whole refueling process. Recall that one unit of time corresponds to one period of the circular orbit of the constellation. Hence, the total amount of fuel to be delivered to the satellites in the constellation including the fuel to be used during the corresponding orbital transfers is 490 units. The spacecraft spends $p_0 = 44.26$ units of fuel to arrive from the higher orbit to the constellation orbit[†] After refueling s_0 returns to its initial orbit by spending $p_f = 6.01$ units of fuel. The mass of the permanent structure for each satellite is $m_{si} = 60$ units and the characteristic constant of the engine is $c_{0i} = 2943$ units for all satellites.

Table 3: Optimal Fuel Consumption for First Step of Mixed Refueling Strategy.

Segment	$t_{ij}^{(1)}$	ΔV_{ij}	Fuel Expense
$i = 1,\ j = 2$	1.9174	0.1822	33.2517
$i = 2,\ j = 3$	1.9174	0.1822	26.8369
$i = 3,\ j = 4$	1.9174	0.1822	20.8556
$i = 4,\ j = 5$	1.9174	0.1822	15.3643
$i = 5,\ j = 6$	1.9174	0.1822	10.2419

Let us consider a constellation with twelve satellites evenly distributed in a circular orbit. The total time allowed for refueling is again $T = 20$ time units. There are eleven rendezvous segments with a single-spacecraft refueling strategy. The optimal time distribution for each of the eleven rendezvous segments and the corresponding fuel consumption are given in Table 2. At the end of this process, each of the six satellites end up with an equal amount of fuel $f_i^+ = 17.31$. The total amount of fuel used during all the transfers is thus $490 - 12 \times 17.31 = 282.28$ units.

[†]Here we assume that the constellation orbit and the higher orbit are coplanar. This is not restrictive. For orbits at different inclinations a plane change may have been considered. However, this extra degree of freedom does not affect the comparison of the two refueling strategies. This is because the fuel of the transfer of s_0 to and from the constellation orbit is part of both refueling strategies, and hence it is not part of the optimization process.

Table 4: Optimal Fuel Consumption for Second Step of Mixed Refueling Strategy. Twelve Satellite Constellation.

Pairs	T	$T^{(1)}/T^{(2)}$	Fuel Expense
(s_1, s_{10})	20.00	10.25/9.75	8.5335
(s_2, s_{11})	18.08	9.33/8.75	9.4564
(s_3, s_{12})	16.17	8.43/7.74	10.6270
(s_4, s_7)	14.25	8.00/6.25	12.0585
(s_5, s_8)	12.33	5.75/6.58	14.1137
(s_6, s_9)	10.41	4.74/5.67	16.8236

For the mixed strategy, there are five rendezvous segments during the first stage, which are all completed within $T^{(1)} = 9.59$ units. The optimal time distribution for

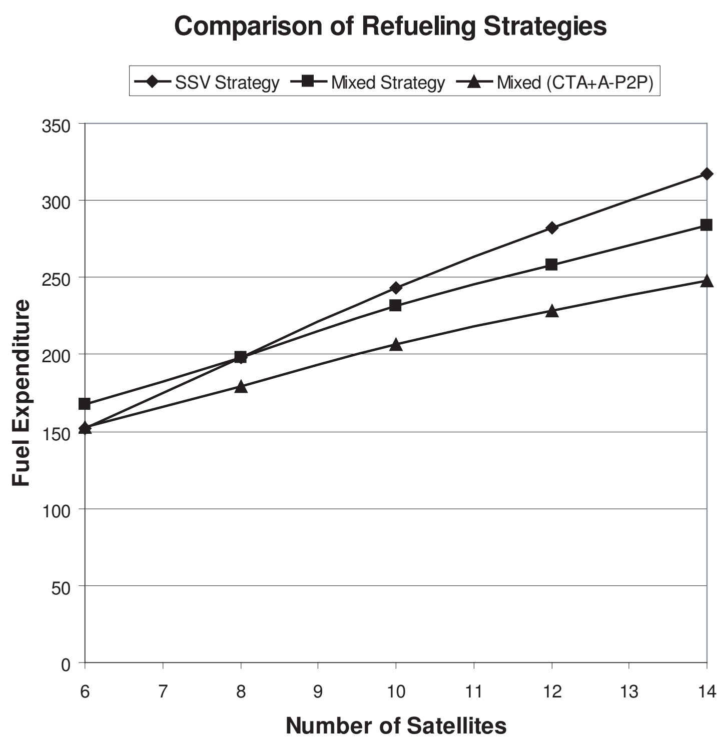


Figure 33: Comparison of Fuel Expenditure of Refueling Strategies.

each of the five rendezvous segments and the corresponding fuel consumption are given in Table 3. The six satellites refueled by s_0 have fuel 55.53 units each before performing the P2P refueling part. The available times for the P2P maneuvers as well as the corresponding fuel consumption are given in Table 4. The final fuel content of the satellites are $f_1(T^+) = f_{10}(T^+) = 23.50, f_2(T^+) = f_{11}(T^+) = 23.04, f_3(T^+) = f_{12}(T^+) = 22.45, f_4(T^+) = f_7(T^+) = 21.74, f_5(T^+) = f_8(T^+) = 20.71, f_6(T^+) = f_9(T^+) = 19.35$. The average amount of fuel in the constellation is 21.80 units. The total amount of fuel burnt using the mixed refueling strategy is $490-12\times 21.80 = 228.4$ units, which is about 19% less than the amount of fuel burnt if the satellites are refueled by a single spacecraft. Clearly, the mixed scenario outperforms the single service vehicle option in this case.

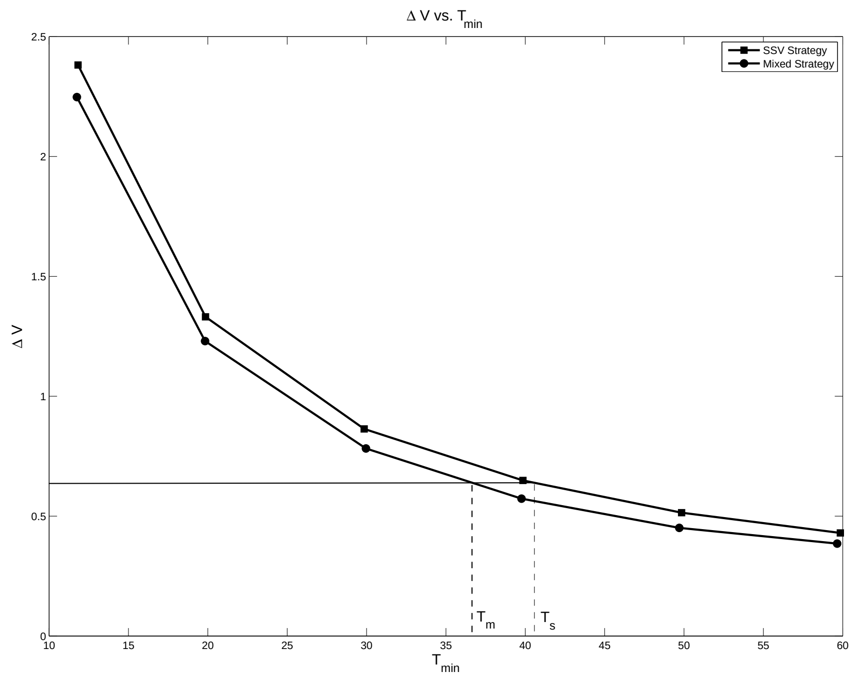


Figure 34: Comparison of Time of Mission for Refueling Strategies.

An elaborate comparison of the refueling strategies is made for varying number of satellites in the constellation. Figure 33 shows the variation of fuel expended during the refueling process with the number of satellites in the constellation. The result clearly demonstrates that the mixed refueling strategy is better than the SSV strategy in terms of fuel expended, with the incorporation of the CTA strtaegy and A-P2P maneuvers. When there are 6 satellites in the constellation, the single-service vehicle strategy is only marginally better. But, with increasing number of satellites, the mixed strategy becomes increasingly better.

Finally, we present a comparison of the two strategies from a different perspective. Suppose, we are given a budget for ΔV, and we wish to answer the question, how fast can we refuel all the satellites for the two strategies? Figure 34 gives a variation of ΔV with the minimum time required to complete the refueling mission, for a constellation of 8 satellites. Let us consider the horizontal line in the figure. The line corresponds to a given budget ΔV_b for the refueling mission. Given this budget, the fastest time we can distribute fuel among all the satellites is T_m for the mixed refueling strategy, and T_s for the SSV refueling strategy. Clearly, $T_m < T_s$. The result demonstrates that, if we are given a total ΔV budget, then the mixed refueling can complete the fuel delivery in a time less than that required by the single service vehicle refueling strategy.

3.7 Summary

In this chapter, we discussed different strategies for refueling a system of several satellites in a circular orbit. In the SSV strategy, one needs to solve an integer program to determine the optimal time for all the transfers. In the case of P2P refueling, one needs to solve a weighted matching problem in order to solve for the P2P maneuvers. Finally, a mixed refueling strategy, that combines the ideas of SSV and P2P refueling, is discussed. In terms of fuel expended during the maneuvers, the mixed refueling strategy is better than a SSV strategy, particularly with increasing number of satellites in the constellation. We provide two cost-reducing measures, namely the Coasting Time Allocation Strategy and Asynchronous P2P maneuvers, in order to further reduce the fuel expenditure in a mixed refueling scenario.

CHAPTER IV

PEER-TO-PEER REFUELING STRATEGY

In the original studies on P2P refueling,[19,72,75] the P2P maneuvers were considered to be means to achieve fuel equalization in the constellation, and the P2P refueling problem was formulated based on this notion. In this chapter, we point out a drawback of such a formulation, and then provide an alternative formulation of the P2P refueling problem. In this alternative formulation, we consider that the satellites are required to maintain a minimum amount of fuel to remain operational. The satellites not meeting this criterion are considered fuel-deficient and those which satisfy the same are considered fuel-sufficient. The objective for the satellites would be to share fuel such that all of them become fuel-sufficient at the end of the P2P maneuvers. We discuss this formulation in detail, and demonstrate with examples, P2P refueling strategy in sample circular constellations. Furthermore, we also extend the problem to allow for cooperative rendezvous between the satellites engaging in a refueling transaction.

4.1 A Practical Drawback of the Fuel Equalization Formulation

As shown in the previous chapter, the optimal P2P maneuvers achieving fuel equalization in a constellation can be obtained by maximizing the cost function $\mathcal{C}_a$. When using the cost function $\mathcal{C}_a$, the expression for the cost of an edge π_{ij} for all $\langle i, j \rangle \in \mathcal{E}_\mathrm{r}$ suggests that the reduced constellation graph $\mathcal{G}_\mathrm{r}$ is, in general, non-bipartite. From a refueling point of view, we are interested in a bipartite matching between the fuel-sufficient and fuel-deficient satellites. However, the objective of fuel equalization does not necessarily lead to a bipartite graph. It can be noted that although there may not exist an edge between two fuel-deficient satellites in the reduced constellation graph (because $\pi_{ij} \leq 0$ for all $i, j \in \mathcal{J}_{d,0}$), there may still be edges between the fuel-sufficient satellites. Now, a question that arises here is whether an optimal P2P assignment consists of an edge between two fuel-sufficient satellites. Before we address this issue, let us define the following terms for convenience. We refer to $\mathcal{M}_p$ as a *bipartite edge set* if there exists no edge $\langle i, j \rangle \in \mathcal{M}_p$ such that $i, j \in \mathcal{J}_{s,0}$. If there exists at least one such edge between fuel-sufficient satellites, we refer to $\mathcal{M}_p$ as a *non-bipartite edge set.* Now, the following proposition gives the condition that guarantees that the optimal P2P assignment as determined by $\mathcal{C}_a$ would be a non-bipartite edge set.

Proposition 2. *The optimal P2P maneuvers determined by maximizing the cost function $\mathcal{C}_a$ consists of a fuel transaction between two fuel-sufficient satellites if the following hold: $|\mathcal{I}_{s,0}| - |\mathcal{I}_{d,0}| \geq 2$, and $\pi_{ij} > 0$ for all $\langle i, j \rangle \in \mathcal{E}_\mathrm{r}$ such that $i, j \in \mathcal{J}_{s,0}$.*

Proof. Consider a bipartite edge set $\mathcal{M}_p$ consisting of $|\mathcal{I}_{d,0}|$ edges. Since an edge between two fuel-deficient satellites does not exist in $\mathcal{G}_\mathrm{r}$, $\mathcal{M}_p$ leaves $|\mathcal{I}_{d,0}|$ fuel-deficient satellites matched with same number of fuel-sufficient satellites. This also means $|\mathcal{I}_{s,0}| - |\mathcal{I}_{d,0}| \geq 2$ fuel-sufficient satellites are left unmatched by $\mathcal{M}_p$. Let $\mathcal{U}$ be the set of indices of orbital slots of the unmatched fuel-sufficient satellites. Clearly, $|\mathcal{U}| \geq 2$. Since $\pi_{ij} > 0$ for all $\langle i, j \rangle \in \mathcal{E}_\mathrm{r}$ such that $i, j \in \mathcal{J}_{s,0}$, there exists an edge in $\langle i, j \rangle \in \mathcal{E}_\mathrm{r}$ for every $i, j \in \mathcal{U}$. Let $\mathcal{E}'$ be the set of edges induced by the set $\mathcal{U}$. Consider any edge $\langle u, v \rangle \in \mathcal{E}'$ and construct the non-bipartite edge set $\mathcal{M}'_p = \mathcal{M}_p \cup \{\langle u, v \rangle\}$. Since $\pi_{uv} > 0$, $\mathcal{M}'_p$ gives a more optimal value of the cost function $\mathcal{C}_a$. Hence, for every bipartite edge set $\mathcal{M}_p$, there exists a non-bipartite edge set $\mathcal{M}'_p$ that gives a more optimal value of the objective function $\mathcal{C}_a$. The optimal P2P assignment therefore has to be a non-bipartite edge set. □

The proposition essentially shows that two fuel-sufficient satellites can pair up in the optimal P2P maneuver. An important question that arises here is whether we want two fuel-sufficient satellites to be involved in a P2P refueling transaction. It may not be practical to use up some fuel in the constellation by having two fuel-sufficient satellites engaged in a refueling transaction. In other words, from a practical point of view, we we do not want a fuel transaction between two fuel-sufficient satellites. Let us illustrate this with the following example.

Example 5. *Optimal P2P assignment consists of a fuel exchange between two fuel-sufficient satellites.*

We consider here a simple example of a circular constellation comprising 8 satellites. For each satellite, mass of permanent structure is considered to be 70 units and the specific thrust of the engine is considered to be 300 sec. The satellites s_1, $s_2, \ldots s_8$ are evenly distributed in the orbit at the slots $\phi_1 = 0, \phi_2 = 45, \ldots, \phi_8 = 315\,\text{deg}$ respectively. They have initial fuel content of $f_1^- = 30$, $f_2^- = 25$, $f_3^- = 28$ and $f_4^- = 30$, $f_5^- = 25$, $f_6^- = 4$, $f_7^- = 6$ and $f_8^- = 4$ units respectively. The mean fuel in the constellation is 19 units and hence the satellite s_1, s_2, s_3, s_4 and s_5 are fuel-sufficient, while s_6, s_7 and s_8 are fuel-deficient. Clearly, $|\mathcal{I}_{s,0}| - |\mathcal{I}_{d,0}| = 2$, satisfying the condition of the proposition. The optimal P2P assignments for this example turns out to be the following: $s_1 - s_7$, $s_2 - s_5$, $s_3 - s_8$, $s_4 - s_8$. Clearly, the fuel-sufficient satellites s_2 and s_5 engage in a refueling transaction, resulting in a fuel consumption of 8.30 units, which is about 33% of the total fuel expenditure (24.85 units) during the overall refueling process. The fuel-sufficient satellites being involved in an impractical fuel exchange resulted in a loss of 8.30 units of fuel that could have otherwise been preserved in the constellation.

This drawback in the formulation of the P2P refueling problem based on the notion of fuel equalization motivates us in resorting to an alternate formulation of the P2P refueling problem based on the notion of achieving fuel sufficiency in the constellation.[20,21,70]

4.2 An Alternative Formulation for the P2P Refueling Problem

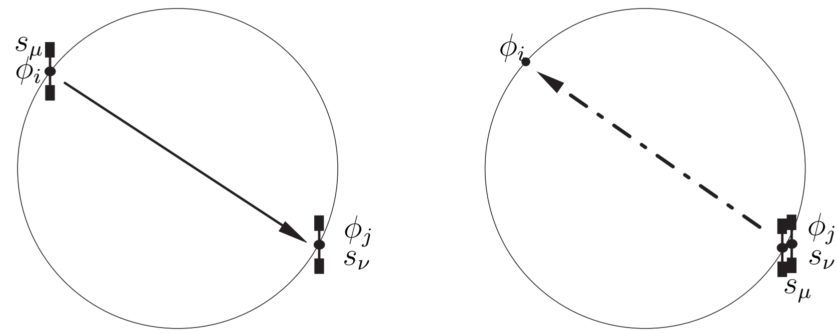
An alternative way of formulating the P2P problem would be to impose a minimum fuel requirement on each satellite in order to remain operational. Let $\underline{f}_i$ denotes this minimum amount of fuel required by satellite s_i, while let $\bar{f}_i$ denote the maximum fuel capacity of the satellite. Hence, fuel-sufficient satellite are those which have at least the requisite amount of fuel and the remaining satellites are fuel-deficient, that is, $\mathcal{I}_{s,t} = \{i : f_{i,t} \geq \underline{f}_i\}$ and $\mathcal{I}_{d,t} = \{i : f_{i,t} < \underline{f}_i\}$. The objective of P2P refueling is therefore to achieve $f_i^+ \geq \underline{f}_i$ for all $i \in \{1, 2, \ldots, n\}$ by expending the minimum amount of fuel during the ensuing orbital maneuvers.

4.2.1 Feasible P2P Maneuver

Let us consider a P2P maneuver between the satellites s_μ and s_ν initially occupying the orbital slots ϕ_i and the satellite s_ν occupy the orbital slot ϕ_j. Note that $s_\mu = \sigma_0(\phi_i)$ and $s_\nu = \sigma_0(\phi_j)$. Without loss of generality, assume s_μ to be a fuel-sufficient satellite and s_ν to be a fuel-deficient satellite, that is, $\mu \in \mathcal{I}_{s,0}$ and $\nu \in \mathcal{I}_{d,0}$. Either of the two satellites may be active during a refueling transaction between the two satellites. Hence, two different refueling transactions are possible for the edge $\langle i, j\rangle \in \mathcal{E}$. In the first case, the fuel-sufficient satellite s_μ is active. Therefore, $\mu \in \mathcal{I}_a \cap \mathcal{I}_{s,0}$ and $\nu \in \mathcal{I}_p \cap \mathcal{I}_{d,0}$. The forward and return trips of the related P2P maneuver are shown in Figure 35. The fuel consumed by the active satellite s_μ to transfer from the orbital slot ϕ_i to the orbital slot ϕ_j is given by:

$$p_{ij}^\mu = \left(m_{\mathrm{s}\mu} + f_\mu^-\right)\left(1 - e^{-\frac{\Delta V_{ij}}{c_{0\mu}}}\right), \tag{123}$$

The fuel content of satellite s_μ after its forward trip (but before fuel exchange takes place) is $f_\mu^- - p_{ij}^\mu$. After the fuel exchange takes place between the two satellites, s_μ performs another orbital maneuver and returns to its original orbital slot ϕ_i. Since the fuel consumption during the maneuver is minimized when the active satellite



(a) Forward Trip (b) Return Trip

Figure 35: P2P Maneuver (s_μ active).

returns to its final slot with exactly the required minimum amount of fuel to remain operational, the amount of fuel consumed during the return trip is given by

$$p^{\mu}_{ji} = \left(m_{\mathrm{s}\mu} + \underline{f}_{\mu}\right) e^{\frac{\Delta V_{ji}}{c_{0\mu}}} \left(1 - e^{-\frac{\Delta V_{ji}}{c_{0\mu}}}\right). \tag{124}$$

In order for satellite s_ν to become fuel-sufficient after the fuel transaction, we must therefore have,

$$\left(f^{-}_{\nu} + f^{-}_{\mu}\right) - \left(\underline{f}_{\mu} + \underline{f}_{\nu}\right) \geq p^{\mu}_{ij} + p^{\mu}_{ji}. \tag{125}$$

If the above condition does not hold, then the P2P refueling transaction is not feasible. Also, if satellite s_μ does not have enough fuel to carry out the orbital transfer during the forward trip, that is, if $p^{\mu}_{ij} \geq f^{-}_{\mu}$, then the P2P refueling transaction is also not feasible.

In the second case, the fuel-deficient satellite s_ν is active. The fuel consumed for the active satellite s_ν to transfer from the orbital slot ϕ_i to the orbital slot ϕ_j is given by

$$p^{\nu}_{ji} = \left(m_{\mathrm{s}\nu} + f^{-}_{\nu}\right) \left(1 - e^{-\frac{\Delta V_{ji}}{c_{0\nu}}}\right). \tag{126}$$

The fuel content of satellite s_ν after its forward trip (but before fuel exchange takes place), is $f^{-}_{\nu} - p^{\nu}_{ji}$. The amount of fuel consumed during the return trip, during which the satellite s_ν travels from the orbital slot ϕ_i to the orbital slot ϕ_j, is given by

$$p_{ij}^{\nu} = \left(m_{s\nu} + \underline{f}_{\nu}\right) e^{\frac{\Delta V_{ij}}{c_{0\nu}}} \left(1 - e^{-\frac{\Delta V_{ij}}{c_{0\nu}}}\right), \tag{127}$$

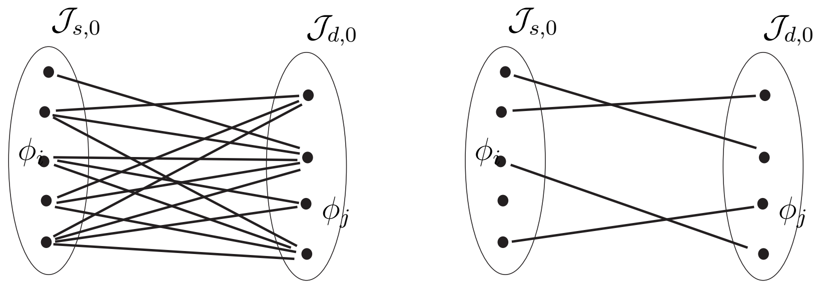
Before the return trip (but after the fuel exchange takes place), the fuel on board satellite s_ν is $\underline{f}_{\nu} + p_{ij}^{\nu}$. The fuel transferred to satellite s_ν during the fuel exchange is $(\underline{f}_{\nu} + p_{ij}^{\nu}) - (f_{\nu}^{-} - p_{ji}^{\nu})$. The fuel on board satellite s_μ after the fuel transaction is $f_{\mu}^{-} - (\underline{f}_{\nu} + p_{ij}^{\nu}) + (f_{\nu}^{-} - p_{ji}^{\nu})$. In order for the satellite s_μ to be fuel-sufficient after the fuel transaction, we must have

$$\left(f_{\mu}^{-} + f_{\nu}^{-}\right) - \left(\underline{f}_{\nu} + \underline{f}_{\mu}\right) \geq p_{ji}^{\nu} + p_{ij}^{\nu}. \tag{128}$$

If the above condition does not hold, then a P2P refueling transaction is not feasible. Also, if the satellite s_ν does not have enough fuel to carry out the orbital transfer during the forward trip, that is, if $p_{ji}^{\nu} \geq f_{\nu}^{-}$, then the P2P refueling transaction is also not feasible.

4.2.2 P2P Constellation Graph

In order to formulate the P2P refueling problem, consider an undirected bipartite graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with the two partitions being $\mathcal{J}_{s,0}$ and $\mathcal{J}_{d,0}$. There exists an edge $\langle i, j \rangle \in \mathcal{J}_{s,0} \times \mathcal{J}_{d,0}$ if the satellites $s_\mu = \sigma_0(\phi_i)$ and $s_\nu = \sigma_0(\phi_j)$ can engage in a P2P refueling transaction such that at the end of the refueling process, both the satellites end up being fuel-sufficient. Let $\mathcal{E} \subseteq \mathcal{J}_{s,0} \cup \mathcal{J}_{d,0}$ be the set of all edges in $\mathcal{G}$. To each edge $\langle i, j \rangle \in \mathcal{E}$, we assign a cost c_{ij} (as in (95)) that equals the fuel expenditure incurred during a P2P refueling transaction between the two. Recognizing that either of the two satellites engaged in a P2P refueling transaction can be the active one, we



(a) Constellation Graph (b) Feasible P2P Solution

Figure 36: P2P Formulation.

define the cost associated with each edge of the graph as follows:

$$c_{ij} = \begin{cases} p_{ij}^{\mu} + p_{ji}^{\mu}, & \text{if } s_\mu \text{ can be active, but } s_\nu \text{ cannot,} \\ p_{ji}^{\nu} + p_{ij}^{\nu}, & \text{if } s_\nu \text{ can be active, but } s_\mu \text{ cannot,} \\ \min\{p_{ij}^{\mu} + p_{ji}^{\mu}, p_{ji}^{\nu} + p_{ij}^{\nu}\}, & \text{if either } s_\mu \text{ or } s_\nu \text{ can be active,} \\ \infty, & \text{if neither } s_\mu \text{ nor } s_\nu \text{ can be active.} \end{cases} \tag{129}$$

Figure 36(a) depicts such the bipartite graph for a constellation of 5 fuel-sufficient and 4 fuel-deficient satellites.

4.2.3 P2P Optimization

We are interested in a set $\mathcal{M}_p \subseteq \mathcal{E}$ of $|\mathcal{I}_{d,0}|$ edges that has minimum total cost and that all fuel-deficient satellites are involved in fuel transactions. Let us also associate with each edge $\langle i,j \rangle \in \mathcal{E}$ a binary variable x_{ij} defined as in (87). We allow one satellite to be involved in at most one P2P maneuver. Therefore, the set of edges included in $\mathcal{M}_p$ cannot share a node. Figure 36(b) depicts a feasible P2P solution in the constellation graph shown in Figure 36(a). We thereby can set up the following optimization problem:

$$(\text{P2P} - \text{IP}) \ \min_{\mathcal{M}_p \subseteq \mathcal{E}} \sum_{\langle i,j \rangle \in \mathcal{E}} c_{ij} x_{ij}, \tag{130}$$

such that

$$\sum_{j \in \mathcal{J}_{d,0}} x_{ij} \leq 1, \text{ for all } i \in \mathcal{J}_{s,0}, \tag{131}$$

$$\sum_{i \in \mathcal{J}_{s,0}} x_{ij} = 1, \text{ for all } j \in \mathcal{J}_{d,0}, \tag{132}$$

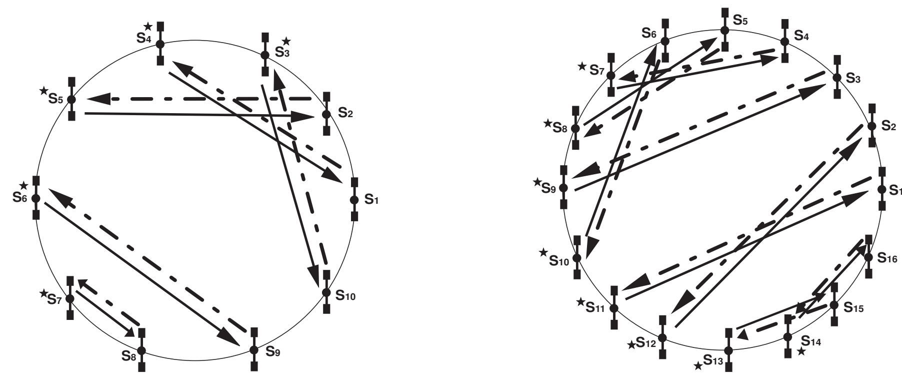
The cost function, given in (130), is the total fuel expenditure corresponding to the P2P maneuvers represented by the edges in $\mathcal{M}_p$. Constraint (131) implies that a fuel-sufficient satellite can be assigned to at most one refueling transaction, while constraint (132) implies that a fuel-deficient satellite has to be assigned to a refueling transaction.

4.3 Numerical Examples: P2P

In this section, we illustrate the P2P refueling scenario with some examples. To this end, let us consider some sample constellations given in Table 5. The optimal assignments for each case can be obtained by solving the optimization problem outlined in the previous section. In particular, we discuss in detail the optimal P2P assignments obtained in the case of constellations C_1 and C_2.

Example 6. *P2P refueling strategy for a constellation of* 10 *satellites.*

Consider the constellation C_1 given in Table 5. This constellation consists of 10 satellites evenly distributed in a circular orbit. The maximum allowed time for refueling is $T = 12$ orbital periods. Each satellite s_i has a minimum fuel requirement of $\underline{f}_i = 12$ units, while the maximum amount of fuel each satellite can hold is $\bar{f}_i = 30$ units. Each satellite has a permanent structure of $m_{si} = 70$ units, and a characteristic constant of $c_{0i} = 2943$ m/s. The indices of the fuel-sufficient satellites are $\mathcal{J}_{s,0} = \{1, 2, 8, 9, 10\}$ and those of the fuel-deficient satelites are $\mathcal{J}_{d,0} = \{3, 4, 5, 6, 7\}$. The optimal P2P assignments obtained after solving the optimization is $s_4 \rightarrow s_1$, $s_5 \rightarrow s_2$, $s_7 \rightarrow s_8$, $s_6 \rightarrow s_9$, $s_3 \rightarrow s_{10}$, and the total fuel consumption for all P2P maneuvers is 26.07 units. This represents 14.48% of the total initial fuel in the constellation. The



(a) Constellation C_1 (b) Constellation C_2

Figure 37: Optimal assignments for P2P refueling.

indices of the active satellites in this case are $\mathcal{J}_a = \{3, 4, 5, 6, 7\}$. Note that $\mathcal{J}_a = \mathcal{J}_{d,0}$, that is, the fuel-deficient satellites are the active ones for the P2P refueling strategy. The optimal P2P assignments are shown in Fig. 37(a). The active satellites are marked by '$\star$'. The forward trips are marked by solid arrows, while the return trips are marked by dotted arrows. The primary reason for fuel-sufficient satellites being the active ones is that their lesser mass compared to their fuel-sufficient counterparts leads to lesser fuel expenditure during the orbital transfers.

Example 7. *P2P refueling strategy for a constellation of* 10 *satellites.*

In this example, we consider the constellation C_2 given in Table 5. This is a constellation of 16 satellites, evenly distributed in a circular orbit. The maximum allowable time for refueling is $T = 30$ orbital periods. Each satellite s_i has a minimum fuel requirement of $\underline{f}_i = 15$ units, a maximum fuel capacity of $\bar{f}_i = 30$ units, permanent structure of $m_{si} = 70$ units, and a characteristic constant of $c_{0i} = 2943$ m/s. The indices of the fuel-sufficient satellites are $\mathcal{J}_{s,0} = \{1, 2, 3, 4, 5, 6, 15, 16\}$, while those of the fuel-deficient satellites are $\mathcal{J}_{d,0} = \{7, 8, 9, 10, 11, 12, 13, 14\}$. The optimal P2P assignments are given by $s_{11} \rightarrow s_1$, $s_{12} \rightarrow s_2$, $s_9 \rightarrow s_3$, $s_7 \rightarrow s_4$, $s_8 \rightarrow s_5$,

$s_{10} \rightarrow s_6$, $s_{13} \rightarrow s_{15}$, $s_{14} \rightarrow s_{16}$ and the total fuel consumption is 37.46 units. This represents 11.71% of the total initial fuel in the constellation. Similar to the previous example, we have that only the fuel-deficient satellites are the active ones, that is, $\mathcal{J}_a = \{7, 8, 9, 10, 11, 12, 13, 14\} = \mathcal{J}_{d,0}$. This is similar to the previous example. The standard P2P assignment for C_2 is shown in Fig. 37(b).

Fig. 6 summarizes the fuel expenditure incurred during the P2P refueling of satellites in the sample constellations. We also indicate the percentage of fuel in the constellation expended to achieve fuel-sufficiency in the constellation. In all these cases, we find that the active satellites carrying out the P2P maneuvers are the fuel-deficient satellites. Of course, the fuel- deficient satellites in all these cases had enough fuel to perform the necessary orbital maneuvers. If any of them were incapable of being the active satellite, a fuel-sufficient satellite has to become the active satellite in its case. In fact, if all the fuel-deficient satellites been empty, all of the P2P maneuvers had to be carried out by the fuel-sufficient satellites. Below, we discuss two cases in which one or more fuel-deficient satellites fail to become the active satellite because of scarcity of initial fuel amount.

Example 8. *P2P refueling strategy when all satellites do not have enough fuel to be active.*

We consider in this example the same constellation $C1$ of Example 6, except for that the fuel-deficient satellites contain 2.0 units of fuel (not sufficient to carry out some of the large-ΔV transfers). This is given by constellation by constellation C_1' in Table 5. Hence, it is not possible for all the fuel-deficient satellites to be the active ones. Under such a consideration, the optimal P2P assignments obtained are: $s_1 \rightarrow s_4$, $s_2 \rightarrow s_5$, $s_6 \rightarrow s_8$, $s_7 \rightarrow s_9$, $s_{10} \rightarrow s_3$. The total fuel expended during the P2P maneuvers is 28.60, which amounts to 15.89% of the initial total fuel in the constellation. Figure 38 shows the optimal P2P maneuvers in the constellation. Note that $\mathcal{J}_{d,0} = \{3, 4, 5, 6, 7\}$ and $\mathcal{J}_a = \{1, 2, 6, 7, 10\}$, so that we no longer have $\mathcal{J}_{d,0} = \mathcal{J}_a$.

The 2.0 units of fuel for satellite s_4, s_5 and s_3 are no longer sufficient to complete the forward trip. Instead, it is possible to carry out the P2P maneuvers by having fuel-sufficient satellites s_1, s_2 and s_{10} to be active.

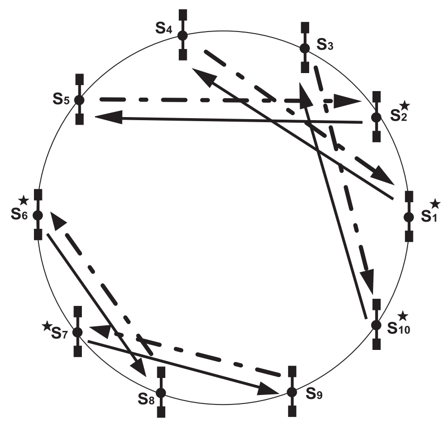


Figure 38: P2P assignments when fuel-deficient satellites do not have enough fuel.

In the P2P refueling strategy discussed so far, we have considered that the satellites, involved in a P2P refueling transaction, engage in a non-cooperative rendezvous, that is, only one of the satellites perform the orbital transfers necessary for refueling. In the remaining part of this chapter, we discuss an extension of the P2P problem, in which we allow both satellites to be active. We call this the Cooperative P2P (C-P2P) refueling strategy. The formulation of the C-P2P strategy is similar to the baseline P2P strategy, as we will show in the next section.

4.4 C-P2P Refueling Strategy

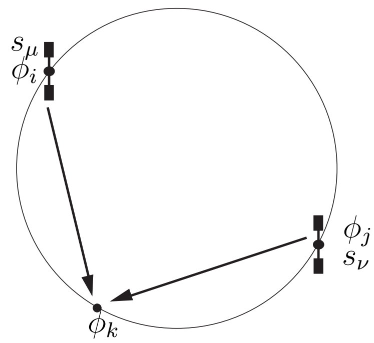
In this section, we formulate the C-P2P refueling problem as an optimization problem over a suitable bipartite constellation graph. Recall that in the C-P2P strategy, we allow cooperative rendezvous between the satellites engaging in a P2P maneuver. To this end, let us consider a set of slots $\Phi' \supseteq \Phi$ on the constellation orbit. These slots are positions where a cooperative rendezvous can take place between two satellites in the constellation. Let $\mathcal{K}$ denote the set of indices for these slots. Now, let us consider a C-P2P maneuver between two satellites $s_\mu = \sigma_0(\phi_i)$ and $s_\nu = \sigma_0(\phi_j)$ occupying the orbital slots ϕ_i and ϕ_j, where $i, j \in \mathcal{J}$. Let these satellites engage in a cooperative rendezvous at the orbital slot ϕ_k, where $k \in \mathcal{K}$. During the first phase of the cooperative P2P maneuver, the two satellites s_μ and s_ν transfer to the orbital slot ϕ_k. After the rendezvous, the satellites s_μ and s_ν are engaged in a fuel exchange and then, in the second phase of the P2P maneuver, the satellites s_μ and s_ν transfer to their original orbital slots ϕ_i and ϕ_j respectively. Without loss of generality, let us assume that s_μ is the fuel-sufficient satellite and that s_ν is the fuel-deficient satellite, that is, $f_\mu^- \geq \underline{f}_\mu$ and $f_\nu^- < \underline{f}_\nu$.

Note that in a non-cooperative P2P maneuver, the amount of fuel exchanged by the two satellites can be determined by the fact that the active satellite returns with just enough fuel to be fuel-sufficient. Unlike the non-cooperative case, the amount of fuel exchanged between the satellites in the cooperative case affects the return trips of both the active satellites. Hence, a natural question that arises here is how to obtain the amount of fuel that must be shared between the two satellites. Of course, the objective is to spend as little fuel during each C-P2P maneuver as possible.

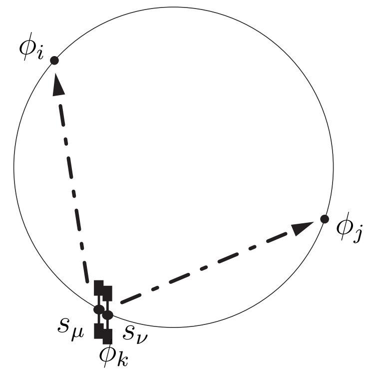
4.4.1 Fuel Expenditure During a C-P2P Maneuver

We now determine the amount of fuel exchange that leads to minimum fuel expenditure during the maneuver. To this end, let us denote by g_μ^ν the amount of fuel that is transferred from satellite s_μ to satellite s_ν. Figure 39 shows the forward and return trips of the C-P2P maneuver. The fuel consumed by the active satellite s_μ to transfer from the orbital slot ϕ_i to the orbital slot ϕ_k is given by:

$$p_{ik}^\mu = \left(m_{\mathrm{s}\mu} + f_\mu^-\right)\left(1 - e^{-\frac{\Delta V_{ik}}{c_{0\mu}}}\right). \tag{133}$$



(a) Forward Trip



(b) Return Trip

Figure 39: C-P2P Maneuver.

Similarly, the fuel expenditure for satellite s_ν to transfer from the orbital slot ϕ_j to the orbital slot ϕ_k is given by:

$$p^\nu_{jk} = \left(m_{s\nu} + f^-_\nu\right)\left(1 - e^{-\frac{\Delta V_{jk}}{c_{0\nu}}}\right). \tag{134}$$

The fuel content of satellite s_μ after its forward trip (but before the fuel exchange takes place) is $f^-_\mu - p^\mu_{ik}$, while that of satellite s_ν is $f^-_\nu - p^\nu_{jk}$. The amount of fuel that s_μ imparts to s_ν is g^ν_μ. Hence, the fuel content of satellite s_μ just after the fuel exchange takes place is $f^-_\mu - p^\mu_{ik} - g^\nu_\mu$, while that of satellite s_ν is $f^-_\nu - p^\nu_{jk} + g^\nu_\mu$. During the return trip, the fuel expenditure of satellite s_μ to transfer from slot ϕ_k to slot ϕ_i is given by

$$p^\mu_{ki} = \left(m_{s\mu} + f^-_\mu - p^\mu_{ik} - g^\nu_\mu\right)\left(1 - e^{-\frac{\Delta V_{ki}}{c_{0\mu}}}\right), \tag{135}$$

while that of satellite s_ν to transfer from slot ϕ_k to slot ϕ_j is given by

$$p^\nu_{kj} = \left(m_{s\nu} + f^-_\nu - p^\nu_{jk} + g^\nu_\mu\right)\left(1 - e^{-\frac{\Delta V_{kj}}{c_{0\nu}}}\right). \tag{136}$$

The final fuel content of satellite s_μ after the cooperative P2P maneuver is given by $f^+_\mu = f^-_\mu - p^\mu_{ik} - g^\nu_\mu - p^\mu_{ki}$, while that of satellite s_ν is given by $f^+_\nu = f^-_\nu - p^\nu_{jk} + g^\nu_\mu - p^\nu_{kj}$. Using the above equations, we have

$$f^+_\mu = \left(m_{s\mu} + f^-_\mu - g^\nu_\mu - p^\mu_{ik}\right) e^{-\frac{\Delta V_{ki}}{c_{0\mu}}} - m_{s\mu}, \tag{137}$$

and

$$f_\nu^+ = \left(m_{s\nu} + f_\nu^- + g_\mu^\nu - p_{jk}^\nu\right) e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} - m_{s\nu}. \tag{138}$$

We therefore have

$$\begin{aligned} f_\mu^+ + f_\nu^+ = {} & \left(m_{s\mu} + f_\mu^- - p_{ik}^\mu\right) e^{-\frac{\Delta V_{ki}}{c_{0\mu}}} - g_\mu^\nu e^{-\frac{\Delta V_{ki}}{c_{0\mu}}} + \left(m_{s\nu} + f_\nu^- - p_{jk}^\nu\right) e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} \\ & + g_\mu^\nu e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} - \left(m_{s\mu} + m_{s\nu}\right). \end{aligned} \tag{139}$$

Minimizing the fuel expenditure during a C-P2P maneuver is the same as maximizing the total fuel content $f_\mu^+ + f_\nu^+$ of the satellites after the maneuver. From the above equation, $f_\mu^+ + f_\nu^+$ is maximized when

$$g_\mu^\nu e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} - g_\mu^\nu e^{-\frac{\Delta V_{ki}}{c_{0\mu}}} = g_\mu^\nu \left(e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} - e^{-\frac{\Delta V_{ki}}{c_{0\mu}}}\right)$$

is maximized. Recall that both satellites need to be fuel-sufficient after the P2P maneuver. Satellite s_μ will be fuel-sufficient if

$$f_\mu^+ \geq \underline{f}_\mu,$$

that is,

$$\left(m_{s\mu} + f_\mu^- - g_\mu^\nu - p_{ik}^\mu\right) e^{-\frac{\Delta V_{ki}}{c_{0\mu}}} - m_{s\mu} \geq \underline{f}_\mu,$$

or

$$g_\mu^\nu e^{-\frac{\Delta V_{ki}}{c_{0\mu}}} \leq \left(m_{s\mu} + f_\mu^- - p_{ik}^\mu\right) e^{-\frac{\Delta V_{ki}}{c_{0\mu}}} - \left(m_{s\mu} + \underline{f}_\mu\right),$$

and hence,

$$g_\mu^\nu \leq \left(m_{s\mu} + f_\mu^- - p_{ik}^\mu\right) - \left(m_{s\mu} + \underline{f}_\mu\right) e^{\frac{\Delta V_{ki}}{c_{0\mu}}}.$$

Also, satellite s_ν will be fuel-sufficient if

$$f_\nu^+ \geq \underline{f}_\nu,$$

that is,

$$\left(m_{s\nu} + f_\nu^- + g_\mu^\nu - p_{jk}^\nu\right) e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} - m_{s\nu} \geq \underline{f}_\nu,$$

or,

$$g_\mu^\nu e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} \geq \left(m_{s\nu} + \underline{f}_\nu\right) - \left(m_{s\nu} + f_\nu^- - p_{jk}^\nu\right) e^{-\frac{\Delta V_{kj}}{c_{0\nu}}},$$

and hence,

$$g_\mu^\nu e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} \geq \left(m_{s\nu} + \underline{f}_\nu\right) - \left(m_{s\nu} + f_\nu^- - p_{jk}^\nu\right) e^{-\frac{\Delta V_{kj}}{c_{0\nu}}}.$$

The conditions of fuel-sufficiency on the satellites provide us with a lower bound $g_\mu^\nu|_\ell$ on the amount of fuel exchange, given by

$$g_\mu^\nu|_\ell = \left(m_{s\nu} + \underline{f}_\nu\right) e^{\frac{\Delta V_{kj}}{c_{0\nu}}} - \left(m_{s\nu} + f_\nu^- - p_{jk}^\nu\right). \tag{140}$$

It also provides an upper bound $g_\mu^\nu|_u$ on the amount of fuel exchange, given by

$$g_\mu^\nu|_u = \left(m_{s\mu} + f_\mu^- - p_{ik}^\mu\right) - \left(m_{s\mu} + \underline{f}_\mu\right) e^{\frac{\Delta V_{ki}}{c_{0\mu}}}. \tag{141}$$

As mentioned already, we need to maximize $g_\mu^\nu \left(e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} - e^{-\frac{\Delta V_{ki}}{c_{0\mu}}}\right)$. This is maximized if

$$g_\mu^\nu = \begin{cases} g_\mu^\nu|_\ell, & e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} < e^{-\frac{\Delta V_{ki}}{c_{0\mu}}}, \\ g_\mu^\nu|_u, & e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} > e^{-\frac{\Delta V_{ki}}{c_{0\mu}}}. \end{cases} \tag{142}$$

Clearly, if $e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} = e^{-\frac{\Delta V_{ki}}{c_{0\mu}}}$, then g_μ^ν can assume any value in the interval $g_\mu^\nu|_\ell \leq g_\mu^\nu \leq g_\mu^\nu|_u$.

To determine the final fuel content of the satellites when the fuel exchange is optimal, we need to consider two cases. If $e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} < e^{-\frac{\Delta V_{ki}}{c_{0\mu}}}$, we have

$$\begin{aligned} f_\nu^+ &= \left(m_{s\nu} + f_\nu^- + g_\mu^\nu|_k - p_{jk}^\nu\right) e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} - m_{s\nu} \\ &= \left(m_{s\nu} + f_\nu^- - \left(m_{s\nu} + f_\nu^- - p_{jk}^\nu\right) - p_{jk}^\nu\right) e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} + \left(m_{s\nu} + \underline{f}_\nu\right) - m_{s\nu} \\ &= \underline{f}_\nu, \end{aligned} \tag{143}$$

which implies that s_ν returns with just enough fuel to be fuel-sufficient. On the other hand, if $e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} > e^{-\frac{\Delta V_{ki}}{c_{0\mu}}}$, we have

$$\begin{aligned} f_\mu^+ &= \left(m_{s\mu} + f_\mu^- - g_\mu^\nu|_u - p_{ik}^\mu\right) e^{-\frac{\Delta V_{ki}}{c_{0\mu}}} - m_{s\mu} \\ &= \left(m_{s\mu} + f_\mu^- - \left(m_{s\mu} + f_\mu^- - p_{ik}^\mu\right) - p_{ik}^\mu\right) e^{-\frac{\Delta V_{ki}}{c_{0\mu}}} + \left(m_{s\mu} + \underline{f}_\mu\right) - m_{s\mu} \\ &= \underline{f}_\mu, \end{aligned} \tag{144}$$

which implies that s_μ returns with just enough fuel to be fuel-sufficient.

If both satellites have the same engine characteristics, then $c_{0\mu} = c_{0\nu}$, and $e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} < e^{-\frac{\Delta V_{ki}}{c_{0\mu}}}$, equivalently, $\frac{\Delta V_{kj}}{c_{0\nu}} > \frac{\Delta V_{ki}}{c_{0\mu}}$, and hence, $\Delta V_{kj} > \Delta V_{ki}$. Similarly, $e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} > e^{-\frac{\Delta V_{ki}}{c_{0\mu}}}$ implies that $\Delta V_{kj} > \Delta V_{ki}$. We can summarize our findings with the following proposition.

Proposition 3. *If two satellites engaging in a cooperative P2P maneuver have engines with the same specific thrust, the optimal fuel exchange takes place when the satellite making the costlier ΔV transfer returns with just enough fuel to be fuel-sufficient.*

4.4.2 C-P2P Optimization Problem

Similar to solving the P2P refueling problem, let us consider the undirected bipartite graph $\mathcal{G}$ with the two graph partitions being the orbital slots of the fuel-sufficient satellites $\mathcal{J}_{s,0}$ and those of the fuel-deficient satellites $\mathcal{J}_{d,0}$. There exists an edge $\langle i, j\rangle \in \mathcal{J}_{s,0} \times \mathcal{J}_{d,0}$ if the satellites $s_\mu = \sigma_0(\phi_i)$ and $s_\nu = \sigma_0(\phi_j)$ can engage in a cooperative or non-cooperative P2P refueling transaction, such that, at the end of the refueling process, both satellites end up being fuel-sufficient. Let $\mathcal{E} \subseteq \mathcal{J}_{s,0} \times \mathcal{J}_{d,0}$ be the set of all edges in $\mathcal{G}$. To each edge $\langle i, j\rangle \in \mathcal{E}$, we assign a cost c_{ij} that equals the fuel expenditure incurred during the cheapest (among all non-cooperative and cooperative) P2P maneuver between the two. Let the satellites $s_\mu = \sigma_0(\phi_i)$ and $s_\nu = \sigma_0(\phi_j)$ be involved in a cooperative rendezvous at the orbital slot $\phi_k \in \Phi'$, where Φ' is the set of all possible orbital slots on the orbit. Note that $\Phi \subseteq \Phi'$. The fuel expenditure incurred during the cooperative maneuver is given by

$$c_{ij}|_{\phi_k} = \left(p^{\mu}_{ik} + p^{\nu}_{jk}\right) + \left(p^{\mu}_{ki} + p^{\nu}_{kj}\right) \tag{145}$$

Note that $\phi_k = \phi_i$ corresponds to a non-cooperative maneuver, in which the satellite s_ν is active, while $\phi_k = \phi_j$ corresponds to a non-cooperative maneuver, in which the satellite s_ν is active. The minimum over all cooperative and non-cooperative fuel expenditures is assigned to be the weight of the edge $\langle i, j\rangle$. Therefore, we have

$$c_{ij} = \min_{\phi_k \in \Phi'} c_{ij}|_{\phi_k} \tag{146}$$

For convenience, let us also define a function $\mathsf{Coop} : \mathcal{E} \mapsto \Phi'$ such that

$$\mathsf{Coop}\,(i, j) = \arg\min_{\phi_k \in \Phi'} c_{ij}|_{\phi_k} \tag{147}$$

Note that if for edge $\langle i, j\rangle$, the cheapest maneuver is non-cooperative, then $\mathsf{Coop}\,(i, j)$ gives the orbital slot of the passive satellite. We are interested in a set $\mathcal{M}_c \subseteq \mathcal{E}$ of $|\mathcal{I}_{d,0}|$ edges that has minimum total cost and such that all fuel-deficient satellites are involved in fuel transactions. Similarly to what we did for the P2P refueling problem, let us also associate with each edge $\langle i, j\rangle \in \mathcal{E}$ a binary variable x_{ij}, defined as

$$x_{ij} = \begin{cases} 1, & \text{if } \langle i, j\rangle \in \mathcal{M}_c, \\ 0, & \text{otherwise.} \end{cases} \tag{148}$$

We can therefore consider the following optimization problem:

$$(\mathrm{CP2P} - \mathrm{IP}) : \min_{\mathcal{M}_c \subseteq \mathcal{E}} \sum_{\langle i,j\rangle \in \mathcal{E}} c_{ij} x_{ij}, \tag{149}$$

such that

$$\sum_{j \in \mathcal{J}_{d,0}} x_{ij} \leq 1, \text{ for all } i \in \mathcal{J}_{s,0}, \tag{150}$$

$$\sum_{i \in \mathcal{J}_{s,0}} x_{ij} = 1, \text{ for all } j \in \mathcal{J}_{d,0}. \tag{151}$$

As before, constraint (150) implies that a fuel-sufficient satellite must be assigned to at most one refueling transaction, while constraint (151) implies that a fuel-deficient satellite has to be assigned to a refueling transaction. However, for the C-P2P problem, we require additional constraints to be imposed. For instance, consider two edges $\langle i, j\rangle, \langle q, r\rangle \in \mathcal{M}_c$. Note that if $\mathsf{Coop}\,(i, j) = \mathsf{Coop}\,(q, r)$, then this implies either one of the following:

i) A cooperative rendezvous corresponding to the two edges occur at the same orbital slot, or

ii) A cooperative rendezvous corresponding to one edge occurs at the slot of the passive satellite corresponding to another edge.

Both cases are impractical and cannot occur physically. Hence, we have to ensure that the following additional constraint also holds:

$$\mathsf{Coop}\,(i, j) \neq \mathsf{Coop}\,(q, r) \;\text{ for all } \langle i, j\rangle, \langle q, r\rangle \in \mathcal{M}_c. \tag{152}$$

The determination of the optimal C-P2P solution requires the minimization of the objective given in (149), subject to the constraints (150)-(152).

4.4.3 Methodology

We can solve the optimization problem given by (149)-(151) to find the set of edges $\mathcal{M}_c$. The set $\mathcal{M}_c$ may or may not be a feasible C-P2P solution, because it may or may not satisfy constraint (152). If it does, then we have the optimal C-P2P solution and we are done. If the constraint (152) is not satisfied, then another bipartite matching problem can be set up in order to yield the optimal (and feasible) C-P2P solution for the same set of satellite pairs (or refueling transactions) given by $\mathcal{M}_c$. We discuss below how this can be achieved.

Let us construct a bipartite graph, with one of the partitions representing the orbital slots given by Φ', and the other partition comprised of nodes representing the

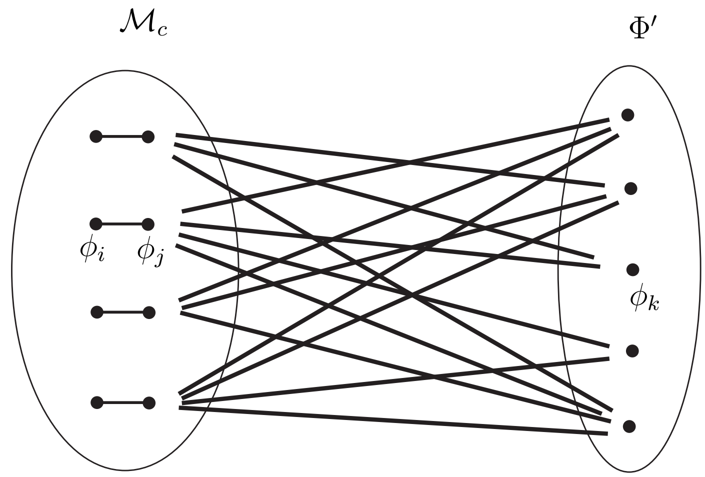


Figure 40: Bipartite graph for determining the C-P2P solution given $\mathcal{M}_c$.

edges given by $\mathcal{M}_c$. Figure 40 depicts such a graph. We say that there exists an edge $< \langle i,j \rangle, \phi_k >$ between $\langle i,j \rangle \in \mathcal{M}_c$ and $\phi_k \in \Phi$, if satellites $s_\mu = \sigma_0(\phi_i)$ and $s_\nu = \sigma_0(\phi_j)$ can engage in a feasible cooperative P2P maneuver at the orbital slot $\phi_k \in \Phi'$, such that at the end of the overall maneuver the satellites return to their original slots with enough amount of fuel to be fuel-sufficient. Let $\mathcal{E}_c$ denote the set of all such edges. We are interested in a set $\mathcal{M}_c \subseteq \mathcal{E}_c$ of edges that assigns to each fuel transaction a slot for cooperative rendezvous and which leads to a feasible C-P2P solution. To this end, let us assign to each edge the binary variable

$$y_{ijk} = \begin{cases} 1, & \text{if} < \langle i,j \rangle, k > \in \mathcal{M}_c, \\ 0, & \text{otherwise.} \end{cases} \tag{153}$$

The following optimization problem yields the optimal C-P2P solution, given the fuel transactions depicted by the infeasible solution $\mathcal{M}_c$:

$$\min_{\mathcal{M}_c \subseteq \mathcal{E}_c} \sum_{< \langle i,j \rangle, \phi_k > \in \mathcal{E}_c} c_{ij}|_{\phi_k}\, y_{ijk}, \tag{154}$$

subject to

$$\sum_{\phi_k \in \Phi'} y_{ijk} = 1, \text{ for all } \langle i,j \rangle \in \mathcal{M}_c, \tag{155}$$

$$\sum_{\langle i,j \rangle \in \mathcal{M}_c} y_{ijk} \leq 1, \text{ for all } \phi_k \in \Phi', \tag{156}$$

Constraint (155) signifies that all fuel transactions need to be assigned a slot for rendezvous, while constraint (156) signifies that an orbital slot can be assigned to at most one refueling transaction. The solution to this optimization problem yields the cheapest feasible C-P2P solution corresponding to the fuel transactions determined by $\mathcal{M}_c$.

4.5 C-P2P Numerical Examples

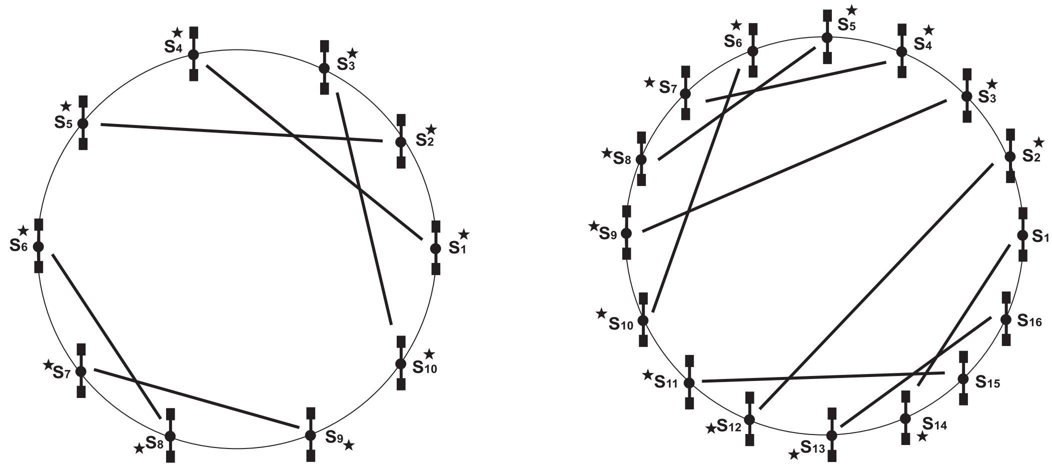
In this section, we will consider sample constellations and will determine the optimal C-P2P refueling strategy for each one of them. We will also compare the total fuel expenditure incurred using C-P2P and P2P refueling of the satellites for the constellations given in Table 5. These numerical examples demonstrate the usefulness of a C-P2P refueling strategy.

Example 9. *C-P2P refueling strategy for constellation* C_1.

For this example the orbital slots for cooperative rendezvous to take place have been assumed to be equally spaced at intervals of 9 deg along the orbit. Hence, there are 40 available slots for the cooperative rendezvous to take place, including the 10 orbital slots occupied originally by the satellites. The optimal assignments obtained from the solution of the optimization problem (CP2P-IP) were found to be non-cooperative. Note that since $\Phi \subseteq \Phi'$, the optimal solution of (CP2P-IP) will be the optimal P2P solution if there exists no cooperative solution that is cheaper than the optimal P2P case. In other words, cooperative maneuvers in cases such as in this example do not help in reducing the fuel expenditure of the overall refueling process.

Example 10. *C-P2P refueling strategy for constellation* C_1'.

As in the previous example, the orbital slots for cooperative rendezvous to take place are equally spaced at intervals of 9 deg along the orbit. The assignments are determined by solving the optimization problem (CP2P-IP), and are given by: $s_1 \leftrightarrow$



(a) Constellation C_1' (b) Constellation C_2

Figure 41: Optimal assignments for C-P2P refueling.

s_4, $s_2 \leftrightarrow s_5$, $s_8 \leftrightarrow s_6$, $s_9 \leftrightarrow s_7$ and $s_{10} \leftrightarrow s_3$. All of these maneuvers are cooperative. For instance, satellites s_1 and s_4 rendezvous at the orbital slot with a lead angle of 54 deg with respect to satellite s_1. Similarly, satellites s_8 and s_6 engage in a cooperative maneuver in which both satellites cooperatively rendezvous at the orbital slot with a lead angle of 27 deg. The solution to the C-P2P integer program yields no conflict that violates the additional constraint. Hence, the above solution corresponds to the optimal C-P2P assignments. The fuel expenditure corresponding to this set of C-P2P assignments is 27.19 units, a reduction of about 8% over the optimal P2P fuel expenditure. This example demonstrates the benefit of allowing satellites to engage in cooperative rendezvous when the fuel-deficient satellites do not have enough fuel to complete the non-cooperative rendezvous. Figure 41(a) shows the optimal C-P2P assignments obtained for this example. An important observation for this example is that for each of the C-P2P maneuvers, the cooperative rendezvous takes place in a slot at which the fuel-deficient satellite arrives by having exhausted almost all of its fuel. In other words, the fuel-deficient satellite moves as close to the fuel-sufficient satellite as it is permitted by its onboard fuel. The final fuel contents of the satellites after the C-P2P maneuvers have taken place are 12.0, 12.0, 13.1, 13.1, 13.1, 12.0, 12.0, 15.5, 15.5 and 12.0 respectively.

Example 11. *C-P2P refueling strategy for constellation C_2.*

Let us now consider the constellation C_2 given in Table 5. The fuel expenditure incurred in the P2P refueling of the satellites in the constellation is 39.67 units. The optimal C-P2P assignments, as determined by solving the (CP2P-IP), are given as follows: $s_1 \leftrightarrow s_{14}$, $s_2 \leftrightarrow s_{12}$, $s_3 \leftrightarrow s_9$, $s_4 \leftrightarrow s_7$, $s_5 \leftrightarrow s_8$, $s_6 \leftrightarrow s_{10}$, $s_{15} \leftrightarrow s_{11}$ and $s_{16} \leftrightarrow s_{13}$. Of these maneuvers, two are non-cooperative, namely the assignments $s_1 \leftrightarrow s_{14}$ and $s_{16} \leftrightarrow s_{13}$. For these, the fuel-deficient satellites have enough fuel to be active. The remaining maneuvers are cooperative. Allowing for cooperative maneuvers reduces the overall fuel expenditure to 36.98 units, which is about 6.8% less than the optimal P2P fuel expenditure. Similarly to the previous example, we have that for the cooperative maneuvers, the fuel-deficient satellites move as close to the fuel-sufficient satellites as permitted by their onboard fuel. Figure 41(b) shows the C-P2P assignments. The final fuel contents of the satellites in the constellation are given by 16.2, 12.3, 12.0, 16.1, 16.1, 14.8, 12.0, 12.0, 12.2, 12.0, 12.0, 12.0, 12.0, 12.0, 14.9 and 16.2 units. The solution generated by the optimization problem (CP2P-IP) does not violate the additional constraint (152). Hence, this is the optimal C-P2P solution. Figure 42 summarizes the results for the sample constellations of Table 5. The optimal P2P and C-P2P fuel expenditure for these constellations are shown. For the constellations C_1 and C_4, the optimal non-cooperative P2P solution is the cheapest way to redistribute fuel in the constellation. For these, the fuel-deficient satellites have enough fuel to complete a non-cooperative rendezvous. Whenever this is not possible, as in case of the remaining constellations, cooperative maneuvers turn out to be beneficial.

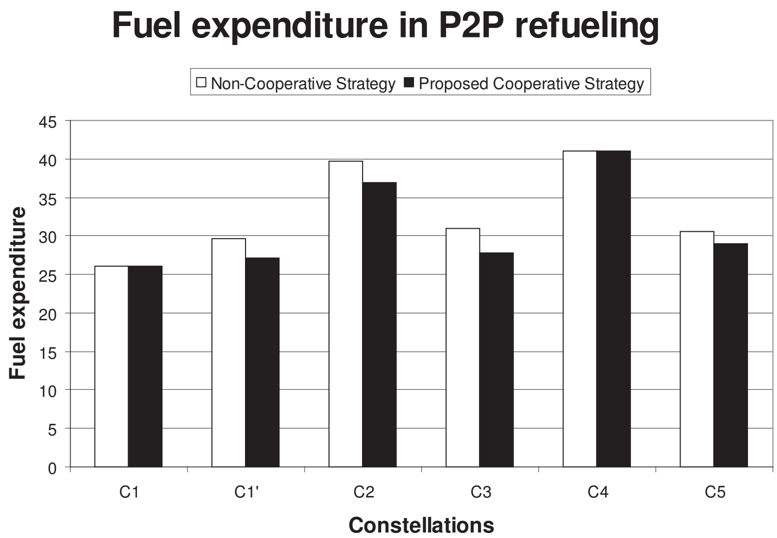
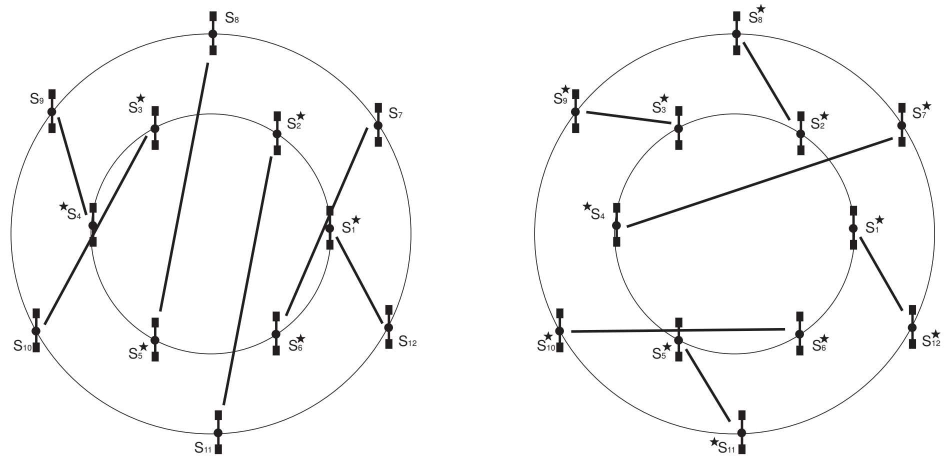


Figure 42: Comparison of P2P and C-P2P refueling strategies for the sample constellations of Table 5.

4.6 Peer-to-Peer Refueling Between Satellites in Different Orbits

In this section, we discuss the P2P refueling strategy for satellites in two different circular orbits. To this end, let us therefore consider a circular constellation consisting of two circular orbits of radii r_1 and r_2, where $r_1 < r_2$. Let the number of satellites in orbit r_1 be denoted by n_1, while that in orbit r_2 be given by n_2. The total number of satellites is therefore $n = n_1 + n_2$. One can consider that the fuel-sufficient satellites are in one orbit, say r_1, while the fuel-deficient satellites are in the other orbit r_2. During a P2P refueling transaction between two satellites $s_\mu = \sigma_0(\phi_i)$ and $s_\nu = \sigma_0(\phi_j)$ occupying the orbital slots ϕ_i on the orbit r_1 and ϕ_j on the orbit r_2, where $i, j \in \mathcal{J}$, we allow these satellites to engage in a cooperative rendezvous at the orbital slot ϕ_{k_r} on an intermediate orbit r. During the first phase of the cooperative P2P maneuver, the two satellites s_μ and s_ν transfer to the orbital slot ϕ_{k_r}. After rendezvous, the satellites s_μ and s_ν are engaged in a fuel exchange and then in the second phase of the P2P maneuver, satellites s_μ and s_ν transfer to their original orbital slots ϕ_i



(a) Non-cooperative Strategy (b) Cooperative Strategy

Figure 43: Optimal assignments for P2P refueling.

and ϕ_j respectively. We will assume that the time allotted for the refueling process allows phase-free Hohmann transfers between the two orbits r_1 and r_2 during the trips. In other words, the satellites engage either in a HHCM or a HPCM rendezvous to exchange fuel between themselves. The optimal refueling strategy can be obtained by employing the methodology illustrated perviously in this chapter.

The next example demonstrates the benefits of cooperative rendezvous for P2P refueling for a large number of satellites in two different coplanar circular orbits.

Example 12. *P2P refueling for a constellation of* 12 *satellites in two circular orbits, each orbit having* 6 *satellites.*

Consider a satellite constellation of two circular orbits, one at an altitude of 1000 Km and the other at an altitude of 1075 Km. The upper orbit has 6 fuel-deficient satellites, while the lower orbit is populated with 6 satellites, all of which are fuel-sufficient. The orbital slots of the satellites in the lower orbit are given by $\Phi_1 = \{0, 60, 120, 180, 210, 270, 330\}$ deg. The fuel content of these satellites are 27, 29, 30, 29.5, 28.5 and 28 units respectively. Similarly, the orbital slots of the satellites in the upper orbit are given by $\Phi_2 = \{30, 90, 150, 210, 270, 330\}$ deg. The fuel content of these satellites are 0.75, 0.70, 0.80, 0.60 and 0.65 units respectively. Each satellite has a minimum fuel requirement of $\underline{f}_i = 12$ units, where $i = 1, \ldots, 12$, while the maximum amount of fuel is $\bar{f}_i = 30$ units. Each satellite has a permanent structure of $m_{si} = 70$ units, and a characteristic constant of $c_{0i} = 2943$ m/s. The indices of the fuel-sufficient satellites are $\mathcal{I}_{s,0} = \{1, 2, 3, 4, 5, 6\}$ and those of the fuel-deficient satellites are $\mathcal{J}_{d,0} = \{7, 8, 9, 10, 11, 12\}$. If all satellites are restricted to engage in non-cooperative rendezvous, then the optimal P2P assignments are $s_1 \leftrightarrow s_{12}$, $s_2 \leftrightarrow s_{11}$, $s_3 \leftrightarrow s_{10}$, $s_4 \leftrightarrow s_9$, $s_5 \leftrightarrow s_8$, and $s_6 - s_7$. The corresponding total fuel expenditure during the refueling process is 12.80 units. The optimal solution is depicted in Fig. 43(a).

If the satellites are allowed to engage in cooperative rendezvous, the optimal C-P2P assignments are $s_1 \leftrightarrow s_{12}$, $s_2 \leftrightarrow s_8$, $s_3 \leftrightarrow s_9$, $s_4 \leftrightarrow s_7$, $s_5 \leftrightarrow s_{11}$, and $s_6 \leftrightarrow s_{10}$. All of these refueling transactions involve cooperative rendezvous. The total fuel expenditure is given by 11.38 units, implying a reduction of fuel consumption by 11% when we allow for cooperative rendezvous between the satellites. Figure 43(b) shows the optimal C-P2P assignments. For instance, satellites s_1 and s_2 meet on the orbit of radius $r = 1.0042$ after both preforming Hohmann transfers. In fact, for all of the refueling transactions, the satellites engage in HHCM rendezvous. In each case, although the allotted time is enough for a non-cooperative Hohmann transfer between the participating satellites, the fuel-deficient satellites do not have enough fuel to complete the non-cooperative Hohmann transfer. Instead, they move as close as possible to the orbit of a fuel-sufficient satellite by expending all their fuel.

4.7 Summary

In this chapter, we discussed the formulation of the P2P refueling strategy, based on the notion of achieving fuel-sufficiency in the constellation. We formulated the problem using a bipartite graph, and outlined the optimization problem that needs to be solved in order to determine the optimal P2P maneuvers. It is observed that in the optimal P2P strategy, the fuel-deficient satellites tend to be active because of their smaller mass leading to lesser fuel expenditure during the orbital transfers. However, the fuel of a fuel-deficient satellite may not be sufficient for it to be active. In such cases, fuel-sufficient satellites may be active. In the baseline P2P strategy, it is assumed that only one of the two satellites, engaging in a P2P maneuver, is active. We extend the P2P problem to the case of a Cooperative P2P (C-P2P) refueling strategy, in which we allow both satellites, participating in a refueling transaction, to be active. We discussed the formulation of the C-P2P strategy, and with the help of numerical examples compared it to the baseline P2P strategy. It has been found that cooperative maneuvers are particularly beneficial when the fuel-deficient satellites have too low fuel to perform the non-cooperative maneuvers. This is a particularly important result for a problem such as refueling, because a refueling mission would be performed at end-of-life of fuel of the satellites, and it is likely that the fuel-deficient satellites would have very low fuel content.

Table 5: Sample Constellations.

Label	Description
C_1	10 satellites, Altitude $= 35,786$ Km, $T = 12$ f_i^-: $30, 30, 6, 6, 6, 6, 6, 30, 30, 30$ $\bar{f}_i = 30$, $\underline{f}_i = 12$, $m_{si} = 70$ for all satellites
C_1'	10 satellites, Altitude $= 35,786$ Km, $T = 12$ f_i^-: $30, 30, 1.5, 1.5, 1.5, 1.5, 1.5, 30, 30, 30$ $\bar{f}_i = 30$, $\underline{f}_i = 12$, $m_{si} = 70$ for all satellites
C_2	16 satellites, Altitude $= 1,200$ Km, $T = 30$ f_i^-: $30, 30, 30, 30, 30, 30, 10, 10, 10, 10, 10, 10, 10, 10, 30, 30$ $\bar{f}_i = 30$, $\underline{f}_i = 15$, $m_{si} = 70$ for all satellites
C_2'	16 satellites, Altitude $= 1,200$ Km, $T = 30$ f_i^-: $30, 30, 30, 30, 30, 30, 1.5, 1.5, 1.5, 1.5, 1.5, 1.5, 1.5, 1.5, 1.5, 1.5$ $\bar{f}_i = 30$, $\underline{f}_i = 15$, $m_{si} = 70$ for all satellites
C_3	12 satellites, Altitude $= 2,000$ Km, $T = 30$ f_i^-: $30, 30, 30, 10, 10, 10, 10, 10, 10, 30, 30, 30$ $\bar{f}_i = 30$, $\underline{f}_i = 15$, $m_{si} = 70$ for all satellites
C_3'	12 satellites, Altitude $= 12,000$ Km, $T = 20$ f_i^-: $25, 25, 25, 25, 25, 25, 2, 2, 2, 2, 2, 2$ $\bar{f}_i = 25$, $\underline{f}_i = 12$, $m_{si} = 75$ for all satellites
C_4	18 satellites, Altitude $= 6,000$ Km, $T = 25$ f_i^-: $25, 25, 25, 25, 25, 25, 25, 25, 25, 6, 6, 6, 6, 6, 6, 6, 6, 6$ $\bar{f}_i = 25$, $\underline{f}_i = 12$, $m_{si} = 75$ for all satellites
C_5	12 satellites, Altitude $= 12,000$ Km, $T = 20$ f_i^-: $25, 25, 25, 25, 25, 25, 8, 8, 8, 8, 8, 8$ $\bar{f}_i = 25$, $\underline{f}_i = 12$, $m_{si} = 75$ for all satellites
C_5'	14 satellites, Altitude $= 1,400$ Km, $T = 35$ f_i^-: $25, 25, 25, 25, 25, 25, 25, 1.5, 1.5, 1.5, 1.5, 1.5, 1.5, 1.5$ $\bar{f}_i = 25$, $\underline{f}_i = 10$, $m_{si} = 75$ for all satellites
C_6	14 satellites, Altitude $= 1,400$ Km, $T = 35$ f_i^-: $25, 25, 25, 25, 25, 25, 25, 8, 8, 8, 8, 8, 8, 8$ $\bar{f}_i = 25$, $\underline{f}_i = 12$, $m_{si} = 75$ for all satellites
C_7	16 satellites, Altitude $= 30,000$ Km, $T = 15$ f_i^-: $10, 10, 10, 10, 10, 10, 10, 10, 28, 28, 28, 28, 28, 28, 28, 28$ $\bar{f}_i = 30$, $\underline{f}_i = 15$, $m_{si} = 70$ for all satellites
C_8	16 satellites, Altitude $= 1,200$ Km, $T = 30$ f_i^-: $30, 10, 30, 10, 30, 10, 30, 10, 30, 10, 30, 10, 30, 10, 30, 10$ $\bar{f}_i = 30$, $\underline{f}_i = 15$, $m_{si} = 70$ for all satellites

Table 6: Fuel expenditures during P2P refueling.

Constellation	Optimal P2P Fuel Expenditure	Percentage of total fuel in constellation
C_1	26.07	14.48%
C_2	37.46	11.71%
C_3	26.73	11.14%
C_4	41.06	14.72%
C_5	23.38	11.81%
C_6	28.77	12.45%
C_7	19.26	6.35%
C_8	9.38	2.93%

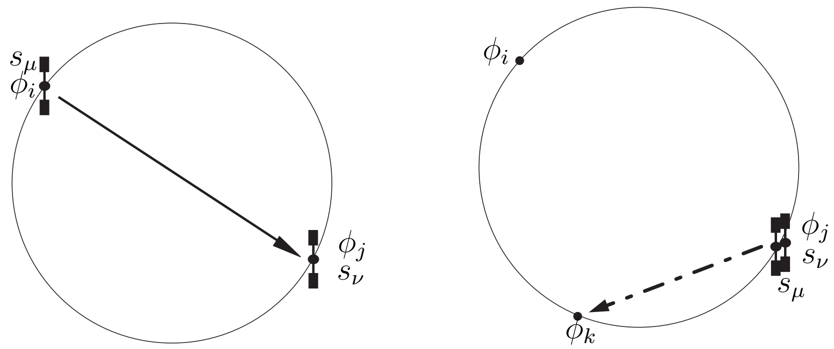
CHAPTER V

EGALITARIAN PEER-TO-PEER REFUELING STRATEGY

In the the P2P refueling strategy discussed so far, we have considered that the active satellites are constrained to return to their original orbital slots. In this chapter, we consider an extension of the baseline P2P refueling problem, in which we remove this constraint. In other words, we allow the active satellites to interchange their orbital slots during their return trips. The underlying assumption is that all satellites are similar and capable of performing the same functions, and hence can replace each other in the constellation. This extension of the P2P refueling problem is referred to as the the Egalitarian P2P (E-P2P) refueling strategy. Note that by E-P2P, we mean a non-cooperative maneuver, that is, only one of the two satellites involved in a fuel exchange is active, and after the fuel exchange it returns to any available orbital slot left vacant by other active satellite. In this chapter, we discuss the formulation of the E-P2P refueling problem, and demonstrate with examples the benefits of this strategy in reducing the fuel expenditure during the refueling process.

5.1 Formulation Using an Undirected Constellation Graph

During an E-P2P maneuver, an active satellite performs an orbital transfer to rendezvous with a passive satellite, exchanges fuel, and then returns back to an orbital position left vacant by another active satellite. Let us consider an E-P2P maneuver between two satellites s_μ and s_ν initially occupying the orbital slots ϕ_i and ϕ_j respectively. Figure 44 depicts the forward and return trips of the E-P2P maneuver in which the satellite s_μ is active. During the forward trip, the active satellite s_μ



(a) Forward Trip

(b) Return Trip

Figure 44: E-P2P Maneuver (s_μ active).

performs an orbital transfer from the slot ϕ_i to the slot ϕ_j occupied by the passive satellite s_μ. During the return trip, the satellite s_μ returns to an orbital slot ϕ_k left vacant by another active satellite. Note that the E-P2P maneuver comprises of three slots, ϕ_i, ϕ_j, and ϕ_k. Hence, an E-P2P maneuver can be represented by a triplet of orbital slots. In this section, we will formulate the E-P2P problem over an undirected tripartite graph.

To this end, let us define a complete tripartite constellation graph $\mathcal{G}$ consisting of three partitions. The first partition consists of nodes that correspond to the elements of the index set $\mathcal{J}_{s,0}$, the second partition consists of nodes that correspond to the elements of the index set $\mathcal{J}_{d,0}$, and the third partition consists of nodes that correspond to the elements of the index set $\mathcal{J}_a$. Therefore, nodes of $\mathcal{G}$ are given by $\mathcal{J}_{s,0} \cup \mathcal{J}_{d,0} \cup \mathcal{J}_a$ and the edges of $\mathcal{G}$ are all edges induced by triplets in $\mathcal{J}_{s,0} \times \mathcal{J}_{d,0} \times \mathcal{J}_a$, that is, $\mathcal{G} = \{\mathcal{J}_{s,0} \cup \mathcal{J}_{d,0} \cup \mathcal{J}_a, \mathcal{J}_{s,0} \times \mathcal{J}_{d,0} \times \mathcal{J}_a\}$.

Let us consider a triplet $(i, j, k) \in \mathcal{J}_{s,0} \times \mathcal{J}_{d,0} \times \mathcal{J}_a$. We say that the triplet (i, j, k) is *feasible* if the satellites $\sigma_0(\phi_i)$ and $\sigma_0(\phi_j)$ can engage in a *feasible* P2P refueling maneuver, that is, the active satellite (which can be either $\sigma_0(\phi_i)$ or $\sigma_0(\phi_j)$)

rendezvous with the passive satellite, exchanges fuel, and then returns to the orbital slot initially occupied by the active satellite $\sigma_0(\phi_k)$, such that both $\sigma_0(\phi_i)$ and $\sigma_0(\phi_j)$ end up being fuel-sufficient at the end of the process. Let $\mathcal{T} \subseteq \mathcal{J}_{s,0} \times \mathcal{J}_{d,0} \times \mathcal{J}_a$ denote the set of all feasible triplets in the constellation graph $\mathcal{G}$.

5.1.1 E-P2P Maneuver Costs and Feasible Triplets

Let us consider a triplet $(i, j, k) \in \mathcal{J}_{s,0} \times \mathcal{J}_{d,0} \times \mathcal{J}_a$ in the constellation graph $\mathcal{G}$. Also, let the satellite s_μ occupy the orbital slot ϕ_i at time $t = 0$ and the satellite s_ν occupy the orbital slot ϕ_j at time $t = 0$. Hence, $s_\mu = \sigma_0(\phi_i)$ and $s_\nu = \sigma_0(\phi_j)$. Without loss of generality, assume s_μ to be a fuel-sufficient satellite and s_ν to be a fuel-deficient satellite, that is, $\mu \in \mathcal{I}_{s,0}$ and $\nu \in \mathcal{I}_{d,0}$. Either of the two satellites may be active during a refueling transaction between the two satellites. Hence, two different P2P refueling transactions are possible for the triplet (i, j, k).

In the first case, the fuel-sufficient satellite is active, that is, satellite s_μ performs the orbital maneuver to rendezvous with the passive satellite s_ν. Therefore, $\mu \in \mathcal{I}_a \cap \mathcal{I}_{s,0}$ and $\nu \in \mathcal{I}_p \cap \mathcal{I}_{d,0}$. After the fuel exchange takes place between the two satellites, s_μ performs another orbital maneuver and moves to the orbital slot ϕ_k initially occupied by the active satellite $\sigma_0(\phi_k)$. Note that $k \in \mathcal{J}_a$ and $k \neq j$. The fuel consumed by the active satellite s_μ to transfer from the orbital slot ϕ_i to the orbital slot ϕ_j is given by:

$$p_{ij}^{\mu} = \left(m_{\mathrm{s}\mu} + f_\mu^-\right)\left(1 - e^{-\frac{\Delta V_{ij}}{c_{0\mu}}}\right), \tag{157}$$

where $m_{\mathrm{s}\mu}$ is the mass of the permanent structure of satellite s_μ, and ΔV_{ij} is the optimal velocity change required for a two-impulse transfer from the orbital slot ϕ_i to the orbital slot ϕ_j. The parameter $c_{0\mu}$ is defined by $c_{0\mu} = g_0 I_{\mathrm{sp}\mu}$, where g_0 is the acceleration due to gravity at the Earth's surface and $I_{\mathrm{sp}\mu}$ is the specific thrust of satellite s_μ. The fuel content of satellite s_μ after its forward trip (but before fuel exchange takes place) is $f_\mu^- - p_{ij}^{\mu}$. Since the fuel consumption during the maneuver is minimized when the active satellite returns to its final slot with exactly the required minimum amount of fuel to remain operational, the amount of fuel consumed during the return trip (during which satellite s_μ travels from ϕ_j to ϕ_k), is given by

$$p_{jk}^{\mu} = \left(m_{s\mu} + \underline{f}_{\mu}\right) e^{\frac{\Delta V_{jk}}{c_{0\mu}}} \left(1 - e^{-\frac{\Delta V_{jk}}{c_{0\mu}}}\right). \tag{158}$$

In (158), ΔV_{jk} denotes the optimal velocity change required for the transfer from the orbital slot ϕ_j to the orbital slot ϕ_k. Before the return trip (but after the fuel exchange takes place), the fuel on board satellite s_μ is $\underline{f}_{\mu} + p_{jk}^{\mu}$. The fuel transferred to satellite s_ν during the fuel exchange is $(f_\mu^- - p_{ij}^\mu) - (\underline{f}_{\mu} + p_{jk}^{\mu})$, assuming that the satellite s_ν has enough fuel capacity to accommodate this amount of fuel. The fuel on board satellite s_ν after it is refueled is $f_\nu^- + (f_\mu^- - p_{ij}^\mu) - (\underline{f}_{\mu} + p_{jk}^{\mu})$. In order for satellite s_ν to become fuel-sufficient after the fuel transaction, we must therefore have,

$$\left(f_\nu^- + f_\mu^-\right) - \left(\underline{f}_{\mu} + \underline{f}_{\nu}\right) \geq p_{ij}^\mu + p_{jk}^\mu. \tag{159}$$

If the above condition does not hold, then the P2P refueling transaction between s_μ and s_ν is not feasible. Also, if satellite s_μ does not have enough fuel to carry out the orbital transfer during the forward trip, that is, if $p_{ij}^\mu \geq f_\mu^-$, then the P2P refueling transaction is also not feasible. Let $c_1(i,j,k)$ denote the cost of a P2P maneuver for the case when the fuel-sufficient satellite is active. Then $c_1(i,j,k)$ is given by the sum of (157) and (158). We therefore have,

$$c_1(i,j,k) = \begin{cases} p_{ij}^\mu + p_{jk}^\mu, & \text{if } p_{ij}^\mu < f_\mu^- \text{ and } p_{ij}^\mu + p_{jk}^\mu \leq \left(f_\mu^- + f_\nu^-\right) - \left(\underline{f}_{\mu} + \underline{f}_{\nu}\right), \\ \infty, & \text{otherwise.} \end{cases} \tag{160}$$

In the second case, the fuel-deficient satellite is active, that is, satellite s_ν performs the orbital maneuver to rendezvous with the passive satellite s_μ. Therefore, $\mu \in \mathcal{I}_p \cap \mathcal{I}_{s,0}$ and $\nu \in \mathcal{I}_a \cap \mathcal{I}_{d,0}$. After a fuel exchange takes place between the two satellites, s_ν performs another orbital maneuver and travels to the orbital slot ϕ_k

initially occupied by the active satellite $\sigma_0(\phi_k)$. Note that $k \in \mathcal{J}_a$ and $k \neq i$. The fuel consumed for the active satellite s_ν to transfer from the orbital slot ϕ_i to the orbital slot ϕ_j is given by

$$p_{ji}^{\nu} = \left(m_{s\nu} + f_{\nu}^{-}\right)\left(1 - e^{-\frac{\Delta V_{ji}}{c_{0\nu}}}\right). \tag{161}$$

The fuel content of satellite s_ν after its forward trip (but before fuel exchange takes place), is $f_\nu^- - p_{ji}^\nu$. The amount of fuel consumed during the return trip, during which the satellite s_ν travels from the orbital slot ϕ_i to the orbital slot ϕ_k, is given by

$$p_{ik}^{\nu} = \left(m_{s\nu} + \underline{f}_{\nu}\right) e^{\frac{\Delta V_{ik}}{c_{0\nu}}} \left(1 - e^{-\frac{\Delta V_{ik}}{c_{0\nu}}}\right), \tag{162}$$

Before the return trip (but after the fuel exchange takes place), the fuel on board satellite s_ν is $\underline{f}_\nu + p_{ik}^\nu$. The fuel transferred to satellite s_ν during the fuel exchange is $(\underline{f}_\nu + p_{ik}^\nu) - (f_\nu^- - p_{ji}^\nu)$. The fuel on board satellite s_μ after the fuel transaction is $f_\mu^- - (\underline{f}_\nu + p_{ik}^\nu) + (f_\nu^- - p_{ji}^\nu)$. In order for the satellite s_μ to be fuel-sufficient after the fuel transaction, we must have

$$\left(f_\mu^- + f_\nu^-\right) - \left(\underline{f}_\nu + \underline{f}_\mu\right) \geq p_{ji}^\nu + p_{ik}^\nu. \tag{163}$$

If the above condition does not hold, then a P2P refueling transaction between s_μ and s_ν is not feasible. Also, if the satellite s_ν does not have enough fuel to carry out the orbital transfer during the forward trip, that is, if $p_{ji}^\nu \geq f_\nu^-$, then the P2P refueling transaction is also not feasible. Let $c_2\,(i,j,k)$ denote the cost of a P2P maneuver for the case when the fuel-deficient satellite is active. Then, $c_2\,(i,j,k)$ is given by the sum of (161) and (162). We therefore have,

$$c_2\,(i,j,k) = \begin{cases} p_{ji}^\nu + p_{ik}^\nu, & \text{if } p_{ji}^\nu < f_\nu^- \text{ and } p_{ji}^\nu + p_{ik}^\nu \leq \left(f_\mu^- + f_\nu^-\right) - \left(\underline{f}_\nu + \underline{f}_\mu\right) \\ \infty, & \text{otherwise.} \end{cases} \tag{164}$$

Of the two possible P2P maneuvers associated with the triplet (i,j,k), the cheaper one is of interest to us. To this end, let the total fuel expenditure incurred in the P2P maneuver associated with the triplet (i, j, k) be given by

$$c(i,j,k) = \begin{cases} c_1(i,j,k), & \text{if } c_1(i,j,k) \leq c_2(i,j,k) \\ c_2(i,j,k), & \text{otherwise.} \end{cases} \tag{165}$$

We therefore associate with each triplet $(i, j, k) \in \mathcal{J}_{s,0} \times \mathcal{J}_{d,0} \times \mathcal{J}_a$ a single P2P maneuver. The set of all feasible triplets can then defined by $\mathcal{T} = \{(i, j, k) \in \mathcal{J}_{s,0} \times \mathcal{J}_{d,0} \times \mathcal{J}_a : c(i, j, k) < \infty\}$. Let now $\mathsf{Act} : \mathcal{T} \mapsto \mathcal{I}$ be a function that returns the index of the orbital slot of the active satellite, that is,

$$\mathsf{Act}(i,j,k) = \begin{cases} i, & \text{if } c_1(i,j,k) \leq c_2(i,j,k) \\ j, & \text{otherwise.} \end{cases} \tag{166}$$

Similarly, let $\mathsf{Pas} : \mathcal{T} \mapsto \mathcal{I}$ be a function that returns the index of the orbital slot of the passive satellite, that is,

$$\mathsf{Pas}(i,j,k) = \begin{cases} j, & \text{if } c_1(i,j,k) \leq c_2(i,j,k) \\ i, & \text{otherwise.} \end{cases} \tag{167}$$

Moreover, the edges induced by the triplets that are not feasible can be removed from the graph $\mathcal{G}$ in order to yield a *reduced constellation graph* $\mathcal{G}_r$. Therefore, $\mathcal{G}_r = \{\mathcal{J}_{s,0} \cup \mathcal{J}_{d,0} \cup \mathcal{J}_a, \mathcal{T}\}$. Henceforth, we restrict our discussion to the reduced constellation graph $\mathcal{G}_r$.

Using equations (157), (158), (161) and (162), we can ascertain the cost of a triplet $(i, j, k) \in \mathcal{T}$ using (165). Notice that the calculation of the optimal costs $\Delta V_{ij}, \Delta V_{ji}$, ΔV_{jk} and ΔV_{ki} in Equations (157), (158), (161) and (162) requires, in general, the solution of the two-impulse multi-revolution Lambert problem.[61]

5.1.2 The Three-Index Assignment Problem

Since our goal is to refuel all fuel-deficient satellites, each of them should be part of a feasible fuel transaction. We therefore seek a set of exactly $|\mathcal{I}_{d,0}|$ feasible triplets

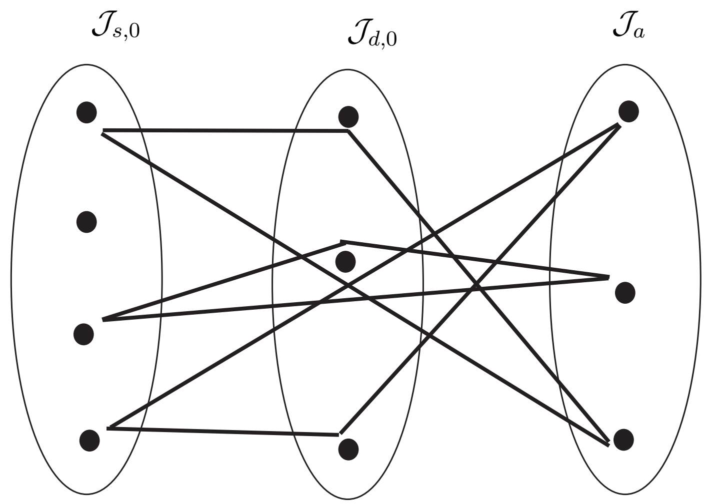


Figure 45: Feasible E-P2P Solution.

$\mathcal{M}_e^* \subseteq \mathcal{T}$ in the reduced constellation graph $\mathcal{G}_r$ such that none of the triplets in $\mathcal{M}_e^*$ share a common vertex or a common edge, and such that the sum of the costs of all these triplets is minimum. To this end, let us define as a *feasible E-P2P solution* a set $\mathcal{M}_e$ of $|\mathcal{J}_{d,0}|$ feasible triplets that has the following properties:

i) An active and a passive satellite should feature in a single E-P2P maneuver, that is, $i \neq i', j \neq j'$ for all triplets $(i, j, k), (i', j', k') \in \mathcal{M}_e$.

ii) The returning positions for all active satellites are distinct, that is, $k \neq k'$ for all triplets (i, j, k), $(i', j', k') \in \mathcal{M}_e$.

iii) The return positions are the slots left vacant by the active satellites.

iv) The orbital slots of the passive satellites cannot be the return positions for any of the active satellites.

Fig. 45 depicts a set of triangles in the constellation graph that corresponds to a feasible E-P2P solution. The cost $\mathcal{C}(\mathcal{M}_e)$ of a feasible E-P2P solution $\mathcal{M}_e$ is defined to be the sum of the cost of all triplets in $\mathcal{M}_e$, that is,

$$\mathcal{C}(\mathcal{M}_e) = \sum_{(i,j,k)\in\mathcal{M}_e} c\,(i, j, k)\,. \tag{168}$$

The *optimal E-P2P solution* is a feasible E-P2P solution $\mathcal{M}_e^*$ that achieves the minimum value of (168) among all feasible set of triplets. That is, $\mathcal{C}(\mathcal{M}_e^*) \le \mathcal{C}(\mathcal{M}_e)$ for all feasible $\mathcal{M}_e \subset \mathcal{T}$.

In order to determine the optimal E-P2P solution, let us therefore consider a set $\mathcal{M}_e \subseteq \mathcal{T}$ that consists of $|\mathcal{I}_{d,0}|$ triplets. To each triplet $(i,j,k) \in \mathcal{T}$, let us associate a binary variable x_{ijk} as follows

$$x_{ijk} = \begin{cases} 1, & \text{if } (i,j,k) \in \mathcal{M}_e, \\ 0, & \text{otherwise.} \end{cases} \tag{169}$$

We can therefore formulate the problem of finding the set of feasible triplets $\mathcal{M}_e \subseteq \mathcal{T}$ that yield the minimum cost as follows

$$\min_{\mathcal{M}_e \subseteq \mathcal{T}} \sum_{(i,j,k)\in\mathcal{M}_e} c\,(i,j,k)\, x_{ijk}, \tag{170}$$

such that

$$\sum_{j\in\mathcal{J}_{d,0}} \sum_{k\in\mathcal{J}_a} x_{ijk} \le 1, \text{ for all } i \in \mathcal{J}_{s,0}, \tag{171}$$

$$\sum_{i\in\mathcal{J}_{s,0}} \sum_{k\in\mathcal{J}_a} x_{ijk} = 1, \text{ for all } j \in \mathcal{J}_{d,0}, \tag{172}$$

$$\sum_{i\in\mathcal{J}_{s,0}} \sum_{j\in\mathcal{J}_{d,0}} x_{ijk} \le 1, \text{ for all } k \in \mathcal{J}_a, \tag{173}$$

$$r \ne \mathsf{Pas}(i,j,k) \text{ for all } (i,j,k)\,,(p,q,r) \in \mathcal{M}_e. \tag{174}$$

Constraint (171) signifies that not all fuel-sufficient satellites have to be part of P2P refueling transactions, because we may have $|\mathcal{I}_{s,0}| > |\mathcal{I}_{d,0}|$. Constraint (172) implies that each fuel-deficient satellite must be part of exactly one P2P fuel transaction. Constraint (173) signifies that each of the slots left vacant by the active satellites needs to be assigned to a P2P refueling transaction. Note that the set of active satellites is not known a priori. We only know that $\mathcal{J}_a \subset \mathcal{I}$. For solving our problem, we use $\mathcal{J}_a = \mathcal{I}$ for the third partition of the constellation. Therefore, not all nodes of the third partition correspond to orbital slots of active satellites. Hence the inequality sign in the constraint (173). Constraint (174) implies that the return orbital slot for the active satellite in a P2P maneuver cannot be the orbital slot of a passive satellite of a different P2P maneuver. To illustrate this, let us consider two triplets $(i, j, k) \in \mathcal{M}_e$ and $(p, q, r) \in \mathcal{M}_e$ such that $i \neq p$, $j \neq q$, $k \neq r$. Without loss of generality, assume $\mathsf{Act}(i, j, k) = i$ and $\mathsf{Act}(p, q, r) = p$. If $r = j$, then the fuel-sufficient satellite $\sigma_0(\phi_p)$ initially occupying orbital slot ϕ_p returns to the orbital slot ϕ_j. However, the orbital slot ϕ_j is not vacant because the satellite $\sigma_0(\phi_j)$ is passive and never leaves its slot. Constraint (174) avoids such infeasible cases. A set of triplets $\mathcal{M}_e \subseteq \mathcal{T}$ satisfying (169), (171)-(174) will be referred to as a *basic feasible solution* for our problem.

It should be mentioned at this point that a few differences emerge between our problem and the standard three-index assignment problem (AP3) discussed in Refs. 2, 7,8,16,25,55,66. First, the AP3 is a matching problem in a complete tripartite graph, whose partitions have the same number of nodes. In the case of the constellation graph $\mathcal{G}$ or the reduced constellation graph $\mathcal{G}_r$, the three partitions do not have the same number of nodes. Secondly, in our problem, we have additional constraints given in (174), which need to be accounted for whenever a basic feasible solution is considered. Nonetheless, in our problem we can readily construct one basic feasible solution without solving a three-index assignment problem. This solution is obtained by solving the P2P refueling problem, while constraining the active satellites to return to their orbital slots after refueling. This problem can be easily solved as a two-index assignment problem.[19,70,72,75]

5.2 Greedy Random Adaptive Search Procedure

In this section, we use a Greedy Random Adaptive Search Procedure to solve the three-index assignment problem, while taking into account the additional constraints in (174). The GRASP has been used to solve the standard AP3, and primarily consists of two phases: a construction phase that builds a basic feasible solution, and a local search phase that locates a solution in the neighborhood of the basic feasible solution with a lower cost. Reference 25 discusses two variants of implementing the construction phase (randomized greedy, maximum regret) as well as two variants of implementing the local search phase (two-exchange neighborhood search, variable depth exchange). We will use the randomized greedy method in the construction phase in order to generate a basic feasible solution, and we will perform a local search using a two-exchange neighborhood.

5.2.1 Construction of a Basic Feasible Solution

The construction phase iteratively builds a feasible solution $\mathcal{M}_e$ by selecting $|\mathcal{I}_{d,0}|$ triplets, one at a time, from a list L of eligible triplets from $\mathcal{T}$. The list L initially consists of all triplets in the reduced constellation graph $\mathcal{G}_r$, that is, $L = \mathcal{T}$, because initially all triplets are eligible for selection during the construction of $\mathcal{M}_e$.

Let $\mathcal{M}_\ell$ denote the constructed solution after the ℓth iteration, where $\ell \leq |\mathcal{I}_{d,0}|$. Initially $M_0 = \varnothing$. Assume $p-1 < |\mathcal{I}_{d,0}|$ triplets have been added after $p-1$ iterations, so the current constructed solution is denoted by $\mathcal{M}_{p-1} = \{(i_\ell, j_\ell, k_\ell) : \ell = 1, 2, \ldots, p-1\}$. The pth triplet needs to be added to $\mathcal{M}_{p-1}$.

A parameter η, known as restricted candidate list parameter, is selected at random from the interval $[0, 1]$ and is used to form a list L_r called the *restricted candidate list* that comprises of the best (in terms of lower cost) candidate triplets available for selection during the current iteration step. The restricted candidate list $L_r \subseteq L$ is defined as

$$L_r = \{(i, j, k) \in L : c\,(i, j, k) \leq \underline{c} + \eta(\bar{c} - \underline{c})\}, \tag{175}$$

where $\underline{c}$ and $\bar{c}$ are given by

$$\underline{c} = \min_{(i,j,k)\in L} c\,(i, j, k) \text{ and } \bar{c} = \max_{(i,j,k)\in L} c\,(i, j, k)\,. \tag{176}$$

The definition of the restricted candidate list given in (175) shows the greedy nature of the algorithm. Only triplets in L having cost less than $\underline{c} + \eta\,(\bar{c} - \underline{c})$ are made eligible for selection. At the pth step the triplet (i_p, j_p, k_p) is chosen at random from L_r, provided it does not violate (174), that is,

$$k_p \neq \mathsf{Pas}(i_\ell, j_\ell, k_\ell) \text{ for all } \ell = 1, 2, \ldots, p-1, \tag{177}$$

and

$$k_\ell \neq \mathsf{Pas}(i_p, j_p, k_p) \text{ for all } \ell = 1, 2, \ldots, p-1. \tag{178}$$

Equation (177) implies that the return orbital slot corresponding to the triplet (i_p, j_p, k_p) cannot be the orbital slot of a passive satellite corresponding to any of the triplets in $\mathcal{M}_{p-1}$, and equation (178) implies that the orbital slot of the passive satellite corresponding to the triplet (i_p, j_p, k_p) cannot be the returning orbital slot corresponding to any of the triplets in $\mathcal{M}_{p-1}$. Once the pth triplet is selected, the set of candidate triplets L must be adjusted to take into account that (i_p, j_p, k_p) is now part of the solution. Therefore, any triplet $(i, j, k) \in L$ with $i = i_p$ or $j = j_p$ or $k = k_p$ is removed from L because any such triplet cannot be selected in the future; otherwise at least one of the constraints (171), (172), or (173) will be violated. Subsequently, the list L is updated accordingly. Finally, $\mathcal{M}_p = \mathcal{M}_{p-1} \cup (i_p, j_p, k_p)$.

The adaptive nature of the GRASP method is due to the fact that once a triplet from L_r is selected for addition to $\mathcal{M}_{p-1}$, all triplets that are made ineligible for addition to $\mathcal{M}_{p+1}$ are removed from L. The probabilistic nature of the algorithm arises from the use of the random parameter η and the random selection of a triplet from the restricted candidate list. In the most simple implementation of the algorithm the value of η is not changed during the construction phase.

5.2.2 Local Search

In the local search phase, the feasible solution from the construction phase is improved upon by searching its neighborhood for a better solution. If an improvement is detected, the solution is updated and a new neighborhood search is initialized. The definition of the neighborhood $\mathcal{N}(\mathcal{M}_e)$ of $\mathcal{M}_e$ is crucial for the performance of the local search. Here we use the 2-exchange neighborhood suggested in Ref. 2. Recall that the basic feasible solution generated by the construction phase consists of $|\mathcal{I}_{d,0}|$ triplets. For convenience, let us denote the triplet (i_ℓ, j_ℓ, k_ℓ) by t_ℓ. Let also $\mathcal{D} = \{1, 2, \ldots, |\mathcal{I}_{d,0}|\}$ denote the index set of triplets in $\mathcal{M}_e$. We can therefore write $\mathcal{M}_e = \{t_\ell : \ell \in \mathcal{D}\}$. We will denote the difference between $t_p, t_q \in \mathcal{M}_e$ by

$$\delta(t_p, t_q) = \{r : t_{p,r} \neq t_{q,r},\ r = 1, 2, 3\}. \tag{179}$$

The distance between the triplets t_p and t_q is then defined as

$$d(t_p, t_q) = |\delta(t_p, t_q)|. \tag{180}$$

Using (180), we can define the 2-exchange neighborhood of the triplet pair $(t_p, t_q) \in \mathcal{M}_e \times \mathcal{M}_e$ as

$$N_2(t_p, t_q) = \{(\tau, \sigma) \in \mathcal{M}_e \times \mathcal{M}_e : d(t_p, \tau) + d(t_q, \sigma) = 2\}. \tag{181}$$

The neighborhood of the solution $\mathcal{M}_e$ consists of the union of 2-exchange neighborhoods of all possible triplet pairs $(t_p, t_q) \in \mathcal{M}_e$, that is,

$$\mathcal{N}(\mathcal{M}_e) = \bigcup_{(t_p, t_q) \in \mathcal{M}_e} N_2(t_p, t_q). \tag{182}$$

During the local search phase the cost of each $\mathcal{M}'_e \in \mathcal{N}(\mathcal{M}_e)$ (validated with respect to the constraints as in (174)) is compared with the cost of $\mathcal{M}_e$. If the cost is lower, then the current search is halted, and a search around the neighborhood of $\mathcal{M}'_e$ is initialized. The local search ends when no neighbor of the current solution has a lower cost.

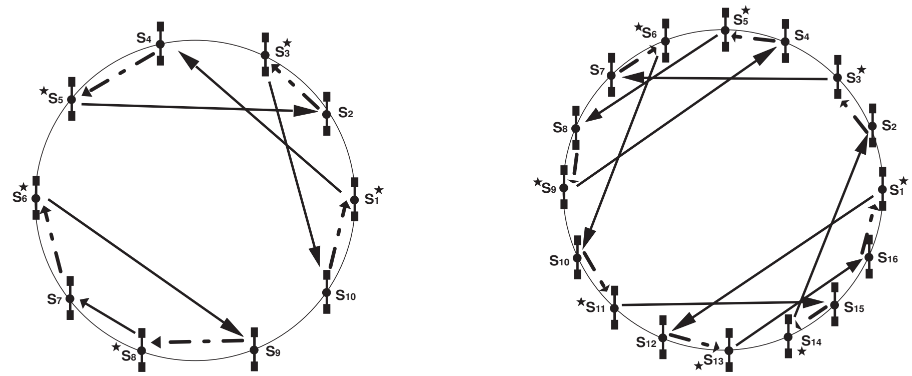
The successive application of the construction phase and the local search phase may generate several local minima. The procedure halts either after the maximum number of iterations is reached, or if a local minimum with a value less than or equal to some pre-specified value is found.

5.3 Numerical Results: GRASP Solution

In this section, we apply the GRASP method in order to determine the optimal assignments required for P2P refueling of sample constellations when the active satellites are not restricted to return to their original orbital slots. We also compare the results against the baseline P2P case, namely when the active satellites are constrained to return to their original orbital slots. With the help of numerical examples, we show how the removal of such a restriction leads to considerable reduction in the fuel expenditure required for the refueling process to be completed. In all examples, we run the GRASP procedure $10,000$ times in order to determine the optimal assignments.

Example 13. *E-P2P refueling strategy in a constellation of* 10 *satellites.*

We revisit Constellation C_1 given in Table 5. It is found that when the active satellites are allowed to interchange their orbital slots, the optimal assignments for E-P2P refueling are $s_8 \to s_7 \to s_6$, $s_6 \to s_9 \to s_8$, $s_3 \to s_{10} \to s_1$, $s_1 \to s_4 \to s_5$, $s_5 \to s_2 \to s_3$. The fuel expenditure during the refueling process is 18.73 units, which is less than the fuel expenditure for the baseline P2P case. This represents 10.41% of the total initial fuel in the constellation, or an improvement of 28% over the standard P2P scenario. Figure 46(a) shows the constellation and the optimal assignments. The active satellites are marked by '$\star$'. The forward trips are marked by solid arrows, while the return trips are marked by dotted arrows. In the optimal assignment produced by the GRASP method, it is observed that each active satellite, after undergoing a fuel transaction with the corresponding passive satellite, returns to an available orbital slot in the vicinity of the passive satellite with which it was involved in the transaction. For instance, satellite s_1 undergoes a fuel transaction with the satellite s_4 and then returns to the orbital slot initially occupied by active satellite s_5. Moving to an orbital slot in the vicinity involves an orbital transfer through a smaller transfer angle and thereby results in a likely lesser fuel expenditure during the



(a) Constellation C_1 (b) Constellation C_2

Figure 46: E-P2P solution determined by GRASP.

return trip. Hence, the active satellites, having the freedom to return to any available orbital slot, opt to move to a nearby orbital slot during the return trip. In the baseline P2P strategy, such freedom is not available, and some of the active satellites have to perform orbital transfers that incur higher cost. Another observation is the fact that some of the active satellites are also fuel-sufficient. Note that satellites s_1 and s_8 are fuel-sufficient and active. For this problem, by having some fuel-sufficient satellites as the active satellites, it is ensured that all active satellites are able to return to the nearest orbital slot, thereby saving fuel during the return trip.

Figure 47 depicts a basic feasible solution generated by the GRASP method along with a local search performed about this solution. Figure 47(a) is the basic feasible solution and corresponds to the assignment $s_4 \rightarrow s_{10} \rightarrow s_1$, $s_1 \rightarrow s_3 \rightarrow s_4$, $s_7 \rightarrow s_8 \rightarrow s_2$, $s_2 \rightarrow s_5 \rightarrow s_7$, $s_6 \rightarrow s_9 \rightarrow s_6$. The cost of this assignment is 24.87 units of fuel. By performing a search in the neighborhood of this solution, another assignment of lower cost, shown in Figure 47(b) is obtained. In this assignment, satellite s_4 returns to the orbital slot initially occupied by s_2 instead of the orbital slot initially occupied by s_1, while satellite s_7 returns to the orbital slot initially occupied by s_1 instead of returning to the orbital slot initially occupied by s_2. The cost of this assignment is 24.77 units

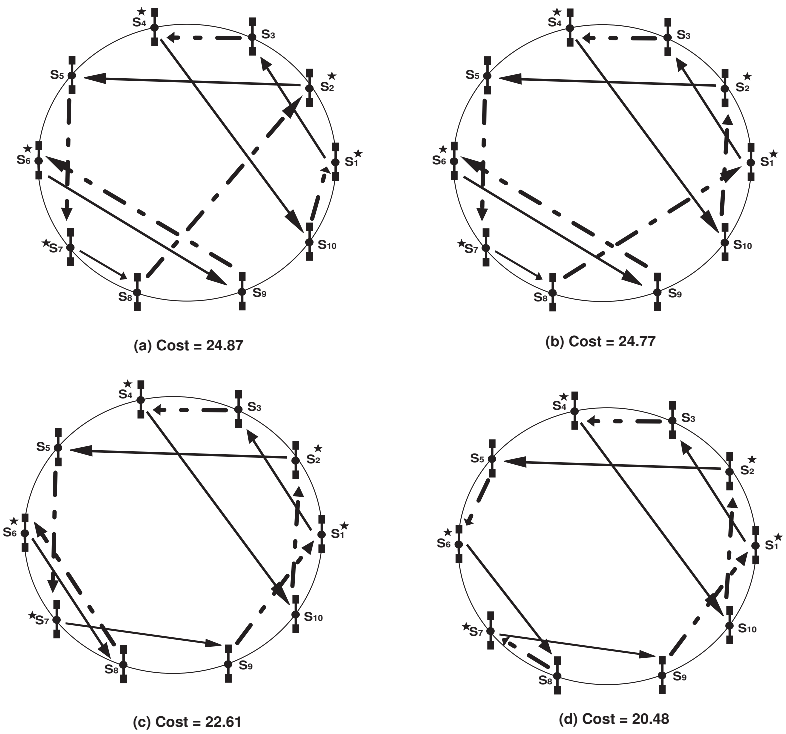


Figure 47: Example of local search of the GRASP method.

of fuel. A local search performed in the neighborhood of this solution yields the assignment shown in Figure 47(c). In this assignment, satellite s_7 rendezvous with s_9 instead of s_8, while satellite s_6 rendezvous with s_8 instead of s_9. The cost of this assignment is 22.61 units of fuel. A search in the neighborhood of this solution now yields yet another assignment shown in Figure 47(d). In this assignment, satellite s_2 returns to the orbital slot initially occupied by s_6 instead of the orbital slot initially occupied by s_7, while satellite s_6 returns to the orbital slot initially occupied by s_7 instead of returning to its original orbital slot. The cost of this solution is 20.48. Finally, a local search in the neighborhood of this solution yields no other cheaper solution, thereby implying that the assignment in Figure 47(d) is a local minimum.

Example 14. *E-P2P refueling strategy in a constellation of* 16 *satellites.*

In this example, we consider again Constellation C_2 given in Table 5. Using the GRASP method to determine the optimal assignments for E-P2P refueling of satellites in this constellation. If the active satellites are allowed to interchange orbital slots, then the optimal assignment for the P2P refueling problem, as determined by the GRASP method, are $s_1 \rightarrow s_{12} \rightarrow s_{13}$, $s_3 \rightarrow s_7 \rightarrow s_6$, $s_5 \rightarrow s_8 \rightarrow s_9$, $s_6 \rightarrow s_{10} \rightarrow s_{11}$, $s_9 \rightarrow s_4 \rightarrow s_5$, $s_{11} \rightarrow s_{15} \rightarrow s_{14}$, $s_{13} \rightarrow s_{16} \rightarrow s_1$, $s_{14} \rightarrow s_2 \rightarrow s_3$. Here, $\mathcal{I}_a = \{1, 3, 5, 6, 9, 11, 13, 14\}$. Relaxing the return orbital position constraint reduces the fuel expenditure to 24.82 units. This represents 7.76% of the total initial fuel in the constellation or an improvement of 33% over the standard P2P scenario. Figure 46(b) shows the constellation and the optimal assignments. The active satellites are marked by '$\star$'. Similarly to Example 13, it is observed that the active satellites, after undergoing fuel transactions with the corresponding passive satellites, return to an available orbital slot in their vicinity. For instance, satellite s_1 undergoes a fuel transaction with satellite s_{12} and returns to the orbital slot occupied by active satellite s_{13}. Also, the active satellites include fuel-sufficient ones. Here the fuel-sufficient satellites s_1, s_3, s_5 and s_6 being active ensures that all active satellites return to the neighboring orbital slot.

The above examples show the benefit of an E-P2P strategy over a P2P strategy. P2P or E-P2P refueling strategy discussed so far has all been formulated on an undirected constellation graph. However, it is more natural to formulate such a problem over a constellation graph with directed edges (also termed arcs), with each arc representing an orbital transfer, the direction being from the initial to final orbital slot of the active satellite during the transfer. We now provide an alternative formulation of the E-P2P problem using a directed graph approach. In the next section, we use a directed graph to construct a constellation network and formulate the E-P2P refueling problem as a minimum cost flow in the network.

5.4 Network Flow Formulation for the E-P2P Problem

In this section, we discuss in detail the network flow formulation for solving the E-P2P problem. As we will show, we can set up a minimum cost flow problem, the solution to which would provide the will correspond to a set of maneuvers for which total ΔV is minimized. We also describe a local search method for improving this solution, in terms of fuel expenditure, by performing a local search similar to the GRASP method.

5.4.1 Constellation Digraph

Let us consider a directed tripartite constellation graph $\mathcal{G}$ with the three partitions being $\mathcal{J}_a, \mathcal{J}_p, \mathcal{J}_r$. Because we do not know a priori which satellites are active, we consider $\mathcal{J}_a = \mathcal{J}_p = \mathcal{J}_r = \mathcal{I}$. On the constellation digraph $\mathcal{G}$ we represent a E-P2P maneuver (i, j, k) by the directed edges (i, j) and (j, k), where $i \in \mathcal{J}_a$, $j \in \mathcal{J}_p$, and $k \in \mathcal{J}_r$. Since a fuel transaction can only be between a fuel-sufficient and a fuel-deficient satellite, we have that either $i \in \mathcal{J}_{s,0}$ and $j \in \mathcal{J}_{d,0}$, or $i \in \mathcal{J}_{d,0}$ and $j \in \mathcal{J}_{s,0}$. Therefore, the set of edges representing all possible forward trips is given by

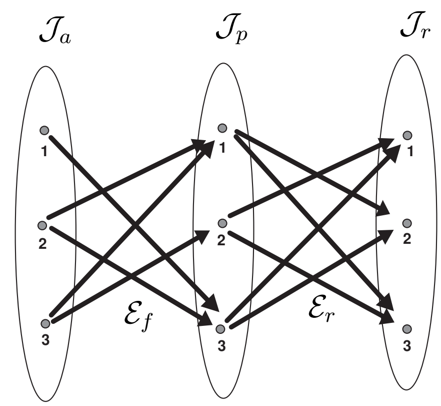


Figure 48: E-P2P Directed constellation graph.

$$\mathcal{E}_f = \{(i, j) : i \in \mathcal{J}_{s,0} \cap \mathcal{J}_a, j \in \mathcal{J}_{d,0} \cap \mathcal{J}_p\} \cup \{(i, j) : i \in \mathcal{J}_{d,0} \cap \mathcal{J}_a, j \in \mathcal{J}_{s,0} \cap \mathcal{J}_p\}. \quad (183)$$

The return maneuver from the orbital slot ϕ_j to the orbital slot ϕ_k, where $k \neq j$, can be represented by a directed edge $(j,k) \in \mathcal{J}_p \times \mathcal{J}_r, j \neq k$. We can therefore denote the set of all possible return trips by

$$\mathcal{E}_r = \{(j,k) : j \in \mathcal{J}_p, k \in \mathcal{J}_r, j \neq k\}. \tag{184}$$

Thus, the set of vertices in the constellation digraph is given by $\mathcal{V} = \mathcal{J}_a \cup \mathcal{J}_p \cup \mathcal{J}_r$, while the set of edges is given by $\mathcal{E} = \mathcal{E}_f \cup \mathcal{E}_r$. Let the constellation digraph be $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Figure 48 shows the digraph for a constellation, with vertices representing orbital slots of satellites and edges representing orbital maneuvers. Note that a pair of directed edges $(i,j) \in \mathcal{E}_f$ and $(\ell, k) \in \mathcal{E}_r$ represents an E-P2P maneuver if and only if $\ell = j$.

5.4.2 Cost Assignment

With each orbital transfer represented by a directed edge $(i,j) \in \mathcal{E}$, we associate a cost c_{ij} as follows

$$c_{ij} = \Delta V_{ij} \text{ for all } (i,j) \in \mathcal{E}, \tag{185}$$

where ΔV_{ij} is the required velocity change for a satellite to transfer from the orbital slot ϕ_i to the orbital slot ϕ_j. Note that the calculation of ΔV_{ij} requires, in general, the solution of a two-impulse multi-revolution Lambert problem.[61]

We should point out here that – ideally – the cost c_{ij} should be the fuel consumption during the transfer. However, the amount of fuel depends on the mass of the satellite performing the transfer, which may not be known a priori. For instance, recall that the edge $(j,k) \in \mathcal{E}_r$ represents a valid return trip for any of the E-P2P maneuvers in which an edge $(i,j) \in \mathcal{E}_f$ represents a forward trip. The set of possible active satellites that can carry out the orbital transfer from the slot ϕ_j to the slot ϕ_k is given by $\{\sigma_0(\phi_i) : (i,j) \in \mathcal{E}_f\}$. For each of these active satellites, the fuel expenditure for the return trip represented by the edge $(j,k) \in \mathcal{E}_r$ is different. Therefore, if fuel expenditure is used to define the cost of edges, no unique value can be assigned to an edge $(j,k) \in \mathcal{E}_r$. This is the reason we use (185) tacitly recognizing the fact that the results will necessarily be suboptimal in terms of actual fuel consumption.

5.4.3 Constellation Network Flow

Given the constellation digraph $\mathcal{G}$, we now set up the constellation network $\mathcal{G}_n$ and show that the E-P2P problem can be formulated as a minimum cost flow problem on the constellation network $\mathcal{G}_n$. To this end, we add a source node s and a sink node t to the constellation digraph $\mathcal{G}$. For all $i \in \mathcal{J}_a$, we also add an arc (s,i) with associated cost $c_{si} = 0$. We denote the set of these arcs by $\mathcal{E}_s$. Similarly, for all $k \in \mathcal{J}_r$, we add an arc (k,t) with associated cost $c_{kt} = 0$. We denote the set of these arcs by $\mathcal{E}_t$. The set of nodes for $\mathcal{G}_n$ is $\mathcal{V}_n = \{s\} \cup \mathcal{V} \cup \{t\}$, while the set of arcs (directed edges) of $\mathcal{G}_n$ is $\mathcal{E}_n = \mathcal{E}_s \cup \mathcal{E} \cup \mathcal{E}_t$. That is, $\mathcal{G}_n = (\mathcal{V}_n, \mathcal{E}_n)$. A depiction of $\mathcal{G}_n$ is given in Figure 49.

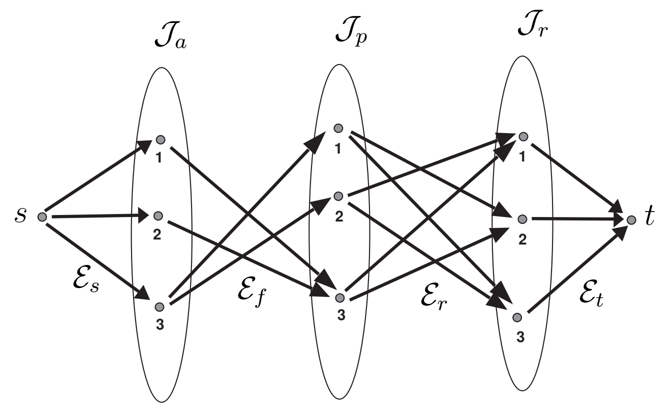


Figure 49: E-P2P Constellation flow network.

Let us now consider a $s \to t$ flow in the network $\mathcal{G}_n$. By a $s \to t$ flow, we mean a flow from the source s to the sink t passing through the nodes $i \in \mathcal{J}_a$, $j \in \mathcal{J}_p$ and $k \in \mathcal{J}_r$ in that order, that is, a flow along the directed path $\{s \to i \to j \to k \to t\}$. Note that the $s \to t$ flow passes through the arcs $(s,i) \in \mathcal{E}_s$, $(i,j) \in \mathcal{E}_f$, $(j,k) \in \mathcal{E}_r$ and $(k,t) \in \mathcal{E}_t$. Of these, the arcs (i,j) and (j,k) constitute an E-P2P maneuver (i,j,k), and the sum of the costs of all these edges is the total cost of the E-P2P maneuver (i,j,k). The remaining arcs (s,i) and (k,t) have zero cost and therefore the cost of a unit flow along the path $\{s \to i \to j \to k \to t\}$ is the total cost of the corresponding E-P2P maneuver. We can therefore associate an E-P2P maneuver with a unique $s \to t$ flow.

5.4.4 Network Flow Minimization Problem

We will now formulate the E-P2P refueling problem as a minimum cost flow problem. It is to be noted here that the integrality property [1] states that if all arc capacities and supplies/demands of the nodes are integers, the minimum cost flow problem has an integral optimal solution. In other words, we can arrive at the optimal solution by considering only integer values of the flow variables. In the formulation of our problem, we will include integer arc capacities and integer supply/demand for each node, and will show that a feasible integral flow in the constellation corresponds to a feasible E-P2P solution $\mathcal{M}_e$. Now let us introduce a flow variable x_{ij} for each edge $(i, j) \in \mathcal{E}_n$. The flow variable x_{ij} equals the amount of flow through the edge (i, j). We consider $x_{ij} \in \{0, 1\}$. Clearly, for each edge (i, j), the capacity which is the maximum amount of flow that is permissible through that edge equals 1. In addition, let b_i denote the amount of supply at node $i \in \mathcal{V}_n$, such that $b_i < 0$ denotes demand at the node. For all nodes $i \in \mathcal{N} \backslash \{s, t\}$, we have $b_i = 0$. For the source and sink nodes, we have $b_s = |\mathcal{I}_{d,0}|$ and $b_t = -|\mathcal{I}_{d,0}|$, respectively. This implies that we wish to send a flow equal to $|\mathcal{I}_{d,0}|$ through the network from the source to the sink, given that no edge allows more than one unit of flow through it.

All nodes in the constellation network $\mathcal{G}_n$ are required to satisfy the usual flow balance equations

$$\sum_{j:(i,j)\in\mathcal{E}_n} x_{ij} - \sum_{j:(j,i)\in\mathcal{E}_n} x_{ji} = b_i \text{ for all } i \in \mathcal{V}_n. \tag{186}$$

However, our initial consideration $\mathcal{J}_a = \mathcal{J}_p = \mathcal{J}_r = \mathcal{I}$ requires the introduction of additional constraints. First, note that $\mathsf{Act}(\mathcal{M}_e) = \mathsf{Ret}(\mathcal{M}_e)$. Hence, if the flow passes through a node $i \in \mathcal{J}_a$, then the flow has to pass through the node $i \in \mathcal{J}_r$.

Moreover, if the flow does not pass through the node $i \in \mathcal{J}_a$, no flow should then pass through $i \in \mathcal{J}_r$. This constraint can be written as

$$x_{si} = x_{it} \text{ for all } i \in \mathcal{J}_a = \mathcal{J}_r. \tag{187}$$

Second, note that $i \in \mathsf{Act}(\mathcal{M}_e)$ implies $i \notin \mathsf{Pas}(\mathcal{M}_e)$. Hence, the network should not allow two $s \to t$ flows, one that passes through node $i \in \mathcal{J}_a$ and the other that passes through $i \in \mathcal{J}_p$. That is, the satellite originally occupying the orbital slot ϕ_i cannot be simultaneously the active satellite and the passive satellite with respect to two different P2P maneuvers. This implies the following constraint

$$x_{sj} + \sum_{i:(i,j)\in\mathcal{E}_f} x_{ij} \leq 1 \text{ for all } j \in \mathcal{J}_p. \tag{188}$$

Finally, given the constellation network $\mathcal{G}_n$, we seek to find the minimum cost flow in the network

$$\text{(EP2P-IP): } \min \sum_{(i,j)\in\mathcal{E}_n} c_{ij}x_{ij} \tag{189}$$

subject to the constraints (186)-(188) and $x_{ij} \in \{0, 1\}$.

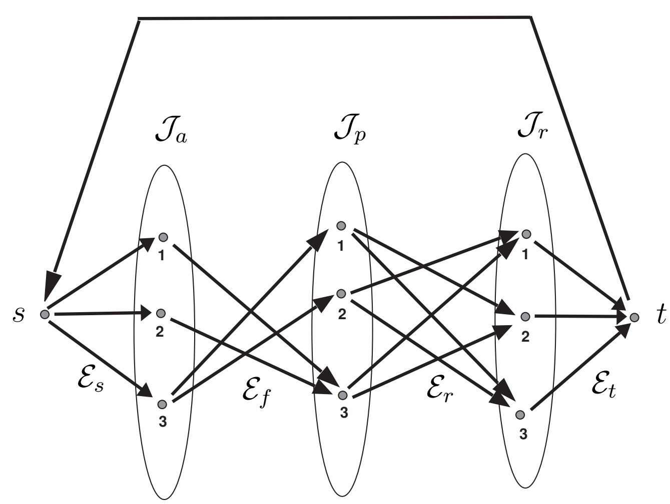
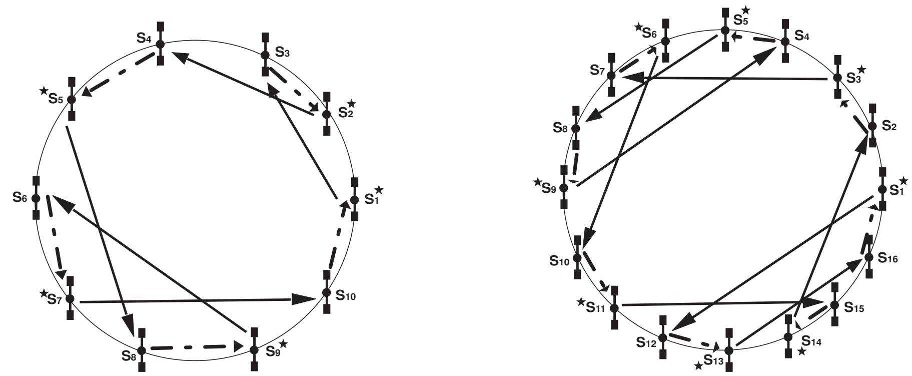


Figure 50: Constellation flow network with an additional (t, s) arc.

Note that in Fig. 49 the source sends a total flow equal to $|\mathcal{J}_{d,0}|$ to the sink via the network. Since the capacity of each edge is unity, an integral flow in the network will be comprised of $|\mathcal{J}_{d,0}|$ flows from s to t. We now show that the network ensures that all fuel-deficient satellites are involved in fuel transactions. The flow from the sink reaches $|\mathcal{J}_{d,0}|$ nodes in $\mathcal{J}_a$. Clearly, these nodes are given by $\mathsf{Act}(\mathcal{M}_e)$ and $|\mathsf{Act}(\mathcal{M}_e)| = |\mathcal{J}_{d,0}|$. The indices of the original orbital slots of the active fuel-sufficient satellites are given by $\mathsf{Act}(\mathcal{M}_e) \cap \mathcal{J}_{s,0}$ and those for the active fuel-deficient satellites are given by $\mathsf{Act}(\mathcal{M}_e) \cap \mathcal{J}_{d,0}$. The set of nodes in $\mathcal{J}_p$ through which the flow passes are given by $\mathsf{Pas}(\mathcal{M}_e)$. Evidently, the indices of the original orbital slots of the passive fuel-sufficient satellites are given by $\mathsf{Pas}(\mathcal{M}_e) \cap \mathcal{J}_{s,0}$, while those of the passive fuel-deficient satellites are given by $\mathsf{Pas}(\mathcal{M}_e) \cap \mathcal{J}_{d,0}$. Because a fuel transaction can only be between a fuel-sufficient and a fuel-deficient satellite, the number of passive fuel-deficient satellites will equal the number of active fuel-sufficient satellites, that is, we have $|\mathsf{Pas}(\mathcal{M}_e) \cap \mathcal{J}_{d,0}| = |\mathsf{Act}(\mathcal{M}_e) \cap \mathcal{J}_{s,0}|$. However, the total number of active satellites is $|\mathcal{J}_{d,0}|$, so that we have $|\mathsf{Act}(\mathcal{M}_e) \cap \mathcal{J}_{s,0}| + |\mathsf{Act}(\mathcal{M}_e) \cap \mathcal{J}_{d,0}| = |\mathcal{J}_{d,0}|$. It follows that $|\mathsf{Act}(\mathcal{M}_e) \cap \mathcal{J}_{d,0}| + |\mathsf{Pas}(\mathcal{M}_e) \cap \mathcal{J}_{d,0}| = |\mathcal{J}_{d,0}|$, which implies that the flow from the source reaches all nodes corresponding to the orbital slots of all fuel-deficient satellites. In other words, all fuel-deficient satellites are involved in a fuel transaction during the E-P2P maneuvers represented by the optimal flow in the network as required by the problem statement.

Remark 1. Note that in the network flow formulation for the problem, the supply or demand at each node representing an orbital slot of a satellite is zero. If we now let $b_s = b_t = 0$, but add an arc (t, s) in the network $\mathcal{G}_n$ and impose a flow $|\mathcal{I}_{d,0}|$ through this arc from the sink to the source, then the problem remains unaltered. All nodes in the augmented network (see Fig. 50) now have zero demand/supply and the flow in the network has to be a circulation. We know that a circulation can always be decomposed into cycles.[1] Hence, the optimal cost flow should be in the form of cycles.

Let the solution obtained by solving (EP2P-IP) be denoted by $\mathcal{M}_{\mathrm{IP}}$ and the corresponding fuel expenditure be denoted by $\mathcal{C}_{\mathrm{IP}}$. Note that the solution has to be



(a) Constellation C_1 (b) Constellation C_2

Figure 51: Optimal E-P2P assignments.

sub-optimal because instead of minimizing the total fuel expenditure, we are minimizing the total ΔV during the E-P2P maneuvers represented by the flow in the constellation network. However, we can perform a local search in the N_2 neighborhood (refer Section 5.2.2) of $\mathcal{M}_{\text{IP}}$ in order to find a solution cheaper in terms of fuel expenditure. We denote by $\mathcal{M}_{\text{H}}$ the final solution that is obtained by the application of the local search method on $\mathcal{M}_{\text{IP}}$.

5.5 Numerical Examples: E-P2P Solution

In this section, we apply our proposed method to determine the optimal assignments for E-P2P refueling of sample constellations given in Table 5. In order to obtain the optimal assignments, the integer program (EP2P-IP) is solved using the binary integer programming solver `bintprog` of MATLAB. This solver uses branch-and-bound to solve integer programs. We also compare the results against the baseline P2P strategy, in which the active satellites are constrained to return to their original orbital slots.

Example 15. *E-P2P refueling strategy in a constellation of* 10 *satellites.*

In this example, we determine the optimal assignments for E-P2P refueling of satellites in constellation C_1 by solving the optimization problem (EP2P-IP). The solution of (EP2P-IP) yields the following optimal assignment for E-P2P refueling: $s_1 \to s_3 \to s_2$, $s_2 \to s_4 \to s_5$, $s_5 \to s_8 \to s_9$, $s_7 \to s_{10} \to s_1$, $s_9 \to s_6 \to s_7$. The fuel expenditure during the E-P2P refueling process is 19.11 units, which is less than the fuel expenditure for the baseline P2P case. This represents 10.62% of the total initial fuel in the constellation. Figure 51(a) shows the optimal assignments for the E-P2P case. Similar to the observations for Example 13, it is observed that each active satellite, after undergoing a fuel transaction with the corresponding passive satellite, returns to an available orbital slot in the vicinity of the passive satellite with which it was involved in the transaction. For instance, satellite s_1 undergoes a fuel transaction with satellite s_3, and then returns to the orbital slot initially occupied by active satellite s_2. Moving to an orbital slot in the vicinity involves an orbital transfer through a smaller transfer angle, and thereby it likely results in a lesser fuel expenditure during the return trip. Another observation that is similar to the solution yielded by the GRASP method is some of the active satellites are also fuel-sufficient. For instance, satellites s_1, s_2 and s_9 are fuel-sufficient and active. Furthermore, note in figure 51(a) that the optimal solution comprises a Hamiltonian cycle $\{s_1 \to s_3 \to s_2 \to s_4 \to s_5 \to s_8 \to s_9 \to s_6 \to s_7 \to s_{10} \to s_1\}$ in the constellation.

Example 16. *E-P2P refueling strategy in a constellation of* 16 *satellites.*

For constellation C_2 given in Table 5, the solution of (EP2P-IP) yields the following optimal assignment for E-P2P refueling: $s_1 \to s_{12} \to s_{13}$, $s_3 \to s_7 \to s_6$, $s_5 \to s_8 \to s_9$, $s_6 \to s_{10} \to s_{11}$, $s_9 \to s_4 \to s_5$, $s_{11} \to s_{15} \to s_{14}$, $s_{13} \to s_{16} \to s_1$, $s_{14} \to s_2 \to s_3$. Here, $\mathcal{J}_a = \{1, 3, 5, 6, 9, 11, 13, 14\}$. Note that this assignment is the same as determined using the GRASP method given in Example 14. Therefore, as before, the fuel expenditure is 24.82 units, that represents 7.76% of the total initial fuel in the constellation. Figure 51(b) shows the constellation and the optimal assignments for the E-P2P case. The active satellites are marked by '$\star$'. Also note that the figure shows that the optimal solution corresponds to three cycles in the constellation, namely, $\{s_1 \to s_{12} \to s_{13} \to s_{16} \to s_1\}$, $\{s_3 \to s_7 \to s_6 \to s_{10} \to s_{11} \to s_{15} \to s_{14} \to s_2 \to s_3\}$ and $\{s_5 \to s_8 \to s_9 \to s_4 \to s_5\}$.

We have also tested the proposed methodology on other constellations as depicted in Table 5. The optimal assignments for these constellations show considerable reduction in fuel consumption against the baseline P2P strategy. For instance, for constellation C_3, the baseline P2P refueling strategy yields an optimal assignment $s_4 \to s_1$, $s_5 \to s_2$, $s_7 \to s_{10}$, $s_6 \to s_3$, $s_{11} \to s_8$, $s_9 \to s_{12}$ with a fuel expenditure of 26.73 units, with the fuel-deficient satellites being the active ones. Our proposed methodology yields the optimal assignment $s_1 \to s_4 \to s_5$, $s_3 \to s_6 \to s_7$, $s_5 \to s_2 \to s_3$, $s_7 \to s_{10} \to s_{11}$, $s_9 \to s_{12} \to s_1$, $s_{11} \to s_8 \to s_9$, that reduces the fuel expenditure to 18.87 units. The optimal solution consists of the Hamiltonian cycle $\{s_1 \to s_4 \to s_5 \to s_2 \to s_3 \to s_6 \to s_7 \to s_{10} \to s_{11} \to s_8 \to s_9 \to s_{12} \to s_1\}$. Similarly, for the other constellations, the fuel expenditure reduces from 41.06 units to 26.26 units in case of C_4, from 28.38 to 18.86 in case of C_5, from 28.77 units to 19.26 units in case with C_6, and from 34.97 units to 22.75 units in case of C_7.

5.5.1 Computational Time

As mentioned before, the E-P2P refueling problem is NP-hard. This means that there currently exists no polynomial-time algorithms for this problem. Although for the instances of P2P refueling, the number of satellites is not huge (there might be 15-20 satellites in one orbit), it is interesting to see how the computational time varies with the number of satellites in a constellation. Figure 52 shows the variation of computational time with number of satellites. It can be seen that the time increases with increasing number of satellites. However, note that this increase is roughly linear, possibly because the constellations considered do not have too many satellites. In the instances, all constellations have less than 20 satellites.

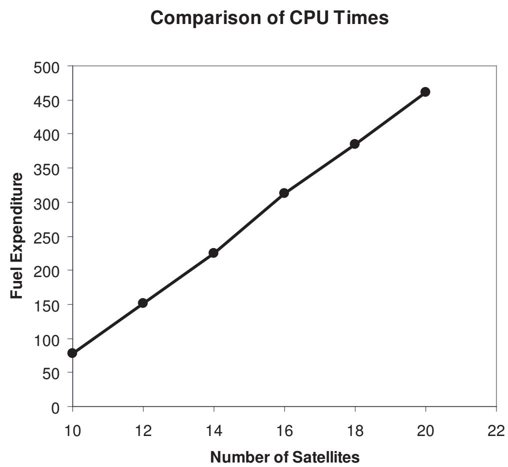
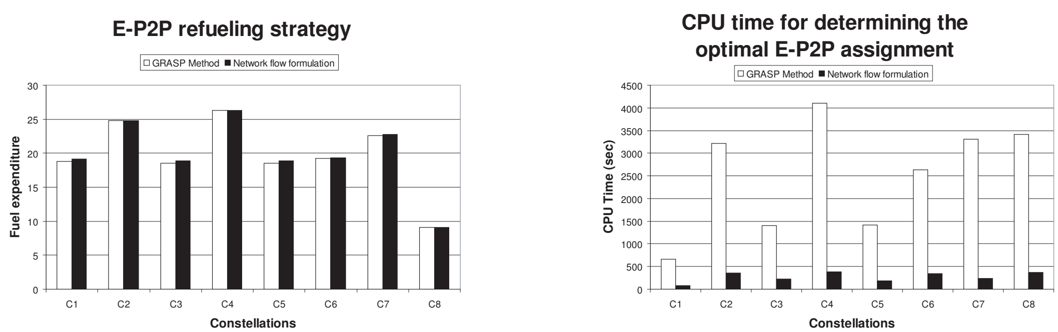


Figure 52: CPU Time vs. Number of Satellites.



(a) Fuel expenditure of optimal solution

(b) CPU time required to compute optimal solution

Figure 53: Comparison with GRASP.

5.5.2 Comparison with results using GRASP

Here, we provide a comparison of the results obtained by solving (EP2P-IP) with those obtained using the GRASP method. Figure 53(a) shows the fuel expenditure incurred in E-P2P refueling of the sample constellations based on the assignments determined by the network flow formulation and the GRASP method. Typically, we find that the solution yielded by the GRASP method are marginally better than those yielded by the network flow formulation. This is encouraging, given the fact that that the network flow formulation minimizes total ΔV rather than actual fuel consumption.

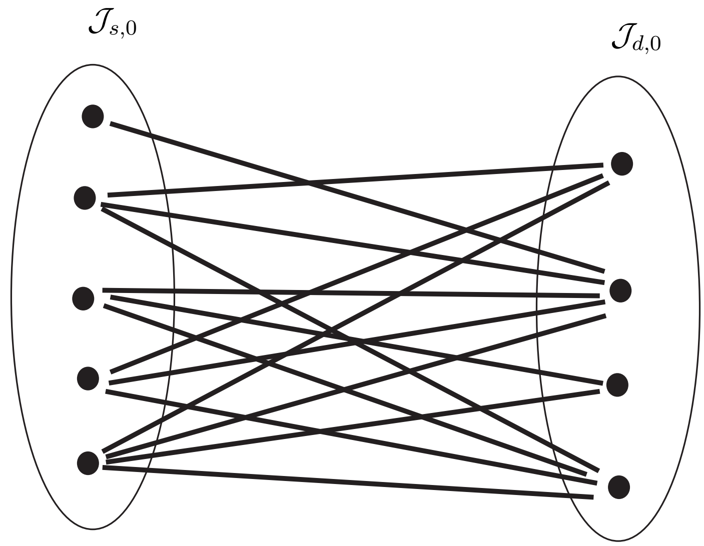


Figure 54: Bipartite Graph for Lower Bound Calculation.

Nonetheless, as shown in Figure 53(b), the network flow formulation generates the solution much faster than the GRASP method. Although, we acknowledge here that the GRASP method has capabilities of parallelization that can speed up the computations for determining the optimal solution.

5.6 Bounds On The Optimal E-P2P Fuel Expenditure

In this section, we provide a measure of the sub-optimality of the solution $\mathcal{M}_H$ by deriving the bounds on the optimal fuel expenditure for E-P2P refueling. We show that the lower bound on the total fuel expenditure $\mathcal{C}\left(\mathcal{M}_e^*\right)$ can be obtained by solving a bipartite assignment problem. To this end, let us consider the bipartite graph $\mathcal{G}_\ell = \{\mathcal{J}_{s,0} \cup \mathcal{J}_{d,0}, \mathcal{E}_\ell\}$ (Figure 54). There exists an (undirected) edge $\langle i, j\rangle$ between two nodes $i \in \mathcal{J}_{s,0}$ and $j \in \mathcal{J}_{d,0}$ if and only if the satellites $s_\mu = \sigma_0(\phi_i)$ and $s_\nu = \sigma_0(\phi_j)$ can engage in a feasible E-P2P maneuver. $\mathcal{E}_\ell$ is the set of all such edges in the graph $\mathcal{G}_\ell$. If the two satellites s_μ and s_ν can engage in a feasible E-P2P maneuver, then the orbital slot ϕ_i is said to be a neighbor of the orbital slot ϕ_j and vice versa. Let $\mathcal{N}(i)$ denote the index set of orbital slots that are neighbors of the orbital slots ϕ_i. Since a fuel transaction can only be between a fuel-sufficient and a fuel-deficient satellite, we have $\mathcal{E}_\ell = \{\langle i,j\rangle : i \in \mathcal{N}(j) \cap \mathcal{J}_{s,0}, j \in \mathcal{N}(i) \cap \mathcal{J}_{d,0}\}$. To each edge $\langle i,j\rangle$, we associate a cost c_{ij}^ℓ that takes into account the fuel expenditure during the forward transfer and the minimum fuel expenditure among all possible return costs. Therefore, if the fuel-sufficient satellite $s_\mu = \sigma_0(\phi_i)$ is active, then the fuel consumption for the related E-P2P maneuver (forward trip + cheapest return trip) is given by

$$c_{ij}^\mu = p_{ij}^\mu + \min_{k \in \mathcal{I}\setminus\{j\}} p_{jk}^\mu. \tag{190}$$

We denote by k_{ij}^μ the index of return slot for which the return cost is minimum. On the other hand, if the fuel-deficient satellite $s_\nu = \sigma_0(\phi_j)$ is active, then the fuel consumption for the related E-P2P maneuver (forward trip + cheapest return trip) is given by

$$c_{ij}^\nu = p_{ji}^\nu + \min_{k \in \mathcal{I}\setminus\{i\}} p_{ik}^\nu. \tag{191}$$

We denote by k_{ij}^ν the index of return slot for which the return cost is minimum. Therefore, the cost of the edge $\langle i,j\rangle \in \mathcal{E}_\ell$ is taken as

$$c_{ij}^\ell = \begin{cases} c_{ij}^\mu, & \text{if } c_{ij}^\mu \leq c_{ij}^\nu, \\ c_{ij}^\nu, & \text{if } c_{ij}^\mu > c_{ij}^\nu. \end{cases} \tag{192}$$

Also, let k_{ij} denote the index of the orbital slot to which the active satellite can return to by spending the minimum amount of fuel. We therefore have

$$k_{ij} = \begin{cases} k_{ij}^\mu, & \text{if } c_{ij}^\mu \leq c_{ij}^\nu, \\ k_{ij}^\nu, & \text{if } c_{ij}^\mu > c_{ij}^\nu. \end{cases} \tag{193}$$

We are interested in a set $\mathcal{M}_\ell \in \mathcal{E}_\ell$ of $|\mathcal{I}_{d,0}|$ edges such that no two edges share the same nodes, that is, $\{i,j\} \cap \{\ell,k\} = \emptyset$ for all $\langle i,j\rangle, \langle \ell,k\rangle \in \mathcal{M}_\ell$. Let us associate with each edge $\langle i,j\rangle \in \mathcal{E}_\ell$ the binary variable x_{ij} given by

$$x_{ij} = \begin{cases} 1, & \text{if } x_{ij} \in \mathcal{M}_\ell, \\ 0, & \text{otherwise.} \end{cases} \tag{194}$$

We now define the following optimization problem on $\mathcal{G}_\ell$:

$$\text{(AP-LB):} \min_{\mathcal{M}_\ell \subseteq \mathcal{E}_\ell} \sum_{\langle i,j \rangle \in \mathcal{E}_\ell} c_{ij}^\ell x_{ij}, \tag{195}$$

subject to

$$\sum_{j \in \mathcal{N}(i)} x_{ij} \leq 1 \text{ for all } i \in \mathcal{J}_{s,0}, \tag{196}$$

$$\sum_{i \in \mathcal{N}(j)} x_{ij} = 1 \text{ for all } j \in \mathcal{J}_{d,0}. \tag{197}$$

Constraint (196) implies that each fuel-sufficient satellite can be assigned to at most one fuel-deficient satellite for refueling purpose, while constraint (197) implies that each fuel-deficient satellite has to be assigned to a fuel-sufficient satellite. Let the optimal solution to the problem (AP-LB) be $\mathcal{M}_\ell^*$ and the optimal value of the objective given in (195) be denoted by $\mathcal{C}_{LB}$. We then have,

$$\mathcal{C}_{\text{LB}} = \sum_{\langle i,j \rangle \in \mathcal{M}_\ell^*} c_{ij}^\ell. \tag{198}$$

We now state the following theorem.

Theorem 1. *The total fuel expenditure $\mathcal{C}(\mathcal{M}_e^*)$ corresponding to the optimal E-P2P solution $\mathcal{M}_e^*$ is bounded below by the optimal value $\mathcal{C}_{\text{LB}}$ of the objective function in the bipartite assignment problem (AP-LB). Also, $\mathcal{C}(\mathcal{M}_e^*)$ is bounded above by the optimal fuel expenditure $\mathcal{C}_{\text{P2P}}$ obtained via P2P refueling. Therefore, $\mathcal{C}_{\text{LB}} \leq \mathcal{C}(\mathcal{M}_e^*) \leq \mathcal{C}_{\text{P2P}}$.*

Proof. The optimal E-P2P solution $\mathcal{M}_e^*$ consists of $|\mathcal{J}_{d,0}|$ triplets. For convenience, let us consider the two mappings $\mathsf{Suff} : \mathcal{T} \mapsto \mathcal{J}_{s,0}$ and $\mathsf{Def} : \mathcal{T} \mapsto \mathcal{J}_{d,0}$ that give the indices of the orbital slots of the fuel-sufficient satellite and fuel-deficient satellite respectively, corresponding to a triplet $(i,j,k) \in \mathcal{T}$. Therefore, $\mathsf{Suff}(\mathcal{M}_e^*)$ corresponds to a set of $|\mathcal{I}_{d,0}|$ distinct nodes in the partition $\mathcal{J}_{s,0}$ of $\mathcal{G}_\ell$, while $\mathsf{Def}(\mathcal{M}_e^*)$ corresponds to all nodes in the partition $\mathcal{J}_{d,0}$ of $\mathcal{G}_\ell$. Since a triplet $(i,j,k) \in \mathcal{M}_e^* \subseteq \mathcal{T}$ corresponds to a feasible E-P2P maneuver, there exists an edge $\langle q,r \rangle \in \mathcal{E}_\ell$ such that $q = \mathsf{Suff}(i,j,k)$

and $r = \mathsf{Def}\,(i,j,k)$. Let us therefore define the mapping $\mathcal{Q} : \mathcal{T} \mapsto \mathcal{E}_\ell$ that gives an edge in $\mathcal{E}_\ell$ for every triplet in $\mathcal{T}$. Now consider the following assignment in $\mathcal{G}_\ell$: $x_{qr} = 1$ for all $\langle q, r\rangle \in \mathcal{Q}(\mathcal{M}_e^*)$ and 0 otherwise. Note that

$$\sum_{r \in \mathcal{N}(q)} x_{qr} = 0 \text{ for all } q \in \mathcal{J}_{s,0} \backslash \mathsf{Suff}\,(\mathcal{M}^*)\,,$$

and

$$\sum_{r \in \mathcal{N}(q)} x_{qr} = 1 \text{ for all } q \in \mathsf{Suff}\,(\mathcal{M}^*)\,.$$

We also have,

$$\sum_{q \in \mathcal{N}(r)} x_{qr} = 1 \text{ for all } r \in \mathsf{Def}\,(\mathcal{M}^*)\,,$$

where $\mathsf{Def}\,(\mathcal{M}^*) = \mathcal{J}_{d,0}$. Hence, the optimal E-P2P solution $\mathcal{M}_e^*$ corresponds to a feasible solution $\mathcal{Q}(\mathcal{M}_e^*)$ for the optimization problem (AP-LB). Hence, we have

$$\sum_{\langle q,r\rangle \in \mathcal{Q}(\mathcal{M}_e^*)} c_{qr}^{\ell} \geq \sum_{\langle q,r\rangle \in \mathcal{M}_\ell^*} c_{qr}^{\ell}. \tag{199}$$

Now, considering $s_\mu = \sigma_0(\phi_i)$, we have the fuel expenditure $\mathcal{C}\,(\mathcal{M}_e^*)$ as

$$\mathcal{C}\,(\mathcal{M}_e^*) = \sum_{(i,j,k) \in \mathcal{M}_e^*} \left(p_{ij}^{\mu} + p_{jk}^{\mu}\right) \geq \sum_{\langle i,j\rangle : (i,j,k) \in \mathcal{M}_e^*} \left(p_{ij}^{\mu} + \min_{k \in \mathcal{I}\backslash\{j\}} p_{jk}^{\mu}\right). \tag{200}$$

Now, consider $\langle q, r\rangle = \mathcal{Q}\,(i,j,k)$. Also, let $s_\alpha = \sigma_0(\phi_q)$ and $s_\beta = \sigma_0(\phi_r)$. Further, note that we have two cases: either $q = i, r = j$, or $q = j, r = i$. In the first case, when the fuel-sufficient satellite is active, $\mu = \alpha$ and the right-hand side of the inequality in (200) reduces to

$$\sum_{\langle i,j\rangle : (i,j,k) \in \mathcal{M}_e^*} \left(p_{ij}^{\mu} + \min_{k \in \mathcal{I}\backslash\{j\}} p_{jk}^{\mu}\right) = \sum_{\langle q,r\rangle \in \mathcal{Q}(\mathcal{M}_e^*)} \left(p_{qr}^{\alpha} + \min_{k \in \mathcal{I}\backslash\{r\}} p_{rk}^{\alpha}\right) = \sum_{\langle q,r\rangle \in \mathcal{Q}(\mathcal{M}_e^*)} c_{qr}^{\alpha}. \tag{201}$$

In the second case, when the fuel-deficient satellite is active, $\mu = \beta$ and the right-hand side of the inequality in (200) reduces to

$$\sum_{\langle i,j\rangle : (i,j,k) \in \mathcal{M}_e^*} \left(p_{ij}^{\mu} + \min_{k \in \mathcal{I}\backslash\{j\}} p_{jk}^{\mu}\right) = \sum_{\langle q,r\rangle \in \mathcal{Q}(\mathcal{M}_e^*)} \left(p_{rq}^{\beta} + \min_{k \in \mathcal{I}\backslash\{q\}} p_{qk}^{\beta}\right) = \sum_{\langle q,r\rangle \in \mathcal{Q}(\mathcal{M}_e^*)} c_{qr}^{\beta}. \tag{202}$$

Using equations (201) and (202) and observing the definition of cost of edges in $\mathcal{E}_\ell$ given by (192), we have

$$\sum_{\langle i,j\rangle:(i,j,k)\in\mathcal{M}_e^*}\left(p_{ij}^{\mu}+\min_{k\in\mathcal{I}\setminus\{j\}} p_{jk}^{\mu}\right)\geq \sum_{\langle q,r\rangle\in\mathcal{Q}(\mathcal{M}_e^*)}\min\{c_{qr}^{\alpha},c_{qr}^{\beta}\}=\sum_{\langle q,r\rangle\in\mathcal{Q}(\mathcal{M}_e^*)}c_{qr}^{\ell}. \quad (203)$$

Using (203), we have from (200),

$$\mathcal{C}\left(\mathcal{M}_e^*\right)\geq \sum_{\langle q,r\rangle\in\mathcal{Q}(\mathcal{M}_e^*)}c_{qr}^{\ell}. \quad (204)$$

Finally, comparing (199) and (204), we have

$$\mathcal{C}\left(\mathcal{M}_e^*\right)\geq \mathcal{C}_{\mathrm{LB}}. \quad (205)$$

For the upper bound, recall that the P2P refueling is a special case of E-P2P and therefore the optimal P2P solution given by $\mathcal{M}_{P2P}$ is a feasible E-P2P solution. Hence,

$$\mathcal{C}\left(\mathcal{M}_e^*\right)\leq \mathcal{C}_{\mathrm{P2P}}. \quad (206)$$

The inequalities (205) and (206) give the desired result. □

Note that an edge $\langle q,r\rangle\in\mathcal{E}_\ell$ corresponds to a feasible triplet (i,j,k_{ij}) in $\mathcal{T}$, where $q=\mathsf{Suff}\,(i,j,k)$ and $r=\mathsf{Def}\,(i,j,k)$. Hence, from the solution $\mathcal{M}_\ell^*$ of the optimization problem (AP-LB), we can construct a set $\mathcal{T}_\ell^*$ of $|\mathcal{J}_{d,0}|$ triplets. In general, this set of triplets $\mathcal{T}_\ell^*$ does not correspond to a feasible E-P2P solution because we may not necessarily have $\mathsf{Act}(\mathcal{T}_\ell^*)=\mathsf{Ret}(\mathcal{T}_\ell^*)$. In other words, either more than one active satellite would compete for the same return position, or an active satellite would try to return to an orbital slot occupied by a passive satellite. In case the condition is met, the set of triplets $\mathcal{T}_\ell^*$ represents a feasible E-P2P solution. This observation along with the Theorem 1 leads to the following corollary.

Corollary 1. *If $\mathsf{Act}(\mathcal{T}_\ell^*)=\mathsf{Ret}(\mathcal{T}_\ell^*)$, then $\mathcal{T}_\ell^*$ is globally optimal solution for the E-P2P problem.*

In case the condition is not met, the set of triplets $\mathcal{T}_\ell^*$ do not correspond to a feasible E-P2P solution. We would use the lower bound given in Theorem 1 in order to estimate the level of sub-optimality of the results obtained by our proposed methodology.

5.6.1 Sub-optimality measure

As already mentioned, the E-P2P solution $\mathcal{M}_H$ given by our proposed methodology is sub-optimal. The fuel expenditure associated with this E-P2P solution is given by $\mathcal{C}(\mathcal{M}_H)$. Considering the bounds given by Theorem 1, we have an estimate of how much sub-optimal these results are. The maximum percentage of sub-optimality of $\mathcal{M}_H$ is given by

$$\eta = \frac{\mathcal{C}(\mathcal{M}_H) - \mathcal{C}_{LB}}{\mathcal{C}_{LB}} \times 100\% \tag{207}$$

5.7 *Sub-optimality of E-P2P solution*

In this section, we will look at the solution of the (AP-LB) and use it to have estimates on the sub-optimality of the E-P2P solution as determined by the GRASP method or the network flow formulation. Typically, the set of E-P2P assignments given by the solution of (AP-LB) does not correspond to a feasible E-P2P solution. For instance, for constellation C_1, the lower bound on the fuel expenditure obtained by solving (AP-LB) is 17.05 units. The solution of (AP-LB) corresponds to the following assignment of satellites for E-P2P refueling: $s_4 \to s_1 \to s_2$, $s_3 \to s_2 \to s_3$, $s_5 \to s_8 \to s_9$, $s_6 \to s_9 \to s_{10}$, $s_7 \to s_{10} \to s_1$. Evidently, this does not correspond to a feasible E-P2P solution. Similarly, for constellation C_2, the solution of (AP-LB) yields the following set of E-P2P maneuvers: $s_{13} \to s_1 \to s_2$, $s_{14} \to s_2 \to s_3$, $s_{10} \to s_3 \to s_4$, $s_9 \to s_4 \to s_5$, $s_8 \to s_5 \to s_6$, $s_7 \to s_6 \to s_7$, $s_{11} \to s_{15} \to s_{16}$, $s_{12} \to s_{16} \to s_1$. Clearly, this also does not correspond to a feasible E-P2P solution. However, there can be cases when the solution of (AP-LB) yields a feasible E-P2P solution. For instance, for constellation C_8, (AP-LB) yields the following set of E-P2P assignments:

$s_{16} \to s_1 \to s_2$, $s_2 \to s_3 \to s_4$, $s_4 \to s_5 \to s_6$, $s_6 \to s_7 \to s_8$, $s_8 \to s_9 \to s_{10}$, $s_{10} \to s_{11} \to s_{12}$, $s_{12} \to s_{13} \to s_{14}$, $s_{14} \to s_{15} \to s_{16}$. Clearly, this is a feasible E-P2P solution and by Corollary 1, this is the globally optimal E-P2P solution. Figure 55 summarizes the results obtained for all the constellations along with the lower and upper bounds for the optimal fuel expenditure for E-P2P refueling. Furthermore,

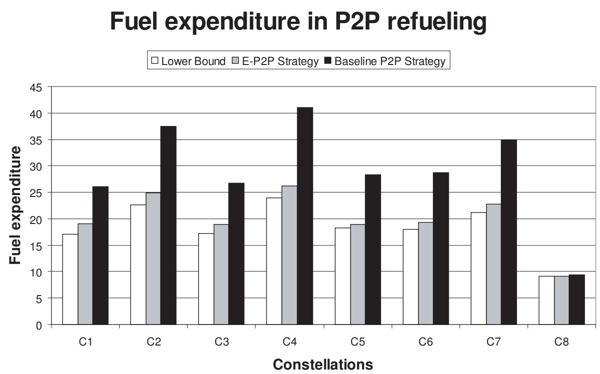


Figure 55: Comparison of E-P2P and baseline P2P refueling strategies.

we can use the measure of sub-optimality defined in the previous section in order to estimate how much sub-optimal the E-P2P solution is.

Table 7: Sub-optimality of results.

Constellation	Sub-optimality of $\mathcal{M}_{\mathrm{H}}$	Sub-optimality of GRASP solution
C_1	12.1%	10.05%
C_2	9.69%	9.69%
C_3	9.26%	7.17%
C_4	9.70%	9.70%
C_5	3.06%	1.09%
C_6	7.23%	7.22%
C_7	7.25%	6.36%
C_8	0.00%	0.00%

5.8 Summary

We investigated the (non-cooperative) Egalitarian P2P (E-P2P) refueling strategy, in which the active satellites are allowed to interchange their orbital slots during their return trips. We provided two formulations for the problem, and pointed out the relative merits of the formulations. The primary benefit of the E-P2P strategy is the significantly reduced fuel expenditure, compared to the P2P strategy. Recognizing the sub-optimality of the E-P2P solution generated by our method, we derive bounds on the optimal fuel expenditure incurred in E-P2P refueling. The lower bound provides a measure of the sub-optimality of the solutions, however the lower bound may or may not correspond to a feasible E-P2P solution. When it does correspond to a feasible solution, the bound is tight and the global optimal E-P2P solution can be obtained by solving the bipartite matching problem used to determine the bound.

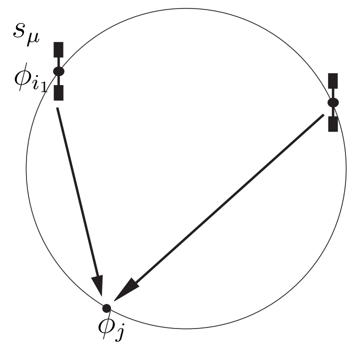
CHAPTER VI

COOPERATIVE EGALITARIAN PEER-TO-PEER REFUELING STRATEGY

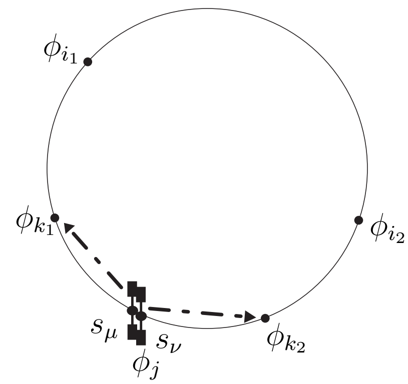
From the discussion in the previous chapters, we have found that the two extensions of the P2P refueling problem, namely the Egalitarian P2P and the Cooperative P2P, help in substantially reducing the fuel expenditure during the refueling process. A natural question that arises is that if both E-P2P and C-P2P strategies are better, why should not they be combined together in a single refueling strategy. In this chapter, we address this question by combining these ideas into what we term the Cooperative Egalitarian P2P (CE-P2P) refueling strategy. We discuss in detail a network flow formulation for the problem, and then compare the various refueling strategies for several constellations.

6.1 CE-P2P Problem Formulation

Let us consider a CE-P2P maneuver between two satellites $s_\mu = \sigma_0(\phi_{i_1})$ and $s_\nu = \sigma_0(\phi_{i_2})$, occupying the orbital slots ϕ_{i_1} and ϕ_{i_2} respectively, where $i_1, i_2 \in \mathcal{J}$. Without loss of generality, assume s_μ to be the fuel-sufficient satellite and s_ν to be the fuel-deficient satellite, that is, $i_1 \in \mathcal{J}_{s,0}$ and $i_2 \in \mathcal{J}_{d,0}$. Let these satellites engage in a rendezvous at the orbital slot ϕ_j, where $j \in \mathcal{J}_c$. After the refueling transaction, the satellites s_μ and s_ν return to the orbital slots ϕ_{k_1} and ϕ_{k_2} respectively, where $k_1, k_2 \in \mathcal{J}_r$. Given $i_1, i_2 \in \mathcal{J}_a$, $j \in \mathcal{J}_c$, and $k_1, k_2 \in \mathcal{J}_r$, we can represent an assignment for a CE-P2P maneuver by (i_1, i_2, j, k_1, k_2). An assignment (i_1, i_2, j, k_1, k_2) is feasible if the satellites s_μ and s_ν engaging in the CE-P2P refueling transaction end up being



(a) Forward Trip



(b) Return Trip

Figure 56: CE-P2P Maneuver.

fuel-sufficient after the maneuver is complete. Let $\mathcal{P}$ denote the set of all feasible CE-P2P assignments in the constellation. Let $\mathcal{M}_{ce} \subseteq \mathcal{P}$ denote the set of $|\mathcal{J}_{d,0}|$ feasible CE-P2P maneuvers such that all fuel-deficient satellites are included in the refueling transactions. The cost of a CE-P2P solution is the total fuel expenditure incurred during all the orbital transfers taking place. Figure 56 depicts the forward and return trips of the CE-P2P maneuver. Let p_{ij}^{μ} denote the fuel used by satellite s_μ during its transfer from the orbital slot ϕ_i to the slot ϕ_j. Therefore, the cost of the CE-P2P solution is given by

$$\mathcal{C}(\mathcal{M}_{ce}) = \sum_{(i_1,i_2,j,k_1,k_2)\in\mathcal{M}_{ce}} p_{i_1j}^{\mu} + p_{i_2j}^{\nu} + p_{jk_1}^{\mu} + p_{jk_2}^{\nu}. \tag{208}$$

Also note that, if $i_1 = j = k_1$ or $i_1 = j = k_1$, for a CE-P2P assignment $(i_1, i_2, j, k_1, k_2) \in \mathcal{P}$ then the assignment represents an E-P2P maneuver (non-cooperative). Let $\mathcal{P}_e$ denote the set of feasible E-P2P maneuvers in the constellation. Clearly, $\mathcal{P}_e \subseteq \mathcal{P}$. Similarly, if $i_1 = k_1$ and $i_2 = k_2$, for the CE-P2P assignment $(i_1, i_2, j, k_1, k_2) \in \mathcal{P}$ then the assignment represents a C-P2P maneuver (non-Egalitarian). Let $\mathcal{P}_c$ denote the set of feasible C-P2P maneuvers in the constellation. Clearly, $\mathcal{P}_c \subseteq \mathcal{P}$. Furthermore, let $\mathcal{M}_{ce}^*$ denote the optimal set of assignments that minimizes the fuel expenditure during CE-P2P refueling. We therefore have

$$\mathcal{C}(\mathcal{M}^*_{ce}) = \min_{\mathcal{M}_{ce}\subseteq\mathcal{P}} \mathcal{C}(\mathcal{M}_{ce}). \tag{209}$$

Similarly, let $\mathcal{M}^*_c \subseteq \mathcal{P}_c$ and $\mathcal{M}^*_e \subseteq \mathcal{P}_e$ denote the optimal set of assignments for C-P2P and E-P2P refueling. We therefore have,

$$\mathcal{C}(\mathcal{M}^*_c) = \min_{\mathcal{M}_{ce}\subseteq\mathcal{P}_c} \mathcal{C}(\mathcal{M}_{ce}), \tag{210}$$

and

$$\mathcal{C}(\mathcal{M}^*_e) = \min_{\mathcal{M}_{ce}\subseteq\mathcal{P}_e} \mathcal{C}(\mathcal{M}_{ce}). \tag{211}$$

6.1.1 CE-P2P Maneuver Costs

Let us consider a CE-P2P maneuver (i_1, i_2, j, k_1, k_2). During the first phase of the maneuver, the two satellites $s_\mu = \sigma_0(\phi_{i_1})$ and $s_\nu = \sigma_0(\phi_{i_2})$ transfer to the orbital slot ϕ_j. The fuel consumed by the active satellite s_μ to transfer from the orbital slot ϕ_{i_1} to the orbital slot ϕ_j is given by

$$p^\mu_{i_1 j} = \left(m_{\mathrm{s}\mu} + f^-_\mu\right)\left(1 - e^{-\frac{\Delta V_{i_1 j}}{c_{0\mu}}}\right), \tag{212}$$

where $m_{\mathrm{sp}\mu}$ denotes the mass of the permanent structure of the satellite s_μ, $c_{0\mu}$ denotes the characteristic constant for the satellite s_μ, and $\Delta V_{i_1 j}$ denotes the optimal velocity change required for the transfer from the slot ϕ_{i_1} to ϕ_j. The characteristic constant is defined by $c_{0\mu} = g_0 I_{\mathrm{sp}\mu}$, where g_0 denote the gravitational acceleration on the surface of the earth, and $I_{\mathrm{sp}\mu}$ denote the specific thrust of the engine of the satellite s_μ. Similarly, the fuel expenditure for satellite s_ν to transfer from the orbital slot ϕ_{i_2} to the orbital slot ϕ_j is given by:

$$p^\nu_{i_2 j} = \left(m_{\mathrm{s}\nu} + f^-_\nu\right)\left(1 - e^{-\frac{\Delta V_{i_2 j}}{c_{0\nu}}}\right). \tag{213}$$

The fuel content of satellite s_μ after its forward trip (but before the fuel exchange takes place) is $f^-_\mu - p^\mu_{i_1 j}$, while that of satellite s_ν is $f^-_\nu - p^\nu_{i_2 j}$. The amount of fuel that s_μ delivers to s_ν is g_μ^ν. Hence, the fuel content of satellite s_μ just after the fuel exchange takes place is $f_\mu^- - p_{i_1 j}^\mu - g_\mu^\nu$, while that of satellite s_ν is $f_\nu^- - p_{i_2 j}^\nu + g_\mu^\nu$. After the fuel exchange, and in the second phase of the P2P maneuver, satellites s_μ and s_ν transfer to the orbital slots ϕ_{k_1} and ϕ_{k_2}, respectively. During the return trip, the fuel expenditure of satellite s_μ to transfer from slot ϕ_j to slot ϕ_{k_1} is given by

$$p_{jk_1}^\mu = \left(m_{\mathrm{s}\mu} + f_\mu^- - p_{i_1 j}^\mu - g_\mu^\nu\right)\left(1 - e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}}\right), \tag{214}$$

while that of satellite s_ν to transfer from slot ϕ_j to slot ϕ_{k_1} is given by

$$p_{jk_2}^\nu = \left(m_{\mathrm{s}\nu} + f_\nu^- - p_{i_2 j}^\nu + g_\mu^\nu\right)\left(1 - e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}}\right). \tag{215}$$

The amount of fuel exchanged affects the return trip fuel expenditure. Following an analysis similar to the one in Ref. 22, it can be shown that the fuel expenditure during the CE-P2P is minimized if the amount of fuel exchanged by the satellites is given by

$$g_\mu^\nu = \begin{cases} g_\mu^\nu|_\ell, & e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}} < e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}}, \\ g_\mu^\nu|_u, & e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}} > e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}}, \end{cases} \tag{216}$$

where,

$$g_\mu^\nu|_\ell = \left(m_{\mathrm{s}\nu} + \underline{f}_\nu\right) e^{\frac{\Delta V_{jk_2}}{c_{0\nu}}} - \left(m_{\mathrm{s}\nu} + f_\nu^- - p_{i_2 j}^\nu\right), \tag{217}$$

and

$$g_\mu^\nu|_u = \left(m_{\mathrm{s}\mu} + f_\mu^- - p_{i_1 j}^\mu\right) - \left(m_{\mathrm{s}\mu} + \underline{f}_\mu\right) e^{\frac{\Delta V_{jk_1}}{c_{0\mu}}}. \tag{218}$$

Also, if $e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}} = e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}}$, g_μ^ν can assume any value in the interval $g_\mu^\nu|_\ell \leq g_\mu^\nu \leq g_\mu^\nu|_u$. For the maneuver to be feasible we must have $g_\mu^\nu|_\ell \leq g_\mu^\nu|_u$, that is, there exists a fuel exchange that would result in both satellites to be fuel-sufficient at the end of the maneuver. Furthermore, for feasibility of the CE-P2P maneuver, we must also have $p_{i_1 j}^\mu < f_\mu^-$ and $p_{i_2 j}^\mu < f_\mu^-$, that is, both satellites must have enough fuel to complete their forward trips.

6.1.2 Constellation Digraph

We can represent a CE-P2P maneuver using a directed graph. To this end, let us define a constellation graph $\mathcal{G}$ consisting of three partitions $\mathcal{J}_a$, $\mathcal{J}_c$ and $\mathcal{J}_r$. The nodes of $\mathcal{G}$ are given by $\mathcal{J}_a \cup \mathcal{J}_c \cup \mathcal{J}_r$. However, we do not know a priori which satellites are active, which are passive, and which slots are used for cooperative rendezvous. That is, we do not know the sets $\mathcal{J}_a$, $\mathcal{J}_c$ and $\mathcal{J}_r$ a priori. We therefore let $\mathcal{J}_a = \mathcal{J}_r = \mathcal{J}$ and $\mathcal{J}_c = \mathcal{J}'$. We will denote an orbital transfer using a *directed* edge, with the direction of edge signifying the direction of the orbital transfer. Let an edge (i, j), where $i \in \mathcal{J}_a$ and $j \in \mathcal{J}_c$, denote a forward trip from the slot ϕ_i to the slot ϕ_j, and let the associated cost for this transfer be denoted by c_{ij}. Let an edge (j, k), where $j \in \mathcal{J}_c$ and $k \in \mathcal{J}_r$, denote a return trip from the slot ϕ_j to ϕ_k, and let the associated cost for this transfer be denoted by c_{jk}. A set of edges (i_1, j), (i_2, j), (j, k_1) and (j, k_2) represents a CE-P2P maneuver. Note that any edge (i, j) having $\phi_i = \phi_j$ does not represent a physical transfer, since it would mean that the active satellite occupies the same orbital slot during its forward/return trip. Naturally, the cost associated with such an edge is zero. Hence, if we have $\phi_{i_1} = \phi_j$ or $\phi_{i_2} = \phi_j$, then the maneuver is actually non-cooperative, because one of the satellites involved in the refueling transaction remains in its orbital slot throughout the maneuver. In other words, our representation of a CE-P2P maneuver allows an E-P2P maneuver to be treated as a special case of a CE-P2P maneuver in which one forward edge and one return edge does not actually represent a maneuver, and each of these edges has a zero cost.

Ideally, the cost of the edges in the graph $\mathcal{G}$ has to be the fuel expenditure during the orbital transfers. However, the calculation of the fuel expenditure is dependent on the mass of the satellite performing the orbital transfer. Since we do not know a priori which satellites are going to pair up for the refueling transactions, the return trip fuel expenditure cannot be uniquely determined for the return trip edges on the graph $\mathcal{G}$. Instead of the fuel expenditure, we can use the velocity change ΔV required

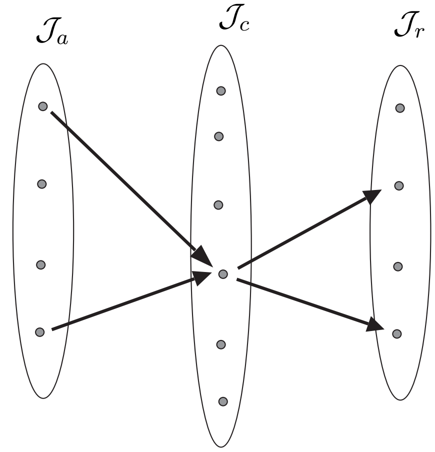


Figure 57: Directed constellation graph.

for the corresponding orbital transfer because the ΔV can be uniquely determined for all edges. The minimization of ΔV would yield sub-optimal results since the true objective is to minimize fuel expenditure. However, it was observed in our numerical simulations that solutions are only marginally sub-optimal when we minimize ΔV. Furthermore, in order to avoid solutions in which a fuel-deficient satellite does not have enough fuel to complete the desired rendezvous, we only allow those forward edges (i, j) in the graph $\mathcal{G}$ for which we have $p_{ij}^{\mu} < f_{\mu}^{-}$, where $s_{\mu} = \sigma_0(\phi_i)$ and $\phi_j \in \Phi'$.

6.1.3 A Network Flow Formulation

We now present a network flow formulation for the solution of CE-P2P problem. We set up a constellation network $\mathcal{G}_n$ using the constellation digraph $\mathcal{G}$. To this end, we add a source node s and a sink node t to the constellation digraph $\mathcal{G}$. For all $i \in \mathcal{J}_a$, we also add an arc (s, i) with associated cost $c_{si} = 0$. We denote the set of these arcs by $\mathcal{E}_s$. Similarly, for all $k \in \mathcal{J}_r$, we add an arc (k, t) with associated cost $c_{kt} = 0$. We denote the set of these arcs by $\mathcal{E}_t$. Let us now consider two $s \to t$ flows in the network $\mathcal{G}_n$, that pass through the same node $j \in \mathcal{J}_c$. A pair of such flows $s \to i_1 \to j \to k_1 \to t$ and $s \to i_2 \to j \to k_2 \to t$ represent a CE-P2P maneuver (i_1, i_2, j, k_1, k_2). The total cost of the flows equal the total ΔV required for all the orbital transfers during a CE-P2P maneuver. We seek $|\mathcal{J}_{d,0}|$ pairs of flows in the constellation network with minimum total cost, such that all flows also pass through all the fuel-deficient satellites in the constellation. Note that each assignment (i_1, i_2, j, k_1, k_2) in a CE-P2P solution $\mathcal{M}_{ce}$ corresponds to a set of edges (s, i_1), (s, i_2), (i_1, j), (i_2, j), (j, k_1), (j, k_2), (k_1, t), and (k_2, t) in $\mathcal{G}_n$. The total cost of these edges is therefore the total ΔV required for all the orbital transfers corresponding to the assignment (i_1, i_2, j, k_1, k_2). Let the set of edges in the network corresponding to all assignments in the CE-P2P solution $\mathcal{M}_{ce}$ be denoted by $\mathcal{M}$. Also, let the set of slots where the cooperative rendezvous takes place corresponding to the solution $\mathcal{M}_{ce}$ be given by $\mathcal{Y}$. Let us now introduce the following decision variables for our optimization problem. Corresponding to each edge (i, j), we introduce a flow variable x_{ij} defined by

$$x_{ij} = \begin{cases} 1, & \text{if } x_{ij} \in \mathcal{M}, \\ 0, & \text{otherwise.} \end{cases} \tag{219}$$

Also, corresponding to each slot for cooperative rendezvous, let us introduce the decision variables y_j, as follows

$$y_j = \begin{cases} 1, & \text{if } j \in \mathcal{Y}, \\ 0, & \text{otherwise.} \end{cases} \tag{220}$$

We need $|\mathcal{J}_{d,0}|$ CE-P2P maneuvers in order to refuel all fuel-deficient satellites. Hence, the total flow that goes out of the source is $2|\mathcal{J}_{d,0}|$ and the flow distributes itself into $|\mathcal{J}_{d,0}|$ fuel-sufficient satellites and $|\mathcal{J}_{d,0}|$ fuel-deficient satellites. Noting that $\mathcal{J}_{s,0} \cup \mathcal{J}_{d,0} = \mathcal{J}$, we have,

$$\sum_{i \in \mathcal{J}} x_{si} = 2|\mathcal{J}_{d,0}|, \tag{221}$$

and

$$\sum_{i \in \mathcal{J}_{s,0}} x_{si} = |\mathcal{J}_{d,0}|. \tag{222}$$

An amount of flow equal to the flow originating from the source must be collected at the sink node, that is,

$$\sum_{k \in \mathcal{J}} x_{kt} = 2|\mathcal{J}_{d,0}|. \tag{223}$$

The flow balance equations at the different nodes yield the following constraints

$$x_{si} = \sum_{j \in \mathcal{J}_c} x_{ij}, \text{ for all } i \in \mathcal{J}_a, \tag{224}$$

$$x_{kt} = \sum_{j \in \mathcal{J}_c} x_{jk}, \text{ for all } i \in \mathcal{J}_r, \tag{225}$$

and

$$\sum_{i \in \mathcal{J}_a} x_{ij} = \sum_{k \in \mathcal{J}_r} x_{jk}, \text{ for all } j \in \mathcal{J}_c. \tag{226}$$

The orbital slots available for return are exactly the orbital slots for the active satellites. Hence, we have,

$$x_{si} = x_{it}, \text{ for all } i \in \mathcal{J}. \tag{227}$$

The total number of slots for rendezvous is the total number of CE-P2P maneuvers, which in turn equals the number of fuel-deficient satellites in the constellation. We therefore have,

$$\sum_{j \in \mathcal{J}_c} y_j = |\mathcal{J}_{d,0}|. \tag{228}$$

If a slot is selected for cooperative rendezvous, two satellites must transfer to that location (unless it is a non-cooperative maneuver). Hence, we have the following constraint:

$$\sum_{i \in \mathcal{J}} x_{ij} = 2y_j, \text{ for all } j \in \mathcal{J}_c. \tag{229}$$

The two satellites transferring to the slot ϕ_j must be a fuel-sufficient and a fuel-deficient satellite. In other words, we have at most one fuel-sufficient satellite ending up in the slot ϕ_j, that is,

$$\sum_{i \in \mathcal{J}_{s,0}} x_{ij} \leq 1, \text{ for all } j \in \mathcal{J}_c. \tag{230}$$

Given the decision variables defined in (219) and (220), and the set of constraints (221)-(230), we are required to minimize the total ΔV for the CE-P2P maneuvers, that is,

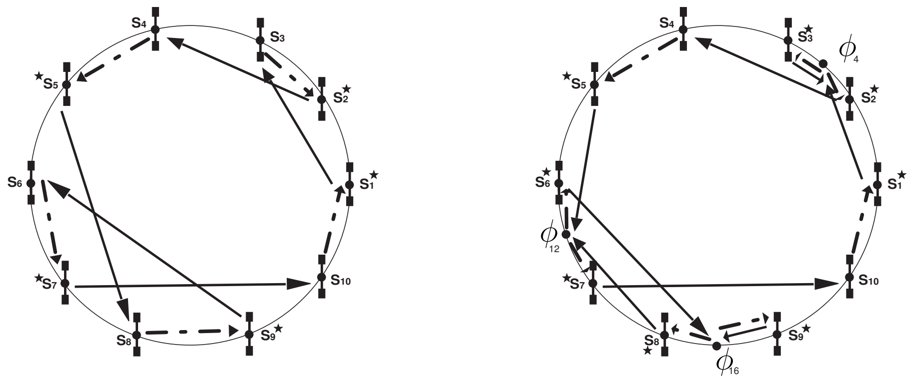
$$\text{(CE-P2P)} \ : \min \sum_{(i,j)\in\mathcal{E}_n} c_{ij}x_{ij}. \tag{231}$$

6.2 CE-P2P Numerical Examples

In this section we discuss a few numerical examples that show the benefit of a cooperative refueling strategy for different satellite constellations. These constellations vary in the number of satellites, the mass and fuel content of the satellites, and the constellation orbit. The details of these constellations are given in Table 5.

Example 17. *CE-P2P strategy for a constellation of* 10 *satellites.*

Let us consider the constellation C_1 given in Table 5. It consists of 10 satellites evenly distributed in a circular orbit. The initial fuel content of the satellites $s_1, s_2, \ldots, s_{10}$ are 30, 30, 6, 6, 6, 6, 6, 30, 30, 30 units respectively. The maximum allowed time for refueling is $T = 12$ orbital periods. Each satellite s_i has a minimum fuel requirement of $\underline{f}_i = 12$ units, while the maximum amount of fuel for each satellite is $\bar{f}_i = 30$ units. Each satellite has a permanent structure of $m_{\mathrm{s}i} = 70$ units, and a characteristic constant of $c_0 = 2943$ m/s. The indices of the fuel-sufficient satellites are $\mathcal{I}_{s,0} = \{1, 2, 8, 9, 10\}$ and those of the fuel-deficient satellites are $\mathcal{I}_{d,0} = \{3, 4, 5, 6, 7\}$. Let Φ' be a set of 20 evenly distributed slots, out of which 10 are occupied by the satellites. We have, $\mathcal{J}' = \{1, 2, \ldots, 20\}$, and the satellites occupy the slots $\mathcal{J} = \{1, 3, \ldots 19\}$ respectively, that is, we have $s_i = \sigma_0(\phi_{2i-1})$ for all $i \in \{1, 2, \ldots, 10\}$. An E-P2P strategy for this constellation yields the following optimal assignments: $s_1 \to s_3 \to s_2$, $s_2 \to s_4 \to s_5$, $s_5 \to s_8 \to s_9$, $s_7 \to s_{10} \to s_1$, $s_9 \to s_6 \to s_7$, where the assignment $s_1 \to s_3 \to s_2$ implies that the satellite s_1 undergoes an orbital transfer to rendezvous with s_3, exchanges fuel, and then returns to the orbital slot



(a) E-P2P strategy

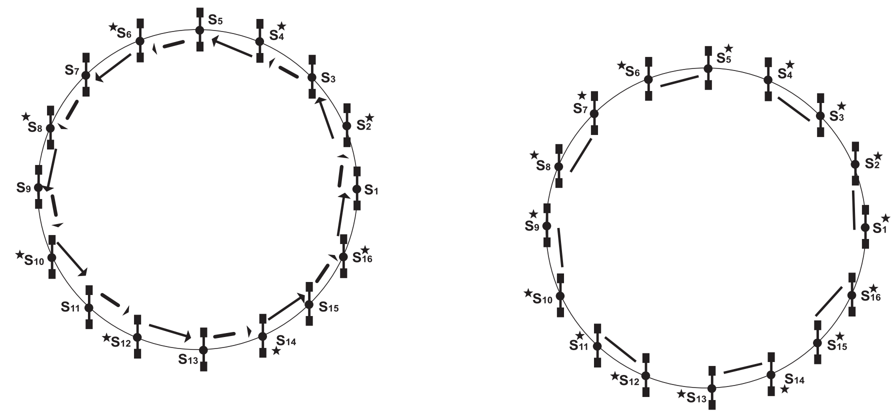
(b) CE-P2P strategy

Figure 58: Optimal assignments.

originally occupied by the satellite s_2. Figure 58(a) depicts these E-P2P maneuvers. The fuel expenditure during the E-P2P refueling process is 19.11 units. This represents 10.62% of the total initial fuel in the constellation. Figure 58(a) shows the optimal assignments for the E-P2P case. A C-P2P strategy for this constellation yields a higher fuel expenditure than the E-P2P case. Let us now consider a CE-P2P strategy for refueling satellites in this constellation. First, let us look at the solution provided by the problem (CE-P2P-LB). The lower bound on CE-P2P expenditure is found to be $\mathcal{C}_{\rm LB} = 17.05$ units. The corresponding optimal matching is the following satellites pairs: $s_1 \leftrightarrow s_4$, $s_2 \leftrightarrow s_3$, $s_8 \leftrightarrow s_5$, $s_9 \leftrightarrow s_6$, and $s_{10} \leftrightarrow s_7$ with their preferred slots for rendezvous being ϕ_1, ϕ_3, ϕ_{15}, ϕ_{17}, and ϕ_{19} respectively. Note that in all of these matchings between the fuel-sufficient and fuel-deficient satellites, the fuel-deficient satellite performs a non-cooperative rendezvous with the corresponding fuel-sufficient satellite. The preferred return locations for these active satellites are ϕ_3, ϕ_7, ϕ_{17}, ϕ_{19}, and ϕ_1 respectively. All these are slots adjacent to the corresponding rendezvous slot. Note that these slots are occupied by the passive satellites and it is not possible for all of the active satellites to return to their most preferred choice of orbital slots. Hence, the solution of (CE-P2P-LB) is not a feasible CE-P2P solution. We therefore solve the optimization problem (CE-P2P) yielding the following assignments: $(s_1, s_3) \to \phi_4 \to (s_2, s_3)$, $s_2 \to s_4 \to s_5$, $(s_5, s_8) \to \phi_{12} \to (s_6, s_7)$, $(s_6, s_9) \to \phi_{16} \to (s_8,\ s_9)$ and $s_7 \to s_{10} \to s_1$. Figure 58(b) depicts this solution. Note that, like the E-P2P case, all active satellites transfer to available slots in the vicinity during their return trips. The fuel expenditure during the cooperative E-P2P refueling process is 18.65 units, which represents 2.5% fuel savings over the E-P2P refueling strategy. This example demonstrates the utility of the CE-P2P refueling strategy in reducing the fuel expenditure incurred during a (non-cooperative) E-P2P strategy or a (non-Egalitarian) C-P2P strategy. The solution determined is potentially sub-optimal. Comparing with the lower bound on fuel expenditure, we have $\eta = 9.38\%$. This means that our solution is at most 9.38% sub-optimal. Furthermore, looking at the optimal CE-P2P solution, we find that two of the maneuvers are actually non-cooperative E-P2P maneuvers. Satellites s_2, s_4 and s_7, s_{10} engage in (non-cooperative) E-P2P maneuvers, while the remaining transactions are all cooperative. Hence, s_4 and s_{10} are the passive satellites for the CE-P2P refueling strategy, that is, they remain in their orbital slots throughout the refueling process.

Example 18. *Global minimum in the case of a constellation of* 16 *satellites.*

Let us consider the constellation C_3 in Table 5 consisting of 16 satellites, evenly distributed in a circular orbit. The fuel content of satellites $s_1, s_2, \ldots, s_{16}$ are 30, 10, 30, 10, 30, 10, 30, 10, 30, 10, 30, 10, 30, 10, 30, 10 respectively. The indices of the fuel-sufficient satellites are $\mathcal{I}_{s,0} = \{1, 5, 7, 9, 11, 13, 15\}$ and those of the fuel-deficient satellites are $\mathcal{I}_{d,0} = \{2, 4, 6, 8, 10, 12, 14, 16\}$. Let us consider Φ' to be a set of 32 orbital slots evenly distributed on the orbit, out of which 16 are initially occupied by the satellites. We therefore have, $\mathcal{J}' = \{1, 2, \ldots, 32\}$. The satellites occupy the slots $\phi_1, \phi_3, \ldots \phi_{31}$ respectively, so that $s_i = \sigma_0(\phi_{2i-1})$ for all $i \in \{1, 2, \ldots, 16\}$. If we solve (CE-P2P-LB), we have the lower bound on the CE-P2P fuel expenditure to



(a) Fuel-deficient satellites can initiate non-cooperative rendezvous

(b) Fuel-deficient satellites cannot initiate non-cooperative rendezvous

Figure 59: Global Minimum for a Constellation of 16 satellites.

be $\mathcal{C}_{\text{LB}} = 9.08$ units of fuel. The optimal matching yielded by (CE-P2P-LB) is the following satellites pairs: $s_1 \leftrightarrow s_{16}$, $s_2 \leftrightarrow s_3$, $s_4 \leftrightarrow s_5$, $s_6 \leftrightarrow s_7$, $s_{10} \leftrightarrow s_{11}$, $s_{12} \leftrightarrow s_{13}$, and $s_{14} \leftrightarrow s_{15}$. For all of these matchings, the fuel-deficient satellite performs a non-cooperative rendezvous with the corresponding fuel-sufficient satellite and returns to an orbital slot previously occupied by a different active satellite. Furthermore, the active satellites rendezvous with their preferred choice of fuel-sufficient satellite in its vicinity, and return to their preferred choice of orbital slots without any conflict. Thus, the solution of (CE-P2P-LB) yields a feasible, and hence the global optimum, CE-P2P solution. Figure 59(a) depicts this global minimum. In particular, we find that the global minimum is also the optimal (non-cooperative) E-P2P solution. The (non-Egalitarian) C-P2P solution has a higher fuel expenditure (10.34 units) in this case.

Example 19. *Fuel-deficient satellites have insufficient fuel to engage in non-cooperative rendezvous.*

Let us consider the constellation C_4 given in Table 5. This is similar to the constellation C_3, except that now the fuel-deficient satellites have much less amount of fuel so that they cannot engage in a non-cooperative rendezvous. If we solve (CE-P2P-LB), the optimal matching obtained is the following set of satellites pairs: $s_1 \leftrightarrow s_2$, $s_3 \leftrightarrow s_4$, $s_5 \leftrightarrow s_6$, $s_7 \leftrightarrow s_8$, $s_9 \leftrightarrow s_{10}$, $s_{11} \leftrightarrow s_{12}$, $s_{13} \leftrightarrow s_{14}$, and $s_{15} \leftrightarrow s_{16}$. The lower bound obtained is $C_{\text{LB}} = 9.48$ units of fuel. In each of these assignments, the fuel-deficient satellite engages in a cooperative rendezvous with a neighboring fuel-sufficient satellite and after undergoing a fuel-exchange, returns to its original orbital slot. For each pair of active satellites engaging in a fuel exchange, the slot for cooperative rendezvous is midway between the original slots of the satellites. In fact, all fuel-deficient satellites rendezvous with their preferred choice of fuel-sufficient satellites and return to their preferred orbital slots, without any conflict. The solution of (CE-P2P-LB) is therefore a feasible CE-P2P solution and, hence, also the global optimal solution. Figure 59(b) depicts the matching between the satellites required

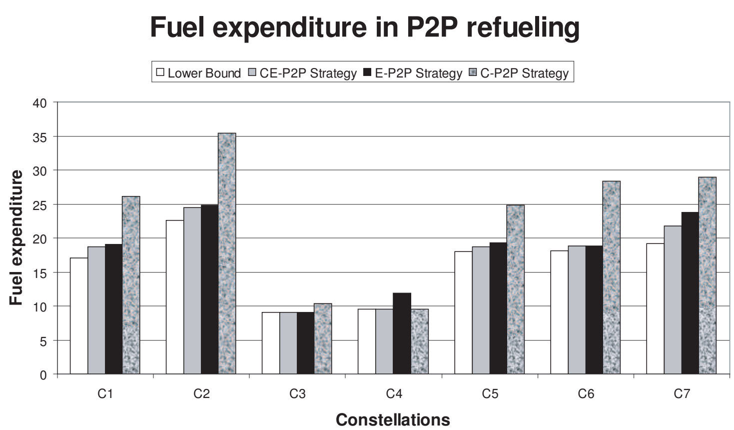


Figure 60: Refueling Expenditures.

for refueling. The global minimum in this case is the optimal C-P2P solution. For this constellation, the (non-cooperative) E-P2P solution has a higher fuel expenditure of 11.85 units.

Figure 6 provides a comparison of the CE-P2P, E-P2P and C-P2P refueling strategies for the constellations depicted in Table 5. It also shows the lower bound given by the (CE-P2P-LB) solution for all constellations. In general, it is observed that

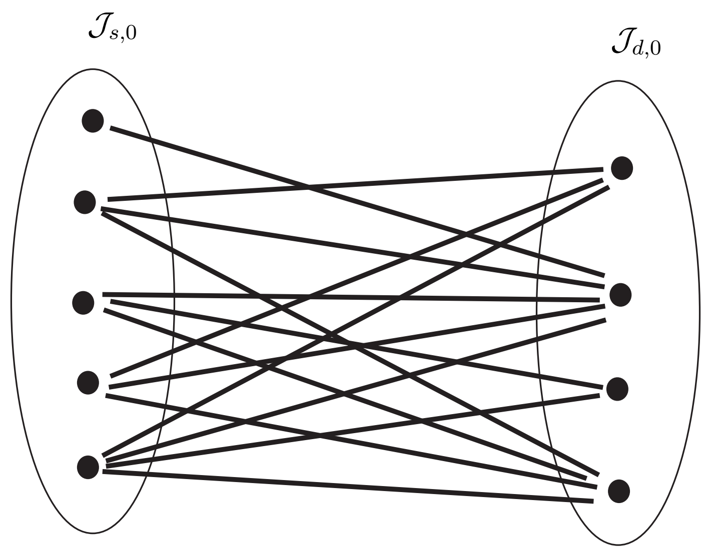


Figure 61: Bipartite Graph for CE-P2P Lower Bound Calculation.

the CE-P2P strategy provides an improvement over either the E-P2P or the C-P2P strategies.

6.3 Bounds On The Optimal Fuel Expenditure

The set of CE-P2P maneuvers obtained by solving the optimization problem (CE-P2P) corresponds to the minimum total ΔV required for the orbital transfers taking place during refueling. Let this solution be denoted by $\mathcal{M}_{ce}^{\mathrm{H}}$. Our true objective is to minimize fuel expenditure, and hence the solution $\mathcal{M}_{ce}^{\mathrm{H}}$ is potentially sub-optimal. In this section, we provide a measure of the sub-optimality of the solution $\mathcal{M}_{ce}^{\mathrm{H}}$ by deriving bounds on the optimal fuel expenditure for CE-P2P refueling. In particular, we show that a conservative lower bound on the total fuel expenditure $\mathcal{C}\left(\mathcal{M}_{ce}^{*}\right)$ can be obtained by solving a bipartite assignment problem. To this end, let us consider the undirected bipartite graph $\mathcal{G}_{\ell} = \{\mathcal{J}_{s,0} \cup \mathcal{J}_{d,0}, \mathcal{E}_{\ell}\}$ (Figure 61). We will represent a P2P maneuver between two satellites by an *undirected* edge in the graph $\mathcal{G}_{\ell}$. In particular, we say that there exists an (undirected) edge $\langle i_1, i_2 \rangle$ between two nodes $i_1 \in \mathcal{J}_{s,0}$ and $i_2 \in \mathcal{J}_{d,0}$ if and only if the satellites s_μ and s_ν, occupying initially the orbital slots ϕ_{i_1} and ϕ_{i_2}, respectively, can engage in a feasible CE-P2P maneuver. By this, we mean the satellites can engage in a rendezvous at a slot ϕ_j, where $j \in \mathcal{J}'$, and return respectively to the orbital slots ϕ_{k_1} and ϕ_{k_2}. The set of all such edges in the graph is given by $\mathcal{E}_\ell = \{\langle i_1, i_2\rangle$: there exists $j \in \mathcal{J}'$, and $\phi_{k_1}, \phi_{k_2} \in \mathcal{J}$ such that either $(i_1, i_2, j, k_1, k_2) \in \mathcal{P}\}$. To each edge $\langle i_1, i_2\rangle$, we associate a cost $c^\ell_{i_1 i_2}$ that takes into account the fuel expenditure during the forward and return trips of the satellites, among all possible slots for cooperative rendezvous and return positions. The minimum fuel consumption for all possible return slots corresponding to the cooperative rendezvous slot ϕ_j, where $j \in \mathcal{J}'$, is given by

$$\left[p^\mu_{i_1 j} + p^\nu_{i_2 j} + \min_{k_1,k_2 \in \mathcal{J}, k_1 \neq k_2} \left(p^\mu_{jk_1} + p^\mu_{jk_2}\right)\right].$$

Therefore, the cost of the edge $\langle i_1, i_2\rangle \in \mathcal{E}_\ell$ is taken as

$$c^\ell_{i_1 i_2} = \min_{j \in \mathcal{J}_c} \left[p^\mu_{i_1 j} + p^\nu_{i_2 j} + \min_{k_1,k_2 \in \mathcal{J}, k_1 \neq k_2} \left(p^\mu_{jk_1} + p^\mu_{jk_2}\right)\right]. \tag{232}$$

It represents the minimum possible fuel expenditure if the satellites s_μ and s_ν engage in a CE-P2P maneuver.

We are interested in a subset $\mathcal{M}_\ell$ of $\mathcal{E}_\ell$ with $|\mathcal{J}_{d,0}|$ edges, such that no two edges share the same node. This ensures that a satellite can be assigned to only one CE-P2P maneuver. Let us associate with each edge $\langle i, j\rangle \in \mathcal{E}_\ell$ the binary variable x_{ij} given by

$$x_{ij} = \begin{cases} 1, & \text{if } x_{ij} \in \mathcal{M}_\ell, \\ 0, & \text{otherwise.} \end{cases} \tag{233}$$

We now define the following optimization problem on $\mathcal{G}_\ell$*:

$$\text{(CE-P2P-LB): } \min \sum_{\langle i,j\rangle \in \mathcal{E}_\ell} c^\ell_{ij} x_{ij}, \tag{234}$$

subject to

$$\sum_{j:\langle i,j\rangle \in \mathcal{E}_\ell} x_{ij} \leq 1 \text{ for all } i \in \mathcal{J}_{s,0}, \tag{235}$$

*CE-P2P-LB stands for CE-P2P - Lower Bound

$$\sum_{i:\langle i,j\rangle \in \mathcal{E}_\ell} x_{ij} = 1 \text{ for all } j \in \mathcal{J}_{d,0}. \tag{236}$$

The constraint (235) implies that each fuel-sufficient satellite can be assigned to, at most, one fuel-deficient satellite, while the constraint (236) implies that each fuel-deficient satellite has to be assigned to a fuel-sufficient satellite. Let the optimal solution to the problem (CE-P2P-LB) be $\mathcal{M}_\ell^*$ and the optimal value of the objective given in (234) be denoted by $\mathcal{C}_{\text{LB}}$. We then have

$$\mathcal{C}_{\text{LB}} = \sum_{\langle i,j\rangle \in \mathcal{M}_\ell^*} c_{ij}^\ell. \tag{237}$$

We now state the following theorem.

Theorem 2. *The total fuel expenditure $\mathcal{C}(\mathcal{M}_{ce}^*)$ corresponding to the optimal CE-P2P solution $\mathcal{M}_{ce}^*$ is bounded below by the optimal value $\mathcal{C}_{\text{LB}}$ of the objective function in the bipartite assignment problem (CE-P2P-LB). Moreover, $\mathcal{C}(\mathcal{M}_{ce}^*)$ is bounded above by the optimal fuel expenditure $\mathcal{C}(\mathcal{M}_e^*)$ obtained via E-P2P refueling or $\mathcal{C}(\mathcal{M}_c^*)$ obtained via C-P2P refueling, whichever is smaller. Therefore, $\mathcal{C}_{\text{LB}} \leq \mathcal{C}(\mathcal{M}_{ce}^*) \leq \min\{\mathcal{C}(\mathcal{M}_e^*), \mathcal{C}(\mathcal{M}_c^*)\}$.*

Proof. The optimal CE-P2P solution $\mathcal{M}_{ce}^*$ consists of $|\mathcal{J}_{d,0}|$ assignments. For an assignment given by $(i_1, i_2, j, k_1, k_2) \in \mathcal{M}_{ce}^*$, the satellites $s_\mu = \sigma_0(\phi_{i_1})$ and $s_\nu = \sigma_0(\phi_{i_2})$ represent the fuel-sufficient and fuel-deficient satellites respectively. Since $\mathcal{M}_{ce}^* \subseteq \mathcal{P}$, s_μ and s_ν can engage in a feasible CE-P2P maneuver, which implies that the edge $\langle i_1, i_2\rangle$ exists in $\mathcal{G}_\ell$. We therefore define the mapping $\mathcal{Q} : \mathcal{P} \mapsto \mathcal{E}_\ell$ that gives an edge in $\mathcal{E}_\ell$ for every assignment in $\mathcal{P}$. For instance, $\mathcal{Q}\,(i_1, i_2, j, k_1, k_2) = \langle i_1, i_2\rangle$. Note that the CE-P2P solution $\mathcal{M}_{ce}^*$ corresponds to $|\mathcal{J}_{d,0}|$ distinct fuel-sufficient and all $|\mathcal{J}_{d,0}|$ fuel-deficient satellites involved in refueling transactions (refer to (221) and (222)). Let us now consider the following assignment in $\mathcal{G}_\ell$: $x_{qr} = 1$ for all $\langle q, r\rangle \in \mathcal{Q}(\mathcal{M}^*)$ and 0 otherwise. For all the $|\mathcal{J}_{d,0}|$ fuel-sufficient satellites included in CE-P2P solution $\mathcal{M}_{ce}^*$, we have

$$\sum_{r:\langle q,r\rangle \in \mathcal{E}_\ell} x_{qr} = 1,$$

whereas for the remaining $|\mathcal{J}_{s,0}| - |\mathcal{J}_{d,0}|$ fuel-sufficient satellites not included in any refueling transaction, we have

$$\sum_{r:\langle q,r\rangle \in \mathcal{E}_\ell} x_{qr} = 0.$$

Combining the above two equations, we have

$$\sum_{r:\langle q,r\rangle \in \mathcal{E}_\ell} x_{qr} \leq 1 \text{ for all } q \in \mathcal{J}_{s,0}.$$

All the fuel-deficient satellites are included in the CE-P2P solution and each of them engages in a refueling transaction with a distinct fuel-sufficient satellite (refer to (221),(222), and (230)). We therefore have,

$$\sum_{q:\langle q,r\rangle \in \mathcal{E}_\ell} x_{qr} = 1 \text{ for all } r \in \mathcal{J}_{d,0}.$$

Hence, the optimal CE-P2P solution $\mathcal{M}_{ce}^*$ corresponds to a feasible solution $\mathcal{Q}(\mathcal{M}_{ce}^*)$ for the optimization problem (CE-P2P-LB). Hence, we have

$$\sum_{\langle q,r\rangle \in \mathcal{Q}(\mathcal{M}_{ce}^*)} c_{qr}^\ell \geq \sum_{\langle q,r\rangle \in \mathcal{M}_\ell^*} c_{qr}^\ell. \tag{238}$$

Now, let us consider the fuel expenditure $\mathcal{C}(\mathcal{M}_{ce}^*)$. We have

$$\begin{aligned}
\mathcal{C}(\mathcal{M}_{ce}^*) &= \sum_{(i_1,i_2,j,k_1,k_2)\in\mathcal{M}_{ce}^*} p_{i_1 j}^\mu + p_{i_2 j}^\nu + \left(p_{jk_1}^\mu + p_{jk_2}^\mu\right) \\
&\geq \sum_{\{i_1,i_2,j\}:(i_1,i_2,j,k_1,k_2)\in\mathcal{M}_{ce}^*} \left[p_{i_1 j}^\mu + p_{i_2 j}^\nu + \min_{k_1,k_2\in\mathcal{J},k_1\neq k_2} \left(p_{jk_1}^\mu + p_{jk_2}^\mu\right)\right] \\
&\geq \sum_{\{i_1,i_2\}:(i_1,i_2,j,k_1,k_2)\in\mathcal{M}_{ce}^*} \left[\min_{j\in\mathcal{J}_c}\left(p_{i_1 j}^\mu + p_{i_2 j}^\nu + \min_{k_1,k_2\in\mathcal{J},k_1\neq k_2} \left(p_{jk_1}^\mu + p_{jk_2}^\mu\right)\right)\right].
\end{aligned} \tag{239}$$

Using (232), we have from (239),

$$\mathcal{C}(\mathcal{M}_{ce}^*) \geq \sum_{\langle q,r\rangle \in \mathcal{Q}(\mathcal{M}_{ce}^*)} c_{qr}^\ell. \tag{240}$$

Finally, comparing Eq. (238) and Eq. (240), we have

$$\mathcal{C}(\mathcal{M}_{ce}) \geq \mathcal{C}_{\text{LB}}. \tag{241}$$

For the upper bound, recall that $\mathcal{P}_c \subseteq \mathcal{P}$ and $\mathcal{P}_e \subseteq \mathcal{P}$. Therefore, from the definition of $\mathcal{C}(\mathcal{M}_{ce}^*)$, $\mathcal{C}(\mathcal{M}_c^*)$ and $\mathcal{C}(\mathcal{M}_e^*)$, given in (209)-(211), we have

$$\mathcal{C}(\mathcal{M}_{ce}^*) \leq \mathcal{C}(\mathcal{M}_c) \text{ and } \mathcal{C}(\mathcal{M}_{ce}^*) \leq \mathcal{C}(\mathcal{M}_e). \tag{242}$$

The inequalities (241) and (242) give the desired result. □

The fuel expenditure associated with the (CE-P2P) solution, obtained by solving the optimization problem (CE-P2P), is given by $\mathcal{C}(\mathcal{M}_{ce}^{\text{H}})$. Since $\mathcal{M}_{ce}^{\text{H}}$ might be a sub-optimal solution, we have $\mathcal{C}(\mathcal{M}_{ce}^{\text{H}}) \geq \mathcal{C}(\mathcal{M}_{ce}^*)$. Considering the bounds given by Theorem 2, we obtain an estimate of sub-optimality of these results. Specifically, we may define the maximum percentage of sub-optimality of $\mathcal{M}_{ce}^{\text{H}}$ by the following expression

$$\eta_{ce} = \frac{\mathcal{C}(\mathcal{M}_{ce}^{\text{H}}) - \mathcal{C}_{\text{LB}}}{\mathcal{C}_{\text{LB}}} \times 100\%. \tag{243}$$

Note that because the solution of the CE-P2P-LB problem may correspond to an infeasible CE-P2P solution, η is a worst case (conservative) estimate of the suboptimality of $\mathcal{M}_{ce}^{\text{H}}$. However, we can guarantee that the solution is no worse than η, but it could also be better. In fact, there are indeed cases in which the solution of the (CE-P2P-LB) does lead to a feasible solution. In such cases, the solution is globally optimal.

6.4 Summary

In this chapter, we studied a Cooperative Egalitarian P2P (CE-P2P) strategy for refueling satellites in a circular constellation. We have presented a network flow formulation for determining the optimal set of CE-P2P maneuvers in the constellation and we computed a lower bound on the fuel expenditure for the optimal set of CE-P2P maneuvers. The bound is determined by solving a bipartite assignment problem, the solution of which may or may not correspond to a feasible CE-P2P solution. In case it does, we have a globally optimal CE-P2P solution. Otherwise, the bound helps in providing an estimate of the sub-optimality of the CE-P2P solution obtained by our proposed methodology. The CE-P2P strategy is found to be a better refueling strategy compared to either a (non-cooperative) Egalitarian P2P (E-P2P) strategy or a (non-Egalitarian) Cooperative P2P strategy (C-P2P). In fact, the CE-P2P strategy allows for the benefits of both Egalitarian P2P refueling and Cooperative P2P refueling. On one hand, active satellites can perform smaller-ΔV (and hence lower fuel expenditure) orbital transfers since they are allowed to return to any available orbital slot. On the other hand, the CE-P2P strategy reduces the fuel expenditure by allowing satellites to engage in cooperative rendezvous. This is particularly advantageous when the fuel-deficient satellite does not have enough fuel to initiate a non-cooperative rendezvous.

CHAPTER VII

CONCLUSIONS AND FUTURE WORK

7.1 Conclusions

Refueling is one of the important operations of on-orbit servicing of space system. This dissertation focusses on the problem of determining the optimal refueling strategy for a system of multiple satellites in a circular constellation. The primary aim of the dissertation is to answer the following question: Given a service vehicle and a certain number of satellites having insufficient amount of fuel, what is the "best way" of planning a refueling mission? Typically, a refueling mission would comprise of several orbital transfers taking place, each of which would consume fuel. Therefore, by "best way" of planning a refueling mission, we mean that we wish to expend the minimum amount of fuel during all orbital maneuvers required for the mission.

We assume that the service vehicle and satellites employ a chemical propulsion system, so that the maneuvers are impulsive in nature. Specifically, we consider that the transfers are time-fixed two-impulse rendezvous. Hence, the optimal trajectory for a single orbital transfer can be obtained by considering the multi-revolution solutions to the Lambert's problem. We studied the problem of rendezvous between two satellites in different circular orbits, by allowing the satellites to engage in a cooperative rendezvous in a different circular orbit. It is found that if time is sufficient for a non-cooperative Hohmann transfer between the satellites, the optimal rendezvous non-cooperative. However, if the time is not sufficient for a non-cooperative Hohmann transfer, but is sufficient for a phase-free Hohmann transfer between the circular orbits, then we find that the optimal solution is a Hohmann-Phasing Cooperative Maneuver. In such a case, one of the satellites perform a Hohmann transfer to the orbit of the other satellite, and the latter satellite just performs a Phasing Maneuver to complete the rendezvous.

The determination of the optimal set of orbital transfers for a complete refueling mission presents a large-scale optimization problem. The conventional notion of refueling is to have a service vehicle visit all fuel-deficient satellites in an optimal sequence. In the strategy, known as the single-service vehicle (SSV) refueling strategy, one needs to solve an integer program to determine the optimal time for all the transfers. An alternative scenario for refueling satellites is the Peer-to-Peer (P2P) refueling strategy, which is a redistribution of fuel within the constellation without the aid of an external service vehicle. During a P2P maneuver, a fuel-sufficient satellite and a fuel-deficient satellite engages in a fuel exchange, after one of them (active) performs the orbital transfer to rendezvous with the other (passive). The active satellite returns to its original position after the refueling process. The optimal set of P2P maneuvers can be determined by solving a bipartite assignment problem. It is observed that typically in the optimal set of P2P maneuvers, the fuel-deficient satellites are active, because they have the lighter mass, and thereby likely to expend less fuel during the maneuvers. However, if the fuel-deficient satellites do not have sufficient fuel to be active, then the fuel-deficient satellites are active.

The P2P comes as a natural choice during the second phase of a mixed refueling strategy, in which the service vehicle delivers fuel to some of the satellites (perhaps, half) in the constellation, and these satellite refueled by the service vehicle engage in P2P maneuvers with the remaining satellites in order to distribute the fuel among them. In terms of the fuel expended during a refueling mission, a mixed refueling strategy is better than a single service vehicle strategy, particularly with increasing number of satellites in the constellation, and/or with decreasing time for the refueling mission. However, the mixed refueling strategy can be improved further. A Coasting Time Allocation algorithm has been implemented to determine the optimal time sharing between the forward and return trips in the P2P maneuver that needs to be completed within a given time. The notion of Asynchronous P2P (A-P2P) maneuvers identifies that in the mixed refueling strategy, a satellite refueled by the service vehicle can start a P2P maneuver immediately after it has been refueled by the service vehicle. The introduction of the CTA strategy and the notion of A-P2P maneuvers substantially decrease the amount of fuel expended during a mixed strategy. One question that arises at this point is: Can we further improve the P2P phase of the mixed refueling strategy?

One extension of the P2P problem is the Egalitarian P2P (E-P2P) strategy, in which we allow the active satellites to interchange their orbital positions during their return trips. The E-P2P problem can be formulated as a three-index assignment problem in an undirected constellation graph. Alternatively, a network flow formulation can be used to solve for the E-P2P maneuvers. However, both methodologies yield sub-optimal solutions. Lower bound on the optimal fuel expenditure incurred during E-P2P refueling is derived, in order to obtain a measure of the sub-optimality of the solutions. However, the lower bound may or may not correspond to a feasible E-P2P solution. When it does correspond to a feasible E-P2P solution, the bound is tight and represents the global optimal E-P2P solution. The E-P2P strategy yields significantly less fuel expenditure, compared to the P2P strategy. This is because all active satellites perform low-ΔV maneuvers in order during their return trips, thus saving a substantial amount of fuel.

Another extension of the P2P problem is the Cooperative P2P (C-P2P) strategy, in which we allow both satellites to be active. The formulation of the C-P2P strategy is similar to the baseline P2P strategy, except for additional constraints that need to be accounted for. Cooperative maneuvers are particularly beneficial when the fuel-deficient satellites have too low fuel to be active. This is particularly important in the case of the refueling problem, because a refueling mission would be performed at end of lifetime of fuel of the satellites, and it is likely that the fuel-deficient satellites would have very low fuel content.

The fact that both E-P2P and C-P2P refueling strategies are better provides the motivation for combining these two ideas into one single strategy, referred to as the Cooperative Egalitarian P2P (CE-P2P) strategy. A network flow formulation of the problem can be used to determine the CE-P2P maneuvers in the constellation. The formulation yields only sub-optimal solutions, and a lower bound on the fuel expenditure during CE-P2P refueling is used to provide a measure of the sub-optimality of the solution. The CE-P2P strategy is found to be a better refueling strategy compared to either a (non-cooperative) Egalitarian P2P (E-P2P) strategy or a (non-Egalitarian) Cooperative P2P strategy (C-P2P). In fact, the CE-P2P strategy allows for the benefits of both E-P2P refueling and C-P2P refueling. On one hand, active satellites can perform smaller-ΔV (and hence lower fuel expenditure) orbital transfers since they are allowed to return to any available orbital slot. On the other hand, the CE-P2P strategy reduces the fuel expenditure by allowing satellites to engage in cooperative rendezvous. This is particularly advantageous when the fuel-deficient satellite does not have enough fuel to initiate a non-cooperative rendezvous.

7.2 Contributions of the Dissertation

We finally conclude by outlining the primary contributions of this dissertation:

- The problem of achieving fuel equalization in a constellation using P2P maneuvers involves the minimization of two conflicting objectives. A rationale is developed to justify the use of a simple cost function that implicitly takes into account both conflicting objectives, and results in a solution that yields a reasonable compromise between the two objectives.

- The problem of cooperative rendezvous between two satellites in different circular orbits is studied with the assumption that each satellite performs two-impulse transfers, and that the terminal orbit of rendezvous is circular. For the time of rendezvous that prohibits a non-cooperative Hohmann transfer between the satellites, but allows for a phase-free Hohmann transfer between the orbits, we characterized the optimal solution as a Hohmann-Phasing Cooperative Maneuver (HPCM).

- Two cost-reducing measures are incorporated in the mixed refueling strategy. The first of them is the development of a Coasting Time Allocation algorithm that optimally divides total P2P time between the forward and return trips of the maneuver. The second is the introduction of the notion of Asynchronous P2P maneuvers. These measures substantially reduce the fuel expended during a mixed refueling strategy.

- The idea of allowing active satellites to interchange their orbital positions were introduced in the form of an Egalitarian Peer-to-Peer (E-P2P) refueling strategy. Two different methodologies are developed to solve for the E-P2P problem. Both methodologies yield sub-optimal solutions. Lower bound on E-P2P fuel expenditure are derived to provide estimates of sub-optimality of the solutions. Finally, it is demonstrated that the E-P2P strategy provides significantly less fuel expenditure, compared to the baseline P2P strategy.

- The idea of cooperative rendezvous is introduced in the problem of P2P refueling. A formulation is developed to solve for the optimal C-P2P strategy. It is shown that the satellites, engaging in a cooperative rendezvous, share fuel in such a way that the satellite, performing the higher-ΔV transfer during the return trip, ends up with just enough fuel to be sufficient at the completion of the maneuver. It is demonstrated that cooperative P2P maneuvers help in reducing the fuel expenditure during refueling if the fuel-deficient satellites have low amount of fuel and cannot perform a non-cooperative P2P maneuver. This is a very important result in the context of refueling, because refueling is an operation that would performed near the end-of-life of fuel of satellites. Hence, most satellites are likely to be having a vary low amount of fuel content. In this case, cooperative strategy would be beneficial. It is found that the fuel-deficient satellite moves, by expending all of its fuel, as close as possible to a fuel-sufficient satellite in order to undergo a fuel exchange.

- The ideas of E-P2P and C-P2P strategies are combined into one single refueling strategy, known as the Cooperative Egalitarian P2P (CE-P2P) strategy. A formulation is developed to solve for the optimal CE-P2P solution. The methodology yields only sub-optimal solution. Hence, a lower bound on the fuel expenditure during CE-P2P refueling is derived. It is demonstrated that the CE-P2P strategy helps in further reduction of fuel expenditure during P2P refueling.

Finally, we conclude with the main result of this dissertation: In terms of fuel-expenditure during a refueling mission, we have

$$\text{Lower Bound} \;\leq\; \text{CE-P2P} \;\leq \begin{bmatrix} \text{E-P2P} \\ \text{C-P2P} \end{bmatrix} \leq\; \text{P2P} \tag{244}$$

7.3 Future Work

Several extensions of the current work are described in this section.

7.3.1 Servicing of Multiple Satellites

We can also extend the refueling problem to the more general problem of servicing, as illustrated in Figure 62. The inner orbit comprises of n_f fuel units and n_o other Orbital Replacement Units (ORUs). The latter units might be upgraded avionics

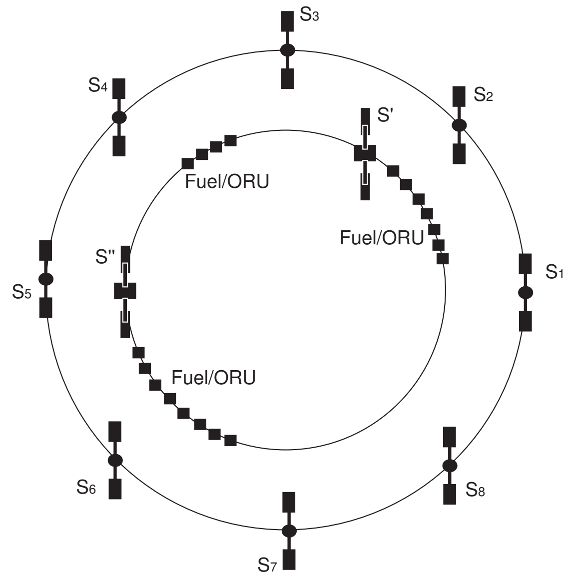


Figure 62: On-Orbit Servicing.

units, or additional payload for all or some of the satellites that need to be serviced. These items need to be delivered to n satellites evenly distributed in the outer orbit. The service vehicle can deliver these units by following a mixed strategy. In such a strategy, the service vehicle delivers the units to part of the satellites and then these satellites distribute the units to the remaining satellites. We assume here that the satellites can hold additional units that need to be delivered to another satellite via a P2P maneuver.

In the mixed strategy we studied, we considered that the service vehicle visits half of the satellites that need to be serviced. However, the optimal number of satellites serviced by the service vehicle might be different, and needs to be investigated. Furthermore, the problem becomes even more interesting when we have more than one service vehicle. The discrete optimization problem in this case becomes even more complex and efficient algorithms need to be developed in order to tackle it.

7.3.2 Low Thrust Servicing

In our study, we have considered that the service vehicle or the satellites employ chemical propulsion system, so that the maneuvers are impulsive in nature. A natural extension of the work is therefore the case when the satellites employ an electric, solar-electric, or ionic propulsion system. In these cases, we have low-thrust maneuvers. The primary benefit of using low-thrust propulsion systems is an efficient usage of propellant. The problem of minimum-fuel, time-fixed low-thrust maneuvers have been studied in the literature, and typically the optimal transfer is determined by solving a non-linear programming problem (NLPs). There have been several studies in the literature that deal with the problem of solving the NLP associated with a low-thrust maneuver.[37,63,67] Generating good guesses for solving the NLPs is a difficult problem, and can be non-intuitive.[67] The biggest challenge in the study of low-thrust servicing missions is that numerous orbital transfer problems need to be solved.

For instance, consider a simple case of 10 satellites in a constellation. A service vehicle visits 5 of these satellites, which engage in low-thrust E-P2P maneuvers with the remaining satellites. There are of course $5 \times 5 = 25$ possible pairings between the satellites. For each pairing, either satellite can be active, and the active satellite can return to any one of $10-1 = 9$ orbital positions. This means there are $25 \times 2 \times 9 = 450$ possible E-P2P maneuvers. Each E-P2P maneuver would comprise of a forward trip and a return trip, and determining the optimal trajectory for each trip would require the solution of a NLP. Hence, a low-thrust E-P2P mission would require the solution of $2 \times 450 = 900$ NLPs. Development of efficient algorithms to tackle such large-scale problem is therefore the prime challenge in solving a low-thrust servicing mission, and presents an interesting research area that needs to be investigated.

7.3.3 Servicing Satellites in Different Planes

In this dissertation, we considered that all the satellites that need to be refueled are on the same plane. However, the studies may be extended to the case of servicing satellites in non-coplanar orbits. In general, plane changes are costly maneuvers, compared to the phasing maneuvers, unless the planes differ by small angle. Hence, if there are many satellites in the same plane, then one may think of dedicating a service vehicle for each plane. If this is not the case, then plane change maneuvers need to be considered. For instance, if each plane has only 3-4 satellites, then we need to consider the fuel expenditure owing to plane changes required for servicing. This can be easily incorporated within the framework developed in this dissertation. Nevertheless, P2P refueling problem incorporating plane change maneuvers need to be studied, and this might be one direction of extending the work of this dissertation.

7.3.4 Servicing Satellites Flying in Formation

We have studied the problem of refueling a system of multiple satellites moving in one or more circular orbits. Of course, the system of multiple satellites we have considered is simple. However, we have seen that even for this simple system, the problem of finding the "best way" of refueling the satellites is challenging, because it involves the solution of a large-scale optimization problem. Extension of the work to consider more complex systems, for instance a system of formation flying spacecraft, seems natural. The dynamics of such a system is described by CW-equations, which are the linearized equations of motion. The optimal transfer from one satellite to another can then be calculated by considering the dynamics of the system as given by the CW-equations.

7.3.5 Optimal Scheduling

In the studies of P2P refueling, we have assumed that there are not any constraint that prohibits a satellite from performing a maneuver. However, in a real-world scenario, there would be restrictions on a satellite from leaving it position. For instance, in case of communication satellites, in order to maintain connectivity in the constellation, two active satellites may not be allowed to perform the orbital maneuvers simultaneously. Hence, two P2P maneuvers, if taking place simultaneously, may violate a connectivity constraint, and thereby lead to a downtime of the constellation. Given a set of P2P maneuvers that need to be executed, the problem of scheduling these maneuvers within a given time, such that the total downtime of the constellation is minimized, is interesting. If there is a sufficiently large time available for a refueling mission, then the maneuvers can take place one by one without any conflict. This would lead to minimum possible downtime in the constellation corresponding to a set of P2P maneuvers. However, given an upper bound on the time, within which all maneuvers need to be scheduled, the problem of minimizing the downtime becomes interesting. Ref. 69 looked at this problem for the baseline P2P strategy. However, the scheduling problem for E-P2P, C-P2P, or the general CE-P2P strategies have not been looked at. Hence, this presents another direction, in which the work can be extended.

7.3.6 Risk Analysis

The risk factor involved in the fuel delivery stage of a servicing mission of a system of multiple satellites has not been analyzed in any work. A single service vehicle refueling mission would involve several maneuvers by the service vehicle and a failure of the service vehicle would lead to the failure of the remaining mission. However, in a mixed refueling strategy, the service vehicle performs less number of manuevers, but there are other satellites performing P2P maneuvers. Because of more number of maneuvers, the chances of a failure occuring might be more. However, if the failure occurs to an active satellite performing a P2P maneuver, it does not affect the remaining mission. Analyzing the risk associated with both refueling missions presents an interesting problem for study.

7.3.7 Stochastic Formulation

The formulations for the various refueling strategies we have discussed are deterministic in nature. A deterministic formulation assumes that we have a good idea of the fuel content of the satellites. But, in general, we may not have an idea of the exact fuel content of the satellites. Particularly, for satellites performing frequent orbital maneuvers as part of mission requirements, the fuel content at a certain time cannot be known exactly in advance, and this causes a hindrance in advanced planning of a refueling mission. Such a situation calls for a stochastic formulation of the problem. At any instance of time t, we need to consider that the fuel content of a satellite follows a probability distribution. With knowledge of such probability distribution, one can only calculate the expected cost of a refueling mission planned for a certain time t. The optimal set of maneuvers can then be determined by minimizing the expected fuel expenditure during a refueling mission. This would lead to solving a stochastic programming problem. This also presents an interesting direction of future work.

7.4 Summary

The dissertation looked at the problem of refueling multiple satellites moving in a circular orbit. The problem is challenging because one has to deal with a large-scale optimization problem in order to decide on the best servicing strategy. In this dissertation, methodologies have been developed in order to determine the optimal set of maneuvers required for such a refueling mission. Several possible extensions of the work have been identified. Overall, it can be concluded that the problem of planning a servicing mission for a system of multiple satellites is a very rich problem, and there are several unexplored areas that remains to be studied.

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