Monitoring Versus Incentives

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Monitoring Versus Incentives

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# SUMMARY

In this thesis, I examine the relationship between principal and agent in a moral-hazard setting where the principal has the ability to monitor the actions of the agent at an interim stage of the project. I show that monitoring can induce the agent to exert higher levels of effort and can result in a reallocation of project payoffs between the two parties. This reallocation is not a one-way street: Situations exist where monitoring encourages greater effort from the agent, resulting in greater project payoffs for both principal and agent. For projects that are characterized as high-risk, high-reward projects where agent involvement is costly, monitoring is often the optimal strategy; this is an explanation for why venture capital type investments are the subject of intense monitoring.

The structure of my model allows the principal to share interim monitoring information with the agent. Thus, the agent can modify effort in later periods conditional on this information. I analyze when it is optimal for the principal to share this information, the impact sharing has on effort levels, the changes in allocation of project payoffs resulting from this sharing and subsequent changes in effort levels, and the impact of this information sharing on risk distribution between the principal and agent.

# CHAPTER 1

## INTRODUCTION TO THESIS

The purpose of this thesis is to examine the effects of monitoring and incentives in a principal-agent relationship with moral hazard. This thesis is comprised of two parts. The first part studies the relationship between principal and agent when monitoring is not implemented or where monitoring is undertaken and the results are shared with the agent when the project is completed. The second part of this thesis extends this analysis to allow the principal to share monitoring results at an interim stage of the project.

Chapter 3 reviews of the literature concerning the principal-agent problem with a focus on the role of monitoring. Due to the common themes of parts 1 and 2, chapter 3 serves as a literature review for both parts.

In chapter 4, I give a general description of the assumptions of the models and an overview of the methodology used to optimize the principal and agent's strategies when monitoring is not implemented or monitoring information is shared at the completion of the project.

The results for the no-monitoring case are considered in chapter 5. The results illustrate situations where the reservation constraint does not bind and the agent extracts rents from the project.[1] Analysis in this chapter will also show that the higher the

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[1] A reservation constraint establishes a baseline level of expected utility that the agent can receive if he decides to forego employment in the principal's project and seek employment elsewhere in the economy. When this constraint binds, the agent extracts no rent from the project. Rent in this context represents the utility in excess of the minimum utility available to the agent through seeking alternative employment in the labor market.

uncertainty of project success, as driven by exogenous factors, the greater the incentive intensity of the contracts negotiated between the two parties.[2]

In chapter 6, the effects of monitoring are considered; in this chapter, communication of monitoring results only occurs at the end of the project. In this case there exists circumstances where monitoring is never the optimal strategy. This occurs in regions where contractual incentives were sufficient to induce the agent to exert effort at the preferred level while receiving expected payoffs that yielded the reservation level. It also occurs in regions where increased project payouts generated from higher levels of agent effort are insufficient to compensate the agent for the cost of this increased effort.

When monitoring does have an impact on the principal-agent relationship, two effects are found to exist. Circumstances exist where monitoring encourages the agent to exert higher levels of effort. There are also circumstances in which payoffs from the project are redistributed between the parties. This redistribution is not a one-way street flowing from the agent to the principal. In situations where the agent is encouraged by monitoring to exert higher levels of effort, there are circumstances where both the principal and agent enjoy the rewards of the extra effort as both parties share in the increased project payouts.

Chapter 9, describes the assumptions of the model and an overview of the methodology used to optimize the principal and agent's strategies when monitoring information can be shared before the completion of the project.

The results for the monitoring case when information can be shared before the completion of the project are considered in chapter 10. These results are considered in

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[2] A contract will be said to have greater incentive intensity than a second contract if an agent receives a higher expected payment from the first contract when his effort level produces the highest expected payoffs for the principal.

terms of the general question as to when it would be advantageous for the principal to agree to share interim-monitoring information with the agent. Unless monitoring is perfect, the estimate of agent effort generated from monitoring will always produce a noisy signal of actual effort. Since there is some probability that the principal will receive the wrong signal, the agent will require a higher level of incentives to compensate for this risk. When communication is introduced into the model, it allows the agent to decide whether it is worthwhile to invest high effort in the second period contingent on the outcome of the period one monitoring; the results of monitoring will inform the agent whether high effort in the future has any chance of being rewarded.

# PART 1

## MONITORING WITH NO COMMUNICATION

CHAPTER 2

PREFACE: MONITORING WITH NO COMMUNICATION

In order to understand the role that monitoring will play in a principal-agent relationship consider the following general setup.[3] The principal employs an agent to manage a project. The success of the project will be dependent on, among other factors, the effort exerted by the agent in his role of manager of the project. The principal's goal is to induce a certain level of effort from an agent in order to maximize the principal's share of project payoffs. If the principal can directly observe the effort of the agent, no moral-hazard issue exists and the principal can achieve his goal through the negotiation of a contract with the agent. However, if the level of agent effort is not directly observable[4], then the principal has two choices.

First, the principal can negotiate with the agent an *incentive contract* based on the payoff from the project. Incentive contracts are used to align the actions of the agent with the desires of the principal. This type of contract pays higher compensation to the agent when the payoff from the project and other information used to establish the contractual payments are at the principal's preferred levels.[5] Unfortunately, the use of incentive contracts will not generally lead to efficient solutions to the principal-agent problem:

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[3] A general description of principal-agent relationships can be found in Sappington (1991).

[4] If there exist exogenous random events that effect the payoff from the project, which are not directly observable by the principal, then the payoff from the project will be a noisy measure of the agent's actions. A classic example of this kind of exogenous random event would be the effects of weather on the crop yields of a farm managed by a tenant on behalf of a landlord. The level of crops produced by the farm is not only dependent on the work of the tenant but is also effected by the weather, which is outside the control of the tenant. If the landlord cannot perfectly assess the weather's effects on crop yields then the yields will also lead to imperfect estimates of the tenant's effort.

[5] For example, with an incentive contract, a manager in charge of sales for a corporation receives a bonus once some target level of sales is exceeded. A definition of incentive contract in the context of the discrete model investigated in this dissertation can be found in chapter 5.

Efficient in the sense that, if the principal could directly observe the agent's effort, then the principal could negotiate a contract with the agent, and the agent could exert a level of effort, that would lead to project payoffs that would be strictly preferred by both principal and agent.[6]

Second, the principal can negotiate with the agent an incentive contract, based on the payoff of the project, and information received from monitoring the agent's actions. According to the *informativeness principal*, total utility should be increased by including in the construction of the incentive contract, signals of agent effort that improve the estimate of this effort. Since monitoring is costly, this may not be the case; the cost of monitoring may out weigh the additional value created by the improved precision of measurement.

The use of monitoring and incentive contracts can reduce the costs associated with moral hazard in the principal-agent relationship. A number of factors including the following will determine the exact mix of monitoring and incentive intensity:[7] The cost and effectiveness of monitoring technologies; the cost of the agent's effort; the impact of the agent's effort on the project's payout, and the impact of exogenous events on the outcome of the project. My research examines the importance of each of these factors on the principal's choice on the level of incentive and monitoring intensity.

When a principal monitors an agent's effort, monitoring can lead to either the redistribution of wealth, changes in the agent's effort levels, or liquidation of the project [see Barry et al., (1990) and Gorman and Sahlman (1989) for a discussion of the role of

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[6] I will demonstrate the existence of inefficient solutions in chapter 5.
[7] Within the context of the model developed in my research, a contract will have a greater incentive intensity than a second contract if an agent receives a higher expected payment from the first contract when his effort level produces the highest expected payoffs for the principal.

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monitoring in venture capital investments].[8] Since the principal will only implement

monitoring if it is in his best interests, a question remains as to whether the agent is

indifferent or adverse to the use of monitoring by the principal. For example, Barney et

al., (1996) and Fried and Hirsch (1994), have shown that entrepreneurs will sometimes

value the monitoring role and non-financial advice supplied by venture capitalists.[9] My

research will show that monitoring will never induce the agent to reduce effort levels,

however, when monitoring induces the agent to increase effort, in some circumstances,

dependent on the parameters of the project, some of the increased payoff from the project

is allocated to the agent. In these cases, even after accounting for the cost of the

additional effort, the agent will improve his share of wealth and extract rents from the

project.[10] This result is reliant on the non-binding of the *reservation constraint*.

Prior research of the principal-agent relationship usually introduces a *reservation*

*constraint* assumption [See Grossman and Hart (1983), Holmstrom (1979), or Ross

(1973)]. A reservation constraint establishes a baseline level of expected utility that the

agent can receive if he decides to forego the offered contract and seek employment

elsewhere in the economy. When this constraint binds,[11] the agent extracts no rent from

the project. This leads to considerable restrictions on the benefits of monitoring since, if

the agent does not extract rent, monitoring cannot reduce the expected utility of the agent,

and thus, monitoring can only change the actions of the agent. The introduction of limited

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[8] Harris and Raviv (1979) show that monitoring is never an optimal strategy when the agent is risk neutral. The results herein demonstrate that this result no longer holds when limited liability is introduced into the model.

[9] Of course, venture capitalists will often replace CEOs of new ventures should results of monitoring suggest this is the optimal strategy. It is unlikely that the entrepreneur would appreciate this form of involvement [Lerner (1995)].

[10] Rent in this context represents the utility in excess of the minimum utility available to the agent through seeking alternative employment in the labor market.

[11] In some prior research the reservation constraint does not bind, see for example Grossman and Hart (1983).

liability in the models developed herein lead to outcomes where the reservation constraint does not bind and thus, redistribution of wealth is a possibility.

The two-period model introduced in this paper allows the principal to monitor during the first period, receiving the results of this monitoring before the second period begins. In the following research, monitoring provides a signal of effort, which the principal uses as a factor to compensate the agent. However, the principal could use the results of monitoring as a basis for liquidating the project if assessments of future success deem this an optimal strategy.[12] In the models developed in my research, liquidation will not be allowed as a strategy for the principal since, given a sufficiently high level of liquidation receipts, the payoff from any project can be improved by liquidation. Thus, any model introducing monitoring and liquidation is acutely dependent on the level of liquidation proceeds, which can be set to dominate any of the other models investigated. In addition, allowing for liquidation complicates the results of the model while obscuring the intricacies of the effects of monitoring. Finally, there are sufficient results from the models without liquidation to contribute to the investigation of the relationship between principal and agent.

The designs of the models presented in my research make several unique contributions to the extant literature. First, it allows for the construction of contracts that can be analyzed directly rather than resorting to comparative static analysis.[13] These constructions use a simplified model to tackle the issues studied by Demougin and Fluet

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[12] The liquidation strategy would be optimal if the liquidation proceeds exceeded the expected payoffs from continuation of the project. Analysis of the use of monitoring to make decisions on abandoning projects in venture-capital investments can be found in Gompers (1995).

[13] When solving the maximization problem, it is common for first-order conditions not to yield explicit solutions to the principal-agent problem. To study the impact of model parameters on the implicit solutions it then becomes necessary to resort to comparative-static techniques. My models produce explicit contractual solutions, which allow for direct investigation.

(2001). Whereas Demougin and Fluet (2001) use comparative static techniques to consider the optimal mix of incentives and monitoring, the solutions in chapters 5 and 6, describe contracts directly, and considers the impact of model parameters on the principal-agent relationship through their effects on these contracts. Specifically, these structures allow for characterization of optimal effort levels for different regions of model parameters. In addition, the ranges of parameters that lead to the optimality of monitoring strategies can be directly established. Finally, this analysis establishes the range of parameters where monitoring is never an optimal strategy.

Second, it allows for solution using a simplex algorithm applied in an algebraic rather than a numerical setting. This approach sidesteps the issues concerning the applicability of a first-order approach by applying the value maximization principal directly to the principal-agent problem [see Grossman and Hart (1983), Jewitt (1988), Mirrlees (1999), and Rogerson (1985) for discussion of the situations where the first-order approach can be used in the analysis of principal-agent relationships].

Third and finally, it allows the principal to share the signals of first-period effort generated by monitoring with the agent, at the end of the first period. The implications of the last point, sharing of monitoring results at an interim stage on the relationship between principal and agent, has not been previously modeled and thus, brings originality to my research. When the principal shares monitoring results at an interim stage, the following questions can be investigated: Is it beneficial for the principal to share the results of monitoring or is it best to keep the agent in the dark? And, if the sharing of monitoring results is possible and desirable from the principal's point of view, what

strategies will the agent employ?[14] My research in part 2 finds that it can be beneficial for the principal to share this interim monitoring information with the agent since it will shift monitoring risk from the agent to the principal.[15] This dimension to my analysis will be the focus of part 2.

Since my research explicitly considers the relationship between incentive contracts and monitoring it is useful to consider the results in the light of actual principal-agent relationships that have been the focus of empirical research. The relationship between venture capitalist and entrepreneur is of particular interest, since the venture capitalist is faced with projects with a significant likelihood of failure,[16] and seeks to control his investments with incentive contracts and monitoring activities.[17] Entrepreneurs who accept venture capital typically have contracts that have high *incentive components* whose payout is dependent on project success. Furthermore, the venture capitalist actively monitors the project and the entrepreneur's success is strongly linked to the success of the project and monitoring results [Sahlman (1990)]. The analysis of monitoring in chapter 6 demonstrates precisely these situations: Monitoring activity by venture capitalists can lead to greater entrepreneurial effort and increase the expected payoff to the entrepreneur. In some cases, the involvement of the principal through monitoring activity increases *incentive intensity* and improves the expected payoffs from the project, and increases the expected payoffs for the entrepreneur even after accounting for the cost of the additional effort.

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[14] In the context of this research, the agent's strategic decision is what level of effort to devote to the project.
[15] Monitoring risk refers to the principal receiving an erroneous signal of agent effort.
[16] Shalman 1990 finds the probability of failure in these relationships to be 34.5%.
[17] The venture capitalists will also stage capital infusions but, as noted earlier, this aspect is outside the scope of the research presented in this paper.

This part of my dissertation is organized as follows. Chapter 3 reviews of the literature concerning the principal-agent problem with a focus on the role of monitoring. In chapter 4, I give a general description of the assumptions of the models and an overview of the methodology used to optimize the principal and agent's strategies. The results for the no-monitoring case are considered in chapter 5. In chapter 6, the effects of monitoring are considered; in this chapter, communication of monitoring results only occurs at the end of the project. Chapter 7 concludes this part.

# CHAPTER 3

# LITERATURE REVIEW

Analysis of the principal-agent relationship finds its roots in the works of Cheung (1969) and Arrow (1970). Cheung succinctly describes the principal-agent problem noting that risk can be reduced either by information collection, diversified through adequate portfolio selection, or shared through contractual means. Much of the early work considered the relationship between manager and worker. For example, Alchian and Demsetz (1972) consider the roll played by the firm in facilitating monitoring to avoid employee shirking. Their results suggest that firms may exist to allow for the monitoring of labor supplied by team members who would individually benefit from shirking. Stiglitz (1975) considered the principal-agent problem in the context of a worker-supervisor relationship within the setting of a firm. Using assumptions regarding the form of the optimal contracts, Stgilitz demonstrates the impact of information, monitoring cost and worker risk aversion on the structure of these contracts.

Investigation of the relationship between external investor and manager within the principal-agent framework was studied by Jensen and Meckling (1975). Jensen and Meckling (1976) also considered the impact of agency costs in the firm and consider the role of contracting in reducing this cost.

Ross (1973) elucidated the principal-agent problem in a general setting and considered contractual forms that induced the same attitude toward risk for the principal and agent thus allowing for contractual solutions to the principal-agent problem. Ross represents one of the earliest works investigating the principal-agent relationship, an

outline of the setup is described below since it introduces and describes the assumptions that are used in my model.

In the situation where the payoff from a project x is a random variable, and the effort exerted by the agent cannot be observed, the single-period principal-agent model with no monitoring can be programmed as follows:

$$\max_{s(x)} E\{V(x - s(x))\} \quad \text{subject to}$$

$$E\{U(a, s(x))\} \geq E\{U(a^*, s(x))\} \quad \text{for all } a^* \in A, \quad \text{and}$$

$$E\{U(a, s(x))\} \geq R \quad \text{for some real number R}$$

In this formulation s(x) represents the contract agreed between principal and agent. The contract between principal and agent can only depend on attributes that can be observed by both principal and agent and is negotiated before commencement of the project. In this program, the only observable attribute is the payoff from the project. In later chapters of my dissertation, the principal will have the ability to spend resources on monitoring and use the signals as to the effort levels of the agent to enhance the contract.

The choice of action of the agent is selected form some set of possible actions A. Once the contract is negotiated between principal and agent the agent chooses an action $a \in A$ so as to maximize his expected utility. In general, the chosen effort level is not observable by the principal, although the principal can choose to spend resources to monitor the agent and obtain an estimate of the actual effort exerted; these situations will be considered in chapter 6 of this dissertation.

The principal and agent are both presumed to act to maximize expected utility. The utility of the principal is represented by the function V. The principal receives the

profit from the project after making the contractual payment s(x) to the agent. In later chapters of my research, any resources devoted to monitoring the actions of the agent will also decrease the expected utility of the principal. The utility of the agent is represented by the function U.

The expectations operator in this program is conditioned on the function f(x, a) representing the probability distribution of the random payoff x. As can be seen from its arguments, the payoff from the project is affected by the action a taken by the agent. To apply certain methods to the solution of program various assumptions are made as to the form of the probability distribution. From an economic standpoint, it would seem reasonable that increased agent effort should improve the payoff of the project in some way. At a basic level for a* > a, it is often assumed that $E_{a*}\{x\} > E_a\{x\}$ where the subscript refers to the effort level for the probability distribution function. Thus, greater effort leads to a higher expected payoff from the project.

The first constraint in the program is often described as the individual rationality constraint since the response by the agent to a particular contract s(x) and a probability function f(x, a) will be to choose the level of effort that will maximize his expected utility. The second constraint is often described as the participation or reservation constraint and represents the effect of a competitive market for the agent's talents. The agent can obtain the level of utility, R, with certainty in the competitive market.

The three classic papers on monitoring in the principal-agent relationship are Harris and Raviv (1978), Holmstrom (1979), and Shavell (1979). These three papers are referred to in the literature as the basis for other studies on monitoring. Each of these papers describe how the use of imperfect monitoring will increase agent risk.

Harris and Raviv (1978) consider conditions under which imperfect monitoring maybe optimal in the principal-agent relationship and consider the impact of monitoring uncertainty on this relationship. Specifically, they show that imperfect monitoring leads to dichotomous contracts; if the monitoring shows that the agent's actions are acceptable, then the agent is rewarded, if not, the agent is dismissed. My research studies the form of these contracts in more detail while also allowing monitoring to be costly.

Holmstrom (1979) studies the impact of external information received on the effort exerted by the agent and considers situations in which this additional information improves both the principal and agent's position. In this research, information is assumed to be obtained without cost. Holmstrom shows that as long as the information is not random, both principal and agent see an increase in value in their relationship. My research considers costly monitoring and studies situations in which monitoring does not improve the position of the agent.

Shavell (1979) considers the impact of agent's risk aversion on the value of monitoring in the principal-agent relationship. Shavell proves, in a similar fashion to Holmstrom (1979), that as long as the information is not random, both principal and agent see an increase in value in their relationship.

My research in this dissertation extends the work on monitoring described in these three papers. My monitoring technologies are costly; this leads to situations where monitoring is too expensive to implement. I construct actual contractual forms and analyze the impact of the model parameters on incentive and monitoring intensity. I explore the impact of monitoring on the principal and agent and describe when monitoring effects effort, expected payouts, and the sharing of risk. Finally, in part 2, I

allow for sharing of interim monitoring results. This has never before, I believe, been

modeled in the principal-agent relationship.

CHAPTER 4

METHODOLOGY: MONITORING WITH NO COMMUNICATION


A two-period model will be developed to consider the relationship between

principal and agent. Three variants of this model will be considered. In the first, the

principal does not monitor the agent and, therefore, the contract between the two parties

is solely based on the payoff from the project. The second model involves monitoring

agent effort during period one. In this model, the results of monitoring will not be shared

with the agent until the end of the project: The signal of agent effort obtained from

monitoring will be incorporated into the contractual payment from the principal to the

agent. In the third model, which will be studied in part 2, monitoring will be undertaken

during the first period and the results of this monitoring will be shared with the agent at

the end of the first period. This final model allows the agent to modify his actions in

period two based on the monitoring results from period one. Once again, the signal of

agent effort will be incorporated into the contractual payment from the principal to the

agent. Certain structures and parameters are common to all three models and are

described and defined in this chapter.

For all three models, it is assumed that the principal has a fully funded project and

at the commencement of the project and these funds will be sufficient through the end of

the second period. The principal is risk neutral and maximizes his expected returns.[18] The

return to the principal will consist of the gross payoff from the project less the contractual

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[18] The assumption that the principal is risk neutral is a common simplification in research on the principal-agent problem. Risk-neutral principals can be thought of as expected profit maximizers. This treatment can be justified through the use by principals of diversification and syndication to reduce risk. For example, venture capital funds will syndicate their investments and, therefore, reduce the risk in their investment portfolios [Lerner (1994a)].

payment to the agent and any resources spent on monitoring. The principal has unlimited liability.

It is assumed that the agent is also risk neutral. The agent will maximize expected net returns, which are defined as contractual payments received from the principal less the agent's cost of effort. It is assumed that the agent brings wealth to the project in the amount of $\upsilon \geq 0$: Any contract agreed with the principal cannot generate losses for the agent in any state of the world that exceeds $\upsilon$. Thus, the agent has limited liability.[19] The amount $\upsilon$ can be thought of as the agent's investment in the project; the principal in effect requires the agent to invest his own capital in the project.[20] This assumption will lead to incentive contracts being the solution to the principal-agent problem analyzed in this dissertation. The outcome is similar to those generated by the assumption of agent's risk aversion: Risk cannot be completely transferred to the agent. In the case of risk aversion, transfer of risk requires the principal to compensate the agent. In the case of limited liability, complete transfer of risk is not possible since limited liability does not allow the agent to absorb unlimited losses, assuming the agent has insufficient wealth. In addition, limited liability brings a real world dimension to my model: When an agent manages a project, the agent will usually have insufficient capital to finance the project, thus the introduction of a principal is a natural result of the agent's limited liability. It is

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[19] The inclusion of a limited liability constraint is common in research that includes risk neutrality for the agent. See Demougin and Fluet (2001), Dmougin and Garvie (1991), Innes (1989) and Sappington (1981) for implementations of limited liability with risk-neutrality, and see Sappington (1991) for an illustration of why the assumption of limited liability with risk neutrality can generate incentive contracts solutions to the principal-agent problem with moral hazard in the same manner as the assumption of risk aversion.
[20] The assumption of limited liability introduces a wealth effect into the models investigated. The agent's decisions on effort levels and the contracts negotiated between the two parties will be directly impacted by the wealth of the agent. Although this will complicate solutions to models, it will also add another dimension to the analysis of the relationship between principal and agent.

assumed that the agent does earn a return on the capital invested in the project.[21] This

assumption will greatly simplify the analysis of the contractual solutions of the models.[22]

The model evolves over two periods. The two parties agree on a contract at time

zero; also, the agent decides on the level of effort to exert on the project during period

one, and the principal decides on the level of monitoring, if any, to use during this

period.[23] If the principal monitors the agent, the principal can estimate at time one the

level of effort exerted by the agent during period one. If the results of monitoring are

shared with the agent at this intermediate stage, then this leads to an expanded choice of

strategic decisions for the agent since his period-two effort can be conditioned on the

outcome of the principal's monitoring results. The project matures at time two and both

parties receive payments from the project.

At time zero, the agent has to decide whether to accept the negotiated contract.[24]

It is assumed that there is a competitive market for the agent's talents and if he does not

accept the current project, he can receive the equivalent of R at time two in the market.[25]

There is no cost to the agent in the assessment of the project before he agrees to accept

the contract. The agent's strategy is to decide whether to accept the contract, and if he

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[21] The limited liability value, $\upsilon$, can be defined as a "with return" value, so that the capital invested by the agent, say $\zeta$, earns a return of $\rho$ so that $\upsilon = (1 + \rho)\zeta$.

[22] It is straightforward to demonstrate that if both parties are risk neutral, then the moral hazard issue can be resolved by the principal *franchising* the project to the agent. In this solution to the principal-agent problem, the agent pays a fixed fee to the principal at the outcome of the project. Through this franchising mechanism, the agent will bear all of the cost of the effects of moral hazard. This solution does not generally exist when the agent has limited liability since, if a low project payout materializes, the agent may have insufficient resources to make the franchise payment. Although the franchise contract is negotiated ex ante, the payments are made ex post.

[23] If the principal does not monitor the agent's effort during period one, the agent can decide on the level of effort in both periods at time zero since no additional information will be available at time one to change the agent's decision.

[24] Throughout the analysis herein, the assumption used in most research and described in Sappington (1991) will be used: It is assumed, that when the agent is indifferent among strategies, the agent will choose the strategy most preferred by the principal.

[25] This reservation constraint ensures that the expected return of the agent must meet or exceed R plus repayment of the limited liability value, $\upsilon$. The returns in both cases are net of the cost of effort.

does so, the level of effort to devote to the project. Since there are two periods, the agent will decide on effort levels in both periods. In the cases where the principal does not monitor, or does monitor at time one and does not share the signal of effort with the agent, the agent can decide on his effort level in both periods at time zero. In this case, there is no new information generated during the project with which the agent can modify his effort decisions in period two. If the principal monitors during period one and shares the signal of effort with the agent at time one then the agent can decide on the level of effort in period two conditioned on this signal. To simplify the analysis it is assumed that the agent cannot quit the project at the end of period one and seek alternative employment: The contract will contain a penalty that will make alternative employment an unacceptable strategy during period two.[26] This means quasi-rent in period two is always positive and thus, the agent will never quit the project at time one and, therefore, will always take the project to completion.[27] The effort exerted by the agent in each period is costly. It is assumed that the cost measured in time two dollars is given by k for high effort and zero for low effort.

The project's payoff will be dependent on the two periods of agent's effort and a random element. There are three possible payoffs from the project at the end of period two. These are given by $\tau + V$, $V$, and $V - \tau$, where $\tau > 0$. It is assumed that the agent can exert one of two levels of effort in each of the periods, high or low. Thus, there are four two-period effort strategies; high-high, high-low, low-high, or low-low. For the project

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[26] In the analysis that follows, this penalty will not explicitly be introduced as it is assumed that the penalty is sufficient to ensure that the agent will never quit at time one.

[27] Rent is defined as the excess earnings an agent receives from taking a job compared to alternative employment. Quasi-rent is the excess earnings an agent makes by not quitting a job. Therefore, if quasi-rent is positive the agent will not resign. This assumption will not be explicitly included in my model.

payoff purposes, it is assumed that the two choices, high-low or low-high lead to the

same probability distribution for the project payoffs.

This probability distribution is described in the following matrix form:

$$
\begin{array}{c}
\text{HH} \\
\text{HL or LH} \\
\text{LL}
\end{array}
\left(
\begin{array}{ccc}
q & \dfrac{1-q}{2} & \dfrac{1-q}{2} \\[2mm]
\dfrac{1-q}{2} & q & \dfrac{1-q}{2} \\[2mm]
\dfrac{1-q}{2} & \dfrac{1-q}{2} & q
\end{array}
\right)
$$

Where row one represents high-high effort, row two high-low effort or low-high

effort, row three low-low effort; column one payoff $\tau + V$, column two payoff V, and

column three payoff $V - \tau$. It is assumed that q>⅓ which implies that the outcomes with

the highest probabilities are on the diagonal of the payoff matrix. Therefore, higher effort

levels result in a greater probability of project success as measured by project payoff. It is

assumed that both principal and agent are aware of the probability distribution and are

aware of the effect of agent's effort on the payoffs from the project before a contract is

negotiated.[28]

When the agent exerts low effort in both periods the expected return from the

project before cost of effort is considered is given by $V - (3q - 1)\tau/2$. This payoff

increases to V as the agent increases effort from low to high in one of the periods, and

$V + (3q - 1)\tau/2$ as the agent increases his effort to high in both periods. Therefore in both

cases, an increase in effort cost of k yields and increase in project payoffs $(3q - 1)\tau/2$.

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[28] Although the probability distribution is known, the realized value of the project's payoffs will not be discovered until the maturity of the project.

The ratio $(3q - 1)\tau/2k$ will be defined as the *return to effort*. This ratio measures the extra

project payoff generated by inducing the agent to increase his effort in one period.

Within this framework, there are a number of different possibilities for monitoring

and communication. Figure 1 describes the different models.
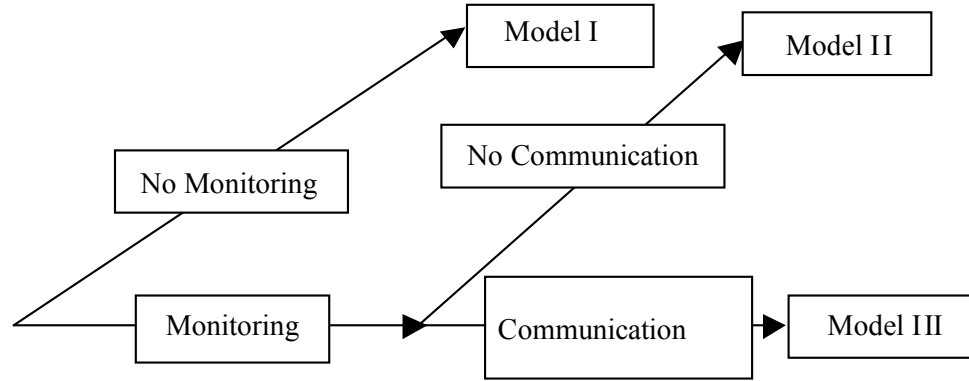


**Figure 1: Overview of Models Investigated**

Model I is the baseline model. In this case, the principal does not monitor. The

contract between the two parties can only be based on the outcome of the project, which

has three possible payouts. Therefore, the contract negotiated between the agent and

principal is of the form $(\sigma_1, \sigma_2, \sigma_3)$. The principal always has the choice to resort to this

format if the expected payoff from model I to the principal dominates expected payoffs

from the other models.

In model II the principal monitors during period one but does not share the

outcome of the monitoring until completion of the project. Since the principal's

monitoring generates a signal of period one effort, this signal is available to supplement

the project payoff information in the contract structure. There is an implicit assumption

that the results of monitoring can be shared with the agent at time two and that the agent

can verify without cost the results of monitoring. This avoids the problem of the principal

falsifying the results of monitoring to his benefit. There is an additional assumption that

the agent cannot change the principal's estimate of effort by supplying information to

him without the principal spending more resources on monitoring that new information.

In this way, the agent cannot supplement the principal's monitoring results without

additional cost being incurred by the principal. The contract will be of the form

$(\sigma_{H1}, \sigma_{H2}, \sigma_{H3}, \sigma_{L1}, \sigma_{L2}, \sigma_{L3})$ where the suffix H and L refers to the estimate of effort

discovered by the principal through the monitoring process during period one. In this

model, because of the lack of communication between the two parties, the agent's

available strategies do not change from the no-monitoring model: The agent will still

have to decide whether to exert high or low effort in the second period without any

additional information.

Model III represents situations in which the principal monitors during period one

and shares the results of the monitoring with the agent at time one. The same assumptions

hold as with model II: The principal can share the monitoring results with the agent

without cost and the agent cannot supplement these monitoring results with additional

information at no cost to the principal. In this case, the agent can modify his second

period effort based on the results of the monitoring. As with model II, the contract will be

of the form $(\sigma_{H1}, \sigma_{H2}, \sigma_{H3}, \sigma_{L1}, \sigma_{L2}, \sigma_{L3})$ where the suffix H and L refers to the estimate

of effort discovered during the monitoring process in the first period. In this model, the

strategies available to the agent have increased in number and sophistication since the

agent can now condition his second-period effort on the outcome of the first-period

monitoring. The question then becomes whether the principal benefits from sharing

information at an interim stage and thus improving the strategic choices of the agent.

In the monitoring models II and III, it is assumed that the principal can monitor

during period one to estimate the agent's effort level in the first period. The amount of

monitoring resources spent will be defined as $m \in [\eta, M)$ where $M > \eta > 0$. The lower

level of monitoring $\eta$ represents a fixed cost of monitoring; this amount must be spent

before any useful signal of effort can be received. The upper level of monitoring, M is the

amount necessary to establish with certainty the true effort level. The probability of

discovering the true effort level will be defined as $p(m)$ where $p(M) = 1 > p(\eta) > \frac{1}{2}$,

$p'(m) > 0$, and $p''(m) \leq 0$.

# CHAPTER 5

## SOLUTIONS WHERE MONITORING IS NOT AVAILABLE

In model I, the principal does not monitor. At time zero, the two parties agree on the contract $(\sigma_1, \sigma_2, \sigma_3)$ where $\sigma_1$ is paid to the agent if the project payoff is $\tau + V$, $\sigma_2$ if the project payoff is $V$, and $\sigma_3$ if the project payoff is $V - \tau$. Due to the limited liability restriction, the construction of the contract must ensure that $\sigma_1, \sigma_2, \sigma_3 \geq 0$. The agent can decide on effort levels for both periods at time zero since the agent does not receive any additional information during either period on which to update his decisions. Therefore, due to the absence of information generated at time one, and since it is assumed the high-low and low-high strategies produce the same probabilities of success, the solutions arising from these two strategies will be identical: To simplify the following analysis, reference to the low-high strategy will be excluded.[29]

To solve this system first, the contracts that will induce the agent to exert a particular level of effort at the minimum of cost to the principal will be established; this step is often described as the *implementation problem*. When the optimal contracts have been established that induce high-high, high-low or low-low effort with the highest expected payoff to the principal, the payoffs across strategies can be compared and the optimal effort levels from the principal's point of view can be found.

---

[29] This simplification will also be applied to the monitoring model where information is not shared at time one since in that case, no additional information will be received by the agent during the project. When monitoring data is shared in model III this argument will no longer be true since the agent can decide on period-two effort at time one and this decision can be dependent on the monitoring information received which is dependent on the effort level during period one.

First, consider the high-high effort strategy. Suppose it is believed that the contract ($\sigma_1$, $\sigma_2$, $\sigma_3$) will induce this action. Given this contract, the expected payout to the agent will be $U_{HH} \equiv q\sigma_1 + (1 - q)(\sigma_2 + \sigma_3)/2 - 2k$.

In order for this contract to compel the agent to exert high effort in both periods, the expected payoff to the agent must be greater than those generated by the other two available agent strategies. Extending the definition above, this will require $U_{HH} > U_{HL}$ and $U_{HH} > U_{LL}$ where:

$$
\begin{pmatrix} q & \dfrac{1-q}{2} & \dfrac{1-q}{2} \\ \dfrac{1-q}{2} & q & \dfrac{1-q}{2} \\ \dfrac{1-q}{2} & \dfrac{1-q}{2} & q \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{pmatrix} - \begin{pmatrix} 2k \\ k \\ 0 \end{pmatrix} = \begin{pmatrix} q\sigma_1 + \dfrac{1-q}{2}(\sigma_2 + \sigma_3) - 2k \\ q\sigma_2 + \dfrac{1-q}{2}(\sigma_1 + \sigma_3) - k \\ q\sigma_3 + \dfrac{1-q}{2}(\sigma_1 + \sigma_2) \end{pmatrix} \equiv \begin{pmatrix} U_{HH} \\ U_{HL} \\ U_{LL} \end{pmatrix}
$$

Therefore, the following constraints must be satisfied:

$\lambda$cons, $\gamma$1cons, $\gamma$2cons, $\gamma$3cons $\geq 0$;     where

$$
\begin{pmatrix} \lambda\text{cons} \\ \gamma1\text{cons} \\ \gamma2\text{cons} \\ \gamma3\text{cons} \end{pmatrix} \equiv \begin{pmatrix} U_{HH} - R - \upsilon \\ U_{HH} - U_{HH} \\ U_{HH} - U_{HL} \\ U_{HH} - U_{LL} \end{pmatrix} = \begin{pmatrix} q\sigma_1 + \dfrac{1-q}{2}(\sigma_2 + \sigma_3) - 2k - R - \upsilon \\ 0 \\ \dfrac{3q-1}{2}(\sigma_1 - \sigma_2) - k \\ \dfrac{3q-1}{2}(\sigma_1 - \sigma_3) - 2k \end{pmatrix}
$$

The first constraint ensures that the contract allows the agent to make at least the reservation payoff when exerting high effort in both periods and ensures that the wealth invested in the project, $\upsilon$, is repaid. The remaining three constraints ensure that the high-high strategy dominates both high-low and low-low strategies. The constraint $\gamma$1cons is included for completeness. There are also three other constraints generated by the limited liability restriction: $\sigma_1$, $\sigma_2$, $\sigma_3 \geq 0$.

Given high-high effort and the contract $(\sigma_1, \sigma_2, \sigma_3)$, the expected payoff to the principal is defined as $V_{HH} \equiv V + (3q - 1)\tau/2 - q\sigma_1 - (1 - q)\sigma_2/2 - (1 - q)\sigma_3/2$.

To find the optimal contract the following program must be solved:

Maximize $V_{HH}$ over values of $(\sigma_1, \sigma_2, \sigma_3)$, subject to

(A1)    $\lambda$cons, $\gamma$1cons, $\gamma$2cons, $\gamma$3cons $\geq 0$;    and

(A2)    $\sigma_1, \sigma_2, \sigma_3 \geq 0$.

To solve this program a simplex algorithm will be employed. The three steps involved are to establish a starting vertex[30], evaluate conditions for each constraint to be dominant at the starting vertex, and then demonstrate that no connected vertices lead to strictly improving payoffs to the principal. The following results are demonstrated for the high-high strategy. The equivalent results are given without proof for the high-low and low-low strategies since the proofs are essentially identical in methodology.

**Result I:** For the high-high strategy, the following contracts are viable since they satisfy the constraints (A1) through (A2).

$$\left\{\frac{2k + R + \upsilon}{q}, 0, 0\right\} \quad \text{given} \quad 0 < k \leq -\frac{(-1 + 3q)(R + \upsilon)}{2(-1 + q)} \quad \text{and;}$$
$$\left\{\frac{4k}{-1 + 3q}, 0, 0\right\} \quad \text{given} \quad k \geq -\frac{(-1 + 3q)(R + \upsilon)}{2(-1 + q)}$$

**Proof I:** See appendix A.

Thus, a set of potential contracts over an exhaustive list of conditions of the model parameters has been established that compel the agent to exert high effort in both periods and satisfy the limited liability requirement. At the overlap of the two conditions:

$$k = -\frac{(-1 + 3q)(R + \upsilon)}{2(-1 + q)},$$

---

[30] A vertex is a contract where constraints bind and no slack exists in the system in the sense that the contract cannot be adapted to increase expect payoff to the principal without violating any constraints.

The contracts are identical. Furthermore, since the principal's payoff is decreasing in payments to the agent, the optimal contract will arise from the binding of some of the constraints (A1) or (A2); there will be no slack in the system.

Next, whether any improvement can be made to the principal's expected payoff by replacing the above contracts by those generated from connected vertices has to be considered. In this context, a connected vertex is formed by changing one of the constraints at the current vertex by one from the set of constraints (A1) or (A2), restricted to those not already binding. Improvement will be defined as strict improvement; it is possible to find connected vertices that yield the same payoff, but only the initial vertex will be considered as the optimal contract in these cases. This simplification will not change the expected payoffs to either party but will simplify notation and the description of results.

The next result is a general result that will hold throughout the remainder of my research. If a viable contract is found at the binding of the reservation constraint, then no improvement in expected payoffs can be found by adopting a contract defined at a connected vertex.

**Result II:** If a viable contract is found at the binding of the reservation constraint, then no improvement can be made to the expected payoffs to either the principal or the agent by using a contract defined at a connected vertex.

**Proof II:** See appendix A.

The proof for the above result is straightforward and relies on the fact that at the binding of the reservation constraint the expected payoff to each party is independent of changes in the contractual components along this constraint. Therefore, if a viable

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contract occurs at the binding of the reservation constraint, this contract is an actual

contract that satisfies the principal's desire for the strategy employed by the agent.

Hence, when we have

$$0 < k \leq -\frac{(-1+3q)(R+\upsilon)}{2(-1+q)},$$

the contract that induces high-high effort from the agent is given by

$$\left\{ \frac{2k+R+\upsilon}{q}, 0, 0 \right\}.$$

If the reservation constraint does not bind potential improvements in expected

payoffs caused by using contracts defined at connected vertices need to considered.

**Result III:** For the high-high strategy, the following contracts are solutions

satisfying the constraints (A1) and (A2).

$$\left\{ \frac{2k+R+\upsilon}{q}, 0, 0 \right\} \quad \text{given} \quad 0 < k \leq -\frac{(-1+3q)(R+\upsilon)}{2(-1+q)} \quad \text{and;}$$

$$\left\{ \frac{4k}{-1+3q}, 0, 0 \right\} \quad \text{given} \quad k \geq -\frac{(-1+3q)(R+\upsilon)}{2(-1+q)}$$

**Proof III:** See appendix A.

Therefore, the optimal contracts that induce the high-high effort strategy by the

agent are:

$$\left\{ \frac{2k+R+\upsilon}{q}, 0, 0 \right\} \quad \text{given} \quad 0 < k \leq -\frac{(-1+3q)(R+\upsilon)}{2(-1+q)} \quad \text{and;}$$

$$\left\{ \frac{4k}{-1+3q}, 0, 0 \right\} \quad \text{given} \quad k \geq -\frac{(-1+3q)(R+\upsilon)}{2(-1+q)}$$

In a similar fashion, the optimal contracts inducing high-low and low-low effort

are given by:

For high – low effort $\left\{0, \dfrac{k + R + \upsilon}{q}, 0\right\}$ given $0 < k \le -\dfrac{(-1 + 3\,q)\,(R + \upsilon)}{-1 + q}$ and;

$\left\{0, \dfrac{2\,k}{-1 + 3\,q}, 0\right\}$ given $k \ge -\dfrac{(-1 + 3\,q)\,(R + \upsilon)}{-1 + q}$

For low – low effort $\left\{0, 0, \dfrac{R + \upsilon}{q}\right\}$ for all values of k > 0.

The optimal contracts are displayed graphically in figure 2.



High-high 2

High-high 1

High-low 2

High-low 1

Low-low

$0 \qquad -\dfrac{(-1 + 3\,q)\,(R + \upsilon)}{2\,(-1 + q)} \qquad -\dfrac{(-1 + 3\,q)\,(R + \upsilon)}{-1 + q}$
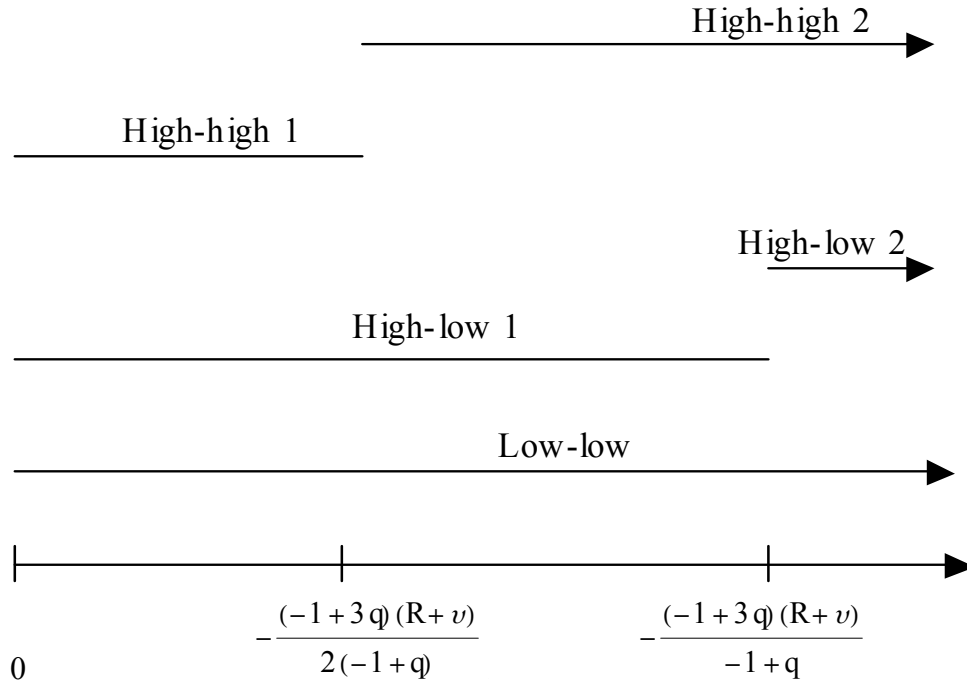
**Figure 2: Overview of Optimal Contracts in the No Monitoring Case**

To understand the overlaps of contracts across strategies, consider for example an agent involved with a project who has a cost of high effort k such that:

$$-\dfrac{(-1 + 3\,q)\,(R + \upsilon)}{2\,(-1 + q)} < k < -\dfrac{(-1 + 3\,q)\,(R + \upsilon)}{-1 + q}$$

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The principal has a choice of inducing high-high, high-low or low-low effort by using contracts:

$$\left\{\frac{4k}{-1+3q}, 0, 0\right\}, \quad \left\{0, \frac{k+R+\upsilon}{q}, 0\right\}, \quad \text{or} \quad \left\{0, 0, \frac{R+\upsilon}{q}\right\}, \quad \text{respectively.}$$

The final solution to the principal-agent problem with no monitoring can be found by answering the question: Which of these contracts leads to the greater expected payoff to the principal?

**Result IV:** The optimal strategies in the no monitoring case are given by the following contracts and conditions.[31]

$$\left\{\frac{2k+R+\upsilon}{q}, 0, 0\right\} \quad \text{given} \quad 0 < k \le -\frac{(-1+3q)(R+\upsilon)}{2(-1+q)} \quad \text{and} \quad \tau \ge \frac{2k}{-1+3q} \quad \text{(N1)}$$

$$\left\{\frac{4k}{-1+3q}, 0, 0\right\} \quad \text{given}$$

$$-\frac{(-1+3q)(R+\upsilon)}{2(-1+q)} \le k \le -\frac{(-1+3q)(R+\upsilon)}{-1+q}$$

$$\text{and} \quad \tau \ge \frac{2(k(1+q)-(-1+3q)(R+\upsilon))}{(1-3q)^2} \quad \text{or}$$

$$k \ge -\frac{(-1+3q)(R+\upsilon)}{-1+q} \quad \text{and} \quad \tau \ge \frac{4kq}{(1-3q)^2} \quad \text{(N2)}$$

$$\left\{0, \frac{k+R+\upsilon}{q}, 0\right\} \quad \text{given} \quad -\frac{(-1+3q)(R+\upsilon)}{2(-1+q)} \le k \le -\frac{(-1+3q)(R+\upsilon)}{-1+q}$$

$$\text{and} \quad \frac{2k}{-1+3q} \le \tau \le \frac{2(k(1+q)-(-1+3q)(R+\upsilon))}{(1-3q)^2} \quad \text{(N3)}$$

$$\left\{0, \frac{2k}{-1+3q}, 0\right\} \quad \text{given} \quad k \ge -\frac{(-1+3q)(R+\upsilon)}{-1+q}$$

$$\text{and} \quad \frac{2(2kq-(-1+3q)(R+\upsilon))}{(1-3q)^2} \le \tau \le \frac{4kq}{(1-3q)^2} \quad \text{(N4)}$$

---

[31] No monitoring contracts are labeled (N1) through (N5). In later chapters, contracts that arise from monitoring but no communication of results will be labeled (M1),… Similarly, contracts that arise from monitoring with communication of results will be labeled (C1),…

$$\left\{0,\, 0,\, \frac{R+v}{q}\right\} \quad \text{given} \quad k>0 \quad \text{and} \quad 0 \le \tau \le \frac{2\,k}{-1+3\,q} \quad \text{or}$$

$$k \ge -\frac{(-1+3\,q)\,(R+v)}{-1+q} \quad \text{and} \quad \frac{2\,k}{-1+3\,q} \le \tau \le \frac{2\,(2\,k\,q-(-1+3\,q)\,(R+v))}{(1-3\,q)^2} \quad \text{(N5)}$$

**Proof IV:** See appendix A.

### 5.1 Analysis of Results when Monitoring is not Implemented

The solutions for the no-monitoring model are displayed graphically in figure 3.
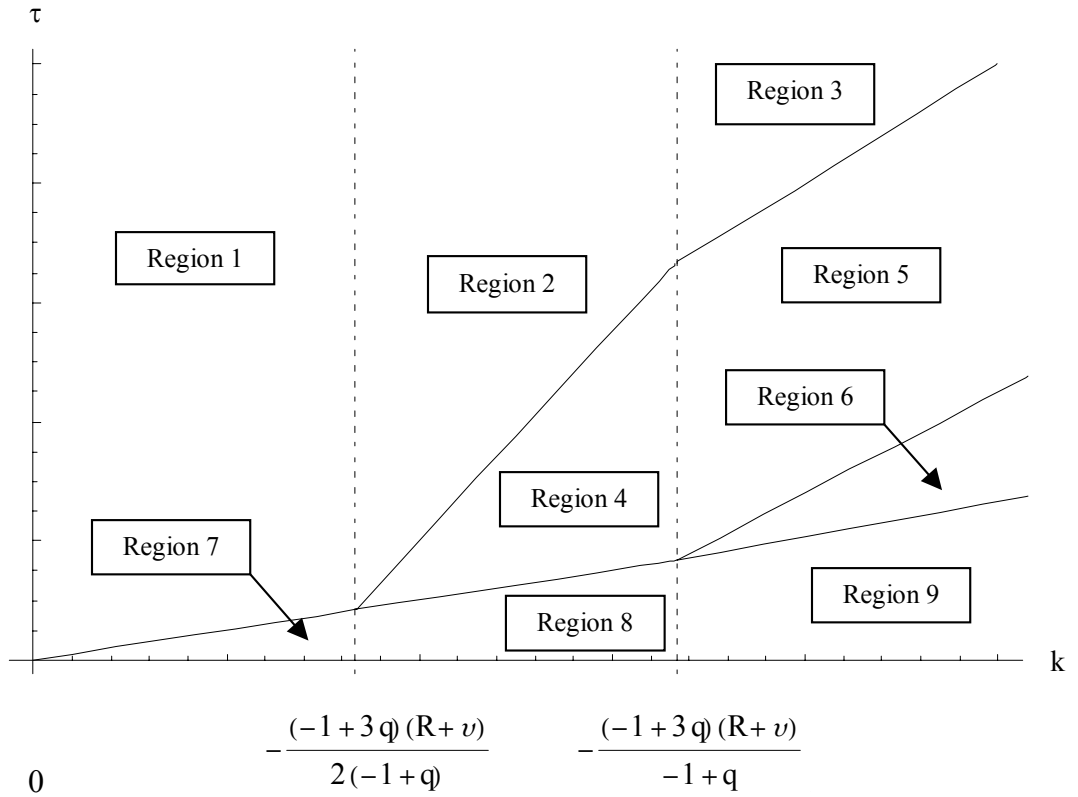


**Figure 3: Graphical Depiction of Regions in the No Monitoring Case**

Both of these gridlines are linearly dependent on the reservation payoff R, and the limited liability value $\upsilon$. In addition, the gridlines are strictly increasing functions of q. The implication of these relationships is described below.

In a descriptive way, the regions in figure 3 can be thought of in terms of high, moderate and low cost of effort, and high, medium and low spread of payoffs. For instance, region 1 describes projects with low cost of high effort and high dispersion. It should be remembered that the categorization of spread and cost of effort into these three groupings is a relative not an absolute measure.

For the no-monitoring case, the *incentive intensity* of the contractual solutions to the principal-agent relationship can be investigated. In general, for two contracts, the first has greater incentive intensity than the second, if increased effort yields a greater increase in payments to the agent from the first contract than from the second contract. Therefore, given two contracts, if the contractual payments for a given level of effort $e_a$, can be described as $C_1(e_a)$ and $C_2(e_a)$, respectively, then for any effort level $e_b > e_a$ contract one will have greater incentive intensity if $C_1(e_b) > C_2(e_b)$.

For the discrete models developed in the no-monitoring case, the contractual solutions have the forms $(C, -\upsilon, -\upsilon)$, $(-\upsilon, C, -\upsilon)$ or $(-\upsilon, -\upsilon, C)$ where $C \geq -\upsilon$. The values C will be defined as the *incentive component* of the contract. If there exists strict inequality, $C > -\upsilon$, then the contract can properly be defined as an *incentive contract*, since the agent will have an incentive to exert effort in the way preferred by the principal. The state in which the incentive component is paid will be defined as the principal's *preferred state*, and the agent's strategy that yields the greatest likelihood of the preferred state occurring will be defined as the agent's *preferred effort level*. In this context, for

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two contracts defined as $(C_1, -\upsilon, -\upsilon)$ and $(C_2, -\upsilon, -\upsilon)$, the first will have greater incentive intensity than the second if $C_1 > C_2$.

**Result V:** For the incentive components of the contractual solutions (N1) through (N5):

(i)     For all contracts, the incentive components are decreasing functions of the probability of success q;

(ii)     for contracts where the reservation constraint binds, the incentive components are increasing functions of the reservation payoff R and the limited liability value $\upsilon$;

(iii)     for all contracts with the exception of (N5), the incentive components are increasing functions of the cost of high effort k; and

(iv)     for contracts where the reservation constraint does not bind, the incentive components are decreasing functions of the limited liability value $\upsilon$.

**Proof V:** See appendix A.

Result V(i) shows that the higher the uncertainty of project success, as driven by exogenous factors, the greater the incentive intensity. Since higher uncertainty reduces the likelihood that the principal rewards the agent for exerting the preferred effort level, the agent has to receive higher payoffs in the preferred states to compensate.

In the cases where the reservation constraint binds, the wealth that the agent brings to the project – the limited liability value $\upsilon$ – allows the principal to coerce the agent into exerting the preferred effort level by taking the amount in the case of unsuccessful project outcomes. Since the agent's expected payoff is at the reservation payment level, the incentive component of these contracts has to compensate the agent

for this appropriation. Result V(ii) states that the incentive intensity of these contracts

increases as the level of limited liability increases. This result stands in contrast to the

cases where the reservation constraint does not bind, since result V(iv) states that the

limited liability value reduces the incentive intensity.

The following result extends result V(iii) across contracts that have the same

preferred effort level.

**Result VI:** For contracts (N1) and (N2), and (N3) and (N4), the incentive

components of the contractual solutions are an increasing function of the cost of high

effort, k.

**Proof VI:** See appendix A.

In regions 1, 4, 6, 7, 8, and 9 in figure 3, the reservation constraint binds,

therefore, the agent extracts no rent from the project. In regions 2, 3, and 5 in figure 3, the

agent extracts rent since the expected payoff earned from the contract is greater than the

reservation payout, R. This situation does not arise due to any scarcity of workers in the

labor market, but is a direct consequence of the desire by the principal for the agent to

exert higher effort than could be induced by lower levels of compensation. In models II

and III, monitoring will be another mechanism available to the principal to ensure the

agent works at the preferred effort level.

Region 1 represents projects where the spread of payoffs from the project is high

and the cost of agent effort is low. This region can be defined in terms of the following

relationships:

$$0 < k \le -\frac{(-1+3q)(R+\upsilon)}{2(-1+q)} \quad \text{and} \quad \tau \ge \frac{2k}{-1+3q}$$

In this region, the optimal strategy for the principal is to induce the agent to exert high effort in the both periods. The incentive contract between the two parties that induces this behavior is of the form:

$$\left\{\frac{2k+R+\upsilon}{q},\ 0,\ 0\right\}$$

With this contract, the agent is only compensated if the project yields a high payoff; in the other two cases, moderate or low payoffs, the agent loses the limited liability value $\upsilon$ – his wealth invested in the project. If the agent exerts high effort in both periods, the expected payoff to the agent from this contract is the reservation value R and repayment of the limited liability value $\upsilon$ and thus, the agent extracts no rents from his employment.

Projects in region 1 can be though of as "caretaker" type employment. The agent is required to be present and needs to exert his effort diligently. If the agent is lax in his role, the projects failure may result and this will have an important impact on the project's payoff. An example of this type of employment would be a security role in an area protecting expensive resources. The work of the personnel does not require extensive training or education and, therefore, the cost of effort is likely to be minor, but its completion is vital to the success of the project.

Since the agent's cost of high effort at its lowest level in region 1, the agent's responsiveness to incentives is high. Thus, the need for aggressive incentives does not arise and, therefore, the incentive intensity in region 1 is low. Furthermore, as the probability of success, q, increases, the slope of the boundary defined by $\tau = 2k/(3q-1)$ decreases and the value of the other boundary of region 1, defined by

k = (3q – 1)(R + υ)/(2(1 – q)) increases. Thus, increasing probability of success leads to lower levels of incentive intensity as region 1 expands its range.

Regions 2 and 3 represent projects where the spread of payoffs from the project is high and the cost of agent effort is moderate and high, respectively. These regions are defined in terms of the following relationships:

$$-\frac{(-1+3\,q)\,(R+\upsilon)}{2\,(-1+q)} \leq k \leq -\frac{(-1+3\,q)\,(R+\upsilon)}{-1+q}$$

$$\text{and}\quad \tau \geq \frac{2\,(k\,(1+q)-(-1+3\,q)\,(R+\upsilon))}{(1-3\,q)^2}\quad \text{for region 2, and}$$

$$k \geq -\frac{(-1+3\,q)\,(R+\upsilon)}{-1+q}\quad \text{and}\quad \tau \geq \frac{4\,k\,q}{(1-3\,q)^2}\quad \text{for region 3.}$$

In these regions, the optimal strategy for the principal is to induce the agent to exert high effort both periods. The contract between the two parties that induces this behavior is of the form:

$$\left\{\frac{4\,k}{-1+3\,q},\, 0,\, 0\right\}$$

With this contract, the principal pays the agent incentive component of the contract if the project yields a high payoff. In the other two cases, the agent loses the limited liability value υ, since this represents the agent's wealth invested in the project. If the agent works at the preferred effort level, his expected payoff from the project will be:

$$\frac{4\,k\,q}{-1+3\,q}$$

In these two regions, the expected payoff to the agent from this contract, net of the cost of effort, is greater than the reservation value R and repayment of the limited liability value υ. Thus, the agent extracts rents from employment in these types of projects and incentive intensity is at its highest.

Projects described as region 2 and 3 type projects can be though of as "entrepreneurial" type employment for the agent. Since for these projects high levels of effort are expensive which could be thought of as paying for the skill and experience of the agent. The employee in this type of position can be thought of as anybody who makes strategic decisions managing projects or businesses. Since participation in these projects is costly to the agent, and since the project payoff is highly dependent on the agent's effort, then the principal needs to offer more than the reservation level of compensation to ensure that the project is given the best chance of success. Due to the high level of payoff dispersion, $\tau$, it is worthwhile for the principal to reward the agent for high effort. Therefore, the incentive intensity is high in these regions.

Region 4 represents projects where the spread of payoffs from the project is moderate and the cost of agent effort is moderate. This region is defined in terms of the following relationships:

$$-\frac{(-1+3\,q)\,(R+\upsilon)}{2\,(-1+q)} \le k \le -\frac{(-1+3\,q)\,(R+\upsilon)}{-1+q}$$

$$\text{and} \quad \frac{2\,k}{-1+3\,q} \le \tau \le \frac{2\,(\,k\,(1+q)-(-1+3\,q)\,(R+\upsilon))}{(1-3\,q)^2}$$

In this region, the optimal strategy for the principal is to induce the agent to exert high effort in the first period and low effort in the second period. The contract between the two parties that induces this behavior is of the form:

$$\left\{0, \frac{k+R+\upsilon}{q}, 0\right\}$$

With this contract, the agent receives the incentive component of the contract if the project yields moderate payoffs. In the other two cases, the agent loses the limited liability value $\upsilon$. Given the optimal strategy of the agent, the expected payoff to the agent

from this contract, net of the cost of effort, is the reservation value R and repayment of

the limited liability value $\upsilon$. Since the cost of effort is moderate in this region, the

incentive intensity of the agent's contract is low.

Region 5 represents projects where the spread of payoffs from the project is

moderate and the cost of agent effort is high. This region can be defined in terms of the

following relationships:

$$k \geq -\frac{(-1+3q)(R+\upsilon)}{-1+q} \quad \text{and} \quad \frac{2(2kq-(-1+3q)(R+\upsilon))}{(1-3q)^2} \leq \tau \leq \frac{4kq}{(1-3q)^2}$$

In this region, the optimal strategy for the principal is to induce the agent to exert

high effort in the first period and low effort in the second period. The contract between

the two parties is of the form:

$$\left\{0, \frac{2k}{-1+3q}, 0\right\}$$

With this contract, the principal will pay the agent the incentive component of the

contract if the project yields moderate payoffs. In the other two cases, the agent loses the

limited liability value $\upsilon$. If the agent works at the preferred effort level, then the expected

payoff to the agent is given by:

$$\frac{2kq}{-1+3q}$$

This expected payoff to the agent from this contract, net of the cost of effort, is

greater than the reservation value R. Thus, the agent extracts rents from employment in

these types of projects.

Projects in regions 4 and 5 can be thought of as managerial/supervisor types of

employment. The moderate spread of project payouts and the relative cost of agent effort

cannot justify the principal to induce the agent to exert high effort in both periods. For

projects in region 5, since the cost of effort is relatively high, the principal has to offer more than the reservation level of compensation.

Regions 6, 7, 8 and 9 represent projects where the spread of payoffs from the project is low and the cost of agent effort is low, moderate and high, respectively. These regions can be defined in terms of the following relationships:

$$k \geq -\frac{(-1+3\,q)\,(R+\upsilon)}{-1+q} \quad \text{and} \quad \frac{2\,k}{-1+3\,q} \leq \tau \leq \frac{2\,(2\,k\,q-(-1+3\,q)\,(R+\upsilon))}{(1-3\,q)^2} \quad \text{for region 6}$$

$$0 < k \leq -\frac{(-1+3\,q)\,(R+\upsilon)}{2\,(-1+q)} \quad \text{and} \quad 0 \leq \tau \leq \frac{2\,k}{-1+3\,q} \quad \text{for region 7}$$

$$-\frac{(-1+3\,q)\,(R+\upsilon)}{2\,(-1+q)} \leq k \leq -\frac{(-1+3\,q)\,(R+\upsilon)}{-1+q} \quad \text{and} \quad 0 \leq \tau \leq \frac{2\,k}{-1+3\,q} \quad \text{for region 8, and}$$

$$k \geq -\frac{(-1+3\,q)\,(R+\upsilon)}{-1+q} \quad \text{and} \quad 0 \leq \tau \leq \frac{2\,k}{-1+3\,q} \quad \text{for region 9}$$

In these regions, the optimal strategy for the principal is to induce low effort from the agent in the first and second periods. The contract between the two parties that induce this behavior is of the form

$$\left\{0,\ 0,\ \frac{R+\upsilon}{q}\right\}$$

With this contract, the agent receives the incentive component of the contract only if low payoffs occur. In the other two cases, the agent loses the limited liability value $\upsilon$. If the agent pursues the preferred effort level, the expected payoff to the agent from this contract is the reservation value R and incentive intensity is at its lowest level.

For these types of project, the cost of effort can be at any level. However, because the dispersion of the project payoffs is low, high effort is not worth paying for at any cost. This type of project can be thought of as a cash-cow type of business: The project pays out regular constant cash flows and work by employees does not make any significant difference to this payout. Examples could include sales and service of a

mature product in a company's product line. This product will eventually be phased out, but due to market saturation, there will be no significant change in cash flow due to worker involvement. Another example would be cleaning services in a fully automated factory. The facility being cleaned diligently will not have a significant impact on the output of the plant.[32]

As described above, each of the contractual solutions in result IV are incentive contracts in the sense that the agent receives the highest payment – the incentive component of the contract – when the outcome of the project is as desired by the principal. This outcome has the greatest chance of occurring when the agent employs the strategy that is optimal from the principal's point of view – the preferred effort level. For the undesired project outcomes, the agent loses the limited liability value. Because of the dichotomous nature of these contracts, the contracts can also be described as efficiency wages: The agent will exert the required effort because the cost of not doing so is prohibitive.

Next, the impact of moral hazard on the relationship between principal and agent can be considered in the context of the no-monitoring case. Suppose no moral-hazard concerns existed in the relationship between principal and agent[33], then the contract between principal and agent could be constructed on two parameters; the outcome of the project, and the actual effort exerted by the agent. Therefore, the contractual solution, in its most general form, will have the following structure,

$\{\sigma_{HH1}, \sigma_{HH2}, \sigma_{HH3}, \sigma_{HL1}, \sigma_{HL2}, \sigma_{HL3}, \sigma_{LL1}, \sigma_{LL2}, \sigma_{LL3}\}$, where $\sigma_{HH1}$ is the contractual

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[32] The maintenance of the assembly line would fall within region 1 since the worker involvement, although not costly to the worker, would be critical to the success of the project.
[33] With no moral hazard, we can assume that the principal can directly observe the level of effort.

payment made to the agent if the project yields the highest payoff and the agent exerts high effort in both periods,… the other contract elements are similarly defined.

Solving the associated maximization problem leads to solutions where the agent exerts high effort in both periods of the project when $\tau \geq 2k/(3q-1)$, and low effort in both periods when $\tau \leq 2k/(3q-1)$. In both cases, the reservation constraint binds and the agent is rewarded with an expected payment, net of cost of effort, of $R + \upsilon$, extracting no rents from the project.

Analysis of these results, and comparisons to the results established above in model I, can be used to understand the cost to the principal and agent of the moral hazard issue. Table 1 shows the change in expected payout, because of the inclusion of moral hazard, to each participant in each of the regions described in the no-monitoring case. The regions in this table are those described in figure 3.

**Table 1: Effects of Moral Hazard**

|  | Change in expected payment to principal | Change in expected payment to agent |
|---|---|---|
| Region 1 | – | – |
| Regions 2 and 3 | $-2(1+q)k/(3q-1)+R+\upsilon$ | $2(1+q)k/(3q-1)-R-\upsilon$ |
| Region 4 | $-(3q-1)\tau/2+k$ | – |
| Region 5 | $-(3q-1)\tau/2+2(2q-1)k/(3q-1)+R+\upsilon$ | $(1-q)k/(3q-1)-R-\upsilon$ |
| Region 6 | $-(3q-1)\tau+2k$ | – |
| Regions 7, 8 and 9 | – | – |

The inclusion of an assumption of moral hazard never results in the principal being strictly better off and, similarly, the agent is never strictly worse off. Therefore, the moral hazard problem works in all cases to the benefit of the agent, allowing him to extract rents from the project in regions 2, 3 and 5. This benefit manifests itself in higher levels of incentive intensity in regions 2, 3 and 5.

Furthermore, the inability of the principal to separate the actions of the agent from the exogenous effects leads to solutions that are not pareto optimal. Consider projects that are described by the interior of region 4. If the agent's effort level could be discovered without cost, a contract could be constructed where the agent is paid the reservation value plus the cost of effort, $R + \upsilon + 2k$, if the agent exerts high effort in both periods, or else the agent loses the limited liability value. In this case, the principal's position will strictly improve whereas the agent's position will remain unchanged. The actual solution in this case is an inefficient allocation in the sense that if the principal could directly observe the agent's effort, then there is a total value maximizing allocation that would be strictly preferred by the principal and agent: Even with the use of incentive contracts efficiency is not achieved.

Certain observations can also be made about the impact of limited liability on the relationship between the principal and agent. Suppose a principal has a project and is faced with a choice between two possible agents. Suppose that the cost of effort for both agents is identical but the first agent has greater personal wealth to invest in the project. The following result shows that the principal never strictly prefers the agent with lower limited liability.

**Result VII:** Given any project, if a principal has a choice between two agents with limited liability values $\upsilon_1 > \upsilon_2 \geq 0$, where the cost of effort for both agents is identical, then the second agent with the lower limited liability is never strictly preferred by the principal.[34]

**Proof VII:** See appendix A.

This result is possible since wealth effects are introduced into the model presented herein through the assumption of limited liability. Agents are differentiated by the varying levels of wealth they can bring to projects through the losses they can endure.

### 5.2 Solution of the Dual Problem

Since the solutions to the principal-agent relationship were found using a linear programming methodology, there exists a dual problem for the original primal problem. The primal problem used to find contractual solutions that induce the entrepreneur to exert high effort in both periods was analyzed above and can be described as:

Max $\mathbf{f.\sigma}$ s.t. $\mathbf{A\sigma} \leq \mathbf{c}$ for $\sigma \in \mathbb{R}^{3+}$ where $\mathbf{f}$, $\sigma$, $\mathbf{A}$ and $\mathbf{c}$ are defined as follows:

$$\mathbf{f} \equiv \begin{pmatrix} -q \\ \frac{1}{2}(-1+q) \\ \frac{1}{2}(-1+q) \end{pmatrix}, \quad \sigma \equiv \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{pmatrix},$$

$$\mathbf{A} \equiv \begin{pmatrix} q & \frac{1-q}{2} & \frac{1-q}{2} \\ \frac{1}{2}(-1+3q) & \frac{1}{2}(1-3q) & 0 \\ \frac{1}{2}(-1+3q) & 0 & \frac{1}{2}(1-3q) \end{pmatrix} \quad \text{and} \quad \mathbf{c} \equiv \begin{pmatrix} 2k+R+\upsilon \\ k \\ 2k \end{pmatrix}$$

The corresponding dual problem is described by:

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[34] Since the reservation constraint requires the agent to receive expected payments at least at the level R plus repayment of the limited liability value $\upsilon$, then the first agent has a tighter reservation constraint.

Min $\mathbf{c}.\boldsymbol{\lambda}$ s.t. $\mathbf{A}^T\boldsymbol{\lambda} \geq \mathbf{f}$ for $\boldsymbol{\lambda} \in \mathbb{R}^{3+}$ where $\boldsymbol{\lambda}$ is defined as:

$$\boldsymbol{\lambda} \equiv \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix}$$

Thus, the dual problem used to find contractual solutions that induce the entrepreneur to exert high effort in both periods is given by:

Minimize $-(2k + R + \upsilon)\lambda_1 - k\lambda_2 - 2k\lambda_3$ over values of $(\lambda_1, \lambda_2, \lambda_3)$, subject to

$$-q\lambda_1 + \frac{1}{2}(1 - 3q)\lambda_2 + \frac{1}{2}(1 - 3q)\lambda_3 \geq -q$$

$$\frac{1}{2}(-1 + q)\lambda_1 + \frac{1}{2}(-1 + 3q)\lambda_2 \geq -\frac{1-q}{2}$$

$$\frac{1}{2}(-1 + q)\lambda_1 + \frac{1}{2}(-1 + 3q)\lambda_3 \geq -\frac{1-q}{2} \qquad \text{and}$$

$\lambda_1, \lambda_2, \lambda_3 \geq 0$.

The solution of dual problem in each case confirms the results of the primal problem analysis and describes shadow prices for the constraints described in the primal problems. In regions 1, 4, 6, 7, 8, and 9 where the reservation constraint was found to bind, the shadow prices for k and $R + \upsilon$ is 1. In other words, market conditions lead to a reduction in the cost of high effort, reservation payment, or level of limited liability then this will lead to a corresponding increase in the expected payoff to the principal.

In the remaining regions, the shadow price for $R + \upsilon$ is always zero. This is not surprising since the reservation constraint does not bind in these regions. In addition, the shadow price for the agent's cost of effort k is:

$$\frac{2q}{-1 + 3q}$$

Thus, a reduction in the agent's cost of effort improves the expected payoff to the principal and this effect is magnified in projects with low levels of the probability of success q.

# CHAPTER 6

# SOLUTIONS WHERE MONITORING IS AVAILABLE BUT

# INFORMATION IS NOT SHARED

If the principal monitors the agent's effort during period one, the incentive contract between the two parties can be constructed to include the results of the monitoring activity. At time zero, the two parties agree on the contract $(\sigma_{H1}, \sigma_{H2}, \sigma_{H3}, \sigma_{L1}, \sigma_{L2}, \sigma_{L3})$, where $\sigma_{H1}$ is paid to the agent if monitoring generates a signal of first period effort being high and the project payoff is $\tau + V$, $\sigma_{H2}$ if monitoring generates a signal of first period effort being high and the project payoff is V, and so on. Due to the limited liability restriction, the structure of the contract must ensure that $\sigma_{H1}, \sigma_{H2}, \sigma_{H3}, \sigma_{L1}, \sigma_{L2}, \sigma_{L3} \geq 0$. Since the new information generated at time one is not shared with the agent, the agent's strategy on both period's effort levels can be considered to be made at time zero.

To solve this system, the contracts that induce a particular strategy from the agent at the minimum of cost to the principal, given a fixed level of monitoring resources m, are established. When the contracts that induce high-high, high-low or low-low effort at the highest payoff to the principal have been constructed, the payoffs across strategies can be compared. As will be seen, for any project given a cost of effort, k, and a level of project payoff spread, $\tau$, either a single contract and monitoring strategy will dominate, or multiple contracts and monitoring strategies will dominate and the single optimal solution will be dependent on the form of monitoring function.

First, consider the principal's strategy that induces high effort in both periods. Suppose it is believed that the contract ($\sigma_{H1}$, $\sigma_{H2}$, $\sigma_{H3}$, $\sigma_{L1}$, $\sigma_{L2}$, $\sigma_{L3}$) will induce this action. Given this contract, and a level of monitoring resources m, the expected payout to the agent will be:

$$U_{HH} \equiv -2k + pq\sigma_{H1} + (1-p)q\sigma_{L1} + \frac{1}{2}(1-q)(p(\sigma_{H2} + \sigma_{H3}) + (1-p)(\sigma_{L2} + \sigma_{L3}))$$

In order for this contract to compel the agent to exert high effort in both periods, the expected payoff to the agent must be greater than those generated by the other two available agent strategies. Extending the definition above, this will require $U_{HH} > U_{HL}$, $U_{HH} > U_{LH}$ and $U_{HH} > U_{LL}$ where:

$$\begin{pmatrix} qp\sigma_{H1} + q(1-p)\sigma_{L1} + \frac{1-q}{2}p(\sigma_{H2} + \sigma_{H3}) + \frac{1-q}{2}(1-p)(\sigma_{L2} + \sigma_{L3}) - 2k \\ qp\sigma_{H2} + q(1-p)\sigma_{L2} + \frac{1-q}{2}p(\sigma_{H1} + \sigma_{H3}) + \frac{1-q}{2}(1-p)(\sigma_{L1} + \sigma_{L3}) - k \\ qp\sigma_{L2} + q(1-p)\sigma_{H2} + \frac{1-q}{2}p(\sigma_{L1} + \sigma_{L3}) + \frac{1-q}{2}(1-p)(\sigma_{H1} + \sigma_{H3}) - k \\ qp\sigma_{L3} + q(1-p)\sigma_{H3} + \frac{1-q}{2}p(\sigma_{L1} + \sigma_{L2}) + \frac{1-q}{2}(1-p)(\sigma_{H1} + \sigma_{H2}) \end{pmatrix} \equiv \begin{pmatrix} U_{HH} \\ U_{HL} \\ U_{LH} \\ U_{LL} \end{pmatrix}$$

In order for the contract and level of monitoring to induce high effort in both periods, the payout matrix and the reservation utility must conform to the following constraints:

$\lambda$cons, $\gamma$1cons, $\gamma$2cons, $\gamma$3cons, $\gamma$4cons $\geq 0$;     where

$$\begin{pmatrix} \lambda\text{cons} \\ \gamma 1\text{cons} \\ \gamma 2\text{cons} \\ \gamma 3\text{cons} \\ \gamma 4\text{cons} \end{pmatrix} \equiv \begin{pmatrix} U_{HH} - R - \upsilon \\ U_{HH} - U_{HH} \\ U_{HH} - U_{HL} \\ U_{HH} - U_{LH} \\ U_{HH} - U_{LL} \end{pmatrix}$$

$$= \begin{pmatrix} qp\sigma_{H1} + q(1-p)\sigma_{L1} + \dfrac{1-q}{2}p(\sigma_{H2}+\sigma_{H3}) + \dfrac{1-q}{2}(1-p)(\sigma_{L2}+\sigma_{L3}) - 2k - R - \upsilon \\ 0 \\ \dfrac{3q-1}{2}p(\sigma_{H1}-\sigma_{H2}) + \dfrac{3q-1}{2}(1-p)(\sigma_{L1}-\sigma_{L2}) - k \\ \dfrac{qp+q+p-1}{2}(\sigma_{H1}-\sigma_{L2}) + \dfrac{qp-2q+p}{2}(\sigma_{H2}-\sigma_{L1}) + \dfrac{1-q}{2}(2p-1)(\sigma_{H3}-\sigma_{L3}) - k \\ \dfrac{qp+q+p-1}{2}(\sigma_{H1}-\sigma_{L3}) + \dfrac{1-q}{2}(2p-1)(\sigma_{H2}-\sigma_{L2}) + \dfrac{qp-2q+p}{2}(\sigma_{H3}-\sigma_{L1}) - 2k \end{pmatrix}$$

The first constraint ensures that the contract allows the agent to make at least the reservation payoff when exerting high effort in both periods, and receives back the wealth invested in the project. The remaining four constraints ensure that the high-high strategy dominates over both high-low, low-high and low-low strategies. The constraint $\gamma 1$cons is included for completeness. There are also six other constraints generated by the limited liability restriction $\sigma_{H1}, \sigma_{H2}, \sigma_{H3}, \sigma_{L1}, \sigma_{L2}, \sigma_{L3} \geq 0$.

Given high effort in both periods and the contract ($\sigma_{H1}, \sigma_{H2}, \sigma_{H3}, \sigma_{L1}, \sigma_{L2}, \sigma_{L3}$) the expected payoff to the principal is:

$$V_{HH} \equiv -m + V + \frac{1}{2}(-1+3q)\,\tau - pq\sigma_{H1} + \frac{1}{2}p(-1+q)(\sigma_{H2}+\sigma_{H3})$$

$$+ (-1+p)\,q\sigma_{L1} - \frac{1}{2}(-1+p)(-1+q)(\sigma_{L2}-\sigma_{L3})$$

To find the optimal contract we have the following program:

Maximize $V_{HH}$ over values of ($\sigma_{H1}, \sigma_{H2}, \sigma_{H3}, \sigma_{L1}, \sigma_{L2}, \sigma_{L3}$), subject to:

(A1)    $\lambda$cons, $\gamma 1$cons, $\gamma 2$cons, $\gamma 3$cons, $\gamma 4$cons $\geq 0$;    and

(A2)    $\sigma_{H1}, \sigma_{H2}, \sigma_{H3}, \sigma_{L1}, \sigma_{L2}, \sigma_{L3} \geq 0$.

The method for finding the optimal contract for each level of monitoring resources, m, is similar to the methods used in the no monitoring case.

**Result VIII:** For the high-high strategy, the following contracts satisfy the constraints (A1) and (A2).

$$\left\{\frac{2k+R+v}{pq}, 0, 0, 0, 0, 0\right\} \text{ given } 0 < k \leq -\frac{(-1+3q)(R+v)}{2(-1+q)} \text{ or}$$

$$\left(\left(\frac{1}{3} < q < \frac{1}{2} \text{ and } -\frac{(-1+3q)(R+v)}{2(-1+q)} < k \leq -\frac{(-1+3q)(R+v)}{2(-1+2q)}\right) \text{ or}\right.$$

$$\left.\frac{1}{2} \leq q < 1 \text{ and } k > -\frac{(-1+3q)(R+v)}{2(-1+q)}\right) \text{ and}$$

$$\left.\frac{(-1+q)(2k+R+v)}{2k(-1+q)-(1+q)(R+v)} \leq p < 1\right\}$$

$$\left\{\frac{2k}{p(-1+3q)}, 0, 0, 0, 0, 0\right\} \text{ given } \frac{1}{3} < q < \frac{1}{2}, \quad k \geq -\frac{(-1+3q)(R+v)}{2(-1+2q)} \text{ and}$$

$$\frac{-1+q}{-3+5q} \leq p < 1$$

$$\left\{\frac{4k}{-1+p+q+pq}, 0, 0, 0, 0, 0\right\} \text{ given } \frac{1}{3} < q < \frac{1}{2}, \quad k > -\frac{(-1+3q)(R+v)}{2(-1+2q)} \text{ and}$$

$$\frac{1}{2} < p \leq \frac{-1+q}{-3+5q} \text{ or}$$

$$\left(\left(\frac{1}{3} < q < \frac{1}{2} \text{ and } -\frac{(-1+3q)(R+v)}{2(-1+q)} < k \leq -\frac{(-1+3q)(R+v)}{2(-1+2q)}\right) \text{ or}\right.$$

$$\left.\frac{1}{2} \leq q < 1 \text{ and } k > -\frac{(-1+3q)(R+v)}{2(-1+q)}\right) \text{ and}$$

$$\left.\frac{1}{2} < p \leq \frac{(-1+q)(2k+R+v)}{2k(-1+q)-(1+q)(R+v)}\right\}$$

Where the monitoring function p is evaluated at the optimal level of monitoring resources given the monitoring function forms.[35]

**Proof VIII:** See appendix A.

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[35] The optimization program finds the optimal strategy for any given level of monitoring resources, $m \in [\eta, M)$. The final stage is to find the level of monitoring, m*, that maximizes the expected payoff to the principal across these optimal strategies. Later in this chapter, forms of monitoring functions will be introduced and the level of optimal monitoring will be found for these classes of functions. This will demonstrate that the process of optimization is viable.

So far, the contracts that induce the agent to exert high effort in both periods along with the parameter values that ensure the constraints (A1) and (A2) are satisfied have been found. For these contracts to be solutions to the optimization problem set out above, two further attributes need to be considered. First, do these parameter values span the entire parameter space, and second, can the expected payoff to the principal be improved by using contracts described at connected vertices.[36]

**Result IX:** The parameter values that ensure the constraints (A1) and (A2) are satisfied for the three contracts span the entire parameter space.

**Proof IX:** See appendix A.

**Result X:** For the three contracts established in result VIII, the expected payoff to the principal cannot be improved by using contracts found at connected vertices.

**Proof X:** See appendix A.

Therefore, the optimal contracts that induce high-high strategy from the agent are:

$$\left\{\frac{2k+R+v}{pq}, 0, 0, 0, 0, 0\right\} \quad \text{given} \quad 0 < k \le -\frac{(-1+3q)(R+v)}{2(-1+q)} \quad \text{or}$$

$$\left(\left(\frac{1}{3} < q < \frac{1}{2} \quad \text{and} \quad -\frac{(-1+3q)(R+v)}{2(-1+q)} < k \le -\frac{(-1+3q)(R+v)}{2(-1+2q)} \quad \text{or}\right.\right.$$

$$\left.\frac{1}{2} \le q < 1 \quad \text{and} \quad k > -\frac{(-1+3q)(R+v)}{2(-1+q)}\right) \quad \text{and}$$

$$\left.\frac{(-1+q)(2k+R+v)}{2k(-1+q)-(1+q)(R+v)} \le p < 1\right\}$$

$$\left\{\frac{2k}{p(-1+3q)}, 0, 0, 0, 0, 0\right\} \quad \text{given} \quad \frac{1}{3} < q < \frac{1}{2}, \quad k \ge -\frac{(-1+3q)(R+v)}{2(-1+2q)} \quad \text{and}$$

$$\frac{-1+q}{-3+5q} \le p < 1$$

$$\left\{\frac{4\,k}{-1+p+q+p\,q},\,0,\,0,\,0,\,0,\,0\right\} \quad \text{given} \quad \frac{1}{3}<q<\frac{1}{2}, \quad k>-\frac{(-1+3\,q)\,(R+\upsilon)}{2\,(-1+2\,q)} \quad \text{and}$$

$$\frac{1}{2}<p\le\frac{-1+q}{-3+5\,q} \quad \text{or}$$

$$\left(\left(\frac{1}{3}<q<\frac{1}{2} \quad \text{and} \quad -\frac{(-1+3\,q)\,(R+\upsilon)}{2\,(-1+q)}<k\le-\frac{(-1+3\,q)\,(R+\upsilon)}{2\,(-1+2\,q)} \quad \text{or}\right.\right.$$

$$\left.\frac{1}{2}\le q<1 \quad \text{and} \quad k>-\frac{(-1+3\,q)\,(R+\upsilon)}{2\,(-1+q)}\right) \quad \text{and}$$

$$\left.\frac{1}{2}<p\le\frac{(-1+q)\,(2\,k+R+\upsilon)}{2\,k\,(-1+q)-(1+q)\,(R+\upsilon)}\right\}$$

In a similar fashion, the following optimal contracts induce high-low, low-high and low-low effort in the first and second periods, respectively:

For high-low effort:

$$\left\{0,\,\frac{k+R+\upsilon}{p\,q},\,0,\,0,\,0,\,0\right\} \quad \text{given} \quad 0<k\le-\frac{(-1+3\,q)\,(R+\upsilon)}{-1+q} \quad \text{or}$$

$$k>-\frac{(-1+3\,q)\,(R+\upsilon)}{-1+q} \quad \text{and} \quad \frac{(-1+q)\,(k+R+\upsilon)}{k\,(-1+q)-(1+q)\,(R+\upsilon)}\le p<1$$

$$\left\{0,\,\frac{2\,k}{-1+p+q+p\,q},\,0,\,0,\,0,\,0\right\} \quad \text{given} \quad k>-\frac{(-1+3\,q)\,(R+\upsilon)}{-1+q} \quad \text{and}$$

$$\frac{1}{2}<p\le\frac{(-1+q)\,(k+R+\upsilon)}{k\,(-1+q)-(1+q)\,(R+\upsilon)}$$

For low-high effort:

$$\left\{0,\,0,\,0,\,0,\,\frac{k+R+\upsilon}{p\,q},\,0\right\} \quad \text{given} \quad 0<k\le-\frac{(-1+3\,q)\,(R+\upsilon)}{-1+q}$$

$$\left\{0,\,0,\,0,\,0,\,\frac{2\,k}{p\,(-1+3\,q)},\,0\right\} \quad \text{given} \quad k>-\frac{(-1+3\,q)\,(R+\upsilon)}{-1+q}$$

For low-low effort:

$$\left\{0,\,0,\,0,\,0,\,0,\,\frac{R+\upsilon}{p\,q}\right\} \quad \text{for all parameters}$$

For each of the effort strategies above, the regions of dominance span the complete parameter spaces. The following result shows that the low-high effort strategies can be discarded.

**Result XI:** The low-high effort strategies are redundant since the expected payoff from the high-low effort strategies is identical.

**Proof XI:** See appendix A.

**Result XII:** The contracts that maximize the expected payout to the principal in the monitoring without communication model are equations (M1) through (M6) below; the corresponding parameter values are included in appendix B:

$$\left\{ \frac{2k + R + \upsilon}{pq}, 0, 0, 0, 0, 0 \right\} \tag{M1}$$

$$\left\{ \frac{2k}{p(-1 + 3q)}, 0, 0, 0, 0, 0 \right\} \tag{M2}$$

$$\left\{ \frac{4k}{-1 + p + q + pq}, 0, 0, 0, 0, 0 \right\} \tag{M3}$$

$$\left\{ 0, \frac{k + R + \upsilon}{pq}, 0, 0, 0, 0 \right\} \tag{M4}$$

$$\left\{ 0, \frac{2k}{-1 + p + q + pq}, 0, 0, 0, 0 \right\} \tag{M5}$$

$$\left\{ 0, 0, 0, 0, 0, \frac{R + \upsilon}{pq} \right\} \tag{M6}$$

**Proof XII:** See appendix A.

In the analysis that follows, the level of *monitoring intensity* will be considered for each contractual solution. For any probability function p(m), a level of monitoring, $p(m_1)$ will be described as strictly more intense than a second level of monitoring $p(m_2)$ if $p(m_1) > p(m_2)$.

Before comparing the contractual solutions in result XII to the no-monitoring contracts, it is worth considering how the parameter values associated with these

solutions compare to the parameter values of the solutions in the no-monitoring case. This comparison is set out in table 2. The column headers for the table are described as follows: "High and High with Reservation" refers to contracts where the agent exerts high effort in both periods and the reservation constraint binds. The other headers are described in a similar manner. The regions in this table are described in figure 3.

**Table 2: Comparison of Regions of Influence of Monitoring Contracts**

| | High and High with Reservation | High and High with No Reservation | High and Low with Reservation | High and Low with No Reservation | Low and Low with Reservation |
|---|---|---|---|---|---|
| Region 1 | XXX | | | | |
| Region 2 | XXX | XXX | | | |
| Region 3 | XXX | XXX | | | |
| Region 4 | XXX | XXX | XXX | | |
| Region 5 | XXX | XXX | XXX | XXX | |
| Region 6 | XXX | XXX | XXX | XXX | XXX |
| Region 7 | | | | | XXX |
| Region 8 | | | | | XXX |
| Region 9 | | | | | XXX |

## 6.1 Comparison of Monitoring Contracts to No Monitoring Contracts

In this section, comparison of monitoring verses no monitoring strategies are made and the circumstances where monitoring is the optimal strategy for the principal is formulated. Firstly, monitoring may induce the agent to exert higher effort. This could lead to higher expected payoffs to the principal only, or to both the principal and agent. In addition, there is a possibility that the principal's position will improve due to this increase in effort but the agent's position may actually worsen as measured by expected payoff due to the cost of this higher effort.

Increase in effort is only possible in regions 4, 5, 6, 7, 8, and 9. In any of these regions, improvement in the expected payoff to the agent is possible. Only in region 5 is there potential for reduction in the expected payoff to the agent since in this region, the agent is extracting rents from the project and there is potential for monitoring to induce the agent to exert higher effort but reduce his expected payoff to the reservation value.

Secondly, monitoring may not result in any change in effort but there could be a shift in expected payoff from the agent to the principal. This outcome is possible in regions 2, 3, and 5 since the agent is receiving expected payoffs in excess of the reservation payment and thus, monitoring may reduce the agent's rent.

Thirdly, monitoring may induce the agent to exert lower effort but, due to a shift in expected payoff to the benefit of the principal, the principal prefers this strategy. This set of circumstances is a possibility in regions 2, 3, and 5, since the agent is receiving expected payoffs in excess of the reservation payment and the agent is exerting effort above the lowest level possible.

Finally, for certain projects monitoring may not be the optimal strategy. The following two results demonstrate the existence of projects where monitoring will leave effort levels unchanged and where the expected payoff to the agent remains at the reservation level. Since there is no improvement in expected payoffs from the project and since there can be no shifting of resources from the agent to the principal, monitoring will not be an optimal strategy for these projects.

**Result XIII:** In region 1, monitoring is never the optimal strategy for the principal.

**Proof XIII:** See appendix A.

In the no-monitoring case, region 1 was described as "caretaker" employment. Since the cost of effort is relatively cheap and the spread of project payouts is relatively high, the principal can motivate the agent by contractual means to exert high effort in both periods. There are sufficient payoffs from a successful project to compensate the agent for his cheap devotion to duty. Monitoring can produce the same results but is not the optimal method when compared to contractual means.

This illustrates the concepts of the *monitoring intensity principle* which states that as incentive intensity[37] increase so should the level of monitoring intensity: If an agent's compensation is sensitive to the level of effort he applies, then it is prudent to measure this effort level accurately. It was established in chapter 5, for region 1 type contractual solutions, incentive intensity is at a low level. Therefore, monitoring the agent's effort during the first period is not an optimal strategy for the principal.

A similar result holds in regions 7, 8 and 9.

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[37] Incentive intensity was defined in chapter 5. The same definition holds in this chapter with minor modifications.

**Result XIV:** In regions 7, 8 and 9, monitoring is never the optimal strategy for the principal.

**Proof XIV:** See appendix A.

In the no-monitoring case, projects in regions 7, 8, and 9 were described as "unskilled" employment. In these regions, the dispersion of project payout is sufficiently low that agent effort is not worth encouraging. Since this can be achieved by incentive contracts, and since the agent need only be compensated at the reservation level, then monitoring is never the optimal strategy in these regions.

Analysis of the results below will show that regions 1, 7, 8 and 9 are the only regions where monitoring is never the optimal strategy.[38] Thus, if for some exogenous reason, the principal is forced to monitor projects that can be described as residing in these regions, then monitoring resources will be devoted to the project at the minimum level required by the exogenous factors driving the monitoring requirement. Since in these regions the agent is receiving the reservation payment, the monitoring cost arising from the exogenously required monitoring will fall entirely on the principal.[39]

The following result shows that projects do not exist where monitoring would leave effort levels unchanged, but expected payoff to the agent would increase from the reservation level to a non-reservation level. If these situations did exist, then monitoring would lead to a decrease in expected payoff to the principal.

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[38] Monitoring may not be the optimal strategy in other regions but this will arise due to the effectiveness of the monitoring technologies available to the principal.

[39] See Walker (2000) for a discussion of the harm caused to the relationship between contractors and local government authorities when monitoring is required by legislation even though it is not justified economically.

**Result XV:** There do not exist projects where monitoring would lead to no change in effort levels, but would lead to an increase in expected payoffs to the agent from the reservation to a non-reservation level.

**Proof XV:** See appendix A.

The proof of this result relies on the parameter conditions for the contractual solutions under scrutiny being empty. It should be noted that even if the intersection of the conditions in result XV were not empty, monitoring would still not be preferred since the principal would not waste resources monitoring if the only outcome would be to increase the expected payoff of the agent while holding the effort level constant.

The following result illustrates situations in which monitoring does not induce any change in effort but there is a shift in expected payoff from the agent to the principal. In the following case, the use of monitoring reduces the agent's expected payoff to the level of the reservation payment and thus, the ability of the agent to extract rents from employment are removed.

**Result XVI:** For contracts (N2) and (N4) there exists monitoring functions that leave the level of effort from the agent unchanged, but this monitoring activity results in a reduction in expected payments to the agent from a non-reservation to a reservation value.

For contract (N2), this can occur only in a subset of the contract's conditions given by:

$\{1/2 \leq q < 1$ and $k \geq (3q - 1)(R + \upsilon)/(1 - q)$, or

$1/3 < q < 1/2$ and $(3q - 1)(R + \upsilon)/(1 - q) \leq k$

$\leq (3q - 1)(R + \upsilon)/[2(1 - 2q)]\}$ and

$\tau \geq 4qk/(3q-1)^2$ or

$(3q-1)(R+\upsilon)/[2(1-q)] \leq k \leq (3q-1)(R+\upsilon)/(1-q)$ and

$\tau \geq [2k(1+q) - 2(3q-1)(R+\upsilon)]/(3q-1)^2.$

For contract (N4), this can occur only in a subset of the contract's conditions given by:

$(3q-1)(R+\upsilon)/(1-q) < k \leq 2(3q-1)(R+\upsilon)/(1-q)$ and

$2[kq - (3q-1)(R+\upsilon)]/(3q-1)^2 \leq \tau \leq 2(k+R+\upsilon)/(3q-1)^2.$

For contract (N2), in the subset of conditions, the monitoring functions that achieve this outcome have the following forms:

$p(m^*) \geq (1-q)(2k+R+\upsilon)/[(1+q)(R+\upsilon) + 2k(1-q)]$

for   $m^* \leq 2(1-q)k/(3q-1) - R - \upsilon.$

For contract (N4), in the subset of conditions, the monitoring functions that achieve this outcome have the following forms:

$$\frac{(1-q)(2k+R+v)}{k(1-q)+(1+q)(R+v)} \leq$$

$$p \leq \frac{(1-q)(-1+3q+2(k+R+v))}{-2k(-1+3q)+(1+q)(-1+3q)+2(1+q)(R+v)} \quad \text{for}$$

$$m \leq \frac{k(1-q)}{-1+3q} - R - v.$$

In both cases, monitoring activity yields the same overall expected payoff to the principal and agent considered in total before monitoring cost, and is an optimal strategy for principal due to transfer of wealth from the agent to the principal.

**Proof XVI:** See appendix A.

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For contracts (N2) and (M1), the principal induces the agent to exert high effort in both periods. With monitoring, however, the cost of monitoring reduces the overall payoff from the project from $V - 2k + (3q - 1)\tau/2$ to $V - 2k + (3q - 1)\tau/2 - m$.

Due to this monitoring activity, the expected payoff from the project to the agent declines from $2k(1 - q)/(3q - 1) - \upsilon$ to $R + \upsilon$.

The contract that induces this effort from the agent in concert with monitoring is given by $\{(2k + R + \upsilon)/(pq) - \upsilon, -\upsilon, -\upsilon, -\upsilon, -\upsilon, -\upsilon\}$. With this contract, the principal pays the agent the incentive component of this contract if the high project payoff occurs and the results from the monitoring are a signal that the agent expended high effort in the first period. In the other five cases, the agent loses the limited liability value $\upsilon$.

This contract is comparable to the non-monitoring contract that induces high effort in both periods, and which leads to the binding of the reservation constraint. This contract, as discussed in chapter 5, has the form $\{(2k + R + \upsilon)/q - \upsilon, -\upsilon, -\upsilon\}$.

When comparing the incentive components of each contract, $(2k + R + \upsilon)/(pq) - \upsilon$ and $(2k + R + \upsilon)/q - \upsilon$, the incentive component from the agent's point of view has been improved on over the comparable incentive component in the no-monitoring case. As the monitoring intensity principle dictates, incentive intensity increases when monitoring is a preferred strategy. This comes at the expense of receiving the higher payout with less frequency. In a sense, monitoring has extended region 1 to include areas of region 2 at the agent's expense.

The expected payoff to the principal, because of this monitoring activity, is given by $V + (3q - 1)\tau/2 - R - 2k - m$. Thus, the expected payoff is not dependent on the probability of discovering the true level of effort, p. In this situation, monitoring is

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optimal if the probability of discovering the true effort level in period one is greater or equal $(1 - q)(2k + R + \upsilon)/[(1 + q)(R + \upsilon) + 2k(1 - q)]$.

Since the expected payoff to the principal is declining as the level of monitoring resources increases, then the principal will only expend sufficient monitoring resources such that $p(m^*) = (1 - q)(2k + R + \upsilon)/[(1 + q)(R + \upsilon) + 2k(1 - q)]$.

The boundary condition in this expression is an increasing function in the agent cost of effort and a decreasing function of the probability of project success and limited liability. Thus, monitoring intensity will increase for projects with low likelihood of success or for agents with high cost of effort or low levels of limited liability.

For an example of a group of monitoring functions that satisfies these conditions, consider monitoring functions of the form $p(m) = am + b$. In general, for a linear function to satisfy the conditions above, it is required that:

$$0 < a < 1/[2(M - \eta)] \quad \text{and} \quad 1/2 - a\eta < b < 1 - aM.$$

In region 2, if we have $M < 2k(1 - q)/(3q - 1) - (R + \upsilon)$, then the following conditions for a and b yield linear monitoring functions that result in monitoring being the optimal strategy in region 2:

$$a < [2k(1 - q) - (3q - 1)(R + \upsilon)]/\{2[2k(1 - q) + (1 + q)(R + \upsilon)](M - \eta)\} \quad \text{and}$$

$$(1 - q)(2k + R + \upsilon)]/[2k(1 - q) + (1 + q)(R + \upsilon)] - aM \leq b$$

$$\leq (1 - q)(2k + R + \upsilon)]/[2k(1 - q) + (1 + q)(R + \upsilon)] - a\eta.$$

This group of linear functions is not an exhaustive collection of linear monitoring functions in region 2 that lead to monitoring being the optimal strategy but, simply an example of classes of such functions. For this group, the optimal level of monitoring is given by $m^* = (1 - q)(2k + R + \upsilon)]/\{a[2k(1 - q) + (1 + q)(R + \upsilon)]\} - b/a$.

**Result XVII:** For contracts (N2) and (N4), there exists monitoring functions that leave the level of effort from the agent unchanged, but this monitoring activity results in a reduction in the agent's expected payoff but not to the level of the reservation payment.

For contract (N2) this can occur only in subsets of the contract's conditions given by:

$1/3 < q < 1/2$, $k \geq (3q - 1)(R + \upsilon)/[2(1 - 2q)]$, and $\tau \geq 4qk/(3q - 1)^2$, or $\qquad$ (X1)

$(3q - 1)(R + \upsilon)/[2(1 - q)] \leq k \leq (3q - 1)(R + \upsilon)/(1 - q)$ and

$\qquad \tau \geq [2k(1 + q) - 2(3q - 1)(R + \upsilon)]/(3q - 1)^2$, or

$\{1/3 < q < 1/2$ and $(3q - 1)(R + \upsilon)/(1 - q) \leq k \leq (3q - 1)(R + \upsilon)/[2(1 - 2q)]$, or

$1/2 \leq q < 1$ and $k > (3q - 1)(R + \upsilon)/(1 - q)\}$ and $\tau \geq 4qk/(3q - 1)^2$, or $\qquad$ (X2)

$1/3 < q < 1/2$, $k > (3q - 1)(R + \upsilon)/[2(1 - 2q)]$, and $\tau \geq 4qk/(3q - 1)^2$. $\qquad$ (X3)

For contract (N4) this can occur only in subsets of the contract's conditions given by:

$(3q - 1)(R + \upsilon)/(1 - q) < k < 2(3q - 1)(R + \upsilon)/(1 - q)$ and

$\qquad 2[qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2 \leq \tau \leq 2(k + R + \upsilon)/(3q - 1)^2$, or $\qquad$ (X4)

$(3q - 1)(R + \upsilon)/(1 - q) < k < 2(3q - 1)(R + \upsilon)/(1 - q)$ and

$\qquad 2(k + R + \upsilon)/(3q - 1)^2 \leq \tau < 4qk/(3q - 1)^2$, or

$1/3 < q < 1/2$, $2(3q - 1)(R + \upsilon)/(1 - q) \leq k \leq (3q - 1)(R + \upsilon)/(2q - 1)$, and

$\qquad 2[qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2 \leq \tau < 4qk/(3q - 1)^2$. $\qquad$ (X5)

For contract (N2), in the subset of conditions, the monitoring functions that achieve this outcome have the following forms:

$p(m^*) \geq (1 - q)/(3 - 5q)$, $m^* \leq 2qk/(3q - 1)$ for (X1),

$p(m^*) \leq (1 - q)(2k + R + \upsilon)/[(1 + q)(R + \upsilon) + 2k(1 - q)]$,

m* ≤ 4q(1 − q)(2p − 1)/[(3q − 1)(q + p + qp − 1) for (X2), or

p(m*) ≤ (1 − q)/(3 − 5q), m* ≤ 4q(1 − q)(2p − 1)/[(3q − 1)(q + p + qp − 1) for (X3).

For contract (N4), in the subset of conditions, the monitoring functions that achieve this outcome have the following forms:

p(m*) ≤ (1 − q)(k + R + υ)/[(1 + q)(R + υ) + k(1 − q)],

m* ≤ 2q(1 − q)(2p − 1)/[(3q − 1)(q + p + qp − 1) for (X4), or

p(m*) ≤ (1 − q)(3q − 1)τ/[(1 + q)(3q − 1)τ − 4qk],

m* ≤ 2q(1 − q)(2p − 1)/[(3q − 1)(q + p + qp − 1) for (X5).

In both cases, monitoring activity yields the same overall expected payoff to the principal and agent considered in total before monitoring cost, and is an optimal strategy for principal due to transfer of wealth from the agent to the principal.

**Proof XVII:** See appendix A.

As an illustration of monitoring functions that fulfill these conditions, consider (X1). In this region, both the monitoring and non-monitoring contracts induce high effort in both periods. With monitoring, however, the overall payoff from the project is reduced by the cost of monitoring from V − 2k + (3q − 1)τ/2 to V − 2k + (3q − 1)τ/2 − m. Due to monitoring, the expected payoff to the agent declines from 2k(1 − q)/(3q − 1) − υ to 2k(1 − 2q)/(3q − 1) − υ.

This represents a loss in expected payoff of 2kq/(3q − 1). The expected payoffs to the principal and agent if monitoring is undertaken are:

2kq/(3q − 1) + V + (3q − 1)τ/2 + υ − m and 2k(1 − 2q)/(3q − 1) − υ, respectively.

The contract that induces this effort from the agent in concert with monitoring is given by {2k/[p(3q − 1)], −υ, −υ, −υ, −υ, −υ}. With this contract, the agent receives the

incentive component if high payoffs occur and the results from the monitoring are a signal that the agent expended high effort in the first period. In the other five cases, the agent loses the limited liability value $\upsilon$.

Notice that in this region, monitoring is optimal if the probability of discovering the true effort level in period one is greater or equal to $(1 - q)/(3 - 5q)$. Since the expected payoff to the principal is declining as the level of monitoring resources increases, the principal will only expend sufficient monitoring resources such that $p(m^*) = (1 - q)/(3 - 5q)$.

As an example of a group of monitoring function that satisfies these conditions, consider the monitoring function of the form $p(m) = am + b$. In general, for a linear function to satisfy the conditions for monitoring functions it is required that:

$$0 < a < 1/[2(M - \eta)] \quad \text{and} \quad 1/2 - a\eta < b < 1 - aM.$$

In this sub region if the parameter conditions are such that the following is true, $1/3 < q < 5/11$ and $M < 2k(1 - q)/(3q - 1) - (R + \upsilon)$, then the following conditions for a and b yield linear monitoring functions that provide for monitoring being the optimal strategy in region 2:

$$(3q - 1)/[2(3 - 5q)(M - \eta)] < a < [2(1 - 2q)]/[(3 - 5q)(M - \eta)] \quad \text{and}$$

$$b < (1 - q)/(3 - 5q) - a\eta.$$

This group of linear functions is not an exhaustive collection of linear monitoring functions in this sub region of region 3 that lead to monitoring being the optimal strategy. For this group, the optimal level of monitoring is given by $m^* = (1 - q)/[(3 - 5q)a] - b/a$.

The previous results illustrate situations where the optimal level of monitoring is found at the boundary of the regions. This does not need to be the case. For (X2) and

(X3), the expected payoff to the principal, if monitoring is undertaken, is given by

$V + (3q - 1)\tau/2 + \upsilon - m - 4kpq/(pq + q + p - 1)$.

For (X4) and (X5), the expected payoff to the principal if monitoring is undertaken is given by $V + \upsilon - m - 2kpq/(pq + q + p - 1)$.

In these regions, the expected payoff to the principal is dependent on the level of monitoring resources spent directly, and indirectly through the probability of discovering the true level of effort due to the monitoring activity. The two terms that are impacted by the level of monitoring resources in the principal's expected payoff are m and $4kpq/(pq + q + p - 1)$ for the first, and $2kpq/(pq + q + p - 1)$ for the second. The derivative of the second term in each case with respect to m is given by,

$4k(1 - q)qp'/(pq + q + p - 1)^2$ and $2k(1 - q)qp'/(pq + q + p - 1)^2$, respectively, where p′ is the derivative of p with respect to m. Since p is assumed to be a strictly increasing function of m, the second term is strictly increasing in m. Therefore, dependent on the form of the monitoring function, interior solutions to the principal's optimization problem may exist in these regions.

The next set of results consider situations in which monitoring induces the agent to exert higher levels of effort. The first result demonstrates increased exertion of effort but expected payments to the agent staying at the reservation level. Since these payoffs take into account the cost of the additional effort, the agent sees no net change in his expected payoffs even though he expends more effort.

**Result XVIII:** For contracts (N3) and (N5), there exists monitoring functions that induce the agent to exert a higher level of effort, but this monitoring activity results in the agent's expected payoff remaining at the reservation payment level.

For contract (N3), this can occur only in subsets of the contract's conditions given by:

$$(3q - 1)(R + \upsilon)/[2(1 - q)] < k \leq (3q - 1)(R + \upsilon)/(1 - q) \text{ and}$$

$$2k/(3q - 1) < \tau \leq 2[(1 + q)k - (3q - 1)(R + \upsilon)]/(3q - 1)^2. \tag{XI1}$$

For contract (N5), this can occur only in subsets of the contract's conditions given by:

$$\{1/3 < q < 1/2 \text{ and } (3q - 1)(R + \upsilon)/(1 - q) < k \leq (3q - 1)(R + \upsilon)/[2(1 - 2q)], \text{ or}$$

$$1/2 \leq q < 1 \text{ and } k > (3q - 1)(R + \upsilon)/(1 - q)\}, \text{ and}$$

$$2k/(3q - 1) < \tau \leq 2[2qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2, \text{ or} \tag{XI2}$$

$$1/3 < q < 1/2, k > (3q - 1)(R + \upsilon)/[2(1 - 2q)], \text{ and}$$

$$2k/(3q - 1) < \tau \leq 2[(1 - q)k - (3q - 1)(R + \upsilon)]/(3q - 1)^2, \text{ or} \tag{XI3}$$

$$\{1/3 < q \leq 3/7 \text{ and } (3q - 1)(R + \upsilon)/(1 - q) < k \leq (3q - 1)(R + \upsilon)/[2(1 - 2q)], \text{ or}$$

$$3/7 < q < 1 \text{ and } (3q - 1)(R + \upsilon)/(1 - q) < k \leq 2(3q - 1)(R + \upsilon)/(1 - q)\}, \text{ and}$$

$$2k/(3q - 1) < \tau \leq 2[2qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2, \text{ or}$$

$$\{1/3 < q < 1/2 \text{ and } 2(3q - 1)(R + \upsilon)/(1 - q) < k \leq (3q - 1)(R + \upsilon)/[2(1 - 2q)], \text{ or}$$

$$1/2 \leq q < 1 \text{ and } k \geq 2(3q - 1)(R + \upsilon)/(1 - q)\}, \text{ and}$$

$$2k/(3q - 1) < \tau \leq 2(k + R + \upsilon)/(3q - 1), \text{ or}$$

$$\{1/3 < q < 3/7 \text{ and } 2(3q - 1)(R + \upsilon)/(1 - q) < k \leq (3q - 1)(R + \upsilon)/(1 - 2q), \text{ or}$$

$$3/7 \leq q < 1/2 \text{ and } (3q - 1)(R + \upsilon)/[2(1 - 2q)] < k < (3q - 1)(R + \upsilon)/(1 - 2q)\}, \text{ and}$$

$$2[(1 - q)k - (3q - 1)(R + \upsilon)]/(3q - 1)^2 < \tau \leq 2(k + R + \upsilon)/(3q - 1), \text{ or}$$

$$1/3 < q < 3/7, (3q - 1)(R + \upsilon)/[2(1 - 2q)] < k < 2(3q - 1)(R + \upsilon)/(1 - q), \text{ and}$$

$$2[(1 - q)k - (3q - 1)(R + \upsilon)]/(3q - 1)^2 < \tau \leq 2[2qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2. \tag{XI4}$$

For contract (N3), in the subset of conditions, the monitoring functions that achieve this outcome have the following forms:

$p(m^*) \geq (1 - q)(2k + R + \upsilon)/[(1 + q)(R + \upsilon) + 2k(1 - q)]$,

$m^* \leq (3q - 1)\tau/2 - k$ for (XI1).

For contract (N5), in the subset of conditions, the monitoring functions that achieve this outcome have the following forms:

$p(m^*) \geq (1 - q)(2k + R + \upsilon)/[(1 + q)(R + \upsilon) + 2k(1 - q)]$,

$m^* \leq (3q - 1)\tau - 2k$ for (XI2),

$p(m^*) \geq (1 - q)(k + R + \upsilon)/[(1 + q)(R + \upsilon) + 2k(1 - q)]$,

$m^* \leq (3q - 1)\tau/2 - k$ for (XI3), or

$(1 - q)(k + R + \upsilon)/[(1 + q)(R + \upsilon) + 2k(1 - q)] \leq$

$p(m^*) \leq (1 - q)[2(k + R + \upsilon) + (3q - 1)\tau]/[(1 + q)(3q - 1)\tau$

$+ 2(3q - 1)k + 2(1 + q)(R + \upsilon)]$,

$m^* \leq (3q - 1)\tau/2 - k$ for (XI4).

**Proof XVIII:** See appendix A.

The next result demonstrates increased exertion of effort by the agent but expected payments to the agent decrease from a non-reservation to a reservation level. Therefore, adoption of monitoring by the principal encourages the agent to exert greater effort and allocates the payoff from the project away from the agent to the principal.

**Result XIX:** For contract (N4), there exists monitoring functions that induce the agent to exert a higher level of effort, and this monitoring activity results in the agent's expected payoff decreasing from a non-reservation to a reservation payment level.

For contract (N4), this can occur only in subsets of the contract's conditions given by:

$\{1/3 < q < 1/2$ and $(3q - 1)(R + \upsilon)/(1 - q) < k \le (3q - 1)(R + \upsilon)/[2(1 - 2q)]$, or

$1/2 \le q < 1$ and $k > (3q - 1)(R + \upsilon)/(1 - q)\}$, and

$$2[2qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2 \le \tau \le 4qk/(3q - 1)^2.$$

For contract (N4), in the subset of conditions, the monitoring functions that achieve this outcome have the following forms:

$p(m^*) \ge (1 - q)(2k + R + \upsilon)/[(1 + q)(R + \upsilon) + 2k(1 - q)]$, and

$m^* \le [(3q - 1)^2\tau + 4(2q - 1)k - 2(3q - 1)(R + \upsilon)]/[2(3q - 1)]$.

**Proof XIX:** See appendix A.

The next result demonstrates increased exertion of effort and an increase in the expected payments to the agent from a reservation to a non-reservation level. In this case, use of monitoring encourages the agent to exert greater effort but also allocates the improvement in expected payoff from the project between the agent and the principal. This improvement is of sufficient size to cover the additional expense of the extra effort; the agent is better off in total from the principal's decision to monitor.

**Result XX:** For contracts (N3) and (N5), there exists monitoring functions that induce the agent to exert a higher level of effort, and this monitoring activity results in the agent's expected payoff improving from a reservation level to a non-reservation level.

For contract (N3), this can occur only in subsets of the contract's conditions given by:

$(3q - 1)(R + \upsilon)/[2(1 - q)] < k \le (3q - 1)(R + \upsilon)/(1 - q)$ and

$2k/(3q - 1) < \tau \le 2[(1 + q)k - (3q - 1)(R + \upsilon)]/(3q - 1)^2.$ (XIII1)

For contract (N5) this can occur only in subsets of the contract's conditions given by:

$1/3 < q < 1/2$, $k \geq (3q - 1)(R + \upsilon)/[2(1 - 2q)]$, and

$2[(1 - q)k - (3q - 1)(R + \upsilon)]/(3q - 1)^2 < \tau \leq 2[2qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2$, (XIII2)

$1/3 < q < 1/2$, $k > (3q - 1)(R + \upsilon)/(1 - 2q)$, and

$\quad 2qk/(3q - 1)^2 < \tau \leq 2[(1 - q)k - (3q - 1)(R + \upsilon)]/(3q - 1)^2$, (XIII3)

$\{1/3 < q < 3/7$ and $(3q - 1)(R + \upsilon)/(1 - q) < k \leq (3q - 1)(R + \upsilon)/[2(1 - 2q)]$, or

$\quad 3/7 \leq q < 1$ and $(3q - 1)(R + \upsilon)/(1 - q) < k \leq 2(3q - 1)(R + \upsilon)/(1 - q)\}$, and

$\quad 2k/(3q - 1) < \tau \leq 2[2qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2$, or

$\{3/7 \leq q < 1/2$ and $2(3q - 1)(R + \upsilon)/(1 - q) < k \leq (3q - 1)(R + \upsilon)/[2(1 - 2q)]$, or

$\quad 1/2 \leq q < 1$ and $k \geq 2(3q - 1)(R + \upsilon)/(1 - q)\}$, and

$\quad 2k/(3q - 1) < \tau < 2(k + R + \upsilon)/(3q - 1)$, (XIII4)

$\{1/3 < q < 3/7$ and $2(3q - 1)(R + \upsilon)/(1 - q) \leq k < (3q - 1)(R + \upsilon)/(1 - 2q)$, or

$\quad 3/7 \leq q < 1/2$ and $(3q - 1)(R + \upsilon)/[2(1 - 2q)] < k < (3q - 1)(R + \upsilon)/(1 - 2q)\}$, and

$\quad 2(k + R + \upsilon)/(3q - 1) \leq \tau \leq 2[2qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2$, or

$1/3 < q < 1/2$, $k > (3q - 1)(R + \upsilon)/(1 - 2q)$, and

$\quad 2k/(3q - 1)^2 \leq \tau \leq 2[2qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2$, (XIII5)

$\{1/3 < q < 3/7$ and $2(3q - 1)(R + \upsilon)/(1 - q) \leq k \leq (3q - 1)(R + \upsilon)/(1 - 2q)$, or

$\quad 3/7 \leq q < 1/2$ and $(3q - 1)(R + \upsilon)/[2(1 - 2q)] < k < (3q - 1)(R + \upsilon)/(1 - 2q)\}$, and

$\quad 2[(1 - q)k - (3q - 1)(R + \upsilon)]/(3q - 1)^2 \leq \tau < 2(k + R + \upsilon)/(3q - 1)$, or

$1/3 < q < 3/7$, $(3q - 1)(R + \upsilon)/[2(1 - 2q)] < k < 2(3q - 1)(R + \upsilon)/(1 - q)$, and

$2[(1 - q)k - (3q - 1)(R + \upsilon)]/(3q - 1)^2 \leq \tau \leq 2[2qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2$, (XIII6)

{3/7 < q < 1/2 and $2(3q - 1)(R + \upsilon)/(1 - q) \le k \le (3q - 1)(R + \upsilon)/[2(1 - 2q)]$, or

    1/2 ≤ q < 1 and $k \ge 2(3q - 1)(R + \upsilon)/(1 - q)$}, and

$$2(k + R + \upsilon)/(3q - 1) \le \tau \le 2[2qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2, \qquad \text{(XIII7)}$$

1/3 < q < 1/2, $k > (3q - 1)(R + \upsilon)/(1 - 2q)$, and

$$2[qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2 < \tau \le 2qk/(3q - 1)^2, \qquad \text{(XIII8)}$$

{1/3 < q < 1/2 and $2(3q - 1)(R + \upsilon)/(1 - q) < k \le (3q - 1)(R + \upsilon)/(1 - 2q)$, or

    1/2 ≤ q < 1 and $k > 2(3q - 1)(R + \upsilon)/(1 - q)$}, and

$$2(k + R + \upsilon)/(3q - 1) < \tau \le 2[2qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2, \text{ or}$$

1/3 < q < 1/2, $k > (3q - 1)(R + \upsilon)/(1 - 2q)$, and

$$2qk/(3q - 1)^2 < \tau < 2[2qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2, \text{ or} \qquad \text{(XIII9)}$$

{1/3 < q < 1/2 and $2(3q - 1)(R + \upsilon)/(1 - q) < k \le (3q - 1)(R + \upsilon)/(1 - 2q)$, or

    1/2 ≤ q < 1 and $k > 2(3q - 1)(R + \upsilon)/(1 - q)$}, and

$$2k/(3q - 1) < \tau \le 2(k + R + \upsilon)/(3q - 1), \text{ or}$$

1/3 < q < 1, $(3q - 1)(R + \upsilon)/(1 - q) < k \le 2(3q - 1)(R + \upsilon)/(1 - q)$, and

$$2k/(3q - 1) < \tau < 2[2qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2. \qquad \text{(XIII10)}$$

For contract (N3), in the subset of conditions, the monitoring functions that achieve this outcome have the following forms:

$$(1 - q)[2(k + R + \upsilon) + (3q - 1)\tau]/[(1 + q)(3q - 1)\tau + 2(3q - 1)k + 2(1 + q)(R + \upsilon)] \le$$

$$p(m^*) \le (1 - q)(2k + R + \upsilon)/[(1 + q)(R + \upsilon) + 2k(1 - q)],$$

$$m^* \le (3q - 1)\tau/2 + R + \upsilon - k(3qp - q - p + 1)/(qp + q + p - 1) \text{ for (XIII1).}$$

For contract (N5), in the subset of conditions, the monitoring functions that achieve this outcome have the following forms:

$p(m^*) \geq (1 - q)/(3 - 5q)$, $m^* \leq (3q - 1)\tau + R + \upsilon - 2qk(3q - 1)$ for (XIII2),

$(1 - q)/(3 - 5q) \leq p(m^*) \leq (1 - q)[(3q - 1)^2\tau - 4qk]/[(1 + q)(3q - 1)^2\tau - 8q(1 - q)k]$,

$\quad m^* \leq (3q - 1)\tau + R + \upsilon - 2qk(3q - 1)$ for (XIII3),

$(1 - q)[2(k + R + \upsilon) + (3q - 1)\tau]/[(1 + q)(3q - 1)\tau + 2(3q - 1)k + 2(1 + q)(R + \upsilon)] \leq$

$\quad p(m^*) \leq (1 - q)(2k + R + \upsilon)/[(1 + q)(R + \upsilon) + 2k(1 - q)]$,

$\quad\quad m^* \leq (3q - 1)\tau + R + \upsilon - 4qpk(pq + p + q - 1)$ for (XIII4),

$(1 - q)(3q - 1)\tau/[(1 + q)(3q - 1)\tau - 4qk] \leq p(m^*) \leq (1 - q)/(3 - 5q)$,

$\quad m^* \leq (3q - 1)\tau + R + \upsilon - 4qpk(pq + p + q - 1)$ for (XIII5),

$(1 - q)[2(k + R + \upsilon) + (3q - 1)\tau]/[(1 + q)(3q - 1)\tau + 2(3q - 1)k + 2(1 + q)(R + \upsilon)] \leq$

$\quad p(m^*) \leq (1 - q)/(3 - 5q)$,

$\quad m^* \leq (3q - 1)\tau + R + \upsilon - 4qpk(pq + p + q - 1)$ for (XIII6),

$(1 - q)(3q - 1)\tau/[(1 + q)(3q - 1)\tau - 4qk] \leq$

$\quad p(m^*) \leq (1 - q)(2k + R + \upsilon)/[(1 + q)(R + \upsilon) + 2k(1 - q)]$,

$\quad\quad m^* \leq (3q - 1)\tau + R + \upsilon - 4qpk(pq + p + q - 1)$ for (XIII7),

$(1 - q)[2(R + \upsilon) + (3q - 1)\tau]/[(1 + q)(3q - 1)\tau - 4qk + 2(1 + q)(R + \upsilon)] <$

$\quad p(m^*) \leq (1 - q)/(3 - 5q)$,

$\quad\quad m^* \leq (3q - 1)\tau/2 + R + \upsilon - 2qpk(pq + p + q - 1)$ for (XIII8),

$(1 - q)[2(R + \upsilon) + (3q - 1)\tau]/[(1 + q)(3q - 1)\tau - 4qk + 2(1 + q)(R + \upsilon)] <$

$\quad p(m^*) \leq (1 - q)(3q - 1)\tau/[(1 + q)(3q - 1)\tau - 4qk]$,

$\quad\quad m^* \leq (3q - 1)\tau/2 + R + \upsilon - 2qpk(pq + p + q - 1)$ for (XIII9), or

$(1 - q)[2(R + \upsilon) + (3q - 1)\tau]/[(1 + q)(3q - 1)\tau - 4qk + 2(1 + q)(R + \upsilon)] <$

$\quad p(m^*) \leq (1 - q)(k + R + \upsilon)/[(1 + q)(R + \upsilon) + k(1 - q)]$,

$m^* \leq (3q - 1)\tau/2 + R + \upsilon - 2qpk(pq + p + q - 1)$ for (XIII10).

**Proof XX:** See appendix A.

The next result demonstrates monitoring activity increasing the level of effort exerted by the agent. The expected payment to the agent changes from a non-reservation level to another non-reservation level. The expected payment to the agent can increase, decrease, or remain the same dependent on the project parameters.

**Result XXI:** For contract (N4), there exists monitoring functions that induce the agent to exert a higher level of effort, and this monitoring activity results in the agent's expected payoff changing from one non-reservation level to another non-reservation level.

This can occur only in subsets of the contract's conditions given by:

$1/3 < q < 1/2$, $k \geq (3q - 1)(R + \upsilon)/[2(1 - 2q)]$, and

$$2[2qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2 \leq \tau \leq 4qk/(3q - 1)^2, \qquad \text{(XIV1)}$$

$\{1/3 < q < 3/7$ and $k \geq 2(3q - 1)(R + \upsilon)/(1 - q)$, or

$3/7 \leq q < 1$ and $k > (3q - 1)(R + \upsilon)/[2(1 - 2q)\}$, and

$$2[2qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2 \leq \tau < 4qk/(3q - 1)^2, \text{ or}$$

$1/3 < q < 3/7$, $(3q - 1)(R + \upsilon)/[2(1 - 2q)] < k < 2(3q - 1)(R + \upsilon)/(1 - q)$, and

$$2(k + R + \upsilon)/(3q - 1) \leq \tau < 4qk/(3q - 1)^2, \qquad \text{(XIV2)}$$

$1/3 < q < 3/7$, $(3q - 1)(R + \upsilon)/[2(1 - 2q)] < k < 2(3q - 1)(R + \upsilon)/(1 - q)$, and

$$2[2qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2 \leq \tau < 2(k + R + \upsilon)/(3q - 1), \qquad \text{(XIV3)}$$

$\{1/3 < q < 3/7$ and $(3q - 1)(R + \upsilon)/(1 - q) < k \leq (3q - 1)(R + \upsilon)/[2(1 - 2q)]$, or

$3/7 \leq q < 1/2$ and $2(3q - 1)(R + \upsilon)/(1 - q) < k < 2(3q - 1)(R + \upsilon)/(1 - q)\}$, and

$$2[2qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2 \leq \tau < 2(k + R + \upsilon)/(3q - 1), \text{ or} \qquad \text{(XIV4)}$$

$\{1/3 < q \le 3/7$ and $(3q - 1)(R + \upsilon)/(1 - q) < k \le (3q - 1)(R + \upsilon)/[2(1 - 2q)]$, or

$3/7 < q < 1$ and $(3q - 1)(R + \upsilon)/(1 - q) < k \le 2(3q - 1)(R + \upsilon)/(1 - q)\}$, and

$2(k + R + \upsilon)/(3q - 1) \le \tau < 4qk/(3q - 1)^2$, or

$\{3/7 < q < 1/2$ and $2(3q - 1)(R + \upsilon)/(1 - q) < k \le (3q - 1)(R + \upsilon)/[2(1 - 2q)]$, or

$1/2 \le q < 1$ and $k > 2(3q - 1)(R + \upsilon)/(1 - q)\}$, and

$$2[2qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2 \le \tau < 4qk/(3q - 1)^2. \hspace{2cm} \text{(XIV5)}$$

The subset of conditions, the monitoring functions that achieve this outcome have the following forms:

$p(m^*) \ge (1 - q)/(3 - 5q)$, $m^* \le (3q - 1)\tau/2$ for (XIV1).

$(1 - q)(3q - 1)\tau/[(1 + q)(3q - 1)\tau - 4qk] \le p(m^*) \le (1 - q)/(3 - 5q)$,

$\quad m^* \le (3q - 1)\tau - 2q(5qp - q - 3p + 1)k/[(3q - 1)(qp + q + p - 1)]$ for (XIV2),

$(1 - q)[2(k + R + \upsilon) + (3q - 1)\tau]/[(1 + q)(3q - 1)\tau + 2(3q - 1)k + 2(1 + q)(R + \upsilon)] \le$

$\quad p(m^*) \le (1 - q)/(3 - 5q)$,

$\quad\quad m^* \le (3q - 1)\tau - 2q(5qp - q - 3p + 1)k/[(3q - 1)(qp + q + p - 1)]$ for (XIV3),

$(1 - q)[2(k + R + \upsilon) + (3q - 1)\tau]/[(1 + q)(3q - 1)\tau + 2(3q - 1)k + 2(1 + q)(R + \upsilon)] \le$

$\quad p(m^*) \le (1 - q)(2k + R + \upsilon)/[(1 + q)(R + \upsilon) + 2k(1 - q)]$,

$\quad\quad m^* \le (3q - 1)\tau - 2q(5qp - q - 3p + 1)k/[(3q - 1)(qp + q + p - 1)]$ for (XIV4),

$(1 - q)(3q - 1)\tau/[(1 + q)(3q - 1)\tau - 4qk] \le$

$\quad p(m^*) \le (1 - q)(2k + R + \upsilon)/[(1 + q)(R + \upsilon) + 2k(1 - q)]$,

$\quad\quad m^* \le (3q - 1)\tau - 2q(5qp - q - 3p + 1)k/[(3q - 1)(qp + q + p - 1)]$ for (XIV5).

**Proof XXI:** See appendix A.

The next result demonstrates monitoring will never lead to a decrease in effort exerted by the agent.

**Result XXII:** Monitoring never results in a decrease in effort exerted by the agent.

**Proof XXII:** See appendix A.

Table 3 summarizes the results established above. Shaded cells represent regions where the mixture of changes in effort level and expected payoff to the agent are not possible. Cells denoted by the contents XXX represent situations where monitoring is the optimal strategy for certain monitoring functions. Blank cells represent situations where there is potential for monitoring to be optimal but the results above find this not to be the case.

**Table 3: Comparison of No Monitoring Contracts to Monitoring Contracts**

| Agent's Position | Higher Effort | | | Same Effort | | | Lower Effort | | |
|---|---|---|---|---|---|---|---|---|---|
| | Better | Same | Worse | Better | Same | Worse | Better | Same | Worse |
| Region 1 | ▒ | ▒ | ▒ | ▒ | ▒ | ▒ | ▒ | ▒ | ▒ |
| Region 2 | ▒ | ▒ | ▒ | ▒ | ▒ | XXX | ▒ | ▒ | |
| Region 3 | ▒ | ▒ | ▒ | ▒ | ▒ | XXX | ▒ | ▒ | |
| Region 4 | XXX | XXX | ▒ | ▒ | ▒ | ▒ | ▒ | ▒ | ▒ |
| Region 5 | XXX | XXX | XXX | ▒ | ▒ | XXX | ▒ | ▒ | |
| Region 6 | XXX | XXX | ▒ | ▒ | ▒ | ▒ | ▒ | ▒ | ▒ |
| Region 7 | | | ▒ | ▒ | ▒ | ▒ | ▒ | ▒ | ▒ |
| Region 8 | | | ▒ | ▒ | ▒ | ▒ | ▒ | ▒ | ▒ |
| Region 9 | | | ▒ | ▒ | ▒ | ▒ | ▒ | ▒ | ▒ |

Consider regions 1, 2, and 3. In these regions, the agent is exerting the maximum level of effort before any monitoring resources are brought to bear on the project. Thus, no increase in effort level is possible. For region 1, since the agent is already exerting maximum effort and receiving the reservation payoff, and since monitoring is costly, there is never any benefit for the principal in monitoring. Whereas, for regions 2 and 3, even though monitoring is costly, there is the potential to redistribute wealth from the agent to the principal. It has been shown in the above results that it is possible for the principal to monitor in regions 2 and 3, and redistribute the wealth from the agent to the principal. It has also been shown that monitoring never influences the agent to reduce the level of effort devoted to the project.

For region 4, in the no-monitoring case, the agent exerts high effort in the first period, and low in the second, and receives an expected payment at the reservation level R. Therefore, the principal will never have the incentive to monitor in order to reduce or leave the effort level unchanged. Furthermore, the principal cannot use monitoring to increase effort and reduce the expected payments to the agent since the agent is already at the reservation level. The only remaining options are to use monitoring to increase effort level and for the principal to either share some of the increased payoff with the agent, or leave the agent's position unchanged. It has been shown by the above results that it is possible for the principal to monitor in region 4, increasing the agent's effort levels. In certain cases, all the gains of this increased effort accrue to the principal, in other cases the gains are shared between the agent and principal.

Region 5 has similarities to region 4, however, the agent receives expected payments that bring his expected payoff above the reservation level R. Therefore, the

principal may have the incentive to monitor in order to reduce or leave the effort level unchanged if there is redistribution of wealth to the benefit of the principal. Furthermore, the principal could use monitoring to increase effort and reduce the expected payments to the agent. It has been shown in the above results, that it is possible for the principal to monitor in region 5, and either leave the agent's effort levels unchanged, or increase the agent's effort levels. In the former situation, the wealth is redistributed from the agent to the principal. In the latter situation, all three opportunities for wealth redistribution exist: Both principal and agent can share in the gains; all benefit accrues to the principal with no overall loss of benefit to the agent; and all benefit accrues to the principal with an overall loss of benefit to the agent.

For region 6, 7, 8, and 9, in the no-monitoring case, the agent exerts low effort in both periods and receives an expected payment that brings his expected payoff to the reservation level R. Therefore, the principal cannot monitor to reduce or leave the effort level unchanged. Furthermore, the principal cannot use monitoring to increase effort and reduce the expected payments to the agent. This is the case since the agent is not extracting rents from these projects. This leaves monitoring as a method to increase effort level and for the principal either to share some of the increased payoff with the agent, or leave the agent's position unchanged. It has been shown in the above results that monitoring is only an optimal strategy for the principal in region 6, where monitoring increases the agent's effort levels. In certain cases, all the gains of this increased effort accrue to the principal, in other cases the gains are shared between the agent and principal.

## 6.2 Solution of the Dual Problem

Since the solutions to the principal-agent relationship were found using a linear programming methodology, there exists a dual problem for the original primal problem. The primal problem used to find contractual solutions that induce the entrepreneur to exert high effort in both periods was analyzed above and can be described as:

Max $\mathbf{f}.\boldsymbol{\sigma}$ s.t. $\mathbf{A}\boldsymbol{\sigma} \leq \mathbf{c}$ for $\boldsymbol{\sigma} \in \mathbb{R}^{6+}$ where $\mathbf{f}$, $\boldsymbol{\sigma}$, $\mathbf{A}$ and $\mathbf{c}$ are defined as follows:

$$\mathbf{f} \equiv \begin{pmatrix} -pq \\ \frac{1}{2}p(-1+q) \\ \frac{1}{2}p(-1+q) \\ -(1-p)q \\ \frac{1}{2}(1-p)(-1+q) \\ \frac{1}{2}(1-p)(-1+q) \end{pmatrix}, \quad \boldsymbol{\sigma} \equiv \begin{pmatrix} \sigma_{H1} \\ \sigma_{H2} \\ \sigma_{H3} \\ \sigma_{L1} \\ \sigma_{L2} \\ \sigma_{L3} \end{pmatrix},$$

$$\mathbf{A} \equiv \begin{pmatrix} pq & \frac{p(1-q)}{2} & \frac{p(1-q)}{2} & q(1-p) & \frac{(1-p)(1-q)}{2} & \frac{(1-p)(1-q)}{2} \\ \frac{(3q-1)p}{2} & \frac{(3q-1)p}{2} & 0 & \frac{(3q-1)(1-p)}{2} & \frac{(3q-1)(1-p)}{2} & 0 \\ \frac{(p-1+q+pq)}{2} & \frac{(p-2q+pq)}{2} & \frac{(2p-1)(1-q)}{2} & \frac{(p-2q+pq)}{2} & \frac{(p-1+q+pq)}{2} & \frac{(2p-1)(1-q)}{2} \\ \frac{(p-1+q+pq)}{2} & \frac{(2p-1)(1-q)}{2} & \frac{(p-2q+pq)}{2} & \frac{(p-2q+pq)}{2} & \frac{(2p-1)(1-q)}{2} & \frac{(p-1+q+pq)}{2} \end{pmatrix}$$

$$\text{and} \quad \mathbf{c} \equiv \begin{pmatrix} 2k+R+\upsilon \\ k \\ k \\ 2k \end{pmatrix}$$

The corresponding dual problem is described by:

Min $\mathbf{c}.\boldsymbol{\lambda}$ s.t. $\mathbf{A}^T\boldsymbol{\lambda} \geq \mathbf{f}$ for $\boldsymbol{\lambda} \in \mathbb{R}^{3+}$ where $\boldsymbol{\lambda}$ is defined as:

$$\boldsymbol{\lambda} \equiv \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{pmatrix}$$

Thus, the dual problem used to find contractual solutions that induce the entrepreneur to exert high effort in both periods is given by:

$$\text{Minimize}\,(-2k - R - \upsilon)\lambda_1 + k(-\lambda_2 - \lambda_3 - 2\lambda_4)\ \text{over values}\ (\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4)\ \text{subject to}$$

$$-\frac{1}{2}p(2q\lambda_1 - \lambda_2 + 3q\lambda_2) + \frac{1}{2}(1 - p - q - pq)(\lambda_3 + \lambda_4) \geq -2pq$$

$$p(-1 + q)\lambda_1 + p(-1 + 3q)\lambda_2 + (-p + 2q - pq)\lambda_3$$
$$+(-1 + 2p)(-1 + q)\lambda_4 \geq -p(1 - q)$$

$$p(-1 + q)\lambda_1 + (-1 + 2p)(-1 + q)\lambda_3 + (-p + 2q - pq)\lambda_4 \geq -p(1 - q)$$

$$2(-1 + p)q\lambda_1 + (-1 + p)(-1 + 3q)\lambda_2 + (p - 2q + pq)(\lambda_3 + \lambda_4) \geq -2(1 - p)q$$

$$-(-1 + p)(-1 + q)\lambda_1 - (-1 + p)(-1 + 3q)\lambda_2$$
$$+(-1 + p + q + pq)\lambda_3 - (-1 + 2p)(-1 + q)\lambda_4 \geq (-1 + p)(1 - q)$$

$$-(-1 + p)(-1 + q)\lambda_1 - (-1 + 2p)(-1 + q)\lambda_3$$
$$+(-1 + p + q + pq)\lambda_4 \geq (-1 + p)(1 - q)\quad\text{and}$$

$$\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4 \geq 0.$$

The solution of dual problem in each case confirms the results of the primal problem analysis and describes shadow prices for the constraints described in the primal problems. For solutions to the primal problem where the reservation constraint was found to bind, the shadow prices for k and $R + \upsilon$ is 1. In other words, market conditions lead to a reduction in the cost of high effort, reservation payment or limited liability level then this will lead to a corresponding increase in the expected payoff to the principal.

In the remaining solutions, the shadow price for $R + \upsilon$ is always zero. This is not surprising since the reservation constraint does not bind in these solutions. For the non-reservation contracts, the shadow prices of the cost of effort are described in table 4.

**Table 4: Shadow Prices of the Cost of Effort**

| Contract | Shadow Price of Cost of Effort |
|----------|-------------------------------|
| M2 | $\dfrac{2q}{-1+3q}$ |
| M3 | $\dfrac{pq}{-1+p+q+pq}$ |
| M5 | $\dfrac{2pq}{-1+p+q+pq}$ |

The shadow prices for the cost of effort for both M3 and M5 are lower than the shadow price when monitoring is not undertaken and the reservation constraint does not bind. Therefore, monitoring lessens the impact of cost of effort on the principal-agent relationship.

## 6.3 Discussion of Empirical Implications

The research presented in this paper can be discussed in the context of venture capital investments in entrepreneurial projects. The wealth of empirical research devoted to this form of investment makes it an ideal topic to illustrate the theoretical work presented in this paper. Whereas venture capital can be thought of in the general context of financing new ventures, in this discussion venture capital will refer specifically to finance supplied by venture capital organizations.

Within the context of my research, the venture capitalist would represent the principal, and the entrepreneur the agent. The venture capitalist is in a position to fund fully the entrepreneur's project, possibly in syndication with other venture capitalist. The two parties must agree on a contract to compensate the entrepreneur for his involvement

with the project and the venture capitalist has to decide on the level of monitoring to use in the assessment of the entrepreneur's actions.

Venture capitalists focus their investments on high-risk projects where the opportunity for high payouts exists, but the probability of failure is high. Huntsman and Hoban (1980) investigation of venture capital investments finds high returns but also found that exclusion of the top 10% of performers leaves the remaining investments as losers on average. Chiampou and Kallett (1989) find venture capital investments return on average 17.5%, which is considered very high. The investments also exhibit high risk measured as standard deviation of returns of 37.6%. The assets of these projects are often intangible and are difficult to value. Sahlman (1990) notes that *the [venture capital] environment is characterized by substantial uncertainty about payoffs... and a high degree of information asymmetry between principals and agents.* In the models presented in this paper, high-risk projects where the opportunity for profits is substantial can be thought of as projects where probability of success, $q$, is low and the dispersion of payouts, $\tau$, is high.

The value of these projects rests with the human capital – the entrepreneurs – running the projects. Gorman and Sahlman (1989) report that venture capitalists identify weak management as the main reason for project failure. In the context of the models presented in this paper, the high value placed on human capital can be thought of as projects where the cost of effort, $k$, is high. This reflects both the time that an entrepreneur must devote to the project and the prior costs associated with the entrepreneur's effort such as securing a good education and acquiring experience.

The venture capitalists actively manage these projects through hands on monitoring and staging of financing [see Gorman and Sahlman (1989), Barry et al., (1990) and Gompers (1995) for a discussion of the role of monitoring in venture capital investments].[40] Rosenstein et al., (1993) find that from the entrepreneur's point of view the venture capitalists are useful in monitoring financial performance, serving as a sounding board and for the recruitment/replacement of the CEO. They add value to projects through monitoring, but will also use monitoring to justify removal of the entrepreneurs when the monitoring results justify the action.

Projects that could be described as a venture capital – entrepreneur type project in the models presented herein, will have high levels of payout dispersion, $\tau$, high cost of agent effort, k, and low levels of the likelihood of project success, q. Somewhat arbitrarily, consider projects with the following parameters:

$1/3 < q < 1/2$, $k \geq (3q - 1)(R + \upsilon)/[2(1 - 2q)]$, and $\tau \geq 4qk/(3q - 1)^2$.

For these projects, there exists a non-reservation, no-monitoring contractual solution of the form $\{4k/(3q - 1) - \upsilon, -\upsilon, -\upsilon\}$. There are also two possible monitoring contractual solutions. The first is of the form $\{2k/[(3q - 1)p] - \upsilon, -\upsilon, -\upsilon, -\upsilon, -\upsilon, -\upsilon\}$ which dominates all other contracts for monitoring functions where

$p(m^*) \geq (1 - q)/(3 - 5q)$ and $m^* \leq 2qk/(3q - 1)$.

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[40] The monitoring role is not the only value-added service provided by the venture capitalist. The venture capitalist also acts as a screen to ensure worthwhile projects are funded. Furthermore, the reputation of the venture capitalist also plays various roles in the relationship between venture capitalist, entrepreneur and outside investors [Hsu (2004), Megginson and Weiss (1991), and Timmons and Bygrave (1986)]. Venture capitalists are also proficient in ensuring that investments that subsequently raise public equity do so at the right time [Lerner (1994b)].

The second has the form $\{4k/(qp + q + p - 1) - \upsilon, -\upsilon, -\upsilon, -\upsilon, -\upsilon, -\upsilon\}$ which dominates all other contracts for monitoring functions where $p(m^*) \leq (1 - q)/(3 - 5q)$ and

$m^* \leq 4q(1 - q)(2p - 1)/[(3q - 1)(q + p + qp - 1)]$.

Each of the contracts is *dichotomous*: If the results of monitoring reveal the action of the agent to be acceptable, the agent is paid according to a predetermined schedule; otherwise, the agent receives a fixed payment. This confirms a result found by Harris and Raviv (1979) that monitoring activity leads to dichotomous contracts: The agent is rewarded if the project is successful and monitoring finds the agent working at the principal's preferred level, else the agent is punished.

For these projects, the monitoring activity does not increase the expected payoff from the project and does not induce the agent to exert higher levels of effort. However, monitoring will redistribute the expected payoffs from the agent to the principal. This agrees with Rosenstein et al., (1993) findings that the involvement of venture capitalists monitoring the performance of the entrepreneur does not improve project performance.

For all three contracts, the incentive component of each contract is decreasing in the likelihood of success, q. In other words, the entrepreneur requires greater incentive intensity when the likelihood of success decreases. In this case incentive intensity will be high when the entrepreneur has a greater impact on project success; projects with low levels of likelihood of success, q, do not respond as effectively to the entrepreneur's efforts. This result agrees with Sahlman (1990) research that shows that low likelihood of success but high levels of entrepreneurial incentives typify venture capital projects. My work also confirms results established by Gompers (1995) that shows that a low likelihood of success in entrepreneurial ventures leads to greater monitoring intensity.

With the two monitoring contracts, the standard deviation of the entrepreneur's contractual payments can be calculated as:

$$\frac{2k\sqrt{\left(\frac{1}{p}-q\right)q}}{-1+3q} \quad \text{and} \quad \frac{4k\sqrt{pq(1-pq)}}{-1+p+q+pq}, \quad \text{respectively.}$$

Both of the expressions are strictly decreasing in both p and q. In other words, as project risk increases greater levels of risk are transferred to the entrepreneur. This confirms a finding by Sahlman (1990).[41]

Kaplan and Strömberg (2004) in their survey of venture capital investments categorize risk into *internal* and *external*. Uncertainty about the entrepreneur's ability, operations of the project being hard to observe, the discretion the entrepreneur has for the use of funds are all described as internal risks. Risks that are common to both venture capitalist and entrepreneur are described as external risk. External risks within the context of my model are captured by the probability of success parameter, q. Internal risks are not explicitly measured within my models. Kaplan and Strömberg find that projects with high external risk are associated with high levels of incentive pay. This is, once again, consistent with my results. This result has been shown to arise in practice many years ago in the context of rice farming in Malaysia. Huang (1973) explains the prevalence of sharecropping, which is a classic form of incentive pay, in farming regions where harvest uncertainty is driven by external parameters such as the weather.

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[41] Sahlman (1990) notes that entrepreneurs in venture capital relationships are undiversified and thus are concerned with total risk rather than market risk.

## 6.4 Impact of Probability of Success, Reservation Level, and Level of Limited Liability

It is useful to consider the impact of underlying project parameters on the optimal strategies for principal and agent. Consider the regions 1, 7, 8, and 9 of figure 3. These regions are defined by the following parameters:

Region 1: $\quad \tau \geq 2k/(3q-1) \quad$ and $\quad k \leq (3q-1)(R+\upsilon)/[2(1-q)]$

Regions 7, 8 and 9 $\quad \tau \leq 2k/(3q-1)$.

From the results presented in this chapter, in these regions, monitoring is never the optimal strategy. Consider the effect of increasing levels of probability of success, q. The boundary, $2k/(3q-1)$ is a decreasing function of q with the limiting value of k for $q = 1$. Thus, as the probability of success increases, the boundary bisects the project space described by the parameters $\tau$ and k. Also, the boundary $(3q-1)(R+\upsilon)/[2(1-q)]$ is an increasing function of the probability of success, the reservation payout and the limited liability value.

Therefore, if it is possible to increase the value of any of these three parameters then there is a greater likelihood that the project will not be subject to monitoring. It may be possible for the principal to alter the operational characteristics of the project to improve the probability of success. It is unlikely that the reservation payout can be changed, as this is an exogenous parameter outside the principal's control. It certainly is possible to vary the limited liability parameter through the recruitment process. Therefore, the choice of an agent who brings a sufficiently large personal investment to the project could remove the need to monitor.

CHAPTER 7

CONCLUDING COMMENTS: MONITORING WITH NO

COMMUNICATION

Part 1 extends research on the relationship between principal and agent where

moral hazard exists and contractual payments to the agent are restricted by a limited

liability constraint. A simple model is used to investigate the impact of incentives and

monitoring on this relationship. Actual contractual forms are derived and their structure is

investigated. Furthermore, the effects of the impact of the communication of monitoring

results at an interim stage are considered.

First, a model was investigated that allowed for only incentive contracts as a

means to influence agent effort. In this case, it was found that the reservation constraint

does not always bind and, therefore, the agent can extract rents from the project. This

occurs for projects where agent involvement is costly and the project yields high payoffs.

In these projects, there are sufficient levels of project payoffs to compensate the agent

with expected payments greater than those, which could be generated by alternative

employment.

Second, monitoring was introduced with no interim communication of results. In

this case there existed certain circumstances where monitoring was never the optimal

strategy regardless on the form of monitoring technologies available. This occurred in

regions where contractual incentives were sufficient to induce the agent to exert effort at

the preferred level while receiving expected payoffs that yielded the reservation level. It

also occurred in regions where increased project payouts generated from higher levels of agent effort are insufficient to compensate the agent for the cost of this increased effort.

When monitoring does have an impact on the principal-agent relationship, two effects are found to exist. Circumstances exist where monitoring encourages the agent to exert higher levels of effort. There are also circumstances in which payoffs from the project are redistributed between the parties. This redistribution is not a one-way street flowing from the agent to the principal. In situations where the agent is encouraged by monitoring to exert higher levels of effort, there are circumstances where both the principal and agent enjoy the rewards of the extra effort as both parties share in the increased project payouts.

# PART 2

# MONITORING WITH COMMUNICATION

# CHAPTER 8

## PREFACE: MONITORING WITH COMMUNICATION

The purpose of part 2 of this dissertation is to extend the analysis of part 1 on monitoring and incentives to allow for monitoring results to be shared by the principal with the agent at an interim stage. Due to the intricacy of the calculations necessary to solve the associated optimization problems and the complexity of the description of the results, I will focus the analysis only on regions of parameters that are of particular interest when investigating venture capital – entrepreneur relationships.

In my analysis, actual contracts will be constructed and investigated and the optimal mix of incentives and monitoring intensity will be established.[42] The effects of sharing interim monitoring information on the principal-agent relationship – distribution of wealth, change in effort levels, and distribution of risk – will also be analyzed. Of particular interest will be the impact of information sharing on the level of risk accepted by the agent: The application of imperfect monitoring leads to increased risk borne by the agent over the risk associated with non-monitoring strategies.[43] Part 2 will study whether risk associated with imperfect monitoring is reduced from the agent's perspective if interim monitoring results are shared.

In order to understand the role that sharing of monitoring results will play in a principal-agent relationship consider the following general setup.[44] The principal employs an agent to manage a project. The success of the project will be dependent on, among

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[42] Definitions of incentive and monitoring intensity can be found in part 1 of this dissertation and in the work that follows.

[43] This result was established in part 1.

[44] A general description of principal-agent relationships can be found in Sappington (1991) and in part 1 of this dissertation.

other factors, the effort exerted by the agent in his role of manager of the project. The principal's goal is to induce a certain level of effort from an agent in order to maximize the principal's share of project payoffs. If the principal can directly observe the effort of the agent, no moral-hazard issue exists and the principal can achieve his goal through the negotiation of a contract with the agent. However, if the level of agent effort is not directly observable[45], then the principal has two choices.

First, the principal can negotiate with the agent an incentive contract based on the payoff from the project.[46] Second, the principal can negotiate with the agent an incentive contract, based on the payoff of the project, and information received from monitoring the agent's actions.

When a principal monitors an agent's effort, monitoring can lead to either the redistribution of wealth, changes in the agent's effort levels, or liquidation of the project. Results from part 1 shows that monitoring will never induce the agent to reduce effort levels, however, when monitoring induces the agent to increase effort, in some circumstances, dependent on the parameters of the project, some of the increased payoff from the project is allocated to the agent. In these cases, even after accounting for the cost of the additional effort, the agent will improve his share of wealth and extract rents from the project.[47] This result is reliant on the non-binding of the reservation constraint.

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[45] If there exist exogenous random events that effect the payoff from the project, which are not directly observable by the principal, then the payoff from the project will be a noisy measure of the agent's actions. A classic example of this kind of exogenous random event would be the effects of weather on the crop yields of a farm managed by a tenant on behalf of a landlord. The level of crops produced by the farm is not only dependent on the work of the tenant but is also effected by the weather, which is outside the control of the tenant. If the landlord cannot perfectly assess the weather's effects on crop yields then the yields will also lead to imperfect estimates of the tenant's effort.

[46] Incentive contracts are used to align the actions of the agent with the desires of the principal; they were formally defined in chapter 4 of part 1.

[47] Rent in this context represents the utility in excess of the minimum utility available to the agent through seeking alternative employment in the labor market.

The two-period model introduced in part 1 of this dissertation and extended in part 2 allows the principal to monitor during the first period, receiving the results of this monitoring before the second period begins. In part 2, monitoring provides a signal of effort, which the principal uses as a factor to compensate the agent and it allows the principal to share the signals of first-period effort generated by monitoring with the agent, at the end of the first period. The implications of the last point, sharing of monitoring results at an interim stage on the relationship between principal and agent, has not been previously modeled and thus, brings originality to my research. When the principal shares monitoring results at an interim stage, the following questions can be investigated: Is it beneficial for the principal to share the results of monitoring or is it best to keep the agent in the dark? And, if the sharing of monitoring results is possible and desirable from the principal's point of view, what strategies will the agent employ?[48] My research finds that it can be beneficial for the principal to share this interim monitoring information with the agent since it will shift monitoring risk from the agent to the principal.[49]

It is clear that the sharing of information does change the nature of the agent's strategies and greatly extends the agent's range of possible actions. This is the case since the agent can condition his post-monitoring effort on the outcome of monitoring. For instance, if the principal monitors and finds the agent exerting effort at a low level, then sharing these findings with the agent may induce him to change his level of effort in the future. There are two assumptions implicit in this discussion. First, the results of monitoring have to be verifiable by the agent and cannot be forged by the principal. This stops the principal reducing contractual payments to the agent based on fictitious

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[48] In the context of this research, the agent's strategic decision is what level of effort to devote to the project.
[49] Monitoring risk refers to the principal receiving an erroneous signal of agent effort.

information on prior effort levels; the monitoring results have to be auditable in some

sense by the agent. Second, even if the agent shares new information with the principal to

correct erroneous signals generated by monitoring, the principal cannot assimilate this

new information into his monitoring results without cost. In other words, all monitoring

has some cost associated with it.

Part 2 is organized as follows. In chapter 9, I give a general description of the

assumptions of the models and an overview of the methodology used to optimize the

principal and agent's strategies. The results are considered in chapter 10. Chapter 11

concludes part 2. A brief review of the literature concerning the principal-agent problem

with a focus on the role of monitoring is discussed in chapter 3 of part 1 of this

dissertation.

CHAPTER 9

METHODOLOGY: MONITORING WITH COMMUNICATION

A two-period model was developed in part 1 to consider the relationship between principal and agent. Two variants of this model were considered in part 1. In the first, the principal did not monitor the agent and, therefore, the contract between the two parties was solely based on the payoff from the project. The second model involved monitoring agent effort during period one. In this model, the results of monitoring are not shared with the agent until the end of the project: The signal of agent effort obtained from monitoring was incorporated into the contractual payment from the principal to the agent. In the third model, which is the focus of part 2, monitoring will be undertaken during the first period and the results of this monitoring can be shared with the agent at the end of the first period. This final model allows the agent to modify his actions in period two, based on the monitoring results from period one. Once again, the signal of agent effort will be incorporated into the contractual payment from the principal to the agent. Certain structures and parameters are common to all three models, and were described in part 1: They will be defined again in this chapter to allow part 2 to be considered in its own regard.

For all three models, it is assumed that the principal has a fully funded project and at the commencement of the project, and these funds will be sufficient through the end of the second period. The principal is risk neutral and maximizes his expected returns.[50] The

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[50] The assumption that the principal is risk neutral is a common simplification in research on the principal-agent problem. See part 1 for further discussion.

return to the principal will consist of the gross payoff from the project less the contractual payment to the agent and any resources spent on monitoring.

It is assumed that the agent is also risk neutral. The agent will maximize expected net returns, which are defined as contractual payments received from the principal less the agent's cost of effort. It is assumed that the agent brings wealth to the project in the amount of $\upsilon \geq 0$: Any contract agreed with the principal cannot generate losses for the agent in any state of the world that exceeds $\upsilon$. Thus, the agent has limited liability.[51] The amount $\upsilon$ can be thought of as the agent's investment in the project; the principal in effect requires the agent to invest his own capital in the project.[52] It is assumed that the agent does earn a return on the capital invested in the project.[53] This assumption will greatly simplify the analysis of the contractual solutions of the models.[54]

The model evolves over two periods. The two parties agree on a contract at time zero; also, the agent decides on the level of effort to exert on the project during period one, and the principal decides on the level of monitoring, if any, to use during this period.[55] If the principal monitors the agent, the principal can estimate at time one the level of effort exerted by the agent during period one. If the results of monitoring are

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[51] The inclusion of a limited liability constraint is common in research that includes risk neutrality for the agent. See Demougin and Fluet (2001), Dmougin and Garvie (1991), Innes (1989) and Sappington (1981) for implementations of limited liability with risk-neutrality, and see Sappington (1991) for an illustration of why the assumption of limited liability with risk neutrality can generate incentive contracts solutions to the principal-agent problem with moral hazard in the same manner as the assumption of risk aversion.

[52] The assumption of limited liability introduces a wealth effect into the models investigated. The agent's decisions on effort levels and the contracts negotiated between the two parties will be directly impacted by the wealth of the agent. Although this will complicate solutions to models, it will also add another dimension to the analysis of the relationship between principal and agent.

[53] The limited liability value, $\upsilon$, can be defined as a "with return" value, so that the capital invested by the agent, say $\zeta$, earns a return of $\rho$ so that $\upsilon = (1 + \rho)\zeta$.

[54] It is straightforward to demonstrate that if both parties are risk neutral, then the moral hazard issue can be resolved by the principal *franchising* the project to the agent.

[55] If the principal does not monitor the agent's effort during period one, the agent can decide on the level of effort in both periods at time zero since no additional information will be available at time one to change the agent's decision.

shared with the agent at this intermediate stage, then this leads to an expanded choice of strategic decisions for the agent since his period-two effort can be conditioned on the outcome of the principal's monitoring results. The project matures at time two and both parties receive payments from the project.

At time zero, the agent has to decide whether to accept the negotiated contract.[56] It is assumed that there is a competitive market for the agent's talents and if he does not accept the current project, he can receive the equivalent of R at time two in the market.[57] There is no cost to the agent in the assessment of the project before he agrees to accept the contract. The agent's strategy is to decide whether to accept the contract, and if he does so, the level of effort to devote to the project. Since there are two periods, the agent will decide on effort levels in both periods. In part 1 where the principal does not monitor, or does monitor at time one and does not share the signal of effort with the agent, the agent can decide on his effort level in both periods at time zero. In this case, there is no new information generated during the project with which the agent can modify his effort decisions in period two. In part 2, the principal can monitor during period one and can share the signal of effort with the agent at time one. If this is the optimal strategy for the principal then the agent can decide on the level of effort in period two conditioned on this signal. To simplify the analysis, it is assumed that the agent cannot quit the project at the end of period one and seek alternative employment: The contract implicitly contains a penalty that will make alternative employment an unacceptable strategy during

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[56] Throughout the analysis herein, the assumption used in most research and described in Sappington (1991) will be used: It is assumed, that when the agent is indifferent among strategies, the agent will choose the strategy most preferred by the principal.

[57] This reservation constraint ensures that the expected return of the agent must meet or exceed R plus the repayment of the limited liability value, $\upsilon$. The returns in both cases are net of the cost of effort.

period two.[58] This means quasi-rent in period two, from remaining employed in the project, is always positive and thus, the agent will never quit the project at time one and, therefore, will always take the project to completion.

The project's payoff will be dependent on the two periods of agent's effort and a random element. There are three possible payoffs from the project at the end of period two. These are given by $\tau + V$, $V$, and $V - \tau$, where $\tau > 0$. It is assumed that the agent can exert one of two levels of effort in each of the periods, high or low. Thus, there are four two-period effort strategies; high-high, high-low, low-high, or low-low. The effort exerted by the agent in each period is costly. It is assumed that the cost measured in time-two dollars is given by k for high effort and zero for low effort. For the project payoff purposes, it is assumed that the two choices, high-low or low-high lead to the same probability distribution for the project payoffs.

This probability distribution is described in the following matrix form:

$$
\begin{array}{c}
HH \\[1em]
HL\,or\,LH \\[1em]
LL
\end{array}
\left(
\begin{array}{ccc}
q & \dfrac{1-q}{2} & \dfrac{1-q}{2} \\[1em]
\dfrac{1-q}{2} & q & \dfrac{1-q}{2} \\[1em]
\dfrac{1-q}{2} & \dfrac{1-q}{2} & q
\end{array}
\right)
$$

Where row one represents high-high effort, row two high-low effort or low-high effort, row three low-low effort; column one payoff $\tau + V$, column two payoff $V$, and column three payoff $V - \tau$. It is assumed that $q > \frac{1}{3}$ which implies that the outcomes with the highest probabilities are on the diagonal of the payoff matrix. Therefore, higher effort levels result in a greater probability of project success as measured by project payoff. It is

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[58] In the analysis that follows, this penalty will not explicitly be introduced as it is assumed that the penalty is sufficient to ensure that the agent will never quit at time one.

assumed that both principal and agent are aware of the probability distribution and are aware of the effect of agent's effort on the payoffs from the project before a contract is negotiated.[59]

When the agent exerts low effort in both periods the expected return from the project before cost of effort is considered is given by $V - (3q - 1)\tau/2$. This payoff increases to V as the agent increases effort from low to high in one of the periods, and $V + (3q - 1)\tau/2$ as the agent increases his effort to high in both periods. Therefore in both cases, an increase in effort cost of k yields and increase in project payoffs $(3q - 1)\tau/2$. The ratio $(3q - 1)\tau/2k$ will be defined as the *return to effort*. This ratio measures the extra project payoff generated by inducing the agent to increase his effort in one period.

Within this framework, there are a number of different possibilities for monitoring and communication. Diagram 1 describes the different models.

Model I is the baseline model. In this case, the principal does not monitor. The contract between the two parties can only be based on the outcome of the project, which has three possible payouts. Therefore, the contract negotiated between the agent and principal is of the form $(\sigma_1, \sigma_2, \sigma_3)$. The principal always has the choice to resort to this format if the expected payoff from model I to the principal dominates expected payoffs from the other models.

In model II, the principal monitors during period one but does not share the outcome of the monitoring until completion of the project. Since the principal's monitoring generates a signal of period one effort, this signal is available to supplement the project payoff information in the contract structure. There is an implicit assumption

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[59] Although the probability distribution is known, the realized value of the project's payoffs will not be discovered until the maturity of the project.

that the results of monitoring can be shared with the agent at time two and that the agent can verify without cost the results of monitoring. This avoids the problem of the principal falsifying the results of monitoring to his benefit. There is an additional assumption that the agent cannot change the principal's estimate of effort by supplying information to him without the principal spending more resources on monitoring that new information. In this way, the agent cannot supplement the principal's monitoring results without additional cost being incurred by the principal. The contract will be of the form $(\sigma_{H1}, \sigma_{H2}, \sigma_{H3}, \sigma_{L1}, \sigma_{L2}, \sigma_{L3})$ where the suffix H and L refers to the estimate of effort discovered by the principal through the monitoring process during period one. In this model, because of the lack of communication between the two parties, the agent's available strategies do not change from the no-monitoring model: The agent will still have to decide whether to exert high or low effort in the second period without any additional information.

Model III, which is the focus of part 2 of this dissertation, represents situations in which the principal monitors during period one and shares the results of the monitoring with the agent at time one. The decision to share the information is made ex ante since it is assumed that the policy of sharing of information is negotiated at the same time as the terms of the contract between the principal and agent are negotiated. The same assumptions hold as with model II: The principal can share the monitoring results with the agent without cost and the agent cannot supplement these monitoring results with additional information at no cost to the principal. In this case, the agent can modify his second period effort based on the results of the monitoring. As with model II, the contract will be of the form $(\sigma_{H1}, \sigma_{H2}, \sigma_{H3}, \sigma_{L1}, \sigma_{L2}, \sigma_{L3})$ where the suffix H and L refers to the

estimate of effort discovered during the monitoring process in the first period. In this model, the strategies available to the agent have increased in number and sophistication since the agent can now condition his second-period effort on the outcome of the first-period monitoring. The question then becomes whether the principal benefits from sharing information at an interim stage and thus improving the strategic choices of the agent.

In the monitoring models II and III, it is assumed that the principal can monitor during period one to estimate the agent's effort level in the first period. The amount of monitoring resources spent will be defined as $m \in [\eta, M)$ where $M > \eta > 0$. The lower level of monitoring $\eta$ represents a fixed cost of monitoring; this amount must be spent before any useful signal of effort can be received. The upper level of monitoring, $M$ is the amount necessary to establish with certainty the true effort level. The probability of discovering the true effort level will be defined as $p(m)$ where $p(M) = 1 > p(\eta) > \frac{1}{2}$, $p'(m) > 0$, and $p''(m) \leq 0$.

CHAPTER 10

SOLUTIONS WHERE MONITORING IS AVAILABLE AND

INFORMATION IS SHARED

In this chapter, the principal's ability to communicate the monitoring results to the agent at the end of period one is considered. At time zero, the two parties agree on the contract ($\sigma_{H1}$, $\sigma_{H2}$, $\sigma_{H3}$, $\sigma_{L1}$, $\sigma_{L2}$, $\sigma_{L3}$), where $\sigma_{H1}$ is paid to the agent if monitoring generates a signal of first period effort being high and the project payoff is $\tau + V$, $\sigma_{H2}$ if monitoring generates a signal of first period effort being high and the project payoff is V, and so on. It is also assumed at time zero that the principal and agent agree as to whether the results of monitoring will be shared at time one. This assumption will simplify the analysis since it will eliminate possible strategies for the principal as to when to share monitoring information. Allowing the principal to decide ex post on whether to share monitoring information at the interim stage would allow for the principal to implement this strategy randomly: The decision to share information could be based on the possible application of mixed strategies. I assume, to allow a simplified model to be studied, that only pure strategies are available to the principal at the interim stage. In this simplified model, the agent will be able to infer the results of monitoring at the interim stage based on whether the principal shares information. Due to the limited-liability restriction, the contract must be structured to ensure that $\sigma_{H1}$, $\sigma_{H2}$, $\sigma_{H3}$, $\sigma_{L1}$, $\sigma_{L2}$, $\sigma_{L3} \geq 0$.

In this model, the strategies available to the entrepreneur have increased in number and sophistication over those available in models I and II since the agent can now condition his second-period effort on the results of the first-period monitoring. The

question now becomes whether the principal benefits from sharing information at an interim stage even though this sharing improves the strategic choices for the entrepreneur.

The following notation will describe the available strategies for the agent – XYZ, where X describes the effort level exerted in the first period; the agent makes the decision on this effort level at time zero. Y describes the effort level in the second period if the monitoring generates an estimate of first period effort as being at the high level. Finally, Z describes the effort level in the second period if the monitoring generates an estimate of first period effort as being at the low level.[60] Therefore, each of the X, Y, and Z can take values of H for high effort level, or L for low effort level. Considering the various mixes of first and second-period strategies and result of monitoring, the following strategies are available to the agent:

HHH ≡ High effort in first period and high effort in second period whatever the
      outcome of monitoring.

HHL ≡ High effort in first period and, if monitoring correctly deduces the period
      one effort as being high, the agent exerts high effort in the second period.
      If the period one monitoring incorrectly deduces the period one effort as
      being low, the agent exerts low effort in the second period.

HLH ≡ High effort in first period and, if the period one monitoring correctly
      deduces the period one effort as being high, the agent exerts low effort in
      the second period. If the period one monitoring incorrectly deduces the

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[60] Both decisions Y and Z can be based on erroneous monitoring signals. For example, if the agent exerts high effort in the first period but monitoring indicates the agent exerted low effort, then his second period strategy will be Z.

period one effort as being low, the agent exerts high effort in the second period.

HLL ≡ High effort in first period and low effort in second period whatever the outcome of monitoring.

LHH ≡ Low effort in first period and high effort in second period whatever the outcome of monitoring.

LHL ≡ Low effort in first period and, if the period one monitoring incorrectly deduces the period one effort as being high, the agent exerts high effort in the second period. If the period one monitoring correctly deduces the period one effort as being low, the agent exerts low effort in the second period.

LLH ≡ Low effort in first period and, if the period one monitoring incorrectly deduces the period one effort as being high, the agent exerts low effort in the second period. If the period one monitoring correctly deduces the period one effort as being low, the agent exerts high effort in the second period.

LLL ≡ Low effort in first period and low effort in second period whatever the outcome of monitoring.

The methodology for finding optimal strategies is identical for the non-communication case. Furthermore, the results are detailed and lengthy and, therefore, the results will for the most part be displayed graphically in figure 4.
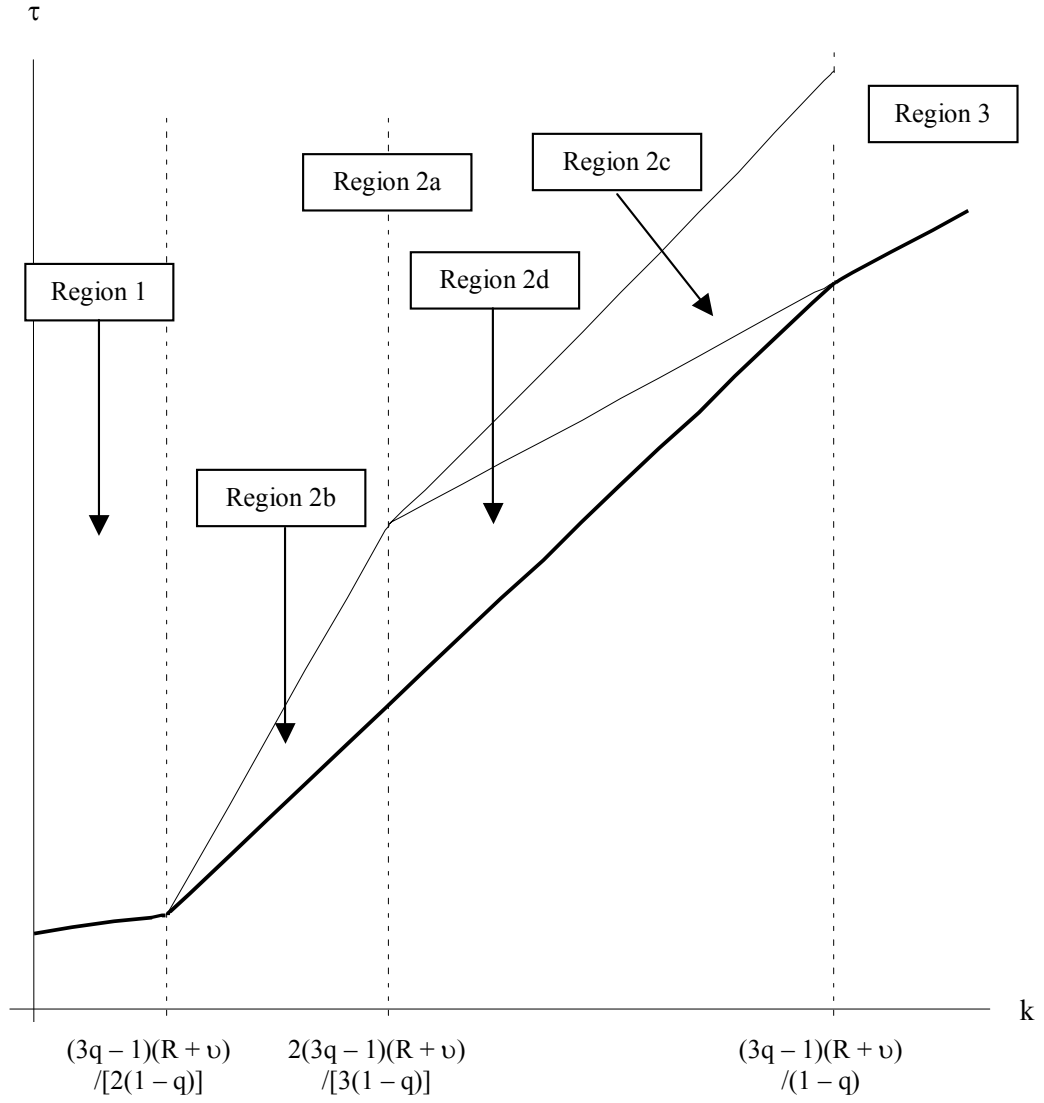
**Figure 4: Analysis of Information Sharing Contracts in Region 2**

The figure shows axes labeled $\tau$ (vertical) and $k$ (horizontal), with regions marked. Along the horizontal axis are the values:

$(3q-1)(R+\upsilon)/[2(1-q)]$

$2(3q-1)(R+\upsilon)/[3(1-q)]$

$(3q-1)(R+\upsilon)/(1-q)$

Labels: Region 1, Region 2a, Region 2b, Region 2c, Region 2d, Region 3

The axes on this chart do not meet at the origin; the focus is on region 2 and axis drawn to scale would obscure the detail of this region. The labels for regions 1 and 3 are included for reference purposes. The bold lines demonstrate the boundaries of regions 1, 2 and 3 in the no-monitoring case.

Instead of solving for all possible combinations of parameters k and $\tau$, the analysis will be restricted to regions 1 and 2 of the non-monitoring case defined by the following conditions:

$$\text{Region 1}: \quad \tau \geq \frac{2\,k}{-1+3\,q} \quad \text{and} \; k \leq \frac{(-1+3\,q)\,(R+\upsilon)}{2\,(1-q)}$$

$$\text{Region 2}: \quad \tau \geq \frac{2\,(k\,(1+q)-(-1+3\,q)\,(R+\upsilon))}{(-1+3\,q)^2}$$

$$\text{and} \; \frac{(-1+3\,q)\,(R+\upsilon)}{2\,(1-q)} \leq k \leq \frac{(-1+3\,q)\,(R+\upsilon)}{1-q}$$

As will be seen from the analysis that follows, region 1 demonstrates situations where monitoring, with or without sharing of information at an interim stage, is not the strategy for the principal that maximizes his expected payout. This will be in contrast to region 2 where sharing of interim monitoring information is sometimes the strategy that maximizes the expected payoff to the principal and can lead to shifting of risk between the two parties.

To solve this system, the contracts that induce a particular strategy from the agent at the minimum of cost to the principal, given a fixed level of monitoring resources m, are established. When the contracts that induce HHH, HHL, HLH, HLL, LHH, LHL, LLH or LLL strategies have been constructed, the payoffs across strategies can be compared. As was seen in the non-communication case in part 1, for any project given a cost of effort, k, and a level of project payoff spread, $\tau$, either a single contract and

monitoring strategy will dominate, or multiple contracts and monitoring strategies will dominate and the single optimal solution will be dependent on the form of monitoring function.

First, consider the principal's strategy that induces high effort in both periods regardless of the signal received from monitoring at time one. Suppose it is believed that the contract ($\sigma_{H1}$, $\sigma_{H2}$, $\sigma_{H3}$, $\sigma_{L1}$, $\sigma_{L2}$, $\sigma_{L3}$) will induce this action. Given this contract, and a level of monitoring resources m, the expected payout to the agent will be:

$$U_{HHH} \equiv -2k + pq\sigma_{H1} + (1-p)q\sigma_{L1} + \frac{1}{2}(1-q)(p(\sigma_{H2}+\sigma_{H3}) + (1-p)(\sigma_{L2}+\sigma_{L3}))$$

In order for this contract to compel the agent to exert high effort in both periods regardless of monitoring results, the expected payoff to the agent must be greater than those generated by the other seven available agent strategies. Extending the definition above, this will require $U_{HHH} > U_{HHL}$, $U_{HHH} > U_{HLH}$,…, and $U_{HHH} > U_{LLL}$ where:

$$
\begin{pmatrix}
-2k + pq\sigma_{H1} - \frac{1}{2}p(-1+q)(\sigma_{H2}+\sigma_{H3}) - (-1+p)q\sigma_{L1} \\
\quad + \frac{1}{2}(-1+p)(-1+q)(\sigma_{L2}+\sigma_{L3}) \\[4pt]
-k(1+p) + pq\sigma_{H1} - \frac{1}{2}p(-1+q)(\sigma_{H2}+\sigma_{H3}) - (-1+p)q\sigma_{L2} \\
\quad + \frac{1}{2}(-1+p)(-1+q)(\sigma_{L1}+\sigma_{L3}) \\[4pt]
k(-2+p) + pq\sigma_{H2} - \frac{1}{2}p(-1+q)(\sigma_{H1}+\sigma_{H3}) - (-1+p)q\sigma_{L1} \\
\quad + \frac{1}{2}(-1+p)(-1+q)(\sigma_{L2}+\sigma_{L3}) \\[4pt]
-k - \frac{1}{2}p(-\sigma_{H1}+q\sigma_{H1}-2q\sigma_{H2}-\sigma_{H3}+q\sigma_{H3}) - (-1+p)q\sigma_{L2} \\
\quad + \frac{1}{2}(-1+p)(-1+q)(\sigma_{L1}+\sigma_{L3}) \\[4pt]
-k - (-1+p)q\sigma_{H2} + \frac{1}{2}(-1+p)(-1+q)(\sigma_{H1}+\sigma_{H3}) + pq\sigma_{L2} \\
\quad - \frac{1}{2}p(-1+q)(\sigma_{L1}+\sigma_{L3}) \\[4pt]
k(-1+p) - (-1+p)q\sigma_{H2} + \frac{1}{2}(-1+p)(-1+q)(\sigma_{H1}+\sigma_{H3}) \\
\quad - \frac{1}{2}p(-1+q)(\sigma_{L1}+\sigma_{L2}) + pq\sigma_{L3} \\[4pt]
-kp + \frac{1}{2}(-1+p)(-1+q)(\sigma_{H1}+\sigma_{H2}) - (-1+p)q\sigma_{H3} + pq\sigma_{L2} \\
\quad - \frac{1}{2}p(-1+q)(\sigma_{L1}+\sigma_{L3}) \\[4pt]
\frac{1}{2}(-1+p)(-1+q)(\sigma_{H1}+\sigma_{H2}) - (-1+p)q\sigma_{H3} \\
\quad - \frac{1}{2}p(-1+q)(\sigma_{L1}+\sigma_{L2}) + pq\sigma_{L3}
\end{pmatrix}
\equiv
\begin{pmatrix}
U_{HHH} \\
U_{HHL} \\
U_{HLH} \\
U_{HLL} \\
U_{LHH} \\
U_{LHL} \\
U_{LLH} \\
U_{LLL}
\end{pmatrix}
$$

In order for the contract and level of monitoring to induce high effort in both periods regardless of monitoring signals, the payout matrix and the reservation utility must conform to the following constraints:

$\lambda$cons, $\gamma$1cons, $\gamma$2cons,…, $\gamma$8cons $\geq 0$;     where

$$
\begin{pmatrix}
\lambda\text{cons} \\
\gamma1\text{cons} \\
\gamma2\text{cons} \\
\gamma3\text{cons} \\
\gamma4\text{cons} \\
\gamma5\text{cons} \\
\gamma6\text{cons} \\
\gamma7\text{cons} \\
\gamma8\text{cons}
\end{pmatrix}
\equiv
\begin{pmatrix}
U_{HHH} - R - v \\
U_{HHH} - U_{HHH} \\
U_{HHH} - U_{HHL} \\
U_{HHH} - U_{HLH} \\
U_{HHH} - U_{HLL} \\
U_{HHH} - U_{LHH} \\
U_{HHH} - U_{LHL} \\
U_{HHH} - U_{LLH} \\
U_{HHH} - U_{LLL}
\end{pmatrix}
$$

$$
=
\begin{pmatrix}
-2k - R - v + pq\sigma_{H1} - \frac{1}{2} p(-1+q)(\sigma_{H2} + \sigma_{H3}) - (-1+p)q\sigma_{L1} \\
\qquad + \frac{1}{2}(-1+p)(-1+q)(\sigma_{L2} + \sigma_{L3}) \\[4pt]
0 \\[4pt]
k(-1+p) - \frac{1}{2}(-1+p)(-1+3q)(\sigma_{L1} - \sigma_{L2}) \\[4pt]
-kp + \frac{1}{2}p(-1+3q)(\sigma_{H1} - \sigma_{H2}) \\[4pt]
-k + \frac{1}{2}p(-1+3q)(\sigma_{H1} - \sigma_{H2}) - \frac{1}{2}(-1+p)(-1+3q)(\sigma_{L1} - \sigma_{L2}) \\[4pt]
-k + \frac{1}{2}(p-2q+pq)(\sigma_{H2} - \sigma_{L1}) + \frac{1}{2}(-1+p+q+pq)(\sigma_{H1} - \sigma_{L2}) \\
\qquad - \frac{1}{2}(-1+2p)(-1+q)(\sigma_{H3} - \sigma_{L3}) \\[4pt]
k(-1-p) + \frac{1}{2}(p-2q+pq)(\sigma_{H2} - \sigma_{L1}) - \frac{1}{2}(-1+2p)(-1+q)(\sigma_{H3} - \sigma_{L2}) \\
\qquad + \frac{1}{2}(-1+p+q+pq)(\sigma_{H1} - \sigma_{L3}) \\[4pt]
k(-2+p) + \frac{1}{2}(p-2q+pq)(\sigma_{H3} - \sigma_{L1}) + \frac{1}{2}(-1+p+q+pq)(\sigma_{H1} - \sigma_{L2}) \\
\qquad - \frac{1}{2}(-1+2p)(-1+q)(\sigma_{H2} - \sigma_{L3}) \\[4pt]
-2k + \frac{1}{2}(p-2q+pq)(\sigma_{H3} - \sigma_{L1}) - \frac{1}{2}(-1+2p)(-1+q)(\sigma_{H2} - \sigma_{L2}) \\
\qquad + \frac{1}{2}(-1+p+q+pq)(\sigma_{H1} - \sigma_{L3})
\end{pmatrix}
$$

The first constraint ensures that the contract allows the agent to make at least the reservation payoff when exerting high effort in both periods, and receives back the wealth invested in the project. The remaining eight constraints ensure that the HHH strategy dominates all other strategies. The constraint $\gamma1$cons is included for completeness. There are also six other constraints generated by the limited liability restriction:

$\sigma_{H1}$, $\sigma_{H2}$, $\sigma_{H3}$, $\sigma_{L1}$, $\sigma_{L2}$, $\sigma_{L3} \geq 0$.

Because the interim monitoring results are shared, there are two additional constraints. Suppose the monitoring process correctly assess the first period effort as high. The contract must ensure that high effort in the second period is still the optimal decision for the agent conditional on this monitoring information.

If the monitoring results confirm high effort in the first period then the agent will be paid $\sigma_{H1}$ if the project pays $\tau + V$, $\sigma_{H2}$ if the project pays $V$, and $\sigma_{H3}$ if the project pays $V - \tau$. If the agent exerts high effort in the second period then the probability of these payoffs occurring are:

$$q, \quad \frac{1}{2}(1-q), \quad \text{and} \quad \frac{1}{2}(1-q), \quad \text{respectively.}$$

If the agent exerts low effort in the second period then the probability of these payoffs occurring are:

$$\frac{1}{2}(1-q), \quad q, \quad \text{and} \quad \frac{1}{2}(1-q), \quad \text{respectively.}$$

Therefore, the following condition will ensure that high effort in the second period is still the optimal decision for the agent conditional on this monitoring information confirming high effort in the first period:

$$-k + q\sigma_{H1} + \frac{1}{2}(1-q)\sigma_{H2} + \frac{1}{2}(1-q)\sigma_{H3}$$

$$\geq \frac{1}{2}(1-q)\sigma_{H1} + q\sigma_{H2} + \frac{1}{2}(1-q)\sigma_{H3}$$

Suppose instead that the monitoring results incorrectly assess the first period effort as low. The contract must ensure that high effort in the second period is still the optimal decision for the agent given this time one information. Using similar arguments to those stated above, the following condition would ensure this is the case:

$$-k + q\sigma_{L1} + \frac{1}{2}(1-q)\,\sigma_{L2} + \frac{1}{2}(1-q)\,\sigma_{L3}$$

$$\geq \frac{1}{2}(1-q)\,\sigma_{L1} + q\sigma_{L2} + \frac{1}{2}(1-q)\,\sigma_{L3}$$

Given high effort in both periods and the contract ($\sigma_{H1}$, $\sigma_{H2}$, $\sigma_{H3}$, $\sigma_{L1}$, $\sigma_{L2}$, $\sigma_{L3}$) the expected payoff to the principal is:

$$V_{HHH} \equiv -m + V + \frac{1}{2}(-1 + 3q)\,\tau - pq\sigma_{H1} + \frac{1}{2}p(-1+q)(\sigma_{H2} + \sigma_{H3})$$

$$+ (-1+p)\,q\sigma_{L1} - \frac{1}{2}(-1+p)(-1+q)(\sigma_{L2} + \sigma_{L3})$$

To find the optimal contract we have the following program:

Maximize $V_{HHH}$ over values of ($\sigma_{H1}$, $\sigma_{H2}$, $\sigma_{H3}$, $\sigma_{L1}$, $\sigma_{L2}$, $\sigma_{L3}$), subject to:

(A1)    $\lambda$cons, $\gamma$1cons, $\gamma$2cons,…, $\gamma$8cons $\geq 0$;

(A2)    $\sigma_{H1}$, $\sigma_{H2}$, $\sigma_{H3}$, $\sigma_{L1}$, $\sigma_{L2}$, $\sigma_{L3} \geq 0$;

(A3)    $-k + \frac{1}{2}(-1 + 3q)\,\sigma_{H1} + \frac{1}{2}(1 - 3q)\,\sigma_{H2} \geq 0$   and

$$-k + \frac{1}{2}(-1 + 3q)\,\sigma_{L1} + \frac{1}{2}(1 - 3q)\,\sigma_{L2} \geq 0$$

The method for finding the optimal contract for each level of monitoring resources, m, is similar to the methods used in the non-communication case and are not repeated in this part of the dissertation.

## 10.1 Region 1

Region 1 represents projects with high spread of project payouts and low cost of agent effort. This region is interesting as it will illustrate values of the parameters of the problem where sharing of interim monitoring information is never the strategy that maximizes the expected payoff to the principal. This will be in contrast to region 2 below

where sharing can be the best strategy, and the sharing of information will shift the risk between the two parties.

In this region, solution of the linear programming methodology described above shows the only optimal strategy for the principal is to induce the agent to exert high effort in the first period and high effort in second period whatever the outcome of monitoring – strategy HHH. With this strategy, the agent's expected compensation is at the reservation level. The form of the contract is:

$$\left\{ \frac{2\,k\,(-1+2\,q+p\,q) + (-1+3\,q)\,(R+v)}{p\,q(-1+3\,q)}, \; 0, \; 0, \; \frac{2\,k}{-1+3\,q}, \; 0, \; 0 \right\}$$

As with the no-communication case, in region 1, the principal can induce the agent to exert high effort in both periods with contractual means only; monitoring is not an optimal strategy. The same is true for the communication case. Since there can be no improvement in expected payoffs from the projects and since there will be no shifting of resources between the agent and principal, monitoring will not be an optimal strategy for these projects.

Suppose the principal is obliged to monitor in this region because of some institutional requirement.[61] Is communication an optimal strategy in these restricted cases? Given identical monitoring functions, the principal will be indifferent between communication and lack of communication. This is the case since in both instances the agent exerts high effort in both periods and receives expected payments at the reservation level. Therefore, there can be no improvement in the principal's position through changes in effort or redistribution of wealth because of communication.

---

[61] For instance, a financial institution maybe required to monitor investments because of regulations or other institutional requirements.

The above conclusion is true since both contracts lead to the same expected

payout to both parties and ensures the agent's limited liability constraint is not violated.

However, there are additional observations that can be made concerning the variance of

the agent's payouts. When interim sharing of monitoring results is agreed on, the contract

is no longer strictly dichotomous; there are payouts in two possible outcomes. This is in

contrast to the non-communication contract, which is shown below:

$$\left\{\frac{2k+R+\upsilon}{pq}, \ 0, \ 0, \ 0, \ 0, \ 0\right\}$$

**Result XXIII:** In region 1, there exist sub regions where the variance of the

agent's payouts is less if a communication strategy is adopted.

**Proof XXIII**: Direct comparison of the two variances confirm this result. The

regions where the variance is lower and higher are set out in appendix C. ∎

Therefore, use of a communication strategy even though it does not change the

expected payouts of the principal or the agent would shift some risk associated with

imperfect monitoring from the agent to the principal.

## 10.2 Region 2

Region 2 represents projects with high dispersion of profits and moderate cost of

agent effort. In this region, solution of the linear programming methodology described

above yields five possible communication contracts. The first contract is the same

contract that induces a HHH strategy by the agent in region 1: It yields the reservation

payment to the agent and induces high effort in both periods regardless of the results of

monitoring communicated at time one. This contract will be defined as HHH1. The

contract dominates other HHH contracts over the second region when:

$$\frac{(-1+q)(2k(-1+2q)+(-1+3q)(R+v))}{2k(-1+q)^2-(1+q)(-1+3q)(R+v)} \leq p(m) < 1$$

The second contract is also a HHH contract but yields a non-reservation expected payout to the agent. This contract will be defined as HHH2. This contract dominates other HHH contracts over the second region when:

$$\frac{1}{2} < p(m) < \frac{(-1+q)(2k(-1+2q)+(-1+3q)(R+v))}{2k(-1+q)^2-(1+q)(-1+3q)(R+v)}$$

The next two contracts are both HHL contracts. The first yields a reservation expected payout to the agent; the other compensates the agent with an expected payout in excess of the reservation payment. These contracts will be defined as HHL1 and HHL2, respectively. Their regions of dominance can be described as follows:

$$-\frac{(-1+3q)(R+v)}{2(-1+q)} \leq k \leq -\frac{2(-1+3q)(R+v)}{3(-1+q)} \quad \text{or}$$

$$-\frac{2(-1+3q)(R+v)}{3(-1+q)} < k \leq -\frac{(-1+3q)(R+v)}{-1+q} \quad \text{and}$$

$$\frac{(1+q)(R+v)-\sqrt{4k^2(-1+q)^2+4k(-1+q)^2(R+v)+(1+q)^2(R+v)^2}}{2k(-1+q)} \leq p(m) < 1$$

For the first, and

$$-\frac{2(-1+3q)(R+v)}{3(-1+q)} < k \leq -\frac{(-1+3q)(R+v)}{-1+q} \quad \text{and}$$

$$\frac{1}{2} < p(m) < \frac{(1+q)(R+v)-\sqrt{4k^2(-1+q)^2+4k(-1+q)^2(R+v)+(1+q)^2(R+v)^2}}{2k(-1+q)}$$

For the second.

The final contract is an HLH contract. With this contract, the reservation constraint does not bind. This contract will be defined as HLH1. Its region of dominance is found to be:

$$-\frac{2\,(-1+3\,q)\,(R+\upsilon)}{3\,(-1+q)} < k \le -\frac{(-1+3\,q)\,(R+\upsilon)}{-1+q} \quad \text{and}$$

$$\frac{1}{2} < p\,(m) < \frac{2\,q-\sqrt{2}\,\sqrt{-(-1+q)\,q}}{-1+3\,q} \quad \text{or}$$

$$-\frac{(-1+3\,q)\,(R+\upsilon)}{2\,(-1+q)} \le k \le -\frac{2\,(-1+3\,q)\,(R+\upsilon)}{3\,(-1+q)} \quad \text{and}$$

$$\frac{X_1 + \sqrt{X_2}}{2\,k\,(-1+q)\,(-1+3\,q)} < p\,(m) \le \frac{2\,q-\sqrt{2}\,\sqrt{-(-1+q)\,q}}{-1+3\,q}$$

Where

$$X_1 = 2\,k\,(-1+q)^2 - (1+q)\,(-1+3\,q)\,(R+\upsilon), \quad \text{and}$$
$$X_2 = 4\,k^2\,(-1+q)^2\,(1-4\,q+7\,q^2)$$
$$+4\,k\,(-1+q)\,(-1+3\,q)\,(1-2\,q+5\,q^2)\,(R+\upsilon) + (1+q)^2\,(-1+3\,q)^2\,(R+\upsilon)^2.$$

To find the dominant communication contracts over the second region the expected payouts to the principal from each of these contracts need to be compared over their common regions of influence. The expected payout to the principal for each of the five contracts is given below:

$$-2\,k - m - R + V + \frac{1}{2}\,(-1+3\,q)\,\tau - \upsilon \quad \text{for} \quad \text{HHH1}$$

$$-m - \frac{2\,k\,q\,(-1+q+4\,p\,q)}{(-1+3\,q)\,(-1+p+q+p\,q)} + V + \frac{1}{2}\,(-1+3\,q)\,\tau \quad \text{for} \quad \text{HHH2}$$

$$-m + k\,(-1-p) - R + V + \frac{1}{2}\,p\,(-1+3\,q)\,\tau - \upsilon \quad \text{for} \quad \text{HHL1}$$

$$-m - \frac{2\,k\,p\,(1+p)\,q}{-1+p+q+p\,q} + V + \frac{1}{2}\,p\,(-1+3\,q)\,\tau \quad \text{for} \quad \text{HHL2}$$

$$-m + \frac{2\,k\,q\,(-1+p-p^2+5\,q-7\,p\,q+3\,p^2\,q)}{(-1+3\,q)\,(p-2\,q+p\,q)} + V$$
$$-\frac{1}{2}\,(-1+p)\,(-1+3\,q)\,\tau \quad \text{for} \quad \text{HLH1}$$

Results XXIV through XXVI establish the conditions necessary for HHH1 to be the preferred strategy.

**Result XXIV:** In region 2, the set of parameter values for which HHH1 is the HHH strategy that maximizes expected payouts to the principal is a subset of parameter values for which HHL1 is the HHL strategy that maximizes expected payouts to the principal.

**Proof XXIV:** The set of parameters where the HHL1 strategy maximizes the expected payout to the principal is a union of two disjoint subsets. These subsets span region 2 when considering the parameters q, k, and $\tau$. In the first of these subsets defined by:

$$-\frac{2(-1+3q)(R+v)}{3(-1+q)} < k \leq -\frac{(-1+3q)(R+v)}{-1+q}$$

There is no restriction of the p parameter. Therefore, to complete the proof it is necessary for the following condition to hold in the second of the disjoint subsets:

$$\frac{(1+q)(R+v) - \sqrt{4k^2(-1+q)^2 + 4k(-1+q)^2(R+v) + (1+q)^2(R+v)^2}}{2k(-1+q)}$$
$$\leq \frac{(-1+q)(2k(-1+2q)+(-1+3q)(R+v))}{2k(-1+q)^2 - (1+q)(-1+3q)(R+v)}$$

Direct calculation shows this to be true within the sub set:

$$-\frac{2(-1+3q)(R+v)}{3(-1+q)} < k \leq -\frac{(-1+3q)(R+v)}{-1+q} \qquad \blacksquare$$

Thus, in the region 2, the set of parameter values for which HHH1 is the HHH strategy that maximizes expected payouts to the principal is a subset of parameter values for which HHL1 is the HHL strategy that maximizes expected payouts to the principal.

Next, I will show that the expected payout to the principal is greater for the contract HHH1 when compared to contract HHL1.

**Result XXV:** In region 2, the expected payout to the principal from contract HHH1 is greater than the expected payout from contract HHL1.

**Proof XXV:** The expected payout to the principal from contract HHH1 less the expected payout from contract HHL1 is given by:

$$-2k - m - R + V + \frac{1}{2}(-1 + 3q)\tau - \upsilon$$

$$+ m - k(-1-p) + R - V - \frac{1}{2}p(-1+3q)\tau + \upsilon$$

$$= \frac{1}{2}(-1+p)(2k-(-1+3q)\tau)$$

This will be strictly positive if $-2k - (1 - 3q)\tau > 0$. This is true in region 2 ∎

Since the set of parameter values for which HHL1 is the HHL strategy that maximizes expected payouts to the principal, and the set of parameters for which HHL2 is the HHL strategy that maximizes expected payouts to the principal are disjoint and span region 2 then from result II, it is unnecessary to compare expected payouts to the principal from contracts HHH1 and HHL2.

**Result XXVI:** In the set of parameter values for which HLH1 is the HLH strategy that maximizes expected payouts to the principal, the expected payout to the principal from contract HHL1 is greater than the expected payout from contract HLH1.

**Proof XXVI:** The expected payout to the principal from contract HHL1 less the expected payout from contract HLH1 is given by:

$$-2k - m - R + V + \frac{1}{2}(-1 + 3q)\tau - \upsilon$$

$$+ m - \frac{2kq(-1+p-p^2+5q-7pq+3p^2q)}{(-1+3q)(p-2q+pq)} - V + \frac{1}{2}(-1+p)(-1+3q)\tau$$

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$$= -\frac{2\,k\left((1-q)\,q + p^2\,q(-1+3\,q) + p\left(-1+3\,q-4\,q^2\right)\right)}{(-1+3\,q)\,(p-2\,q+p\,q)}$$
$$-R + \frac{1}{2}\,p(-1+3\,q)\,\tau - \upsilon$$

Direct calculation establishes that this is strictly positive over the set of parameter values for which HHH1 is the HHH strategy that maximizes expected payouts to the principal. ∎

Thus, result XXVI establishes that contract HHL1 is preferred to contract over the set of parameter values for which HHH1 is the HHH strategy that maximizes expected payouts to the principal. Therefore, results XXIV, XXV, and XXVI establish that contract HHH1 is the preferred communication contract in region in the region of dominance of HHH1. Thus, this contract will dominate other communication contracts over the entire region when:

$$p(m) \geq \frac{(-1+q)\,(2\,k\,(-1+2\,q) + (-1+3\,q)\,(R+\upsilon))}{2\,k\,(-1+q)^2 - (1+q)\,(-1+3\,q)\,(R+\upsilon)}$$

The other four communication contracts will be dominant in sub regions of region 2 in the cases where:

$$p(m) < \frac{(-1+q)\,(2\,k\,(-1+2\,q) + (-1+3\,q)\,(R+\upsilon))}{2\,k\,(-1+q)^2 - (1+q)\,(-1+3\,q)\,(R+\upsilon)}$$

To find the preferred communication contracts in this region for the four contracts it is necessary to find the intersection of the regions of dominance of the remaining four contracts and then compare expected payouts to the principal offered by each viable contract in the resulting sub regions. Using similar methods to those applied in results XXIV, XXV, and XXVI the conditions for the remaining four contracts to be optimal are set out in appendix D.

For the remaining four contracts, the set of parameters over which they are the preferred principal strategy are easier to interpret graphically as a full description using equations is cumbersome, as can be seen from appendix D. The first of these contracts also induces a HHH strategy by the agent but results in a non-reservation expected payoff to the agent. The second of these contracts induces a HHL strategy by the agent and results in a reservation expected payoff to the agent. The third of these contracts also induces a HHL strategy by the agent and results in a non-reservation expected payoff to the agent. The fourth of these contracts induces a HLH strategy by the agent and results in a non-reservation expected payoff to the agent. A summary of the regions of dominance of these four contracts is described in table 5. In this table for each column

$$\Pi \equiv (1 - q)[2(1 - 2q)k - (3q - 1)(R + \upsilon)]/[2k(1 - q)^2 - (1 + q)(3q - 1)(R + \upsilon)].$$

In addition, the contracts are described in terms of the sub-regions of region 2 described in figure 4.

**Table 5: Comparison of Regions of Dominance of Monitoring Contracts with Communication**

|  | Communication Contracts when $p(m^*) \geq \Pi$ | Communication Contracts when $p(m^*) < \Pi$ |
|---|---|---|
| Region 2a | HHH with binding reservation constraint. | HHH with non-binding reservation constraint. |
| Region 2b | HHH with binding reservation constraint. | HHH with non-binding reservation constraint, or HHL with binding reservation constraint. |
| Region 2c, $1/3 < q < 1/2$ | HHH with binding reservation constraint. | HHH with non-binding reservation constraint, or HLH with non-binding reservation constraint. |
| Region 2c, $1/2 \leq q < 1$ | HHH with binding reservation constraint. | HHH with non-binding reservation constraint, or HHL with non-binding reservation constraint. |
| Region 2d, $1/3 < q < 1/2$ | HHH with binding reservation constraint. | HHH with non-binding reservation constraint. |
| Region 2d, $1/2 \leq q < 1$ | HHH with binding reservation constraint. | HHH with non-binding reservation constraint, or HHL with non-binding reservation constraint. |

The interesting comparisons arise when the principal can decide on whether to communicate the results to the agent at the interim stage. Referring back to the non-communication case, region 2 projects were dominated by two monitoring contracts. Both contracts induced the agent to exert high effort in both periods. The first contract resulted in the binding of the reservation constraint and dominated for monitoring functions where the optimal level of monitoring is such that:

$$p(m) \geq \frac{(-1+q)(2k+R+v)}{2k(-1+q)-(1+q)(R+v)}$$

The second contract resulted in the non-binding of the reservation constraint and dominated for monitoring functions where the optimal level of monitoring is such that:

$$p(m) < \frac{(-1+q)(2k+R+v)}{2k(-1+q)-(1+q)(R+v)}$$

**Result XXVII:** In region 2, communication is never an optimal strategy for projects with monitoring functions where the optimal level of monitoring is such that:

$$p(m) \geq \frac{(-1+q)(2k+R+v)}{2k(-1+q)-(1+q)(R+v)}$$

**Proof XXVII:** Direct observation shows that the second non-communication contract does not fulfill this condition. Thus, it remains to show that the first non-communication dominates the five communication contracts in the region defined by:

$$p(m) \geq \frac{(-1+q)(2k+R+v)}{2k(-1+q)-(1+q)(R+v)}$$

Direct calculation shows that all communication contracts fulfill this condition with the exception of HHL2.

For HHH1 the expected payout to the principal is given by:

$$-2k-m-R+V+\frac{1}{2}(-1+3q)\tau-v$$

This is identical to the expected payout from the non-communication contract.

For the other three communication contracts, direct calculation shows that the expected payout to the principal will be less than that earned from the non-communication contract in the respective regions of dominance. ∎

Result XXVII shows that the first non-communication contract dominates all communication contracts over its region of dominance. For the second, the analysis set out in result XXVIII below show conditions for communication to be optimal:

**Result XXVIII:** In region 2 for projects with monitoring function optimal solutions of the form $p(m^*) < (1 - q)(2k + R + \upsilon)/[2k(1 - q) + (1 + q)(R + \upsilon)]$, the HHH non-reservation communication strategy is dominated by the second non-communication contract. All other communication strategies in this region – the third communication contract that induces a HHL strategy by the agent and results in a reservation expected payoff to the agent; the fourth communication contract that induces a HHL strategy by the agent and results in a non-reservation expected payoff to the agent; and the fifth contract that induces a HLH strategy by the agent and results in a non-reservation expected payoff to the agent – are all dominant over the same sub regions described in the communication analysis. These results are summarized in table 5.

**Proof XXVIII:** The expected payout from contract HHH2 minus the expected payout from the second non-communication contract is given by:

$$V + \frac{1}{2}(-1 + 3q)\tau - m - \frac{2kq(-1 + q + 4pq)}{(-1 + 3q)(-1 + p + q + pq)}$$

$$-V - \frac{1}{2}(-1 + 3q)\tau + m + \frac{4kpq}{-1 + p + q + pq}$$

$$= \frac{2k(-1 + 2p)(-1 + q)q}{(-1 + 3q)(-1 + p + q + pq)}$$

This will be strictly positive when $1 - p - q - qp > 0$, but since $\frac{1}{2} < p < 1$ and $1/3 < q < 1$, then HHH2 is strictly dominated by the second non-communication contract.

For the remaining three contracts, the same methodology yields the solutions set out in appendix E. ∎

In a sense, the ability to communicate alters the marginal projects – those projects that are described as region 2 but are close to the boundary with region 4; these projects have a payout dispersion that is high but a small reduction in this dispersion level would convert these projects to moderate payout dispersion project found in region 4. Without communication, projects that are described in region 2 are subject to incentives and monitoring levels that induce the agent to exert high effort in both periods; projects that are described in region 4 are subject to incentives and monitoring levels that induce the agent to exert high effort in the first period and low effort in the second period. For those projects that lie close to the boundary of these two regions, communication of monitoring results at the interim stage allows the agent to change effort levels in the second period; thus, communication allows for an intermediate strategy for the agent. The principal benefits since the agent is encouraged to exert lower effort and this requires a lower transfer of wealth from the principal to the agent. The agent benefits because of a reduction in effort level when the principal receives an unfavorable signal from monitoring activity.

Further analysis can be undertaken to understand which party benefits from communication and what are the mechanisms that generate this benefit. To study this aspect of the relationship, the analysis will be completed for HHL2 as the analysis is the

simplest and similar conclusions can be drawn from the analysis of the other contracts. The following result is true for both HHL1 and HHL2:

**Result XXIX:** In region 2, the expected payout to the agent is always strictly less if communication is the principal's preferred strategy and HHL1 and HHL2 are the optimal contracts.

**Proof XXIX**: Communication is a preferred strategy in regions where the second non-communication contract dominates other non-communication contracts. This contract rewards the agent with an expected payout in excess of the reservation payout. Since HHL1 yields an expected payout equal to the reservation payout then if HHL1 is the preferred contract, the expected payout to the agent is strictly less if communication is implemented.

For the contract HHL2, the expected payout to the agent from the second non-communication contract less the expected payout from the communication contract is given by:

$$\frac{2\,k\,(-1+p)\,(-1+q)}{-1+p+q+pq} - \frac{k\left(-1+p^2\right)(-1+q)}{-1+p+q+pq} = -\frac{k\,(-1+p)^2\,(-1+q)}{-1+p+q+pq}$$

This is strictly positive since $1 - p - q - qp < 0$. ∎

When the HHL2 communication contract is the optimal solution to the principal-agent relationship, there are many effects on the agent. With communication, this contract dominates the non-communication, non-reservation strategy inducing high effort in both periods. The details of the five contracts and their expected payouts to both parties are set out in table 6.

**Table 6: Comparison of Contracts and Expected Payouts in Region 2**

| Strategy | Contract | Expected Payout – Principal | Expected Payout – Agent |
|---|---|---|---|
| High – high no communication first contract | $\left\{\dfrac{2k}{p(-1+3q)}, 0, 0,\right.$ $\left. 0, 0, 0\right\}$ | $-m+\dfrac{2kq}{1-3q}+V+\dfrac{1}{2}(-1+3q)\tau$ | $\dfrac{k(2-4q)}{-1+3q}$ |
| High – high no communication second contract | $\left\{\dfrac{4k}{-1+p+q+pq}, 0, 0,\right.$ $\left. 0, 0, 0\right\}$ | $-m-\dfrac{4kpq}{-1+p+q+pq}$ $+V+\dfrac{1}{2}(-1+3q)\tau$ | $\dfrac{2k(-1+p)(-1+q)}{-1+p+q+pq}$ |
| HHL with binding reservation constraint | $\left\{\dfrac{k(1+p)+R+\upsilon}{pq}, 0, 0,\right.$ $\left. 0, 0, 0\right\}$ | $-m+k(-1-p)-R+V$ $+\dfrac{1}{2}p(-1+3q)\tau-\upsilon$ | $R+\upsilon$ |
| HHL with non-binding reservation constraint | $\left\{\dfrac{2k(1+p)}{-1+p+q+pq}, 0, 0,\right.$ $\left. 0, 0, 0\right\}$ | $-m-\dfrac{2kp(1+p)q}{-1+p+q+pq}$ $+V+\dfrac{1}{2}p(-1+3q)\tau$ | $\dfrac{k(-1+p^2)(-1+q)}{-1+p+q+pq}$ |
| HLH with non-binding reservation constraint | $\left\{0, \dfrac{2k}{-1+3q}, 0,\right.$ $\left.\dfrac{2(k-5kq+4kpq)}{(-1+3q)(p-2q+pq)}, 0, 0\right\}$ | $\dfrac{2kq(-1+p-p^2+5q-7pq+3p^2q)}{(-1+3q)(p-2q+pq)}$ $+V+\dfrac{1}{2}(1-p)(3q-1)\tau-m$ | $\dfrac{k(1-q)(2p-p^2-2q-2pq+3p^2q)}{(3q-1)(p-2q+pq)}$ |

Consider the variance of the payout to the agent, where payout is measured after consideration of effort exerted. To calculate these values for the two contracts it is necessary to consider the payout in each possible contractual state and its associated probability; the information is set out in table 7.

**Table 7: Payouts from the Second Non-Communication Strategy and from the Communication Strategy HHL2**

No Communication

| Payout to Agent, Net of Effort | Probability |
|---|---|
| $\dfrac{4k}{-1+p+q+pq}-2k$ | $qp$ |
| $-2k$ | $1-qp$ |

With Communication

| Payout to Agent, Net of Effort | Probability |
|---|---|
| $\dfrac{2k(1+p)}{-1+p+q+pq}-2k$ | $pq$ |
| $-2k$ | $p(1-q)$ |
| $-k$ | $1-p$ |

The variance for the non-communication and communication contract payouts are respectively:

$$\frac{16k^2\,p\,q\left(1-3\,p\,q+2\,p^2\,q^2\right)}{\left(-1+p+q+p\,q\right)^2} \quad \text{and}$$

$$\begin{aligned}
\big(k^2\,p\big(&-1+12\,q+2\,p^4\left(-1+q\right)^2q-9\,q^2+2\,q^3+p\left(3-6\,q-17\,q^2+8\,q^3\right)\\
&+p^3\left(1-2\,q-7\,q^2+8\,q^3\right)+p^2\left(-3+10\,q-11\,q^2+12\,q^3\right)\big)\big)\big/\left(-1+p+q+p\,q\right)^2
\end{aligned}$$

**Result XXX:** In the sub region of region 2, where the communication strategy HHL2 is the optimal contracts amongst all monitoring strategies, there exist regions where the variance of the agent's payouts are less than if a non-communication strategy is adopted.

**Proof XXX**: Direct comparison of the two variances described above in the region of dominance of strategy HHL2 confirm this result. The regions where the variance is lower are set out in appendix E. ∎

Therefore, in some circumstances the use of communication reduces the variance of the payouts to the agent. Therefore, the additional risk, assumed by the agent due to the noise associated with the monitoring signal can be reduced by the communication of monitoring results at the interim stage. This reductions comes at a cost, however; the reduction in the expected payout to the agent. In order to understand whether the trade-off is worthwhile – lower variance but lower expected payouts – changes in the coefficient of variation of project payouts will be considered. The coefficient of variation is defined as:

Standard deviation of payouts / expected value of payouts to the agent.

**Result XXXI:** In the sub-region of region 2, where the communication strategy HHL2 is the optimal contracts amongst all monitoring strategies, there exist regions where the coefficient of variation of the agent's payouts is greater than if a non-communication strategy was adopted.

**Proof XXXI**: Direct comparison of the two coefficients of variation in the region of dominance of strategy HHL2 confirm this result. The regions where the coefficients of variation is higher and lower are set out in appendix D. ∎

These results can be considered in terms of the general question as to when it would be advantageous for the principal to agree to share interim-monitoring information with the agent. Unless monitoring is perfect, the estimate of agent effort generated from monitoring will always produce a noisy signal of actual effort. Since there is some probability that the principal will receive the wrong signal, the agent will require a higher level of incentives to compensate for this risk.[62] When communication is introduced into the model, it allows the agent to decide whether it is worthwhile to invest high effort in the second period contingent on the outcome of the period one monitoring; the results of monitoring will inform the agent whether high effort in the future has any chance of being rewarded. So for example, suppose the preferred effort level is for the agent to exert high effort in both periods. If at time one, the monitoring results produce the erroneous signal that the agent exerted low effort during that period the agent can now change his strategy in the second period. This will benefit both principal and agent since some of the risk associated with monitoring is shifted from the agent to the principal.

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[62] This will be true even though the agent is risk neutral. Because of limited liability, the non-incentive components of the contract result in the agent losing his limited liability value, $\upsilon$. Therefore, if the probability of receiving the incentive component is reduced because of imperfect monitoring, the value paid to the agent in that state will need to be increased to ensure the agent remains *incentivised* to work at the level preferred by the principal.

Since the principal does not have limited liability, the risk has no impact on the

principal.[63]

Since the solutions to the principal-agent relationship were found using a linear

programming methodology, there exist dual problems for the original primal problems.

For example, the primal problem used to find contractual solutions that induce the

entrepreneur to exert high effort in both periods is described by the following program:

Minimize:

$$(-2k - R - v)\lambda_1 + k(-1 + p)\lambda_2 - kp\lambda_3 - k\lambda_4 - k\lambda_5$$
$$+ k(-1 - p)\lambda_6 + k(-2 + p)\lambda_7 - 2k\lambda_8 - k\lambda_9 - k\lambda_{10}$$

over values of $(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9, \lambda_{10})$ subject to

$$-2pq\lambda_1 - p(-1 + 3q)(\lambda_3 + \lambda_4) +$$
$$(-1 + p + q + pq)(-\lambda_5 - \lambda_6 - \lambda_7 - \lambda_8) + (1 - 3q)\lambda_9 \geq -2pq$$

$$(p - 2q + pq)(-\lambda_5 - \lambda_6) + (-1 + q)(p\lambda_1 - \lambda_7 + 2p\lambda_7 - \lambda_8 + 2p\lambda_8)$$
$$+ (-1 + 3q)(p\lambda_3 + p\lambda_4 + \lambda_9) \geq -(1 - q)p$$

$$p(-1 + q)\lambda_1 + (-1 + 2p)(-1 + q)(\lambda_5 + \lambda_6) - (p - 2q + pq)(\lambda_7 + \lambda_8) \geq -(1 - q)p$$

$$2(-1 + p)q\lambda_1 + (-1 + p)(-1 + 3q)(\lambda_2 + \lambda_4)$$
$$+ (p - 2q + pq)(\lambda_5 + \lambda_6 + \lambda_7 + \lambda_8) + (1 - 3q)\lambda_{10} \geq -2(1 - p)q$$

$$-(-1 + p)(-1 + 3q)(\lambda_2 + \lambda_4) + (-1 + p + q + pq)(\lambda_5 + \lambda_7) + (-1 + q)(-(-1 + p)\lambda_1$$
$$- (-1 + 2p)(\lambda_6 + \lambda_8)) + (-1 + 3q)\lambda_{10} \geq -(-1 + p)(-1 + q)$$

$$-(-1 + p)(-1 + q)\lambda_1 - (-1 + 2p)(-1 + q)(\lambda_5 + \lambda_7)$$
$$+ (-1 + p + q + pq)(\lambda_6 + \lambda_8) \geq -(-1 + p)(-1 + q)$$

$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9, \lambda_{10} \geq 0$.

The solution of dual problem in each case confirms the results of the primal

problem analysis in regions 1 and 2 and describes shadow prices for the constraints

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[63] Of course, the agent's still faces risk from his incentive contract as this is necessary to ensure the agent exerts effort at the preferred level.

described in the primal problems. For solutions of the dual problem where the reservation constraint was found to bind, the shadow prices for k and $R + \upsilon$ is 1. In other words, market conditions lead to a reduction in the cost of high effort, reservation payment, or level of limited liability then this will lead to a corresponding increase in the expected payoff to the principal.

In the remaining solutions, the shadow price for R is always zero. This is not surprising since the reservation constraint does not bind in these solutions. For the HHL contract with non-binding reservation constraint, the shadow price for the agent's cost of effort k is:

$$\frac{2pq}{(1+p)(-1+p+q+pq)}$$

Thus, a reduction in the agent's cost of effort improves the expected payoff to the principal. This shadow price is strictly decreasing in both parameters. Therefore, projects with high levels of probability of success q, or monitoring intensity see less improvement in expected payoff to the principal from a reduction in the agent's cost of effort, compared to projects with lower levels of the two parameters.

For the HLH contract with non-binding reservation constraint, the shadow price for the agent's cost of effort k is:

$$\frac{2(2p-1)(1-q)q}{(1-p)(3q-1)(p-2q+pq)} \quad \text{when} \quad -\frac{-5+q+\sqrt{17-18q+q^2}}{2(1+q)} \leq p < \frac{2q}{1+q}$$

For all other values of the probability, p, the shadow price is:

$$\frac{2q(p^2(1-3q)-2q+4pq)}{(2-3p+p^2)(-1+3q)(p-2q+pq)}$$

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The shadow prices for the cost of effort for both HHL2 and HLH1 are lower than the shadow price when communication is not undertaken. Therefore, communication lessens the impact of cost of effort on the principal-agent relationship.

# CHAPTER 11

# CONCLUDING COMMENTS: MONITORING WITH COMMUNICATION

Part 2 of this thesis extends research on the relationship between principal and agent where moral hazard exists and contractual payments to the agent are restricted by a limited liability constraint. A simple model was used to investigate the impact of incentives and monitoring on this relationship. Actual contractual forms were derived and their structure was investigated. Furthermore, the effects of the impact of the communication of monitoring results at an interim stage was considered.

When the principal is in a position to share the results of monitoring with the agent at an interim stage of the project, thus allowing the agent to modify his future effort, I find that sharing of interim information is an optimal strategy for "marginal" projects. Marginal projects are those that reside on the boundary between regions of projects that have differing levels of effort in the no-monitoring cases. The principal benefits since the agent is encouraged to exert lower effort and this requires a lower transfer of wealth from the principal to the agent. The agent benefits because of a reduction in effort level when the principal receives an unfavorable signal from monitoring activity.

Since sharing of interim monitoring results can lead to reduced effort in the second period, and leads to contracts that payout in more than on sate of the world, the risk associated with monitoring can be shifted from the agent to the principal. Since the principal has unlimited liability this is beneficial to the agent.

# APPENDIX A

# PROOF OF RESULTS

**Proof I:** First, the contracts satisfy the limited liability constraints. For the two contracts, this will be the case if $[2k + R + (1 - q)\upsilon]/q \geq -\upsilon$ and $4k/(3q - 1) - \upsilon \geq -\upsilon$. This is the case for both contracts given the possible values of the parameters of the model. Next, both contracts satisfy constraints $\lambda$cons, $\gamma$1cons, $\gamma$2cons, $\gamma$3cons $\geq 0$. For the first contract since:

$\lambda$cons $= q[2k + R + (1 - q)\upsilon]/q - 2(1 - q)\upsilon/2 - 2k - R = 0$,

$\gamma$1cons $= 0$,

$\gamma$2cons $= [(3q - 1)(R + \upsilon) - 2k(1 - 2q)]/(2q)$, and

$\gamma$3cons $= [(3q - 1)(R + \upsilon) - 2k(1 - q)]/(2q)$.

For $\gamma$2cons, the constraint will be satisfied if:

$1/3 < q < 1/2$ and $k \leq (3q - 1)(R + \upsilon)/[2(1 - 2q)]$, or $1/2 \leq q < 1$.

For $\gamma$3cons, the constraint will be satisfied if:

$1/3 < q < 1$ and $k \leq (3q - 1)(R + \upsilon)/[2(1 - q)]$.

These two conditions hold when $k \leq (3q - 1)(R + \upsilon)/[2(1 - q)]$.

Similarly, the second contract requires the condition $k \geq (3q - 1)(R + \upsilon)/[2(1 - q)]$. ∎

**Proof II:** Consider the payoff to the agent at a contract for which the reservation constraint binds. By definition, the expected payoff to the agent is R. Any connected vertex will also arise at the binding of the reservation constraint. Since the expected payoff is not dependent on the individual contractual elements, connected vertices do not

lead to a strict improvement in expected payoff to the agent. Since both principal and agent maximize expected payoffs then there can be no strict improvement in payoffs at viable contracts where the reservation constraint binds. ∎

**Proof III:** For the first contract, the reservation constraint binds and thus, by result II, the contract cannot be improved on. For the second contract, the $\gamma 3 cons$ constraint binds. Therefore, to find a contract at a connected vertex the contract at the connected vertex will also arise from the binding of the $\gamma 3 cons$ constraint.[64] Any connected vertex will be found by increasing values of the second and third contractual payment at the expense of the first. Therefore, to demonstrate that there can be no strict improvement in expected payoff to the principal by using a contract at a connected vertex, it will suffice to demonstrate that a strict improvement in expected payoff to the principal does not arise from increasing the values of the second and third contractual payment at the expense of the first along the binding constraint.

Suppose the second and third contractual payments change by the values $\delta_2 > 0$ and $\delta_3 > 0$, respectively. Since the $\gamma 3 cons$ constraint still binds, the first contractual payment will transition to a new value $\sigma_1$, given by $(3q - 1)[\sigma_1 - (\delta_3 - \upsilon)]/2 = 0$, or $\sigma_1 = (\delta_3 - \upsilon)$. This will yield a new contract: $\{\delta_3 - \upsilon, \delta_2 - \upsilon, \delta_3 - \upsilon\}$.

For this new contract, the expected payoff to the principal is:

$$V_{HH} = V + (3q - 1)\tau/2 - q(\delta_3 - \upsilon) - (1 - q)(\delta_2 - \upsilon)/2 - (1 - q)(\delta_3 - \upsilon)/2.$$

From direct observation, it can be seen that the expected payoff is a decreasing function of $\delta_2$ and $\delta_3$. Thus, there is no strict improvement in payoffs by using contracts at a connected vertex. ∎

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[64] Other constraints will also bind at a connected vertex, but the $\gamma 3 cons$ constraint will certainly bind.

**Proof IV:** The methodology will be demonstrated for one contract. Similar methods can be used to prove the results for the other contracts but are not shown for brevity.

Consider the first contract $\{[2k + R + (1 - q)\upsilon]/q, -\upsilon, -\upsilon\}$. For this contract to dominate other potential contracts, the expected payoff to the principal has to be greater than the expected payoff for both the third and fifth contract since their domains overlap.

For the expected payoff of the first to dominate the third:

$$V + (3q - 1)\tau/2 - [2k + R + (1 - q)\upsilon] + (1 - q)\upsilon \geq V - [k + R + (1 - q)\upsilon] + (1 - q)\upsilon$$

Therefore, the necessary condition for the first contract to dominate the third is

$$\tau \geq k/(3q - 1).$$

For the expected payoff of the first to dominate the fifth:

$$V + (3q - 1)\tau/2 - [2k + R + (1 - q)\upsilon] + (1 - q)\upsilon$$

$$\geq V - (3q - 1)\tau/2 - [R + (1 - q)\upsilon] + (1 - q)\upsilon$$

Therefore, the necessary condition for the first contract to dominate the fifth is

$\tau \geq 2k/(3q - 1)$. Combining this condition with the condition above and the original condition on the first contract, $k \leq (3q - 1)(R + \upsilon)/[2(1 - q)]$, yields the conditions necessary for the first contract to be optimal:

$$\tau \geq 2k/(3q - 1) \text{ and } k \leq (3q - 1)(R + \upsilon)/[2(1 - q)].$$

The conditions for the other contracts can be established in a similar manner. ∎

**Proof V:** The results can be shown directly by differentiating each of the incentive components by the parameters under consideration. ∎

**Proof VI:** The incentive component of the contractual solution (N1) is:

$$[2k + R + (1 - q)\upsilon]/q, \text{ and for N(2), } 4k/(3q - 1) - \upsilon.$$

From direct observation, both incentive components are strictly increasing functions of the cost of high effort, k.[65]

In figure 3, the boundary $k = (3q - 1)(R + \upsilon)/[2(1 - q)]$ separates projects with contractual solutions of the form (N1) and (N2), with lower values of high effort yielding solutions (N1), and higher values, (N2). By direct substitution, the incentive components of both contracts are identical at this boundary. Therefore, for contracts (N1) and (N2), the incentive components of the contractual solutions are an increasing function of the cost of high effort, k.

A similar argument holds for the pair of contracts (N3) and (N4).                    ■

**Proof VII:** First, the conditions in result IV can be rearranged to see the impact of the limited liability parameter $\upsilon$. Consider the condition $0 \leq \tau \leq 2k/(3q - 1)$. This condition represents the combined regions 7, 8 and 9 for all values of the limited liability constraint $\upsilon$. The expected payoff to the principal in this region is given by $V - (3q - 1)\tau/2 - R$. Since this is independent of the level of limited liability parameter then the principal will be indifferent between the two agents.

Next, consider the conditions $0 < k \leq (3q - 1)R/[2(1 - q)]$ and $\tau \geq 2k/(3q - 1)$. This is a sub region of region 1 for all values of the limited liability constraint $\upsilon$. The expected payoff to the principal in this region is given by $V + (3q - 1)\tau/2 - R - 2k$. Once again, this is independent of the level of the limited liability parameter; therefore, the principal will be indifferent between the two agents.

Next, consider the conditions:

$$(3q - 1)R/[2(1 - q)] \leq k \leq (3q - 1)R/(1 - q) \text{ and } \tau \geq 2k(1 + q)/(3q - 1)^2.$$

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[65] As confirmed by result V(iii) above.

This region is contained in regions 1 and 2 above. For $\upsilon \geq 2k(1 - q)/(3q - 1) - R$, the region is contained in region 1, and for $0 \leq \upsilon \leq 2k(1 - q)/(3q - 1) - R$, the region is contained in region 2. Therefore, if $\upsilon_1 > \upsilon_2 \geq 2k(1 - q)/(3q - 1) - R$, the principal must choose between two agents that will yield expected payoffs independent of the level of limited liability:

$$V + (3q - 1)\tau/2 - R - 2k.$$

Once again, this is independent of the level of the limited liability parameter; therefore, the principal will be indifferent between the two agents.

If $0 \leq \upsilon_2 < \upsilon_1 \leq 2k(1 - q)/(3q - 1) - R$, the principal must choose between two agents that will yield expected payoffs of the form:

$$V + (3q - 1)\tau/2 - 4kq(3q - 1) + \upsilon_1 \text{ or, } V + (3q - 1)\tau/2 - 4kq(3q - 1) + \upsilon_2.$$

Therefore, the principal will strictly prefer the agent with the higher limited liability.

If $0 \leq \upsilon_2 < 2k(1 - q)/(3q - 1) - R < \upsilon_1$, the principal must choose between two agents that will yield expected payoffs of:

$$V + (3q - 1)\tau/2 - R - 2k \text{ and } V + (3q - 1)\tau/2 - 4kq(3q - 1) + \upsilon_2.$$

For the second agent to be strictly preferred the following condition would need to hold: $\upsilon_2 > 4kq(3q - 1) - R - 2k$. This will require both:

$$4kq(3q - 1) - R - 2k \geq 0 \quad \text{and} \quad 4kq(3q - 1) - R - 2k < 2k(1 - q)/(3q - 1) - R.$$

Consider the first condition. This requires $k \geq (3q - 1)R/[2(1 - q)]$ which is consistent with the structure of this sub region. Consider the second condition. This simplifies to $R > R$, which is a contradiction.

The results for the remaining sub regions can be proved in a similar fashion. ∎

**Proof VIII:** The proof will be demonstrated for contract (C2): This will demonstrate the methodology used in proving the results for all three contracts but will occupy the least space in this paper.

First, it needs to be shown that the contract (C2) satisfies the limited liability constraints, (A2). Since, given the restrictions placed on q we have $2k/[(3q-1)p] > 0$, the result follows immediately.

Next, it needs to be shown that the contract (C2) satisfies reservation constraint, $\lambda_{cons} \geq 0$. The following is the expected payoff to the agent, assuming that the contract (C2) induces high effort in both periods:

$$2k(1-2q)/(3q-1) - \upsilon.$$

The second contract will satisfy the limited liability constraint when:

$$2k(1-2q)/(3q-1) - \upsilon \geq R.$$

Since the coefficient of k is required to be positive for this relationship to be true, then the second contract will satisfy the limited liability constraint when:

$$1/3 < q < 1/2 \text{ and } k \geq (3q-1)(R+\upsilon)/[2(1-2q)].$$

It should be noted that this constraint binds only when $k = (3q-1)(R+\upsilon)/[2(1-2q)]$.

Next, it needs to be shown that the contract (C2) satisfies the constraint, $\gamma_2 cons \geq 0$. The following is the expected payoff to the agent, assuming that the contract (C2) induces high effort in the first period and low in the second, $2k(1-2q)/(3q-1) - \upsilon$. Since this is identical to the agent's expected payoff from exerting high effort in both periods, the constraint $\gamma_2 cons \geq 0$ is satisfied. Furthermore, the constraint binds for all permissible values of the parameters.

Next, it needs to be shown that the contract (C2) satisfies the constraint, $\gamma 3cons \geq 0$. The following is the expected payoff to the agent, assuming that the contract (C2) induces low effort in the first period and high in the second, $k(1 - q - 2qp)/[(3q - 1)p] - \upsilon$. Therefore, the second contract will satisfy the constraint when:

$$2k(1 - 2q)/(3q - 1) - \upsilon \geq k(1 - q - 2qp)/[(3q - 1)p] - \upsilon.$$

This simplifies to $k(1 - q)(2p - 1)/[(3q - 1)p] \geq 0$.

It can be seen immediately, given the restrictions placed on q, p and k, this constraint is satisfied and indeed, this constraint does not bind.

Finally, it needs to be shown that the contract (C2) satisfies the constraint, $\gamma 4cons \geq 0$. The following is the expected payoff to the agent, assuming that the contract (C2) induces low effort in both periods $k(1 - q)(1 - p)/[(3q - 1)p] - \upsilon$. Therefore, the second contract will satisfy the constraint when:

$$2k(1 - 2q)/(3q - 1) - \upsilon \geq k(1 - q)(1 - p)/[(3q - 1)p] - \upsilon.$$

This simplifies to $k(q + 3p - 5pq - 1)/[(3q - 1)p] \geq 0$. Since $k, (3q - 1)p > 0$, the second contract will satisfy the constraint $\gamma 4cons \geq 0$ when $(q + 3p - 5pq - 1) \geq 0$. For this condition to be met the following has to be true:

$$3 - 5q > 0 \text{ and } 0 < 1 - q \leq (3 - 5q)/2 \text{ and } 1/2 < p < 1, \quad \text{or}$$

$$3 - 5q > 0 \text{ and } (3 - 5q)/2 < 1 - q < 3 - 5q \text{ and } (1 - q)/(3 - 5q) \leq p < 1.$$

For $1 - q \leq (3 - 5q)/2$ to be consistent would require $1 - 3q \geq 0$. Therefore, if the second contract is going to satisfy constraint $\gamma 4cons \geq 0$ then it will require:

$$3 - 5q > 0 \text{ and } (3 - 5q)/2 < 1 - q < 3 - 5q \text{ and } (1 - q)/(3 - 5q) \leq p < 1.$$

The conditions $3 - 5q > 0$ and $(3 - 5q)/2 < 1 - q < 3 - 5q$ yield the following relationships:

$3/5 > q$, $q > 1/3$, and $1/2 > q$.

Hence, contract (C2) satisfies constraint $\gamma4cons \geq 0$ when:

$1/3 < q < 1/2$ and $(1 - q)/(3 - 5q) \leq p < 1$.

Hence, considering all constraints (A1) and (A2), contract (C2) will be permissible if:

$1/3 < q < 1/2$ and $k \geq (3q - 1)(R + \upsilon)/[2(1 - 2q)]$      from constraint $\lambda cons \geq 0$, and

$1/3 < q < 1/2$ and $(1 - q)/(3 - 5q) \leq p < 1$      from constraint $\gamma4cons \geq 0$.

The intersection of these two sets of conditions is given by:

$1/3 < q < 1/2$ and $k \geq (3q - 1)(R + \upsilon)/[2(1 - 2q)]$, and $(1 - q)/(3 - 5q) \leq p < 1$.

The conditions for contracts (C1) and (C3) can be found in a similar fashion.    ∎

**Proof IX:** The union of the regions of influence of (C2) and (C3) is given by:

$1/3 < q < 1/2$, $(3q - 1)(R + \upsilon)/[2(1 - q)] < k \leq (3q - 1)(R + \upsilon)/[2(1 - 2q)]$, and

$(1 - q)(2k + R + \upsilon)/[2k(1 - q) + (1 + q)(R + \upsilon)] \leq p(m^*)$,     or

$1/2 \leq q < 1$, $k > (3q - 1)(R + \upsilon)/[2(1 - q)]$, and

$(1 - q)(2k + R + \upsilon)/[2k(1 - q) + (1 + q)(R + \upsilon)] \leq p(m^*)$,     or

$1/3 < q < 1/2$ and $k \geq (3q - 1)(R + \upsilon)/[2(1 - 2q)]$.

The union of this combined region and the region of influence of (C1) is given by:

$1/3 < q < 1$ and $k > 0$, and $1/2 < p(m) < 1$.    ∎

**Proof X:** For the first contract, the reservation constraint binds. Therefore, for any

connected vertex, the reservation constraint will also bind. The expected payoff to the

principal, when the reservation constraint binds, is given by $V + (3q - 1)\tau/2 - 2k - R - m$.

Since this is independent of the individual contractual payments, there is no strict

improvement to the expected payoff to the principal by using contracts at connected

vertices.

As can be seen from the detailed workings of proof IX, the second contract arises from the binding of the $\gamma 2\text{cons} \geq 0$ constraint. Therefore, the contract at the connected vertex will also arise from the binding of the $\gamma 2\text{cons} \geq 0$ constraint. Suppose the second, third, fourth, fifth and sixth contractual payments are changed by the values $\delta_2 > 0$, $\delta_3 > 0$, $\delta_4 > 0$, $\delta_5 > 0$ and $\delta_6 > 0$, respectively. Since the $\gamma 2\text{cons}$ constraint still binds, the first contractual payment will change to a new value $\sigma_{H1}$, given by:

$$(3q - 1)[p\sigma_{H1} - p\delta_2 + (1 - p)\delta_4 - (1 - p)\delta_6 + p\upsilon)]/2 - k = 0.$$

Therefore, we have a new contract $\{\sigma_{H1}, \delta_2 - \upsilon, \delta_3 - \upsilon, \delta_4 - \upsilon, \delta_5 - \upsilon, \delta_6 - \upsilon\}$ where;

$$\sigma_{H1} = \delta_2 - (1 - p)(\delta_4 - \delta_5)/p + 2k/[p(3q - 1)] - \upsilon.$$

For this new contract, the expected payoff to the principal is:

$$V + (3q - 1)\tau/2 - 2kq/(3q - 1) - m$$

$$- (1 + q)p\delta_2/2 - (1 - q)p\delta_3/2 - (1 + q)(1 - p)\delta_5/2 - (1 - q)(1 - p)\delta_6/2.$$

From direct observation it can be seen that the expected payoff is not a strictly increasing function of $\delta_2$, $\delta_3$, $\delta_4$, $\delta_5$, or $\delta_6$. Thus, there is no strict improvement in payoffs by using contracts at connected vertices.

A similar result can be shown to hold for contract 3. ∎

**Proof XI:** The expected payoff to the principal from contracts (C4) and (C6) are identical:

$$V - k - R - m.$$

Therefore, contract (C6) is redundant in the region containing the intersection of the two contracts' regions of dominance. Since the region of dominance of contract (C6) is entirely contained in the region of dominance of contract (C4), contract (C6) is redundant.

Comparing the expected payoff to the principal from contracts (C4) and (C7) requires the following condition for contract (C7) to dominate (C4), $k < (3q - 1)(R + \upsilon)/(1 - q)$. This is outside the region of dominance of contract (C7).

Finally, comparing expected payoff to the principal from contracts (C5) and (C7) requires the following condition for contract (C7) to dominate (C4), $-(2p - 1)(1 - q) > 0$. Since this condition is false, the low-high effort strategies are redundant. ∎

**Proof XII:** The methodology used in this proof is identical to the equivalent result for the non-monitoring case (result IV in chapter 5) and is not repeated for this proof. ∎

**Proof XIII:** This is the case since in this region the agent can be induced to exert high effort in both periods and accept the reservation payment as an expected payoff through contractual means alone. Because monitoring is costly, and could not induce any greater effort or shift resources for the benefit of the principal, and since both participants maximize expected payoffs, it follows that monitoring is not an optimal strategy in this region. ∎

**Proof XIV:** In regions 7, 8 and 9, the principal's optimal strategy is to encourage the agent to exert low effort in both periods both the monitoring and no monitoring case (see results IV and XII). This is the optimal strategy in both cases since the spread $\tau$ is sufficiently low and paying the agent for higher effort would not maximize the principals expected project returns. Furthermore, from result IV, the principal can extract the preferred effort level from the agent and pay him the reservation payment through contractual means. Because monitoring is costly and could not induce any greater effort or shift resources for the benefit of the principal, and since both participants maximize expected payoffs, it follows that monitoring is not an optimal strategy in this region. ∎

**Proof XV:** If these projects existed, they would be found at the intersection of the parameter conditions of (N1), and (M2) or (M3), or (N3) and (M5). Direct comparison of the conditions for these contracts shows that these intersections are empty. ∎

**Proof XVI:** The proof involves finding the intersection of the conditions for contracts (N2) and (M1), and (N4) and (M4) and establishes the sub regions in which the expected payoff to the principal from monitoring dominates the expected payoff to the principal from not monitoring. Since this can be done in a straightforward (albeit long-winded manner) the details are not included for this proof. ∎

**Proof XVII:** The proof is identical to the methodology applied to result XVI. ∎

**Proof XVIII:** The proof is identical to the methodology applied to result XVI. ∎

**Proof XIX:** The proof is identical to the methodology applied to result XVI. ∎

**Proof XX:** The proof is identical to the methodology applied to result XVI. ∎

**Proof XXI:** The proof is identical to the methodology applied to result XVI. ∎

**Proof XXII:** The proof is identical to the methodology applied to result XVI. ∎

APPENDIX B

CONDITIONS FOR SOLUTIONS WHERE MONITORING IS

AVAILABLE BUT RESULTS ARE NOT SHARED

The following relationships set out parameter values for contracts (M1) through (M6) described in result XII.

$\{(2k + R + \upsilon)/pq - \upsilon, -\upsilon, -\upsilon, -\upsilon, -\upsilon, -\upsilon\}$    given

$\quad 0 < k \le (3q - 1)(R + \upsilon)/[2(1 - q)]$, and $\tau \ge 2k/(3q - 1)$ or

$\{1/3 < q < 1/2, (3q - 1)(R + \upsilon)/[2(1 - q)] < k \le (3q - 1)(R + \upsilon)/[2(1 - 2q)]$, or

$\quad 1/2 \le q < 1, k > (3q - 1)(R + \upsilon)/[2(1 - q)]\}$, $\tau \ge 2k/(3q - 1)$, and

$\quad (1 - q)(2k + R + \upsilon)/[2k(1 - q) + (1 + q)(R + \upsilon)] \le p < 1.$    (M1)

$\{2k/[(3q - 1)p] - \upsilon, -\upsilon, -\upsilon, -\upsilon, -\upsilon, -\upsilon\}$    given

$\quad 1/3 < q < 1/2, \ k > (3q - 1)(R + \upsilon)/[2(1 - 2q)],$

$\quad\quad \tau \ge 2[k(1 - q) - (3q - 1)(R + \upsilon)]/(3q - 1)^2$, and $(1 - q)/(3 - 5q) \le p < 1$, or

$\quad 1/3 < q < 1/2, k > (3q - 1)(R + \upsilon)/(1 - 2q),$

$\quad\quad 2kq/(3q - 1)^2 \le \tau < 2[k(1 - q) - (3q - 1)(R + \upsilon)]/(3q - 1)^2$, and

$\quad\quad (1 - q)/(3 - 5q) \le p \le (1 - q)[4kq - (3q - 1)^2\tau]/[8k(1 - q) - (1 + q)(3q - 1)^2\tau].$    (M2)

$\{4k/(qp + q + p - 1) - \upsilon, -\upsilon, -\upsilon, -\upsilon, -\upsilon, -\upsilon\}$    given

$\quad \{(3q - 1)(R + \upsilon)/[2(1 - q)] < k \le (3q - 1)(R + \upsilon)/(1 - q)$, and

$\quad\quad 2k/(3q - 1) \le \tau < 2[k(1 + q) - (3q - 1)(R + \upsilon)]/(3q - 1)^2$, or

$\quad 1/3 < q < 1/2, (3q - 1)(R + \upsilon)/(1 - q) < k \le (3q - 1)(R + \upsilon)/[2(1 - 2q)]$, and

$\quad\quad 2k/(3q - 1) \le \tau < 2(k + R + \upsilon)/(3q - 1)$, or

$1/2 \le q < 1$, $k > (3q - 1)(R + \upsilon)/(1 - q)$, and

$2k/(3q - 1) \le \tau < 2(k + R + \upsilon)/(3q - 1)\}$, and

$(1 - q)[(3q - 1)\tau + 2(k + R + \upsilon)]/\{(1 + q)[(3q - 1)\tau + 2(R + \upsilon)] - 2k(3q - 1)\}$

$\le p \le (1 - q)(2k + R + \upsilon)/[2k(1 - q) + (1 + q)(R + \upsilon)]$, or

$\{(3q - 1)(R + \upsilon)/[2(1 - q)] < k \le (3q - 1)(R + \upsilon)/(1 - q)$, and

$\tau \ge 2[k(1 + q) - (3q - 1)(R + \upsilon)]/(3q - 1)^2$, or

$1/3 < q < 1/2$, $(3q - 1)(R + \upsilon)/(1 - q) < k \le (3q - 1)(R + \upsilon)/[2(1 - 2q)]$,

and $\tau \ge 4kq/(3q - 1)^2$, or

$1/2 \le q < 1$, $k > (3q - 1)(R + \upsilon)/(1 - q)$, and $\tau \ge 4kq/(3q - 1)^2\}$, and

$1/2 < p \le (1 - q)(2k + R + \upsilon)/[2k(1 - q) + (1 + q)(R + \upsilon)]$, or

$1/3 < q < 1/2$ and $\{k > (3q - 1)(R + \upsilon)/(1 - 2q)$, and

$2kq/(3q - 1)^2 \le \tau < 4kq/(3q - 1)^2$, or

$(3q - 1)(R + \upsilon)/[2(1 - 2q)] < k < (3q - 1)(R + \upsilon)/(1 - 2q)$, and

$2kq/(3q - 1)^2 \le \tau < 4kq/(3q - 1)^2\}$, and

$(1 - q)(3q - 1)\tau/[(1 - q)(3q - 1)\tau - 4qk] \le p \le (1 - q)/(3 - 5q)$, or

$1/3 < q < 1/2$, $(3q - 1)(R + \upsilon)/[2(1 - 2q)] < k < (3q - 1)(R + \upsilon)/(1 - 2q)$,

$2[k(1 - q) - (3q - 1)(R + \upsilon)]/(3q - 1)^2 \le \tau < 2(k + R + \upsilon)/(3q - 1)$, and

$(1 - q)[(3q - 1)\tau + 2(k + R + \upsilon)]/\{(1 + q)[(3q - 1) + 2(R + \upsilon)] - 2k(3q - 1)\}$

$\le p \le (1 - q)/(3 - 5q)$, or

$\{1/3 < q < 1/2$ and $(3q - 1)(R + \upsilon)/(1 - q) < k \le (3q - 1)(R + \upsilon)/[2(1 - 2q)]$, or

$1/2 \le q < 1$ and $k > (3q - 1)(R + \upsilon)/(1 - q)\}$,

$2(k + R + \upsilon)/(3q - 1) \le \tau < 4kq/(3q - 1)^2$, and

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$(1 - q)(3q - 1)\tau/[(1 + q)(3q - 1) - 4qk]$

$\leq p \leq (1 - q)(2k + R + \upsilon)/[2(1 - q)k + (1 + q)(R + \upsilon)]$, or

$1/3 < q < 1/2$, $k > (3q - 1)(R + \upsilon)/[2(1 - 2q)]$, $\tau \geq 4kq/(3q - 1)^2$, and

$1/2 < p \leq (1 - q)/(3 - 5q)$. (M3)

$\{-\upsilon, (k + R + \upsilon)/pq - \upsilon, -\upsilon, -\upsilon, -\upsilon, -\upsilon\}$ given

$(3q - 1)(R + \upsilon)/[2(1 - q)] < k \leq (3q - 1)(R + \upsilon)/(1 - q)$,

$2k/(3q - 1) \leq \tau < 2[k(1 + q) - (3q - 1)(R + \upsilon)]/(3q - 1)^2$, and

$1/2 < p \leq (1 - q)[(3q - 1)\tau + 2(k + R + \upsilon)]/\{(1 + q)[(3q - 1)\tau + 2(R + \upsilon)] - 2k(3q - 1)\}$, or

$\{1/3 < q < 1/2$, $(3q - 1)(R + \upsilon)/(1 - q) < k \leq (3q - 1)(R + \upsilon)/[2(1 - 2q)]$, and

$2k/(3q - 1) \leq \tau \leq 2(k + R + \upsilon)/(3q - 1)$, or

$1/3 < q < 1/2$, $(3q - 1)(R + \upsilon)/[2(1 - 2q)] < k < (3q - 1)(R + \upsilon)/(1 - 2q)$, and

$2[k(1 - q) - (3q - 1)(R + \upsilon)]/(3q - 1)^2 < \tau \leq 2(k + R + \upsilon)/(3q - 1)$, or

$1/2 \leq q < 1$, $k > (3q - 1)(R + \upsilon)/(1 - q)$, and

$2k/(3q - 1) \leq \tau \leq 2(k + R + \upsilon)/(3q - 1)\}$, and

$(1 - q)(k + R + \upsilon)/[k(1 - q) + (1 + q)(R + \upsilon)] < p$

$\leq (1 - q)[(3q - 1)\tau + 2(k + R + \upsilon)]/\{(1 + q)[(3q - 1)\tau + 2(R + \upsilon)] - 2k(3q - 1)\}$, or

$1/3 < q < 1/2$, $k > (3q - 1)(R + \upsilon)/[2(1 - 2q)]$,

$2k/(3q - 1) \leq \tau \leq 2[k(1 - q) - (3q - 1)(R + \upsilon)]/(3q - 1)^2$, and

$(1 - q)(k + R + \upsilon)/[k(1 - q) + (1 + q)(R + \upsilon)] \leq p < 1$. (M4)

$\{-\upsilon, 2k/(qp + q + p - 1) - \upsilon, -\upsilon, -\upsilon, -\upsilon, -\upsilon\}$

$(3q - 1)(R + \upsilon)/(1 - q) < k < 2(3q - 1)(R + \upsilon)/(1 - q)$,

$2[2qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2 \leq \tau < 2(k + R + \upsilon)/(3q - 1)$, and

$$1/2 < p \leq (1-q)(k+R+\upsilon)/[k(1-q)+(1+q)(R+\upsilon)], \text{ or}$$

$$\{(3q-1)(R+\upsilon)/(1-q) < k < 2(3q-1)(R+\upsilon)/(1-q), \text{ and}$$

$$2(k+R+\upsilon)/(3q-1) \leq \tau < 4qk/(3q-1)^2, \text{ or}$$

$$1/3 < q < 1/2, \; 2(3q-1)(R+\upsilon)/(1-q) \leq k \leq (3q-1)(R+\upsilon)/(1-2q), \text{ and}$$

$$2[2qk-(3q-1)(R+\upsilon)]/(3q-1)^2 \leq \tau < 4qk/(3q-1)^2, \text{ or}$$

$$1/2 \leq q < 1, \; k \geq 2(3q-1)(R+\upsilon)/(1-q), \text{ and}$$

$$2[2qk-(3q-1)(R+\upsilon)]/(3q-1)^2 \leq \tau < 4qk/(3q-1)^2\}, \text{ and}$$

$$1/2 < p \leq (1-q)(3q-1)\tau/[(1+q)(3q-1)\tau-4qk], \text{ or}$$

$$\{1/3 < q < 1/2, \; 2(3q-1)(R+\upsilon)/(1-q) \leq k \leq (3q-1)(R+\upsilon)/(1-2q), \text{ and}$$

$$2(k+R+\upsilon)/(3q-1) < \tau < 2[2qk-(3q-1)(R+\upsilon)]/(3q-1)^2, \text{ or}$$

$$1/3 < q < 1/2, \; k > (3q-1)(R+\upsilon)/(1-2q), \text{ and}$$

$$2kq/(3q-1)^2 < \tau < 2[2qk-(3q-1)(R+\upsilon)]/(3q-1)^2, \text{ or}$$

$$1/2 \leq q < 1, \; k > 2(3q-1)(R+\upsilon)/(1-q), \text{ and}$$

$$2kq/(3q-1)^2 < \tau < 2[2qk-(3q-1)(R+\upsilon)]/(3q-1)^2\}, \text{ and}$$

$$(1-q)[(3q-1)\tau+2(R+\upsilon)]/\{(1+q)[(3q-1)\tau+2(R+\upsilon)]-4qk\} \leq$$

$$p \leq (1-q)(3q-1)\tau/[(1+q)(3q-1)\tau-4qk], \text{ or}$$

$$\{(3q-1)(R+\upsilon)/(1-q) < k \leq 2(3q-1)(R+\upsilon)/(1-q), \text{ and}$$

$$2k/(3q-1) \leq \tau < 2[2qk-(3q-1)(R+\upsilon)]/(3q-1)^2, \text{ or}$$

$$1/3 < q < 1/2, \; 2(3q-1)(R+\upsilon)/(1-q) < k \leq (3q-1)(R+\upsilon)/(1-2q), \text{ and}$$

$$2k/(3q-1) \leq \tau \leq 2(k+R+\upsilon)/(3q-1), \text{ or}$$

$$1/2 \leq q < 1, \; k > 2(3q-1)(R+\upsilon)/(1-q), \text{ and}$$

$$2k/(3q-1) \leq \tau \leq 2(k+R+\upsilon)/(3q-1)\}, \text{ and}$$

$$(1 - q)[(3q - 1)\tau + 2(R + \upsilon)]/\{(1 + q)[(3q - 1)\tau + 2(R + \upsilon)] - 4qk\} \leq$$

$$p \leq (1 - q)(k + R + \upsilon)/[k(1 - q) + (1 + q)(R + \upsilon)], \text{ or}$$

$$1/3 < q < 1/2, \ k > (3q - 1)(R + \upsilon)/(1 - 2q),$$

$$2[qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2 \leq \tau \leq 2kq/(3q - 1)^2, \text{ and}$$

$$(1 - q)[(3q - 1)\tau + 2(R + \upsilon)]/\{(1 + q)[(3q - 1)\tau + 2(R + \upsilon)] - 4qk\} \leq$$

$$p \leq (1 - q)/(3 - 5q) \tag{M5}$$

$$\{-\upsilon, -\upsilon, -\upsilon, -\upsilon, -\upsilon, (R + \upsilon)/pq - \upsilon\}$$

$$0 \leq \tau \leq 2k/(3q - 1), \text{ or}$$

$$k > (3q - 1)(R + \upsilon)/(1 - q), \ 2k/(3q - 1) < \tau < 2[qk - (3q - 1)(R + \upsilon)]/(3q - 1)^2, \text{ and}$$

$$\tfrac{1}{2} < p \leq (1 - q)[(3q - 1)\tau + 2(R + \upsilon)]/\{(1 + q)[(3q - 1)\tau + 2(R + \upsilon)] - 4qk\}. \tag{M6}$$

REGIONS WHERE THE VARIANCE OF PAYOUTS FOR THE AGENT

CHANGES WITH COMMUNICATION IN REGION 1

**Variance strictly decreases with the adoption of communication in the following**

**sub-region of region 1:**

$$\frac{1}{3} < q \le \frac{1}{2}, \ k \le -\frac{(-1+3q)(R+v)}{2(-1+q)} \text{ and } \frac{1}{2} < p < 1, \quad \text{or}$$

$$\frac{1}{2} < q \le \frac{5}{8}, \ k < -\frac{(-1+2q)(-1+3q)(R+v)}{(-2+3q)(-1+4q)} \text{ and}$$

$$\frac{1}{2} < p < \frac{k(-2+5q)+(-1+3q)(R+v)}{2q(3k(-1+2q)+(-1+3q)(R+v))} \quad \text{or}$$

$$\frac{1}{2} < q \le \frac{5}{8}, \ -\frac{(-1+2q)(-1+3q)(R+v)}{(-2+3q)(-1+4q)} \le k \le -\frac{(-1+3q)(R+v)}{2(-1+q)} \text{ and}$$

$$\frac{1}{2} < p < 1 \quad \text{or}$$

$$\frac{5}{8} < q < 1, \ k \le -\frac{(-1+3q)(R+v)}{2(-1+q)} \text{ and } \frac{1}{2} < p < \frac{k(-2+5q)+(-1+3q)(R+v)}{2q(3k(-1+2q)+(-1+3q)(R+v))}$$

**Variance strictly increases with the adoption of communication in the following sub-**

**region of region 1:**

$$\frac{1}{2} < q \le \frac{5}{8}, \ k < -\frac{(-1+2q)(-1+3q)(R+v)}{(-2+3q)(-1+4q)} \text{ and}$$

$$\frac{k(-2+5q)+(-1+3q)(R+v)}{2q(3k(-1+2q)+(-1+3q)(R+v))} < p < 1 \quad \text{or}$$

$$\frac{5}{8} < q < 1, \ k \le -\frac{(-1+3q)(R+v)}{2(-1+q)} \text{ and } \frac{k(-2+5q)+(-1+3q)(R+v)}{2q(3k(-1+2q)+(-1+3q)(R+v))} < p < 1$$

CONDITIONS FOR SOLUTIONS WHERE MONITORING IS

IMPLEMENTED AND RESULTS ARE SHARED, STRATEGIES HHH2,

HHL1, HHL2 AND HLH1

**For HHH2**

$$\frac{1}{3} < q < \frac{1}{2}, \; -\frac{2\,(-1+3\,q)\,(R+v)}{3\,(-1+q)} < k < -\frac{(-1+3\,q)\,(R+v)}{-1+q}, \; \tau \geq \frac{4\,k\,q}{(-1+3\,q)^2} \; \text{and} \; \frac{1}{2} < p < P[4] \; \text{or}$$

$$\frac{1}{3} < q < \frac{1}{2}, \; -\frac{2\,(-1+3\,q)\,(R+v)}{3\,(-1+q)} < k \leq K[1], \; T[1] < \tau < \frac{4\,k\,q}{(-1+3\,q)^2} \; \text{and} \, P[1] \leq p < P[4] \; \text{or}$$

$$\frac{1}{3} < q < \frac{1}{2}, \; K[1] < k < -\frac{(-1+3\,q)\,(R+v)}{-1+q},$$

$$\frac{2\,(k\,(1+q)-(-1+3\,q)\,(R+v))}{(-1+3\,q)^2} \leq \tau < \frac{4\,k\,q}{(-1+3\,q)^2} \; \text{and} \, P[1] \leq p < P[4] \; \text{or}$$

$$-\frac{(-1+3\,q)\,(R+v)}{2\,(-1+q)} < k \leq -\frac{2\,(-1+3\,q)\,(R+v)}{3\,(-1+q)}, \; \tau \geq -\frac{2\,(k\,(-3+q)+2\,(-1+3\,q)\,(R+v))}{(-1+3\,q)^2} \; \text{and}$$

$$\frac{1}{2} < p < \frac{2\,k\,(-1+q)\,(-1+2\,q)+(-1+q)\,(-1+3\,q)\,(R+v)}{2\,k\,(-1+q)^2-(1+q)\,(-1+3\,q)\,(R+v)} \; \text{or}$$

$$-\frac{(-1+3\,q)\,(R+v)}{2\,(-1+q)} < k \leq -\frac{2\,(-1+3\,q)\,(R+v)}{3\,(-1+q)},$$

$$\frac{2\,(k\,(1+q)-(-1+3\,q)\,(R+v))}{(-1+3\,q)^2} \leq \tau < -\frac{2\,(k\,(-3+q)+2\,(-1+3\,q)\,(R+v))}{(-1+3\,q)^2} \; \text{and}$$

$$P[2] \leq p < \frac{2\,k\,(-1+q)\,(-1+2\,q)+(-1+q)\,(-1+3\,q)\,(R+v)}{2\,k\,(-1+q)^2-(1+q)\,(-1+3\,q)\,(R+v)} \; \text{or}$$

$$-\frac{2\,(-1+3\,q)\,(R+v)}{3\,(-1+q)} < k \leq -\frac{(-1+3\,q)\,(R+v)}{-1+q}, \; \tau \geq T[2] \; \text{and}$$

$$P[4] \leq p < \frac{2\,k\,(-1+q)\,(-1+2\,q)+(-1+q)\,(-1+3\,q)\,(R+v)}{2\,k\,(-1+q)^2-(1+q)\,(-1+3\,q)\,(R+v)} \; \text{or}$$

$$-\frac{2\,(-1+3\,q)\,(R+v)}{3\,(-1+q)} < k \leq -\frac{(-1+3\,q)\,(R+v)}{-1+q}, \; \frac{2\,(k\,(1+q)-(-1+3\,q)\,(R+v))}{(-1+3\,q)^2} \leq \tau < T[2] \; \text{and}$$

$$P[2] \leq p < \frac{2\,k\,(-1+q)\,(-1+2\,q)+(-1+q)\,(-1+3\,q)\,(R+v)}{2\,k\,(-1+q)^2-(1+q)\,(-1+3\,q)\,(R+v)} \quad \text{or}$$

$$\frac{1}{2} \le q < 1, \quad -\frac{2(-1+3q)(R+v)}{3(-1+q)} < k \le -\frac{(-1+3q)(R+v)}{-1+q}, \quad \frac{4kq}{(-1+3q)^2} < \tau < T[2] \text{ and } \frac{1}{2} < p \le P[3] \text{ or}$$

$$\frac{1}{2} \le q < 1, \quad -\frac{2(-1+3q)(R+v)}{3(-1+q)} < k \le -\frac{(-1+3q)(R+v)}{-1+q},$$

$$\tau \ge T[2] \text{ and } \frac{1}{2} < p \le P[4]$$

**For HHL1**

$$-\frac{(-1+3q)(R+v)}{2(-1+q)} < k \le -\frac{2(-1+3q)(R+v)}{3(-1+q)},$$

$$\frac{2(k(1+q)-(-1+3q)(R+v))}{(-1+3q)^2} \le \tau < -\frac{2(k(-3+q)+2(-1+3q)(R+v))}{(-1+3q)^2}$$

$$\text{and } \frac{1}{2} < p < P[2] \text{ or}$$

$$-\frac{2(-1+3q)(R+v)}{3(-1+q)} < k \le -\frac{(-1+3q)(R+v)}{-1+q}, \quad \frac{2(k(1+q)-(-1+3q)(R+v))}{(-1+3q)^2} \le \tau < T[2]$$

$$\text{and } P[4] \le p < P[2]$$

**For HHL2**

$$\frac{1}{2} \le q < 1, \quad -\frac{2(-1+3q)(R+v)}{3(-1+q)} < k \le -\frac{(-1+3q)(R+v)}{-1+q},$$

$$\frac{2(k(1+q)-(-1+3q)(R+v))}{(-1+3q)^2} \le \tau \le \frac{4kq}{(-1+3q)^2} \text{ and } \frac{1}{2} < p \le P[4] \text{ or}$$

$$\frac{1}{2} \le q < 1, \quad -\frac{2(-1+3q)(R+v)}{3(-1+q)} < k \le -\frac{(-1+3q)(R+v)}{-1+q},$$

$$\frac{4kq}{(-1+3q)^2} < \tau < T[2] \text{ and } P[3] < p \le P[4]$$

**For HLH1**

$$\frac{1}{3} < q < \frac{1}{2}, \quad -\frac{2(-1+3q)(R+v)}{3(-1+q)} < k \le K[1], \quad \frac{2(k(1+q)-(-1+3q)(R+v))}{(-1+3q)^2} \le \tau \le T[1]$$

$$\text{and } \frac{1}{2} < p < P[4] \text{ or}$$

$$\frac{1}{3} < q < \frac{1}{2}, \quad -\frac{2(-1+3q)(R+v)}{3(-1+q)} < k \le K[1], \quad T[1] < \tau < \frac{4kq}{(-1+3q)^2} \text{ and } \frac{1}{2} < p < P[1] \text{ or}$$

$$\frac{1}{3} < q < \frac{1}{2}, \quad K[1] < k < -\frac{(-1+3q)(R+v)}{-1+q}, \quad \frac{2(k(1+q)-(-1+3q)(R+v))}{(-1+3q)^2} \le \tau < \frac{4kq}{(-1+3q)^2}$$

$$\text{and } \frac{1}{2} < p < P[1]$$

The parameters K[1] through P[4] are defined in appendix H.

# CONDITIONS FOR SOLUTIONS WHERE MONITORING IS

# IMPLEMENTED AND RESULTS CAN BE SHARED

**For the second non-communication contract**

$$-\frac{(-1+3\,q)\,(R+\upsilon)}{2\,(-1+q)} < k \le -\frac{2\,(-1+3\,q)\,(R+\upsilon)}{3\,(-1+q)},$$

$$\frac{2\,(k\,(1+q)-(-1+3\,q)\,(R+\upsilon))}{(-1+3\,q)^2} \le \tau < -\frac{2\,(k\,(-3+q)+2\,(-1+3\,q)\,(R+\upsilon))}{(-1+3\,q)^2} \text{ and}$$

$$P[5] \le p \le \frac{(-1+q)\,(2\,k+R+\upsilon)}{2\,k\,(-1+q)-(1+q)\,(R+\upsilon)} \text{ or}$$

$$-\frac{(-1+3\,q)\,(R+\upsilon)}{2\,(-1+q)} < k \le -\frac{2\,(-1+3\,q)\,(R+\upsilon)}{3\,(-1+q)}, \ \tau \ge -\frac{2\,(k\,(-3+q)+2\,(-1+3\,q)\,(R+\upsilon))}{(-1+3\,q)^2} \text{ and}$$

$$\frac{1}{2} < p \le \frac{(-1+q)\,(2\,k+R+\upsilon)}{2\,k\,(-1+q)-(1+q)\,(R+\upsilon)} \text{ or}$$

$$-\frac{2\,(-1+3\,q)\,(R+\upsilon)}{3\,(-1+q)} < k < -\frac{(-1+3\,q)\left(7+11\,q+\sqrt{1-6\,q+73\,q^2}\right)(R+\upsilon)}{8\,(-1-2\,q+3\,q^2)},$$

$$\frac{2\,(k\,(1+q)-(-1+3\,q)\,(R+\upsilon))}{(-1+3\,q)^2} \le \tau < T[3] \text{ and} P[5] \le p \le \frac{(-1+q)\,(2\,k+R+\upsilon)}{2\,k\,(-1+q)-(1+q)\,(R+\upsilon)} \text{ or}$$

$$-\frac{2\,(-1+3\,q)\,(R+\upsilon)}{3\,(-1+q)} < k < -\frac{(-1+3\,q)\left(7+11\,q+\sqrt{1-6\,q+73\,q^2}\right)(R+\upsilon)}{8\,(-1-2\,q+3\,q^2)}, \ \tau \ge T[3] \text{ and}$$

$$P[6] \le p \le \frac{(-1+q)\,(2\,k+R+\upsilon)}{2\,k\,(-1+q)-(1+q)\,(R+\upsilon)} \text{ or}$$

$$-\frac{(-1+3\,q)\left(7+11\,q+\sqrt{1-6\,q+73\,q^2}\right)(R+\upsilon)}{8\,(-1-2\,q+3\,q^2)} \le k \le -\frac{(-1+3\,q)\,(R+\upsilon)}{-1+q},$$

$$\tau \ge \frac{2\,(k\,(1+q)-(-1+3\,q)\,(R+\upsilon))}{(-1+3\,q)^2} \text{ and}$$

$$P[6] \le p \le \frac{(-1+q)\,(2\,k+R+\upsilon)}{2\,k\,(-1+q)-(1+q)\,(R+\upsilon)} \text{ or}$$

$$\frac{1}{2} \le q < 1, \quad -\frac{2(-1+3q)(R+\upsilon)}{3(-1+q)} < k < -\frac{(-1+3q)\left(7+11q+\sqrt{1-6q+73q^2}\right)(R+\upsilon)}{8(-1-2q+3q^2)},$$

$$T[3] \le \tau < \frac{4kq}{(-1+3q)^2} \quad \text{and} \quad \frac{-\tau+4q\tau-3q^2\tau}{-4kq-\tau+2q\tau+3q^2\tau} \le p \le P[6] \text{ or}$$

$$\frac{1}{2} \le q < 1, \quad -\frac{(-1+3q)\left(7+11q+\sqrt{1-6q+73q^2}\right)(R+\upsilon)}{8(-1-2q+3q^2)} \le k < -\frac{(-1+3q)(R+\upsilon)}{-1+q},$$

$$\frac{2(k(1+q)-(-1+3q)(R+\upsilon))}{(-1+3q)^2} \le \tau < \frac{4kq}{(-1+3q)^2} \quad \text{and} \quad \frac{-\tau+4q\tau-3q^2\tau}{-4kq-\tau+2q\tau+3q^2\tau} \le p \le P[6] \text{ or}$$

$$\frac{1}{2} \le q < 1, \quad -\frac{2(-1+3q)(R+\upsilon)}{3(-1+q)} < k < -\frac{(-1+3q)(R+\upsilon)}{-1+q}, \quad \tau \ge \frac{4kq}{(-1+3q)^2} \text{ and}$$

$$\frac{1}{2} < p \le P[6] \text{ or}$$

$$\frac{1}{3} < q \le \frac{1}{2}, \quad -\frac{2(-1+3q)(R+\upsilon)}{3(-1+q)} < k \le K[1], \quad T[1] < \tau < \frac{4kq}{(-1+3q)^2} \text{ and} \frac{1}{2} < p \le P[1] \text{ or}$$

$$\frac{1}{3} < q \le \frac{1}{2}, \quad -\frac{2(-1+3q)(R+\upsilon)}{3(-1+q)} < k \le K[1], \quad \frac{2(k(1+q)-(-1+3q)(R+\upsilon))}{(-1+3q)^2} \le \tau \le T[1]$$

$$\text{and} \frac{1}{2} < p < P[6] \text{ or}$$

$$\frac{1}{3} < q \le \frac{1}{2}, \quad K[1] < k < -\frac{(-1+3q)(R+\upsilon)}{-1+q}, \quad \frac{2(k(1+q)-(-1+3q)(R+\upsilon))}{(-1+3q)^2} \le \tau < \frac{4kq}{(-1+3q)^2}$$

$$\text{and} \frac{1}{2} < p \le P[1]$$

**For HHL1**

$$-\frac{(-1+3q)(R+\upsilon)}{2(-1+q)} < k \le -\frac{2(-1+3q)(R+\upsilon)}{3(-1+q)},$$

$$\frac{2(k(1+q)-(-1+3q)(R+\upsilon))}{(-1+3q)^2} \le \tau < -\frac{2(k(-3+q)+2(-1+3q)(R+\upsilon))}{(-1+3q)^2} \text{ and}$$

$$\frac{1}{2} < p < P[5] \text{ or}$$

$$-\frac{2(-1+3q)(R+\upsilon)}{3(-1+q)} < k < -\frac{(-1+3q)\left(7+11q+\sqrt{1-6q+73q^2}\right)(R+\upsilon)}{8(-1-2q+3q^2)},$$

$$\frac{2(k(1+q)-(-1+3q)(R+\upsilon))}{(-1+3q)^2} \le \tau < T[3] \text{ and} P[6] \le p < P[5]$$

**For HHL2**

$$\frac{1}{2} \le q < 1, \quad -\frac{2(-1+3q)(R+v)}{3(-1+q)} < k < -\frac{(-1+3q)\left(7+11q+\sqrt{1-6q+73q^2}\right)(R+v)}{8(-1-2q+3q^2)},$$

$$T[3] \le \tau < \frac{4kq}{(-1+3q)^2} \quad \text{and} \quad \frac{1}{2} < p < \frac{-\tau+4q\tau-3q^2\tau}{-4kq-\tau+2q\tau+3q^2\tau} \quad \text{or}$$

$$\frac{1}{2} \le q < 1, \quad -\frac{2(-1+3q)(R+v)}{3(-1+q)} < k < -\frac{(-1+3q)\left(7+11q+\sqrt{1-6q+73q^2}\right)(R+v)}{8(-1-2q+3q^2)},$$

$$\frac{2(k(1+q)-(-1+3q)(R+v))}{(-1+3q)^2} \le \tau < T[3] \quad \text{and} \quad \frac{1}{2} < p \le P[6] \quad \text{or}$$

$$\frac{1}{2} \le q < 1, \quad -\frac{(-1+3q)\left(7+11q+\sqrt{1-6q+73q^2}\right)(R+v)}{8(-1-2q+3q^2)} \le k < -\frac{(-1+3q)(R+v)}{-1+q},$$

$$\frac{2(k(1+q)-(-1+3q)(R+v))}{(-1+3q)^2} \le \tau < \frac{4kq}{(-1+3q)^2} \quad \text{and} \quad \frac{1}{2} < p < \frac{-\tau+4q\tau-3q^2\tau}{-4kq-\tau+2q\tau+3q^2\tau}$$

**For HLH1**

$$\frac{1}{3} < q \le \frac{1}{2}, \quad -\frac{2(-1+3q)(R+v)}{3(-1+q)} < k \le K[1], \quad T[1] < \tau < \frac{4kq}{(-1+3q)^2} \quad \text{and} \quad P[1] < p < P[6] \quad \text{or}$$

$$\frac{1}{3} < q \le \frac{1}{2}, \quad K[1] < k < -\frac{(-1+3q)(R+v)}{-1+q}, \quad \frac{2(k(1+q)-(-1+3q)(R+v))}{(-1+3q)^2} \le \tau < \frac{4kq}{(-1+3q)^2}$$

$$\text{and} \quad P[1] < p < P[6] \quad \text{or}$$

$$\frac{1}{3} < q \le \frac{1}{2}, \quad -\frac{2(-1+3q)(R+v)}{3(-1+q)} < k \le -\frac{(-1+3q)(R+v)}{-1+q}, \quad \tau \ge \frac{4kq}{(-1+3q)^2}$$

$$\text{and} \quad \frac{1}{2} < p < P[6]$$

The parameters K[1] through P[6] are defined in appendix H.

# REGIONS WHERE THE VARIANCE OF PAYOUTS FOR THE AGENT

# CHANGES FOR CONTRACT HHL2 COMPARED TO THE SECOND

# NON-COMMUNICATION STRATEGY

**Variance decreases in the following sub-region of region 2:**

$$\frac{1}{2} \leq q \leq Q[1], \quad -\frac{2(-1+3q)(R+v)}{3(-1+q)} < k < -\frac{(-1+3q)\left(7+11q+\sqrt{1-6q+73q^2}\right)(R+v)}{8(-1-2q+3q^2)},$$

$$T[3] \leq \tau < \frac{4kq}{(-1+3q)^2} \quad \text{and} \quad \frac{1}{2} < p < \frac{-\tau+4q\tau-3q^2\tau}{-4kq-\tau+2q\tau+3q^2\tau} \quad \text{or}$$

$$Q[1] < q < 1, \quad -\frac{2(-1+3q)(R+v)}{3(-1+q)} < k \leq K[2], \quad T[3] \leq \tau < \frac{4kq}{(-1+3q)^2} \quad \text{and}$$

$$\frac{1}{2} < p < \frac{-\tau+4q\tau-3q^2\tau}{-4kq-\tau+2q\tau+3q^2\tau} \quad \text{or}$$

$$Q[1] < q < 1, \quad K[2] < k < -\frac{(-1+3q)\left(7+11q+\sqrt{1-6q+73q^2}\right)(R+v)}{8(-1-2q+3q^2)},$$

$$T[3] \leq \tau < T[4] \quad \text{and} \quad \frac{1}{2} < p < P[7] \quad \text{or}$$

$$Q[1] < q < 1, \quad K[2] < k < -\frac{(-1+3q)\left(7+11q+\sqrt{1-6q+73q^2}\right)(R+v)}{8(-1-2q+3q^2)},$$

$$T[4] \leq \tau < \frac{4kq}{(-1+3q)^2} \quad \text{and} \quad \frac{1}{2} < p < \frac{-\tau+4q\tau-3q^2\tau}{-4kq-\tau+2q\tau+3q^2\tau} \quad \text{or}$$

$$\frac{1}{2} \leq q \leq Q[1], \quad -\frac{2(-1+3q)(R+v)}{3(-1+q)} < k < -\frac{(-1+3q)\left(7+11q+\sqrt{1-6q+73q^2}\right)(R+v)}{8(-1-2q+3q^2)},$$

$$\frac{2(k(1+q)-(-1+3q)(R+v))}{(-1+3q)^2} \leq \tau < T[3] \quad \text{and} \quad \frac{1}{2} < p \leq P[4] \quad \text{or}$$

$$Q[1] < q < 1, \quad -\frac{2(-1+3q)(R+\upsilon)}{3(-1+q)} < k < K[2],$$

$$\frac{2(k(1+q)-(-1+3q)(R+\upsilon))}{(-1+3q)^2} \le \tau < T[3] \quad \text{and} \quad \frac{1}{2} < p \le P[4] \quad \text{or}$$

$$Q[1] < q < 1, \quad K[2] \le k < -\frac{(-1+3q)\left(7+11q+\sqrt{1-6q+73q^2}\right)(R+\upsilon)}{8(-1-2q+3q^2)},$$

$$\frac{2(k(1+q)-(-1+3q)(R+\upsilon))}{(-1+3q)^2} \le \tau < T[3] \quad \text{and} \quad \frac{1}{2} < p < P[7] \quad \text{or}$$

$$\frac{1}{2} \le q \le Q[1], \quad -\frac{(-1+3q)\left(7+11q+\sqrt{1-6q+73q^2}\right)(R+\upsilon)}{8(-1-2q+3q^2)} \le k < -\frac{(-1+3q)(R+\upsilon)}{-1+q},$$

$$\frac{2(k(1+q)-(-1+3q)(R+\upsilon))}{(-1+3q)^2} \le \tau < \frac{4kq}{(-1+3q)^2}$$

$$\text{and} \quad \frac{1}{2} < p < \frac{-\tau+4q\tau-3q^2\tau}{-4kq-\tau+2q\tau+3q^2\tau} \quad \text{or}$$

$$Q[1] < q < 1, \quad -\frac{(-1+3q)\left(7+11q+\sqrt{1-6q+73q^2}\right)(R+\upsilon)}{8(-1-2q+3q^2)} \le k < K[3],$$

$$\frac{2(k(1+q)-(-1+3q)(R+\upsilon))}{(-1+3q)^2} \le \tau < T[4] \quad \text{and} \quad \frac{1}{2} < p < P[7] \quad \text{or}$$

$$Q[1] < q < 1, \quad -\frac{(-1+3q)\left(7+11q+\sqrt{1-6q+73q^2}\right)(R+\upsilon)}{8(-1-2q+3q^2)} \le k < K[3],$$

$$T[4] \le \tau < \frac{4kq}{(-1+3q)^2} \quad \text{and} \quad \frac{1}{2} < p < \frac{-\tau+4q\tau-3q^2\tau}{-4kq-\tau+2q\tau+3q^2\tau} \quad \text{or}$$

$$Q[1] < q < 1, \quad K[3] \le k < -\frac{(-1+3q)(R+\upsilon)}{-1+q},$$

$$\frac{2(k(1+q)-(-1+3q)(R+\upsilon))}{(-1+3q)^2} \le \tau < \frac{4kq}{(-1+3q)^2}, \quad \text{and}$$

$$\frac{1}{2} < p < \frac{-\tau+4q\tau-3q^2\tau}{-4kq-\tau+2q\tau+3q^2\tau}$$

**Variance increases in the following sub-region of region 2:**

$$Q[1] < q < 1, \ K[2] < k < -\frac{(-1+3q)\left(7+11q+\sqrt{1-6q+73q^2}\right)(R+v)}{8(-1-2q+3q^2)},$$

$$T[3] \le \tau < T[4] \quad \text{and} \quad P[7] \le p < \frac{-\tau+4q\tau-3q^2\tau}{-4kq-\tau+2q\tau+3q^2\tau} \quad \text{or}$$

$$Q[1] < q < 1, \ K[2] \le k < -\frac{(-1+3q)\left(7+11q+\sqrt{1-6q+73q^2}\right)(R+v)}{8(-1-2q+3q^2)},$$

$$\frac{2(k(1+q)-(-1+3q)(R+v))}{(-1+3q)^2} \le \tau < T[3] \quad \text{and} \quad P[7] \le p \le P[4] \quad \text{or}$$

$$Q[1] < q < 1, \ -\frac{(-1+3q)\left(7+11q+\sqrt{1-6q+73q^2}\right)(R+v)}{8(-1-2q+3q^2)} \le k < K[3],$$

$$\frac{2(k(1+q)-(-1+3q)(R+v))}{(-1+3q)^2} \le \tau < T[4] \quad \text{and}$$

$$P[7] \le p < \frac{-\tau+4q\tau-3q^2\tau}{-4kq-\tau+2q\tau+3q^2\tau}$$

The parameters Q[1] through P[7] are defined in appendix H.

REGIONS WHERE THE COEFFICIENT OF VARIATION OF PAYOUTS

FOR THE AGENT CHANGES FOR CONTRACT HHL2

**Coefficient of Variation decreases in the following sub-region of region 2:**

$$\frac{1}{2} \le q \le Q[2], \quad -\frac{2(-1+3q)(R+v)}{3(-1+q)} < k < -\frac{(-1+3q)\left(7+11q+\sqrt{1-6q+73q^2}\right)(R+v)}{8(-1-2q+3q^2)},$$

$$T[3] \le \tau < \frac{4kq}{(-1+3q)^2} \quad \text{and} \quad \frac{1}{2} < p < \frac{-\tau+4q\tau-3q^2\tau}{-4kq-\tau+2q\tau+3q^2\tau} \quad \text{or}$$

$$Q[2] < q < 1, \quad -\frac{2(-1+3q)(R+v)}{3(-1+q)} < k \le K[4], \quad T[3] \le \tau < \frac{4kq}{(-1+3q)^2} \quad \text{and}$$

$$\frac{1}{2} < p < \frac{-\tau+4q\tau-3q^2\tau}{-4kq-\tau+2q\tau+3q^2\tau} \quad \text{or}$$

$$Q[2] < q < 1, \quad K[4] < k < -\frac{(-1+3q)\left(7+11q+\sqrt{1-6q+73q^2}\right)(R+v)}{8(-1-2q+3q^2)},$$

$$T[3] \le \tau < T[5] \quad \text{and} \quad \frac{1}{2} < p < P[8] \quad \text{or}$$

$$Q[2] < q < 1, \quad K[4] < k < -\frac{(-1+3q)\left(7+11q+\sqrt{1-6q+73q^2}\right)(R+v)}{8(-1-2q+3q^2)},$$

$$T[5] \le \tau < \frac{4kq}{(-1+3q)^2} \quad \text{and} \quad \frac{1}{2} < p < \frac{-\tau+4q\tau-3q^2\tau}{-4kq-\tau+2q\tau+3q^2\tau} \quad \text{or}$$

$$\frac{1}{2} \le q \le Q[2], \quad -\frac{2(-1+3q)(R+v)}{3(-1+q)} < k < -\frac{(-1+3q)\left(7+11q+\sqrt{1-6q+73q^2}\right)(R+v)}{8(-1-2q+3q^2)},$$

$$\frac{2(k(1+q)-(-1+3q)(R+v))}{(-1+3q)^2} \le \tau < T[3] \quad \text{and} \quad \frac{1}{2} < p \le P[4] \quad \text{or}$$

$$Q[2] < q < 1, \quad -\frac{2(-1+3q)(R+v)}{3(-1+q)} < k < K[4], \quad \frac{2(k(1+q)-(-1+3q)(R+v))}{(-1+3q)^2} \le \tau < T[3] \quad \text{and}$$

$$\frac{1}{2} < p \le P[4] \quad \text{or}$$

$$Q[2] < q < 1, \ K[4] \le k < -\frac{(-1+3\,q)\left(7+11\,q+\sqrt{1-6\,q+73\,q^2}\right)(R+\upsilon)}{8\,(-1-2\,q+3\,q^2)},$$

$$\frac{2\,(k\,(1+q)-(-1+3\,q)\,(R+\upsilon))}{(-1+3\,q)^2} \le \tau < T[3] \quad \text{and} \quad \frac{1}{2} < p < P[8] \quad \text{or}$$

$$\frac{1}{2} \le q \le Q[2], \ -\frac{(-1+3\,q)\left(7+11\,q+\sqrt{1-6\,q+73\,q^2}\right)(R+\upsilon)}{8\,(-1-2\,q+3\,q^2)} \le k < -\frac{(-1+3\,q)\,(R+\upsilon)}{-1+q},$$

$$\frac{2\,(k\,(1+q)-(-1+3\,q)\,(R+\upsilon))}{(-1+3\,q)^2} \le \tau < \frac{4\,k\,q}{(-1+3\,q)^2} \quad \text{and}$$

$$\frac{1}{2} < p < \frac{-\tau+4\,q\,\tau-3\,q^2\,\tau}{-4\,k\,q-\tau+2\,q\,\tau+3\,q^2\,\tau} \quad \text{or}$$

$$Q[2] < q < 1, \ -\frac{(-1+3\,q)\left(7+11\,q+\sqrt{1-6\,q+73\,q^2}\right)(R+\upsilon)}{8\,(-1-2\,q+3\,q^2)} \le k < K[5],$$

$$\frac{2\,(k\,(1+q)-(-1+3\,q)\,(R+\upsilon))}{(-1+3\,q)^2} \le \tau < T[5] \quad \text{and} \quad \frac{1}{2} < p < P[8] \quad \text{or}$$

$$Q[2] < q < 1, \ -\frac{(-1+3\,q)\left(7+11\,q+\sqrt{1-6\,q+73\,q^2}\right)(R+\upsilon)}{8\,(-1-2\,q+3\,q^2)} \le k < K[5],$$

$$T[5] \le \tau < \frac{4\,k\,q}{(-1+3\,q)^2} \quad \text{and} \quad \frac{1}{2} < p < \frac{-\tau+4\,q\,\tau-3\,q^2\,\tau}{-4\,k\,q-\tau+2\,q\,\tau+3\,q^2\,\tau} \quad \text{or}$$

$$Q[2] < q < 1, \ K[5] \le k < -\frac{(-1+3\,q)\,(R+\upsilon)}{-1+q},$$

$$\frac{2\,(k\,(1+q)-(-1+3\,q)\,(R+\upsilon))}{(-1+3\,q)^2} \le \tau < \frac{4\,k\,q}{(-1+3\,q)^2} \quad \text{and}$$

$$\frac{1}{2} < p < \frac{-\tau+4\,q\,\tau-3\,q^2\,\tau}{-4\,k\,q-\tau+2\,q\,\tau+3\,q^2\,\tau}$$

**Coefficient of Variation increases in the following sub-region of region 2:**

$$Q[2] < q < 1, \ K[4] < k < -\frac{(-1+3\,q)\left(7+11\,q+\sqrt{1-6\,q+73\,q^2}\right)(R+\upsilon)}{8\,(-1-2\,q+3\,q^2)},$$

$$T[3] \le \tau < T[5] \quad \text{and} \quad P[8] < p < \frac{-\tau+4\,q\,\tau-3\,q^2\,\tau}{-4\,k\,q-\tau+2\,q\,\tau+3\,q^2\,\tau} \quad \text{or}$$

$$Q[2] < q < 1, \ K[4] < k < -\frac{(-1+3\,q)\left(7+11\,q+\sqrt{1-6\,q+73\,q^2}\right)(R+\upsilon)}{8\,(-1-2\,q+3\,q^2)},$$

$$\frac{2\,(k\,(1+q)-(-1+3\,q)\,(R+\upsilon))}{(-1+3\,q)^2} \le \tau < T[3] \quad \text{and} \quad P[8] < p \le P[4] \quad \text{or}$$

$$Q[2] < q < 1, \quad -\frac{(-1+3q)\left(7+11q+\sqrt{1-6q+73q^2}\right)(R+v)}{8(-1-2q+3q^2)} \le k < K[5],$$

$$\frac{2(k(1+q)-(-1+3q)(R+v))}{(-1+3q)^2} \le \tau < T[5] \quad \text{and} \quad P[8] < p < \frac{-\tau+4q\tau-3q^2\tau}{-4kq-\tau+2q\tau+3q^2\tau}$$

The parameters Q[2] through P[8] are defined in appendix H.

# APPENDIX H

# DEFINITION OF PARAMETERS USED IN APPENDIX D THROUGH G

Q[1] is the fourth root of the equation

$$4 - 39\,q + 511\,q^2 - 2448\,q^3 + 6186\,q^4 - 4527\,q^5 - 4157\,q^6 + 3446\,q^7 + 1312\,q^8$$

Q[2] is the second root of the equation

$$-2 + 23\,q - 173\,q^2 + 208\,q^3 + 1148\,q^4 - 3545\,q^5 + 1067\,q^6 - 446\,q^7 + 2136\,q^8$$

K[1] is the third root of the equation

$$4\,k^4\,(-1+q)^5\,(-1-12\,q+9\,q^2) + k^3\,(-1+q)^3\,(-15-64\,q+346\,q^2-264\,q^3+45\,q^4)(R+v)$$
$$-3\,k^2\,(-1+q)^3\,(-1+3\,q)\,(-7-25\,q+27\,q^2+13\,q^3)(R+v)^2$$
$$-k\,(-1+q)\,(-1+3\,q)^2\,(13+26\,q-8\,q^2+6\,q^3+11\,q^4)(R+v)^3$$
$$-(1+q)^2\,(-1+3\,q)^3\,(3+q^2)(R+v)^4$$

K[2] is the third root of the equation

$$2\,k^3\,(-1+q)\,(-4+7\,q)\,(1-2\,q+6\,q^2+4\,q^3)$$
$$-2\,k^2\,(-4+27\,q-62\,q^2+53\,q^3-34\,q^4-32\,q^5+16\,q^6)(R+v)$$
$$-k\,(-2+13\,q-27\,q^2+31\,q^3-q^4+2\,q^5+32\,q^6)(R+v)^2$$
$$-2\,q(2+q)\,(1+q^2)(1-q+2\,q^2)(R+v)^3$$

K[3] is the third root of the equation

$$k^3\,(-1+q)^3\,(-4-41\,q-135\,q^2-115\,q^3-33\,q^4+112\,q^5)$$
$$+k^2\,(-1+q)^2\,(-1+3\,q)\,(-12-79\,q-129\,q^2-29\,q^3-135\,q^4+24\,q^5)(R+v)$$
$$-4\,k\,(-1+q)\,(-1+3\,q)^2\,(3+9\,q+2\,q^2+16\,q^3+6\,q^4+6\,q^5)(R+v)^2$$
$$-2\,(2+q)\,(-1+3\,q)^3\,(1+q^2)(1-q+2\,q^2)(R+v)^3$$

K[4] is the third root of the equation

$$2\,k^3\,(-1+q)^2\,(-1+4\,q)\,(-2+q+4\,q^2)$$
$$-2\,k^2\,(-1+q)^2\,(-2+11\,q-7\,q^2+6\,q^3+8\,q^4)(R+v)$$
$$-k\,(-1+8\,q-18\,q^2+34\,q^3-17\,q^4+18\,q^5+8\,q^6)(R+v)^2$$
$$+2\,(-1+q)\,q(1+q^2)(1-q+2\,q^2)(R+v)^3$$

And K[5] is the third root of the equation

$$k^3(-1+q)^3(-2-17q-47q^2-47q^3-139q^4+132q^5)$$
$$-k^2(-1+q)^2(1+q)(-1+3q)(6+25q+20q^2+29q^3+12q^4)(R+v)$$
$$-2k(-1+q)(1+2q)(-1+3q)^2(3+4q^2+2q^3+3q^4)(R+v)^2$$
$$-2(-1+3q)^3(1+q^2)(1-q+2q^2)(R+v)^3$$

$$T[1] = \left(-2k^3(-1+q)^3(-1-10q+3q^2)-k^2(-1+q)(-5-16q+66q^2-16q^3+3q^4)(R+v)\right.$$
$$+2k(-1+q)(1+q)(2+q-10q^2+3q^3)(R+v)^2+(1+q)^4(-1+3q)(R+v)^3+\left.\sqrt{X_1}\right)$$
$$/\left(2(1-4q+3q^2)(k^2(-1+q)^2(1+3q)+2k(1-5q^2)(R+v)-(1+q)^2(-1+3q)(R+v)^2)\right)$$

Where $X_1$ is defined as:

$$\left(4k^2(-1+q)^2+4k(-1+q)^2(R+v)+(1+q)^2(R+v)^2\right)$$
$$x\left(-k^2(-1+q)^3(-1+3q)+2k(-1+q-3q^2+3q^3)(R+v)+(1+q)^3(-1+3q)(R+v)^2\right)^2$$

$$T[2] = \frac{1}{4(1-3q)^2 q(R+v)}$$
$$\left(4k^2(-1+q)^2+(1-4q+3q^2)(R+v)\left((R+v)+q(R+v)\right.\right.$$
$$+\left.\sqrt{4k^2(-1+q)^2+4k(-1+q)^2(R+v)+(1+q)^2(R+v)^2}\right)$$
$$+2k\left((2-7q+10q^2+3q^3)(R+v)\right.$$
$$\left.\left.-(-1+q)\sqrt{4k^2(-1+q)^2+4k(-1+q)^2(R+v)+(1+q)^2(R+v)^2}\right)\right)$$

$$T[3] = \frac{2k(1+q)+(1+q)(R+v)+\sqrt{4k^2(-1+q)^2+4k(-1+q)^2(R+v)+(1+q)^2(R+v)^2}}{2(-1+3q)}$$

T[4] is the third root of

$$8k^3q^2(-1-4q-9q^2+2q^3)+2k^2q(-1+3q)(1+27q+7q^2+33q^3+4q^4)\tau$$
$$-2kq(-1+3q)^2(9+q+13q^2+7q^3+6q^4)\tau^2+(2+q)(-1+3q)^3(1+q^2)(1-q+2q^2)\tau^3$$

T[5] is the third root of

$$8k^3q^2(-1+4q-9q^2+2q^3)+2k^2q(1+q)^2(-1+3q)(1+q+4q^2)\tau$$
$$-2kq(-1+3q)^2(3+q+3q^2+3q^3+2q^4)\tau^2$$
$$+(-1+3q)^3(1+q^2)(1-q+2q^2)\tau^3$$

P[1] is the second root of

$$p^3(1+q)(4kq+\tau-2q\tau-3q^2\tau)+p^2(-8kq-\tau+q\tau+5q^2\tau+3q^3\tau)$$
$$+4k(-1+q)q-2p(-1+q)q(6k+\tau-3q\tau)$$

$$P[2] = \frac{1}{(-1+2q+3q^2)(2k+\tau-3q\tau)}\Big(2kq(1+q)+(R+v)-2q(R+v)$$
$$-3q^2(R+v)-\tau+6q\tau-9q^2\tau+\sqrt{X_2}\Big)$$

Where $X_2$ is defined as:

$$-4k^2(-1+4q-3q^2-6q^3+2q^4)+(1-3q)^2((1+q)(R+v)+q(-1+3q)\tau)^2$$
$$-4k(-1+2q+3q^2)((1-3q+4q^2)(R+v)+q^2(-1+3q)\tau)$$

$$P[3] = \frac{-2kq(1+q)+\tau-6q\tau+9q^2\tau+\sqrt{X_3}}{(-1+3q)(-4kq+(-1+2q+3q^2)\tau)}$$

Where $X_3$ is defined as:

$$q(4k^2q(5-14q+13q^2)-4k(1-3q)^2(1-3q+4q^2)\tau+(1-3q)^4q\tau^2)$$

$$P[4] = \frac{(1+q)(R+v)-\sqrt{4k^2(-1+q)^2+4k(-1+q)^2(R+v)+(1+q)^2(R+v)^2}}{2k(-1+q)}$$

$$P[5] = \frac{2kq+(1-3q)\tau-(1+q)(R+v)+\sqrt{X_4}}{(1+q)(2k+(1-3q)\tau)}$$

Where $X_4$ is defined as:

$$4k^2+(q(-1+3q)\tau+(1+q)(R+v))^2-4k(q(-1+3q)\tau+(-1+q+2q^2)(R+v))$$

$$P[6] = \frac{(1+q)(R+v)-\sqrt{4k^2(-1+q)^2+4k(-1+q)^2(R+v)+(1+q)^2(R+v)^2}}{2k(-1+q)}$$

P[7] is the second root of

$$1+4q+9q^2-2q^3+p(-2+10q-22q^2-10q^3)+p^3(2q-4q^2+2q^3)+p^2(1-11q^2+10q^3)$$

And P[8] is the second root of

$$1-4q+9q^2-2q^3+p(-2+10q+2q^2-10q^3)+p^2(1-11q^2-6q^3)+p^3(2q-4q^2+2q^3)$$

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