INCLUDING SEVERE UNCERTAINTY INTO ENVIRONMENTALLY BENIGN LIFE CYCLE DESIGN USING INFORMATION-GAP DECISION THEORY

A Dissertation
Presented to
The Academic Faculty

By

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In Partial Fulfillment
Of the Requirements for the Degree
Doctor of Philosophy in the
George W. Woodruff School of Mechanical Engineering

Georgia Institute of Technology

April, 2008

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This work is dedicated to my parents,

For their unconditional love and support.

ACKNOWLEDGEMENTS

This thesis exists in large part because of the significant contributions of numerous educators, colleagues, friends, and family, all of whom deserve my acknowledgement and my sincere thanks.

I thank my advisor, Dr. Bert Bras, for his guidance, insight, and feedback, his remarkable Dutch directness, and his firm encouragement. I hope in the future to be able to utilize the training he has provided me in the area of environmentally conscious design and manufacturing. My tremendous thanks goes to Dr. Chris Paredis, who provided invaluable guidance and intellectual stimulation in the area of decision making and uncertainty formalisms. The more mathematical contributions in this thesis were *directly* catalyzed by his involvement. Thanks also goes to Chris for a summer of financial support. I would like to thank the other members of my thesis committee, Dr. Janet Allen, Dr. Leon McGinnis, and Dr. Jean-Lou Chameau for their fresh perspectives, constructive criticism, tact and *patience*, and time spent reviewing my work. Dr. Chameau deserves additional appreciation for supporting me for several semesters using funds made available through the Hightower Chair at the Georgia Institute of Technology. Dr. Chameau stuck with me as a committee member as he ascended through the ranks of academia, and for that I am quite appreciative. I also recognize major support as a GRA through NSF grant CMMI-0522116.

Thanks must go out to Dr. Yakov Ben-Haim, the creator of info-gap theory, for opening up an important line of research, for his imagination in going outside of the bounds of the status quo, and for his willingness to personally communicate with me and provide feedback on my ideas back in the early stages of my work.

I delight at the opportunity to thank the many members of Georgia Tech for both provoking discussion and being great friends. It was a pleasure to be in the esteemed company of fellow SRL members Hae-Jin Choi, Scott Cowan, Carolyn Conner, Rich Malak, Tarun Rathnam, Steve Rekuc, Nate Rolander, and Andrew Schnell. These are but a few SRLiens with whom I interacted and learned. In the Sustainable Design and Manufacturing Program, I'd like to thank Dr. Carol Carmichael, Tina Guldberg, and the many SDM students over the years, especially my cohorts in EBDM on my Ph.D. journey, John Reap and Felipe Roman. Sophie Govetto deserves a special nod for planting the seeds that would eventually result in my major thesis example problem. Special recognition goes to Jason Aughenbaugh also for his contributions to the various articles we worked on together, and his critical insight into info-gap theory.

Next, Matt Chamberlain, Marco Fernandez, Benay Sager, and Christopher Williams have been excellent friends to lean on, research with, and even be a groomsman alongside of. You guys continue to inspire me. Additionally, local friends Jack Zentner, Janel Nixon, Sarah Keith, and Mike Zentner have made Atlanta a great place to be. I look forward to many years of friendship with all of these fine folks.

My girlfriend, Sharon Carroway, was a source of support, constancy, patience, nourishment, comfort, laughs, and *love* throughout the entire thesis process. Sharon, I could not have done this without you, and I can't wait to make it up to you!

Lastly, I give my utmost thanks to my family. Their love and encouragement have been the foundation for my successes in life. My parents, as people who find great

satisfaction in being educators and scientists, have provided the example that has been and will always be a huge inspiration on my career.

I am fortunate beyond words to have such wonderful people in my life. Thanks again to you all.

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GLOSSARY

Certainty: the condition of knowing everything necessary to choose the course of action whose outcome is most preferred.

Ignorance: inability to recognize (much less quantify) the existence of uncertainty; an unknown unknown.

Imprecision: the gap between the presently available state of information and a state of precise information; i.e., uncertainty that can be reduced by gathering information. Elsewhere referred to as "epistemic uncertainty" or "reducible uncertainty".

Indeterminacy: Inability to make a decision or identify the most preferred choice, perhaps due to propagated uncertainty; uncertainty about some state or condition that is affected by the unknown outcomes of indeterminate actors or events earlier in a causal chain.

Info-gap uncertainty: the gap between what is known and what needs to be known in order to make a fully competent decision.

Info-gap uncertainty parameter: α, unknown size of the discrepancy (error) between a known nominal estimate and an unknown actual value of a variable or model having info-gap uncertainty.

Irreducible uncertainty: inherent randomness, recurrence of chance events, e.g., the numbers that come up when rolling dice. Also called aleatory uncertainty.

Model: a mathematical representation, useful to a design and/or analysis activity. See system model and uncertainty model.

Parameter: One variable, or input, into a system model or reward function. The parameter itself may be an output of another function.

Performance function: mathematical expression of some goal that a design is to achieve, e.g., profit maximization or waste minimization. Akin to an objective function in optimization. The term reward function is used in descriptions of information-gap decision theory and can be considered a specific type of system model.

Preferredness: The preference ranking assigned to an alternative. There are situations demonstrated in this thesis where preferredness is indeterminate.

Preferredness Switch Point: A preference ranking change at some value of critical reward, as seen on a robustness-performance ($\hat{\alpha}-r_c$) plot.

Representativeness: the degree to which a nominal value or model taken from one situation is valid or matches reality in a new situation.

Robustness: (Info-gap definition) immunity to failure; (Taguchi definition) minimized variation in a response variable, given variation in an input parameter.

Selection decision: choice of a design from a field of alternatives, described by a system model(s), which aims to meet certain targets and does not violate defined constraints.

System model: A mathematical relationship between input and output parameters, e.g., a cause-effect model, behavioral model, mechanics model, logistics model, etc.

Uncertainty: the gap between certainty and the decision maker's present state of information

Uncertainty model: A mathematical representation of uncertainty for either a system model or the parameters within that model. Some choices for modeling the uncertainty of a parameter include probabilistic, fuzzy (if membership is uncertain or linguistically vague), information gap, etc.

Uncertainty parameter: See: Info-gap uncertainty parameter.

LIST OF SYMBOLS

a_n	scaling reference elicited using BBSE; used to determine s_n
p	betting price; also the magnitude of a probability elicited subjectively
q	a design option or design variable
$\hat{q}$	a design option or design variable that is most preferred from a set of alternatives because it affords the most robustness
r_c	critical reward; satisficing level of reward that must be met or exceeded
s_n	scaling factor; scales an info-gap to a "typical" reference size
u	variable whose uncertainty can be modeled as an info-gap
u_n	one of several info-gap uncertain variables, $n = 1 \ldots N$
$\tilde{u}$	nominal estimate for a variable whose uncertainty is modeled as an info-gap
$\mathcal{U}$	an info-gap model of uncertainty
α	info-gap uncertainty parameter; the unknown horizon of uncertainty around $\tilde{u}$
α_n	info-gap uncertainty parameter for u_n; one of several info-gaps $n = 1 \ldots N$
$\hat{\alpha}$	robustness to info-gap uncertainty, α
$\hat{\alpha}_n$	robustness to info-gap uncertainty, α_n, one of several info-gaps $n = 1 \ldots N$

LIST OF ABBREVIATIONS

BBSE	Bet-Based Scaling Elicitation
DA	Decision Analyst
EBDM	Environmentally Benign Design and Manufacture
EFS	Equal Fractional Scaling
ESRT	Equal Scale Robustness Trade-off
IGDT	Info-Gap Decision Theory
IBI	Interval of Bet Imprecision
IPI	Interval of Preferredness Indeterminacy
ISI	Interval of Scaling Imprecision
ITS	Imprecise Trade-off Specification (as in: ITS sector)
LCA	Life Cycle Assessment
MQ	Motivating Question
OMQ	Overarching Motivating Question
PSP	Preferredness Switch Point
RA	Risk Assessment
SEC	Steel Easy Change (a type of oil filter)
TASO	Take-Apart, Spin On (a type of oil filter)
TS	Trade-off Specification (as in: TS line)
UFS	Unequal Fractional Scaling

SUMMARY

Due to increasing interest in sustainable development, today's engineer is often tasked with designing systems that are environmentally benign over their entire life cycles. Unfortunately, environmental assessments commonly suffer from significant uncertainty due to lack of information, particularly for time-distant life cycle aspects. Under severe uncertainty, traditional uncertainty formalisms require more information than is available. However, a recently devised formalism, information-gap decision theory (IGDT), requires no more information than a nominal estimate; error bounds on that estimate are unknown. The IGDT decision strategy, accordingly, favors the design that is robust to the most estimation error while still guaranteeing no worse than some "good enough" critical level of performance. In some cases, one can use IGDT to identify a preferable design option without needing more information or more complex uncertainty analysis.

In this dissertation, IGDT is investigated and shown to enhance decision support for environmentally benign design and manufacturing (EBDM) problems. First, the applicability of the theory to EBDM problems is characterized. Conditions that warrant an info-gap analysis are reviewed, the insight it can reveal about design robustness is demonstrated, and practical limitations to its use are revealed. Second, a new mathematical technique is presented that expands capabilities for analyzing robustness to multiple info-gap uncertainties simultaneously. The technique elicits scaling factors more rigorously than before and allows one to imprecisely express their beliefs about info-gap scaling. Two examples problems affected by info-gaps are investigated: oil filter selection and remanufacturing process selection. It is shown that limited

information about uncertainty can, in some cases, indeed enable one to identify a most preferable design without requiring more information.

CHAPTER 1:

INTRODUCTION

In this thesis, a relatively new uncertainty formalism, *information-gap decision theory* (IGDT), is investigated and shown to enhance decision support for environmentally benign design and manufacturing (EBDM). IGDT is applied to several problems featuring severely deficient information about uncertainty. Additionally, a new technique for modeling and analyzing the effects of multiple uncertainties is presented, validated, and tested out.

In this chapter, the purpose and direction of the thesis is introduced. In Section 1.1, motivation for using IGDT is established and the context in which it is considered is explained. In Section 1.2, the specific "Motivating Questions" to investigate are presented along with corresponding assertions about their Answers, which are to be explained and defended in the remainder of the thesis. In Section 1.3, the intellectual contributions to be presented, tested, and defended are summarized. In Section 1.4, an explanation of the approach to validation and testing is provided. Finally, in Section 1.5, an overview to the story and content of the entire thesis is presented.

1.1 Context and Motivation

The research in this thesis is motivated by the prevalence of severe uncertainty in EBDM. The general context of EBDM is introduced in the next subsection, followed by an overview of the problem of uncertainty in that context.

1.1.1 Environmentally Benign Design and Manufacturing

Companies are becoming more and more concerned with the environment because a growing number of people, including consumers, are realizing that there is a cost to society that results from environmental impact. All products and processes affect in some way our environment during their life-span. In Figure 1.1, a schematic representation of a product's life-cycle is given. Materials are mined from the earth, air and sea, processed into products, and distributed to consumers for use, represented by the flow from left to right in the top half of Figure 1.1.

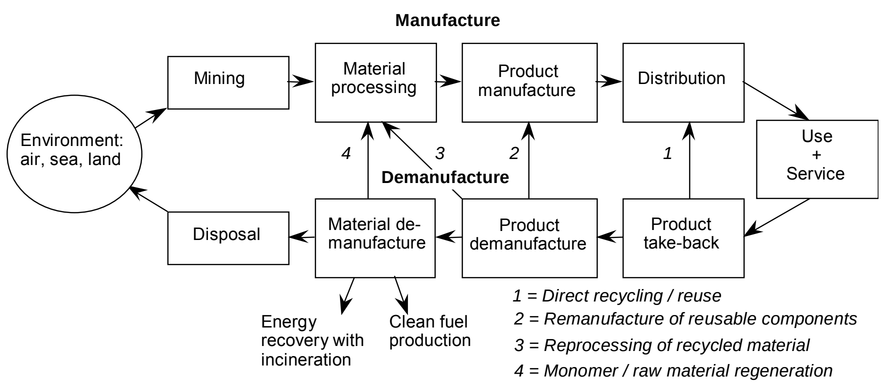

Figure 1.1: A Generic Representation of a Product's Life-Cycle (Bras 1997)

In general, a company's environmental impact comes from (excessive or wasteful) consumption of natural resources and emissions of pollutants to air, water, and land. Recognition of the negative effects of air emissions has led, among others, to the Clean Air Act and Corporate Average Fuel Economy legislation in the United States.

The emergence of product take-back directives in Europe and Japan has forced manufacturers to include product recycling and reuse considerations in their designs, as represented in the lower half of Figure 1.1 (EU 2000, 2003). This has lead to initiatives for product take-back and "demanufacture" (a phrase used to characterize the process opposite to manufacturing necessary for recycling materials and products) (Gutowski et al. 2001, Allen et al. 2002). It is therefore not surprising that there is a growing interest in Environmentally Benign Design and Manufacture (EBDM), defined by NSF as "a system of goals, metrics, technologies, and business practices that address the long term dilemma for product realization: how to achieve economic growth while protecting the environment?" (Bras et al. 2006). A key characteristic of life-cycle issues in EBDM is that only very limited information and knowledge is available, resulting in large uncertainty. A product and its embodied materials may interact with a global ecosystem over a very long time-horizon, and its impact on the environment depends to a large extent on the future behavior of stakeholders (e.g., consumers, service personnel, and policy makers).

1.1.2 Critical Issue: Inherent Uncertainty in the Product Life-Cycle

Designers, engineers, managers, and companies alike are faced with new and emerging issues around product life-cycles that they have never faced before. Design performance is influenced both by product attributes as well as life cycle activities and circumstances outside the control of the company designing the product. Such circumstances, which link product configuration choices to environmental impact, for instance, are connected in a chain of cause and effects, as shown in Figure 1.2, where a design feature on a transmission is shown to relate, remotely, to the dispersion of automatic transmission fluid (ATF) waste into the environment.

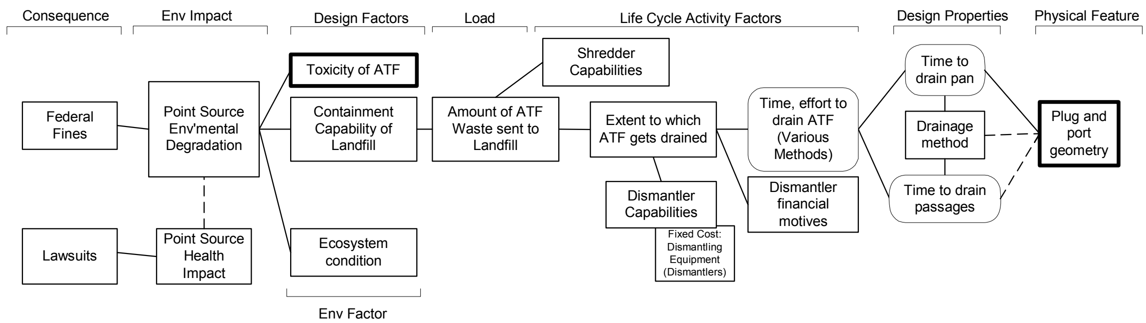

Figure 1.2: Cause and effect parameters in a transmission's product life cycle, only concerning the end-of-life path of transmission fluid.

An evaluation of all of the loads and impacts along these chains has traditionally been addressed with life cycle assessment (LCA) methods, as described in (Keoleian et al. 1994, Vigon et al. 1994, Wrisberg and Udo de Haes 2002, Keoleian and Kar 2003). Although existing LCA case studies commonly use deterministic parameters in their assessments, many researchers are starting to account for the large amount of uncertainty in LCA information (Huijbregts 1998, Björklund 2002, Matthews et al. 2002, Ross et al. 2002, von Bahr and Steen 2004). A common theme in the literature is that the trust in LCA results is undermined by the large simplifications and unconfirmed assumptions in life cycle models. These assumptions often have to be made due to the scarcity of data about a variety of LCA aspects, such as generation of environmental loads (e.g., energy use, resource extraction, and waste production over the life cycle), dispersion of loads (by region as point or non-point source), and impact of loads on ecosystem or human health. With limited information, statistical characterizations of uncertainty are not feasible.

Quite commonly, as will be discussed in Section 2.2.4, designers must rely on information or models that are known to be significantly uncertain, but that uncertainty has not been quantified. This uncertainty occurs when there is only scarce information available, or when information is available but taken from different design scenarios where "the past is a weak indication of the future" (Ben-Haim 2006). This type of uncertainty will be referred to as "severe uncertainty" or "severely deficient information" in this thesis. Under severe uncertainty, there is nothing available to describe the uncertain variable or model other than a *nominal estimate*, with upper and lower error bounds on either side of that estimate unknown. This nominal estimate may be a rough approximation, a comparable baseline from a similar life cycle design problem, a constant or value from a handbook, etc. In some cases, significant effort or expertise is required to quantify the uncertainty; in other cases, the uncertainty is simply not quantifiable in the time frame in which a design decision must be made.

Several typical strategies tend to be employed when validated uncertainty characterizations are non-existent and severe uncertainty is present. Most often in EBDM, rather arbitrary ±10% parameter variation is assumed, and judgments are made based considering the sensitivity of the results (Björklund 2002). In other cases, especially when more easily measurable performance aspects like cost and quality are being considered, uncertain EBDM aspects that are unregulated may simply be ignored out of convenience.

Another course of action is to rely on unwarranted assumptions that fill in missing information, perhaps in order to use an uncertainty formalism that requires more information about uncertainty. Several studies have quantitatively investigated how use of probabilistic methods under very limited statistical data can lead to inaccurate estimates of the probability of failure of safety-critical designs (Ben-Haim and Elishakoff

1990) or designs with a greater chance of severe failure (Aughenbaugh and Paredis 2006). The assumption (or interpolation) of more information than is available can lead to risky results. So, the question arises, when information and/or characterizations of uncertainty are sparse, to what extent can we—and should we—use *only* that information in decision support analyses?

1.1.3 An Emerging Approach to Decisions under Severely Deficient Information

One answer is to search for decisions that are robust to lack of information. When information is severely uncertain, a decision maker may want to make a decision that will yield a reasonably satisfactory result over a large range of realizations of the uncertain parameters. Information-gap decision theory (IGDT), developed by Ben-Haim (Ben-Haim 2006), is one means of identifying which designs have performance that is immune to the effects of uncertainty. In choosing to use IGDT, one essentially asks, "How wrong can a model and/or its parameters be without jeopardizing the quality of decisions that are based on this model?" A detailed introduction to and evaluation of IGDT is presented in Chapter 3, but a brief overview is presented next.

In IGDT, it is assumed that a decision maker has available a nominal, but suspect, estimate of an uncertain quantity. The decision maker wishes to analyze his options without any further assumptions about uncertainty, as further information is unavailable. In response, IGDT presents an approach to making design decisions when there is an *info-gap*, that is, a gap of unknown size between the uncertain quantity's true value (which could be known but is not) and the available nominal estimate. This concept is illustrated for an uncertain quantity in Figure 1.3.

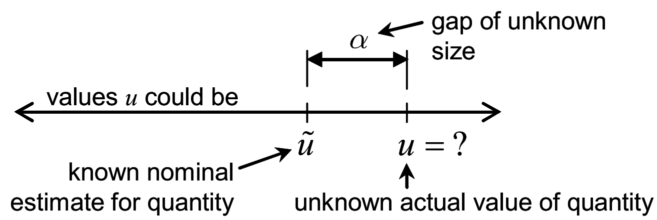

Figure 1.3: Simple representation of info-gap uncertainty (Reproduced from (Duncan et al. 2007))

IGDT models the size of this gap as an unspecified uncertainty parameter, α. The design decision maker confronts this gap by employing a performance-satisficing (rather than the traditional performance-maximizing) decision policy that seeks to maximize robustness[1] to uncertainty. This requires the decision maker to specify a satisficing, critical performance level—a "good enough", minimally acceptable level of performance that is guaranteed to be exceeded—and accordingly choose the design that, subject to this minimum requirement, allows for the largest information gap, i.e., the largest α. Thus, when using info-gap theory to confront a design problem with severe uncertainty, one adopts the mindset that "good enough" performance is acceptable and that the design that guarantees *at least* that performance under the most uncertainty is preferable.

A trade-off often exists between demand for critical performance and the amount of robustness that can be achieved. Given the severe state of uncertainty, one may wish to compare this robustness-performance trade-off for each design option. Such a trade-off is depicted conceptually in Figure 1.4 for two design alternatives. As seen in the figure, if a decision maker is willing to settle for a critical performance below 2.4 units,

[1] In Section 3.2.3, the info-gap definition of robustness will be shown to be different than another definition, where robustness involves minimizing the *variation* in performance (outputs) caused by uncertain input parameters.

as long as that performance is *guaranteed* to be met or exceeded, Design 1 would be preferable as it offers greater robustness to info-gap uncertainty. So, preference ranking for design alternatives depends on one's trade-off preference, which is assumed to not be known or to be difficult to express *a priori*. Inspection of robustness curves can help one induce their preference for this trade-off.

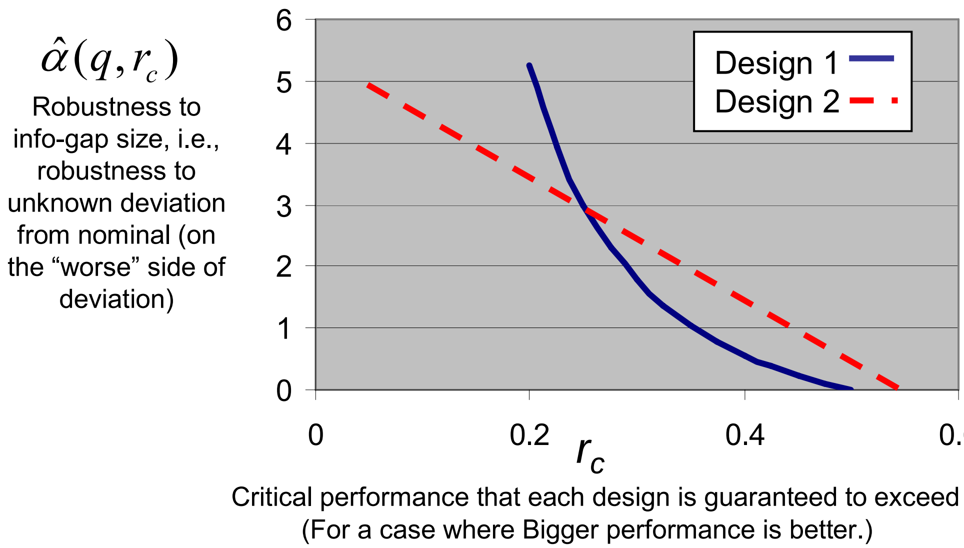

Figure 1.4: Trade-offs between robustness to info-gaps and critical performance.

To summarize, not only is the modeling of uncertainty unique; the mode of assessing robustness, inducing trade-off preferences, and identifying most-preferred designs also differs from other formalisms. These differences are considered in depth in Section 3.6.

Because IGDT is a relatively new decision formalism, it has not seen wide application. It receives mention—in passing—in more recent overviews and/or comparisons of uncertainty formalisms. Usually, these reviews offer no critical investigation of the way IGDT models uncertainty, measures robustness, and influences preference rankings. Intuitively, there would seem to be a match between the needs of EBDM analyses and the decision support potential of IGDT, but until now this has remained unexplored.

IGDT is aligned with a main theme of EBDM research—to get as much design knowledge as possible out of deficient information, even if that requires a different approach to evaluating the acceptability of performance. EBDM practitioners commonly lack information characterizing the uncertainty of some aspect of a system's life cycle, but in some cases acceptable design decisions can be made without knowing more. As Ben-Haim asserts, "a little information can go a long way, especially when it is not corrupted by unwarranted or unjustifiable assumptions" (Ben-Haim 2006). IGDT analyses can uncover important weaknesses or opportunities in different designs' capacity to provide robustness to faulty data. If severe uncertainty never favors a decision alternative, it can be removed from further consideration; if uncertainty always favors one alternative, it can be chosen without further collection of information. As this thesis reveals, other similar design insights can also be gained from IGDT analyses.

1.2 Motivating Questions

With the context now explained, the general question to be answered by this thesis can be stated as such:

<u>Overarching Motivating Question (OMQ)</u>: How should one represent and analyze severe uncertainty inherent in product life-cycle information to provide better decision support for environmentally benign design and manufacture?

Answer[2] to OMQ: A decision maker should apply info-gap decision theory to determine whether preferable design choices can be identified without requiring more information about the life-cycle than is available.

As mentioned at the end of Section 1.1.2, relying on unwarranted information or assumptions to remedy a severe lack of information may in some cases lead to bad decisions. Instead, we advocate the application of IGDT to EBDM design problems to determine whether decisions can be reached without more complex, information-demanding methods for design under uncertainty.

The Overarching Question is answered in this thesis by applying IGDT to problems and discussing the particular design insight gained for each. The following two specific secondary Motivating Questions, however, lead to the more tangible and substantial contributions.

1.2.1 Motivating Question 1: IGDT Applicability

Because there have been no IGDT applications for life cycle design problems in the literature (other than ones by the author), the appropriateness and usefulness for EBDM has not been well characterized. In fact, IGDT applicability *in general* is not immediately clear from the limited examples in the literature. This is mainly because IGDT is a relatively new formalism. Thus, the first Motivating Question concerns IGDT applicability:

Motivating Question 1 (MQ1): How should we determine when IGDT will be most appropriate for supporting EBDM decisions?

[2] Note that we propose *Answers to advance* rather than Hypotheses, which would need to be refutable. The challenge of devising clear hypotheses that can be scientifically refutable is a defining characteristic of design research such as this, as discussed by (Aughenbaugh 2006).

A more informal way to ask this question is: if "a little information can go a long way", when is this the case, and how "far" can we take it? Like most formalisms for decisions under uncertainty, IGDT cannot be applied to all scenarios, and when it can, it doesn't always provide useful design insight. In addition, as the number of uncertainties and design alternatives under consideration scale up, info-gap analysis becomes increasingly difficult to carry out and interpret. This is mostly due to the way that IGDT's unique satisficing approach is implemented, as is explained in Chapter 3. Cognizant of these gaps, we propose the following Answer:

<u>Answer to MQ1:</u> An evaluation of the usage conditions and decision support capabilities of IGDT, as well as the needs and characteristics of archetype EBDM problems, can be used to establish a set of guidelines for screening applicability.

These guidelines are for the most part a checklist of criteria, to be presented in Section 4.1. The elements in the guidelines are a compilation of observations about the structure of different info gap problems as well as lessons learned from actual applications.

The answer to MQ1 may not seem particularly aggressive—can guidelines alone assure that IGDT is *most* appropriate, as MQ1 asks? Ideally, one would want an experimental head-to-head comparison of the "value" of design information yielded from an info-gap analysis versus that of competing uncertainty formalisms. However, a review of the underlying assumptions and mathematics of IGDT versus other uncertainty formalisms in Section 3.5 reveals that they cannot be compared in terms of value of information. This is because they rely on different information sources and decision rules. So, a design problem will be *eligible* for solution using IGDT based on the state of information about uncertainty. Given only that information state, IGDT is distinct enough, with a clearly different approach to modeling uncertainty and assessing robustness, that it is not in direct competition with other uncertainty formalisms. If further information (either measured or subjective) is available, another uncertainty formalism should be used.

Consistent with the fact that the inputs and outcomes of an info-gap analysis do not compare to other uncertainty and decision formalisms, IGDT differs because it is not meant to be a purely normative decision theory. As Ben-Haim states (p.3, (Ben-Haim 2006)):

> *…info-gap theory is not a closed computational methodology. Rather, [its] quantitative assessments assist the decision maker to evaluate options … and to evoke and evolve preferences in light of the analysis of uncertainties, expectations, and demands.*

The details implications of this decision support perspective to EBDM are discussed in Chapter 3, after example problems in Chapters 4 and 6, and in the discussion at the conclusion of this thesis.

1.2.2 Motivating Question 2: Extension of Methods

The other main thesis objective focuses on the modeling and analysis of multiple info-gaps (henceforth referred to as "multi-gaps"). The current approach in the literature uses *scaling factors* to scale (or map) the relative sizes of different info-gaps to a single parameter representing the "overall" gross uncertainty that designs might face (Ben-Haim and Laufer 1998). Different designs are ranked by the size of their robustness to this gross measure of overall uncertainty. Because of this, the choice of scaling factors can have a significant influence on what robustness is expected of different design alternatives.

The current multi-gap technique explains how to *use* scaling factors but offers little substantial guidance about how to *determine* them; it is simply suggested that they might be available as pre-existing "prior knowledge" (Ben-Haim and Laufer 1998). In addition, scaling factors are quantified *precisely* even though they are associated with severely uncertain variables.

These facts provide motivation for an improved technique for capturing and utilizing what a decision maker knows about the relative scales of multi-gaps. Robustness calculations based on information that is more representative of what the designer knows will in turn lead to better decision support. From these ideas, a Motivating Question follows:

> <u>Motivating Question 2 (MQ2)</u>: For problems affected by multiple uncertainties, how should scaling factors be elicited in a rigorous fashion that allows for imprecision in those factors?

In response, we propose a novel technique for eliciting and modeling potentially available information about the relative scales of different info-gaps:

> <u>Answer to MQ2:</u> One's beliefs about info-gap scaling should be elicited in the form of subjective probabilities, which are revealed through one's betting behavior. The method for eliciting subjective probabilities allows for imprecise expression of one's knowledge about scaling, which in some cases causes indeterminacy in the preference rankings for design alternatives.

The foundations of this answer are the subject of Chapter 5. It is demonstrated that a subjective probability, when elicited through a betting scenario involving the info-gap uncertainty of interest, can serve as a reference point for establishing relative info-gap scales. In the IGDT analysis and decision making, the subjective probability is not actually used as a probability, only as a reference point for one's belief about info-gap scale.

The other benefit of using a subjective probability as a reference point is that it can be elicited imprecisely. The upper and lower bounds on the precision with which one can identify a scaling factor correspond to upper and lower *previsions*, a concept from subjective probabilities that can also be revealed by betting behavior. This concept is explained in Section 5.3.1. The ability to quantify imprecision adds another dimension by which a decision maker can express the quality of their understanding of info-gap scale. In other words, more information can be captured about how rough one's knowledge of uncertainty scale is. Scaling imprecision can be propagated into the info-gap analysis and assessed, as will be discussed in Chapters 5 and 6. It will be shown that scaling imprecision does not necessarily prevent a decision from being made.

To generalize, MQ2 is motivated by the idea that more information about info-gaps leads to robustness that is more consistent with the decision maker's understanding uncertainty. This may not mean *more* robustness but should mean *more accurate* robustness. Accordingly, more informed mapping of relative info-gap scales to the gross uncertainty parameter will ensure that trade-offs between competing robustnesses are consistent with the decision maker's rough understanding of uncertainty.

1.3 An Overview of Contributions

The contributions in this thesis correspond to the Answers to the two secondary Motivating Questions in the previous section. The first contribution is the guidelines for when to use IGDT in EBDM. The guidelines serve as a checklist: what information sources does the decision maker have? How firm are their preferences? What type of model for performance is available, and what sort of performance does it measure? The guidelines also include a review of the modes of decision support that IGDT could potentially provide.

The second contribution is obviously more mathematical and thus more generalizable. Ben-Haim's existing technique for using scaling factors to map several uncertainties onto a single parameter will be leveraged, but the addition of a rigorous technique for eliciting those factors is new, as is the means of modeling and propagating imprecision when eliciting scaling. The betting-based scaling technique is presented in Chapter 5 as part of an *overall approach* to assessing how the combined effects of multiple info-gaps influence preference rankings for design alternatives. The systematic approach will be embodied as a decision tree. The goals of the approach are to (1) identify what information about scaling is available and (2) analyze what type of bearing that information has on decision making. In some cases, very little information will need to be known about scaling to identify a preferred design; in others, the lack of information can be shown to prevent decision making altogether.

1.4 A Plan for Validation and Verification

The initial steps towards validation that this thesis provides differs for the two main Answers to the Motivating Questions, since they differ greatly in their aims. The "story" of how we provide support towards validity in this thesis is as follows for each Answer. The two main example problems used in our validation work are an oil filter selection design problem (Duncan et al. 2006, Duncan et al. 2007) introduced in Section 4.2 and a remanufacturing process selection problem (Duncan et al. 2007) that is the sole focus of Chapter 6.

For Answer 1, in which guidelines of applicability are derived, validation requires that we:

- Carefully review the characteristics and decision support needs of a set of EBDM archetypes (Section 2.1). The type of EBDM problems to be solved are summarized in Section 2.1.3.
- Examine each part of the structure of info-gap modeling, decision rules, and trade-off analysis (Section 3.4), to ensure IGDT's internal logical consistency (Section 3.5),
- Compare this structure to other formalisms for decisions under uncertainty and explain why a direct comparison is infeasible (Section 3.6),
- Summarize when EBDM archetype characteristics are seen to match IGDT capabilities (Section 4.1),
- Apply info-gap to the basic oil-filter problem to demonstrate and generalize its usefulness and limitations, which can be translated into guideline information (Section 4.3),
- Explain how the guidelines verify that the remanufacturing process example problem is worthy of an IGDT analysis (Section 6.2.1),
- Discuss whether the guidelines can be applicable to a wider class of EBDM problems (Section 7.2).

Because these steps are not experimental in nature nor based on rigorous logic, they do not necessarily validate that the guidelines are *fully* general or complete for all types of EBDM problems. Nonetheless, the guidelines are based on a careful assessment of needs, structure, and capabilities, and they do provide a valuable starting point for considering whether to apply IGDT to EBDM problems. Such a starting point has been non-existent prior to this thesis.

Answer 2 involves synthesis of existing mathematical methods with the intent to extend the capabilities of multi info-gap analyses; therefore, it can be validated with more rigor than Answer 1. Validation requires that we:

- Verify the need for new methods by critically examining the indeterminacy that multiple uncertainties can cause when trying to rank design alternatives by their "overall" robustness (Section 5.1.1),
- Evaluate the information demands, assumptions, and function of Ben-Haim's scaling technique, in order to demonstrate when its assumptions can be restrictive or where its elicitation methods could be expanded on (Section 5.2.1),
- Review the theoretical rigor of using a betting scenario to elicit subjective beliefs (Section 5.3.1),
- Tie the new concept of imprecision in multi-info-gap scaling to the more rigorous, previously examined idea of imprecision in subjective probabilities (Section 5.3.1),
- Argue that a subjective probability, when tied to an operationalized betting scenario, is relevant as a reference point for calibrating or "mapping" the scales of different info-gaps (Section 5.3.2),

- Test the validity of the components of a new technique for "bet-based scaling elicitation" on the basic oil-filter problem and two uncertainties (Section 5.4),
- Explain and defend heuristics for making decisions when there is imprecision in scaling elicitation has propagated into indeterminacy in design preference rankings (Section 5.4.4),
- Explain mathematically how the new scaling technique applies generally to problems with more than two uncertainties, for either precise or imprecise scaling (Sections 5.2.4.2 and 5.4.5),
- Apply the new scaling technique to the more advanced remanufacturing process design problem and argue the value of the design insight gained (Section 6.2.2).

Notice that no hard comparisons are proposed to measure the "value" of the new scaling elicitation techniques, as compared to old methods which incorporate less information or which do no explain where scaling information comes from. It will be assumed that any technique that incorporates new information, as long as it is elicited rigorously, will provide robustness to a more accurate representation of what a decision maker knows about uncertainty, thus making the outcomes of assessments more valuable.

Verification of the outputs of info-gap analyses (i.e., that trade-off plots are accurate, that robustness sizes are indeed "maximum") will rely on the fact that all of the functions considered are linear or at least monotonic, and that all of the info-gaps are based on simple interval structures. Discussion of the challenges of verification for more complex functions will be provided in Chapter 7.

1.5 Organization of This Thesis

This thesis is divided into seven chapters. In this chapter, we have posed motivating questions and answers as well as a plan for validating the contributions that emerge from them. In Chapter 2, we review the structure and capabilities of various uncertainty formalisms, the structure and needs of EBDM, and the extent to which various uncertainty formalisms have been successfully adopted in EBDM. In Chapter 3, we evaluate the structure of IGDT, explain the decision support that it provides, and compare both of those to other uncertainty formalisms. In Chapter 4, we formulate guidelines for when IGDT will be useful to EBDM, and use those guidelines as rationalization for an info-gap analysis on an oil filter design selection problem. In Chapter 5, we review how current techniques analyze robustness to multiple uncertainties, extend existing methods that map multiple uncertainties to a gross parameter of uncertainty, and test these methods out on the oil filter example problem. In Chapter 6, we apply both the screening guidelines and the new multi-info-gap scaling technique to a comprehensive example problem involving remanufacturing process selection. In Chapter 7, we revisit the motivating questions, review novel contributions, and suggest future directions for research.

CHAPTER 2:

EBDM AND THE PROBLEM OF DEFICIENT INFORMATION

In this chapter, the problem of deficient life cycle information, as it affects the field of environmentally benign design and manufacturing (EBDM), is considered. First, in Section 2.1, the meaning of EBDM is explained, and the scope that will be considered within EBDM in this thesis is bounded. Section 2.2 consists of a literature review of the causes and types of information limitations in environmental life-cycle assessments. In Section 2.3, it is shown that information limitations make it problematic to apply various existing uncertainty formalisms to EBDM decisions.

2.1 The EBDM Context

In this section, we move from general definitions of EBDM to the specific context and types of problems to be considered in this thesis.

2.1.1 Definition and Scope

Environmentally Benign Design and Manufacture[3] (EBDM) is defined by NSF as "a system of goals, metrics, technologies, and business practices that address the long term dilemma for product realization: how to achieve economic growth while protecting the environment?" EBDM is comparable to other approaches to reducing negative environmental impact as shown in Figure 2.1, which distinguishes differences in their organizational and temporal spans (Coulter et al. 1995). EBDM is often acknowledged to

[3] The terms Environmentally Benign Design and Manufacture (EBDM) appears to simply be a recent update of the term Environmentally Conscious Design and Manufacture (ECDM). They will be referred to interchangeably in this thesis.

be generally equivalent to Design for Environment (Ashley, 1993, Fiksel, 1996a, Navin-Chandra, 1991), Life-Cycle Design (Alting and Joergenson, 1993, EPA, 1993), and Green Design (Congress, 1992), since they are all “practices that are intended to yield products whose aggregate environmental impact is as small as possible” ((Glantschnig 1994) via (Emblemsvag and Bras 2001)). Per the classification in Figure 2.1, EBDM is situated within the larger effort of sustainable development and utilizes techniques from environmental engineering and pollution prevention.

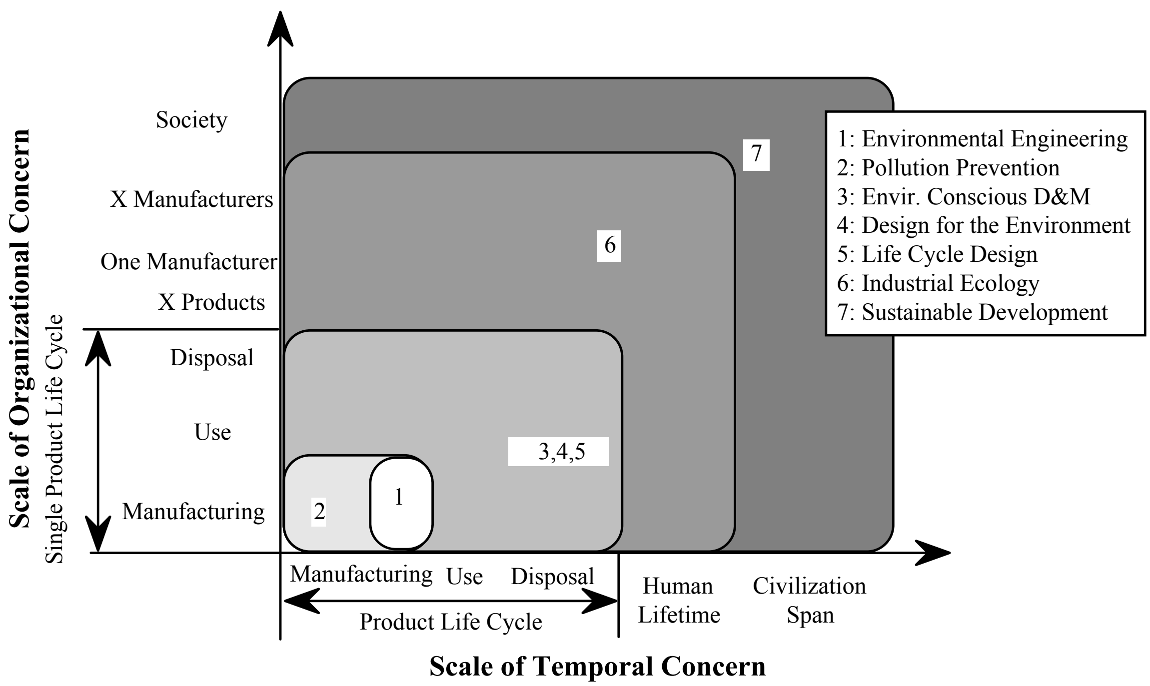

Figure 2.1: Environmental and Temporal Scale of Environmental Impact Reduction Approaches (from (Coulter et al. 1995))

It is generally agreed that environmental considerations cover a product’s entire life cycle and that a holistic, systems-based view provides the largest capability for reducing environmental impact of both products and associated processes (Bras 1997, Congress, 1992, EPA 1993).

2.1.2 From Environmental Assessments to Environmentally Benign Decisions

Environmental Life Cycle Assessment (LCA) is the method used to assess environmental aspects and impacts of either products and service systems (ISO 2006). LCA considers environmental impacts generated by all parts of a product's life cycle, from acquisition of materials through manufacture to recovery or disposal (Figure 1.1). One conducts an LCA by progressing through four distinct though interdependent phases: goal and scope definition, inventory analysis, impact assessment, and interpretation. Depending on the product system being evaluated and on how one scopes the assessment, the environmental performance dimensions measured by LCA can be numerous, and the spatial and temporal scales considered can be wide reaching. As such, "full", high quality LCA for complex systems is limited by a host of problems summarized in Table 2.1 and discussed at length in (Reap et al. 2008). All of the problems shown in the table can, on one way or another, be a significant root cause of uncertainty and/or lack of information, which is discussed at length in Section 2.2 of this chapter.

Table 2.1: Categories of Current Limitations to Environmental Life Cycle Analysis

LCA Phase	Problem
Goal and Scope	Functional Unit Definition
	Boundary Selection
	Social and Economic Impacts
	Alternative Scenario Considerations
Inventory	Allocation
	Local Technical Uniqueness
Impact Assessment	Impact Category & Methodology Selection
	Spatial Variation
	Local Environmental Uniqueness
	Dynamics of the Environment
	Time Horizons
Interpretation	Weighting and Valuation
	Uncertainty in the Decision Process
All	Data Availability and Quality

LCA is an *assessment framework*; the ISO standards that define it offer no specific guidance on how to actually design a more environmentally benign product. Design guidance in the form of prescriptive rules and suggested metrics is instead supplied by Design for Environment (or any of the equivalent approaches of Figure 2.1). In turn, these design approaches can be strengthened by a made more comprehensive using a formalized design approach like (Pahl and Beitz 1996).

In this thesis, however, the focus is solely on the analysis and decision making portion of engineering design. The more "creative" part of the design process is assumed to already be done and to have generated a set of design concepts. The focus of this thesis is analyzing the performance of a set of design alternatives to determine which is best. That is, a *preference ranking* is to be assigned over the design alternatives. It is assumed that a decision would select the most preferred alternative. Thus, in this thesis, settling on a preference ranking will be considered as a primary input to decision making.

The components of a design decision can be represented by an influence diagram (McGovern et al. 1993, Clemen 1996). A typical influence features design alternatives input on one end, which connect to and influence a network of simulation and analysis models, which combine with uncertainties and preferences to influence an objective function on the terminal end of the diagram. The influence diagram concept has been tailored to general aspects of environmentally benign life cycle analysis in Figure 2.2. This can be thought of as decision analysis that includes EBDM objectives and includes as much of the lifecycle and its upstream and downstream impacts (per Figure 1.1) as possible.

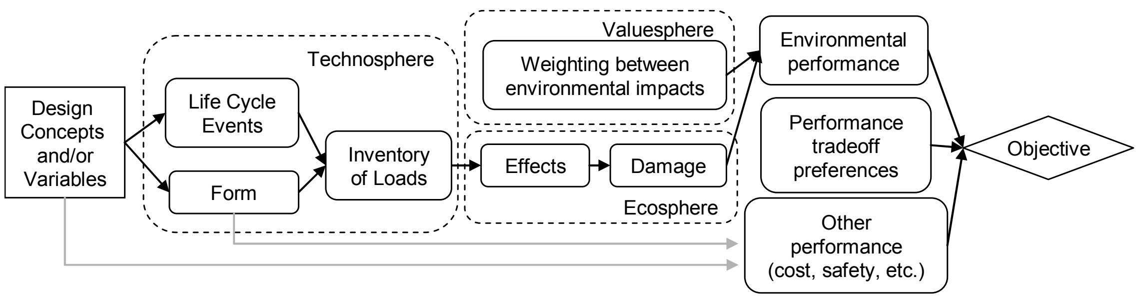

Figure 2.2: The components of an environmental analysis (from (Duncan et al. 2006))

Decompositions similar to Figure 2.2 have been proposed (for example (Hofstetter 1998, Lu and Gu 2003)), though none are identical in form or scope to the structure presented here. Components are grouped, as indicated by dashed-lines in the figure, using Hofstetter's concept of "spheres" of knowledge and reasoning about environmental evaluation (Hofstetter 1998)}. These spheres correspond to stages in LCA (left column in Table 2.1) as noted:

- Technosphere: description of the product and its life cycle and an inventory of loads (e.g., emissions). The aggregate environmental loads created by the technosphere correspond to the output of a Life Cycle Inventory (LCI) in LCA.
- Ecosphere: modeling of changes to the environment. The aggregate environmental impacts suffered by the ecosphere correspond to the output of Impact Analysis (IA) in LCA.
- Valuesphere: modeling of the perceived seriousness or importance of changes to the environment. Using the set of values contained in the valuesphere to weight the impacts on ecosphere results in an environmental performance measure similar to the outcome of an LCA.

Information about uncertainty can also be factored into a decision analysis. An *uncertainty formalism* involves:

- modeling the uncertainty mathematically,
- analyzing how its size and effects propagate through the chain of analysis models in the influence diagram, and
- interpretation of the influence of the propagated uncertainty on decisions.

Different uncertainty formalisms that receive wide use will be reviewed in Section 3.2, and sources of severe uncertainty in EBDM are reviewed in Section 2.2. But at this point in the thesis, all that is needed is a simple review of areas where uncertainty tends to manifest in the different "spheres" of Figure 2.2.

In the technosphere, uncertainty might exist about the *form* (e.g., volume, mass, and material content) for any given concept and design variables. Information about what materials are used can be limited, especially when suppliers provide subcomponents. It will be assumed that in most cases, uncertainty about form is reduced during the design process. There can be considerable uncertainty about the *life cycle events* (e.g., frequency of service, properties of the material-cycling, energy-supply infrastructures, customer usage behavior, and actual disposal paths). Uncertainty in the *inventory of loads*, in turn, is dependent on the uncertainty in form and life cycle events.

Ecosphere components typically involve considerably more uncertainty. Environmental *effects* (e.g., ozone layer depletion, carcinogenesis, and toxic stress) are related to inventory first through fate analyses, and then exposure and effect analyses (Goedkoop and Spriensma 2001). Fate analyses are simpler for point-source loads, but become complex for products that are sold, used, and disposed of over a wide spatial and temporal range. Exposure and effect analyses are data intensive, involve simplified models, and may have limited applicability depending on how actual conditions deviate or fluctuate. Similar forms of uncertainty affect analysis of *damage*, e.g., ecosystem or human health impairment, and resource depletion with respect to available reserves.

Uncertainty also arises in the valuesphere. The valuesphere attempts to model the decision maker's preferences. This involves somehow relatively weighting different environmental impacts, e.g., what amount of non-renewable resource depletion is equivalent to species loss. Weights between environmental impacts and other design goals, e.g., cost and reliability, might also be included. Many factors add to valuesphere uncertainty, including lack of information about values, failure to reach consensus, and the potential for values to shift in the future.

Multiple objectives can conflict or trade off across a set of decision options: one option dominates the others for one objective but is itself dominated for another objective. To identify the most preferable decision option, one relatively weights the importance or value of different objectives and aggregates those weighted values into a single composite score. For LCA, this requires quantifying and comparing the value of different environmental impacts even when their units and scales differ. Several groups have proposed weighting methods to measure environmental impact as a single score, e.g., the Eco-indicator 99 (EI 99) impact assessment method in which particular scores, measured in millipoints (mPt), are assigned to specific materials and processes (Goedkoop and Spriensma 2001). ISO, on the other hand, promotes transparency in LCA results and discourages the use of single score indicators without also presenting the data and weighting behind them. Whatever the case, a multi-criteria decision between alternatives ultimately requires some form of weighting to be applied, whether done explicitly (like EI 99) or implicitly (observing different scores and coming to a conclusion about preferredness). The challenge of weighting environmental performance, especially comparing it to other factors like cost and quality, are numerous (Reap et al. 2008).

2.1.3 Scope of EBDM Considered in this Thesis

In this thesis, the applicability of IGDT will be explored for basic EBDM problems of the following nature:

- Consideration of information deficient, time-distant life cycle aspects,

- Single-score environmental performance measuring, both using Eco-Indicator 99 and monetary measures,
- Uncertainties in life-cycle design aspects from each of the “spheres” of Figure 2.2 (though ecosphere and valuesphere aspects will be combined into a single indicator, as explained in Section 4.2.2.)
- Multiple severe uncertainties,
- Selection between alternative design options.

Different combinations of these dimensions will be used.

2.2 Limits to LCA Data Availability and Quality

Having reviewed main LCA phase, attention is now paid to reasons for uncertainty in those phases, as cited by a wide range of literature. In environmental life cycle design, a significant source of uncertainty is data or models that are of poor quality (Reap et al. 2008). In her survey of approaches to improve reliability, Björklund generally identifies the main types of uncertainty due to data quality: badly measured data ('data inaccuracy'), data gaps, unrepresentative (proxy) data, model uncertainty, and uncertainty about LCA methodological choices (Björklund 2002). Specific instances of these data quality limitations are next discussed, grouped by those that are general, those that specifically affect life cycle inventory (i.e., the aspects in the “technosphere” portion of the life cycle in Figure 2.2) and those particular to impact assessment (i.e., the “ecosphere”).

2.2.1 General Problems Limiting Data Quality

A number of general reasons explain the existence of poor or unavailable data. Data and models alike can fail to accurately represent the full spatial and temporal scope chosen in the initial phase of an LCA. Data can be effectively unobservable during the time period devoted to conducting an LCA. For example, consider product recovery infrastructure models and scenarios, which will be the focus of Chapter 7. Uncertainty may also arise when different data sources measuring the same quantity conflict (Finnveden 2000, Björklund 2002). Standardized databases of LCA data are sought to reduce the burdens of data collection; yet, easily accessible, peer-reviewed data sets remain absent (UNEP 2003). There are few established, standardized or consistent ways to assess and maintain data quality (Vigon and Jensen 1995). Regarding LCA databases, Bare and coauthors identify a fundamental conflict between the sophistication of the data and the variety of categories that the data covers (Bare et al. 1999).

2.2.2 Data Quality in Life Cycle Inventories (Technosphere)

Some barriers to data collection are specific to life cycle inventory (LCI) analysis. In general, the literature tends to agree that data for life cycle inventories is not widely available nor of high quality (Ayres 1995, Ehrenfeld 1997, Owens 1997). Data collection costs can be prohibitively large, e.g., when sub-metering must be implemented in an industrial facility, when data must be gathered from the field or when data must be frequently collected to remain relevant (Maurice et al. 2000). In other cases, data exists outside of the LCA practitioner's organization, e.g., when withheld upstream or downstream by suppliers or other partners who have concerns (potentially valid) that sharing inventory data might reveal confidential information related to their competitive advantage (Ayres 1995). When available, external data can be of unknown quality. When data is not measured by the organization conducting the LCA, the accuracy, reliability, collection method and frequency of measurement may not be known and the limits of the data cannot necessarily be deduced (Lee et al. 1995). As a result, uncertainty distributions or even upper and lower bounds are commonly unavailable (Owens 1997). Furthermore, mass balances are often not performed, or are performed incorrectly (Ayres 1995). Data also can become outdated, compiled at different times corresponding to different materials produced over broadly different time periods (Jensen et al. 1997). LCI data may be unrepresentative because it is taken from similar but not identical processes, is based on assumptions about technology levels, or uses averages, all of which may be features of database values (Björklund 2002). During inventory analysis data with gaps are sometimes ignored, assumed or estimated (Graedel 1998, Lent 2003). Also, practitioners may extrapolate data based on limited data sets (Owens 1997). In fairness, it should be noted that ISO LCA standards require a company to document its data sources (ISO 2006, 2006), addressing many of the concerns raised in publications written in the late 1990s. Still, companies not complying with ISO might take these shortcuts, limiting data quality.

2.2.3 Data Quality in Environmental Impact Assessment (Ecosphere)

Probably the most serious data and model quality limitations affect the impact assessment stage, as there tend to be large discrepancies between a characterization model and the corresponding environmental mechanism (ISO 2000, 2006). The most fundamental barrier to model quality are limits to available scientific knowledge (ISO 2000, 2006). New chemicals constantly appear on the industrial market with poor models or measures of the mechanisms that disperse them into the environment. Finnveden points towards this being the case with dioxins (Finnveden 2000). Even if dispersion models exist, fate still is often be ignored in calculations of impact (Bare et al.

1999). Besides dispersion, the threshold levels that would create environmental damages may not be modeled or measured (Owens 1997) or may be represented using reduced order models, such as with linear dose-response curves (Bare et al. 1999). Even if thresholds are known, they might not apply to any particular locale or time period, or they might be affected by synergistic combinations of chemicals (Bare et al. 1999, Björklund 2002). To summarize the fundamental problem of modeling to an appropriate level of comprehensiveness, especially for environmental impact assessment, Bare and coauthors note that it is hard to know "where to draw the line between sound science and modeling assumptions" (Bare et al. 1999).

2.2.4 Challenge: Estimates with Unknown Uncertainty

Info-gap uncertainty can be seen to affect environmentally benign life cycle design in many of the information deficient scenarios mentioned in the previous three subsections. EBDM practitioners commonly have access to data or models that have unknown bounds on their accuracy. Major examples of this are reviewed as follows. Time-distant life cycle aspects of new products are estimated based on observations of older, different products. Environmental impact models that are specific to one geographic region can deviate to an unknown extent when used to model a region on another continent, and so forth. In fact, in general, the more precise a set of data is, the less widely applicable it is! So, it is not difficult to argue that info-gaps exist in EBDM, but what sorts of problems exist that could be resolved using IGDT?

2.3 Unresolved Problems Applying Uncertainty Formalisms to EBDM Decisions

We next review limitations of environmental life cycle assessment (Reap et al. 2008). These limitations extend to environmentally benign life cycle design. First, we review reasons for information limitations in practice. Then, we explain how lack of information reduces the quality of LCA and, in some cases, the tractability of problem solving in general. Lastly, the aspects that IGDT has potential to improve are discussed. Admittedly, the following review will document numerous problems relating to uncertainty in LCA that no existing formalisms—IGDT included—can resolve.

Whether the desired outcome of an LCA is a simple benchmark or a more involved recommendation of action, its reliability depends on appropriate consideration of uncertainty. One evaluates the effects of uncertainty using two main classes of techniques that will be referred to repeatedly in this section. The first, *uncertainty analysis*, models uncertainties in the inputs to an LCA and propagates them to results. For comparative LCAs, this can reveal whether there are significant differences between decision alternatives. The second type, *sensitivity analysis*, studies the effects of arbitrary changes in inputs on LCA outputs. This helps to identify the most influential LCA inputs when their uncertainty has yet to be or cannot be quantified.

The ISO LCA series of standards briefly mentions these two techniques but provides little guidance (ISO 2000, 2000, 2006) as to when or how procedurally to apply them. In response, LCA researchers and practitioners have proposed or adopted different variations of these techniques (Björklund 2002, Lloyd and Ries 2007). Choosing one can be difficult, especially for predictive assessments, comparative assessments of complex systems, or assessments with broad scope. Even when an appropriate choice is apparent, in practice, there are still many hurdles to using them.

In this section, problems associated with evaluating uncertainty in LCA fall into four categories:

- Modeling of uncertainty,
- Incorporation of multiple uncertainties,
- Completeness of analysis, and
- Cost of analysis.

These general problem areas are considered in depth in the next four subsections respectively. A review of the major concerns raised are used later in Section 2.3.5 as a basis for evaluating the general case for info-gap theory. In the context of this chapter, it serves a thesis goal of characterizing the overall problems that uncertainty creates when one tries to assess how environmentally benign a product or system is from a broad life cycle perspective.

2.3.1 Appropriate Representation of Uncertainty

Mathematically representing the variety of uncertainty types identifiable in LCA is often not straightforward. Probability distributions can be used to represent random variability in input parameters, which is arguably the type of uncertainty most familiar to LCA practitioners. Probability distributions have been used in a variety of LCA uncertainty analysis methods (Huijbregts 1998, Björklund 2002, Ciroth et al. 2004) and applied to numerous case studies (Maurice et al. 2000, McCleese and LaPuma 2002, Geisler et al. 2004). However, Björklund observes that *few classical statistical analyses in the LCA literature describe their data sources or assumptions or reveal how* *probability distributions were determined* (Björklund 2002). So, EBDM is affected by variability, but practitioners commonly lack data sets to characterize it. Probability distributions can, alternatively, be defined based on subjective expert estimates rather than data sets; in fact, this option is used for a majority of LCI data (Björklund 2002). In such cases, sensitivity analysis should be used to examine the sensitivity of conclusions to estimates and assumptions about probability distributions. Lack of credible expertise about distributions can reduce confidence in the results of uncertainty analysis. In general, probabilistic methods don't receive wide use partially because practitioners commonly lack the information required by those formalisms. Or, they rely on assumptions about information that undermine their trustworthiness.

Apart from variability, another chief source of uncertainty in information or models relates to their *representativeness*, commonly limited due to missing or incomplete data (Weidema and Wesnæs 1996). In response, a variety of mathematical formalisms, also surveyed by Björklund (2002), have been proposed for use in uncertainty analysis. Analysis of intervals (modeled based on expertise) can be effective but is not widely adopted because it is considered "pessimistic" (Björklund 2002). Several newer uncertainty formalisms have also begun to be examined to address uncertainty in representativeness. These include possibility distributions (Benetto et al. 2005), upper and lower bounds with no distributional information (Pohl et al. 1996), and fuzzy intervals (Pohl et al. 1996, Gonzalez et al. 2002, Güereca et al. 2006, Sadiq and Khan 2006). These studies have been for hypothetical cases and have not been applied in practice. The lack of means to *procedurally* define the information required by these forms of uncertainty is a potential problem identified by (Aughenbaugh 2006). We believe this exact reason deters practitioners from utilizing these newer formalisms. In response, Aughenbaugh and coauthors have considered analyzing life cycle design problems using probability bounds analysis because it *does* have a procedural definition (Aughenbaugh et al. 2006).

Lastly, some types of uncertainty, such as that due to LCA methodological choices, cannot be represented using any uncertainty formalism. The typical approach is to use sensitivity analysis, which may include analyzing different design scenarios (Björklund 2002, Lloyd and Ries 2007).

2.3.2 Incorporating Different Uncertainties into Overall Performance Measures

Problems also become apparent when one attempts to aggregate, for decision purposes, the influence that multiple heterogeneous uncertainty types have on LCA results. This is particularly problematic for comparative LCA, where the goal is identification of the best performing alternative, even for a single environmental performance dimension. In best case scenarios where all input uncertainty can be represented by probability distributions, uncertainty can be propagated to outputs using well established techniques. From there, a decision maker can compare statistical differences or expected (i.e., average) environmental performance.

However, in one LCA alone, it is possible that one or more uncertainty representations other than probability distributions are warranted due to the sparsity or non-probabilistic nature of available information. Unfortunately, combination of different uncertainty formalisms is often mathematically impossible and, when feasible, not theoretically sound, though this capability is being pursued by some researchers (Joslyn and Booker 2004). This prevents the incorporation of all uncertainty types into a single propagated result, even for one environmental performance dimension. Given the convenience of such single 'scores,' practitioners might be tempted to model all uncertainty information using a single formalism unjustifiably, either relying on unwarranted assumptions or ignoring available data. Even though ISO mandates that assumptions be documented (ISO 2006), detecting whether or not such assumptions lead to an unreliable decision could be difficult for a complex assessment case.

In some cases, qualitative information may be available to describe the degree of representativeness of uncertain quantities or models. Examples of this metadata (i.e., data about data) include dimensions such as age of the data, the geographical area to which it applies and technology assumptions. Researchers have proposed formalizing these metadata types as data quality indicators (DQI) (Weidema and Wesnæs 1996), though opinions differ as to how to incorporate them. ISO 14041 (ISO 1998) only recommends providing such metadata alongside LCA results for transparency purposes or to guide which alternative scenarios to analyze using sensitivity analysis as defined by ISO. Weidema and Wesnæs have proposed a method for transforming DQI scores to probability distributions using pre-defined, default distributions (Weidema and Wesnæs 1996); however, these conversions are subjectively defined.

To summarize, the fundamental problem is a tradeoff between two aspirations. The first is the (idealistic) motivation to utilize as much available information (qualitative or quantitative) about uncertainty—and as few unwarranted assumptions about that information—as possible. The conflicting aspiration is to factor all uncertainty models, however heterogeneous in form, into an efficient, rational decision-making process. To date, there are no frameworks for uncertainty analysis in LCA that guide characterization of this tradeoff to make the assessment as comprehensive as possible yet still tractable in terms of decision making.

2.3.3 Completeness and Conclusiveness of LCA Uncertainty Analyses

Intuitively, limitations in the comprehensiveness of an uncertainty analysis can considerably affect the quality of LCA conclusions and recommendations. The level of completeness achievable is proportional to the scope defined for a particular LCA, e.g., the time and geographical boundaries chosen. For complex products with long lifetimes, a 'complete' characterization of uncertainty might only be possible over a timescale that is too small to be of use to the practicing organization.

In addition, the degree to which comprehensiveness can be achieved (e.g., direct data collection, quantification of uncertainty in representativeness, model validation, etc.) varies across the phases of an LCA. For instance, developing models and characterizing uncertainty tends to be harder for impact assessment than for life cycle inventory (Owens 1997) and, likewise, harder for some indicator categories than others (ISO 2000, 2006).

Even if one can achieve comprehensiveness in some portions of an LCA uncertainty analysis, the severe uncertainty and data limitations of other more difficult portions can dominate LCA outcomes and lead to inconclusive outcomes (ISO 2000, Björklund 2002, ISO 2006). In response, practitioners can be tempted to characterize more readily quantifiable uncertainty and fail to acknowledge (or even know about) the existence of other uncertainty (Finnveden 2000). Such partial uncertainty analyses may generate false confidence in the reliability of results (Bare et al. 1999).

For comparative LCA, a converse problem also arises: modeling uncertainty in all LCA phases comprehensively and conservatively can lead to inconclusiveness. A complete representation of uncertainty may entail wide probability distributions or broad intervals of imprecision, propagating to results to make the alternatives under consideration indistinguishable. In fact, Finnveden argues that, from a scientific perspective, "it can in general not be shown that one product is environmentally preferable to another one, even if this happens to be the case" (Finnveden 2000).

The above problems of LCA have also been considered in the field of risk assessment (RA). Risk assessments (or analyses) in general tend to rely on specific models of the mechanisms related to risks, usually valid for a specific place and time (Morgan et al. 1990, Dekay et al. 2002). In contrast, LCA tends to include multiple impacts over different temporal or spatial scales, often with simplified models or assumptions. From the RA perspective, the lack of "spatial, temporal, dose-response, and threshold information" in LCA makes its results overly conservative, since it implies that all environmental burdens will affect sites that are sensitive to adverse impact (Owens 1997). Direct comparisons between RA and LCA have clarified where the techniques are compatible or overlap or where their respective practitioners could learn from each other (Cowell et al. 2002, Hofstetter et al. 2002, Matthews et al. 2002). However, limited conclusiveness in results remains a problem in LCA due to its often wide scope.

2.3.4 Resource Intensiveness of LCA Uncertainty Analysis

Lastly, the analysis of data quality in LCA, including sensitivity analysis and uncertainty analysis, incurs costs that can be daunting to practitioners. Deriving probability distributions through statistical analysis requires significant collection of test data. Alternatively, subjective distributions can be defined based on expertise, but better data requires more knowledgeable and expensive experts. The costs of characterizing uncertainty (by whatever means) is generally not quantified nor discussed in LCA research or practitioner communities, nor are techniques or frameworks that guide efficient data gathering. Admittedly, such issues would be hard to generalize for all LCA types.

2.3.5 Summary: From Problems to Motivation

For the problems in the previous four subsections, here is a preview of how (if possible) they motivate our research:

- Modeling of uncertainty: Practitioners are reluctant to use or rely on uncertainty formalisms that require more information than is available. A formalism that requires only sparse information but still offers some decision making power is needed.
- Incorporating different uncertainty types into a single decision: Any new decision formalisms probably won't help here, but it might be useful to practitioners to have multiple means by which to evaluate uncertainty. In fact, some advocate that "uncertainty should be examined from more than one viewpoint, with more than one tool, to avoid model / tool (software) myopia" (Regan et al. 2002).
- Completeness and conclusiveness of uncertainty analysis: The "completeness" part of this problem relates to fundamental limitations created by the abstraction of real systems to evaluation models in design. This problem is made more "wicked" by the wide scope that LCA attempts. It may be difficult for any theory to quantify the error created by abstraction. The "conclusiveness" problem is similarly wicked.
- Cost of analysis: Methods are needed that can utilize, as far as possible, whatever information is available, using analysis that is as simple as possible.

So, info-gap theory should be investigated since it proposes to make conservative assessments of severe uncertainty, doing more with less.

2.4 What Has Been Presented and What Is Next

So, severe uncertainty *does* exist in EBDM. It appears to fit the form of info-gaps, at least as far as they have been discussed at this point in the thesis. And, other uncertainty models aren't adopted or trusted when they assume too much. However, the fact that info-gaps exist does not verify that IGDT will be useful. Thus, in the next chapter we introduce info-gap theory and the info-gap analysis procedure in full, and we explain how they relate and compare to other uncertainty formalisms.

CHAPTER 3:

EXPLANATION AND EVALUATION OF INFO-GAP THEORY

In this chapter, the goals are to explain the info-gap uncertainty formalism and to show that its purpose and structure differs from that of other uncertainty formalisms. First, in Section 3.1, a general discussion of uncertainty and its different forms is provided for context. In Section 3.2, different formalisms for making decisions under uncertainty are reviewed. This in turn provides a reference point for a conceptual overview of IGDT in Section 3.3. The mathematical components of a generic info-gap analysis are presented in greater detail in Section 3.4, and an overall assessment of its validity is offered in Section 3.5. In Section 3.6, IGDT is compared, from several different perspectives, to the uncertainty formalisms first presented in Section 3.2. IGDT is not found be in direct competition with these alternative formalisms.

3.1 The Nature and Sources of Uncertainty

Before reviewing the various approaches to including uncertainty into design decisions, a discussion of the nature and sources of uncertainty is warranted. In Section 3.1.1, a general definition of uncertainty is provided, followed by a review of different uncertainty types in Section 3.1.2.

3.1.1 Definition of Uncertainty

Broadly defined, uncertainty is a some form of lack of knowledge, for instance, about the true value of a quantity, true form of a model, appropriateness of a modeling or methodological decision, etc. Uncertainty can be defined with respect to *certainty*, which is "the condition of knowing everything necessary to choose the course of action whose outcome is most preferred" (Nikolaidis 2005, Aughenbaugh et al. 2006). Following this, *uncertainty* is "the gap between certainty and the decision maker's present state of information" (Nikolaidis 2005, Aughenbaugh et al. 2006). This is nearly equivalent to Ben-Haim's definition of info-gap uncertainty introduced in Section 1.1.3. However, the nuances of what *type* of uncertainty is being modeled sets different uncertainty formalisms apart.

3.1.2 Types of Uncertainty

Beyond the general definition for uncertainty, different specific forms of uncertainty have been identified. As recently reviewed by (Choi 2005, Nikolaidis 2005, Thunnissen 2005), numerous uncertainty taxonomies exist that aim to categorize and delineate the various types. There tends to be disagreement and/or overlap in the content of these taxonomies. This is largely due to differences in their basic definitions (e.g., what "ambiguity" is), their conceptual modes of classification (e.g., by causes of uncertainty, by "nature" of uncertainty, by information available for quantifying uncertainty, etc.), and even their intent (e.g., prescriptive, "diagnostic", or in some cases, some motive that clearly stated). Because of this disagreement, no attempt is made in this thesis to choose and adhere to one "best", most complete taxonomy.

Nevertheless, several main types of uncertainty are repeatedly acknowledged in the literature. The following list—which is not meant to be exhaustive—is assembled from a parts of several taxonomies (Wynne 1992, Nikolaidis 2005, Thunnissen 2005):

- Irreducible uncertainty – inherent randomness, recurrence of chance events, e.g., the numbers that come up when rolling dice. Also called aleatory uncertainty.
- Imprecision – the gap between the presently available state of information and a state of precise information[4]; i.e., uncertainty that can be reduced by gathering information. Choice of this term (as opposed to "epistemic uncertainty" or "reducible uncertainty") follows the reasoning used in Section 2.3.4 of (Aughenbaugh 2006). Imprecision may be due to conflicting information sources.
- Linguistic ambiguity – vagueness in assigning a numerical value to some term or proposition (Joslyn and Booker 2004).
- Indeterminacy – Inability to make a decision or identify the most preferred choice, perhaps due to propagated imprecision; uncertainty about some state which is dependent on the unknown outcomes of indeterminate actors or events earlier in a causal chain. One form of indeterminacy will be shown in Section 5.1.2 to affect info-gap analyses having multiple info-gap uncertainties.
- Ignorance – inability to recognize (much less quantify) the existence of uncertainty; an unknown unknown.

The first three uncertainty types in the above list are the ones most commonly handled by the mathematical uncertainty formalisms introduced in the next section. Indeterminacy is included in the list as a different type because it relates to simulation outputs and/or decisions[5]. Finally, ignorance is included simply to acknowledge the most

[4] Note that "precise information" as used here is *not* the same as "certainty", as defined in Section 3.1.1. One can have precise information about other forms of uncertainty. For instance, one may precisely know the odds of a coin toss, but still not have certainty about what its outcome will be for any one event.
[5] The term "decision" here could be interpreted very broadly, e.g., a problem-framing decision about what some true state of the world is.

severe form of uncertainty, one which cannot be measured, simulated, or reduced *a priori*.

3.2 Existing Uncertainty Formalisms in the Context of Decision Making

To provide a reference by which to compare info-gap theory, a variety of uncertainty formalisms, all of which have received much more attention in the literature, are next discussed. Every complete formalism has three main aspects:

- Formulation, or modeling of what is known about uncertainty. An overview of different formalisms' uncertainty models, which correspond to the uncertainty types identified in the previous section, is provided in Section 3.2.1.
- Analysis, or computation mathematics that allow the uncertainty to be propagated through the influence diagram of Figure 2.2. Analysis techniques for each formalisms will be assumed to be rigorous enough that the details need not be discussed in this section. An overview of the pluses and minuses of each formalism is presented, however, in Section 3.2.2.
- Interpretation, or using the output of analyses to determine a preference ranking over design options that takes uncertainty into consideration in some way. Interpretation for two main formalisms is discussed in Section 3.2.3. This provides a reference point for explaining how IGDT includes of uncertainty into design decisions later in Section 3.3.

Finally, a review of the shortcomings of various uncertainty formalisms in light of the data limitations identified in Chapter 2 are presented in Section 3.2.4.

3.2.1 Models for Uncertainty

Different uncertainty formalisms represent different types of uncertainty; therefore, their uncertainty models are expressed differently mathematically and they have different information requirements. The general[6] shapes of and relationships between the representations most commonly referenced in the literature are shown in Figure 3.1, taken from (Hemez 2002). Mathematical details will not be explained here; a comprehensive and critical review can be found in (Aughenbaugh 2006).

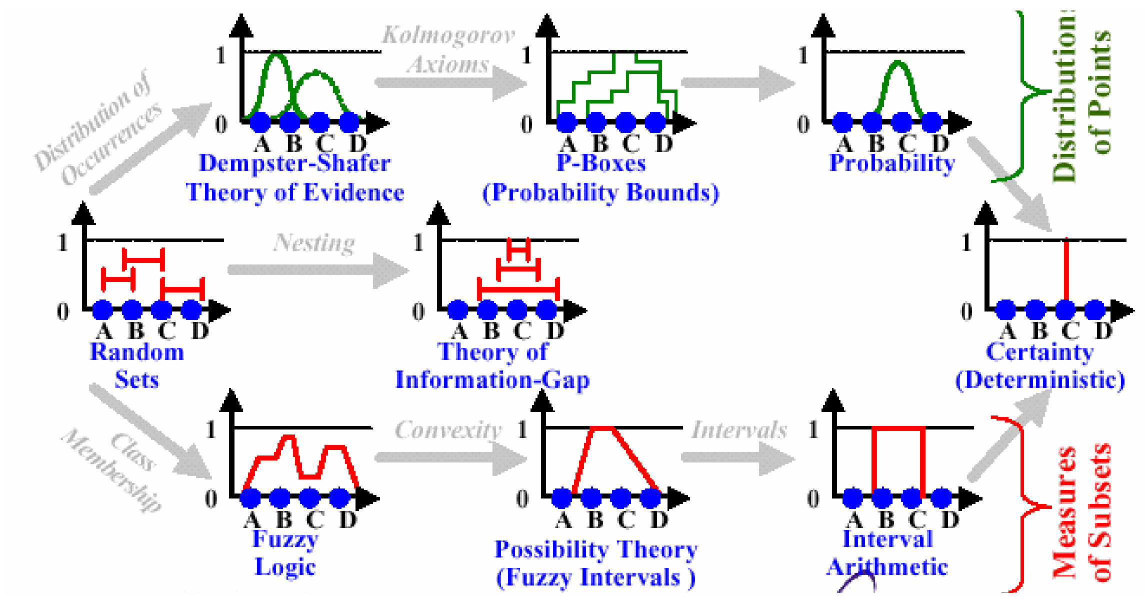

Figure 3.1: Different models of uncertainty and their relationships, from (Hemez 2002)

Generally, it can be said that all uncertainty models have the following properties which characterize them:

[6] This figure is meant to simply illustrate the basic structural differences between different uncertainty models. The relationships (i.e., arrows) in this figure are not comprehensive; for a full theoretical discussion of the similarities, differences, and relationships between uncertainty formalisms, refer to (Joslyn and Booker 2004).

- structure (i.e., applicability to a phenomenon, e.g., random variability can be represented as a lottery),
- size (i.e., quantified parameters like bounds, mean, variance, etc.), and
- distribution of occurrences (i.e., uncertainty measures that are normalized mathematical functions)

In Section 3.4.1, info-gap models will be explained in terms of these properties.

Irreducible uncertainty is best represented in stochastic terms, e.g., probability density functions. In traditional statistical decision theory (Berger 1985), it is assumed that all uncertainties can be characterized using precise probability distributions (left side of Figure 3.1). Various assumptions or scenarios can lead to these probabilities, such as large historical databases, well-elicited beliefs, or well-founded prior distributions. The information requirements (or assumptions) are stronger for probabilities than for the other methods.

Imprecision is best represented in terms of intervals (Kearfott and Kreinovich 1996, Kreinovich et al. 1999, Muhanna and Mullen 2004). In interval methods, exact bounds on the uncertainties are required, but there is no measure of where a value lies within those bounds. An interval is represented on a CDF plot in the middle of Figure 3.1; it is shown as a box because of imprecision.

When sufficient information to support probability density functions (PDFs) is lacking, characterizing their imprecision is important because otherwise PDFs may mischaracterize the objectively available information and result in flawed decisions. To address this problem, Ferson and Donald created probability bounds analysis (PBA), a method that represents uncertainty using upper and lower cumulative probability distributions (Ferson and Donald 1998, Aughenbaugh and Paredis 2006). These structures, called probability boxes or just p-boxes, capture both variability and imprecision by generalizing both probability theory and interval methods (right side of Figure 3.1). Because a p-box is more general than both interval methods and probability, its information requirements are as low as the lowest of the two, but its applicability (without throwing information away) is greater than either.

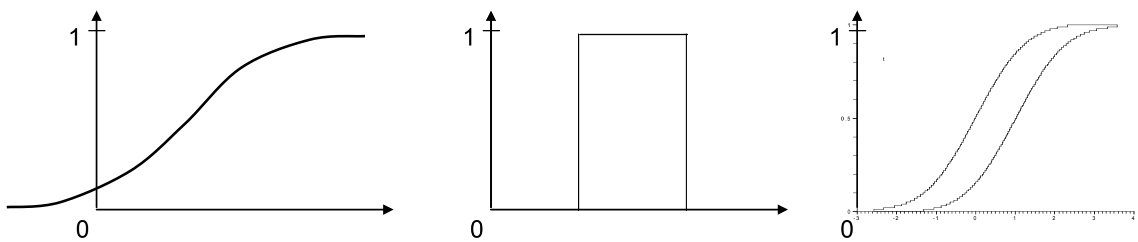

Figure 3.2: Comparison of cumulative distributions of a precise probability, an interval, and a p-box.

Other uncertainty formalisms also exist (Joslyn and Booker 2004). Linguistic ambiguity in set membership (Zadeh 1965), as well as the possibility or necessity of event outcomes (Zadeh 1978), can be modeled by fuzzy membership functions. Additionally, the belief and plausibility (also fuzzy measures) of outcomes, based on sets of evidence, can be modeled with evidence theory (Shafer 1976). These formalisms have come under some criticism because of they lack operational definitions; that is, it is unclear how a designer would procedurally measure those uncertainties (Aughenbaugh 2006). Regardless, the intent in this thesis is not to consider uncertainty types that could be modeled by fuzzy membership functions.

Ideally, a decision maker should choose the uncertainty representation most appropriate for the available information and the assumptions that he or she is willing to make. A good decision requires using all of the information that is available and not using or assuming information that is not present (Du et al. 2005, Aughenbaugh and Paredis 2006). This perspective motivates a consideration of info-gap theory; it will be shown in Section 3.5 to require less information than all other formalisms.

3.2.2 Approaches to Analyzing the Effects of Input Uncertainty on Performance

The preceding uncertainty models can be used to quantify the effects of uncertain inputs on output uncertainty. The plusses and minuses of various approaches are summarized by Table 3.1, adapted from (Ferson 2002). The uncertainty assessment methods shown in the table are limited to ones commonly used in life cycle assessments. As should be intuitive, the choice of an approach is driven largely by available information.

Table 3.1: Review and Comparison of Risk Assessment Approaches, adapted from (Ferson 2002)

Assessment Type	Pluses	Minuses
Deterministic	• Simple	• Does not express reliability of results
What-if and sensitivity analysis	• General and flexible • Works for all uncertainty	• Cumbersome to design and implement • Computationally expensive, sometimes impossible • Hard to explain when elaborated
Worst case analysis	• Accounts for uncertainty by being conservative • Useful in a screening assessment	• Level of conservatism not consistent from analysis to analysis • Impossible to compare risks from different analyses • Possibly hyperconservative • Results in regulation that is unfair and burdensome to industry • Estimates may be biased
Interval Analysis	• Simple and easy to explain • Generalizes and refines worst case scenarios • Works no matter where uncertainty comes from • Especially useful in a screening assessment	• Ranges grow very quickly • Often too conservative • No exact value, but exact bounds
Monte Carlo analysis (probability)	• Simple to implement • Simple to explain • Characterizes impacts of all possible magnitudes • Can use information about correlations among variables	• Requires much empirical info or assumptions • Analysts need to guess some things • Routine assumptions lead to non-protective conclusions • Confounds ignorance with variability • May be inappropriate for non-statistical uncertainty • May not be acceptable to merge subjective estimates from different sources
Fuzzy arithmetic (possibility)	• Computations are simple and easy to explain • Doesn't require detailed empirical information • Doesn't require knowledge of dependencies or correlations among variables	• Not clear it's acceptable to merge numbers whose conservativisms are different
Probability bounds analysis	• Handles uncertainty about parameter values, distribution shapes, dependencies, and model form • Bounds get narrower with better empirical info • Bounds are rigorous	• Displays must be cumulative • Must truncate infinite tails • Optimal bounds are expensive to compute when parameters are repeated

Note that sensitivity analysis has also been included in Table 3.1. In basic decision analysis, nominal values are assumed to be known, and the problem is solved using these values. Then a standard sensitivity analysis (such as with a tornado diagram (Clemen 1996)) is performed to explore the effects of uncertainty (in the form of bounded intervals of fixed size around the nominal values). Input variations are not necessarily related to available information about uncertainty; they are simply applied for all uncertain variables to gauge which is most influential on outputs. Generally this method assumes independence between uncertain quantities and irrelevance of higher-order interactions.

3.2.3 Including Uncertainty into Design Decisions

Continuing the discussion of uncertainty formalisms, it is helpful to explain conceptually how their uncertainty models propagate to outputs, and how decisions are made given those outputs. The next series of graphs relates the form of input uncertainty to the form of output uncertainty using a single continuous design variable and utility as a performance measure.

Inputs in the form of probability distributions propagate to output utilities that are also distributions, as shown in Figure 3.3. Maximizing the expectation of the output distribution yields a design optimal per von Neumann-Morgenstern expected utility theory (von Neumann and Morgenstern 1944).

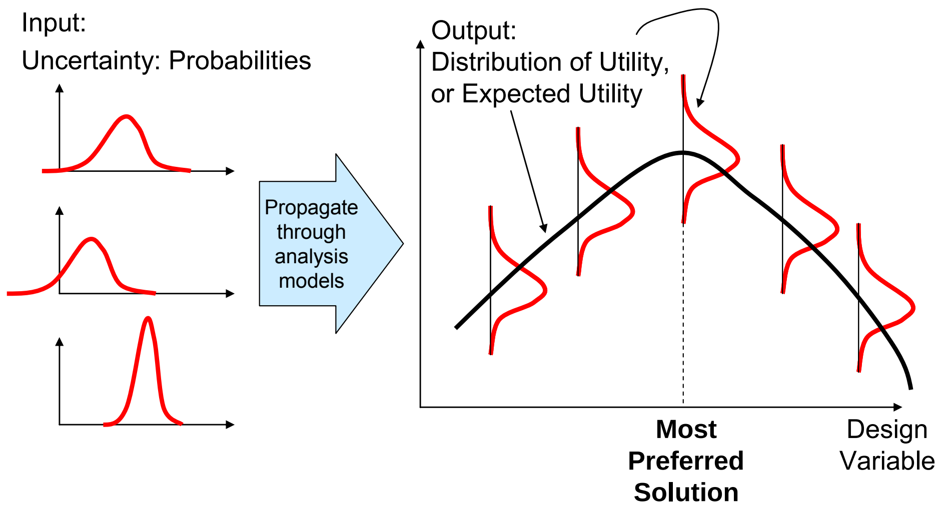

Figure 3.3: Propagating probability distributions to a performance measure.

A similar propagation applies to interval uncertainties, however the output takes the form of best case and worst case performance curves. A number of decision policies could be applied to yield a decision given this output. One heuristic option is maximizing the performance under worst case conditions, a maxi-min policy, depicted in Figure 3.4.

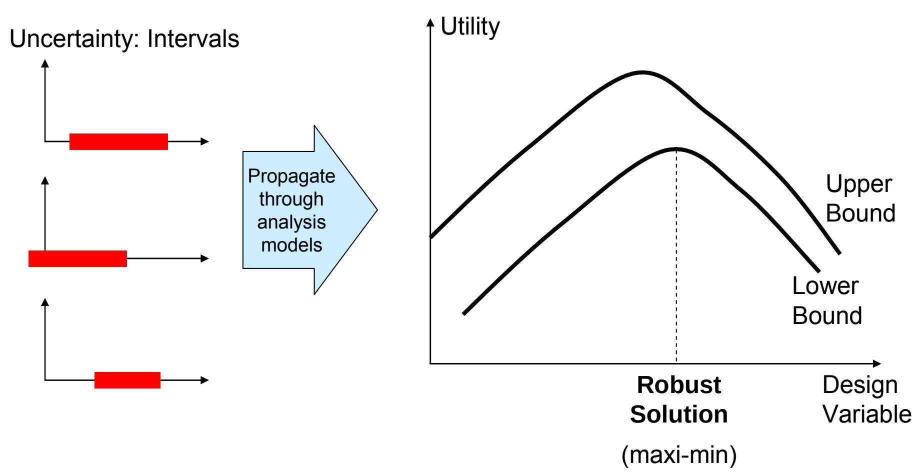

Figure 3.4: Propagating intervals to a performance measure.

These conceptual plots are revisited as a basis for comparison when introducing the info-gap decision strategy in Section 3.3.

Another approach to designing for robustness to uncertainty, rooted in the methods of Taguchi[7], is to minimize the degree to which uncertain inputs ("noise factors") create variations on performance (Phadke 1989). This type of robustness is depicted in Figure 3.5, where the preferable design ("*b*"), offers the same average performance but with less variation (Chen et al. 1996).

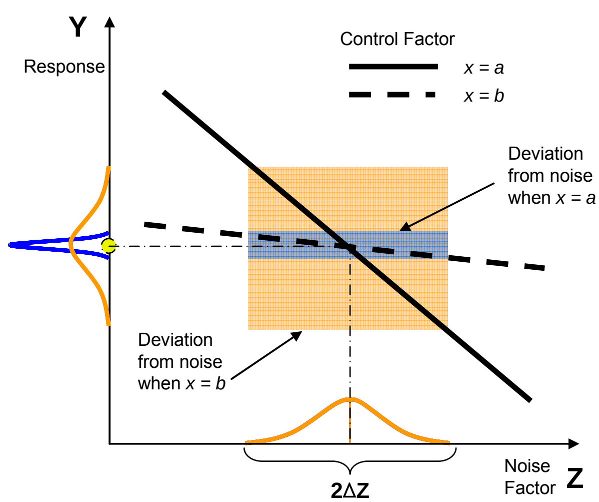

Figure 3.5: "Type I" Robust Design (Taguchi Robustness), adapted from (Chen et al. 1996).

This concept of robustness can utilize either probability distributions or intervals, as long as the parameters of those uncertainty models are fixed *a priori* to analysis. As will be discussed in Section 3.6.2, the above approach to (and concept of) robustness is

[7] For brevity, this perspective of robustness will be referred to in this thesis as "Taguchi robustness", even though more recent work has gone beyond—and improved upon—Taguchi's original work.

fundamentally different than the one used by IGDT. In fact, their definitions of robustness differ; from the Taguchi robustness perspective, info-gap robustness may appear closer to reliability. Thus, info-gap theory does not seek to replace or compete with design for robustness in the Taguchi sense.

3.2.4 Limitations of the Preceding Uncertainty Formalisms

The preceding formalisms are assumed to be sufficient for normative decision making when enough information is available to specify the parameters of their respective uncertainty models[8]. Info-gap theory is not intended to replace or compete with these formalisms in such cases. However, as was mentioned in Section 2.2, the gathering the required information is too demanding for some LCA and EBDM applications. (In Chapter 4, specific examples of when info-gap models are warranted are provided.) The main motivation for an investigation of info-gap is the desire to for a means to represent and assess the effects of sparse uncertainty without requiring a normalized distribution nor static interval bounds.

3.3 Conceptual Overview of Info-Gap Decision Theory

Before discussing any mathematics, an understanding of the "big picture" of info-gap theory is useful. The following short overview explains what info-gap uncertainty is, how robustness to that uncertainty is achieved, and how that robustness is used to establish a preference ranking between design options.

Info-gap theory, developed by Ben-Haim and explained in depth in his 2006 book, is an approach to analyzing options and making decisions under sparse information

[8] Further critical evaluation of possible problematic aspects of well-established uncertainty formalisms is not pursued in this thesis because IGDT does not aim to improve those problems; rather, IGDT applies under conditions when others do not.

about uncertainty. IGDT has steadily evolved over 15 years from a body of work on convex set-based models of uncertainty (Ben-Haim and Elishakoff 1990; Ben-Haim 1999; Ben-Haim 2000). The foundations of IGDT are explained in detail in (Ben-Haim 2006). A summary follows.

To establish a "vocabulary" for the overview, the four basic inputs to an info-gap analysis are:

- u, the uncertain variable for which some nominal $\tilde{u}$ is available[9]
- q, some design variable(s), or a member of a set of design alternatives Q
- $R(q,u)$, a performance (or "reward") model of system response whose output is a performance attribute of interest.
- r_c, a critical value of performance that must be guaranteed (met or exceeded); alternatively: a failure criterion.

These components are used to formulate:

- $\hat{\alpha}(q,r_c)$, the *info-gap robustness function*, which outputs the largest info-gap uncertainty that a design option q can endure and still deliver a performance no worse than r_c.

The meaning of these different entities, as well as the general relationships between them, are found in the following narrative.

Generally speaking, an information[10] gap is the disparity between what *is known* and what *could be known* (Ben-Haim 2006). In the simplest[11] example of an info-gap

[9] Note that the use of the tilde (˜) above a variable denotes a nominal value for that variable. The tilde here should not be confused with the same notation used for fuzzy sets.
[10] Note that Ben-Haim "use[s] the term 'information' in its broadest lexical meaning, referring not only to facts or data, but also knowledge and understanding" (p. 1, Ben-Haim 2006). (Footnote cont. next page→)

uncertainty, there exists some discrepancy between an uncertain quantity's available (but suspect) nominal value[12], $\tilde{u}$, and the quantity's true value, u, which could be known but is not (Ben-Haim 2006). This gap may be due to either ignorance or extremely rare events. The deficient information state of an info-gap is depicted conceptually in Figure 1.3.

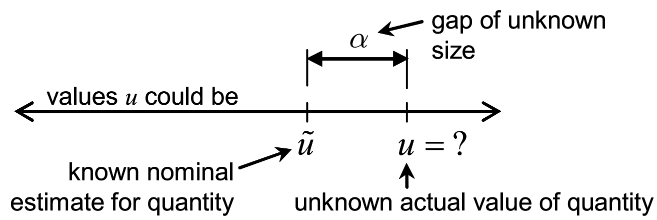

Figure 3.6: Simple representation of info-gap uncertainty (Reproduced from (Duncan et al. 2007))

Because the size of this gap is unknown, it is represented mathematically using an *uncertainty parameter*, α, sometimes referred to as the *horizon of uncertainty*. This parameter actually reflects two different types of uncertainty:

- Unknown location of the actual value for u within any horizon of uncertainty, α
- Unknown horizon of uncertainty, α

When info-gap uncertainty exists, the decision maker cannot or does not wish to assume more information than is available, so α remains unspecified, a free parameter. To our knowledge, no other uncertainty formalisms allow any of their parameters or coefficients (e.g., mean, standard deviation, interval bounds, etc.) to remain unspecified before use in analysis or optimization.

Though the usage of this term might cause some confusion, it is meant to simplify terminology given that, for instance, the "info" lacking in an info-gap may be knowledge about the form of a model.
[11] More complex info-gap models are briefly discussed in Section 3.4.1.2.
[12] "Value", in this case, means "quantity" and not "worth".

For comparison to earlier uncertainty models, an info-gap model can be characterized using the three properties introduced in Section 3.2.1:

- Structure: the "horizon of uncertainty" is of the form of an interval (Ben-Haim 2006), or in terms of Section 3.1.2, a severe form of imprecision. Some info-gap models can have more of a structure (see Section 3.4.1.2) than just α, but, as Ben-Haim notes, "An info-gap model separates what we know (structure) from what we don't know", which is some aspect of *size*, represented by α.
- Size: Because α remains unspecified, the interval size is unknown. To reiterate, this sets it apart from the previous formalisms.
- Distribution of occurrences: Info-gap uncertainty does not measure any distributions. (This property is shared by intervals.)

The full family of nested sets created by the free uncertainty parameter α can be viewed in Figure 3.7. Notice that the sets are centered on the nominal $\tilde{u}$; α grows equally in both directions away from the nominal[13]. This property is explained further in Section 3.4.1.

[13] Contrast this to Figure 3.6, which only depicts the deviation *above* the nominal.

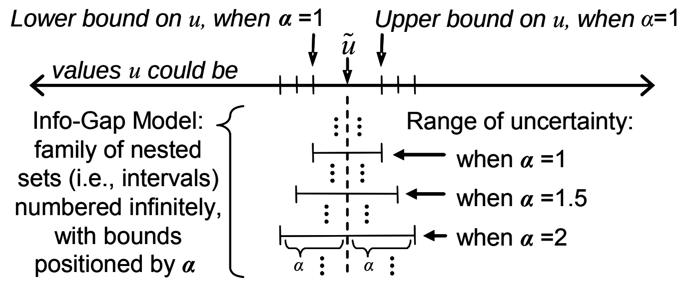

Figure 3.7: Representing unbounded uncertainty as an α–parameterized family of nested sets (Reproduced from (Duncan et al. 2006))

For a fictitious (yet illustrative) problem, the influence that an info-gap model has on performance *R(q,u)* can be visualized in Figure 3.8. Compare this plot to the version affected by static interval uncertainty in Figure 3.4. Each value that the uncertainty parameter α could take corresponds to an upper and lower performance curve, $\bar{U}$ and $\underline{U}$. The inner pair of curves in the Figure occur where α=1, the outer pair where α=2. Since α is unbounded[14] and continuous, there are an infinite number of these performance curves.

[14] Actually, α is not necessarily *truly* unbounded. For almost any severely uncertain variable or model, there is surely *some* bound that would never be practically exceeded. But, it is assumed that the span over which α could be realized is considerably large and uncertain, and the performance effects of α being any value in the range of those sizes is unknown.

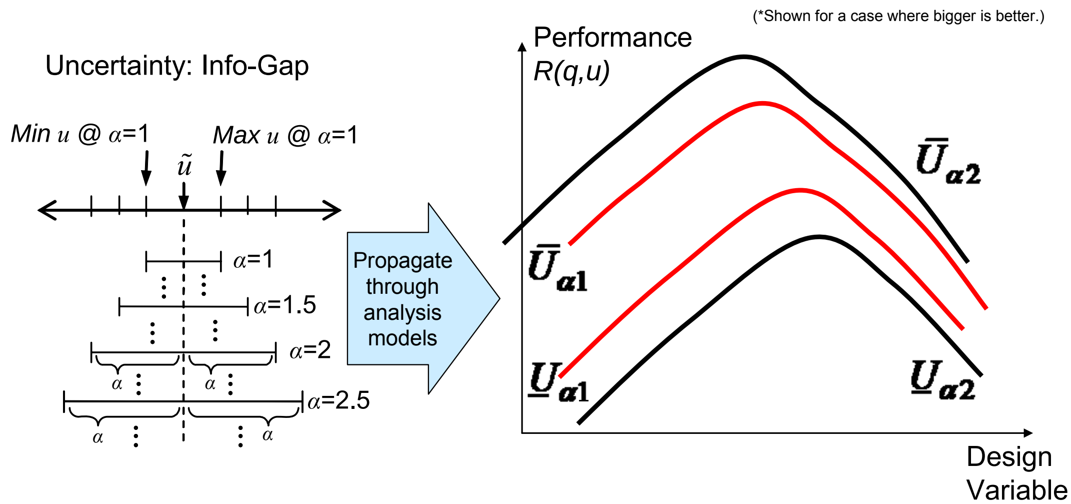

Figure 3.8: The effects of an info-gap uncertainty on performance ("reward")

So, what good is leaving uncertainty unbounded in this manner? Using IGDT, one seeks decision options that are *robust* to info-gap uncertainty. A robust decision yields a reasonably satisfactory system performance over a large range of realizations of the uncertain parameter α. To achieve robustness, system performance, *R(q,u)*, is not optimized but instead *satisficed*. Satisficing generally means accepting "good enough" performance in order to afford the achievement of other objectives, especially when only idealized models or limited information is available (Simon 1947; Ben-Haim 2006). In the specific case of IGDT, one satisfices performance to increase immunity (i.e., robustness) to error estimating $\tilde{u}$. To satisfice performance *R(q,u)*, one must specify some level of *critical performance*, r_c, to be exceeded by all design alternatives.

The *info-gap robustness*, $\hat{\alpha}(q, r_c)$, of a design alternative q is the largest horizon of uncertainty, α, that the design can withstand while still guaranteeing better than critical performance r_c. In other words, info-gap robustness is the greatest amount of error in the nominal that a design can endure and still perform at least as well as r_c (but not necessarily optimally). This is expressed mathematically as the robustness function, $\hat{\alpha}(q, r_c)$, a guide to which is offered in Section 3.4.3.

The meaning of info-gap robustness can be better understood using Figure 3.9. For the continuous design variable q, there are two values, q_A and q_B, whose info-gap robustness is quantified. The info-gap robustness of each design, $\hat{\alpha}(q, r_c)$, is the largest value for α that can be sustained by each design without making the Performance drop below the critical performance r_c. For q_A, α can grow as big as $\alpha = \alpha_1$ and still guarantee the critical reward, so its info-gap robustness is $\hat{\alpha}(q_A, r_c) = \alpha_1$. For q_B, α can grow as big as $\alpha = \alpha_2$ and still guarantee the critical reward .

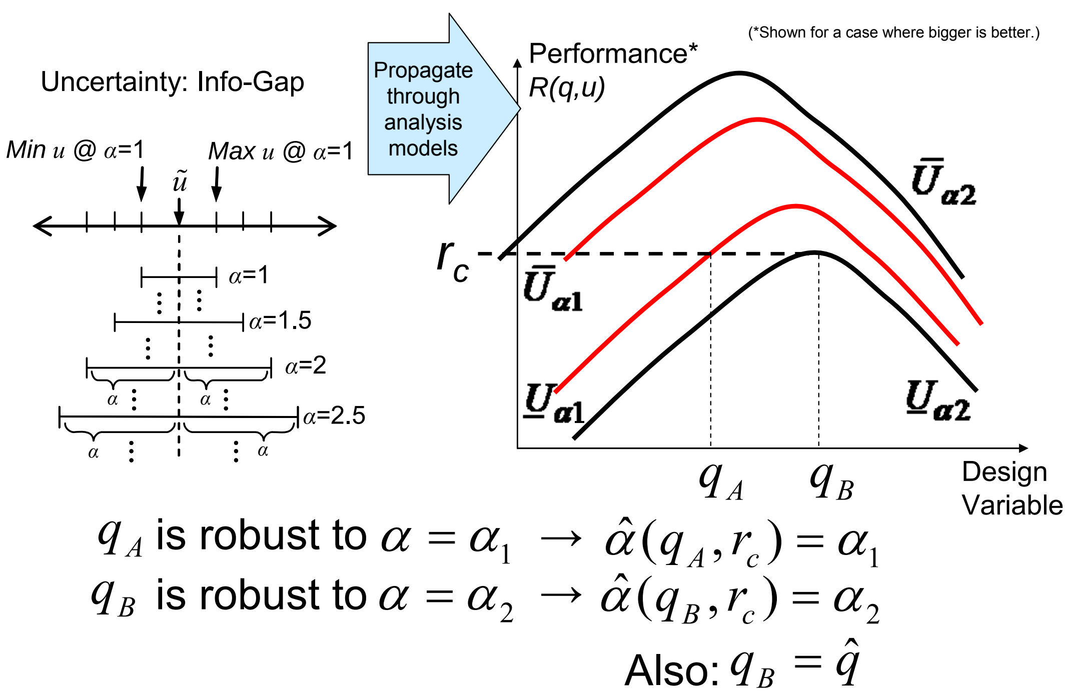

Figure 3.9: The info-gap robustness of two different values for the design variable *q*.

Info-gap robustness can be used as a measure by which to assign preference rankings on a set of design alternatives q. Because $\hat{\alpha}(q_B, r_c) > \hat{\alpha}(q_A, r_c)$, from the perspective of info-gap theory q_B is preferred over q_A. Furthermore, the *robust-satisficing decision rule* used in IGDT prefers the design with the largest info-gap robustness at a given r_c, i.e., $\hat{\alpha}(q, r_c)$ is maximized over the set of design options. The most preferred design is designated $\hat{q}$. In Figure 3.9, $\hat{\alpha}(q_B, r_c)$ is the maximum info-gap robustness possible because q_B is the design with maximum performance on the curve whose peak intersects r_c. Thus, $q_B = \hat{q}$ and is most preferred for that particular value of r_c. A more mathematically formal version of this statement is presented in Section 3.4.

Settling for a less demanding r_c often affords a design more robustness to info-gap uncertainty. By graphically plotting robustness versus the critical performance, one can view a tradeoff, henceforth denoted as the "$\hat{\alpha}(q, r_c)$–r_c trade-off". For the running example from Figure 3.9, the trade-off between *maximum* info-gap robustness and r_c is depicted in Figure 3.10. Note that, because this is a design problem with a continuous variable, every point on the trade-off line corresponds to a different value for $\hat{q}$. In other words, $\hat{q}$ can be a function $\hat{q}(r_c)$.

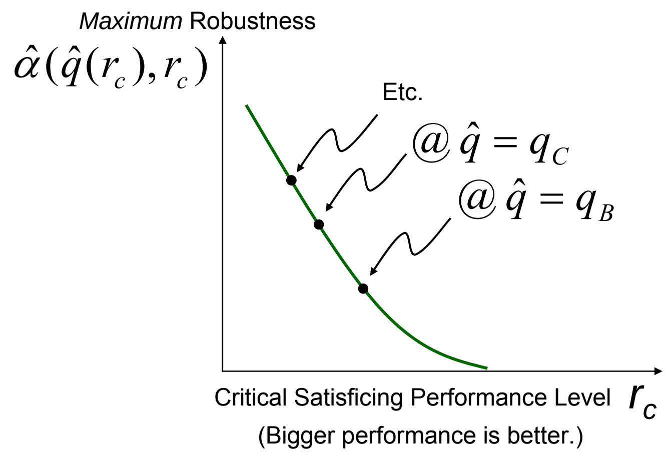

Figure 3.10: Trade-off between robustness and critical performance for continuous design variable *q*.

The $\hat{\alpha}(q, r_c)$–r_c trade-off curves take on a slightly different form for design problems that select between discrete alternatives. In these selection cases, the robustness for <u>each</u> alternative can be plotted over <u>all</u> r_c. This is the case in Figure 1.4, repeated again for convenience in Figure 3.11 below. The *maximum* info-gap robustness, $\hat{\alpha}(\hat{q}(r_c), r_c)$, is the highest $\hat{\alpha}(q, r_c)$ value at each r_c. The design with the maximum info-gap robustness at a particular value for r_c is the most preferred design, per the robust-satisficing decision rule. A plot of maximum info-gap robustness for the same example as in Figure 3.11 is shown in Figure 3.12. In some cases, as in Figure 3.12 at r_c=2.4, the design alternative (1 or 2) that has the maximum info-gap robustness can switch. This results in a *preference ranking change* (later denoted a Preferredness Switch Point, or PSP) at some r_c. In other words, if a decision maker is willing to settle for a critical performance at or below 2.4 units, as long as that performance is *guaranteed* to be met or exceeded, Design 1 would be preferable as it offers greater robustness to info-gap uncertainty.

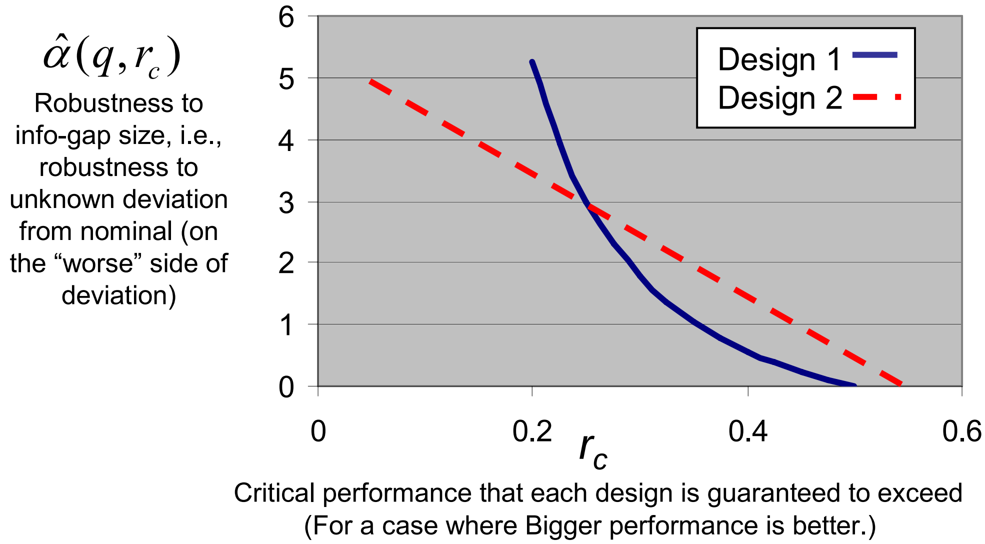

Figure 3.11: Trade-off between info-gap robustness and r_c for two discrete alternatives.

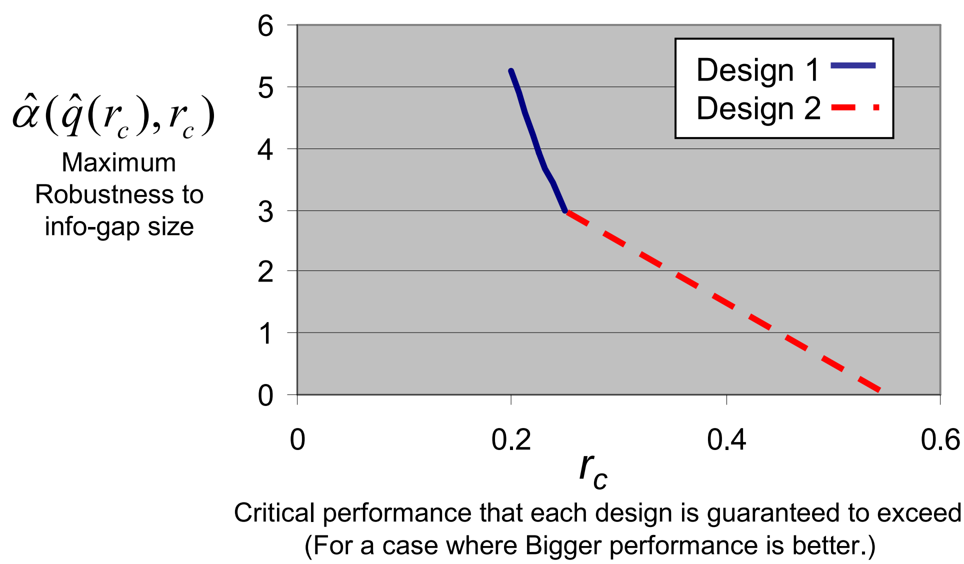

Figure 3.12: Trade-off between *maximum* info-gap robustness and r_c for two discrete alternatives.

So, preference ranking for design alternatives depends on one's trade-off preference, which is assumed to not be known or to be difficult to express *a priori*. Inspection of robustness curves can help one induce their preference for this trade-off.

3.4 Mathematical Components of a Basic Info-Gap Analysis

Now that a high-level overview of IGDT has been provided, a more detailed explanation is warranted for the four basic IGDT input components and the robustness function. For each, the basic mathematical structure, variety in modeling forms, and qualifications on their use are next provided in respective subsections.

3.4.1 Info-Gap Models of Uncertainty

An info-gap model $\mathcal{U}$ of the uncertainty in u is represented mathematically as family of nested, convex sets centered around the nominal $\tilde{u}$. The size of each set in the family is characterized by the uncertainty parameter, α, with the widths of each member progressively expanding outward above and below the nominal at a distance $\alpha \geq 0$. According to Ben-Haim, an info-gap model's set structure models "clustering of events rather than frequency of reoccurrence, likelihood, plausibility, or possibility" (Ben-Haim 2006). The specific structure of the info-gap model depends on what type of model u itself is (i.e., a constant or a function) and on available knowledge about the form in which uncertainty grows around the nominal.

3.4.1.1 Uncertain Variables

When u is a constant or single variable, the structure of the info-gap is relatively simple. In the case from Figure 1.3, u is an uncertain variable for which one has an estimate $\tilde{u}$ but cannot quantify the discrepancy, α, between the actual u and $\tilde{u}$. The horizon of uncertainty α in this case may be interpreted as bounding approximation error in $\tilde{u}$, or any other source of variation that one cannot bound or otherwise quantify due to lack of information.

For the case of Figure 1.3, the severe uncertainty of u is represented mathematically by a "interval bounded" info-gap model:

$$\mathcal{U}(\alpha,\tilde{u}) = \left\{u : \left|u-\tilde{u}\right| \le \alpha\right\}, \alpha \ge 0 \tag{3.1}$$

This model is depicted in Figure 3.13, which shows that the nested set model captures two levels of uncertainty: interval uncertainty within any set of fixed α, and unknown horizon of uncertainty α. Note that the units of α are the same as those of u and $\tilde{u}$.

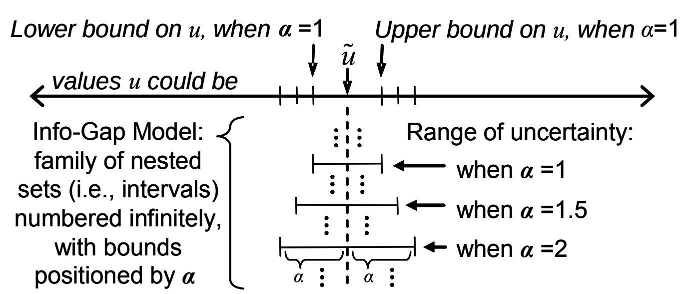

Figure 3.13: Representing unbounded uncertainty as an α–parameterized family of nested sets (Reproduced from (Duncan et al. 2006))

The info-gap model in Eq. (3.1) may also be expressed in simpler, equivalent notation:

$$u-\alpha \ \le \ u \ \le \ u+\alpha, \quad \alpha \ge 0 \tag{3.2}$$

In some situations, it may be more intuitive to express variation α as *fractional deviation*, that is, as a percentage of the nominal. This has the following form:

$$\mathcal{U}(\alpha,\tilde{u}) = \left\{ u : \frac{|u-\tilde{u}|}{\tilde{u}} \le \alpha \right\}, \alpha \ge 0 \tag{3.3}$$

which is equivalent to:

$$\tilde{u}(1-\alpha) \le u_n \le \tilde{u}(1+\alpha), \quad \alpha \ge 0 \tag{3.4}$$

The α parameter of a fractionally defined info-gap is unitless, expressed as a percentage. This type of normalized info-gap structure is shown to be useful for analyzing multiple info-gaps in Chapter 4 of this thesis.

3.4.1.2 Uncertain Functions

When *u* is a function rather than a constant, there is considerably more variety in the way that an info-gap model can be structured. As listed in Section 2.5 of (Ben-Haim 2006), info-gap models for uncertain functions include:

- Uniform bounded: similar in form to Eq. (3.1) except that *u* is replaced with a function *u(t)*.
- Envelope bounded: Similar to uniform bounds, except the parameter α is multiplied by (and thus modulated by) $\psi(t)$, which modifies the shape of the deviation as some function of *t*. It is usually loosely suggested that $\psi(t)$ comes from some available "prior information".
- Energy bounded: bounds the integral of a quadratic function; appropriate for uncertain dynamic phenomena.
- Fourier bounded: useful for uncertain spectral information

While the details of these modeling options are not important to this thesis, it should be stressed that each of these different model types are parameterized by α. Thus, each of these model types' unique mathematical structure modifies or modulates how α affects whatever particular phenomena is uncertain.

3.4.2 Performance Function

The performance function (i.e., performance model) is not particularly unique in IGDT compared to other uncertainty formalisms; it simply must be a function of the design variables q and the uncertainty u. The output of the performance function does however need to be expressible with certainty[15]. Critical performance, r_c, is the satisficed level of critical performance that must be exceeded, as explained in Section 3.3. When a larger (rather than smaller) reward $R(q,u)$ is desirable, the critical value r_c is defined such that for all q and u, the critical satisficing constraint requires:

$$R(q,u) \geq r_c \tag{3.5}$$

Alternatively, when smaller performance is better:

$$R(q,u) \leq r_c \tag{3.6}$$

The design variable q can be either continuous (which will yield trade-off plots that look like Figure 3.10) or discrete (which have trade-off plots of the form of Figure 3.11). If info-gap uncertainty affects the performance of each member of a set of discrete alternatives in different ways, then the alternatives do not need to share the same performance function. However, the output of each separate $R(q_i,u)$ must be measured using the same units. This is the case for the example problem discussed in Chapter 4.

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[15] The value is certain, but may change depending on the value that α takes.

All of the design alternatives, if affected by other uncertainties other than info-gaps (e.g., aspects not yet specified in the early design stages), should be affected equally by those other uncertainties so that they can be considered independently of the info-gaps.

3.4.3 The Info-Gap Robustness Function

From the main IGDT components, *u*, *R(q, u)*, and r_c, a robustness function $\hat{\alpha}(q, r_c)$ can be defined that maximizes the size that the uncertainty parameter α can take and still satisfy the critical constraint of Eq. (3.5). This constraint is embedded into the *robustness function*, defined mathematically as an optimization problem:

$$\hat{\alpha}(q, r_c) = \max\left\{\alpha : \left(\min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u)\right) \geq r_c\right\} \tag{3.7}$$

The info-gap robustness $\hat{\alpha}(q, r_c)$ is "the maximum tolerable α so that all *u* [in the info-gap model's family of sets] up to uncertainty size α satisfy the minimum requirement for survival" (Ben-Haim 2006). Stated more simply, the robustness function is the greatest value of the uncertainty parameter for which the performance requirement is not violated. Eq. (3.7) is formulated for cases where larger values of performance are better. If smaller performance is better:

$$\hat{\alpha}(q, r_c) = \max\left\{\alpha : \left(\max_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u)\right) \leq r_c\right\} \tag{3.8}$$

The "hat" on the symbol for robustness, $\hat{\alpha}$, distinguishes it from the uncertainty size α. The actual value of α is unknown, but one can still determine how much robustness, $\hat{\alpha}$, to unknown uncertainty bounds can be achieved by choosing a satisficing design rather than a risky, reward-optimizing design.

The info-gap model, parameterized from its center, has two ends of interest for each set in the family, as seen in Figure 3.13. The focus of this thesis will only be on the bound that creates the worst consequence to performance. However, IGDT can consider the "favorable" end of the interval (i.e., α on the other side of the nominal) when using an *opportuneness function* (Ben-Haim 2006). Ben-Haim provides examples where robustness and opportuneness grow at different rates, which might differently influence a decision maker who is a risk taker rather than risk averse. These scenarios are not typical, however.

To review, the typical steps to finding a satisficing, robust-optimal design using IGDT include translating the severely uncertain information into an info-gap model, defining the reward function, $R(q,u)$, choosing a critical level of guaranteed performance, r_c, and finding the robust-satisficing design, $\hat{q}(r_c)$. If the requirement for critical performance is flexible, one can take the additional step of plotting the relationship between r_c and $\hat{q}(r_c)$. Additional explanation of IGDT is done via example in Section 4.2.

3.4.4 Evaluating Design Rankings and Weighing Trade-offs

The design q that yields the largest robustness $\hat{\alpha}(q,r_c)$ for a given r_c is the "robust-satisficing" design, denoted $\hat{q}(r_c)$. In mathematical terms:

$$\hat{q}_c(r_c) = \arg\max_{q \in Q} \left(\hat{\alpha}(q,r_c) \right) \tag{3.9}$$

For a given choice of r_c, $\hat{q}(r_c)$ is the most preferable design; however, changing r_c will in some cases switch $\hat{q}(r_c)$. This switch is gradual over a range of continuous design variables, as is demonstrated in the problem in Figure 3.10. For design problems involving selection, however, the preference ranking switches at a specific value for r_c, as shown in Figure 3.11. Whereas Ben-Haim refers to this as a "preference reversal", we refer to it as a Preferredness Switch Point (PSP), since "reversal" implies a change between 2 options and there could be more than one switch point if more than one design is considered (e.g., the problem in Chapter 6).

Robustness-reward curves like Figure 3.10 or Figure 3.11 are a way to assess info-gaps with the intent of forming or confirming preferences for trade-offs, which in turn can induce a preference ranking. The visual nature of these curves enables one to weigh robustness versus performance using "gut" reaction, rather than express preferences *a priori*, which could be difficult to do for a satisficed level of performance. As Ben-Haim notes, "It often happens that a decision maker chooses both r_c and the optimal action $\hat{q}(r_c)$ in an iterative (and introspective) fashion from consideration of the robustness function" $\hat{\alpha}(q, r_c)$. This process will be narrated for the two example problems in this thesis.

Lastly, it is important to distinguish between selection problems where a single design $\hat{q}(r_c)$ dominates preference rankings for all choices of r_c, versus cases where $\hat{q}(r_c)$ switches somewhere over the r_c range. Some $R(q,u)$ models, when affected by changing uncertainty, will not respond differently for different design alternatives. Thus, the design that performs optimally under no uncertainty will also perform better than any other alternative no matter what the uncertainty turns out to be. A conceptual example of this is provided in Figure 3.14, which features a design problem of selecting between two design alternatives, where Design A always dominates. Finding that one design dominates may seem like a trivial case, but it is actually ideal because no further uncertainty assessment is needed to make a decision.

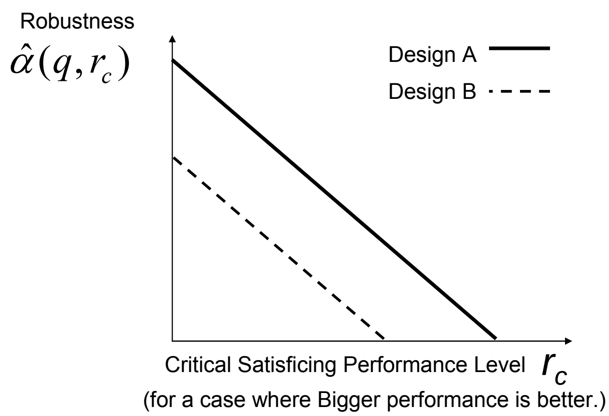

Figure 3.14: Trivial design problem; robustness-performance trade-off does not influence decision

On the other hand, when a preferredness change does occur somewhere within a range of potentially acceptable r_c levels (as in Figure 3.11), one must reflect on how much robustness one thinks they will need. This essentially involves settling one's preferences for a robustness-reward tradeoff. Given that the size of the uncertainty is unknown, making such a trade-off essentially involves a *gamble*[16]. This mode of reflecting on the decision is the way IGDT facilitates decision making without requiring more information than is available. Ben-Haim has considered some of the implications of and approaches to "gambling" in this sense (Ben-Haim 2006).

While it may be difficult to completely generalize whether a preferredness switch will occur for a given selection problem, a few observations can be made. Robustness curves are likely to cross and create a PSP when:

- the uncertainty model $\mathcal{U}$ is somehow a function of the design options q,

[16] The term gambling, here, is not related to probabilities.

- the reward function *R(q,u)* is a structurally different equation for different design options *q*. (This is the case for the example problems in this thesis.)

There may be other observable instances that can help a designer screen whether a an info-gap analysis will influence preference rankings of design alternatives.

3.5 Commentary on the Internal Validity of IGDT

Can one trust the results of an info-gap analysis? As mentioned at the beginning of Section 3.2, the principal components of an uncertainty formalism are its *formulation* (i.e., the soundness of its models, and how the uncertainty is measured), its *analysis* (i.e., techniques for calculating, combining, or propagating uncertainty outputs), and its interpretation (i.e., its inference scheme, how one reaches a decision). Each of these three aspects is next considered separately for info-gap theory.

First, the formalism's uncertainty models should accurately represent uncertainty, preferably using an operational definition, i.e., a set of operations, or a procedure by which the uncertainty can be measured. Info-gap uncertainty models are sparse and straightforward: they consist of what you do know (a nominal model or parameter whose representativeness is in question) and the unknown degree of error around that nominal. Because the uncertainty parameter is undefined, one does not need an operational definition for it. The axioms underlying the properties of info-gap models are discussed briefly in Section 3.6.3.

The computational techniques of an info-gap analysis consist of whatever means are used to solve the optimization problem embedded in the robustness function of Eq. (3.7). Finding this solution returns the robustness for a decision option at some level r_c. In this thesis, exhaustive searches are used to calculate info-gap robustness. The computational effort is relatively low for problems featuring monotonic performance functions and interval-bounded info-gaps of the type in Eq. (3.1). All of the examples in this thesis bear these basic features.

Finally, the robust-satisficing decision rule (introduced in Section 3.3) must be deemed acceptable by a decision maker if info-gap theory is to be used. The strategy of satisficing performance to gain robustness to uncertainty should seem relatively intuitive. This rule is compared to the rules of other decision formalisms in Section 3.6.2.

A decision maker using IGDT is also able to assess the trade-off between their choice of satisficing r_c level and the corresponding robustness that is achievable at that level. (This was depicted in Figure 3.10 and Figure 3.11.) Settling on a trade-off means determining how much robustness one thinks they will need; given that the size of the uncertainty is unknown, this effectively involves a *gamble*. This judgment of how much robustness is adequate is not guaranteed to guard against a bad decision (based on a bad trade-off), for instance, if the true error in the nominal turns out to be greater than the most preferred design's info-gap robustness. This does not imply that the results of an info-gap analysis are invalid; it does mean, however, that the decision support afforded by IGDT is less powerful than normative methods.

3.6 Comparing IGDT to Other Uncertainty Formalisms

Further comparison of IGDT versus other decision formalisms is now warranted. The key differences in structure, decision rules, and decision axioms are examined in this section. Additionally, consideration is given to whether the value of IGDT versus other formalisms could be comparable to problems of the same type.

3.6.1 Information Requirements

IGDT information requirements are low and fundamentally different that those of other formalisms. An info-gap model requires a nominal model or nominal piece of starting information, but if sample data are available, they should be used to define a probability distribution, or p-box if that sample is small. An info-gap is comparable to a standard interval in that both represent imprecision; however, the starting information is different because an info-gap requires a nominal and nothing else and an interval requires no nominal but bounds.

Building on the discussion in Section 3.2.1, the uncertainty that an info-gap represents is different than the uncertainty models of other formalisms. As Ben-Haim states, "We can rank degrees of information gap in terms of the size of the uncertainty parameter α, but this is much weaker information than probability or possibility where the distribution functions indicate recurrence-frequency or plausibility" (Ben-Haim 2006). In other words, one may have the rough notion that one size of α is "much" bigger than another, but the likelihood or possibility of either of those values for α are not known.

3.6.2 Differences in Decision Rules

Besides identifying what information is available to characterize uncertainty, one must consider what rule (i.e., what strategy) one wishes to adopt to include the effects of uncertainty into decisions. What decision rule options are feasible for any particular problem depends on the information available about uncertainty. Given imprecise outputs, one may prefer to maximize the performance under worst case (maxi-min, as in Figure 3.4) or best case (maxi-max) conditions, or any of a number of other heuristics.

Given probabilistic uncertainty on outputs, one may prefer to maximize expected utility (as in Figure 3.3) or minimize the variation on outputs (as in Figure 3.5) per robust design in the Taguchi sense. Choice of one of these decision rules depends on the goals of the decision scenario.

The decision rule in info-gap theory—to maximize the size that error in the nominal value could take, constrained by some critical level of performance that the decision maker must specify—differs from previous decision rules. A robust-satisficing decision is motivated by scarcity of information. It requires acceptance of a more conservative concept of desired performance: acceptable rather than optimal. Also, because decision results using this rule can be sensitive to the r_c level chosen, a decision maker can evaluate the trade-off between info-gap robustness and r_c to help confirm or evolve their choice of an r_c level. This step is *central* to analysis of decisions in IGDT; it is recommended that, under severe uncertainty, a decision *not* be made using only a single fixed level of r_c. This aspect of catalyzing preferences and calibrating one's sensitivity to risk is not observed to be as prominent when using other decision rules. Accordingly, info-gap does not require that such preferences be specified firmly *before* uncertainty analysis.

As such, info-gap decision theory is meant to fill a different role than other uncertainty formalisms. It is a last resort (or preliminary assessment) when information is, for whatever reason, too limited to enable the parameters of other uncertainty models to be specified without unwarranted assumptions. It seems intuitively reasonable that one would, as IGDT requires, be willing to trade good-enough performance in exchange for extra allowance for error in estimates. IGDT does not guarantee the optimal performance; rather; it guards against choice of decisions that quickly underperform in the presence of uncertainty.

3.6.3 Rationality and Axiomatic Underpinnings of Decisions

From a normative standpoint, a decision theory can be judged as trustworthy if its information models, methods for elicitation, and decision rules are founded on mathematically sound axioms (Hazelrigg 2003). Aughenbaugh, for instance, argues for the use of imprecise probabilities based on these grounds (Aughenbaugh 2006). Also, many decision makers, especially economists, cite the dependability of expected utility theory based on the strength of both Kolmogorov's axioms (Kolmogorov 1956) and the von Neumann-Morgenstern (vNM) axioms (von Neumann and Morgenstern 1944).

IGDT has axioms for the structure and behavior of its *models* (Ben-Haim 1999); however, it lacks a normative decision axiom as strong as the vNM axioms. This is mostly because preferences for robustness are (or can be) formed based on inspection of robustness curves (e.g. Figure 3.10 or Figure 3.11). As Hall observes, "[info gap decision theory] is not dogmatic in being derived from axioms of rationality and leading to prescribed behaviour, though it certainly has a rationale that is justifiable as far as available information will allow" (Hall 2003). And, as Ben-Haim himself defends:

> *If "rationality" means choosing an action which maximizes the best estimate of the outcome, as is assumed in much economic theory, then info-gap robustness is not rational. However, in a competitive environment, survival dictates rationality…If rationality means selecting reliable means for achieving essential goals, then info-gap robust-satisficing is extremely rational."* (Ben-Haim 2006)

This idea seems reasonable, though it warrants future discussion, perhaps on a philosophical level.

3.6.4 IGDT "Performance" Versus Other Formalisms

Aughenbaugh and Paredis devised a scheme for comparing the average performance of two different uncertainty models: a precise normal fit distribution versus an imprecise p-box (Aughenbaugh and Paredis 2006). Each model utilized the exact same test samples drawn from a "truth" probability distribution known only by an "omniscient supervisor". This test revealed that which model performs better depended on the number of samples available to describe a random variable, material strength. Could a similar test be applied to IGDT versus other formalisms?

Unfortunately, no. As explained in Section 3.6.1, info-gaps and other uncertainty models can't utilize the exact same information; doing so either throws away info (to make it usable in an info-gap) or requires unavailable assumptions for the other (to fill in missing data such as bounds or distribution parameters). Once one starts to use information in the form of samples, probability is automatically favored. Moreover, the decision rule for determining the most preferred design differs for info-gap (as indicated throughout Section 3.4), also discouraging comparability. Info-gap robustness, and, accordingly, design preference ranking based on that, depends (structurally) on a choice of r_c, the satisficing level of critical performance. Choosing an r_c level would require extra input than would be used in another uncertainty formalism.

3.7 What Has Been Presented, and What is Next

In this chapter, the general concept of uncertainty has been presented along with different models and approaches to including it into design decision making. From this reference point, IGDT can be seen to be unique in the way that it models severely deficient information about uncertainty and assesses the robustness of designs from a satisficing perspective. Thus, info-gap theory is meant to complement existing uncertainty formalisms, providing a means for analysis of the decision-implications of uncertainty when information about that uncertainty is too sparse to warrant another formalism. Additionally, IGDT uses much different information, and its performance (i.e., success in determining "good" decisions under uncertainty) cannot be experimentally compared to other uncertainty models if the same starting information is to be used.

Since IGDT has been identified as uniquely applicable depending on what is known about uncertainty, the next task is to explore its generality to handle various scenarios of EBDM problems that involve severe uncertainties.

CHAPTER 4:

USAGE GUIDELINES FOR APPLYING IGDT TO EBDM

At this point in the thesis, the problems and needs created by EBDM information deficiency have been reviewed in Chapter 2, and the structure, information requirements, and usage of IGDT have been examined in Chapter 3. The current chapter synthesizes what has been learned so far into a set of guidelines for applying IGDT to EBDM. A first attempt at proposing guidelines is made in Section 4.1. An oil filter selection design problem is posed in Section 4.2, and we use it to consider how the guidelines would help one decide to apply IGDT. In Section 4.3, the design problem is solved using IGDT and design insight is gained and discussed. Finally, the design guidelines are updated using usability lessons learned during the application of info-gap to the problem.

4.1 Proposal of an Initial Set of Usage Guidelines

Different components of an info-gap analysis (per Section 3.4) can apply to EBDM problems in different foreseeable ways. The following section makes an initial attempt at a set of guidelines for understanding the applicability of IGDT to an EBDM problem. These guidelines combine findings from the survey of EBDM uncertainties (Chapter 2) with insight about formulating the components of an info-gap problem (Chapter 3). System performance, uncertainty type, and one's willingness to satisfice performance to gain robustness to uncertainty are all considered as follows.

4.1.1 Selection between Design Alternatives

Generally, EBDM design problems can either consist of selection between discrete alternatives (i.e., choice between design options), specification of some value of a continuous design variable (e.g., specifying the thickness of a beam), or some combination of the two. Conceptual examples of robustness-performance curves for a selection problem and for a continuous design problem were illustrated in Figure 3.11 and Figure 3.10, respectively. For a continuous design problem, switches in which design is most preferred occur at specific points (a PSP) rather than gradually, as in continuous design problems. Thus, a decision maker will not need to be as precise when specifying a robustness performance trade-off for a selection problem. In addition, the techniques to be presented in Chapter 5, which enable analysis of multiple info-gaps, are difficult to apply to design problems with continuous variables. Thus, a decision maker will probably have most success applying IGDT to EBDM design problems limited to selection between discrete design alternatives.

4.1.2 Info-gap Uncertainties in EBDM

Variables and models alike can be info-gaps; in this section we will use the term "models" generally to describe both. Phenomena that are info-gap uncertain tend to be models dependent on numerous compounded uncertainties, time-distant aspects, and/or system complexities. They can fall in any of the "spheres" of influence discussed in Section 2.1.2. Some general groupings for info-gap uncertain models include:

- <u>Estimates of unknown validity</u>: variables that are specific to (or have been measured from) one system but whose validity in representing a different system is unknown. Examples include models of:

  - Machine performance from one factory, applied to estimate the performance of a new factory's machines, which may be a new or different technology.
  - Environmental (impact) performance indicators from an LCA database (or handbook) whose quantities are derived in an undocumented fashion or whose embedded valuation applies to geographic regions with different environmental priorities.
- Time-sensitive models: models that may be applicable now but will deviate in validity by some unknown amount in the future. Examples include models of:
  - Ecological impact models that could change with the climate or geographical changes.
- Human behavior: economic demand, consumption habits, regulatory levels
- Unobserved socio-techno interactions: customer usage trends, market success of technologies, destination of products at end of life, product "hacking", terrorism.
- Natural systems behavior: climate variables, storm loads, actual environmental responses to pollutants, resource availability.
- Composite uncertainties: Several separate uncertainties combine together and are considered one overall severe uncertainty.

4.1.3 Performance Functions *R(q,u)* in EBDM

Different considerations about the performance function include what it measures and what structure it takes.

4.1.3.1 Units of Environmental Performance

Units that *R(q,u)* might be include:

- Single score all-in-one indicators: These performance measures come from impact assessment methods for life-cycle analysis in which particular scores are assigned to specific materials and processes. An example is Eco-Indicator 99, measured in millipoints (Goedkoop and Spriensma 2001). This is explained more in Section 4.2.2.
- Composite weighted sums for single impact types: Global Warming Potential, eutrophication, etc.
- Specific impact oriented metrics: Various measures of eco-system damage.
- Energy measures: Energy consumption, total embedded energy, or even exergy
- Waste measures: Waste production, the Waste Index (Emblemsvag and Bras 2001)
- Monetary measures: valuation of the effects of environmental impact, e.g. by emissions taxes or credits, resource consumption, etc.

4.1.3.2 Structure of Performance Functions

As mentioned at the end of Section 3.4.4, selection problems (i.e., a choice between design alternatives) are more likely to feature ranking switches (PSPs) when the following is true of the performance function:

- the uncertainty model $\mathcal{U}$ is somehow a function of the design options q,
- the $R(q,u)$ is a structurally different equation for different design options q.

An example of the second item above is a classic payback scenario. This is when a design option that has a high up-front cost but low continuing cost eventually "pays back" and achieves savings if utilized for a long enough period of time. Different design alternatives may perform differently overall depending on the timescales involved. This case will be demonstrated in the oil filter selection example problem in this chapter.

4.1.4 Satisficing Behavior in EBDM

Like for all info-gap problems, one must be willing to satisfice environmental performance to gain robustness to uncertainty. Compared to financial measures, environmental performance dimensions may be less intuitive for people to relate to or express preferences for in terms of "value"; thus, the ability to analyze robustness-performance trade-offs to help induce or catalyze one's preferences is useful.

As far as choices for satisficing targets, regulation thresholds could feasibly be used as a level of "critical performance" (r_c) under conditions of severe uncertainty. This should only be used for critical satisficing performance, however, not to design to the lowest standard.

4.2 Introduction to the Test Example Problem: Oil Filter Selection

The following oil filter problem has been used to test out the applicability of IGDT (Duncan et al. 2006, Duncan et al. 2008).

About 250 million light duty oil filters are discarded (and not recycled) in the United States each year, while about 250 million more are recycled (FMC 2002). The environmental impact of these filters can be substantial, as disposable filters contain large amounts of steel, aluminum, or plastic, depending on the style of filter.

In this example, it is assumed that an automobile manufacturer wants to reduce the environmental impact of oil filters from its cars by designing a more environmentally benign filter. Naturally, some simplifications and assumptions are introduced in the problem. For example, the exact parameters for the problem are chosen to be realistic, but they do not represent hard, real-world data. Consequently, the emphasis is not on the actual decision outcome (i.e., the chosen filter), but rather on the decision and analysis *process*.

4.2.1 Types of Oil Filters

In this simplified model, shown in Figure 4.1, an oil filter is comprised of five main components: housing, top cap, filter, inner support, and bottom cap. support, and bottom cap. The housing, top cap, and bottom cap make up the *casing*, and the inner support and filter make up the *cartridge*.

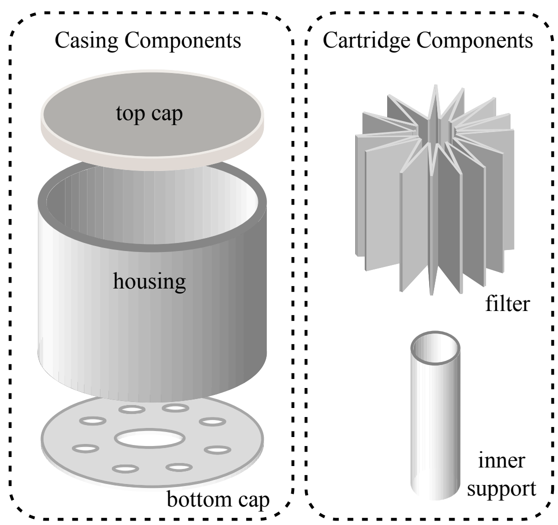

Figure 4.1: Oil Filter Schematic Diagram (from (Aughenbaugh et al. 2006))

Two different types of oil filters are considered, as summarized in Table 4.1. The dimensions of all components have been specified for the appropriate balance of strength and weight and are therefore fixed.

Table 4.1: Types of Oil Filters

Filter	Material	Discarded parts
SEC	Steel	All (casing and cartridge)
TASO	Aluminum	Cartridge only

Engineers have developed two competing concepts for the new design. The first filter considered is the steel easy change (SEC) filter. For an SEC filter, the structural components are made of steel and the filter of cellulose. The entire filter is designed to be replaced at once; it is simply unscrewed from the engine and the discarded or recycled. The second type of filter is the take-apart spin-on (TASO) filter. A TASO filter's structural elements are made out of aluminum and when the filter is replaced, only the cartridge is discarded; the casing is designed to last for the lifetime of the engine and is reused when the filter is changed. The environmental performance of both alternatives is considered over a vehicle's entire lifetime, which relates to F, the total number of filters used over the lifetime.

4.2.2 Environmental Impact Calculation

It is assumed that the primary environmental impact of an oil filter arises due to the construction, transportation, and disposal of the casing and cartridge components shown in Figure 4.1. Other substances, such as oil residue and rubber seals are generally equivalent in both filter types, and therefore do not contribute to the selection decision.

Eco-indicator 99 is an impact assessment method for life-cycle analysis in which particular scores, measured in millipoints (mPt), are assigned to specific materials and processes (Goedkoop and Spriensma 2001). In this example problem, only those environmental impacts that increase in direct proportion to mass are considered. For simplicity, these impacts-per-mass for different stages of the life cycle (mining, processing, disposal, recycling, etc.) will be summed for each component and referred to, for simplicity, as Eco-indicator *rates*, or simply *ecorates*, subsequently.

In the Eco-indicator methodology, the ecorates are tabulated as shown in by considering the three spheres of knowledge and reasoning noted in Section 2.1.2. The rates are tabulated for various products or by-products of manufacturing processes and product life-cycles. For each of the items in a potential load inventory (part of the technosphere), there are associated environmental effects in the ecosphere. For example, the release of CFCs into the environment depletes the ozone layer. In some cases these effects are clearly understood, and in other cases there is uncertainty as to how strong the effects are.

Each effect, in turn, has particular *damages* associated with it. These damage estimates are often more uncertain than the effects. For example, consider the current debate surrounding the damages that result from increased greenhouse gas emissions—how much are they damaging the ecosystem? There is not universal agreement, and hence significant uncertainty, as to the true damages.

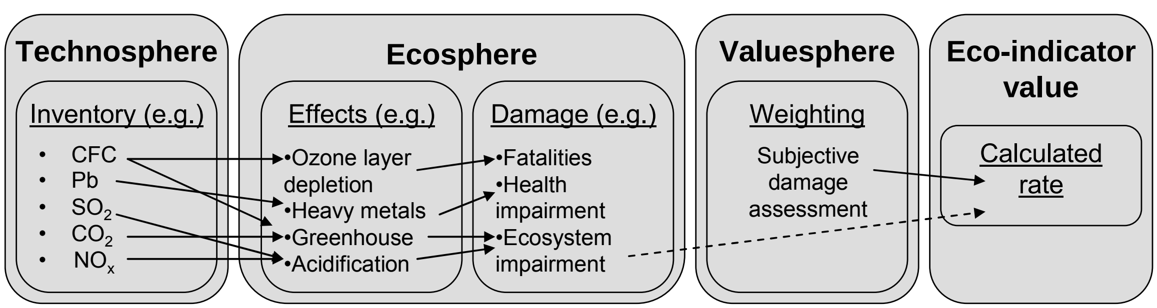

Figure 4.2: Eco-indicator calculation. Adapted from (Goedkoop and Spriensma 2001)

Once the ecosphere aspects are modeled, one evaluates how much he or she actually cares about these damages relative to other damages. This is a valuesphere question. The value that someone or some society places on a particular damage can vary with factors such as culture, religion, and geographic location. For example, assuming greenhouse gas emissions are causing global warming and raising sea levels, how much does one care about these damages compared to damages caused by acid rain?

The Eco-indicator methods condense ecosphere and valuesphere information for individual materials on a per-mass basis. In order to calculate the actual impact of a process or product, the total mass of materials present—the inventory or technosphere information—must be determined. In this example, we assume that the filter casings and cartridges can each be parameterized per filter, and thus the impact of each can be summarized with one mass and ecorate. The specific assumed data are shown in Table 4.2. The impact I_c of a given component c can be calculated as:

$$I_c = mass_c \cdot ecorate_c \tag{4.1}$$

The total environmental impact over a vehicle's lifetime depends on the number of filters F used, which is categorized as a life cycle event in the terminology of Figure 2.2. The quantity F is uncertain because not every vehicle is in service for the same number of miles, and car owners change the filters with difference frequencies. When using LCA in practice it is important to communicate fully to decision makers what masses and remaining assumptions were used in determining the Eco-indicator scores. However, because this thesis is not intended to be an actual recommendation of a filter, the rest of this step will be skipped for brevity.

Table 4.2: Mass and impact-per-mass for all components

		mass, kg	ecorate, millipoints/kg
TASO	Cartridge	$m_{cr,T}$=0.071	$e_{cr,T}$=5.07
	Casing	$m_{cs,T}$=1.841	$e_{cs,T}$=17.10
SEC	Cartridge	$m_{cr,S}$=0.075	$e_{cr,S}$=5.50
	Casing	$m_{cs,S}$=0.817	$e_{cs,S}$=1.78

Assembling these components into equations for environmental performance, the total impacts of the filters are:

$$\begin{aligned} I_{TASO} &= I_{casing} + \left(I_{cartridge} \cdot F\right) \\ &= (m_{cs,T} \cdot e_{cs,T}) + (m_{cr,T} \cdot e_{cr,T} \cdot F) \end{aligned} \tag{4.2}$$

$$\begin{aligned} I_{SEC} &= \left(I_{casing} + I_{cartridge}\right) \cdot F \\ &= (m_{cs,S} \cdot e_{cs,S} + m_{cr,S} \cdot e_{cr,S}) \cdot F \end{aligned} \tag{4.3}$$

An essential difference between the environmental impact of the designs is their casings: the TASO incurs a high one-time load whereas the SEC incurs a smaller load every time the filter is changed. For small F, the SEC filter has a smaller impact, but as F increases, the impact of replacing the casing with the SEC filter will exceed the one-time impact of the TASO's casing. The TASO casing has a higher impact because it is contains more material—it is built to last as long as the car's engine—than the SEC filter and because the material is aluminum, which is more resource intensive per unit weight than steel. In contrast, the SEC filter is made of steel (with a lower impact per mass) and contains less material since its lifetime is shorter.

4.2.3 Considering IGDT Applicability to the Problem

The oil filter selection problem introduced in this section can be used to explore the application of IGDT to various scenarios regarding the uncertainty in the ecorates and the number of filters used over the vehicle's lifetime. This is justified by the following reasoning.

The Eco-indicator construct provides a baseline for comparing the environmental impact of different materials across all of the spheres. However, the uncertainties in the ecosphere (effects and damages) and valuesphere combine to yield a very large uncertainty in the ecorates. If one does not know on what assumptions Eco-indicator scoring is based, one does not know how applicable those values are to different geographic regions or different social value sets. This falls under the category "estimates of unknown validity" from the Guidelines in Section 4.1.2.

The example also assumes severely deficient information regarding the average number of filters used over an engine's life, *F*. Despite recommendation by the manufacturer of a particular mileage period between filter changes, uncertainty about the frequency with which the customers will actually change their filters, coupled with uncertainty about the average lifetime of their cars, makes the actual average number of lifetime filter changes severely uncertain. This uncertainty is a combination of categories from the Guidelines in Section 4.1.2: "Human Behavior", "Estimates of Unknown Validity", and "Composite Uncertainty".

Next, the performance functions *I* are different for the two design alternatives. In addition, their structure are that of a pay-back scenario, as mentioned in 4.1.3.2. It will be seen that these two facts mean that the environmental performance relates differently to the two info-gap uncertain variables.

Finally, the decision maker wishes to evaluate the selection decision without collecting further information, and decides to use the IGDT approach to do so. She is willing to take the attitude that settling for some guaranteed lower-bound on performance is acceptable and preferable to risky, (but higher) optimized performance that relies on the veracity of unfounded assumptions about the uncertainty. Accordingly, the decision maker seeks the design alternative with maximum robustness to the unknown gap between the unknown *actual* variables and their nominal *estimate*. The desire to maximize the size to which the discrepancy can grow is subject to a satisficing critical constraint that defines a largest environmental load that can be accepted, one that is sub-optimal with respect to the solution with no uncertainty, yet "good enough" given its robustness to uncertainty.

4.3 Oil filter Design Decisions with Info-Gap Theory

In the following series of examples, info-gap models and analyses of uncertainty will be explained for increasingly complicated situations. It is shown that different analysis approaches require different considerations and demands on the decision maker to form preferences for tradeoffs in critical performance versus robustness to uncertainty. The examples include, progressively:

- Section 4.3.1: One uncertainty that affects both design alternatives. Specifically, the number of filters F used over the vehicle's lifetime.
- Section 4.3.2: One uncertainty that has the same units and type but a different nominal for each alternative. Specifically, the *ecorate* of the casing material for each alternative is considered uncertain.

4.3.1 Example 1: Uncertain Number of Filters Used in Lifetime

For this first example problem, an info-gap analysis will be explained in detail. Subsequent examples are variants of this problem, so only their formulation differences and results will be presented.

4.3.1.1 Info-gap model

It is assumed that the design firm has experience making filters for vehicles owned by customers in the industrial sector who schedule regular maintenance and change filters with predictable frequency. On average, those customers use 17 filters over the life of an engine. However, the design company wishes to expand its business with a new filter design targeting the public sector. Customers in that sector are expected to have less predictable maintenance behavior, and the degree to which their change frequency will deviate from that of industrial customers is unknown.

Thus, the info-gap model for this example can be specified with the knowledge that:

- The nominal value of oil filters used over an engine's lifetime is $\tilde{F} = 17$, taken from information on maintenance rates in the industrial sector.
- The growth of deviation around nominal can be expressed mathematically as a simple interval bounded info-gap.

Combining the uncertainty parameter, α, with this sparse information, the info-gap model for lifetime filter usage is:

$$F(\alpha, \tilde{F}) = \left\{ F : \left| F - \tilde{F} \right| \le \alpha \right\},\ \alpha \ge 0 \tag{4.4}$$

The form of this particular info-gap model can also be expressed more simply:

$$\tilde{F} - \alpha \leq F \leq \tilde{F} + \alpha \tag{4.5}$$

4.3.1.2 Reward function and satisficing critical value

The other two components needed for an info-gap decision analysis are the reward function and satisficing critical value for performance. The reward functions for environmental impact, based on Table 4.2 and Eqs. (4.2) and (4.3), of the TASO and SEC designs are respectively:

$$R(q_1, u) = I(TASO, F) = 31.48 + (0.36 \cdot F),\ \text{[mPt]} \tag{4.6}$$

$$R(q_2, u) = I(SEC, F) = (1.46 + 0.41) \cdot F,\ \text{[mPt]} \tag{4.7}$$

The designer chooses a critical value of $I_{critical}$=40mPt, which is the highest level of environmental impact deemed tolerable. In this problem, the decision maker seeks to minimize impact, so the inequality in Eq. (3.6) is used and the critical constraint is given as:

$$I(alt, F) \leq I_{critical} \tag{4.8}$$

For convenience, the variable *alt* is used to represent the discrete design alternatives, TASO and SEC.

4.3.1.3 Info-gap robustness function

Of main interest in an info-gap analysis is what largest amount of robustness to uncertainty, $\hat{\alpha}(q, r_c)$, is achievable. This robustness is the largest amount of uncertainty α that can be sustained by a design alternative q while still guaranteeing, at worst, achievement of the chosen critical performance level r_c. Expressed in the form of Eq. (3.8), the info-gap robustness for this example is:

$$\hat{\alpha}(alt, I_{critical}) = \max\left\{\alpha : \max_{F \in U(\alpha,\tilde{F})} I(alt, F) \le I_{critical}\right\} \tag{4.9}$$

For this particular problem, finding the expression for $\hat{\alpha}$ for either design alternative is relatively simple. First, the uncertain variable F in Eqs. (4.6) and (4.7) is replaced with $\tilde{F} + \alpha$, the side of the parameterized info-gap model associated with worse performance, e.g.:

$$I(TASO, F) = 31.48 + (0.36 \cdot (\tilde{F} + \alpha)) \tag{4.10}$$

With this equation form, one can solve for α and calculate $\alpha(alt, I_{critical})$, equivalent in this case to info-gap robustness, $\hat{\alpha}(alt, I_{critical})$. When the reward function, info-gap model, and/or design space q assume more complicated forms, the optimization problem embedded in Eq. (4.9) can be more difficult to solve.

For the critical level $I_{critical}$=40mPt, $\hat{\alpha}(TASO, 40mPt)$=6.7 filters and $\hat{\alpha}(SEC, 40mPt)$=4.5 filters. One design is preferable to another when it can assure performance at or better than the critical requirement *amidst a greater amount of deviation α*. In this case, the TASO is "robust-optimal" and preferred to the SEC because the TASO filter can meet the critical impact constraint for a larger amount of uncertainty than the SEC filter can.

4.3.1.4 Analysis of Robustness-Performance Tradeoff

Analysis of preference for the trade between robustness and critical (acceptable) performance is facilitated by examining a tradeoff plot. This plot is shown in Figure 4.3

and discussed next. Critical levels of performance can be chosen along the horizontal axis, with the corresponding robustness found as the vertical distance from the horizontal axis to the performance line for each design alternative.

The designer, not knowing the estimation error α, is tasked with choosing a point on the horizontal axis corresponding to his or her demanded level of satisficing performance. In some applications, the r_c value may be strongly dictated by external factors. In other applications, the decision maker has the flexibility to relax their choice of critical performance level in order to gain more robustness. The decision maker can explore this tradeoff graphically in Figure 4.3 by examining the maximum robustness achievable for different values $I_{critical}$. In this example, the design having *maximum* robustness is the one whose performance function plot is the highest at a given critical performance level.

The plot in Figure 4.3 is instrumental in understanding how design preference changes as the demand for minimally acceptable performance is relaxed further away from the performance-optimal level. For example, at critical satisficing level discussed earlier, $I_{critical}$=40mPt, it can be seen that $\hat{\alpha}(TASO)$=6.7 filters and $\hat{\alpha}(SEC)$=4.5 filters. If an impact as aggressively low as 31.7mPt were demanded, only the SEC would satisfy the constraint, and even then, there would be no tolerance for error, α, in estimating the number of filter changes. Thus, $\hat{\alpha}(SEC, 31.7mPt)$=0. Until the critical requirement is relaxed (i.e., moved to the right on the axis) as far as 37.6mPt, SEC is still the only viable option, with its tolerance for error growing linearly. At $I_{critical} = 37.6mPt$, TASO is now a viable design, but offers no info-gap robustness. TASO's robustness eventually overtakes SEC at $I_{critical}$=39mPt, where the performance lines cross in Figure 4.3 and the preferred design changes. The decision maker must explore these tradeoffs and determine what feasible combinations of robustness and critical performance are preferable.

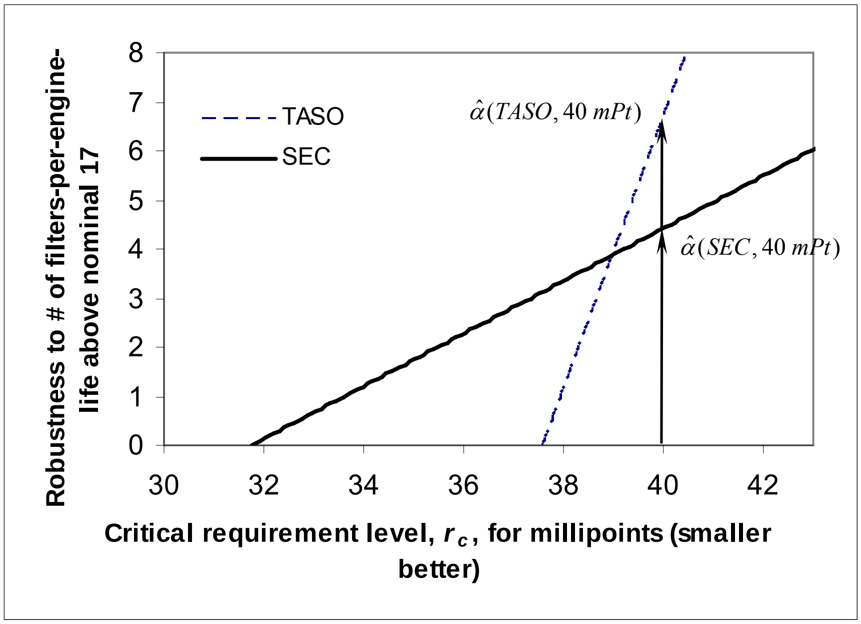

Figure 4.3: Info-gap robustness versus Environmental impact

The following knowledge is gained in this simple example:

- If the decision maker can accept a worst-case environmental impact as high (which indicates worse performance in this example) as 39mPt, then the TASO design is preferable because it can endure the highest amount of error above the nominal guess and still satisfy the performance constraint. Moreover, the rate at which info-gap robustness is gained with incremental relaxation of the $I_{critical}$ demand (i.e., the line slope) is faster for TASO than SEC, making TASO even more attractive past 39mPt of demand.

- If there were no uncertainty, SEC would outperform TASO by a difference of 5.9 mPt; however, if *in reality* the deviation above the nominal estimate of 17 filter changes grew as high as 6 changes, for a total of 23 changes, TASO would then instead outperform SEC by 3.2 mPt.
- The designer, not knowing what the uncertain variable actually will be, can use the info-gap analysis and plot in Figure 4.3 to get a handle on what a decision change entails under the satisficing decision rule. If it seems reasonable that the average number of filter changes could deviate more than 4 above the estimate of 17, and that a relaxed demand of 7.3 mPt is reasonable, then the designer should choose the more robust TASO. Past that decision-switch point, the TASO option takes advantage of greater robustness-per-incremental-relax-in-demand, as indicated by TASO line's flatter slope. It is up to the decision maker to sort out his or her preference for robustness versus guaranteed achievement of, at worst, some critical level of performance.

4.3.2 Example 2: Uncertain environmental impact rates-per-material

In this section, an info-gap analysis is performed assuming extreme uncertainty in the ecorates for the filter casings. The rationale for considering ecorates as extremely uncertain was discussed in Section 4.2.2. In this section, the previously unknown number of filters will be considered known in order to isolate the effects of uncertainty in the estimates of the casing ecorates. Similarly, the ecorates for the cartridges will be assumed known in order to facilitate illustration of the main ideas. An illustration of the more complicated case of multiple uncertainties is postponed until the next Chapter.

Whereas the lifetime number of filters affected both TASO and SEC alternatives, the ecorates, while having similar properties, have different nominal values for each design alternative because each is made from a different material. The values for ecorates, which were previously exact, are now used as the nominal values, i.e., $\tilde{e}_{case,TASO}$=17.1 mPt/kg and $\tilde{e}_{case,SEC}$=1.78 mPt/kg. We note that the units of the two are the same, and thus they can be expressed using a common α. (Note: One should only model in this fashion if it is believed that each uncertainty deviates in a similar scale from their nominals, which are different.) Using the interval bounded form as before:

$$e_{case}(\alpha, \tilde{e}_{material}) = \left\{ e_{material} : \left| e_{material} - \tilde{e}_{material} \right| \leq \alpha \right\},\ \alpha \geq 0 \tag{4.11}$$

Or, more simply stated, for the side of the info-gap model that creates worse performance:

$$e_{material} \leq \tilde{e}_{material} + \alpha \tag{4.12}$$

Substituting $e_{material}$ with $\tilde{e}_{material} + \alpha$ in the original performance function of Eqs. (4.6) and (4.7), a new plot of robustness to error in estimating $e_{material}$ is presented in Figure 4.4.

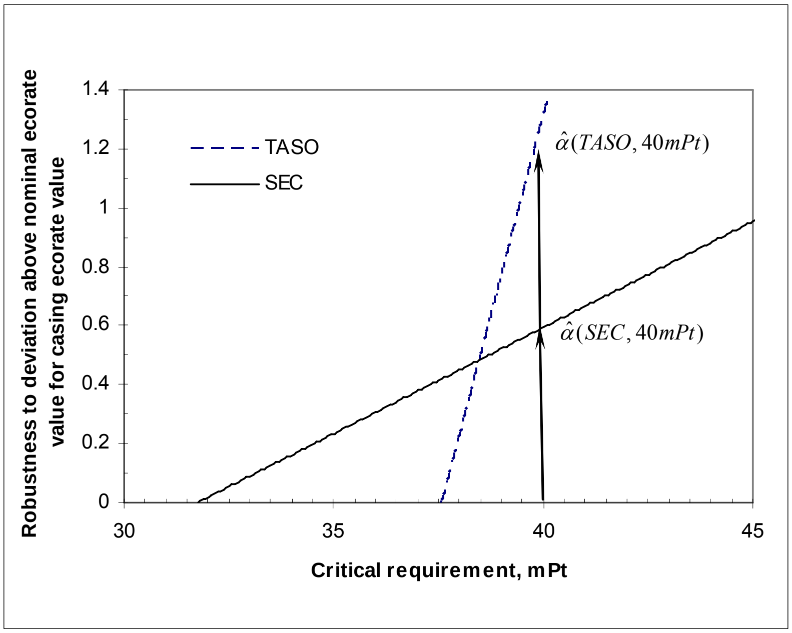

Figure 4.4: Info-gap robustness to $e_{material}$ versus satisficing critical level of environmental impact

In the analysis of Section 4.3.1, each design alternative endured the same uncertain quantity, F, so α was clearly the same for each alternative. This made comparisons between the robustness of the SEC filter and TASO filter straightforward.

In Figure 4.4, consider the comparison of the robustness of the TASO and SEC filters for a critical impact of 40mPt. At this critical impact, the TASO filter allows for a larger α than SEC. However, a particular α for SEC is not necessarily equivalent to the same α for TASO. To clarify, as defined in Figure 4.4, the units of the two α's are the same, but the meaning is not necessary equivalent. For example, is a deviation of 1mPt/kg from the nominal value of 17.1 mPt/kg for the TASO casing the same as a 1mPt/kg deviation from the nominal value of 1.78mPt/kg for the SEC casing? We believe this to be a highly restrictive assumption because the underlying causes of the uncertainty could be different.

An alternative way to compare two uncertainties with different nominal values would be to use the percent deviations (Eq. (3.3)) from the nominal. However, this still assumes that the uncertainties tend to deviate in the same percentages in reality. These restrictive demands on assumptions are examined in more detail in the next Chapter, in which both the ecorates for the casings and the number of filters F are assumed to be uncertain.

4.3.3 Discussion of Oil Filter Problem

In certain situations, the info-gap design analysis approach can eliminate the need for further data collection by facilitating decision making under extreme uncertainty (i.e., when estimation error cannot be quantified). For instance, if a switch in design choice (e.g., from SEC to TASO) requires a small sacrifice in guaranteed performance yet affords a reasonably large amount of extra robustness to error in a nominal estimate, one could decide to switch their choice without collecting more information.

4.4 An Updating of the Guidelines: Lesson Learned About Usability

Applying IGDT to the preceding example problem has provided insight that can be used as future guidelines. Most importantly, it was found that making assessments of robustness-performance trade-offs with either performance or uncertainty measured in a single-score environmental performance indicator can be difficult. This can reduce the precision with which one could make a trade-off and determine which design has the most preferable robustness characteristics.

4.4.1 Measuring Performance with Composite Indicators Can Be Hard to Judge

The IGDT approach requires that the decision maker be able to evaluate the acceptability of some satisficing level of critical performance in light of the corresponding gain in robustness to an info-gap of unknown size. In the examples in the previous Section, we assumed that the decision maker could state a preference for some acceptable size in the Eco-indicator 99 measure of environmental impact, specifically $I_{critical} = 40\text{mPt}$. Although Eco-indicator scoring is grounded in reality, with one millipoint corresponding to 1/1000000 of the environmental load of a European citizen over 1 year, the Eco-indicator 99 construct was primarily developed to compare options relatively, not absolutely (Goedkoop and Spriensma 2001). Whether or not a decision maker would find it reasonable to state one's preference for an absolute millipoint score with that reference point in mind is debatable.

4.4.2 Some Info-Gap Sizes are Harder to Assess

Similarly, IGDT requires a decision maker to have a *relative* understanding of the magnitude of deviation around an uncertain quantity's nominal estimate, but not all uncertainty severities are equally easy to assess. In this example, it is probably easier to understand the severity of error in number of lifetime filter changes in Section 4.3.1 than to understand the severity of particular errors in an ecorate in the analysis of Section 4.3.2. This problem was compounded in Section 4.3.2 because there were uncertain ecorates whose actual values differ for different materials. Difficulty assessing the severity of an uncertainty makes trading off critical performance to gain robustness difficult, perhaps prohibitively so.

4.5 What Has Been Presented, and What is Next

In this chapter, we have proposed what a set of guidelines would look like for gauging the applicability of IGDT to an EBDM problem. The guidelines were used to determine that the oil filter problem was a potential candidate. An info-gap analysis was performed on the oil filter problem for two different uncertain variables of much different type. Design insight and lessons learned were gleaned from solving the example problem, and used to qualify the usability of Eco-Indicator scoring in info-gap problems.

The next chapter uses the oil filter selection problem as a platform to examine the effects that multiple uncertainties have when trying to evaluate info-gap robustness and settle on preferences for a robustness-performance trade-off.

CHAPTER 5:

ASSESSING MULTIPLE INFO-GAP UNCERTAINTIES

So far in this thesis, we have reviewed, for general cases, the conditions and assumptions that a decision maker would need to accept when deciding to use info-gap; we've outlined when IGDT's satisficing strategy will be valuable; and, we've explained what all this means in light of specific aspects of EBDM. In this chapter we consider decision scenarios affected by multiple info-gap uncertainties, which we will call "multi-gaps" for brevity. In Section 5.1, we illustrate how the existence of multi-gaps can create indeterminacy in preference rankings. In Section 5.2, we review the assumptions and implications of prior techniques that analyze multi-gaps through the use of scaling factors. In Section 5.3, we leverage existing techniques and propose Bet-Based Scaling Elicitation (BBSE), a new, more rigorous technique for eliciting scaling factors. After applying BBSE to an example problem in Section 5.4, we discuss its implications of the newly presented techniques in Section 5.5.

5.1 Motivation

We now review the mission of IGDT—that is, to evaluate how satisficing performance affords robustness to info-gaps—in the context of multiple uncertainties. This section illustrates the effects that multiple info-gaps have on determining design preferredness, and clarifies what the challenges are and where gaps in previous capabilities exist.

5.1.1 The Added Complexity of Assessing Multiple Info-Gaps

As demonstrated in Section 3.3, for problems with only one info-gap uncertainty, the decision analyst (DA) is interested in assessing how much robustness is achievable for a range of satisficing critical performance levels, r_c. From there, preference rankings can be induced for a set of design option q. For multiple info-gap uncertainties (multi-gaps), the DA's interest remains fundamentally the same: How much robustness to *each* uncertainty can *simultaneously* be achieved for various levels of r_c, for the different design options q? How "good" are the robustnesses that can be achieved, and what $\hat{q}$ should be chosen accordingly?

Assessing robustness to multi-gaps becomes more complicated when robustnesses "compete". A decision maker naturally wants to maximize the size to which each robustness, $\hat{\alpha}_n$, can allowably grow. However, when growth in each α separately results in worse performance, a trade-off exists between the simultaneously achievable maxima for each $\hat{\alpha}_n$. This trade-off resembles Pareto optimality: robustness to one uncertainty cannot be further increased without causing a corresponding loss in robustness to another uncertainty. An example of this trade-off is shown in Figure 5.1, which is introduced in detail in Section 5.1.2.

Competition between robustnesses adds complexity to the trade-offs being assessed. Before, for one info-gap uncertainty, the decision analyst would assess the adequacy of a design's robustness in light of the r_c value on which it depends. For multi-gaps, however, the analyst assesses not only the "size" of *each* robustness $\hat{\alpha}_n(r_c, q)$, but also the trade-offs between simultaneously achievable robustnesses, all of which are dependent on r_c. This not only makes eliciting $\hat{\alpha}(q, r_c)$–r_c trade-off preferences more complicated, but also, as will be shown in the next section, can create ranges of r_c where one cannot identify a most preferred design $\hat{q}$ without additional information.

5.1.2 Exploration of an Example

The above situation can be illustrated using a three dimensional plot based on the EBDM example of oil filter design originally introduced in Section 4.2. The plot is shown in Figure 5.1. The plot is of robustness to the two uncertainty parameters of the following info-gap models:

$$F(\alpha_F, \tilde{F}) = \left\{ F : \left| F - \tilde{F} \right| \leq \alpha_F \right\}, \ \alpha_F \geq 0 \tag{5.1}$$

$$e_{case}(\alpha_e, \tilde{e}_{material}) = \left\{ e_{material} : \left| e_{material} - \tilde{e}_{material} \right| \leq \alpha_e \right\}, \ \alpha_e \geq 0 \tag{5.2}$$

The performance equations (4.6) and (4.7) and the robustness functions of the type (4.9) are used again for the 3-D plot. The two intersecting surfaces shown are the robustnesses, $\hat{\alpha}_{ecorate}(r_c, q)$ and $\hat{\alpha}_{filterlife}(r_c, q)$, which are functions of critical performance r_c (expressed as a "milliPoint" environmental score) for the two design alternatives, q_1 = TASO (solid shading) and q_2 = SEC (grid shading). Close inspection will reveal that the 2-D plot traced out in the $\hat{\alpha}_{filterlife}(r_c, q) - r_c$ plane is the same as in Figure 4.3; likewise, the 2-D plot traced out in the $\hat{\alpha}_{ecorate}(r_c, q) - r_c$ plane is the same as in Figure 4.4.

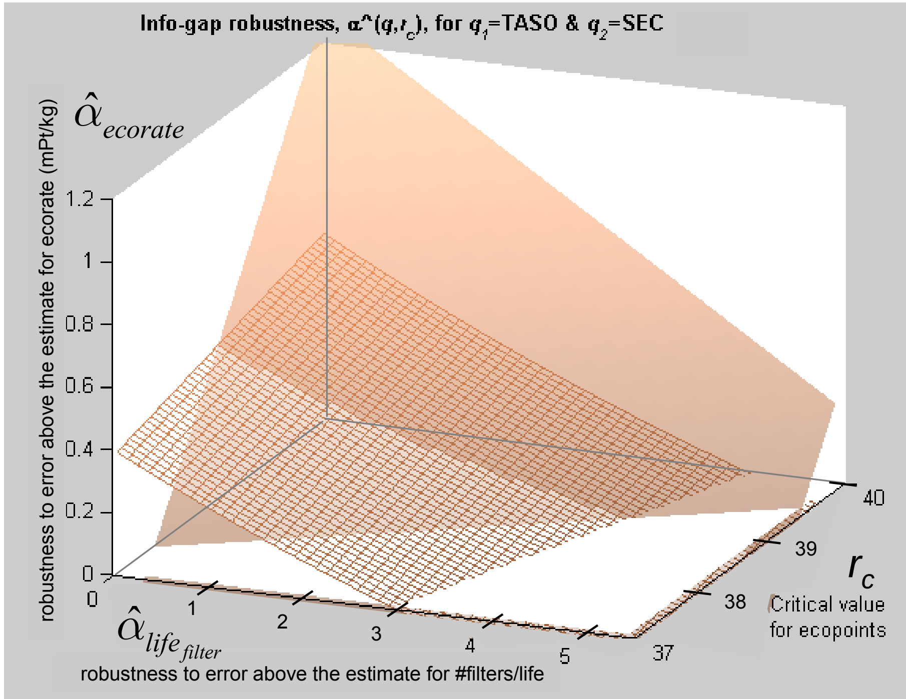

Figure 5.1: Plots of the info-gap robustness of two different filter designs

Both robustnesses $\hat{\alpha}_n$ depend on r_c, but the sizes that they can *simultaneously* be trade off. This trade-off can be viewed on a 2-D plane taken from the 3-D plot at a fixed r_c, as shown on the right side of Figure 5.2. The 2-D plot is also shown overlaid onto the 3-D plot on the left side of the same figure; a green[17] dotted box/border denotes the plane with which the 2-D plot is coincident. The 2-D plot is called a *competing robustness plot*[18] because it shows competition between robustnesses. The red-solid and blue-

[17] When reading this thesis on a black-and-white printout, it is helpful to know that the green line will always be dotted, corresponding to an edge of a plane of constant r_c; a black line will always be solid, corresponding to the robustness of the SEC design option; a blue line will always be dashed, corresponding to the robustness of the TASO design option; and a red line will always be dot-dashed, corresponding to the trade-off specification (TS) line, which is introduced in Section 5.2.3.

[18] Plotting of "competing robustnesses" requires the reward function, *R(q,u)*, to be solved for α, thereby yielding the robustness function, $\hat{\alpha}(r_c, q)$. Furthermore, since the plot involves the dependency of $\hat{\alpha}_1$ on $\hat{\alpha}_2$, the function becomes $\hat{\alpha}_1(r_c, q, \alpha_2)$. Solving for this functional form may be difficult in many scenarios. The competing robustness plots are provided in this section for the purposes of illustration of the concepts involved. Later it will be shown that they are not actually used in (continues on next page →)

dashed *robustness trade-off lines* shown are the maximum simultaneously achievable robustnesses to each uncertainty, for each design, for a specific r_c. In the 2-D plot, in the two locations where the trade-off lines intersect the plot axes, robustness to one uncertainty is completely maximized, and there is no robustness to the other uncertainty. In this particular problem, each uncertainty has significant influence on performance, thus neither can grow exceptionally large (compared to the other) simultaneously.

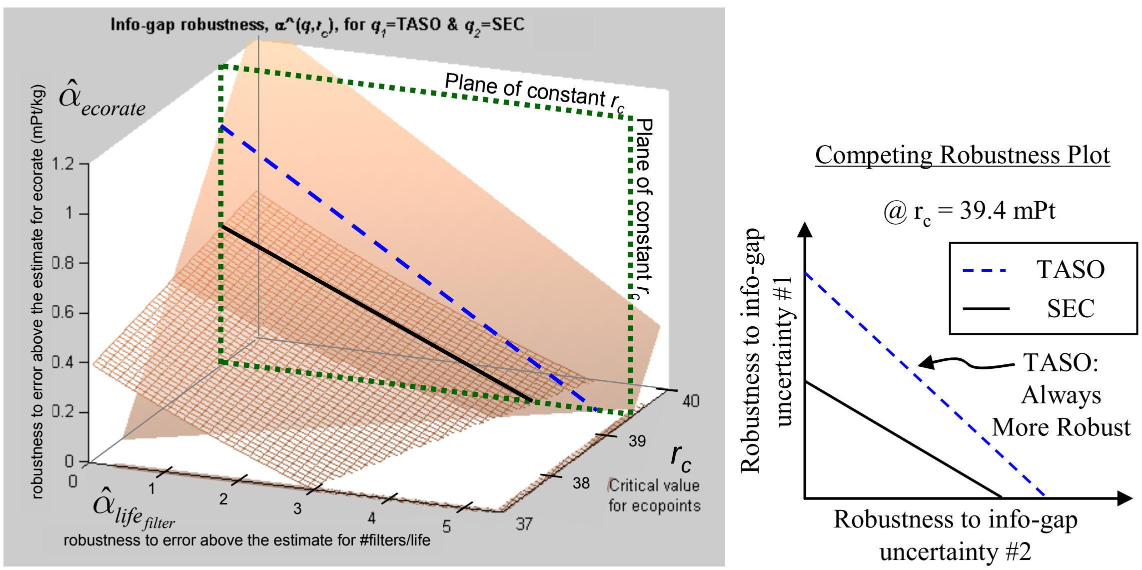

Figure 5.2 : A slice of the 3-D plot at a specific r_c, showing TASO's dominance.

When one design's robustness trade-off line is furthest from the origin, that design's robustness to multiple uncertainties exceeds, or *dominates*, that of all other designs. For some choices of r_c, such as in Figure 5.2 for r_c = 39.4 mPt, one design *always* dominates by providing more robustness for any way that competing robustness $\hat{\alpha}_n$ trade off. In that case, for that specific r_c, a single most preferable choice $\hat{q}$ exists no

multi-gap assessments to support elicitation of preferences for trade-off between robustness and performance.

matter how the robustnesses trade off, i.e., no matter the position along the robustness trade-off lines.

There are some values for r_c (i.e., points along the r_c axis), however, where no design completely dominates for all robustness trade-offs. That is, $\hat{q}$ cannot be determined uniquely without knowing how to trade-off between each competing robustness $\hat{\alpha}_n$.

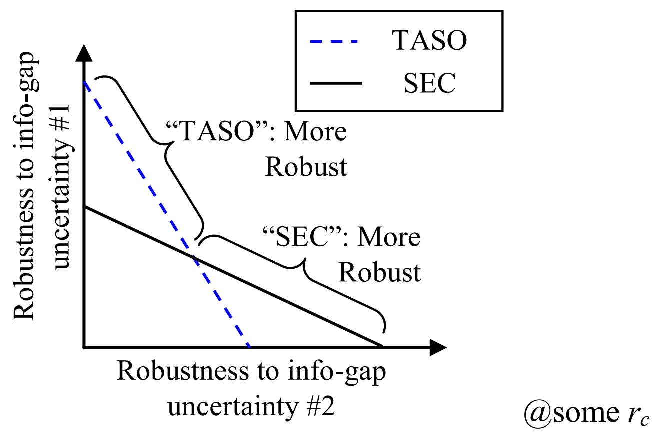

Figure 5.3 : Preference indeterminacy: A most preferred *q* cannot be determined without more info.

Such values for r_c lie inside an interval [$r_{c,low}$, $r_{c,high}$] along the range of r_c in which no most preferred design $\hat{q}$ can be determined without additional information about how to trade off between competing robustnesses. This line has not been observed in other literature; it will be henceforth be called an *interval of preferredness*[19] *indeterminacy* (IPI). To put this concept into perspective, recall that before when evaluating the

[19] Our choice of the term "preferredness" may seem awkward in comparison to the term "preference", which might seem equivalent. However, "preferredness" will be used here to stay consistent with the theme that one's preference for maximum uncertainty to info-gaps does not change; however, which design is most preferred (i.e., preferredness) *could* change depending on how one would trade between competing robustnesses. Within the IPI, preferredness is indeterminate.

$\hat{\alpha}(q, r_c)$–r_c trade-off for *one* uncertainty, there was a distinct preferredness switch point (PSP, also called a design preference ranking switch earlier in Section 3.4.4), at some level of critical performance r_c, where the most preferred design switched. An IPI, on the other hand, occurs due to multiple uncertainties; the bounds of the IPI correspond to the PSPs found when considering the different uncertainties individually. This is depicted in Figure 5.4, where the cross-over points (highlighted with circles) from Figure 4.3 and Figure 4.4 are traced to the bounds of the IPI. The IPI is highlighted on the r_c axis as a bold line segment with large dots at the IPI boundaries (the IPI is shown again on the right axis just for clarity).

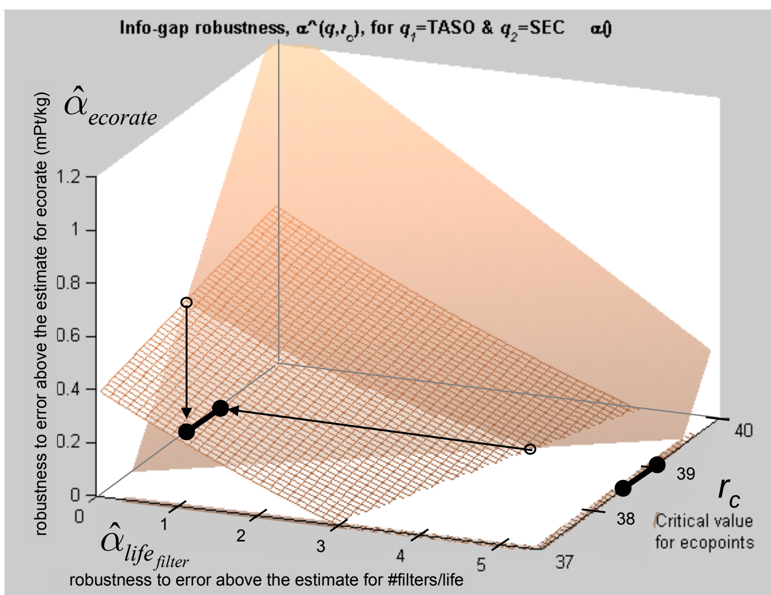

Figure 5.4 : Correspondence between IPI and PSPs from earlier single-info-gap examples.

The IPI is a *range* of r_c levels for which the most preferred design is *indeterminate* without somehow specifying a trade-off. For the filter example, values of

r_c = 39.0, 38.7, and 38.5 mPt are on the IPI's high bound, interior, and low bound, respectively, as shown in Figure 5.5, Figure 5.6, and Figure 5.7.

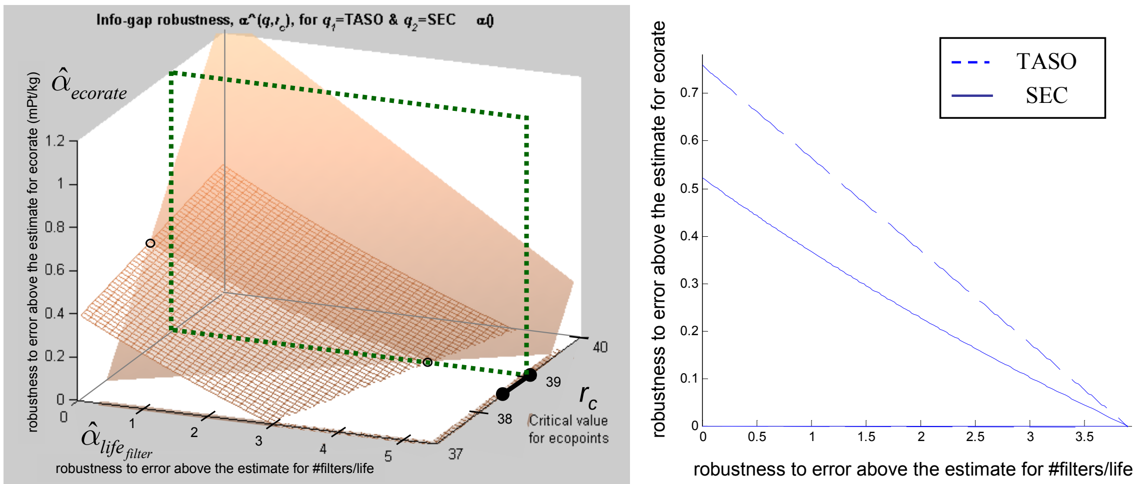

Figure 5.5 : A slice of the 3-D plot at r_c = 39 mPt, showing TASO's complete dominance.

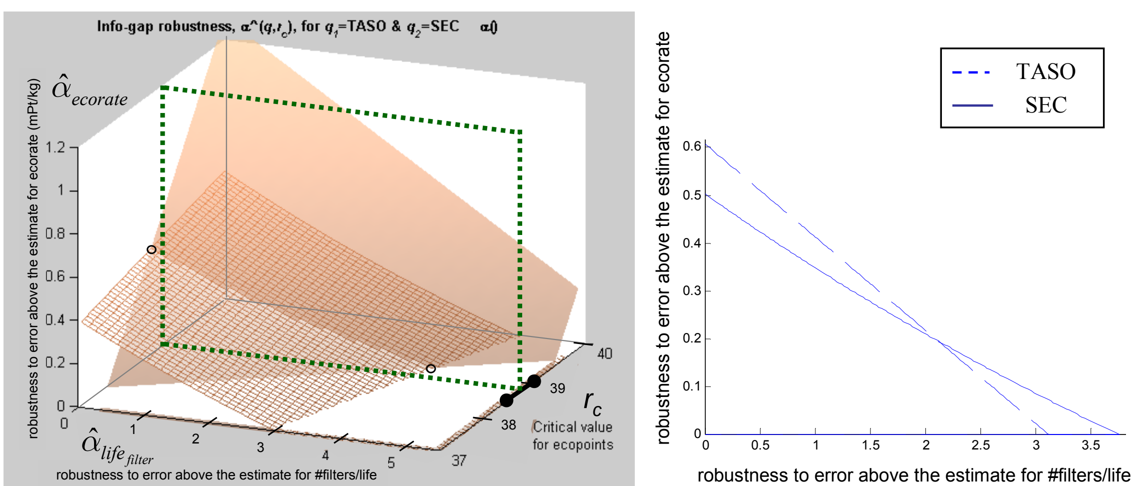

Figure 5.6 : A slice of the 3-D plot at r_c = 38.7 mPt, showing a transition in dominance.

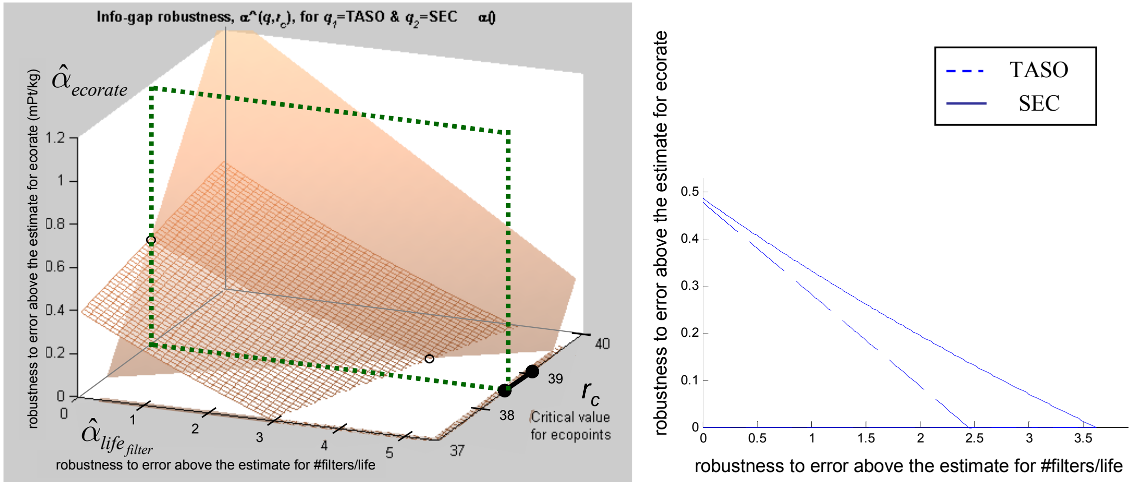

Figure 5.7 : A slice of the 3-D plot at r_c = 38.5 mPt,, showing SEC's complete dominance.

For any $r_c \leq 39$ mPt (as in Figure 5.2, for instance), TASO is the clear choice because it dominates for any trade-off between robustnesses. At 39 mPt, as in Figure 5.5, SEC has caught up partially; its robustness line intersects the TASO line on the $\hat{\alpha}_{life_{filter}}$ axis. This intersection at the axis means that the SEC option has caught up in its ability to provide robustness to error in the *filter life* estimate, but only for the trivial trade-off where there is no robustness to error in the *ecorate* estimate. The r_c = 39 mPt value at which this first intersection occurs is one end of the IPI: the value of r_c where TASO is always most preferred except for at the intersection point, where it is preferred *at least as much* as SEC.

In Figure 5.6, for r_c = 38.7 mPt, inside the IPI, determining the most preferred design $\hat{q}$ now requires the DA to trade somehow between achieving robustness to one uncertainty or the other. If the DA is unable to specify a trade-off, then a most preferred design cannot be identified.

In Figure 5.7, created by the plane where r_c = 38.5 mPt, SEC now becomes the sole choice for any trade-off between competing robustnesses. This is the low end of the IPI. Past this boundary, i.e., for critical performance more aggressive than (i.e., smaller than) 38.5 mPt, SEC will always be most preferred.

Finally, it should be noted that an IPI can be calculated for more than 2 uncertainties. Note that in Figure 5.4 the bounds of an IPI correspond to the PSPs found when considering the different info-gap uncertainties individually. For n uncertainties, the PSP created by each info-gap separately can be plotted along the r_c axis. The extreme high and low switch points along that axis bound the IPI for multiple uncertainties.

5.1.3 Categorization of Different Multi-Gap Scenarios

A conceptual guide to the different multi-gap analysis situations mentioned so far, and how they relate to the sections that follow is depicted in Figure 5.8. This decision tree also addresses the question, "How should one systematically evaluate the effects of multiple info-gaps, gradually adding information or assumptions until indeterminacy (as created by an IPI) is resolved?" One begins at the top of the tree and moves downward, gradually adding onto overall analysis of info-gap uncertainty and, if needed, eliciting and incorporating new information as necessary.

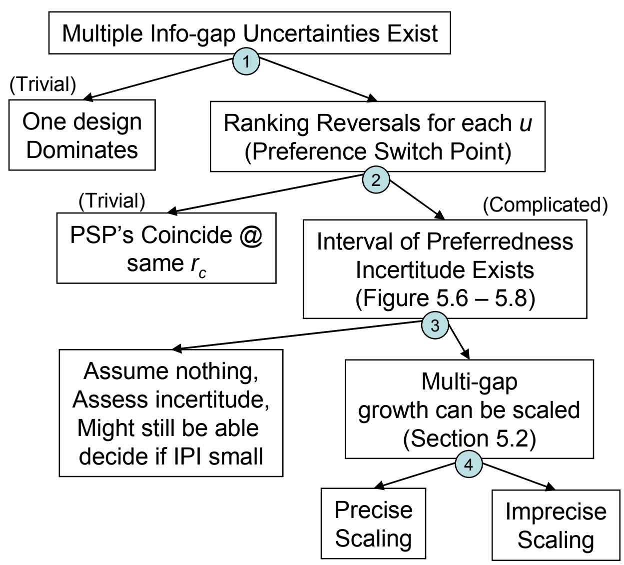

Figure 5.8 Different multi-gap assessment techniques, and their novelty

In Branch Point 1 of the Figure, one plots robustness functions for each different info-gap uncertainty separately to determine if there is a PSP for the designs separately. If there are no PSPs, multiple info-gaps exist, but one design option $\hat{q}$ always provides the most robustness to every info-gap, no matter the r_c value, and no matter the trade-off between competing robustnesses. This is a trivial case but is sometimes seen.

For Branch Point 2, if there are PSPs for each info-gap and the PSP locations do not coincide at the same value for r_c, this means that the combined effects of multiple uncertainties create an IPI. For a given r_c inside the IPI, no single design alternative always provides the most robustness to each uncertainty for all possible trade-offs between those competing robustnesses. No preferred option $\hat{q}$ can be identified unless one can specify how to trade between robustnesses.

In Branch Point 3, if the IPI is narrow enough along the range of r_c, a decision still may be possible because the range over which preferredness is indeterminate is negligible. In this case, it does not greatly matter how one trades off between competing robustnesses, thus no extra information about equivalent uncertainty scales or preferences for trade-offs need to be elicited.

In the case where the IPI is not narrow (Branch Point 4), which will be the concern of the remainder of this chapter, the IPI may span a great enough range of r_c values that a trade-off between robustnesses needs to be specified.

5.2 Multi-Gap Scaling: Concept and Benefits

This section explains and demonstrates the concept of scaling multi-gaps to a single baseline uncertainty parameter as a means of resolving indeterminacy in preference rankings. In Section 5.2.1, an existing technique for scaling is explained generally. In Section 5.2.2, two specific ways to apply the technique are reviewed and examined for weaknesses. In Section 5.2.3, a more in depth explanation is provided as to how scaling resolves the indeterminacy (IPI) identified in Section 5.1. In Section 5.2.4, the general concept of imprecise scaling and its usefulness in *partially* reducing the IPI is explained. It should be noted that, in this section, demonstration is limited to how to *apply* normalized α parameters to assessments; a discussion of how to actually *elicit* normalized α parameters is postponed until Section 5.3.

5.2.1 Scaling Factors: An Existing Technique

Some precedence for multi-gap scaling exists in the literature. In an example problem (Ben-Haim and Laufer 1998), Ben-Haim and Laufer offer "uncertainty weights" as a modeling option for defining a vector of info-gap uncertainties (i.e., multi-gaps). We will instead use the term *scaling factors* for these weights[20]. The mathematical form of a vector u of info-gap models that include scaling factors can be generalized as:

$$\mathcal{U}(\alpha,\tilde{u}) = \left\{ u : \frac{|u_n - \tilde{u}_n|}{\tilde{u}_n} \le s_n\alpha,\ n = 1,\ldots,N \right\},\ \alpha \ge 0 \tag{3}$$

or, in simpler terms:

$$\tilde{u}_n - s_n\tilde{u}_n\alpha \ \le\ u_n \ \le\ \tilde{u}_n + s_n\tilde{u}_n\alpha, \quad \alpha \ge 0 \tag{4}$$

where s_n is a unitless scaling factor that modifies the magnitude of α to be of appropriate scale for each uncertain variable u_n in the vector $\mathcal{U}$ (Ben-Haim and Laufer 1998). Although s_n is a scaling factor, it can also be thought of as an indicator or relative measurement of the decision analyst's confidence in the nominal. This confidence is a *belief*, perhaps based on prior evidence or perhaps not, that maps to α. If one s_n is small relative to others then the analyst has relatively more confidence in his/her assessment of that uncertain quantity.

[20] The term "weights" often is used in discussions about *preference* modeling, so use of that term will be avoided during the discussion of *uncertainty* or *belief* modeling. In this situation, calibration of scale seems to be a more accurate description of the function provided by Ban-Haim and Laufer's "uncertainty weights". This is purely a semantic choice.

Because α is fractional variation[21] from nominal (see Eq. (3.3)), it is a unitless percentage. The term *baseline* α will henceforth be used to refer to this α because, effectively, it parameterizes the info-gap uncertainty of multiple u_n. The baseline α can also be thought of as the "overall" level of gross uncertainty that the design alternatives could face, as was discussed before in Section 1.2.2.

"Separate" uncertainty parameters α_n can also be defined:

$$\alpha_n = s_n \alpha \, , \quad n = 1 \, , \, \dots \, , N \tag{5}$$

Like the α on which it depends, α_n is a fractional variation from nominal, so it is also a unitless percentage.

The structure of Eq. (3) creates a *mapping* between different info-gaps via the baseline α, which effectively imposes a trade-off between competing robustnesses. This mapping is depicted conceptually for the oil filter problem in Figure 5.9, where scaling factors for info-gap uncertainties in *ecorate* and *filter life* are each shown to correspond to a particular value of the baseline α.

[21] Again, "variation" refers to discrepancy between the known $\tilde{u}$ and the unknown u; it does *not* refer to the probabilistic concept of variance.

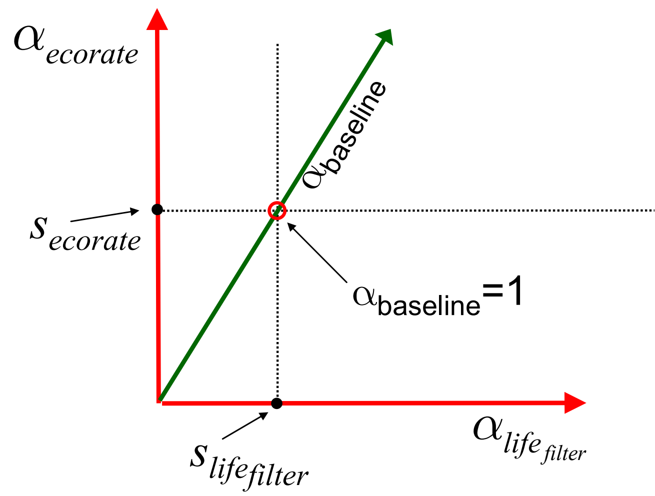

Figure 5.9: Mapping the relative scales of two info-gaps to the baseline *α* measure of gross system uncertainty.

With a scale mapping applied, the trade-off between info-gap robustnesses $\hat{\alpha}_n$ and critical reward r_c can be analyzed using the shared baseline $\hat{\alpha}$. This makes it easier to identify $\hat{\alpha} - r_c$ trade-off *trends* that would be difficult to assess when considering each robustness separately (as was done in Figure 5.1, for instance). The concept of using the shared $\hat{\alpha}$ as a proxy can be seen in Figure 5.10, which graphically depicts the results of scaling in the right half of the figure.

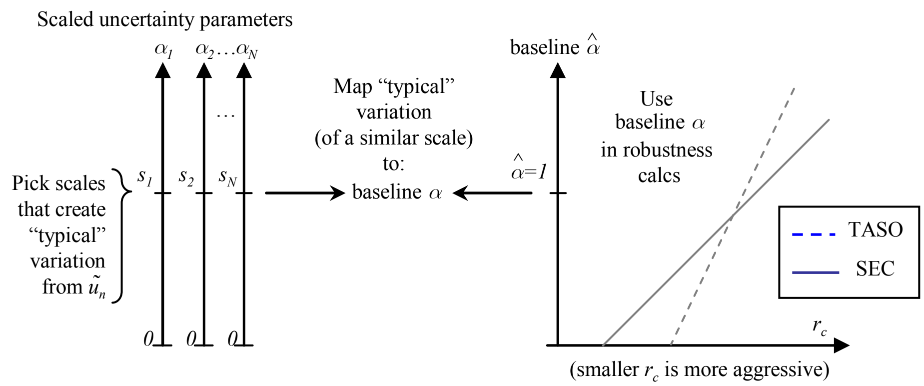

Figure 5.10 : Scaling to combine multiple α_n into a baseline α for use in assessing $\hat{\alpha} - r_c$ trade-off trends for n info-gap uncertainties.

This type of bulk assessment of the effects of all uncertainties (via mapping their scales to a single baseline α) is a heuristic, necessary because so little information about uncertainty is available. This heuristic is non-ideal but provides some insight into how immune a design is to the overall gross uncertainty that a design might face.

5.2.2 Details and Limitations of Two Ways to Use Scaling Factors

Next, two different general classes of techniques that use scaling factors are discussed: equal fractional scaling and unequal fractional scaling. Particular attention is paid to their limitations at the end of each subsection.

5.2.2.1 Equal Fractional Scaling (EFS)

In the case where fractional variation α_n is of equal scale for all u_n, (e.g., α=10% fractional variation from nominal $\tilde{u}_1$ is of equivalent scale to 10% fractional variation from nominal $\tilde{u}_2$), the scaling factor s is equal to 1 for all n. (Because s is a factor of scale, if all s_n = 1, then no scaling factor is actually necessary.) We will refer to this modeling choice as *equal fractional scaling* (EFS). In cases where the analyst has no understanding of how to scale α_n sizes to each other, one could assume that EFS reflects this ignorance; it requires no more information than the nominal $\tilde{u}_n$. Besides cases where one does not know how to scale, EFS has also been used in examples in the literature where u_n with different $\tilde{u}_n$ sizes all have a common bound, i.e., zero. This is the case in the endangered species example of (Regan 2006), where the viability of each of a set of preservation options was severely uncertain but could only drop as low as u=0.

EFS can be applied to the oil filter example discussed earlier in Section 5.1.2. First, a normalized info-gap model of the type of (3) is needed:

$$\mathcal{U}(\alpha,\tilde{u}) = \left\{ u : \frac{|u_n - \tilde{u}_n|}{\tilde{u}_n} \leq \alpha,\ n = \text{ecorate, filterlife} \right\}, \alpha \geq 0 \tag{5.6}$$

This is then utilized in the following robustness function, repeated from Eq .:

$$\hat{\alpha}_{fractional}(alt, I_{critical}) = \max \left\{ \alpha : \max_{u_n \in \mathcal{U}(\alpha,\tilde{u}_n)} I(alt,u) \leq I_{critical} \right\} \tag{5.7}$$

The formerly three-dimensional visualization of trade-offs between $\hat{\alpha}_{ecorate}$, $\hat{\alpha}_{life_{filter}}$, and r_c (as visualized in Figure 5.5 through Figure 5.7) is condensed to two dimensions: between $\hat{\alpha}_{fractional}$ and r_c. Also, the IPI seen before in Figure 5.5 is reduced to a switch point at r_c = 46.5 mPt. This is depicted in Figure 5.11.

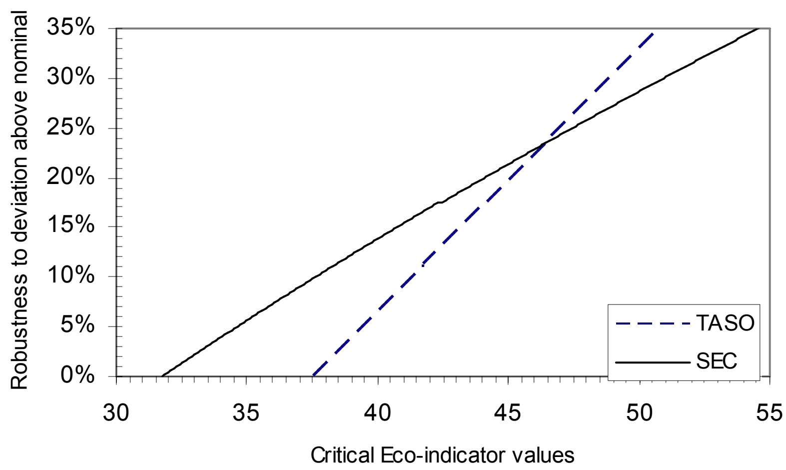

Figure 5.11 : An $\hat{\alpha} - r_c$ trade-off plot created by Equal Fractional Scaling (EFS)

While the EFS assumption makes trading off convenient, non-equal scaling is probably inappropriate for the oil filter example given that the uncertain quantities involved are of heterogeneous type.

5.2.2.2 Unequal Fractional Scaling (UFS)

Unequal fractional scaling (UFS) is the case where one or more s_n equals something other than 1, requiring that all s_n be specified. Thus, UFS requires more information that EFS. This technique has been applied by Ben-Haim in a project scheduling example problem (Section 3.2.6 of (Ben-Haim 2006)). In the example, scaling factors are chosen from “rough information about the relative variability of the different tasks” that comprise the schedule. No further explanation of elicitation is provided.

Using UFS, one could decide to use the shared baseline α parameter to assess $\hat{\alpha}(q, r_c)$–r_c trade-offs in two dimensions in the same way that was explained for EFS. However, with UFS, because the scaling factors s_n is embedded within the info-gap model definition of Eq. (3), the baseline α is only a *proxy* for the actual variations α_n. The decision analyst must be sure to understand the equivalence of the "size" of this baseline. One technique for calibrating this "size" is to set α=1 as a "typical" variation. If a more direct understanding of the $\hat{\alpha}$ vs. r_c trade-offs are needed, the analyst may wish to convert a particular proxy $\hat{\alpha}$ size to one or more of the equivalent $\hat{\alpha}_n$ and compare those to r_c. These concerns are better illustrated by the example problems of Section 5.3.3.2. Like EFS, UFS collapses the visualization of $\hat{\alpha}$ vs. r_c trade-offs into a two dimensional plot, as can be seen in Figure 5.25.

5.2.3 How Scaling Factors Specify Trade-offs Between Competing Robustnesses

When applied to info-gap models, scaling factors map a trade-off between competing robustnesses and, in turn, collapse the IPI presented previously in Section 5.1.2 into a PSP. This can be visualized on "competing robustness" plots first shown in Figure 5.3, used to introduce the IPI. Given scaling factors, a *trade-off specification line*[22] (TS line) can be plotted that maps multi-gap scale equivalence for different values of the baseline α. This line is parameterized by the baseline α, as depicted by the points in the line in Figure 5.12. The distance along the TS line corresponds to the size of the traded-off robustnesses; the slope of the line depends on the values of the scaling factors.

When a trade-off between robustnesses is specified, a most preferred design ($\hat{q}$) can be determined, also depicted in Figure 5.12. As explained before in Section 5.1.2, a design q on a competing robustness plot dominates other designs when its robustness trade-off line is furthest away from the plot origin. To update this idea, the design whose

[22] This trade-off specification line should not be confused with the robustness trade-off line introduced earlier, which reflects the set of all possible trade-offs between competing robustnesses. The TS line will be shown as a red dot-dashed line.

robustness trade-off line crosses the TS line furthest away form the origin offers the most robustness to each info-gap for the scaling factors (i.e., robustness trade-off) specified.

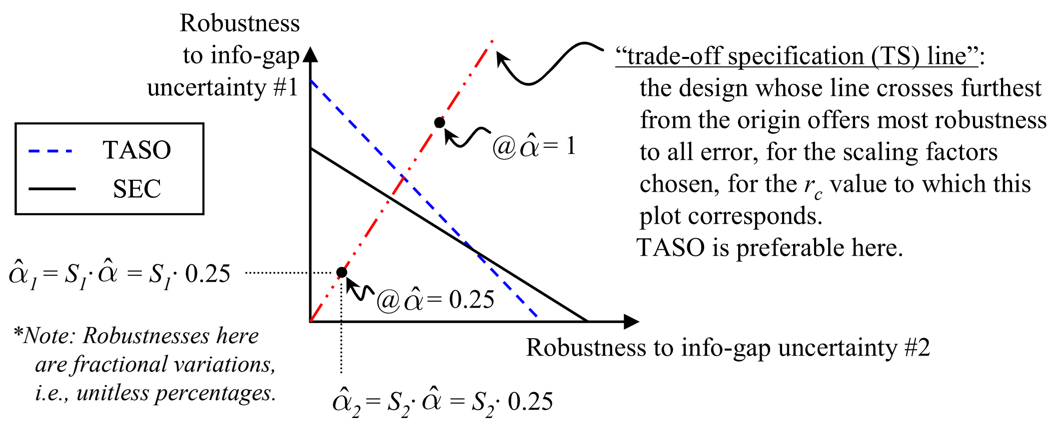

Figure 5.12 : Plot of competing robustnesses at some r_c, showing dominance along a "TS line".

When the relative sizes of robustnesses are scaled, a PSP can be found on the r_c axis. This is illustrated in Figure 5.13 for the previous oil filter example. The competing robustness plot that corresponds to that PSP will show the TS line passing through the intersection of the robustness trade-off lines of the designs q. This is shown in the right half of Figure 5.13.

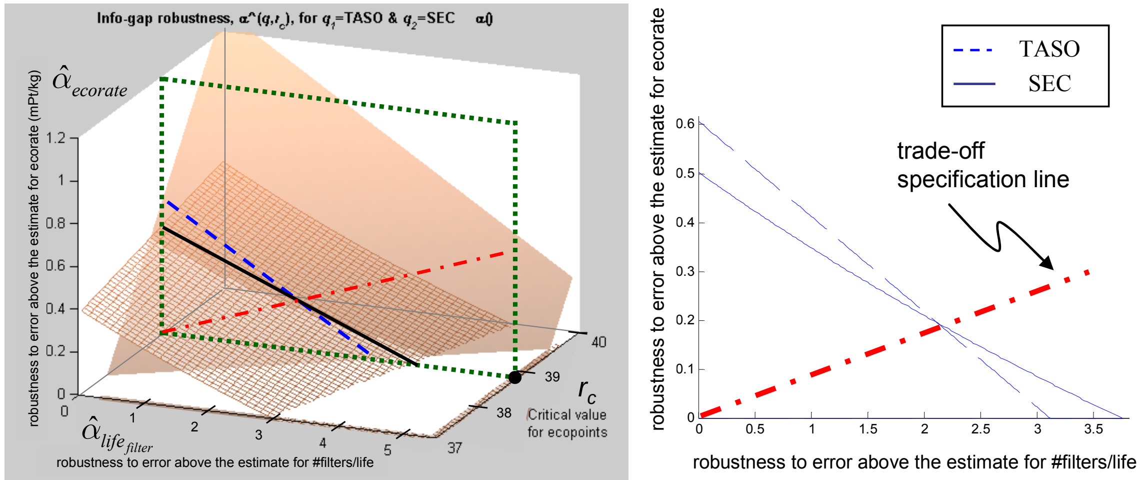

Figure 5.13 : A slice of the 3-D plot at r_c = 38.7 mPt, showing a switch in dominance.

In the oil filter example, the PSP is found (calculated) to be located at r_c = 38.7 mPt, within the former IPI of Figure 5.5. For the robustness scaling specified, any other value for r_c besides the PSP will show one design dominating, looking similar to the scenario in Figure 5.12.

The TS line has been introduced in this section strictly for the purposes of illustrating the benefits of scaling and reinforcing the concepts with respect to the three dimensional plots introduced in Section 5.1.2. In the example in Section 5.4, plotting this line is not necessary to assess trade-offs. Rather, the preference ranking indeterminacy (IPI) or PSP that it generates along the r_c axis is of greater interest. The TS line concept is, however, used further in the section that follows to help explain the concept of imprecise scaling.

5.2.4 Imprecise Scaling

The preceding discussions have progressed under the assumption that scaling factors can always be determined precisely; however, this is not always the case. In

Section 5.3.2, an elicitation method will be introduced that allows for imprecision. In anticipation of this, we propose that scaling factors can in sometimes only be known to lie within a *interval of scaling imprecision*, or ISI. Generally, for N info-gap uncertainties, an ISI assumes the mathematical form:

$$s_n \in [\, s_{n,low}, s_{n,high} \,], \quad n = 1 \,, \ldots, N \tag{8}$$

Consider the case with two info-gap uncertainties parameterized respectively by α_1 and α_2, which per Eq. (5) are scaled to a baseline α by scaling factors s_1 and s_2. If the scaling factors are imprecise: $s_1 \in [\, s_{1,low}, s_{1,high} \,]$ and $s_2 \in [\, s_{2,low}, s_{2,high} \,]$. Any scaling value contained within the first interval could—consistent with the analyst's limited understanding of scaling—be of equivalent scale to any value contained in the second interval.

5.2.4.1 Illustrating the Imprecise Scaling Concept for *N*=2 Info-Gap Uncertainties

The effects of this imprecision can be explained graphically for N=2 info-gaps using competing robustness trade-off plots. As shown in Figure 5.14, an *imprecise trade-off specification sector* (ITS sector) is formed that contains all TS lines consistent with the set of all combinations of elements from intervals s_1 and s_2. The ITS sector is bounded by the TS lines generated when the high and low scaling factors for different α_n are combined. These are called *extreme pairings* of imprecise scaling factor bounds. For example, for N=2 the pairing of $s_{1,low}$ with $s_{2,high}$ generates a TS line that is one border of the sector, whereas pairing $s_{1,high}$ with $s_{2,low}$ creates a border at the opposite extreme, as shown in Figure 5.14.

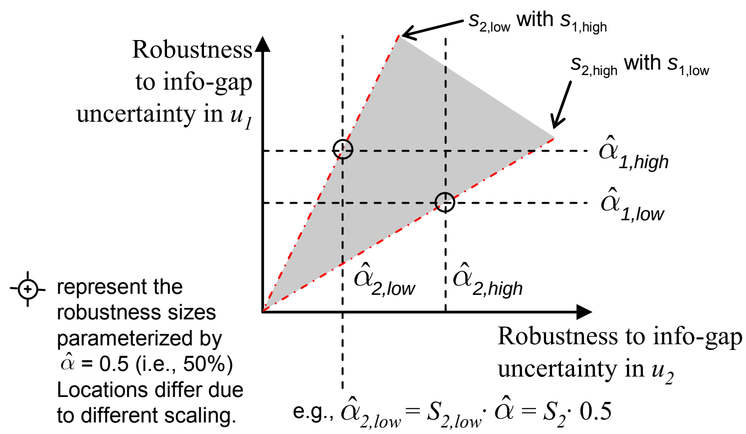

Figure 5.14 : ITS sector, bounded by TS lines formed by scaling factors from opposite extremes.

The imprecision introduced by the ITS sector creates a corresponding interval of preferredness indeterminacy (IPI) along the range of r_c. For instance, if the TS line from Figure 5.13 were treated as the lower (i.e., most clockwise) boundary of an ITS sector, the corresponding switch point on the r_c axis (previously shown in the left side of Figure 5.13) now becomes the lower bound of an IPI. Figure 5.13 is updated in Figure 5.15 to reflect this concept. The upper (i.e., most counterclockwise) boundary of the ITS sector corresponds to the other IPI bound, also shown on the r_c axis in Figure 5.15.

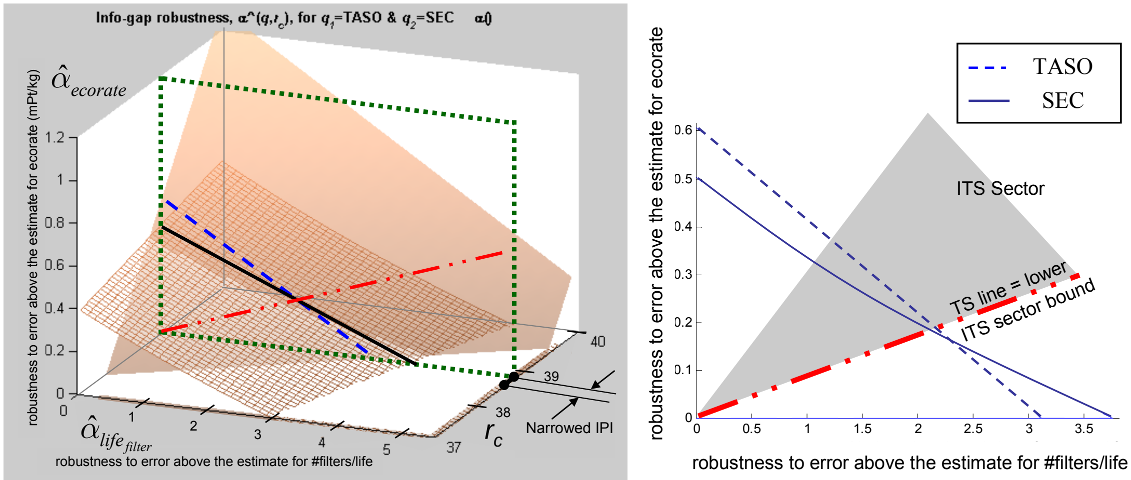

Figure 5.15 : ITS sector, and the relationship between one of its bounds to the bounds of an IPI.

Note that the IPI created by scaling imprecision is narrower than the IPI that resulted from no normalization, such as that of Figure 5.5 through Figure 5.7. This means that if one can narrow, to some extent, the range of imprecision to which different α_n can be scaled (and over which robustnesses could be traded), then the IPI will be narrowed accordingly. This also means that not specifying *any* trade-off scaling at all is the same as maximum imprecision.

Table 5.1 summarizes the different degrees of precision with which multi-gap scaling can be applied. The columns of the table include the types of information that could be available to for use in specifying a trade-off between competing $\hat{\alpha}_n$, how that is reflected as values for s_n, the graphical interpretation of that, and the effects on IPI size. The different graphical interpretations are also summarized conceptually in Figure 5.16, with the numbered graphical elements corresponding to the numbered rows in Table 5.1.

Table 5.1: Different levels of scaling info and their implications

Info available about scaling (and, accordingly, robustness trade-offs)	**Scaling factors s_n**	**TS line location on a competing robustness plot**	**Decision's dependency on r_c level?**
1. None available (or none used)	Unknown	Could be anywhere in quadrant	Widest interval of preferredness indeterminacy (IPI)
2. Imprecision in scaling narrowed to some degree	Known within $[s_{n,low}, s_{n,high}]$	Inside an ITS sector	Narrowed IPI
3. Precise scaling; constant with growth of baseline α	Known precisely	Static TS line with constant slope	Certitude; IPI collapses into PSP

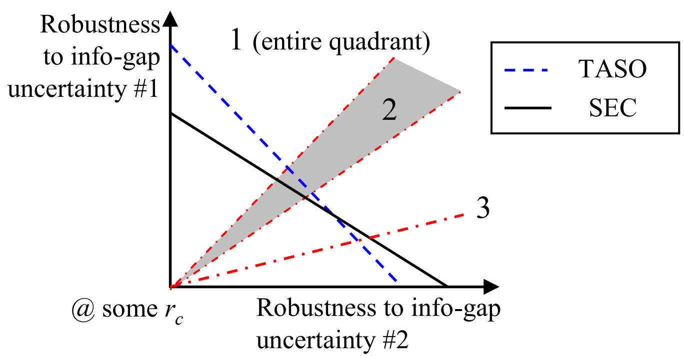

Figure 5.16 : Graphical depiction of different normalization options.

The preceding graphical explanations are meant to elucidate the concepts involved; however, in practice, a different approach is used to perform a multi-gap analysis affected by imprecise scaling. Robustness-performance ($\hat{\alpha}(q, r_c)$–r_c) trade-off curves should be generated using each extreme pairing of imprecise scaling factor bounds. For example, one robustness curve would use $s_{1,low}$ with $s_{2,high}$; a second curve would use $s_{1,high}$ with $s_{2,low}$. The curves generated at the extremes of pairings can then be superimposed to view how scaling imprecision creates indeterminacy in preference rankings. An example of superimposing robustness curves is presented in Section 5.4.4, along with a suggested heuristic for making a decision given the resulting indeterminacy.

5.2.4.2 Imprecise Scaling Concept for *N*>2 Info-Gap Uncertainties

For *N*>2 number of info-gap uncertainties, the additional extreme pairings of imprecise scaling factor bounds ($s_{n,low}$, $s_{n,high}$) must also be considered. This creates $2^{(N\text{-}1)}$ unique extreme pairings. For instance, for N=3: the four pairings are:

$s_{1,high}$ with $s_{2,low}$ with $s_{3,high}$
$s_{1,low}$ with $s_{2,high}$ with $s_{2,high}$
$s_{1,high}$ with $s_{2,low}$ with $s_{3,low}$
$s_{1,low}$ with $s_{2,high}$ with $s_{2,low}$

To display the extremes in indeterminacy that result, one would need to plot $2^{(N\text{-}1)}$ robustness curves for each design alternative in a selection problem. Clearly, this becomes difficult to manage quite quickly. More about the complexities and limitations of problems with both *N*>2 info-gaps *and* imprecise scaling factors are discussed in Section 5.4.5, after an example with *N*=2 info-gaps is presented in Section 5.4.4.

5.3 Bet-Based Scale Elicitation (BBSE)

Now that the meaning, usage, and benefits of multi-gap scaling have been introduced, an explanation about how to elicit scaling factors s_n is warranted. A novel *bet-based scale elicitation* (BBSE) technique is next presented to improve scaling factor elicitation, which was found to be lacking in rigor in pervious sections. BBSE is built upon a technique for eliciting subjective probabilities; however, the numbers elicited are used to establish scaling factors rather than probabilities. This section presents the betting foundations and assembles the steps for scaling.

The overall BBSE strategy can be broken down into two main steps:

- Step 1: eliciting scaling factors, and
- Step 2: mapping to them to the baseline α.

More specifically, the scales of each info-gap are calibrated to a comparable reference for belief (betting); then, that calibration reference is mapped to a baseline α, making the scaling factors usable in Eq. (3). Then, the analyst proceeds with unequal fractional scaling (Section 5.2.2.2) in its usual procedure. The two main steps are elaborated next, starting with a discussion of betting, which is the foundation of Step 1.

5.3.1 Foundation: Using Betting Behavior to Reveal Belief about Uncertainty

The BBSE approach leverages a technique for eliciting subjective probabilities. A recent review of the subjective interpretation of probability is provided in Section 3.3.3.3 of (Aughenbaugh 2006), which is paraphrased as follows. The fundamental principle is that *a person's belief about probability is reflected by (and revealed through) their betting behavior*. This belief is *based* on some combination of knowledge, assumptions, preferences, and even biases, and can be easily *expressed* through the action taken when a bet is presented. It is argued that subjective probabilities are appropriate in cases where there are few or no sample data points available to quantify uncertainty, or when the uncertain event is one that simply does not repeat.

A definition of belief that relates betting to subjective probabilities was conceived by (de Finetti 1980), discussed in (Hajek 2003), and clarified in (Aughenbaugh 2006). A simpler version of this definition, appropriate for the purposes of this chapter, can be stated as such:

> An individual's degree of belief in outcome *X* is *p* if and only if *p* units of currency ($0<p<1$) is the price, known as a *fair price*, at which he or she is indifferent between buying or selling a bet that pays:
>
> - 1 unit of currency if outcome *X* occurs, and
> - 0 units of currency if *X* does not occur.
>
> Assuming that the person is risk neutral (i.e., is neither more averse nor more attracted to risk as the stakes change) and that there are no endowment effects in this problem, *p* then represents the individual's probability that outcome *X* will occur.

Under these conditions, the probability, *P(X)*, of the event *X* occurring, where $0 \leq P(X) \leq 1$, is given directly by the fair price the individual would be willing to pay to enter the bet. For example, purchasing a bet for \$0.10—not a large wager—implies that the individual does not find the likelihood of the event (and, by extension, winning a bet that it will occur) to be very probable. This betting behavior reveals a belief in a probability $P(X)=0.10$ that the event will occur.

The theoretical strengths of the subjective interpretation of probability are argued by (Aughenbaugh 2006), using criteria for rigor developed by (Walley 1991). Two key aspects exist that imply rigor. First, the subjective belief is *operationalized*, i.e., observable through the betting activity. Rigor in elicitation is achieved because the concept of the uncertainty measure is synonymous with a corresponding set of operations, eliminating ambiguity as to what is actually being measured. Second, the subjective interpretation is *behavioral* because it has implications concerning the individual's behavior in actual decision making, i.e., agreeing or not to the terms of a bet.

The subjective interpretation also has its limitations, which Aughenbaugh also reviews (Aughenbaugh 2006). First, betting behavior depends not just on probabilities but also preferences, which may be influenced by the individual's current state of wealth. This problem motivates the use of relatively small bet prices and payouts between \$0 and \$1. Lastly, people have been observed to be inherently limited in their abilities to assess probabilities, whatever the means used (Tversky and Kahneman 1974, Kahneman et al. 1982). Overall, we contend that these limitations affect *any* form of elicitation that one would turn to when statistical sampling is not an option. Therefore, the rigor added by bet-based elicitation makes it better than any other technique in situations of sparse information.

5.3.2 Bet-Based Scaling Elicitation (BBSE): A Novel Premise

BBSE is devised to take advantage of the premise that betting at a certain bet price p corresponds to a level of belief p about uncertainty[23]. BBSE uses the value p not as a probability but rather as a reference point for size; α_n scale will be mapped to α based on p.

There are two participants in BBSE: the *Subject*, whose beliefs about scaling are being elicited, and the *Elicitor*, who is the questioner and whose job may be automated. The object in BBSE is to elicit a value $\alpha_n = a_n$ such that the Subject would agree to buy the bet at price p; that is, a_n makes p become a *fair* price. Thus, a_n is a scaling reference.

[23] Henceforth, when p or the bet price is discussed, it will be implied that these are the same as a level of probability p consistent with price p.

(As indicated by Figure 5.17, a_n is the distance above/below $\tilde{u}_n$ per the usual info-gap uncertainty model structure, see: Section 3.4.1.) This is achieved by the general bet structure outlined in Figure 5.17. The Subject's choice of a_n will depend on the win/loss stakes involved as well as his or her own understanding of uncertainty.

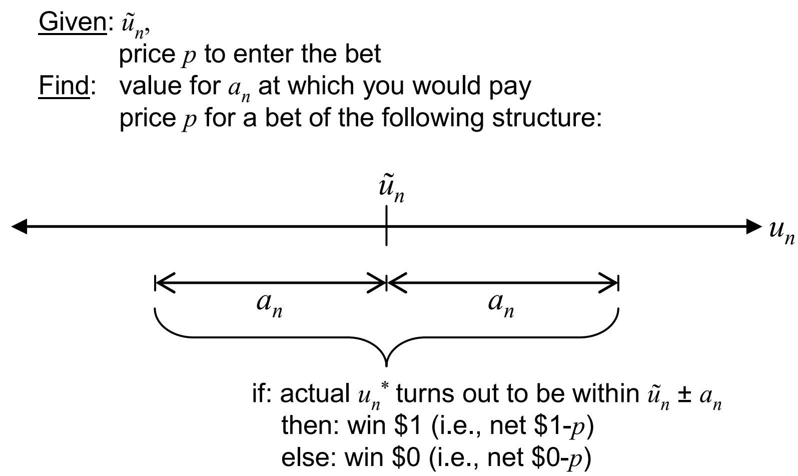

Figure 5.17 : Bet structure used in BBSE

For example, if a Subject is presented with $\tilde{u}_1$ = 17 and bet price of p = \$0.90, suppose a value of α_1 = 5.5 is elicited, as is depicted in Figure 5.18. Based on the subjective probability interpretation upon which BBSE is founded, the Subject believes that there is a 90% that the actual value of u_1 will fall between 11.5 and 22.5.

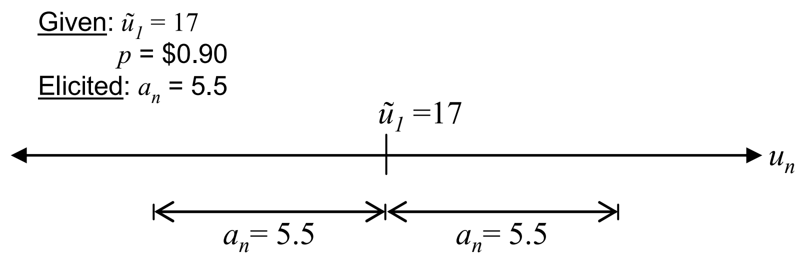

Figure 5.18 : Outcome of an elicitation scenario

However, rather than use this information as a probability in the traditional manner, BBSE merely employs it as a reference "size": α_1=5.5 is now scaled to the value p=\$0.90 and, accordingly, a "size" of p=0.90. For each of the n info-gaps in a given multi-gap problem, there is a value for a_n that has a "size" p=0.90. From this idea, the main assumption connecting betting to scaling can be established:

Any a_n that makes p a fair price will be of comparable scale to any other a_n that does the same.

An expanded explanation of how this assumption relates to scale mapping is provided later in Section 5.3.3.2. (Later in Section 5.3.3.2 it is shown that the an gotten from p is mapped to a value of baseline; this allows calculation of the factors that were originally discuss, e.g., in Eq. (3). Skipping ahead to Figure 5.21 shows the "big picture" of the overall scaling scheme.)

Compared to Ben-Haim's method of multi-gap scaling, which estimates "uncertainty weights" in an ad hoc fashion, BBSE offers improved elicitation rigor. This rigor is entirely based on the betting structure that BBSE borrows from the subjective interpretation of probability introduced in Section 5.3.1. In BBSE terms, the Subject's belief in the size of different values for α_n is observable through the operationalized betting activity. This size, which is actually a subjective probability, is then used to establish scale. Also, BBSE is behavioral because it corresponds to an individual's behavior in actual decision making, i.e., agreeing or not to the terms of a bet. Whether these qualities imbues BBSE with *enough* rigor to be *completely* dependable could be questionable. To this point we simply reply that *any* elicitation rigor—as long as it is valid—is preferable to no rigor. We operate under the assumption that the rigor that Walley has applied to subjective probability elicitation is theoretically sound (Walley 1991).

Imprecision, in the context of betting, is the last concept to explain before the BBSE steps can be elaborated on. In some cases, the Subject's knowledge about info-gap scales may be limited in a way that makes a_n imprecise. A precise a_n is simply a single quantity, one at which p is a fair price to purchase the bet. If a_n is imprecise, it lies within an interval that will be referred to as the *interval of bet imprecision*, or IBI. For a_n values within the bounds of the IBI, the Subject cannot decide whether or not to take the bet at price p. Above the upper bound of the IBI, the Subject knows they *would not* buy the bet. Below the lower bound they know they *would* buy it. This scenario is depicted graphically in Figure 5.19. Imprecision in betting corresponds to the idea of upper and lower previsions for subjective probabilities as explained by (Walley 1991).

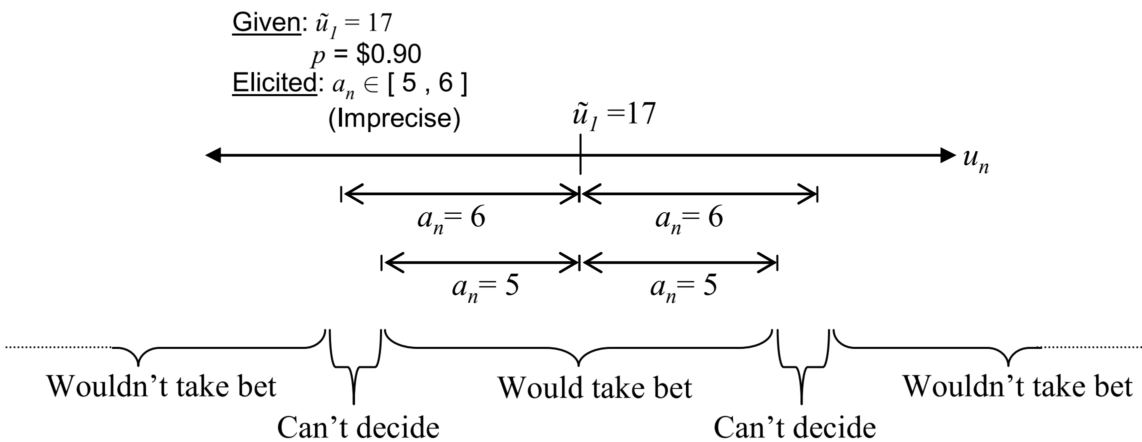

Figure 5.19 : Betting behavior when elicitation is imprecise.

When an IBI exists, the conversion from a_n to s_n in BBSE Step 2 creates the interval of scaling imprecision (ISI) introduced earlier in Section 5.2.4.

5.3.3 Bet-Based Scaling Elicitation: Steps

The three main steps introduced briefly at the beginning of the BBSE section are next presented in further detail.

5.3.3.1 <u>Step 1: Elicitation</u>

The general bet-purchasing scenario is presented to the Subject, per Figure 5.17:

- Parameters for the bet:
    - The nominal estimate, $\tilde{u}_n$
    - A price p to buy the bet, somewhere between \$0 and \$1. It is suggested that p be set at \$0.90.
- The Subject is told that a value a_n is sought such that:
    - If the actual u_n (unknown) turns out to be within $\tilde{u}_n \pm a_n$, the bet pays \$1.
    - If outside these bounds, the bet pays \$0.

Depending on what form is more understandable by the subject, the value for a_n to be elicited may be either fractional (a unitless percentage of the nominal) or non-normalized (having units). Elicitation may be done either *directly* or *indirectly*, which are described next as options A and B:

1A) Direct elicitation:

- The Subject is forced to buy a bet at price p. They must outright pick the a_n which makes p a fair price. They may determine that a_n is either precise or that it lies within an IBI with bounds defined by the Subject.

—or—

1B) Indirect elicitation: (basically, a bracketing method)

- The Subject is asked to respond "Yes" or "No" to whether they would accept a bet having price p and some a_n value assigned by the Elicitor. This bet offering is then repeated, each time featuring a new a_n value that is strategically chosen by the Elicitor to narrow the bounds on the IBI. This strategy is depicted in Figure 5.20, which is labeled to correspond to the following steps:

  I) Commonly, the Elicitor begins by asking whether the Subject would buy a bet where $\alpha_n = a_n = 0$. To this the Subject will usually reply "No". (A reply of "Yes" would suggest that the Subject is completely sure that the nominal estimate is correct, which will most often not be the case for a variable with severe uncertainty.) This becomes the first lower bound on the a_n IBI.

  II) The Elicitor picks some large value for a_n to try to find an upper bound for the IBI. The new bet is presented to the Subject. If the Subject replies "No", then they believe that a_n lies outside of the current guess, and the interval is not large enough. In response to a "No" here, a new bet with a larger value for a_n should be offered until the Subject provides a "Yes" response. In the case of Figure 5.20, Bet B2 receives a "Yes" response and the Elicitor can continue.

III) At this point the Elicitor knows extreme upper and lower bounds within which a_n is known to lie and can thus start halving the IBI to try to narrow these bounds. This is done by picking a point near the middle of the current IBI and using it as the a_n in the next round of bet offering. If the Subject's response to any bet offering is ever "No", then the next bet's a_n must be greater than the a_n of the current bet. Conversely, if the response is ever "Yes", then the next bet's a_n must be less than the a_n of the current bet. As this step is repeated, the bounds of the IBI are narrowed in the fashion depicted in Figure 5.20, as the Subject's answers for Bets B3-B6 are "Yes", "No", "Yes", and "No", respectively.

IV) The series of bet offerings stops when one of the following stopping condition is met:

- The Subject is unable to reply "Yes" or "No" to bets because their sense of scale is not precise enough.

End) After betting stops, the Subject can choose whether to use the last two elicited bounds to define an IBI for a_n, or else specify a precise a_n using the midpoint between those bounds.

BBSE Step 1 is repeated for all u_n (i.e. $n = 1, \ldots, N$), using the same p, so that all a_n are elicited. If not expressed as percentages (i.e., as fractional variation from nominal), each a_n will need to be normalized by their respective $\tilde{u}_n$. This makes them usable for scaling per Eqs. (3) and (4).

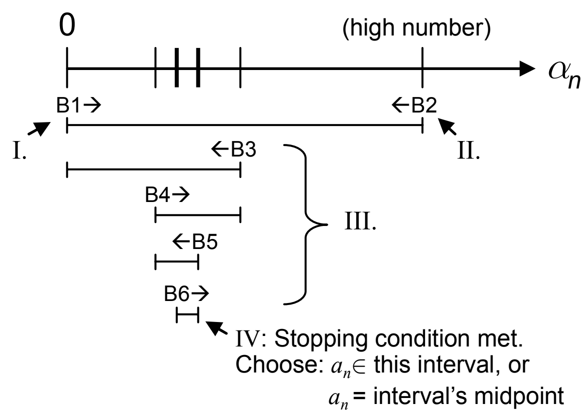

Figure 5.20 : Indirect elicitation offers progressive bets that gradually narrow the IBI

As should be apparent, indirect elicitation (option B) spreads elicitation over a greater number of questions, placing lower information demands (i.e., answers in "Yes" or "No" form rather than a quantity) on the Subject for each of those questions. This option is better for Subjects who have less knowledge about the uncertain quantities involved, or who are less confident in their ability to quantify a_n, especially when their knowledge is very imprecise. More discussion about the appropriateness of direct or indirect scaling elicitation will be provided in the discussion section of this chapter, as well as after they are applied to the example problems featured in this thesis.

5.3.3.2 Step 2: Calibrating p to some value of the baseline α

Per Step 2 of BBSE, the Decision Analyst must associate or "map" p to some specific value of the baseline α. This calibration choice is left to the Decision Analyst's discretion, but it is recommended that α=1 (where is α defined fractionally) be considered the "typical" horizon of uncertainty, and that the p used in betting be associated with α=1, as is depicted in Figure 5.21. It is important to keep in mind that scaling between the bet price p and the baseline α are *only* known for that calibration point. Any other fraction (or multiple) of the baseline α (e.g., 0.5·α) is *not* necessarily calibrated to the same fraction (or multiple) of the bet price (e.g., 0.5·p), as denoted by "No!" in Figure 5.21. This is because calibration is not guaranteed to scale linearly between α and p; all that is known is that they are equal at the assigned calibration point.

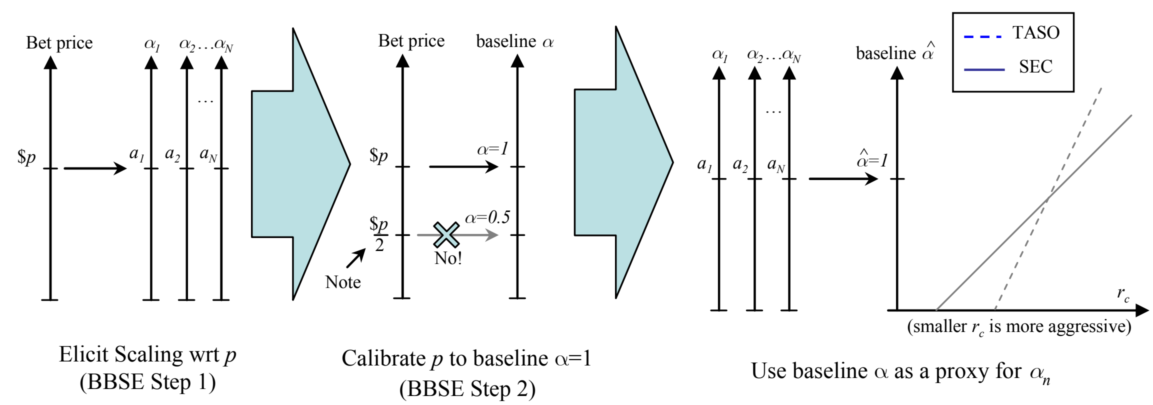

Figure 5.21 : Calibrating *p* to the baseline α

Once a baseline α has been matched to the value of p used in betting, Eq. (5) can be used to derive scaling factors s_n from the a_n. Consistent with the structure of the scaled info-gap model in Eq. (3), each s_n is constant for all values of the baseline α. But, up to this point in the chapter, scaling elicitation has only involved one value of *p*, and thus only a single point to which multi-gap scales are calibrated. How can one assure constant scaling for all sizes that the uncertainty parameters α_n could take? The solution is to use the following assumption:

> The value of *p* corresponds to a point on a *belief distribution*[24] for each info-gap, as illustrated in Figure 5.22. If every info-gap's belief distribution is assumed to be of the same shape, then only one value of *p* needs to be used in the elicitation of a_n.

Using this assumption in conjunction with knowledge that every distribution is centered around $\tilde{u}_n$ (as enforced by the symmetry built into the definition of info-gap models), one can be sure that scaling of "size" is constant for all values of the baseline α.

Note that, although the S-shape of the CDF in Figure 5.22 suggests a normal distribution, the actual shape is not known. Not knowing the distribution shape is not a problem for multi-gap scaling, as long as it is still acceptable to assume that every info-gap uncertainty's belief distribution has the same shape.

[24] To reiterate, this distribution is only used for the purposes scaling, not for use as an actual subjective probability distribution.

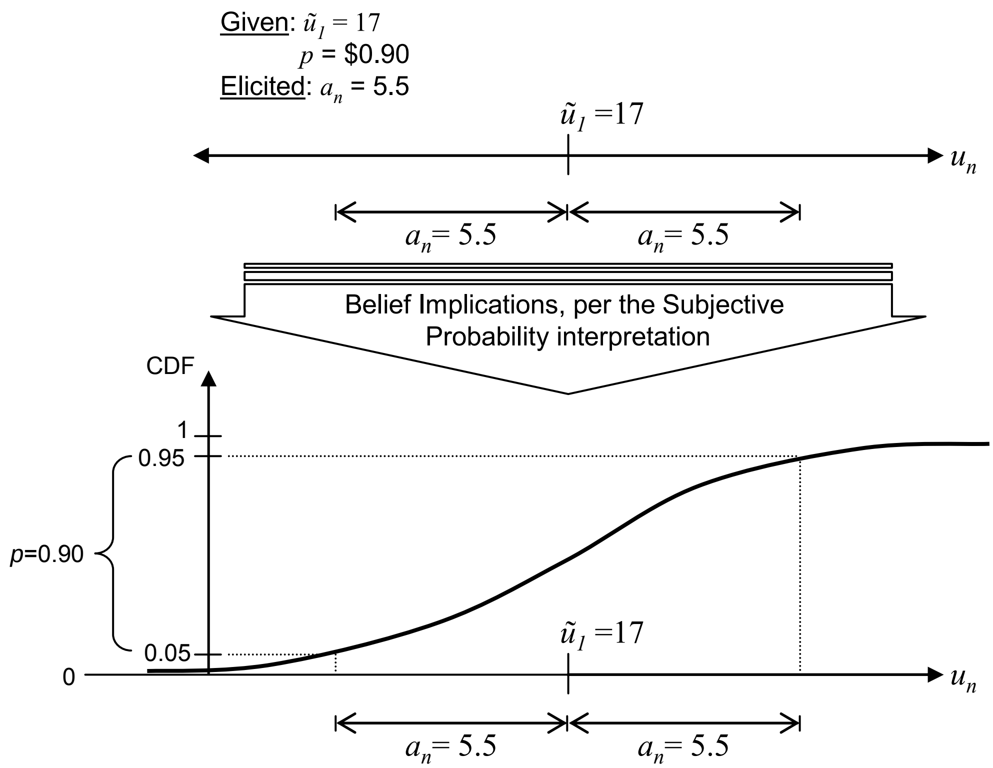

Figure 5.22 : Subjective belief distribution implied by the Subject's response to betting.

Whatever the calibration choice, both the bet level p and the value of the baseline α value to which it corresponds should be made transparent to the Decision Maker. Converting the baseline α axis into the equivalent units of one or more α_n provides this transparency. This assures that, when assessing the $\hat{\alpha} - r_c$ trade-off, the decision maker understands the actual magnitude of the baseline α as it serves its proxy role. If the decision maker possesses this understanding, then the final decision analysis should not be significantly sensitive to the choice of bet level p nor corresponding baseline α, both of which ideally should be no more than artifacts of the BBSE scaling technique.

5.4 Applying BBSE to the Oil Filter Example Problem

The BBSE steps can be assembled and applied to the oil filter example problem. Recall that, up to this point, $\hat{\alpha} - r_c$ trade-off plots have been generated for the case of: a single info-gap uncertainty (Figure 4.3, Section 4.3.1); the separate effects of a second info-gap uncertainty (Figure 4.4, Section 4.3.2); the combined effects of two info-gaps, with visualization in three dimensions (Figure 5.5 through Figure 5.7, Section 5.1.2); and the combined effects of two info-gaps, with Equal Fractional Scaling (EFS) applied (Figure 5.11, Section 5.2.2.1). In this section, a multi-gap assessment with BBSE is provided starting with problem formulation and ending with a 2-D $\hat{\alpha} - r_c$ trade-off plot of the combined effects of two info-gaps, with Precise BBSE applied. To orient the reader, the map of multi-gap assessment options is presented again, this time noting the branches to which each of the following subsections correspond.

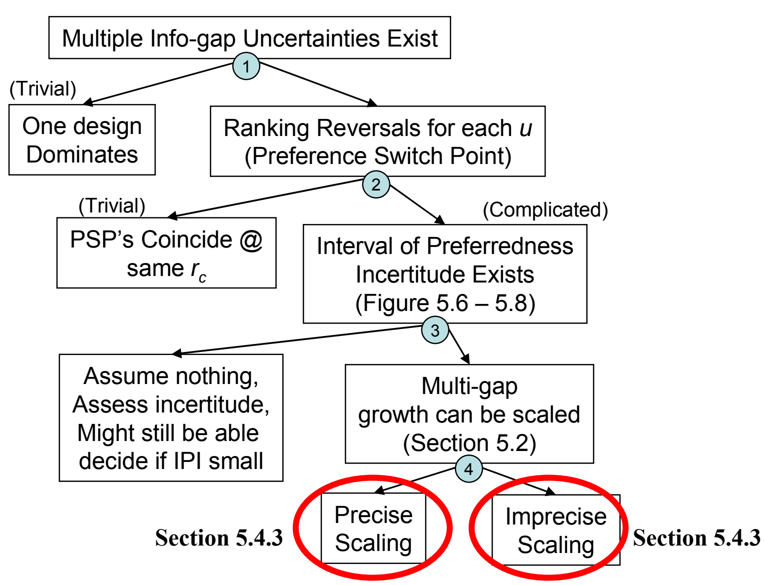

Figure 5.23 Different multi-gap assessment techniques, and their novelty

5.4.1 Problem Formulation

EFS can be applied to the oil filter example discussed earlier in Section 5.1.2. First, a normalized info-gap model of the type of (3) is needed, *this time* with scaling factors added in:

$$\mathcal{U}(\alpha,\tilde{u}) = \left\{ u : \frac{|u_n - \tilde{u}_n|}{\tilde{u}_n} \le s_n\alpha,\ n = \text{ecorate, filterlife} \right\}, \alpha \ge 0 \tag{5.9}$$

This is combined with the performance functions from Chapter 3 (Eqs. (4.2) and (4.3)), repeated here for convenience:

$$\begin{aligned} I_{TASO} &= I_{casing} + \left(I_{cartridge} \cdot F \right) \\ &= (m_{cs,T} \cdot e_{cs,T}) + (m_{cr,T} \cdot e_{cr,T} \cdot F) \end{aligned} \tag{5.10}$$

$$\begin{aligned} I_{SEC} &= \left(I_{casing} + I_{cartridge} \right) \cdot F \\ &= (m_{cs,S} \cdot e_{cs,S} + m_{cr,S} \cdot e_{cr,S}) \cdot F \end{aligned} \tag{5.11}$$

The uncertain variables F and e_{cs} in Eqs. (5.10) and (5.11) are replaced with the info-gap models of Eq. (5.9). This is then utilized to generate the following robustness function, repeated from Eq . (4.9):

$$\hat{\alpha}_{fractional}(alt, I_{critical}) = \max \left\{ \alpha : \max_{u_n \in \mathcal{U}(\alpha,\tilde{u}_n)} I(alt,u) \le I_{critical} \right\} \tag{5.12}$$

But first, the scaling factors s_{filter} and $s_{ecorate}$ need to be determined to be able to solve for the info-gap robustness for each design alternative.

5.4.2 Elicitation of Scaling Factors

Begin BBSE for the filters option:

- Given:
  - Nominal estimate for oil filters used per engine life: $\tilde{F}$=17 filters/life
  - Bet price, p = \$0.90
  - Bet payout:
    - \$1 if truth is inside 17± a_{filter}
    - \$0 if outside
  - Find: $\alpha_{filter} = a_{filter}$ such that p is a fair price to the Subject
- Bets presented, with responses:

  B1) Q: Would you pay \$0.90 when α_{filter} = 0? A: No.

  B2) Q: Would you pay \$0.90 when α_{filter} = 17? A: Yes.

  B3) Q: Would you pay \$0.90 when α_{filter} = 8? A: Yes.

  B4) Q: Would you pay \$0.90 when α_{filter} = 4? A: No.

  B5) Q: Would you pay \$0.90 when α_{filter} = 6? A: Yes.

  B6) Q: Would you pay \$0.90 when α_{filter} = 5? A: No.
- Subject decides she cannot be more precise than a whole number, so a_{filter} is known to be within IBI bounds [5,6] filters/life. The Subject is offered the choice to choose a precise midpoint value of a_{filter} = 5.5 filters/life, and she accepts.

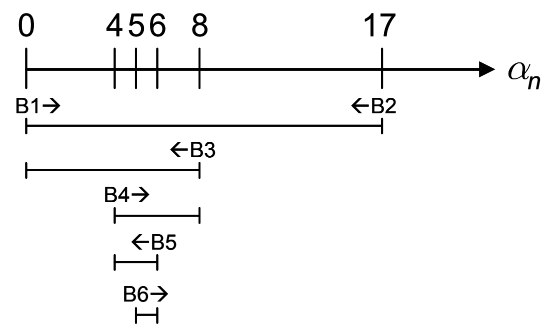

Figure 5.24 : Using a series of bet offerings to narrow down the value of a_n.

When BBSE is applied to the uncertain estimate of the ecorate for the filter casing, the elicited result is a precise value of $a_{ecorate}$ = 70%. Unlike filter life, the Subject finds it easier to express their beliefs about ecorate scale as a fractional percentage from nominal. These values will be calibrated to a baseline α value of 1. Thus the scaling factor output of BBSE is: s_{filter} = 5.5 filters/life and $s_{ecorate}$ = 70%.

5.4.3 Weighing Trade-offs between Robustness and Critical Performance

The results of BBSE are utilized to create info-gap models per Eq. (3). Following the standard technique for defining the robustness function and plotting robustness versus performance, the plot in Figure 5.25 can be generated.

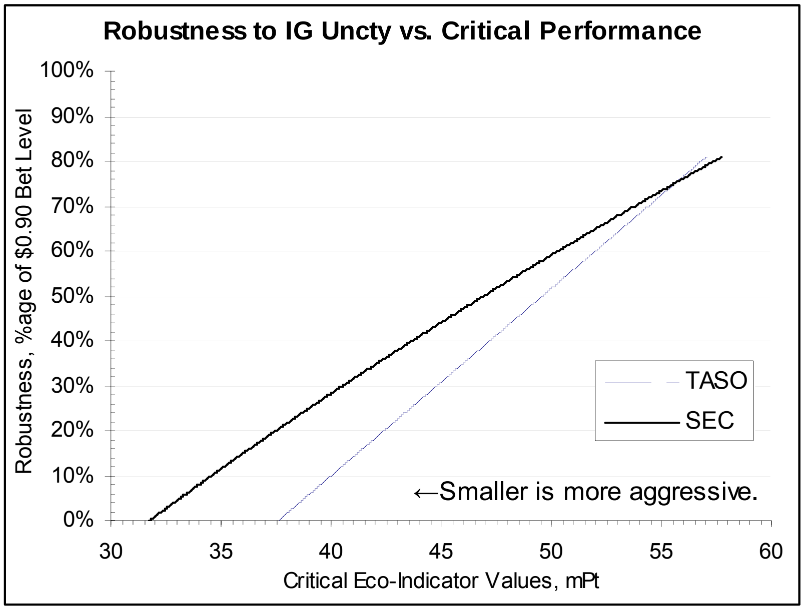

Figure 5.25 : A $\hat{\alpha} - r_c$ trade-off plot that combines the effects of two info-gaps affecting filter choice.

In Figure 5.25, the results of Unequal Fractional Scaling (UFS) are shown, appearing different than those of Equal Fractional Scaling (EFS) plotted in Figure 5.11. Notice that with precise BBSE applied, the r_c value at which preferences switch is now 55.7 mPt, a difference of 9.2 mPt greater (i.e., worse) than before. This happens at 75.2% of the calibration points (in the figure), i.e., the scaling factor values found when p = \$0.90. Because the calibration values are a_{filter} = 5.5mPt (or, expressed fractionally: $a_{filter}/\tilde{u}_n$ =5.5/17) and $a_{ecorate}$ = 70%, the crossover point occurs at robustnesses of $\hat{\alpha}_{filter} = 4.1$ filters/life above nominal and $\hat{\alpha}_{ecorate} = 52.6\%$ above nominal. Given the similar, shallow slopes of the two robustness lines, one would probably prefer the SEC option unless it is suspected that more robustness is needed, or that greater than 55.7mPt is a reasonable amount of guaranteed performance to sacrifice. In summary, the trade-offs between competing robustnesses more accurately account for what the Subject knows about uncertainty. The shift in the preferredness switch point could influence decision making accordingly.

5.4.4 Multi-Gap Assessment with Imprecise BBSE

Now assume that the Subject is unable to precisely determine scaling factors at the conclusion of BBSE. The elicited intervals of scaling imprecision (ISI) are: $s_{filter} \in [4.5, 8]$ filters per engine life (fpel) and $s_{ecorate} \in [40\%, 90\%]$. As explained in Section 5.2.4 and depicted in Figure 5.14, mixing high and low scaling factors into pairs generates the bounds an interval of preferredness imprecision (IPI). Pairing $s_{filter,low}$ with $s_{ecorate,high}$ generates the $\hat{\alpha} - r_c$ plot[25] in Figure 5.26; whereas, pairing $s_{filter,high}$ with $s_{ecorate,low}$ results in the plot in Figure 5.27. Note that although the vertical axes in both plots span 0% to 100%, these are percentages of scaling factors which differ between the two plots as indicated.

[25] Note that, because the inclusion of imprecision results in an increased number of different curves to plot (as was explained at the end of Section 5.2.4.1), the curve color/type convention established in Footnote 17 cannot be adhered to for Figure 5.26 through Figure 5.28.

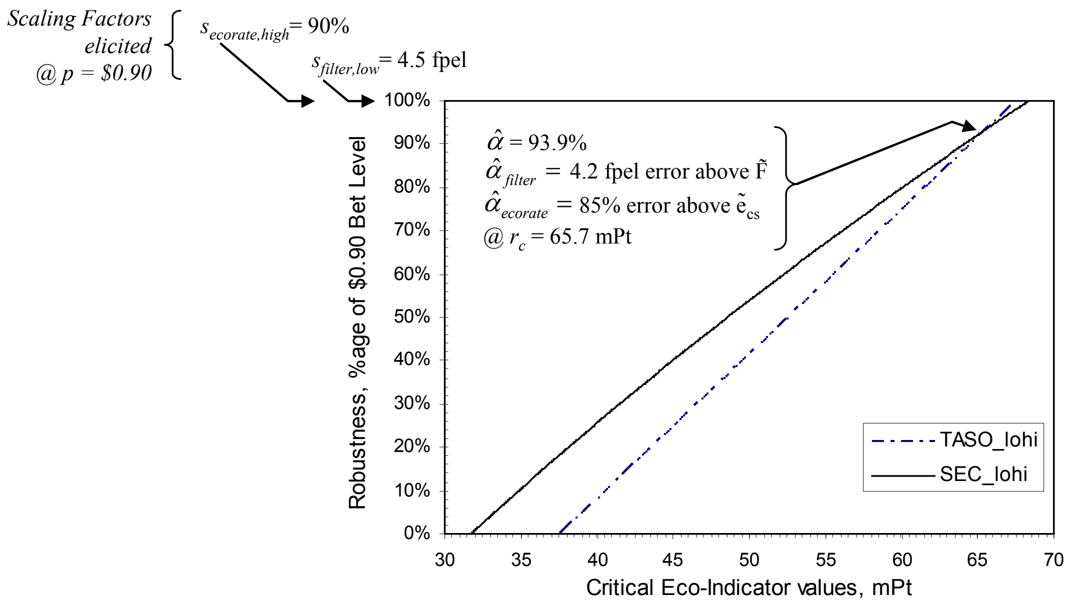

Figure 5.26 : A $\hat{\alpha} - r_c$ trade-off plot showing part of imprecise BBSE: "low" s_{filter} & "high" $s_{ecorate}$.

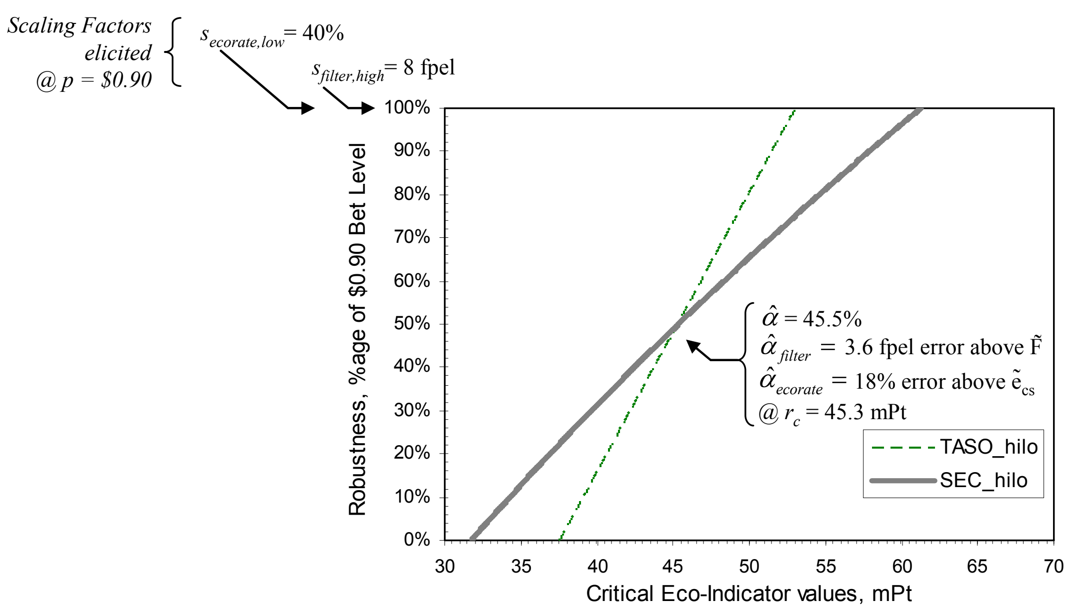

Figure 5.27 : A $\hat{\alpha} - r_c$ trade-off plot showing part of imprecise BBSE: "high" s_{filter} & "low" $s_{ecorate}$.

At these extremes of scaling, the preferredness switch points are located at r_c = 65.7mPt and 45.3mPt, bounding an IPI. Between these two points, it is not known which design alternative dominates in its capacity to provide robustness. In other words, preference rankings within that IPI are indeterminate. For instance, consider r_c = 50mPt: for the "low/high" combination of scaling in Figure 5.26, SEC provides more robustness; whereas, for the "high/low" combination in Figure 5.27, TASO provides more robustness.

Different scaling combinations could also be combined into a single plot, as in Figure 5.28. Keep in mind that this plot incorporates the (imprecise) mapping of beliefs about scaling equivalencies. Scaling factors are not transparent in this plot, other than the annotation made to the line key at bottom right.

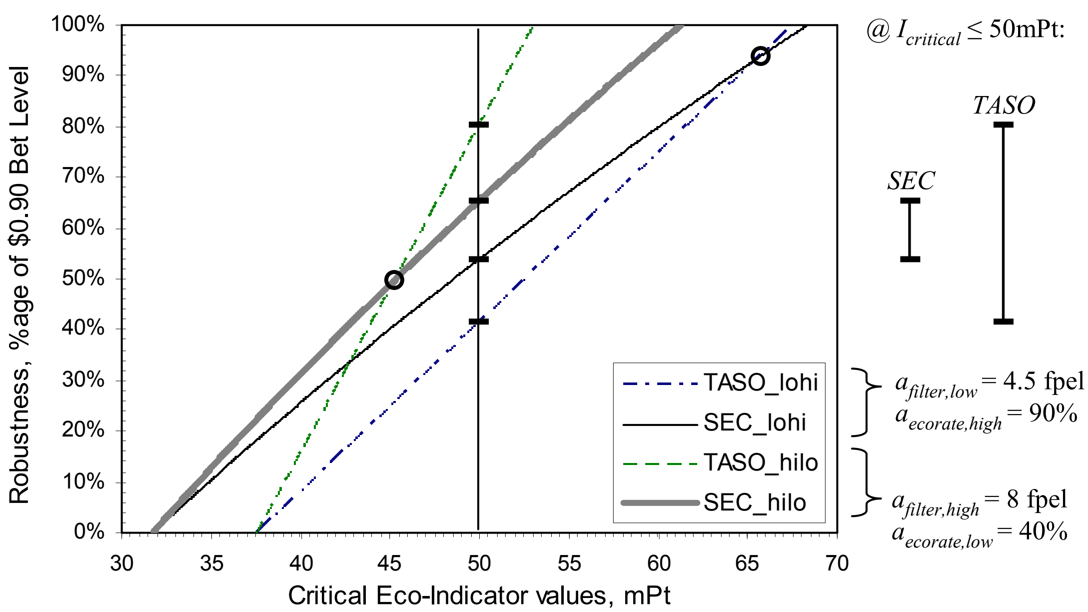

Figure 5.28 : A $\hat{\alpha} - r_c$ trade-off plot combining the effects of two info-gaps whose scaling is imprecise.

Within the IPI, imprecision prevents one from determining which design offers the most robustness (in terms of the baseline α to which all α_n are mapped). This is the case at r_c= 50mPt, where the imprecise robustness ranges spanned (on the vertical axis) due to scaling extremes overlap, as shown in the right side of Figure 5.28. To resolve this, one could adopt a heuristic decision policy to reach a solution. For instance, under a "maxi-min" policy, one would prefer the option that has the highest lower bound; in this case, SEC.

The preceding methods all help a designer to understand the implications that scaling different uncertainty parameters α_n to a single baseline α has on one's ability to achieve preference rankings on designs. The new capability to express imprecision about this scaling allows one to get a sense of what decision making power might be available under deficient information. This is akin to a sensitivity analysis, but with actual elicited information on the bounds of scaling imprecision.

5.4.5 Analysis with Imprecise BBSE and N>2 Info-Gap Uncertainties

For N>2 uncertainties, an info-gap analysis affected by imprecise scaling factors is still carried out in generally the same way as it is when there are two info-gaps, as in Section 5.4.4. The aspects of interest in the analysis are still:

1) the range of r_c where preference rankings are indeterminate (i.e., the IPI), and
2) for any specific value of r_c (e.g., in the example problem where r_c=50mPt), the size of imprecision in determining what robustness could be (as in the vertical intervals on right side of Figure 5.28), and whether that imprecision range overlaps with that of other design alternatives.

However, additional curves must be superimposed onto the combined robustness plots of the type seen in Figure 5.28. Because of imprecision, a curve must be generated for each extreme pairing of imprecise scaling bounds, as explained in Section 5.2.4.2. The IPI generated by $N>2$ info-gaps will accordingly grow to the size of the extreme high and low PSPs generated over the set of all extreme pairings. Likewise, for any specific value of r_c within the IPI, the interval of imprecision in determining what robustness could be (i.e., aspect of interest #2) is also bounded by the extreme high and low robustnesses found over the set of all extreme pairings. Applying imprecise scaling to an analysis of $N>2$ info-gap uncertainties is left to future work. However, it seems reasonable to expect that the combined effects of all scaling imprecision would result in exceedingly large intervals of preference-ranking indeterminacy, and consequently discourage decision making.

5.5 Discussion

The preceding sections raise a variety of points for discussion. In Section 5.5.1, the prospects of applying BBSE to functions with info-gap uncertainty are briefly considered. In Section 5.5.2, a discussion is presented regarding whether or not it is worthwhile to expend extra effort to improve the accuracy of scaling by using non-constant scaling factors. The trustworthiness of using scaling factors to simplify multi-gap assessments is considered in Section 5.5.3.

5.5.1 Eliciting Scaling Factors for Functions with Info-Gap Uncertainty

Though the new methods presented in this chapter are intuitive for variables (or constants) having info-gap uncertainty, they do not extend as easily to functions with info-gap uncertainty. Ben-Haim proposes an “envelope bounded” info-gap model for uncertain functions (originally mentioned in Section 3.4.1.2), which has an effect analogous to scaling factors (Ben-Haim 2006). However, it is unknown whether scaling envelopes could be elicited through a betting scenario. This is left to future work, as the scope of this thesis only includes uncertain life-cycle variables.

5.5.2 Whether or Not to Consider Non-Constant Scaling Factors

As described thus far, BBSE elicits constant s_n scaling factors, but non-constant scaling could conceivably be appropriate in some situations. For instance, if the extent to which an unknown u_n could deviate from nominal $\tilde{u}_n$ is bounded by some physical limit (e.g., 0, for a variable that must be non-negative), then the perceived “size” or scale of uncertainty α_n might accelerate approaching the hard limit.

Whatever the motivation, non-constant scaling would affect trade-offs between competing robustnesses by resulting in a TS-line that is curved instead of straight, as depicted in Figure 5.29, a modified version of Figure 5.12. This in turn would create a different preference switch point (PSP) location than the one that would be found using constant s_n.

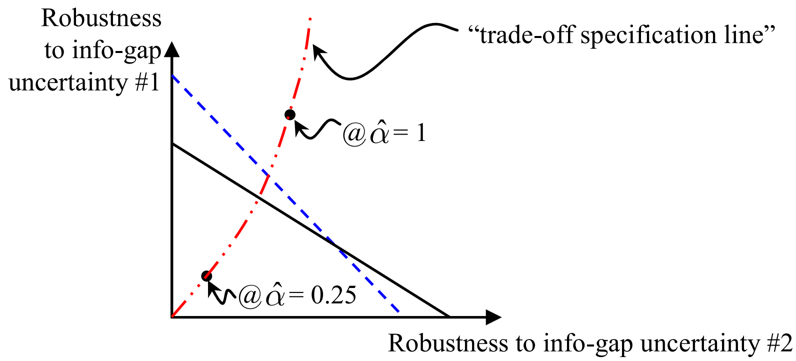

Figure 5.29 : Non-constant scaling factors can cause the TS-line to be non-linear.

Unfortunately, just because one can conceptualize non-constant scaling doesn't mean it can be practically elicited. Two possible ways can be thought of to define non-constant scaling factors, and each has significant limitations. In the first, the decision analyst simply skips BBSE altogether and declares non-constant "uncertainty weights" (as Ben-Haim does for constant weights). This requires modification of Eq. (3) to make each s_n a function of the baseline α. This route to modeling is not operational like BBSE is, and we expect that in most cases it would demand knowledge unavailable to a decision analyst faced with severe uncertainty.

In the other approach, if one were to conduct multiple BBSE iterations using different values of p (e.g., p=\$0.90, then p=\$0.75, etc.), the resulting scaling might be non-constant. This would be the case when the a_n scaling reference values elicited at different p reveal that the corresponding size belief distributions for different info-gaps are not of congruent shape. In response, one would need to formulate non-constant s_n functions from the a_n values using some means of interpolation, which to date has not been conceived. Modeling effort is another concern. Just *one* BBSE iteration could be time consuming depending on the number of info-gap uncertain variables involved; iterating BBSE for multiple *p*-value references compounds this effort.

Instead of eliciting non-constant scaling factors, one could theoretically bound the non-linear tradeoffs they create using constant scaling factors that are imprecise, per Eq. (8). This is conceptually depicted in Figure 5.30, which is an updated version of Figure 5.29. As explained in Section 5.1.2, this imprecise trade-off specification results in an interval of preferredness indeterminacy (IPI).

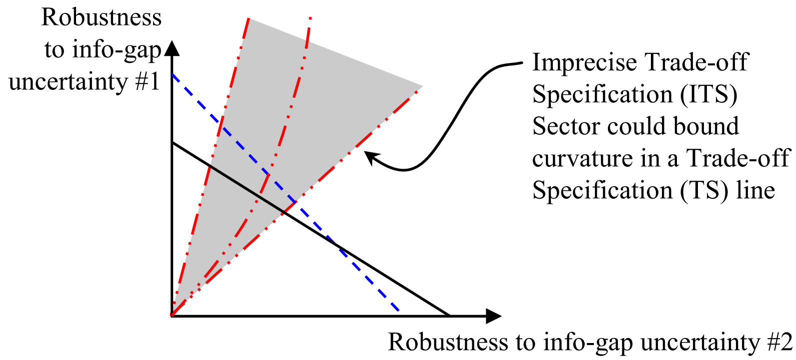

Figure 5.30 : Non-constant scaling factors can cause the TS-line to be non-linear.

Unfortunately, no operational way has been conceived to elicit $s_{n,low}$ or $s_{n,high}$ values that would bound this error. Contrast this with the elicitation of *constant* scaling factors in BBSE Step 1, where imprecision can be quantified in an operational manner through progressive bet offerings.

In summary, non-constant scaling factors are difficult to elicit precisely, and there is no known technique for bounding their imprecision. If the accuracy of non-constant scaling factors cannot be trusted, then they can't be trusted to increase the accuracy of Equal-Scaled Robustness Trade-offs. Thus, we do not encourage their use.

5.5.3 How Trustworthy are the Results of Scaling Elicitation?

BBSE incorporates new scaling information, elicited through relatively rigorous means, into Ben-Haim's pre-existing scheme (in Section 5.2.1) for trading off between competing multi-gap robustnesses. The available information consistent with a Subject's beliefs about scaling may be very rough; therefore, supporting techniques also enable the modeling of imprecision and offer rules for making decision amidst that imprecision. Beyond what has been outlined for BBSE, we do not suggest further extensions to scaling elicitation without some new rigorous basis.

Can one determine absolutely the trustworthiness of BBSE outputs and their effects on assessments? We cannot make this claim, at least not absolutely, given the sparsity of information involved. Recall that IGDT is meant to be a "process of exploration" of options, rough information sources, and their implications, not a closed form decision method (Ben-Haim 2006). With this in mind, it is recommended that sensitivity checks be performed (some of which is achieved by representing imprecision in scaling). A decision analyst should use caution recommending a decision if trade-off preferences turn out to be sensitive to the structure of the problem or to various assumptions employed. However, any further quantification and analysis of uncertainty about info-gap uncertainty (i.e., meta-uncertainty) in scaling is generally not fruitful.

Alternatively, we reiterate that when using IGDT one should be on the lookout for scenarios where good decisions can be reached with as little info (or as few restrictive assumptions) as possible. Sometimes, simple analyses uncover important weaknesses or opportunities in different designs' capacity to provide robustness to faulty data. We have provided a few examples of when this might be the case. Future examples may reveal other instances where IGDT, applied as a “first cut” at analyzing a problem, is enough to help a decision maker develop preferences without further effort.

5.6 What Has Come Before and What is Next

In this chapter, we have presented a variety of techniques that allow one to elicit and introduce information, either precise or imprecise, about one’s beliefs about the scales of severe uncertainty. Several of these techniques are applied in the next section to a remanufacturing facility design problem.

CHAPTER 6:

A REMANUFACTURING PROCESS EXAMPLE PROBLEM

In this chapter, the IGDT techniques presented in this thesis are applied to a more elaborate example problem. The problem involves *selection* of the types of technologies and number of stations to be used in a remanufacturing process. The profitability of the manufacturing process is affected by severe uncertainty in (1) the demand for remanufactured parts and (2) the cost penalties of carbon emissions. The problem features significantly more discrete design alternatives to consider than before as well as a reward function with discontinuities. The example problem again takes the form of a pay-back scenario, as expensive machines that are more energy efficient are seen to recoup their capital costs with increased throughput.

In Section 6.1, the details of the remanufacturing process design problem are presented. In Section 6.2, the effects of the two different info-gap uncertainties will be assessed separately and then together using precise bet-based scaling elicitation.

6.1 Remanufacturing Problem Scenario

In this example, it is assumed that the process designer for a remanufacturing firm wants to reduce the environmental impact of its facility. For the subprocesses under consideration, the major environmental loads are energy consumption and greenhouse gas (carbon) emissions. Assigning these loads a monetary cost in US$ has the effect of reducing waste when designing to maximize profit[26]. Other solid wastes are considered negligible.

Naturally, some simplifications and assumptions are introduced in the problem. For example, the exact parameters for the problem are chosen to be realistic, but they do not represent hard, real-world data. Consequently, the emphasis is not on the actual decision outcome (i.e. the chosen station configuration), but rather on the decision and analysis process.

6.1.1 Problem Scenario

An engineering manager of an upcoming remanufacturing facility is tasked with choosing the type and amount of technology needed for three different subprocesses: sorting, cleaning, and drying. These processes prepare a used part for machining and other refurbishing. A flow diagram of the process and the subprocesses of interest is depicted in Figure 6.1.

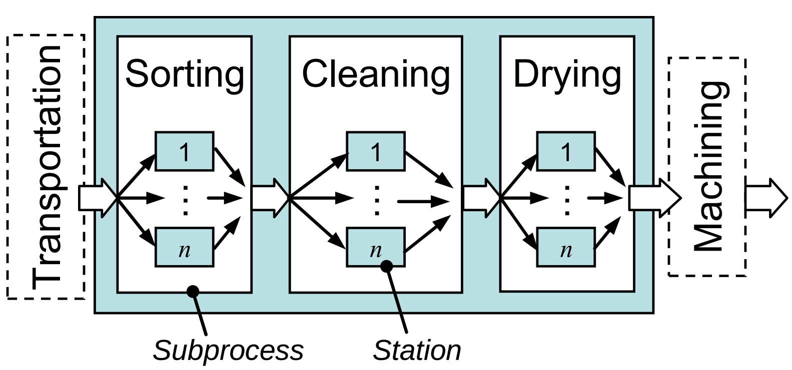

Figure 6.1: Process diagram, with arrows denoting flow of parts.

[26] It is acknowledged, however, that a manufacturer could spend more money on renewable electricity sources and reduce their burden on the environment. We do not take this into consideration. All electricity consuming technologies in the manufacturing line *are* assumed to use the same "grid" electricity.

Although the overall remanufacturing process also involves collection and machining of parts, these aspects are outside the scope of consideration. The selection decision of interest consists of specifying what type of station (e.g., pure manual labor, a machine, labor with machine assistance, etc.) will be used in each of the three subprocesses, as well as what number of stations will be used for each subprocess. It is assumed that the type of stations used in any subprocess must all be the same. A "design alternative", q, is any unique combination of type and number of machines for each subprocess. For instance, a process with 1 automated sorting station, 3 ultrasonic cleaning stations, and 2 gas drying stations would constitute one design alternative. The types and maximum number of stations for each subprocess are shown in Table 6.1.

Table 6.1: Types of stations for different subprocesses.

Sorting **Max stations = 3**	**Cleaning** **Max stations = 3**	**Drying** **Max stations = 3**
Fully Manual	Batch Aqueous	Gas
Manual Assisted by Electrostatic Pen	Ultrasonic	Electric
Automated	Conveyor Spray	Ambient

Across the different subprocesses, each station type has a different maximum throughput that it can handle, as well as different variable (i.e., per-part) costs and fixed costs. Variable costs (US$/part) are incurred by labor, energy, and penalties due to carbon emissions. Fixed costs (US$/hr) are due to overhead from machine capital costs, as well as the energy and carbon emission overheads of continuously running machines (e.g., energy losses in an oven).

For different station types, trends exist in the magnitude of the constants for capacity and fixed and variable costs. In general, stations that rely on machines and automation tend to have high capacity and high capital costs, but lower variable costs. Alternatively, stations based primarily or wholly on manual labor have low capital costs, but also lower throughput and high labor costs. Additionally, machines with high capital costs tend to have lower energy consumption and accordingly, lower carbon emissions.

6.1.2 Performance Function: Profit

Profit depends on the station-specific variables mentioned in the previous section. The total profit per hour (US$/hr), *P*, that the firm can make using a design alternative *q* is:

$$P(q) = \left(throughput_q \cdot (price_{part} - c_{variable_q}) \right) - c_{fixed_q} \tag{6.1}$$

where *throughput* is the number of parts remanufactured per hour; $price_{part}$ is the firm's selling price for each remanufactured part (US$/part); $c_{variable}$ is the total variable costs of remanufacturing; c_{fixed} is the total fixed costs associated with all machines in every subprocess. The *q* subscript on the throughput, variable cost and fixed cost denotes that they are each dependent on the specific design alternatives.

Throughput is governed by the equation:

$$throughput_q = \min\left[\min(capacity_{i,q}), demand \right] \tag{6.2}$$

where $capacity_{i,q}$ is combined maximum throughput of all stations in any one subprocess *i* for design alternative *q*, and *demand* (later denoted as *D*) is the market demand for remanufactured parts per hour. In Eq. (6.2), the subprocess with the smallest capacity constrains the throughput of the entire remanufacturing process. Alternatively, throughput is equal to demand when demand is smaller than the capacity of any subprocesses.

For the variable cost component in Eq. (6.1):

$$c_{variable_q} = c_{part} + \sum_i \left(c_{labor_{i,q}} + c_{energy_{i,q}} + c_{carb_{i,q}} \right) \tag{6.3}$$

where each c is a per-part cost, including the cost that the remanufacturing firm paid for the used part (assumed constant), as well as the variable costs mentioned in the previous section. The station dependent variable costs in Eq. (6.3) are calculated as follows:

$$c_{labor_{i,q}} = \left(\frac{cost}{manhour} \right) \cdot \left(\frac{manhours}{part} \right)_{i,q} \tag{6.4}$$

$$c_{energy_{i,q}} = \left(\frac{cost}{kWh} \right) \cdot \left(\frac{kWh}{s} \right) \cdot \left(\frac{s}{part} \right)_{i,q} \tag{6.5}$$

$$c_{carb_{i,q}} = \left(\frac{cost}{lb_{carb}} \right) \cdot \left(\frac{lb_{carb}}{part} \right)_{i,q} \tag{6.6}$$

Note that each of the above equations depend on the type of stations associated with q, but not the number of stations.

Having established the model for variable costs, the equation for total fixed costs is now needed:

$$c_{fixed_q} = \sum_i \left[(n_{stat_{i,q}}) \cdot \left[c_{capital_{i,q}} + c_{enoverhd_{i,q}} + c_{carboverhd_{i,q}} \right] \right] \tag{6.7}$$

where $n_{stat,i,q}$ is the number of stations for a particular subprocess and design alternative, and the fixed costs are due to equipment capital costs, and energy and carbon emission overhead, as mentioned in the previous section. Equations for fixed costs are:

$$c_{capital_{i,q}} = \left(\frac{cost_{capital}}{hours_{lifetime}} \right)_{i,q} \tag{6.8}$$

$$c_{enoverhd_{i.q}} = \left(\frac{cost}{kWh} \right) \cdot \left(kW_{overhead} \right)_{i,q} \tag{6.9}$$

$$c_{carboverhd_{i,q}} = \left(\frac{cost}{lb_{carb}} \right) \cdot \left(\frac{lb_{carb,overhead}}{hr} \right)_{i,q} \tag{6.10}$$

The end result of Equations (6.2) through (6.10) is that profit, given by Eq. (6.1), is dependent on market demand *and* design alternatives (i.e., type and number of stations).

6.2 IGDT Analysis

The station selection problem will now be used to explore the application of IGDT to a scenario with severe uncertainty: first in market demand and, later, in the costs of carbon emissions. The decision maker wishes to evaluate the selection decision without evaluating the market space further, and decides to use the IGDT approach to do so.

The decision maker takes the attitude that settling for some guaranteed lower-bound on Profit is acceptable and preferable to risky, (but higher) optimized Profit that relies on the veracity of unfounded assumptions about how uncertain the nominal estimate is. Accordingly, the decision maker seeks the design alternative with maximum robustness to the unknown gap between the unknown *actual* Profit and a nominal *estimate*. The desire to maximize the size to which this discrepancy can grow is subject to a satisficing critical constraint that defines a smallest critical Profit that can be accepted, one that is sub-optimal with respect to what would be the best solution under no uncertainty, yet “good enough” given its robustness to uncertainty.

6.2.1 IGDT Analysis of One Uncertainty: Demand

To analyze the robustness of different designs to info-gaps, the standard procedure is followed: an available nominal estimate is translated into an info-gap model, a robustness function is formulated and solved, and trade-offs between critical performance and info-gap robustness are assessed to identify a design with a preferable amount of robustness.

6.2.1.1 Info-Gap Model

The info-gap model for this example can be specified with the knowledge that:

- The nominal value for average demand is $\tilde{D} = 175\, parts/hr$, taken from a previous year’s demand for a similar product.
- The growth of deviation around nominal can be expressed mathematically as a simple, uniformly-bounded interval.

Combining the uncertainty parameter, α, with this sparse information, the info-gap model, $\mathfrak{D}$, for average demand is:

$$\mathfrak{D}(\alpha, \tilde{D}) = \left\{ D : \left| D - \tilde{D} \right| \le \alpha \right\},\ \alpha \ge 0 \tag{6.11}$$

The effective mathematical meaning of this particular info-gap model can also be expressed more simply:

$$\tilde{D} - \alpha \le D \le \tilde{D} + \alpha \tag{6.12}$$

For the problem at hand, an unexpected drop in demand is worse for overall profit; therefore, $\tilde{D}-\alpha$ is the side of the parameterized info-gap boundary that is of most interest.

6.2.1.2 Reward function and satisficing critical value

The other two components needed for an info-gap decision analysis are the reward function and satisficing critical level for performance. Combining Eqs. (6.1) and (6.2), the reward function for Profit, which is dependent on the design alternative, q, and the uncertain demand, D, is:

$$P(q,D)=\left[\min\left[\min(capacity_{i,q}),D\right]\times(price_{part}-c_{variable_q})\right]-c_{fixed_q} \qquad (6.13)$$

Next, the critical constraint inequality in Eq. (3.5) becomes, for this example:

$$P(q,D)\geq P_{critical} \qquad (6.14)$$

For now, it will be assumed that the decision maker can choose a critical profit, $P_{critical}$=\$90, i.e., the lowest level of Profit deemed tolerable. As will be seen later, the decision maker may wish to adjust his or her choice of this value once the tradeoffs between critical performance and robustness to severe uncertainty are illustrated graphically.

6.2.1.3 Info-gap robustness function

Of main interest in an info-gap analysis is what largest amount of robustness to uncertainty, $\hat{\alpha}(q,r_c)$, is achievable. This robustness is the largest amount of uncertainty α that can be sustained by a design alternative q while still guaranteeing, at worst, achievement of the chosen critical performance level r_c. Expressed in the Eq. (3.7) form, the info-gap robustness for this example is:

$$\hat{\alpha}(q, P_{critical}) = \max\left\{\alpha : \left(\min_{D \in \mathfrak{D}(\alpha, \tilde{D})} P(q, D)\right) \geq P_{critical}\right\} \quad (6.15)$$

Carrying out the optimization to find info-gap robustness $\hat{\alpha}(q, P_{critical})$ appears daunting at first glance, especially given that Eq. (6.13) is embedded inside Eq. (6.15) as *P(q,D)*. However, the simple, linear, uniform bounds of the info-gap model $\mathfrak{D}$ makes the relationship between $\hat{\alpha}(q, P_{critical})$ and $P_{critical}$ mathematically equivalent[27] to the relationship between α and *P(q,D)*. In other words, for this simple case, if the (x, y) point pair (α, *P(q,D))* can be calculated, their values can be swapped (i.e., ordered (y, x)) to yield the point pair corresponding to ($P_{critical}$, $\hat{\alpha}(q, P_{critical})$). This is advantageous because *P(q,D)* can be easily calculated as a function of α. To do this, one must substitute $\mathfrak{D}$ (the info-gap model for *D* defined in Eq. (6.11)) for *D* in the equation for Profit, Eq. (6.13). Then, $\mathfrak{D}$ itself can be substituted with $\tilde{D} - \alpha$, which is the boundary from Eq. (6.12) associated with worse performance. For the example at hand, the result of the substitution of D with $\tilde{D} - \alpha$ is:

$$P(q, D(\alpha)) = \left[\min\left[\min(capacity_{i,q}), (\tilde{D} - \alpha)\right] \times (price_{part} - c_{variable_q})\right] - c_{fixed_q} \quad (6.16)$$

So, if one exhaustively calculates *Profit* for all *q* and α, one can accordingly obtain and plot the robustness function $\hat{\alpha}(q, Profit_{critical})$ for all designs *q* and all critical

[27] This shortcut may not be valid for more complicated info-gap models.

levels of Profit. This has been done for the facility design problem, as shown in Figure 6.2.

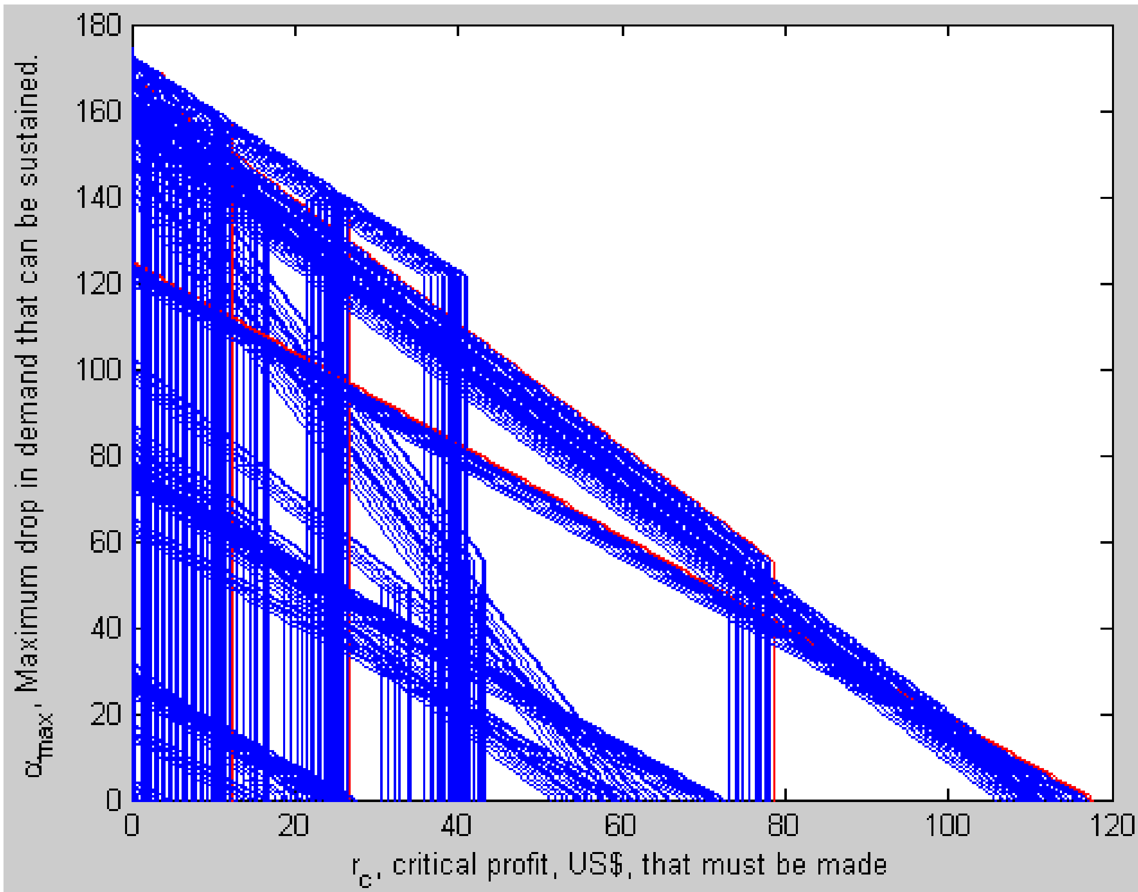

Figure 6.2: Robustness to drops in demand, versus Critical profit, for all design alternatives.

Due to the large combinatorial number of design alternatives in Figure 6.3, it is difficult to ascertain which design alternatives are best. (This problem will be remedied in the following section.) However, some general behaviors can be seen in the graph. Generally, the robustness-performance tradeoff for each design is in the form of a line moving from the bottom right up to the top left. This is intuitive, as a reduction in one's expectation for minimally acceptable profit (i.e., $P_{critical}$) should allow for more robustness to drops in demand, i.e., movement up the vertical axis. The problem takes the form of the pay-back problem described in Section 4.1.3.2. Different design alternatives have different slopes, and some fall off sharply, sometimes before the critical profit is very large. This precipitous drop occurs when a design alternative's capacity is exceeded by demand. In such a scenario, the alternative cannot generate any greater profit because it cannot increase throughput because of limited capacity.

6.2.1.4 Analysis of Robustness-Performance Tradeoff

To aid identification of which design alternatives are most preferred over different ranges of $P_{critical}$, an algorithm based on Eq. (3.9) was written to log which alternatives, $\hat{q}(P_{critical})$, have the maximum info-gap robustness for different critical Profit levels. The tradeoff lines of designs that dominate for at least part of the range of $P_{critical}$ (i.e., the horizontal axis) are shown in Figure 6.3. Notice that they outline the dense collection of curves from Figure 6.2. The subscript numerals place the $\hat{q}(P_{critical})$ in order of decreasing critical Profit level. The intervals for which each preferred design dominates are shown by double arrows.

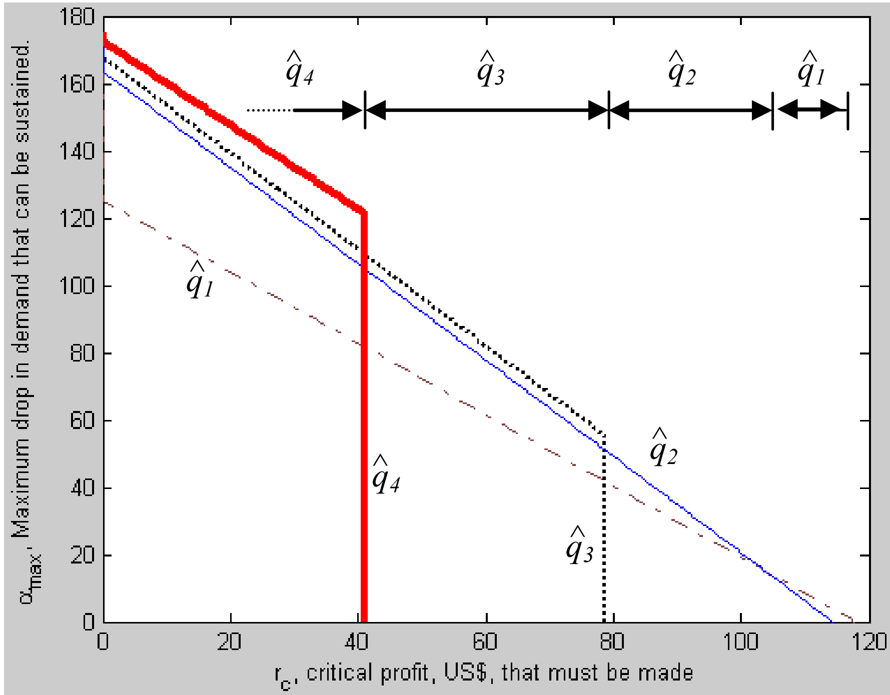

Figure 6.3: Robustness to drops in demand, versus critical profit, for several most preferred designs.

The particular design alternative (i.e., the type and number of machines) to which each $\hat{q}(P_{critical})$ line corresponds is shown in Table 2, along with exact numerical ranges of $P_{critical}$ and $\hat{\alpha}(q, P_{critical})$ over which they dominate in Figure 6.3. The point where dominance changes ($P_{critical}$ = \$105, \$79, and \$41) are preference switch points (PSPs).

Table 6.2: Types of stations for different subprocesses.

q_i	Sort type	Sort # of stns	Clean type	Clean # of stns	Dry Type	Dry # of stns	Critical Profit Range	Info-Gap Robustness Range
1	Pen	1	Batch	1	Gas	1	\$117 to \$105	0 to 15
2	Pen	1	Batch	1	Elec	2	\$105 to \$79	15 to 52
3	Pen	1	Batch	1	Elec	1	\$79 to \$41	52 to 109
4	Pen	1	Batch	1	Amb	3	\$41 to \$0	109 to 140

From Table 6.2, it can be seen that the sort and clean subprocesses are not affected by severe uncertainty in demand. The drying subprocess was affected, as it is the most energy and carbon emission intensive. It also makes sense that the high volume, high capital cost machines allow for the most profit with small allowances for drops in demand, yet are dominated by a low volume, low capital station type for higher levels of uncertainty.

But the question remains, by what procedure does a decision maker go about eliciting their preference for a tradeoff between robustness and critical profit using Figure 6.3? Critical levels of performance along the horizontal axis can be weighed against the corresponding info-gap robustness, graphed by the tradeoff line for each design.

The designer, not knowing the estimation error α, is tasked with choosing a point on the horizontal axis corresponding to his or her demanded level of satisficing performance. In some applications, the r_c value may be strongly dictated by external factors. In other applications, the decision maker has the flexibility to relax their choice of critical performance level in order to gain more robustness. The decision maker can explore this tradeoff graphically in Figure 6.3 by examining the maximum info-gap robustness achievable for different values of $Profit_{critical}$. In this example, the design having *maximum* robustness is the one whose tradeoff line is the highest at a given critical performance level.

The plot in Figure 6.3 is instrumental in understanding how design preference changes as the demand for minimally acceptable profit is relaxed further away from the performance-optimal level. For example, at critical satisficing level discussed in Section 6.2.1.2, $P_{critical}$=\$90, it can be seen that $\hat{q}_2$ is most preferred and is robust to a drop of roughly 40 units below the nominal demand. If profit as aggressively high as \$117 were demanded (accepting no worse), only $\hat{q}_1$ would satisfy the constraint, and even then, there would be no tolerance for error, *α*, in estimating demand. Thus, $\hat{\alpha}(q_2, P_{critical} = \$117) = 0$. If one could accept a guarantee of profit no worse than \$105, $\hat{q}_2$ becomes more favorable from the info-gap robustness-maximizing perspective. The decision maker must explore these tradeoffs and determine what feasible combinations of robustness and critical performance are preferable.

6.2.1.5 Insight Gained in the Info-Gap Analysis

The following knowledge is gained in this simple example:

- If the decision maker can accept profit that would never be worse than \$105, design $\hat{q}_2$ is preferable because it can endure the highest amount of error below the nominal guess and still satisfy the performance constraint. Moreover, the rate at which info-gap robustness is gained with incremental relaxation of the $P_{critica}l$ demand (i.e., the line slope) is faster for $\hat{q}_2$ than $\hat{q}_1$, making $\hat{q}_2$ even more attractive past \$105 of demand.

- Though it provides the most robustness between \$79 and \$41 of critical profit, $\hat{q}_3$ never provides significantly more robustness than $\hat{q}_2$, and would not provide any profit above \$79 due to its limited throughput capacity.
- $\hat{q}_4$, with its cheap, yet slow and labor intensive ambient drying stations, becomes superior should one be willing to accept very low levels of profit in exchange for robustness to very high drop in demand. Notice, however, that $\hat{q}_4$ has very limited capacity, limiting its ability to provide profit if the drop in demand is not very high.
- The designer, not knowing what the uncertain variable actually will be, can use the info-gap analysis and plot in Figure 6.3 to get a handle on what a decision change entails under the satisficing decision rule. It is up to the decision maker to sort out his or her preference for robustness versus guaranteed achievement of, at worst, some critical level of performance.

6.2.2 IGDT Analysis with Two Uncertainties: Demand and Emission Pricing

A second uncertain variable, the cost per ton of carbon emissions, can also be modeled as an info-gap.

6.2.2.1 <u>Info-Gap Models</u>

Carbon emission cost can be represented simply as a price per ton. This assumes that the price is set in a "carbon emissions trading" market, perhaps in a government mandated cap-and-trade system (Labatt and White 2007). A characteristic of carbon emissions prices, as indicated by historical evidence in Figure 6.4, is potential volatility in pricing (The Economist 2006). Note that we apply a carbon cost only to processes that generate carbon at the facility. Upstream sources of carbon at electricity plants are assumed to be accounted in some other fashion and, perhaps, passed along in the cost of electricity.

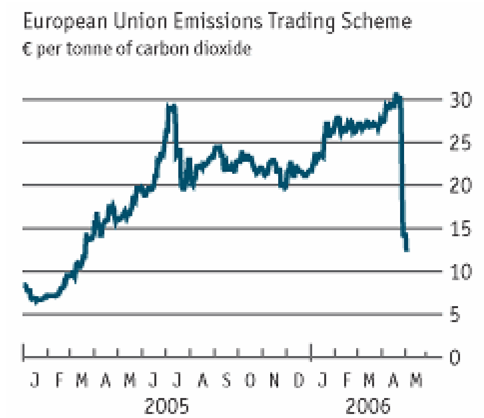

Figure 6.4: An example of volatility in carbon pricing (from (The Economist 2006)).

The info-gap model for this example can be specified with the knowledge that:

- An estimated nominal value for carbon cost is $C_{carbon} = \$5.50 / ton$, taken from the best estimate from a panel of experts.
- The growth of deviation around nominal can be expressed mathematically as a simple, uniformly-bounded interval.

Combining the uncertainty parameter, α, with this sparse information, the info-gap model, $\mathfrak{C}$, for carbon cost is:

$$\mathfrak{C}(\alpha, \tilde{C}_{carbon}) = \left\{ C_{carbon} : \left| C_{carbon} - \tilde{C}_{carbon} \right| \le \alpha \right\}, \ \alpha \ge 0 \qquad (6.17)$$

The effective mathematical meaning of this particular info-gap model can also be expressed more simply:

$$\tilde{C}_{carbon} - \alpha \le C_{carbon} \le \tilde{C}_{carbon} + \alpha \qquad (6.18)$$

For the problem at hand, an unexpected rise in cost is worse for overall profit; therefore, $\tilde{C}_{carbon} + \alpha$ is the side of the parameterized info-gap boundary that is of most interest.

The info-gap model for uncertainty in demand, *D*, will be the same used in Section 6.2.1.1.

6.2.2.2 Precise Bet-Based Scaling Elicitation

Using direct elicitation per Section 5.3.3.1, the Subject reveals her belief about scaling for each info-gap uncertainty as such:

- She would pay \$0.90 to enter a bet that pays \$1 if a_{carbon} were set equal to **400%** of the nominal for C_{carbon}. That is, she believe with a subjective, precise probability of 0.9 that carbon price increases would not grow above an additional 4 x \$5.50 = \$22 per ton.
- She would pay \$0.90 to enter a bet that pays \$1 if a_{demand} were set equal to **50%** of the nominal for *D*. That is, she believe with a subjective, precise probability of 0.9 that demand for remanufactured parts would not drop below 0.5 x 175 = 87.5 parts/hr.

Per Step 2 of BBSE, the scaling references a_n are calibrated to a baseline level of 1, making the scaling factors $s_{carbon} = 4$ and $s_{demand} = 0.5$.

6.2.2.3 Plotting and Analysis of Robustness-Performance Tradeoff

With the scaling factors determined, a combined uncertainty model is defined:

$$\mathcal{U}(\alpha,\tilde{u}) = \left\{ u : \frac{|u_n - \tilde{u}_n|}{\tilde{u}_n} \le s_n \alpha,\ n = \text{carbon, demand} \right\}, \alpha \ge 0 \qquad (6.19)$$

This model, along with the performance function Eq. (6.13), are used in the robustness function, Eq. (6.15). The resulting plot, showing only the designs with maximum robustness, appears in Figure 6.5.

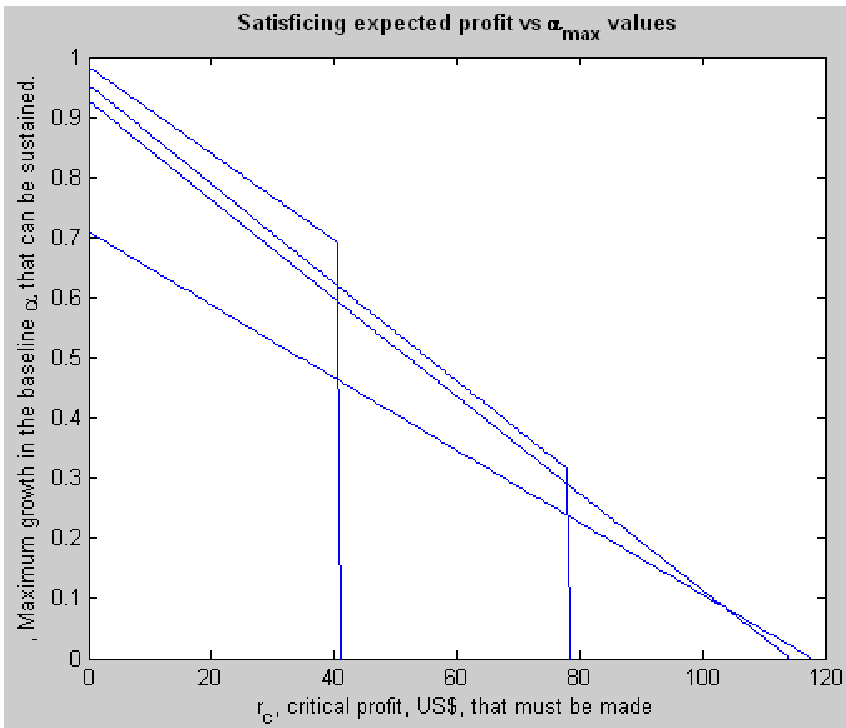

Figure 6.5: Robustness to baseline α, versus critical profit, for several most preferred designs.

Comparison to Figure 6.3 reveals that the graphs are nearly identical, showing the same designs as being preferred for the same ranges. Why is this?

It turns out that the influence of carbon costs on even the designs that emit the most carbon dioxide (i.e., natural gas heated ovens for the drying process) are very small.

A sensitivity analysis reveals this. In Figure 6.6, the designs with maximum info-gap robustness to demand drop, originally see in Figure 6.3 are plotted[28] for two different values of the carbon cost. The outer line is for nominal $C_{carbon} = \$5.50/ton$ and the inner line is *ten times* that cost. For the scaling chosen, carbon cost does not significantly influence design choice. This is indicated by the two small circles, which represent the switch point between $\hat{q}_1$ and $\hat{q}_2$, which was of interest in Figure 6.3. The switch point is insensitive to the size of carbon cost.

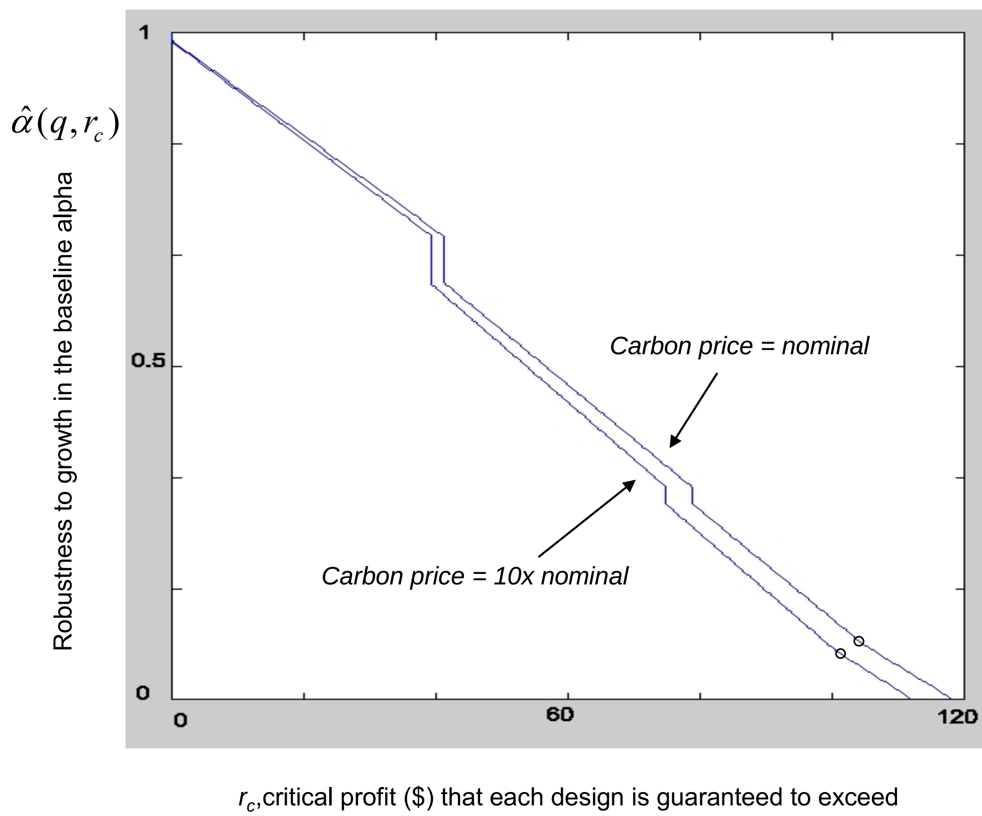

Figure 6.6: Sensitivity to two different extreme costs of carbon: nominal, and 10x nominal.

[28] These plots show only the maximum robustness "contour", i.e., only the portion of a design's robustness curve where that design has maximum robustness is plotted, in the same fashion as in **Figure** 3.12

For the sake of verifying the effects that carbon pricing has on preference ranking, observe the plot in Figure 6.7 which is similar to Figure 6.6, except that the inner line represents 50 times the nominal carbon cost.

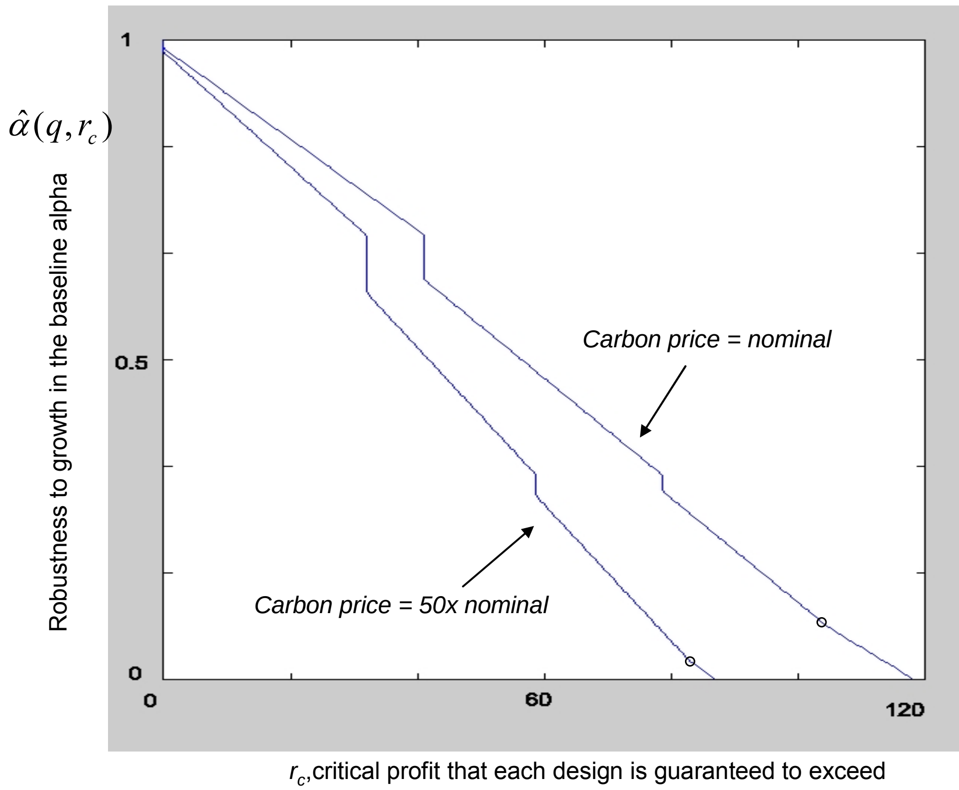

Figure 6.7: Sensitivity to two different extreme costs of carbon: nominal, and 50x nominal.

It can be seen that, for the case of expensive carbon emission penalties (inner curve), the design $\hat{q}_1$ (seen before in Table 6.2), offers the most robustness for a much smaller range of r_c compared to the outer curve. This is because $\hat{q}_1$ utilizes natural gas fired drying and is affected aversely as the increase in carbon costs rise.

Because of the miniscule influence of carbon for the example problem as defined, imprecise BBSE is not applied, as the intervals of preferredness indeterminacy (IPI) would be negligibly narrow.

6.2.2.4 Engineering Insight Gained

Using a remanufacturing scenario, we have analyzed a design problem having complicated tradeoffs between fixed and variable costs. Similar tradeoffs existed in the previous oil filter example (Duncan et al. 2006), but the facility subprocesses created significantly more design alternatives to consider, as well as discontinuities that affected how the tradeoff lines were interpreted. We have identified another situation where the info-gap design analysis approach could feasibly enable a decision using no more than a sparse nominal value for demand, formulated into a simple info-gap model. Specifically, it was observed that a switch in design choice from $\hat{q}_1$ to $\hat{q}_2$ requires a small sacrifice in guaranteed profit levels, yet affords a reasonably large amount of extra robustness to error in a nominal estimate.

We also saw that, for this problem, the effects of uncertain carbon pricing did not affect the preference rankings significantly compared to the case that considered uncertain demand only. A simple sensitivity analysis was employed to verify the small influence of carbon pricing.

6.3 What Has Been Presented and What is Next

The preceding example problem has revealed further insight into how info-gap theory could be applied to practical problems. In the next Chapter, we look back on the questions answered in this thesis, and consider the overall motivating question about the general worth of IGDT to EBDM design problems.

CHAPTER 7:

CLOSURE

In this chapter, the findings and insight gained from the preceding chapters are collected and used to support a discussion of the overall relevance of the work. First, in Sections 7.1 through 7.3, the motivating thesis questions, along with their answers, are reviewed and discussed using supporting points from relevant chapters. For the contributions associated with each question, commentary on the value and wider usefulness of the contributions is provided. Suggestions for future work are offered in Section 7.5, followed in Section 7.6 by parting thoughts on the direction that future researchers might be inspired to take to confront severe uncertainty.

7.1 Review of Overarching Motivating Question

The work in this thesis was inspired by a relatively general question:

Overarching Motivating Question (OMQ): How should one represent and analyze severe uncertainty inherent in product life-cycle information to provide better decision support for environmentally benign design and manufacture?

While there may be numerous answers to this question, which depend on the definition of "severe uncertainty", the answer explored in this thesis was:

Answer to OMQ: A decision maker should apply info-gap decision theory to determine whether preferable design choices can be identified without requiring more information about the life-cycle than is available.

It has been demonstrated that IGDT offers a cheap, yet relatively coarse means for identifying what options are most immune to the effects of uncertainty of unknown size, under the condition that one is willing to settle for performance which is lesser, yet guaranteed. We advocate that it *should* be applied if all one has is a nominal estimate of unknown variability or unknown validity. The analysis effort involved is not significant and can reveal when an option will always provide the most robustness (and thus be preferable) or never provide enough (and be eliminated from further consideration). This means IGDT can be used to screen whether more information about uncertainty even needs to be collected or not. It was shown, for example in the remanufacturing example problem in Figure 6.3, that, if a relatively small sacrifice in demand for performance is made, designs can be identified that can withstand more error than the performance-optimal option. *This is arguably the most valuable type of finding from an info-gap analysis.* If this is the case, and if less than optimal performance—if guaranteed—is acceptable, one should take the more robust choice. This involves a satisficing focus on *survival*, rather than a risky demand for the optimal. Of course, one's definition of survival is based on subjective preferences; therefore, the ability to induce preferences for a trade-off, as enabled by plots of the robustness function, is vital. Thus, not only does an info-gap analysis reveal when decisions can be made using only sparse information (instead of subjective uncertainty estimates or other assumptions); it also postpones the formation of preferences for trade-offs between robustness and performance until they can be viewed directly on a trade-off plot.

Another argument for the use of IGDT is that the analysis it entails is accomplished rather cheaply. For systems with monotonic, continuous performance functions, the analysis involved does not require sophisticated search techniques to find maximum robustness. In fact, an easy-to-compute plot of system performance *R(q,u)* versus uncertainty size α can be directly viewed as a robustness function plot simply be reinterpreting its axes as r_c and $\hat{\alpha}$. (An example of this short-cut was shown in Section 6.2.1.3.)

In summary, IGDT is not guaranteed to reveal an obviously preferable option in cases where one does not exist, but the chance that it could lead to a decision without requiring more information makes it worth trying. This is especially true given the usually simple equations in EBDM problems, which make the cost of an info-gap analysis low.

7.2 Motivating Question 1 and its Corresponding Contribution

Though it has been demonstrated that IGDT *can* be effective, more examples characterizing when it is most useful are always needed. This motivated the first subquestion in the thesis:

<u>Motivating Question 1 (MQ1)</u>: How should we determine when IGDT will be most appropriate for supporting EBDM decisions?

<u>Answer to MQ1:</u> An evaluation of the usage conditions and decision support capabilities of IGDT, as well as the needs and characteristics of archetype EBDM problems, can be used to establish a set of guidelines for screening applicability.

Situations where IGDT is more useful are identified and evaluated qualitatively in discussions in Chapter 4. These situations are as follows:

- When the uncertainties involved fall into any of the categories or are of any of the types reviewed in Section 4.1.2, including: estimates of unknown validity; time-sensitive models; human behavior; unobserved socio-techno interactions; natural systems behavior; and composite uncertainties.
- When design preference rankings switch over the range of r_c. This happens when the design that is performance-optimal achieves robustness (as r_c is relaxed) at a slower rate than another design that it outperforms under no uncertainty. (For instance, see Figure 4.3.) An example of this scenario was seen in the form of a "pay-back" problem structure described in Section 4.1.3.2, seen in both the oil filter and remanufacturing examples.
- When a design problem involves selection between alternatives, as opposed to a continuous variable whose value must be specified. IGDT is more interesting for selection problems because the range of r_c over which different discrete alternatives dominate is wider and easier to weigh because there are distinct switch points. Selection problems of this type are more often in the earlier stages of design, where there is also usually less information available.
- When the measure of performance is a more direct indicator of value (e.g., monetary, or measurement of energy consumption) as opposed to a single-unit composite score of performance, which may not have much meaning by itself (see Section 4.4).

- When an understanding of error size is easier to interpret. For instance, it is easier to physically understand the size of the number of filters used over a lifetime than to understand the size of a single-unit score of environmental performance (e.g., from a database value in Section 4.3.2).

While these guidelines seem intuitively valid, they are also very general. It would be strong to label any of them "archetypes", they are more like starting points for reference. They are a useful start to characterizing when to use IGDT and what one might get out of an info-gap analysis. But, like any modeling or decision approach, expertise identifying when it will be useful is only gained when it has been applied to a wide number of problems. The applicability and usefulness of IGDT is still very context dependent. Reading Chapters 3 and 4 of this thesis equips a person with examples that could help them better recognize applicability when encountering new uncertainties, new performance function types, etc.

7.3 Motivating Question 2 and its Corresponding Contribution

The second subquestion was oriented towards the assessment of multiple uncertainties, a key demand for EBDM:

Motivating Question 2 (MQ2): For problems affected by multiple uncertainties, how should scaling factors be elicited in a rigorous fashion that allows for imprecision in those factors?

Answer to MQ2: One's beliefs about info-gap scaling should be elicited in the form of subjective probabilities, which are revealed through one's betting behavior. The method for eliciting subjective probabilities allows for imprecise expression of one's knowledge about scaling, which in some cases causes indeterminacy in the preference rankings for design alternatives.

It is demonstrated in Chapter 5 that subjective probabilities, when elicited through a rigorous betting method outlined in Section 5.3, could be used as a reference point for scaling. But is this more rigorous method better? This is difficult to test; in Section 5.3.1, it is explained that subjective probabilities are an *operationalized* way to measure belief about scale, and that this measure is transferable between different info-gaps. A decision maker can review the widely accepted theoretical foundations already laid out by Walley in the field of imprecise probabilities (Walley 1991) in order to decide whether they trust betting as a reference. Betting provides a more rigorous specification of scaling factors; we argue that any increase in rigor is worthwhile as long as the means of establishing that rigor is cheap. Specifically, the bracketing method (of Section 5.3.3.1) used to determine scaling factors in the betting scenario is easy to implement and places low cognitive demands on the person whose beliefs are being elicited.

The ability to represent imprecision in the Subject's beliefs, newly offered in this thesis, is a way to conservatively introduce new information into the process of scaling between multiple uncertainties. The propagation of this imprecision to ranges where preference ranking was indeterminate (the "interval of preferredness indeterminacy", for instance in Figure 5.15) follows a guiding theme of this thesis: to see whether or not decisions can be made using only limited information. No attempts were made to simulate or measure through experimentation how a real decision maker would react to intervals of indeterminate choice, however.

There are fundamental accuracy limitations to the general approach to scaling info-gaps that is used in this thesis. The technique of mapping all uncertainty parameter sizes onto a single gross proxy measure for the "overall" uncertainty (Section 5.2) is a rather coarse assessment of info-gap uncertainty. However, most aspects of the modeling of uncertainty as an info-gap are of rough accuracy. The investment of significant effort into any one aspect of the modeling or analysis is still prone to being overshadowed by the fact that the bounds on uncertainty are completely unknown. We contend, however, that the more rigorous means of eliciting scaling factors (Section 5.3.3) and assessing their imprecision (Section 5.2.4) is of a low enough cost to be warranted in practice. More information about info-gaps leads to robustness that is more consistent with the decision maker's understanding of uncertainty.

In summary, three new sub-contributions in the area of analyzing robustness to multiple uncertainties are presented in this thesis. All of these are mathematically general and can be applied to any IGDT problem, including non-EBDM ones. The contributions are:

- The concept (from Section 5.1.2) that an *interval of indeterminacy* (IPI) can exist when one is unable to scale different info-gaps with respect to each other, and that this interval is in some cases small enough to not require that scaling be specified. For selection problems, calculation of the size of this interval can scale up to $n>2$ info-gap uncertainties as described at the end of Section 5.1.2.

- The addition of rigor to the elicitation of scaling factors, but at a low cost, in Section 5.3. This technique also scales up to any number of info-gap uncertainties; the BBSE steps would simply be repeated for each uncertain variable. However, as acknowledged in Section 5.5.1, BBSE only applies to uncertain quantities, not uncertain models that assume a functional form.
- The ability, through the new rigorous method also based in betting, to represent imprecision when specifying scaling factors, in Section 5.4.4. Making decisions under scaling imprecision for $N>2$ info-gap uncertainties was explained to be possible in Section 5.4.5, but conjectured to lead to increasingly indeterminate preference rankings over design alternatives.

An additional smaller contribution is the systematic procedure for identifying when to apply these three multi-gap techniques, as outlined in the decision tree of Figure 5.8.

7.4 A Review of Validation Steps

The bulleted lists in Section 1.4 (separated for the two Motivating Questions) contained steps that were to be followed to (1.) verify capability gaps to be filled in the thesis and (2.) move towards validation of the new contributions meant to fill those capability gaps. The latter of those two types of steps are now summarized, recognizing what forms of validation were used, as well as what the major findings were. An overview for Motivating Question 1 is provided in Table 7.1.

Table 7.1 : Validation Steps and Findings for Motivating Question 1

Mode: Evaluation of IGDT and Review of Applicability to EBDM		
Thesis Section	Activity	Validation Type
3.4 & 3.5	Argued IGDT's internal logical consistency *Finding: judged to be sound*	Review of axioms and logic: for info-gap models, analysis, and decision rules
3.6	Compared IGDT to other uncertainty formalisms *Finding: direct comparison is infeasible, as each have different starting info requirements*	Critical Review: Compared requirements and assumptions of major uncertainty formalisms' models, analysis, and decision rules
4.1	Summarized when EBDM archetype characteristics are seen to match IGDT capabilities *Finding: initial list of usage guidelines, previously non-existent*	Critical Review: info-gap information requirements, and corresponding info-availability in selected EBDM scenarios
4.3	Applied info-gap to the basic oil-filter problem to find usefulness and limitations *Finding: Further guideline information*	Demonstration; Qualitative Discussion of plusses & minuses

To reiterate, the lack of any quantitative, directly comparative studies between info-gap and other uncertainty formalisms reflects the fact that IGDT is suited to an entirely unique class of problems. Thus, given any specific set of starting information about uncertainty in a decision problem, info-gap theory IGDT is not in direct competition with alternative uncertainty formalisms.

An overview for Motivating Question 2 is provided in Table 7.2.

Table 7.2 : Validation Steps and Findings for Motivating Question 2

Mode: Synthesis of Existing Techniques to Provide New Capabilities		
Thesis Section	Activity	Validation Type(s)
5.3.1	Evaluated the betting scenario used to elicit subjective beliefs	Critical evaluation of pre-existing work by Walley; Review of Walley's criteria for elicitation rigor
5.3.2	Defended subjective probability as a reference point for "mapping" the scales of different info-gaps	Logical explanation: pluses, minuses, and underlying assumptions of the technique reviewed
5.3.2	Grounded the new concept of imprecision in multi-info-gap scaling to the more rigorous, previously examined idea of imprecision in subjective probabilities	Critical discussion: pre-existing work by Walley (upper and lower previsions)
5.4.3	Tested bet-based scaling with precise elicitation. *Finding: assumed to better reflect the decision maker's info about uncertainty, but no solid proof due to the nature of severe uncertainty*	Demonstration: on the basic oil-filter problem and two uncertainties; Quantitative Comparison: to existing scaling methods that were applied in Section 5.2.2.1
5.4.4	Tested bet-based scaling with imprecise elicitation. *Finding: assumed to better reflect the decision maker's info about uncertainty, but no solid proof due to the nature of severe uncertainty*	Demonstration: on the basic oil-filter problem and two uncertainties
5.4.4	Proposed heuristics for assigning preference rankings given indeterminacy caused by imprecision in scaling elicitation	Demonstration; logical explanation
5.5.3	Reviewed assumptions and "correctness" of elicited scaling factors	Qualitative Discussion
6.2.2	Tested bet-based scaling with precise elicitation *Finding: indeterminacy (IPI) was negligibly small, so scaling was not needed.* *Follow-up: a sensitivity analysis technique was created and applied.*	Demonstration; Qualitative Discussion

To reiterate, no quantitative comparisons were used to measure the “value” of the new scaling elicitation techniques as compared to old methods. This is because, for the old methods, there is no explain where scaling information comes from, so the same starting information about scaling cannot be used as a way to compare both methods. It is assumed that any technique that incorporates new information, as long as it is elicited rigorously, will provide robustness to a more accurate representation of what a decision maker knows about uncertainty, thus making the outcomes of assessments more valuable.

7.5 Future Work

Three main areas for future work are discussed next: exploring further applicability to EBDM, testing IGDT on more complex problem types, and using IGDT as a component in a framework for uncertainty management.

7.5.1 Future EBDM Application Areas

Further understanding of the full range of applicability and potential limitations of IGDT in EBDM scenarios can be gained by trying out more problems. A clear set of principles that guide applicability and predict usefulness will probably not be practical. Rather, a wider set of example problems from which similar new problems could be identified is probably more useful. We note that in the initial phases of Ben-Haim’s establishment of info-gap theory, he paid close attention to theoretical foundations, all the while sticking to relatively academic problems. His more recent phase of research activities (as evidenced by recent publications, workshops, and seminars) has transitioned to trying to apply IGDT to as wide of a scope of problems as possible. Certainly there is much more ground to be covered in EBDM applications, as info-gaps specific to the ecosphere and valuesphere portions of Figure 2.2 have not yet even been seriously considered, even though they contain probably the most severe uncertainty.

7.5.2 Greater Problem Complexities

The examples in this thesis do not go beyond considering more than two info-gaps at a time. The BBSE techniques of Section 5.3 can scale up to accommodate more uncertainties, however, for imprecise scaling, the combined effects of imprecision can create intervals of ranking indeterminacy that are quite wide.

Of greater interest would be exploration of the practicality of considering multiple reward functions at once. There is already an established technique for accommodating multi-criterion problems (Ben-Haim 2006), but Ben-Haim has left explorations of its practicality as something for other researchers to explore.

Considering more complex performance functions, as well as info-gap uncertainties in models, could be explored. We expect that the computation of the robustness function $\hat{\alpha}(q, r_c)$ can be difficult for systems with non-continuous *R(q,u,)* functions. In this thesis, all of the problems have been solved using exhaustive searches. Future work would use more efficient search algorithms to generate robustness-performance frontiers.

7.5.3 Towards Using IGDT in a Larger Uncertainty Management Framework

IGDT certainly has its own place within the field of uncertainty formalisms. How could it compliment these other formalism, and what place would it serve in a larger framework for uncertainty management? IGDT could serve as a screening method, applicable in much earlier stages of uncertainty analysis. However, we do not see it playing as strong of a complimentary role as, e.g., probability bounds analysis plays for traditional probability theory.

7.6 Parting Thoughts

In this thesis it is shown how info-gap decision theory can be used to include severely deficient information into environmentally benign life cycle design. For two major examples, we have demonstrated the practicality of using minimal information (suspect nominal estimates and imprecise scaling factors) to make support decisions without needing to collect more information. In some cases, a preferable design option can be identified, in other cases there exists indeterminacy in preference rankings. Before an info-gap analysis, it is hard to distinguish which will be the case. However, this thesis demonstrates a variety of modes in which IGDT can be tested and extended, providing much needed examples and discussion in the IGDT research domain. It is hoped that future researchers will observe these steps and become inspired to take steps of their own.

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