Airline Integrated Planning and Operations

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Airline Integrated Planning and Operations

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SUMMARY

Large-scale optimization methods are commonly used in solving complex airline problems. The airline planning process is traditionally a sequential decision-making process, including schedule development, fleet assignment, aircraft maintenance routing and crew scheduling. In order to pursue more profitability, there are increasing needs to integrate planning functions and operational considerations into planning process. In response, the objective of this work is to integrate fleet assignment (FAM) with crew scheduling, and, in addition, to provide solutions robust to real time operations. The three challenges of this work are 1) to understand the influence of fleet assignment on the performance of crew scheduling; 2) to address the crew scheduling problem in the integrated model; and 3) to achieve robustness. To deal with these challenges, specific contributions have been made in the following five areas:

(1) Schedule analysis on crew friendliness

Schedule analysis is studied to evaluate the crew friendliness of a schedule for a given fleet. Two types of analysis methods are proposed. In the Global Analysis method, average duty time can be obtained, which is a key factor to evaluate performance of a pairing solution. In Pattern Analysis, typical and ideal duty patterns for each type of schedule are identified. Based on schedule analysis, guidelines are given for making small modifications on a given schedule when the analysis indicates problems. Schedule analysis also shows the importance and advantages in integrating fleet assignment and crew planning.

(2) Duty flow model for crew pairing problem

Schedule analysis demonstrated how the number of duties and duty patterns affect crew pairing solution. A duty flow model is proposed for solving the crew pairing problem. In this model, duties are chosen to partition the flights based on observations of duty features to constitute good pairing solutions from schedule analysis. The main objective is to reduce the number of duties. A duty network is created to connect duties into pairings. Although it is not designed to find the optimal crew pairing solutions, this model can efficiently find suboptimal legal pairing solutions. This makes it very promising in an integrated planning or real-time recovery framework.

(3) Robust crew scheduling method

A new robust crew scheduling method based on station purity is investigated. The method aims at increasing crew swapping options in operation by limiting the number of crew bases serving a given station. Spoke stations are classified into naturally pure spokes and mixed spokes. Spoke purity plans are generated for each spoke and used to formulate the robust crew scheduling model. Pairings are composed of hub-to-hub strings embedded in the spoke plans. Computational results show that with little or no extra cost, more robust crew pairing solutions can be obtained using this method. It is also found that the size and solution times of the robust crew scheduling model are much smaller than that of the traditional set partitioning model.

(4) Integrated fleet assignment and crew scheduling

Based on the insight of imposing station purity, fleet purity and crew base purity are combined, and an integrated fleet assignment and crew robust planning approach is proposed. The impacts of crew base purities and fleet purities on FAM profit, crew scheduling, and computational efficiency are investigated. It is found that adding crew base purity can avoid locked rotations in FAM solutions, and significantly reduce the solution time. By imposing station purity, an integrated planning model which integrates fleet assignment and crew connections is proposed. Computational results show that this method can work efficiently in solving industrial size problems.

(5) Airline integrated recovery method

The approach of integrating fleet assignment and crew scheduling can also be applied to airline integrated recovery. The major challenge in integrated recovery is to stop the ripple effect caused by disruptions. We attempt to address the recovery scope in an integrated recovery framework. The main strategy is to define different recovery sets for schedule change, aircraft rerouting, and crew rerouting. Based on this recovery scope, a new integrated recovery model and Bender's decomposition solution approach is proposed. In the integrated recovery model, the duty flow model is combined with FAM in the master problem. The duty network is now built in a dated version and is crew-specific because of the features of recovery. Instead of enumerating aircraft routings, a multicommodity network flow model is adopted to model the aircraft maintenance routing, which can reduce the number of variables without losing maintenance considerations. In Bender's decomposition method, feasibility cuts coming from aircraft maintenance routing problems are generated and returned to the master problem.

CHAPTER 1

INTRODUCTION

1.1 Airline planning and operation process

Airline schedule planning stage includes a sequence of decision-making phases:

(1) Schedule Design – Build flight schedule structure consisting of routes and frequencies to attract the profitable market, and schedule flights to meet these routes and frequencies;

(2) Fleet Assignment (FAM) – Determine what type or size of aircraft should be assigned to each flight in order to maximize profit;

(3) Aircraft Maintenance Routing – Determine how to route aircraft to ensure the satisfaction of maintenance requirements;

(4) Crew Scheduling – Determine which crew need to be assigned to each flight in order to minimize crew cost.

After the fleet assignment is determined, the yield/revenue management process maximizes revenue by allowing an airline to selectively accept and reject reservation requests based on their relative value.

Because of the complexities of each problem and the limitations of computer resources, the above planning process is currently done sequentially. At the operation stage, the same situation remains. Recovery actions to get back to the original schedule upon disruptions include (1) Change the flight schedule by delaying or cancelling flights; (2) Refleet and/or reroute the aircraft to cover the disrupted schedule; (3) Reroute the crew to cover the modified schedule and match the aircraft refleeting change; (4) Re-accommodate disrupted passenger itineraries.

By making decisions sequentially, there is limited information shared and little interaction between the processes. However, the downstream decisions have big impacts on the upstream decisions. The benefits of integrated planning and operation will change the decision making process in airlines. Integration over functions as well as the timeline has become an active research topic in the field of airline operations research.

Integrated planning and integrated recovery are intended to integrate the functional phases at planning stage and operation stage respectively. Robust planning is to make decisions beneficial to the operations at the planning stage. It can be considered as integration over the timeline. Since a complete model integrating all functions or integrating over both functions and timeline is not computationally attainable, alternative integrating strategies are investigated to achieve partial integration. These integrating strategies include (1) integrating schedule design and fleet assignment (Rexing et al. 2000, Lohatepanont and Barnhart 2004); (2) integrating fleet assignment and aircraft maintenance routing (Clarke et al. 1996, Barnhart et al. 1998b); (3) integrating aircraft routing and crew pairing (Klabjan et al. 2002, Cohn and Barnhart 2003, Cordeau et al. 2001, 2005); (4) integrating fleet assignment and crew scheduling (Clarke et al. 1996, Barnhart et al. 1998a, Sandhu and Klabjan 2004); (5) integrating fleet assignment and revenue management (Jacobs et al. 1999, Barnhart et al. 2002b, Smith 2004). In parallel, research on robust planning include (1) robust fleet assignment (Ageeva 2000, Rosenberger 2004, Smith 2004, Johnson 2005); (2) robust aircraft routing (Kang and Clarke 2003, Lan et al. 2003); (3) robust crew scheduling (Yen and Birge 2006, Schaefer et al. 2005, Ehrgott and Ryan 2002, Chebalov and Klabjan 2006). Work on integrated recovery includes the integrated recovery framework and a Bender's decomposition scheme proposed by (Lettovsky 1997).

1.2 Overview of the contents

The objective of our research is to develop efficient robust/integrated planning and integrated recovery models and their corresponding solution methodologies. Our focus is to integrate fleet assignment and crew scheduling, and, in addition, to provide solutions robust to real-time operations. The three challenges of this work are 1) to understand the influence of fleet assignment on the performance of crew scheduling; 2) to address the crew scheduling problem in the integrated model; and 3) to achieve robustness. This dissertation discusses how to solve these challenges in the following chapters.

In Chapter 2, schedule analysis is studied to evaluate the crew friendliness of a schedule for a given fleet. Two types of analysis methods are proposed. In the Global Analysis method, average duty time can be obtained, which is a key factor to evaluate the performance of a pairing solution. In Pattern Analysis, typical and ideal duty patterns for each type of schedule are identified. Based on schedule analysis, guidelines are given for making small modifications on a given schedule when the analysis indicates problems. Schedule analysis also shows the importance and advantages in integrating fleet assignment and crew planning.

In Chapter 3, a duty flow model for solving the crew pairing problem is proposed. Schedule analysis demonstrated how the number of duties and duty patterns affect crew pairing solution. A duty flow model is proposed for solving the crew pairing problem. In this model, duties are chosen to partition the flights based on observations of duty features to constitute good pairing solutions from schedule analysis. The main objective is to reduce the number of duties. A duty network is created to connect duties into pairings. Although it is not designed to find the optimal crew pairing solutions, this model can efficiently find suboptimal legal pairing solutions. This makes it very promising in an integrated planning or real-time recovery framework. An integrated fleet assignment and crew scheduling model based on the duty flow model is constructed and used to solve the case study problem in Chapter 2, profit improvement is obtained.

In Chapter 4, a robust crew scheduling method is discussed. A new robust crew scheduling method based on station purity is investigated. The method aims at increasing crew swapping options in operation by limiting the number of crew bases serving a given station. Spoke stations are classified into naturally pure spokes and mixed spokes. Spoke purity plans are generated for each spoke and used to formulate the robust crew scheduling model. Pairings are composed of hub-to-hub strings embedded in the spoke plans. Computational results show that with little or no extra cost, a more robust crew pairing solutions can be expected. It is also found that the size and solution times of the robust crew scheduling model are much smaller than that of the traditional set partitioning model.

In Chapter 5, integrated fleet assignment and crew robust planning is studied. Based on the insight of imposing station purity, fleet purity and crew base purity are combined, and an integrated fleet assignment and crew robust planning approach is proposed. The impacts of crew base purities and fleet purities on FAM profit, crew scheduling, and computational efficiency are investigated. It is found that adding crew base purity can avoid locked rotations in FAM solutions, and significantly reduce the solution time. By imposing station purity, an integrated planning model which integrates fleet assignment and crew connections is proposed. Computational results show that this method can efficiently solve industrial size problems.

In Chapter 6, airline integrated recovery method is discussed. The idea of integrating fleet assignment and crew scheduling can also be applied to airline integrated recovery. The major challenge in integrated recovery is to stop the ripple effect caused by disruptions. We attempt to address the recovery scope in an integrated recovery framework. The main strategy is to define different recovery sets for schedule change, aircraft rerouting, and crew rerouting. Based on this recovery scope, a new integrated recovery model and Bender's decomposition solution approach is proposed. In the integrated recovery model, the duty flow model is combined with FAM in the master problem. The duty network is now built in a dated version and is crew-specific because of the features of recovery. Instead of enumerating aircraft routings, a multicommodity network flow model is adopted to model the aircraft maintenance routing, which can reduce the number of variables without losing maintenance considerations. In Bender's decomposition method, feasibility cuts coming from aircraft maintenance routing problems are generated and returned to the master problem.

Chapter 7 concludes this dissertation by providing a brief discussion on the benefits of the various models developed and avenues for future research in the area of integrated planning and recovery in airlines.

CHAPTER 2

SCHEDULE ANALYSIS ON CREW FRIENDLINESS

The crew pairing problem is to construct a set of pairings that cover the schedule of a specific aircraft fleet at minimum cost. After solving numerous crew pairing problems, we found that for some problems, very good pairing solutions with low pay and credit can be obtained, while for the others, the pairing solutions are poor and hard to be improved by means of computational efforts. Therefore, it is necessary to explore the bottleneck for getting good pairing solutions.

Two types of analyzing methods, called global analysis and pattern analysis respectively, are developed. In global analysis, we attempt to get the number of planes and crews needed to overnight at each station. Then, we estimate the number of duties needed in the pairing solution to obtain the information about average duty time. By comparing the average duty time with the parameter of average duty guarantee in the pairing cost function, we are able to evaluate performance of the crew solution. In pattern analysis, by further analyzing the two different types of crew overnights, i.e., legal short rests and midday breakouts, we can get the patterns of duties and pairings, as well as the number of pairings needed in the pairing solution. Ultimately, the pattern analysis can provide clues to repair the schedule in a reasonable range for improving crew friendliness.

Based on schedule analysis, we found two main types of schedules that may have difficulties to find good pairing solutions. Although both types can have very low average duty time, they have different duty patterns. For one type, the number of duties required by crew overnights is much greater than the number of duties required by plane rotation. For the other type, vice versa, the number of duties required by plane rotation is much greater than the number of duties required by crew overnights. Thus, a balanced version would be preferable in terms of crew friendliness. A methodology to repair both types of schedule is proposed. In addition, our schedule analysis shows the advantage of integrated FAM and crew planning, which gives a balanced schedule for each aircraft fleet.

This chapter is organized into five sections. Section 2.1 reviews the fundamental concepts of crew pairing. Section 2.2 introduces the two schedule analysis methods. Section 2.3 discusses the method to repair the different types of schedule. Section 2.4 presents the results of case studies and highlights the advantages and importance of integrated FAM and crew planning. Section 2.5 summarizes our contributions in this chapter.

2.1 Fundamentals of crew pairing problem

In our study, we consider the domestic daily crew pairing problem, that is, to construct a set of pairings that cover the collection of domestic flights flown by a given aircraft type in one day at minimum cost. A crew pairing is a sequence of flights that starts at a crew base and ends at that same crew base. It can span several days in which crew members will rest, usually overnight, at some location other than where they reside. The periods between rests in a pairing are called duties. Duties can be thought of as a day's assignment. Note however that a duty may last only a few hours and a rest can occur in the middle of the day.

The Federal Aviation Administration (FAA) and contractual restrictions define the structure and cost of legal duties and pairings. A duty begins with a briefing, typically 45 minutes, and ends with a debriefing, typically 15 minutes. These times are part of the duty and must be counted in the elapsed time of the duty. Each duty period must satisfy work rules limiting the maximum number of flights in a duty period, the maximum flying time per duty period, the maximum elapsed time of a duty period, and the minimum and maximum sit time between consecutive flights in a duty period. The cost of a duty, expressed in hours is the maximum of three quantities: the actual flying time in the duty period; a fraction, coeff1, times the elapsed time of the duty; and a guaranteed minimum number of hours, called minimum duty guarantee. The cost can be expressed as:

$$\text{duty cost} = \max\{\sum \mathit{blocktime}, \text{coeff1*elapsed time, min duty guarantee}\}. \quad (2.1)$$

Legal pairings may be composed of up to a maximum number of duties. A pairing must allow a minimum number of hours of rest between duties. A complicated FAA rule for pairings is the so called “8-in-24”. For instance, if the flying time in 24 consecutive hours is larger than 8 hours, while the consecutive hours of rest in that 24 hours is less than 9 hours, than a compensatory rest of at least 10 hours is needed for the next rest. Modeling the problems on a daily basis asks no leg repetition in a pairing, since each flight in each pairing must be covered exactly once for each day. The cost of a pairing in hours is the maximum of three quantities: the sum of the costs of the individual duties that make up the pairing; a fraction, coeff2, times the total elapsed time of the pairing, TAFB – Time Away From Base; and an average duty guarantee times the number of duties in the pairing. The cost can be expressed as:

$$\text{pairing cost} = \max\{\sum \mathit{duty}\cos t, \text{coeff2*TAFB, \#duties*average duty guarantee}\}. \quad (2.2)$$

The quality of a crew pairing solution is evaluated by pay and credit, defined as:

$$\text{pay\&credit} = \frac{\sum pairing\cos t - total\ blocktime}{total\ blocktime}. \tag{2.3}$$

Because of the cost structure for duties and pairings, a lower bound on the cost of a given schedule is the total number of hours of flying in the schedule. Pay and credit tells the percentage of excess cost for the pairings. In this chapter, reasons intrinsic to schedules for causing pay and credit will be explored.

2.2 Analyzing methodology

Anbil et al (1992) pointed out three main causes of pay and credit for pairings: (1) long or frequent sits within a duty; (2) long overnight rests between duties, and (3) “deadheading”. According to the definitions of duty and pairing cost in Equation (2.1) and (2.2), long sits and long overnight rests will result in a domination of the second items in the cost definitions. Given a set of schedules, it would be beneficial to observe the station activities for determining the maximum sit time and the number of long overnight rests needed to connect the flights. If long sit time or long overnight rests are not allowed, deadheading would be necessary. Usually it is relatively easy to identify the reasons that cause poor crew solutions when the second item dominates the crew cost and results in high pay and credit. When the third item dominates the crew cost, however, it is difficult to understand the reason just from observing the schedule. The following analysis methods are proposed due to this situation.

2.2.1 Global analysis

According to the cost function defined in Equation (2.2), in a poor pairing solution, we frequently found that the average duty guarantee dominates among the three items.

Thus, it becomes crucial to know the average duty time that a schedule actually has. For getting this, we need to estimate the number of duties. There are two ways to estimate the number of duties.

In the first method, the minimum number of planes needed to fly these flights can be calculated from the schedule. Dividing total blocktime by the minimum number of planes gives the plane average flying time. If the duration of a plane rotation is long, e.g. greater than 8 hours (the maximum blocktime for a duty period), at least two duties are needed for one plane rotation. Generally, large plane average flying time means long plane rotations. In such case, 2*{the minimum number of planes} gives a good estimation of the number of duties. If the plane average flying time is small, it is possible that the duration of some plane rotation is less than 8 hours, and the plane rotation can be covered by a single duty. In this case, we can determine the plane rotation by First-In-First-Out heuristic, and obtain an estimated number of short plane rotations. By taking this approach, a more precise estimation of the number of duties can be obtained. We denote this first estimation of the number of duties as NumDuty(Aircraft).

In the second method, from the schedule and legality rules about the overnight rest time, we can find the minimum number of crew overnight rests needed at each station. Then, the total number of crew overnight rests can be calculated. Knowing the maximum length of a pairing, we can obtain the number of duties from the number of crew overnights. For instance, the maximum length of pairing is 3 means that a pairing consists of at most 3 duties, as well as 2 overnight rests. The value of {the total number of crew overnights}*3/2 is the second way to estimate the minimum number of duties needed. We denote this second estimation of the number of duties as NumDuty(Crew).

The number of duties needed should take the maximum of these two estimations. That is, NumDuty=max{NumDuty(Aircraft),NumDuty(Crew)}. Dividing the total blocktime by NumDuty will give us the average duty time. If the average duty time from the schedule is much smaller than the average duty guarantee in the pairing cost function, a poor pairing solution will be anticipated.

The difference between these two estimations generates two different types of schedule. For the first type, called **Type I schedule**, the number of duties required by crew overnights is much greater than the number of duties required by plane rotation, i.e., NumDuty(Crew)>>NumDuty(Aircraft). For the second type, called **Type II schedule**, the number of duties required by crew overnights is much smaller than the number of duties required by plane rotation, i.e., NumDuty(Crew)<<NumDuty(Aircraft). Both types could cause poor performance of the pairing solution. A balanced version would be preferable in terms of the crew friendliness. Examples for both types will be illustrated in case studies in Section 2.4.

2.2.2 Pattern analysis

Based on the information of crew overnight rests and the number of planes overnighting at the crew bases, we can analyze the patterns of duties and pairings. First we classify the starting and ending styles of duties. From duty styles, we can obtain the pairing patterns.

Crew overnight rests consist of legal short rests, midday breakouts, and double overnight rests. Legal short rest means that an evening arrival flight, A, legally connects to a morning departure flight B. Double overnight rest means that if there is not enough rest time between A and B, and there is no midday breakout choice, the crew has to wait for one more day to leave. Midday breakouts mean that if there is not enough rest time between A and B, instead of a double overnight, it would be better for the pairing solution that a pair of midday flights (arrival flight C and departure flight D) is used to connect with A and B. In other words, one crew arrives in the middle of the day via C and leaves the next morning via B, the other crew arrives in the evening via A and leaves in the middle of the next day via D, as shown in Figure 2.1. It also shows that a pair of midday breakout gives a pair of rests of style AM-AM and PM-PM simultaneously. In contrast, a legal short rest is in style of PM-AM. In this chapter, we omit the analysis about double overnight rests since we discuss the cases in which the third cost item becomes dominant.

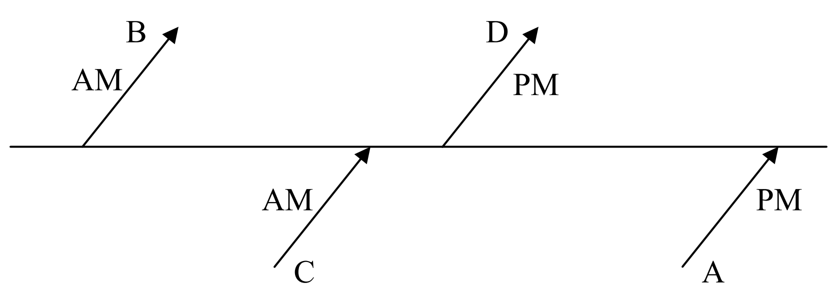


Figure 2.1: Midday breakouts

Table 2.1 gives the notations related to the definition of duty patterns. In Table 2.1, the AM or PM starting and ending of a duty is not the absolute morning flights or evening flights. For crew-base stations, AM or PM is dependent on whether it is flown by the overnight plane. For non-crewbase stations, AM or PM is determined by whether it is flown by the overnight plane, as well as by how the midday breakout pair of flights is chosen. We assume that for the crew, there is no overnight rest allowed at other crew bases. Then, a duty can only have Rest(AM)-start after a legal short rest PM-AM, or the AM-AM rest in a midday breakout. So, the number of duties that have Rest(AM)-start is:

#Rest(AM)-start = # midday breakouts + # legal short rests (2.4)

Similarly, we can have the followings:

#Rest(PM)-start = # midday breakouts (2.5)

#Rest(PM)-end = # midday breakouts + # legal short rests (2.6)

#Rest(AM)- end = # midday breakouts (2.7)

Table 2.1: Notations related to define duty patterns

Notation	Meaning
CB(AM)-start	Duty starts from CB using the overnight plane
CB(PM)-start	Duty starts from CB without using the overnight plane
Rest(AM)-start	Duty starts from other stations using the morning flights
Rest(PM) -start	Duty starts from other stations using the afternoon flights
CB(AM)-end	Duty ends at CB without using the overnight plane
CB(PM)-end	Duty ends at CB using the overnight plane
Rest (AM)-end	Duty ends at other stations using the morning flights
Rest (PM)-end	Duty ends at other stations using the afternoon flights.

For crew-base stations, since there is no overnighting crew, a duty starts at a crew base can either use the overnight plane or wait for an inbound plane. According to the convention we defined for AM/PM at crew bases, we have,

#CB(AM)-start = # planes overnight at the crewbase (2.8)

#CB(PM)- end = # planes overnight at the crewbase (2.9)

The total number of pairings is #CB(AM)-start + #CB(PM)-start. The total number of duties is SUM{#Rest(AM)-start, #Rest(PM)-start, #CB(AM)-start, #CB(PM)-start}.

2.2.2.1 Duty pattern for Type I schedule

A typical duty pattern for Type I schedule is shown in Figure 2.2. In Type I schedule, the number of duties is greater than 2*#planes. Thus, besides the "half-day" duties, we may expect some very short duties, illustrated as dash-dot lines in Figure 2.2.

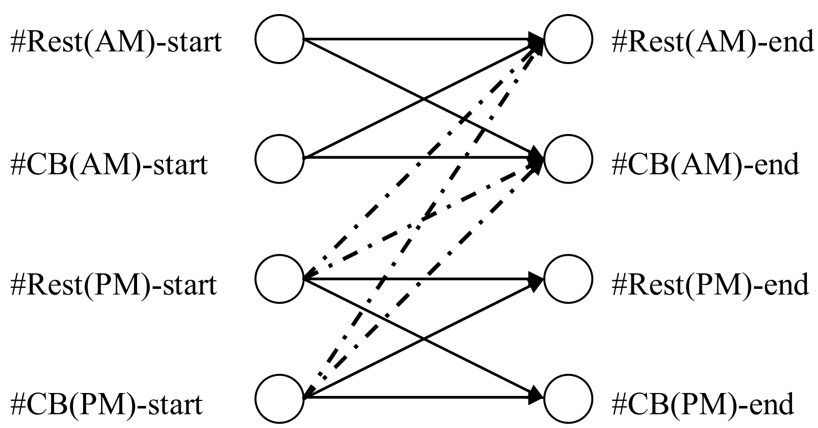


Figure 2.2: Typical duty pattern for Type I schedule

2.2.2.2 Duty pattern for Type II schedule

A typical duty pattern for Type II schedule is shown in Figure 2.3. In Type II schedule, due to short plane rotations that can be covered by a single duty, there will be AM-PM duties, as illustrated by dash-dot lines in Figure 2.3.

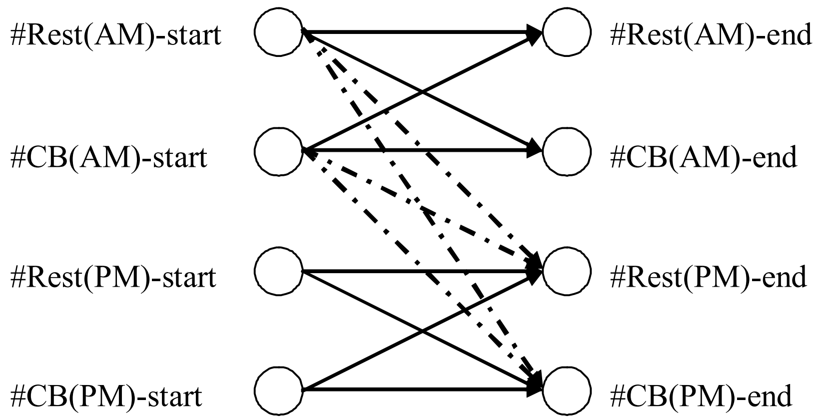


Figure 2.3: Typical duty pattern for Type II schedule

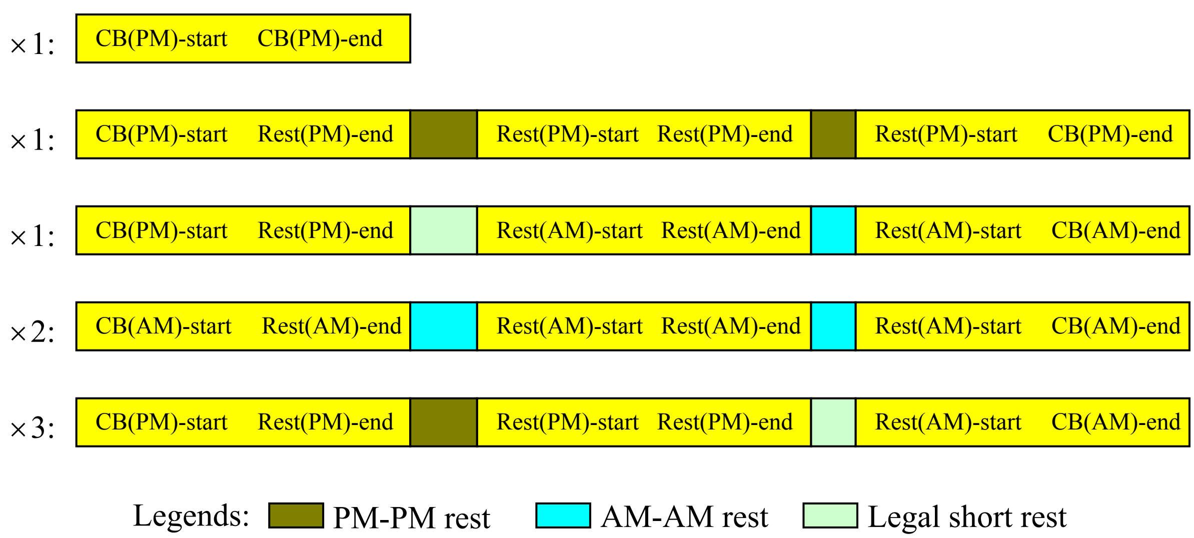


Figure 2.4: An example of pairing patterns

2.2.2.3 Pairing pattern

By extending the duty pattern into layers, pairing pattern can be obtained. Figure 2.4 shows an example of pairing patterns. The maximum length of a pairing is 3 duties. The total number of legal short rests and midday breakouts, as well as the number of different types of duties should be consistent with those in the duty pattern.

2.3 Schedule adjustment

If the pairing solution is poor, it is very likely that the schedule has very low average duty time. To improve, it is necessary to reduce the total number of duties. It is noted that considering sequential decision-making in airline planning, a reduced number of duties would also benefit the crew rostering process by making the days off requirements for crew easier to satisfy. For the two different types of schedule, we have two corresponding methods to adjust.

2.3.1 Type I schedule

From the duty pattern of Type I schedule, it is observed that there are very short PM-AM duties. To reduce the number of duties, we want to eliminate such short duties. Figure 2.5 shows the ideal duty pattern for Type I schedule. From Figure 2.5, we have the following formula,

pairings = # legal short rests + 2 * # planes overnight at CB (2.10)

Usually it is needed to increase items at the right hand side: {# legal short rests} and {# planes overnight at CB}. We anticipate the crew friendliness of the schedule be improved by retiming the schedule in a small range, as well as by adding or dropping a small number of legs.

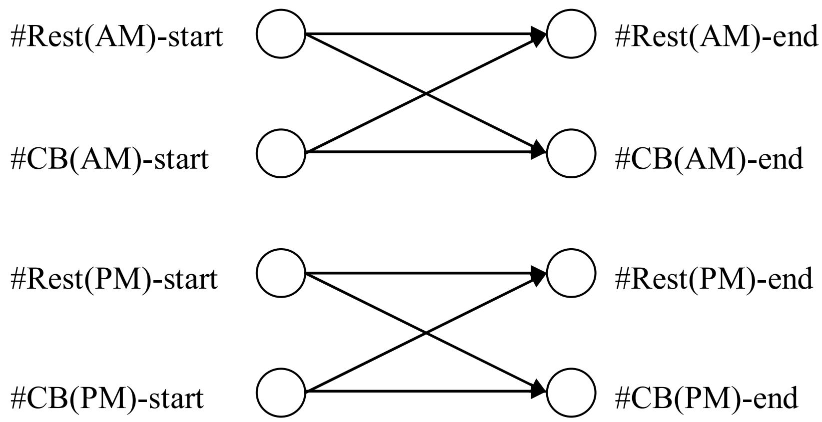


Figure 2.5: Ideal duty pattern for Type I schedule

At the station where we want to change a midday breakout to legal short rest, we can try to make the early evening arrival flight arrive earlier, and/or make the late morning departure flight depart later. However, we avoid changing the number of planes overnight at this station.

If there are long sits or long rests, we may add a pair of flights in between. An extreme example is the lonely double overnight. At the station where we want to move a plane overnight to some crew base, we can add 2 legs. Let one leg depart after the early evening arrival flight, at the same time, let the other leg arrive before the late morning departure flight, as shown in Figure 2.6. This will not save the total number of crew overnights, but it will not add crew overnights either. Since adding new legs may violate crew base balance constraint, it might be necessary to drop some legs.

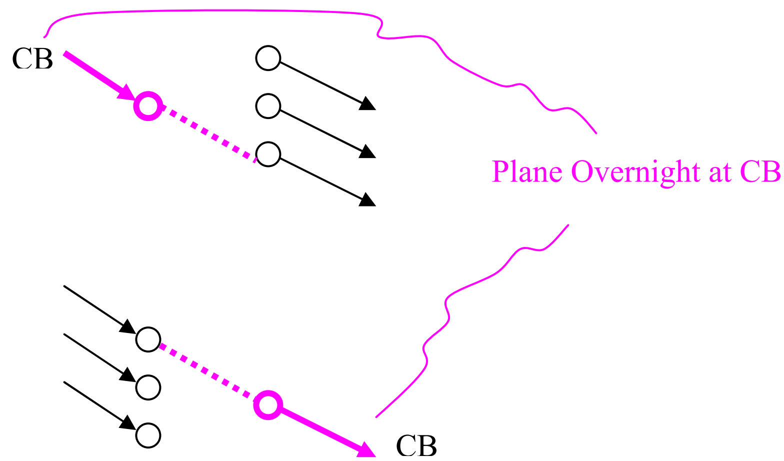


Figure 2.6: Move plane overnight to crewbase by adding new legs

2.3.2 Type II schedule

For Type II schedule, to reduce the number of duties, we want to increase the number of short plane rotations so that a plane rotation that needs at least two duties can be covered by one duty. Specifically, we want to have more duties with (AM)-start and (PM)-end. The ideal duty pattern for Type II schedule is similar to the typical pattern for Type II schedule as shown in Figure 2.3. But, the total number of duties and the number of duties with (AM)-start and (PM)-end will be reduced. It can be achieved by retiming some flights. A good try will be to move the first flight in the plane rotation later, also move the last flight in the plane rotation as early as possible so as to shorten the plane rotation.

2.4 Case study

A set of 164 flight legs for two fleet types is analyzed, in which Fleet 1 schedule has 102 legs, Fleet 2 schedule has 62 legs. Using our schedule analysis, these two subsets of schedule are classified as Type I schedule and Type II schedule respectively. Schedule adjustment methods are investigated for both schedules.

2.4.1 Fleet I schedule

In Fleet I example, Table 2.2 shows that NumDuty(Crew) = 26, NumDuty(Aircraft) = 22, and NumDuty(Crew) > NumDuty(Aircraft). According to the previous definition, this is a Type I schedule. The number of duties needed for the original schedule is 26, which determines the average duty time to be 238 minutes, much smaller than the average duty guarantee of 270 minutes. This explains the high pay and credit of the optimal pairing solution for the original schedule. On the other hand, 26 duties implies at least 9 pairings in the pairing solution, provided that the maximum length of pairing is three. The original Fleet I schedule cannot produce the ideal duty pattern shown in Figure 2.5 since

legal short rests + 2 * # planes overnight at CB = 3+2*1 = 5.

The original schedule has a small number of planes overnighting at the crew base, and a relatively large number of crew midday breakouts. To improve, it is necessary to decrease the number of duties, increase the number of planes overnighting at crew bases, and increase the number of legal short rests. By changing the midday breakouts to legal short rests, we can increase the number of legal short rests, as well as reduce the total number of crew overnight rests, resulting in reduced number of duties. In addition, the method of adding legs as shown in Figure 2.6 can be adopted to move one plane overnight from one station to the crew base. To improve the original Fleet I schedule, two legs are added (creating one more crew overnighting at the CB and one less midday breakout), and one leg is retimed (removing another midday breakout). Solving the crew pairing problem for the new schedule, zero pay-and-credit can be achieved. Clearly, the adjustments made a tremendous improvement. Even with the cost for the two newly added legs, the pairing cost for the new schedule is still significantly smaller than the original cost. In addition, this adjustment did not violate the crew base balance constraints.

We compare the new schedule with the original schedule in terms of duty and pairing patterns. The comparison of the duty patterns is shown in figure 2.7. The new schedule achieved the ideal duty pattern for Type I schedule. Therefore, it can be concluded that the duty pattern illustrated in Figure 2.5 is appealing indeed.

Table 2.2: Fleet I schedule analysis

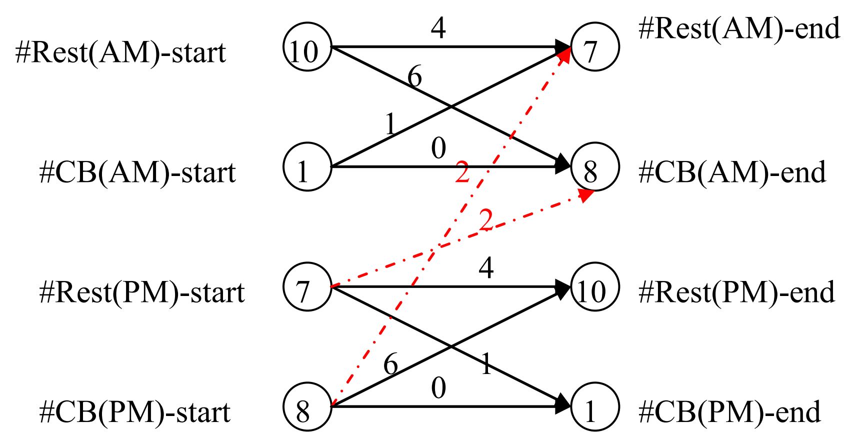
(Original schedule / New schedule)

Stations	Crewbase?	# planes overnight		# crews overnight		# midday breakouts		# legal short rests	
FAT	---	2	2	3	3	1	1	1	1
LAX	Yes	1	2	---	---	---	---	---	---
MRY	---	2	2	3	3	1	1	1	1
PSP	---	0	0	0	0	0	0	0	0
SAN	---	2	1	4	2	2	1	0	0
SBA	---	2	2	4	3	2	1	0	1
SBP	---	2	2	3	3	1	1	1	1
Total		11	11	17	14	7	5	3	4

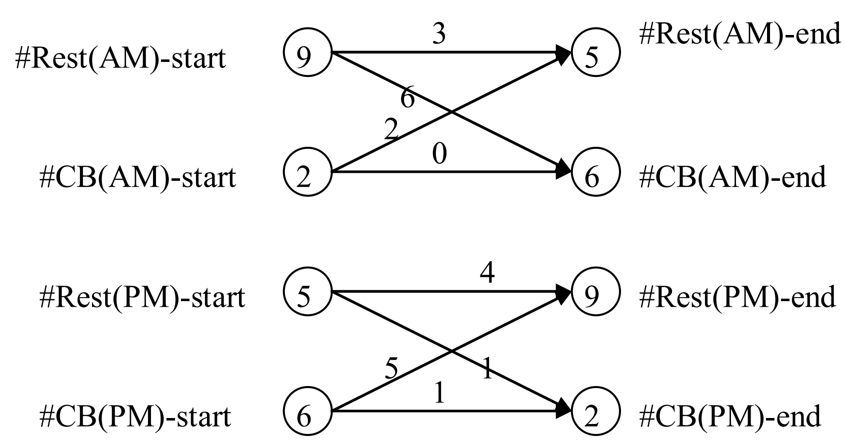
Plane average flying time = Total blocktime / # planes = 9.382 (hrs)

NumDuty = max{ NumDuty(Aircraft), NumDuty(Crew) } = max{22,26} = 26
NumDuty = max{ NumDuty(Aircraft), NumDuty(Crew) } = max{22,21} = 22

Average duty time = Total blocktime/NumDuty = 238 min Average duty time = Total blocktime/NumDuty = 281 min	Average duty guarantee = 270 min	
Midday breakouts percentage:	41.2%	35.7%
Pairing solution cost in Pay & Credit:	13.37%	0.00%
Pairing solution cost in minutes:	7,020	6,302



(a) duty pattern for the original schedule



(b) duty pattern for the new schedule

Figure 2.7: Case study: Fleet I Schedule

2.4.2 Fleet II schedule

In this example, as shown in Table 2.3, NumDuty(Crew) < NumDuty(Aircraft). This is a Type II schedule. The average plane flying time is not large and there might be some long duties covering the whole plane rotations. So, NumDuty(Aircraft) is 17, instead of 18 (which is 2*#planes). The average duty time is comparable to the average duty guarantee. Correspondingly, the pay and credit of the original pairing solution is not very poor.

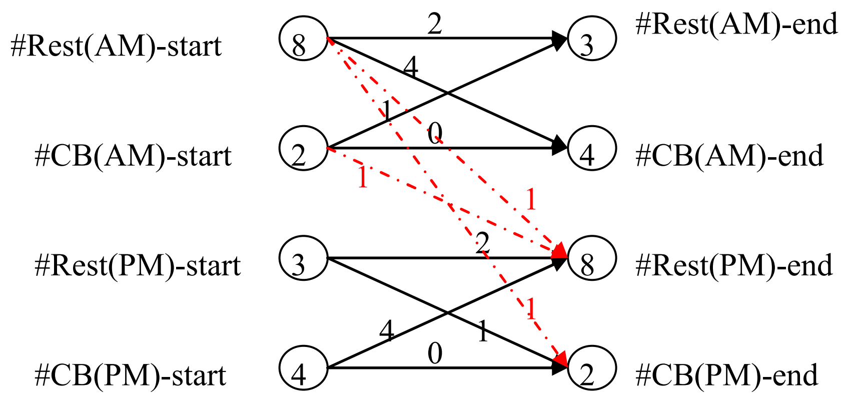
To improve, we make early evening arrival flights arrive earlier, and late morning departure flights depart later, resulting in more planes with shorter plane rotations that can be covered by one single duty. As a result, the number of duties can be reduced. Besides, a few small changes on departure times are made in order to create more day connections. The pay-and-credit is reduced from 2.19% to 1.65%. In addition, TAFB (Time Away From Base) in the pairing solution is reduced apparently after the schedule adjustment.

Table 2.3: Fleet II schedule analysis

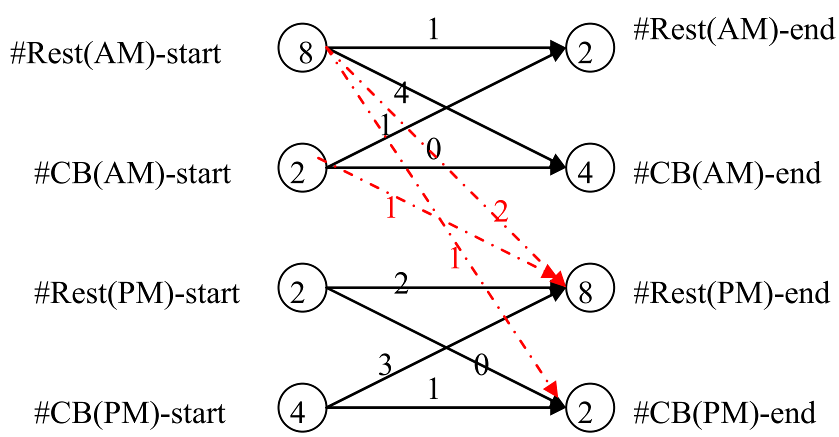
(Original schedule / New schedule)

	Crewbase?	# planes overnight		# crews overnight		# midday breakouts		# legal short rests	
LAX	Yes	2	2	---	---	---	---	---	---
SAN	---	1	1	1	1	0	0	1	1
SBP	---	1	1	1	1	0	0	1	1
SFO	---	1	1	1	1	0	0	1	1
SJC	---	3	3	5	5	2	2	1	1
SNA	---	1	1	1	1	0	0	1	1
Total		9	9	9	9	2	2	5	5
Plane average flying time = Total blocktime / # planes = 8.767 (hrs)									
# duties needed =17 (#crew RON in pairing solution = 11, #midday=3) # duties needed =16 (#crew RON in pairing solution = 10, #midday=2)									
Average duty time = 278 Average duty time = 296				Average duty guarantee = 270 min					
Midday breakouts percentage:				22.2%			22.2%		
Pairing solution cost in Pay & Credit:				2.19%			1.65%		
Pairing solution cost in minutes:				4,837			4,812		
Total TAFB of the pairing solution:				17,055			15,624		

Duty pattern of the original schedule for Fleet II are compared to that of the modified schedule in Figure 2.8. It can be seen that the number of duties with (AM)-start and (PM)-end is increased from 3 in the original schedule to 4 in the modified schedule. The total number of duties is decreased from 17 to 16 with the schedule adjustments.



(a) duty pattern for the original schedule



(b) duty pattern for the new schedule

Figure 2.8: Case study: Fleet II Schedule

2.4.3 Advantages of integrated fleet assignment and crew planning

Schedule analysis highlights the advantages and importance of integrated planning in two ways. First, it suggests that small schedule adjustments may have significant impact on crew scheduling. Second, by performing schedule analysis on combined schedule of different fleet types, it shows the necessity of integrating fleet assignment and crew planning. To demonstrate, we continue the case study by applying schedule analysis to the combined schedule of the two fleet types.

Table 2.4: Schedule analysis on combined schedule

		Fleet I			Fleet II			Combined		
	CB	# ron plane	# ron crews	mid-day	# ron plane	# ron crews	mid-day	# ron plane	# ron crews	mid-day
FAT	--	2	3	1				2	3	1
LAX	Y	1	---	---	2	---	---	3	---	---
MRY	--	2	3	1				2	3	1
PSP	--	0	0	0				0	0	0
SAN	--	2	4	2	1	1	0	3	5	2
SBA	--	2	4	2				2	4	2
SBP	--	2	3	1	1	1	0	3	3	1
SFO	--				1	1	0	1	1	0
SJC	--				3	5	2	3	3	2
SNA	--				1	1	0	1	1	0
Total		11	17	7	9	9	2	20	26	9
Plane ave. flying time		9.382 hrs			8.767 hrs			9.105 hrs		
# duties needed		Max{26,22}=26			17 between{14,18}			Max{40, 39} = 40		
Ave duty time		238			278			273		

Note that the characteristics of Type I and Type II schedules are opposite of each other. By integrating fleet assignment and crew planning, balanced resource utilization can be expected. In Table 2.4, the results show that the number of duties (i.e. 40) required by the plane rotation, and the number of duties (i.e. 39) required by crew overnights are very close. By further performing pattern analysis on the combined schedule, an ideal duty pattern for Type I schedule is achieved. Therefore, a more crew-friendly solution can be expected by integrating fleet assignment and crew planning.

2.5 Summary

Solving the crew scheduling problem is a time consuming process. However, schedule analysis, without intensive computational efforts, can evaluate the crew friendliness of a schedule and suggest how to improve when the performance is poor. The schedule analysis methods discussed in this chapter are most suitable to schedules in which the average duty guarantee is the dominating item among the three measures in the pairing cost function. If the TAFB item plays a dominant role in cost because of the existence of a large amount of long sits and lonely double overnights, to repair these schedules, we suggest reducing the lonely double overnights and fixing the long sits.

Two schedule analysis methods are proposed. In Global Analysis method, average duty time is obtained as a key factor to evaluate the performance of the pairing solution. Accordingly, two types of problem schedules are defined. In Pattern Analysis, Type I and Type II schedules are further discussed. The typical and ideal duty patterns for each type of schedule are identified. Based on pattern analysis, guidelines to repair each of the schedules are proposed. Case study confirms the efficiency of the methods. It also shows the importance and advantages to perform integrated fleet assignment and crew planning.

CHAPTER 3

DUTY FLOW MODEL FOR CREW SCHEDULING

3.1 Introduction

Schedule analysis demonstrates how the number of duties and duty patterns affect crew pairing solutions. In this chapter, a duty flow model for solving crew pairing problems is proposed, which chooses good duties to partition the flights. The main purpose is to find the least number of duties that can cover the flights and give a good pairing solution. A duty network is built to connect duties into pairings. Although this model is not designed to find the optimal crew pairing solution, it can efficiently find suboptimal legal pairing solutions, which makes it promising in an integrated planning or real-time recovery framework. The other advantages of this model include: (1) For some carriers, such as some airlines in Asia or some low-cost carriers, crew are not paid by pay and credit, but by days of work, which is roughly the number of duties. Duty flow model is more suitable for this cost structure than the traditional crew pairing model. (2) A reduced number of duties would benefit the crew rostering process by making the days off requirements for crew easier to satisfy, because rosters are based on days of work. Section 3.2 gives a brief review of the previous work on crew pairing scheduling optimization. The duty flow model is introduced in Section 3.3, with computational results discussed in Section 3.4. In Section 3.5, an integrated fleet assignment and crew scheduling model based on the duty flow model is constructed and used to solve the case study problem in Chapter 2, and profit improvement is obtained.

3.2 Previous work on crew scheduling

The crew scheduling problem is to find a minimum cost assignment of flight crews to a given flight schedule. The flight schedule considered includes all flight legs assigned to a single aircraft type, which is the output of the upstream decisions, i.e., schedule development and fleet assignment. The crew scheduling problem is typically broken into two sequentially solved subproblems, the crew pairing problem and the crew rostering problem. The crew pairing problem generates minimum cost pairings that cover the collection of flight legs in the schedule, while the crew rostering problem combines the pairings into month-long crew schedules and assigns the month-long plans to individual crew members. Usually the overall quality of a crew scheduling solution is evaluated by pay and credit of the crew pairing solution, which is the excess cost of crew beyond the required flying hours. The current research on the airline crew scheduling problem focuses on seeking more efficient solution approaches as well as integrating the crew problem with other scheduling problems.

A review of previous work on crew scheduling can be found in (Arabeyre et al. 1969), (Etschmaier and Mathaisel 1985), and (Barnhart et al. 2002a). More recent survey papers on overall airline scheduling process can be found in (Barnhart et al. 2003), (Clarke and Smith 2004), and (Barnhart and Cohn 2004).

Because of complicated restrictions, regulatory requirements, and nonlinear cost structures defined on legal duties and pairings, the set partitioning model is a powerful and efficient way to model and solve the crew pairing problem. There is one binary decision variable for each possible pairing, and the objective function is defined to minimize the cost of the selected pairings such that for each flight, exactly one pairing containing that flight is chosen. By defining pairings as variables, explicit formulation of complicated working rules can be avoided, also nonlinear pairing cost can be computed up in front so that nonlinearity can be excluded from the model. The set partitioning model for the crew pairing problem is formulated as:

$$\begin{aligned} \min \quad & \sum_{p \in P} c_p X_p, \\ s.t. \quad & \sum_{p: l \in p} X_p = 1, \quad \forall l \in L, \\ & X_p \in \{0,1\}, \quad \forall p \in P \end{aligned} \tag{3.1}$$

Where $X_p = 1$ if pairing p is in the solution, and 0 otherwise. L is the set of flight legs and P is the set of pairings. A column p has a 1 in row l if flight l is flown by pairing p, c_p is the cost of pairing p.

The drawback of this modeling approach is that there are potentially many billions of possible crew pairings, especially in hub-and-spoke networks. Pairings must be either enumerated or generated dynamically by column generation. Pairing enumeration can be accomplished by first enumerating all the possible duties for the schedule and then enumerating all the possible ways to form pairings from the duties. Both duty and pairing enumeration can be accomplished by a depth-first-search approach. A local optimization approach is adopted in (Anbil et al. 1991) and (Gershkoff 1989). Anbil et al. (1992) find an optimal solution over a large subset of the possible pairings to the LP relaxation. Five and a half million feasible pairings were enumerated. Hu and Johnson (1999) present compact storage scheme and primal–dual subproblem simplex method to speed up solving the linear programming relaxation problem. The other approach uses constrained shortest path methods on specially structured networks to price out attractive pairings, as in (Lavoie et al. 1988) and (Barnhart et al. 1994). Two alternative network representations, i.e., the time-space and the time-line network were investigated by (Barnhart et al. 1994). In both networks, for each duty there is one arc corresponding to it. The tail node of a duty arc represents the first flight in the duty and the head node of a duty arc represents the last flight in the duty period. Shaw (2003) propose hybrid column generation method which combines delayed column generation and compact storage for enumerated extended duties. To obtain the integer solution, a branch-on-follow-ons branching rule is typically adopted (Ryan and Foster 1981, Chu et al. 1997).

Vance et al. (1997) present a duty-period-based formulation and prove that its LP relaxation provides a stronger bound than the traditional set partitioning model does. In their formulation, the decision process is broken into two stages. First, a set of duty periods is selected to partition the flight segments. Second, a set of pairings is selected to partition these duty periods.

Barnhart et al. (1998a) develop an approximate duty-based model for the crew pairing problem. At the core of their model is a time-line network, called the duty network. The pairing cost is underestimated by the total time-away-from-base cost. For long-haul crew scheduling, the duty-based model has significantly reduced number of columns and is much easier and faster to solve than the conventional crew pairing problem.

The most recent research on crew scheduling includes integrating the crew problem with other airline scheduling problems. Barnhart et al. (1998a) investigate an integrated approximate model for fleet assignment and crew pairing optimization. Cohn and Barnhart (2003), Cordeau et al. (2001, 2005) present models integrating aircraft maintenance routing and crew scheduling. Clarke et al. (1996) take into account the crew factors in the fleet assignment model. Sandhu and Klabjan (2004) propose a model that considers fleeting, aircraft rotation, and crew pairing simultaneously. In their work, pairings are generated by delayed column generation and the aircraft rotation problem is captured by the plane count constraints.

An important research direction in crew planning is to integrate crew pairing and crew rostering into one crew planning problem that provides the best rosters, both from a cost and a quality point of view (Kohl and Karisch 2004).

3.3 Duty flow model for crew scheduling

Instead of considering crew scheduling as choosing pairings to partition the scheduled flights, we choose duties to partition the flights based on observations of duty features to constitute good pairing solution from schedule analysis. The main objective of this model is to reduce the number of duties. In Chapter 2, we have explained how the number of duties affects the quality of crew pairing solutions. In addition, the objective function of the duty flow model will penalize poor duties, in particular those that are short or have long sit-times.

Two different models were investigated. The first model formulates Duty Partition plus Duty Pattern. We apply the duty pattern result obtained from schedule analysis to the duty partition model by exactly limiting the number of various duty types, such as (Rest(AM)-start, Rest(AM)-end). The purpose is to study whether the duty pattern can help find a set of good duties that will eventually constitute a legal and good pairing solution. Computational results show that this model can give the right number of duties. Nevertheless, it does not guarantee the duty solution can be grouped into legal pairings. Therefore, it can be concluded that the ideal duty pattern is a necessary condition for the existence of a good pairing solution, but not sufficient for finding a good pairing solution.

The second model formulates Duty Partition plus Pairing Pattern, called **Duty Flow Model**. The selected duties are guaranteed to constitute a legal pairing solution by means of a duty connection network. Pairing legalities such as 8-in-24 rule are considered when constructing the duty connection network. The duty flow model is more precise in the sense of finding pairing solutions since it conforms to pairing pattern. This model is not designed to find the optimal crew pairing solution, but it can quickly find a suboptimal legal pairing solution, which makes it promising in an integrated planning or real-time recovery framework. In this section, we focus on introducing the duty flow model.

3.3.1 Generate smart duties

There are many ways to partition duties. In the duty flow model, the objective is to reduce the total number of duties in the pairing solution. The solution will naturally choose duties that start from the morning flights flown by overnighting aircraft and end at the evening flights which lead to aircraft overnights, if the elapsed time and flying time of these duties are legal. If not, it is preferred that one plane rotation is split into two duties, so that one of them includes the first flight in the plane rotation, while the other includes the last flight in the plane rotation. Consequently, most of the duties in a good pairing solution either start from the morning flights by overnighting aircraft or end at the evening flights which lead to aircraft overnight, or both. To find the corresponding morning flights and evening flights, we only need to examine the "overnight island". The concept of "islands" was first introduced by Hane et al. (1995) in the fleet assignment timeline network. We try to identify the morning beginning leg and evening ending leg from the island that includes plane overnight. In our case, this feature is applicable to both hubs and spokes.

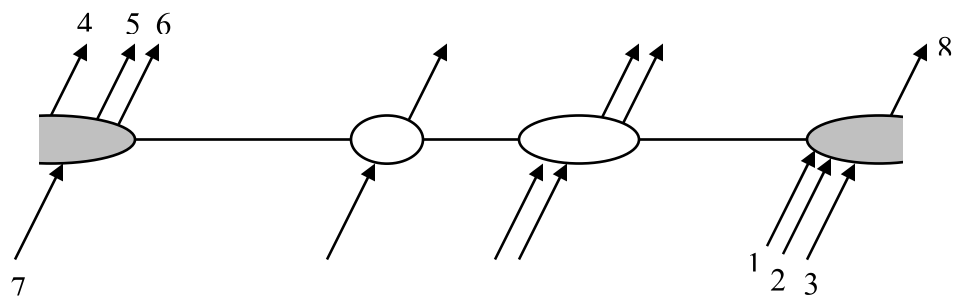


Figure 3.1: How to identify beginning and ending leg set

Figure 3.1 shows an example. The oval with shade represents the overnight island. There are two actual beginning legs and two actual ending legs since two airplanes overnight at this station. Leg 4 must be a beginning leg, while only one of leg 5 and leg 6 can be beginning leg, since one of them has connection with the incoming flight 7. Two of the legs among 1,2,3 will be ending legs, since one of them need to connect to leg 8.

3.3.2 Duty Connection Network

The duty connection network is constructed to ensure that the chosen duties can be grouped into legal pairings. Although the solution is about duties, there is a legal pairing solution embedded in the network. Three duty pairings are taken into account in this context. Pairings with more duties, say 4, can be constructed in a similar way.

The duty connection network depicted in Figure 3.2 contains three types of arcs: duty arcs, rest arcs and dummy arcs. There are 6 types of nodes: source node s, sink node t, crew base beginning leg nodes (CBiBEG), crew base ending leg nodes (CBiEND), rest of stations beginning leg nodes (RestBEG), and rest of stations ending leg node (RestEND). Using flights that start or end a duty instead of duties as node set greatly reduces the size of the network. From the enumerated duty set D, the node sets of CBiBEG, CBiEND, RestBEG, and RestEND can be easily obtained. The nodes of RestBEG and RestEND are shared by multiple commodities (crewbases), resulting in a multi-commodity network. The nodes are connected correspondingly by duty arcs, rest arcs and dummy arcs, where duty arcs connect CBiBEG with CBiEND, CBiBEG with RestEND, RestBEG with CBiEND, and RestBEG with RestEND; Rest arcs connect RestEND with RestBEG; Dummy arcs connect source/sink node to CBiBEG/CBiEND respectively.

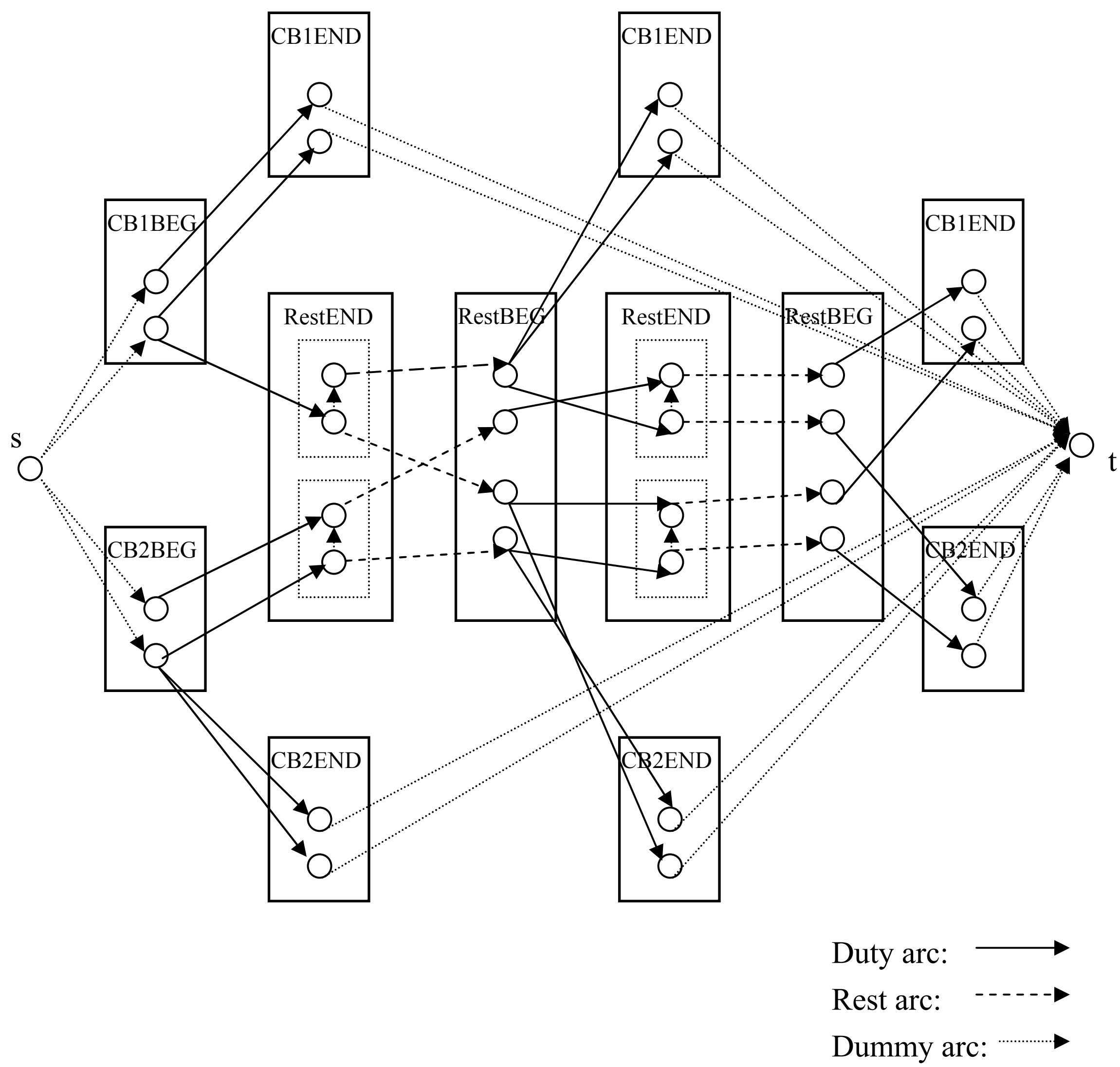


Figure 3.2: Duty connection network

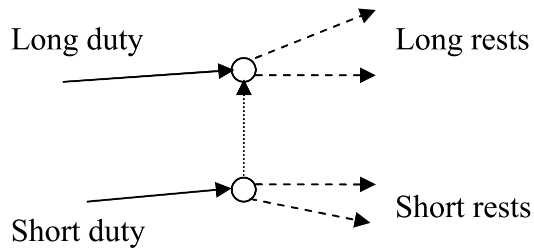


Figure 3.3: Method to avoid 8-in-24 violations

It is noted that there is a dummy arc lying between the leg node pairs in RestEND. This is designed to guarantee no 8-in-24 violations in the embedded pairing solution. Each leg in RestEND corresponds to two nodes. The first node connects to long overnight rest arcs, such as AM-AM rests or PM-PM rests in midday breakouts. The second node connects to short overnight rest arcs, such as PM-AM legal short rests. If a duty that ends at this leg is long, the corresponding duty arc should be connected to the first node, as shown in Figure 3.3. This mechanism ensures that two long duties will never be connected by a short rest, so that 8-in-24 rule will not be violated. The way to split single node into two due to short vs. long overnight rests is similar to that in (Barnhart et al. 1994) for long-haul crew scheduling, although the motivations are different. In this context, the main concern is to avoid 8-in-24 violations. Another legality rule on daily pairing problem is no leg repetition. This can be guaranteed by the duty partition constraints.

3.3.3 The Formulation

The formulation of the duty flow model with pairing patterns is given by:

Minimize:

$$\sum_{d \in D} (1 + P_d) X_d \tag{3.2}$$

Subject to:

$$\sum_{d: l \in d} X_d = 1, \qquad \forall l \in L, \tag{3.3}$$

$$\sum_{a \in O(n)} Z_{a,cb} = \sum_{a \in I(n)} Z_{a,cb}, \qquad \forall n \in N, \forall cb \in CB, \tag{3.4}$$

$$\sum_{cb} \sum_{a: a=d} Z_{a,cb} = X_d, \qquad \forall d \in D, \tag{3.5}$$

$$X_d \in \{0,1\}, \qquad \forall d \in D, \tag{3.6}$$

$$Z_{a,cb} \in \{0,1\}, \qquad \forall a \in A, \forall cb \in CB, \tag{3.7}$$

Where $X_d = 1$ if duty d is chosen, and 0 otherwise; $Z_{a,cb}$=1 if arc a in the duty connection network is chosen and assigned commodity of crew base cb, 0 otherwise; L is the set of flight segments; D is the set of enumerated duties; N is the node set of the duty connection network; A is the arc set of the duty connection network; CB is the set of crew bases. The duty connection network is a multi-commodity flow network, where crew bases are the commodities. s is the source node of the network, and t is the sink node of the network. O(n) and I(n) represent the sets of out-going arcs and incoming arcs of node n, respectively.

Each duty variable has cost coefficient 1 since the main objective is to reduce the total number of duties. P_d is a penalty function defined to promote good duties in the solution. If duty cost is larger than the duty total block time,

$P_d = \overline{P}_d + 0.1 \cdot Blk_d / Elap_d + 0.01;$

Otherwise,

$P_d = \overline{P}_d + 0.1 \cdot Blk_d / Elap_d .$

The ratio of duty total block time to duty elapsed time, $Blk_d / Elap_d$, is used to evaluate the duty efficiency. $\overline{P}_d$ vs. duty total block time is a piecewise linear function, defined as shown in Figure 3.4 to discourage short duties. Duty total block time is compared with the minimum duty guarantee, and the average duty guarantee defined in the duty and pairing cost function. Bigger penalties are defined for duties with shorter effective flying time. Based on this definition, short duties and duties with long sit-times will be penalized.

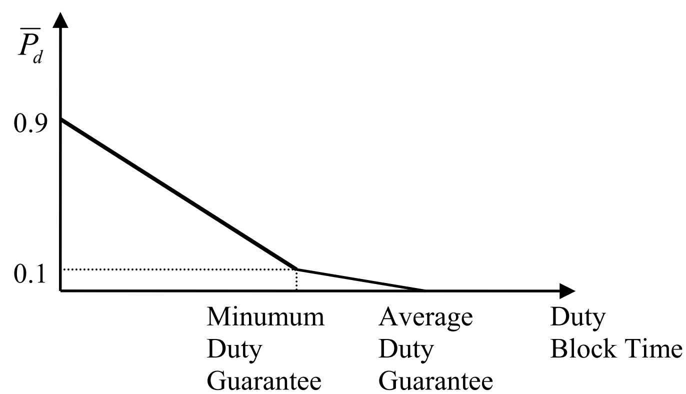


Figure 3.4: Definition of $\overline{P}_d$

Expressions (3.3)-(3.5) give the constraints of this model. Expression (3.3) enforces that each flight is covered by exactly one duty. Expression (3.4) gives constraints related to the duty connection network. For each node in the network, flow conservation constraints are required for separate commodities. It is noted that we can fix a lot of variables to 0, i.e., set $Z_{a,cb} = 0$, if a is the input or output arc of a node in CBiBEG or

CBiEND, while cb is not the corresponding ith crew base. Expression (3.5) ensures that each chosen duty can only appear once in the pairing solution embedded in the network.

Note that crew availability balance constraints can be easily included in this model by simply adding up the flying hours of duty arcs assigned to each crew base.

3.4 Computational Results

Computational experiments were performed using flight schedule data from a US domestic carrier. In Table 3.1, computational results for seven test cases are listed, which include the size of the schedule, the number of duties enumerated, total block time of the schedule, number of crew bases, number of nodes and arcs in the duty connection network, CPU time for solving the duty flow model, CPU time for solving the crew pairing model, pay & credit result of the duty flow model, and the optimal pay & credit result from the traditional crew pairing model. Our results show that the duty flow model can find good legal pairing solutions very quickly, indicating this is a good approximation method. Comparing the CPU time of the two methods, in general, the duty flow model can be solved one hundred times faster than the traditional crew pairing model. Most importantly, the branching and bound process for solving the duty flow model can generally find the first integer solution very quickly. Table 3.2 compares the actual number of duties in the crew pairing solution from our duty flow model and the traditional crew pairing set partitioning model. It is found that solutions from the duty flow model need less or equal number of duties. As previously discussed, a reduced number of duties would benefit the crew rostering process by making the days off requirements for crew easier to satisfy. Such duty solution can also save crew cost for certain airlines in which crew are paid by days of work.

Table 3.1: Computational results of duty flow model

Problem	1	2	3	4	5	6	7
Flights	62	62	102	104	219	219	350
Duties	706	987	5,530	3,332	8,514	7,945	16,428
Total block time	4,734	4,734	6,192	6,302	19,347	22,039	41,625
Crew Bases	1	1	1	1	4	4	3
Nodes	352	352	512	507	1,096	1,100	1,756
Arcs	1,456	1,768	6,790	4,287	10,886	11,343	21,001
Time (sec)	0.82	1.57	6.03	1.21	22.36	248.84	261.45
Time for Opt. (sec)	77	141	6,530	83	2,619	68,549	5,926
Pay & Credit	3.86%	2.05%	14.60%	0.77%	28.31%	7.02%	2.49%
Optimal Pay & Credit	2.19%	1.65%	13.37%	0.00%	26.16%	4.93%	0.44%

Table 3.2: Comparison of number of duties in the pairing solution

Problem	1	2	3	4	5	6	7
Duty flow	16	16	26	22	66	77	120
Traditional	17	16	26	22	73	79	127

3.5 Integrate fleet assignment and crew pairing

3.5.1 The integrated model

We propose an integrated planning model which integrates fleet assignment and crew scheduling. For the fleet assignment modeling, a formulation similar to the basic FAM formulation introduced in (Hane et al. 1995) is adopted. The FAM formulation maximizes operating profit: revenue minus operating cost. Timeline network is defined for each station, fleet type combination. Ground arcs are used to track the number of planes on the ground. There are two sets of variables: flight-fleet assignment variables and ground arc variables. The three sets of constraints are:

Cover constraints – every flight must be assigned to one and only one fleet type;

Balance constraints – ensure flow conservation in the time line network;

Plane count constraints – the total number of planes in the air and on the ground can not exceed the available fleet size.

Thus, the basic FAM is formulated as:

Maximize:

$$\sum_{f\in F}\sum_{i\in L}(R_{f,i}-C_{f,i})x_{f,i} \tag{3.8}$$

Subject to:

$$\sum_{f\in F}x_{f,i}=1, \forall i\in L \tag{3.9}$$

$$y_{f,h,t^-}+\sum_{i\in I(f,h,t),i\in L}x_{f,i}-y_{f,h,t^+}-\sum_{i\in O(f,h,t),i\in L}x_{f,i}=0, \forall f,h,t \tag{3.10}$$

$$\sum_{h\in S}y_{f,h,t_m}+\sum_{i\in CL(f)}x_{f,i}\le N_f, \forall f\in F \tag{3.11}$$

$$x_{f,i}\in\{0,1\}, \forall f\in F, \forall i\in L \tag{3.12}$$

$$y_{f,h,t}\ge 0, \forall f,h,t \tag{3.13}$$

The definitions of the sets appeared in the formulation are:

S: Set of stations, indexed by h.

F: Set of fleet types, indexed by f.

L: Set of flight legs, indexed by i.

CL(f): Set of flight legs crossing the counting line flown by fleet f.

I(f,a,t): Set of flight legs inbound to {f,a,t}.

O(f,a,t): Set of flight legs outbound from {f,a,t}.

The decision variables are:

$x_{f,i}$ $= \begin{cases} 1, \text{if leg i} \in L \text{ is assigned fleet f} . \\ 0, \text{otherwise.} \end{cases}$

y_{f,h,t^-}: The number of aircraft on the ground for fleet type f, at airport h, on the ground arc just prior to time t.

y_{f,h,t^+}: The number of aircraft on the ground for fleet type f, at airport h, on the ground arc just following time t.

The parameters defined in the model include:

$R_{f,i}$: Revenue for flight leg i if it is assigned fleet type f.

$C_{f,i}$: Cost for flight leg i if it is assigned fleet type f.

N_f: The number of aircraft available of fleet type f.

For crew scheduling, the duty flow model introduced in Section 3.4 is adopted. However, different duty set, denoted as D^f , is defined for each fleet type f. Correspondingly, the duty connection network is also fleet specific, with node set denoted as N^f. The decision variables $Z_{a,cb,f}$ are defined on different duty connection networks for different fleets. The integrated FAM and crew planning model has the following objective function:

$$Maximize \sum_{f \in F} \sum_{i \in L} (R_{f,i} - C_{f,i}) x_{f,i} - \sum_{f \in F} \sum_{d \in D^f} G \cdot (1 + P_d) X_d \tag{3.14}$$

in which, cost on crew is added to (3.8). Besides trying to reduce the total number of duties, G defined as the average duty guarantee, is multiplied with the original objective function of the duty flow model, to make this cost item more comparable to the real crew cost.

The constraints related to duty flow model are added to the FAM model. Equation (3.15) provides a duty to cover the flight legs. Equations (3.16) are the flow balance constraints for each duty connection network. Some variables of $Z_{a,cb,f}$ can be fixed as zero if an arc in the network is not possible to be covered by some crew bases. Equation (3.17) implies that any duty chosen can only appear once in the pairing solution embedded in the networks.

$$\sum_{d:i\in d, d\in D^f} X_d = x_{f,i}, \forall i \in L, \forall f \in F \tag{3.15}$$

$$\sum_{a\in O(n)} Z_{a,cb,f} = \sum_{a\in I(n)} Z_{a,cb,f}, \quad \forall n \in N^f, \forall cb \in CB, \forall f \in F \tag{3.16}$$

$$\sum_{cb} \sum_{a:a=d} Z_{a,cb,f} = X_d, \quad \forall d \in D^f, \forall f \in F \tag{3.17}$$

Overall, the integrated model consists of (3.14), (3.9)~(3.13), and (3.15)~(3.17).

3.5.2 Computational results

For the case study problem discussed in Section 2.4, we solve the fleet assignment and crew scheduling problem simultaneously on the combined schedule. Table 3.3 provides the computational results. It shows that the airline resources are utilized in a more balanced way, compared to the original solution. The savings on crew cost amounts to (71148-68358)*365 = 1.018 million dollars per year on this small problem with 164 flights. The actual profit saving is over 1.8 million dollars a year for this schedule. In addition, using the duty flow model, the total number of duties to cover this schedule is reduced by 3.

Table 3.3: Performance of the integrated model

	Integrated model	Original
Number of duties	39	42
Fleet I pay & credit	3.21%	2.19%
Fleet II pay & credit	5.27%	13.37%
Total pairing cost ($/day)	68,358	71,148
Actual profit ($/day)	342,459	337,445

The integrated model consists of 328 flight-fleet variables, 254 ground arc variables, and 47,513 duty variables for each fleet type. It took 1962 seconds to solve the MIP problem using ILOG CPLEX 9.0 on a Pentium 4 processor. It is noted that the number of duties enumerated on the combined schedule, 47,513, is much larger than the number of duties enumerated on schedules of different fleets (5,530 for Fleet I, and 706 for Fleet II). This explains the second challenge of integrated FAM and crew planning, i.e., the problem size is greatly enlarged due to scheduling crew on the whole schedule. In the following two chapters, we will discuss methods to improve crew modeling in a way that problem sizes are reduced and good crew solutions are kept. These methods include spoke connection plan, crew base purity, and crew connection modeling.

3.6 Summary

A duty flow model is proposed for solving the crew pairing problem. This model is not designed to find the optimal crew pairing solution, but it can quickly find a very good legal pairing solution. To meet with the real-time requirements in recovery or to deal with the computational tractability in integrated planning, the duty flow model is a good alternative. An integrated fleet assignment and crew scheduling model based on the duty flow model is constructed and used to solve the case study problem in Chapter 2. Profit improvement is obtained. In Chapter 6, we will extend this method to solve integrated recovery problem.

CHAPTER 4

ROBUST AIRLINE CREW SCHEDULING BASED ON STATION PURITY PLANS

4.1 Introduction

In reality, delays and disruptions are pervasive in airline operations. A flight delay results in an increased pairing cost and potentially into calling on duty a reserve crew. Robust airline crew scheduling aims to find crew schedules that perform well in operations.

We propose a new robust crew scheduling method, which aims at increasing crew swapping options in operation. In our method, the scale of the robust crew scheduling model is even smaller than that of the traditional set partitioning crew pairing model. The work in this chapter is inspired by the concepts of station purity and station decomposition proposed by Smith (2004). In the fleet assignment problem, station purity ensures that the number of fleet types serving a given station does not exceed a specified limit. Limiting the number of different fleets or families serving a station creates more opportunities for aircraft swaps and crew swaps to cover operational disruptions.

A move-up crew is not only required to be fleet family compatible, but also required to come from the same crew base. Based on this consideration, we apply the station purity idea to crew scheduling problem. By limiting the number of crew bases serving a specific station, we can expect a better crew swapping opportunities in operation. The nature of the hub-and-spoke network ensures that the proposed method is rational and feasible. In fact, most spoke stations are only connected to a few crew bases nearby. It is not recommended to send crews from crew bases far away to visit the spoke. The benefit is the quick recovery from disruption by deadheading them home fast. Therefore, we impose crew base purity to stations that are not crew bases. The station decomposition scheme is embedded because of the different treatment to crewbases and non-crewbases.

After a review of the previous work on robust crew scheduling in Section 4.2, we introduce how to generate spoke plans satisfying crew base purity, and then propose the corresponding robust crew scheduling model and related algorithms in Section 4.3. Computational results on real airline data are presented in Section 4.4. It shows that with little or no extra cost, more robust pairing solutions can be expected. In addition, the new method greatly reduces the size of the crew pairing problem, which allows for solving larger crew pairing problem with explicit enumerated pairings.

4.2 Previous work

Optimized solutions are rarely executed as planned. Adverse weather, mechanical failures, crew sickness, and air traffic control will cause necessary changes to the schedule, leading to significantly increased costs. Moreover, the success of applying optimization models at the planning stage causes tightened schedules of resources, resulting in less slack in the schedule to adsorb the disruptions. Robust airline crew scheduling aims to find crew schedules that perform well in operations.

Previous research on robust airline crew scheduling includes stochastic and deterministic methods. (Yen and Birge 2006) propose a stochastic programming approach to the airline crew scheduling problem. They consider a stochastic crew scheduling model and devise a solution methodology for integrating disruptions in the evaluation of crew schedules. The goal is to use that information to find robust solutions that better withstand disruptions. They formulate the crew scheduling problem as a two-stage stochastic binary optimization problem with recourse. (Schaefer et al 2005) propose a stochastic extension to the deterministic crew scheduling problem. They modify the coefficient vector of the objective function to reflect the expected cost of each decision variable rather than deterministic cost. Monte Carlo simulation is used to estimate the operational cost. The simulation does not account for disruption interactions between potential crew schedules. (Ehrgott and Ryan 2002) propose a bi-criteria optimization model. The measure called "non-robustness" is evaluated for each pairing, based on the effect of potential delays within the pairing. The non-robustness measure is then treated as a second objective. This model tends to get pairing solutions with long connection time. (Chebalov and Klabjan 2006) present a deterministic method that addresses robustness by considering crews that can be swapped in operations. They add a second objective for maximizing the number of move-up crew to the traditional crew scheduling model. A move-up crew is a crew that is ready to fly a different flight, which means it is ready to fly, it is from the same crew base, and it has the same number of days till the end of the pairing. A review on robust crew scheduling can be found in (Ball et al. 2006).

4.3 Robust crew scheduling model

4.3.1 Crew spoke plan

Different from the fleet assignment problem, in crew pairing problem, a spoke plan includes not only crew base assignment, but also flight connections. In the following, we assume all crew bases are hubs, and non-crewbase stations are spokes.

4.3.1.1 Spoke connection plan

In our model, each connection plan is a perfect matching from incoming legs to outgoing legs. Perfect matching enumeration algorithm is developed to obtain the connection plans. The algorithm is based on depth-first search. A controlling parameter is used to ensure the enumerated perfect matchings having small ground time. Figure 4.1 illustrates how to generate the spoke connection plan from its schedule.

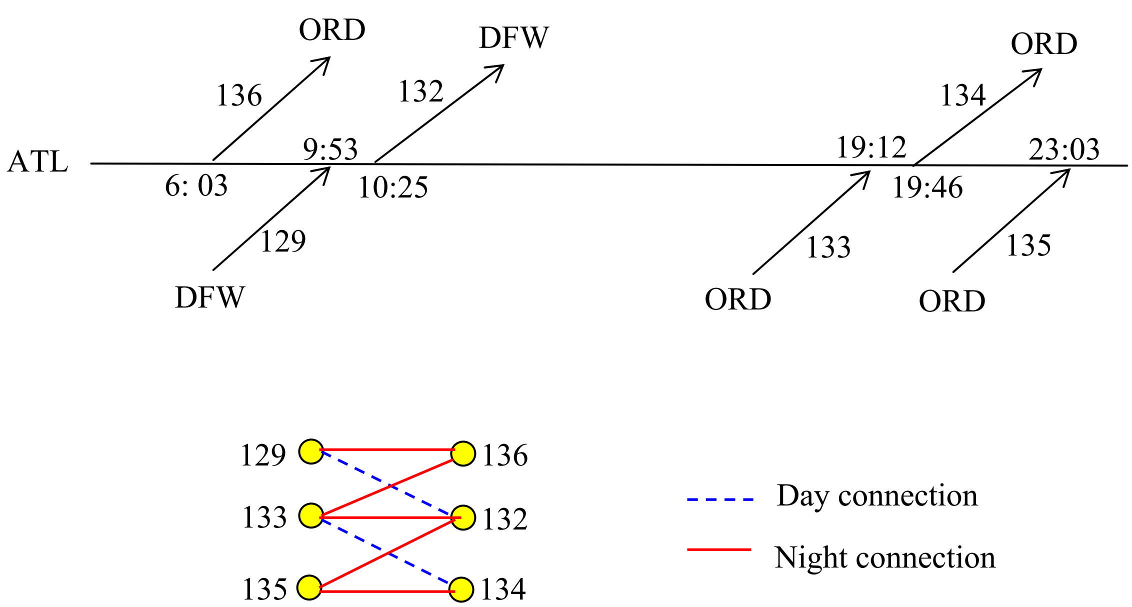


Figure 4.1: Example of spoke connection plan

A bipartite graph is constructed with one node set defined as incoming legs and the other node set as outgoing legs. Arc is added between two nodes if the connect time of the corresponding two legs satisfies legal day or night connection condition. The result of matching enumeration algorithm gives three connection plans:

(129 – 136, 133 – 134, 135 – 132),

(129 – 132, 133 – 136, 135 – 134),

and (129 – 134, 133 – 136, 135 – 132).

(129 – 136, 133 – 132, 135 - 134) is eliminated since it has large ground time.

4.3.1.2 Perfect matching enumeration algorithm

Data structure and depth-first searching algorithm are designed to enumerate perfect matchings for the bipartite graph. Specifically, two matrices are used to store the searching process. Matrix STATE stores the status of edges at each depth, while matrix CHOICE stores the list of edges to be explored at each depth. There are three types of status for edges, 1 means the edge is in the current matching, -1 means the edge is disabled from consideration, and 0 means the edge is not visited yet. The algorithm is listed as below. Figure 4.2 illustrated how the data structure works. It provides very compact storage scheme for the depth-first searching process.

Algorithm: Enumerate perfect matchings for bipartite graph

Input: A bipartite graph B=(V,U,E).

Output: Sets of perfect matchings

Main program:

For edge e in E, set status as 0;

Set depth=1;

Get the first vertex v in V;

Run EnumMatch;

Program EnumMatch:

```
Get all edges incident with v with status 0, store in vector Choice(v)
If |Choice(v)|=0  return;
For j =1 to |Choice(v)|
    If depth < |V|
        Get the jth edge e in Choice(v), and set the status as 1.
        Check other edges incident with end vertices of e, and change their
        status from 0 to -1.
        depth=depth+1;
        Set v = the (depth)-th vertex in V;
        Run EnumMatch;
        depth = depth -1;
    Else
        The set of edges with status 1 gives a perfect matching;
        Update current minimum weight;
        Check if total weight is smaller than Ratio times the current minimum
        weight. If then, store the matching.
    Endif
Endfor
```

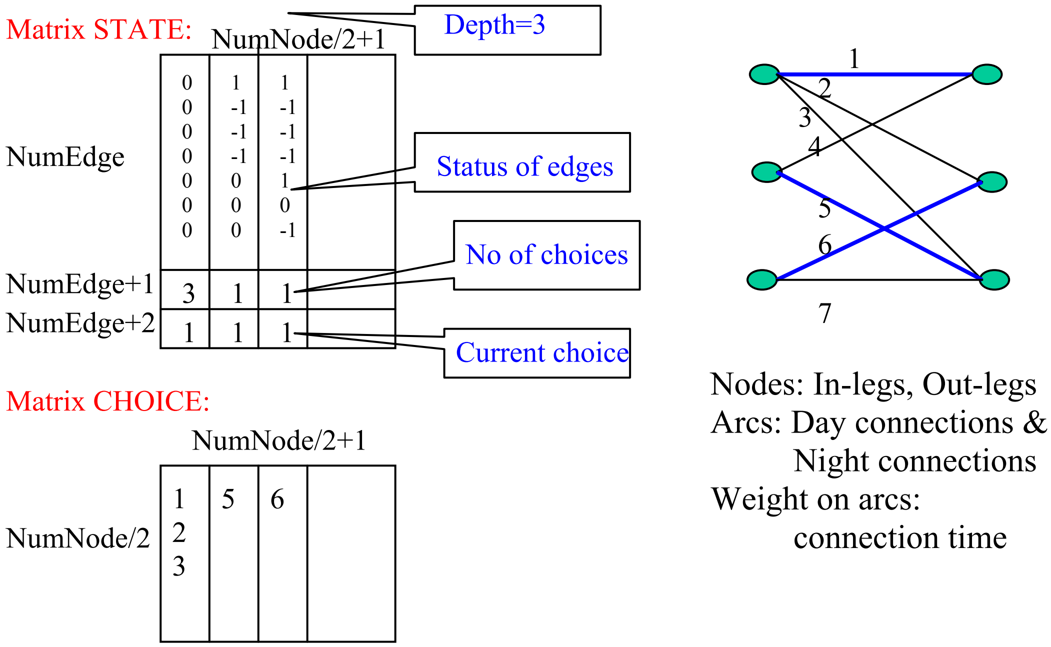


Figure 4.2: Data structure for perfect matching enumeration

4.3.1.3 Crew base assignment and crew base purity

Each connection in the spoke connection plan is a hub-to-hub string. It turns out that the pairings in our model are composed of hub-to-hub strings and hub-to-hub legs (if there is any). Therefore, as to crew base assignment, we need to assign the same crew base to the legs in a hub-to-hub string. In the following, four concepts on crew base purity will be introduced to define the types of purity plans.

Naturally Pure Spoke: all of its incoming/outgoing flights are from/to the same crew base. If a spoke is not a naturally pure spoke, we call it **Mixed Spoke**.

Pure Connection: a connection that the origin and destination stations are the same crew base.

Overnight Purity: only one crew base is used to cover all the night connections at a spoke.

Leg Pure Plan: the schedule is divided according to crew base isolation, and different crew bases are used to cover the corresponding subsets of schedule.

Table 4.1: Spoke purity plan options for crew base assignment

<table>
<tr><th></th><th>Plan 1</th><th>Plan 2</th><th>Plan 3</th><th>Plan 4</th></tr>
<tr><td>Keep naturally pure spoke?</td><td>√</td><td>√</td><td>√</td><td>√</td></tr>
<tr><td colspan="5">For mixed spokes:</td></tr>
<tr><td>Keep pure day connection?</td><td>√</td><td>√</td><td>√</td><td rowspan="4">leg pure plan</td></tr>
<tr><td>Keep pure night connection?</td><td></td><td>√</td><td></td></tr>
<tr><td>Overnight purity?</td><td>√</td><td></td><td>√</td></tr>
<tr><td>Evenly distributed crew base assignment</td><td></td><td></td><td>√</td></tr>
</table>

We have studied four spoke purity plans, listed in Table 4.1. All plans keep naturally pure spokes. For mixed spokes, all plans keep pure day connection. Thus, the difference between each plan lies in how to deal with purity in night connections. Plan 4, currently used in industry, is a leg pure plan since it works on separated schedule for different crew base. Plan 2 keeps pure night connections. For non pure connections, all possible combinations are considered. Plan 1 keeps overnight purity which is strict and hard to be satisfied. In Plan 3, strict overnight purity is relaxed so that multiple crew bases can cover the night connections, but an evenly distributed assignment is preferred.

4.3.2 Model formulation

Spoke plans are generated according to schemes of spoke purity. A spoke plan is composed of strings and crew base assignments on the strings. After generating the spoke plans, string set for different crew base is extracted. Pairings for different crew base are enumerated based on these string sets and hub-to-hub legs (if there is any). The robust crew pairing model based on spoke plans can be formulated as the following MIP problem.

4.3.2.1 Sets

S: Set of spoke stations, indexed by s.

CB: Set of crew bases, indexed by cb.

$Plan$: Set of spoke plans of spoke s, indexed by pl.

$Pair$: Set of pairings of crew base cb, indexed by pa.

$String$: Set of strings in the plans that is assigned crew base cb, indexed by str.

F: Set of hub-to-hub flights, indexed by f.

4.3.2.2 Decision variables

$$x_{pl}: \quad = \begin{cases} 1, \text{if plan pl is chosen for its corresponding spoke station.} \\ 0, \text{otherwise.} \end{cases}$$

$$y_{pa}: \quad = \begin{cases} 1, \text{if pairing pa is selected in the pairing solution.} \\ 0, \text{otherwise.} \end{cases}$$

4.3.2.3 Parameters and data

CP_{pa}: Pairing cost of pairing pa.

$$\alpha_{str,cb,pl} \quad = \begin{cases} 1, \text{if string str in plan pl is assigned crew base cb.} \\ 0, \text{otherwise.} \end{cases}$$

$$\beta_{str,cb,pa} \quad = \begin{cases} 1, \text{if string str is in pairing pa, and pa} \in \text{Pair(cb).} \\ 0, \text{otherwise.} \end{cases}$$

$$\gamma_{f,pa} \quad = \begin{cases} 1, \text{if flight f is in pairing pa.} \\ 0, \text{otherwise.} \end{cases}$$

$BT_{pl,cb}$: Total block time of flights assigned to crew base cb in plan pl.

LB_{cb} : Lower bound of available flying hours for crew base cb.

UB_{cb} Upper bound of available flying hours for crew base cb.

4.3.2.4 Formulation

Minimize:

$$\sum_{pa \in Pair} CP_{pa} y_{pa} \tag{4.1}$$

Subject to:

$$\sum_{pl \in Plan(s)} x_{pl} = 1, \ \forall s \in S \tag{4.2}$$

$$-\sum_{pl} \alpha_{str,cb,pl} x_{pl} + \sum_{pa} \beta_{str,cb,pa} y_{pa} = 0, \ \forall str \in String(cb), \forall cb \in CB \tag{4.3}$$

$$\sum_{pa} \gamma_{f,pa} y_{pa} = 1, \ \forall f \in F \tag{4.4}$$

$$LB_{cb} \le \sum_{pl} BT_{pl,cb} x_{pl} \le UB_{cb} \tag{4.5}$$

$$x_{pl} \in \{0,1\}, \forall pl \in Plan \tag{4.6}$$

$$y_{pa} \in \{0,1\}, \forall pa \in Pair \tag{4.7}$$

The objective is to minimize the total pairing cost. There are three sets of constraints. The first set is plan convexity constraints, as shown in Equation (4.2). For each spoke, one and only one plan is to be chosen. The second is string (leg) cover constraints. For each string in the chosen spoke plan, one and only one pairing is to be chosen to cover, see Equation (4.3). Similarly, for each hub-to-hub flight, one and only one pairing is to be chosen to cover, see Equation (4.4). The last set of constraints is crew base balance constraints. This is restricted by the crew availability at each crew base, as shown in Equation (4.5).

4.3.3 Follow-on fixing to get integer solution

Follow-on fixing is a good heuristic method to obtain integer solutions for crew pairing problem in traditional set partitioning formulation. Here, it is modified to facilitate spoke-plan-based crew pairing model.

Any string str can be described by a triple: str={leg1, leg2, cb}. From the LP solution, we have,

$$\sum_{pa \ni str} y_{pa} = \sum_{pl \ni str} x_{pl} \leq \sum_{pl \in Plan(s)} x_{pl} = 1 .$$

The follow-on fixing is in fact a hub-to-hub-string fixing in this context. To apply the follow-on fixing to this spoke-plan-based crew pairing model, we fix those strings with $\sum_{pa \ni str} y_{pa} > 0.9$. An alternative method is to fix those strings with $\sum_{pa \ni str} y_{pa} = 1$, and one additional string with the biggest $\sum_{pa \ni str} y_{pa}$ among those that are strictly smaller than 1, as mentioned in (Shaw 2003). After fixing, the restricted LP problem is resolved.

Assuming str is the string to be fixed, follow-on fixing is to delete the columns that conflict with string str. This includes,

- Delete plan columns

IF $pl \in Plan(\mathrm{s})$, AND $str \notin pl$, THEN plan pl is deleted.

- Delete pairing columns

To delete pairing columns, first we define the conflict set of str.

For $str' \in \bigcup_{cb \in CB} String(cb)$,

IF $leg1(str') = leg1(str)$ AND ($leg2(str') \neq leg2(str)$ OR $cb(str') \neq cb(str)$)

OR

$leg2(str') = leg2(str)$ AND ($leg1(str') \neq leg1(str)$ OR $cb(str') \neq cb(str)$)

THEN $str' \in Conflict(str)$.

Next, pairing pa is deleted if it includes a string $str' \in Conflict(str)$.

4.3.4 Algorithm

The overall algorithm for the proposed robust crew scheduling method is as follows.

Step1: At each spoke, create bipartite graph of legal connections, enumerate matchings by depth-first searching algorithm, and only keep the matchings with small ground time controlled by a ratio parameter.

Step2: For each matching, generate crew base assignment plan according to different plan scheme. Spoke plans are obtained.

Step3: Extract string set for each crew base from the spoke plans.

Step4: Enumerate pairings for each crew base based on its string set.

Step5: Formulate the crew pairing model based on spoke plans, and solve its LP relaxation.

Step6: Use follow-on fixing to find integer solution.

4.4 Computational results and discussion

4.4.1 Result of various plans

A schedule of 136 legs is studied. The data come from a major US domestic carrier. In this schedule, there are two crew bases and twenty spokes, in which ten of the spokes are naturally pure, and the other ten are mixed spokes. Applying different spoke plan schemes, robust crew pairing model is solved. It is known that without adding spoke purity plan, the optimal pairing solution of this schedule is 0% and 0.02% for the cases without or with crew base balance constraints, respectively. The results in Table 4.2 show how adding spoke purity affects the solution quality. Although Plan 1 is the most robust in operation, it is rather strict. Without crew base balance constraints, Plan 1 gives a pairing solution with higher pay and credit. With crew base balance constraints, however, it cannot give a feasible solution. Plan 2 gives the best pay and credit among the four plans. The performance of the pairing solution obtained in Plan 2 is almost the same as that of the model without imposing any purity. Plan 4 is the method currently used in the industry. It has the least number of plans enumerated. Plan 3 is a good trade-off between robustness and optimality.

Table 4.2: Computational results of various types of plans

	Plan 1	Plan 2	Plan 3	Plan 4
# cb1 Pairings	19,490	18,492	19,490	13,068
# cb2 Pairings	275,479	267,433	275,479	303,962
# of plans	5,710	24,815	35,474	73
# cb1 strings	144	124	144	57
# cb2 strings	170	145	170	70
Without crew base balance constraints				
LP solution	14,658	14,463	14,468	14,480
IP solution	14,687	14,465	14,480	14,480
Pay & credit	1.55%	0.01%	0.12%	0.12%
With crew base balance constraints				
LP solution	Infeasible	14,463	14,468	14,480
IP solution	Infeasible	14,468	14,480	14,480
Pay & credit	----	0.03%	0.12%	0.12%

Table 4.3 shows the impact of crew base purity on the crew pairing solution using the following statistics, which provides a way of evaluating robustness.

CS – total number of crew-base/station combinations in pairing solution which sums up the number of crew bases serving each station;

CSO – total number of crew-base/station/overnight combinations which counts for the number of overnights from different crew bases at different stations;

CSLO – total number of crew-base/station/lonely-overnight combinations which counts for the number of lonely overnights from different crew bases at different stations;

CSD – total number of crew-base/station/day-visit combinations which counts for the number of day visits from different crew bases at different stations;

CSLD – total number of crew-base/station/lonely-day-visit combinations which counts for the number of lonely day visits from different crew bases at different stations.

Table 4.3: Impact of crew base purity

	No Plan	Plan 1	Plan 2	Plan 3	Plan 4
CS	33	31	33	33	31
CSO	14	10	14	14	14
CSLO	3	2	3	2	3
CSD	30	30	30	30	28
CSLD	19	17	19	16	17

The results in Table 4.3 correspond to the solutions in Table 4.2 of the cases without considering crew base balance constraints. These data provide a measure of the robustness. A plan with good robustness should have reduced number of lonely overnight and/or crewbase/station combinations in the solution. Table 4.3 shows that Plan 1 is the most robust, while Plan 3 and Plan 4 are robust by having less CSLO and CSLD, or less CS and CSD. There is no difference in these statistics between Plan 2 and the case without purity plan. From Table 4.2 and Table 4.3, it can be concluded that achieving robustness will influence the quality of crew solution to some extent, but the influence can be controlled in reasonable scope by choosing appropriate purity plans.

4.4.2 Problem size

As previously mentioned, the connections at spokes are in fact hub-to-hub strings. In (Shaw 2003), the advantages of hub-to-hub strings in constructing pairings are investigated in detail. Here, the number of strings is further reduced because the strings are extracted from good spoke connection plans, which explains why our method tends to have smaller problem size than the traditional crew pairing model. Therefore, our method can solve larger crew pairing problem with the same computing resources.

For a small star network schedule with 102 legs, the number of pairings generated by traditional legal adjacency rule method is 30-50 times more than that of the spoke plan method, as shown in Table 4.4.

Table 4.4: Comparison of # of pairings enumerated

	# pairings enumerated by legal adjacency rule	# pairings enumerated by spoke plans
Schedule 1	37,334,559	1,466,864
Schedule 2	39,559,877	791,236

4.4.3 Computational efficiency on finding integer solutions

Using the follow-on fixing heuristics, the computational efficiency is significantly improved, compared to ILOG cplex MIP solver. Table 4.5 shows the comparison of the computer times as well as the values of the integer solutions between different methods. The time listed for follow-on fixing includes the CPU times spent on both solving LP and follow-on fixing process. Note that the computer time is dramatically reduced from several ~ ten hours down to several ~ ten minutes in follow-on fixing. Because it is a heuristic method, the optimality might be sacrificed to some extent. Table 4.5 compares the total pairing cost of the integer solutions obtained by both methods.

Table 4.5: Efficiency improvements by follow-on fixing

	Plan 2 (Without crew base balance constraints)	Plan 2 (With crew base balance constraints)	Plan 3 (Without crew base balance constraints)	Plan 3 (With crew base balance constraints)
Time by cplex (seconds)	36,012	35,421	9,600	10,075
Time by follow-on fixing (seconds)	LP: 14.74 IP: 363	LP: 25 IP: 339	LP: 46 IP: 10.7	LP: 23.47 IP: 717
Integer solution by cplex	14,465	14,468	14,480	14,480
Integer solution by follow-on fixing	14,465	14,482.5	14,505.5	14,509

4.5 Summary

An efficient robust crew scheduling method is proposed. The robustness comes from crew base purity plan imposed at the spokes. Specifically, spoke purity plans for crew are investigated, a robust crew pairing model is proposed, and an efficient follow-on fixing heuristic algorithm is presented to solve the model. Computational experiments on real airline data show that:

- With little or no extra cost, more robust crew pairing solution can be expected;
- The proposed model has much smaller problem size than the traditional crew pairing set partitioning model;
- The follow-on fixing heuristic is efficient.

CHAPTER 5

INTEGRATED FLEET AND CREW ROBUST PLANNING IMPOSING STATION PURITY

5.1 Introduction

In the future, airline schedule planning will be done in a more integrated fashion in an effort to improve operational efficiencies (Clarke and Smith 2004). In this chapter, we present our contributions in integrating fleet assignment and crew scheduling, with an effort to improve the crew friendliness of the FAM solution and achieve robustness for operational efficiencies. There are three challenges in this work: 1) Understanding the influence of fleet assignment on the performance of crew scheduling. This includes what can lead to poor crew solution, for instance, crew double overnight rests. 2) Addressing the crew scheduling problem in the integrated model. The crew problem becomes much harder in the integrated model, since it is needed to work on the whole schedule, instead of the sub-schedules for different fleet types. Thus, computational tractability is an important issue to be considered. 3) Achieving robustness in an integrated framework, so that the resulted plans are robust for both aircraft and crew recovery in operations.

Previous studies to address crew scheduling in FAM formulations or to integrate fleet assignment with crew scheduling optimization include (Clarke et al. 1996), (Barnhart et al. 1998a), (Smith 2004, Smith and Johnson 2006), and (Sandhu and Klabjan 2004). (Clarke et al. 1996) was the first attempt to address crew scheduling issues in FAM formulation. In their model, a cost is added on each lonely double overnight and the optimization model is then used to balance the costs between lonely double overnights and fleeting. The number of lonely double overnights is counted by defining legal rest arcs and midday time window in the timeline network. The actual number of lonely double overnights for a fleet at a station is the number of crews without a legal rest minus the number of midday departures assigned to that fleet. Barnhart et al. (1998a) propose an integrated approximate model for fleet assignment and crew pairing optimization which combines the basic FAM and a duty-based model for crew pairing problem. (Smith 2004) proposes a robust fleet assignment model imposing station purity. The crew, maintenance, and operational issues can be addressed simultaneously through station purity, limiting the number of fleets or crew-compatible families that can serve each station. Adding fleet purity can greatly reduce planned crew costs, maintenance costs, and improve robustness. However, it has significant negative impact on the computational efficiency of FAM. To improve the computational efficiency, a column generation solution approach called station decomposition was proposed (Smith 2004). (Sandhu and Klabjan 2004) propose an integrated planning model which integrates fleeting, aircraft routing and crew pairing simultaneously. Crew pairings are modeled explicitly and the aircraft rotation problem is captured by the plane count constraints. The reported computing environment consists of a cluster of 27 dual 900 MHz Itanium 2 processors. For a test case with 942 flights and 4 fleet families, it costs 34 hours by Lagrangian approach and 29 hours by Benders decomposition to solve the integrated planning model, which shows that the integrated planning problem is truly computational intensive.

In our proposed integrated fleet and crew robust planning method, both fleet purity

(Smith 2004) and crew base purity (as discussed in Chapter 4) are considered. In Chapter 4, we have showed that adding crew base purity will not significantly degrade the quality of crew solution in robust crew scheduling. In the meanwhile, crew base purity can reduce the size of the crew scheduling problem dramatically. Work in (Smith 2004) showed that adding fleet purity can greatly reduce planned crew costs, maintenance costs, and improve robustness, but has negative impact on the computational efficiency of FAM. Therefore, in this chapter, we first discuss how different fleet and crew base purity work together to affect FAM profit, crew solution quality, and the computational efficiency. Then, we propose an integrated fleet assignment and crew connection model imposing station purity which can further improve crew solution without sacrificing FAM profit or computational efficiency. Instead of modeling crew pairings or duties explicitly, this model integrates fleet assignment with crew connections to avoid the curse of dimensionality. Solving this model provides both fleet assignment solution and a pseudo crew pairing solution. Legal pairings can be obtained by solving the decomposed sub-problems. This method is tested by solving an industrial size problem with 1388 daily flights.

5.2 Station Purity

We extend the station purity idea proposed by Smith (2004) (originally refers to fleet purity only) to include both fleet purity and crew base purity. In this section, we demonstrate how to define fleet purity and crew base purity together at stations. Based on the robust FAM model proposed in (Smith 2004) and a test scenario from real airline schedule, we study how fleet and crew base purity work together to impact the FAM profits, crew solution, and computational efficiency.

5.2.1 Fleet purity and crew base purity

Station purity ensures that the number of fleet types or fleet families and the number of crew bases serving a station are limited. With station purity, we can not only create more flexibility in crew scheduling to improve crew solution, but also create more opportunities for aircraft and crew swapping to cover operational disruptions.

Crew base purity is determined by the adjacency graph of the flight network. As discussed in Chapter 4, in hub-and-spoke network, most spoke stations are only connected to a few crew bases nearby. Note that we have defined **naturally pure spoke** and **mixed spoke** depending on if a spoke is connected to only one crew base or more. For a naturally pure spoke, its crew base purity is restricted as 1, and the specific crew base, to which it connects, is assigned to this spoke. For a mixed spoke, we usually need to assign all the connected crew bases. Otherwise, we may lose feasible solutions. For a crew base station, we allow other crew bases within distance of 2 in the adjacency graph (i.e. through non-stop or 1-stop flight) to have day visit at this station. Besides the crew base purity level defined on stations, we adopt **leg pure** idea discussed in Chapter 4 to further restrict the crew base assignment possibilities at mixed spokes by using the corresponding crew base to cover the flights going to or coming from the specific crew base station.

Fleet purity level is related to both the size of the station and the crew base purity. In our model, two fleet purity schemes are defined.

(1) **Family purity scheme**

- For crew bases or large spokes:

Penalties on fleet family and station pairs are defined in the objective function, instead of adding hard constraints on station fleet family purity level.

- For small or medium sized spokes

  Hard constraints on fleet family purity at these stations are defined to be 1.

(2) **Fleet purity scheme**

- For crew bases or large spokes:

  Penalties on fleet type and station pairs are defined in the objective function, instead of adding hard constraints on station fleet type purity level.

- For small or medium sized spokes
  - ✧ Mixed spokes: Hard constraints on fleet family purity are defined to be 1.
  - ✧ Naturally pure spokes: Hard constraints on fleet type purity are defined to be 1.

5.2.2 Scenarios

A flight schedule with an industrial problem size is used to test the impact of station purity as well as the integrated fleet and crew planning model. It is based on a daily schedule of one major US domestic carrier. The scenario flight network is constructed by deleting parts of the schedule that are quite isolated from the rest of the network. Consequently, there are 1388 daily flights and six crew bases in this schedule. The fleet consists of 3 fleet families, JET-1, JET-2 and TURBO. Table 5.1 shows the size of the fleets. JET-1 is the largest fleet and includes three crew-compatible sub-fleets. One of the main characteristics of this flight network is that the fleets are distributed among crew bases, resulting in 15 different fleet types in the problem. It is noted that there are quite a few regional airlines or commuter airlines operate with this characteristic, i.e., fleets are divided among hubs or crew bases, and crew serve the aircrafts from their own base.

Table 5.1: Fleet family and size

Fleet family	Size
JET-1	165
JET-2	24
TURBO	12

In our tests, large spokes are defined as having more than 40 daily operations, medium spokes are defined as having from 21 to 40 daily operations, and small spokes are defined as having less than 20 daily operations. Based on these criteria, 98 small and medium spokes are imposed fleet purity constraints. The number of naturally pure spoke is 66 out of the total 100 spokes, and most of the mixed spokes only connect to two crew bases. The scale of the problem is restricted within a reasonable range because of the large proportion of small spokes and naturally pure spokes. Table 5.2 lists the statistics related to station purity, and Figure 5.1 shows the adjacency graph of the flight network for the testing schedule.

Table 5.2: Scenario data

Number of		Scenario
	Cities	106
	Flights	1,388
	Fleet types	15
	Fleet families	3
	Crew bases	6
	Large spokes	2
	Medium spokes	13
	Small spokes	85
	Naturally pure spokes	66
Mixed spokes	Connect to 2 crew bases	27
	Connect to 3 or more crew bases	7

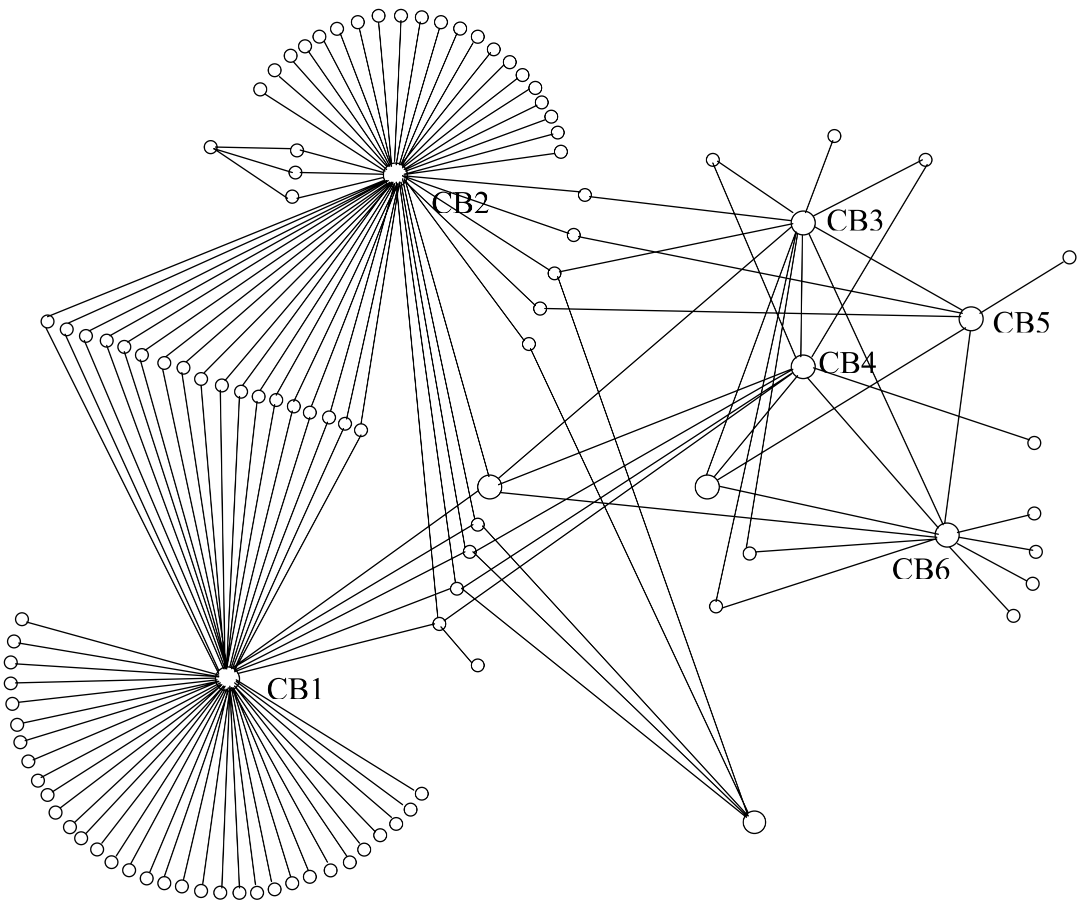


Figure 5.1: Adjacency graph of the flight network

Nine purity scenarios are designed to study the impact of station purities. For fleet purity, three cases are considered. In the first case, no purity is imposed on fleet type or fleet family, as in the traditional FAM formulation. The second case is called “Family purity”, and the third case is called “Fleet purity”, as described in Section 5.2.1 respectively. Similarly, for crew base purity, three cases are considered. In the first case, no crew base purity is imposed. The second case is called “CB purity 1”, in which crew base purity as defined in Section 5.2.1 are imposed on all naturally pure spokes, mixed spokes and crew bases. Besides, leg pure plan is imposed on those mixed spokes connecting to more than two crew bases, and a bonus is used in the objective function to encourage leg pure plans at other stations. The third case is referred to "CB purity 2", which is based on the second case. The only difference is that, in the third case, leg pure plan is also imposed on the mixed spokes between crew bases CB1 and CB2. The combination of these settings gives nine different purity scenarios.

5.2.3 Impact of station purity

The impact of fleet and crew base purities is tested using the robust FAM model proposed by Smith (2004). The crew base purity is implemented by restricting the legal flight-fleet assignments. In the scenario problem, fleets are distributed among crew bases, and crews are encouraged to fly their own fleets. Therefore, crew base purity implies that some flight-fleet assignments are disabled. The proposed method can also be applied to general problems without this characteristic, as long as the plane count is restricted on the real fleet types instead of the fleets distributed at the crew bases. To implement fleet purity, an auxiliary variable $w_{f,s}$ is defined to indicate whether fleet or fleet family f serves station s in the FAM solution. Based on $w_{f,s}$, fleet/family purity constraints and penalties on fleet/family and station pairs can be defined. A detailed robust FAM model is described in (Smith 2004), and (Smith and Johnson 2006).

Table 5.3 gives the problem size of the robust FAM model under different station purity scenarios. It is shown that adding crew base purity can reduce the size of the problem, and adding fleet purity has larger problem size than adding family purity. Table 5.4 shows the robust FAM results for different station purity scenarios. By adding "Family purity", FAM profit decreases by 2.9% in average, while CPU time increases by

300 times in average. By adding "Fleet purity", FAM profit decreases by 3.6% in average, while CPU time increases by 900 times in average. For this problem, the negative impact of fleet purity on the computational efficiency of FAM is more severe than that reported in (Smith 2004). However, by adding crew base purity, FAM profit only decreases by 0.6% in average. The most notable benefits of adding crew base purity is the improvement of computational efficiency. By adding "CB purity 1", the CPU time is 16 times faster in average. By adding "CB purity 2", the CPU time is 10 ~ 100 times faster. Figure 5.2 and Figure 5.3 clearly illustrate the impact of station purity on CPU time as well as on FAM profit.

Table 5.3: Problem sizes

	No CB purity				CB purity 1				CB purity 2			
	No	Combo*	Rows	Cols	No	Combo	Rows	Cols	No	Combo	Rows	Cols
No fleet purity	1	9,026	7,589	15,212	4	6,684	6,352	11,576	7	5,530	5,644	9,732
Family purity	2	9,026	25,741	16,803	5	6,684	19,827	13,184	8	5,530	16,804	11,322
Fleet purity	3	9,026	43,793	17,473	6	6,684	33,195	13,851	9	5,530	27,864	11,988

*Combo refers to legal flight-fleet assignments

Table 5.4: Robust FAM results

	No CB purity			CB purity 1			CB purity 2		
	No.	Time*	Profit**	No.	Time	Profit	No.	Time	Profit
No fleet purity	1	44.3	6.54	4	3.3	6.48	7	4.03	6.476
Family purity	2	15,463	6.33	5	548	6.30	8	1,683.6	6.30
Fleet purity	3	38,756	6.29	6	6,573	6.26	9	232.2	6.25

* in CPU seconds
** in million dollars

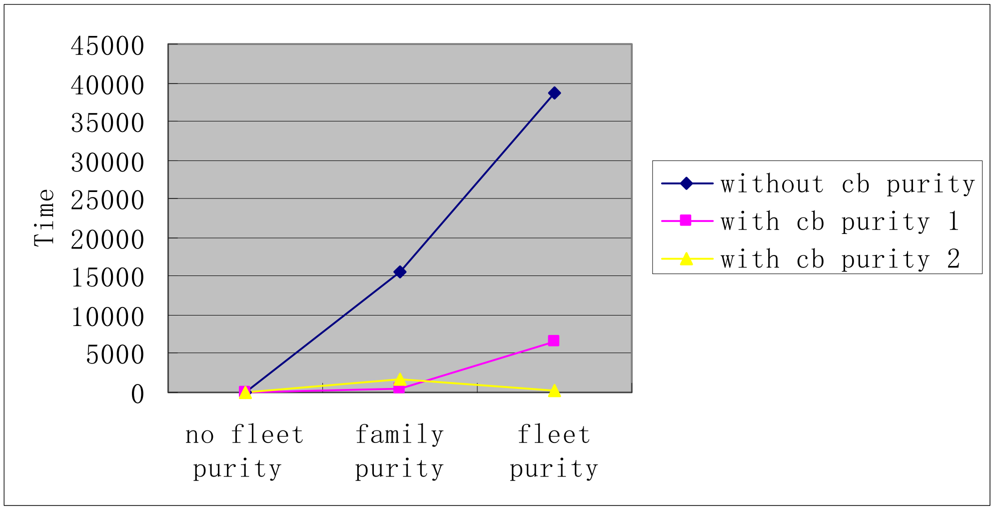


Figure 5.2: Impact of station purity on runtime

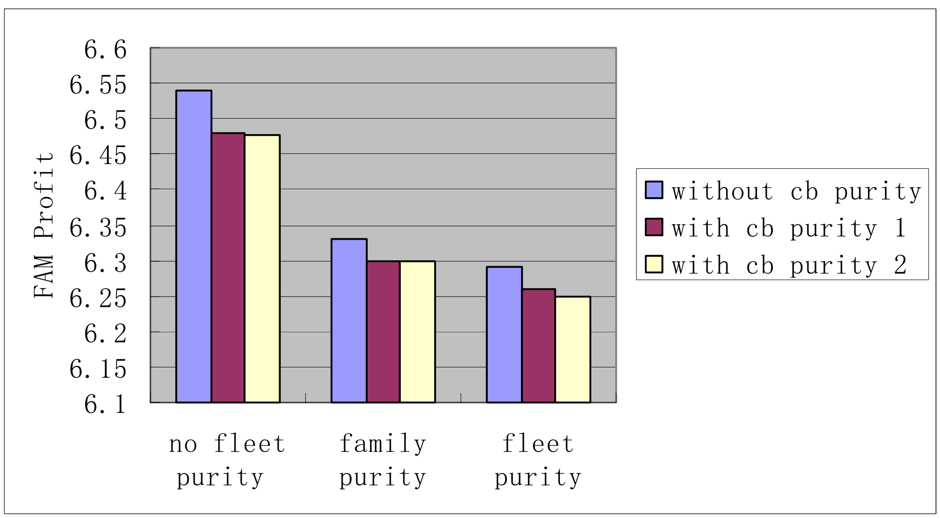


Figure 5.3: Impact of station purity on FAM profit

Note that the FAM profit decrease caused by adding family or fleet purity is quite big for this testing schedule. In this problem, there are one large fleet family and two small fleet families. The reason of FAM profit loss is that by setting the fleet/family purity level as 1, only a small number of stations can be served by the two small fleet families. The fleet/family purity constraints can be relaxed somewhat by setting the fleet/family purity level as 2. Further computational experiments show that FAM profit only decreases by 0.09% with family purity level set as 2 for the cases of “Family purity”; and FAM profit decreases by 0.13% with family and fleet type purity level set as 2 for the cases of “Fleet purity”. In practice, compromise would be necessary for balancing the benefits of robustness and loss of FAM profit by adjusting the fleet purity level.

Taking the FAM solution from the robust FAM model, crew scheduling problems are solved for each fleet using the duty flow model proposed in Chapter 3. It is found that, without imposing crew base purity, the FAM solution tends to produce locked rotation, which means that the flights covered by some aircrafts are separate from the flights covered by other aircrafts in the fleet. Locked rotations are unacceptable to most airlines (Clarke et al. 1997). The crew scheduling problem for a FAM solution with locked rotations is not feasible. Deadheads are required to lead crews to those separate routes. Adding crew base purity is an effective way to avoid the locked rotations, since it encourages crew and fleet not to fly far from its base station. The definition of crew base purity on naturally pure spokes and mixed spokes, as well as the hard or soft constraints on leg pure plan, can altogether limit the stations that a fleet can visit lie within distance of 2 to its base station in the adjacency graph. That means the stations that a fleet can visit are connected by a non-stop or 1-stop itinerary to its base station. The resulted sub-network for each fleet tends to be connected.

The crew problems are solved for all three scenarios of “CB purity 1”. Table 5.5 shows crew solution performance under different fleet purity in terms of actual pairing cost in minutes, excess cost in Pay & credit, total number of duties in the solution, and the maximum sit time needed to produce a feasible crew solution. By adding fleet purity, the total number of fleet-station combinations and lonely fleets are reduced. Hence the crew lonely double overnights can also be reduced, which has significant effect on the crew performance. Table 5.6 compares the number of crew double overnights influenced by different fleet purities. The original schedule includes 3 double overnight rests intrinsically. It is shown that Solution 6 (Fleet purity) has the best crew results. Solution 5 (Family purity) has better crew results than Solution 4, but is not as good as Solution 6. This is because, in the case of "Family purity", there are more chances for lonely crew base visit than in the case of "Fleet purity", which may cause more crew double overnight rests. In addition, the lonely crew base visits at stations are not preferable in operational side.

Table 5.5: Crew solution results

	Solution No.	Pairing cost*	Pay & credit	Number of duties	Maximum Sit time*
No fleet purity	4	130,038.45	4.95%	378	506
Family purity	5	127,413.8	2.83%	394	293
Fleet purity	6	126,653.35	2.22%	392	250

* in minutes

Table 5.6: Impact of fleet purity on crew lonely double overnights

	Sol No.	Family station pairs	Fleet Station pairs	Family-station (FS) Singleton	Crew lonely double ovn caused by FS singletons	Crewbase-station singleton	Double ovn in crew solution
No fleet purity	4	160	308	33	9	15	21
Family purity	5	110	241	9	0	16	9
Fleet purity	6	111	184	9	0	12	6

To sum up, the following conclusions can be made:

(1) Adding crew base purity can avoid locked rotations in FAM solution. It will not degrade the FAM profit significantly. Most importantly, the CPU time needed to solve the robust FAM model can be reduced by 10~100 times.

(2) By adding fleet/family purity, the crew solution is improved by 2~3% in pay-and-credit, which gives a saving of crew scheduling cost by up to 5~8 million dollars per year.

The value of purity also includes the savings of $500,000 per year for each family-station pair (Smith and Johnson 2006), and the savings in aircraft maintenance cost and operational costs.

5.3 Integrated FAM and crew connection model

Imposing station purity to FAM formulation can improve crew scheduling, because FAM solutions with fleets serving a smaller number of stations with greater frequency provide more flexibility for crew assignment to reduce crew cost. In the above computational experiments, the best crew solution obtained for the scenario schedule has six double overnights. In addition to the three intrinsic double overnights of the schedule, three more double overnights were caused by fleet assignment. There were also some long sit times in the crew solution. These findings suggested that further research on integrated fleet and crew planning is necessary.

Addressing the crew scheduling problem in the integrated model is perhaps the most challenging task in integrated planning. Crew scheduling for separate fleets is already computationally expensive. In an integrated model, the crew problem becomes much harder to solve, since it is needed to work on the whole schedule. In the previous scenario schedule, the number of duties enumerated on the whole schedule would exceed two millions. It is extremely computationally expensive to model pairings or duties explicitly in the integrated model. Therefore, we integrate fleet assignment with crew connections in our integrated planning model. This model maintains the characteristics and benefits of station purity and connection plans as previously discussed.

5.3.1 Model formulation

Day and night connections are enumerated a priori based on the minimum and maximum sit or rest time, which constitute the connection variables. At spokes, a flight has either day connection or night connection, while at crew bases, a flight can either be the beginning/ending of a pairing or have day connections. Our model provides fleet assignment solution and a pseudo crew pairing solution. Based on crew base and fleet division provided by the solution, legal pairings can be obtained by solving the decomposed sub-problems.

5.3.1.1 Sets

L: Set of legs in the schedule, indexed by i.

S: Set of stations, indexed by s.

CB: Set of stations that are crew bases.

F: Set of fleet types, indexed by f.

CF: Set of compatible crew base and fleet pairs, indexed by (c,f).

CL(f): Set of flight legs crossing the counting line flown by fleet f.

I(f,a,t): Set of flight legs inbound to {f,a,t}.

O(f,a,t): Set of flight legs outbound from {f,a,t}.

5.3.1.2 Decision variables

$$x_i^{(c,f)} = \begin{cases} 1, \text{if leg i} \in L \text{ is assigned fleet f and crew base c.} \\ 0, \text{otherwise.} \end{cases}$$

y_{f,h,t^-}: The number of aircraft on the ground for fleet type f, at airport a, on the ground arc just prior to time t.

y_{f,h,t^+}: The number of aircraft on the ground for fleet type f, at airport a, on the ground arc just following time t.

$$w_{f,s} = \begin{cases} 1, \text{if fleet f} \in \text{F serves station s} \in \text{S in the solution.} \\ 0, \text{otherwise.} \end{cases}$$

fs : Total number of fleet-station combinations in the solution.

$$x_{day(j,i)}^{(c,f_j,f_i)} = \begin{cases} 1, \text{if day connection (leg j, leg i) is assigned crew base c, fleet type } f_j, f_i. \\ 0, \text{otherwise.} \end{cases},$$

$$x_{nite(j,i)}^{(c,f_j,f_i)} = \begin{cases} 1, \text{if night connection (leg j, leg i) is assigned crew base c, fleet type } f_j, f_i. \\ 0, \text{otherwise.} \end{cases}$$

$$x_{start,i}^{(c)} = \begin{cases} 1, \text{if departure leg i at crew base c is the begining of a pairing.} \\ 0, \text{otherwise.} \end{cases}$$

$$x_{end,i}^{(c)} = \begin{cases} 1, \text{if arrival leg i at crew base c is the end of a pairing.} \\ 0, \text{otherwise.} \end{cases}$$

5.3.1.3 Parameters and data

$R_{f,i}$: Revenue for flight leg i if it is assigned fleet type f.

$C_{f,i}$: Cost for flight leg i if it is assigned fleet type f.

$B_{f,i}$: Bonus for flight leg i if it is assigned fleet type f.

$Ctim_{day(i,j)}$: Connection time for day connection day(i,j), in minutes.

$Ctim_{nite(i,j)}$: Connection time for night connection nite(i,j), in minutes..

N_f: The number of aircraft available of fleet type f.

$I_{s,i} = \begin{cases} 1, \text{if flight i serves station s.} \\ 0, \text{otherwise.} \end{cases}$

SP_s: Fleet purity level at station s.

5.3.1.4 Formulation

Maximize:

$$\sum_{(c,f)}\sum_{i\in L}(R_{f,i} - C_{f,i} + B_{f,i})x_i^{(c,f)} - \sum_{day(i,j)} Ctim_{day(i,j)} x_{day(i,j)}^{(c,f_i,f_j)} - \sum_{nite(i,j)} Ctim_{nite(i,j)} x_{nite(i,j)}^{(c,f_i,f_j)} - P * fs \quad (5.1)$$

Subject to:

$$\sum_{(c,f)} x_i^{(c,f)} = 1, \forall i \in L \quad (5.2)$$

$$y_{f,h,t^-} + \sum_{(c,f)\ni f}\ \sum_{i\in I(f,h,t), i\in L} x_i^{(c,f)} - y_{f,h,t^+} - \sum_{(c,f)\ni f}\ \sum_{i\in O(f,h,t), i\in L} x_i^{(c,f)} = 0, \forall f,h,t \quad (5.3)$$

$$\sum_{h\in H} y_{f,h,t_m} + \sum_{(c,f)\ni f}\ \sum_{i\in CL(f), i\in L} x_i^{(c,f)} \le N_f, \forall f \in F \quad (5.4)$$

$$w_{f,s} \ge I_{s,i} x_i^{(c,f)}, \forall i \in L, \forall (c,f) \quad (5.5)$$

$$\sum_{f\in F} w_{f,s} \le SP_s, \forall s \in S \quad (5.6)$$

$$fs = \sum_{f\in F}\sum_{s\in S} w_{f,s} \quad (5.7)$$

$$x_i^{(c,f)} = \sum_j \sum_{f_j} x_{day(i,j)}^{(c,f,f_j)} + \sum_j \sum_{f_j} x_{nite(i,j)}^{(c,f,f_j)}, \forall (c,f), \forall \text{ incoming } i \in L \text{ at station } s \notin CB \quad (5.8)$$

$$x_i^{(c,f)} = \sum_j \sum_{f_j} x_{day(j,i)}^{(c,f_j,f)} + \sum_j \sum_{f_j} x_{nite(j,i)}^{(c,f_j,f)}, \forall (c,f), \forall \text{ outgoing } i \in L \text{ at station } s \notin CB \quad (5.9)$$

$$x_i^{(c,f)} = \sum_j \sum_{f_j} x_{day(i,j)}^{(c,f,f_j)} + x_{end,i}^{(c)}, \forall (c,f), \forall \text{ incoming } i \in L \text{ at crewbase c} \quad (5.10)$$

$$x_i^{(c,f)} = \sum_j \sum_{f_j} x_{day(j,i)}^{(c,f_j,f)} + x_{start,i}^{(c)}, \forall (c,f), \forall \text{ outgoing } i \in L \text{ at crewbase c} \quad (5.11)$$

$$x_i^{(c,f)} = \sum_j \sum_{f_j} x_{day(i,j)}^{(c,f,f_j)}, \forall (c,f), \forall \text{ incoming } i \in L \text{ at station } s \in CB \text{, but } s \neq c \tag{5.12}$$

$$x_i^{(c,f)} = \sum_j \sum_{f_j} x_{day(j,i)}^{(c,f_j,f)}, \forall (c,f), \forall \text{ outgoing } i \in L \text{ at station } s \in CB \text{, but } s \neq c \tag{5.13}$$

$$x_i^{(c,f)} \in \{0,1\}, \forall (c,f), \forall i \in L^h \tag{5.14}$$

$$y_{f,h,t} \geq 0, \forall f, h, t \tag{5.15}$$

$$w_{f,s} \in \{0,1\}, \forall f \in F, s \in S \tag{5.16}$$

$$fs \geq 0 \tag{5.17}$$

$$x_{day(j,i)}^{(c,f_j,f)}, x_{nite(j,i)}^{(c,f_j,f)}, x_{start,i}^{(c)}, x_{end,i}^{(c)} \in \{0,1\} \tag{5.18}$$

Constraints (5.2) – (5.4), corresponding to those in the basic FAM formulation, are cover constraints, station fleet balance constraints, and plane count constraints respectively. The only difference is that for each flight, not only a fleet type but also a crew base is assigned here. Constraints (5.5) – (5.7) are related to fleet purity, which are same as those defined in the robust FAM model (Smith 2004). Variables $w_{f,s}$ are defined to obtain the information whether fleet type is used to serve station s in the solution. Equation (5.6) defines hard constraints on fleet/family purity. Variable *fs* summarizes total number of fleet/family station combinations in the solution. Constraints (5.8) – (5.13) are related to crew connections. Equations (5.8)-(5.9) are applied to stations that are not crew bases. Equation (5.8) implies that each incoming flight leg to such a station has either a day connection or night connection. Equation (5.9) implies that each outgoing flight leg from such a station has either a day connection or night connection. The constraints (5.10) to (5.13) are applied to stations that are crew bases. It means a departure flight at crew base c is either the beginning of a pairing or it has day connections. Accordingly, an arrival flight at crew base c is either the end of a pairing or it has day connections. If crews from one crew base visit another crew base, they can only have day connections there.

5.3.2 Computational results

5.3.2.1 Algorithm/Implementation

In preprocessing, legal fleet and crew base assignments are generated for flights and connections based on the scenarios of station purities. For each station, a set of crew base stations is defined to identify which crew bases are allowed to serve this station. Each crew can only fly the fleets from their own base. Once the crew base purity for each station is determined, there are compatible pairs of fleets and crew bases to be assigned to flights. As to connections, by checking the arrival station and the departure station of a connection, only the crew bases in the intersection set of the two corresponding crew base sets are allowed to serve this connection. This scheme allows crew from one crew base to visit another crew base, but then come back to its own connected stations.

The computational experiments are conducted on a Pentium 4 processor (1.83GHz, 1.5G RAM) using ILOG CPLEX 9.0. The models are formulated in ILOG Concert 2.0. The LP problem is solved by dual steepest-edge simplex. In order to get integer solution, flight variables and connection variables with value ≥ 0.99 are fixed to 1. As a result, related connection variables with different fleet or crew base assignment can be fixed to 0. The resulted MIP problem is then solved by CPLEX's branching and bound process.

5.3.2.2 Results

The integrated fleet assignment and crew connection model is applied to solve the schedule described in Section 5.2.2. The size of the MIP model and the performance of this method are summarized in Table 5.7. Four purity scenarios are tested. The indices of these scenarios are in subsequent order with the earlier scenarios introduced in Section 5.2.2. Scenario 10 has "Fleet purity" (same meaning as) defined in Section 5.2.1, and basic crew base purity including 1) crew base purity imposed on all naturally pure spokes, mixed spokes and crew bases; 2) leg pure plan on those mixed spokes connecting to more than two crew bases except the stations that will generate double overnights. Scenario 11 has "Fleet purity", crew base purities as in scenario 10, and bonus defined in the objective function to encourage leg pure plan at other stations. Scenario 12 has "Fleet purity", crew base purities as in scenario 11, and leg pure plan on parts of the mixed spokes between crew bases CB1 and CB2. Different from scenario 12, scenario 13 has "Family purity" instead of "Fleet purity".

Table 5.7 shows that the CPU time to solve the connection model is reduced dramatically as the purity scenarios improved. Scenario 13 can be solved in thirteen minutes, which is about 100 times faster than solving scenario 10. Moreover, comparing to the results in Table 5.4, the FAM profit didn't degrade because of adding crew connection variables and constraints.

The decomposed crew scheduling problems are solved for scenario 12 and scenario 13 respectively. Table 5.8 shows crew solution performances, and Table 5.9 compares the number of crew double overnights influenced by different fleet purities. For both scenarios, there are no more double overnights besides the ones intrinsic to the original schedule. Moreover, the maximum sit time is 3 hours as required. "Fleet purity" as in scenario 12 gives better crew solution. In scenario 13 - the "Family purity" case, there are more chances for lonely crew base visit, although they are not lonely double overnight rest now, it is not as flexible for crew scheduling or crew recovery as the "Fleet purity" case. However, the "Family purity" gives better FAM profit and has better computational efficiency. A compromise will be necessary.

Table 5.7: Results of the integrated model

No.	Fleet purity type	Crew base purity	rows	cols	Time**			Profit*	Gap ***
					LP	IP	Total		
10	Fleet	base	45,571	195,494	826	74,977	75,803	6.28	2.93
11	Fleet	Bonus on leg pure	45,571	195,494	764	9,089	9,853	6.26	2.30
12	Fleet	Forced leg pure	42,274	183,323	491	1,811	2,302	6.29	1.50
13	Family	Forced leg pure	30,238	182,656	229	554	783	6.34	0.68

* FAM profit in million dollars
** in CPU seconds
***MIP gap (in percent) returned by CPLEX.

Table 5.8: Crew solution of the integrated model

	Solution No.	Pairing cost	Pay & credit	Number of duties	Maximum Sit time
Fleet purity	12	126,370.35	1.99%	380	[20,180]
Family purity	13	126,772.75	2.114%	392	[20,180]

Table 5.9: Statistics of the integrated model

	Sol No.	Family station pairs	Fleet Station pairs	Family-station (FS) Singleton	Crew lonely double ovn caused by FS singletons	Crewbase-station singleton	Double ovn in crew solution
Fleet purity	12	118	183	10	0	12	3
Family purity	13	118	243	10	0	16	3

5.4 Summary

In this chapter, we investigate the integrated fleet and crew robust planning method imposing station purity. The impacts of crew base purity and fleet purity on FAM profit, crew scheduling, and computational efficiency are thoroughly tested and validated from our analysis. Adding crew base purity can avoid locked rotations in FAM solutions, and significantly decrease the solution time by 10~100 times. Adding fleet purity can improve planning quality in crew scheduling and maintenance. Both crew base purity and fleet purity together will improve robustness in the planning solution. Due to the importance of good crew connections and spoke plans to the crew pairing solution, we propose an integrated planning model which integrates fleet assignment and crew connections. This model performs well in solving industrial size problem. With properly defined station purities, the scenario schedule with 1388 daily flights can be solved within 13 minutes, and the solution quality improvements are significant.

CHAPTER 6

INTEGRATED RECOVERY

6.1 Introduction

Optimization at the planning stage tends to tightly couple resources, such as aircraft and crew. In the previous chapters, we have discussed robust planning methods that produce plans less vulnerable to disruptions. During the day of operations, disruptions, such as inclement weather, mechanical problems, sick crew, air traffic control and ground delay program of FAA, frequently jeopardize the execution of planned schedules. Therefore, it is important to have a good recovery method that can capitalize on the benefits of proposed robust plans.

Without taking into account crew feasibility and crew friendliness, the schedule of fleet assignment and aircraft routing could cause poor overall solutions. Integrated recovery is an appealing answer which can capture the availability of all three resources: aircraft, crew and available seats. However, each resource is scheduled separately because of different sets of rules. This leads to a "snowball effect" when a flight does not operate as scheduled. The disruption will propagate through the network. In integrated recovery, a small disruption could result in a huge disrupted leg set. In this chapter, we intend to address the recovery scope in an integrated recovery framework. Based on the recovery scope, a new integrated recovery model is then proposed. Finally, some crucial preprocessing issues are discussed.

6.2 Literature review on airline recovery

6.2.1 Flight rescheduling and aircraft rerouting

Early airline recovery research focused on flight rescheduling and aircraft rerouting.

Teodorovic and Gubernic (1984) discuss how to reroute a reduced number of planes to operate the existing network when one or more aircraft were taken out of operation. The objective is to minimize the total passenger delay. They use a lexicographic optimization model and the branch and bound method to solve the problem. Flight delays and aircraft rerouting are the main strategies considered.

In (Teodorovic and Stojkovic 1990), when one or more aircrafts are taken out of operation, a new schedule and a routing for the remaining aircrafts are designed to minimize the total number of cancelled flights as well as minimize total passenger delays. A dynamic-programming based heuristic algorithm is adopted to solve the lexicographic optimization problem. In their study, flight delays and cancellations, as well as aircraft rerouting are modeled.

Teodorovic and Stojkovic (1995) design a new airline schedule & aircraft rotation when there is a disturbance in carrying out the planned airline schedule. Legs are grouped together in the crew rotations and crew rotations are grouped together in the aircraft rotations. A heuristic FIFO method is used to generate the crew rotations. Since it is not guaranteed that crews can go back to their crew bases, deadheads may be necessary. This work is the first attempt to integrate crew rotation with aircraft rescheduling.

Jarrah et al. (1993) present an overview of a decision support framework for airline flight cancellations and delays at United Airlines. Their underlying solution methodology is based on network flow theory. Flight delays, flight cancellations, swapping aircrafts, and spare aircrafts are the main recovery strategies.

Yan and Yang (1996) are the first to combine delays and cancellation into one minimum cost network flow model. They model the problem as network flow problem with side constraints. Lagrangian relaxation with the subgradient method is applied.

Yan and Tu (1997) build a multi-commodity network flow model to formulate a multifleet routing and multistop flight scheduling problem when schedule disturbance happens.

Clarke (1997) proposes a comprehensive framework for reassigning operational aircraft to scheduled flights in the aftermath of irregularities. The decision model allows for multiple-fleet-type aircraft swapping, as well as flight delays and cancellations during rescheduling, and incorporates the impact of air-traffic-control flow management initiatives and crew availability.

Thengvall et al. (2001) present three multi-commodity network-type models for determining a recovery schedule for all aircraft operated by a large carrier following a hub closure. The first is a pure network with side constraints, the second is a generalized network, and the third is a pure network with side constraints in which the time horizon is discretized. Each model allows for cancellations, delays, ferry flights, and substitution between fleets and subfleets. (Thengvall et al 2003) propose a bundle algorithm to solve the multicommodity network model.

Cao and Kanafani (1997) present a 0-1 quadratic programming model for addressing cancellations and delays.

Rosenberger (2003) presents a set packing model that reschedules legs and reroutes aircraft by minimizing an objective function involving rerouting and cancellation costs. Alternative maintenance feasible routings for disrupted aircraft rotations are enumerated up in front. A heuristic is developed for selecting aircrafts to be rerouted. The model is finally revised to minimize crew and passenger disruptions. This method is good for modeling maintenance feasibility.

6.2.2 Crew rescheduling

Stojkovic et al. (1998) address operational airline crew scheduling problem. The problem consists of modifying, as necessary, personalized planned monthly assignments of airline crew members during day-to-day operations. It requires covering, at minimal cost, all flight segments from a given time period with available crew while minimizing the disturbances of crew members. An optimization approach is proposed for the problem in which the flight schedule is fixed and represents input data. The problem is mathematically formulated as a set partitioning type problem, and a column generation method embedded in a branch and bound search tree has been implemented to solve it.

Lettovsky et al. (2001) propose a new solution framework for airline crew recovery. It provides, in almost real time, a recovery plan for reassigning crews to restore a disrupted crew schedule. Preprocessing techniques are applied to extract a subset of the schedule for rescheduling. A fast crew-pairing generator is built, which enumerates feasible continuations of partially flown crew trips. Several branching strategies are presented that allow fast generation of integer solutions.

Yu et al. (2003) introduce the CrewSolver decision-support system developed by CALEB Technologies for Continental Airlines to generate globally optimal, or near optimal, crew-recovery solutions. It is reported that since its implementation, the system has successfully dealt with several high-profile events. In each case, Continental recovered quickly and obtained overall benefits worth millions of dollars.

6.2.3 Integrated recovery

(Lettovsky 1997) is the first systematic work on airline integrated recovery. An integrated recovery framework and a Bender's decomposition scheme are proposed. In the model, there are 4 sets of decision variables: (1) if assign certain equipment type to certain flight segment; (2) if certain pairing is chosen for the crew being considered; (3) if certain routing is chosen for the aircraft being considered; (4) for the rescheduled flights, how many seats are assigned to certain itinerary to do passenger recovery. In the Bender's decomposition scheme, the linking variables are the first set of variables.

(Ball et al. 2006) provide a comprehensive survey on airline irregular operations and control.

6.3 Scope of recovery

Most optimization based recovery methods adopt a predefined time window and a potential resource set to limit the scope to reschedule. It is critical to find a good recovery scope which is able to provide a reasonably good recovery solution, and also to assure that the formulated problem is computationally tractable in real-time.

The difficulty of integrated recovery lies in handling aircraft and crew concurrently. For example, if we only consider aircraft recovery, for a small disruption, pushback or short cycle cancellation heuristics will work well regarding to the maintenance requirements. However, to take into account crew recovery at the same time, the delay or cancellation decisions might result in crew pairings inapplicable. To recover the disrupted crew pairing, it might be necessary to delay or cancel more flights or swap with another crew pairing, perhaps of another fleet type. This will cause more plane rotations being disrupted. The chained effect significantly complicates the problem. To address this issue, we propose a new approach to determine the recovery scope in integrated recovery framework. The idea is to define different recovery sets for schedule change, aircraft rerouting, and crew rerouting.

A time window (TW) for aircraft recovery is defined as a time interval between the start of the disruption and the end of the day. If no feasible solution exists, its length can be extended to the end of next day. For crew recovery, no exact time window is defined. Instead, a level is specified in which the crew schedule needs to be recovered. Eventually, crew schedule needs to be recovered in the rostering level. However, in order to make an immediate response to the disruptions and assign every crew a legal job to do, a tactical recovery solution in the level of pairings or duties would be more practical. Gao and Kalyta (2006) introduce the work being done at Sabre Airline Solutions about a tactical crew recovery module based on duty recovery, which aims to find a quick duty recovery solution in minutes. This will leave 12 to 24 hours time duration to conduct a complete recovery in the rostering level. The method discussed in this chapter can be applied to both pairing recovery and duty recovery depending on the size of problem which can be solved in expected time durations.

Recovery is event driven. In our method, four types of incidents are taken into account: (1) A scattered set of delayed flights caused by some reasons; (2) Unscheduled maintenance of some aircrafts, each has an unavailable time window; (3) Crew not available; (4) Airport service rate change, such as airport closure caused by severe weather condition. For each case, there is a leg set L0 that has been disrupted. If several incidents happened at the same time, the disrupted leg set would take the union of several sets. In summary, L0 is the union of (1) legs misconnected in the routings of aircraft or crew; (2) Legs in the repair time window of the aircrafts; (3) Legs in the pairing (duties) of the unavailable crew within TW; (4) Legs departing or landing in the service rate changing time window.

From the disrupted leg set L0, disrupted aircraft set A1 and crew set C1 can be identified. An alternative aircraft set A2 and crew set C2 need to be chosen heuristically to make feasible and better recovery decisions. Table 6.1 defines the leg sets that will be used in the context. There are four types of leg sets to be rescheduled. On L1-1 (the first type of leg sets), full set of reschedule can be conducted, i.e., delay, cancellation, fleet type change, as well as aircraft and crew rerouting. On L1-2 (the second type), the reschedule actions could be fleet type change, aircraft, and crew rerouting. On L3-1 and L5 (the third and fourth type), the schedule and fleet type are not allowed to change, and only aircraft or crew need to be rerouted accordingly. Figure 6.1 illustrates the above idea, which is based on the following observations.

Case 1: within the same fleet, if there is no delay/cancellation decision, crew and aircraft recovery can be done separately.

Case 2: within the same fleet, if delay/cancellation is allowed, interactions between aircraft and crew rotations have to be considered.

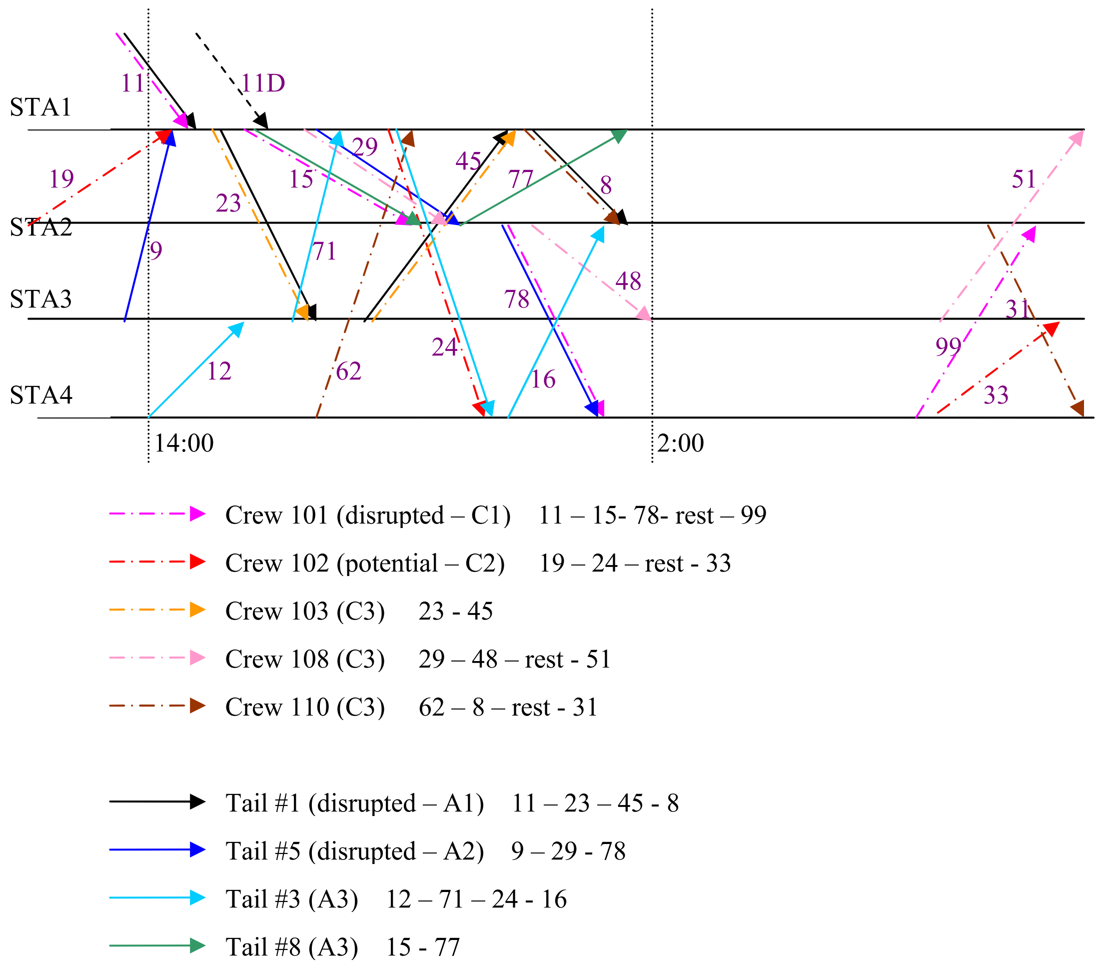
Case 3: among multiple fleets, no matter whether there is delay/cancellation or not, interactions between aircraft and crew rotations have to be taken into account.

In conclusion, we have defined different recovery sets for schedule change, aircraft rerouting, and crew rescheduling, namely L_s, L_a, and L_c. Apparently, $L_a \supseteq L_s$ and

$L_c \supseteq L_s$. For the legs in L_a, but not in L_s, the fleet assignment and crew assignment will not change; Vice versa, for the legs in L_c, but not in L_s, the fleet assignment and aircraft assignment will not change.

Table 6.1: Leg sets for recovery

Leg sets	Meaning	Recovery actions
L0	Disrupted legs	
L1	legs in the rotations of A1,A2,C1,C2 in TW (where A1: set of disrupted aircrafts, A2: set of potential aircrafts, C1: set of disrupted crews, C2: set of potential crews)	
L1-1	legs in the rotations of A1,C1 in TW	can change fleet, can be cancelled, can have delay options, aircraft, crew rerouting
L1-2	legs in the rotations of A2, C2 in TW	can change fleet aircraft, crew rerouting
L2	legs in crew rotation C1,C2 beyond TW	
L3	legs in rotation of A and C in TW exclude those in L1 (where A: aircraft set for legs in L1, C: crew set for legs in L1)	
L3-1	legs in rotation of A in TW exclude those in L1	need to assign planes from the same fleet type as before to cover them
L3-2	legs in rotation of C in TW exclude those in L1	
L4	legs in rotation of C beyond TW (L2 $\subseteq$ L4)	
L5	L3-2 $\cup$ L4	need to assign crews from the same fleet type as before to cover them



Ls: 11, 15, 78, 24, 23, 45, 8, 29, 78

La: Ls and 77, 12, 71,16

Lc: Ls and 48, 62, 99, 33, 51, 31

Figure 6.1: Recovery scope consisting leg set, aircraft set and crew set

(Scenario: Flight 11 arrives late, L0={11}.)

6.4 New airline integrated recovery framework

Based on our recovery scope, an integrated recovery method is proposed. The recovery functions include schedule recovery, fleet reassignment, aircraft rerouting, and crew rescheduling. The approach of integrating fleet assignment and crew scheduling can also be applied to airline integrated recovery. Our method incorporates schedule adjustment into a dated FAM plus crew model in which crew rescheduling is represented by duty partition or duty flow model. The differences between the integrated recovery model and the integrated planning model lie in that, in recovery 1) since various delayed options or even cancellations for the legs need to be taken into account, schedule adjustment must be modeled; 2) the legs being picked up in the recovery set are dated, instead of daily; 3) crew recovery is specific to individual crew member. These features lead to two changes in constructing the duty connection network for each fleet type. First, because the flights are dated, there are different duty sets for different days. It is noted that the first day duty set must include legal partial duties for those disrupted crews. Second, because recovery is crew-specific, the commodities defined in the duty connection network are no longer crew bases, but specific crews. The timeline network for fleet assignment also becomes dated. Hence there is no circular arc any more connecting the beginning and the end of day. At the end of the recovery time window, the number of planes at each station of any particular fleet type is expected to go back to the level in the original schedule.

About aircraft maintenance routing, instead of enumerating aircraft routings, a multicommodity network flow model is used for the aircraft maintenance routing, which can reduce the number of variables without losing maintenance considerations. At the end of the recovery period, in order to go back to the original aircraft routing, the demand for the number of aircrafts for each fleet type at each station is known. With this information, the multi-commodity flow conservation constraints can be formulated.

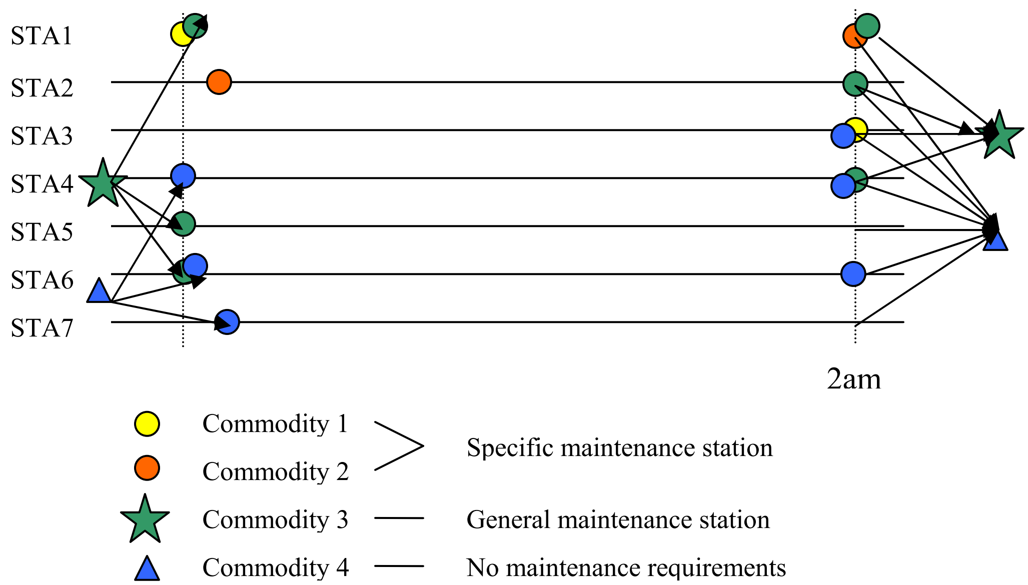


Figure 6.2: Schematic of multi-commodity flow network for aircraft maintenance routing

In aircraft maintenance routing, for each fleet type, there are aircrafts with different kinds of maintenance requirements. For example, there are 3 different types of maintenance requirements: (1) Aircrafts that need maintenance at some specific stations; (2) Aircrafts that need maintenance at the end of TW, but no specific station requirement; (3) Aircrafts that have no maintenance requirements. The number of aircrafts for each maintenance type at each station should meet the demand. Figure 6.2 illustrates the aircraft maintenance routing network. After solving the integrated recovery model, a flow decomposition procedure is needed to get the routings for each aircraft with satisfied maintenance requirements.

The interaction between aircraft routing and crew scheduling is due to crew short connect. Usually, the minimum sit time for crew is 45 minutes, while the minimum turn time for aircraft is 30 minutes. Short connect means that the connection time for crew can be less than 45 minutes, but this requires crew to follow the route of the same aircraft. In our model, issues with crew short connect are considered.

6.4.1 Sets

C: Crew set, indexed by c.

F: Fleet set, indexed by f.

A(f): Aircraft set for fleet type f.

L_s: Set of flights that would be rescheduled.

L_a: Set of flights that would have aircraft reassigned.

L_c: Set of flights that would have crew reassigned.

S: Set of stations, indexed by h.

$D(l)$: Delayed alternatives of leg l in L_s.

I(f,h,t): Set of flight legs inbound to $\{f,h,t\}$.

O(f,h,t): Set of flight legs outbound from $\{f,h,t\}$.

D^f: Duty set of fleet f, indexed by d.

N^f: Node set of the duty network of fleet f.

I(n): Incoming arc set of node n $\in N^f$.

O(n): Outgoing arc set of node n $\in N^f$.

SC: Set of short connects.

K_f: Set of commodities (aircraft maintenance type) of fleet type f, indexed by k_f.

6.4.2 Parameters

$R_{f,i}$: Revenue for flight leg i if it is assigned fleet type f.

$C_{f,i}$: Cost for flight leg i if it is assigned fleet type f.

c_l: Cost for cancellation, $\forall l \in L_s$.

P_d: Penalty defined on duties.

$N_{f,h,T}$: Expected number of aircraft of fleet type f at station h at time T.

$n_{k_f hT}$: Expected number of aircraft of maintenance type k in fleet type f at station h at time T.

$\beta_{f,j}$: $= \begin{cases} 1, \text{if flight } j \in L_c \setminus L_s \text{ was flown by fleet f originally.} \\ 0, \text{otherwise.} \end{cases}$

$\gamma_{f,j}$: $= \begin{cases} 1, \text{if flight } j \in L_a \setminus L_s \text{ was flown by fleet f originally.} \\ 0, \text{otherwise.} \end{cases}$

$\delta_{(i,j),f}$: $= \begin{cases} 1, \text{if short connect (i, j) was flown by fleet f.} \\ 0, \text{otherwise.} \end{cases}$

6.4.3 Decision variables

$x_{f,i}$ $= \begin{cases} 1, \text{if leg i is assigned fleet f, } \forall i \in D(l), l \in L_s \\ 0, \text{otherwise.} \end{cases}$

κ_l $= \begin{cases} 1, \text{if leg l is cancelled, } \forall l \in L_s \\ 0, \text{otherwise.} \end{cases}$

y_{f,h,t^-}: The number of aircrafts on the ground for fleet type f, at airport h, on the ground arc just prior to time t.

y_{f,h,t^+}: The number of aircrafts on the ground for fleet type f, at airport h, on the ground arc just following time t.

X_d $= \begin{cases} 1, \text{if duty d is chosen in the solution for fleet f. } \forall d \in D^f, f \in F \\ 0, \text{otherwise.} \end{cases}$

$Z_{a,c,f} \quad = \begin{cases} 1, \text{if arc a in crew c's duty network is selected.} \\ 0, \text{otherwise.} \end{cases}$

$s_{(i,j),f} \quad = \begin{cases} 1, \text{if short connect (i, j) is flown by fleet type f.} \\ 0, \text{otherwise.} \end{cases}$

$Y_{k,i} \quad = \begin{cases} 1, \text{if leg i is flown by aircraft k.} \\ 0, \text{otherwise.} \end{cases}$

g_{kht^-t}: Ground arc variables with aircraft $k \in K$ at station h and $[t^-, t]$ the time interval covered by the arc (in the maintenance routing network)

6.4.4 Formulation

The integrated recovery model can be formulated as:

$$Maximize \sum_{f \in F} \sum_{l \in L_s} \sum_{i \in D(l)} (R_{f,i} - C_{f,i}) x_{f,i} - \sum_{l \in L_s} c_l \kappa_l - \sum_{f \in F} \sum_{d \in D^f} G \cdot (1 + P_d) X_d \tag{6.1}$$

Subject to:

(part 1: FAM + crew)

$$\sum_{f \in F} \sum_{i \in D(l)} x_{f,i} + \kappa_l = 1, \forall l \in L_s \tag{6.2}$$

$$y_{f,h,t^-} + \sum_{i \in I(f,h,t), i \in D(l), l \in L_s} x_{f,i} + \sum_{i \in I(f,h,t), i \in L_a \setminus L_s} 1 - y_{f,h,t^+} - \sum_{i \in O(f,h,t), i \in D(l), l \in L_s} x_{f,i} - \sum_{i \in O(f,h,t), i \in L_a \setminus L_s} 1 = 0, \quad \forall f, h, t \tag{6.3}$$

$$y_{f,h,T^+} = N_{f,h,T}, \forall f \in F, h \in S \tag{6.4}$$

$$\sum_{f \in F} \sum_{i \in L_w} x_{f,i} \le \max_w, \forall \mathrm{w} \in \mathrm{W_h}, \forall h \in S \tag{6.5}$$

$$\sum_{d: i \in d, d \in D^f} X_d = x_{f,i}, \forall i \in D(l), \forall l \in L_s, \forall f \in F \tag{6.6}$$

$$\sum_{d: j \in d, d \in D^f} X_d = \beta_{f,j}, \forall j \in L_c \setminus L_s, \forall f \in F \tag{6.7}$$

$$\sum_{a \in O(n)} Z_{a,c,f} = \sum_{a \in I(n)} Z_{a,c,f}, \quad \forall n \in N^f, \forall c \in C, \forall f \in F \tag{6.8}$$

$$\sum_{c} \sum_{a: a=d} Z_{a,c,f} = X_d, \quad \forall d \in D^f, \forall f \in F \tag{6.9}$$

$$\sum_{d \in D^f} \delta_{(i,j),d} X_d = s_{(i,j),f}\,, \quad \forall f \in F, \forall (i,j) \in SC \tag{6.10}$$

(part 2: maintenance routing)

$$x_{f,i} = \sum_{k \in A(f)} Y_{k,i}\,, \quad \forall f \in F, \forall i \in D(l), \forall l \in L_s \tag{6.11}$$

$$\sum_{k \in A(f)} Y_{k,j} = \gamma_{f,j}\,, \ \forall j \in L_a \setminus L_s, \forall f \in F \tag{6.12}$$

$$Y_{k,i} - Y_{k,j} \le 1 - s_{(i,j),f}\,, \ \forall (i,j) \in SC, \forall k \in A(f) \tag{6.13}$$

$$Y_{k,j} - Y_{k,i} \le 1 - s_{(i,j),f}\,, \ \forall (i,j) \in SC, \forall k \in A(f) \tag{6.14}$$

$$\sum_{i \in \delta_t^+} Y_{k,i} + g_{kht^-t} - \sum_{i \in \delta_t^-} Y_{k,i} - g_{khtt^+} = 0, \quad \forall k, \forall h \in S, \forall t \tag{6.15}$$

$$\sum_{k \in k_f} g_{khTT^+} = n_{k_f hT}\,, \quad \forall k_f \in K_f, \forall h \in S \tag{6.16}$$

The decision variables $x_{f,i}, y_{f,h,t^-}, X_d$ are flight-fleet assignment variables, ground arc variables in the fleet assignment time-line network, and duty variables, respectively. Because the commodities on the duty connection network now are specific crews, $Z_{a,c,f}$ is defined on specific crew. Binary variable κ_l represents whether leg $l \in L_s$ will be cancelled.

The first group of constraints is related to fleet assignment and crew pairing. Equation (6.2) means each leg $l \in L_s$ is either delayed or cancelled. Equation (6.3) is the balance constraint for each fleet type. Note that legs in $L_a \setminus L_s$ must be covered by an aircraft from the same fleet type. Equation (6.4) requires that at the end of the recovery time window, the number of planes of any fleet type at each station should be brought back to the value in the original schedule. Constraint (6.5) models service rate change at each station.

Equation (6.6) implies if $x_{f,i}$=1, there must be a duty with corresponding fleet type to cover it. Equation (6.7) implies that for legs in $L_c \setminus L_s$, if the pre-assigned fleet type is f, there must be a duty with corresponding fleet type to cover it. Equations (6.8) and (6.9) have the same meanings as in Section 3.3. They are related to the duty connection networks. Equation (6.10) extracts information about short connects from the duty solution. Same aircraft constraint needs to be imposed on those short connects embedded in the duty solution.

The second group of constraints is related to aircraft maintenance routing. Constraint (6.11) means that for each leg assigned fleet type f, an aircraft of fleet type f need be chosen. Constraint (6.12) requires that the legs in $L_a \setminus L_s$ must be covered by an aircraft from the same fleet type. Constraint (6.13) and (6.14) requires that short connections in the duty solution must be flown by the same aircraft. Constraint (6.15) is flow conservation in the aircraft routing network. Equation (6.16) restricts the number of aircrafts for each type of maintenance to meet the demand at the end of the recovery window.

The above model is suitable for crew pairing recovery. But, it can also be used for duty recovery by simply taking out the constraints (6.8) and (6.9), which are related to duty connection network. Certainly the definitions of let sets Ls, La, and Lc will change accordingly. The model assures that every crew considered is assigned a legal duty.

6.4.5 Bender's decomposition

Bender's decomposition method will be applied to solve the integrated recovery model. Jarrah (1993) discuss whether to do crew rotations or aircraft rotations first. Their computational study shows that when aircraft rotations were organized first and crew availability and working time conditions were not checked, it frequently happened that the obtained aircraft rotations were not feasible due to crew working time constraints. Therefore, it takes longer computer time if do aircraft rotation first.

Crew cost is mostly related to different paths due to the pay-and-credit cost structure. As to aircraft routings, however, no apparent cost structure is defined since our main concern is the maintenance feasibility. Thus, in our integrated recovery model, crew duty flow (or partition) is combined with fleet assignment in the master problem. Using the flight-fleet assignment variables as linking variables, Bender's decomposition can be applied. If the aircraft maintenance routing subproblem is infeasible, a set of feasibility cuts caused by infeasible maintenance will be returned to the master model.

6.5 Preprocessing issues

We need to pay attention to several preprocessing issues: (1) How to determine potential aircraft or crew set to provide swap options; (2) How to determine flight delay options efficiently. The reason to raise these questions is to find a good recovery solution from a small scale of alternatives.

6.5.1 Determine potential swap options

Four controlling parameters are used to obtain the potential resource set. They are predefined time window T1, T2, MAXNUM – the maximum cardinality of the potential set, and NSTAGE – the maximum number of downline stages.

Figure 6.3 demonstrates how to select potential aircraft swapping set. If a delayed leg causes aircraft misconnection, the aircrafts, which fly a flight that departs in time interval [new_arr_delayed, new_arr_delayed +T2] and have a predecessor flight arriving in time interval [dep_missed – T1, dep_missed], are selected into the potential aircraft set for swapping.

For all effective candidates, high priority is given to aircrafts with the same fleet type. If the number of potential aircrafts is smaller than MAXNUM, aircrafts from other fleet types will be brought in. But, a reasonable amount of potential crew from that fleet type needs to be included in the potential crew set. Note that the potential set could be empty. For example, at a sparsely visited spoke, there might be no route to swap. In such case, other recovery solutions can be appealed to, such as delay, cancellation or calling spare planes.

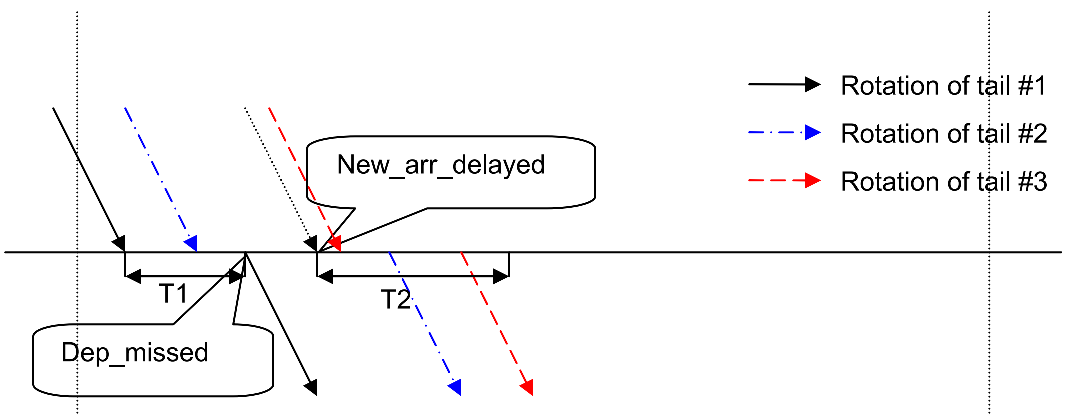


Figure 6.3: Demonstration of potential swap options

(Tail #2 is the effective swap candidate. Note: The time of the incoming arrows is ready time)

Rosenberger (2003) propose a method to obtain the potential swapping options by finding shortest directed cycle from the interaction graph. This method can be classified as cyclic swaps. In contrast, our method is limited 2-swaps.

When looking for potential set, it is necessary to take into account the legality rules (maintenance for aircraft, FAA legality rules for crew). For example, we want to avoid a hot plane as potential, as well as a crew who has flied enough time. Potential swap set for crew can be obtained using a method similar to that for aircraft. The difference is that if the station is a crew base, the disrupted crew will be sent home. Lettovsky (1997) present an alternative recovery solution. First, delay the misconnected leg (e.g. as early as possible to connect the delayed leg), and then consider to do swap at the arrival station of this successor leg. This idea can be used here to find the potential swap set if the delayed successor leg misconnects with its successor. In our method, a parameter NSTAGE is used to limit the stages of this downline effect until the misconnection is absolved.

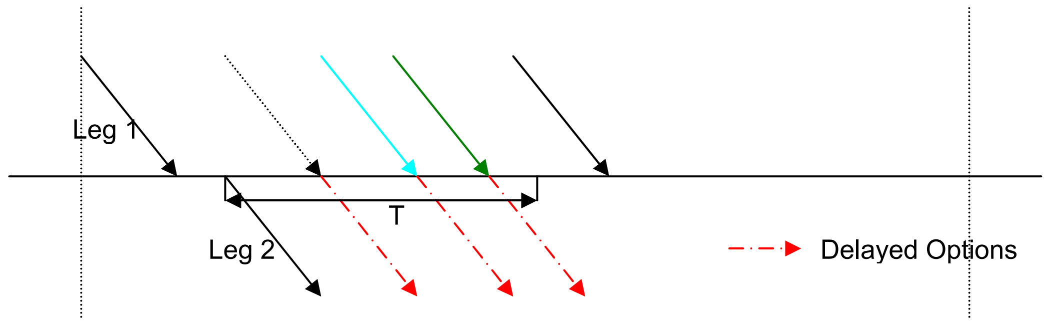
6.5.2 Flight delay options

(1) Delay options due to aircraft or crew misconnection

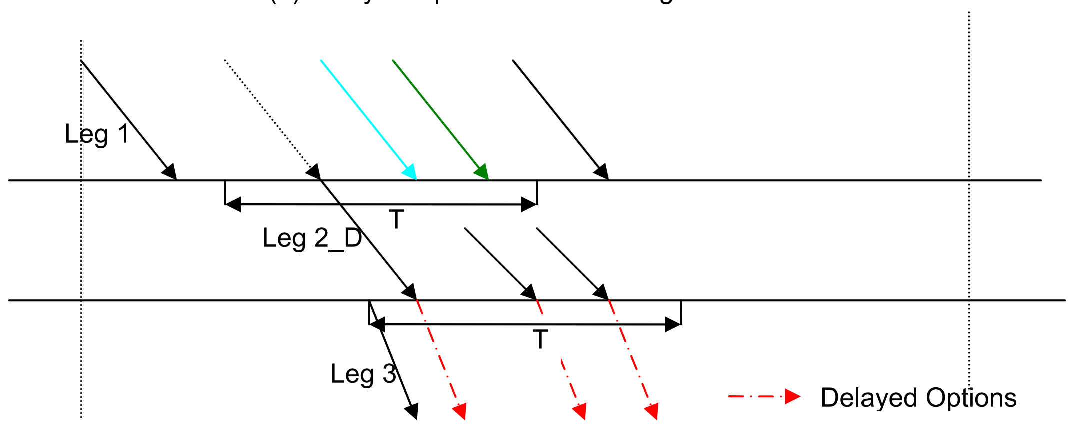
For all misconnections, no matter it is crew rotation or aircraft rotation, delay options can be determined by the method: predefined time window + MAXNUM+NSTAGE, as shown in Figure 6.4. Delay options are considered only if there is a connection within the time window and the maximum number of options doesn't exceed MAXNUM.

(2) Delay options due to passenger misconnection

There might be several itineraries misconnected due to one delayed leg. A threshold value is used to judge if the delay option will be added according to passenger itinerary misconnection. The delayed departure time for the successor leg in itinerary is a summation of the actual arrival time of the delayed leg and the minimum passenger connection time.



(a) Delayed Options at the 0th stage



(b) Delayed Options at the 1st stage

Figure 6.4: Flight delay options

6.5.3 Get delay and swap options together

It would be attractive if delay and swap options can be determined at the same time for the sake of compatible delay and swap options, as well as a reduced number of options. A bipartite graph at each station is constructed, similar to (Jarrah 1993). One node set represents aircraft nodes, the other represents flight nodes. Forward arcs – from aircraft nodes to flight nodes -- denote the original rotation, while backward arcs show candidate aircrafts for a flight. The stations are connected up. Using the original schedule, as well as using supply or demand nodes to show resource deficiency/return, a set of paths that connect supply/demand nodes can be found. Each path provides a feasible recovery solution consisting swaps and delays. Compared to (Jarrah 1993), the advantages of our implementation are: (1) A set of good and feasible paths are generated, instead of just the shortest one; (2) The airport network is taken as an inseparable system, instead of separate stations; (3) By building similar network, both aircraft and crew recovery options can be obtained. In addition, if deadhead legs are added into flight nodes, the crew network can provide options for enumerating pairings with deadhead. Reserve crew and spare planes can be easily modeled in this network.

6.6 Summary

In this chapter, integrated airline recovery method is proposed. Important preprocessing issues are discussed, such as how to control the propagation of disruptions, how to make a reasonable recovery scope, how to obtain potential swapping aircraft and crew set, and how to determine the flight delay options. A new integrated recovery framework is proposed. Duty flow model is combined with FAM in the master problem. The duty network is built in dated version and crew-specific. Instead of enumerating aircraft routings, a multicommodity network flow model is used for the aircraft maintenance routing, which can reduce the number of variables without losing maintenance considerations. In Bender's decomposition method, feasibility cuts about aircraft routings are generated and returned to the master problem.

CHAPTER 7

CONCLUSIONS AND FUTURE RESEARCH

7.1 Conclusions

Integration over business functions as well as the timeline is critical in finding new profits in airline planning and operation process. This dissertation investigates how to integrate fleet assignment and crew scheduling in order to produce more robust solutions to real time operations.

Chapter 2 provides a way of evaluating the crew friendliness of a schedule for a given fleet without intensive computational efforts. It facilitates the evaluation of the schedule development and the FAM solution with respect to crew scheduling issues. Based on schedule analysis, guidelines are suggested for making small modifications on a given schedule when the analysis indicates problems. Schedule analysis also shows the importance and advantages in integrating fleet assignment and crew planning.

The duty flow model described in Chapter 3 provides a way of approximating the crew pairing problem to deal with larger size crew scheduling problem in integrated planning or recovery within allowable time durations. The integrated fleet assignment and crew scheduling framework based on duty flow model is proposed and computational results indicate profit improvement.

Chapter 4 extends the station purity idea proposed by Smith in robust fleet assignment to crew base purity and robust crew scheduling. The proposed method can increase crew swapping options in operation by limiting the number of crew bases serving a given station. Computational results show that with little or no extra cost, a more robust crew pairing solution can be expected. It is also found that the scale of the robust crew scheduling model is much smaller than that of the traditional set partitioning model.

Chapter 5 provides a model to perform robust and integrated fleet assignment and crew planning based on both fleet purity and crew base purity. Our analysis shows that (1) Adding fleet purity can significantly improve planning quality and robustness in crew scheduling, maintenance, and operation; (2) Crew base purity can help avoid locked rotations in FAM solution, improve crew robustness, and significantly reduce the solution time; (3) The influence of fleet purity on FAM profit can be reduced by adjusting the fleet purity level, and a station purity scheme combining fleet purity and crew base purity can overcome the negative impacts (of adding fleet purity) on the computational efficiency. An integrated planning model which integrates fleet assignment and crew connections is proposed imposing station purity. Computational results show that this method is very efficient in solving industrial size problems.

Chapter 6 discusses important issues in integrated recovery. Recovery scope for the integrated recovery framework is proposed. Heuristics on selecting flight delaying options, move-up crews, and potential swapping aircrafts are discussed. Following the idea of integrating FAM and crew, a new integrated recovery model and Bender's decomposition solution method is proposed. Duty flow model is combined with FAM in the master problem, while feasibility cuts about aircraft routings are generated from solving aircraft routing subproblem and returned to the master problem.

7.2 Future research

It is important to understand how to limit the flight retiming options for integrating fleet assignment and schedule development (Rexing et al. 2000). Schedule analysis can provide efficient options to repair the schedule, resulting in reduced size of the problem when integrating schedule development and crew/fleet together. Moreover, the principles discussed in Section 2.3 for schedule adjustment are heuristic. It provides us options on retiming of legs, adding legs etc. However, there are some other issues need to be taken into account, such as the effect of retiming on revenue, or on passenger itinerary. In addition, it is necessary to restrict the range of schedule adjustment and to generate a new schedule without violating plane count constraints. Considering all these factors, a MIP model would help to make decisions on schedule repair and eventually provide an optimized solution. Thus, more research needs to be done on applying schedule analysis results to integrating fleet assignment and/or crew pairing with rescheduling options.

Given the success in implementing models in Chapter 4 and Chapter 5, a more promising and integrated model that includes aircraft maintenance routing can be achieved. Consequently, a comprehensive robust integrated planning model will be obtained, in which the aircraft maintenance routing can be modeled by the multicommodity flow network as discussed in Chapter 6.

The tactical integrated recovery framework proposed in Chapter 6 does not incorporate the concerns about robustness. The station purity strategies discussed in Chapter 5 can also be applied to the integrated recovery model for a robust rescheduling solution. Additional research will also be required to address the scalability of the presented approach for different scales of disruptions.

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