

ABSTRACT

CAI, HAN. Mobile Ad-hoc Networks: Mobility-induced Metrics, Performance Analysis, and System Design. (Under the direction of Professor Do Young Eun).

Mobile Ad-hoc Network (MANET), a type of self-configuring wireless ad-hoc network, comprises of mobile elements equipped with wireless communication devices. The mobility pattern of mobile nodes and the packet forwarding strategy crucially decide MANET performance. The mobility pattern affects system performance through mobility-induced metrics such as contact time and inter-meeting time. These metrics are critical in determining the MANET performance, as well as choosing various scheduling/forwarding algorithms.

In this dissertation, we study the effect of mobility patterns on the MANET performance through the mobility-induced metrics, e.g., inter-meeting time. The inter-meeting time is typically assumed to be exponentially distributed in MANET performance studies. However, recent empirical results disclose clear power-law behavior of inter-meeting time distribution. This outright discrepancy potentially undermines our understanding of the performance tradeoffs in MANET obtained under the assumed inter-meeting time with exponential distribution, and thus calls for further study on the power-law (or more generally, non-exponential) inter-meeting time including its fundamental cause, mobility modeling, and its effect.

We first prove that the *finite/infinite domain with respect to the time scale of interest* critically decides the exponential/power-law tail of the inter-meeting time distribution. We then show a *convex ordering* relationship among inter-meeting times of various mobility models indexed by their degrees of correlation, which is in good agreement with the ordering of network performance under a set of mobility

patterns whose inter-meeting time distributions have power-law ‘head’ followed by exponential ‘tail’. Finally, we analyze various characteristics of the relative mobility of a random pair of nodes in MANET to show that they produce inter-meeting time with different *aging* properties. The aging property allows us to establish for the first time that the approach based on exponential inter-meeting time assumption can always under-estimate or over-estimate the actual system performance, under stochastic mobility patterns with specific aging properties. Our results also provide theoretic guidelines on how to exploit the memory structure toward better design of protocols under general mobility.

Mobile Ad-hoc Networks: Mobility-induced Metrics, Performance
Analysis, and System Design

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DEDICATION

To my parents

To my husband *Tao*

BIOGRAPHY

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Chapter 1

Introduction

Mobile Ad-hoc Network (MANET) is a type of self-configuring wireless ad-hoc network. MANET comprises of mobile elements equipped with wireless communication devices. One example for MANET is pocket switched network [48, 22, 49], where the mobile elements are people with communication devices such as laptop, personal digital assistant, etc. Another example is vehicle ad-hoc network [16] where the mobile elements are vehicles which are able to communicate with each other or with roadside equipment. MANET can be modeled as a time-varying graph [6], where the node is mobile element, and the link is the time-varying communication links among mobile elements which can be established at a given time instance subject to some constraints, e.g., signal-to-interference-noise ratio [43, 27].

While the network topology (e.g., nodes, links, data flows, etc.) is affected by the mobility, so as the packet transmission among mobile nodes in the network [41, 6, 39, 86]. When the nodes are mobile in a comparatively slower time scale than that of a packet transmission session (duration of such session depends on the node density,

mobility, etc.), the localization of the nodes is the key [42, 83, 51] to establish the end-to-end path. A more common type of MANET is a network in which the mobile nodes are intermittently connected [90, 91] and the graph is disconnected. In this case, the conventional routing schemes fails [78, 56] since the setup of end-to-end path before data transmission becomes impossible. In consequence, the packet can only be transmitted by exploring the opportunity brought by the mobility, especially in sparse mobile networks or delay/disruption tolerant networks (DTNs) [32, 62, 71, 17, 67, 48].

The performance of intermittently connected MANET [72, 23, 90, 91, 53], is crucially determined by the mobility pattern of mobile nodes and the packet forwarding strategy. Mobility patterns in various scenarios can be very different. Hence, the studies in MANET have taken two systematic approaches in parallel: one is based on real traces, the other is based on synthetic mobility models that abstract out key mobility characteristics. These two approaches are complimentary to each other: the former is more realistic, while the latter facilitates tractable performance analysis as well as system design over different mobility settings in a controlled and repeatable manner. With increasing demand for the deployment of MANET, a large set of mobility models have been proposed in the literature. Examples include Random Waypoint mobility model (RWP) [15, 24, 36, 54, 55, 65], Random Direction mobility model (RDM) [81], Random Walk mobility model (RWM) [26] and its many derivatives such as Brownian motion model (BMM) on a sphere [35, 85], *i.i.d.* mobility model [70, 97], Reference Point Group Mobility (RPGM) [47], Manhattan (MH) [6] among others. (See [20] for a survey.) These mobility models have been widely used for predicting or estimating key performance metrics of various network protocols,

such as the capacity of MANET [41, 2], capacity-delay trade-offs [86, 35, 84, 97, 70], and the performance of relaying algorithms [41, 85, 22].

The mobility pattern affects system performance through mobility-induced metrics [6, 82, 7] such as contact time and inter-meeting time. In order to successfully transfer data from a mobile node to another, the mobile node needs to first wait until it ‘sees’ other mobile node (until it gets inside the transmission range of the other node) for data-relay, and then will be able to relay the data during the period it maintains the connection with the other node. The former metric is called the *inter-meeting time* of the two nodes [22, 46, 95, 48], and the latter is called the *contact time* [48, 62]. These two metrics are critical in determining the delay and capacity of the network, as well as choosing various scheduling/forwarding algorithms. In particular, the inter-meeting time of two nodes is a major component of the end-to-end delay in MANET [45, 40, 41, 84, 85, 86], as it denotes how long it takes to encounter the other mobile node to have any chance to relay/forward the data for communications. Larger inter-meeting time leads to larger end-to-end delay. Synthetic mobility models have been used for deducing properties of the inter-meeting time of a pair of mobile nodes, which in turn is used as an input to the analysis and prediction of network performance [52, 40, 44, 90].

In this dissertation, we study the effect of mobility patterns on the MANET performance through the mobility-induced metrics, e.g., inter-meeting time. We start from the characteristic of inter-meeting time and find that the controversial tail behavior of inter-meeting time distribution critically depends on the boundary of mobility models under consideration. We then discuss the relationship between the size of the boundary and the relevant timescale of interest and their effect on

the inter-meeting time distribution. Such relationship is further extended by us to study the stochastic anatomy of inter-meeting time distribution. Particularly, we find a stochastic ordering relationship between inter-meeting time distributions originated from mobility patterns with different degrees of correlation. Such ordering relationship can be applied to predict the performance comparison of forwarding algorithms. Last, we disclose the relationship between various mobility characteristics and a critical stochastic property, aging property of the inter-meeting times. The aging property allows us to establish for the first time that the approach based on exponential inter-contact time assumption can either always under-estimate or always over-estimate the actual system performance, under different stochastic mobility patterns indexed by their aging properties. Our results on aging properties also provide theoretic guidelines on how to exploit the memory structure toward better design of protocols under general mobility.

1.1 Crossing Over the Bounded Domain: From Exponential To Power-law Inter-meeting Time

There have been several studies on the characteristics of the inter-meeting time in the literature. For example, authors in [41, 84, 40] assume that the inter-meeting time is exponentially distributed so as to make their analysis tractable. This assumption is supported by numerical simulations [85, 87, 40] based on random waypoint mobility model (RWP) [55, 15, 24, 65, 36, 54, 20]. Further, there have been also some theoretical results showing that the first and second moments of the inter-meeting

time are bounded above [87, 86] under Brownian motion model on a sphere [35, 85].

However, recent empirical results on the inter-meeting time via real, extensive mobility traces [22, 46, 48, 95] in fact indicate that the tail behavior of the inter-meeting time is far from being exponential, but is close to a power-law [22, 25, 48]. For example, in [48], four distinct sets of data are used to analyze the inter-meeting time. Two of them test WiFi networks (large area) containing thousands of nodes over three or four months (UCSD [73], Dartmouth [46]); another two use a Bluetooth network [48] containing hundreds of nodes in an office/lab environment (small area) over several days. All of these invariably show that the distribution of the inter-meeting time T follows a power-law over a wide range of timescale, i.e., $\mathbb{P}\{T > t\} \sim t^{-\alpha}$ for some constant $\alpha > 0$. More strikingly, there it was also shown that the power-law exponent α is typically less than 1, making even the first moment of the inter-meeting time infinite. This is in sharp contrast to all the results in the current literature where an exponential inter-meeting time distribution is either assumed or numerically verified through various current mobility models.

Above discrepancy between the recent empirical data and the simulation results based on current mobility models has motivated several studies in the literature. [22] investigates the effect of power-law inter-meeting time distribution on the system performance (capacity and delay) and calls for new mobility models to produce power-law inter-meeting time; [71] studies the effect of infrastructure and multi-hop transmission on networks with power-law inter-meeting time and calls for new forwarding algorithm to effectively utilize communication opportunities such as the existence of low-delay multi-hop path; [11] proposes a method to generate power-law inter-meeting time using one dimensional random walk model. Most recently, [58]

has found that although the inter-meeting time distribution follows a power-law over a wide range of time scale, there exists a characteristic time, on the order of about half a day, beyond which the inter-meeting time distribution decays exponentially. While these works provide some answers to the characteristics and effect of power-law inter-meeting time, the following question still remains: *What is the fundamental reason for this discrepancy?* Answer to this question is very important¹ because the disclosure of the fundamental reasons will provide essential guidelines on all related studies in MANETs – modeling of power-law inter-meeting time, analysis of its effect [22, 71], capacity-delay trade-offs [41, 35, 84, 97, 70, 86], and the design and performance analysis of relaying algorithms [41, 22].

We study the fundamental reason for the *exponential/power-law behavior* of inter-meeting time distribution by first showing that the *finite boundary* is one of the key aspects that give rise to the universal appearance of the exponential inter-meeting time distribution in the literature. Indeed, almost all the current mobility models have boundaries or “end-of-the-world”. For example, in the definition of RWP [15, 24, 54, 55, 20], the boundary is necessary because whenever the node changes direction and/or speed, it selects the destination ‘uniformly’ over some bounded simulation area. As for random direction mobility model (RDM) [81] and *i.i.d.* mobility model [97, 70, 20], a boundary is incorporated into their definitions. Similarly, for random walk mobility model (RWM) [26] or Brownian motion on a sphere [85, 35], the boundary is also enforced as all the mobile nodes are constrained

¹Even with the recent claim in [58] saying that the tail of inter-meeting time distribution is still exponential after the characteristic time around 12 hours, note that the delay of interest in most of the network scenarios is within this time scale over which the inter-meeting time distribution exhibits power-law behavior [22, 58].

to move on a sphere or inside a boundary. Here the *boundary* may be of different types including ‘reflective’ or ‘wrapping’ as defined in [20], which converts a 2-D rectangular area into a torus-shaped area.

In particular, we rigorously prove that the tail of the inter-meeting time between two independent mobile nodes² decays *at least exponentially fast* for any RWP model and for any RWM (or RDM) model as long as the boundary is finite, i.e., the mobility model is defined on a bounded domain. Our assumptions are quite general in that they are satisfied by nearly all the current mobility models in the literature. Our results readily show the finiteness of all the moments of the inter-meeting time (much stronger than the finiteness of only the first and second moments as shown in [87]), and provide analytical support on the simulation-based studies of the inter-meeting time under the current mobility models with boundaries [85, 87, 40].

Our theoretical results imply that the finite boundary, among others, is the key determining factor of the exponential decay in the inter-meeting time distribution, and thus potentially undermines our understanding of the performance of MANET. Moreover, we prove that removing the boundary from 2-D RWM readily gives the power-law distribution of the inter-meeting time.³ We also discuss the relationship between the size of the boundary and the relevant timescale of interest and their effect on the inter-meeting time distribution. Simulation results are provided to support our theoretical analysis and discussion.

²Mobile nodes may be dependent if they belong to the same ‘group’ and follow similar paths, or belong to different groups whose pathways rarely overlap. The inter-meeting time in such cases may show very different characteristics.

³Similar suggestion has been made in [11] via 1-D random walk, but our example in Section 3.2 is in 2-D.

1.2 Stochastic Anatomy of Inter-meeting Time

While there is now strong evidence showing that the inter-meeting time distribution has a mixture of power-law and exponential behavior in reality [58] with partial explanation for its root from first part of work in this dissertation, the following issue still remains unsolved: the effect of mobility patterns on the network performance such as network capacity or the end-to-end delay of forwarding algorithms through the mobility-induced inter-meeting time *distribution*.

The problem of the impact of mobility patterns on network performance is not new. However, it is facing new challenge due to the discovery of the mixture of power-law and exponential behaviors in the inter-meeting time distribution. While [58] maintains that current mobility models can still generate this mixture behavior, due to the complexity of the problem at hand, the study of effect of mobility patterns on the inter-meeting time distribution is largely limited to the estimation of the average inter-meeting time (first-order behavior) [40, 93, 58], or two extreme cases, exponential and power-law, separately. Under exponential inter-meeting time distribution, the performance of forwarding algorithms (e.g., epidemic routing [98], its K-hop variants [40], spray and wait [92], etc.) has been widely studied based on Markovian analysis [88, 40, 92, 89, 44, 90, 91]. or so-called the mean-field approach based on ordinary differential equation (ODE) [44, 102]. On the other hand, under the assumption that inter-meeting time distribution has power-law tail, [22] analyzes the performance of generic two-hop forwarding algorithms and draws rather pessimistic conclusions, i.e., the mean end-to-end delay of any two-hop forwarding algorithm is infinite if the inter-meeting time between two nodes has power-law tail

with infinite mean. The only exception is the very recent work [3] on the analysis and comparison of two-hop routing algorithms under exponential and hyper-exponential inter-meeting time distribution.

We investigate how the stochastic nature of the mobility patterns affects the whole *distribution* of the inter-meeting time. In contrast to existing works focusing largely on the mean inter-meeting time under a given specific mobility model, we directly quantify the impact of statistical mobility patterns on the *shape* of the inter-meeting time distribution, which in turn critically affects the performance of forwarding/routing algorithms running on top of the underlying nodes' mobility pattern. Motivated by recent findings in the first part of work in this dissertation as well as [58] that shows clear transition behavior of the inter-meeting time distribution from power-law to exponential (which is called ‘dichotomy’ in [58]), our approach in this dissertation is to first divide the whole distribution of the inter-meeting time into *head* and *tail*, which is separated by the critical timescale τ_0 , beyond which the inter-meeting time distribution shows exponential decay (tail).

Specifically, we first study the scaling relationship between τ_0 , the size of the domain, and the statistical nature of general mobility patterns such as the degree of correlations or tendency of preserving the same direction in the mobility pattern. Second, for the ‘head’ of the inter-meeting time distribution ($t < \tau_0$), we prove via correlated random walk models that the inter-meeting time of mobile nodes with stronger correlations in their paths is *stochastically larger* [74] than that with weaker correlations. This stochastic ordering result, along with our scaling relationship between τ_0 and the degree of correlations, provides quantitative understanding of how different mobility models lead to different *shapes* of the inter-meeting time

distribution and enables us to compare different degrees of power-law behavior in the head of the inter-meeting time. Third, we establish an *invariance property* for a class of mobility models. Based on reformulation of Kac’s recurrence theorem [57, 59, 30, 8] for any stationary ergodic process, we prove that the average inter-meeting time is invariant with respect to the degree of correlations in the mobility models. More interestingly, we also show that this invariance result holds not only for the inter-meeting time but also for many other variants of the contact-based metrics including contact time, inter-hitting time, inter-any-contact time, both under Boolean (distance-based) model and under physical interference models. The invariance result thus tightly binds the head and the tail of the inter-meeting time distribution and imposes a unifying constraint in that the mean inter-meeting time (and its variants) remains the same under a given domain size, *regardless of* the shape of the distribution and the location of τ_0 . Last, we discuss the impact of our results on the study of mobility metrics and on the performance analysis of forwarding/routing algorithms in contact-based MANETs.

1.3 Aging Rules: What Does the Past Tell About the Future in Mobile Ad-Hoc Networks?

In spite of the discovery of the non-exponential behavior of the inter-meeting time, the classical approach based on exponential inter-meeting time assumption is still very popular due to the complexity of analyzing performance under non-exponential inter-meeting time. The coexistence of two approaches based on ex-

ponential and non-exponential inter-meeting time seems reasonable in view of two contradictory goals – realism and simplicity (or analytical tractability). Nevertheless, up till now, works based on these two approaches are conducted in separate ways, and this raises the following question: how much sacrifice have we made for simplicity?

We analyze various characteristics of the relative mobility of a random pair of nodes in MANET to show that they produce inter-meeting time with different *aging* properties. Particularly, by exploring the memory structure of the inter-meeting time, we characterize how the past experience of a mobile node (e.g., the time elapsed since the latest meeting between the node and its target node, called the *age*) relates to its future behavior (e.g., how long it will take to meet the same target node again, called the *residual lifetime*). Due to the memoryless property of exponential inter-meeting time, the memory structure in contact dynamics has seldom been analytically studied. In contrast to existing works on the explanation and modeling of non-exponential inter-meeting time, our work here focus on *opportunities* brought by the non-exponential (memory) structure of the inter-meeting time.

Specifically, by first fixing one node and changing the law of random walks on directed graphs with dynamic topologies modeling random environments, we analyze four classes of stochastic mobility patterns and show that they produce inter-meeting times with constant/decreasing/increasing failure rate and new-better-than-used property. We then consider the case when both nodes are mobile and use simulation results to disclose that the inter-meeting times under Random Waypoint (RWP) models [55, 20] and random walk (RW) mobility models [26] have increasing and decreasing failure rates, respectively. After that, we show *convex order-*

ing relationships between *exponential and non-exponential* inter-meeting times with different aging properties, which allow us to establish for the first time that the approach based on exponential inter-meeting time assumption can either *always under-estimate* or *always over-estimate* the actual network performance depending on positive or negative aging properties of underlying non-exponential mobility patterns. Our results on aging properties rigorously state how we should develop forwarding/routing algorithms that exploit the ‘memory’ in the right way.

1.4 Outline

The rest of the dissertation is organized as follows. Chapter 2 first discusses related work on empirical observation of the power-law inter-meeting time distribution, and then provides several definitions and technical results to be used later on in both parts of our work in this dissertation. In Chapter 3, we study the exponential/power-law behavior of inter-meeting time. Chapter 4 provides our analysis on the stochastic anatomy of inter-meeting time distribution under general mobility models. Chapter 5 presents our study on the aging property of inter-meeting time and its applications in performance analysis as well as system design. We finally conclude in Chapter 6.

Chapter 2

Background

In this chapter we first discuss the empirical observation of power-law behavior of inter-meeting time. Then, we collect a set of distinct mobility models to be used for our study. At last, we give basic assumptions, notations and several definitions of the metric of interest that will be used in the dissertation. We also provide some basic properties associated with those definitions and notations.

2.1 Empirical Observation of Power-law

The power-law behavior of the inter-meeting time distribution has been reported through various data sets under different scenarios. In this section, we provide brief summary on how those data sets have been collected and interpreted in the context of the inter-meeting time distribution, while referring to [48, 22, 71] for more details.

Most data sets available for inter-meeting time study in the literature can be classified into access point (AP) based or Bluetooth-based ones. AP-based data

sets [73, 46] keep track of the time instants at which a mobile node gets associated with an AP and typically record activities (AP logs) of a large number of mobile nodes for a long period of time. Two nodes are assumed to ‘meet’ only if both of them are in the range of the same AP. In view of the inter-meeting time of mobile nodes, some inaccuracies are inevitable in these AP-based data sets since (i) two nodes within the range of each other may not be within the range of the same AP, i.e., the duration of the inter-meeting time could be overestimated, and (ii) two nodes within the range of the same AP may be out of their communication ranges as well, i.e., the inter-meeting time could also be underestimated.

On the other hand, Bluetooth-based data sets record time instants at which a node is in contact with other nodes, along with the ID of the node in contact [48, 22, 71]. While the number of nodes and the duration of the activities are typically smaller than those of AP-based ones,¹ Bluetooth-based data provide more accurate information on the inter-meeting time, as they are free of any reference to APs and directly keep track of contact/inter-contact events. Nevertheless, in all AP-based and Bluetooth-based data sets, the power-law behavior of the inter-meeting time has been shown to persist over from several hours up to even days.

2.2 Mobility Models

First, we provide definitions for the mobility models used in Chapter 4.

Correlated Random Walk on Grid (CRW) [9]: In each time step, a mobile node

¹For example, the UCSD data set [73] includes more than 195 thousands contacts collected from 273 participating devices over 16 days, while the Infocom 2005 Bluetooth-based data set includes about 7 thousands contacts collected from 41 dedicated iMote devices over 3 days.

moves to one of its 2(4) neighbor sites on 1-D(2-D) grid. The initial direction of the node's step is randomly chosen from right and left (1-D case), or from right, left, up and down (2-D case). Then, the node takes a step in the chosen direction. After that, at each time slot, the node takes a step in the same direction with probability p , opposite direction with probability q and turns to other directions with probability r .² Hence, in 1-D case, $p + q = 1$, while in 2-D case, $p + q + 2r = 1$. Boundary condition is wrapping around.

Random Direction Mobility Models (RD) [81, 20]: a mobile node first chooses a uniform random direction, then moves until it hits the boundary of the domain in that direction.³ After that, it chooses another random direction and the whole process repeats itself.

Isotropic Random Walk (IRW) [26, 20]: a mobile node first selects a random step-length L , a speed v from some well-defined distribution, and a direction ϕ taken uniformly and randomly from $[0, 2\pi)$. Then, it moves according to the chosen velocity for L with angle ϕ , and upon its completion of the step, the whole procedure repeats independently of all others. Note that by taking suitable step-length distribution, IRW can generate many different mobility patterns. For instance, Brownian motion mobility model can be approximated by taking very small step-lengths over small time intervals.

Remark 1 *It is known that the stationary distributions of node positions in the*

²Our definition is a bit different from [9], where a node takes step at time instance governed by a Poisson process.

³In fact, there are two variants of RD [20]. Here we describe the first variant; the second variant of RD belongs to the IRW described next.

steady-state for all the above mobility models exist and are given by a uniform distribution over Ω [75, 10, 37, 12, 9]. There are also many other mobility models whose stationary distributions of the node position are all uniform [12, 75]. In order to focus on the effect of mobility pattern on the network performance, in this dissertation, we do not consider the pause of mobile nodes, unless otherwise specified.

□

In what follows, we will introduce random walks on directed graphs, which are widely used to model the system dynamics in many fields where the topology critically affects the system performance (e.g., social science [1], physics [28], etc.). This class of mobility models will be used in Chapter 5.

A finite digraph (graph with directed edges [100]) $\mathcal{G}$ is described by $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, W\}$ where $\mathcal{V} = \{0, 1, \dots, N\}$ ($N < \infty$) is a vertex set, $\mathcal{E}$ is an edge set such that $(i, j) \in \mathcal{E}$ if and only if a directed edge from vertex i to j exists ($i, j \in \mathcal{V}$), and $W = \{w_{ij}\}$ is $(N+1) \times (N+1)$ matrix with edge weight $0 < w_{ij} < \infty$ if $(i, j) \in \mathcal{E}$ and $w_{ij} = 0$ otherwise. Throughout the paper, we assume that the graph $\mathcal{G}$ has no multiple edges. A *directed* path of length m from vertex i to vertex j ($i = k_0, k_1, \dots, k_{m-1}, k_m = j$) ($k_r \in \mathcal{V}$ for $r = 0, \dots, m$) exists if for each $r = 0, \dots, m - 1$, $(k_r, k_{r+1}) \in \mathcal{E}$. An *undirected* path between i and j exists if, for each r , at least one of (k_r, k_{r+1}) and (k_{r+1}, k_r) belongs to $\mathcal{E}$. In our study of MANET, we find the following definitions useful.

Definition 1 *A graph $\mathcal{G}$ is connected if for any $i, j \in \mathcal{V}$, there exists a directed path from i to j .*

Definition 2 *A graph $\mathcal{G}$ is weakly connected if for any $i, j \in \mathcal{V}$, there exists an undirected path between i and j .*

In MANETs such as pocket switched networks [22, 48], usually there is a ‘two-way’ path connecting any two different sites, i.e., $\mathcal{G}$ is connected, while in vehicular ad-hoc networks, there might be only ‘one-way’ path connecting two different sites (e.g., the ‘one-way’ roads), for which $\mathcal{G}$ is weakly connected. Clearly, any connected graph $\mathcal{G}$ is also weakly connected, but the converse is generally not true.

We define a vertex weight matrix D as follows:

Definition 3 *For $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, W\}$, the vertex weight matrix $D = \{d_{ij}\}$ is $(N+1) \times (N+1)$ matrix with $d_{ii} = \sum_{k=0}^N w_{ik}$ (sum of i^{th} row elements of W) and $d_{ij} = 0$ for $i \neq j$.*

For a mobile node C following random walk on $\mathcal{G}$ (written as RWG), at each time step, the probability for C walking from vertex (or site) i to site j is $p_{ij} = w_{ij}/d_{ii}$, which is well defined when $\mathcal{G}$ is connected, i.e., $d_{ii} > 0$. (From now on, we will assume graph $\mathcal{G}$ is connected, unless otherwise specified.) Since D is a diagonal matrix with positive elements on the diagonal, we can set D^{-1} as the diagonal matrix with diagonal elements $1/d_{ii}$. Then, the random walk on graph $\mathcal{G}$ (or the position of the walker $C(t)$ at time step t) can be described by a Markov chain (MC) with state space $\mathcal{V} = \{0, 1, \dots, N\}$ and transition matrix $P = D^{-1}W = \{p_{ij}\}$.

Remark 2 Conversely, for any MC with transition matrix P , we can always find a RWG model with edge weight matrix W (and thus D constructed from W) such

that $P = D^{-1}W$.⁴ In other words, the set of random walk on graph (RWG) mobility models is equivalent to the set of all Markov chain mobility models.

2.3 Definitions and Properties

2.3.1 Contact-based Metrics

Consider a set of mobile nodes $\{A, B_1, B_2, \dots, B_N\}$ following some mobility models in a common domain Ω . For a node A , let $A(t) \in \Omega$ be the position of node A at time t . Similarly for $B_i(t)$.

We define by $\mathcal{N}_A(t) \subset \Omega$ the *contact set* of node A at time t , i.e., a node X can communicate with A at time t if and only if $X(t) \in \mathcal{N}_A(t)$. Exact definition of the contact set varies depending on the context: (i) If we consider A 's contact with an arbitrary set of nodes $\Theta = \{B_i, i \in \mathcal{I}\}$ for some index set $\mathcal{I}$ under *Boolean* model with communication range d , then the contact set of Θ becomes

$$\mathcal{N}_\Theta(t) = \bigcup_{i \in \mathcal{I}} \{x \in \Omega : \|x - B_i(t)\| \leq d\}. \quad (2.1)$$

(ii) If we consider contact of node A under *SINR* (Signal-to-Interference-Noise Ratio) model [43, 27], then the contact set of A at time t will be in the form of

$$\mathcal{N}_A(t) = \left\{ x \in \Omega : \frac{P\|x - A(t)\|^{-\alpha}}{N_0 + \sum_i P\|x - B_i(t)\|^{-\alpha}} \geq \beta \right\} \quad (2.2)$$

for some suitable threshold β (e.g., the minimum signal-to-interference noise ratio required for successful decoding at the receiver) and path loss exponent $\alpha \in [2, 4.5]$, i.e., node X can communicate with node A at time t ($X(t) \in \mathcal{N}_A(t)$) if and only if the channel condition between two nodes (given by SINR) is good enough.

⁴The simplest way is to set $D = I$ (identity matrix) and $W = P$.

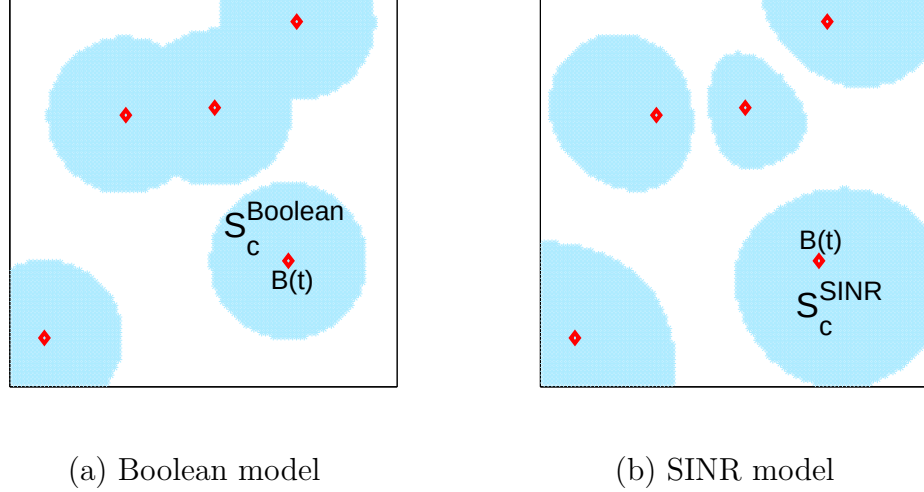


Figure 2.1: Contact sets (possibly time-varying) $S_c^{Boolean}(\cdot)$ and $S_c^{SINR}(\cdot)$ for a node B under Boolean and SINR interference models, respectively. Positions of nodes in (a) and (b) are the same.

Figure 2.1 shows examples of contact set for a node B under Boolean model (a) and SINR model (b).

Remark 3 *Boolean interference model is reasonable if the density of the mobile nodes is not high and the interference from other mobile nodes are negligible. This is especially the case in sparse mobile networks or delay/disruption tolerant networks (DTNs) [32, 62, 71, 17, 67, 48]. In case of high level of density of mobile nodes, inter-meeting time based on such distance-based interference model might be rather optimistic and would lead to underestimation of the actual inter-meeting time, since two nodes may not be able to communicate due to interference even when they are nearby. Still, even for such a highly populated MANET, inter-meeting time analysis based on Boolean model can also be applied to different mobile ‘groups’ whose density is usually much lower than that of mobile nodes.*

Definition 4 The inter-meeting (or inter-contact) time of node A with a set of nodes Θ is defined by⁵

$$T_I \triangleq \inf_{t>0} \{t : A(t) \in \mathcal{N}_\Theta(t)\} \quad (2.3)$$

given that

$$A(0^-) \in \mathcal{N}_\Theta(0^-) \text{ and } A(0) \notin \mathcal{N}_\Theta(0). \quad (2.4)$$

If we replace $\mathcal{N}_\Theta(t)$ in (5.1) and (2.4) by $\mathcal{N}'_\Theta(t) = \Omega \setminus \mathcal{N}_\Theta(t)$, we obtain the *contact time* (or contact duration) of node A with a set of nodes Θ . Similarly, if the set of nodes in Θ are static (not moving), then we call the inter-meeting time as *inter-hitting time*. Further, if we remove the condition in (2.4), i.e., we start measuring the time until next encounter to Θ from a randomly chosen time instant $t = 0$, then T_I becomes the *first passage time* (FPT) of node A to the set Θ .

Definition 5 Let $\{S(t) \in \Omega\}_{t \geq 0}$ be a collection of subsets of Ω . The first passage time (FPT) T_F of $A(t)$ to $\{S(t)\}$ is defined as

$$T_F \triangleq \inf_{t>0} \{t : A(t) \in S(t)\}.$$

given that $A(0) \notin S(0)$. □

In other words, the FPT T_F corresponds to the residual life time of the inter-meeting time [22, 58]. When the inter-meeting time T_I has a finite mean, the distribution of T_F is given by

$$\mathbb{P}\{T_F > t\} = \frac{1}{\mathbb{E}\{T_I\}} \int_t^\infty \mathbb{P}\{T_I > s\} ds, \quad (2.5)$$

⁵Note that our definition includes the pairwise inter-meeting time and the aggregate inter-contact time [58] by appropriately choosing the set Θ .

i.e., T_F has the *equilibrium* distribution of T_I . Finally, we note that all these metrics can be similarly defined under a discrete time setting. For instance, for the inter-meeting time between nodes A and B , T_I becomes

$$T_I = \min_{t \geq 0} \{t : A(t) \in \mathcal{N}_B(t), A(t+1) \notin \mathcal{N}_B(t+1)\}$$

given that $A(0) \notin \mathcal{N}_B(0)$ and $A(1) \in \mathcal{N}_B(1)$.

2.3.2 Useful Properties

As we will be interested in the behavior of the tail of the inter-meeting time distribution, it is advantageous to introduce the following definition.

Definition 6 *Let $\mathcal{P}$ be a collection of positive random variables whose complementary distribution decay at least exponentially fast. Specifically, we write $X \in \mathcal{P}$ if there exists a constant $c > 0$ such that*

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \log \mathbb{P}\{X > t\} = -c < 0. \quad (2.6)$$

Note that (2.6) implies $\mathbb{P}\{X > t\} \leq \exp(-ct + o(t))$ where $o(t)/t \rightarrow 0$ as $t \rightarrow \infty$. Or, equivalently, $\mathbb{P}\{X > t\} = \exp(-c[t + o(t)/t]) \leq Ke^{-ct}$ for some $K > 0$ and for all sufficiently large t , indicating that the tail of X decays at least exponentially fast. With this definition, we immediately obtain the following properties that will be useful later.

(P1) If $X, Y \in \mathcal{P}$, then $aX + bY \in \mathcal{P}$ for any positive constants a, b .

(P2) If $X \in \mathcal{P}$, then $X_e \in \mathcal{P}$, where X_e has the equilibrium distribution of X , i.e.,

$$F_e(x) = \mathbb{P}\{X_e \leq x\} = \int_0^x (1 - F(u)) du / \mathbb{E}\{X\}, \quad (2.7)$$

where $F(x) = \mathbb{P}\{X \leq x\}$.

Property P1 can be seen by noting that for any (possibly dependent) random variables X and Y ,

$$\mathbb{P}\{aX + bY > t\} \leq \mathbb{P}\{X > t/(2a)\} + \mathbb{P}\{Y > t/(2b)\}. \quad (2.8)$$

Thus, if $X \in \mathcal{P}$ with constant $c_1 > 0$ and $Y \in \mathcal{P}$ with $c_2 > 0$, it follows that

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \log \mathbb{P}\{X + Y > t\} = -\min \left\{ \frac{c_1}{2a}, \frac{c_2}{2b} \right\} < 0.$$

P2 follows since

$$\begin{aligned} \mathbb{P}\{X_e > t\} &= \frac{1}{\mathbb{E}\{X\}} \int_t^\infty \mathbb{P}\{X > x\} dx \\ &\leq \frac{1}{\mathbb{E}\{X\}} \int_t^\infty K e^{-cx} dx = \frac{K}{c\mathbb{E}\{X\}} e^{-ct} \end{aligned}$$

for all sufficiently large t . Further, $X \in \mathcal{P}$ ensures that all the moments of X are finite, thus its moment generating function is well-defined, i.e., $\mathbb{E}\{e^{\theta X}\} < \infty$ for some $\theta > 0$.

2.4 Aging Property of Random Variables

For a positive random variable denoting certain duration of an event such as the inter-meeting time in MANET or the system (component) lifetime in reliability engineering [38, 66], a simple but powerful way to characterize how its past experience relates to the future behavior (memory structure) is through its aging property. For instance, for a given component with its age $t > 0$ (has been operational since t seconds ago), what is its residual life (remaining time until failure)? Similarly, given

that the latest contact with a certain node took place t seconds ago, how long does it take until encounter to the same node again? To characterize the aging property, we need the following definition:

Definition 7 [80, 66] *For a discrete random variable $X > 0$ with distribution $F(t) = \mathbb{P}\{X < t\}$ ($t = 1, 2, \dots$), for each t such that $\bar{F}(t) = 1 - F(t) > 0$, the failure/hazard rate function of $F(t)$ is defined by*

$$r(t) \triangleq \mathbb{P}\{X = t\} / \bar{F}(t) = \mathbb{P}\{X = t\} / \mathbb{P}\{X \geq t\}. \quad (2.9)$$

X is an increasing/decreasing failure rate (IFR/DFR) random variable if $r(t)$ is increasing/decreasing function of t .

The failure rate of a continuous lifetime random variable X can be similarly defined as in Definition 7. Note that from (2.9) the failure rate $r(t)$ can be looked as the conditional probability that a component with lifetime X fails at time t , given that it has not failed till time $t - 1$. When X is geometrically distributed with $\mathbb{P}\{X = t\} = p(1 - p)^{t-1}$ for some $p \in (0, 1)$, it follows that $r(t) = p$, i.e., X has constant failure rate (CFR). Conversely, if a discrete random variable X has CFR, then its distribution is necessarily geometric or equivalently memoryless (in continuous time case, exactly exponential).

Denote by X_t the residual lifetime of X at time $t \geq 0$. Then, its ccdf $\bar{F}_t(\tau) = \mathbb{P}\{X_t \geq \tau\}$ (called survival function at time t) is given by

$$\bar{F}_t(\tau) = \mathbb{P}\{X - t \geq \tau | X \geq t\} = \bar{F}(t + \tau) / \bar{F}(t), \quad (2.10)$$

for all t such that $\bar{F}(t) > 0$. Then, it is well known that a distribution F has IFR/DFR if and only if $\bar{F}_t(\tau)$ is decreasing/increasing in t for any given $\tau > 0$ [66].

This means that for DFR $F(t)$ and for any $t_1 \leq t_2$, $\bar{F}_{t_1}(\tau) \leq \bar{F}_{t_2}(\tau)$ for all $\tau > 0$. (The inequality is reversed for IFR $F(t)$.) This is equivalent to $X_{t_1} \leq_{st} X_{t_2}$ [80], i.e., $\mathbb{E}\{f(X_{t_1})\} \leq \mathbb{E}\{f(X_{t_2})\}$ for any increasing (non-decreasing) function f . For this reason, a DFR random variable X is said to have ‘negative aging’ property. (The residual life becomes stochastically larger as the age t increases.) Similarly for an IFR random variable X , the residual life X_t at age t is stochastically decreasing in t (positive aging). The positive/negative aging property can also be defined in weaker sense:

Definition 8 [80, 66] *For a discrete random variable $X > 0$ with cdf $\bar{F}(t) = \mathbb{P}\{X \geq t\}$ ($t = 1, 2, \dots$), X is called new better than used (written as $X \in \text{NBU}$), if*

$$\bar{F}(t + \tau) \leq \bar{F}(t)\bar{F}(\tau). \quad (2.11)$$

If the inequality in (2.11) is reversed, X is called new worse than used (written as $X \in \text{NWU}$).

When $\bar{F}(\tau) > 0$, (2.11) is equivalent to $\bar{F}(t + \tau)/\bar{F}(t) \leq \bar{F}(\tau)$. Also, from (2.10), if X has IFR, then $\bar{F}_t(\tau) = \bar{F}(t + \tau)/\bar{F}(t)$ is decreasing in t for each $\tau > 0$. Hence, from Definition 8, if X has IFR, $X \in \text{NBU}$. Similarly, if X has DFR, $X \in \text{NWU}$.

Chapter 3

Crossing Over the Bounded Domain: From Exponential To Power-Law Inter-meeting time

The inter-meeting time is typically assumed to be exponentially distributed in many performance studies of MANET or numerically shown to be exponentially distributed under most existing mobility models in the literature. However, recent empirical results show otherwise: the inter-meeting time distribution in fact follows a power-law. This outright discrepancy potentially undermines our understanding of the performance tradeoffs in MANET obtained under the exponential distribution of the inter-meeting time, and thus calls for further study on the power-law inter-meeting time including its fundamental cause, mobility modeling, and its effect. In this chapter, we disclose that the controversial tail behavior of inter-meeting time distribution critically depends on the boundary of mobility models under consider-

ation.

We start from mobility models with finite boundaries, i.e., $\Omega \in \mathbb{R}^2$ is bounded, in Section 3.1.1 and 3.1.2. We rigorously prove that mobility models such as random waypoint mobility models (RWP) and random walk models (RWM), defined on a bounded domain Ω necessarily result in an asymptotic exponential tail of the inter-meeting time under Boolean interference model defined in Section 2.3.1. The mobility models such as RWP, RWM, and RDM under consideration have very different mobility patterns in a bounded domain, but the exponential behavior of their inter-meeting time distribution is universal. Hence, we discuss the relationship between finite boundary and exponential tail of inter-meeting time distribution in Section 3.2.1. After that, in Section 3.2.2, we consider the case of unbounded Ω (e.g., $\Omega = \mathbb{R}^2$), and show through a class of simple isotropic random walks in an open-space without boundary (i.e., $\Omega = \mathbb{R}^2$) that the power-law decay of the inter-meeting time will arise. We then discuss the relationship between the size of the boundary and the relevant timescale of interest and their effect on the inter-meeting time distribution, the heterogeneous modeling approach toward power-law like inter-meeting time distribution, as well as the effect of nodes' pause time on their inter-meeting time distribution in Section 4. Simulation results are provided to support our analysis and discussion in Section 3.4.

3.1 Inter-meeting Time with Exponential Tail

In this section we are interested in mobility models with finite boundaries, i.e., $\Omega \in \mathbb{R}^2$ is bounded, and rigorously prove that mobility models such as random

waypoint mobility models (RWP) and random walk models (RWM), defined on a bounded domain Ω necessarily result in an asymptotic exponential tail of the inter-meeting time. We will consider the case of unbounded Ω (e.g., $\Omega = \mathbb{R}^2$) later in Section 3.2.

3.1.1 RWP Models

In RWP model [50], a node first selects a random waypoint as its destination uniformly in a convex bounded region Ω , and a speed V uniformly from $[v_{min}, v_{max}]$ ($0 < v_{min} \leq v_{max}$), and then moves to its destination at its chosen speed. After it reaches the destination, it pauses for a random amount of time and then selects a new destination and speed, and repeat the whole procedure independently. Denote the time instant at which a node pauses and chooses a new destination as *joint time* and the path of a node consisting of a straight line segment as one *leg* [50]. The speed of node A is chosen uniformly from $[v_{min}^A, v_{max}^A]$. Let τ_n^A be the duration of the n^{th} pause time of A. Similarly we define τ_n^B for node B.

We first consider the traditional RWP model where all the pause times are zero and mathematically prove that the inter-meeting time decays at least exponentially fast. Later in this section, we generalize the result to include the case of non-zero random (possibly infinite) pause time. Throughout the section we assume that nodes A and B are independent, but we do allow that the speed and pause time distributions are different for different nodes.

RWP with Zero Pause Time: First, consider the zero pause time case. Let $\tau_n^A = \tau_n^B = 0$ for all $n = 1, 2, \dots$. Then we have the following result.

Proposition 1 *There exists constant $c > 0$ such that*

$$\mathbb{P}\{T_I > t\} \leq e^{-ct},$$

for all sufficiently large t . □

Proof. Let $D < \infty$ be the ‘diameter’ of Ω or the maximum length of a line segment (leg) in Ω . Let

$$\zeta = 2D / \min\{v_{min}^A, v_{min}^B\} > 0,$$

which is well defined since $v_{min} > 0$ and $D < \infty$. Note that ζ is the longest time it takes for both nodes to finish two legs. In other words, during the time interval of ζ , both nodes must have jumped at least twice. Hence, ζ can be interpreted as a *renewal* interval for nodes A and B since for any $t > 0$, $\{A(s); s \leq t\}$ and $\{A(u + \zeta); u \geq t\}$ are independent (similarly for node B), i.e., after ζ seconds, nodes A and B completely forget where they were ζ seconds ago [50].

Let π_A and π_B be the stationary node distributions of A and B, respectively, and let $X(t) = (A(t), B(t)) \in \Omega^2$. Without loss of generality, we assume that the initial distribution of $X(t)$ is chosen as its stationary distribution $\pi_A \times \pi_B$. One immediate consequence is that the joint node distribution of A and B becomes stationary, i.e., $\mathbb{P}\{A(t) \in S_1, B(t) \in S_2\} = \pi_A(S_1)\pi_B(S_2)$ for all $t > 0$ and $S_1, S_2 \subset \Omega$.

Let $\mathcal{E}_t = \{\|A(t) - B(t)\| > d\}$ be the event that two nodes are ‘out-of-range’ at time t . From the stationarity assumption, $\mathbb{P}\{\mathcal{E}_t\}$ does not depend on t . Since the stationary node distribution of the RWP is non-zero on any point of Ω [50], for any given $B(t) = B \in \Omega$, we see that there exists a positive constant f (may depend

on d and the shape of Ω) such that $\pi_A(\mathcal{N}_B(t)) \geq f > 0$ where $\mathcal{N}_B(t)$ is from (2.1).

Thus, by taking average again with respect to π_B , we have

$$\mathbb{P}\{\mathcal{E}_t\} \leq 1 - f \triangleq \rho < 1. \quad (3.1)$$

Observe that for the inter-meeting time T_I , we have

$$\{T_I > t\} \equiv \cap_{s=0+}^t \mathcal{E}_s \subset \cap_{s \in \mathcal{I}_t} \mathcal{E}_s \quad (3.2)$$

for any subset (index set) $\mathcal{I}_t \subset (0, t]$. Now, we choose the index set $\mathcal{I}_t$ as $\mathcal{I}_t = \{\zeta, 2\zeta, \dots, \lfloor \frac{t}{\zeta} \rfloor \zeta\}$. Since ζ is the ‘renewal’ interval for both nodes, it follows that $X(t_i)$ ($i = 1, 2, \dots$) are all independent whenever $t_i \in \mathcal{I}_t$ and $t_i \neq t_j$. Thus, $\mathcal{E}_{t_i}$ for different t_i in the index set $\mathcal{I}_t$ are also independent. Hence, from (3.2),

$$\mathbb{P}\{T_I > t\} \leq \mathbb{P}\left\{\bigcap_{s \in \mathcal{I}_t} \mathcal{E}_s\right\} = \prod_{s \in \mathcal{I}_t} \mathbb{P}\{\mathcal{E}_s\} \leq \rho^{\lfloor \frac{t}{\zeta} \rfloor} \sim e^{-ct}, \quad (3.3)$$

where $c = -(\log \rho)/\zeta > 0$ from $0 < \rho < 1$ and $0 < \zeta < \infty$. This completes the proof. $\square$

Remark 4 *Proposition 1 can be immediately extended to the case of non-zero pause time as long as it is bounded above. If $\max\{\tau_n^A, \tau_n^B\} \leq M < \infty$, we just need to redefine the renewal interval as $\zeta' = \zeta + 2M$ in the above proof.*

RWP with Random Pause Time: We now consider RWP models with non-zero random pause time. Let V^A, V^B and τ^A, τ^B be the speed and pause time of two independent nodes A and B, respectively. Similarly as before, we define a sequence of ‘renewal’ intervals for node A as

$$T_n^A \triangleq \frac{D}{V_{2n-1}^A} + \tau_{2n-1}^A + \frac{D}{V_{2n}^A} + \tau_{2n}^A, \quad \text{for } n = 1, 2, \dots,$$

where $D < \infty$ is the diameter of Ω . In words, T_n^A is the longest time it takes for node A to finish two legs including two pause times. Then, as before, for each T_n^A , $\{A(s); s \leq t\}$ and $\{A(u + T_n^A); u > t\}$ are independent. Since $\{T_n^A, n \geq 1\}$ are *i.i.d.* (so are $\{\tau_n^A, n \geq 1\}$), it follows that $\{T_n^A, n \geq 1\}$ are also *i.i.d.* Similarly define T_n^B for node B.

Define $X(t) \triangleq (A(t), B(t)) \in \Omega^2$ and assume that the system is in the steady-state with its stationary distribution. Then, along the same line in the proof of Proposition 1, we have for some $0 < \hat{\rho} < 1$,

$$\mathbb{P}\{T_I > t\} \leq \hat{\rho}^{|\mathcal{I}_t|}, \quad (3.4)$$

where $|S|$ is the cardinality of a set S and $\mathcal{I}_t = \{t_1, t_2, \dots, t_{|\mathcal{I}_t|}\} \subset (0, t]$ is an index set with the following property:

$$\{X(t_i), i = 1, 2, \dots, |\mathcal{I}_t|\} \text{ are independent.} \quad (3.5)$$

The difficulty here lies in the fact that, in contrast to Proposition 1, there is no finite ζ such that $X(t)$ and $X(t + \zeta)$ become independent. In addition, even if T_n^A forms a ‘renewal’ sequence for $A(t)$ (T_n^B for $B(t)$), this does not mean that $X(t)$ is also ‘renewing’ on any such sequence. For instance, after a ‘renewal’ of node A, it could be that the pause time of node B is exceptionally long so that the node A still has to renew itself possibly an infinite number of times before it ‘sees’ a renewal of node B.

In what follows, we will construct a random index set $\mathcal{I}_t$ satisfying (3.5). For notational simplicities, we use T^A and T^B to denote the distribution of T_n^A and T_n^B , T_e^A and T_e^B to denote the equilibrium distribution of T^A and T^B (see (2.7)), respectively, whenever no ambiguity arises.

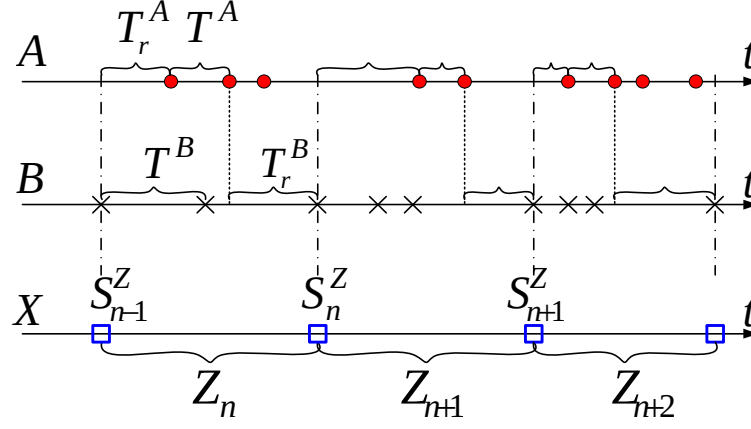


Figure 3.1: Illustration for the construction of Z_n . Start from S_{n-1}^Z , first skip two consecutive ‘renewal’ points of node A and then wait for the next following renewal point of node B. This defines Z_n . T_r^A and T_r^B are ‘residual life’ of node A and B, respectively.

Set $t = 0$ as one of the ‘renewal’ point of node B. We will inductively construct a sequence of Z_n , $n = 1, 2, \dots$, where each Z_n is a sum of non-overlapping T_k^B . Define

$$S_n^Z = \sum_{k=1}^n Z_k$$

and $S_0^Z = 0$. Starting from S_{n-1}^Z ($n \geq 1$), we first skip two consecutive ‘renewal’ points of node A and then mark the next following renewal point of node B. Let t' be this marked point. Then, we set $Z_n \triangleq t' - S_{n-1}^Z$ and $t' = S_n^Z$. See Figure 3.1 for illustration. Notice that Z_n consists of three parts: (i) the residual life of node A (T_r^A) evaluated at S_{n-1}^Z (time from S_{n-1}^Z until the first renewal point of node A), (ii) a renewal interval of length T_k^A for some k , and (iii) the residual life of node B (T_r^B) evaluated at the second renewal point of node A (from the second renewal point of node A until S_n^Z). From this construction, we have the following:

Claim 1: $\{Z_{2k-1}\}_{k \geq 1}$ are independent. So are $\{Z_{2k}\}_{k \geq 1}$.

Claim 1 immediately follows since for each n , the first component of Z_{n+2} (residual life of node A) and the last component of Z_n (residual life of node B) are disjoint and separated apart by at least one renewal interval of node A and B (see Figure 3.1).

To proceed, we need the following technical assumption.

Assumption 1 *Let $\mu_k = \mathbb{E}\{Z_k\} < \infty$. Then,*

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \mu_{2k-1} = \tilde{\mu}_1 < \infty. \quad (3.6)$$

Further, there exists $\theta_0 > 0$ such that for each $\theta \in [0, \theta_0)$,

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \log \mathbb{E} \left\{ e^{\theta(Z_{2k-1} - \mu_{2k-1})} \right\} = \Lambda_1(\theta) < \infty, \quad (3.7)$$

where $\Lambda_1(\theta)$ is well-defined and differentiable at $\theta = 0$. Similarly for $\tilde{\mu}_2$ and $\Lambda_2(\theta)$ with indices in (3.6) and (3.7) replaced by even numbers. $\square$

Remark 5 *Assumption 1 is quite reasonable since (3.6) and (3.7) require that $\{Z_{2k-1}\}$ ($k \geq 1$) have similar distribution and the moment generating function of each zero-mean random variables $Z_k - \mu_k$ be finite, only in the asymptotic sense. For example, if each of the three components belongs to $\mathcal{P}$,¹ then from property (P1), it readily follows that $Z_n \in \mathcal{P}$, which guarantees the existence of the moment generating function. One special case can be where T^A and T^B are both exponentially distributed, for which T_r^A and T_r^B are also exponentially distributed (due to memoryless property) and thus Assumption 1 is automatically satisfied.* $\square$

We now state our main result.

¹If $T^A, T^B \in \mathcal{P}$ and the system is in the steady-state that $T^A \stackrel{d}{=} T_e^A$ (same for T^B), then properties (P1) and (P2) ensure that $Z \stackrel{d}{=} T_e^A + T^A + T_e^B \in \mathcal{P}$.

Theorem 1 *Under Assumption 1, we have $T_I \in \mathcal{P}$, i.e., the inter-meeting time of nodes A and B decays at least exponentially fast.* $\square$

Proof. Similarly as in the proof of Proposition 1, we take one ‘snapshot’ of $X(t)$ per each renewal interval Z_{2n} ($n = 1, 2, \dots$). To be specific, we construct a random index set $\mathcal{I}_t = \{t_1, t_2, \dots\}$ as follows: choose $t_1 \in [S_1^Z, S_2^Z]$, $t_2 \in [S_3^Z, S_4^Z]$, and in general, $t_j \in [S_{2j-1}^Z, S_{2j}^Z]$. Repeat this procedure until S_{2n}^Z becomes larger than t . Then, for any $i < j$, $[t_i, t_j]$ contains at least one full renewal interval of node A and B, thus $X(t_i)$ and $X(t_j)$ are independent. In this way, it is straightforward to see that $\{X(t_i), i = 1, 2, \dots\}$ are independent (and actually identically distributed from the stationarity of $X(t)$).

Let $N(t) \triangleq |\mathcal{I}_t|$ be a counting process that tells us the number of independent ‘snapshots’ of $X(u) = (A(u), B(u))$ during $[0, t]$. By conditioning on $N(t)$ and from (3.4), we have

$$\mathbb{P}\{T_I > t\} \leq \mathbb{E}\{\hat{\rho}^{N(t)}\} = \mathbb{E}\{e^{-\xi N(t)}\}, \quad (3.8)$$

where $\xi = -\log \hat{\rho} > 0$. Note that, for any constant $\beta > 0$,

$$\begin{aligned} \mathbb{E}\{e^{-\xi N(t)}\} &= \mathbb{E}\{e^{-\xi N(t)} 1_{\{N(t) \leq \beta t\}}\} + \mathbb{E}\{e^{-\xi N(t)} 1_{\{N(t) > \beta t\}}\} \\ &\leq \mathbb{P}\{N(t) \leq \beta t\} + e^{-\xi \beta t}. \end{aligned} \quad (3.9)$$

A closer look at the way we construct $\mathcal{I}_t$ reveals that

$$\{N(t) \leq \beta t\} \equiv \{S_{2\beta t}^Z \geq t\}$$

assuming that βt takes on integer value without loss of generality. Thus, for all

sufficiently large t , we have

$$\begin{aligned} \mathbb{P}\{N(t) \leq \beta t\} &= \mathbb{P}\{S_{2\beta t}^Z \geq t\} = \mathbb{P}\{S_{2n}^Z \geq \frac{n}{\beta}\} \text{ (set } n = \beta t) \\ &\leq \mathbb{P}\left\{\sum_{k=1}^n Z_{2k-1} \geq \frac{n}{2\beta}\right\} + \mathbb{P}\left\{\sum_{k=1}^n Z_{2k} \geq \frac{n}{2\beta}\right\} \end{aligned} \quad (3.10)$$

where (3.10) is from (2.8) with $a = b = 1$. For the first term in (3.10), note first that from the assumption in (3.6), for any given $\epsilon > 0$, we have

$$\left| \frac{1}{n} \sum_{k=1}^n \mu_{2k-1} - \tilde{\mu}_1 \right| < \epsilon \quad (3.11)$$

for all sufficiently large n . We then choose $\beta > 0$ such that

$$\alpha \triangleq \frac{1}{2\beta} - (\tilde{\mu}_1 + \epsilon) > 0. \quad (3.12)$$

Then, from (3.11) and (3.12), it follows that

$$\frac{1}{2\beta} = \alpha + \tilde{\mu}_1 + \epsilon > \alpha + \frac{1}{n} \sum_{k=1}^n \mu_{2k-1} \quad (3.13)$$

for all sufficiently large n .

Rewriting the first term in (3.10) gives, for any $\theta > 0$ and for all sufficiently large n ,

$$\mathbb{P}\left\{\sum_{k=1}^n Z_{2k-1} \geq \frac{n}{2\beta}\right\} \leq \mathbb{P}\left\{\sum_{k=1}^n (Z_{2k-1} - \mu_{2k-1}) \geq \alpha n\right\} \quad (3.14)$$

$$\leq e^{-\theta \alpha n} \mathbb{E}\left\{\exp\left(\theta \sum_{k=1}^n [Z_{2k-1} - \mu_{2k-1}]\right)\right\} \quad (3.15)$$

$$= \exp\left(-n\left[\alpha\theta - \frac{1}{n} \sum_{k=1}^n \log \mathbb{E}\{e^{\theta(Z_{2k-1} - \mu_{2k-1})}\}\right]\right), \quad (3.16)$$

where (3.14) is from (3.13), (3.15) is from Markov's inequality, and (3.16) is from independence of $\{Z_{2k-1}\}_{k \geq 1}$ (Claim 1). We then take log in both sides of (3.16),

divide by n , and take limit with respect to n . By optimizing the resulting expression over $0 < \theta < \theta_0$, from the assumption in (3.7), we have

$$\lim_{n \rightarrow \infty} \frac{1}{n} \log \mathbb{P} \left\{ \sum_{k=1}^n Z_{2k-1} \geq \frac{n}{2\beta} \right\} \leq -I(\alpha) \quad (3.17)$$

where $I(x) = \sup_{0 < \theta < \theta_0} [x\theta - \Lambda_1(\theta)]$. Note that $I(x)$ is a well-defined convex function from Assumption 1. Further, since $\Lambda_1(0) = 0$ and $\lambda'_1(0) = 0$ (from $\mathbb{E}\{Z_k - \mu_k\} = 0$), we have $I(\alpha) > 0$ from our choice of $\alpha > 0$ in (3.12). Similarly, we can repeat the same lines for the second term in (3.10). This completes the proof of Theorem 1. $\square$

Remark 6 *The exponential tail behavior of the inter-meeting time of two nodes moving according to RWP model has been mentioned in [87]. However, while [87] provides only simulation results, we rigorously prove the results. Our theoretical result is quite general in that it applies to not only the bounded pause time case, but also the random, unbounded pause time case (see Assumption 1). Moreover, two users under discussion are not required to have the same stationary node distribution.*

3.1.2 Random Walk Models (RWM)

In the current literature studying the RWM [35, 86], commonly a discrete-time, discrete-space 2-D model is used: A square (the bounded area) is divided into multiple sub-squares, called cells, and time is divided into slots of equal duration. At each time slot a node is in and can be only in one cell. The initial position of a node is uniformly chosen from all cells. At the beginning of each time slot, the node jumps from its current cell to one of its adjacent cells with equal probability. Moreover, the boundary can be wrapping or reflecting (see Figure 3.2).

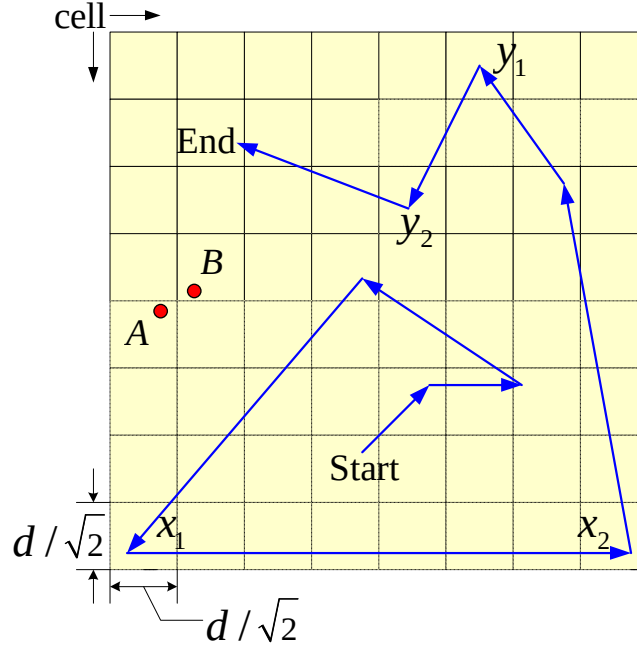


Figure 3.2: A trajectory of our RWM model: The node can jump from a cell to any other cell with certain probability, and the boundary condition can be arbitrary ($x_1 \rightarrow x_2$: wrapping; $y_1 \rightarrow y_2$: reflecting).

In this section, we consider a more general RWM. The bounded area is assumed to be a unit square. Divide the square into $N = 1/[\frac{d}{\sqrt{2}} \times \frac{d}{\sqrt{2}}]$ sub-squares (cell)², each of which with area $\frac{d}{\sqrt{2}} \times \frac{d}{\sqrt{2}}$ as shown in Figure 3.2. The initial position of a node is chosen uniformly from N cells. We number the cells by $1, 2, \dots, N$: the cell on the n^{th} row and m^{th} column is assigned the number $i = (n - 1)\sqrt{N} + m$, where $n, m = 1, 2, \dots, \sqrt{N}$ and $i = 1, 2, \dots, N$. At the beginning of each time slot, the node jumps from the i^{th} cell to the j^{th} cell with probability $p(i, j) \in [0, 1]$. This model is very general in the sense that: (i) in any position, the node can jump to any other cell (including the current cell) with any preassigned probability, which obviously

²We assume $\sqrt{N}$ is an integer without loss of generality.

includes the model in [35, 86] where the node can only jump to one of its adjacent cells with equal probability, and (ii) we allow any arbitrary boundary condition including wrapping, reflecting, etc., by assigning different jumping probabilities for the cells on the boundary. (See Figure 3.2 for example.)

This general RWM can be studied using a Markov Chain (MC) with N states and its transition matrix $P = \{p_{ij}\}$ where $p_{ij} = p(i, j)$, i.e., the probability of jumping from state i to j . In the following, we will give our main result on the inter-meeting time of a pair of nodes, in which only one of the two nodes is required to follow the RWM described above and the mobility model of the other node can be completely arbitrary.

To avoid trivialities, the MC is assumed to be irreducible, i.e., any single node in this finite system can go to anywhere in a finite number of steps. Still, this alone is not sufficient to establish a well-defined mobility model. For example, define a mobility model such that the node can only jump from cell i to cell $i + 1$ (when $i = N$, the user can only jump to cell 1). Obviously this MC is irreducible, but it is not a good mobility model. The introduction of a mobility model is to emulate the mobility pattern in real life in an abstract and simple way. In reality, if an obstacle is present, the moving object (people, bus, etc.) will simply bypass it and continue the movement in the system, rather than get stuck on it.

From the viewpoint of graph theory, the mobility model can be looked as a directed graph. Each cell corresponds to a vertex, and there is a directed edge from vertex i to vertex j if and only if $p_{ij} > 0$. Hence, the irreducibility actually ensures that the graph is connected. However, how strong is the connectivity? One fundamental way to test it is to delete one vertex; if the graph is still connected

after the deletion of any one vertex, then we say that the graph is 2-connected. For example, the mobility model example given above is connected, but not 2-connected. In view of this, we impose the following assumption to avoid trivial models.

Assumption 2 *After we delete any single state from the MC model, the resulting state space is still a communicating class.* $\square$

Note that we are interested in the tail behavior of the inter-meeting time, so we surely want to rule out any possibility that two nodes never meet. For example, if nodes A and B somehow *conspire together* not to meet each other forever (e.g., node B always move ‘parallel’ to node A), the inter-meeting time becomes infinite with probability 1. To rule out this possibility, we also need the following technical assumption.

Assumption 3 *For any possible trajectory of node B, node A eventually meets node B with positive probability. Specifically, there exists $M < \infty$ such that $\mathbb{P}\{T_I > M\} < 1$ regardless of the trajectory of node B, i.e., $\mathbb{P}\{T_I > M\} < 1$ is true for any trajectory.* $\square$

Remark 7 *Assumptions 2 and 3 ensure that (i) there exist at least two different paths between any two cells and (ii) it is impossible for node B to get away from node A forever even if it chooses its ‘optimal’ path to stay away from node A (no conspiracy). Note that all existing versions of RWM [20, 35, 86] readily satisfy Assumptions 2 and 3.* $\square$

To proceed, we present the following definitions and lemma:

Definition 9 Let $\|\cdot\|_\infty$ be the norm of a matrix $K = \{k_{ij}\}$ defined by $\|K\|_\infty \triangleq \max_i \sum_j |k_{ij}|$, i.e., the largest absolute row sum. For any k -dimensional vector $\vec{x} = \{x_1, \dots, x_k\}$ and $n \in \mathbb{N}$, we define n -norm of $\vec{x}$ as $\|\vec{x}\|_n = (\sum_{i=1}^k |x_i|^n)^{1/n}$. $\square$

Definition 10 A finite set of matrices $C_m = \{c_{ij}^m\} \in \mathbb{R}^{N \times N}$ where $m \in \{1, \dots, L\}$ is said to be a sub-stochastic matrix set if there exists a finite integer M such that for any integer set $\{h_1, \dots, h_M\}$ with $h_i \in \{1, \dots, L\}$ and $i \in \{1, \dots, M\}$, $\|C_{h_1} \cdots C_{h_M}\|_\infty < 1$. $\square$

Lemma 1 Let $\{P_m\} \in \mathbb{R}^{N \times N}$ ($m = 1, 2, \dots, M$) be a sub-stochastic matrix set. Then, for any index set $\mathcal{I} = \{n_1, n_2, \dots\}$ where $n_i \in \{1, \dots, M\}$ and $i \in \mathbb{N}$, there exist constants $c > 0$ and $0 < q < 1$ (independently of the choice of $\mathcal{I}$) such that for any $\vec{x} \in \mathbb{R}^{1 \times N}$, the product of any k ($k \in \mathbb{N}$) matrices $A(\mathcal{I}, k) = P_{n_1} P_{n_2} \cdots P_{n_k}$ satisfies

$$\|\vec{x}A(\mathcal{I}, k)\|_n \leq cq^k \|\vec{x}\|_n \quad \text{for all sufficiently large } k.$$

Proof. See Appendix. $\square$

We now state our main result.

Theorem 2 Suppose that node A moves according to the RWM described above satisfying Assumptions 2 and 3. Then, there exists constant $\gamma > 0$ such that $\mathbb{P}\{T_I > t\} \leq e^{-\gamma t}$ for all sufficiently large t .

Remark 8 *We require that only one of the nodes (node A) moves according to the RWM. This means that the other node (node B) can move according to any other arbitrary mobility models defined on a bounded domain such as RWP, random direction models, etc., as long as two nodes eventually meet with non-zero probability.*

Proof. Since node B moves according to some arbitrary mobility model, the only information we know about node B's movement is its path-wise position at $t = 0, 1, 2, \dots$. Let $\mathcal{T}$ be the set of all possible trajectories of node B satisfying Assumptions 3. Without loss of generality, assume A and B are in the communication range just before time 0 and $A(0) = 1 \neq B(0)$. (If $A(0) \neq 1$, simply renumber the cells.) Let $T_F^{\mathcal{B}}$ be the FPT of A to the trajectory $\mathcal{B} \in \mathcal{T}$ as in Definition 5. Then, we have

$$\mathbb{P}\{T_I > t\} = \sum_{\mathcal{B} \in \mathcal{T}} \mathbb{P}\{T_F^{\mathcal{B}} > t\} \mathbb{P}\{\mathcal{B}\} \quad \text{for all } t \geq 1. \quad (3.18)$$

In the following, we fix $\mathcal{B} \triangleq \{B(t) = k_t, t \in \mathbb{N}\}$ ($k_t \in \{1, 2, \dots, N\}$) to show that $\mathbb{P}\{T_F^{\mathcal{B}} > t\}$ is *uniformly upperbounded* by $c \exp(-\gamma t)$ where $c > 0$ and $\gamma > 0$, i.e., for any $\mathcal{B} \in \mathcal{T}$, $\mathbb{P}\{T_F^{\mathcal{B}} > t\} \leq c \exp(-\gamma t)$. For the simplicity of expression, we use T_F in stead of $T_F^{\mathcal{B}}$ because now we are discussing fixed trajectory $\mathcal{B}$.

Let $P = \{p_{ij}\}$ be the transition matrix of the Markov chain for $A(t)$ (position of node A at time t) where p_{ij} is the probability of jumping from cell i to cell j in one step. We then define a set of matrices $\{P_i, i = 1, 2, \dots, N\}$ by setting the i^{th} column of P to 0 and keeping all the other elements the same.

From $A(0) = 1$, the initial state vector becomes $\vec{x}_0 = [1 \ 0 \ \dots \ 0]$. Then $\mathbb{P}\{T_F > 1\}$ is the probability that node A jumps from state 1 to any state other than $B(1) =$

k_1 , i.e.,

$$\mathbb{P}\{T_F > 1\} = \sum_{1 \leq j \leq N, j \neq k_1} p_{1j}. \quad (3.19)$$

Define a matrix $Q^{(1)} \in \mathbb{R}^{N \times N}$ as $Q^{(1)} = P_{k_1}$ and denote by $q_{ij}^{(1)}$ the i^{th} row and j^{th} column element of $Q^{(1)}$. Then, from (3.19), it follows that $\mathbb{P}\{T_F > 1\} = \sum_{j=1}^N q_{1j}^{(1)} = \vec{x}_0 Q^{(1)} \mathbf{1}$, where $\mathbf{1} = [1 \ 1 \ \dots \ 1]^T$ is the N -dimensional column vector whose elements are all 1. Following the similar line, we define $Q^{(t)} = Q^{(t-1)} P_{k_t}$ for all $t \geq 2$. Then we can show by induction that

$$\mathbb{P}\{T_F > t\} = \sum_{j=1}^N q_{1j}^{(t)} = \vec{x}_0 Q^{(t)} \mathbf{1}, \quad (3.20)$$

where $q_{1j}^{(t)}$ is the first row and j^{th} column element of the following matrix $Q^{(t)}$:

$$Q^{(t)} = P_{k_1} P_{k_2} \cdots P_{k_t}, \quad t \geq 1. \quad (3.21)$$

Consequently, if we can prove that for any given trajectory $\mathcal{B}$ of node B, there exists constant $c < \infty$ and $\gamma > 0$ (independently of $\mathcal{B}$) such that $\vec{x}_0 Q^{(t)} \mathbf{1} \leq ce^{-\gamma t}$ for all sufficiently large t , then from (3.18) and (3.20), we are done.

In what follows, we will show that $\{P_i, i = 1, \dots, N\}$ is a sub-stochastic matrix set as defined in Definition 10. From (3.20), $\vec{x}_0 Q^{(t)} \mathbf{1}$ is the sum of the first row of $Q^{(t)}$, or equivalently, the probability that node A starts from cell 1 ($A(0) = 1$) and never meets node B (with the trajectory $\mathcal{B}$) up to time t . Similarly, it can be shown that the sum of the i^{th} row of $Q^{(t)}$ is the probability that node A starts from cell i ($A(0) = i$) and never meets B with the trajectory $\mathcal{B}$ up to time t . Thus, from Assumption 3, there exists $0 < M < \infty$ such that for any $t \geq M$, $\sum_j q_{ij}^{(t)} = \mathbb{P}\{T_F > t\} \leq \mathbb{P}\{T_F > M\} < 1$, for any trajectory $\mathcal{B}$ and any $i \in \{1, 2, \dots, N\}$. Subsequently,

for all $t \geq M$,

$$\|P_{k_1}P_{k_2}\dots P_{k_t}\|_\infty = \|Q^{(t)}\|_\infty = \max_i \sum_j |q_{ij}^{(t)}| \quad (3.22)$$

$$= \max_i \sum_j q_{ij}^{(t)} < 1, \quad (3.23)$$

where (3.22) is from (3.21) and $\|\cdot\|_\infty$ norm is from Definition 9, and the equality in (3.23) is from $q_{ij}^{(t)} \geq 0$.

Note that (3.23) is true for any trajectory $\mathcal{B}$ of node B. In particular, choose $\mathcal{B} = \{B(t) = k_t\}$ where $k_t \in \{1, 2, \dots, N\}$ and $t \in \{1, \dots, M\}$ and M is the finite integer as in Assumption 3. Hence, from Definition 10, $\{P_i, i = 1, \dots, N\}$ is a sub-stochastic matrix set.

In consequence, from Lemma 1, there exists constant $c > 0$ and $0 < q < 1$ such that

$$\|\vec{x}_0 Q^{(t)}\|_n \leq cq^t \|\vec{x}_0\|_n \quad (3.24)$$

for any n -norm, where $\vec{x}_0 = [1 \ 0 \ \dots \ 0]$. Fix $\|\cdot\|_1$ in (3.24) as in Definition 9. Then, $\|\vec{x}_0 Q^{(t)}\|_1 = \sum_j |q_{1j}^{(t)}| = \sum_j q_{1j}^{(t)} = \vec{x}_0 Q^{(t)} \mathbf{1}$ and $\|\vec{x}_0\|_1 = \|[1 \ 0 \ \dots \ 0]\|_1 = 1$. Hence, for all sufficiently large t , from (3.24), we have

$$\vec{x}_0 Q^{(t)} \mathbf{1} = \sum_{j=1}^N q_j^{(t)} \leq cq^t = c \exp(-\gamma t) \quad (3.25)$$

where $\gamma = -\ln q > 0$. Note that (3.25) holds good for any trajectory $\mathcal{B}$ and the constants c, q are independent of $\mathcal{B}$. Hence, from (3.18) and (3.20), we are done. $\square$

Through space/time quantization, Theorem 2 can also be applied to any other continuous-time, continuous-space models (e.g., random direction model (RDM), etc) provided that both the time it takes for a node to walk from one cell to another

and the pause time are bounded. For any continuous-space analogue of our model, note that whenever nodes A and B are in the same cell, they are ‘in-range’ (the maximum distance between A and B is no larger than d). Note also that even when they meet in the continuous-space model, they may not in the quantized discrete-space model. For example, nodes A and B in Figure 3.2 are within a distance of d , but they are in different cells, i.e., they do not meet in the discrete-space model. In consequence, the inter-meeting time in a continuous-space model is upper-bounded by that in the discrete-space (quantized) model. Hence, Theorem 2 readily shows that the tail of the inter-meeting time of a continuous-space model also decays at least exponentially fast.

3.2 From Exponential to Power-law Inter-meeting Time

3.2.1 Finite Boundary and Exponential Tail

So far we have proven that the inter-meeting time of two independent mobile nodes with mobility models such as RWP, RWM, and RDM in a bounded domain has at most exponential tails. These models are clearly different in defining each node’s mobility pattern. In fact, for other mobility models in a bounded domain (e.g., Brownian motion on a sphere, i.i.d. mobility model on a square, etc.), we can also show similar exponential behavior of the inter-meeting time. This observation along with our theoretical results assert that the exponential tail of the inter-meeting time seems a universal property and the empirically observed power-law decay of the

inter-meeting time will not arise by simply tweaking the aforementioned mobility models.

What is common in all these mobility models? The finite boundary. Suppose that two mobile nodes (without pause) have not met for a long time (e.g., several hours). If there is no boundary, intuitively, it is more likely that they are moving toward different directions, hence chances are that they will not meet for the next several hours and the inter-meeting time is prolonged. In other words, the inter-meeting time has very strong memory and may result in a power-law type distribution.

However, when there is a finite boundary, after a long time (much longer than a typical amount of time for a node to hit the boundary), it forces the mobile nodes to bounce back once being hit, which tends to reset the nodes' location and 'erase' the memory in the inter-meeting time. This nearly memoryless behavior caused by the existence of the finite boundary tends to shorten the inter-meeting time and actually results in an asymptotic exponential tail.

Remark 9 *While Theorems 1 and 2 in Section 3 clearly indicate that finite boundaries have significant impact on the tail behavior of the inter-meeting time distribution, we point out that they both require mobile nodes to be independent and the moment generating function of the pause time to be finite. (See Remarks 5 and 7.) In other words, in the presence of non-negligible dependency among mobile nodes (e.g., they belong to the same group following similar paths all the time or totally different groups whose typical pathways do not overlap) or significant amount of pause time (e.g., pause time following power-law with infinite mean), the inter-meeting*

time could be power-law type even under a finite boundary. Another exception could be the case where there exists a considerable amount of correlations in the trajectory of a mobile node (e.g., the Gauss-Markov mobility model [68]). The effect of finite boundary on the inter-meeting time distribution under such spatial-temporal correlations is beyond the scope of this dissertation. As to the effect of power-law pause time, see Section 4 for discussion.

3.2.2 Infinite Domain and Power-law Tail

In what follows, we will show through a class of simple isotropic random walks in an open-space without boundary (i.e., $\Omega = \mathbb{R}^2$) that the power-law decay of the inter-meeting time will arise by simply removing the boundary. We consider only a discrete-time model here, but similar results also hold for its continuous-time analogue, which we omit due to the space constraint.

In a two-dimensional (2-D) discrete-time isotropic random walk model, at the beginning of each time slot, the node chooses a random direction uniformly from $[0, 2\pi]$, travels for a random length R which is chosen from $(0, \infty)$ following certain distribution, then the process repeats itself. Denote by R_k the length of the k^{th} step and θ_k the random angle uniformly distributed over $[0, 2\pi]$. Then, the position of node A at time t ($t = 1, 2, \dots$) can be written as

$$A(t) = \sum_{k=1}^t \phi^A(k) = \sum_{k=1}^t R_k^A \exp(i\theta_k^A) \in \mathbb{R}^2, \quad (3.26)$$

where $A(0) = \mathbf{0}$. Since both sequences $\{R_k^A\}$ and $\{\theta_k^A\}$ are *i.i.d.* and independent from each other, $\phi^A(k) = R_k^A \exp(i\theta_k^A)$ ($k = 1, 2, \dots$) are also *i.i.d.* Thus, $A(t)$ is a sum of *i.i.d.* vectors, i.e., a random walk in 2-D. (Similarly for node B.)

The following result will be used in our proof of the main result in this section.

Theorem 3 [*Sparre-Andersen (S-A) Theorem in [5, 79]*]: *For any one-dimensional discrete time random walk process starting at $x_0 \neq 0$ with each step chosen from a continuous, symmetric but otherwise arbitrary distribution, the First Passage Time Density (FPTD) to the origin asymptotically decays as $\sim t^{-3/2}$ with the number of steps t .*

Remark 10 *The only assumption required in S-A Theorem is that each step of the random walker is chosen from a continuous (the probability of choosing a specific step length is zero) and symmetric distribution (at each step, the random walker goes left or right with equal probability). Denote the first passage time (FPT) as T_F , then by S-A Theorem, the probability density function of T_F decays as $\sim t^{-3/2}$, which means that the complementary cumulative distribution function (ccdf) of T_F decays as $\mathbb{P}\{T_F > t\} \sim t^{-1/2}$. The result is applicable for any initial position of the node except the origin.*

Now we present our main result on the inter-meeting time of two isotropic random walkers.

Theorem 4 *Suppose that two independent nodes A and B move according to the 2-D isotropic random walk model described above. Then, there exists constant $C > 0$ such that the inter-meeting time T_I of nodes A and B satisfies*

$$\mathbb{P}\{T_I > t\} \geq Ct^{-1/2}, \text{ for all sufficiently large } t. \quad (3.27)$$

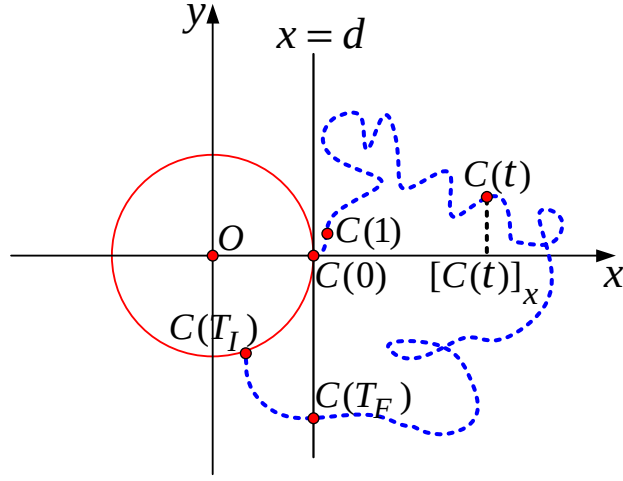


Figure 3.3: T_F is a lower bound on the inter-meeting time T_I as $C(t)$ must cross the line $x = d$ before crossing the circle of radius d .

Proof. Define by $C(t) = A(t) - B(t) \in \mathbb{R}^2$ the difference vector between the position of node A and B at time t . Assume $\|C(0)\| = d$ and $\|C(1)\| > d$. Then, the inter-meeting time T_I becomes $T_I := \inf_{t>0} \{t : \|C(t)\| \leq d\}$.

Draw an x -axis connecting $C(0)$ and the origin as shown in Figure 3.3 and also y -axis accordingly. Let $[C(t)]_x$ be the projection of $C(t)$ onto the x -axis. Then, clearly, $[C(0)]_x = d$ and $[C(1)]_x > d$.³ Define by T_F the FPT of $[C(t)]_x$ to $x = d$ in 1-D (the vertical line tangent to the circle at $(d, 0)$). Suppose that the two nodes meet for the first time at $t = T_I$ since $t = 0$, or equivalently, $C(t)$ crosses the circle for the first time at $t = T_I$ after $t = 0$. Then, it is clear that $C(t)$ must have crossed the vertical line $x = d$ before it crosses the circle. In other words, we have $T_I \geq T_F$, and hence

$$\mathbb{P}\{T_I > t\} \geq \mathbb{P}\{T_F > t\}, \text{ for all } t > 0. \quad (3.28)$$

³This is always possible by making one time step small enough such that when the node gets out of the circle at $t = 1$, it is located outside that tangential line at $x = d$.

Observe that

$$C(t) = \sum_{k=1}^t \phi^A(k) - \sum_{k=1}^t \phi^B(k) = \sum_{k=1}^t \left(R_k^A e^{i\theta_k^A} - R_k^B e^{i\theta_k^B} \right),$$

Thus, we have

$$[C(t)]_x = \sum_{k=1}^t (R_k^A \cos \theta_k^A - R_k^B \cos \theta_k^B)$$

where R_k^A, R_k^B are all *i.i.d.* and so are θ_k^A, θ_k^B . In other words, $[C(t)]_x$ is one-dimensional random walk (sum of *i.i.d.* random variables) with each step distributed as $R_x = R^A \cos \theta^A - R^B \cos \theta^B$. Clearly, $R_x \stackrel{d}{=} -R_x$, so it is symmetric. Also, the distribution of R_x is continuous because the uniform distribution is continuous. Hence, by Theorem 3 and Remark 10,

$$\mathbb{P}\{T_F > t\} \sim t^{-1/2}.$$

In view of (3.28), this completes the proof. $\square$

The result in (3.27) is close to the empirical result reported in [48] that the coefficient of the power-law tail is nearly 0.4 (see Figure 4 in [48]). Recall that in Section 3, mobility models are analyzed through an irreducible Markov Chain (MC) by quantizing the space and time. Whenever there is a finite boundary, since the communication range d of two nodes is non-zero, we can always divide the system (bounded area) into a set of finite cells, each with diameter d . Hence, the MC has finite states. This finiteness of the state-space along with the irreducibility condition implies that the MC is positive recurrent [14]. In other words, starting from any initial position, the MC will return to a given state infinitely often, and the mean return time is finite. Further, Assumptions 2 and 3 guarantee that the MC will visit any arbitrary *moving set of states* also infinitely often and the mean return time

to this moving set is finite. In fact, our result in Theorem 2 shows not only the mean of this return time (to a moving set) is finite, but its distribution is at most exponential, making all the moments also finite.

In contrast, however, when the node walks in an open space without boundary, the corresponding MC has an infinite number of states. As a result, even if the chain is recurrent, it may take infinitely long on average to return to a given state (null recurrence), which clearly rules out the possibility of an exponential tail of the inter-meeting time. In some sense, different recurrence properties in a bounded/unbounded space provide a quick explanation on the completely distinct tail behaviors of the inter-meeting time in these two situations.

3.3 Discussion

3.3.1 Scaling the Size of the Space

From Sections 3 and 3.2, we see that the finite boundary plays a key role in generating the exponentially decaying inter-meeting time or the FPT, and by removing the boundary, the power-law inter-meeting time distribution can be observed. However, the question is: *How to decide whether a domain is virtually bounded or not?* In fact, it is hard to believe that this change happens abruptly only at the infinity. If we set the boundary for RWM in Section 3 large enough such that the mobile node rarely hits the boundary under the timescale of interest, it is almost equivalent to the random walk in an open-space or we can say there is practically no boundary. In this regard, the question is: where does the transition from exponential to

power-law happen? Or, equivalently, how do we choose (or scale) the size of the space (boundary) to observe the power-law inter-meeting time distribution over a given timescale?

The answer lies in the interaction between the timescale under discussion and the size of the boundary. As an example, consider a 2-D random walk model in (3.26) (or equivalently we can take 2-D Brownian motion in a continuous time case). Note that $\mathbb{E}\{A(t)\} = 0$ for all t from symmetry. However, observe that

$$\mathbb{E}\{|A(t)|^2\} = \sum_{k,l=1}^t \mathbb{E}\{R_k R_l e^{i(\theta_k - \theta_l)}\} = \sum_{k=1}^t \mathbb{E}\{R_k^2\} = \sigma^2 t$$

from $\mathbb{E}\{e^{i\theta}\} = 0$ and the independence of the random variables for $k \neq l$. Thus, its standard deviation or the *average displacement* of node A during time t scales as $O(\sqrt{t})$.⁴ In other words, after a duration of time t , a node will typically travel a distance of $O(\sqrt{t})$ from its original position.

Suppose that the maximum timescale of interest is t_0 , i.e., we are interested in system dynamics only over $[0, t_0]$, and the ‘radius’ of the bounded space is r_0 . When $r_0 \ll \sqrt{t_0}$, the node will hit the boundary (thus ‘feel’ it) many times and a mobility model with a finite boundary is in effect. Hence all the results in Section 3 naturally apply. In contrast, when $r_0 \gg \sqrt{t_0}$, the node will almost never hit the boundary and we can say that the node is moving around practically in an unbounded space. Hence in this case results similar to those in Section 3.2 apply.

For example, Figure 3.4 shows sample trajectories of five mobile nodes following the standard 2-D Brownian motion, observed over time duration (a) $[0, 10^4]$ and (b)

⁴We implicitly assume that $\sigma^2 = \mathbb{E}\{R^2\} < \infty$. If $\mathbb{E}\{R^2\} = \infty$, then under constant velocity, the mobile node tends to ‘spread out’ quicker and its average displacement will grow faster than $O(\sqrt{t})$.

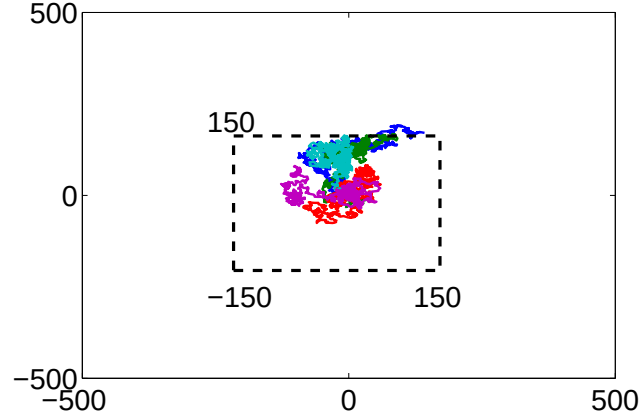
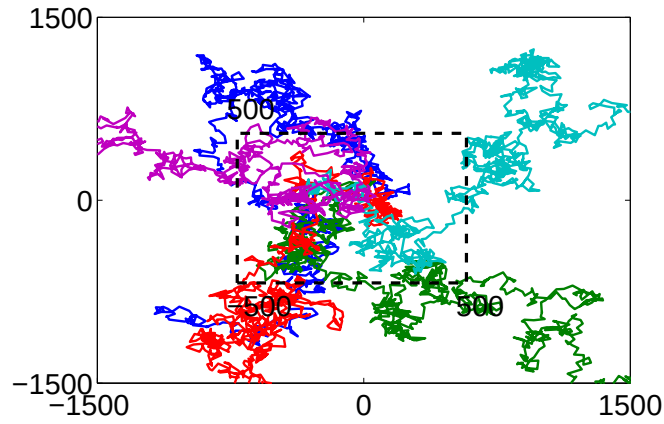
(a) $[0, 10^4]$ (second)(b) $[0, 10^6]$ (second)

Figure 3.4: Sample trajectories of five nodes following standard 2-D Brownian motion observed over time duration (a) $[0, 10^4]$ (second) and (b) $[0, 10^6]$ (second). All nodes start from the origin at $t = 0$. The average displacement of each node scales as $O(\sqrt{t})$ from both (a) and (b).

$[0, 10^6]$ (seconds) and drawn over different $x-y$ ranges. As the timescale increases by 100 times, the average displacement of each node grows approximately by $\sqrt{100} = 10$ times. This figure also provides an answer to whether a given domain is essentially bounded or not. For instance, if the time scale of interest is $[0, 10^4]$ seconds, the mobile node rarely hits the boundary of $\Omega' = [-500m, 500m] \times [-500m, 500m]$, thus Ω' is essentially unbounded. When the time scale of interest is around 10^6 second, the mobile node now hits the boundary of Ω' frequently as seen in Figure 3.4(b), so it is essentially bounded.

In order to better grasp the idea of the space-time dynamics, let us take an example of the so-called *i.i.d.* mobility model and study different ways of scaling the size of the boundary as a function of time and their impact on the distribution of the inter-meeting time. In the *i.i.d.* mobility model, a node moves in a $D \times D$ square consisting of $N = D^2$ unit cells. At the start of each time slot, the node jumps to one of N cells with equal probability independently of everything else. Note that this is equivalent to setting $p_{ij} = 1/N$ in the RWM in Section 3.1.2. The two nodes ‘meet’ whenever they are in the same cell, and at any given time, this happens with probability $1/N = 1/D^2$. Suppose now that the length of the boundary is a function of time, i.e., $D = D(t)$. When $D(t)$ grows over time (i.e., $D(t) \rightarrow \infty$ as $t \rightarrow \infty$), note that

$$\begin{aligned} \mathbb{P}\{T_I > t\} &= [1 - 1/D^2(t)]^t \\ &= [1 - 1/D^2(t)]^{D^2(t) \cdot \frac{t}{D^2(t)}} \sim e^{-g(t, N)}, \end{aligned} \quad (3.29)$$

where $g(t, N) \triangleq t/D^2(t)$. Hence, the scaling function $g(t, N)$ completely determines the tail behavior of the inter-meeting time. For example, when $g(t, N) \sim \beta t$ or

$D(t) = O(1)$, we have $\mathbb{P}\{T_I > t\} \sim e^{-\beta t}$ as expected. When $g(t, N) \sim \alpha \log t$, we get $\mathbb{P}\{T_I > t\} \sim e^{-\alpha \log t} = t^{-\alpha}$. In other words, we get the desired power-law behavior when the length of the boundary is scaled as $D(t) \sim \sqrt{t/(\alpha \log t)}$. This is also consistent with the aforementioned discussion that the average displacement scales as $O(\sqrt{t})$. By maintaining the size of the space as $D(t) = O(\sqrt{t/(\alpha \log t)})$ or larger, we can ensure that the ‘world’ is expanding at about the same rate (or larger) as the node, so the node rarely feels the existence of the boundary. If $D(t) \ll O(\sqrt{t/(\alpha \log t)})$, the size of the world grows much slower than the average displacement of a node, so the node inevitably hits the boundary many times and the non-power-law or exponential behavior of the inter-meeting time starts to kick in.

This first-order scaling rule via $g(t, N)$ also provides a way to choose a ‘right’ size of the world in the random walk model for the study of MANET. For instance, if one is interested in some performance metric that makes sense only over a certain time period, e.g., hours, days, or even weeks (could be possible in delay-tolerant network settings), then the scaling function $g(t, N)$ tells us approximately how to set the size of the boundary in which the MANET is operating so as to make the inter-meeting time power-law distributed. Another possible way is to dynamically increase the size of the boundary $D(t)$ as time goes on to control the ‘frequency’ of hitting the boundary as desired.

3.3.2 Power-Law vs. Heterogeneous Approach

The power-law behavior of the inter-meeting time distribution reported in [22, 48] is from the ‘aggregate’ measurements, in which all the inter-meeting time samples for all possible pairs of nodes are taken into account over a given measurement period. An analytically feasible way to model this behavior is to assume that all mobile nodes are *i.i.d.* [22], i.e., the inter-meeting time of any pair of nodes follows the same distribution. This applies to the scenarios where the (individual) inter-meeting time of a given pair is close to the ‘aggregate’ inter-meeting time as evidenced by the in-depth study in [58].

There are also different voices, e.g., [25] claimed that the power-law behavior observed from these aggregated inter-meeting time samples can also be a result of the heterogeneity in the inter-meeting time of different pairs where the ‘individual’ inter-meeting time of a given pair still follows an exponential distribution. This type of approach has been widely used in teletraffic engineering [33] to represent power-law decaying correlations as a weighted sum of exponential functions to facilitate Markovian analysis.⁵ This approach, however, is unable to reflect the recent findings in [58] where the ‘individual’ inter-meeting times for most pairs of nodes (always more than a half) do exhibit power-law behavior over a wide range of timescale. Nevertheless, the existence of heterogeneity itself in the nodes’ behavior and their inter-meeting time (in terms of different ‘contact rates’ under the assumption of exponential distribution or Poisson contacts) as pronounced in [25, 31] suggests that

⁵For any completely monotone function [34] (including all the power-law functions) $f(t)$, with $(-1)^n f^{(n)}(t) \geq 0$ for all t and $n = 1, 2, \dots$, there exists $h(s) \geq 0$ such that $f(t) = \int_0^\infty e^{-st} h(s) ds$. Thus, by suitably choosing the ‘weights’ $h(s)$, a given power-law can be represented as a weighted sum of exponentials $\{e^{-st}\}$.

performance analysis based on a homogeneous setting with *i.i.d.* nodes be taken with great care.

We here emphasize that our focus in this dissertation is on the inter-meeting time distribution of *any given pair* of two nodes. Thus, our results can be applied to homogeneous setting with *i.i.d* nodes as well as any heterogeneous setting in which some fraction of pairs of nodes have power-law like inter-meeting times while others have exponential inter-meeting times with different ‘contact rates’. For instance, our observation on space-time scaling in Section 3.3.1 suggests that we can control the shape of the ‘individual’ inter-meeting time distribution to be either an exponential or a mostly power-law by choosing appropriate domain size with respect to a given time scale. Specifically, an exponential-like inter-meeting time distribution can be generated by pairs of nodes in the same ‘social group’ whose *moving domain* $\Omega(t)$ is essentially bounded with respect to the time scale under consideration and nodes in that group rarely cross over the boundary of $\Omega(t)$. Note that the boundary here is not physical but should be interpreted as a virtual one.⁶ Similarly, for certain pair of nodes that are loosely coupled, i.e., they reside in a much larger moving domain with weak correlation or they belong to totally different social groups, the only (virtual) boundary that ‘push’ them back is from their own daily movement pattern (e.g., people returning home after working 8 hours or buses returning to their origins) and thus their inter-meeting time distribution will follow a power-law until they hit this ‘virtual boundary’ over a much longer period.

⁶The whole group (e.g., students in the same grade and same department following similar class schedules in a campus) may move toward the same destination while their tendency to maintain contact with other nodes in the same group will create ‘virtual boundary’ to ‘force’ them back whenever they are about to leave the group’s trajectory.

3.3.3 Effect of Pause Time

Our analysis so far can also be employed to study the effect of the transmission range d on the contact time distribution. In fact, the contact time can be looked as the inter-meeting time with $\mathcal{N}_B(t)$ in equation (2.1) replaced by its complement $\mathcal{N}_B^c(t) = \Omega \setminus \mathcal{N}_B(t)$. Consequently, the size of boundary and the transmission range can be used to control the inter-meeting time and the contact time distribution, respectively. In contrast, the pause time of mobile node affects both the contact and inter-meeting time distribution. Clearly, as stated in Remark 9, the power-law pause time with infinite mean makes both the contact and inter-meeting time have infinite means. On the other hand, when the pause time has all the finite moments, Theorem 1 guarantees that the inter-meeting time distribution on a bounded domain should be of exponential type.

In this section, we briefly discuss the effect of power-law pause time with finite mean on both the contact and inter-meeting time distribution. The heavy-tailed nature of the pause time distribution has been discovered in recent empirical studies [64, 99]. Specifically, we consider the pause time τ :

$$\mathbb{P}\{\tau > t\} = \min\{1, (t/\tau_{\min})^{-\mu}\}, \quad \mu > 1. \quad (3.30)$$

Clearly, the power-law pause time of node A will contribute to the inter-meeting time between A and B whenever A pauses for a long time outside of B 's transmission range, thus making the inter-meeting time power-law distributed as well.

In order to quantitatively capture the extent to which a given power-law affects the contact and inter-meeting time distribution, we consider two independent mobile nodes following RWP defined on $200m \times 200m$ square, where each node has

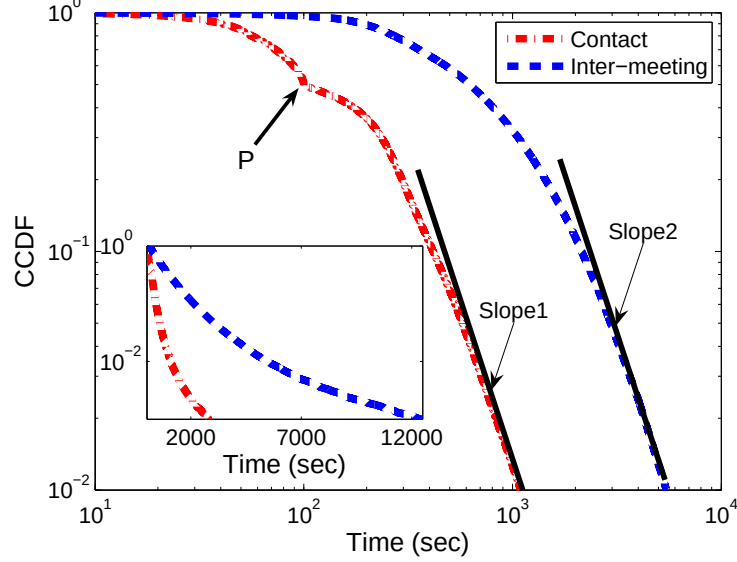


Figure 3.5: CCDF of the contact and inter-meeting time for RWP model with power-law pause time on a log-log scale. We have set $\tau_{\min} = 400/3$ and $\mu = 1.5$ in (3.30) so that $\mathbb{E}\{\tau\} = 400$ seconds and the average step length is about $104.2m$ [50]. Under $v = 1m/s$, this leads to $400/(400 + 104.2) = 79.3\%$ pause ratio, which is 79.01% from our simulation. Both the contact and inter-meeting time exhibit power-law behavior at around timescale 10^3 . The inset in the bottom left shows the same curves on a semi-log plot to clearly show the heavier-than-exponential behavior. P is the point where the effect of pause time starts to kick in.

transmission range $d = 50m$ and pauses for τ seconds after each step. Figure 3.5 shows that the CCDF of both the contact and inter-meeting time follow power-law distribution at around timescale 10^3 . Further, it also indicates that the exponents of the power-law distributions of contact and inter-meeting time are both around 1.5 (slope1 and slope2 are almost the same on a log-log scale), matching the exponent in the power-law pause time. This is in contrast to the empirical results in [22] showing that there exists a sizable difference between the two exponents in the power-law distributed contact and inter-meeting time (The difference is about 0.9

for Bluetooth-based dataset, and is more than 1.3 in WiFi datasets). Hence, in view of our discussion above, the power-law pause time *alone* is not enough to capture the varying degrees of power-law behavior in the contact and inter-meeting time, and our space-time scaling for the domain size discussed in Section 3.3.1 should be in effect to properly capture the stronger power-law behavior in the inter-meeting time distribution over an appropriate range of timescale.

3.4 Simulation

In this section we provide simulation results to support our theoretical results in Sections 3 and 3.2, as well as the discussion on scaling the size of the space in Section 4. We here consider RWP and random walk model (RWM) with/without boundary. In any case, the speed of a node is chosen uniformly from 1 m/sec to 1.68 m/sec with a mean value of 1.34 m/sec [64], since this is known as the mean walking speed of human beings. The transmission range is set to 50 meters, and we have square domain for all mobility models with finite boundary in our simulations.

As mentioned before, the interaction between the timescale under discussion and the size of the boundary is essential in determining the tail behavior of the inter-meeting time. To see this interaction, we fix the simulation time period and change the size of the boundary using RWP and RWM (including both bounded and essentially unbounded domain cases) models to observe different tail behaviors of the inter-meeting time.

Figure 3.6 shows the complementary distribution (CCDF) of the inter-meeting time $\mathbb{P}\{T_I > t\}$ on a semi-log scale for RWP models under varying sizes of the

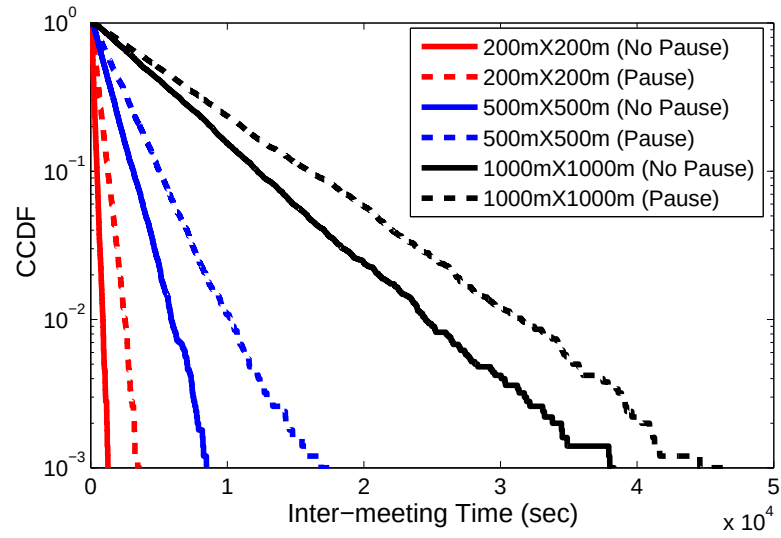


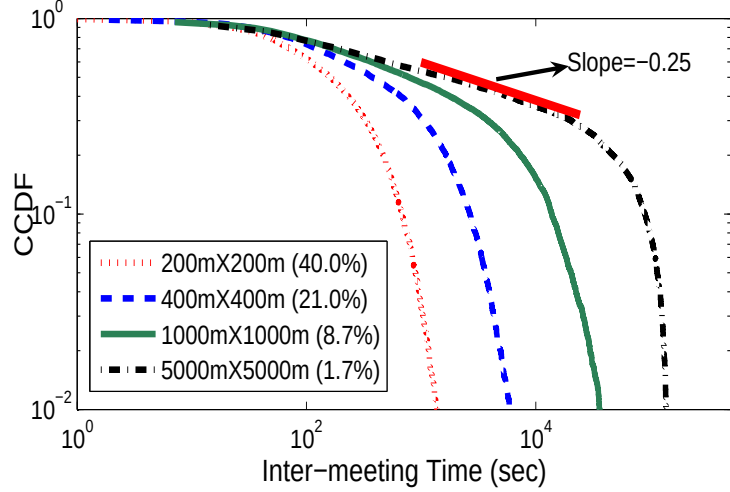
Figure 3.6: CCDF of the inter-meeting time for RWP models under different sizes of the domain: “Pause” means an exponentially distributed pause time with a mean value of 3 minutes, and “No pause” means zero pause time. Figure is drawn on a semi-log scale. In any case, the slope increases for larger size of the domain.

domain. We set the simulation period as 40 hours,⁷ and simulate zero pause time case ('no pause') as well as an exponentially distributed pause time with mean of 3 minutes ('pause'). Note that no matter how large the boundary is, the inter-meeting time always has an exponential tail, which is in good agreement with our theoretical results in Proposition 1 and Theorem 1. Recall from Section 4 that in case of *i.i.d.* mobility models, the slope of the exponential tail is given by $g(t, N)/t = 1/D^2$. In other words, the slope of the tail in a semi-log scale is inversely proportional to the area of the bounded domain (D^2). Similarly, even for RWP models, we notice from Figure 3.6 that the product of the slope of the tail and the area of the domain (ranging from 200×200 up to 1000×1000 square meters) remains almost constant, which implies that the intuition obtained from *i.i.d.* mobility models can also be applied to RWP models.

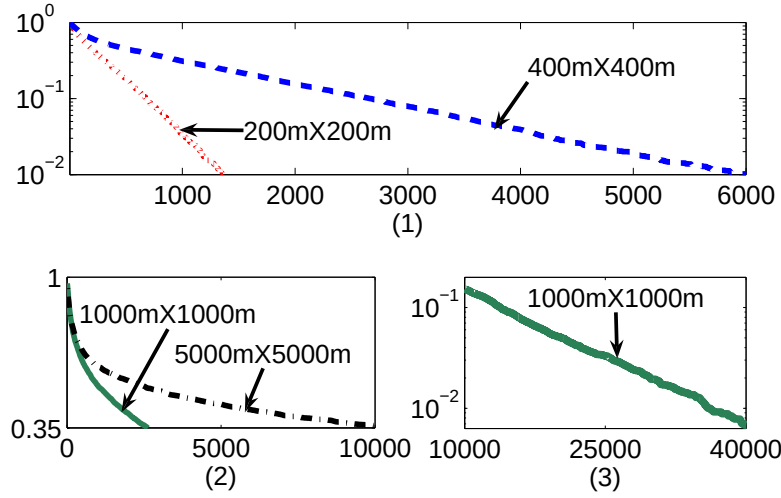
Figure 3.7 shows CCDF of the inter-meeting time on a log-log scale for random walk models (RWM) under different sizes of the domain, where the boundary is always reflective. To observe the transition of inter-meeting time behavior from an exponential type to a power-law more clearly, we draw the simulation results both on a log-log scale in Figure 3.7(a) and on a semi-log scale in Figure 3.7(b). In this case, we set the simulation period to 40 hours.⁸ Here, we use a very simple and widely used random walk model where a node changes directions uniformly from $[0, 2\pi]$ every 50 seconds [20]. For each size of the domain simulated, we calculate

⁷The timescale which matters in real network is commonly in the order of hours (less than one day). Hence, we choose 40 hours, which is long enough to observe the accurate tail behavior of the inter-meeting time in this timescale.

⁸Since $\sqrt{144000} * 1.35 \approx 512$ (144000 seconds are 40 hours, 1.35 is the square root of the second moment of step length), we expect that the two smallest domains can be looked as bounded, whereas the other two can be looked as essentially unbounded from our discussion in Section 4.



(a) Log-Log scale



(b) Semi-Log scale

Figure 3.7: CCDF of the inter-meeting time for RWM models under different sizes of the domain on a log-log scale (a) and on a semi-log scale (b). The hitting frequency to the boundary decreases as the size of the domain increases. For 5000×5000 square meters size, a node rarely hits the boundary and thus moves like in an unbounded space and produces power-law behavior of the inter-meeting time, as seen in (a). Exponential behavior of the inter-meeting time distribution under smaller domains (e.g., $200m \times 200m$, $400m \times 400m$) is clearly shown in (b)(1) (straight line on a semi-log scale). Power-law type behavior starts to appear in mid-to-large domain size as shown in (b)(2).

the hitting frequency defined as the ratio between the number of steps within which the node hits the boundary and the total number of steps. For example, when the size of domain is 200×200 , the hitting frequency is quite high (40.0%), thus the inter-meeting time behaves like an exponential (The CCDF of inter-meeting time is a straight line on a semi-log scale, as shown in Figure 3.7(b)(1).) as expected from our theoretical results. In contrast, when the size of the domain is very large (5000×5000 square meters), the hitting frequency is as low as 1.7%, which means that it is practically an unbounded domain for the duration of the timescale (40 hours). Hence, as Theorem 4 shows, the inter-meeting time follows a power-law distribution. (The CCDF of inter-meeting time is a straight line on a log-log scale, as Figure 3.7(a) shows.) Further, the slope of this CCDF on a log-log scale is shown to be around 0.25, which is also consistent with our findings in Section 3.2. In addition, for a large domain size (e.g., $1000m \times 1000m$), the power-law behavior is dominant over up to $O(10^4)$ seconds, followed by a sharp decrease beyond that timescale. As shown in Figure 3.7(b)(3), beyond 10^4 seconds, the inter-meeting time distribution decays exponentially. This behavior is also consistent with the measurement studies in [48, 58], and is well expected from our theoretical results in Sections 3–4.

Chapter 4

Stochastic Anatomy of

Inter-meeting Time Distribution

under General Mobility Models

In Chapter 3, we study the exponential/power-law behavior of inter-meeting time distribution. Our analysis based on different mobility models reveals the relationship between finite boundary and exponential behavior of inter-meeting time, as well as that between infinite domain and power-law behavior of inter-meeting time. The finiteness of the domain is strongly correlated with the timescale under discussion. Our discussion on the relationship between domain size and time scale then strongly suggests a mixture (power-law and exponential) behavior of inter-meeting time distribution. This mixture behavior presents new challenge to the problem of mobility modeling and its effect on the network performance. Existing studies on this problem via the average inter-meeting time become insufficient when the

inter-meeting time distribution starts to deviate from exponential one. This insufficiency necessarily leads to the increasing difficulty in the performance analysis of forwarding algorithms in MANET. Consequently, in this chapter, we investigate how the stochastic nature of the mobility patterns affects the whole *distribution* of the inter-meeting time, as well as the system performance.

In Section 4.1, we identify the critical timescale in the inter-meeting distribution, at which the transition from power-law to exponential takes place, in terms of the domain size and the statistics of the mobility pattern. In Section 4.2, we prove that *stronger correlations* in mobility patterns lead to *heavier* (non-exponential) ‘head’ of the inter-meeting time distribution. In Section 4.4, we prove that there exists an *invariance* property for several contact-based metrics such as inter-meeting and contact time under both distance-based (Boolean) and physical interference (SINR) based models. Lastly, we discuss the effect of correlation in mobility patterns on system performance in Section 4.4.

4.1 Head/Tail of Inter-meeting Time: A Closer Look at Dichotomy

Recent studies in [58] have pointed out that the inter-meeting time distribution is largely characterized by first a power-law until certain time scale (*head* of the inter-meeting time), beyond which the inter-meeting time distribution becomes of exponential type (*tail* of the inter-meeting time). Specifically, in [58], this dichotomic behavior is empirically observed and emphasized via a large set of real traces. Our

discussion in Section 3.3.1 suggests, via a simple *i.i.d.* uniform mobility model, that the transition from exponential to power-law behavior of the inter-meeting time arises from the coupled dynamics between the relevant time scale of interest and the size of the bounded domain. In this section, we give a closer look at this transition and quantify the timescale for transition in terms of the size of the domain and certain key characteristics of general mobility patterns, thereby providing precise distinction between the head and tail of the inter-meeting time distribution for any general mobility models under arbitrary sized domain.

To this end, first note that for a given mobility model $\mathcal{M}$ and the size of the domain¹ (‘diameter’ of the domain D), the node will forget where it was after it hits the boundary and bounces back to the ‘center’ of the domain. In other words, the timescale beyond which the inter-meeting time becomes of exponential type (tail) is on the same order of the typical amount of time it takes for the node to travel D distance. We call this timescale as *regenerative period* and write as

$$\tau_0 = \tau_0(\mathcal{M}, D)$$

to explicitly denote that it depends on the size of the domain as well as the characteristics of the mobility pattern. The concept of regenerative period is similar to the characteristic time of the inter-meeting time proposed in [58] except that here τ_0 is adopted to study the inter-meeting time distribution of general mobility models under arbitrary domain size. In light of the results in our previous chapters, for a given mobility model and under a given finite domain, the ‘head’ of $\mathbb{P}\{T_I > t\}$ is

¹This should be interpreted as a virtual boundary as in our previous chapters in that mobile nodes tend to return to where they belong after reaching out beyond this virtual domain, either by actual reflection (wrapping around) as typically assumed in mobility models or by quasi-periodic returning behavior in reality.

largely of power-law type for $t < \tau_0$ (essentially unbounded domain) while its ‘tail’ becomes exponential for $t > \tau_0$.

Figure 4.1(a) shows $\mathbb{P}\{T_I > t\}$ of two *i.i.d.* mobile nodes under Boolean model with communication range $d = 50m$, each of which follows IRW with constant speed 1 m/s whose step-length distribution is exponential with mean $100m$. As expected, the ‘head’ of the inter-meeting time distribution displays power-law behavior and is prolonged as the domain size increases. In order to grasp the relationship among τ_0 , D , and $\mathcal{M}$, we consider the average displacement of a mobile node A at time t , which is given by $\sigma(t) = \sqrt{\mathbb{E}\{|A(t)|^2\}}$. From the aforementioned reasoning, we set $\sigma(\tau_0) = D$. Since $\sigma(t) \sim \sqrt{t}$ for the IRW with exponential step-length distribution, it follows that $\tau_0 \propto D^2$ for this mobility model. To quantitatively observe this scaling behavior, we consider $\mathbb{P}\{T_I > \tau_0 t\}$, a rescaled version of the ccdf of the inter-meeting time where the time scale is *normalized* such that the typical amount of time to travel D is equal to one time unit, regardless of the domain size. Figure 4.1(b) shows rescaled versions of $\mathbb{P}\{T_I > \eta t\}$ where η is chosen to be proportional to D^2 . As clearly can be seen, the timescales beyond which the inter-meeting time distribution shows exponential decay all coincide, which asserts that τ_0 is indeed proportional to D^2 under the given IRW model.

Next, to quantify the impact of mobility pattern on the location of τ_0 , we fix the domain size to $D = 2000m$ and consider a set of IRW models, but now with different mean step-length $\lambda = 10, 50, 100m$. Since the speed is always kept to 1 m/s , longer mean step-length naturally translates into stronger tendency of moving straight or preserving the same direction, i.e., stronger correlation over short time scale (until the current step finishes).

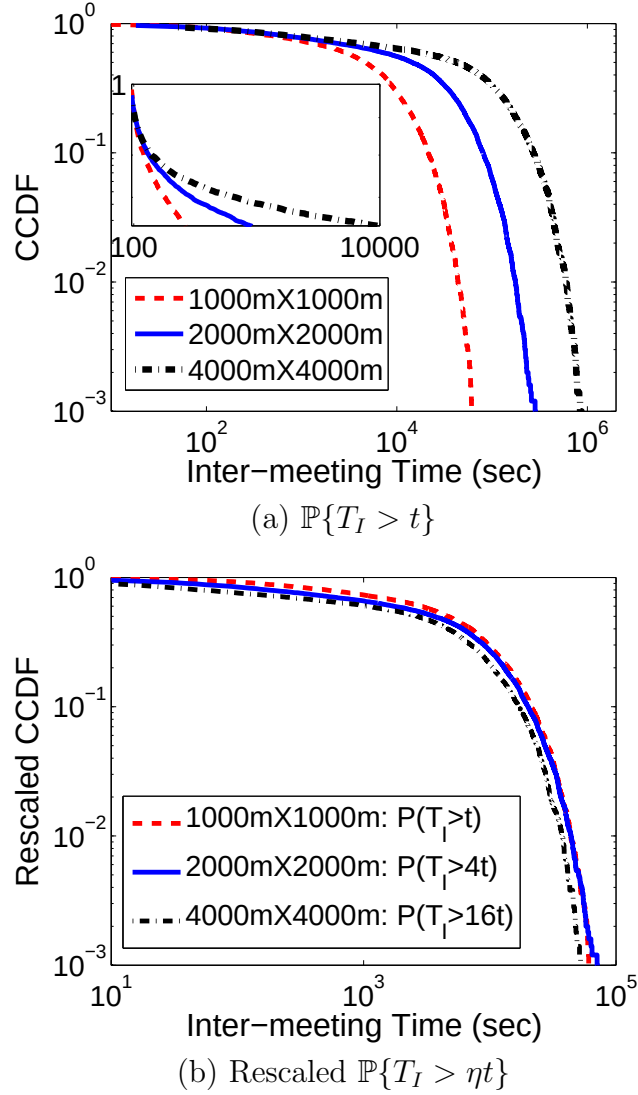


Figure 4.1: Effect of domain size on the regenerative period τ_0 . IRW defined in Section 2.2 is used. Step length distribution is exponential with mean $100m$ and speed is set to constant ($1m/s$). Communication range $d = 50m$. Simulation time: 10^6 seconds. (a) $\mathbb{P}\{T_I > t\}$ in log-log scale; the inset is drawn in linear-log scale to show the power-law behavior more clearly. (b) shows the rescaled complementary cumulative distribution function (ccdf), i.e., $\mathbb{P}\{T_I > \eta t\}$ with factors $\eta = 1, 4, 16$, proportional to D^2 .

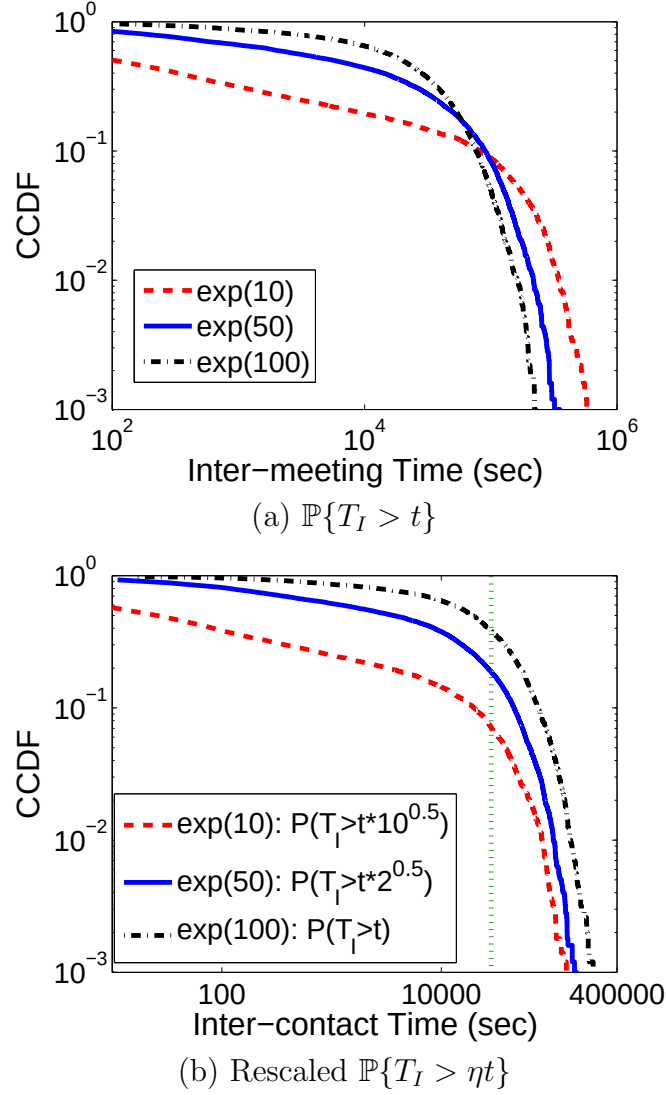


Figure 4.2: Effect of ‘correlation’ of the mobility model on the regenerative period τ_0 . Domain size is fixed to $2000m \times 2000m$. We use IRW models with exponential step-length distribution with means $\lambda = 10, 50, 100$ to represent different degrees of ‘correlation’ (stronger tendency of moving straight for longer duration) (a) $\mathbb{P}\{T_I > t\}$ in log-log scale; (b) shows the rescaled cdf, $\mathbb{P}\{T_I > \eta t\}$ with factors $\eta = \sqrt{10}, \sqrt{2}, 1$, such that $\eta^2 \lambda = \text{Const}$.

Figure 4.2(a) shows $\mathbb{P}\{T_I > t\}$ in a log-log scale for different mean step-lengths. We notice that larger step-lengths (stronger tendency of moving straight) make τ_0 smaller. This is well expected, since larger step-lengths mean that the mobile node will ‘hit’ the boundary earlier. Similarly as before, to quantify the scaling behavior with respect to different mean step-lengths, we consider how $\sigma(t)$ scales as the mean step-length increases under a given domain size. While exact computation of $\sigma(t)$ is quite involved, the scaling behavior can be obtained in a relatively simple manner as follows. Suppose $N(t)$ is the number of steps generated over $[0, t]$ under a given IRW model with mean step-length λ . Then, the position of the node at time t is approximated by $A(t) \approx \sum_{i=1}^{N(t)} L_i e^{j\phi_i}$ where L_i ($i = 1, 2, \dots$) are *i.i.d.* step-lengths with $\mathbb{E}\{L_i\} = \lambda$ and ϕ_i are *i.i.d.* angles uniformly over $[0, 2\pi]$. Thus, $\sigma^2(t) \approx \mathbb{E}\{|\sum_{i=1}^{N(t)} L_i e^{j\phi_i}|^2\}$. Now, suppose that we increase the mean step-length λ to $k\lambda$ ($k > 1$) and let $\hat{L}_i$ be the exponential random variable with mean $k\lambda$. Then, approximately, there will be $N(t)/k$ number of steps generated over $[0, t]$, while the mean of each step-length becomes k times larger, i.e., $\mathbb{E}\{\hat{L}_i\} = k\lambda$. Since L_i is exponentially distributed, this means that $\mathbb{E}\{\hat{L}_i^2\} = k^2\mathbb{E}\{L_i^2\}$. Thus, we have

$$\hat{\sigma}^2(t) \approx \mathbb{E}\left\{\left|\sum_{i=1}^{N(t)/k} \hat{L}_i e^{j\phi_i}\right|^2\right\} \approx k\sigma^2(t).$$

Hence, we have $\hat{\sigma}(t) \approx \sqrt{k}\sigma(t)$, i.e., the average displacement of mobile nodes with k times larger mean step-length grows $\sqrt{k}$ times faster, which implies that the regenerative period (τ_0) becomes $1/\sqrt{k}$ times smaller. In other words, $\tau_0^2 k$ always remains constant. As Figure 4.2(b) shows, after properly rescaling the ccdf $\mathbb{P}\{T_I > \eta t\}$ for different mean step-lengths λ with η chosen such that $\eta^2 \lambda$ remains constant throughout, the regenerative periods τ_0 are all aligned around the same value.

The observations made from Figures 4.1 and 4.2 suggest that τ_0 , the timescale that separates the head and tail of the inter-meeting time distribution, increases as the domain size increases and decreases as the degree of correlations (tendency of moving straight) increases, whereby the exact scaling relationship can be made through the average displacement $\sigma(t)$. Our finding here also matches with empirically observed exponential behaviors of the inter-meeting time over almost all time scale under several popular mobility models such as Random Waypoint (RWP) or RD, since these models have either very short regenerative period (for RWP) or very strong tendency of moving straight (RD), which both contribute to small τ_0 , thus making the inter-meeting time distribution exponential virtually everywhere.²

While the tail behavior of the inter-meeting time distribution has received much attention in the literature, the quantitative analysis of the ‘head’ of $\mathbb{P}\{T_I > t\}$ in terms of mobility patterns has been uncharted territory; the only available result in the literature is that the head of $\mathbb{P}\{T_I > t\}$ basically follows a power-law distribution from measurement study [58] and our analysis in previous chapters, without any attempt to characterize or compare its ‘shape’ for different mobility models. Therefore, in the next section, we turn our attention to the behavior of the head of the inter-meeting time distribution, i.e., $\mathbb{P}\{T_I > t\}$ for $t < \tau_0$ over which the node rarely hits the boundary and thus the domain is essentially unbounded.

²We note that this is one of the key reasons why there have been many results assuming pure exponential distribution of inter-meeting time (thus leading to either Markovian or ODE based analysis), since those models indeed have exponential inter-meeting time distribution throughout and thus the theoretical results *do* match with the simulation in their setup, until recent empirical measurement of inter-meeting time reveals a different story.

4.2 Head of Inter-meeting Time: Stochastic Ordering

In this section, we show how correlations in mobility patterns affect the shape of the head of the inter-meeting time distribution. As mentioned earlier, in this regime, the mobile node practically resides in an unbounded domain (as it rarely hits the boundary). Consider first the inter-meeting time T_I of two independent mobile nodes A and B in an *unbounded* domain, each of which follows correlated mobile trajectory. Then, the difference $C(t) = A(t) - B(t)$ is also correlated over time t and the inter-meeting time of A and B reduces to the inter-hitting time of C to the origin. In order to properly capture the effect of correlations in the mobile trajectories on the inter-meeting time distribution in this regime, we employ a simple, yet effective CRW model introduced in Section 2.2 defined on 1-D grid (unbounded) and consider its inter-hitting time to origin. Recall that for 1-D CRW model, larger p means stronger correlation (stronger tendency to follow the same direction) between adjacent steps.

Define $\rho = p - q$ ($p + q = 1$). To avoid trivialities, assume $-1 < \rho < 1$: when $\rho = -1$, the mobile node always bounces back and forth between two adjacent sites; when $\rho = 1$, it always goes straight line following the same direction. Note that $\rho = 0$ ($p = q = 0.5$) corresponds to a simple random walk. Let $X_t = \pm 1$ be the direction (step) of node C . Then, it follows that $\mathbb{P}\{X_{t+1} = X_t\} = p$, $\mathbb{P}\{X_{t+1} = -X_t\} = q$, and the position of node C at time t is given by $S(t) = \sum_{k=1}^t X_k$. Note here that when $\rho \neq 0$, $S(t)$ itself is not a Markov chain, but a partial sum of Markov chain X_t . Assume that node C is initially at the origin, i.e., $S(0) = 0$ and we use $X_0 = \pm 1$

to denote the initial condition for the direction, e.g., $X_0 = -1$ means that C 's first step, X_1 , will be left (right) with probability p (q).

To proceed, we need the following definition:

$$\mathcal{W}_t \triangleq \sum_{k=1}^t 1_{\{\mathcal{E}(k)\}}, \quad (4.1)$$

where $1_{\{\cdot\}}$ is the indicator function and $\mathcal{E}(k) = \{S(k) > 0\} \cup \{\{S(k) = 0\} \cap \{S(k-1) > 0\}\}$ denotes the event that mobile node is on the *positive side* at time k , i.e., the mobile node is on site $S(k) > 0$ or on $S(k) = 0$ with $S(k-1) > 0$. Hence, $\mathcal{W}_t$ counts the number of steps the node is on the positive side out of t steps. From this definition, clearly $0 \leq \mathcal{W}_t \leq t$ and $t - \mathcal{W}_t$ is the number of steps the node is on the *negative side* out of t steps. The event $\{S(k) = 0\}$ is special here; $\{S(k) = 0\}$ alone cannot determine which side the node belongs to at time k . If the node has jumped from site 1 to 0, i.e., $S(k-1) = 1$, then we say that the node is on the positive side at time k ; or else, it is on the negative side.

From (4.1) and $S(0) = 0$, we have

$$\mathbb{P}\{T_I > t\} = \mathbb{P}\{\mathcal{W}_t = t | X_0 = 1\} = \mathbb{P}\{\mathcal{W}_t = 0 | X_0 = -1\}. \quad (4.2)$$

To see the first equality in (4.2), note that from $X_0 = 1$ and $S(0) = 0$, we can define a virtual state $S(-1) = S(0) - X_0 = -1$. Then, $\mathbb{P}\{\mathcal{W}_t = t | X_0 = 1\}$ is the probability that node C starts from site -1 and goes right to the origin and then never returns back to -1 within t steps. Note that in any realization of $\{\mathcal{W}_t = t\}$, if there exists $0 < k \leq t$ with $S(k) = 0$, then the node must be on the positive side at time k , since $S(k-1) = -1$ immediately implies $\mathcal{W}_t < t$. From the definition of the inter-hitting time in Section 2.3.1, this is exactly $\mathbb{P}\{T_I > t\}$.³ Similarly, the second equality in

³Since all sites are symmetric, the inter-hitting times of node C to any site (not necessarily the

(4.2) can also be derived by considering inter-hitting to site 1, with similar virtual state $S(-1) = 1$ and the event that node C always stays on the negative side up to time t .

Define $r(i, m, t) = \mathbb{P}\{\mathcal{W}_t = m | X_0 = i\}$ ($t = 0, 1, \dots, i = \pm 1$), with $r(i, 0, 0) = 1$ and $\mathcal{W}_t$ is given by (4.1). Then, from (4.2)

$$\mathbb{P}\{T_I > t\} = r(1, t, t) = r(-1, 0, t). \quad (4.3)$$

We only need to compute either $r(1, t, t)$ or $r(-1, 0, t)$. To this end, for $t \in \mathbb{Z}^+$ and $i, j = \pm 1$, we define

$$\begin{aligned} p(i, j, t) &= \mathbb{P}\{S_t = 0, X_t = j | X_0 = i\}, \\ f(i, j, t) &= \mathbb{P}\{S_t = 0, S_{t-1} \neq 0, \dots, S_1 \neq 0, X_t = j | X_0 = i\}, \\ p(i, j, 0) &= 1_{\{i=j\}}, \quad f(i, j, 0) = 0. \end{aligned} \quad (4.4)$$

Here, $p(i, j, t)$ is the probability that node C returns to the origin at time t , and $f(i, j, t)$ is the probability of the *first* return to the origin at time t (after getting out of it). The event $\{S_t = 0, X_t = j | X_0 = i\}$, i.e., return to the origin with direction j at time $t > 0$ from initial direction i , can be decomposed into (i) the first return to the origin with direction j from initial direction i at some time $1 \leq k \leq t$ and (ii) a return to the origin with direction j from initial direction j . Hence, $p(i, j, t)$ and $f(i, j, t)$ satisfy the following recursion [79]

$$p(i, j, t) = \sum_{k=1}^t f(i, j, k) p(j, j, t-k) + 1_{\{t=0\}} p(i, j, t). \quad (4.5)$$

Similarly, suppose now that node C starting from the origin does not always stay on either positive or negative side until a given time t . Then, it must have returned

origin) must have the same distribution.

to the origin at some earlier time l ($0 < l < t$). Thus, the event of node C's stay on the positive side for m out of t steps ($0 < m < t$) can be decomposed into (i) the first return to the origin with direction $j = \pm 1$ at time l from initial direction i and (ii) the node stays on the positive side for $m1_{\{j=1\}} + (m-l)1_{\{j=-1\}}$ out of $t-l$ steps. Hence, we have another recursive relationship between $r(i, m, t)$ and $f(i, j, l)$ for $0 < m < t$ as follows:

$$\begin{aligned} r(i, m, t) &= \sum_{l=1}^m f(i, -1, l) r(-1, m-l, t-l) \\ &\quad + \sum_{l=1}^{t-m} f(i, 1, l) r(1, m, t-l). \end{aligned} \quad (4.6)$$

For simple (independent) random walk, similar recursion relationship exists [79], although many other nice properties of simple random walk do not apply to CRW model here.

From the ‘convolution-like’ structures in (4.5) and (4.6), it is convenient to use generating functions. Define

$$\begin{aligned} P(i, j, z) &= \sum_{t=0}^{\infty} p(i, j, t) z^t, \quad F(i, j, z) = \sum_{t=0}^{\infty} f(i, j, t) z^t \\ R(i, z, \phi) &= \sum_{m=0}^{\infty} \sum_{t=m}^{\infty} r(i, m, t) z^m \phi^{t-m}. \end{aligned} \quad (4.7)$$

Then, multiplying both sides of (4.5) by z^t leads to

$$P(i, j, z) = 1_{\{i=j\}} + F(i, j, z) P(i, j, z). \quad (4.8)$$

Similarly, by multiplying $z^m \phi^{t-m}$ in both sides of (4.6) and considering the case of

$m = 0$, t (since (4.6) holds only under $0 < m < t$), we obtain from (4.7)

$$\begin{aligned} & R(-1, z, \phi) - R(-1, z, 0) - R(-1, 0, \phi) + 1 \\ &= [R(-1, z, \phi) - R(-1, z, 0)] F(-1, -1, z) \\ &+ [R(1, z, \phi) - R(1, 0, \phi)] F(-1, 1, \phi), \end{aligned} \quad (4.9)$$

which corresponds to the initial condition $i = -1$ (i.e., $X_0 = -1$). Similarly, we can obtain another equation like (4.9) starting from $i = 1$.

We collect several useful properties for $R(i, z, \phi)$:

Lemma 2 *We have $R(-1, z, 0) = R(1, 0, z)$, $R(-1, 0, z) = R(1, z, 0)$, $R(i, z, z) = 1/(1 - z)$ and*

$$(1 + \rho) [R(-1, z, 0) - 1] = (1 - \rho) [R(1, z, 0) - 1]. \quad (4.10)$$

Proof. See Appendix. □

We are now ready to state our main result in this section.

Theorem 5 *Let T_I be the inter-hitting time of a mobile node C to the origin in an unbounded 1-D grid. Then, for any given $t > 0$, $\mathbb{P}\{T_I > t\}$ is an increasing function of $\rho \in [0, 1)$.*

Proof. We will first compute $P(i, j, z)$, the generating function of $p(i, j, t)$. Then, we can obtain $\mathbb{P}\{T_I > t\} = r(-1, 0, t)$ from (4.8), (4.9) and Lemma 2. Define

$$U(\theta) = \begin{bmatrix} \theta^{-1}p & \theta q \\ \theta^{-1}q & \theta p \end{bmatrix}, \quad (4.11)$$

and let $U^t(\theta)_{ij}$ be the element on the i^{th} row and j^{th} column of $U^t(\theta)$. For notational convenience, we set $i, j = \pm 1$ and let $-1, 1$ denote the first and the second row (column), respectively. Then it follows that

$$U^t(\theta)_{ij} = \sum_{k=-\infty}^{\infty} \mathbb{P}\{S_t = k, X_t = j | X_0 = i\} \theta^k.$$

From (4.4), $p(i, j, t) = \mathbb{P}\{S_t = 0, X_t = j | X_0 = i\}$ is equal to the coefficient of θ^0 in $U^t(\theta)_{ij}$. Hence, $P(i, j, z)$ equals to the coefficient of θ^0 in $\sum_{t=0}^{\infty} z^t U^t(\theta)_{ij}$. Now, consider

$$M(z) = \begin{bmatrix} P(-1, -1, z) & P(-1, 1, z) \\ P(1, -1, z) & P(1, 1, z) \end{bmatrix},$$

which is equal to the coefficient of θ^0 in $\sum_{t=0}^{\infty} z^t U^t(\theta) = [I - zU(\theta)]^{-1}$. For example, we can write

$$P(-1, -1, z) = 1 + \frac{1 - \rho z^2}{(1 - z^2)f(z, \rho)}, \quad f(x, y) \triangleq \sqrt{\frac{1 - x^2 y^2}{1 - x^2}}. \quad (4.12)$$

Recall that $r(-1, 0, t)$ (or $r(1, t, t)$) is equal to $\mathbb{P}\{T_I > t\}$. Hence, we only need to derive $R(-1, 0, \phi)$.

$$\begin{aligned} R(-1, 0, \phi) &= \sum_{m=0}^{\infty} \sum_{t=m}^{\infty} r(-1, m, t) 0^m \phi^{t-m} \\ &= \sum_{t=0}^{\infty} r(-1, 0, t) \phi^t \quad (\text{since } 0^m = 0 \text{ for } m \neq 0.) \end{aligned} \quad (4.13)$$

Set $z = \phi$ in (4.9), from (4.8), (4.12), and Lemma 2, we get

$$R(-1, 0, \phi) = 1 + (1 + \phi) [f(\phi, \rho) - 1] / (\phi(1 - \rho)), \quad (4.14)$$

where $f(\cdot, \cdot)$ is defined in (4.12). From (4.13), $r(-1, 0, t)$ is the coefficient of ϕ^t in $R(-1, 0, \phi)$. Since $r(-1, 0, 0) = 1$ from initial condition, in what follows, we only consider $t \geq 1$.

From Newton's generalized binomial theorem, we have

$$\sqrt{1-\rho^2\phi^2} = \sum_{m=0}^{\infty} b_m(-\rho^2\phi^2)^m, \quad \frac{1}{\sqrt{1-\phi^2}} = \sum_{m=0}^{\infty} c_m(-\phi^2)^m,$$

where $b_m = d_m^{1/2}$, $c_m = d_m^{-1/2}$ with the *generalized binomial coefficient*

$$d_m^r = \frac{1}{m!} \prod_{i=0}^{m-1} (r-i) \quad (d_0^r = 1).$$

Now, for any given $0 \leq \rho < 1$, expand $f(\phi, \rho) = \sum_{t=0}^{\infty} a_t(\rho)\phi^t$ where

$$a_{2k}(\rho) = \sum_{l=0}^k (-1)^k b_l c_{k-l} \rho^{2l}, \quad a_{2k+1}(\rho) = 0, \quad (k \geq 0). \quad (4.15)$$

$a_{2k+1}(\rho) = 0$ for any $k \geq 0$, since $f(\phi, \rho)$ depends only on ϕ^2 for any given ρ .

Set $\rho = 1$ in $f(\phi, \rho)$, then we have

$$f(\phi, 1) = \sqrt{(1-\phi^2)/(1-\phi^2)} = 1 = \sum_{t=0}^{\infty} a_t(1)\phi^t.$$

Since this is true for all $|\phi| < 1$, we have $a_t(1) = 0$ for any $t > 0$. From (4.15), this leads to $\sum_{l=0}^k (-1)^k b_l c_{k-l} = 0$, i.e., $b_0 c_k = -\sum_{l=1}^k b_l c_{k-l}$. Consequently,

$$\begin{aligned} a_{2k}(\rho) &= (-1)^k b_0 c_k + \sum_{l=1}^k (-1)^k b_l c_{k-l} \rho^{2l} \\ &= \sum_{l=1}^k (-1)^{k+1} b_l c_{k-l} (1 - \rho^{2l}). \end{aligned} \quad (4.16)$$

From (4.13), $\mathbb{P}\{T_I > t\} = r(-1, 0, t)$ is the coefficient of ϕ^t in $R(-1, 0, \phi)$. In light of (4.14), we have

$$\mathbb{P}\{T_I > t\} = r(-1, 0, t) = (a_t(\rho) + a_{t+1}(\rho)) / (1 - \rho).$$

(4.15) denotes that only one of $a_t(\rho)$, $a_{t+1}(\rho)$ is non-zero. Suppose $t = 2k$ w.l.o.g.

Then, from (4.16),

$$\mathbb{P}\{T_I > t\} = r(-1, 0, t) = \frac{a_t(\rho)}{1 - \rho} = \sum_{l=1}^k (-1)^{k+1} b_l c_{k-l} \frac{1 - \rho^{2l}}{1 - \rho}.$$

From direct computation, it is straightforward to see that $(-1)^{k+1}b_l c_{k-l}$ is always positive for any $1 \leq l \leq k$. Therefore, since $(1-\rho^{2l})/(1-\rho)$ is increasing in $\rho \in [0, 1)$, $\mathbb{P}\{T_I > t\}$ is also increasing in ρ and we are done. $\square$

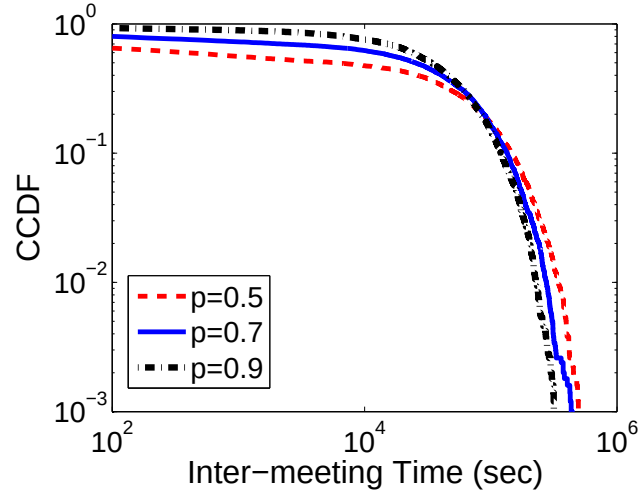
Theorem 5 says that the ‘head’ of the inter-meeting time distribution becomes heavier as the degree of correlation increases. While we provide rigorous proof via 1-D model, the same observation holds for 2-D as can be seen from Figure 4.3, where larger p (thus stronger tendency to keep the same direction) leads to heavier head of the inter-meeting time distribution $\mathbb{P}\{T_I > t\}$ until τ_0 .

In a broader context, Theorem 5 also implies that the head of the inter-meeting time becomes stochastically larger [74] as the tendency of moving straight becomes stronger. Figure 4.2(a) reveals that this is indeed the case: larger mean step-length in IRW models makes $\mathbb{P}\{T_I > t\}$ larger for $t < \tau_0$.

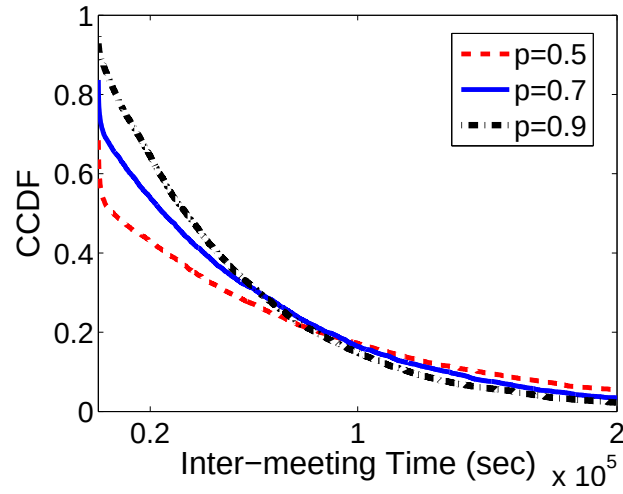
4.3 Invariance Property of Contact-based Metrics

In this section we show the *invariance property* of contact-based metrics including the inter-meeting, contact, and inter-hitting time. In particular, the mean of these contact-based metrics does not depend on the degree of correlation ρ .

Consider three independent mobile nodes A , B and C following CRW in a bounded domain starting from their stationary distribution, i.e., uniform over the bounded domain. In contrast to the previous section, we now consider a bounded domain and the inter-meeting time distribution over all time t . For 1-D CRW on



(a) log-log



(b) linear-linear

Figure 4.3: Effect of correlations of the mobility model on the ccdf of inter-meeting time. CRW on 2-D grid (200×200) defined in Section 2.2 is used. $p = 0.5, 0.7, 0.9$ cases are simulated with $q = r = (1 - p)/3$. (a) $\mathbb{P}\{T_I > t\}$ in a log-log scale; (b) $\mathbb{P}\{T_I > t\}$ in a linear-linear scale.

a ring, we assume N is odd to avoid trivial situation where two nodes never meet if they start from adjacent sites. For the same reason, we also assume $\sqrt{N}$ is odd when 2-D CRW on $\sqrt{N} \times \sqrt{N}$ finite square grid torus is under consideration.

To set the stage for the invariance results, consider a stationary 0 – 1 valued process $\{Y_t\}$ ($t = 0, 1, 2, \dots$) with its stationary probability measure $\mathbb{P}$. Define *recurrence times* [8] $\{n_l\}_{l \geq 1}$ of the state 1 for $\{Y_t\}$ as follows:

$$\begin{aligned} T_1 &\triangleq \min\{t \geq 0 : Y_t = 1\}, \\ T_k &\triangleq \min\{t > T_{k-1} : Y_t = 1\} \quad (k \geq 2), \\ n_l &\triangleq T_{l+1} - T_l. \end{aligned} \tag{4.17}$$

Here, T_k denotes the k^{th} occurrence time instance of state 1. For example, for a sequence $\{Y_t\} = \{110010\dots\}$, $T_1 = 0$, $T_2 = 1$, $T_3 = 4$ and so on. Accordingly, the recurrence times are $n_1 = 1$ and $n_2 = 3$. In this definition, $n_1 = 1$ denotes that the first recurrence time to state 1 is 1. This is somewhat different from what we commonly define the recurrence to state 1, which is usually the return to state 1 *conditioning* on starting from state 1 and subsequently getting out of state 1. For example, $n_2 = T_3 - T_2$ in the above example means the return to state 1 at time $t = 4$ conditioning on starting from state 1 at time $t = 1$ and subsequently getting out of state 1 at time $t = 2$.

The following will be used in the proof of our main result.

Theorem 6 [8] *For a stationary 0 – 1 process $\{Y_t\}$ ($t = 0, 1, 2, \dots$) with*

$$\lim_{n \rightarrow \infty} \mathbb{P}\{Y_0 = 0, Y_1 = 0, \dots, Y_n = 0\} = 0, \tag{4.18}$$

the average recurrence time n_1 conditioning on $Y_0 = 1$ and $Y_1 = 0$ satisfies

$$\mathbb{E}\{n_1|Y_0 = 1, Y_1 = 0\} = 1 + \frac{1 - \mathbb{P}\{Y = 1\}}{\mathbb{P}\{Y = 1\} - \mathbb{P}\{Y_0=1, Y_1=1\}},$$

where Y denotes a random variable with distribution $\mathbb{P}$.

Remark 11 Kac's recurrence theorem [57, 30] relates the average of n_1 conditioning on $\{Y_0 = 1\}$, i.e., $\{T_1 = 0\}$, to the probability of a specific event, e.g., $Y_t = 1$. In particular, Kac's recurrence theorem gives $\mathbb{E}\{n_1|Y_0 = 1\} = 1/\mathbb{P}\{Y = 1\}$. Here, $\mathbb{E}\{n_1|Y_0 = 1\} = \mathbb{E}\{T_2 - T_1|Y_0 = 1\} = \mathbb{E}\{T_2|Y_0 = 1\}$. In other words, $1/\mathbb{P}\{Y = 1\}$ is the expected time interval between two adjacent recurrences of state 1. Theorem 6 reformulates Kac's Recurrence Theorem by relating the average of n_l to the probability of events $\{Y_t = 1\}$ and $\{Y_t = 1, Y_{t+1} = 1\}$. Specifically, Theorem 6 gives $\mathbb{E}\{n_1|Y_0 = 1, Y_1 = 0\}$, the expected time interval between two recurrences of state 1 given that Y_t first gets out of state 1 (and subsequently moves into state 0 before returning to 1 again). Note that if the state 1 is always of 'short' duration in $\{Y_t\}$, i.e., there is no consecutive 1 in the sequence, $\mathbb{P}\{Y_0 = 1, Y_1 = 1\} = 0$, and thus $\mathbb{E}\{n_1|Y_0 = 1, Y_1 = 0\} = 1/\mathbb{P}\{Y = 1\}$. In this case, Theorem 6 reduced to the original Kac's recurrence theorem. $\square$

We now present the main result in this section.

Theorem 7 Let A, B be two independent mobile nodes following 1-D CRW on a ring with N sites (N odd), and C be a static node sitting on an arbitrary site on this ring. Let (i) T_H be the inter-hitting time of node A to C ; (ii) T_I be the inter-meeting time of nodes A and B ; (iii) T_C be the contact time of nodes A and B .

Then, the average of T_H , T_I and T_C are all invariant with respect to the correlation coefficient $\rho \in (-1, 1)$. Specifically,

$$\mathbb{E}\{T_H\} = N - 1, \quad (4.19)$$

$$\mathbb{E}\{T_I\} = 2(N - 1), \quad (4.20)$$

$$\mathbb{E}\{T_C\} = 2. \quad (4.21)$$

Remark 12 For a simple random walk on 1-D ring, i.e., $\rho = 0$, [58] shows (4.19)⁴ and (4.20). N is assumed to be even in [58] while we require it to be odd. This is because in [58] two nodes start from the same site, while in our case two nodes start from anywhere and the inter-meeting time starts once they meet (which is guaranteed only under odd N). We here emphasize that our Theorem 7 holds for CRW with any degree of correlation ρ while [58] only considers a simple random walk. It is also interesting to note that stronger correlations, i.e., larger ρ , lead to heavier head of the inter-meeting time as shown in Section 4.2, but at the same time Theorem 7 says that correlations do not affect the mean of the inter-meeting time defined on a bounded domain. $\square$

Proof. Without loss of generality, assume nodes A , B , C start from their stationary distribution, i.e., uniform over the domain [9]. Let $A(t)$, $B(t) \in \{1, 2, \dots, N\}$ be the position of nodes A and B at time t ($t = 0, 1, 2, \dots$). Since node C is static, we use $C(0)$ to denote its position.

⁴The average hitting time given in [58] is N , since [58] includes the last time slot when node A hits C in the definition of the hitting time, while we do not. Hence, there is a difference of 1 between their result and (4.19).

Inter-hitting time T_H : Define Z_t as

$$Z_t = 1_{\{A(t)=C(0)\}}. \quad (4.22)$$

Since $\{A(t)\}$ is stationary, the sequence $\{Z_t\}$ is also stationary. In order to apply Theorem 6, we need to show that Z_t satisfies (4.18). To see this, note that while the CRW $A(t)$ itself is not Markov for $\rho \neq 0.5$, $\Gamma_A(t) \triangleq \{A(t), X(t)\}$ (augmented with the direction $X(t)$ of the CRW $A(t)$) becomes a Markov chain with $\mathcal{T} = \{1^-, 1^+, 2^-, 2^+, \dots, N^-, N^+\}$ as the state space, where m^-, m^+ means that node is at site m with direction $-1, +1$, respectively. Since this chain is aperiodic and irreducible and has finite states ($2N$), it is also ergodic. Thus, the chain $\{\Gamma_A(t)\}$ will visit any set of state infinitely often, including $\{C(0)^-, C(0)^+\}$, which asserts that the condition in (4.18) is satisfied.

Now, define $\{T_k\}_{k \geq 1}$ and recurrence times $\{n_l\}_{l \geq 1}$ for sequence $\{Z_t\}$ by replacing Y with Z in (4.17). Then, conditioning on $Z_0 = 1$ and $Z_1 = 0$, $n_1 - 1$ gives the inter-hitting time T_H of node A to C . Thus, from Theorem 6,

$$\mathbb{E}\{T_H\} = [1 - \mathbb{P}\{Z = 1\}] / [\mathbb{P}\{Z = 1\} - \mathbb{P}\{Z_0 = 1, Z_1 = 1\}],$$

where Z is a random variable with the same distribution as Z_t . Observe that (i) $\mathbb{P}\{Z = 1\} = 1/N$ since $C(0)$ is randomly chosen from N sites (each site has equal probability $1/N$) and (ii) $\mathbb{P}\{Z_0 = 1, Z_1 = 1\} = \mathbb{P}\{Z_1 = 1 | Z_0 = 1\} \mathbb{P}\{Z_0 = 1\} = 0$ since once node A meets C , A leaves site $C(0)$ at the next time slot. Therefore, (4.19) follows.

Inter-meeting time T_I : The proof is quite similar to the case of inter-hitting time as above. First, define $W_t = 1_{\{A(t)=B(t)\}}$. From the stationarity of $A(t)$ and

$B(t)$ and from their independence, W_t is also stationary. To check the condition in (4.18), similarly as before, construct a chain $\Sigma(t) \triangleq \{\Gamma_A(t), \Gamma_B(t)\}$ with state space $\mathcal{T} \times \mathcal{T}$ (total $(2N)^2$ states). From the independence of $\Gamma_A(t)$ and $\Gamma_B(t)$ and since odd N ensures the chain is aperiodic, it follows that $\Sigma(t)$ is also ergodic. Thus, following the same arguments in the proof for inter-hitting time, we can show that W_t also satisfies the condition in (4.18).

Now, we have

$$\mathbb{E}\{T_I\} = \frac{1 - \mathbb{P}\{A(0) = B(0)\}}{\mathbb{P}\{A(0) = B(0)\} - \mathbb{P}\{A(0) = B(0), A(1) = B(1)\}}. \quad (4.23)$$

Note that $\mathbb{P}\{A(0) = B(0)\} = 1/N$ (since $A(0)$ and $B(0)$ are independent and uniform over N sites). Further, since the direction of a node (A or B) is ± 1 with equal probability under the stationary distribution, it follows that $\mathbb{P}\{A(1) = B(1) \mid A(0) = B(0)\} = 1/2$ and thus (4.20) holds.

Contact time: Set $V_t = 1 - W_t$. Then, the average contact time can be calculated as a ‘dual’ of the average inter-meeting time in (4.23).

$$\begin{aligned} \mathbb{E}\{T_C\} &= \frac{1 - \mathbb{P}\{A(0) \neq B(0)\}}{\mathbb{P}\{A(0) \neq B(0)\} - \mathbb{P}\{A(0) \neq B(0), A(1) \neq B(1)\}} \\ &= \frac{1 - (N-1)/N}{(N-1)/N - (N-1)/N \times (1 - 1/(2N-2))} = 2. \end{aligned} \quad (4.24)$$

In (4.24), $\mathbb{P}\{A(1) = B(1) \mid A(0) \neq B(0)\} = 1/(2N-2)$ since conditioning on $A(0) \neq B(0)$, $A(1) = B(1)$ is true only if there is one and only one site between $A(0)$ and $B(0)$ (with probability $2/(N-1)$ and both nodes A, B jump to it at time $t = 1$ (with probability $1/4$). $\square$

Much in the same way as in Theorem 7, we can show the invariance property for a set of 2-D CRW models as follows, whose proof is omitted for brevity.

Proposition 2 *For 2-D CRW model with $p+q+2r=1$, the average of T_H , T_I and T_C are all invariant with respect to p , q and r . Specifically,*

$$\mathbb{E}\{T_H\} = N - 1, \quad \mathbb{E}\{T_I\} = 4(N - 1)/3, \quad \mathbb{E}\{T_C\} = 2.$$

Recall that the main ingredient for the proof of Theorem 7 is to construct stationary sequence $\{Z_t\}$, $\{W_t\}$, $\{V_t\}$ from $A(t), B(t)$ and show that they all satisfy the condition in (4.18). Suppose now that all the mobile nodes are independent and have stationary distribution for their positions. Let $\Phi(t) = \{A(t), B_1(t), \dots, B_N(t)\}$ be the collection of all the positions of mobile nodes in the steady-state. Then, since $f(\Phi(t))$ is also stationary for any measurable function f , we can readily extend the invariance property as in Theorem 7 to at least the following two more general cases:

General inter-contact to set Θ : For $t \geq 0$, define $W_t^{any} = 1_{\{A(t) \in \mathcal{N}_\Theta(t)\}}$, where $\mathcal{N}_\Theta(t)$ is the contact set of an arbitrary set of node $\Theta = \{B_i, i \in \mathcal{I}\}$ at time t defined in (2.1).

General SINR interference model: For $t \geq 0$, define $W_t^{SINR} = 1_{\{B(t) \in \mathcal{N}_A(t)\}}$, where $\mathcal{N}_A(t)$ is the contact set of node A at time t under SINR interference model in (2.2).

Now, recall from the end of Remark 11 that if the duration of contact is short enough when compared to the typical duration of inter-contact,⁵ the average statistics of the above general contact-based metric can be computed *only* from the stationary distribution of the mobile nodes, independently of their detailed mobility structure or the degree of correlations. Since there are many mobility models whose stationary distributions are identical (e.g., RD or IRW with general step-length distribution), our observation suggests that we can enjoy a great deal of simplification

⁵This is a reasonable assumption in sparse MANET with low-density mobile nodes or delay tolerant networks.

as long as the average statistics are concerned. However, there is a caveat; the system must be in stationary regime before we compute those average statistics. Take a sparse network with large domain for example, then unless the initial positions of all the mobile nodes are close to their stationary distributions, it may take very long for them to converge their stationary distribution⁶ and it could be that we want packets to be forwarded/routed to other nodes *before* the convergence to stationary regime occurs. Our observation of this invariance property thus suggests an interesting relationship between the role of stationarity of the system (or the assumption that system is in the steady-state) and the first-order statistics of contact-based metrics.

Table 4.1: Average inter-meeting/inter-any-contact time (Boolean)

	IRW (10)	IRW (50)	IRW (100)	RD
$200m \times 200m$	456	443	448	451
$500m \times 500m$	2103	2115	2111	2123
$1000m \times 1000m$	8375	8403	8241	8401

(a) Average inter-meeting time (Boolean)

	IRW (10)	IRW (50)	IRW (100)	RD
$200m \times 200m$	83	80	78	80
$500m \times 500m$	357	345	362	366
$1000m \times 1000m$	1557	1501	1526	1453

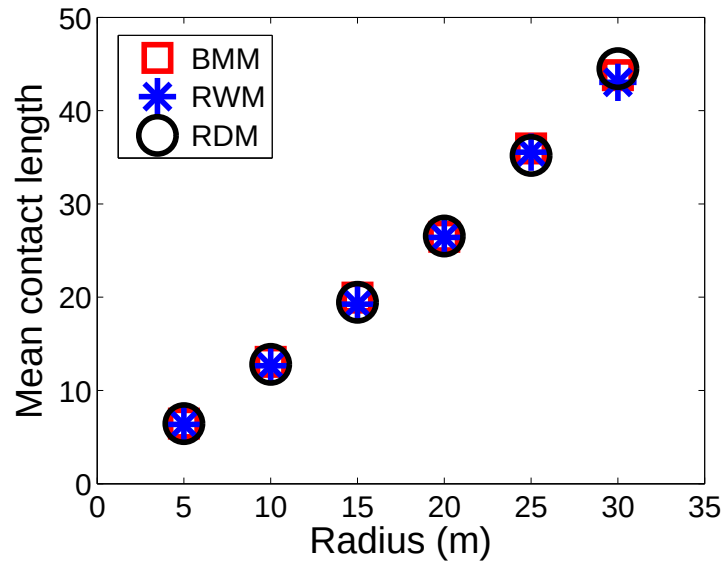
(b) Average inter-any-contact time (Boolean)

Table 4.1 shows the average inter-meeting/inter-any-contact time in seconds un-

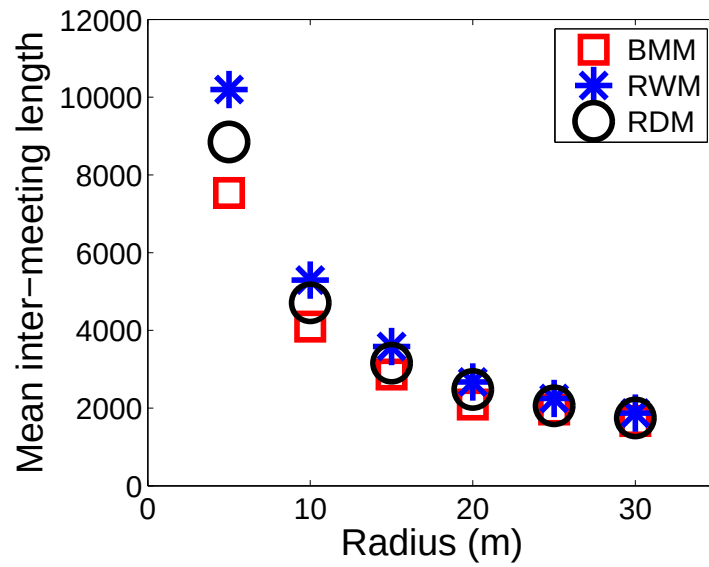
⁶This is *mixing time* in the Markov process literature [30].

der the same mobility models used in Figure 4.2 defined over three different domain sizes. We use Boolean model for contact as defined in (2.1) with communication range set to $d = 50m$. In addition to the set of IRW models with different mean step-lengths, we have also run RD model (see Section 2.2) to represent very strong degree of correlation in the mobility pattern, since under RD model a mobile node always goes straight until it hits the boundary. For inter-meeting time, we consider a pair of mobile nodes. For inter-any-contact time, we consider the inter-contact of a given mobile node A to a set of five given mobile nodes $\{B_i\}$ ($i = 1, 2, \dots, 5$). Table 4.1 clearly shows that, under a given domain size, the average inter-meeting and inter-any-contact time do not depend on mobility patterns with different degrees of correlation, as expected from Theorem 7.

In Table 4.1, the invariance property of the average inter-meeting/inter-any-contact time is shown under fixed communication range $d = 50m$ for IRW models with *exponentially distributed* step length. In fact, such invariance property is true for any given communication range and for even more general mobility models. Figure 4.4 shows that the average contact/inter-meeting time under Boolean interference model is invariant with respect to different random walk mobility patterns. Particularly, IRW models whose step length distributions are constant (mean: 8m), exponential (mean: 40m) and uniform (in $(0, 40)m$) are used. The domain size is set to be $400m \times 400m$. In addition to the invariance property, we can also see that the average contact time is linearly increasing with communication range d ($d = 5, 10, 15, 20, 25, 30m$) (Figure 4.4(a)), and the average inter-meeting time is inversely proportional to the communication range d (Figure 4.4(b)).



(a) Average contact time



(b) Average inter-meeting time

Figure 4.4: Average contact and inter-meeting time under Boolean model.

Table 4.2: Average contact/inter-meeting time (SINR)

SNR(dB)	IRW (10)	IRW (50)	IRW (100)	RD
0	11.9	12.4	12.3	12.1
10	21.5	21.1	21.4	20.8
20	35.0	33.2	34.8	33.1

(a) Average contact time (SINR)

SNR(dB)	IRW (10)	IRW (50)	IRW (100)	RD
0	514.6	552.3	537.1	522.8
10	303.0	327.4	322.4	302.4
20	190.1	207.5	192.1	187.5

(b) Average inter-meeting time (SINR)

Table 4.2 shows the average contact/inter-meeting time of a pair of nodes in seconds under the same mobility models as in Table 4.1. We now use SINR interference model for contact as defined in (2.2), with parameters $\alpha = -4$, $\beta = 5 (\approx 7dB)$. Domain size is set to $400m \times 400m$ throughout. We define $SNR = P/N_0$ as the value measured at distance $d_0 = 15m$.⁷ We vary the signal-to-noise ratio such that $SNR \in \{0, 10, 20\}dB$. As expected from our Theorem 7 and its extension, the average contact and inter-meeting time remain invariant with respect to the mobility patterns with different degrees of correlation.

Figure 4.4 shows a relationship between the communication range d and the average contact/inter-meeting time, e.g., the average contact time is proportional to d (Figure 4.4(a)). By comparing (2.1) and (2.2), the communication range d in

⁷ d_0 varies according to the communication channel quality requirement.

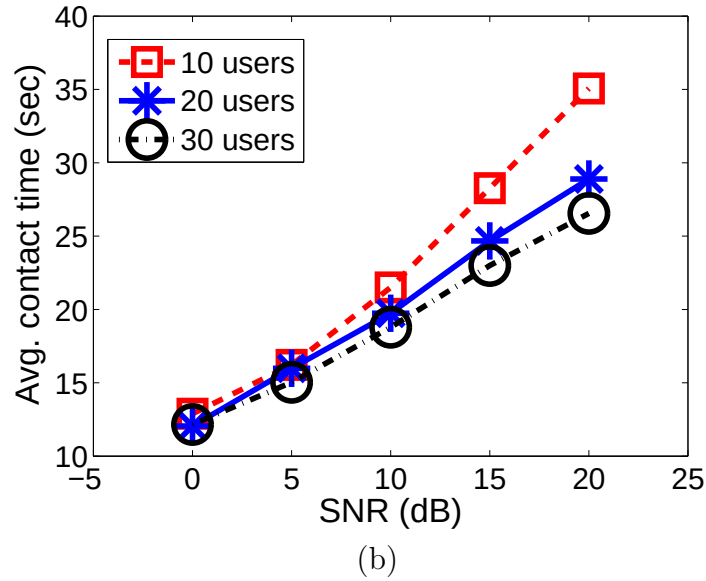
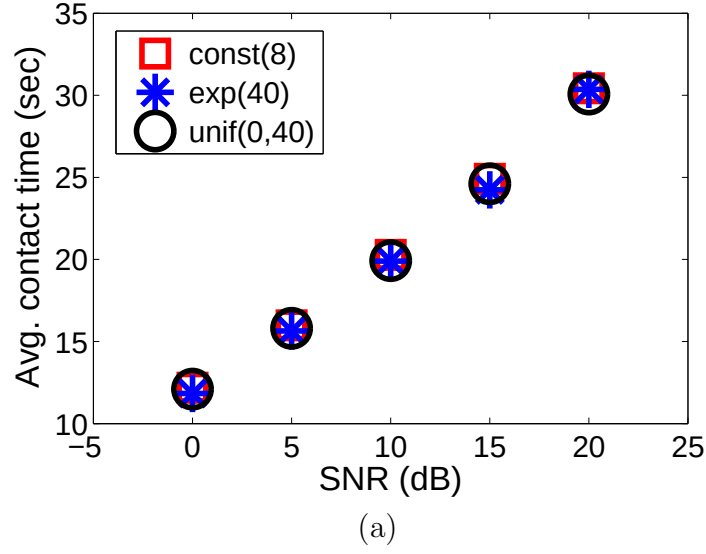


Figure 4.5: (a) Average contact time (20 nodes): the effect of the user-interference on the average contact time under SINR model. (b) Avg. contact time vs. # of nodes and $\text{SNR} = P/N_0$: the collective effect of random noise interference and user-interference on the average contact time under SINR model. IRW mobility model with exponentially distributed step length (mean: 40m) is used.

Boolean model can be looked as the collective effect of path loss coefficient α , random noise level N_0 , and the signal-to-interference threshold β in the SINR model. In this sense, Table 4.2 shows the invariance property of the average contact/inter-meeting time with respect to different mobility models with varying degrees of correlation, under given *random noise interference*. Compared to the Boolean interference model, the SINR model introduces an additional source of interference, i.e., the interference from other mobile nodes (user-interference) in the system. Figure 4.5 shows the effect of *user-interference* on the invariance property of the average contact time (a), as well as the *collective* effect of random noise interference and user-interference (b). The simulation setting is the same as that in Figure 4.4. Different user-interference levels are achieved by changing the number of nodes in the system. Specifically, as the number of node becomes larger, the interference from all other nodes, i.e., $\sum_i P \|x - B_i(t)\|^{-\alpha}$ in (2.2), becomes larger. As Figure 4.5(a) shows, when the number of nodes in the system is changed to 20 (in Table 4.2, this number is 10), the invariance property for the average contact time with respect to different mobility models still holds. The collective effect of two sources of interference defined in the SINR model, i.e., random noise interference and user-interference, is shown in Figure 4.5(b). For instance, the curve for the average contact time versus $\text{SNR} = P/N_0$ becomes more convex for lower user-interference level, e.g., red dashed line (10 users) is more convex than the blue solid line and the black dash-dot line.

Remark 13 [93] shows through direct computation that the average inter-meeting time of IRW model under Boolean contact model does not depend on its step-length distribution. In contrast, in this dissertation, we consider CRW to explicitly capture

the degree of correlations in the mobility patterns and rigorously prove the invariance property for not only the inter-meeting time of a given pair, but a set of very general contact-based metrics such as contact time and inter-any-contact time. Note that our results apply equally to Boolean as well as SINR interference model for contact, which is made possible by utilizing the powerful family of Kac's recurrence theorem. Finally, as mentioned earlier, we note that IRW models with longer mean step-lengths can be interpreted as models with stronger correlations in the mobility pattern (stronger tendency to preserve the same direction). In view of this, Tables 4.1, 4.2 confirm that our invariance results hold true for a broader class of mobility models whenever the concept of correlations in mobility patterns can be applied. $\square$

4.4 Ordering of Forwarding Performance Through Correlation in Mobility

Having discovered a number of stochastic properties of contact-based metrics under general mobility models, we now turn our attention to their implication on the performance of network protocols over mobile nodes. First, recall that the invariance property of average statistics for contact-based metrics holds when the system is in stationary regime. Suppose that the system is already in the steady-state, e.g., the initial positions of mobile nodes are drawn from their stationary distributions. Then, the following question naturally arises: will the invariance property lead to the same (or similar) system performance?

The answer to this question is yes when the inter-meeting time is exponentially

distributed, or equivalently, when the regenerative period τ_0 is very small, which is the case for RWP and RD models. More precisely, for a class of mobility models whose τ is small enough, the invariance result tells us that their inter-meeting time *distributions* are all the same, which will lead to similar network performance. This is also supported by results in [40, 92, 44, 102, 90], in that the average inter-meeting time critically determines the performance of forwarding algorithms.

However, when the inter-meeting time distribution severely deviates from exponential,⁸ which is the case in reality as shown in [22, 58], the ‘head’ of the distribution starts to kick in with non-negligible τ_0 . In particular, our results in Sections 4.2 and 4.3 suggest that *stronger correlations* lead to *heavier head* of the cdf of the inter-meeting time, which must be offset by *lighter tail* so as to maintain the same average⁹ from invariance property for any given domain size. This corresponds to a *less dangerous* ordering [74]:

Definition 11 *X is said to be less dangerous than Y if: (i) $\mathbb{E}\{X\} \leq \mathbb{E}\{Y\}$, and (ii) their CCDF $\mathbb{P}\{X > t\}$ and $\mathbb{P}\{Y > t\}$ have a unique intersection $t_0 \in \mathbb{R}$ such that $\mathbb{P}\{X > t\} \geq \mathbb{P}\{Y > t\}$ for all $t < t_0$ and $\mathbb{P}\{X > t\} \leq \mathbb{P}\{Y > t\}$ for all $t \geq t_0$.*

□

For example, in view of the invariance result, Figure 4.2(a) shows that the inter-meeting time of IRM with exponential step-length distribution of mean $100m$ (say, T_{100}) is less dangerous than the inter-meeting time of IRM with exponential step-length distribution of mean $10m$ (say, T_{10}). Figure 4.3(a) also shows that there

⁸We mean the whole shape of the distribution, not just the tail, since the tail of the inter-meeting has been shown to be always exponential [58, 18].

⁹Note that $\mathbb{E}\{T_I\} = \int_0^\infty \mathbb{P}\{T_I > t\} dt$.

exists a dangerous ordering for CRW models as well. Our next step is to employ the following property [74]:

Proposition 3 *If X is less dangerous than Y and $\mathbb{E}\{X\} = \mathbb{E}\{Y\}$, then $X \leq_{cv} Y$, i.e., $\mathbb{E}\{\varphi(X)\} \leq \mathbb{E}\{\varphi(Y)\}$ for all convex function φ . $\square$*

Thus, from our invariance property, we have $T_{I100} \leq_{cv} T_{I10}$. The convex ordering relationship can be used to facilitate system performance analysis. For example, consider the first passage time (FPT) (or the residual/remaining inter-meeting time) defined in (2.5). FPT is an important mobility metric to evaluate the system performance [22, 58, 3]. As a simple illustration, consider a single packet from a source that is being broadcast to all other N nodes in the network (similar to the epidemic routing [98, 102] but without recovery process). Let t_1 and t_2 ($t_1 < t_2$) be the time instants at which the number of ‘infected’ nodes (those that have already received a copy of the packet) becomes N_1 and $N_1 + 1$, respectively. Thus, $t_2 - t_1$ is the amount of time it takes to increase the number of infected nodes by 1. It is also the residual inter-contact time of the set of infected nodes $\mathcal{B}$ at time t_1 to the set of uninfected nodes $\bar{\mathcal{B}}$ at the same time instant, i.e., $t_2 - t_1$ is the minimum residual inter-contact time of a sequence of residual inter-contact times between one node, say, $A \in \mathcal{B}$ and another node $C \in \bar{\mathcal{B}}$. Such two nodes A and C may or may not have met before t_1 . However, when we look at their residual inter-contact time at t_1 , as long as the initial positions of all nodes are independent, t_1 is a randomly accessed time instant. In other words, $t_2 - t_1$ precisely becomes the first passage time to a set of nodes, whose statistics are governed by the equilibrium distribution as in (2.5). Although this argument does not apply precisely when there are many

packets being transmitted at the same time, we expect that FPT still plays a major role¹⁰ in the performance study of network protocol under various configurations, as long as transmissions of different packets do not heavily interfere with each other.¹¹

To numerically support the aforementioned argument, we present *ns-2* simulation results in Figure 4.6(a) and (b) for the message delivery ratio of epidemic routing protocol [98] under a set of IRW mobility models as in Figure 4.2(a). The domain size is set to $500m \times 500m$ (a) and $2000m \times 2000m$ (b). Clearly, different degrees of correlation lead to very different network performance. In this figure, 600 and 6000 seconds are chosen such that the message delivery ratios of one mobility pattern (here we choose $\exp(100)$) in (a) and (b) at the chosen time instants are the same. Note that the difference in network performance is amplified with larger domain size.

While the average inter-meeting time is invariant as seen in Table 4.1, the average FPT could be adopted to predict the performance ordering shown in Figure 4.6. Figure 4.7 shows the average FPT under a set of IRW mobility models as in Figure 4.2(a). Clearly, mobility patterns with different degrees of correlation leads to different average FPT. Particularly, mobility pattern with weaker correlation has larger average FPT, and the difference is increased as the domain size increased. Under the $2000m \times 2000m$ domain, the average FPT for IRW model with mean step length $10m$ is more than 2.5 times of that for IRW model with mean step length $100m$. This explains why the difference in network performance shown in Figure 4.6 is amplified with larger domain size.

¹⁰Note that if the inter-meeting time is exponentially distributed, so is the FPT with the same mean.

¹¹e.g., when incoming traffic density is low, which is typical for MANET.

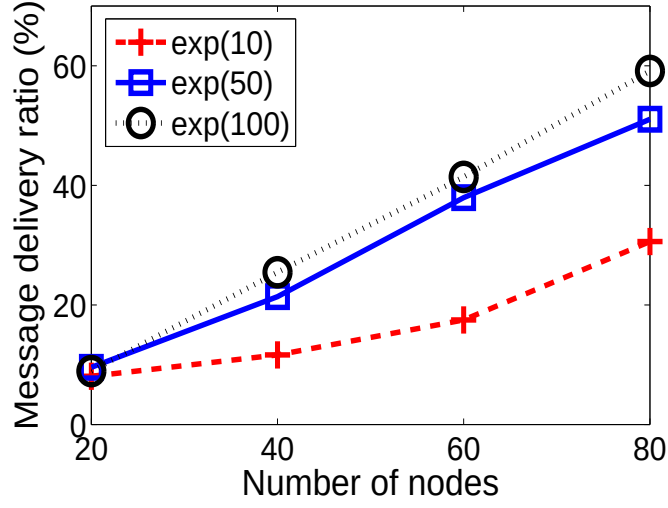
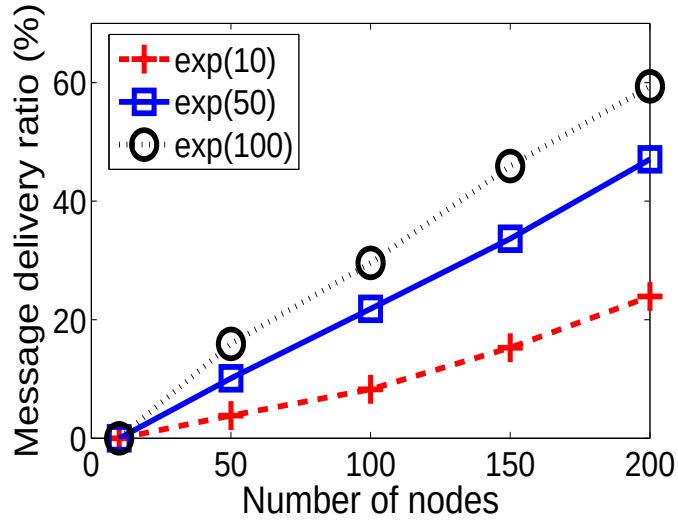
(a) $500m \times 500m$ (b) $2000m \times 2000m$

Figure 4.6: Effect of correlation in mobility patterns on system performance. Simulation setting is the same as Figure 4.2(a) and Table 4.1 with $500m \times 500m$ domain (a) and $2000m \times 2000m$ domain (b). A fixed amount of data (1800 packets) are transmitted using epidemic routing protocol. We use *ns-2* with different number of nodes, and measure the message delivery ratio at 600 seconds in (a) and 6000 seconds in (b).

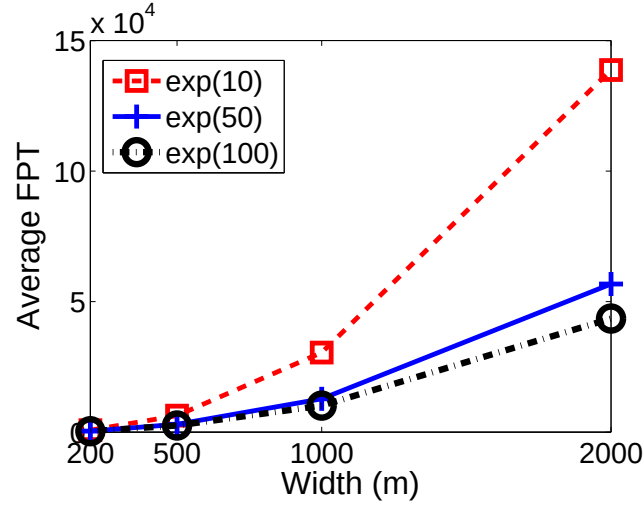


Figure 4.7: Effect of correlation in mobility patterns on the average FPT. Simulation setting is the same as Figure 4.2(a) and Table 4.1 with $200m \times 200m$, $500m \times 500m$, $1000m \times 1000m$, $2000m \times 2000m$ domain. The average FPT with x-axis be the width of the domain is plotted in the figure.

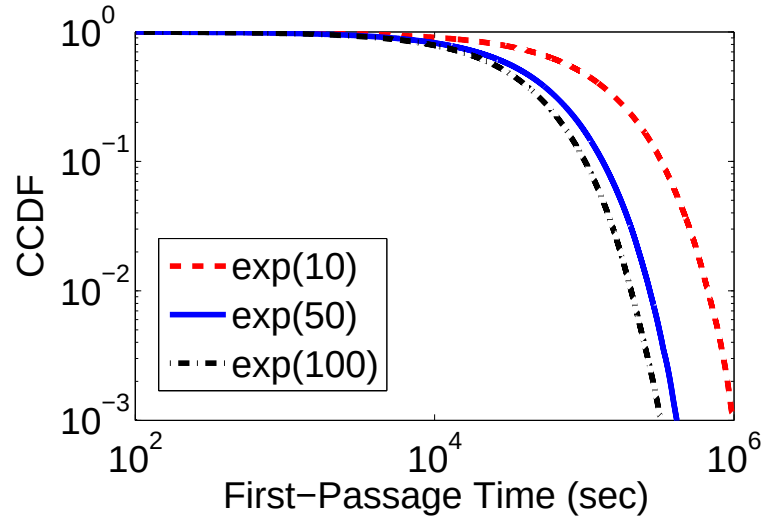


Figure 4.8: Effect of correlation in mobility patterns on the CCDF of FPT. Simulation setting is the same as Figure 4.2(a) and Table 4.1. The CCDF of FPT for $2000m \times 2000m$ domain is shown.

In fact, for inter-meeting time T_{I1} and T_{I2} satisfying $T_{I1} \leq_{cv} T_{I2}$, from (2.5),

$$\mathbb{P}\{T_{F1} > t\} = \frac{\mathbb{E}\{[T_{I1} - t]^+\}}{\mathbb{E}\{T_{I1}\}} \leq \frac{\mathbb{E}\{[T_{I2} - t]^+\}}{\mathbb{E}\{T_{I2}\}} = \mathbb{P}\{T_{F2} > t\} \quad (4.25)$$

for any given $t > 0$, where the second inequality is from $\mathbb{E}\{T_{I1}\} = \mathbb{E}\{T_{I2}\}$ and that the function $\varphi(x) = [x - t]^+$ is convex function of x for any fixed $t > 0$. (4.25) is equivalent to $T_{F1} \leq_{st} T_{F2}$, i.e., T_{F1} is stochastically smaller than T_{F2} [74]. In other words, we have $\mathbb{E}\{\psi(T_{F1})\} \leq \mathbb{E}\{\psi(T_{F2})\}$ for any non-decreasing function ψ . Figure 4.8 clearly shows that (4.25) holds for the mobility models under discussion, as expected.

Chapter 5

Aging Rules: What Does the Past Tell About the Future in Mobile Ad-Hoc Networks?

While the convex ordering relationship among inter-meeting times revealed in Chapter 4 provides a way to compare the system performance while the degrees of correlation in the underlying mobility pattern are comparable, the relationship between this new method based on non-exponential inter-meeting time and the traditional method based on exponential inter-meeting time still remains to be a mystery. Moreover, with the disclosure on the fundamental reason for the exponential/power-law behavior of inter-meeting time distribution in Chapter 3 as well as that on the performance analysis based on non-exponential inter-meeting time in Chapter 4, new and more interesting questions are: can we use the non-exponential nature of the real inter-meeting time to guide the system design? How? In this chapter,

we propose to utilize the opportunities brought by the non-exponential (memory) structure of the inter-meeting time. By exploring such memory structure, we are able to not only ascertain whether the analysis based on exponential inter-meeting assumption over-estimate or under-estimate the real system performance, but also provide guideline for the design of MANET forwarding strategies.

In Section 5.1, we analyze aging property of inter-contact time under four classes of stochastic mobility patterns as well as some existing mobility models in terms of its failure rate. In Section 5.2, we show how to compare approaches based on non-exponential and exponential inter-contact time in the sense of stochastic ordering. In Section 5.3, we discuss how our results on aging properties can be exploited toward better design of forwarding/routing algorithms in MANET.

5.1 Mobility Patterns with CFR/DFR/IFR Inter-meeting Time

As mentioned in Section 2.4, the aging property of a random variable can be properly captured by its failure rate. To show the relationship between the various characteristics of relative mobility of a pair of nodes and the corresponding inter-meeting times with different aging properties, in this section, we first fix one node and show four classes of stochastic mobility patterns that give rise to inter-meeting time with constant/decreasing/increasing failure rates and new-better-than-used property. We then study the aging property of the inter-meeting times of a pair of mobile nodes following two popular mobility models in MANET study, i.e. RWP

and RW, through simulations.

Consider a mobile node C following the class of RWG models on $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, W\}$ as defined in Section 2.2. Let $C(t) \in \mathcal{V}$ be C 's position at time $t \geq 0$. The *inter-meeting time* T_I between node C and any given site $j \in [0, N]$ is defined by

$$T_I^{(j)} = \min_{t \geq 0} \{t : C(t-1) \neq j, C(t) = j\}, \quad (5.1)$$

given that $C(0) = j$ and $C(1) \neq j$. In general, $T_I^{(j)} \neq T_I^{(k)}$ for $j \neq k$ in the RWG model and such distinction will be important in describing various topologies with different degrees of ‘popularity’ of sites. Since we can always renumber the vertex (site), without loss of generality, we consider only $T_I^{(0)}$ and omit the superscript, unless otherwise specified.

5.1.1 Mobility Patterns with CFR Inter-meeting Time

When T_I has CFR, its distribution is exactly geometric (or exponential in continuous time case). A well-known mobility pattern producing memoryless inter-meeting time is the so-called *i.i.d.* mobility pattern, as shown in Figure 5.1(a). In this model, the mobile node chooses any site randomly and uniformly at each time step, and then jump to it. This corresponds to RWG model with $w_{ij} = w > 0$ for all i, j (complete graph with equal edge weights). We will present a more general class of mobility patterns but still with memoryless property all the time. This class of CFR mobility patterns here will be useful in understanding the asymptotic behavior of the failure rate of any general mobility model and also constructing DFR/IFR mobility patterns later on.

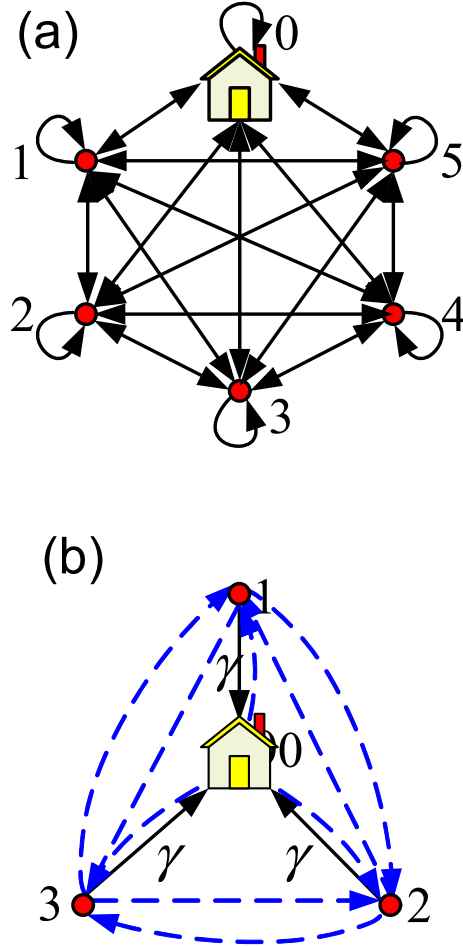


Figure 5.1: Examples of mobility patterns with CFR T_I (site 0 is the ‘home’ vertex). (a) *i.i.d.* mobility pattern, $p_{ij} = 1/6$ for all $i, j \in \mathcal{V}$; (b) $p_{10} = p_{20} = p_{30} = \gamma \in (0, 1)$ (in solid lines) and for all other $i, j \in \mathcal{V}$, $p_{ij} \in (0, 1)$ and $\sum_{j \in \mathcal{V}} p_{ij} = 1$.

Definition 12 [21] *The spectral radius $\rho(B)$ of $n \times n$ matrix B with its eigenvalues λ_i ($i = 1, 2, \dots, N$) is defined as*

$$\rho(B) = \max_i |\lambda_i|. \quad (5.2)$$

Now, we define a class of mobility models $\mathcal{M}_{C1}$ as:

Definition 13 *The class of mobility models $\mathcal{M}_{C1}$ are random walks on a 2-connected digraphs [100] $\mathcal{G}$ satisfying either one of the following:*

- **In-home condition:** *when mobile node C is at site $i \in \mathcal{V} - \{0\}$, it jumps to site 0 with probability $\gamma \in (0, 1)$.*
- **Out-home condition:** *when mobile node C is at site 0, it jumps to site $i \in \mathcal{V} - \{0\}$ with probability p_{0i} satisfying*

$$\vec{p}_0^{\text{out}} P^* = \rho(P^*) \vec{p}_0^{\text{out}}, \quad (5.3)$$

where $\vec{p}_0^{\text{out}} = [p_{01}, \dots, p_{0N}]$ with $\sum_{k=1}^N p_{0k} = 1$, P^* is $N \times N$ matrix obtained by deleting the first row and the first column of matrix P defined in Section 2.2, and $\rho(P^*)$ is the spectral radius of P^* as in Definition 12.

Remark 14 In a 2-connected graph $\mathcal{G}$ in Definition 13, for any two different sites $i, j \in \mathcal{V}$, there exist at least two paths from i to j . This corresponds to a ‘self-recovery’ feature, i.e., when one vertex of 2-connected $\mathcal{G}$ becomes unavailable (due to some emergency or failure), mobile nodes are still able to bypass this site to visit other parts of $\mathcal{G}$. Moreover, $\vec{p}_0^{\text{out}}$ in (5.3) is called quasi-stationary distribution [60].

We now have the following theorem:

Theorem 8 *Under the class of mobility models $\mathcal{M}_{C1}$, the inter-meeting time T_I has constant failure rate.* $\square$

Proof. See Appendix. $\square$

Remark 15 As mentioned in Section 2.4, T_I has CFR is equivalent to that T_I is geometrically distributed, hence satisfies the memoryless property. Or equivalently, the survival function at time t , i.e., $\bar{F}_t(\tau) = \mathbb{P}\{T_I \geq t + \tau | T_I \geq t\}$ is invariant with respect to t . Also, mobility patterns in Figure 5.1(b) satisfy the in-home condition in Definition 13, hence the corresponding T_I has CFR.

The ‘in-home’ condition in Definition 13 implies that all other sites $\mathcal{V} - \{0\}$ are essentially the same to site 0, regardless of their internal transition probabilities. Hence, when the recurrence to site 0 is considered, there will be no difference whether the mobile node is at site i or site j as long as $i, j \in \mathcal{V} - \{0\}$. Thus, the inter-meeting time is CFR or memoryless as expected. The ‘in-home’ condition can also be looked as an evidence for a ‘small world’ around site 0, where from any site other than 0, site 0 can be reached with the same probability $\gamma > 0$.

Under the ‘out-home’ condition in Definition 13, before returning back to site 0, the conditional distribution of $\{1, \dots, N\}$ (given that the node has not visited site 0) always remains the same. In fact, when the graph $\mathcal{G} \setminus \{0\}$ (after removal of site 0 and all its edges from $\mathcal{G}$) is *non-bipartite* [100], it can be shown that $\bar{p}_0^{\text{out}}$ is the stationary distribution of random walk on this resulting graph $\mathcal{G} \setminus \{0\}$. In other words, if $\pi_i(t)$ is the probability that the node is at site i at time t , $[\pi_1(t), \dots, \pi_N(t)]$ converges to $\bar{p}_0^{\text{out}}$ under a conditioning event that the node never visits site 0. Consequently, Theorem 8 also implies that for general mobility patterns (not limited to

$\mathcal{M}_{C1}$), the failure rate of the inter-meeting time eventually converges to a constant,¹ i.e., the inter-meeting time is asymptotically CFR. This is also in consistent with previous studies showing the exponential tail behavior of the inter-meeting time distribution [18, 58, 77].

5.1.2 Mobility Pattern with DFR Inter-meeting Time

In this section we show that a *time reversibility* property in mobility patterns is a main factor that leads to inter-meeting time with DFR. Since most components are from existing studies [61, 4] (and references therein), we will just briefly explain key steps of existing results and add some missing links to show the relationship between the time reversibility property in mobility patterns and inter-meeting time with DFR.

First, consider a class of random walks on an *undirected*, connected, and non-bipartite graph $\mathcal{G}$. The edge weight matrix $W = \{w_{ij}\}$ of an undirected graph $\mathcal{G}$ is symmetric, i.e., $w_{ij} = w_{ji}$. When this graph $\mathcal{G}$ is connected and non-bipartite, a mobile node B 's position at time t (i.e., $B(t)$) is irreducible and positive recurrent MC and thus it has unique stationary distribution $[\pi_0, \dots, \pi_N] = [d_{00}/d, \dots, d_{NN}/d]$, where d_{ii} is given in Definition 3 and $d = \sum_{i=0}^N d_{ii}$. Further, from $w_{ij} = w_{ji}$, it is easy to see that for any $i, j \in \{0, 1, \dots, N\}$, $\pi_i p_{ij} = \pi_j p_{ji}$, i.e., $B(t)$ is a time reversible MC [80]. The time reversible MC can also be similarly defined in a continuous time setting.

Now, consider a time-reversible continuous-time MC $B(t)$ on states $\{0, 1, \dots, N\}$

¹Here, conditions such as 2-connected and non-bipartite graph $\mathcal{G} \setminus \{0\}$ are sufficient to ensure the convergence.

with transition rate matrix $Q = \{q_{ij}\}$. Define $(N+1) \times (N+1)$ matrix $U = \{u_{ij}\}$ with $u_{ii} = \sqrt{\pi_i}$ and $u_{ij} = 0$ for $i \neq j$. Since the graph $\mathcal{G}$ is connected, $\pi_i > 0$, and U^{-1} is well-defined. Note that the matrix $S = UQU^{-1}$ is a real symmetric matrix from the time reversibility. Thus, the spectral representation theorem [21] allows to write $S = V\Lambda V^T$, where V is orthonormal (i.e., $VV^T = I$) and $\Lambda = \{\lambda_{ij}\}$ is a diagonal matrix with λ_{ii} being the eigenvalues of matrix S (also eigenvalues of matrix Q). In consequence, $S^{(t)} \triangleq \exp(St) = V \exp(\Lambda t) V^T$ and $Q^{(t)} \triangleq \exp(Qt) = U^{-1} S^{(t)} U = U^{-1} V \exp(\Lambda t) V^T U$. This implies that the transition function [61] can be written as

$$\begin{aligned} f_{ij}(t) &= \mathbb{P}\{B(t) = j | B(0) = i\} = Q_{ij}^{(t)} \\ &= \sum_{k=0}^N a(i, k) a(j, k) \exp(-\lambda_{kk} t), \end{aligned} \quad (5.4)$$

where $a(i, k), a(j, k)$ are real numbers. Thus, $f_{ii}(t)$ is complete monotone. Here, a function $f : [0, \infty) \rightarrow [0, \infty)$ is said to be *completely monotone* (CM) [61] if $f(t) = \int_0^\infty e^{-t\tau} \gamma(d\tau)$, where γ is a probability measure on $[0, \infty)$. This complete monotonicity of $f_{ii}(t)$ can be extended to a set $\mathcal{S}$, i.e., $f_{\mathcal{S}\mathcal{S}}(t) = \mathbb{P}\{B(t) \in \mathcal{S} | B(0) \in \mathcal{S}\}$ is also complete monotone. In this set-up, it is shown [4] that the first hitting time T_F of time-reversible MC $B(t)$ to the state 0, starting from the initial distribution $\mathbb{P}\{B(0) = j\} = q_{0j} / \sum_{k \neq 0} q_{0k}$ ($j \neq i$), is completely monotone. Specifically, [4] modified the original MC by making the state 0 as an absorbing state and setting $\mathcal{S} = \{1, \dots, N\}$. Then, $\mathbb{P}\{T_F > t\} = f_{\mathcal{S}\mathcal{S}}(t)$, i.e., T_F has CM distribution.

We now return to the case of inter-meeting time. Let $\mathcal{M}_{C2}$ denote the class of random walks on an *undirected*, connected, and non-bipartite graph $\mathcal{G}$. Consider a mobile node C under $\mathcal{M}_{C2}$ in continuous time setting² with transition rate matrix

²Our focus here is to find characteristics of the relative mobility that produce contact dynamics

$Q' = \{q'_{ij}\}$. Then, upon leaving state 0, node C arrives at state j ($j \neq 0$) with probability $q'_{0j} / \sum_{k \neq 0} q'_{0k}$. Therefore, the inter-meeting time T_I of node C has the same distribution as the aforementioned T_F . Hence, T_I is also CM. The failure rate function of a positive random variable with complete monotone density (or cdf) is known to be decreasing [66]. In other words, T_I has DFR.

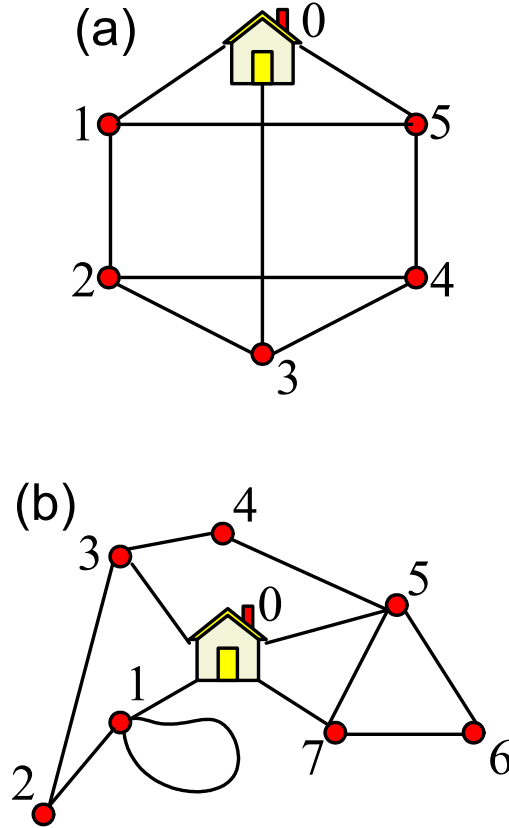


Figure 5.2: Examples of mobility models belonging to $\mathcal{M}_{C2}$. In (a) and (b), the weight of each edge can be any arbitrary positive number.

with certain aging property (here, DFR). We choose the continuous time setting only for convenience borrowing results in [4]. Same results can be also stated in the discrete time setting but with more involved notations.

The class of stochastic mobility patterns $\mathcal{M}_{C2}$ is quite large, as it includes random walks on any connected, non-bipartite and undirected graph $\mathcal{G}$. Figure 5.2(a) and (b) show examples of RWG mobility models belonging to $\mathcal{M}_{C2}$. Clearly, the graphs in both Figure 5.2(a) and (b) are connected with undirected edges with arbitrary positive weights. The non-bipartiteness follows from the existence of a closed path with odd length [100] (e.g., $0 - 1 - 5 - 0$) in Figure 5.2(a) and of a self-loop (site 1) in Figure 5.2(b).

5.1.3 Mobility Pattern with IFR Inter-meeting Time

As mentioned in Section 2.2, some MANETs may have ‘one-way’ feature, i.e., while there is a (directed) path connecting site i to site j , a (directed) path from site j to i may not exist. Reasons for this ‘one-way’ feature are not only from geographical constraints (e.g., one-way street or bus route) [16, 101], but also from social behavior. For example, in a park with several attractions/sites (e.g., Magic Kingdom park at Walt Disney World), although there are likely to be two-way connections between any two sites in the park, visitors usually take ‘streamlined’ path (tour-like) over multiple sites one by one, often bypassing less popular sites for efficiency. We first show a class of stochastic mobility patterns that formalize this ‘tour’-like path, leading to IFR inter-meeting time.

To proceed, we need the following definitions:

Definition 14 [63] *A nonnegative $N \times N$ matrix $R = \{r_{ij}\}$ ($N \geq 2$) is called totally positive of order 2, denoted by $R \in \text{TP}_2$, if determinants of all 2×2 submatrices of*

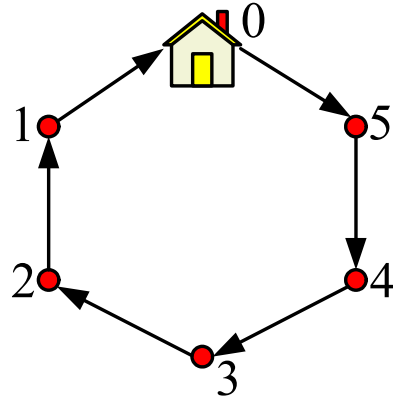
$R, \begin{bmatrix} r_{ij} & r_{ij'} \\ r_{i'j} & r_{i'j'} \end{bmatrix}$, where $i < i'$ and $j < j'$ ($i, j, i', j' \in [1, N]$), are nonnegative.

We define a class of models $\mathcal{M}_{C3}$ as follows:

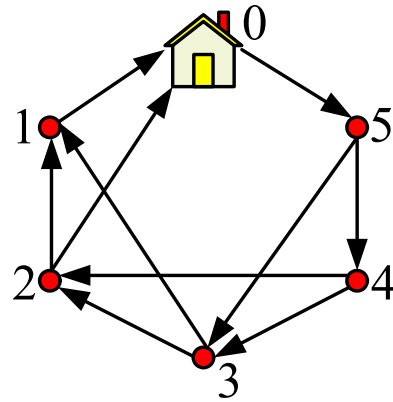
Definition 15 *The class of mobility models $\mathcal{M}_{C3}$ are random walks on weakly connected digraph $\mathcal{G}$ satisfying $p_{0N} = 1$ and either one of the following:*

- **Traffic-circle condition:** *when mobile node C is at site $i \in \mathcal{V} - \{0\}$, it chooses one of sites in $\{0, 1, \dots, i - 1\}$ randomly and uniformly, and then jumps to it.*
- **Total-positivity condition:** *$Q \in \text{TP}_2$, where Q is obtained by replacing the first row of matrix P with $[1, 0, \dots, 0]$.*

Remark 16 The condition $p_{0N} = 1$ sets a primary direction for the mobility patterns in $\mathcal{M}_{C3}$. In particular, define a directed route $\mathcal{R}$ starting at site 0. Since $p_{0N} = 1$, the route $\mathcal{R}$ visits sites $N, N - 1, \dots, 1$ sequentially, often skips some sites in the middle, and finally returns to site 0. In this ‘traffic-circle’ condition, mobile nodes following $\mathcal{R}$ generally take a tour and never go backward between two adjacent visits to the site 0, but they can take short-cuts. As will shown later in the proof of Theorem 9, this ‘traffic-circle’ condition is a special case of the total-positivity condition. However, we list the former separately, since the characteristics of mobility patterns in this case is more intuitive.



(a) Periodic walk



(b) Tour walk

Figure 5.3: Examples of mobility patterns belonging to $\mathcal{M}_{C3}$ in Definition 15.

Figure 5.3 shows two examples of mobility patterns satisfying the total-positivity condition in Definition 15. Specifically, the Q matrix of mobility patterns in Figure 5.3(b) is:

$$Q^{(b)} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ p_2 & 1-p_2 & 0 & 0 & 0 & 0 \\ 0 & p_3 & 1-p_3 & 0 & 0 & 0 \\ 0 & 0 & p_4 & 1-p_4 & 0 & 0 \\ 0 & 0 & 0 & p_5 & 1-p_5 & 0 \end{bmatrix}, \quad (5.5)$$

where $0 < p_i < 1$ ($i = 2, \dots, 5$). The Q matrix of mobility patterns in Figure 5.3(a) equals to $Q^{(b)}$ with $p_i = 0$. It follows that the matrix $Q^{(b)}$ is TP_2 for all choices of $p_i \in [0, 1]$ (see Definition 14).

We now have the following theorem:

Theorem 9 *Under the class of mobility models $\mathcal{M}_{C3}$, the inter-meeting time T_I has increasing failure rate.* $\square$

Proof. See Appendix. $\square$

In fact, mobility patterns in Figure 5.3(a) and (b) as well as those satisfying the ‘traffic-circle’ condition in Definition 15 have a common ‘self-avoidance’ property, i.e., mobile nodes following them never go backward to the sites it has already visited between two consecutive visits to site 0. However, $\mathcal{M}_{C3}$ (specifically, TP_2) also includes mobility patterns that are not self-avoidance type. A simple example can be seen by slightly modifying the tour walk in Figure 5.3 such that when mobile node C is at site 1, it can choose to jump to site 0 with probability $p_1 \in (0, 1)$, or

stay at site 1 with probability $1 - p_1$. After this change, the second row of $Q^{(b)}$ in (5.5) becomes $[p_1, 1 - p_1, 0, \dots, 0]$ and all other rows remain the same. As long as p_1 and p_2 are chosen such that $p_1 \geq p_2$, the modified $Q^{(b)}$ is still TP_2 , thereby giving IFR inter-meeting time from Theorem 9. In what follows, we will present an even more general class of mobility models $\mathcal{M}'_{C3}$, which includes $\mathcal{M}_{C3}$ as a subset and allow ‘going backwards’ as long as its tendency to go backwards is getting weaker. To proceed, we need the following definitions:

Definition 16 [63] *For any two nonnegative vectors $\vec{x} = [x_1, x_2, \dots, x_n]$ and $\vec{y} = [y_1, y_2, \dots, y_n]$, we write $\vec{x} \leq_{st} \vec{y}$ if $\sum_{i=1}^k x_i \geq \sum_{i=1}^k y_i$ for any $1 \leq k \leq n - 1$ and $\sum_{i=1}^n x_i = \sum_{i=1}^n y_i$.*

Definition 17 [63] *A matrix $R = \{r_{ij}\}$ ($i, j \in \{1, \dots, n\}$) is stochastically monotone if*

$$\vec{r}_1 \leq_{st} \vec{r}_2 \leq_{st} \dots \leq_{st} \vec{r}_n, \quad (5.6)$$

where $\vec{r}_k = [r_{k1}, r_{k2}, \dots, r_{kn}]$ ($k = 1, 2, \dots, N$).

Any matrix $R \in \text{TP}_2$ is stochastically monotone [63]. For example, the matrix $Q^{(b)}$ in (5.5) is TP_2 , hence is stochastically monotone.

Consider route $\mathcal{R}$ as in Remark 16, i.e., $\mathcal{R}$ starts at site 0, goes over sites $N, N - 1, \dots, 1$ sequentially and occasionally skips sites in the middle, and ends at site 0. Consider a stochastic monotone Q and a mobile node at site i ($i \in \{N, N - 1, \dots, 1\}$), occasionally going backward with probability $p_i^{(back)} = \sum_{j=i}^N p_{ij}$ on $\mathcal{R}$. From Definitions 16 and 17, $\sum_{j=i}^N p_{ij} \leq \sum_{j=i}^N p_{(i+1)j}$. In other words, $p_i^{(back)} \leq p_j^{(back)}$ for any $i < j$. Since site i is more ‘forward’ on $\mathcal{R}$ than site j when $i < j$, under

the stochastic monotone matrix Q , the mobile node C 's tendency to go backward becomes weaker and weaker as it proceeds on $\mathcal{R}$.

We below show that if the Q matrix in Definition 15 is stochastically monotone but not necessarily TP_2 (thus a weaker condition), then the inter-meeting time T_I is not necessarily IFR, but NBU.

Now we define a class of models $\mathcal{M}'_{C3}$ as follows:

Definition 18 *The class of mobility models $\mathcal{M}'_{C3}$ are random walks on weakly connected digraph $\mathcal{G}$ satisfying $p_{0N} = 1$ and either one of the following:*

- **Progressive-route condition:** *when mobile node C is at site $i \in \mathcal{V} - \{0\}$, it chooses one of sites in $0, 1, \dots, i - 1$ randomly with probability $(1 - (N - i + 1)p_i)/i$, or one of sites in $i, i + 1, \dots, N$ randomly with probability p_i , and then jump to it, where p_i is any number satisfying $p_i \in [0, 1/(N - i + 1))$ and*

$$Np_1 \leq (N - 1)p_2 \leq \dots \leq 2p_{N-1} \leq p_N. \quad (5.7)$$

- **Stochastic-monotone condition:** *Q is stochastically monotone, where Q is from replacing the first row of matrix P by $[1, 0, \dots, 0]$.*

Note that the ‘progressive-route’ condition in Definition 18 allows some probability of going backwards and as a special case, it becomes the traffic-circle condition in Definition 15 when $p_i = 0$ for all $i \in [1, N]$. Note however that it is not a special case of the total-positivity condition in Definition 15. Still, as will be shown later in the proof of Theorem 10, the progressive-route condition is a special case of the stochastic-monotone condition. We then have the following theorem:

Theorem 10 *Under $\mathcal{M}'_{C3}$, $T_I \in \text{NBU}$.* $\square$

Proof. See Appendix. $\square$

Remark 17 Since the class of mobility models $\mathcal{M}'_{C3}$ is a superset of $\mathcal{M}_{C3}$, Theorem 10 implies that under $\mathcal{M}_{C3}$, $T_I \in \text{NBU}$.

In Section 5.1.1 to 5.1.3, we have proposed three classes of stochastic mobility models $\mathcal{M}_{C1}$, $\mathcal{M}_{C2}$, and $\mathcal{M}_{C3}$, each of which abstracts out common characteristics such as ‘small world’ property for $\mathcal{M}_{C1}$, time reversibility’ for $\mathcal{M}_{C2}$, or ‘traffic-circle’ property for $\mathcal{M}_{C3}$. In addition, each of these classes of models possesses very strong aging property in that the failure rate is monotone for all $t > 0$. There are cases whose failure rates are non-monotone and hence clearly the union of the aforementioned four classes cannot cover the entire spectrum of all possible mobility patterns. Still, we point out that each of these classes is general enough. For instance, the class of mobility models $\mathcal{M}_{C2}$ that include all random walks on undirected graphs, share the property of time-reversibility – which by no means is a small class, thereby leading to DFR inter-meeting time.

5.1.4 Aging Properties in Existing Models

In this section, we consider the inter-meeting times between a pair of independent mobile nodes following RWP models [55, 20] and RW mobility models [26], which are widely used in protocol design and performance analysis/comparison in MANET. The relative mobility of a pair of mobile nodes, even when they both follow simple mobility patterns, can become unwieldy for rigorous stochastic analysis.

Nevertheless, based on our analytical results in Sections 5.1.1–5.1.3, we will discuss the similar features between the relative mobility under RWP/RW model and one of the four classes of stochastic mobility models (e.g., $\mathcal{M}_{C2}$ or $\mathcal{M}_{C3}$) to establish the aging property of the corresponding inter-meeting times.

Under RWP model, a mobile node first selects a random waypoint as its destination uniformly in the domain, and then moves to its destination. After it reaches the destination, it selects a new destination, and repeat the whole procedure independently. Consider two independent mobile nodes A and B following RWP model on the same domain. The inter-meeting time between A and B is equivalent to the inter-meeting time of mobile node C satisfying $C(t) = A(t) - B(t)$ to a static reference site 0.³ The farther nodes A and B are away from each other, the higher the chance that they are taking opposite directions (away from the center) at the current step. Thus, in the next step, they are likely to choose their destinations in each other's side and cross each other's path. In other words, the farther node C is away from site 0, the stronger tendency it has to visit site 0. This is to some extent similar to the *traffic circle* property of $\mathcal{M}_{C3}$, and we expect that the inter-meeting time under RWP model would have IFR.

Under RW model, a node chooses a direction randomly and uniformly from $[0, 2\pi)$, and then moves on a straight line for a time period S (random variable) with constant speed. In this case, the *uniform* stationary distribution for both node position (i.e., $\pi_i = \pi_j$, where i, j are different positions on the domain) and node direction (i.e., $p_{ij} = p_{ji}$) [76, 37, 13] readily leads to the time reversibility property (i.e., $\pi_i p_{ij} = \pi_j p_{ji}$). This property is preserved under two independent mobile nodes

³ $A(t), B(t), C(t)$ are the positions of nodes A, B, C at time t .

following RW model. Thus, we expect that the inter-meeting time under RW model would have DFR.

Figure 5.4 shows the survival function at time t (i.e., $\bar{F}_t(\tau)$) in (a) and failure rate function $r(t)$ in (b) of the inter-meeting time under RWP and RW models. Under each model, 10^5 inter-meeting time samples between a pair of i.i.d. mobile nodes with constant speed 1m/s on a $(1000m)^2$ domain are collected. The communication range is 50m. The time period S in RW model is exponentially distributed with average 100 seconds. Under both RWP and RW models, the tail of inter-meeting time cdf is known to decay exponentially [18, 58, 87, 40], which implies that the failure rate of the inter-meeting time eventually converges to a constant, or equivalently, as τ increases, $\bar{F}_t(\tau)$ converges to the same constant under different t (See also Section 5.1.1). This can be seen in the inset in Figure 5.4(a). To better capture the aging property in smaller time scale, we plot $\bar{F}_{10}(\tau)$ and $r(t)$ for $\tau \in (0, 2000)$, to show the difference between two curves more clearly. As expected, Figures 5.4(a) and (b) show that the inter-meeting time under RWP/RW model has IFR/DFR.

5.2 Performance Comparison under Exponential and Non-exponential Inter-meeting Time

So far we have considered various classes of stochastic mobility patterns that lead to different aging properties. In this section, we show that our results can be applied to compare approaches based on exponential and those based on non-exponential inter-meeting time in MANET study.

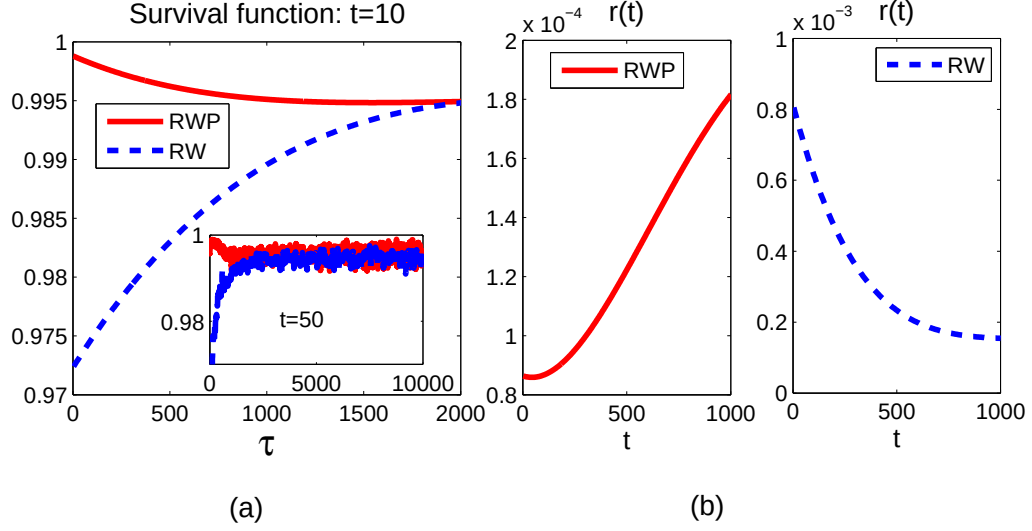


Figure 5.4: Survival function at time t (a) and failure rate function $r(t)$ (b) of the inter-meeting time under RWP and RW models. (a) shows $\bar{F}_t(\tau)$ with $t = 10$ and the case of $t = 50$ is plotted in the inset. (Small fluctuations in the curve caused by the sampling interval effect are eliminated when $t = 10$.)

When T_I has DFR, for larger age (time elapsed since the most recent meeting between a pair of mobile nodes A and B under consideration), the residual lifetime (the remaining time till A and B meet again) of T_I is stochastically larger. In this case, T_I has ‘negative aging’ property. Correspondingly, IFR/CFR inter-meeting time T_I has positive/zero aging property. The stochastically increasing/decreasing property of the remaining contact time is indeed very strong as it requires *monotone* increasing/decreasing failure rate for *all* time t . Sometimes, we only need negative/positive aging property in a weaker sense. For instance, under the class of stochastic mobility patterns $\mathcal{M}'_{C3}$ in Section 5.1.3, the inter-meeting time $T_I \in \text{NBU}$. From Definition 8, the residual lifetime of T_I at age $t > 0$ is stochastically smaller

than T_I itself (at age $t = 0$). Note that this corresponds to the stochastic ordering relationship between the residual lifetime at age t and 0, while for the case of IFR/DFR, the relationship is between the residual lifetime at any age t_1 and t_2 ($t_1, t_2 \geq 0$) with monotone stochastic ordering indexed by age t . In fact, for any positive random variable $X \in \text{NBU}$, the following result proves to be very useful:

Proposition 4 [80] *For a new better than used (NBU) discrete positive random variable X , define its exponential counterpart X_e as a positive random variable satisfying $\mathbb{P}\{X_e > x\} = e^{-x/\mathbb{E}\{X\}}$ ($x \geq 0$). Then, for any convex function $\psi(\cdot)$,*

$$\mathbb{E}\{\psi(X)\} \leq \mathbb{E}\{\psi(X_e)\}, \text{ denoted by } X \leq_{cv} X_e, \quad (5.8)$$

i.e., X is smaller than X_e (exponential random variable with the same mean as X) in convex ordering. The inequality in (5.8) is reversed if X is new worse than used (NWU). $\square$

Considering that IFR/DFR implies NBU/NWU, the result in Proposition 4 can be used to establish a convex ordering relationship between the IFR/DFR intermeeting time T_I and its exponential counterpart. For example, denote T_I under $\mathcal{M}_{C2}, \mathcal{M}_{C3}, \mathcal{M}'_{C3}$ by $T_I^{\mathcal{M}_{C2}}, T_I^{\mathcal{M}_{C3}}, T_I^{\mathcal{M}'_{C3}}$, respectively, and let the corresponding exponential counterparts be $T_e^{\mathcal{M}_{C2}}, T_e^{\mathcal{M}_{C3}}, T_e^{\mathcal{M}'_{C3}}$. Then, from the results in Section 5.1 and (5.8), we have

$$T_I^{\mathcal{M}_{C2}} \geq_{cv} T_e^{\mathcal{M}_{C2}}, \quad (5.9)$$

$$T_I^{\mathcal{M}_{C3}} \leq_{cv} T_e^{\mathcal{M}_{C3}} \quad \text{and} \quad T_I^{\mathcal{M}'_{C3}} \leq_{cv} T_e^{\mathcal{M}'_{C3}}. \quad (5.10)$$

Further, as seen in Section 5.1.4 and following the similar line as above, we have

$$T_I^{(RWP)} \leq_{cv} T_e^{(RWP)} \quad \text{and} \quad T_I^{(RW)} \geq_{cv} T_e^{(RW)}. \quad (5.11)$$

Remark 18 [19] shows a convex ordering relationship indexed by different degrees of correlation in the class of RW type mobility models, in which the inter-meeting time has always DFR according to our study here. In contrast, we provide much more general classes of mobility patterns with various aging properties and a way to compare the non-exponential inter-meeting times with different aging properties with the corresponding exponentially distributed inter-meeting time with the same mean.

How does the convex ordering result in (5.9)–(5.11) impact system performance? Consider the following simple scenario: a set of *i.i.d.* mobile nodes following some given stochastic mobility pattern with certain aging property. Let T be the inter-meeting time between a pair of these mobile nodes. As has been done frequently, suppose that the inter-meeting time is now *assumed* to be exponentially distributed with the same mean $\mathbb{E}\{T\}$, and let T_e denote this exponential random variable. Then, if all nodes are initially located independently according to their stationary distribution, the time to forward/copy a packet from a node A to any other node B upon encounter (link delay) becomes the residual lifetime of the inter-meeting time T between A and B at equilibrium [22, 58] (denoted by T^F), i.e.,

$$\mathbb{P}\{T^F > t\} \triangleq \frac{1}{\mathbb{E}\{T\}} \int_t^\infty \mathbb{P}\{T > s\} ds = \frac{\mathbb{E}\{[T - t]^+\}}{\mathbb{E}\{T\}}.$$

Note that for the exponential random variable T_e , we have

$$\mathbb{P}\{T_e^F > t\} \triangleq \frac{1}{\mathbb{E}\{T_e\}} \int_t^\infty \mathbb{P}\{T_e > s\} ds = \mathbb{P}\{T_e > t\}.$$

Thus, from the convexity of $\psi(x) = [x - t]^+$ in x for any $t \geq 0$ and from (5.9)–(5.11),

we have, for each $t \geq 0$

$$\mathbb{P}\{T^F > t\} \geq \mathbb{P}\{T_e^F > t\} = \mathbb{P}\{T_e > t\}, \quad (5.12)$$

for all mobility patterns with DFR or NWU. Or, equivalently, for any non-decreasing function $\varphi(\cdot)$, we have $\mathbb{E}\{\varphi(T^F)\} \geq \mathbb{E}\{\varphi(T_e)\}$. In other words, in this case, the analysis assuming exponentially distributed inter-meeting time results in *under-estimation* of the real system performance (in the sense of stochastic ordering in the link delay) when the real underlying mobility pattern is any of $\mathcal{M}_{C2}$ or RW models. Similarly, in case of IFR or NBU (e.g., $\mathcal{M}_{C3}, \mathcal{M}'_{C3}$ or RWP models), the inequality in (5.12) is reversed and the analysis assuming exponentially distributed inter-meeting time results in *over-estimation* of the real system performance (in the sense of stochastic ordering in the link delay). We expect that this stochastic comparison result forms an analytic foundation toward the design and comparison of forwarding/routing protocols over mobile nodes with general non-exponential inter-meeting time by analytically comparing its performance with that of the much simpler case – ‘exponentialized’ inter-meeting time.

Lastly, we want to point out that although (5.11) shows that the inter-meeting time T_I under RWP model is smaller than its exponential counterpart in convex ordering, the distribution of T_I is still quite close to an exponential distribution. For example, the survival function under RWP model in Figure 5.4(a) is almost invariant with respect to τ for large τ . Also, Figure 5.4(b) shows that compared to the case of RW model, $r(t)$ under RWP model varies over much smaller range (order of 10^{-4} compared to 10^{-3} in RW case over the same time interval). Thus, we expect that the resulting performance under RWP models will be better than the

case of pure exponential inter-meeting time, but quite close. However, (5.11) also applies to any other mobility patterns (e.g., vehicles' movement on routes, visitors' tour in a park, etc.) under the class of $\mathcal{M}_{C3}$ and $\mathcal{M}'_{C3}$, in which the distribution of inter-meeting time can be much different from exponential distribution, leading to bigger 'gap' in the aforementioned stochastic ordering.⁴

5.3 Application of Aging property in Forwarding

In Section 5.2, we show how the aging property in mobility pattern leads to performance comparison between approaches based on exponential and non-exponential inter-meeting time. In this section, we discuss how the aging property can be *actively exploited* toward better design of forwarding/routing algorithms over mobile nodes.

For simplicity of exposition, here we denote by $Age(A, B)$ the time elapsed since the latest encounter between nodes A and B and by $Res(A, B)$ the remaining time until nodes A and B meet again (residual lifetime). This age information $Age(A, B)$ between two mobile nodes has been utilized in wireless sensor networks [42, 83, 51] to estimate the change in the relative positions of nodes A and B and thus change in the overall network topology caused by nodes' mobility. Similarly, this age information has been partially exploited in MANET forwarding/routing algorithm design [29, 94]. For example, [29] proposes an efficient route discovery strategy from the source node $S = R_0$, through searching for a series of intermediate nodes $R_1, R_2, \dots, R_n$ to the destination node D with the property that for any $i \in [0, n-1]$, $Age(R_{i+1}, D) \leq$

⁴A simple example is the periodic walk in Figure 5.3(a), under which the inter-meeting time cdf is a step function.

$Age(R_i, D)$. [29] then shows that as $Age(A, B)$ increases, the empirical conditional mean of the distance between A and B also increases and finally converges to a constant after certain time $T_{\mathcal{M}}$, which depends on the underlying mobility pattern $\mathcal{M}$ of mobile nodes A and B . [94] present mobility assisted routing algorithms, under which only a small number of copies of a packet are allowed in the network and each copy, if currently resides in node A , can be further relayed to other node B upon encounter only when $Age(B, D) \leq Age(A, D) - t_0$ with some constant parameter $t_0 > 0$ of their choice. This algorithm is shown to perform well under RW mobility models.

Compared to the existing works showing that the smaller $Age(A, B)$ indicates the smaller mean distance between A and B before certain time $T_{\mathcal{M}}$ (from now on, the time scale of our discussion is between time 0 and $T_{\mathcal{M}}$), our results shows that smaller $Age(A, B)$ can lead to either stochastically smaller or larger residual lifetime $Res(A, B)$ when the inter-meeting time has DFR or IFR. For instance, under DFR mobility patterns, we have

$$Age(A, D) \leq Age(B, D) \Rightarrow Res(A, D) \leq_{st} Res(B, D), \quad (5.13)$$

and in this case a mobile node ‘closer’ in distance to the destination D indeed will meet it in shorter time. However, under IFR mobility patterns such as RWP model or $\mathcal{M}_{C3}$ in Section 5.1.3, we have

$$Age(A, D) \leq Age(B, D) \Rightarrow Res(A, D) \geq_{st} Res(B, D), \quad (5.14)$$

which implies that a mobile node that is closer to D in distance will now spend

stochastically longer time to meet the destination.⁵ In other words, while indeed successively finding ‘better’ nodes (in terms of smaller residual lifetime with D) under DFR type mobility patterns, the same algorithm based on age information as above may choose ‘worse’ set of nodes for IFR type mobility patterns. In fact, the simulation results in [94] have shown that while their forwarding algorithm improves the system performance in terms of packet delay under RW mobility model, the same algorithm then leads to worse performance under RWP mobility model. The authors in [94] discussed that the inferior performance under RWP mobility model might be caused by the ‘high mobility’, still assuming that the age information should be utilized in the same way as it was for RW mobility models, and they regarded the age information not so much helpful (due to ‘high mobility’) for RWP mobility models. From our results, however, the true reason for this performance discrepancy under RWP and RW models should be that RWP mobility model has IFR property, while RW mobility model has DFR property, as shown in Figure 5.4. In this regard, our results suggest that existing forwarding algorithms need to be modified to adapt to different aging properties of inter-meeting times, so as to correctly predict the future dynamics (residual lifetime) based on the past events (age information) in an attempt to exploit the non-exponential (memory) structure. Further, we expect that not only the ordering in (5.13) and (5.14) but also the ‘range’ over which the failure rate function $r(t)$ fluctuates, can be utilized to predict how much improvements the forwarding with aging information will bring.

⁵An intuitive explanation is that under IFR mobility patterns, a mobile node A with small $Age(A, D)$ (‘closer’ to D) is most likely taking the direction that is away from D due to its tour (or traffic circle) type mobility pattern and it will take longer to come back and meet D again.

Chapter 6

Conclusion

In Chapter 3, we have proven that one of the fundamental reasons, for the discrepancy on the tail behavior of inter-meeting time between the recent empirical data and the theoretical/simulation results based on most of the current mobility models, is the *finite boundary*. We have also shown that simply removing the boundary can quickly change the inter-meeting time distribution from exponential to power-law by studying a simple random walk in an open space. Further, our guidelines on scaling the size of the domain also help better understand the true role of the boundary. Our theoretical results and findings provide guideline on mobility modeling, performance analysis, and protocol design, to survive the ‘curse’ of the power-law distribution of the inter-meeting time in MANET.

In Chapter 4, we studied the effect of correlation in mobility patterns on the inter-meeting time distribution. We quantified the time scale for transition in inter-meeting time distribution with respect to both domain sizes and correlation in mobility patterns. We then studied the head part of inter-meeting time and proved

that stronger correlation in mobility patterns leads to heavier head of inter-meeting time. Further, we derived invariance property for contact-based metrics such as inter-meeting time, contact time and inter-any-contact time under both Boolean and SINR interference models. Numerical findings support our theoretical analysis and also suggest a convex ordering relationship between inter-meeting time distributions produced by mobility patterns with different degrees of correlation. We expect that our study of stochastic properties of contact-based metrics under general mobility models will provide a first step toward detailed performance analysis of various routing/forwarding algorithms and shed light on better design of network protocols under realistic mobility patterns.

In Chapter 5, by exploring the *memory* structure of contact dynamics, we showed that recently discovered non-exponential inter-contact time can bring tremendous *opportunities* to the study in MANET if correctly exploited. Under four classes of stochastic mobility patterns, we mathematically proved that they produce inter-contact times with constant/decreasing/increasing failure rate as well as new-better-than-used property. We then presented convex ordering relationships between inter-contact times with different aging properties, based on which we compared two approaches using non-exponential and exponential inter-contact time. We also discussed the implication of our results on forwarding/routing algorithm design. We expect that our work in this chapter will form a rigorous foundation toward analysis and design of MANET protocols over non-exponential contacts via stochastic ordering and thus bridge the gap between two parallel bodies of approaches focusing on realism or mathematical simplicity.

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APPENDIX

Appendix A. Proof

In this appendix, we show the proof of Lemma 1 to 2, as well as that of Theorem 8 to 10.

Proof of Lemma 1

In view of Proposition 2 in [69] and $\|P_{k_1}P_{k_2}\cdots P_{k_M}\|_\infty < 1$ for any index set $\{k_1, k_2, \dots, k_M\}$ ($k_i \in \{1, \dots, N\}$, $i \in \{1, \dots, M\}$) and $M < \infty$, the following condition is satisfied by $A(\mathcal{I}, k)$:

$$\lim_{k \rightarrow \infty} A(\mathcal{I}, k) = \mathbf{0}, \text{ for all } \mathcal{I} = \{n_1, n_2, \dots\} \quad (\text{A.1})$$

where $n_l \in \{1, 2, \dots, M\}$ for all $l \in \mathbb{N}$.

Define a discrete-time system whose state at time t is given by a $1 \times N$ vector $\vec{X}(t)$ where $\vec{X}(t) = \vec{X}(t-1)P_{n_t} = \cdots = \vec{X}(0)A(\mathcal{I}, t) = \vec{x}_0 A(\mathcal{I}, t)$. Relation (A.1) means that the system is absolutely asymptotic stable (Section 3.1.1 in [96]), which is equivalent to saying that the system is absolutely exponentially stable (Definition 3.1 in [96]), i.e., for any $\vec{x}_0$, there exists constant $c > 0$, $0 < q < 1$ such that for all sufficiently large k ,

$$\|\vec{x}_0 A(\mathcal{I}, k)\| \leq cq^k \|\vec{x}_0\|, \quad (\text{A.2})$$

where $\|\cdot\|$ is any vector norm. Specifically, (A.2) is true for n -norms $\|\cdot\|_n$. $\square$

Proof of Lemma 2

From the discussion right after (4.2), we have for any $n \in \{0, 1, 2, \dots\}$, $r(-1, n, n) = r(1, 0, n)$. Hence,

$$\begin{aligned} R(-1, z, 0) &= \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} r(-1, m, n) z^m 0^{n-m} \\ &= \sum_{m=0}^{\infty} r(-1, m, m) z^m \\ &= \sum_{n=0}^{\infty} r(1, 0, n) z^n = R(1, 0, z). \end{aligned} \tag{A.3}$$

(A.3) is true since $r(-1, m, n) z^m 0^{n-m} = 0$ when $n \neq m$. Similarly, $R(-1, 0, z) = R(1, z, 0)$. Hence, $R(-1, z, 0) = R(1, 0, z)$ and $R(-1, 0, z) = R(1, z, 0)$ hold.

For any $i \in \{-1, 1\}$ and $n \in \{0, 1, 2, \dots\}$, since we always have $0 \leq m \leq n$, we have

$$\sum_{m=0}^n r(i, m, n) = 1.$$

Then,

$$R(i, z, z) = \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} r(i, m, n) z^n = \sum_{n=0}^{\infty} z^n = \frac{1}{1-z},$$

i.e. $R(i, z, z) = 1/(1-z)$ is true.

To show (4.10), since $S(0) = 0$, for $t \geq 1$, if $\mathcal{W}_t = 0$, in t steps, the node is always on the negative side. Hence, its first step X_1 must be -1 , i.e., left. Consequently,

$$\mathbb{P}\{\mathcal{W}_t = 0 | X_0 = 1\} = \mathbb{P}\{\mathcal{W}_t = 0, X_1 = -1 | X_0 = 1\}.$$

From $r(1, 0, t) = \mathbb{P}\{\mathcal{W}_t = 0 | X_0 = 1\}$,

$$\begin{aligned} r(1, 0, t) &= \mathbb{P}\{\mathcal{W}_t = 0 | X_0 = 1, X_1 = -1\} \mathbb{P}\{X_1 = -1 | X_0 = 1\} \\ &= \mathbb{P}\{\mathcal{W}_t = 0 | X_0 = 1, X_1 = -1\} \cdot q. \end{aligned}$$

Similarly

$$r(1, t, t) = \mathbb{P}\{\mathcal{W}_t = t | X_0 = 1, X_1 = 1\} \cdot p.$$

By symmetry, we have

$$\mathbb{P}\{\mathcal{W}_t = 0 | X_0 = 1, X_1 = -1\} = \mathbb{P}\{\mathcal{W}_t = t | X_0 = 1, X_1 = 1\}.$$

Since $r(-1, t, t) = r(1, 0, t)$, we have for $t \geq 1$,

$$p \cdot r(-1, t, t) = q \cdot r(1, t, t). \quad (\text{A.4})$$

Note that when $t = 0$, $r(-1, t, t) = 1$. Then, from (A.3),

$$R(-1, z, 0) - 1 = \sum_{m=1}^{\infty} r(-1, m, m) z^m. \quad (\text{A.5})$$

Similarly, from $r(1, t, t) = 1$, we have

$$R(1, z, 0) - 1 = \sum_{m=1}^{\infty} r(1, m, m) z^m. \quad (\text{A.6})$$

From (A.4), (A.5) and (A.6),

$$p [R(-1, z, 0) - 1] = q [R(1, z, 0) - 1].$$

Lastly, from $p = (1 + \rho)/2$ and $q = (1 - \rho)/2$, (4.10) follows. $\square$

Proof of Theorem 8

From Section 2.4, we only need to show that $\bar{F}_t(\tau)$ in (2.10) does not depend on t . Since $\bar{F}_t(\tau) = \bar{F}_t(1)\bar{F}_{t+1}(1) \cdots \bar{F}_{t+\tau-1}(1)$, it suffices to show that $\bar{F}_t(1) = \mathbb{P}\{T_I \geq t + 1 | T_I \geq t\}$ does not depend on t . Observe that

$$\begin{aligned} \mathbb{P}\{T_I \geq t + 1 | T_I \geq t\} &= 1 - \mathbb{P}\{T_I = t | T_I \geq t\} \\ &= 1 - \mathbb{P}\{C(t) = 0 | T_I \geq t\} \\ &= 1 - \sum_{i=1}^N \mathbb{P}\{C(t-1) = i | T_I \geq t\} \cdot p_{i0}. \end{aligned} \quad (\text{A.7})$$

Under the ‘in-home’ condition in Definition 13, $p_{i0} = \gamma$ ($\gamma \in (0, 1)$). Hence, from (A.7), $\mathbb{P}\{T_I \geq t + 1 | T_I \geq t\} = 1 - \gamma$ does not depend on t .

As for the ‘out-home’ condition in Definition 13, first note that when $\mathcal{G}$ is 2-connected, P^* is irreducible. Since all elements of P^* are non-negative, by Perron–Frobenius Theorem [21], $\vec{p}_0^{\text{out}} > 0$ in (5.3) exists. Now, define a matrix Q by replacing the first row of matrix P with $[1, 0, \dots, 0]$ and consider another mobile node B whose mobility pattern is defined by the transition matrix Q and position at time t by $B(t)$. Note that 0 is an absorbing state for node B , hence the probability that B has never visited site 0 during time instances $1, 2, \dots, t - 1$ is simply $\mathbb{P}\{B(t - 1) \neq 0\}$.

If $C(0) = 0$, then at time $t = 1$, C leaves site 0 ($\sum_{k=1}^N p_{0k} = 1$) and arrives at site $i \in [1, N]$ with probability p_{0i} . Suppose at time $t = 1$, node B is located at site i (with probability p_{0i}). Then, the distributions of $B(t)$ and $C(t)$ from $t \geq 1$ until they visit site 0 again are identical. Thus, we have

$$\begin{aligned} \mathbb{P}\{C(t - 1) = i | T_I \geq t\} &= \mathbb{P}\{B(t - 1) = i | B(t - 1) \neq 0\} \\ &= q_i(t - 1) / [1 - q_0(t - 1)], \end{aligned} \quad (\text{A.8})$$

where $q_j(t) = \mathbb{P}\{B(t) = j\}$. Finally, note that $\vec{p}_0^{\text{out}}$ in (5.3) is quasi-stationary distribution [60] in that when $\vec{p}_0^{\text{out}}$ is node B ’s initial distribution, $\frac{q_i(t)}{1 - q_0(t)}$ is invariant with respect to t . Hence, from (A.7) and (A.8), $\mathbb{P}\{T_I \geq t + 1 | T_I \geq t\}$ does not depend on t and this completes the proof. $\square$

Proof of Theorem 9

We will first show that the traffic-circle condition in Definition 15 implies the total-positivity condition, i.e. $Q = \{q_{ij}\} \in \text{TP}_2$. From Definition 14, we only need

to show that

$$q_{ij}q_{i'j'} - q_{ij'}q_{i'j} \geq 0, \quad (\text{A.9})$$

for all $i, j, i', j' \in [0, N]$ satisfying $i < i'$ and $j < j'$. Since $q_{00} = 1$ and $q_{0j} = 0$, (A.9) trivially holds when $i = 0$. When $i > 0$, if $j' \geq i$ or $j \geq i'$, then at least one of $q_{ij'} = 0$ and $q_{i'j} = 0$ is true, thus (A.9) automatically holds. Therefore, we only need to consider the case when $j < j' < i < i'$. However, from the traffic-circle condition, we have $q_{ij} = q_{ij'} = 1/i$ (note that $i > 0$), and $q_{i'j'} = q_{i'j} = 1/i'$. In other words, $q_{ij}q_{i'j'} - q_{ij'}q_{i'j} = 0$ and (A.9) still holds. In consequence, we have shown that $Q \in TP_2$ under traffic-circle condition.

Now, similar to the proof of Theorem 8, consider another mobile node B whose mobility pattern is defined by transition matrix Q . If $C(0) = 0$, then $C(1) = N$ from $p_{0N} = 1$ in Definition 15. Suppose that at time $t = 1$ node B is also at site N . Then, as before, distributions of nodes B and C 's trajectories until they visit site 0 again are identical, and thus T_I has the same distribution as that of the first hitting time T_F of mobile node B from site N to site 0. From Theorem 3.21 in [63], T_F has IFR. This completes the proof. $\square$

Proof of Theorem 10

We will first show that under the progressive-route condition in Definition 18, the $(N+1) \times (N+1)$ matrix Q is stochastically monotone, where $Q = \{q_{ij}\}$ is from replacing the first row of transition matrix P by $[1, 0, \dots, 0]$. Let the row vectors of matrix Q be

$$\vec{q}_i = [q_{i0}, q_{i1}, \dots, q_{iN}], \quad i = 0, 1, \dots, N.$$

From Definition 17, we only need to show that

$$\vec{q}_0 \leq_{st} \vec{q}_1 \leq_{st} \dots \leq_{st} \vec{q}_N. \quad (\text{A.10})$$

Since $\vec{q}_0 = [1, 0, \dots, 0]$, clearly $\vec{q}_0 \leq_{st} \vec{q}_1$ (note that the sum of all entries in $\vec{q}_1$ is 1).

For any $i \in [1, N]$, from the progressive-route condition in Definition 18, we have

$$\sum_{j=1}^k q_{ij} = \begin{cases} (k+1)(1 - (N-i+1)p_i)/i & \text{if } k \in [0, i-1], \\ 1 - (N-k)p_i & \text{if } k \in [i, N]. \end{cases}$$

Therefore, for any $i \in [1, N]$, when $k \in [0, i-1]$,

$$\begin{aligned} \sum_{j=1}^k q_{ij} &= (k+1)(1 - (N-i+1)p_i)/i \\ &\geq (k+1)(1 - (N - (i+1) + 1)p_{i+1})/i \\ &\geq (k+1)(1 - (N - (i+1) + 1)p_{i+1})/(i+1) = \sum_{j=1}^k q_{(i+1)j}, \end{aligned} \quad (\text{A.11})$$

where (A.11) is from (5.7), i.e., $(N-i+1)p_i \leq (N - (i+1) + 1)p_{i+1}$. When $k = i$,

$$\sum_{j=1}^k q_{ij} = 1 - (N-i)p_i \geq 1 - (N-i)p_{i+1} = \sum_{j=1}^k q_{(i+1)j}.$$

Similarly, when $k \in [i, N]$, $\sum_{j=1}^k q_{ij} \geq \sum_{j=1}^k q_{(i+1)j}$ from $p_i \leq p_{i+1}$. Hence, (A.10) holds and the matrix Q is stochastically monotone.

Now, again, similar to the proof of Theorem 9, we can define another mobile node B whose mobility pattern is defined by transition matrix Q , and T_I has the same distribution as that of the first hitting time T_F of mobile node B from site N to the site 0. From Theorem 3.20 in [63], $T_F \in \text{NBU}$ and this completes the proof.

□