

ABSTRACT

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Financial markets in different countries undergo crises at one point in time or another. These crises can have different causes but they could affect other markets due to trade relations and capital mobility. Some crises affect markets in other countries more than what market fundamentals would dictate. We will model this phenomenon, also defined as contagion, using two approaches viz., one-factor model and volatility spillover, and compare these approaches.

**Contagion in Financial Markets:
Two Statistical Approaches**

by

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Two Statistical Approaches

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Harshavardhana K. Rao

To my parents especially my mom who wanted me to be a doctor.

Biography

Harshavardhana Rao was born in 1977 in Chennai, India. He obtained his B.Sc (Statistics) from Loyola College, Chennai, India in 1998 and his M.Stat from North Carolina State University, Raleigh, NC in 2001.

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Part I

Introduction

From time to time a country may undergo a crisis in its financial market. While the crisis may be of its own doing, sometimes market fundamentals indicate a stable economy and still we can observe a fall in the stock market levels. In certain situations, we see crises in one country followed almost immediately by crises in other countries, near and far. Each country's economy is strongly influenced by trade and capital inflow and outflow from other countries. This leads to a mechanism wherein a crisis in one country "flows" into other economies.

How much do trade and capital flows and other variables affect the ability to spread crises from one country to another? A study of these international financial crises leads us to believe that there is a transmission mechanism which depends on trade and exchange rate fundamentals and other macroeconomic variables. Since many times crises occur when they are not expected, or where the effects seem to be greater than expected, we shall analyze this unexpected reaction of markets. This phenomenon is also referred to as **financial contagion** by many economists.

We shall analyze this mechanism using statistical methodology. The paper is organized as follows. In Chapter 1 we look at the different models used to analyze contagion. Chapter 2 charts a brief history of contagion in the literature. In Chapter 3 we study in detail the one-factor model. Section 3.1 delves into the identifiability of the model. In Section 3.2 and Section 3.3, we scrutinize two strategies to handle the one-factor model. Chapter 4 uses a conditionally-heteroscedastic approach to analyzing contagion. In Section 4.1 we analyze the complexities of using a conditionally

heteroscedastic approach to modeling volatility of multivariate data. In Section 4.2, we consider one such approach to analyzing volatility of stock market returns. In Section 5.1 we consider the similarities and differences of the two approaches. In Section 5.2 we investigate the presence of conditional heteroscedasticity in the one-factor model. Finally in Chapter 6, we state our conclusions and propose future work.

Chapter 1

Contagion

Stock markets in different countries tend to exhibit movement in the same direction. This movement leads us to believe that stock markets around the world are strongly dependent on each other. During periods of crisis, this dependency could be magnified, due to market fundamentals such as trade and capital mobility. This phenomenon results in crises in countries where we don't expect one, or where the effects of crises seem to be stronger than one would expect. This effect is referred to by many economists as financial contagion and is the focus of many recent studies.

According to a study by Pericoli and Sbracia (2003), contagion has quite a few definitions. We will take a detailed look at all the definitions, and then build statistical models based on those definitions.

The basic premise for contagion as laid out in definition 1 by Pericoli and Sbracia is predicated on a significant increase in probability of crisis in one country conditional

on the occurrence of a crisis in another country. A crisis can be of many types, viz. default by the country on its debt, devaluation of currency, a significant drop in the country's stock market, etc. This definition, however, does not lead to an effective statistical model, since we would need to define a crisis, describe contagion according to this crisis, and then measure it effectively.

One way to look at periods of crises as suggested by Pericolli and Sbracia is to consider the volatility of assets during a period of crisis. During times of crises, we can expect volatility to be at a higher level than during a calm period. This idea leads us to two other definitions for contagion.

- Contagion is a significant increase in co-movements of prices and quantities across markets, conditional on a crisis occurring in one market or group of markets.
- Contagion occurs when volatility “spills over” from the crisis country to the financial markets of other countries.

The above definitions are based on volatility in a loose sense. The first definition actually looks at the covariance or to be more specific the correlation and variances of returns of some assets. A significant increase in the correlation and variances would be a sign of contagion. The second definition considers the effect of an increase in volatility in the crisis country on the volatility levels of the second country. In other words, we would like to know if higher volatility levels in the crisis country leads to

increased volatility in the second country.

We can consider statistical models based on these definitions, since we can view the stock markets of these countries as the base assets. We can measure levels and volatility of the stock market based on the indices of the respective countries.

In Chapter 3, we suggest a likelihood-based technique to analyze the model as suggested by the first definition and in Chapter 4 we look at a model, based on the second definition of contagion, which uses GARCH, a methodology used to model non-constant variances, to describe volatility levels. In Chapter 5 we compare the properties of the two models to infer whether they are testing for contagion in the same manner.

Chapter 2

Previous Work

The word contagion takes its roots from medical literature where it is used to describe the spread of diseases by direct or indirect contact. It is aptly used to describe the spread of financial crises from one country to another, with or without economic fundamentals dictating the possibility of such a spread.

The concept of financial contagion in the literature is equivocal, i.e., there are multiple definitions of the term. The statistical tools used to analyze the presence of contagion depend on the definition and hence are subject to debate. In recent years there has been an effort to classify the different statistical models based on the definitions, and hence study the properties of these models. However very little has been said about the difference between these definitions and what they bring to the models.

Pericolli and Sbracia (2003) and Dungey, Fry, González-Hermosillo, and Martin

(2003) have conducted extensive studies of the literature in this field. They detail the spread of the concept of financial contagion, its influences and the methodology used to model this phenomenon. Pericolli and Sbracia in particular consider different definitions of contagion in financial literature and define statistical models based on these definitions of contagion. These definitions help us to chart the growth of contagion in the literature.

Dungey et al. studied the growth of financial contagion. They look at in particular the one-factor model charting the growth of the statistical analyses based on this model. They tie in the different analyses based on this methodology. However they do not tie in the class of models based on volatility spillovers with the models based on the one-factor approach.

The term “contagion” in financial literature started to emerge around the late 80’s and early 90’s. The stock market crash of 1987 was the starting point of studies exploring the spread of the crises from the US to other countries. King and Wadhwani (1990) and Engle, Ito, and Lin (1990) look at the concept of crises spreading into other markets, and modeled this phenomenon. They considered the spread of crises from market to market looking at them from different perspectives. King and Wadhwani looked at the stock market crisis of 1987, and how it spread through different financial markets. Their approach was based on market fundamentals, using economic theory to model this episode of contagion. They consider a one-factor model approach to model contagion.

Engle et al. modeled intra-day volatility of exchange rates, testing for the existence of volatility spillovers from other exchange rates against only the presence of autocorrelation. They described the existence of volatility spillovers as meteor showers and the presence of autocorrelation only as heat waves. They describe this event using a GARCH model for the volatility of the exchange rates, and use the exchange rates from other markets to investigate whether they influence current exchange rates.

These two papers emerged as the seminal papers in dealing with how crises spread from one financial market to another. King and Wadhwani's paper appears to lay the ground work for the common factor model. This apparently led to different approaches to analyze this model, as indicated by the work of Forbes and Rigobon and Corsetti, Pericoli, and Sbracia.

Engle et al. (1990)'s paper, on the other hand, is the foundation for the spillover approach. His work led to many papers testing for factors that affect volatility. Edwards looked at the spread of interest rate volatility from Mexico to Argentina and Chile using a spillover model. He modeled interest rates with a specific country factor and defined contagion to be any unexplained residual terms.

There are a couple of other approaches to defining and modeling contagion. Pericoli and Sbracia also define contagion based on changing regimes.

Another interesting development was the use of SWARCH (Switching ARCH) to model periods of different volatility. Edwards and Susmel analyzed the interest rates of five countries to explore the grouping of high interest rate volatility in different

countries during the same period.

Many other analyses of the presence of contagion in financial markets have been carried out, based on co-movements or excess volatility. Pericoli and Sbracia (2003) and Dungey et al. (2003) are excellent sources of techniques pursued in these areas.

Another good reference on contagion, its related transmission mechanisms and techniques to prevent contagion is a paper by Dornbusch, Park, and Claessens (2000). They study causes of contagion and evaluate approaches to reducing the risk of contagion.

Part II

Alternative Statistical Analyses

Chapter 3

Structural Change in a Common-Factor Model

The first definition of contagion is based on excess co-movements of stock market returns. Contagion is defined as follows

“Contagion is a significant increase in co-movements of prices and quantities across markets, conditional on a crisis occurring in one market or a group of markets.”

The one-factor model was constructed with this definition of contagion in mind. We will use the one-factor model to test for the presence of contagion using two different techniques for estimation, viz., Generalized Method of Moments and Maximum Likelihood.

3.1 The model

The general one-factor model as suggested by Corsetti et al. (2003) is

$$\begin{aligned} r_1 &= \alpha_1 + \beta_1 \times F + \epsilon_1 \\ r_2 &= \alpha_2 + \beta_2 \times F + \epsilon_2 \end{aligned} \tag{3.1}$$

In this model we have the two returns r_1 and r_2 which could be the returns on two individual stock markets. Also, α_1 and α_2 are the intercepts and F is the common factor driving both markets. We could interpret F as the global market and the ϵ 's are the shocks that drive the individual countries' stock market returns.

From these expressions we can understand that the returns from the two markets are related. This would imply that an increase in correlation is not necessarily proof of presence of contagion. This is because of the fact that the common factor could be the entity increasing the correlation. During a crisis period, an increase in co-movement is merely an indication of inter-dependence rather since both the markets depend on F .

A specific shock to a country results in an increase in the volatility of the returns on the stock market. During a time of crisis, we expect the volatility of the returns of the stock market in the crisis country to increase. We shall assume that it increases by a factor of $(1 + \delta)$, i.e. if C denotes the period in which country 1 has a crisis, then

$$\text{Var}(r_1|C) = (1 + \delta)\text{Var}(r_1)$$

Thus δ represents the size of the crisis. This does not necessarily imply the

presence of contagion, since F is common to both r_1 and r_2 and so an increase in correlation could be explained by the presence of the common factor term.

One way to model contagion is to introduce a structural breakdown in the model. In this case, we could model it by an increase in the value of parameter β_1 . We will assume that, if contagion occurs, we will have β_1 increase by a factor of $(1 + \eta)$.

Our test case will be the Hong-Kong and Singapore market indices, viz., the Hang Seng and the Straits Times. On October 20th, 1997, the Hong-Kong stock market saw a significant drop in its levels, which affected many other markets around the world. We shall look at the Singapore market in greater detail to study the relationship between the two markets. Figure 3.1 shows the movements of the returns of the stock markets in the two markets prior to and after the crisis of October 1997. These returns were calculated as follows. The stock market levels (daily) were multiplied by the exchange rate of that day. The returns were obtained as the difference of the log levels of the current and one-day old values. The stock market levels were obtained from Yahoo! Finance and the exchange rates were obtained from Pacific Exchange Rate Service. The purpose of the conversion from local currency to dollar values was to help compare and to make the perspective that of a US investor.

Figure 3.1 displays the returns of the Hong Kong stock market index Hang Seng in bold and the Singapore stock market index Strait Times in a lighter shade during 1997. The vertical black line is the date of the crisis October 20th, 1997. We can see how the Singapore market index, the Strait Times, follows the Hong Kong market

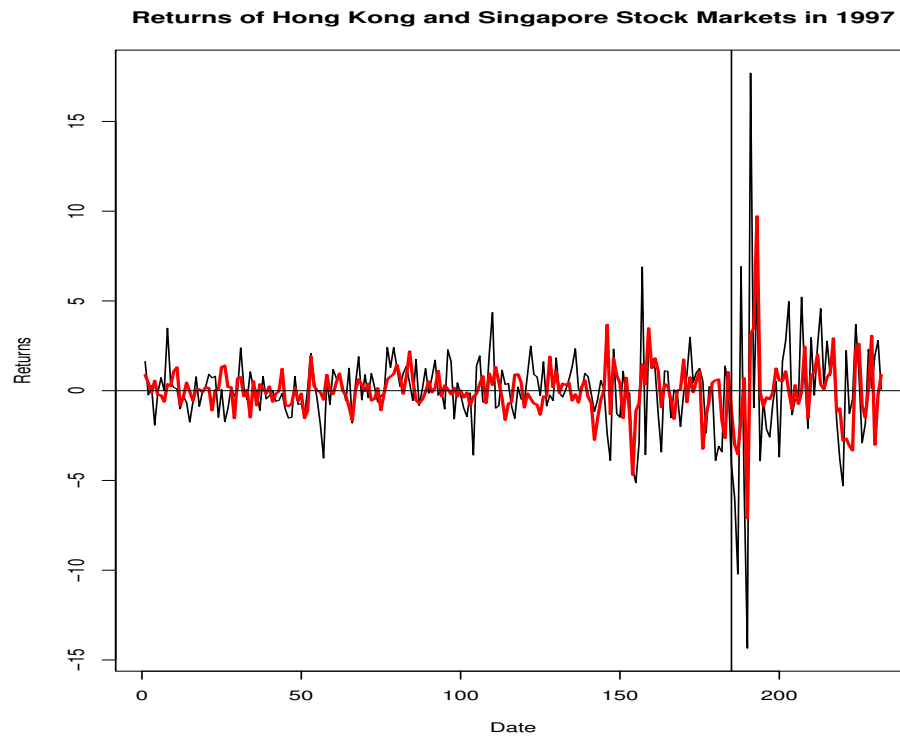


Figure 3.1: Daily Returns of the Hong Kong and Singapore stock market indices, Hang Seng and Straits Times : Red - Singapore, Black - Hong Kong, the black vertical line indicating the date of the crisis.(Source: Yahoo! Finance)

index during the three months following the crisis. We will use this data set in all the modeling that we do.

3.1.1 A closer look at the One-Factor Model

The one-factor model is driven by the variance-covariance matrix of the log-returns of the two countries in question. We have six pieces of information viz., the variances and the covariances of the returns in the two markets in the pre-crisis and post -crisis periods.

We will define inter-dependence as the lack of structural breakdown of the parameters. This definition indicates that an increase in correlation and/or variances can be explained completely by the presence of the common factor. We will then define contagion to be the existence of a structural breakdown viz., the parameter β_1 increases by a factor of $(1 + \eta)$. We can hence test for the presence of contagion by using the following hypothesis $H_0 : \eta = 0$ v/s $H_a : \eta > 0$. The model basically has six parameters of interest under the alternative hypothesis of $H_a : \eta > 0$. The key point is that a structural breakdown during the crisis period leading to a positive η implies that one of the causes of an increase in correlation is contagion.

Another reason for contagion in the one-factor model could be the presence of a country-specific shock which becomes regional during the time of a crisis. This model is also referred to as the two-factor model. We then define the model to be

$$\begin{aligned} r_1 &= \alpha_1 + \beta_1.F + (\nu + \epsilon_1) \\ r_2 &= \alpha_2 + \beta_2.F + \theta.\nu + \epsilon_2 \end{aligned} \tag{3.2}$$

This model under the null hypothesis of $H_0 : \theta = 0$ is the same as the one-factor model where we can look at $\nu + \epsilon_1$ as ϵ_1 . In this case the second common factor is ν and would become common in the presence of contagion. Our test for contagion would involve θ , i.e., testing $H_0 : \theta = 0$ but since we have only six degrees of freedom, the model is over-parametrized having 7 parameters, including σ_ν , the variance of the second common factor. We could argue for the case where $\sigma_\nu = 1$ also, but then we will be giving a lower bound to the variance of $(\nu + \epsilon_1)$. As such this model is not

easily analyzed using maximum likelihood techniques.

Let us make a few simplifying assumptions.

1. $\alpha_1 = \alpha_2 = 0$, i.e. the intercept term is 0.
2. F has a normal distribution with mean 0 and standard deviation 1.
3. ϵ_1 and ϵ_2 have normal distributions with mean 0 and standard deviations σ_1 and σ_2 respectively.
4. F , ϵ_1 and ϵ_2 are independent of each other.

Assumptions 1 and 2, even though simplifying, can be justified since we cannot estimate β_1 and β_2 independent of the variance of F and we can use mean-differenced values to remove the intercepts α_1 and α_2 . Assumption 4 is required to make sure that there is only one factor driving the correlation of the returns of the two markets. We do note that the assumption of normality is over simplifying. Based on these assumptions we can write our model in the following manner

$$\begin{aligned} r_1 &= \beta_1 \cdot F + \epsilon_1 \\ r_2 &= \beta_2 \cdot F + \epsilon_2 \end{aligned} \tag{3.3}$$

during the calm period and

$$\begin{aligned} r_1 &= (\beta_1 \cdot (1 + \eta) \cdot F + \epsilon_1) \cdot \sqrt{(1 + \delta)} \\ r_2 &= \beta_2 \cdot F \cdot \sqrt{(1 + \delta)} + \epsilon_2 \end{aligned} \tag{3.4}$$

during the crisis period, when we expect contagion. If we calculate the variances and the covariances between the two returns during the non-crisis and crisis periods we get

$$\Sigma = \begin{pmatrix} \beta_1^2 + \sigma_1^2 & \beta_1\beta_2 \\ \beta_1\beta_2 & \beta_2^2 + \sigma_2^2 \end{pmatrix}$$

as the variance-covariance terms in the non-crisis period and

$$\Sigma_C = \begin{pmatrix} \beta_1^2(1+\eta)^2(1+\delta) + \sigma_1^2(1+\delta) & \beta_1(1+\eta)\beta_2(1+\delta) \\ \beta_1(1+\eta)\beta_2(1+\delta) & \beta_2^2(1+\delta) + \sigma_2^2 \end{pmatrix}$$

as the variance-covariance terms in the crisis period.

3.1.2 Problems in the Model

The common-factor model used to analyze the returns of two countries has some inherent defects. One of the key shortcomings of this model is its lack of identifiability.

The pre-crisis part of the model has 4 parameters and only the variance-covariance matrix of the returns as our information set. This implies that this model cannot be identified by just the pre-crisis data. When we can analyze the two periods together, we need the variance-covariance matrices in the two periods to be different. If they are not, we are back to the pre-crisis situation and we cannot identify the model. Even in the case where the two variance-covariance matrices are different, we will show that the parameter estimates need not be unique in the case of $H_a : \eta > 0$.

This estimation problem leads to confusion in interpretation of the estimated parameters. It is very difficult to analyze the parameters and their estimates, since

we have two values satisfying all the conditions of the model and yet the difference in magnitude suggests that they cannot have the same interpretation. We shall show why these shortcomings could have serious consequences for interpreting the parameters in this model.

If the components of Σ and Σ_C were defined as follows

$$\Sigma_{(11)} = a \quad \Sigma_{(12)} = b \quad \Sigma_{(22)} = c$$

$$\Sigma_{C(11)} = d \quad \Sigma_{C(12)} = e \quad \Sigma_{C(22)} = f$$

we have the following identities from Appendix A, viz.,

$$\beta_2 = \frac{b}{\beta_1}$$

$$\sigma_1^2 = a - \beta_1^2$$

$$\sigma_2^2 = \frac{c\beta_1^2 - b^2}{\beta_1^2}$$

$$\delta = (f - c) \frac{\beta_1^2}{b^2}$$

$$\eta = \left(\frac{e.b}{b^2 + (f - c)\beta_1^2} \right) - 1$$

Solving the above (see Appendix A) simultaneously, we find β_1 must satisfy the 6 degree polynomial

$$-(f - c)^2 \beta_1^6 + (a(f - c)^2 - 2b^2(f - c))\beta_1^4 + (e^2 b^2 + 2ab^2(f - c) - b^4 - db^2(f - c))\beta_1^2 + ab^4 - db^4 = 0 \quad (3.5)$$

The above equation has only 3 unique roots, since it's a polynomial of degree 3 in β_1^2 , and $\beta_1 \beta_2 = b$. Also, since we expect the variance of the returns in the non-crisis country to also increase, we expect $f > c$ and so we expect the coefficient on β_1^6 to be negative indicating that the function is going to $-\infty$ as β_1 increases. We also expect

the variance of the returns of the crisis country to be greater in the crisis period than in the non-crisis period, leading to $d > a$, which implies that the constant term is also negative. This indicates that the function has either two unique real roots in β_1^2 or no real roots. We simulated values for a, b, c, d, e and f , based on the above conditions, and chose values of the parameter estimates which satisfied the requirements of the parameters. Out of a simulation of 100,000 values, we found 20 cases where the parameter estimates all satisfied the following conditions.

1. $\beta_1^2 > 0$ and $\beta_2^2 > 0$ (which is satisfied automatically)
2. $\sigma_1^2 > 0$ and $\sigma_2^2 > 0$
3. $\delta > 0$ and $\eta > 0$ (since δ is a variance inflation parameter and η is the factor by which β_1 increases)

An example of one such variance-covariance matrix is given below.

$$\Sigma = \begin{pmatrix} 18.77 & 7.51 \\ 7.51 & 27.76 \end{pmatrix}$$

$$\Sigma_C = \begin{pmatrix} 34.08 & 13.48 \\ 13.48 & 31.19 \end{pmatrix}$$

The corresponding parameter estimates, (which make sense in a statistical and economical sense!) are presented in table 3.1

We have two sets of parameter estimates for a given set of variance-covariance matrices. We cannot use the parameter estimate of η to say anything about the size

Parameter Estimates for variance-covariance matrix						
Estimates	β_1	β_2	σ_1	σ_1	δ	η
Solution 1	9.68	5.83	9.09	21.94	0.59	0.13
Solution 2	11.99	4.70	6.78	23.06	0.73	0.04

Table 3.1: Table of Parameter Estimates for the given Σ and Σ values.

of the effect of contagion. Also, we cannot use the parameter estimate of δ which estimates the size of the crisis, (it is the multiplier on the volatility of the returns after crisis). Since we have two estimates of δ there is no clear indication as to what exactly is the increase in volatility. However, since we use maximum likelihood, we can still test for the existence of contagion. The inference from the parameter estimates will not be correct since we have multiple values for each estimate.

The other defect with this model is that it fails to take into consideration the fat-tailed distribution of the returns of stock markets. We will test for this possibility and also introduce fat-tailed distributional assumptions to the model.

3.1.3 Testing for Contagion

In any case we shall look at the analysis done by Corsetti et al. (2003) using Fisher's z-transformation. We will then give another approach based on Maximum Likelihood to test for contagion.

3.2 Fisher's z-transformation

Corsetti et al. (2003) use the one-factor model to test for contagion. Their approach uses Fisher's z-transformation to estimate their test statistic ϕ , which is based on $H_0 : \eta = 0$. The value of ϕ (shown in Appendix I of Corsetti et al. (2003) is)

$$\phi(\lambda_j, \lambda_j^C, \delta, \rho) \equiv \rho \left[\left(\frac{1 + \lambda_j}{1 + \lambda_j^C} \right)^2 \frac{1 + \delta}{1 + \rho^2 \left[(1 + \delta) \frac{1 + \lambda_j}{1 + \lambda_j^C} - 1 \right] (1 + \lambda_j)} \right]^{1/2} \quad (3.6)$$

Here ϕ is the theoretical correlation which is then compared with the actual observed correlation during the crisis period. The observed correlation before the crisis is ρ and j refers to the crisis country. If ρ^C , the correlation during the crisis period is greater than the estimated value of ϕ , we will assume that to be a sign of contagion. In this model, λ is the ratio of the volatilities of the common factor and the error, i.e.,

$$\lambda = \frac{\text{var}(\epsilon_1)}{\text{var}(\beta_1 F)} = \frac{\sigma_1^2}{\beta_1^2} \quad (3.7)$$

We shall assume that the shock affects both the common factor and the country-specific shock equally, which implies that the variance of the common factor and the variance of the crisis country's shocks increase by the same factor of $(1 + \delta)$. Then

$$\lambda^C = \frac{\text{var}(\epsilon_1)}{\text{var}(\beta_1 F)} = \frac{(1 + \delta)\sigma_1^2}{(1 + \delta)\beta_1^2} = \frac{\sigma_1^2}{\beta_1^2} = \lambda \quad (3.8)$$

Under this assumption we get a test for $H_0 : \eta = 0$, based on

$$\phi(\lambda, \lambda, \delta, \rho) = \rho \left[\frac{1 + \delta}{1 + \delta \rho^2 (1 + \lambda)} \right]^{1/2} \quad (3.9)$$

It has to be noted that ϕ is not a true correlation coefficient in the sense of correlation calculated from values of two variables and as such does not have the sampling distribution of a sample correlation coefficient. It is an estimate of what the correlation coefficient should be in the absence of contagion. This is an important assumption since the test for existence of contagion is based on correlation coefficients.

3.2.1 Estimation

Corsetti et al. (2003) do not try to estimate the size of the contagion effect, since they do not have a specific alternative. They only test for the presence of contagion and have the test setup for different types of contagion, viz., a structural breakdown in the factor loading on the crisis country, or country specific shocks becoming regional or common. This test does not depend on the specification of the alternative. In the case of the two-factor model from Equation 3.2 it is clear that the null hypothesis of $H_0 : \eta = 0$ and $H_0 : \theta = 0$ (which is the null hypothesis of the two-factor model) can be both evaluated with the same Fishers-test statistic.

3.2.2 Testing for Contagion

The test for contagion is based on the Fisher's z-transformation

$$z(\hat{\rho}) = \frac{1}{2} \ln \frac{1 + \hat{\rho}}{1 - \hat{\rho}}, \quad (3.10)$$

where $\hat{\rho}$ is the estimated correlation coefficient. The comparison of the two correlations, viz., the estimated and the calculated (under $H_0 : \eta = 0$) is done assuming

independence of the pre-crisis and post-crisis samples. Stuart and Ord (1991) and 1994 show that the difference of the transformed estimated correlations of the two samples is approximately

$$N\left(0, \frac{1}{n_1 - 3} + \frac{1}{n_2 - 3}\right)$$

where n_1 and n_2 are the two sample sizes. Under the assumption that $\lambda = \lambda^C$, Corsetti et al. (2003) derive the threshold value $\bar{\lambda}$ for some fixed confidence level as

$$\bar{\lambda} = \begin{cases} \left\{ \left[\hat{\rho}^{\frac{\hat{\omega}+1}{\hat{\omega}-1}} \right]^2 (1 + \hat{\delta}) - 1 \right\} \frac{1}{\hat{\delta}\hat{\rho}} - 1 & \text{if } \hat{\omega} > 1 \\ +\infty & \text{if } \hat{\omega} \leq 1 \end{cases}$$

where $\hat{\omega} = \exp[2(z(\hat{\rho}) - l\sigma_z)]$, $\sigma_z = \left(\frac{1}{n_1-3} + \frac{1}{n_2-3}\right)^{1/2}$, l is the constant based on the required confidence level and $\hat{\delta}$ is the estimated increase in variance from the pre-crisis to the post-crisis period of the returns. Corsetti et al. note that since $\hat{\delta}$ is used in calculating $\hat{\rho}^C$ and in calculating $\hat{\phi}$, the assumption of independence does not hold. They adjust for this by calculating the significance level based on observed correlations from Monte-Carlo simulations.

Given the value of $\bar{\lambda}$ and λ_j they reject the null hypothesis of interdependence if $\lambda_j > \bar{\lambda}$. Under this testing framework they reject the null hypothesis of interdependence for 5 out of 17 countries whereas Forbes and Rigobon (2002) reject the null in 1 case out of 17.

3.3 Maximum Likelihood: Likelihood Ratio Tests

We start again with our basic one-factor model. We will state our assumptions again for completeness.

1. $\alpha_1 = \alpha_2 = 0$, i.e. the intercept term is 0.
2. F has a normal distribution with mean 0 and standard deviation 1.
3. ϵ_1 and ϵ_2 have normal distributions with mean 0 and standard deviations σ_1 and σ_2 respectively.
4. F , ϵ_1 and ϵ_2 are independent of each other.

Based on these assumptions we can write our model in the following manner.

$$r_1 = \beta_1.F + \epsilon_1$$

$$r_2 = \beta_2.F + \epsilon_2.$$

Since we have F , ϵ_1 and ϵ_2 are normally distributed, it follows that the returns are also normally distributed. We can use a multivariate normal distribution to describe the vector of returns as follows.

$$\mathbf{r} \sim N(\mathbf{0}, \Sigma)$$

where \mathbf{r} is the vector of returns and Σ is given by

$$\begin{pmatrix} \beta_1^2 + \sigma_1^2 & \beta_1\beta_2 \\ \beta_1\beta_2 & \beta_2^2 + \sigma_2^2 \end{pmatrix}$$

during the pre-crisis period. Note that the value of Σ will change in the crisis period and the value will depend on whether we have contagion or not.

Using the above notation we can calculate the correlation during the non-crisis period. We have

$$\begin{aligned}\rho &= \frac{Cov(r_1, r_2)}{\sqrt{Var(r_1).Var(r_2)}} \\ &= \frac{\beta_1\beta_2}{\sqrt{(\beta_1^2 + \sigma_1^2)(\beta_2^2 + \sigma_2^2)}}\end{aligned}$$

In our model, when country 1 is affected by a crisis we have an increase in the variance of r_1 by a factor of δ . If we assume that the factor of increase in variance is equal between the common factor and the country-specific error term then we have

$$Var(r_1|C) = (1 + \delta)Var(r_1) = \beta_1^2(1 + \delta) + \sigma_1^2(1 + \delta)$$

This would imply that the variance of r_2 would increase, since we assume that the variance of the common factor increases,

$$Var(r_2|C) = \beta_2^2(1 + \delta) + \sigma_2^2.$$

And in turn the covariance between r_1 and r_2 will also increase to

$$Cov(r_1, r_2|C) = \beta_1\beta_2(1 + \delta).$$

We have the variance of

$$\Sigma_C = \begin{pmatrix} \beta_1^2(1 + \delta) + \sigma_1^2(1 + \delta) & \beta_1\beta_2(1 + \delta) \\ \beta_1\beta_2(1 + \delta) & \beta_2^2(1 + \delta) + \sigma_2^2 \end{pmatrix}$$

We can hence calculate the correlation between r_1 and r_2 in the period of crisis.

$$\begin{aligned}\rho_C &= \frac{Cov(r_1, r_2|C)}{\sqrt{Var(r_1|C).Var(r_2|C)}} \\ &= \frac{\beta_1\beta_2(1+\delta)}{\sqrt{((\beta_1^2 + \sigma_1^2)(1+\delta))(\beta_2^2(1+\delta) + \sigma_2^2)}}\end{aligned}$$

There are a few points to note.

1. Index returns are typically long-tailed and/or heteroscedastic. This fact implies that normality may not be the best assumption.
2. We will use a working likelihood approach. We can study how this model performs and we will explore the presence of heteroscedasticity.

A working likelihood approach is a good idea when we are not sure about the data generating mechanism which in other words is the true model. Blume (2004) examines the concept of a working likelihood in a normal regression model. Blume details the benefits and drawbacks from this approach. In general, working models can be chosen based on predictive ability, or so that they characterize the statistical evidence correctly. We shall hence begin with the normal regression model and comprehend what the model means.

3.3.1 Contagion according to the model

We can now define contagion, given that we have a model for the correlation after a crisis event that increases volatility beyond what this model suggests. One

way to define contagion would be to use a model that changes either β_1 or β_2 or a model which might introduce more covariance if the country specific error terms were somehow correlated. Due to identifiability restrictions (we have only 6 pieces of information), we cannot differentiate between the different types of contagion that might occur. We restrict our model to the one where β_1 increases to $\beta_1(1 + \eta)$ in the post-crisis period. If $\eta = 0$ that leads us back to the no contagion model. We have our variance-covariance matrix as

$$\Sigma_C = \begin{pmatrix} \beta_1^2(1 + \eta)(1 + \delta) + \sigma_1^2(1 + \delta) & \beta_1(1 + \eta)\beta_2(1 + \delta) \\ \beta_1(1 + \eta)\beta_2(1 + \delta) & \beta_2^2(1 + \delta) + \sigma_2^2 \end{pmatrix}$$

Note that we can now assign our null hypothesis to be $H_0 : \eta = 0$ and our alternative to be $H_a : \eta > 0$. When H_0 is true we get no contagion since our model reduces to the one of interdependence whereas if H_0 was not true then we have contagion. We can hence calculate the correlation between r_1 and r_2 in the period of crisis under the alternative hypothesis.

$$\begin{aligned} \rho_C &= \frac{Cov(r_1, r_2|C)}{\sqrt{Var(r_1|C) \cdot Var(r_2|C)}} \\ &= \frac{\beta_1(1 + \eta)\beta_2(1 + \delta)}{\sqrt{((\beta_1^2(1 + \eta) + \sigma_1^2)(1 + \delta))(\beta_2^2(1 + \delta) + \sigma_2^2)}} \end{aligned}$$

3.3.2 Likelihood for η

The likelihood-ratio test statistic may be used to test the hypothesis $H_0 : \eta = 0$ v/s $H_a : \eta > 0$. In Corsetti et al. (2003) the authors use a direct correlation testing approach. They use the Fisher's z-transformation to compare the hypothesized correlation with the observed correlation. However they do agree that they violate an

assumption of independent samples by using $\hat{\delta}$ in both the samples. Moreover they do not have a specific alternative to describe contagion and so they have the flexibility of just testing for interdependence.

We will use the likelihood based approach to testing this problem because we can assume that the returns are a random sample of observations. The likelihood is constructed based on the multivariate normal distribution. We have the returns of the two countries during the pre-crisis and the post-crisis periods. The multivariate normal distribution is given by Hinkley, Reid, and Snell (1991)

$$f(r) = \frac{1}{(2\pi)^p} \frac{1}{\sqrt{|\Sigma|}} e^{\{-0.5(r'_t \Sigma^{-1} r_t)\}}$$

where p is the order of the normal distribution. Since we have a bivariate normal distribution we plug $p = 2$.

The likelihood is given by

$$L(\Sigma) = \left(\frac{1}{(2\pi)}\right)^{np} \left(\frac{1}{\sqrt{|\Sigma|}}\right)^n e^{\{-0.5 \sum_t (r'_t \Sigma^{-1} r_t)\}}$$

and so -2 times the log likelihood is

$$-2l(\Sigma) = np(\ln(2\pi)) + n(\ln(|\Sigma|)) + \sum_t (r'_t \Sigma^{-1} r_t)$$

If $\lambda = -2\{l(\Sigma|H_0) - l(\hat{\Sigma}_{MLE})\}$, then λ has a χ^2_1 distribution.

3.3.3 Estimation

The results of the analysis of Hong Kong market returns and Singapore market returns (as indicated in Figure 3.1) are presented below. The data for the Hong Kong

market is based on the Hang Seng Index and the data for the Singapore market is based on the Straits Time index. The returns were calculated as the differences of the logs of consecutive values after being converted from the local currency to dollar values using daily exchange rate data (Source:Pacific Exchange Rate Service).

The statistical analysis was done using R. The values of -2 log likelihood were calculated and minimized to give us the parameter estimates. The estimates under the null hypothesis are presented in Table 3.2.

β_1	β_2	σ_1	σ_2	δ	η
0.7345	0.7333	1.3693	0.7186	2.0844	0

Table 3.2: Parameter estimates under the null hypothesis $H_0 : \eta = 0$.

The value of -2 log likelihood is 1684.288. The estimates under the alternative hypothesis are presented in Table 3.3

β_1	β_2	σ_1	σ_2	δ	η
0.4589	0.9569	1.4845	0.3068	1.7271	1.2763

Table 3.3: Parameter estimates under the alternative hypothesis $H_a : \eta > 0$.

Note that the estimates of β_1 and σ_1 are sign invariant, i.e., we can use the absolute values of these estimates here.

The value of -2 log likelihood in this case is 1681.879 and this implies that the

null hypothesis of interdependence is not rejected. (Since the value of $\lambda = 1684.288 - 1681.879 = 2.409$ and the χ^2 critical value for 1 df at $\alpha = 5\%$ is 3.84). Since we fail to reject H_0 , we fail to reject the null hypothesis of no contagion. There is no evidence of excessive co-movement of stock-market prices in between the stock-markets of Hong Kong and Singapore.

3.3.4 Profile Likelihood

In many likelihood analyses we are interested in only one of the many parameters that need to be estimated. In that case the other parameters are referred to nuisance parameters. Estimating the nuisance parameters is not really necessary as we are not really interested in their estimates. We can use the profile likelihood technique to look at the one parameter of interest.

The profile likelihood technique is based on maximizing the likelihood at each value of the parameter of interest. From Severini (2000), we have $p(y; \theta)$ as the density of y . Suppose $\theta = (\psi, \lambda)$ where λ is the nuisance parameter. The profile likelihood of ψ , given by $L_p(\psi) = L(\hat{\theta}_\psi)$, where $\hat{\theta}_\psi$ denotes the maximum likelihood estimate of θ for a fixed value of ψ , i.e. $\hat{\theta}_\psi$ can be denoted by $(\psi, \hat{\lambda}_\psi)$ where $\hat{\lambda}_\psi$ is the maximum likelihood estimate of λ for a fixed value of ψ . We can then define the likelihood ratio statistic for testing $\psi = \psi_0$ to be

$$W = 2[l_p(\hat{\psi}) - l_p(\psi_0)]$$

where $\hat{\psi}$ is the profile maximum likelihood estimate and $l_p(\psi) = \log L_p(\psi)$ is the

profile log likelihood function.

We are interested in testing $H_0 : \eta = 0$ and hence we are concerned only with η . Therefore, the other parameters viz., $\beta_1, \beta_2, \sigma_1, \sigma_2$ and δ are nuisance parameters in this regard. In this case our likelihood ratio statistic

$$W = 2[l_p(\hat{\eta}) - l_p(\eta_0)]$$

where $\eta_0 = 0$ under $H_0 : \eta = 0$. Figure 3.2 shows us the profile of -2log-likelihood of η .

From the figure we can see that there are multiple minima for -2 log likelihood of η . Since we have two minima, we have two estimates for η , the contagion parameter. The statistical aspect of the issue does not differentiate between the two estimates of η . However the economic issues lead us to one estimate. We expect that contagion will cause an increase in the correlation and hence we expect the value of η to be greater than 0. This leads us to choosing the positive value for our estimate of η .

The profile likelihood estimates and likelihood values are similar to those obtained using straight maximum likelihood estimation techniques. Under $H_0 : \eta = 0$, we observe that the maximized likelihood values for both methods are the same, and so on for $H_a : \eta > 0$. The profile likelihood technique reinforces the fact that there are two estimates for the values of η .

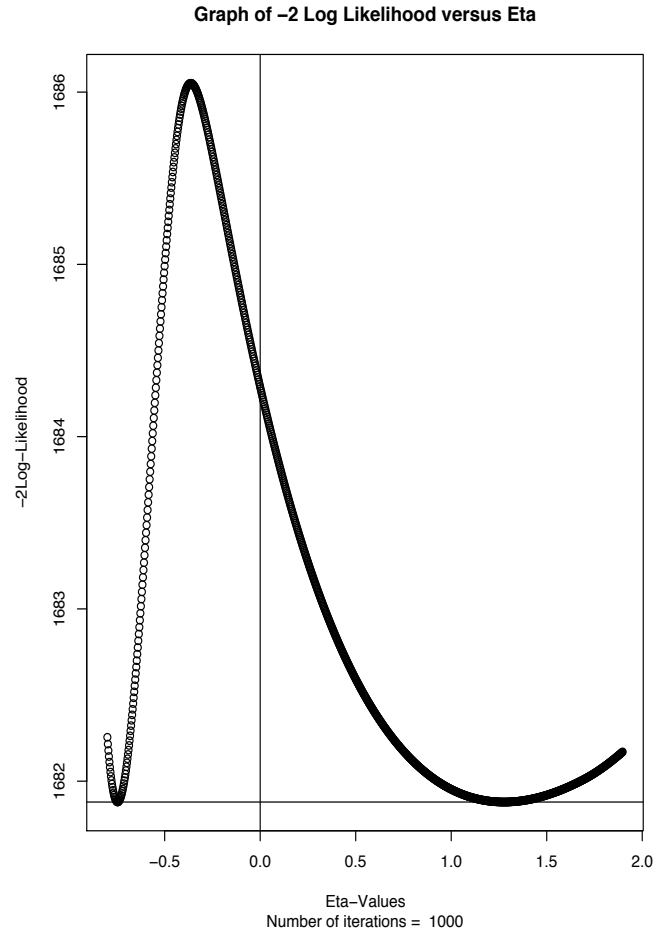


Figure 3.2: Profile of -2 Log Likelihood(η) for the Returns of the two Countries.

3.3.5 Adjusting the Profile Likelihood

The profile likelihood technique is very useful to obtain estimates of parameters using a likelihood approach. If we needed to estimate ψ in (ψ, λ) we calculate the maximum likelihood estimators of λ keeping ψ fixed and we obtain $\hat{\theta}_\psi = (\psi, \hat{\lambda}_\psi)$. We obtain the resulting log profile likelihood $L_p(\psi) = L(\hat{\lambda}_\psi)$.

We have to recognize the fact that $L_p(\psi)$ is not a true likelihood function in the

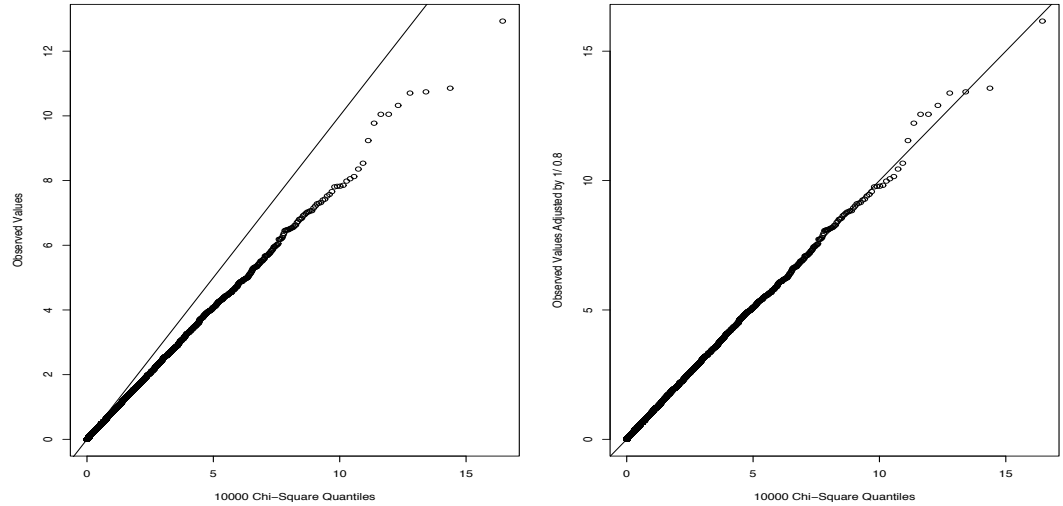


Figure 3.3: Effect of Multiplier on the Q-Q Plot

sense of being based on the density function of a random variable.

We use a simple adjustment technique similar to Bartlett Adjustment (Hinkley et al. 1991; p.249) based on multiplying the likelihood to satisfy certain conditions on the score function of the test statistic. This is an exploratory analysis, and we obtain the multiplier from simulations, using the Q-Q plot to help determine the best value for the multiplier.

We generated values based on the null hypothesis estimates in Table 3.2. The null hypothesis values had $\eta = 0$. The data was then used to calculate the maximum likelihood estimates and the value of the likelihood ratio test. The likelihood ratios

were plotted against $\chi^2(1)$ quantiles to generate a Q-Q plot. The ideal Q-Q plot is a straight line angled at 45° from the origin. Figure 3.3 gives an idea how the Q-Q plots look before and after the adjustment.

To adjust the Q-Q plot we divided the values of the maximized likelihood by values ranging from 0.7 to 1. The value of 0.80 seemed to be the best adjustment factor. We used 10,000 simulated sets of data in calculating the adjustment factor. The value of the adjusted likelihood ratio test is $2.409/0.8=3.011$. This value is still not significant at the 5% level.

We can also use the modified profile likelihood from Severini (2000). This approach is based on approximating a marginal or conditional likelihood function and is computationally intensive when one considers models with a large number of parameters. We did not follow up on this approach.

3.3.6 Some Remarks

The one-factor model is used to test for the presence of contagion. In this regard, some of the key assumptions may not hold viz., normality may not hold. We used a working model to describe the likelihood using the assumption of normality and we tested for contagion. Since we used a working likelihood we explored adjusting the likelihood by using techniques similar to Bartlett's adjustment. The working likelihood does give us a reasonable technique to test for the presence of contagion. However we will need to also consider the possibility that normality may not be the

right assumption. We will look at that possibility in Chapter 5.

Chapter 4

Volatility Spillover

We will use the second definition of contagion to help build the model to test for the presence of contagion in Singapore's stock market returns during the crisis of 1997. The second definition of contagion is as follows.

“Contagion occurs when volatility spills over from the crisis country to the financial markets of other countries.”

Volatility is another expression for the variance of a time series process. One of the standard assumptions for a linear model is that the variance of the error or shocks that drive the process is constant. Another assumption is that the errors or shocks that drive the process are independent. The assumption of constant variance is not satisfied all the time. Models have been designed to take into account increasing/decreasing variances of the errors/shocks that drive the process.

Time series processes in the finance industry have tended to exhibit variances that

change instantly. Most statistical models do not account for variances that change dynamically. A lot of work has been done in the past to model variances mostly in the financial world.

4.1 The model

To construct a model based on this definition of contagion we have to model the volatility of the returns of the two countries. The approach to modeling volatility of stock market returns has been well documented and we will use a conditional heteroscedastic approach to model the volatility of the returns from the Hong Kong and Singapore stock markets. Heteroscedastic variances imply that the variances are not constant. A conditional heteroscedastic approach to modeling variances would suggest that the variances are modeled in a time series fashion, wherein they would depend on either previous observations or previous values of variance. Stock market returns tend to have fat-tailed distributions, and are not normally distributed. However, using a conditional heteroscedastic approach takes into account the fat-tailed nature of these distributions.

The conditional heteroscedastic model comes from Engle (1982) and is widely used to describe fat-tailed observations in time series models. The model uses variances which are conditioned on past values of the series, and hence can effectively model values with non-constant variances. This model is also alluded to by its acronym ARCH which stands for Auto-Regressive Conditional Heteroscedasticity. Bollerslev

in 1986 extended this model to use variances conditional on past observations and also past variances. These models are widely used in finance to model stock market returns and exchange rates. Bollerslev's model is designated as GARCH for Generalized Auto-Regressive Conditional Heteroscedasticity.

We investigated the returns of the stock markets individually in both the countries viz., Hong Kong and Singapore, for GARCH. The variances of the returns from both countries stock markets did show strong heteroscedastic nature.

The GARCH(p, q) model for a value X_t is as follows (Chan 2002; p.105)

$$\begin{aligned} X_t &= \sigma_t \epsilon_t, & \epsilon_t &\sim N(0, 1), \\ \sigma_t^2 &= \alpha_0 + \sum_{i=1}^p \beta_i \sigma_{t-i}^2 + \sum_{j=1}^q \alpha_j X_{t-j}^2 \end{aligned} \quad (4.1)$$

where X_t are the demeaned returns of the stock market indices pre-multiplied by the exchange rates.

We can observe from the manner in which the model for σ_t^2 is specified that the current observation's variance depends on past variances and squares of past observed values. Conditions on α 's and β 's need to be imposed for Equation 4.1 to be well defined. Since finding exact conditions for a general GARCH(p, q) can be tricky, one has to resort to a full case by case study. Nelson (1990) contains a glimpse of the technical aspect of this problem. A GARCH(1,1) model was fit to returns from both the countries. The parameter estimates and their standard errors are given in Table 4.1 and Table 4.2.

GARCH model estimates - Hong Kong			
	α_0	α_1	β_1
Estimates	0.2566	0.2403	0.7096
Standard Errors	(0.1128)	(0.527)	(0.0564)

Table 4.1: Estimates from Univariate GARCH(1,1) fit for Hong Kong Stock Market.

GARCH model estimates - Singapore			
	α_0	α_1	β_1
Estimates	0.0666	0.2522	0.7511
Standard Errors	(0.0282)	(0.0614)	(0.0484)

Table 4.2: Estimates from Univariate GARCH(1,1) fit for Singapore Stock Market.

All the GARCH parameter estimates are significant at least at the 5% level. The Jacque Bera test of normality for the residuals still rejects H_0 : that the residuals are normally distributed. On the other hand the Box-Ljung test rejects the null hypothesis of serial auto-correlation.

The residual plot, however, in Figure 4.1 shows how the model explains some of the non-constant nature of the volatility Hong Kong's stock market.

The residual plots in Figure 4.2 also indicate how the GARCH model rationalizes the fluctuating behavior of the volatility in Singapore's stock market.

The above GARCH models do account for the presence of conditional heteroscedasticity in these data, but to test for volatility spillover we need a mechanism which will demonstrate the influence of one country's volatility on the other. To explore this

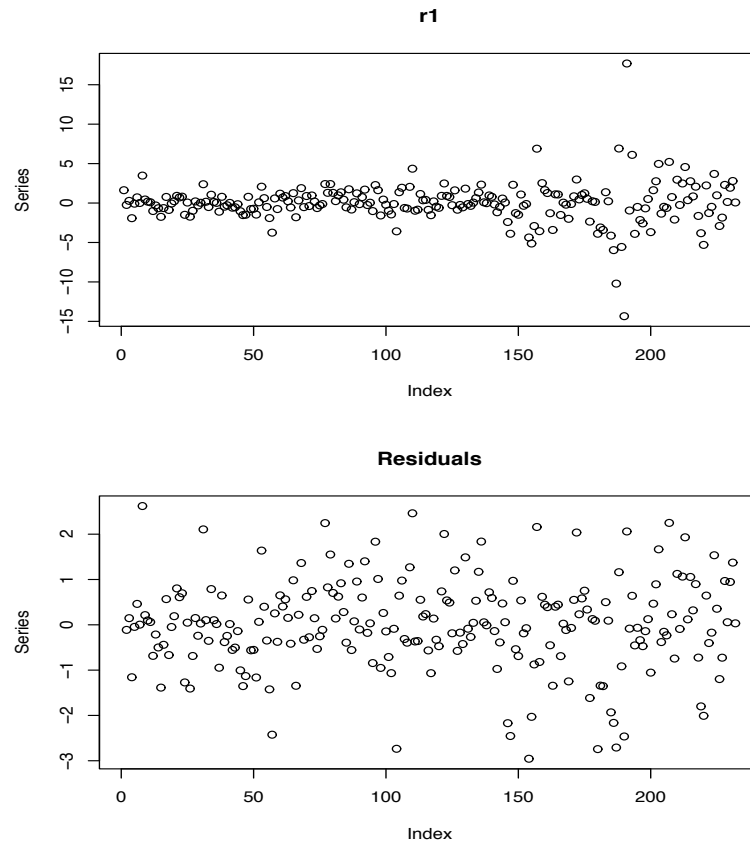


Figure 4.1: Scatter Plot of $r1$ values and residuals from GARCH(1,1) fit

idea we need to use a multivariate GARCH model where there is a flow of volatility from one component to another. Multivariate GARCH is used to model variances or volatilities of multiple stock market returns or exchange rate series among many time series.

The basic form of the multivariate GARCH(p,q) model for a series X_t as given by Chan (2002) is based on the vech notation of a matrix. The vech operator is defined

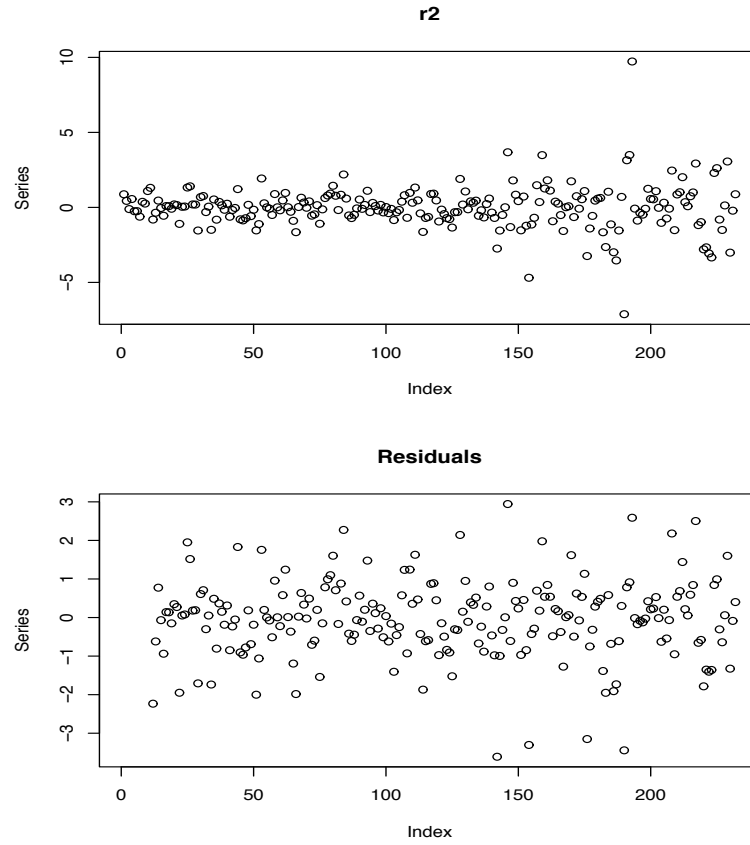


Figure 4.2: Scatter Plot of r_2 values and residuals from GARCH(1,1) fit

as follows.

$$\text{vech} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{22} \\ a_{32} \\ a_{33} \end{pmatrix}$$

The vech operator is applied to symmetric matrices so that we can separate the elements of the matrices.

Let $X_t = H_t^{1/2}Z_t$ where $E(X_t X_t' | \mathcal{F}_{t-1}) = H_t$ and $\mathcal{F}_{t-1} = \sigma(X_{t-1}, X_{t-2}, \dots)$ is the sigma field generated by the past information until time t . Then the GARCH(p, q) structure for the variance gives us

$$\text{vech}(H_t) = A_0 + \sum_{i=1}^p A_i \text{vech}(X_{t-i} X_{t-i}') + \sum_{j=1}^q B_j \text{vech}(H_{t-j})$$

The main requirement of this model is H_t be positive definite. This condition is not satisfied easily with this specification of the model. There is another problem when using the model as specified. The model is not tractable. The number of parameters in the model for the case where we have 2 countries and a GARCH(1,1) model is 21. When the number of countries is 3 the number of parameters to be estimated increases to 108.

We use a factor model to analyze the returns of the stock markets of Hong Kong and Singapore. Pericolli and Sbracia suggested the model based on the definition of contagion using volatility spillovers.

$$R_t = A + B f_t + U_t, \quad U_t \sim (0, \Sigma_t) \quad (4.2)$$

where $R = [r_1, r_2]'$ is the vector of returns, $A = [\alpha_1, \alpha_2]'$ is a vector of constants, B denotes a matrix of factor loadings and $f_t = [f_1, f_2]'$ is a vector of global factors. The vector of country specific shocks $U = [u_1, u_2]'$ has a covariance matrix given by Σ .

To model the variances of the returns Σ , we use the following GARCH model.

$$\Sigma_t = C' C + D' \Sigma_{t-1} D + E' U_{t-1} U_{t-1}' E$$

This GARCH structure is a special case of the model suggested by Engle and Kroner (1995).

$$\Sigma_t = C'C + \sum_{k=1}^K D'_k \Sigma_{t-1} D_k + \sum_{k=1}^K E'_k U_{t-1} U'_{t-1} E_k \quad (4.3)$$

where C , D_k and E_k are matrices of constants. This model is also referred to as the BEKK model and was given by Engle and Kroner, and is a commonly used multivariate GARCH model. The model described by Equation 4.3 has a vech representation and most GARCH models in the vech representation can be rewritten using the BEKK structure, typically with $K = 1$. However, there are a few which cannot be rewritten in the BEKK form. We typically use $K = 1$ to help make the model tractable. The case of $K = 1$ gives the GARCH structure that we use in our model. This model has nice properties, the main one being that with very mild restrictions on the matrices C , D and E , we can guarantee a positive definite covariance matrix for the country specific shocks. The BEKK structure has more tractability. In the case of 2 countries there are 11 parameters to be estimated and in the case of 3, we need to estimate 24 parameters. The case of $K > 1$ is needed when we cannot replicate the multivariate GARCH model with just one set of D 's and E 's.

We can test for volatility spillovers by looking to see how shocks and volatility levels from the crisis country affect volatility levels for the non-crisis country.

4.1.1 Model Specifications

A close examination of the model suggested by Pericolli and Sbracia for testing for the existence of contagion in the volatility spillover aspect leads to a couple of observations.

The first observation is based on the existence of the common factor which is independent of the country specific shocks. These country specific shocks are modeled with a multivariate GARCH structure. This implies that the past shocks and past variances influence the current observations and their variances and covariances. The past values of the common factor do not affect the present values of the country specific shocks since they are independent of each other. This leads to the issue of how the model can “remember” the past shocks but not the past common factor values. One way to correct for this is to model the past observed values of f , the common factor into the variance-covariance model for the country-specific shocks. The other approach is to ignore the existence of the common factor completely and model the variance of the returns of the stock markets of the two countries using multivariate GARCH techniques.

The second approach has definite advantages over the first one, since we do not observe the common factor nor the country-specific shocks individually. This fact leads us to the second observation about the initial model suggested by Pericolli and Sbracia. The complexity of modeling unobserved components of the model, when we consider that we need to have not only estimates of the parameters of the model,

but we need to filter the values of f , the common factor and U 's, the country specific shocks, is very high. Sentana and Fiorentini (2001) look at estimating conditional heteroscedastic factor models, but most such procedures require the existence of shocks which are independent of each other, which when studying for the existence of spillover, may not be possible.

Based on these two observations we consider the following model for modeling the returns of the stock markets.

$$\begin{aligned} R_t &= U_t, & U_t &\sim (0, \Sigma_t) \\ \Sigma_t &= C'C + D'\Sigma_{t-1}D + E'U_{t-1}U_{t-1}'E \end{aligned} \tag{4.4}$$

where C , D and E are matrices of constants, and the only requirement is that C be lower triangular. The key difference between Equations 4.3 and 4.4 is the absence of the common factor from Equation 4.2. The vector of constants can be assumed to be zero, since we use de-meaned returns in the data.

4.2 BEKK structure for volatility

To understand how volatility could spillover from one country to another we will take a closer look at the model of the error variance. The GARCH error model has a BEKK structure and this model was chosen because it requires very few conditions to maintain positive definiteness and the model is tractable. The other advantage of the BEKK model is that it can be estimated using widely available computer software.

We can get parameter estimates for the model from SAS and S-Plus.

The general BEKK structure for GARCH(1,1) is given by Engle and Kroner

$$\Sigma_t = C'C + \sum_{k=1}^K D'_k \Sigma_{t-1} D_k + \sum_{k=1}^K E'_k U_{t-1} U'_{t-1} E_k$$

This general model encompasses most GARCH representations. We will however restrict our attention to the case where $K = 1$, which gives

$$\Sigma_t = C'C + D'\Sigma_{t-1}D + E'U_{t-1}U'_{t-1}E$$

The conditions for identifiability of this model from Engle and Kroner (1995) are as follows.

1. The diagonal elements of C are restricted to be positive.
2. The values of d_{11} and e_{11} are also restricted to be positive.

The diagonal elements of C are restricted to be positive since they emerge as squared terms in $C'C$. The reason for restricting d_{11} to be positive is that we cannot differentiate between d_{11} and $-d_{11}$. The same holds for e_{11} .

After expansion of $D'\Sigma_{t-1}D$ and $E'U_{t-1}U'_{t-1}E$ we get

$$\Sigma_t = C'C + \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} + \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$$

where, from Appendix B we have

$$g_{11}=d_{11}^2\sigma_{11,t-1} + 2d_{11}d_{21}\sigma_{21,t-1} + d_{21}^2\sigma_{22,t-1}$$

$$g_{12}=d_{11}d_{12}\sigma_{11,t-1} + (d_{11}d_{22} + d_{12}d_{21})\sigma_{12,t-1} + d_{21}d_{22}\sigma_{22,t-1}$$

$$g_{21}=g_{12}$$

$$g_{22}=d_{12}^2\sigma_{11,t-1} + 2d_{12}d_{22}\sigma_{21,t-1} + d_{22}^2\sigma_{22,t-1}$$

and

$$h_{11}=e_{11}^2u_{1,t-1}^2 + 2e_{11}e_{21}u_{1,t-1}u_{2,t-1} + e_{21}^2u_{2,t-1}^2$$

$$h_{12}=e_{11}e_{12}u_{1,t-1}^2 + (e_{12}e_{21} + e_{11}e_{22})u_{1,t-1}u_{2,t-1} + e_{21}e_{22}u_{2,t-1}^2$$

$$h_{21}=h_{12}$$

$$h_{22}=e_{12}^2u_{1,t-1}^2 + 2e_{12}e_{22}u_{1,t-1}u_{2,t-1} + e_{22}^2u_{2,t-1}^2$$

Since we are looking at the flow of volatility from country 1 to country 2, we need to check for feedback from last period's volatility levels of country 1 to this period's volatility levels of country 2, and from last period's shocks from country 1 to this period's volatility levels of country 2, viz., we need to test $H_0 : d_{12} = e_{12} = 0$. We shall test for contagion in the form of volatility spillover using this specification.

4.2.1 Testing for Spillover

Our analysis was done using Proc Varmax in SAS. The data of returns is the same as in Figure 3.1. The estimates of the BEKK model are given in Table 4.3 with their corresponding standard errors.

There a few noteworthy observations. The estimates of the matrix C are not significantly different from 0. The estimates of d_{21} , e_{11} and e_{22} are also not significantly

Coefficients for C			
c_{11}	c_{12}	c_{21}	c_{22}
0.0334	0.5221	0	0.1009
(6.7161)	(0.4003)		(19.9592)
Coefficients for D			
d_{11}	d_{12}	d_{21}	d_{22}
0.5902	0.0305	0.2111	-0.2037
(0.1320)	(0.1306)	(0.0659)	(0.1135)
Coefficients for E			
e_{11}	e_{12}	e_{21}	e_{22}
0.0129	-1.3157	-0.5352	0.0236
(17.0669)	(0.1489)	(0.0672)	(0.2191)

Table 4.3: Table of BEKK Parameter Estimates

different from 0. The Proc used the Newton-Raphson optimization technique.

Proc Varmax has a test statement to test combinations of parameters. Since we are interested in testing for spillover of volatility, we tested $H_0 : d_{12} = e_{12} = 0$. The corresponding χ^2 statistic has 2 degrees of freedom and a value of 93.72 with a corresponding P -value less than 0.0001. We reject H_0 to conclude that there is spillover from the volatility of Hong-Kong's stock market into Singapore's stock market.

Chapter 5

Relationship Between The Two Approaches

5.1 Two Definitions

From Pericolli and Sbracia (2003) we can see that contagion has many definitions and that leads one to wonder if these different definitions leading to different tests are really testing the for the same thing. We looked at the following two definitions of contagion.

- Contagion is a significant increase in co-movements of prices and quantities across markets, conditional on a crisis occurring in one market or group of markets.
- Contagion occurs when volatility spills over from the crisis country to the fi-

nancial markets of other countries.

These two definitions led to two approaches for testing for the presence of contagion, modeled using the one-factor model and the BEKK form of the multivariate GARCH structure. The manner of testing for contagion in these two approaches seem different but they are based on variances and covariances of the returns of the stock markets of the two countries. This similarity in the two approaches directs us to the question “Are these two concepts answering the issue of lack of contagion in the same manner?”

We will use the following idea to compare the approaches. The one-factor approach has two time periods, viz., a pre-crisis period and a post-crisis period. The variance-covariance matrix of returns changes from pre-crisis to post-crisis, and this change is what we model in the one-factor model. The conditional heteroscedasticity approach basically has many time periods and we model the change in variance for all time periods. If at the time of crisis, the variance-covariance matrix in the one-factor model changes in the same manner as the change in the conditional heteroscedastic approach, when we observe a large innovation in the crisis country (corresponding to a crisis), then we could compare the two approaches.

To use this idea, we consider the variance-covariance matrices from the one-factor model before and after the crisis. We compare the variances and covariances from this model with the variances and covariances obtained when we observe a large innovation in the conditional heteroscedastic approach and the change in the pre-crisis and post-

crisis variances due to that.

The pre-crisis and post-crisis variance-covariance matrices are given by

$$\Sigma = \begin{pmatrix} \beta_1^2 + \sigma_1^2 & \beta_1\beta_2 \\ \beta_1\beta_2 & \beta_2^2 + \sigma_2^2 \end{pmatrix} = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \quad (5.1)$$

as the variance-covariance terms in the non-crisis period.

$$\Sigma_C = \begin{pmatrix} \beta_1^2(1 + \delta) + \sigma_1^2(1 + \delta) & \beta_1\beta_2(1 + \delta) \\ \beta_1\beta_2(1 + \delta) & \beta_2^2(1 + \delta) + \sigma_2^2 \end{pmatrix} = \begin{pmatrix} d & e \\ e & f \end{pmatrix} \quad (5.2)$$

is the variance-covariance terms in the crisis period when we model under $H_0 : \eta = 0$.

From Appendix A we have the following values for the parameter η .

$$\eta = \frac{e.b}{b^2 + (f - c)\beta_1^2} - 1$$

Under $H_0 : \eta = 0$ we get

$$\begin{aligned} eb &= b^2 + (f - c)\beta_1^2 \\ \Rightarrow eb - b^2 &= (f - c)\beta_1^2 \\ \Rightarrow \frac{b(e - b)}{f - c} &= \beta_1^2 \end{aligned} \quad (5.3)$$

Also from Appendix A we have

$$(\beta_1^2 + \sigma_1^2)(1 + \delta) = d \quad (5.4)$$

and since we have values for these parameters, we can substitute it in Equation 5.4

to get the following.

$$\Rightarrow a \left(1 + \frac{(f - c)b(e - b)}{b^2(f - c)} \right) = d$$

$$\begin{aligned}
\Rightarrow \frac{ae}{b} &= d \\
\Rightarrow ae &= bd \\
\Rightarrow \frac{b}{a} &= \frac{e}{d}
\end{aligned} \tag{5.5}$$

From Equation 5.5 we see that for interdependence to hold (as opposed to contagion, according to the one-factor model), the ratio of the covariance of the returns and the variance of the returns in the crisis country in the two periods has to be the same.

If we perform a regression with the returns of the crisis country being the explanatory (independent) variable and the returns of the non-crisis country being the response (dependent) variable, the estimate of the coefficient of the returns of the crisis country is given as the ratio of the covariance of the two returns and the variance of the returns of the crisis country. From Equation 5.5 the estimates of the regression coefficients of a conditional regression of the returns of country 2 on the returns of country 1 need to be a constant for the two time periods, viz., the pre-crisis and post-crisis periods. This implies that there is constant regression in the two time periods and the regression parameter does not change. We will exploit this relationship among the parameters to explore the relationship between the one-factor model and the conditional heteroscedasticity approach.

Specifically, we look for conditions under which the conditional variance matrices in the Conditional Heteroscedasticity approach change in the same manner as the unconditional variance matrices in the structural change (one-factor) approach.

The variance-covariance matrix for the vector of returns $(r_1, r_2)'$ in the conditional heteroscedastic model is given by

$$\Sigma_t = C'C + D'\Sigma_{t-1}D + E'U_{t-1}U_{t-1}'E$$

The individual components of the variance-covariance matrix are given by

$$\begin{aligned}\sigma_{t,11} &= c_{11}^2 + d_{11}^2\sigma_{t-1,11}^2 + 2d_{11}d_{21}\sigma_{t-1,12} + d_{21}^2\sigma_{t-1,22}^2 + e_{11}^2u_{t-1,1}^2 \\ &\quad + 2e_{11}e_{21}u_{t-1,1}u_{t-1,2} + e_{21}^2u_{t-1,2}^2 \\ \sigma_{t,12} &= c_{11}c_{12} + d_{11}d_{12}\sigma_{t-1,11}^2 + (d_{21}d_{12} + d_{11}d_{22})\sigma_{t-1,12} + d_{21}d_{22}\sigma_{t-1,22}^2 \\ &\quad + e_{11}e_{12}u_{t-1,1}^2 + (e_{21}e_{12} + e_{11}e_{22})u_{t-1,1}u_{t-1,2} + e_{21}e_{22}u_{t-1,2}^2 \\ \sigma_{t,22} &= c_{12}^2 + c_{22}^2 + d_{12}^2\sigma_{t-1,11}^2 + 2d_{12}d_{22}\sigma_{t-1,12} + d_{22}^2\sigma_{t-1,22}^2 + e_{12}^2u_{t-1,1}^2 \\ &\quad + 2e_{12}e_{22}u_{t-1,1}u_{t-1,2} + e_{22}^2u_{t-1,2}^2\end{aligned}$$

To model the change in the variance-covariance matrix from pre-crisis to post-crisis period, we consider an innovation of 1 for $u_{t-1,1}$ and an innovation value of zero for $u_{t-1,2}$. This is just to indicate the difference between a large innovation and a small one. Using this definition of a large innovation we have the following variance-covariance matrix for the conditional heteroscedastic model.

$$\begin{aligned}\sigma_{t,11} &= c_{11}^2 + d_{11}^2\sigma_{t-1,11}^2 + 2d_{11}d_{21}\sigma_{t-1,12} + d_{21}^2\sigma_{t-1,22}^2 + e_{11}^2 \\ \sigma_{t,12} &= c_{11}c_{12} + d_{11}d_{12}\sigma_{t-1,11}^2 + (d_{21}d_{12} + d_{11}d_{22})\sigma_{t-1,12} + d_{21}d_{22}\sigma_{t-1,22}^2 \\ &\quad + e_{11}e_{12} \\ \sigma_{t,22} &= c_{12}^2 + c_{22}^2 + d_{12}^2\sigma_{t-1,11}^2 + 2d_{12}d_{22}\sigma_{t-1,12} + d_{22}^2\sigma_{t-1,22}^2 + e_{12}^2\end{aligned}$$

Let the crisis time period be defined to be T . Then the pre-crisis variance-covariance matrix Σ and post-crisis variance-covariance matrix Σ_C are

$$\Sigma = \begin{pmatrix} \sigma_{T-1,11} & \sigma_{T-1,12} \\ \sigma_{T-1,21} & \sigma_{T-1,22} \end{pmatrix}$$

and

$$\Sigma_C = \begin{pmatrix} \sigma_{T,11} & \sigma_{T,12} \\ \sigma_{T,21} & \sigma_{T,22} \end{pmatrix}$$

We would like to compare these values with the variance-covariance matrices of the one-factor model in Equations 5.1 and 5.2 and exploit the property of constant regression. Hence from Equation 5.5 we need

$$\frac{\sigma_{T-1,12}}{\sigma_{T-1,11}} = \frac{\sigma_{T,12}}{\sigma_{T,11}} \quad (5.6)$$

This leads to

$$\frac{\sigma_{T-1,12}}{\sigma_{T-1,11}} = \frac{c_{11}c_{12} + d_{11}d_{12}\sigma_{T-1,11}^2 + (d_{21}d_{12} + d_{11}d_{22})\sigma_{T-1,12} + d_{21}d_{22}\sigma_{T-1,22}^2 + e_{11}e_{12}}{c_{11}^2 + d_{11}^2\sigma_{T-1,11}^2 + 2d_{11}d_{21}\sigma_{T-1,12} + d_{21}^2\sigma_{T-1,22}^2 + e_{11}^2} \quad (5.7)$$

which from Appendix C gives

$$\begin{aligned} & -d_{11}d_{12}\sigma_{T-1,11}^4 - (c_{11}c_{12} + e_{11}e_{22})\sigma_{T-1,11}^2 \\ & + (c_{11}^2 + e_{11}^2)\sigma_{T-1,12} + 2d_{11}d_{22}\sigma_{T-1,12}^2 \\ & + (d_{11}^2 - d_{12}d_{21} - d_{11}d_{22})\sigma_{T-1,11}^2\sigma_{T-1,12} \\ & + d_{21}(d_{21}\sigma_{T-1,12} - d_{22}\sigma_{T-1,11}^2)\sigma_{T-1,22}^2 = 0 \end{aligned} \quad (5.8)$$

These equations do not give any reasonable method of adjusting the GARCH parameters. The only reasonable conclusion we can draw is that the condition is very difficult to satisfy.

To model the phenomenon of absence of contagion we have to make sure that regression parameters stay constant. To achieve this our variance model needs to have constant ratio of σ_{12} over σ_{11} . This would entail the creation of a new heteroscedastic model where only σ_{22} is free, keeping the other terms pegged to the constant ratio of $\frac{\sigma_{12}}{\sigma_{11}}$.

5.2 Conditional Heteroscedasticity in One-Factor Model

In Section 3.3 we considered a one-factor model approach to analyzing contagion in the structural breakdown form. We used maximum likelihood techniques and estimated the model. We assumed that the common factor and the innovation terms were independent and identically distributed. We will test for this assumption by implementing the following.

We will examine another approach to considering conditional heteroscedasticity in analyzing the contagion models. We shall scrutinize the one-factor model from Section 3.3 with the idea of introducing non-normality in the form of non-constant

variance.

$$\begin{aligned} r_1 &= \beta_1 \times F + \epsilon_1 \\ r_2 &= \beta_2 \times F + \epsilon_2 \end{aligned} \tag{5.9}$$

The assumptions of the one-factor model Equation 5.9 required the common factor F to be normal with mean 0 and variance 1. We consider a different approach exploring for the presence of conditional heteroscedasticity in F , or even in the error terms, ϵ_1 and ϵ_2 , which are also assumed to be normally distributed. To explore this option we decided to filter out the estimates of F and ϵ_1 and ϵ_2 and analyze them for conditional heteroscedasticity.

We follow the approach of Diebold and Nerlove (1989) in analyzing the model. Diebold and Nerlove examined exchange rates from 7 countries, using a factor analysis and Kalman filter approach. We use the same approach, but since our model has only returns from 2 countries, we do not have the degrees of freedom to analyze the model in their approach.

The joint distribution of $(r_1, r_2, F, \epsilon_1, \epsilon_2)'$ is multivariate normal with mean $(0, 0, 0, 0, 0)'$ and variance

$$\begin{pmatrix} \beta_1^2 + \sigma_1^2 & \beta_1\beta_2 & \beta_1 & \sigma_1^2 & 0 \\ \beta_1\beta_2 & \beta_2^2 + \sigma_2^2 & \beta_2 & 0 & \sigma_2^2 \\ \beta_1 & \beta_2 & 1 & 0 & 0 \\ \sigma_1^2 & 0 & 0 & \sigma_1^2 & 0 \\ 0 & \sigma_2^2 & 0 & 0 & \sigma_2^2 \end{pmatrix}$$

under the calm period. Under $H_0 : \eta = 0$, and in the crisis period the variance is

$$\begin{pmatrix} (\beta_1^2 + \sigma_1^2)(1 + \delta) & \beta_1\beta_2(1 + \delta) & \beta_1(1 + \delta) & \sigma_1^2(1 + \delta) & 0 \\ \beta_1\beta_2(1 + \delta) & \beta_2^2(1 + \delta) + \sigma_2^2 & \beta_2(1 + \delta) & 0 & \sigma_2^2 \\ \beta_1(1 + \delta) & \beta_2(1 + \delta) & 1 + \delta & 0 & 0 \\ \sigma_1^2(1 + \delta) & 0 & 0 & \sigma_1^2(1 + \delta) & 0 \\ 0 & \sigma_2^2 & 0 & 0 & \sigma_2^2 \end{pmatrix}$$

To get the estimates of $(F, \epsilon_1, \epsilon_2)$, we consider their conditional expectation, viz., $E((F, \epsilon_1, \epsilon_2)' | (r_1, r_2)')$. The conditional expectation can be obtained in the following manner:

$$\begin{aligned} E((F, \epsilon_1, \epsilon_2)' | (r_1, r_2)') &= E(X | R) \\ &= Cov(X, R) * (Var(R))^{-1} * R \end{aligned}$$

where $X = (F, \epsilon_1, \epsilon_2)'$ and $R = (r_1, r_2)'$.

Since we don't know these parameters, we will use the parameter estimates. We can filter out the values for F , ϵ_1 and ϵ_2 . We will fit GARCH(1,1) models to these filtered values. The output of the GARCH fit for F , the common factor is given in Tables 5.1 and 5.2.

The parameter estimates for the GARCH(1,1) models are significant for the most part, with only α_0 and α_1 not being significant at least at the 5% level. This indicates that there is at least some evidence of conditional heteroscedasticity which has not been addressed in previous work on the one-factor model.

Parameters	α_0	α_1	β_1
Estimates	0.0223 (0.0158)	0.1282 (0.0639)	0.8412 (0.0743)

Table 5.1: GARCH Model for F in pre-crisis period

Parameters	α_0	α_1	β_1
Estimates	1.1041 (1.1326)	0.3534 (0.2527)	0.5666 (0.2111)

Table 5.2: GARCH Model for F in post-crisis period

The strongest evidence for the GARCH model for the common factor comes from the fitting of the GARCH model for the common factor from the combined periods. All the parameters of the GARCH(1,1) fit are significant. The values of Table 5.3 contain the parameter estimates of the GARCH(1,1) fit of F in the two periods together, with their corresponding standard errors. We have to note that the stationarity conditions for the GARCH(1,1) model. viz., $\alpha_0 + \beta_0 < 1$ is violated here.

Parameters	α_0	α_1	β_1
Estimates	0.0301 (0.0126)	0.3559 (0.0685)	0.7019 (0.0364)

Table 5.3: GARCH Model for F in combined periods

Furthermore, the residuals from the GARCH(1,1) fit show no heteroscedasticity

or long-tailed nature as evidenced by the graphs in Figures 5.1 and 5.2.

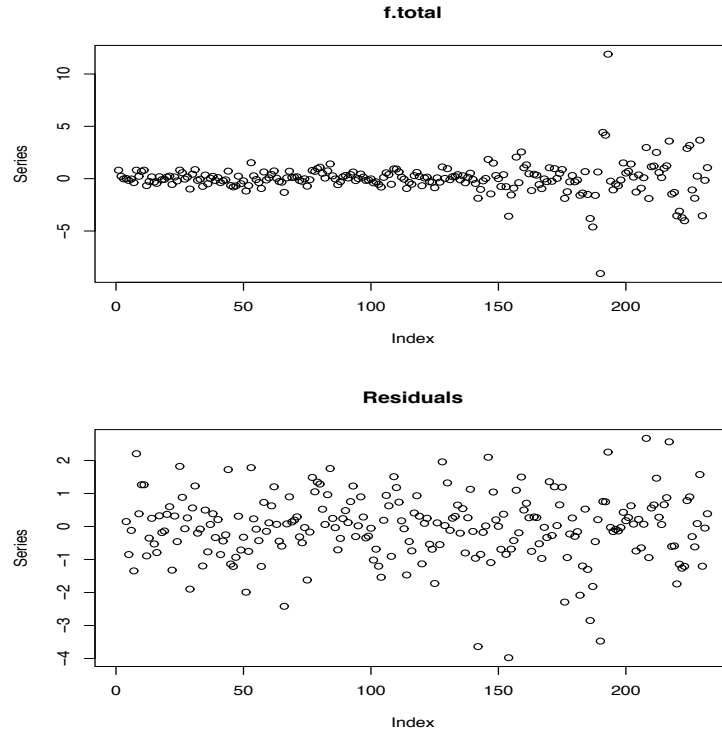


Figure 5.1: Scatter Plot of combined F values and residuals from GARCH(1,1) fit

The ACF's in Figure 5.2 indicate that the residuals are very close to being white noise. The GARCH(1,1) fit does account for the long tailed nature of the returns.

We also modeled the error terms ϵ_s for GARCH(1,1). They do not seem to exhibit any conditional heteroscedasticity and the parameters are not significant at all. This indicates that the common factor term takes into account most of the non-constant variance that is characteristic of most financial data.

The one-factor model does seem to be a good fit for the data. The common factor

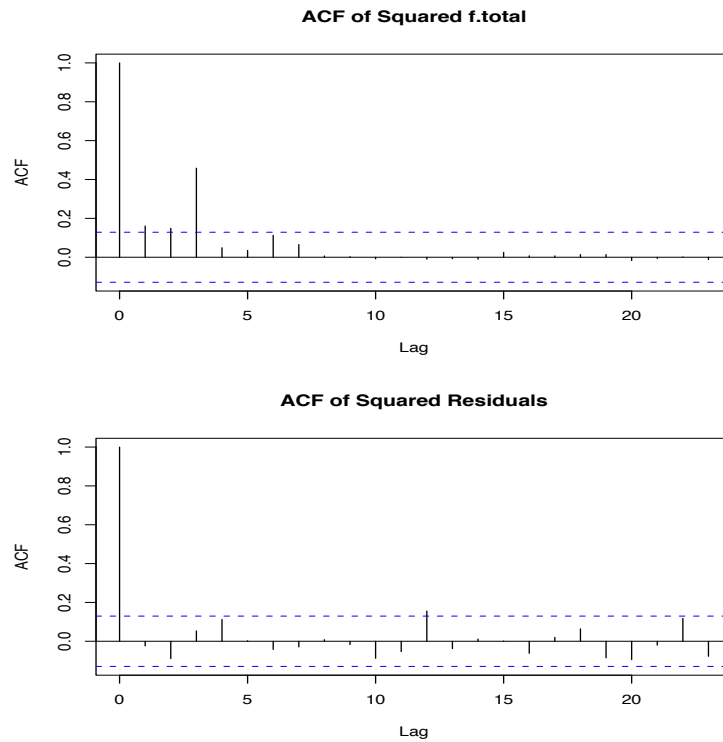


Figure 5.2: ACF of combined F values and residuals from GARCH(1,1) fit

exhibits conditional heteroscedasticity, which takes into account the non-constant volatility of market returns. The countries' own stock markets contribute through the ϵ_s , and they do not demonstrate conditional heteroscedasticity. The one-factor model with this change is a good fit, and testing for contagion in this scenario is an improvement, as it takes into account the non-constant nature of the volatility and still we can model interdependence and test for contagion.

Part III

Summary

Chapter 6

Conclusions and Future Work

Contagion is still not a very well defined concept. There have been many attempts at defining and modeling contagion. Pericolli and Sbracia and Dungey et al. (2003) and Dornbusch et al. (2000) have compiled the different viewpoints on contagion and suggested models based on these viewpoints. Pericolli and Sbracia (2003) have consolidated the literature to suggest a couple of definitions and statistical models to go along with these definitions. We have considered two definitions of contagion and analyzed their corresponding models. It is hence of interest to examine the differences in these models and therefore in the definitions themselves.

The two approaches we considered are the structural break in a one-factor model and the volatility spillover model. We have shown that the one-factor model has some inherent identifiability problems. However we could still test for contagion based on maximizing the likelihood. By failing to reject the null hypothesis of no contagion we

have concluded that there was no contagion present in between the Hong Kong and Singapore stock markets, as any increase in correlation is explained by the common factor. We have also shown that the common factor model in the one-factor approach has conditional heteroscedasticity, which will rationalize the presence of long-tailed observations.

The volatility spillover approach uses a multivariate GARCH structure for the error. We used a BEKK model to analyze the flow of volatility from Hong Kong's market into Singapore. We established the existence of spillover of volatility.

It has to be noted that these two approaches differed in their conclusions about the existence of contagion. In the common factor approach the data did not exhibit significant evidence of a structural breakdown of the parameters (which is the sign of presence of contagion according to the one-factor model) and did exhibit significant evidence that there was spillover of volatility from Hong Kong to Singapore (which is the sign of presence of contagion from the second approach). This in due course led to the question of whether the two approaches were testing for contagion in the same manner.

We hence looked at the relationship between the two approaches. We observed fundamental differences in their approaches, as one models the regression parameters as a constant (based on the definition of inter-dependence), and the other approach works based on the change of volatility from one period to another, and the factors affecting that change. This fact explains the basis for the apparent contradictory

conclusions of the two concepts of contagion.

We could build a conditional-heteroscedasticity model which models the regression parameters as a constant. We can also consider a volatility-spillover model, which works on the same approach as the common factor model, with the exception that the model lets volatility flow from the crisis country to the non-crisis country, but not the other way. This approach will be explored in future work.

Lastly, from Sentana (1998), we can construct a model with a GARCH structure for the variance of the returns which has the same properties as the one-factor model in which we are interested. We can then study the properties of the one-factor model with conditionally heteroscedastic factors and the volatility spillover models. This should lead to new insight into the different definitions of the concept of financial contagion. This approach will also be examined in future research in this area.

Part IV

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Part V

Appendix

Appendix A

Identifiability Restrictions

In Chapter 3, we looked at the identifiability issue in the common-factor model. In the common-factor model we have two periods viz., the calm or non-crisis and the crisis periods. The identifiability problem basically implies that we could have one set of observed values and two sets of different estimates which lead to confusion in interpreting the values of the estimates.

The variance-covariance matrices under the non-crisis and crisis periods are as follows. For the non-crisis period we have

$$\Sigma = \begin{pmatrix} \beta_1^2 + \sigma_1^2 & \beta_1\beta_2 \\ \beta_1\beta_2 & \beta_2^2 + \sigma_2^2 \end{pmatrix}$$

and for the crisis period under the alternative of $H_a : \eta > 0$, we have

$$\Sigma_C = \begin{pmatrix} \beta_1^2(1+\eta)(1+\delta) + \sigma_1^2(1+\delta) & \beta_1(1+\eta)\beta_2(1+\delta) \\ \beta_1(1+\eta)\beta_2(1+\delta) & \beta_2^2(1+\delta) + \sigma_2^2 \end{pmatrix}$$

Since we also defined our variance covariance matrices under the non-crisis to be

$$\Sigma = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

and crisis periods to be

$$\Sigma_C = \begin{pmatrix} d & e \\ e & f \end{pmatrix},$$

we get the following six equations by equating the parameters and the observed values.

$$\beta_1^2 + \sigma_1^2 = a \quad (\text{A.1})$$

$$\beta_1\beta_2 = b \quad (\text{A.2})$$

$$\beta_2^2 + \sigma_2^2 = c \quad (\text{A.3})$$

$$\beta_1^2(1 + \eta)(1 + \delta) + \sigma_1^2(1 + \delta) = d \quad (\text{A.4})$$

$$\beta_1(1 + \eta)\beta_2(1 + \delta) = e \quad (\text{A.5})$$

$$\beta_2^2(1 + \delta) + \sigma_2^2 = f \quad (\text{A.6})$$

From Equation A.1 we get

$$\sigma_1^2 = a - \beta_1^2 \quad (\text{A.7})$$

From Equation A.2 we get

$$\beta_2 = b/\beta_1 \quad (\text{A.8})$$

From Equations A.3 & A.8 we get

$$\sigma_2^2 = c - \beta_2^2 = c - (b/\beta_1)^2 = \frac{c\beta_1^2 - b^2}{\beta_1^2} \quad (\text{A.9})$$

From Equations A.6 A.8 & A.9 we get

$$(b/\beta_1)^2(1+\delta) + \frac{c.\beta_1^2 - b^2}{\beta_1^2} = f \Rightarrow \frac{b^2}{\beta_1^2} + \delta \frac{b^2}{\beta_1^2} + c - \frac{b^2}{\beta_1^2} = f \Rightarrow \delta = (f - c) \frac{\beta_1^2}{b^2} \quad (\text{A.10})$$

From Equations A.5 A.8 & A.10 we get

$$\beta_1(1+\eta)(b/\beta_1)(1+\frac{(f-c)\beta_1^2}{b^2}) = e \Rightarrow (1+\eta) = \frac{e.b}{b^2 + (f-c)\beta_1^2} \Rightarrow \eta = \frac{e.b}{b^2 + (f-c)\beta_1^2} - 1 \quad (\text{A.11})$$

From Equations A.4 A.7, A.10 & A.11 we have

$$\begin{aligned} d &= \left(\beta_1^2 \cdot \frac{(e.b)^2}{b^2 + (f-c)\beta_1^2} + (a - \beta_1^2) \right) \left(1 + \frac{(f-c)\beta_1^2}{b^2} \right) \\ \Rightarrow d &= \left(\frac{e^2.b^2 + (a - \beta_1^2)(b^2 + (f-c)\beta_1^2)(b^2 + (f-c)\beta_1^2)^2}{b^2(b^2 + (f-c)\beta_1^2)} \right) \\ \Rightarrow db^4 + db^2(f-c)\beta_1^2 &= e^2b^2\beta_1^2 + (a - \beta_1^2)(b^4 + (f-c)^2\beta_1^4 + 2b^2(f-c)\beta_1^2) \\ \Rightarrow db^4 + db^2(f-c)\beta_1^2 &= e^2b^2\beta_1^2 + ab^4 + a(f-c)^2ab^2(f-c)\beta_1^2 - b^4\beta_1^4 - (f-c)^2\beta_1^6 - 2b^2(f-c)\beta_1^4 \end{aligned}$$

And we get

$$-(f-c)\beta_1^6 + (a(f-c)^2b(f-c))\beta_1^4 + (2ab^2(f-c) - b^2 + e^2b^2 + db^2(f-c))\beta_1^2 - (db^4 - ab^4) = 0 \quad (\text{A.12})$$

We used the polyroot function in R to give us the roots for 100000 simulated values for a, b, c, d, e and f . Out of the 100000 values of the roots (i.e., β_1^2), we found 20 which had multiple positive values for $\beta_1^2, \beta_2^2, \sigma_1^2, \sigma_2^2, \delta$ and η .

Appendix B

BEKK Expansions

The BEKK model has the following form for the variance-covariance matrix.

$$\Sigma_t = C'C + D'\Sigma_{t-1}D + E'U_{t-1}U_{t-1}'E \quad (\text{B.1})$$

where, in the bivariate case, C is a lower triangular matrix having 3 parameters, and D and E are 2 2x2 square matrices with 8 parameters, leaving us with 11 parameters to estimate.

To understand how volatility and the shocks from the crisis country, viz., country 1 flow into the non-crisis country (country 2), we will expand the variance-covariance matrix Σ_t . We just need to look at $D'\Sigma_{t-1}D$ and $E'U_{t-1}U_{t-1}'E$, since C is a constant.

In any case we have

$$C'C = \begin{pmatrix} c_{11} & 0 \\ c_{12} & c_{22} \end{pmatrix} \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix} = \begin{pmatrix} c_{11}^2 & c_{12}^2 \\ c_{12}^2 & c_{12}^2 + c_{22}^2 \end{pmatrix}$$

$$\begin{aligned}
D'\Sigma_{t-1}D &= \begin{pmatrix} d_{11} & d_{21} \\ d_{12} & d_{22} \end{pmatrix} \begin{pmatrix} \sigma_{t-1,11} & \sigma_{t-1,12} \\ \sigma_{t-1,21} & \sigma_{t-1,22} \end{pmatrix} \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix} \\
&= \begin{pmatrix} d_{11}\sigma_{t-1,11} + d_{21}\sigma_{t-1,21} & d_{11}\sigma_{t-1,12} + d_{21}\sigma_{t-1,22} \\ d_{12}\sigma_{t-1,11} + d_{22}\sigma_{t-1,21} & d_{12}\sigma_{t-1,12} + d_{22}\sigma_{t-1,22} \end{pmatrix} \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix} \\
&= \begin{pmatrix} d_{11}^2\sigma_{t-1,11} + 2d_{11}d_{21}\sigma_{t-1,21} + d_{21}^2\sigma_{t-1,21} & d_{11}d_{12}\sigma_{t-1,11} + d_{12}d_{21}\sigma_{t-1,21} \\ & +d_{11}d_{22}\sigma_{t-1,12} + d_{21}d_{22}\sigma_{t-1,22} \\ d_{11}d_{12}\sigma_{t-1,11} + d_{12}d_{21}\sigma_{t-1,21} & d_{12}^2\sigma_{t-1,11} + 2d_{12}d_{22}\sigma_{t-1,21} + d_{22}^2\sigma_{t-1,22} \\ +d_{11}d_{22}\sigma_{t-1,12} + d_{21}d_{22}\sigma_{t-1,22} & \end{pmatrix} \\
E'U_{t-1}U'_{t-1}E &= \begin{pmatrix} e_{11} & e_{21} \\ e_{12} & e_{22} \end{pmatrix} \begin{pmatrix} u_{t-1,1} \\ u_{t-1,2} \end{pmatrix} \begin{pmatrix} u_{t-1,1} & u_{t-1,2} \end{pmatrix} \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix} \\
&= \begin{pmatrix} e_{11} & e_{21} \\ e_{12} & e_{22} \end{pmatrix} \begin{pmatrix} u_{t-1,1}^2 & u_{t-1,1}u_{t-1,2} \\ u_{t-1,1}u_{t-1,2} & u_{t-1,2}^2 \end{pmatrix} \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix} \\
&= \begin{pmatrix} e_{11}u_{t-1,1}^2 + e_{21}u_{t-1,1}u_{t-1,2} & e_{11}u_{t-1,1}u_{t-1,2} + e_{21}u_{t-1,2}^2 \\ e_{12}u_{t-1,1}^2 + e_{22}u_{t-1,1}u_{t-1,2} & e_{12}u_{t-1,1}u_{t-1,2} + e_{22}u_{t-1,2}^2 \end{pmatrix} \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix} \\
&= \begin{pmatrix} e_{11}^2u_{t-1,1}^2 + 2e_{11}e_{21}u_{t-1,1}u_{t-1,2} + e_{21}^2u_{t-1,2}^2 & e_{11}e_{12}u_{t-1,1}^2 + (e_{11}e_{22} + e_{12}e_{21})u_{t-1,1}u_{t-1,2} \\ & +e_{21}e_{22}u_{t-1,2}^2 \\ e_{11}e_{12}u_{t-1,1}^2 + (e_{11}e_{22} + e_{12}e_{21})u_{t-1,1}u_{t-1,2} & e_{12}^2u_{t-1,1}^2 + 2e_{12}e_{22}u_{t-1,1}u_{t-1,2} + e_{22}^2u_{t-1,2}^2 \\ +e_{21}e_{22}u_{t-1,2}^2 & \end{pmatrix}
\end{aligned}$$

We are interested in testing for volatility spillovers from country 1 to country 2, since when country 1 has a crisis, we would like to see if it affects the volatility of country 2. To this end, we will look at the volatility term of country 2 to test for the presence of volatility from country 1.

The volatility for country 2 is given by $\sigma_{t,22}$ and is given by B.2

$$\sigma_{t,22} = c_{12}^2 + c_{22}^2 + d_{12}^2 \sigma_{t-1,11}^2 + 2d_{12}d_{22}\sigma_{t-1,12} + d_{22}^2 \sigma_{t-1,22}^2 + e_{12}^2 u_{t-1,1}^2 + 2e_{12}e_{22}u_{t-1,1}u_{t-1,2} + e_{22}^2 u_{t-1,2}^2 \quad (\text{B.2})$$

For no volatility spillover from country 1 to country 2 we need to make sure that $\sigma_{t-1,11}$ and $u_{t-1,1}$ do not appear in the above expression. If d_{12} and e_{12} are equal to 0, then the expression for $\sigma_{t,22}$ reduces to B.3

$$\sigma_{t,22} = c_{12}^2 + c_{22}^2 + d_{22}^2 \sigma_{t-1,22}^2 + e_{22}^2 u_{t-1,2}^2 \quad (\text{B.3})$$

Hence to test for the absence of volatility spillover we test the null hypothesis

$$H_0 : d_{12} = e_{12} = 0$$

Appendix C

Constant Regression in GARCH

We considered the effect of constant regression on the GARCH factors in the BEKK model. We have from Equation 5.7 that the regression coefficients of series 2 on series 1 is the same at times $T - 1$ and T if and only if

$$\frac{\sigma_{T-1,12}}{\sigma_{T-1,11}^2} = \frac{c_{11}c_{12} + d_{11}d_{12}\sigma_{T-1,11}^2 + (d_{21}d_{12} + d_{11}d_{22})\sigma_{T-1,12} + d_{21}d_{22}\sigma_{T-1,22}^2 + e_{11}e_{12}}{c_{11}^2 + d_{11}^2\sigma_{T-1,11}^2 + 2d_{11}d_{21}\sigma_{T-1,12} + d_{21}^2\sigma_{T-1,22}^2 + e_{11}^2} \quad (\text{C.1})$$

$$\begin{aligned} \Rightarrow & \quad c_{11}^2\sigma_{T-1,12} + (d_{11}^2\sigma_{T-1,11}^2 + 2d_{11}d_{21}\sigma_{T-1,12} + d_{21}^2\sigma_{T-1,22}^2)\sigma_{T-1,12} + e_{11}^2\sigma_{T-1,12} \\ & = \quad c_{11}c_{12}\sigma_{T-1,11}^2 + (d_{11}d_{12}\sigma_{T-1,11}^2 + (d_{21}d_{12} + d_{11}d_{22})\sigma_{T-1,12} + d_{21}d_{22}\sigma_{T-1,22}^2)\sigma_{T-1,11}^2 \\ & \quad + e_{11}e_{12}\sigma_{T-1,11}^2 \\ \Rightarrow & \quad c_{11}^2\sigma_{T-1,12} + d_{11}^2\sigma_{T-1,11}^2\sigma_{T-1,12} + 2d_{11}d_{21}\sigma_{T-1,12}^2 + d_{21}^2\sigma_{T-1,22}^2\sigma_{T-1,12} + e_{11}^2\sigma_{T-1,12} \\ & = \quad c_{11}c_{12}\sigma_{T-1,11}^2 + d_{11}d_{12}\sigma_{T-1,11}^4 + (d_{21}d_{12} + d_{11}d_{22})\sigma_{T-1,12}\sigma_{T-1,11}^2 + d_{21}d_{22}\sigma_{T-1,22}^2\sigma_{T-1,11}^2 \\ & \quad + e_{11}e_{12}\sigma_{T-1,11}^2 \end{aligned}$$

$$\begin{aligned}
\Rightarrow 0 &= c_{11}^2 \sigma_{T-1,12} + d_{11}^2 \sigma_{T-1,11}^2 \sigma_{T-1,12} + 2d_{11}d_{21} \sigma_{T-1,12}^2 + d_{21}^2 \sigma_{T-1,22}^2 \sigma_{T-1,12} \\
&\quad + e_{11}^2 \sigma_{T-1,12} - c_{11}c_{12} \sigma_{T-1,11}^2 - d_{11}d_{12} \sigma_{T-1,11}^4 - (d_{21}d_{12} + d_{11}d_{22}) \sigma_{T-1,12} \sigma_{T-1,11}^2 \\
&\quad - d_{21}d_{22} \sigma_{T-1,22}^2 \sigma_{T-1,11}^2 - e_{11}e_{12} \sigma_{T-1,11}^2 \\
\Rightarrow 0 &= (c_{11}^2 + e_{11}^2) \sigma_{T-1,12} + (d_{11}^2 - d_{21}d_{12} - d_{11}d_{22}) \sigma_{T-1,11}^2 \sigma_{T-1,12} + 2d_{11}d_{21} \sigma_{T-1,12}^2 \\
&\quad + d_{21}^2 \sigma_{T-1,22}^2 \sigma_{T-1,12} - (c_{11}c_{12} + e_{11}e_{12}) \sigma_{T-1,11}^2 - d_{11}d_{12} \sigma_{T-1,11}^4 \\
&\quad - d_{21}d_{22} \sigma_{T-1,22}^2 \sigma_{T-1,11}^2 \\
\Rightarrow 0 &= -d_{11}d_{12} \sigma_{T-1,11}^4 - (c_{11}c_{12} + e_{11}e_{22}) \sigma_{T-1,11}^2 + (c_{11}^2 + e_{11}^2) \sigma_{T-1,12} \\
&\quad + 2d_{11}d_{22} \sigma_{T-1,12}^2 + (d_{11}^2 - d_{12}d_{21} - d_{11}d_{22}) \sigma_{T-1,11}^2 \sigma_{T-1,12} \\
&\quad + d_{21}(d_{21} \sigma_{T-1,12} - d_{22} \sigma_{T-1,11}^2) \sigma_{T-1,22}^2
\end{aligned} \tag{C.2}$$

Equation C.2 was previously shown in Equation 5.8.