Abstract

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According to classic economic theory, if global capital markets are fully integrated then arbitrage should force real interests rates to be equal among countries. However, a large body of empirical evidence suggests that this parity is not achieved. One potential factor in this failure of economic theory is that international capital transactions are not frictionless. The costs involved in the purchase or sale of a capital asset imply a neutral area in which arbitrage is not profitable and rates freely deviate from equality. This paper uses a variety of econometric methods to investigate the behavioral characteristics of real interest rates and determine the existence and form of transactions costs.

This work is divided into six sections. Section One introduces the concept of capital market integration and details the current state of the literature on the topic. Section Two describes the data used in the analyses. Section Three uses a non-parametric regression method to analyze bilateral real interest rate relationships. Section Four examines these relationships in a multivariate setting, employing a method which incorporates non-linear behavior. Section Five investigates the data for structural instability. Section Six briefly summarizes and concludes the paper.

NON-LINEAR ASPECTS OF CAPITAL MARKET INTEGRATION

by

ANTHONY JOSEPH MANCUSO

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Dedication

This dissertation is dedicated to my parents, Martin and Peggy Mancuso, and to the many surrogate "parents" I have been fortunate to have in my life. Their love and inspiration are a debt that can never be repaid.

Biography

Anthony Joseph Mancuso was born at Eglin A.F.B, FL on March 11, 1971 to Martin and Peggy C. Mancuso. After graduating from Northern Durham High School in Durham, NC, he enrolled at Appalachian State University. In 1993, he graduated with a Bachelor's of Science degree in Business Administration. In 2001, he received a Master of Statistics from North Carolina State University.

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0.1 Introduction and Background

Are real interest rates equal across countries? Over the past 15 years, much research has been undertaken in an attempt to answer that question. The chapters which follow also address this topic – an indication that a definitive answer has remained elusive. While some progress has been made, the question remains largely unanswered.

Before moving on, it is advantageous to formally define real interest parity (RIP). Assume that uncovered interest parity (UIP) holds, that is that

$$R_{1,t} - R_{2,t} = E_t(\Delta s_t) \tag{1}$$

where $R_{1,t}$ is the domestic nominal interest rate, $R_{2,t}$ is a foreign nominal interest rate, s_t is the exchange rate in terms of unit of domestic currency per unit of foreign currency, and $E_t(\Delta s_t)$is the expected change in s_t, conditional on current information. Intuitively, Equation (1) states that differences in the nominal interest rate reflect expected changes in the exchange rate, and that these rates adjust to equalize the return on domestic and foreign assets.

Another fundamental relationship between open economies is purchasing power parity (PPP), which can be stated as

$$E_t(\pi_{1,t}) - E_t(\pi_{2,t}) = E_t(\Delta s_t) \tag{2}$$

where $\pi_{1,t}$ and $\pi_{2,t}$ are the domestic and foreign inflation rates, respectively. Equation (2) means that differences in expected price levels are offset by expected changes in the exchange rate, such that a unit of the domestic currency can purchase the same bundle of goods and services in either country.

Using Equations (1) and (2), RIP can be found as

$$R_{1,t} - R_{2,t} = E_t(\pi_{1,t}) - E_t(\pi_{2,t})$$

$$R_{1,t} - E_t(\pi_{1,t}) = R_{2,t} - E_t(\pi_{2,t})$$

$$r_{1,t} = r_{2,t} \tag{3}$$

where the final step utilizes Fisher's relationship. The derivation emphasizes two points. First, RIP is a joint hypothesis of UIP and PPP[1]. Second, RIP governs the behavior of *ex ante* interest rates. Estimating these rates, which are unobserved, is the subject of Section 0.2.

While an explicit derivation serves a purpose, sometimes it is useful to consider ideas on a less rigorous level. Informally, the concept behind RIP is simple: if capital and goods markets are integrated, arbitrage in goods and financial instruments (embodied by PPP and UIP respectively) will force real interest rates to equalize [2]. This simplicity, however, belies the importance of the concept. A tendency for world rates to equalize circumscribes the ability of economic policy makers to achieve domestic economic goals by affecting the real interest rate. A simple hypothesis with important practical implications would seem to be an ideal candidate for economic analysis.

Unfortunately, a survey of the literature is likely to be a disappointing experience. Hundreds of papers address the topic, each different in both data and method. Such heterogeneity makes comparisons difficult and firm conclusions hard to find. Despite

[1] Another maintained assumption is that the risk premia are equal in both countries.

[2] The validity of the arbitrage argument is not as straight-forward as it might appear. In the derivation above, inflation rates are derived from domestic price indexes. Differences in the composition of these indexes make the exact nature and comparability of the real interest rates uncertain. However, most authors recognize that while this is a liability, there are few credible alternatives.

the variety, however, it is possible to outline broad themes and movements which have appeared over the years.

Early investigators of RIP, such as Mishkin (1984), Mark (1985), and Cumby and Mishkin (1986), applied least squares methods to bilateral regressions of one real interest rate on the another. These authors were looking for strict equality of rates, and they tended to reject RIP. It was quickly realized, however, that these simple approaches were less than satisfactory. First, real interest rates tend to be non-stationary, which makes least squares estimation unreliable. Second, the maintained hypothesis of a linear specification makes inferences less certain. Finally, and most importantly, it was realized that capital market integration does not necessarily mean strict equality. Rates may deviate from equality, but still move together over time and, therefore, still afford little latitude for domestic policy makers to act.

With this final insight firmly in mind, subsequent authors applied Granger causality tests to the problem. Here, the hypothesis was not strict equality between rates, but rather that movements in one rate are reflected by movements in others. Examples of this work are Swanson (1987) and Karfakis and Moschos (1990). By and large, these studies were more supportive of capital market integration, but they were not without their failings. First, if rates are cointegrated, relationships should be tested not with a VAR form, but rather with an error correction model (ECM) (see Engle and Granger (1987)). Second, many of these analyses used bivariate methods. While informative, such methods are susceptible to omitted variable bias.

In response to these issues, a third movement arose which utilized ECM and related co-integration methods to analyze real interest rate systems. Goodwin and

Grennes (1994) applied both bivariate and multivariate cointegration tests and found strong evidence for integration. Chinn and Frankel (1995) and Monadjemi (1998) also found support for this weaker concept of parity. However, these results are still less than completely satisfactory, as they do not offer precise explanations of interest rate movements.

The three chapters of this thesis investigate further the issue of real interest rate parity, and the more general issue of capital market integration. By applying both novel statistical techniques and theoretical insight, the analysis intends to clarify the relationships among these important economic variables.

0.2 Data

Generally speaking, the data comprise weekly observations on *ex ante* real interest rates. Although not as common as monthly data in the literature, weekly data offer two significant advantages. First, weekly data have better "resolution." Analyses performed with monthly data are unable to detect dynamics which occur on a shorter time scale. Of course, economic conditions can change dramatically within a given month, while many international financial transactions can occur within days, if not instantly. Therefore, it seems likely that weekly data provide a more complete picture of financial relationships. The second and less obvious advantage relates to recent work by Taylor (2001), who showed that aggregation can bias the results of certain types of estimations. The greater the degree of aggregation, the larger the inaccuracy. The use of weekly data diminishes this effect.

It is legitimate to question the need to estimate unobserved *ex ante* interest rates when the use of *ex post* rates is a common feature of the literature. The following simple example illustrates the advantage of *ex ante* rates. Consider the *ex post* Fisher relationship

$$R = r_p + \pi$$

where r_p is the *ex post* real interest rate. Similarly, the *ex ante* real interest rate can be defined by

$$r_a = R - E_t(\pi)$$

Thus, a simple estimate of the r_a is

$$r_a = r_p + (E_t(\pi) - \pi)$$

or

$$r_a = r_p + \epsilon$$

where ϵ is the error in the inflation forecast. Invoking rational expectations, the forecast error is assumed to be uncorrelated with current information. Thus, the estimate of r_a is simply r_p.

While intuitively appealing, this practice is not without its flaws. As will be seen below, survey data indicates that inflation forecast error is not likely to be uncorrelated with current information. Although this failing can be addressed in estimation (e.g., modeling errors as an MA process), doing so makes the test for RIP joint with the form of the solution. In contrast, constructing the *ex ante* interest rates does not require explicit modeling of the inflation forecast errors.

Six countries have been selected for this study: Canada, Germany, Japan, Switzerland, the United Kingdom (U.K.), and the United States (U.S.). As many of the analyses are performed bilaterally with the U.S., these nations were chosen for their importance as U.S. trading partners (Canada, Japan), their importance in world financial markets (Switzerland) or both (Germany, U.K.)[3]. The data cover the period from the first week of 1979 to August of 2001. Not every analysis utilizes the full range of data, however – exceptions will be noted as they occur.

[3]In order to control the source of any differences in the real interest rates considered, it is necessary that the countries in the data set be as homogenous as possible. Therefore, only OECD countries are considered in this study. Therefore, the results below should not be extended unconditionally to less-developed countries

0.2.1 Nominal Interest Rates

While the ultimate focus of this work is the behavior of *ex ante* real interest rates, these variables are not directly observed. Instead, they must be derived from other, more readily accessible data. The first component of a real interest rate is a nominal interest rate. For this purpose daily, mid-market, 12-month Eurocurrency rates were taken from the *Financial Times*[4]. These daily values were converted to weekly averages, which are graphed in Figures 1 through 3.

Upon quick visual inspection, two features become apparent. First, most countries have seen a decline in nominal interest rates over the span of the data. Japan has seen the most pronounced decrease, with nominal rates almost at zero. This is due to the persistent deflation evident since the late 1990's.

Second, all of the nominal rates exhibit a relative increase during the late 1980's and early 1990's. This period coincides with German reunification which helps provide an explanation for this feature of the data.

Figures 4 through 6 plot the differences between the U.S. nominal rate and each of the foreign nominal rates. Note that during this time the American rate tended to be higher than those of Japan, Germany, and Switzerland, but lower than those of Canada and the U.K.

It would be tempting to draw conclusions regarding capital market integration from these graphs, but such conjecture would be ill-advised. Most economic theories stress that real interest rates are the relevant metrics for measuring capital market

[4]Fujii and Chinn (2000) note that the longer the term of the interest rates considered (e.g., 5 and 10 year instruments), the more supportive of RIP analyses tend to be. Although this issue will not be addressed in this work, it is conceded that the robustness of these results should be further investigated in this regard.

integration. While there are measures which do use nominal interest rates, such as covered or uncovered interest parity, these also incorporate exchange rate behavior. Therefore, without additional information any judgment would be premature.

0.2.2 *Ex Ante* Inflation Rates

Method

Once a suitable nominal interest rate has been specified, an *ex ante* real interest rate can be obtained by incorporating expected changes in the price level. The issue is to estimate agents' expectations. The approach employed here is due to Frankel (1982)[5].

Frankel noted that the expected interest rate at any time t can be thought of as a weighted average between the instantaneous short-term rate and one with an infinite term, both evaluated at time 0. This can be codified as

$$R_t = (1 - \exp(-\delta t))(\pi_0^e + r) + \exp(-\delta t)R_0 \tag{4}$$

where R_t is a short-term interest rate, π_0^e is long-term expected inflation, r is the steady-state real interest rate (which is assumed to be constant), R_0 is an instantaneous short-term rate, and δ is a constant. Thus, a bond issued at time 0 with a maturity of τ can be written as the average of all the intermediate rates plus a liquidity premium (κ_τ):

$$R_0^\tau = \tau^{-1} \int_0^\tau R_t dt + \kappa_\tau \tag{5}$$

Integration of Equation (5) gives

$$R_0^\tau = (1 - \omega_\tau)(\pi_0^e + r) + \omega_\tau R_0 + \kappa_\tau \tag{6}$$

[5]For an earlier implementation of this method, see Al-Awad and Goodwin (1998).

where

$$\omega_\tau = \frac{1 - \exp(-\delta t)}{\delta \tau} \tag{7}$$

Choosing two liquidities allows Equation (6) to be solved for π_0^e in terms of δ and two κ_τ.

$$\pi_0^e = \frac{\omega_1 R_2 - \omega_2 R_1}{\omega_1 - \omega_2} - \frac{\omega_1 \kappa_2 - \omega_2 \kappa_1}{\omega_1 - \omega_2} - r \tag{8}$$

This leads naturally into estimating π_0^e by

$$\hat{\pi}^e = \frac{\omega_1 R_2 - \omega_2 R_1}{\omega_1 - \omega_2} \tag{9}$$

The only barrier to calculating this value is the unknown δ (a component of ω). Notice from Equation (4) that δ represents the speed at which rates converge to their steady-state value, an interpretation which suggests estimating δ by regressing the real interest rate on its own lagged values. This seems like a circular argument, however, as the real interest rate depends on π^e, which in turn depends on δ. Fortunately, Frankel showed that the regression is equivalent to regressing, the difference between the two nominal rates, $R_1 - R_2$, on its own lagged values. The coefficient of this regression is equal to $\exp^{-\delta/n}$, where n is the number of observations per year.

The measure in Equation (9) differs from the true value in Equation (8) by a constant equal to the omitted terms. Frankel shows that $\hat{\pi}^e$ performs better than investor surveys, even without accounting for the constant. For those so inclined, however, he does provide a means for estimating this value. Note that Equation (8) and Equation (9) can be combined as

$$\pi_0^e - \hat{\pi}^e = \frac{\omega_1 \kappa_2 - \omega_2 \kappa_1}{\omega_1 - \omega_2} - r \tag{10}$$

Assuming that agents are rational, this suggests estimating the constant as the average difference between the actual inflation rate and the $\hat{\pi}^e$. Therefore, the final method is to estimate $\hat{\pi}^e$ from Equation (9), and then subtract the average expectation error.

In order to implement the method above, two additional data series are needed. One is a second nominal interest rate. This study uses three-month nominal Eurocurrency rates. Once again, these data are weekly averages, taken from the *Financial Times*. To implement the last step, a measure of the price level is needed. There are several alternative measures, such as the consumer price index (CPI), the wholesale price index (WPI), and various price indexes found in the Penn tables. However, to ensure consistency with the nominal interest rate the selected measure will be converted to weekly frequency via cubic spline. Therefore, the frequency of the data need to be considered and many of the available measures are quarterly or annual. Based on this rationale, price information is taken from the International Monetary Fund's *International Financial Statistics*, which are available monthly.

Validation

Frankel's method, while fairly simple, is still an estimation technique. Therefore, its results should be analyzed before being accepted as part of the analysis. Figures 7 through 9 plot the *ex ante* inflation rates. At first glance, the series seem to be of a reasonable scale and exhibit trends that are consistent with historical evidence. As with the nominal interest rates in Section 0.2.1, *ex ante* inflation increases in all countries around the time of German reunification, although to varying degrees. Also, the variance and magnitude of expected inflation appears to have moderated over time. This behavior is congruous with recent developments in monetary policy, which place inflation as a primary target variable.

Since no obvious objections arise, the next step is to compare the *ex ante* inflation measure with the actual *ex post* inflation. If agents are rational, then they should not make systematic errors in their inflation predictions. From an empirical standpoint, rationality makes inferences more sure and help alleviate fears of bias arising from use of Frankel's method.

Figures 10 to 12 graph the difference between the *ex ante* and *ex post* inflation rates for each country. Obviously, agents make frequent mistakes in their estimates, sometimes egregious ones. However, they do not seem to make them systematically – sometimes over-estimating, sometimes under-estimating with no clear tendency to err in a particular direction.

Unfortunately, strict rationality also implies that the forecast errors are not serially correlated, a condition which is obviously violated by Frankel's measure[6] This situation, however, is not unique in the literature. Paquet (1992) notes that such serial correlation is often found in the difference between actual inflation and survey forecasts. More recently Berk (1999), uses Dutch consumer survey data and finds that there is a stationary, but not orthogonal, forecast error. Therefore the serial correlation apparent in the forecast error, while troubling from a theoretical standpoint, is consistent with empirical observation[7]. Therefore, rather than test for strict

[6]Some authors, such as Cukierman and Meltzer (1982) and Batchelor and Dua (1987), have suggested that serially correlated forecast errors are consistent with rationality. This work will adhere to the more generally accepted notion of strict rationality and will not address this issue.

[7]It should be noted that the source of the serial correlation is indeterminate. One possibility is that in converting the monthly CPI figures to weekly values, a temporal dependence is created. Further work with Frankel's methodology would be useful in clarifying this issue.

rationality, simple unbiasedness will be considered instead.

Consider that if estimates of inflation are unbiased, then in the following equation

$$\pi_{ante,t} = \beta_0 + \beta_1 \pi_{post,t} + \epsilon_t \tag{11}$$

it should be that $\beta_0 = 0$ and $\beta_1 = 1$. A reasonable approach would be to estimate this equation and test the validity of the restriction. Before doing that, however, it is necessary to evaluate the time series properties π_{ante} and π_{post}. Tables 1 and 2 provide augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) statistics for the levels of the *ex ante* and *ex post* inflation rates, respectively.

Table 1: Stationarity tests for *ex ante* inflation rates

	ADF		PP	
Country	Statistic	p-value	Statistic	p-value
Canada	-1.62	0.4732	- 1.38	0.5918
Germany	-1.54	0.5116	-1.31	0.6290
Japan	-0.74	0.8340	-0.64	0.8599
Switzerland	-1.97	0.2997	-1.86	0.3504
U.K.	-1.82	0.3687	-1.43	0.5710
U.S.	-1.69	0.4369	-1.43	0.5676

In all cases and for both test statistics, the *ex ante* inflation rates are non-stationary. As for the *ex post* rates, those for Canada, Germany and the United

Table 2: Stationarity tests for *ex post* inflation rates

	ADF		PP	
Country	Statistic	p-value	Statistic	p-value
Canada	-1.50	0.5348	-0.96	0.7687
Germany	-2.03	0.2730	-0.98	0.7626
Japan	-4.13	0.0010	-1.49	0.5386
Switzerland	-5.55	0.0001	-1.61	0.4775
U.K.	-5.05	0.0001	-1.45	0.5603
U.S.	-2.58	0.0971	-1.43	0.5697

States appear to be non-stationary (at the 5% level). For the remaining countries, the evidence is mixed with the ADF statistics refuting the unit root hypothesis and the PP statistics supporting it. More information is needed to make a determination, and it comes in the form of the autocorrelation functions (ACFs) for these series. These graphs, which are shown in Figures 13 to 15, all exhibit a slow decay which is characteristic of a non-stationary series. Based on this display, and bearing in mind the PP results above, it would seem reasonable to conclude that all the data series are non-stationary. Therefore, standard testing procedures are invalid and an alternative method for evaluating the *ex ante* inflation rates is needed.

When both sides of an equation are non-stationary, a natural way to approach model testing is to use co-integration. Consider imposing the restriction $\beta_0 = 0$ and $\beta_1 = 1$[8], such that Equation (11) becomes

$$\pi_{ante,t} = \pi_{post,t} + \epsilon_t \tag{12}$$

If the two inflation rates are first-difference stationary (i.e., I(1)) and cointegrated, then the difference between the two should be stationary. In particular, this difference should be stationary around a mean of zero. Finding this to be true would imply that the *ex ante* inflation rate differs from the *ex post* inflation rate by a zero mean error. Thus, the former is an unbiased estimator of the latter and Frankel's method can be used with confidence.

In order for cointegration to be an option, both inflation measures must be I(1). Tables 3 and 4 present unit root tests for the first-differences of the *ex ante* and *ex*

[8]Essentially, this is imposing a known cointegration vector of (1, -1) on the system. See, again, Paquet (1992)

Table 3: Stationarity tests for first difference of *ex ante* inflation rates

	ADF		PP	
Country	Statistic	p-value	Statistic	p-value
Canada	-20.79	0.0001	-28.01	0.0001
Germany	-19.86	0.0001	-26.41	0.0001
Japan	-20.08	0.0001	-30.73	0.0001
Switzerland	-21.07	0.0001	-28.41	0.0001
U.K.	-21.22	0.0001	-28.44	0.0001
U.S.	-19.65	0.0001	-25.06	0.0001

Table 4: Stationarity tests for first difference of *ex post* inflation rates

	ADF		PP	
Country	Statistic	p-value	Statistic	p-value
Canada	-13.58	0.0001	-16.58	0.0001
Germany	-12.87	0.0001	-15.92	0.0001
Japan	-18.26	0.0001	-11.22	0.0001
Switzerland	-24.14	0.0001	-10.15	0.0001
U.K.	-15.28	0.0001	-9.71	0.0001
U.S.	-13.96	0.0001	-14.08	0.0001

post inflation rates, respectively. All series appear to be I(1).

Having established that both series are I(1), the final step in this process is to test for the stationarity of the difference between the *ex ante* and *ex post* inflation measures. Table 5 presents unit root statistics for the difference between the two inflation measures. Clearly, the residuals are stationary (around a mean of zero) and the process outlined in Section 0.2.2 provides a reasonable measure of *ex ante* inflation.

0.2.3 Real Interest Rates

The next step is to evaluate the real interest rates implied by the nominal interest and inflation rates described above. As a start, consider the *ex post* real interest

Table 5: Stationarity tests for difference between *ex ante* and *ex post*

	ADF		PP	
Country	Statistic	p-value	Statistic	p-value
Canada	-3.90	0.0001	-3.35	0.0009
Germany	-3.39	0.0007	-2.77	0.0056
Japan	-2.90	0.0037	-2.25	0.0239
Switzerland	-4.38	0.0001	-2.83	0.0046
U.K.	-4.76	0.0001	-3.04	0.0024
U.S.	-3.33	0.0009	-2.51	0.0119

rates, which are shown in Figures 16 to 18. Notice that there are periods of negative real returns, primarily early in the data set. These values roughly coincide with those periods when expectations were the least accurate (refer to Figures 10 to 12) and are the result of agents grossly underestimating the true inflation rate.

As noted above, much of the analysis will revolve around bilateral comparisons between U.S. and foreign *ex ante* real interest rates. To provide a comparison for these series, which will be discussed below, Figures 19 to 21 present the differential between US and foreign *ex post* real interest rates. Obviously, these series indicate that marked deviations from strict equality are common. However, it is not unreasonable to imagine that the centers of the distributions are fairly close to 0, a fact which would represent a small measure of support for *ex post* RIP.

Finally, the *ex ante* real interest rates are graphed Figures 22 through 24. Probably the most striking feature of these plots is that they suggest that these rates could be stationary around some long-run value (more so if early values are excluded). Also, note that German fiscal policy during reunification did not have as much of an impact on *ex ante* real rates as on the nominal rates presented earlier.

Figures 25 to 27 graph the differences between the U.S. real rate and each of the foreign rates and provide a preliminary glimpse into the nature of their relationships. First, the deviations are less pronounced for these *ex ante* rates than for the *ex post* rates above. Second, although the differentials appear to be stable in some long-run sense, the long-run equilibrium differential does not appear to be zero. Discerning the exact form of these relationships is the subject of the following sections.

The fact that there are non-zero differentials which persist begs for an answer. However, one is not readily forthcoming. It is tempting to attribute the differences to differences in savings rates between the two countries. Based on the graphs, the U.S. savings rate should be less than those for Germany, Japan and Switzerland, but greater than those for Canada and the U.K. According to Marquis (2002), from 1980 to 2001, the U.S. savings rate averaged 8%. Consistent with the graphs, the savings rate for Germany and Japan were more, averaging 12% and 13% respectively. However, the Canadian rate was actually higher as well, averaging 16%.

There are other possible explanations. For example, the differences could reflect differences in risk premia or tax policies between countries. Or, perhaps, the feature arises from differences in the formation and productivity of capital. Whatever the cause, it is obvious that these persistent differences are not easily explained, and the question is deferred to another time.

0.3 Bivariate Analyses

0.3.1 Introduction

Implicitly, Equation (3) assumes that there are no transactions costs and that any difference, no matter what the size, between a domestic and a foreign real interest rate is arbitraged away. Transactions costs alter these dynamics. If the potential profit available from an arbitrage opportunity is less than the cost of realizing that profit, then the transaction will not take place. If, however, the profit is greater than the cost of capitalizing on it, then arbitrage will occur and rates will converge. Even if agents have unencumbered access to all capital markets, transactions costs delineate an area in which rates have no tendency to equalize[9]. Econometric techniques which do not account for this behavior will reject incorrectly a hypothesis of integration.

Despite the serious consequences of ignoring these non-linearities, this topic only recently has received significant attention in the literature. Michael, Nobay and Peel (1997), and Balke and Fomby (1997) note how transactions costs can effectively bias cointegration tests against a finding of integration and offer ways of accounting for these neutral bands. Using these techniques, Sarantis (1999) and Taylor and Peel (2000) both found significant non-linearities in exchange rates, which as noted above, have a significant impact on RIP. Baum, *et. al.* (2001) find similar non-linear behavior in deviations from PPP, which is a crucial component of RIP. Using threshold techniques, Nakagawa (2002) investigates the relationship between real interest rates and the real exchange rate. Finally, Peel and Taylor (2002), testing

[9]Dumas (1992), using a general equilibrium framework, confirms that transactions costs are likely to give rise to just such behavior.

the Keynes-Einzig hypothesis, find evidence of a persistent 0.5% neutral zone in deviations from covered interest parity (CIP), which is a close relative of RIP.

The primary contribution of this chapter is the implementation of a non-parametric regression technique to study non-linearities in real interest rate relationships. This method, which has yet to be applied in this area, provides more flexibility than other available methods and should result in additional insights. It should be noted that all analyses in this section are performed on a subset of the data which begins with November 1979. This is to avoid potential effects from a change in Federal Reserve operating procedures.

0.3.2 Statistical Methods

Co-integration Analysis

A multitude of methods for testing for cointegration have been developed. The most traditional uses the fact that if two variables are I(1) (i.e., first-difference stationary), and cointegrated, then the residuals from the regression

$$y_t = x_t{\prime}\beta + \epsilon_t$$

will form a stationary series. Thus, any test of the stationarity of the residuals will also be a test of cointegration. This analysis will use the modified two-step procedure of Engle and Granger (1987), where an augmented Dickey-Fuller test is performed on the residual series.

Alternative tests for cointegration can be based on the properties of error correction models (ECM). Consider the following equation for a VAR system

$$\mathbf{Z}_t = \Phi_1 \mathbf{Z}_{t-1} + \Phi_2 \mathbf{Z}_{t-2} + \epsilon_t \tag{13}$$

where $\mathbf{Z}_t$ is an nx1 vector of observations, Φ_1 and Φ_2 are parameter matrices, and ϵ_t is a white noise error vector. If all the variables in $\mathbf{Z}_t$ are I(1) then Equation (13) can be rewritten as

$$\Delta \mathbf{Z}_t = \Gamma \Delta \mathbf{Z}_{t-1} + \Pi \mathbf{Z}_{t-2} + \epsilon_t \tag{14}$$

with $\Gamma = -(I - \Phi_1)$ and $\Pi = -(I - \Phi_1 - \Phi_2)$. In general, for an n-dimension ECM, full cointegration implies rank$(\Pi) = n - 1$. Johansen (1988) and Johansen and Juselius (1990) derive two likelihood ratio tests based on this logic. The test utilized here, known as the maximal eigenvalue test, tests for exactly $n - 1$ co-integrating vectors. Critical values for this test statistic will be taken from MacKinnon, Haug, and Michelis (1999), which are more accurate than those provided in Johansen and Juselius.

It has been suggested that the two-step Dickey-Fuller test has greater power than the maximal eigenvalue test (see Gregory (1994)). However, the relative power of these tests seems to fluctuate from situation to situation, and Gregory's analysis was done without benefit of the improved critical values of MacKinnon *et. al.*. Therefore, both tests will be performed and both sets of statistics reported.

Both the Engle-Granger two-step approach and the maximal eigenvalue method will be applied to all possible US/foreign *ex ante* real interest rate pairs. While these analyses are not the heart of the chapter, the results they supply will provide a means to compare the general properties of this data set to others used in the literature.

Benchmark Analyses

Inference using non-parametric regressions must be done graphically; the very nature of the estimator means there are no parameters on which to base hypothesis tests of the usual form. In order to clarify the non-parametric method, a parametric technique will be implemented and used for comparison. A description of this analysis is presented first, for two reasons. First, the presence of parameters eases discussion of econometric issues. Second, highlighting the advantages of the non-parametric method is more readily accomplished if the parametric approach is fully understood.

Consider the following autoregression of interest rate differentials

$$r_{1,t} - r_{2,t} = \beta_0 + \beta_1 \left(r_{1,t-1} - r_{2,t-1}\right) + \epsilon_t \tag{15}$$

where ϵ_t is a white noise error term. In this specification, the parameter β_1 is a measure of the persistence of deviations from RIP. If β_1 is close to zero, then a given series quickly converges to its long-run equilibrium value, absent any future shocks. As β_1 approaches 1, deviations are more and more persistent and convergence is slower.

A useful representation can be derived by subtracting $r_{1,t-1} - r_{2,t-1}$ from each side of Equation (15) and rearranging

$$\left(r_{1,t} - r_{2,t}\right) - \left(r_{1,t-1} - r_{2,t-1}\right) = \beta_0 + \left(\beta_1 - 1\right)\left(r_{1,t-1} - r_{2,t-1}\right) + \epsilon_t$$

or, letting $z_t = r_{1,t} - r_{2,t}$ and $\rho = \beta_1 - 1$,

$$\Delta z_t = \beta_0 + \rho z_{t-1} + \epsilon_t \tag{16}$$

Despite differing in form, Equation (16) expresses the same intuition as Equation (15). Consider the situation when z_{t-1} is large, i.e., there is a large difference between the interest rates. If the parity relationship holds closely, then β is close to 0 and ρ is almost -1. In this case, the change in the differential is almost equal to the negative of the differential, which means that deviations from parity close almost instantly. A similar argument could be used to show that the two equations are consistent in other cases as well.

At this point, the response is assumed to be the same no matter the size of the differential. If transactions costs are important however, there is no reason for this to be true. In particular, small differentials, which offer relatively small profit potential, may be closed more slowly than large ones, if at all. In this scenario, there are two distinct regimes: one for "large" differentials and one for "small" differentials. The threshold autoregression (TAR) accounts for this behavior by expanding Equation (16) as

$$\Delta z_t = \beta_0 + \tau_t \rho_1 z_{t-1} + (1 - \tau_t)\, \rho_2 z_{t-1} \epsilon_t \tag{17}$$

where

$$\tau_t = \begin{cases} 1 & |(r_{1,t-1} - r_{2,t-1}) - \mu_{1,2}| \geq \delta \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

In this specification, τ_t is an indicator function, which will take on values of 0 or 1. Intuitively, if the absolute difference between $r_{1,t-1}$ and $r_{2,t-1}$ differs from its long-run value, $\mu_{1,2}$, by more than some threshold value, δ, then $\tau_t = 1$ and ρ_1 is the relevant parameter. Alternatively, if the absolute deviation is less than the threshold value, $\tau_t = 0$ and ρ_2 governs the relationship.

The most pressing statistical matter is the determination of δ. This will be accomplished by an iterative method, searching across a grid of feasible values at regular intervals. The estimate, $\hat{\delta}$, is chosen to maximize the Wald statistic which tests the hypothesis that there is no difference between the in-band and out-of-band parameter estimates. That is, the *de facto* objective function is

$$(\rho_1 - \rho_2)'[\Sigma]^{-1/2}(\rho_1 - \rho_2) \tag{19}$$

where $\Sigma = Var(\rho_1 - \rho_2)$.

Because it is essentially an order statistic, the statistic in Equation (19) has a non-standard distribution. Therefore, its significance will be tested using a simulation method developed by Hansen (1996).

The results of the estimation of Equation (17), will be reported in two ways. First, half-lives, one of the most common ways results are expressed in this field, will be calculated and compared with those found in the literature. This measure, which is merely the amount of time it takes for half of a given difference to dissipate, is readily derived from Equation (17) as

$$\text{half-life} = \frac{\ln(.5)}{\ln(\rho + 1)}$$

Second, the relationship implied by the parameter values will be displayed graphically along with the non-parametric relationship for comparison purposes.

It should be noted at this point that Equation (18) illustrates some of the failings of the TAR model. First, the TAR is a piecewise linear technique, where different lines are fit to different sections of the data. Second, the estimated relationship is assumed to be symmetric. A large positive value of z_{t-1} is assumed to cause the same response as a large negative value. It is in response to these flaws that the non-parametric analysis is undertaken[10].

[10] Other threshold autoregressive techniques, such as the smooth transition autoregression (STAR), may not suffer from these shortcomings. However, the non-parametric method presented below effectively nests these specifications. Therefore, the TAR is presented as the benchmark for its ease of implementation.

Looking at the graphs of the *ex ante* real interest rates in Section 0.2, it is not readily apparent that threshold effects are present. Fortunately, Terasvirta (1994) provides a likelihood ratio test of a linear specification versus non-linear alternatives[11]. The test procedure begins with the estimation of the auxiliary regression

$$e_{z,t} = \beta 0 + \sum_{i=1}^{p} \beta_{1,i} z_{t-i} z_{t-1} + \sum_{i=1}^{p} \beta_{2,i} z_{t-i} z_{t-1}^2 + \sum_{i=1}^{p} \beta_{3,i} z_{t-i} z_{t-1}^3 \tag{20}$$

where, as before, $z_t = r_{1,t} - r_{2,t}$ and $e_{z,t}$ are the residuals of a pth order auto-regression of z_t. Terasvirta shows that the null hypothesis

$$\text{H: } \beta_{1,i} = \beta_{2,i} = \beta_{3,i} = 0$$

$$\forall\ i = 1, 2, \ldots, p$$

is also a test for the sufficiency of the linear model. The autoregressive order of the models, p, will be selected using the Schwartz Bayes Criterion (SBC), while the lag variable is assumed to be z_{t-1}.

Non-Parametric Regression

Non-parametric regression allows the modeling of complex relationships without the need to assume a functional specification. With the conditional expectation able to assume any value, this technique effectively nests not only the TAR, but also other similar techniques (e.g., ESTAR or LSTAR). This flexibility makes the non-parametric regression a compelling choice for examining real interest rate dynamics.

In choosing a non-parametric method, it is important to be cognizant of the nature of the data and of the relationship in question. As seen above, the TAR

[11]More exactly, the alternative hypothesis is a logistic smooth autoregressive (LSTAR) model. Terasvirta also presents a test statistic which has the exponential smooth threshold autoregressive (ESTAR) as the alternative. However, since the non-parametric technique presented below is able to mimic either alternative model, it is sufficient to reject the linear specification, without considering the particular form of the alternative.

benchmark model is a piecewise linear estimator. One way to consider estimating this relationship is to fit a locally weighted line, i.e., to let

$$m(x) = \min_{\alpha,\beta} \sum_t K(\frac{x_t - x}{b})(y_t - (\alpha + \beta x_t))^2 \tag{21}$$

where y is a dependent variable and x is a scalar explanatory variable.

The key feature of Equation (21) is the kernel function $K(\frac{x_t - x}{b})$. Based on the logic that observations x_t far from a given x provide less information than those nearby, $K(\bullet)$ weights observations based on their proximity to x. The bandwidth parameter, b, controls how rapidly the weights decay to 0.

Fortunately, the minimization problem in Equation (21) has a closed form solution. Let

$$S_{n,j} = \sum_t K(\frac{x_t - x}{b})(x_t - x)^j$$

and

$$w_t = K(\frac{x_t - x}{b})[S_{n,2} - (x_t - x)S_{n,1}]\ (S_{n,0}S_{n,2} - S_{n,1}^2)$$

then

$$\hat{y}_t = \sum_{t=1}^{n} w_t y_t \tag{22}$$

is known as the local linear regression estimator (LLRE), attributable to Fan (1993).

In non-parametric regression, bandwidth selection is critical. If data arise from an experimental design, or are relatively evenly spread throughout the range, a constant bandwidth may be satisfactory. However, if the data are not spatially homogeneous, a constant bandwidth is likely to result in a curve which is either oversmoothed (i.e., shows too little detail) or undersmoothed (i.e., shows too much detail). Fan and

Gijbels (1995) endogenize the choice of bandwidth, and allow for variation across the sample, by the following method.

First, the range of x is divided into sections of equal length, which results in L subintervals with

$$L = n/10log(n)$$

where n is the total number of observations. If necessary, L is rounded to the nearest integer value.

Next, a constant bandwidth is determined for each subset of the data. In this step, each interval is considered independently and the optimal bandwidth is strictly for the data on this interval. Bandwidths on each subset are optimized using cross-validation. In cross-validation, for a given bandwidth, b, the predicted value $\hat{y}_t$ is found by computing a non-parametric regression without using the tth observation. This procedure is repeated for each observation in the interval and the mean squared prediction error (MSPE) is defined as

$$MSPE(b) = n_l^{-1} \sum_{t=1}^{n_l} (y_t - \hat{y}_t)^2$$

where n_l is the number of observations in the lth interval. This measure is a function of the bandwidth, as changing b will alter the predicted values which are used to calculate the MSPE. The bandwidth which minimizes the MSPE is considered optimal. To ease the computational burden, this optimization will be performed using the Nadaraya-Watson estimator

$$\hat{y}_t = \sum_{t=1}^{n_l^t} \frac{K(\frac{x_t - x_l}{b}) y_t}{\sum_{t=1}^{n_l^t} K(\frac{x_t - x_l}{b})} \tag{23}$$

where $\sum_{i=1}^{n_l^t}$ expresses the exclusion of the tth observation.

The optimization process is repeated for each interval and all the observations in a given interval are "assigned" the same value for their bandwidth. This results in a bandwidth step function. As a final step, this function is smoothed using the Nadaraya-Watson estimator from Equation (23), with the ultimate result that every observation has a unique bandwidth which reflects the density of observation in its vicinity. These are the bandwidths used to calculate the LLRE in Equation (22). To look for statistical significance, 95% confidence bands will be calculated following Härdle (1990).

One final issue is the choice of the kernel function. The Epanechnikov kernel, which is defined as

$$K(x) = \begin{cases} 3/4(1 - (\frac{x}{b})^2) & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

has been shown to be the minimum variance kernel (Härdle (1990)), and will be used.

The LLRE will be applied in a non-parametric autoregression (NPAR). That is, the y_t and x_t of Equation (21) will be Δz_t and z_{t-1} respectively, where these data variables retain their previous definitions. To analyze the LLRE results, the resulting curve will be graphed with the TAR of Section 0.3.2 for comparison.

0.3.3 Results

Co-Integration Results

As noted in Section 0.2.3, the preliminary indication is that the *ex ante* real interest rates are stationary. This would be an interesting finding in and of itself, as the vast majority of studies find these rates to be non-stationary. However, in the current context the primary significance is that the co-integration analyses proposed above would have to be amended. Table 6 provides ADF and PP tests for the stationarity of the *ex ante* real interest rates.

Table 6: Stationarity tests for *ex ante* real interest rates

	ADF		PP	
Country	Statistic	p-value	Statistic	p-value
Canada	-4.94	< 0.0001	-4.71	0.0001
Germany	-4.57	0.0002	-4.36	0.0005
Japan	-4.90	< 0.0001	-5.21	< 0.0001
Switzerland	-5.24	< 0.0001	-5.08	< 0.0001
U.K.	-3.92	0.0021	-3.84	0.0027
U.S.	-5.18	< 0.0001	-4.52	0.0003

Evidently, the *ex ante* real interest rates are stationary. Therefore, a new co-integration strategy is required. Suppose that the components of the real interest rate (the 12-month nominal interest rate and the *ex ante* inflation rate) rates are I(1). Then, for each US/Foreign rate pair co-integration analysis could be performed on the four variables (one interest rate and one inflation rate for each country) as a system.

As the *ex ante* inflation rate was found to be I(1) in Section 0.2 (see Tables 1 and 3), only the twelve-month nominal interest rate must be tested. Table 7, provides ADF and PP statistics.

In all cases and for both statistics, the hypothesis of a unit root is not rejected. This finding leads to Table 8, which displays unit root tests for the first difference of the 12-month nominal rate series. A unit root is strongly rejected in all cases, leading to the conclusion that the 12-month nominal interest rates are I(1).

Based on the unit root tests above, it is possible to continue with the maximal

Table 7: Stationarity tests for nominal interest rates

	ADF		PP	
Country	Statistic	p-value	Statistic	p-value
Canada	-1.36	0.6059	-1.08	0.7246
Germany	-1.43	0.5717	-1.19	0.6808
Japan	0.8149	0.41	-0.52	0.8858
Switzerland	-2.05	0.2647	-1.90	0.3305
U.K.	-1.56	0.5009	-1.13	0.7044
U.S.	-1.69	0.4336	-1.40	0.5856

Table 8: Stationarity tests for first-difference of nominal interest rates

	ADF		PP	
Country	Statistic	p-value	Statistic	p-value
Canada	-20.31	< 0.0001	-26.07	< 0.0001
Germany	-21.00	< 0.0001	-24.73	< 0.0001
Japan	-17.98	< 0.0001	-24.74	< 0.0001
Switzerland	-19.82	< 0.0001	-26.10	< 0.0001
U.K.	-20.33	< 0.0001	-26.12	< 0.0001
U.S.	-18.84	< 0.0001	-24.32	< 0.0001

Table 9: Maximal eigenvalue tests for four-variable systems

	No. of Co-integrating Relationships			
Country	$r = 0$	$r = 1$	$r = 2$	$r = 3$
Canada	139.57†‡	34.58†‡	12.07	4.06
Germany	66.94†‡	26.64†‡	6.49	2.43
Japan	32.40†‡	30.88†‡	10.99	1.98
Switzerland	45.29†‡	31.62†‡	12.02	5.56
U.K.	48.09†‡	27.95†‡	10.19	2.50

† significant JJ critical value

‡ significant versus MHM critical value

eigenvalue tests of the four-variable systems. Table 9 shows the results of these tests. In all cases, the null hypothesis is that there are exactly r co-integrating relationships versus the alternative of $r + 1$ relationships.

Significance is tested at the 5% level versus both the Johansen and Juselius (JJ) and MacKinnon *et. al.* (MHM) critical values. The qualitative results of the comparisons are identical because, as can be seen in Table 10, the actual critical values vary only slightly.

Table 10: Maximal eigenvalue test critical values

	JJ	MHM
$r = 0$	28.14	28.58
$r = 1$	22.00	22.30
$r = 2$	15.67	15.88
$r = 3$	9.24	9.17

As noted in Al-Awad and Goodwin (1998), if there are 3 co-integrating relationships between the 4, then there is some type of equilibrium involving all four variables. This finding would be evidence of a stable relationship between the *ex ante* real interest rates, although its exact nature would depend on the exact values found in the co-integration vector(s). In this light, Table 9 provides weak support for RIP, at best. In all cases, the maximal eigenvalue test indicates the presence of 2 co-integrating relationships, which leaves the degree of integration an open question.

The results of the integration testing are consistent with the literature. Using a similar, but more limited, data set Al-Awad and Goodwin also found two co-integrating vectors between the U.S. and Germany, the U.S. and Switzerland, and the U.S. and the U.K. However, they did find that the relationships between the U.S. and Canada, and the U.S. and Japan could be characterized with 3 vectors. Chinn and Frankel (1995), studying the countries of the Pacific Rim, determine that while there is evidence that real interest rates do influence each other, "real interest rate parity appears to be a rare phenomenon." Monadjemi (1998) discovered asymmetries in the causality between the U.S., U.K., and Dutch real interest rates, which would preclude the existence of RIP, but not capital market integration.

0.3.4 Benchmark Analysis Results

Since the co-integration results fail to provide definitive evidence of a strong relationship between *ex ante* real interest rates, the TAR is considered next. Table 11 shows the results of Terasvirta's non-linearity test.

Table 11: Non-linearity tests

Country	Statistic	p-value
Canada	119.82	< 0.0001
Germany	1045.1	< 0.0001
Japan	713.18	< 0.0001
Switzerland	706.94	< 0.0001
U.K.	887.81	< 0.0001

Table 12: Auto-regression results

Country	Constant	ρ_{AR}	AR half - life
U.S.-Canada	-0.0694	-0.0789	8.4338
	(0.0105)	(0.0113)	
U.S.-Germany	0.0047	-0.0324	21.0450
	(0.0031)	(0.0074)	
U.S.-Japan	0.0403	-0.0388	17.5158
	(0.0090)	(0.0030)	
U.S.-Switzerland	0.0565	-0.0317	21.5174
	(0.0134)	(0.0073)	
U.S.-U.K.	-0.0178	-0.0391	17.3787
	(0.0051)	(0.0080)	

Numbers in parentheses are standard errors.
Half-lives are in weeks.

From the table, it is clear the linear model is rejected in favor of one incorporating threshold effects. Therefore, the next step is to implement the TAR estimation. It is worth re-iterating that the TAR is an intermediate model; it serves primarily as a benchmark for the non-parametric auto-regressive models to be presented later. First, consider Table 12, which presents the results of "naive" auto-regressions of the type in Equation (15).

The key feature of Table 12 is the size of the half-lives. In general, the convergence implied by these half-lives is slower than observation and experience would support. By these half-lives, 99% of a given disparity dissipates in as few as 12 months (Canada) or as many as 32 months (Switzerland). It seems unlikely that Figures 25 and 26 result from a process where real interest rate differentials persist for a year or more. Indeed, many of the deviations appear to revert to the mean differential fairly quickly, typically in a matter of a few months.

Although they may be seen as somewhat implausible, such large values are not

Table 13: Threshold auto-regression results

Country	ρ^{in}	ρ^{out}	Out-of-band half-life	Threshold	% Out-of-Band	Wald
U.S.-Canada	-0.0606	-0.1069	6.1310	0.6824	80.25	9.1744†
	(0.0128)	(0.0146)				
U.S.-Germany	-0.0020	-0.0407	16.6817	0.4532	19.66	4.9553
	(0.0155)	(0.0082)				
U.S.-Japan	-0.0169	-0.0878	7.5427	1.7315	5.17	26.4081†
	(0.0089)	(0.0123)				
U.S.-Switzerland	-0.0218	-0.0527	12.8030	1.4641	81.10	9.4677†
	(0.0079)	(0.0100)				
U.S.-U.K.	-0.0540	-0.0270	25.3240	1.1031	5.85	3.4446
	(0.0114)	(0.0104)				

Numbers in parentheses are standard errors and half-lives are in weeks.
† indicates significance at the 5% level.

uncommon in studies which do not account for threshold effects. For example, Phylaktis (1999) reports convergence times which range from 55 months to 13 months, depending on the country and time period considered.

Table 13 shows how accounting for threshold effects affects the analysis. Large deviations from parity are corrected much more rapidly than was found above. In particular, the half-life for a large differential between the U.S. and Swiss real interest rates falls from 21 weeks to 12 weeks. The half-life for a large differential between the U.S. and the Japanese real interest rates falls from 17 weeks to 7.5 weeks. This case also illustrates how the large half-lives arise. Most of the observations, roughly 95%, fall within a neutral band. Within this band, the autoregressive process exhibits little, if any, tendency to converge. When the typical AR model is applied, this large mass of observations dominates the analysis. The TAR model separates the groups and paints a more detailed picture of the convergence process. It is interesting to note that in the U.S.-Canada and U.S.-Switzerland relationships, more than 80% of the observations are out-of-band. The differential for these two countries exhibits a larger variance than is found in the other countries in the sample. Therefore, these are more likely to "jump" out-of-band than is the U.S.-Japan differential.

The half-life estimates are somewhat smaller than those found in Nakagawa (2002), who found convergence toward equilibrium ranging from 10 weeks for to 47 weeks. Overall, however, the estimates are reassuringly similar in magnitude. Estimates of the size of the neutral band, however, are noticeably larger than previous work would indicate. For example, Al-Awad (1997) found transactions costs in the area of 0.1%. By comparison, the thresholds in Table 13 range from 0.45% to 1.7%. The reason for this discrepancy is not immediately clear, particularly since the half-life estimates are consistent with the literature.

When considering the Wald tests for the significance of the threshold, another puzzle arises. While the Hansen test procedure supports the TAR for Canada, Japan, and Switzerland, it fails to reject the linear model for Germany and the U.K. This is clearly at odds with the non-linearity tests presented in Table 11. One possibility is that Hansen's method simply lacks power relative to Terasvirta's test statistic. Whatever the cause, in response to these results AR models will be utilized as the benchmarks for these countries. Note that for the U.S.-U.K. relationship, since the AR is not rejected, the in-band and out-of-band parameters are not significantly different from each other. Therefore, the fact that $|\rho^{in}| > |\rho^{out}|$ is not of importance.

0.3.5 Non-Parametric Regression Results

Earlier it was noted that the TAR is only one type of threshold model. In fact, it is perhaps the most restrictive representative model in that family. The non-parametric auto-regressions (NPAR) illustrate interesting features of the data that the TAR is simply unable to discern.

Consider Figure 28. The solid line represents the relationship implied by the TAR model, while the predicted values from the NPAR are denoted by circles[12]. However, there are some significant differences between the two. In particular, the out-of-band behavior (the section farthest to the left) is more complicated than the TAR indicates. Also, the convergence is slower than is implied by the TAR. To see this, begin at the 0 on the y-axis and trace a horizontal line until it intersects the NPAR line. This occurs at roughly -1.0 on the x-axis. This implies that there is a stable differential when the Canadian *ex ante* real interest rate is roughly 1 percentage point larger than its U.S. counterpart. Moving to the left of that point, both the TAR and NPAR become positive. This indicates that when the differential becomes more negative in this period, the change in that differential is positive, that is the rates move toward each other in the next period. However, the size of the change in the differential is less for the NPAR than for the TAR, which implies a slower convergence. It should be noted that the dramatic fall in the NPAR line (on the right of the curve) is due to relatively few observations. However, since in the NPAR technique fitted values depend on the value of their neighboring observations, the

[12]Because of the high data density, the confidence intervals were virtually indistinguishable from the conditional mean function. As they did not affect the implications of the analysis, they have been omitted from the figures that follow.

method is fairly robust in the presence of outliers. Therefore, the observed feature should not be dismissed out-of-hand.

The U.S.-Germany relationship is shown in Figure 29. As noted above, this figure uses the AR model as a benchmark. It is apparent, however, that the AR specification does not accurately reflect the true relationship between these rates. Using the same process as before, it can be seen that the U.S.-Germany differential is stable when it is slightly more than zero. Near this point, the AR and the NPAR track closely. As the difference between the two grows, however, a disparity emerges. While this is consistent with threshold behavior, the form of the NPAR makes it impossible to test the hypothesis directly.

Compared to the TAR, the NPAR suggests slower convergence when the U.S. rate becomes "too small" relative to the German rate. Yet, the curve indicates faster convergence when the U.S. rate is "too large" relative to the German rate. This feature not captured by the AR, which imposes the same regime over the whole range of the data, and is an interesting asymmetry. Note that this feature may also offer an explanation for why Hansen's method failed to support the non-linearity test results. As noted above, the TAR is a symmetric specification, which assumes the same relationship holds on either side of the band. However, the NPAR suggests that this is not the case. It is conceiveable that the TAR methodology ultimately selects an "average" of the positive and negative out-of-band parameters, which effectively cancel each other out. The result is that both the in-band and out-of-band parameters reflect slow convergence behavior and Hansen's method fails to reject the linear model.

Figure 30 depicts the U.S.-Japan relationship. Notice that, in contrast to the TAR, the NPAR indicates that convergence occurs within the band. Considering the out-of-band observations, the MPAR suggest slower convergence. However, the larger the differential the more rapidly it is closed. This observation again emphasizes how the piecewise linearity of the TAR fails to discern potentially important behaviors.

The U.S.-Switzerland relationship is presented in Figure 31. As was seen in the U.S.-Japan discussion, the in-band relationship exhibits more convergence than suggested by the TAR. The NPAR line is surprisingly straight and the slight curve it does have could be due to a small contingent of large observations visible on the right. This semi-linearity leads to slower relative convergence for out-of-band deviations.

The final relationship is that between the U.S. and the U.K. real interest rates, which is found in Figure 32. Like the U.S.-Germany relationship, the benchmark here is the simple AR model. And, just as in that case, the quality of the AR approximation deteriorates as the magnitude of the differential increases. This is suggestive of threshold behavior. Interestingly, the asymmetric convergence found in the U.S.-Germany relationship (relative to the AR) is also present. Notice that only two country pairs have observations on both sides of the neutral band, and that both of them exhibit this feature. Additionally, these two country pairs are the only two with inconsistencies between the Terasvirta test for non-linearity and Hansen's method.

0.3.6 Conclusion

There are several conclusions to be drawn from the analyses presented. From the TAR analysis, it can be inferred that important threshold effects exist in the relationship between *ex ante* real interest rates. These effects, if ignored, can dramatically affect the statistical inferences and bias results against capital market integration. The NPAR technique reveals that this convergence displays marked asymmetries. Additionally, these would not be adequately described by a simple TAR model. Indeed, it is not clear how accurately other modeling techniques, such as the ESTAR model, would model the data.

The ultimate object of any study, however, is not limited to econometric results. At least as important is what those results imply about economic reality. In Section 0.1, it was noted that strict RIP limits the ability of economic policy makers to attain their domestic economic goals. The weaker form of economic integration discovered here suggests that while there is some freedom, it is not absolute. Policy actions that engender large differentials between real interest rates could induce arbitrage which might work contrary to policy goals.

Another economic issue highlighted by the statistical work is the asymmetry found in the U.S.-Germany and U.S.-U.K. relationships. Recall that in these cases a differential in favor of the foreign country (i.e., one which would provoke capital flows to the foreign country) seems to elicit less of a response than an equivalent differential in favor of the U.S. This suggests that investors take advantage of profit opportunities more readily when they involve the purchase of U.S. financial instruments. Such behavior could be due to differences in information; perhaps U.S. investment opportunities are more readily known. Another possibility is that there is a bias toward U.S. investment among agents who participate in international capital markets, which could be due to the size and perceived security of the U.S. market. As a final hypothesis, the feature could arise from imbalances in capital market restrictions whereby investment in non-U.S. capital markets is relatively more difficult than investing in the U.S. market[13]. By this reasoning, when the U.S. real interest rate is high, agents are more able to invest in the U.S. capital market than in the German or U.K. capital markets and the asymmetry results. All of these suggestions are, of course, highly speculative and future research could be directed at explaining this unusual feature of the data.

[13]While it is true that most of these restrictions were eliminated by the early to mid-1980's, they were in place for a substantial portion of the time span of the data.

0.4 Multivariate Analyses

0.4.1 Introduction

The analysis of the previous chapter focused primarily on bilateral relationships. From a statistical standpoint this approach was advantageous, as many of the procedures employed are noticeably simpler in a univariate framework. However, the economics of these models is somewhat restrictive. Essentially, a univariate model assumes that the response of a given interest rate to an innovation in a second interest rate is independent of any movement that innovation may engender in other capital markets. Since most capital markets are subject to multiple influences, this is a questionable assumption.

To illustrate the deficiency, consider three economies (A,B, and C) which engage in trade in financial goods. When the system is in equilibrium, the real interest rates are assumed to be in equal ($r_A = r_B = r_C$). Now, suppose that there is an exogenous increase in r_A. In response, capital flows from both B and C to A. These capital movements decrease the amount of capital available in B and C. As a result, both r_B and r_C increase. If B and C are identical, then the increases in their interest rates will be the same. Therefore, the relative return on financial instruments between these two countries will be unchanged. However, if there are differences in the way the two economies react to the exogenous increase in r_A, then the trade dynamics between B and C will also be affected. Therefore, if economies are mutually integrated, a shock to one differential (or rather one set of differentials) is reflected potentially in all the differentials.

Some authors have made efforts to consider the interactions among multiple interest rates. The first such attempts were vector autoregressions (VAR), such as those of Swanson (1987) and Karfakis and Moschos (1990). More recent work has applied the method of Johansen (1988) and Johansen and Juselius (1990) and performed co-integration studies of groups of interest rates. Representative examples of this line of investigation are Goodwin and Grennes (1994), Chinn and Frankel (1995) and Monadjemi (1998)[14].

Regardless of the specific approach taken, prior studies share a significant failing: they do not account for transactions costs. As was shown in Section 0.3 transactions costs are known to induce neutral bands where arbitrage may not occur. Ignoring these bands can result in unreliable parameter estimates and, therefore, suspect inferences.

This chapter endeavors to expand the current understanding of the linkages among international capital markets by applying a multivariate threshold model to *ex ante* real interest rates. To the author's knowledge, this marks the first time the group dynamics of these interest rates have been investigated in the context of threshold behavior. The results are expected to reveal that significant cross-country effects do exist and, therefore, that univariate methods do not adequately describe the interest rate relationships among the countries considered.

[14] It should be noted that because they make different assumptions about the time series properties of the data, the VAR and co-integration approaches are exclusive of each other.

0.4.2 Statistical Methods

Vector Autoregression

Before implementing the multivariate threshold method, it is prudent to perform a preliminary multivariate analysis. This work serves two purposes. First, comparison of these results to studies in the literature can provide confidence in the data and functional form. Second, a simpler specification can function as a benchmark which helps to highlight any changes in inference attributable to the new method.

Since the *ex ante* real interest rates in the data set are known to be stationary (see Section 0.3.3), a logical choice for a preliminary analysis is the vector autoregression (VAR). In general terms, a VAR can be written as

$$\mathbf{x}_t = \Gamma_0 + \Gamma_1 \mathbf{x}_{t-1} + \mathbf{\Sigma}_t \tag{24}$$

In this notation, $\mathbf{x}_t$ is a $q \times 1$ vector of stationary variables, Γ_0 is a $q \times 1$ coefficient matrix, Γ_1 is a $q \times q$ coefficient matrix, and $\mathbf{\Sigma}_t$ is a $q \times 1$ random vector.

Because the focus of the chapter is how interest rates converge toward equilibrium, it is natural to consider a VAR based on *ex ante* real interest rates differentials. As opposed to the previous analyses which used the differential between the U.S. and a single foreign rate, the VAR can accommodate larger sets of differentials. The key issue is to determine which rate differentials to estimate together. To maintain some semblance of similarity to the work above, the system will comprise five variables, each being a differential between the U.S. *ex ante* real interest rate and a foreign interest rate of the same types. That is, set in the framework of Equation (24)

$$\mathbf{x}_t = \begin{pmatrix} (r_{US,t} - r_{Can,t}) \\ (r_{US,t} - r_{Ger,t}) \\ (r_{US,t} - r_{Jap,t}) \\ (r_{US,t} - r_{Swi,t}) \\ (r_{US,t} - r_{UK,t}) \end{pmatrix}$$

With such a complex system, half-lives of the type calculated in Section 0.3 may not adequately summarize the dynamics. To more fully represent the results, impulse response curves will be created. Impulse response curves graphically trace the progression of an innovation from time t to equilibrium, based on the assumption that no other disturbances arise. By using this technique, the impact of all the variables on convergence behavior can be accounted for.

To compare the results from the VAR analysis to those from Section 0.3, half-lives will be calculated. The simplest method is based on the impulse response data curve data. For a given impulse response curve, reversion to the long-run mean is assumed to be replicable by an AR(1) process. Using the impulse response curve data, the AR parameter can be found and, consequently, the half-life can be calculated. As an example, consider a data series $(c_0, c_1, \ldots, c_T)$, which has been created as above. For simplicity, c_t is assumed to be a zero-mean series. To derive a half-life, it is presumed that the data series can be modeled as an AR(1) process, so that

$$c_t = \lambda c_{t-1}$$

From the properties of an AR(1) series, it is possible to derive λ as follows

$$c_T = \lambda^T c_0$$

$$\lambda^T = \frac{c_T}{c_0}$$

$$\lambda = \left(\frac{c_T}{c_0}\right)^{1/T}$$

With λ in hand, an implicit half-life can be found as before using the formula from Section 0.3. Note that the derivation is predicated on the impulse response curve data series exhibiting monotonic reversion characteristics. If the actual behavior is more complicated, then the implied half-life is a less than ideal way to summarize the reversion dynamics.

To produce innovations, each of the *ex ante* real interest rates (including the U.S. rate) will be altered in turn. For each of these, the effect on all 5 differentials will be presented. Therefore, this single model will result in 6 sets of impulse response curves, each comprising 5 graphs. That is, an initial change in the Canadian real interest rate will produce one set of curves, while a change in the German real interest rate, which could engender different behavior, will produce a second set.

Multivariate Threshold Method

Once the benchmark analysis is complete, the next step is to estimate the multivariate threshold model. This work utilizes the method of Tsay (1998). Essentially, Tsay takes advantage of the theoretical similarity between testing for thresholds and testing for structural breaks to develop a feasible econometric method. To illustrate, start with a simple, univariate threshold model

$$x_t = \beta_0 + \tau_t \rho_1 x_{t-d} + (1 - \tau_t)\, \rho_2 x_{t-d} + \epsilon_t \tag{25}$$

where

$$\tau_t = \begin{cases} 1 & z_{t-d} \geq \delta \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

In Equation (26), the variable z_{t-d}, which may or may not equal x_{t-d}, is known as the threshold (or forcing) variable. The value d is the threshold lag or the delay[15].

In this simple case, the data set consists of two data series: x, which is the regressor, and z, which is the threshold variable. Implementation of Tsay's method begins by sorting this data set by the threshold variable[16]. In this new data set the threshold variable can be treated as a time index. Therefore, procedures intended to detect structural breaks, when applied to this amended data set, can be used to find thresholds. The value of z_{t-d} at which a "structural" break occurs is actually a threshold value, and corresponds to δ in Equation (26).

To better understand the concept, consider the following simple data set.

t	x_t	x_{t-1}	z_{t-1}
1	3	6	3
2	5	3	4
3	6	5	7
4	5	6	6
5	2	5	4
6	7	2	8
7	6	7	7
8	9	6	2
9	8	9	1
10	5	8	9

where the variables are defined as in the above example. Sorting by the threshold variable (z_{t-1}), results in the new data set

[15] To promote consistency with the analysis of Section 0.3, d will be set to 1.

[16] Note that sorting the data set does not affect the time subscripts in the model. Therefore, the autoregressive characteristics remain unchanged.

t	x_t	x_{t-1}	z_{t-1}
9	8	9	1
8	9	6	2
1	3	6	3
2	5	3	4
5	2	5	4
4	5	6	6
3	6	5	7
7	6	7	7
6	7	2	8
10	5	8	9

Notice that even though the data set has been rearranged, x_t and x_{t-1} have not been changed for a given observation. Therefore, the parameter estimates will be unaffected by the reorganization. Consider the implications of applying a method for determining structural breaks to this second data set, but using z_{t-1} as the index rather than the observation number. Using such a method, one might find a structural break between the observation which has $z_{t-1} = 3$ and the observation which has $z_{t-1} = 4$. This would imply that the autoregressive behavior of the system is different when $z_{t-1} \leq 3$, as opposed to when $z_{t-1} > 3$. This is the very idea which motivates the TAR model. Therefore, by sorting the data applying a structural break technique, thresholds can be discerned.

Tsay develops a criterion for selecting an estimate of δ that leads to consistent parameter estimates. Continuing to refer to the simple example of Equation (25), let $\Sigma(z_{t-d})^-$ be the covariance matrix from a regression based on those observations with values of the threshold variable less than or equal to z_{t-d}. Similarly, let $\Sigma(z_{t-d})^+$ be the covariance matrix from a regression based those observations with values of the threshold variable greater than z_{t-d}. Then, for a given d, the threshold is chosen as

$$\hat{\delta} = \text{argmin}_{z_{t-d}}[\text{tr}\Sigma(z_{t-d})^{-} + \text{tr}\Sigma(z_{t-d})^{+}] \tag{27}$$

where tr signifies the trace operator. The multivariate threshold procedure will be applied to the model of *ex ante* real interest rate differentials found in the benchmark analysis.

A crucial element in Tsay's method is the choice of threshold variable. Since the data will be sorted on the basis of this variable, only scalar variables can be considered. An obvious candidate would be to use the bilateral difference between the U.S. and a single foreign interest rate. Further reflection, however, suggests that this would not be a suitable choice.

To illustrate, consider how the analysis would proceed if the threshold variable were the U.S.-Canada real interest rate differential. Application of Tsay's technique would result in a model with two sets of parameters, one which is applicable when this differential is "large" and another when it is "small." As a next step, this model would be used to create impulse response curves by "shocking" the Canadian rate by some amount (typically, 2 times its standard deviation) and setting all the other variables in the system to zero. This will produce a set of predicted values that are then fed recursively into the system, producing the data for the impulse response curves. The difficulty arises in determining the parameter matrix appropriate for calculating the next predicted value. In this example, whether the system is in-band or out-of-band depends only on the value of the U.S.-Canada differential. Another variable in the system, say the difference between the U.S. and the German real interest rate, could grow arbitrarily large (or small) and have no impact on the choice of parameter set. Therefore, using a single differential as the threshold variable does not adequately recognize the importance of the other variables in the system.

As an alternative, consider setting the threshold variable equal to an index of all the deviations. That is, let

$$z_{t-d} = \sum_i |(r_{US,t-d} - r_{i,t-d}) - \mu_{US,i}| \tag{28}$$

where $i =$ Canada, Germany, Japan, Switzerland, U.K.. This measure addresses the failing of the bilateral differential by including all of the deviations. Therefore, the selection of the parameter matrix is based on all the differentials. It should be noted that the threshold variable could exceed the threshold even if none of the individual elements did, which strays from the original arbitrage interpretation of the threshold. Also, by only including 5 of the 30 bilateral relationships it is possible to form from the data, the model does not capture all the arbitrage opportunities available. In light of these facts, a better interpretation of z_{t-d} might be that it represents the degree to which global capital markets are in disequilibrium.

0.4.3 Results

VAR results

The logical departure point for discussing the results is the impulse response curves derived from the VAR model. Each of the following figures illustrates the convergence behavior of the system for positive and negative shocks to one of the *ex ante* real interest rates. Because the foreign rates are subtracted from the U.S. rate, however, the interpretations of "positive" and "negative" are not entirely straight-forward. For example, in Figure 33 the positive shock represents an decrease in the Canadian *ex ante* real interest rate, because that decreases the term being subtracted and results in a positive shock to the differential. The converse is also true.

The most readily apparent feature of these graphs is that the interest rate differentials return to their long-run equilibrium values, represented by the dotted lines. Also, note that there is evidence of "overshooting" in several of the relationships, particularly that between the U.S. and Canada. That is, after the initial shock, the differential approaches the long-run equilibrium, but continues to grow (in absolute terms) for a time, then over the course of many weeks returns to the equilibrium value. Regardless of the direction of the innovation in the Canadian real interest rate, the convergence dynamics are the same.

The following table presents the half-lives of the autoregressive relationships implied by the impulse response curve. Notice except for the U.S.-Canada relationship, the differential from the long-run equilibrium experiences a large increase before beginning to converge. This behavior runs counter to the assumptions made above which enabled the calculation of an implied half-life. The half-lives are still presented,

Table 14: VAR with shock to Canadian *ex ante* real interest rate

	Implied Half-Life	
Country	Positive Shock	Negative Shock
U.S.-Canada	11.3489	13.2054
U.S.-Germany	41.2172	37.3792
U.S.-Japan	46.6497	34.1847
U.S.-Switzerland	38.2722	36.5546
U.S.-U.K.	28.9335	21.5886

however, they should be interpreted cautiously.

When compared to the half-lives from Section 0.3.4, the figures in Table 14 are uniformly larger. The differences range from a relatively minor 28.9335 versus 25.3240 for the U.S.-U.K. relationship to a dramatic 46.6497 versus 7.5427 for the U.S.-Japan relationship. Since these values are difficult to interpret, it is probably best to base inferences on the impulse response curves.

Figure 34 presents the behavior of the system in response to shocks to the German *ex ante* real interest rate. Notice that in this case, the U.S.-German differential does not exhibit any overshooting, but instead returns monotonically to its long run equilibrium. The half-life is found to be about 20 weeks. This is similar to the one found in Section 0.3.

There are two striking differences between Figures 33 and Figure 34. First, not all of the interest rate relationships return to their long-run equilibria. For both Japan and Switzerland, any shock to the German interest rate results in a permanent increase in the differential with the U.S. Both seem to respond to a change in the German interest rate by increasing. However, the magnitude of the change is extremely small. For Japan, the difference is approximately 0.025%, and the one for

Table 15: VAR with shock to German *ex ante* real interest rate

	Implied Half-Life	
Country	Positive Shock	Negative Shock
U.S.-Canada	59.7472	162.4599
U.S.-Germany	23.6349	13.2425
U.S.-U.K.	46.7918	408.1031

Switzerland is even smaller. Therefore, it is legitimate to question the economic significance of these results. Second, the responses are not symmetric; the differentials respond differently based on the direction of the change in the German real interest rate.

Table 15 presents the implied half-lives for these differentials. Since the U.S.-Japan and U.S.-Switzerland differentials do not converge to their long-run equilibrium values, half-lives do not hold any real meaning for these relationships. Therefore, they are not presented in the table below.

As with the previous set of half-lives, 4 of the 5 sets of impulse response curves initially exhibit an increase in the deviation from the long-run equilibrium. The two which are included in Table 15 have much larger implicit half lives than found in Section 0.3. The U.S.-German differential, however, appears to converge monotonically. Appropriately, the implied half lives for this relationship (23.6349 and 13.2425) are fairly close to the half-life from the bilateral analysis (16.6817).

Figure 35 illustrates the behavior of the interest rate differentials in response to innovations in the Japanese *ex ante* real interest rate. Similar to the U.S.-Germany differential, the U.S.-Japan differential also returns monotonically to its long-run equilibrium value. Among the other countries, the U.S.-Canada and U.S.-U.K. differ-

Table 16: VAR with shock to Japanese *ex ante* real interest rate

	Implied Half-Life	
Country	Positive Shock	Negative Shock
U.S.-Canada	28.4216	29.3730
U.S.-Japan	19.1889	19.8656
U.S.-U.K.	185.8761	12.4283

entials also converge, however, the U.S.-Japan and the U.S.-Switzerland differentials do not. This same behavior was noted when Germany was the catalyst for change, which leads to the suspicion that these three countries form some type of subgroup.

Much like what was found in the results for an innovation in the Canadian interest rate, there is evidence of overshooting in the differentials between the U.S. and Canada, Germany, and Switzerland. In fact, the U.S.-Germany and U.S.-Switzerland differentials pass the long-run equilibrium twice before settling upon a different value. Once again, it should be noted that, ultimately, these series exhibit very small deviations from the long-run equilibrium which may not be significant.

Table 16 presents the implied half-lives for the system based on changes to the Japanese real interest rate. Notice that for reasons outlined above, the calculated half-lives for the U.S.-Germany and U.S.-Switzerland differentials are omitted.

Since the convergence behavior for the interest rate differentials generally is not smooth in this case, the results in Table 16 are similar to previous results for implied half-lives. Notice that the U.S.-Japan differential does exhibit monotonic convergence. As might be expected, the implied half-lives of roughly 19 weeks for this series is closer to the result from Section 0.3 than for either of the other two series presented.

The behavior of the system in response to a change in the Swiss *ex ante* real interest rate is shown in Figure 36. As with the U.S.-Germany differential with the U.S.-Switzerland differential converges monotonically. Also, in a further illustration of what has been seen above, overshooting is apparent in the U.S.-Japan and U.S.-U.K. differentials. And, as might be expected, the German and Japanese differentials do not converge to precisely the long-run equilibrium.

The half-lives implied by this set of impulse response curves can be found in Table 17. The half-lives for the U.S.-Japan and U.S.-German differentials are not presented, as these series do not converge.

Table 17: VAR with shock to Swiss *ex ante* real interest rate

	Implied Half-Life	
Country	Positive Shock	Negative Shock
U.S.-Canada	26.5665	24.2320
U.S.-Switzerland	21.6545	11.8497
U.S.-U.K.	44.0321	111.5766

Similar to the implied half-lives for the systems which shock the U.S.-German and U.S.-Japan differentials, the U.S.-Switzerland differential displays monotonic convergence while the other differentials in the set do not. Once again, the implied half-lives for this differential are closer to their bilateral counterpart than are the U.S.-Canada and U.S.-U.K. half-lives.

The results from innovations to the U.K. *ex ante* real interest rate are presented in Figure 37. The differential with the U.S. converges rather quickly to its long-run equilibrium, although it appears to slightly overshoot it long-run equilibrium. All the other differentials converge as well[17], however none of the relationships equilibrate

[17]For the U.S.-Germany and U.S.-Switzerland differentials, the convergence is not clear at 100

directly; overshooting is present in all.

The half-lives implied by this set of impulse response curves can be found in Table 18.

Table 18: VAR with shock to U.K. *ex ante* real interest rate

	Implied Half-Life	
Country	Positive Shock	Negative Shock
U.S.-Canada	28.1340	19.0397
U.S.-Germany	26.3796	-98.9241
U.S.-Japan	103.8770	137.4934
U.S.-Switzerland	44.0183	-151.0957
U.S.-U.K.	15.4942	13.3695

The pattern seen in all of the half-life sets thus far is repeated in this table. Note that the negative half-lives for the U.S.-Germany and U.S.-Switzerland relationships are a result of the slow convergence for those two series. Over the range of observations used to calculate the implied half-life (observations 1 to 100) these series have not converged sufficiently to result in a positive half-life.

Figure 38 represents the effects of innovations to the U.S. real interest rate. Notice that because the U.S. rate enters as a positive term in all the differentials, a positive shock indicates a positive change to the U.S. rate. Also, the U.S. rate is a component of all the differentials, therefore all of the initial periods exhibit a change.

As might be expected, the graphs indicate a high degree of integration between the countries. Each differential returns to its equilibrium and does so without large deviation. Also, there is evidence of overshooting in all the graphs, with this feature being most prominent in the differentials with the Germany, Japan, and the U.K.

periods from the initial shock. Graphs which present the curve out 300 periods, which are not presented, confirmed that these series do converge.

The half-lives implied by this set of impulse response curves can be found in Table 19.

Table 19: VAR with shock to U.S. *ex ante* real interest rate

	Implied Half-Life	
Country	Positive Shock	Negative Shock
U.S.-Canada	9.0915	11.7263
U.S.-Germany	19.4622	16.1886
U.S.-Japan	19.2762	17.4614
U.S.-Switzerland	18.1428	14.9087
U.S.-U.K.	13.2956	9.7186

All of the series exhibit convergence speeds which are close to the bilateral calculations. Two notable exceptions are the U.S.-Japan relationship, which had a faster bilateral convergence of approximately 7.5 weeks, and the U.S.-U.K. relationship, which converged much more slowly in the bilateral context (approximately 25 weeks).

Multivariate Threshold Results

Having developed a baseline of VAR responses, the next step is to examine how the inclusion of thresholds alters the behavior of the system. Figure 39 shows the impact of a change in Canadian real interest rate. The most noticeable change is that the differentials are less monotonic than before. Several series, for example the U.S.-Switzerland differential approach the long-run equilibrium but retreat from it before equilibrium is attained. Overall, the impression is one of a sine wave of decreasing amplitude. As will be seen, this general comment applies to all the subsequent charts as well.

Another prominent feature is that none of the differentials return to the long-run equilibrium value[18]. Additionally, the new stable point is larger than the long-run equilibrium differential, except for the U.S.-Canada differential, which is less than before. While this may seem odd, it is in fact consistent with Figures 25 to 27 (which show the real interest rate differentials) and the concept of thresholds. It is plausible that when the differentials are in the band on the basis of the index, that they "float" toward the threshold. Any movement large enough to move the threshold variable "out-of-band" is quickly reduced.

As a final difference between Figure 39 and its VAR analog, there is a marked asymmetry between the behavior induced by a positive and a negative shock. A positive shock, which represents a decrease in the Canadian real interest rate, engenders much larger movements in all the differentials. A discussion of what this result might signify is deferred to the conclusion.

Figure 40 displays the graphs for changes in the German *ex ante* real interest rate. As with the previous figure, none of the differentials return to their long-run equilibria. The same pattern as above is evident, with only the U.S.-Canada differential settling at a value less than its long-run equilibrium. In contrast to the previous figure, a negative shock (a positive change in the German interest rate) leads to much larger intermediate changes than a positive shock (a negative change in the German interest rate).

The behavior of the system in response to changes in the Japanese real interest rate is shown in Figure 41. This set of graphs mimics those above, with the differentials ultimately reaching a non-equilibrium position for all the country pairs.

[18]It will be seen that none of the differentials converge to their long-run equilibria. Therefore, no half-lives will be presented in association with these estimations.

As before, only the U.S.-Canada series settles to a point greater than its long-run equilibrium value. Also, note that a negative change in the Japanese real interest rate engenders larger responses than a positive shock of the same size.

An interesting feature is that the series are more synchronized than in any of the previous graphs. While it is tempting to search for an economic rationale for this behavior, it is possible that the root cause is statistical. The 2 standard deviation shock to the Japanese *ex ante* real interest rate may not "push" the system far enough out of equilibrium to be subject to the out-of-band behavior. Therefore, all that is presented is the in-band behavior, which does not reflect international capital flows as much as internal conditions, which affect both differentials the same. As support for this supposition, note from Figure 23 that the Japanese real interest rate appears to be more stable than the others in the sample.

Figure 42 traces innovations in the Swiss *ex ante* real interest rate through the system. As has been seen consistently throughout the analysis, none of the differentials return to their long-run equilibria. Also, the two series move together to a large degree, resembling the results from Figure 41. The explanation offered above also applies here, and the relative stability of the Swiss real interest rate (see Figure 23) offers support. In another parallel to the Japanese results, decreases in the Swiss real interest rate generate larger responses.

Figure 43 shows the behavior of the system for changes in the U.K. *ex ante* real interest rate. The themes discussed in association with the previous charts are apparent here. The differentials do not return to the long-run equilibrium. Also, as in the differentials between the U.S.-Canada, U.S.-Japan, and U.S.-Switzerland relationships, movements in response to a decrease in the U.K. real interest rate are larger than those for an increase the this rate.

Figure 44, which shows the effect of innovations in the U.S. real interest rate, falls squarely in line with the previous analysis. As before, none of the differentials return to the long-run equilibrium. Also, positive shocks to the system produce larger disturbances than negative shocks. Recall, however, that the U.S. real interest is a positive term in the differentials. Therefore, this result implies that it is a positive, not a negative, change that has more impact.

0.4.4 Conclusion

From a statistical standpoint, the main results of this analysis are clear. Typical VAR analysis leads to the conclusion that there is convergence in these series. By adding a threshold, two changes become apparent. First, the differentials do not adjust to the long-run equilibrium value, although both positive and negative shocks converge to the same value. Second, convergence, to the degree it is present, is not smooth. Differentials can vary widely and even overshoot their targets before settling to a stable point. Admittedly, such heterodox ideas need to be tested more rigorously before being put forth as fact. For example, confidence intervals for the impulse response curves would provide assurance that the non-convergence results are more than a statistical artifact. Additionally, shocking all series simultaneously, rather than an individual series, could ensure the robustness of the results. Finally, as will be noted in the final chapter, the bilateral *ex ante* real interest rate relationships are not stable throughout the length of the data set. Therefore, a repeating the analysis on an appropriate subset of the data could provide confidence in the results.

Economic conclusions are more difficult to draw. It seems apparent that there is capital market integration. However, it is also true that this integration is not as complete or as simple as would be implied by classical economic theory. However, two prominent economic features can also be discerned from the results.

The first is the asymmetry in the responses. Recall that a negative change in the Canadian, Japanese, Swiss, and U.K. real interest rate has the greater impact, while for Germany and the U.S. the opposite is true. Roughly speaking, this implies that during periods when capital is moving into Germany and the U.S. global capital markets fluctuate more than when these countries are sending capital abroad. Hypothetically, this suggests that these two countries are disproportionately attractive destinations for foreign investment. It also suggests that performing bilateral analyses with Germany as the base could provide interesting results.

The argument in the previous paragraph assumes that causality runs from changes in the real interest rate to changes in capital flows. Although this assumption is common in international economics, it should be noted that, in reality, the real interest rate and the volume of capital flows are jointly determined. Therefore, the root cause of the above asymmetry could be an exogenous change in the demand and/or supply of financial instruments, which impacts the real interest rate. For example, if the demand for German bonds were to decrease, the real interest rate would increase. However, the number of bonds exchanged would actually fall, which runs counter to the example above. This suggests that there is information in capital flows which may be useful in formulating a hypothesis to explain the asymmetry.

The other noteworthy economic feature is the overshooting found in the data.

This analysis is not designed to detect the underlying cause of this behavior. It is clear, however, that after an initial shock capital markets feedback to each other in complicated, and sometimes destabilizing, ways. This further indicates the weakness of bilateral studies.

0.5 Structural Instabilities in Real Interest Rate Relationships

0.5.1 Introduction

Most analyses of RIP posit a single relationship which holds over the entire sample. However, it is possible, perhaps even probable, that there are periodic changes in the relationships among interest rates. If these structural changes are not accounted for, rejections of the null hypothesis are rejections of the hypothesis that no *single* relationship exists, but offer no information on the possibility that multiple relationships exist. Therefore, as with transactions costs above, ignoring structural instability may lead to spurious rejections of RIP.

Recall Figure 25 from Section 0.2. Notice that both series spend time above and below their respective long-run mean differentials. These intervals could correspond to periods of serial correlation, but also could represent changes in the relationship between the two interest rates.

To date, structural instability has not been a prominent feature of the RIP literature. Caporale and Grier (2000), using the method of Bai and Perron (1998) show that changes in the American political regime have a significant effect on the domestic real interest rate. Other papers, such as Fountas and Wu (1999) and Wu and Fountas (2000), use the work of Gregory and Hansen (1996) to show that real interest rate in Europe and the United States exhibit common stochastic trends, accounting for a single structural break. Felmingham *et. al* (2000), apply this same procedure to real interest rates and find evidence for cointegration in real interest rates between Australia and several important trading partners.

This chapter advances the literature in two ways. First, the econometric test used for revealing structural breaks is new to this field, and offers improvements over the more prevalent approaches. Second, the analysis will simultaneously account for structural instability with non-linearities induced by transactions costs. To the author's knowledge this is the first time such an analysis has been attempted, and will be the most thorough analysis of non-linear interest rate relationships to date.

0.5.2 Statistical Methods

Structural Break Determination

There is no universally accepted method for determining when and if structural breaks occur. Among the contenders, however, the likelihood ratio test of Bai (1999) has several advantages. First, unlike some other methods such as Gregory and Hansen (1996), this procedure identifies multiple breaks. Second, the procedure permits testing breaks without conditioning on the previous breaks, such as in Bai and Perron (1998). Finally, the asymptotic distribution of the test is known analytically, which obviates the need for extensive simulation (again, see Gregory and Hansen (1996)).

Consider the model

$$y_t = x_t{\prime}\beta + \epsilon_t \tag{29}$$

where x_t is a qx1 vector and ϵ_t is a white noise error term. Normally, this form would be assumed to hold for the entire span of the data set. However, if there are

m breaks in a data set, then the model would actually be

$$\begin{gathered} y_t = x_t{\prime}\beta_1 + \epsilon_t \\ y_t = x_t{\prime}\beta_2 + \epsilon_t \\ \vdots \\ y_t = x_t{\prime}\beta_{m+1} + \epsilon_t \end{gathered} \tag{30}$$

with each equation estimated over the observations in only one of the $m+1$ subintervals.

Let λ_i be the ith breakpoint in the data set. Then, a partition of the data set is defined as

$$\Lambda_{\pi,m} = \{(\lambda_1, \lambda_2, \ldots, \lambda_m) : \lambda_i - \lambda_{i-1} \geq \pi T \quad i = 1, 2, \ldots, m\} \tag{31}$$

where T is the total number of observations and π is a positive number such that $\pi \in (0, 1)$. Equation (31) is simply defining the minimum interval size.

To understand how the method is implemented, consider that any m-partition delineates $m+1$ intervals of the data. Suppose we were to estimate a model on each of those intervals and find the sum of squared residuals for each model. That is for $i = 1, 2, \ldots, m$, calculate (SSR) be

$$\mathrm{SSR}_i = \min_{\beta_i} \sum_{t=\lambda_{(i)}+1}^{\lambda_{(i+1)}} (y_t - x_t{\prime}\beta_i)^2 \tag{32}$$

After minimizing the (SSR) for each individual model, define the following quantity

$$\mathrm{S}_T(\Lambda_{\pi,m}) = \sum_{i=1}^{m+1} \mathrm{SSR}_i \tag{33}$$

This is the sum of squared residuals for the model under the assumption that the m-partition which was used to define the current intervals is the true partition of the data. The dependence on $\Lambda_{\pi,m}$ arises from the fact that the partition determines the subintervals of the data, which affects the sum of squared residuals for each subinterval and, consequently, the overall value.

Now, define the optimal value of $\mathrm{S}_T(\Lambda_{\pi,m})$ as

$$\mathrm{S}_T(\Lambda^*_{\pi,m}) = \min_{\{\lambda_i\}} \sum_{i=1}^{m+1} \mathrm{SSR}_i \tag{34}$$

That is, the best m-partition of the data is the one that minimizes the overall sum of squared residuals. The likelihood ratio test statistic developed by Bai (1999), uses the test statistic

$$\mathrm{B}_T(m^0+1|m^0) = \frac{\mathrm{S}_T(\Lambda^*_{\pi,m^0}) - \mathrm{S}_T(\Lambda^*_{\pi,m^0+1})}{\frac{1}{T}\mathrm{S}_T(\Lambda^*_{\pi,m^0+1})} \tag{35}$$

to test the hypothesis

$$\begin{array}{l} \text{H: } m = m^0 \\ \text{A: } m = m^0 + 1 \end{array} \tag{36}$$

Note that the test also is consistent versus A: $m > m^0$. In practice, $\mathrm{S}_T(\Lambda^*_{\pi,m})$ is found by using a grid search method and iterating over the possible values of $\mathrm{S}_T(\Lambda_{\pi,m})$.

Under the null hypothesis, the statistic B_T has a limiting distribution such that[19]

$$\lim_{T\to\infty} \Pr(\mathrm{B}_T(m^0+1|m^0) \geq c) = 1 - \prod_{i=1}^{m^0+1} [1 - G_i(c)] \tag{37}$$

with

$$G_i(c) = \frac{c^{q/2}\exp(-c/2)}{2^{q/2-1}\Gamma(q/2)} \left[\left(1 - \frac{q}{2}\right)\ln(1-\eta_i)\eta_i^{-1} + \frac{2}{c} + o(c^{-2})\right] \tag{38}$$

$$\eta_i = \frac{T\pi}{\lambda_i - \lambda_{i-1}} \tag{39}$$

The expression in Equation (37) can be used to find critical values of any desired size and perform inference on the significance of the additional breakpoint.

[19]The expression for $G_i(c)$ was originally derived in DeLong (1981).

Models

In the discussion above, a simple linear model was estimated on each subinterval. To fulfill the intent of the chapter a different model, one which accounts for threshold effects, will be used. On the basis of Section 0.3, the non-parametric autoregression would seem like an obvious choice. However, estimation of this model is inconsistent with the Bai procedure[20]. Therefore, the model fit to each subsection is the threshold autoregression (TAR) which served as the benchmark analysis in Section 0.3. That is the model is

$$\Delta z_t = \beta_0 + \tau_t \rho_1 z_{t-1} + (1 - \tau_t)\, \rho_2 z_{t-1} \epsilon_t \tag{40}$$

where

$$\tau_t = \begin{cases} 1 & |(r_{1,t-1} - r_{2,t-1}) - \mu_{1,2}| \geq \delta \\ 0 & \text{otherwise} \end{cases} \tag{41}$$

and $z_t = r_{1,t} - r_{2,t}$ represents the difference between two *ex ante* real interest rates. For this work, the bilateral differences between the U.S. and each of the other countries in the data set will be considered.

Use of this model complicates the analysis. First, a dual grid search must be performed – one over the possible break points and one over the potential threshold values. Second, once the break points have been determined and models have been fit to each subset of the data, the significance of the threshold effects must be tested using Hansen's procedure. Despite these difficulties, however, the TAR does meet the primary criteria: the final model is the one that minimizes the squared error.

At this point, the scarcity of results in the literature makes it difficult to deter-

[20]More exactly, the model chosen must be estimated by minimizing the squared error. Unfortunately, it is possible to fit a non-parametric autoregression such that the squared error is zero, in which case the Bai statistic is undefined.

mine *a priori* what can be expected from this analysis. Economists generally hold, however, that global capital markets have become more integrated over the past 20 years (see, for example, Obstfeld and Taylor (2002)). Therefore, while the specific form of the models will change over time, the progression should indicate a trend toward more accessible capital markets.

0.5.3 Results

Although the method above is fairly straight-forward, it produces a rather large set of output. In an effort to provide order to the discussion, the results will be presented on a country-by-country basis first. Then, the overall results will be discussed in a separate section which refers to all the countries.

Canada

Consider Table 20, which lists the results of the Bai procedure for the U.S.-Canada real interest rate relationship.

Table 20: Bai test results - Canada

No. of Breaks		Test
Null	Alt.	Statistic
0	1	4.7531

† significant at the 5% level.
‡ significant at the 10% level.

The Bai test statistic fails to reject the hypothesis of no structural breaks. Subsequently, the threshold model is fit to the entire span of the data. Although this analysis appears in Section 0.3.3, it is repeated here for convenience.

Table 21 presents the results for the TAR model on this data set. The Wald

Table 21: Regimewise Hansen tests - Canada

Regime	Wald	p-value
1	9.1744	0.0120

The Wald statistic tests the equality of the in-band and out-of-band autoregressive parameters

statistic is significant at the 5% level, indicating that threshold effects are present. The next section presents that model.

Regime 1: January 8, 1979 to August 20, 2001

The Canadian data calls for a TAR model, which is presented below.

$$\Delta z_t = \underset{(0.0115)}{-0.0546} + \tau_t \underset{(0.0146)}{(-0.1069)} z_{t-1} + (1-\tau_t) \underset{(0.0128)}{(-0.0606)} z_{t-1}$$

Threshold = 0.6824
In-Band Half-Life = 11.0879
Out-of-Band Half-Life = 6.1310

As was noted previously, the half-lives for this specification are faster than those found in the literature. Additionally, there is evidence of rapid convergence both in and out-of-the band. However, the threshold is larger, by an order of magnitude, than those suggested by previous work. Again, for a more thorough discussion of this model, see Section 0.3.4.

Germany

The next section investigates the bilateral relationship between the U.S. and German *ex ante* real interest rates. Following the structure of the previous section, Table 22 presents the results of the Bai procedure.

Unfortunately, the results are not definitive. Using the 5% critical value leads to the conclusion that there are two regimes (i.e., one break) present in the data.

Table 22: Bai test results - Germany

No. of Breaks Null	Alt.	Test Statistic
0	1	38.3986†
1	2	11.5806‡
2	3	1.4932

† significant at the 5% level.
‡ significant at the 10% level.

However, use of the 10% critical value indicates three regimes (i.e., two breaks). In the interest of prudence, both possibilities will be considered.

Case 1 (One Break)

If there truly is only one structural break in the data series, then there are two regimes. The Wald statistics for threshold effects in the two regimes are shown in Table 23.

Table 23: Regimewise Hansen tests - Germany (1)

Regime	Wald	p-value
1	7.5227	0.0680
2	1.7400	0.7580

The Wald statistic tests the equality of the in-band and out-of-band autoregressive parameters

As if things were not complicated enough, the Wald for the first regime presents additional uncertainty. The statistic is not significant at the 5% level, which would indicate the absence of threshold effects. However, in light of the results of the previous section, it seems rash to discount the existence of thresholds on the basis of a statistic that is significant at the 10% level. Therefore, both an AR and a TAR will be fit to this subset of the data. As for the second regime, threshold effects clearly are not present.

Regime 1: January 8, 1979 - April 26, 1982

On the basis of the Wald statistic above, two models are fit to this regime, an AR and a TAR. The AR model is found to be

$$\Delta z_t = \underset{(0.0204)}{0.0245} + \underset{(0.0238)}{(-0.0493)} \; z_{t-1}$$

$$\text{Half-Life} = 13.7103$$

Alternatively, the TAR

$$\Delta z_t = \underset{(0.0203)}{0.0104} + \tau_t \; \underset{(0.0233)}{(-0.0492)} \; z_{t-1} + (1-\tau_t) \; \underset{(0.1448)}{(0.3427)} \; z_{t-1}$$

$$\begin{gathered}\text{Threshold} = 0.0452 \\ \text{In-Band Half-Life} = N/A \\ \text{Out-of-Band Half-Life} = 13.7389\end{gathered}$$

Interestingly, the two models suggest similar convergence speeds for large deviations, approximately 14 weeks. Of course, the TAR modeling process results in a second half-life being estimated for small deviations. In this case, the parameter value suggests that no convergence occurs inside the band.

At this point, an obvious exercise would be to search for economic events which could serve to explain the hypothesized change in the U.S.-German interest rate relationship. However, there is a serious *caveat* to this method of inquiry. It is deductive reasoning, and it is the antithesis of the scientific method. Given a hypothesis, picking and choosing from the data to find support is often an easy matter. This point notwithstanding, it is important to place the supposed break dates in an historical context. The intent, however, is not to identify the root cause of a break, but rather to present evidence that the timing is feasible.

So, what can said about this particular break point, April 26, 1982? In June of this year, the mark was re-valued versus several major currencies. For example, the mark was adjusted by 10.5% versus the French franc, by 7% versus the Italian lira and by 4.5% versus the other EMS currencies. While these adjustments do not explicitly involve the U.S., it is conceivable that the change in the relative values of these currencies affected the patterns of investment and trade.

Regime 2: May 3, 1982 - August 20, 2001

For this interval, the model was plainly an AR.

$$\Delta z_t = \underset{(0.0021)}{-0.0020} + \underset{(0.0070)}{(-0.0254)} \; z_{t-1}$$

$$\text{Half-Life} = 26.9412$$

The half-life for all deviations in this regime is just short of 27 weeks, which is much larger than the AR fit to the first regime (26.9 weeks versus 13.7 weeks). Given the evidence in favor of increasing capital market integration (see Obstfeld and Taylor (2002)), it seems unlikely that the relationship has changed from one AR specification to a slower one. It seems more likely that there existed a threshold model in the first regime, which then changed to a model without threshold effects.

Case 2 (Two Breaks)

The previous mini-section presented one of two possible partitions of the data for the U.S.-Germany relationship. The second partition consists of two breaks. The Hansen tests for threshold effects in the 3 regimes implied by these breaks are presented in Table 24.

In a frustratingly familiar scenario, the Wald statistics fail to provide conclusive results for the presence of thresholds in the first and second regimes. In the third

Table 24: Regimewise Hansen tests - Germany (2)

Regime	Wald	p-value
1	7.6456	0.0550
2	5.5402	0.0980
3	4.6208	0.1990

The Wald statistic tests the equality of the in-band and out-of-band autoregressive parameters

regime, however, a non-threshold model is indicated.

January 8, 1979 - May 17, 1982

As noted above, the estimation for this regime consists of an AR model

$$\Delta z_t = \underset{(0.0200)}{0.0243} + \underset{(0.0236)}{(-0.0493)} \; z_{t-1}$$

$$\text{Half-Life} = 13.7103$$

and a TAR model

$$\Delta z_t = \underset{(0.0199)}{0.0104} + \tau_t \underset{(0.0231)}{(-0.0493)} \; z_{t-1} + (1-\tau_t) \underset{(0.1436)}{(0.3427)} \; z_{t-1}$$

$$\text{Threshold} = 0.0408$$
$$\text{In-Band Half-Life} = N/A$$
$$\text{Out-of-Band Half-Life} = 13.7103$$

These models are virtually indistinguishable from their counterparts in the one break case. This is because this time period contains only two additional weeks of data. For details, please refer to the previous discussion.

May 24, 1982 - April 1, 1985

The Wald test for the second regime also indicated both an AR and a TAR. The AR model is

$$\Delta z_t = \underset{(0.0084)}{-0.0021} + \underset{(0.0311)}{(-0.0698)} \; z_{t-1}$$

$$\text{Half-Life} = 9.5797$$

Notice that, at less than 10 weeks, this half-life is the smallest seen so far. The TAR model is found to be

$$\begin{array}{ccccccccccc} \Delta z_t & = & -0.0065 & + & \tau_t & (-0.1110) & z_{t-1} & + & (1-\tau_t) & (0.0595) & z_{t-1} \\ & & (0.0085) & & & (0.0352) & & & & (0.0628) & \end{array}$$

Threshold = 0.2725
In-Band Half-Life = N/A
Out-of-Band Half-Life = 5.8912

There are two important features to note for this specification. First, the threshold value is dramatically smaller than those found in Section 0.3, although it is still somewhat larger than might be expected. Second, large deviations are closed at a relatively rapid pace, just short of 6 weeks.

Can the break point – April 1, 1985 – be justified to the minimal standard set above? In May of 1985, the Bundesbank eliminated provisions which regulated access to the German capital market. It is easy to see how relaxing these restrictions would make arbitrage easier and/or more profitable and therefore alter the relationship between the U.S. and the German interest rates.

April 8, 1985 - August 20, 2001

For the final subset of the data, the Wald test firmly rejected the TAR model in favor of an AR specification. The AR was estimated as

$$\begin{array}{cccccc} \Delta z_t & = & -0.0022 & + & (-0.0183) & z_{t-1} \\ & & (0.0019) & & (0.0063) & \end{array}$$

Half-Life = 37.5293

As was seen in the one break case, the half-life implied by this model is larger than either of the out-of-band half-lives from the TAR models or those from the AR models. However, in an AR process all deviations are eventually arbitraged away, whereas in the TAR models small deviations can persist indefinitely. Therefore, one could argue that, categorically, an AR model is an indication of a higher degree of integration than a TAR model. Given the (admittedly anecdotal) evidence for increasing capital market integration over this period, perhaps it makes most sense to consider only the TAR models in the first two regimes. Even this explanation, however, is not without its pitfalls. The threshold for the first regime is markedly smaller than that for the second (0.0484 versus 0.2725), a result which is inconsistent with increasingly open capital markets. Overall, it is difficult to create a reasonable "narrative" from the two break case, which probably makes the one break alternative more attractive.

Comparing these results with those of Section 0.3 highlights the effects of structural instability. By breaking up the data set, two very different functional forms were found. Note that the TAR of the first subset of the data implies faster convergence than the AR from that section (13 weeks versus 21 weeks). However, the AR of the second subset of the data, with a half-life of approximately 27 weeks shows slower convergence. This result should not be unexpected, however; The situation is analogous to the way the TAR separates two different types of responses. The fast convergence in the first regime moderates the slower convergence in the second, resulting in an intermediate value for specifications estimated on the entire data set.

Japan

Compared to the minor chaos of the U.S.-Germany example, the results for testing for structural breaks in the U.S.-Japan interest rate relationship are refreshingly clear. Table 25 presents the Bai test results.

The Bai procedure indicates one break in the data series. Therefore, models must

Table 25: Bai test results - Japan

No. of Breaks		Test
Null	Alt.	Statistic
0	1	37.5897†
1	2	5.6884

† significant at the 5% level.

be estimated on two subsets of the data. Table 26 presents the Wald test statistics for the two regimes.

Table 26: Regimewise Hansen tests - Japan

Regime	Wald	p-value
1	7.8348	0.0650
2	8.5171	0.0210

The Wald statistic tests the equality of the in-band and out-of-band autoregressive parameters

In the first regime, the test procedure does not clearly resolve to either a TAR or an AR model. However, there is unequivocally a threshold in the second regime.

Regime 1: January 8, 1979 - May 25, 1981

Following the precedent set previously, both an AR and a TAR model will be fit to the first regime. The AR model is given as

$$\begin{array}{ccccc} \Delta z_t & = & \underset{(0.0449)}{0.0591} & + & \underset{(0.0269)}{(-0.0382)} \; z_{t-1} \end{array}$$

$$\text{Half-Life} = 17.7964$$

At first glance, this specification seems to imply a fairly rapid convergence between the U.S. and the Japanese real interest rates. However, neither the intercept nor the AR coefficient are significantly different from 0. Accordingly, the differential between the U.S. and the Japanese real interest rate follows a random walk during this period.

The TAR model is estimated as

$$\Delta z_t = \underset{(0.0489)}{-0.0031} + \tau_t \underset{(0.0428)}{(-0.1332)} z_{t-1} + (1-\tau_t) \underset{(0.0314)}{(0.0106)} z_{t-1}$$

Threshold = 1.8084
In-Band Half-Life = N/A
Out-of-Band Half-Life = 4.8490

The most striking feature of this model is the speed of convergence outside of the neutral band. If half of a given deviation dissipates in less than 5 weeks, that would imply a high degree of integration. Of course, the strength of this claim is tempered by the size of the threshold. As noted before, a threshold of this size is larger than those typically found in the literature.

The period surrounding this breakpoint was one of important structural changes in the Japanese economy. The Foreign Exchange and Foreign Trade Control Law, which was passed in December of 1980, required relaxation of capital controls. Also, at the end of May of 1981, tariffs were eliminated or reduced on 215 products. During the same month, the banking laws were substantially revised. As a result, banks could trade in public sector bonds and foreign banks were allowed greater access to Japanese capital markets. All of these events suggest that the economy became dramatically more open to foreign involvement around this time.

Regime 2: June 1, 1981 - August 20, 2001

On the basis of the Wald test statistics in Table 26, a TAR is fit to this subset of the data

$$\Delta z_t = \underset{(0.0091)}{0.0388} + \tau_t \underset{(0.0082)}{(-0.0582)} z_{t-1} + (1-\tau_t) \underset{(0.0089)}{(-0.0377)} z_{t-1}$$

Threshold = 0.4233
In-Band Half-Life = 18.0371
Out-of-Band Half-Life = 11.5597

This model has several noteworthy characteristics. First, convergence is present in the band, which was not the case in Section 0.3.4. Although the out-of-band half life is larger than in Section 0.3.4 the threshold, while still large by some standards, is greatly reduced. A comparison with the models from the first regime, however, does not yield such unambiguous results, particularly when considering the two TAR models. Although the first TAR model has a larger threshold (1.8084 versus 0.4233), the out-of-band half life is significantly smaller for the earlier example (5 weeks versus 11.5 weeks). But, the model estimated on the second regime exhibits convergence in the band, while the one for the first regime does not. For those seeking the most rational story, the strongest case is for the relationship to proceed from a random walk in the first period to a TAR in the second. Even if the first period model is a TAR, however, it could be argued that the smaller threshold and the convergence for deviations of all size indicate a stronger bond in the later time frame.

Switzerland

Table 27 shows the results of the Bai test procedure for the U.S.-Switzerland *ex ante* real interest rate relationship.

Table 27: Bai test results - Switzerland

No. of Breaks		Test
Null	Alt.	Statistic
0	1	30.0816†
1	2	14.2859†
2	3	4.6488

† significant at the 5% level.

Clearly, there are two breaks which delineate three regimes. To determine the type of models to be fit, consider the Hansen test results in Table 28.

Table 28: Regimewise Hansen tests - Switzerland

Regime	Wald	p-value
1	4.4512	0.2350
2	12.1638	0.0050
3	4.0936	0.2480

The Wald statistic tests the equality of the in-band and out-of-band autoregressive parameters

The first period calls for an AR model, as does the third. The statistic for the second regime suggests a threshold model for that regime.

Regime 1: January 8, 1979 - December 28, 1981

The AR model estimated is

$$\begin{array}{rcccc} \Delta z_t & = & \underset{(0.0578)}{0.0655} & + & \underset{(0.0241)}{(-0.0315)} \; z_{t-1} \end{array}$$

$$\text{Half-Life} = 21.6563$$

Although this model would seem to indicate a reasonable amount of integration, neither statistic is significantly different from zero. Therefore, the U.S.-Switzerland interest rate differential, like the U.S.-Japan differential, follows a random walk in this period.

During this time period, the Swiss economy was being affected by a rapidly depreciating franc relative to the U.S. dollar. Therefore, in the middle of 1981, the Swiss central bank abandoned its traditional *laissez faire* approach and dramatically tightened the monetary base. However, it was not until the end of the year that the money supply decreased and net foreign assets did not begin to increase until the fourth quarter.

Regime 2: January 4, 1982 - April 30, 1984

Based on the Wald test statistic above, a TAR model was estimated on this time interval.

$$\Delta z_t = \underset{(0.0656)}{0.3106} + \tau_t \underset{(0.0319)}{(-0.1324)} z_{t-1} + (1-\tau_t) \underset{(0.0373)}{(-0.1800)} z_{t-1}$$

Threshold = 0.2731
In-Band Half-Life = 3.4928
Out-of-Band Half-Life = 4.8805

The threshold for this model is fairly small, at least when compared to that from Section 0.3.4 (1.46% versus 0.27%). However, the half-lives are incongruous, as the in-band half-life is less than the out-of-band value. There is no reason for this to occur, and the only consolation is that in absolute terms, the two differ by only about 9 days. However, it is hard to reconcile these relative values with a plausible economic reality.

The most prominent changes in the Swiss economic landscape during this period were alterations to the definition of money. First, certain private accounts were changed to savings accounts, effectively reducing the M1 money supply by 3% and the M2 money supply by 1.5%. Second, foreign currency held by Swiss citizens was removed from the definition of the money supply, causing reductions of 15% in M2. Finally, central bank statistics were modified to encompass the monetary base of Liechtenstein, which uses the Swiss franc as its official currency. This final change increased M1 by 2.3% and M2 by 4.65%. For the purposes of this analysis, it is important to keep a strict accounting of the final percentage changes in the measures of money supply. Instead, it should be noted that the definitions were in flux, and therefore a key monetary policy target was changing. This could be expected to have an impact on Swiss inflation and exchange rates, which would affect the U.S.-Switzerland real interest rate differential.

Regime 3: May 7, 1984 - August 20, 2001

For the final regime, Hansen's procedure recommended an AR model

$$\Delta z_t = \underset{(0.0140)}{0.0515} + \underset{(0.0081)}{(-0.0304)} \; z_{t-1}$$

$$\text{Half-Life} = 22.4525$$

This model has a significant coefficient on the lagged differential. Relative to the out-of-band parameter in Section 0.3.4, this half-life suggests slower convergence.

While the inconsistency in the second regime's TAR model is troubling, broadly speaking the behavior of the U.S.-Switzerland real interest rate differential follows a reasonable pattern over the course of the data. The models suggest that the two capital markets have become more integrated over time, progressing from a random walk in the first period to a model without transactions costs (i.e., an AR model) in the final period.

U.K.

The Bai procedure results for the final relationship, that between the U.S. and the U.K., are found in Table 29.

Table 29: Bai test results - U.K.

No. of Breaks		Test
Null	Alt.	Statistic
0	1	38.0749†
1	2	18.6717†
2	3	approx. 0

† significant at the 5% level.

From the statistics presented, there appear to be two break points, demarcating three regimes. Table 30 shows the results of Hansen's test for threshold effects.

Table 30: Regimewise Hansen tests - U.K.

Regime	Wald	p-value
1	2.0930	0.7030
2	6.8547	0.0540
3	2.6865	0.4280

The Wald statistic tests the equality of the in-band and out-of-band autoregressive parameters

Once again, the Wald tests report some ambiguity regarding the choice of model in the second regime. As before, both an AR and a TAR will be estimated for these data. For the first and third regimes, the results are definitive and favor an AR specification.

Regime 1: January 8, 1979 - June 22, 1981

The AR model for the first regime is

$$\Delta z_t = \underset{(0.0268)}{0.0035} + \underset{(0.0265)}{(-0.0299)} \; z_{t-1}$$

$$\text{Half-Life} = 22.8339$$

As might be expected on the basis of the discussion of previous sections, the coefficient on the lagged differential is not significantly different from zero. Therefore, as noted above, the differential between the U.S. and the U.K. interest rates can be considered a random walk.

The most significant event in the vicinity of the break point occurred in August of 1981. In this month, the Bank of England began open market operations to maintain short term interest rates within a range of values. However, this value was not disclosed to the public. There are also several changes in the restrictions on bank behavior. For example, the requirement that banks keep 1.5% of their liabilities in Bank of England, non-interest bearing accounts was eliminated.

Regime 2: June 29, 1981 - December 3, 1984

Since the Wald test failed to clearly distinguish between the AR and the TAR models on this subset of the data both will be estimated. The first is the AR

$$\begin{array}{rcccccc} \Delta z_t & = & -0.0609 & + & (-0.1047) & z_{t-1} \\ & & (0.0154) & & (0.0248) & \end{array}$$

$$\text{Half-Life} = 6.2674$$

Evaluated on its own merits, this specification suggests a high degree of integration between U.S. and U.K. capital markets. Not only are there no significant transactions costs, but differentials are closed fairly rapidly, with a half-life of slightly more than 6 weeks.

The alternative model, the TAR, is estimated as

$$\begin{array}{rccccccccccc} \Delta z_t & = & -0.0551 & + & \tau_t & (-0.1044) & z_{t-1} & + & (1-\tau_t) & (0.1338) & z_{t-1} \\ & & (0.0153) & & & (0.0244) & & & & (0.0943) & \end{array}$$

$$\begin{array}{c} \text{Threshold} = 0.0213 \\ \text{In-Band Half-Life} = N/A \\ \text{Out-of-Band Half-Life} = 6.2864 \end{array}$$

Note that the threshold value is more in line with those found in the literature. Also, the out-of-band half-life indicates that the U.S.-U.K. interest rate differential converges quickly to its long-run equilibrium value.

The estimated breakpoint occurs during a period when the British government was opening the economy to both internal and external competition. The limit on outside ownership of stock exchange firms was being phased out, and was to be finally eliminated by 1985. Also, many of the state-owned enterprises were being dismantled and privatized. For example, in November of 1984 shares of British Telecom shares were made available to the public. Another major business concern, British Airways, was to follow in early 1985.

Regime 3: December 10, 1984 - August 20, 2001

The final model to be fit is an AR specification

$$\Delta z_t = \underset{(0.0042)}{-0.0113} + \underset{(0.0075)}{(-0.0252)} \; z_{t-1}$$

$$\text{Half-Life} = 27.1578$$

This model suggests that the two capital markets are integrated, however, not as closely as might be expected when compared to previous periods and other countries. As an illustration, if the AR model is the true model for the second regime, then it takes 21 weeks longer for an interest rate differential to disappear in the third regime (6.2 versus 27.2). It is difficult to imagine this to be the case, in light of the stylized fact of increasing capital market integration over time. However, a logical case can be made for such a change. If the true model in the second regime is a TAR, then the overarching theme is consistent with increasing integration. In this case, the the relationship goes from a random walk (i.e., no convergence) to a threshold model with strong out-of-band convergence, to an AR model where transactions costs are negligible.

Note that the half-life in this final analysis is significantly slower than that found in Section 0.3.4 (27 weeks versus 17 weeks). As noted above, this is most likely a consequence of the inclusion of data points from two distinct regimes. The observations in the second subset of the data, which reflect a period of quick convergence for large differentials, increase the estimated convergence speed for models utilizing on the full data set.

0.5.4 Conclusion

The final step is to attempt to draw all of the individual analyses into something resembling a coherent whole. To aid in this discussion, consider the Figure 45. In this graph, the interest rate relationship between a given country and the U.S. is depicted as a horizontal line. Breaks in the relationship, as determined by Bai's method, are represented by a slight shift in the line.

From the chart it is apparent that for most of the early part of the data series was marked by instability in the European financial markets, at least as characterized by their bilateral relationships with the U.S. With the exception of Canada, all of the countries have a break point within the span of a year (May 25, 1981 to April 26, 1982). Many of these are related to efforts to open capital markets (Germany and Japan) or with changes in policy (Switzerland and the U.K.). Two of the countries also had second breaks around mid to late 1984. These were due to different reasons, however, with capital controls being relaxed in the U.K. and changes begin made to monetary definitions in Switzerland. In general, these results accord well with the relaxation of capital controls that occurred during this time period, which provides more confidence in the method and its conclusions.

Equally as striking is the relative stability of the system in the time period since the mid-1980's. This era includes several significant events: the dissolution of the Soviet Union, the creation of a unified Germany, and the announcement of a unified European currency[21]. None of these events appear to have affected the fundamental

[21] It is important to note that the actual introduction of the Euro was not included as a possible break point, even though the data is present. This is because Bai's procedure requires a minimum regime size to estimate the model. The date for the change to the Euro occurs within one of those regimes.

relationships presented.

The strongest conclusion, perhaps, is that capital markets have become more integrated over time. Some relationships proceed from a random walk to a model with some type of convergence (e.g., the U.S.-Japan and U.S.-Switzerland pairs). For others, neutral bands derived from transactions costs dissipate and all real interest rate differentials are subject to arbitrage (e.g., the U.S.-Germany and U.S.-U.K. pairs. Whatever the exact form, ultimately capital markets are more integrated in the final subset of the data than in the first.

Finally, the results sound a note of caution for empirical researchers. In this field, it is fashionable to use extremely long data sets, some times spanning hundreds of years. However, to do so risks combining observations that arise from markedly different data generating process and, therefore, biasing the results.

0.6 Closing Comments

If there is one overriding conclusion to be drawn from this work, it is that *ex ante* real interest rates are not equal. However, it does appear that, at least among the capital markets considered, there is a significant degree of integration. The analyses above have shown that there are significant threshold effects in the relationships between these rates and that the dynamics these thresholds engender can be quite complicated. Finally, the interest rate relationships are not necessarily stable over time, showing a clear movement toward increasing integration in the past 30 years.

While these results are important in their own right, at some point it is necessary to move beyond descriptive work and to attempt to explain these features. Future work could be directed toward a number of areas. For example, a key component of the analysis is the derivation of the *ex ante* real interest rates. Although the method outlined above was shown to be reasonable, it would be interesting to know if the results differ based on how these rates are created. Such a study could help pinpoint the role of inflation expectations in creating the observed threshold behavior. Also, the results of this work provide useful benchmarks for testing economic models. Models that are unable to replicate these behaviors could be modified until their implications are consistent with these econometric results. In this manner, the above work can form a foundation for creating models which accurately reflect economic reality and, therefore, improve economists' understanding of the world economy.

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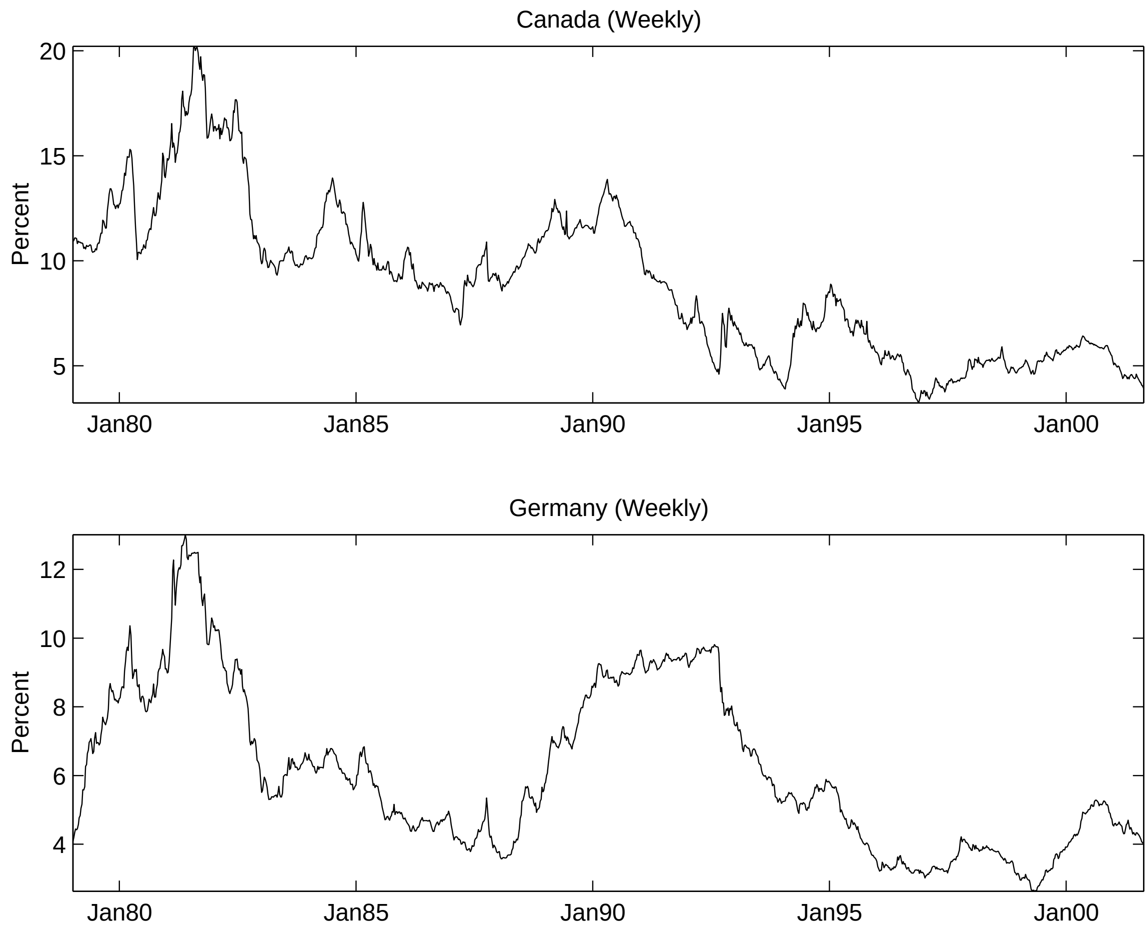

Figure 1: 12-month nominal interest rate

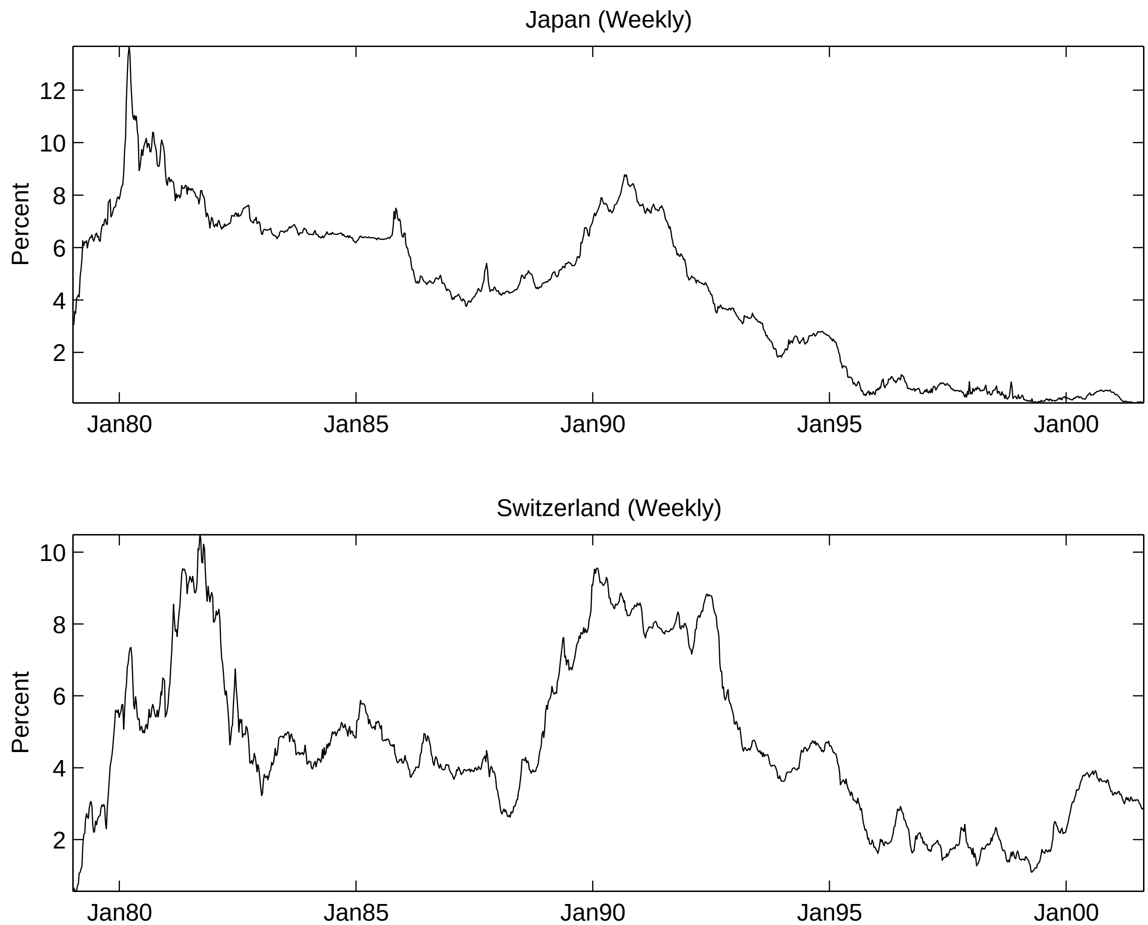

Figure 2: 12-month nominal interest rate

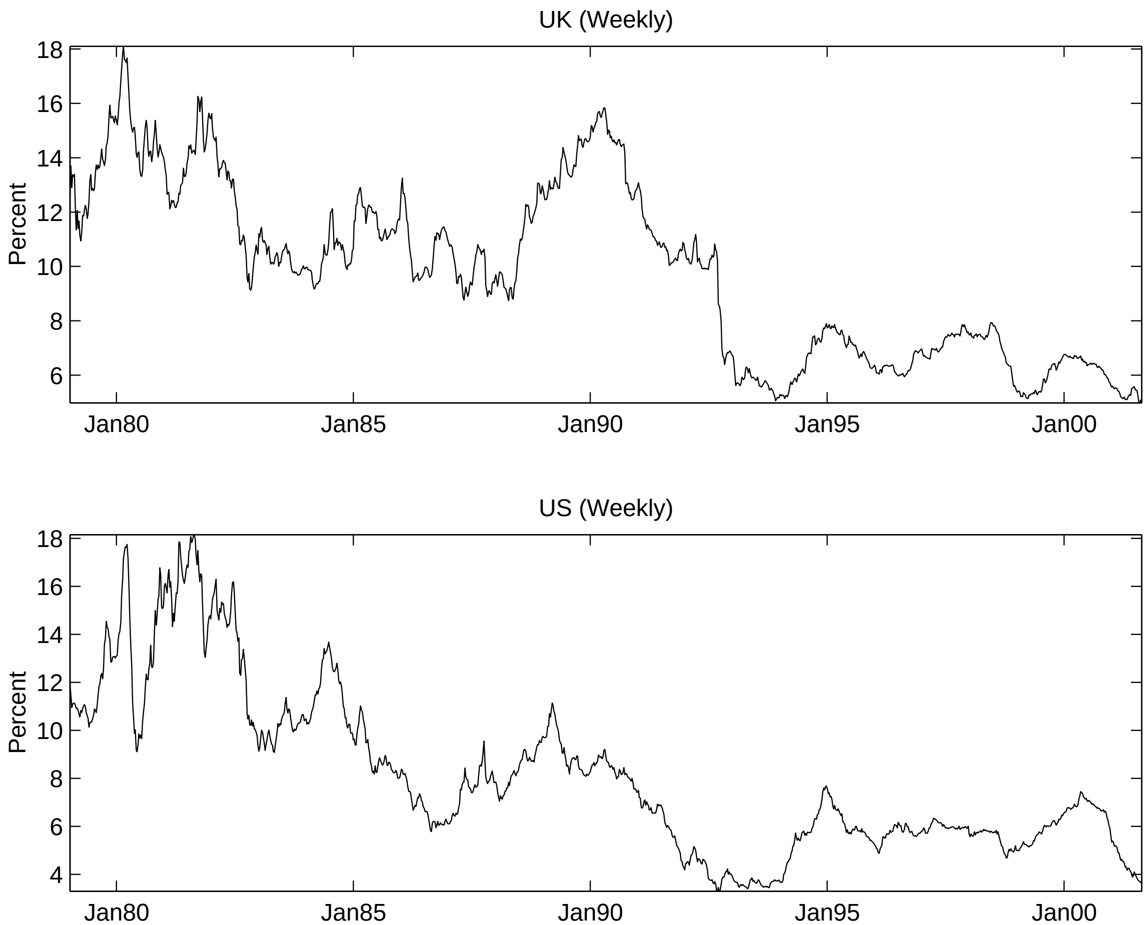

Figure 3: 12-month nominal interest rate

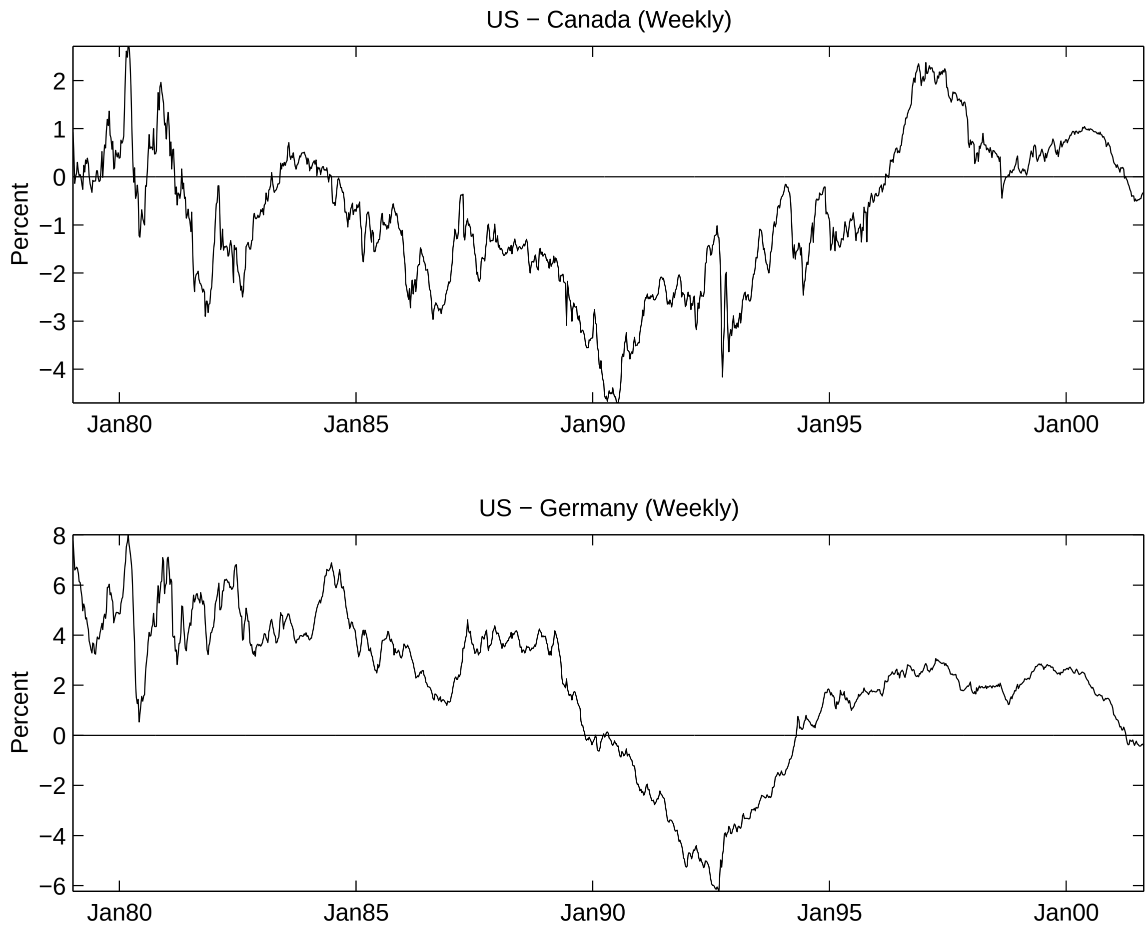

Figure 4: Nominal interest rate differential

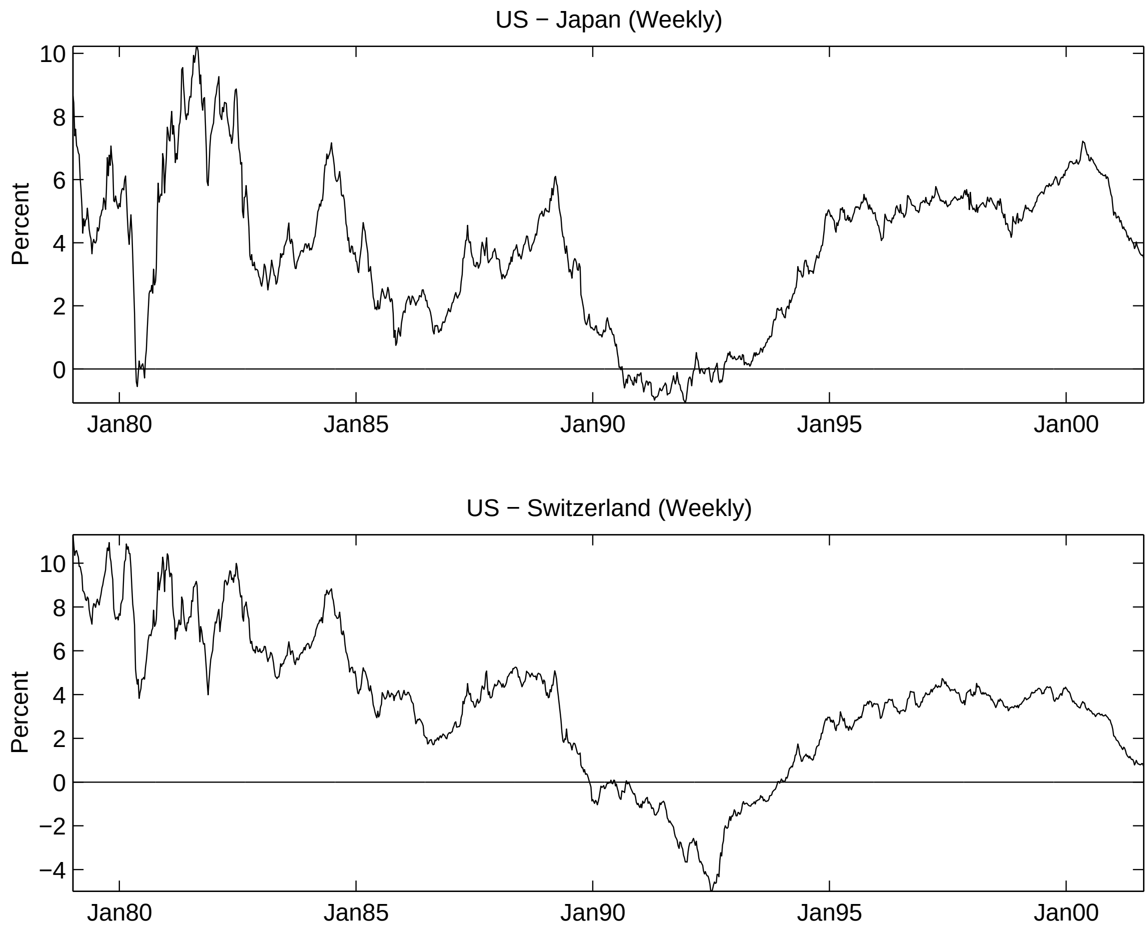

Figure 5: Nominal interest rate differential

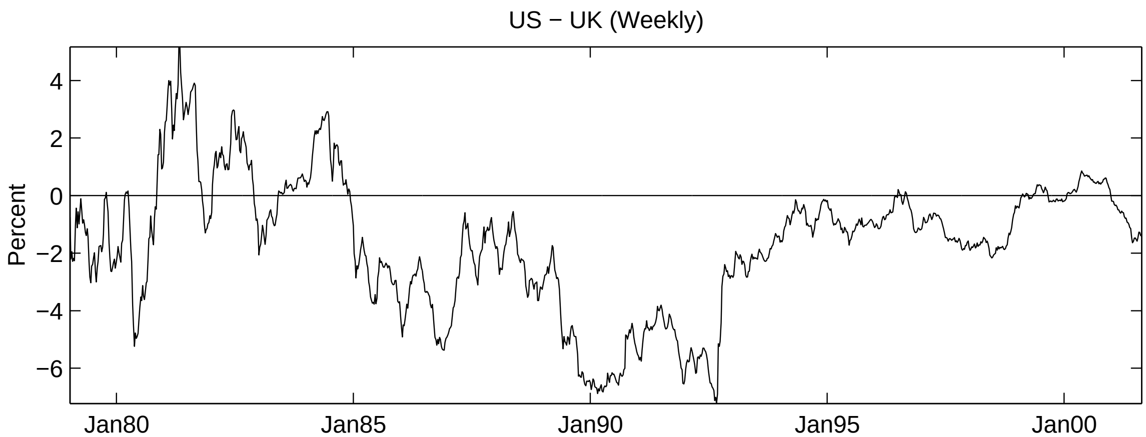

Figure 6: Nominal interest rate differential

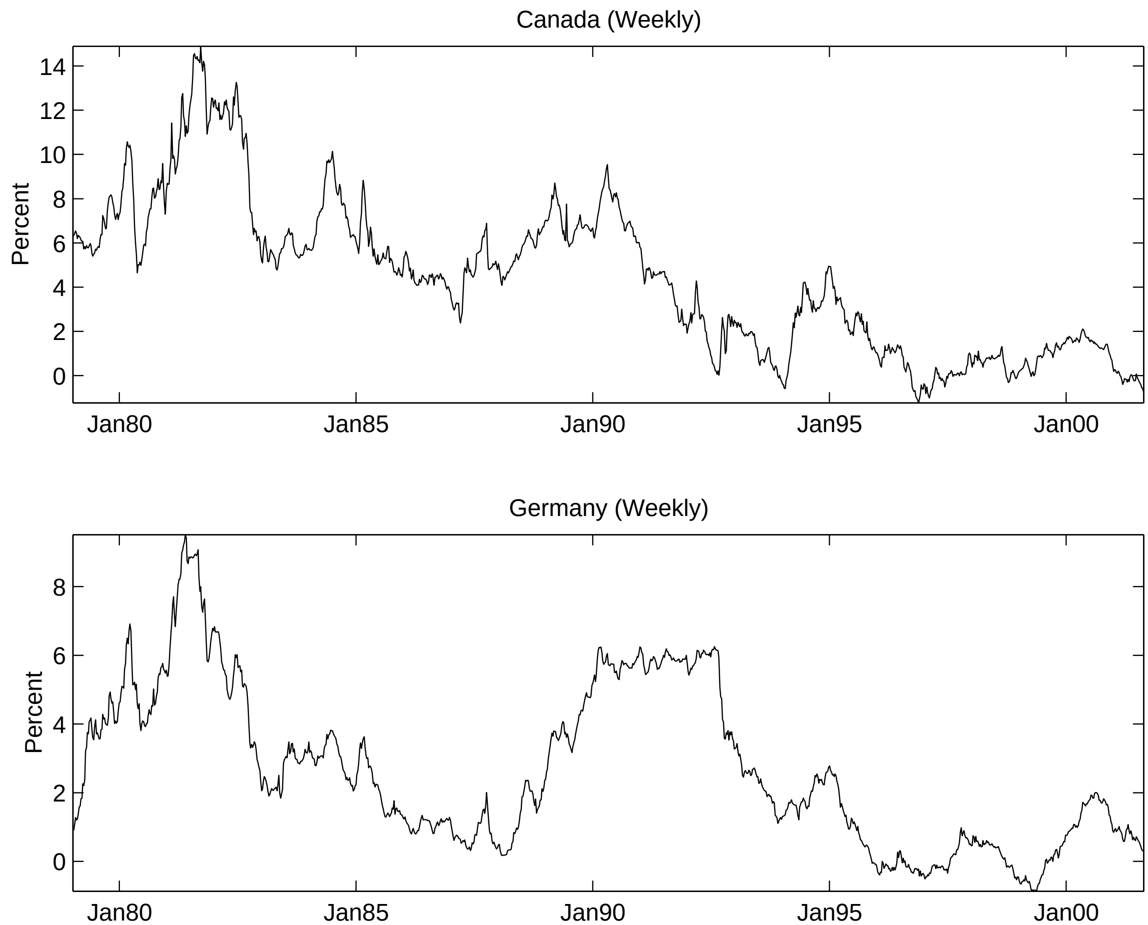

Figure 7: *Ex ante* inflation

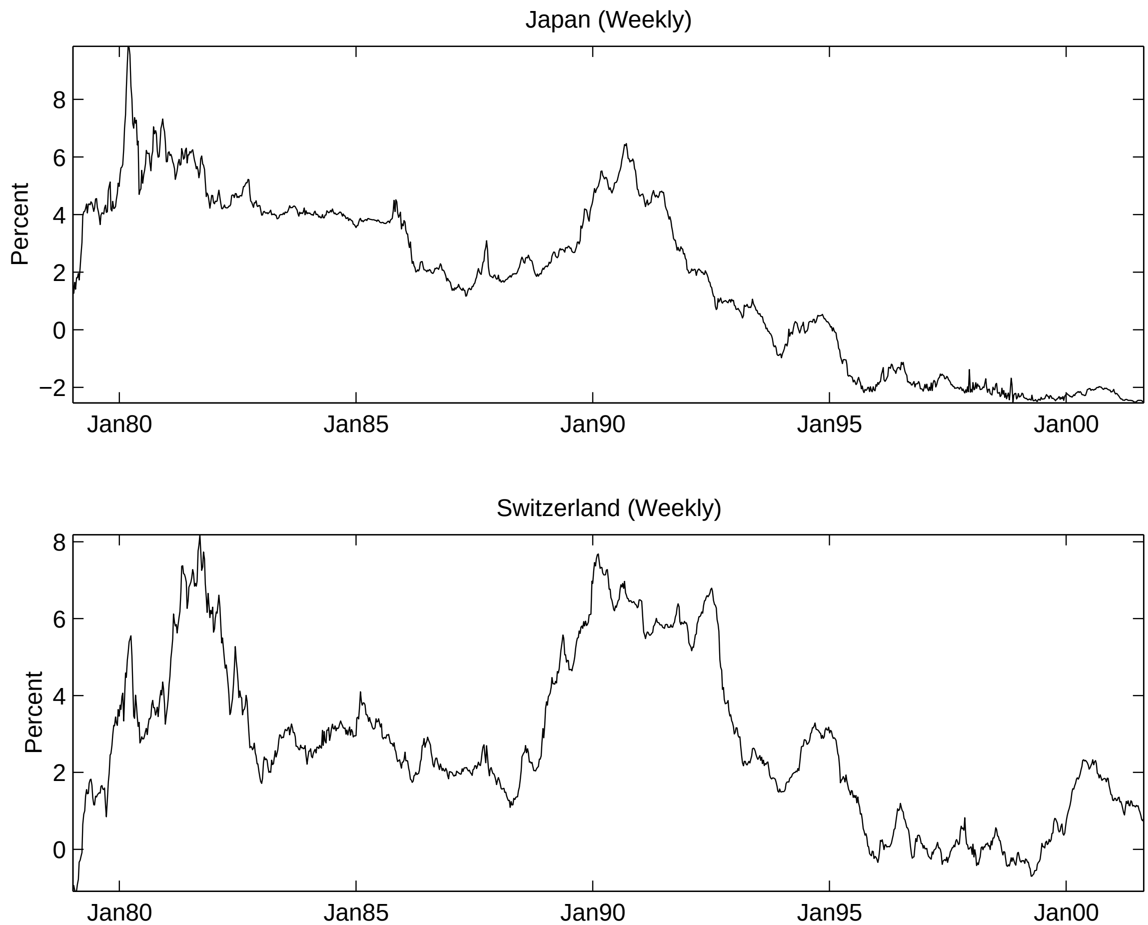

Figure 8: *Ex ante* inflation

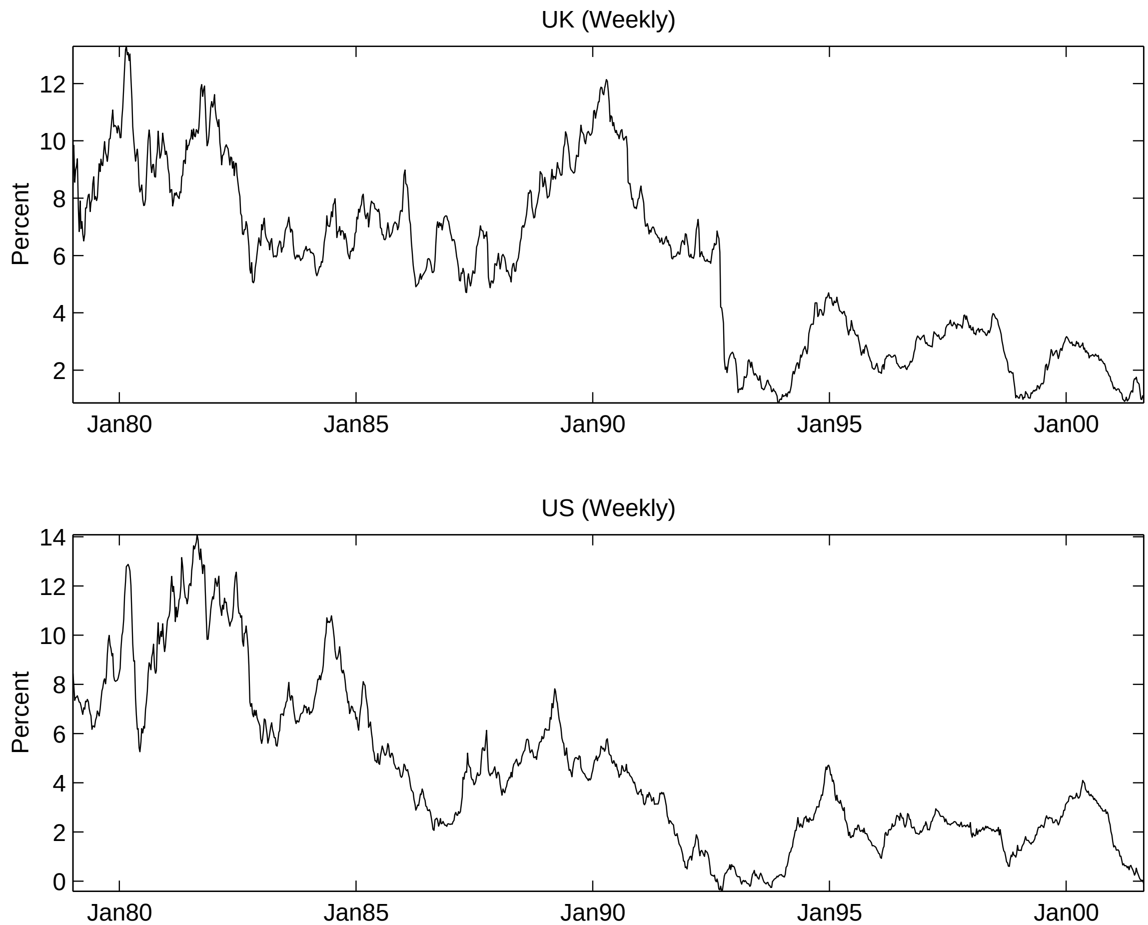

Figure 9: *Ex ante* inflation

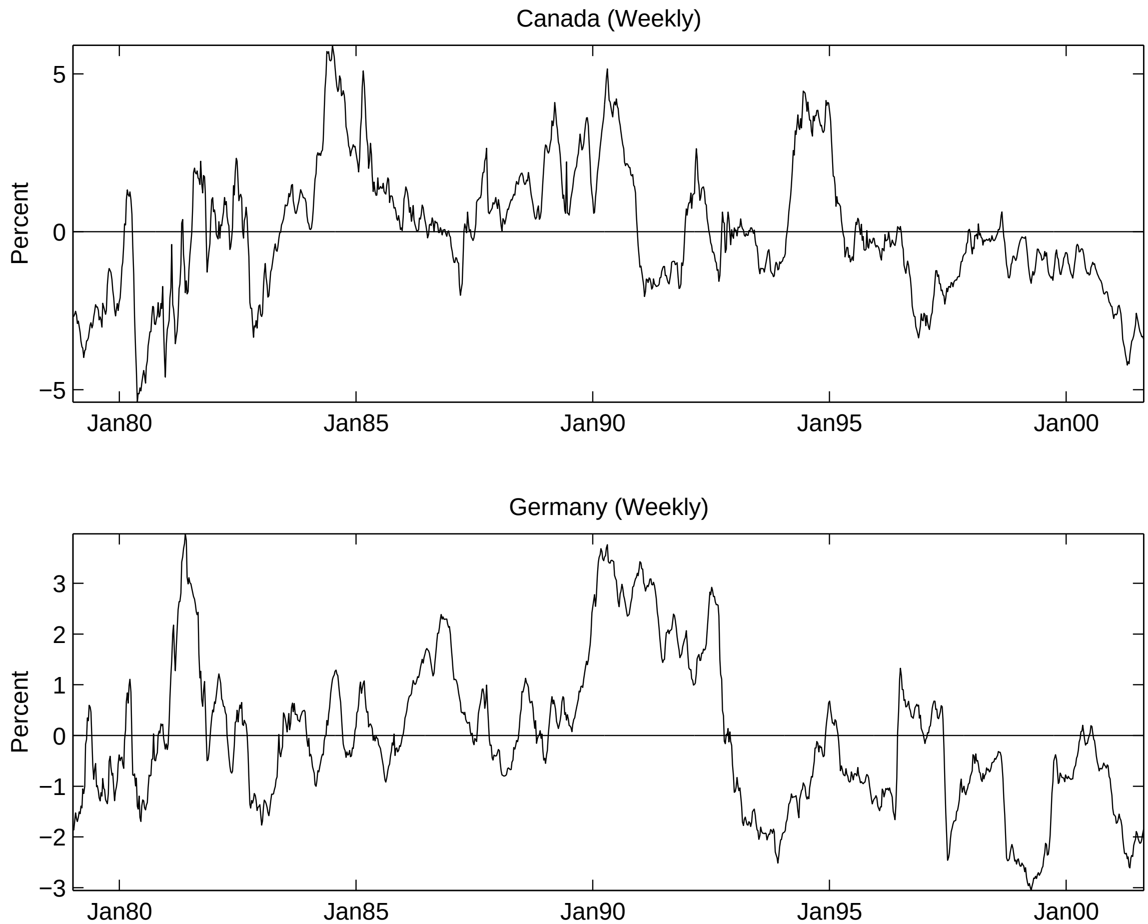

Figure 10: Inflation forecast error (*ex ante* - *ex post*)

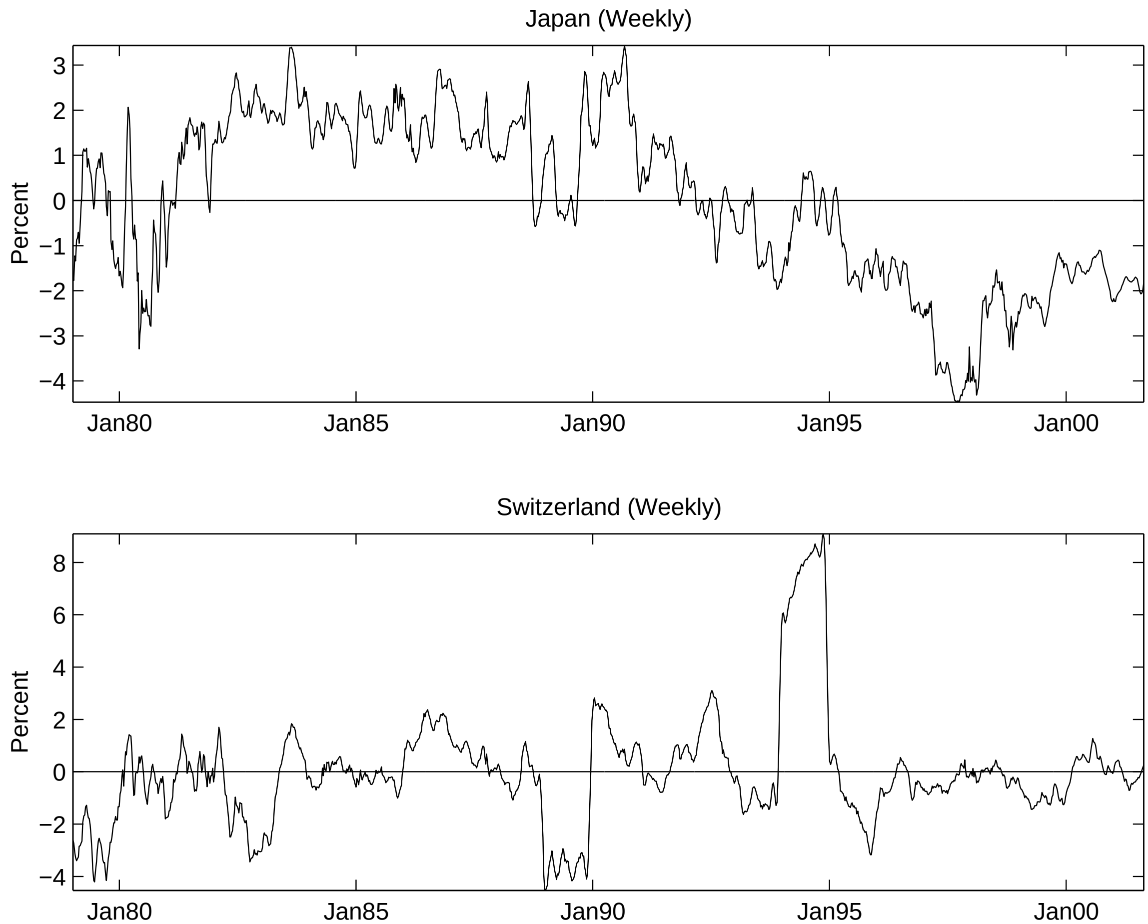

Figure 11: Inflation forecast error (*ex ante* - *ex post*)

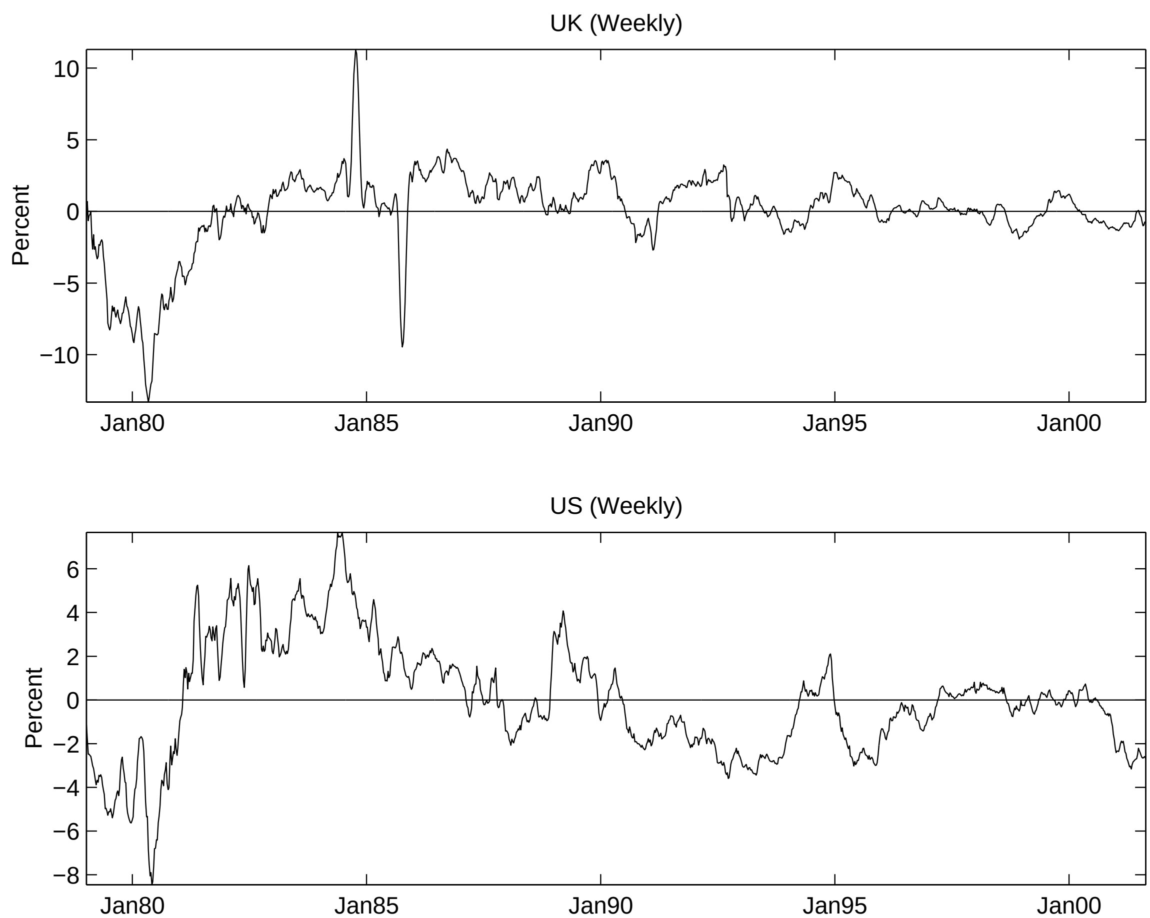

Figure 12: Inflation forecast error (*ex ante* - *ex post*)

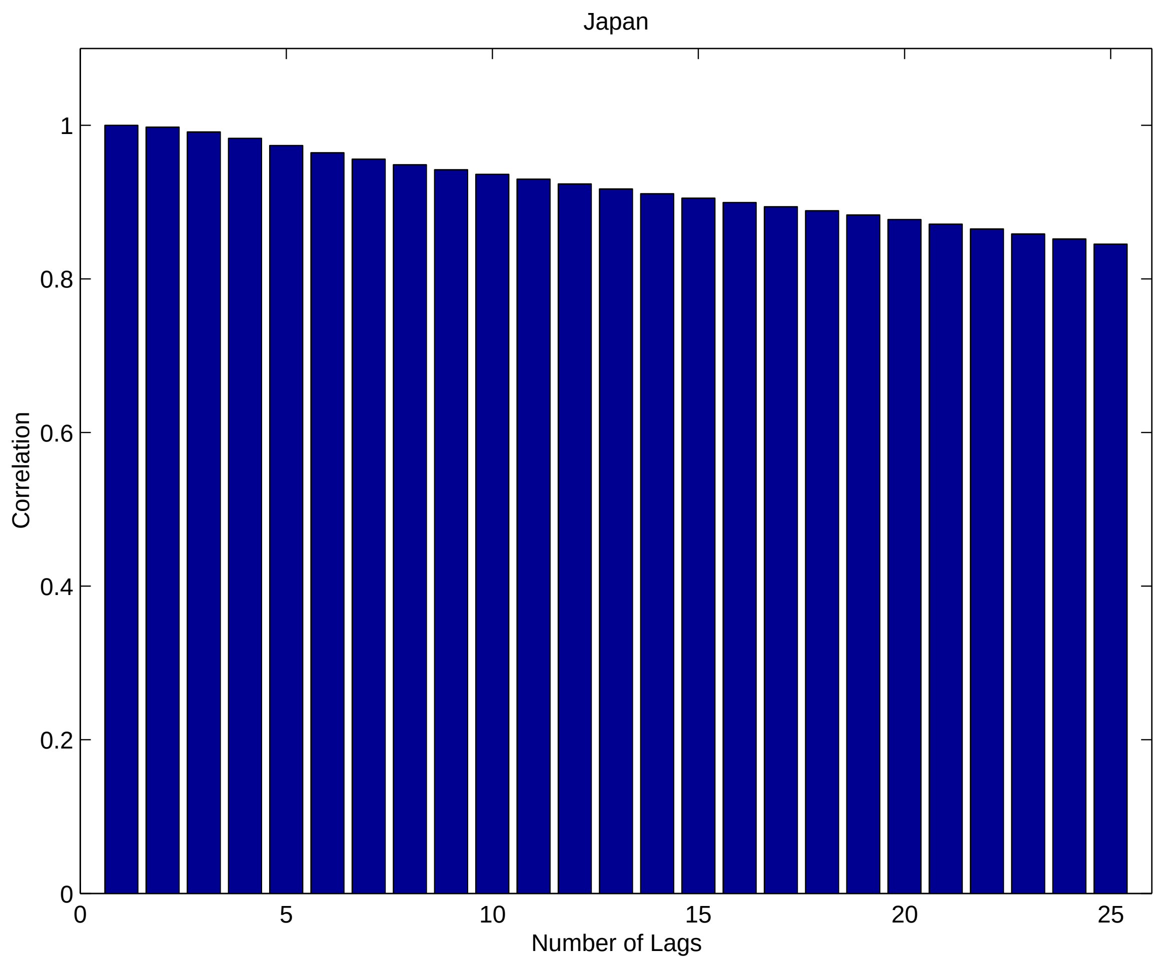

Figure 13: Autocorrelation function (*ex post* inflation)

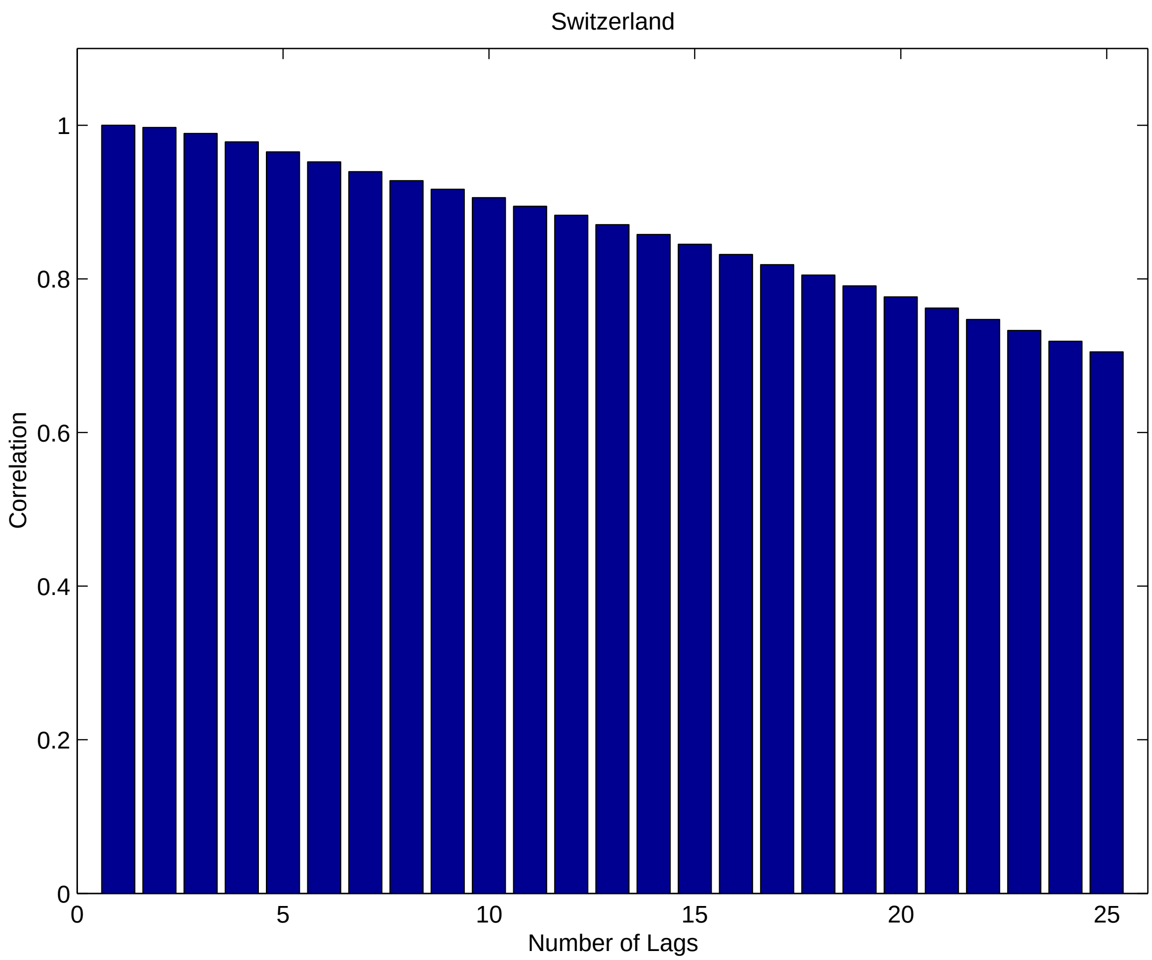

Figure 14: Autocorrelation function (*ex post* inflation)

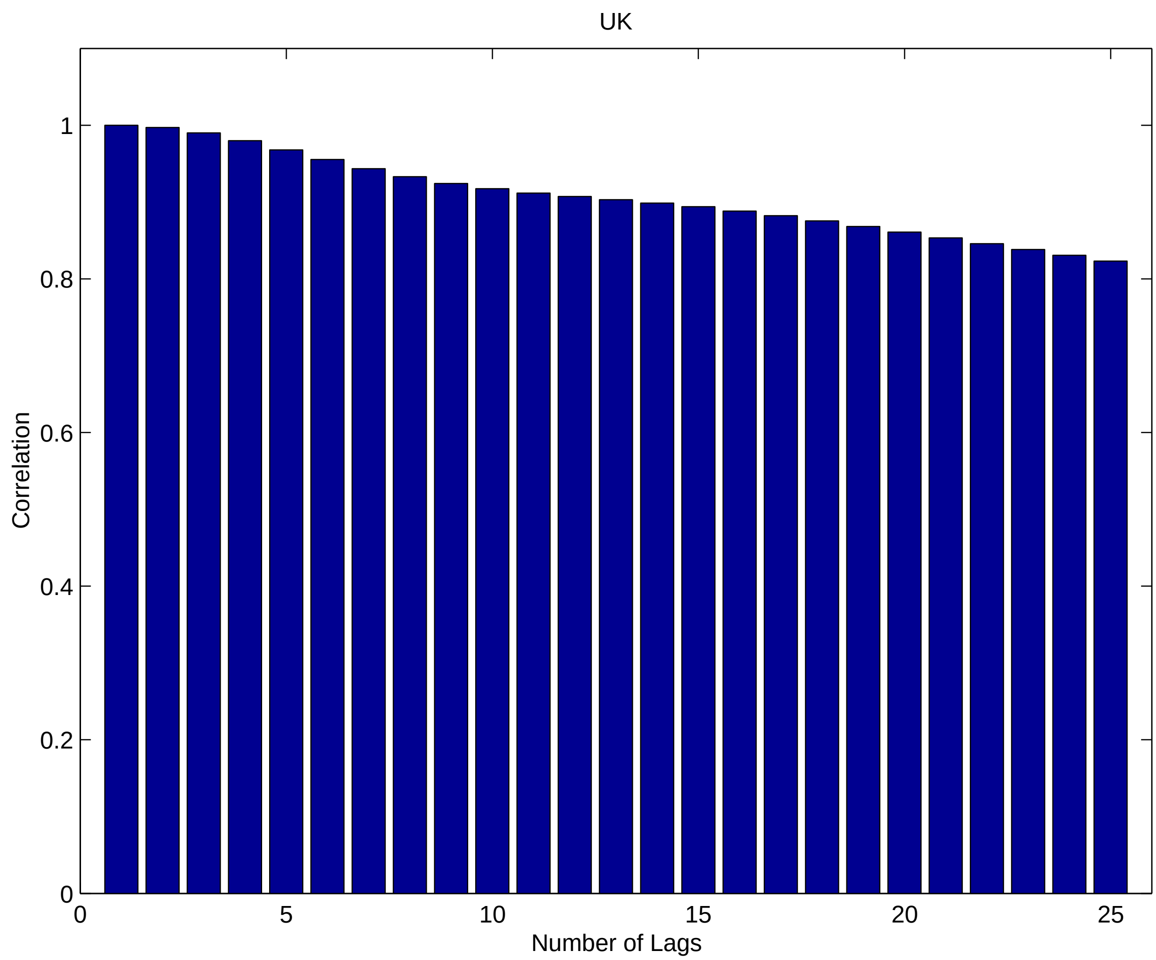

Figure 15: Autocorrelation function (*ex post* inflation)

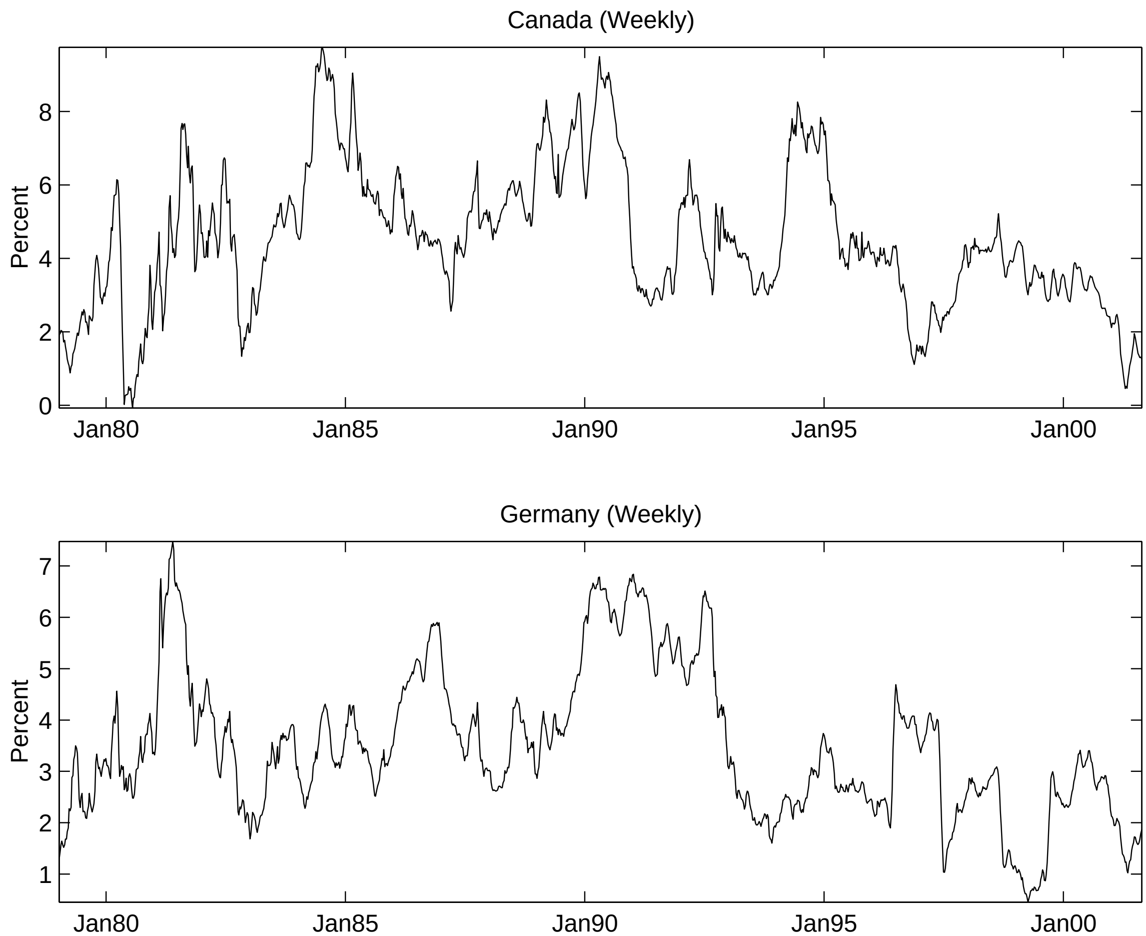

Figure 16: 12-month *ex post* real interest rate

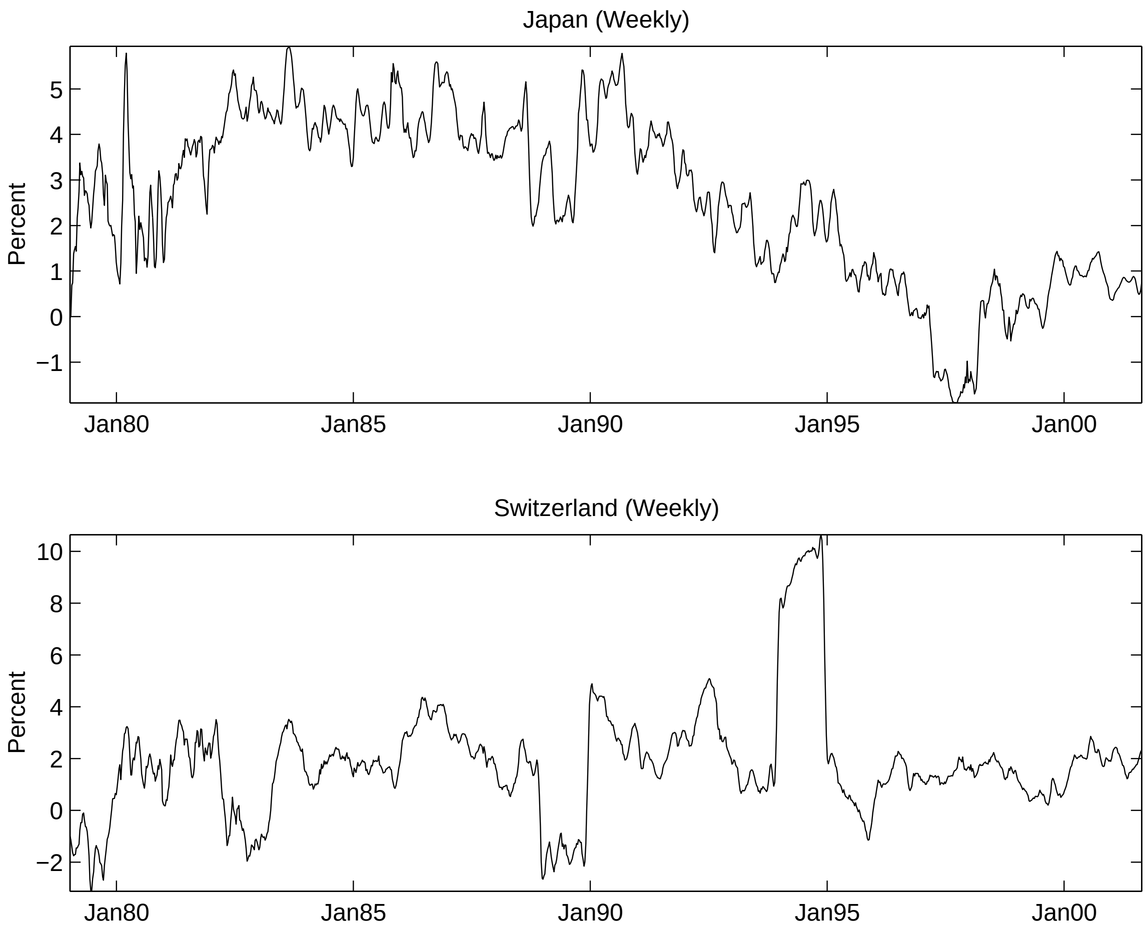

Figure 17: 12-month *ex post* real interest rate

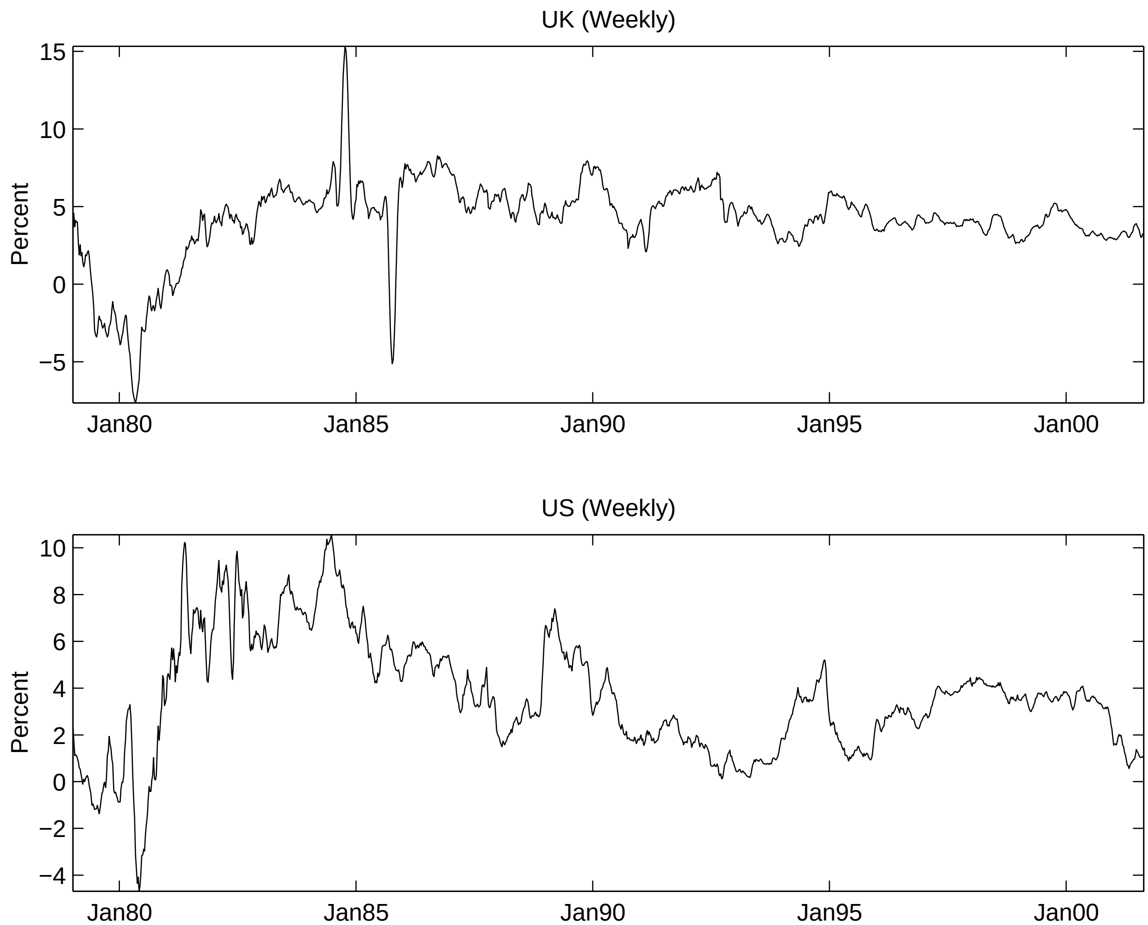

Figure 18: 12-month *ex post* real interest rate

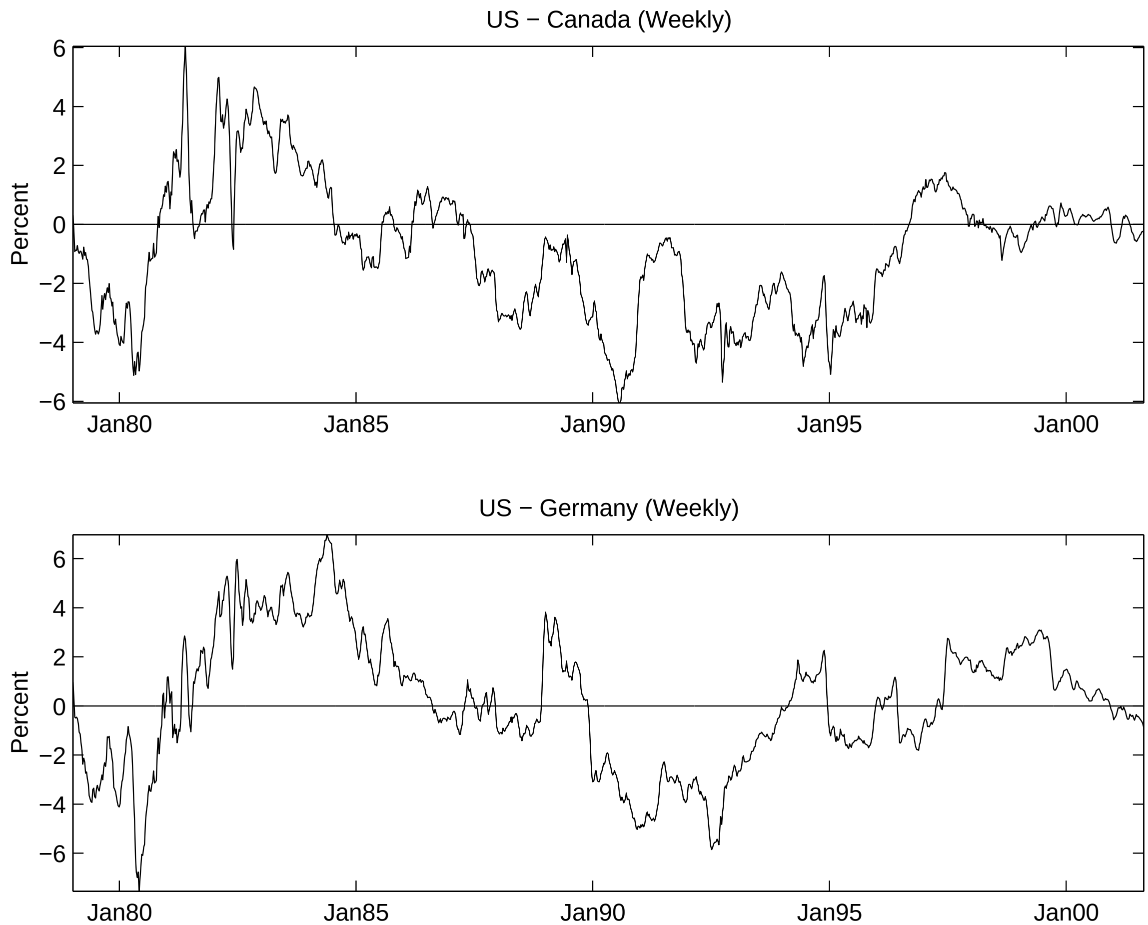

Figure 19: 12-month *ex post* real interest rate differential

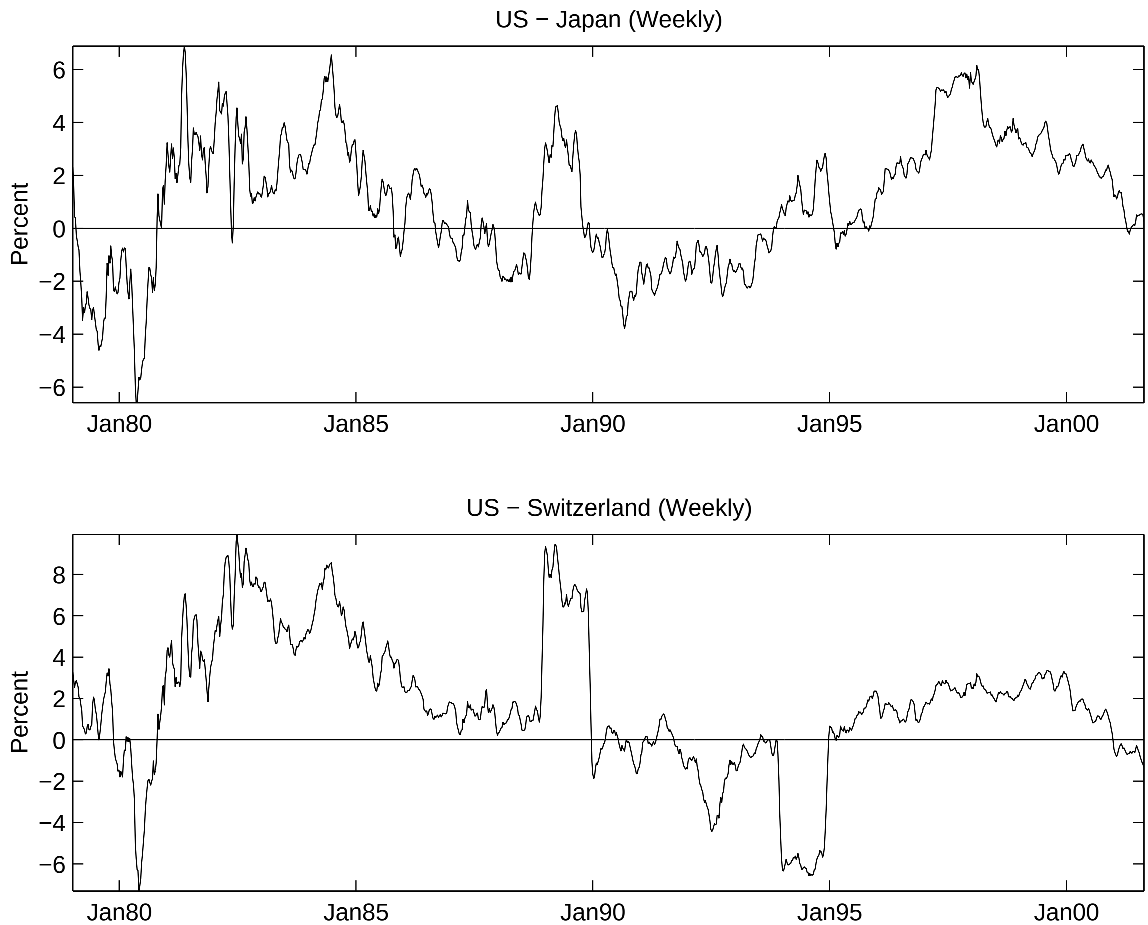

Figure 20: 12-month *ex post* real interest rate differential

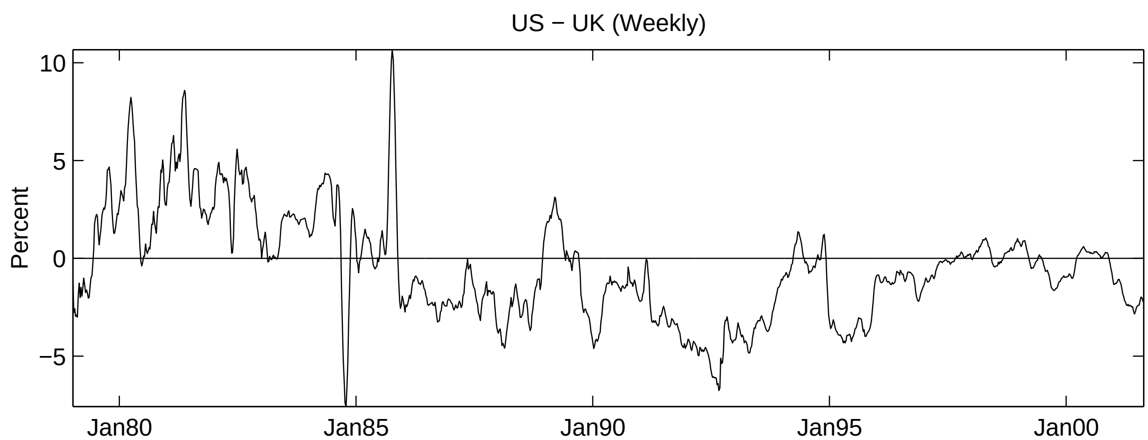

Figure 21: 12-month *ex post* real interest rate differential

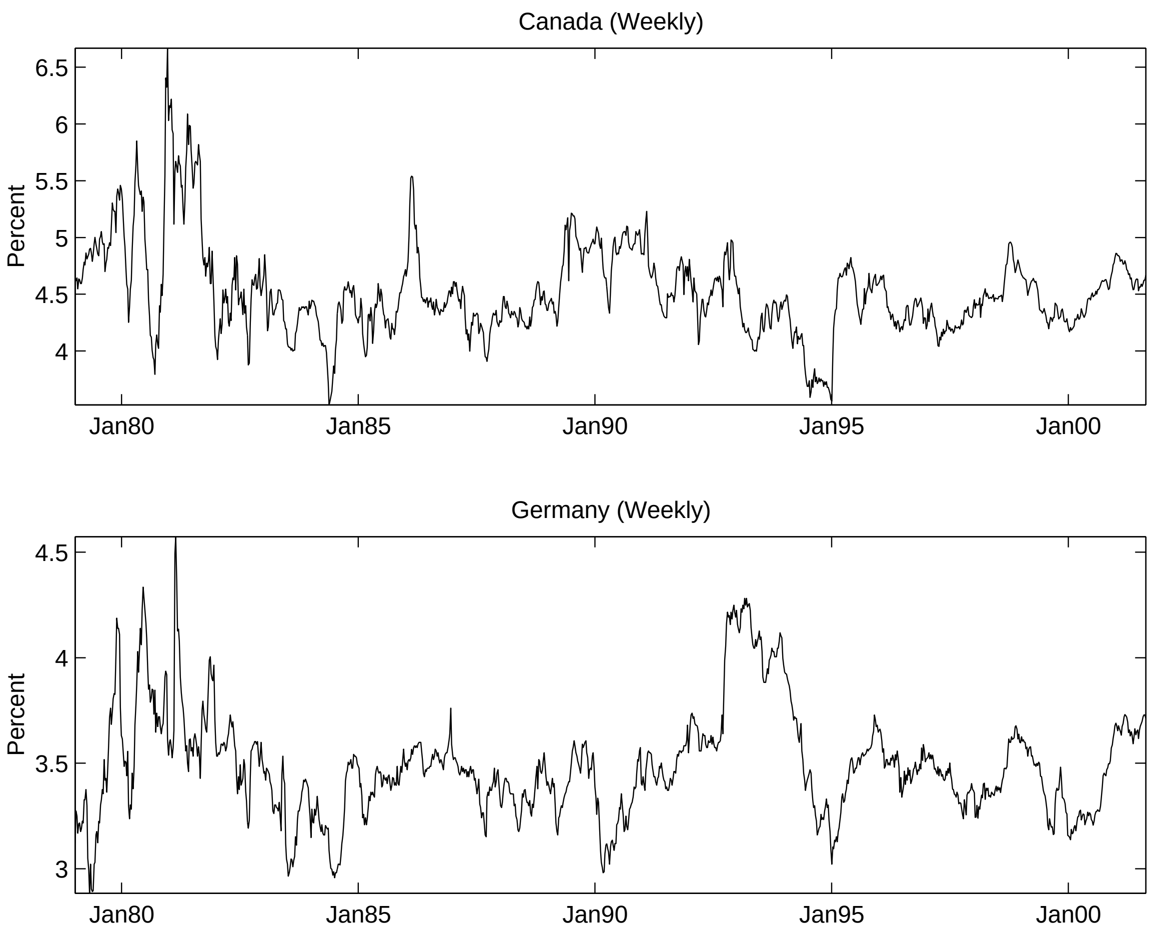

Figure 22: 12-month *ex ante* real interest rate

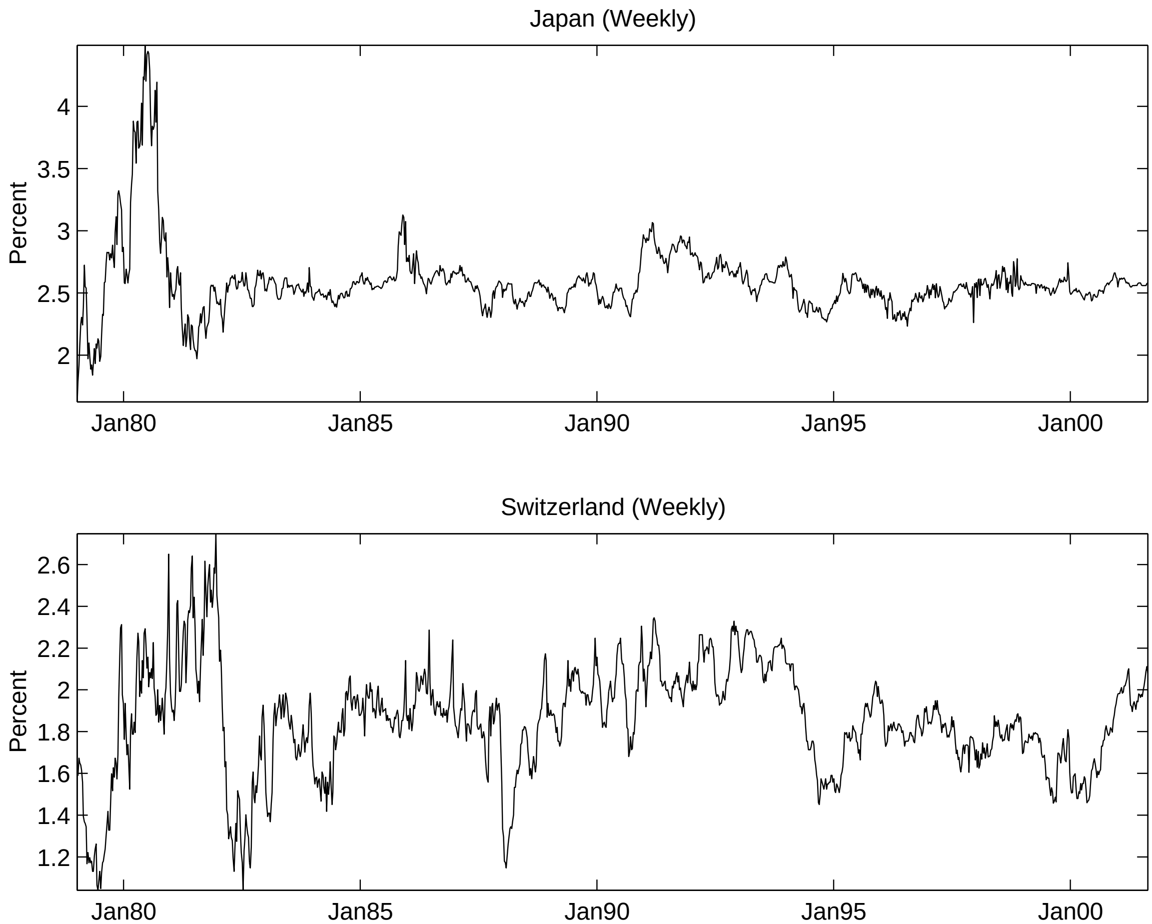

Figure 23: 12-month *ex ante* real interest rate

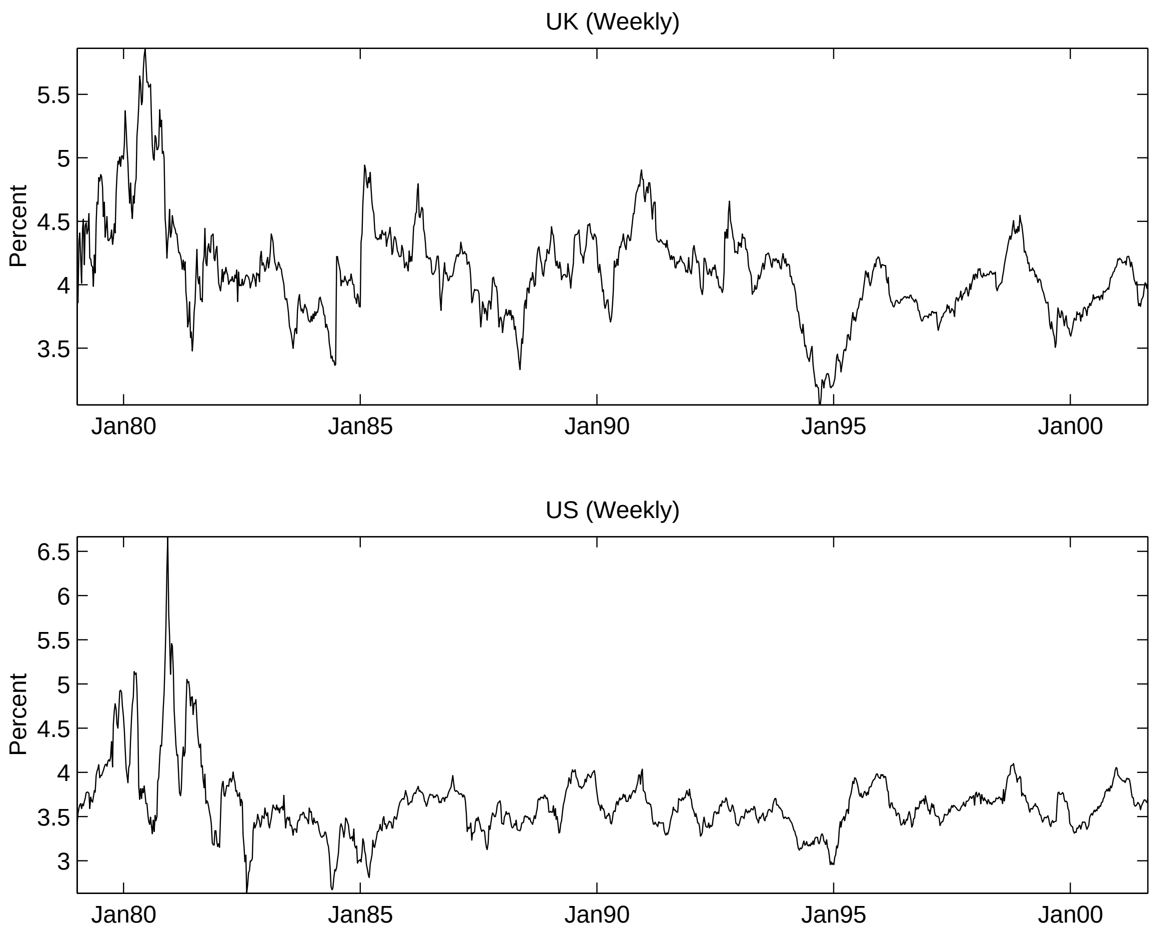

Figure 24: 12-month *ex ante* real interest rate

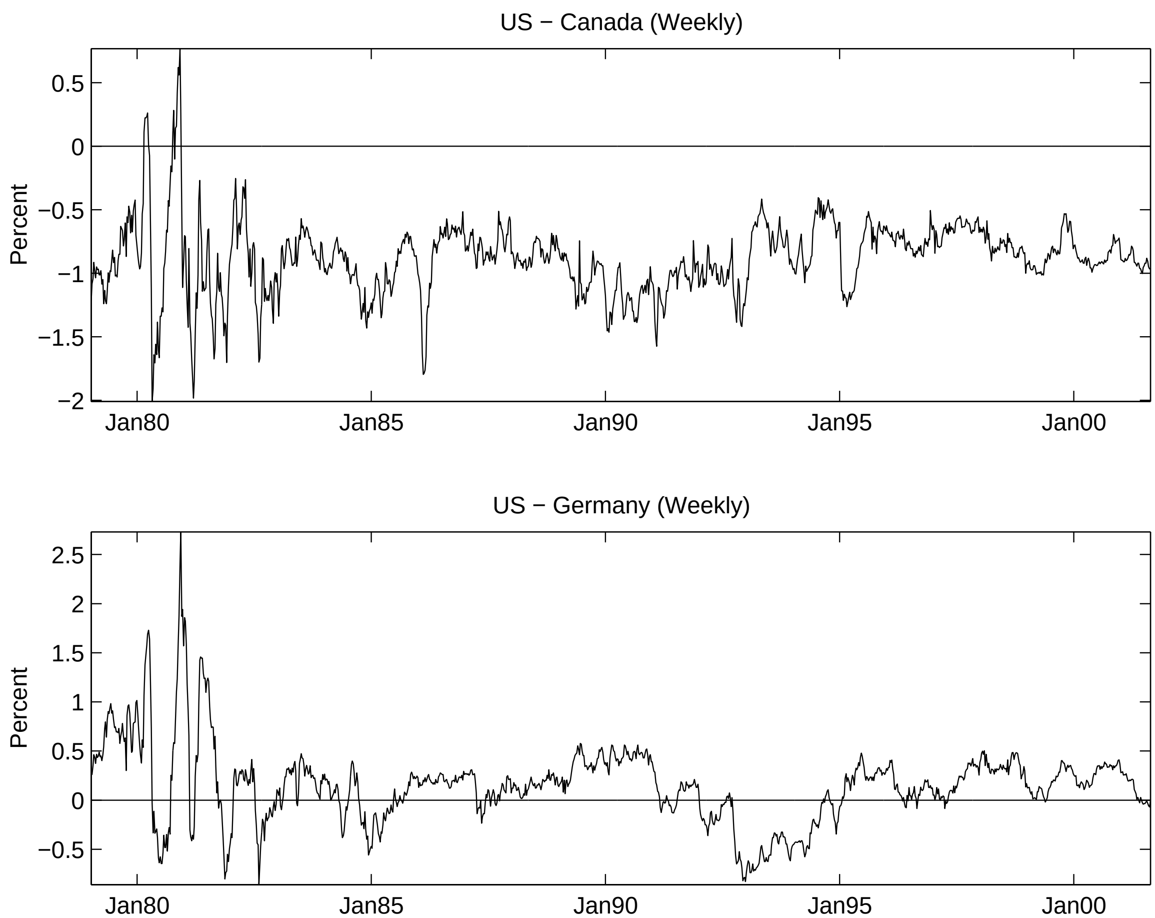

Figure 25: 12-month *ex ante* real interest rate differential

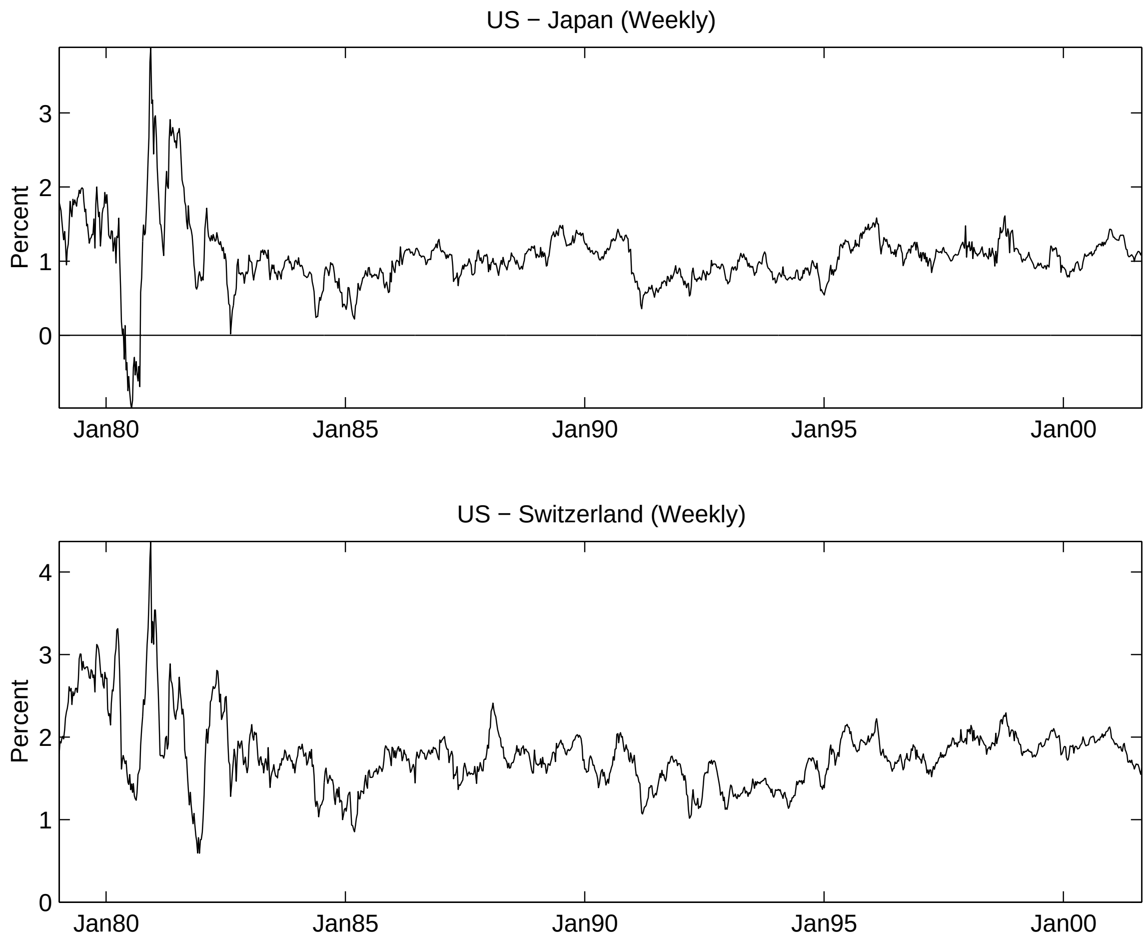

Figure 26: 12-month *ex ante* real interest rate differential

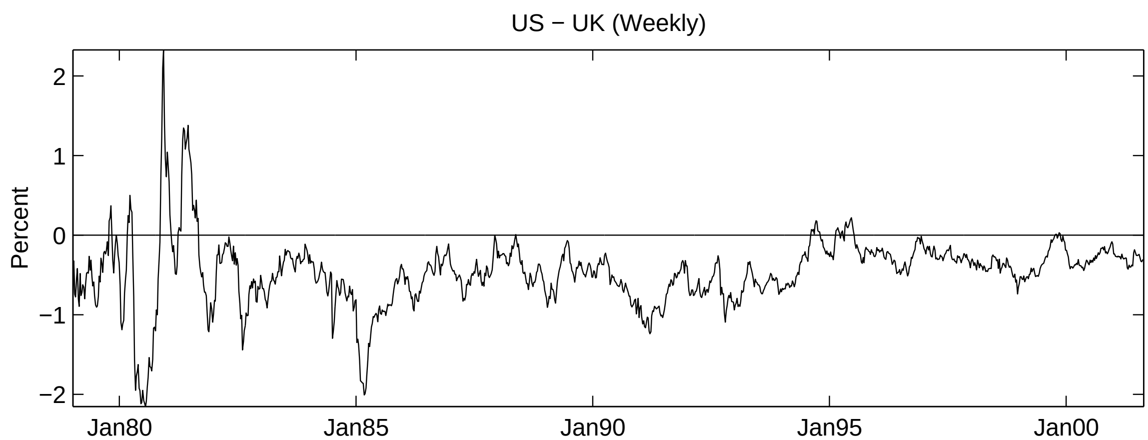

Figure 27: 12-month *ex ante* real interest rate differential

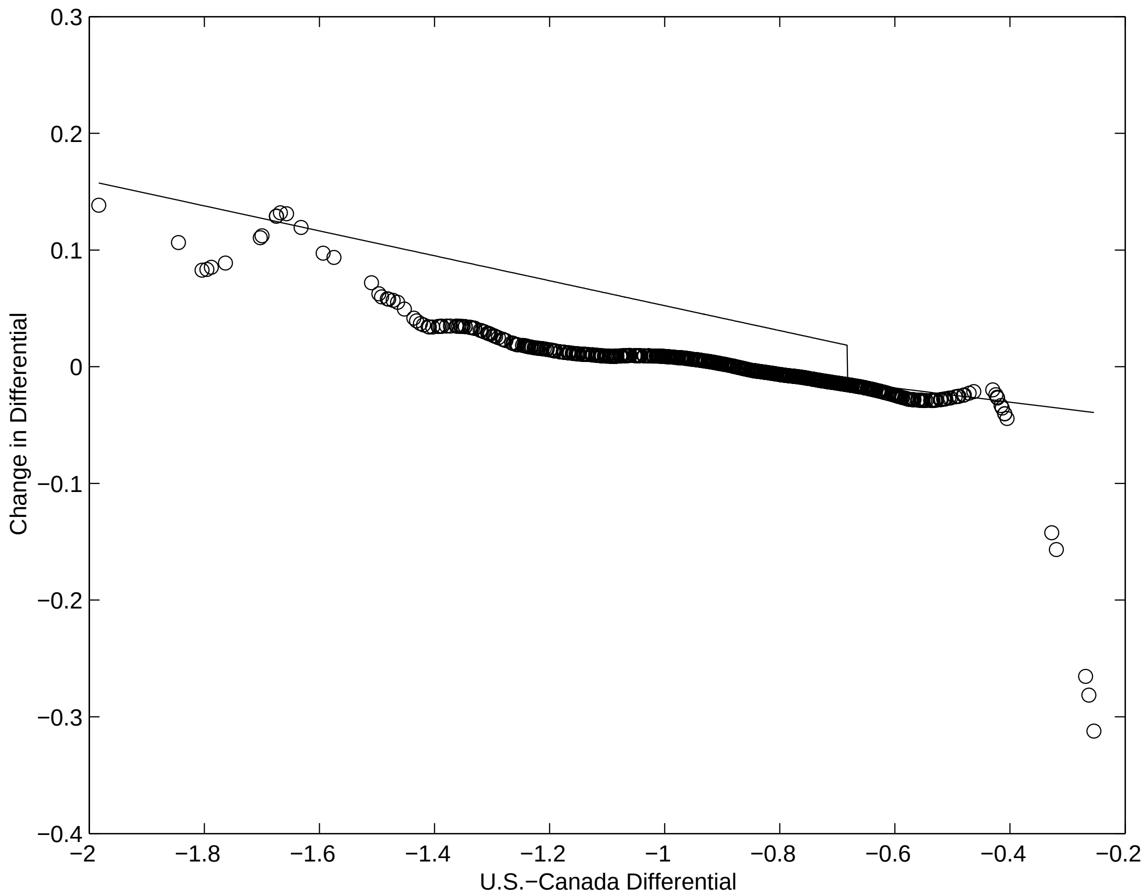

Figure 28: NPAR Model of U.S.-Canadian Real Rate Relationship

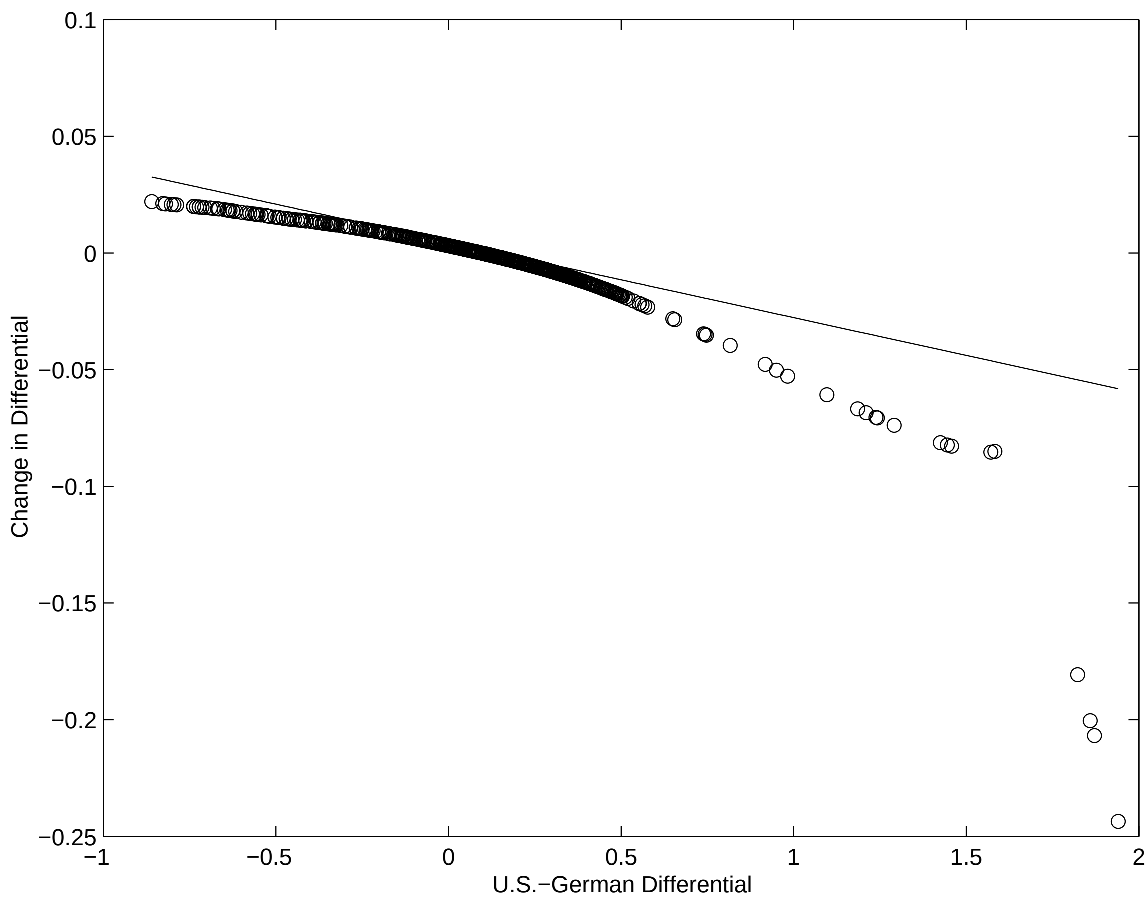

Figure 29: NPAR Model of U.S.-German Real Rate Relationship

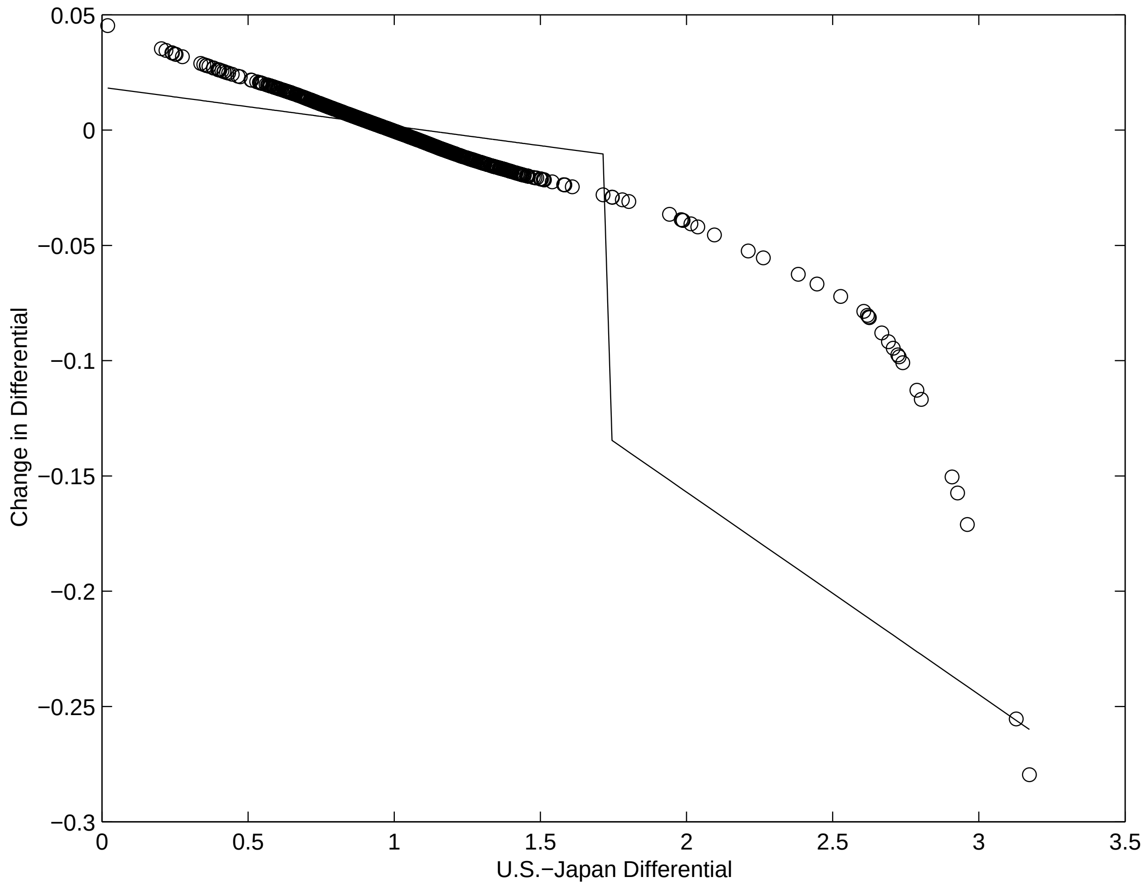

Figure 30: NPAR Model of U.S.-Japan Real Rate Relationship

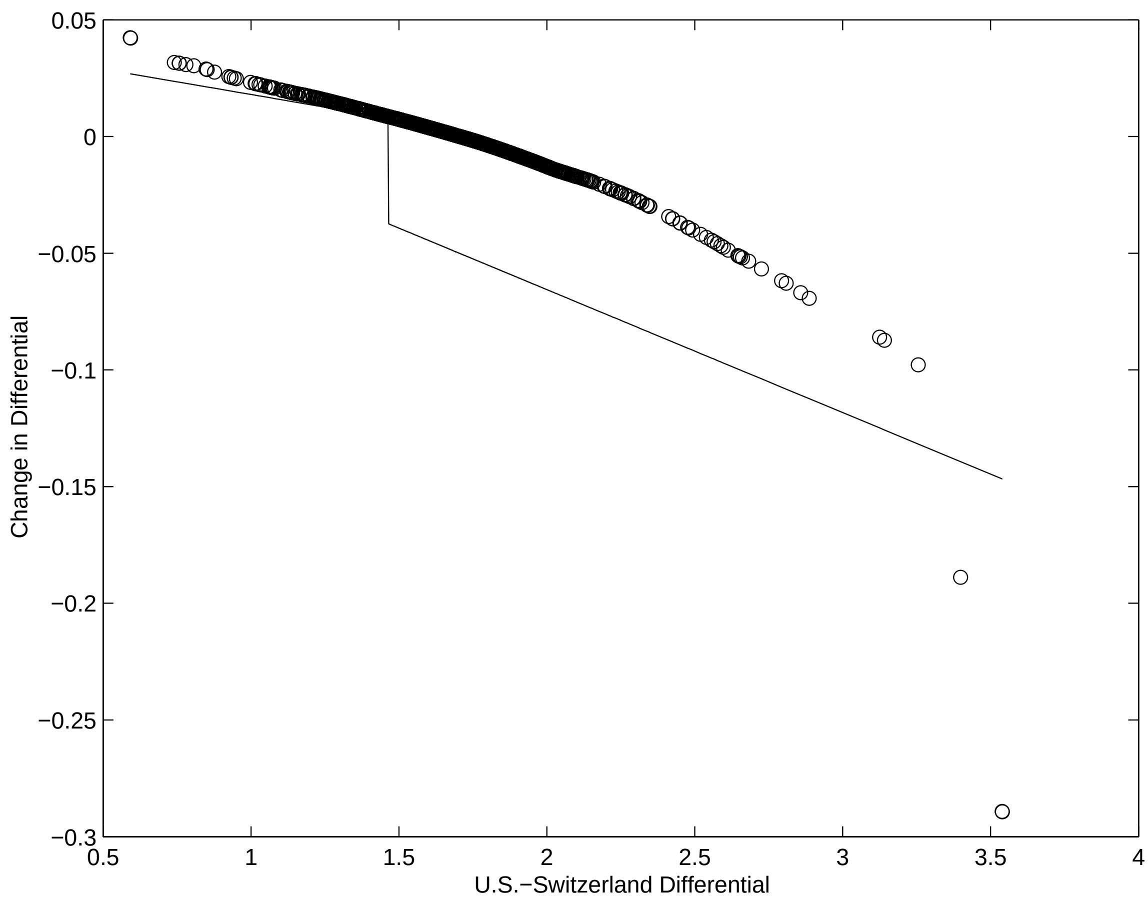

Figure 31: NPAR Model of U.S.-Swiss Real Rate Relationship

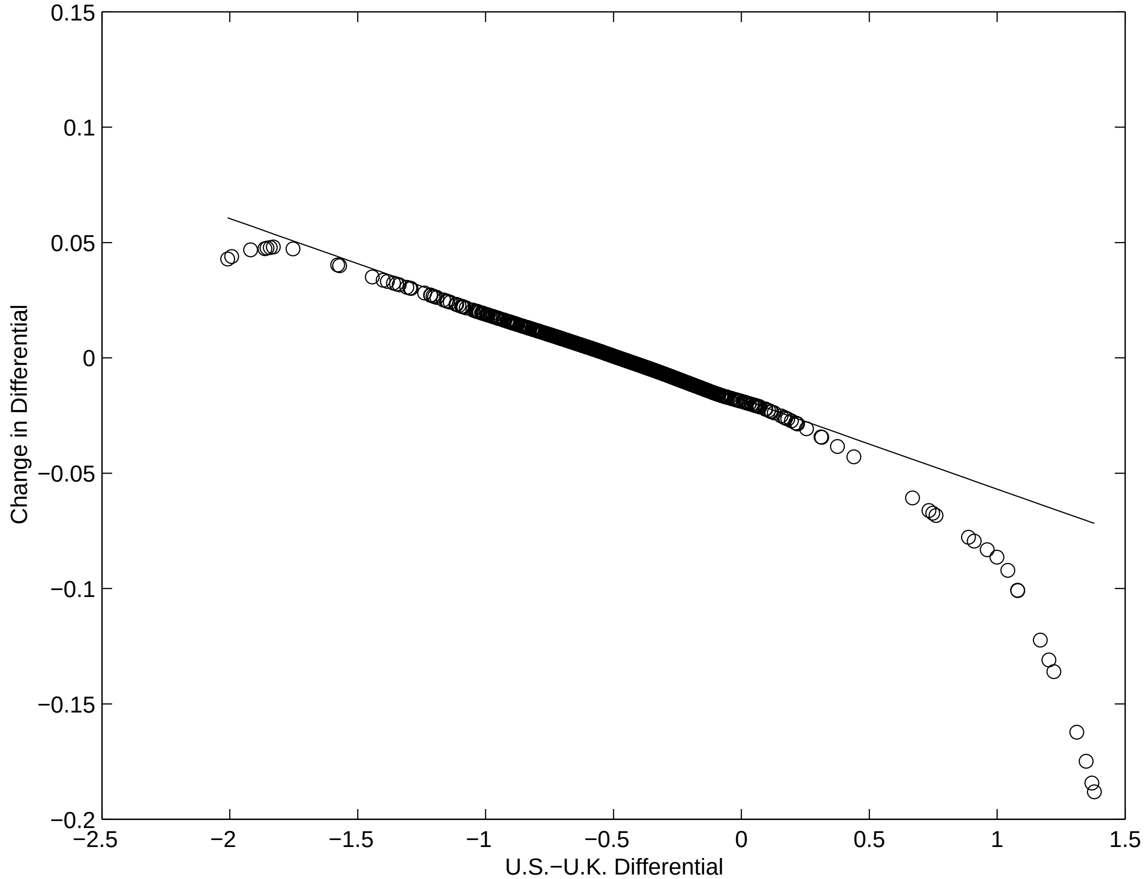

Figure 32: NPAR Model of U.S.-U.K. Real Rate Relationship

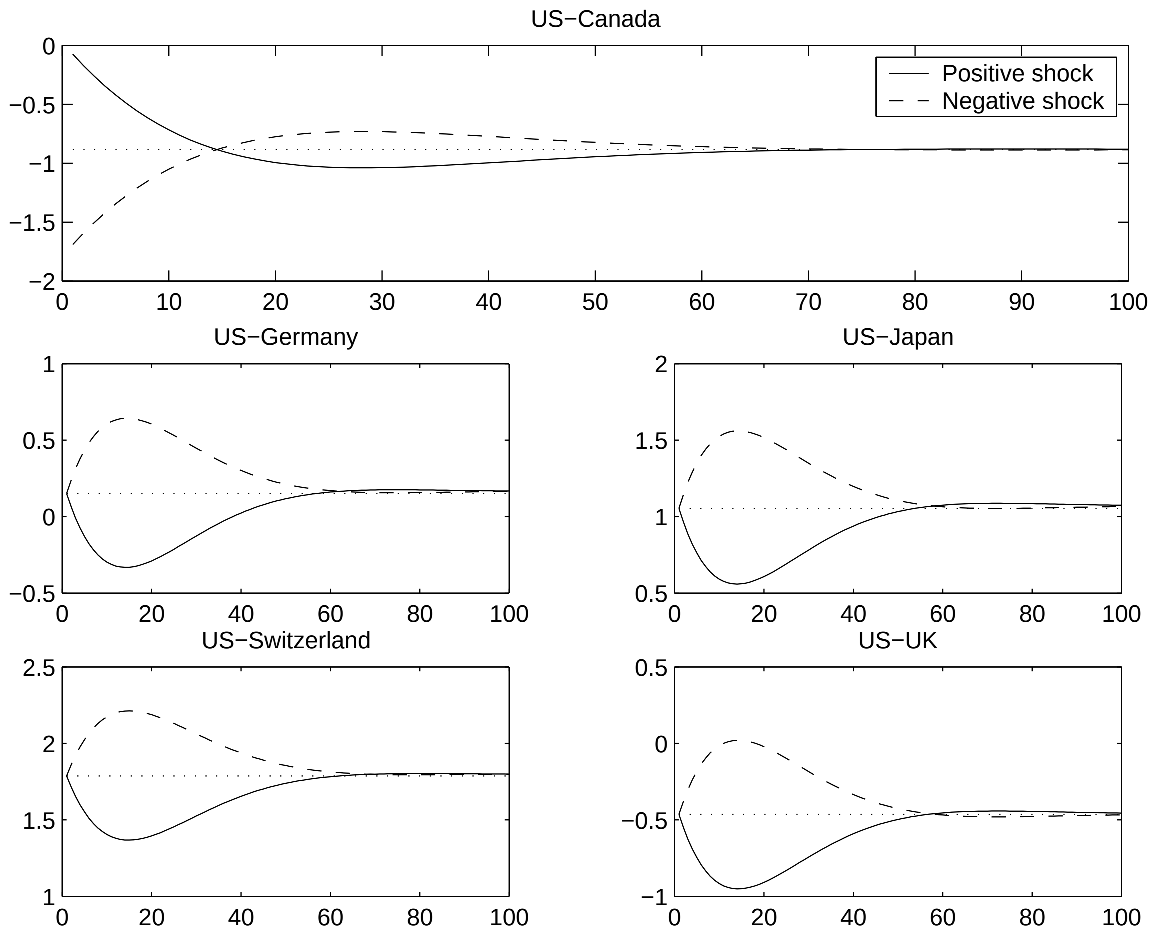

Figure 33: VAR with shock to Canadian *ex ante* real interest rate

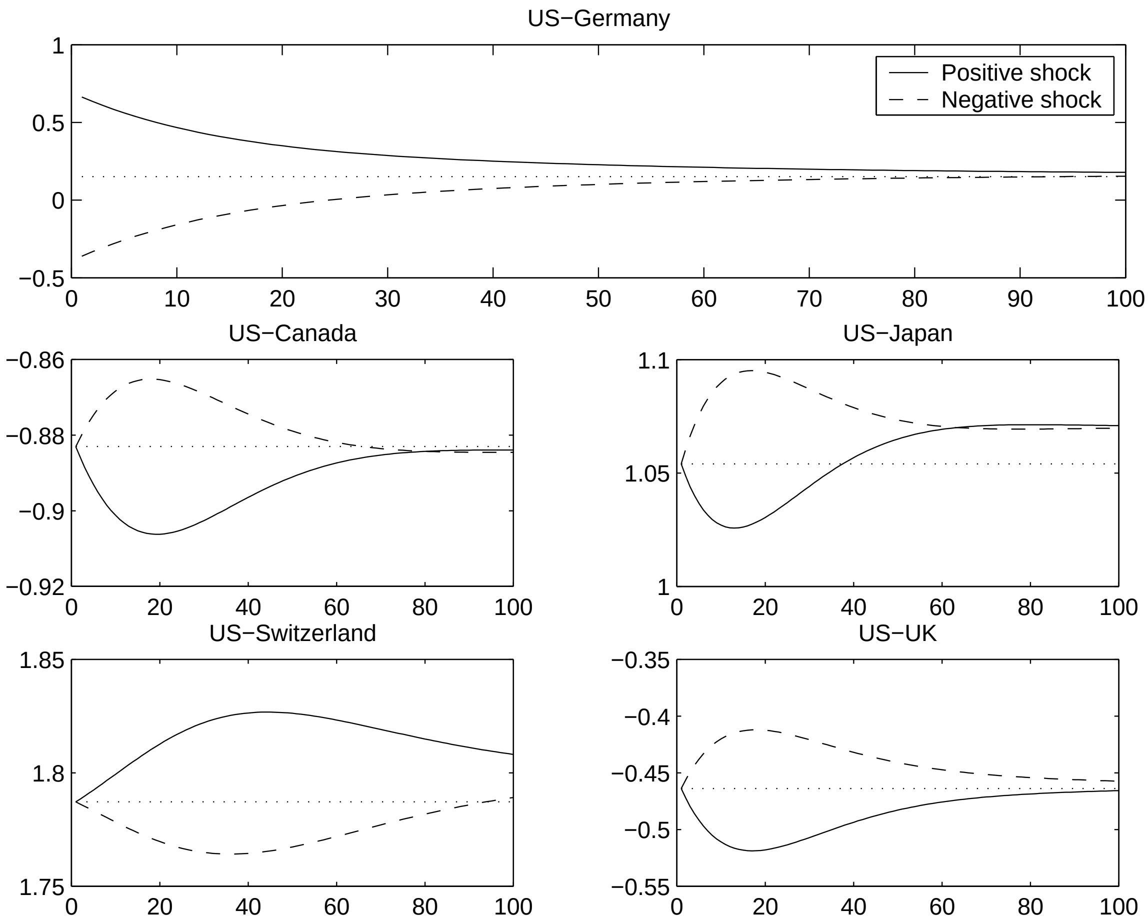

Figure 34: VAR with shock to German *ex ante* real interest rate

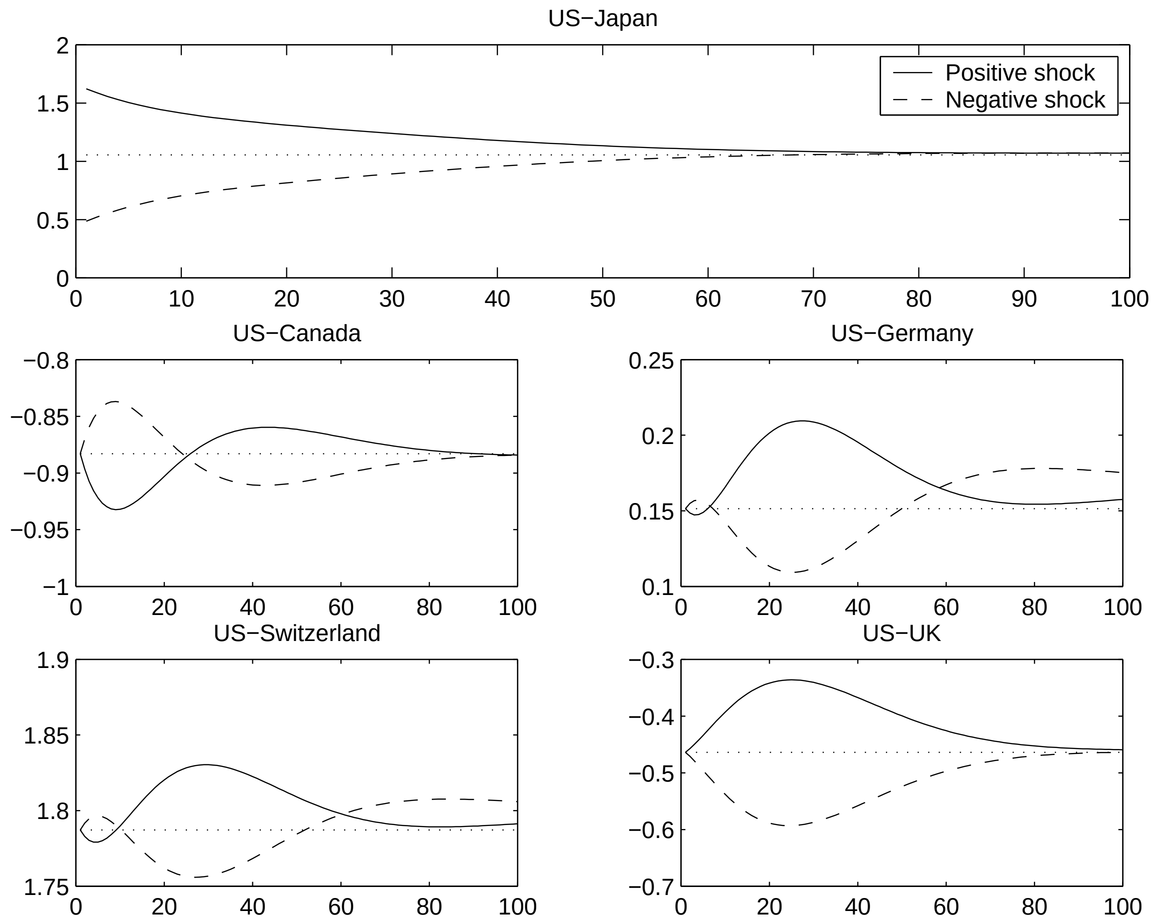

Figure 35: VAR with shock to Japanese *ex ante* real interest rate

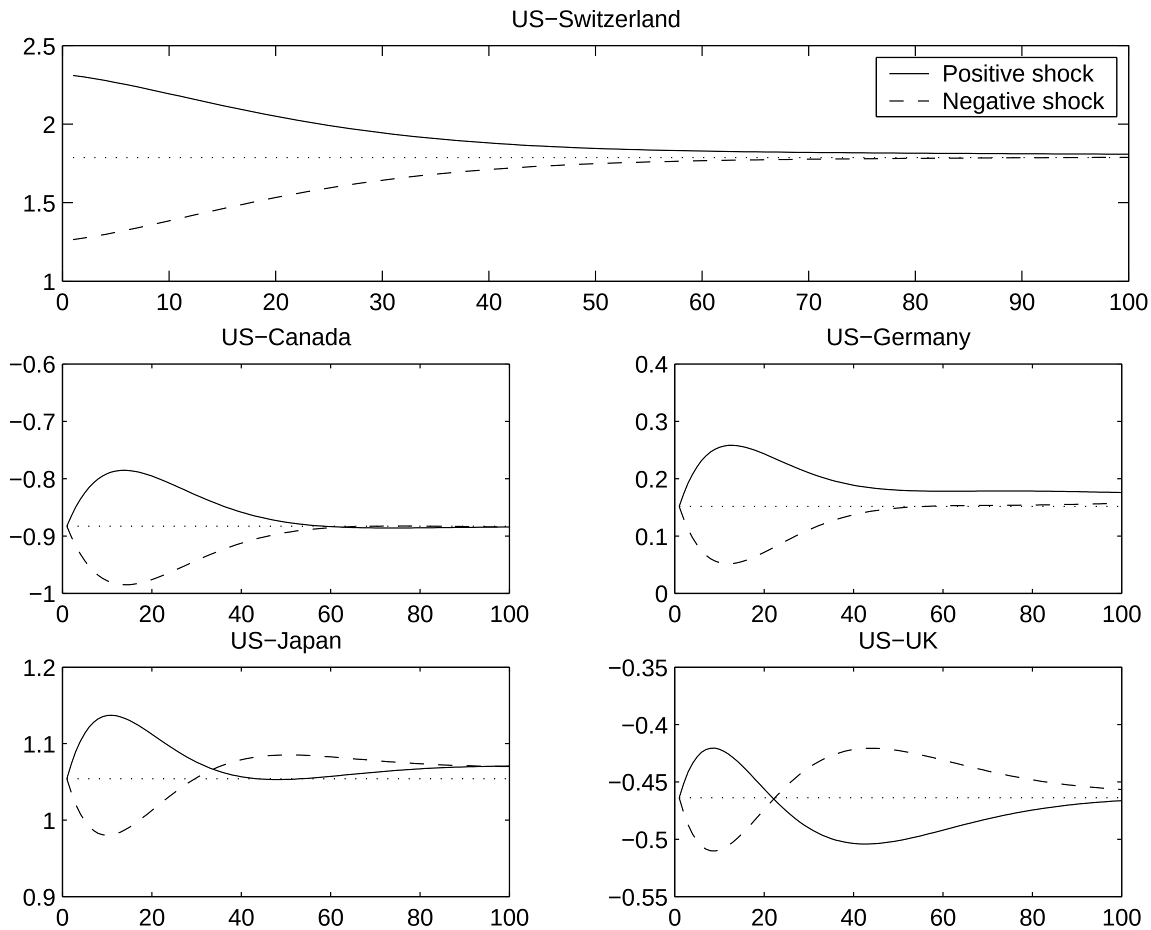

Figure 36: VAR with shock to Swiss *ex ante* real interest rate

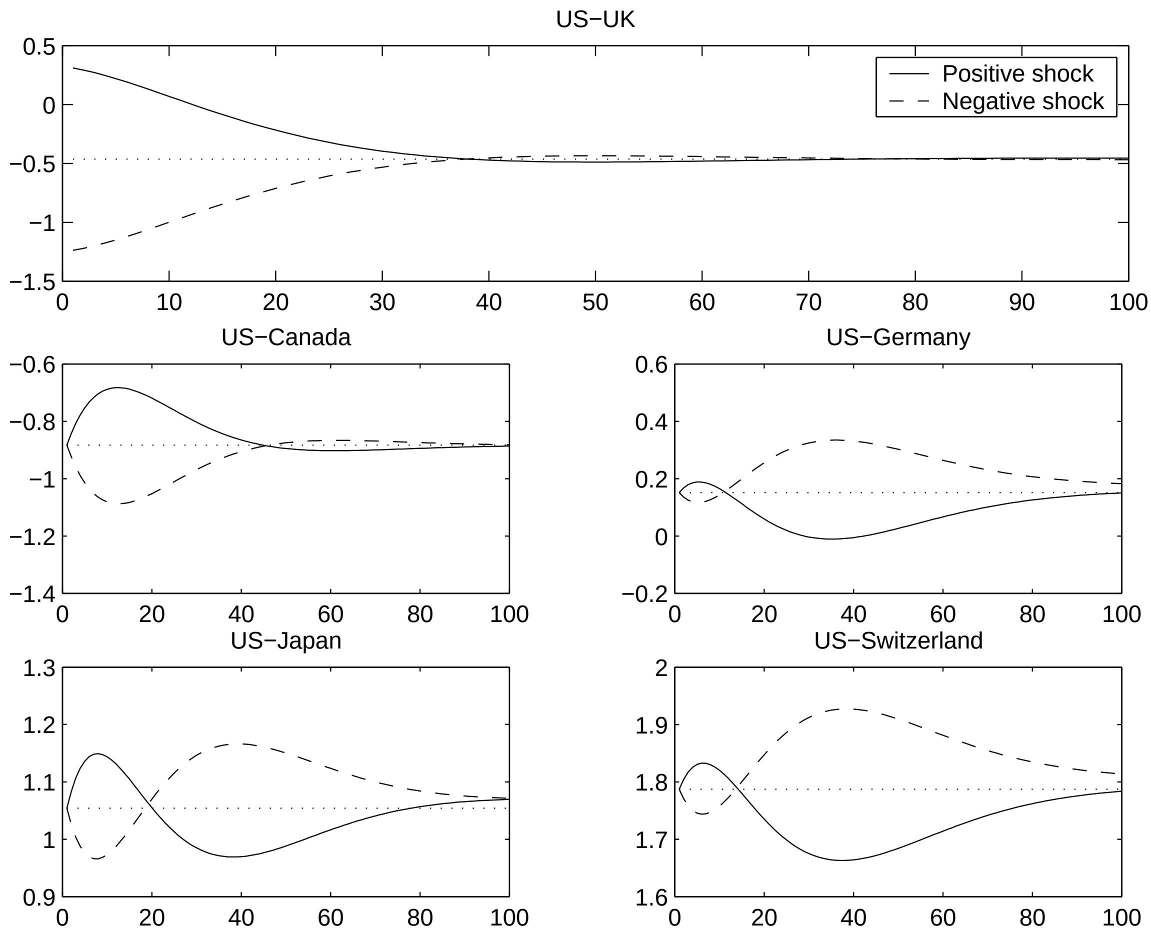

Figure 37: VAR with shock to U.K. *ex ante* real interest rate

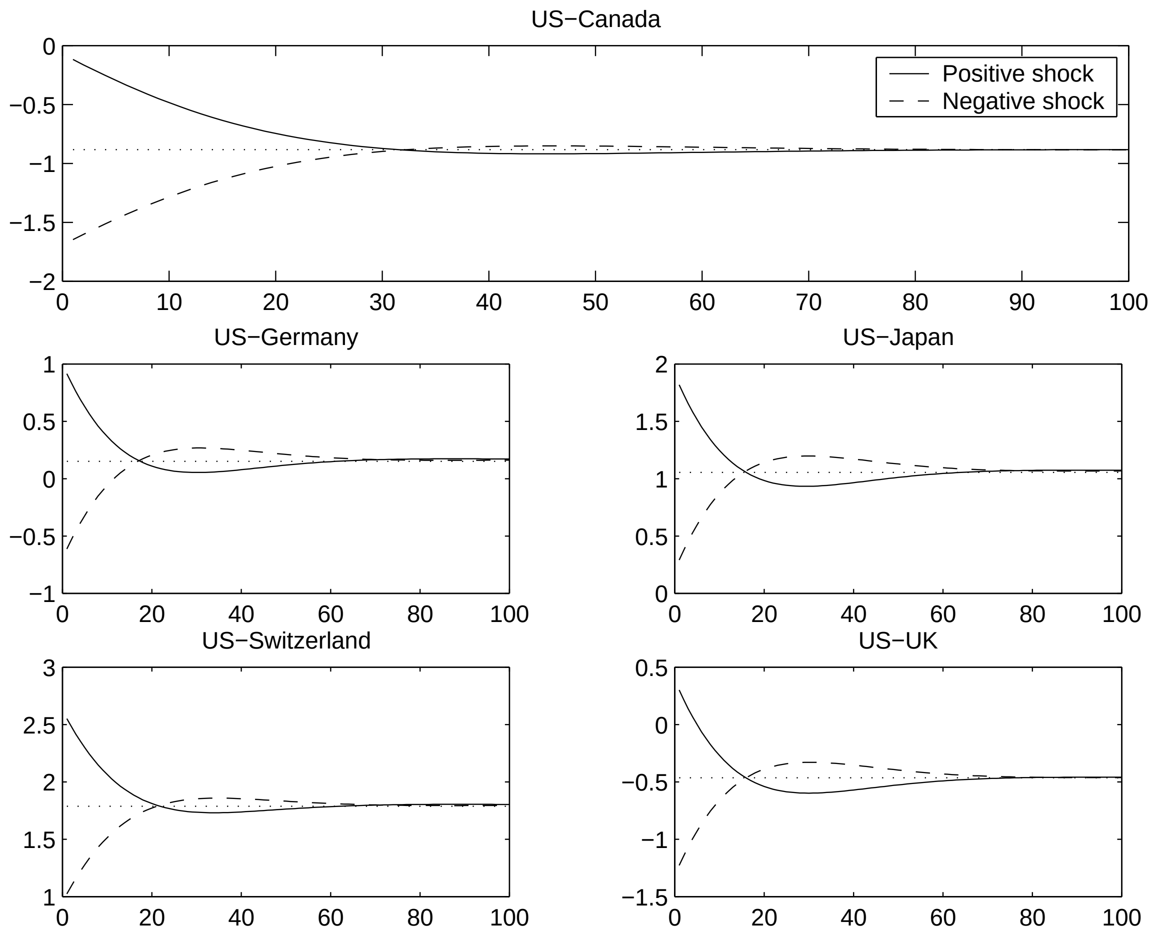

Figure 38: VAR with shock to U.S. *ex ante* real interest rate

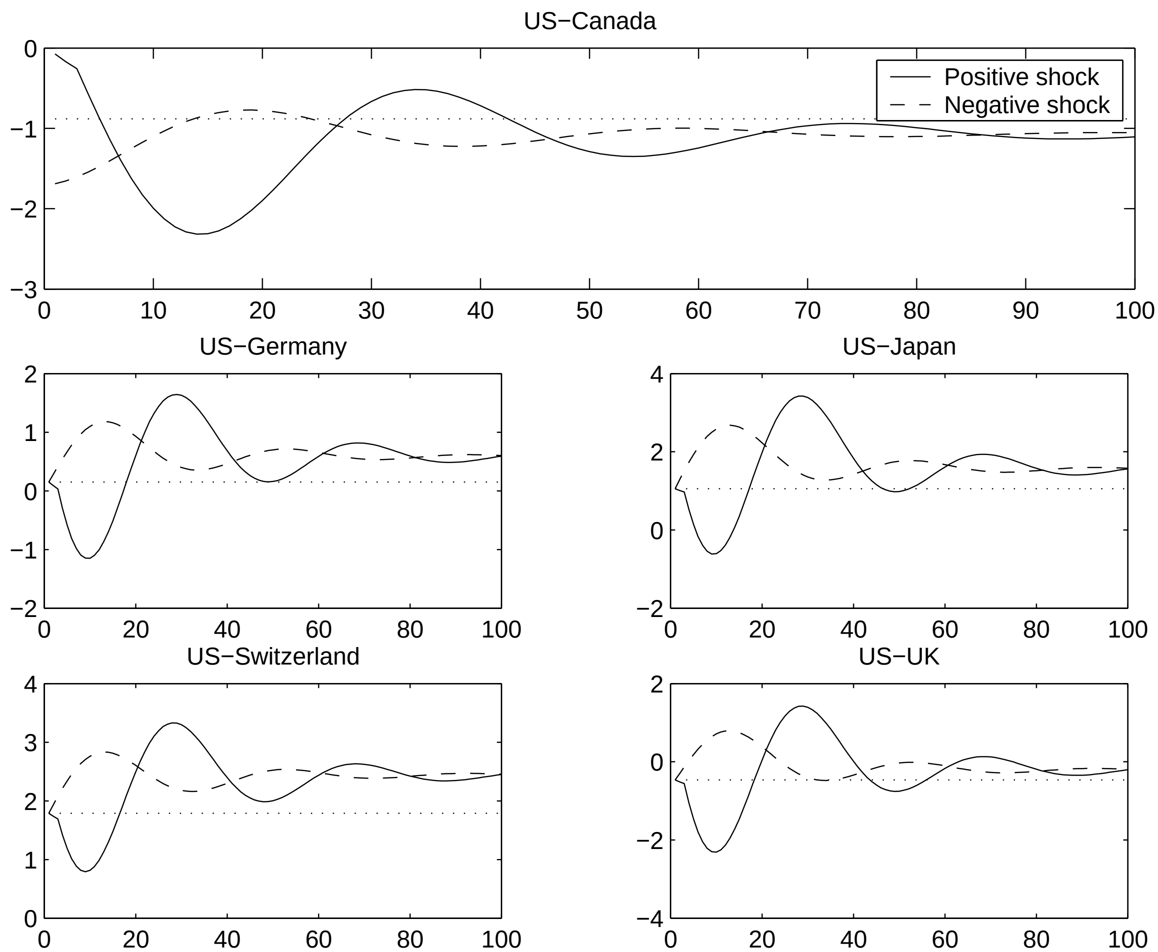

Figure 39: TVAR with shock to Canadian *ex ante* real interest rate

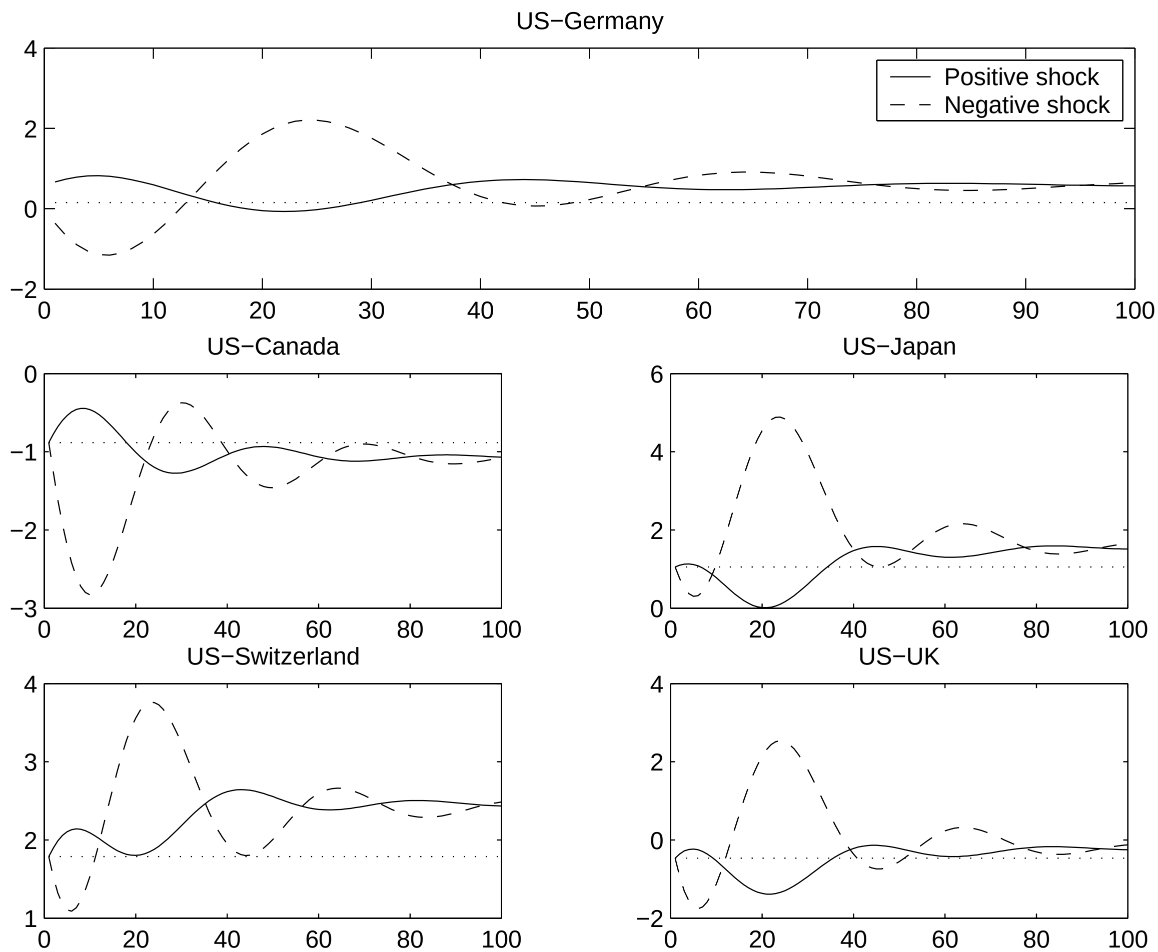

Figure 40: TVAR with shock to German *ex ante* real interest rate

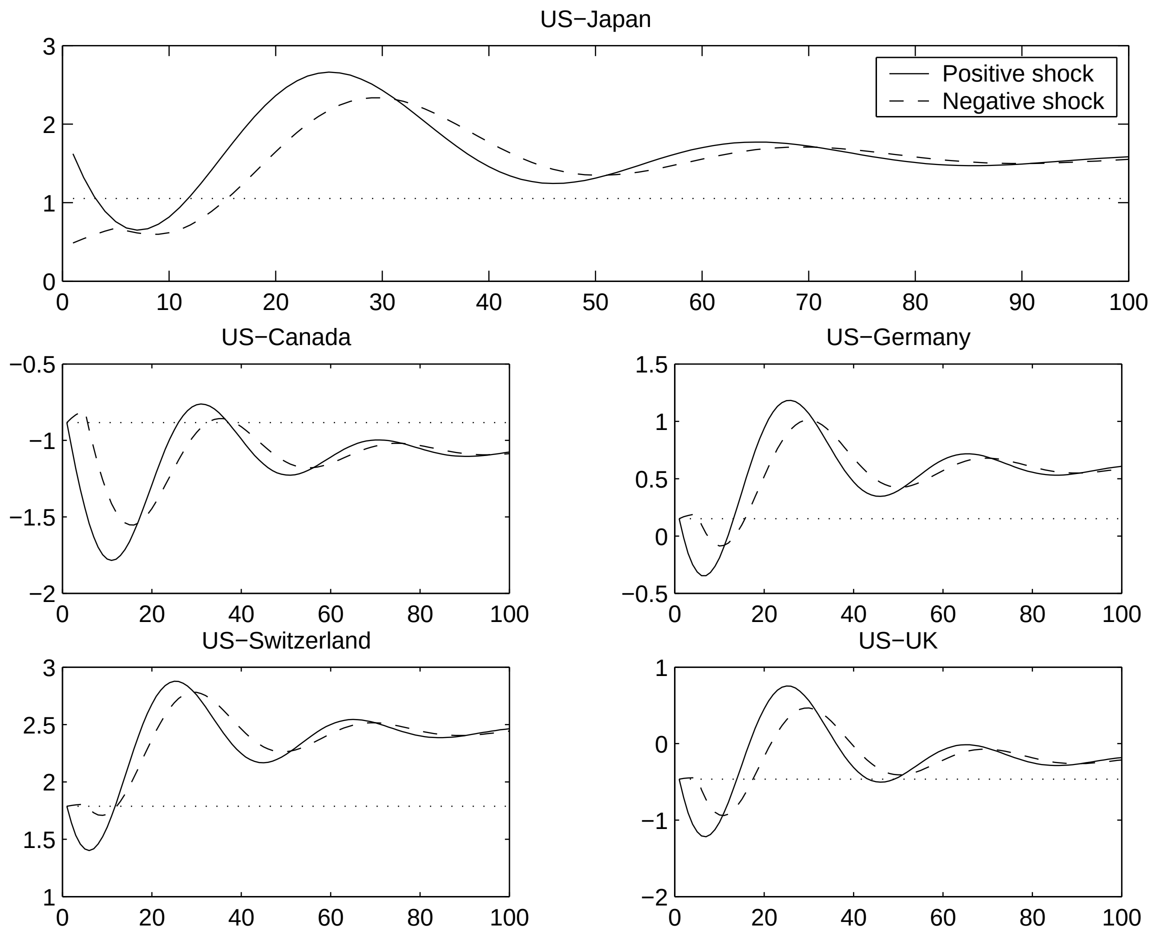

Figure 41: TVAR with shock to Japanese *ex ante* real interest rate

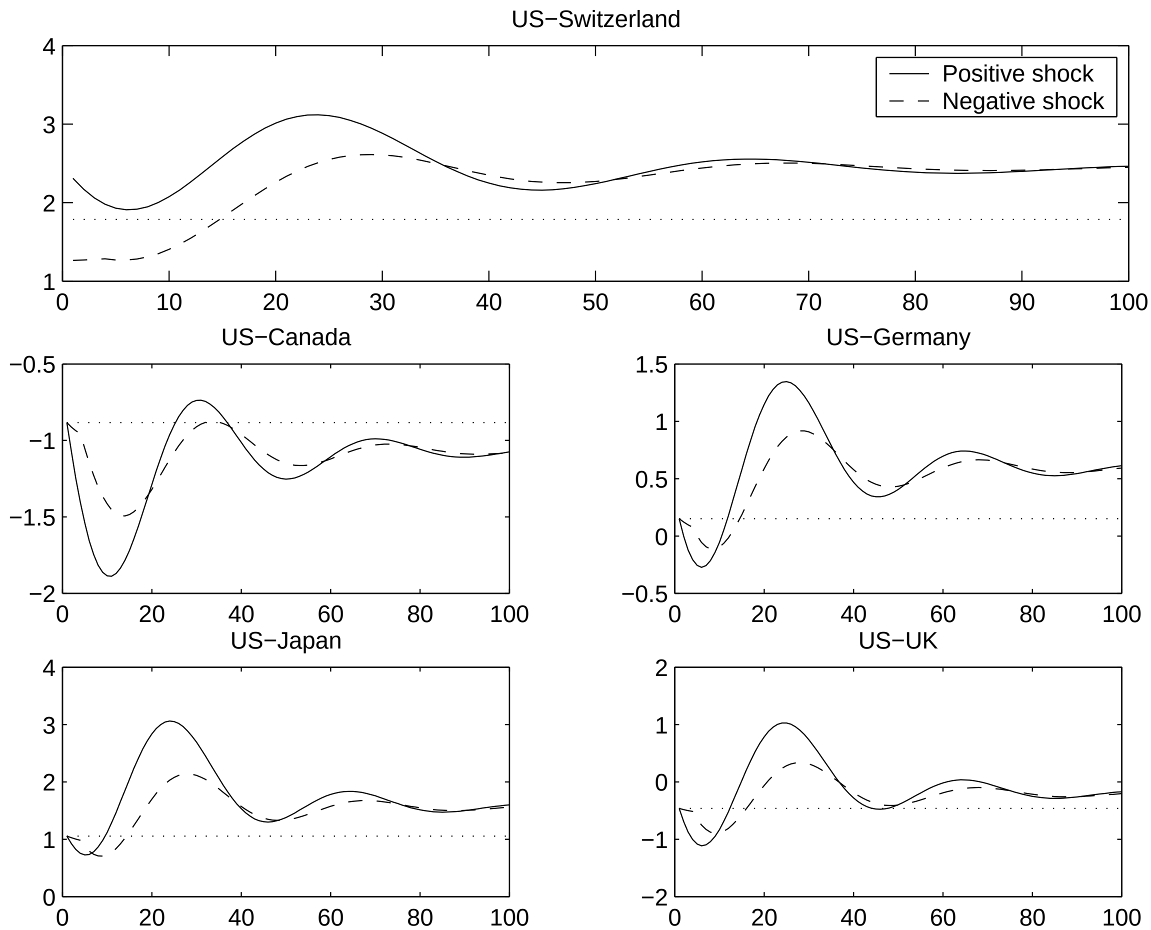

Figure 42: TVAR with shock to Swiss *ex ante* real interest rate

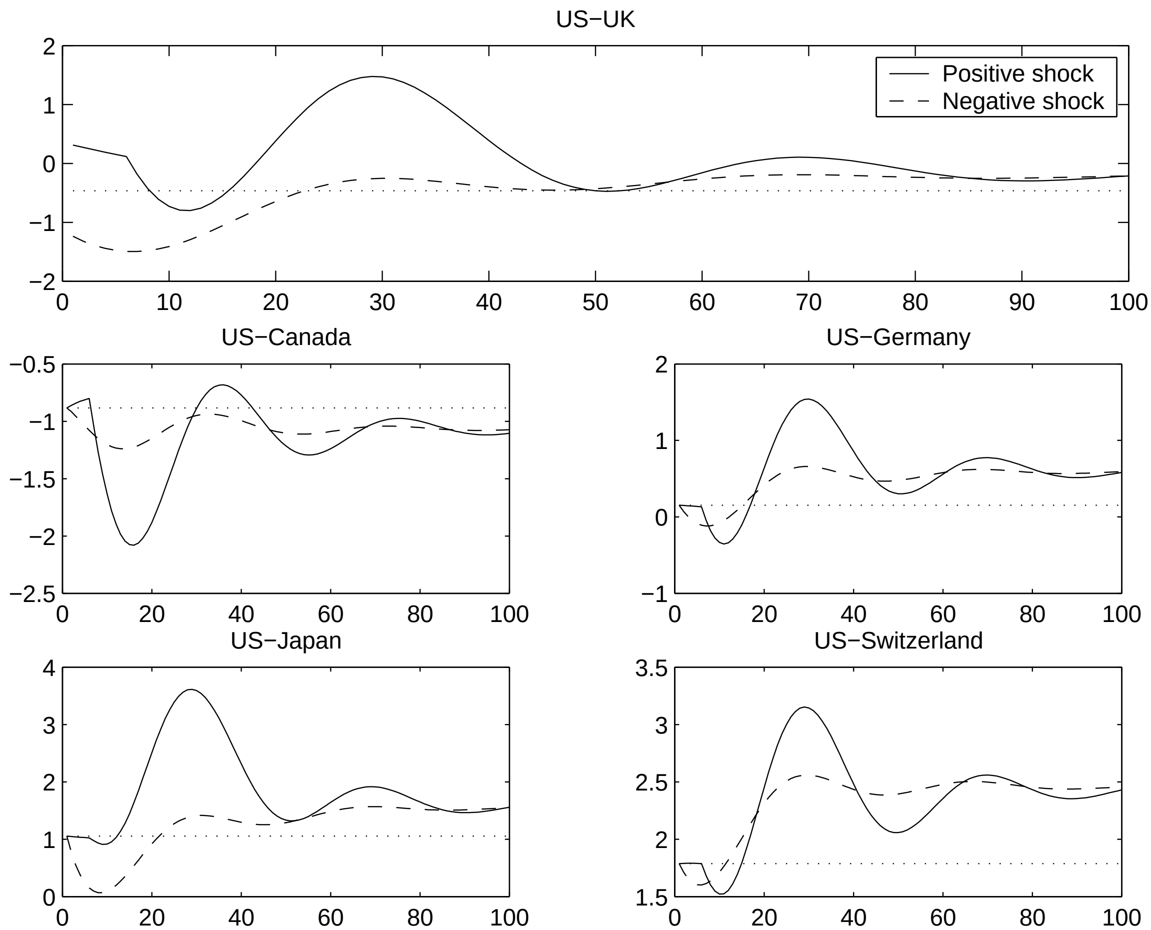

Figure 43: TVAR with shock to U.K. *ex ante* real interest rate

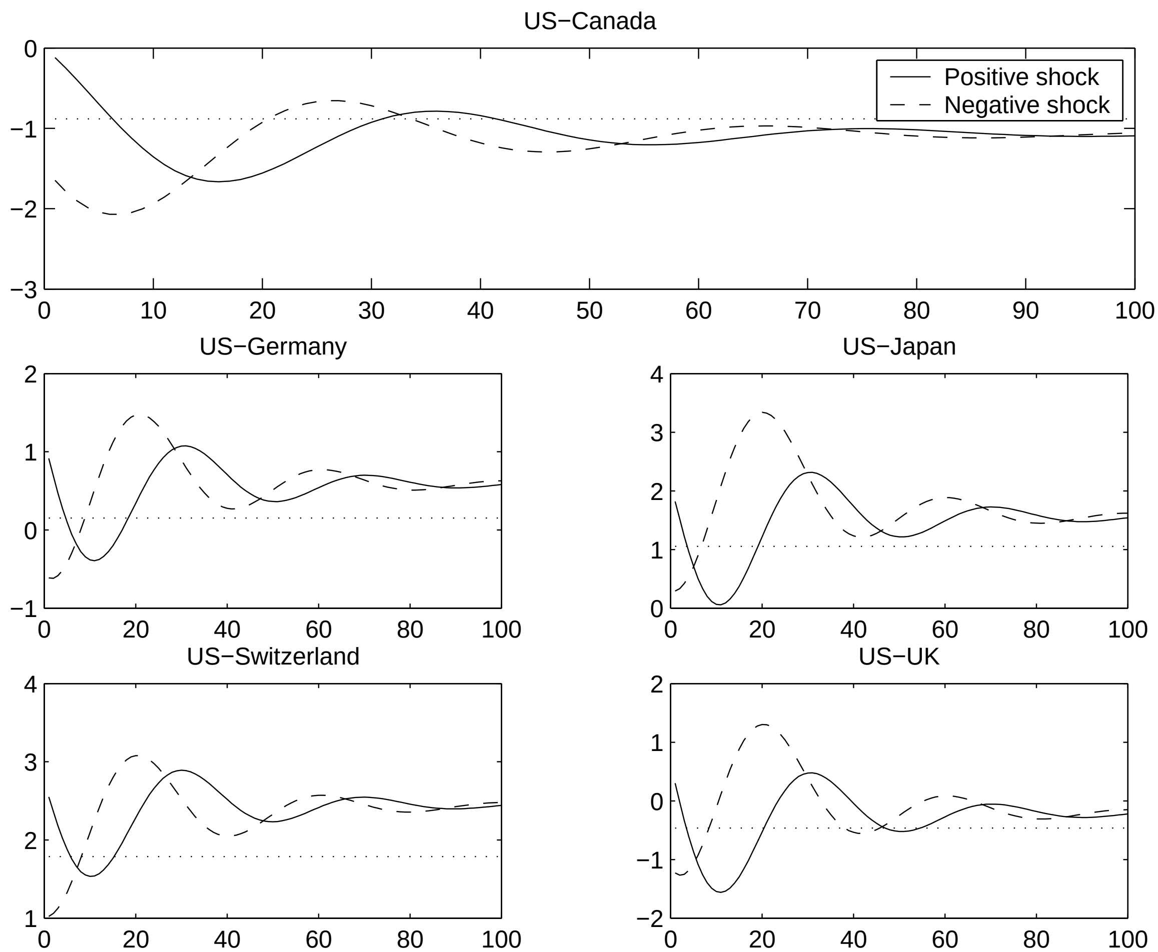

Figure 44: TVAR with shock to U.S. *ex ante* real interest rate

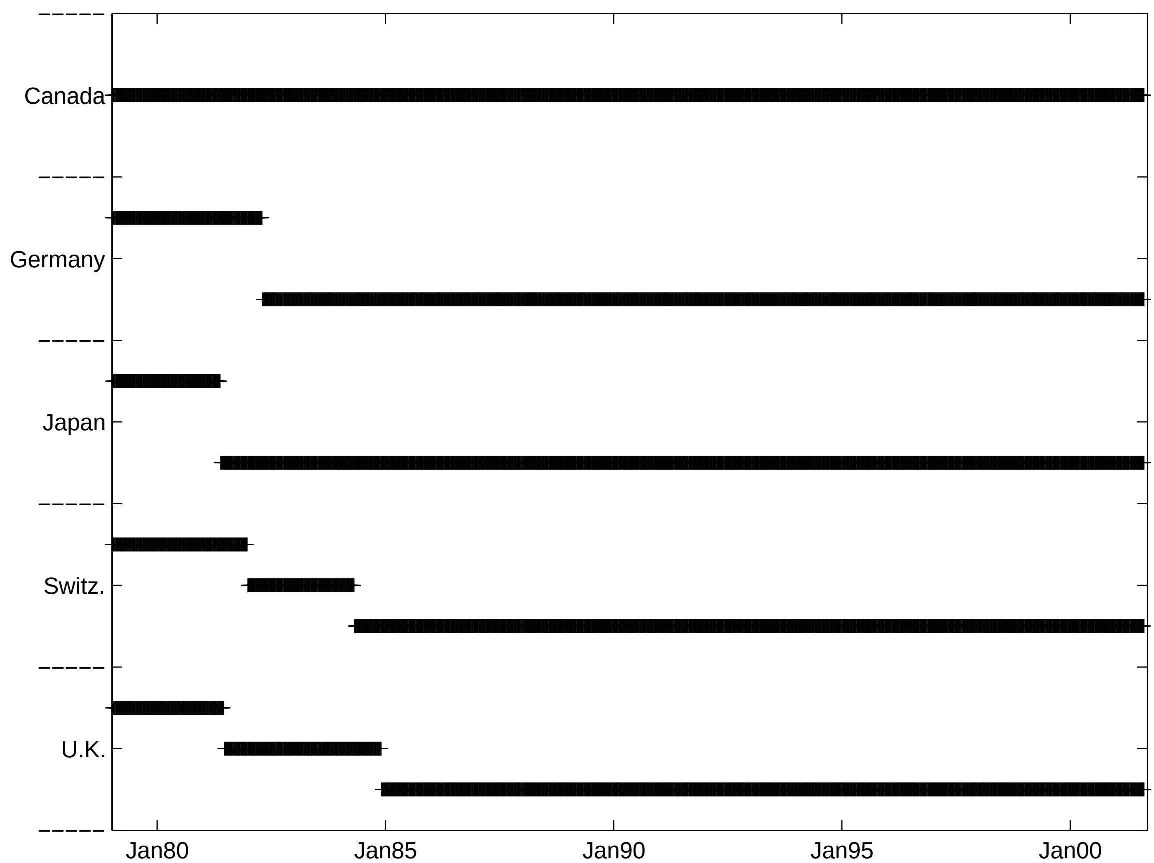

Figure 45: Summary of Likely Structural Breaks