

ABSTRACT

OVIEDO, PEDRO MARCELO. Intermediation of International Capital Inflows and Macroeconomic Fluctuations in Emerging Market Economies (Under the direction of John J. Seater and Enrique G. Mendoza).

This study investigates the macroeconomic implications of financial intermediation of international capital inflows in a small open economy (SOE). The interplay between financial intermediation and macroeconomic fluctuations is studied under alternative representations of the relationship between international lenders, banks, and domestic borrowers, following two typical approaches in the banking literature. Under the industrial organization approach, a model with neoclassical banks is unable to reproduce the large output swings associated with capital outflows observed in actual emerging economies. Furthermore, the volatility of domestic financial variables is consistent with actual statistics only when banks' supply of funds has a finite elasticity. Modelling banks under the incomplete information approach permits incorporation of aggregate risk into the analysis. Under this setting, banking crises are driven by fundamentals, and both capital inflows and country-specific interest rates are endogenous and important in explaining domestic fluctuations. The study ends with a detailed explanation of how to use numerical methods to solve linearized models like the model with neoclassical banks.

**Intermediation of International Capital Inflows and Macroeconomic
Fluctuations in Emerging Market Economies**

by

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To my beloved wife, Pauli, for her love and unconditional support.

Biography

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Introduction

This dissertation investigates the role of banks that intermediate inflows of foreign capital into a small open economy (SOE). The study provides a quantitative analysis of the macroeconomic implications of financial intermediation under alternative representations of the relationship between international lenders, banks, and domestic borrowers.

The dissertation research was motivated by the contrast between two key predictions of standard business cycle models of a SOE and the actual sources of macroeconomic instability in emerging countries. First, in the standard model domestic fluctuations are almost neutral to international-interest-rate shocks. Second, when the SOE has access to a frictionless world capital market, capital inflows are demand determined. However, the sudden capital outflows affecting emerging markets seem to indicate that the behavior of investors bears a closer look, as does the alternative mechanisms through which the cost of international capital impacts on the macroeconomic developments of emerging economies.

Banks are modelled following two typical approaches in the banking literature, namely, the industrial organization approach and the asymmetric information approach. In the first chapter, profit-maximizing neoclassical banks employ capital and labor to produce intermediation services. Banks are the only domestic agents with access to international financial markets. They borrow from foreign creditors and lend to domestic households and firms. Households borrow to smooth consumption and firms to finance their working capital. The quantitative analysis of the stochastic dynamic general equilibrium allocations of the economy indicates that the model

with neoclassical banks is unable to reproduce the large output swings associated with capital outflows observed in actual emerging economies. Also, the volatility of domestic financial variables is consistent with actual statistics only when the banks' supply of funds has a finite elasticity.

In the second chapter, banks are modelled following the incomplete information setting. This setting permits the incorporation of non-diversifiable aggregate risk (i.e. macroeconomic risk) into the analysis. Agency costs arise at the level of the firms and at the level of the bank, and therefore international investors have to 'monitor a monitor'. The optimal funding mechanism to bring working capital from abroad is a two-sided debt contract between firms, banks, and international investors. Banks are risky because their portfolio return hinges upon the macroeconomic performance of the economy and a sufficiently adverse productivity shock can trigger a financial crash. Under this setting, banking crises are driven by fundamentals and *macroeconomic risk*. The model is consistent with the empirical evidence on banking crises because either a slowdown of the economy or a world interest-rate hike tends to breed banking sector problems. Furthermore, the model predicts that capital flows and the country-specific interest rates are endogenous and important factors in domestic fluctuations. This is because, in a world of forward looking economic agents, the aggregate risk affects business cycles inasmuch as all agents incorporate the endogenously determined probability of a crisis into their economic decisions.

The third chapter offers a detailed discussion of how to use numerical techniques to solve linear difference rational expectation macroeconomic models. A set of Matlab functions accompany this chapter. The distinguishing characteristic of this chapter, vis-à-vis other numerical recipes available elsewhere, is the minimization of the computational task necessary to analyze an economic model and the step-by-step documentation of the process involved in solving and analyzing the linear version of a dynamic economic model. Linear dynamic rational expectation models arise from the linearization of dynamic models around a particular stationary point, typically the non-stochastic steady state.

Chapter 1

Capital Flows and Financial Intermediation in a Small Open Economy: Business Cycles with Neoclassical Banks

1.1 Introduction

This chapter investigates the quantitative importance of exogenous fluctuations in capital inflows for the business cycles of a small open economy (SOE). Both the cost of international financing and international capital flows are crucial for the macroeconomic performance of emerging economies. In the 1990's several developing countries faced sudden capital outflows with devastating consequences for their real economies. After the 1994 Mexican devaluation and the Russian default and Asian crises in 1997-

1998, many countries experienced how the capital inflows that shrank interest rates and fuelled economic expansions at the beginning of the 90's, then flew out giving rise to deep recessions, unemployment, and financial turmoil. Figure 1.1 shows the three-month Argentinean interest rate, and a GDP index for the period 1982-1999. The contemporaneous correlation between the two variables is equal to -0.781. A similar pattern seems to relate the interest rate and output in other developing countries like Mexico and Brazil. Considering that international capital inflows enlarge the lending capacity of domestic banks, it is not less surprising the positive co-movement between bank loans and GDP (see Figure 1.2); the contemporaneous correlation equal of these two variables is equal to 0.64.

Event though referring to interest-rate shocks and capital flows as if they were completely independent of domestic *economic* developments may sound unrealistic, there are reasons to believe so under some circumstances. On one hand, there are several *non-economic* domestic factors (e.g. political events) that affect country-specific risk premiums. The recent winding path of the Argentine bond returns which were moving in accordance with the political turmoil in the country is a clear example. On the other hand, both the financing conditions and the availability of external financial capital in emerging countries are, to a large extent, independent of *any* domestic event. This hypothesis has been supported by Calvo, Leiderman and Reinhart (1993) and Calvo, Leiderman and Reinhart (1996). After studying the capital inflows to Latin America at the beginning of the 90's, they conclude that much of these inflows were driven by factors external to the region such as the macroeconomic strength in the developed economies and changes in the regulation of their capital markets. Similarly, Corbo and Hernández (2001) indicate that the size of overall capital flows depend on factors internal to industrial economies, while the distribution of the flows among developing countries hinges on country specific factors. Furthermore, Calvo and Mendoza (2000) demonstrate how likely is a scenario where international investors take portfolio decisions following the 'market' rather than assessing countries' fundamentals. Therefore, contagion effects might also produce large capital flows in globalized markets regardless of the undergoing conditions in a particular country.

In the standard small-open-economy RBC model (as in Mendoza (1991a)), a

change in the international interest rate affects production through the supply of factor inputs. First, labor supply decisions are subject to intertemporal substitution. Second, a change in bond prices makes households vary their consumption path and reallocate their savings between physical capital and international bonds. In this framework, Mendoza (1991a) shows the neutrality of this model with respect to interest-rate fluctuations when it is calibrated to the Canadian economy.^{1 2}

To add a mechanism through which interest-rate disturbances become a source of macroeconomic volatility, Neumeyer and Perri (2001) propose modifying the standard model to introduce a demand for working-capital. Since firms have to pay for the use of factors of production before getting their sale proceeds, the interest rate is part of the cost of employing inputs. The effect of interest-rate shocks on production is the same as the one that Christiano (1991) and Christiano and Eichenbaum (1992) introduce to explain the liquidity effect induced by money inflows in a closed economy. On the one hand, a change in the domestic interest rate (due to a liquidity effect in one case and to an international interest-rate shock in the other) affects input supplies through both assets and intertemporal substitution. On the other hand, the demand for inputs is also affected, since the interest rate becomes part of the cost of employing factors of production.

Neumeyer and Perri (2001) assume that firms fund their production process by directly placing bonds in free-access world capital markets, so that working capital is modelled as a factor of production coming directly from overseas. A subtle analysis of the nature of working capital reveals that its introduction in a macroeconomic model may deserve a deeper analysis. First, working capital is associated with a short-term loan, the typical financial service banks offer to firms, and not with a long-term international bond. Second, these loans are monitoring-intensive and so it is more plausible they would be granted by domestic banks.

¹The utility index in Mendoza (1991a) rules out the intertemporal substitution in labor.

²Following a non-standard procedure where a model economy is used to back out the shocks consistent with the actual evolution of the (model) endogenous variables, Blankenau, Kose and Yi (2001) find that world-interest-rate shocks are important to explain Canadian business cycles. Correia, Neves and Rebelo (1995) finds the same results for the Portuguese economy using another version of this RBC model.

The banking system is a central element of the process of financial intermediation in most developing economies, and domestic capital markets play an almost insignificant role in the borrow-lending process of these emerging economies.³ Beck, Demirgüç-Kunt and Levine (1999) show that while private bonds' market capitalization is around 4% of the GDP, total private credit from financial intermediaries is equal to 20% of the GDP in low and lower-middle income countries. In high income countries these ratios rise to 20% and 60%, respectively.

This chapter studies the business fluctuations of a SOE in which a neoclassical banking system intermediates the inflows of foreign capital and firms have to finance their working capital. The model provides an analytical framework to study the interactions between the financial and non-financial sectors in emerging economies.

Although in a frictionless Arrow-Debreu economy, the form of financial intermediation is inessential for real variables, the banking literature maintains that there exist a broad array of issues which render essential the role of banks (see Freixas and Rochet (1997)). These issues give rise to two paradigms to model the role of banks: the industrial organization approach and the asymmetric approach. The former approach, which considers that banks provide differentiated services whose tangible counterparts are the financial transactions, is used in this chapter. The transformation of financial securities and the exclusive access to international financial markets are the reasons for banks to exist in the model. Under the asymmetric information approach banks overcome the informational asymmetries that preclude the existence of complete markets.⁴ Although the validity of the informational approach is not neglected, this chapter of the dissertation is aimed at exploring the role of financial intermediaries under the industrial organization approach. The model can be considered as a benchmark to compare the dynamic properties of other models where the financial system becomes the mechanism to solve informational problems.

Banks are modelled following Freixas and Rochet (1997, chap. 3). The banking sector is perfectly competitive and banks face two constraints, a financial or balance

³This fact has been early documented by Gurley and Shaw (1960). They also observed that in the earlier stages of financial development, commercial banking is the main form of intermediation.

⁴The second chapter of this work appeals to the asymmetric information approach to discuss the role of aggregate-credit risk and banking crises in the business cycles of emerging countries.

sheet constraint, and a technology constraint Sealey and Lindley (1977). The technological constraint dictates that real resources must be used up in the process of granting a loan. Banks produce loans employing labor and capital along with specific ‘banking skills’. Banks maximize profits and their revenues come from the intermediation margin. The balance sheet constraint assures that what banks lend in the domestic credit market is what they borrow from abroad. Thus, financial decisions are not independent from production decisions but they are made jointly.

Banks issue an internationally traded bond and the proceeds are lent to other agents in the economy: firms and households. Firms must pay ‘a fraction’ of the factors of production they employ before realizing their sales and hence have a demand for working capital. Households use bank loans to smooth consumption and to change the stock of capital they are renting to firms and banks. Thus, from the standpoint of households, bank loans play the role international bonds do in the standard model.

The main findings of this part of the dissertation can be summarized as follows. First, adding working capital needs to the RBC model of small open economies is not enough to break the neutrality of business cycles to interest-rate shocks. Second, only when the supply of credit is not infinitely elastic does the model produce a volatility of domestic finance consistent with actual statistics. Also, the standard small-open-economy RBC model, even when augmented to include neoclassical banks, is unable to reproduce the effects that capital outflows have in actual economies.

Several papers have studied the relationship between financial intermediaries and firms in macroeconomics employing the industrial organization approach. King and Plosser (1984) modify the RBC model to add a banking sector which provides transaction services to study money-output correlations. Contrary to King and Plosser (1984) where banking services are another input of the production function, in this dissertation chapter they are treated as working capital. In Díaz-Giménez, Fitzgerald and Alvarez (1992) banks intermediate among agents of a closed economy. Households borrow from banks to finance the purchases of houses and they lend to banks to save for retirement. The banks in Díaz-Giménez et al. (1992) use real resources to produce both deposits and loans and operate a constant return technology so that interest-rate spreads are independent of the resources being intermediated. Inas-

much as banking skills are an input in fixed supply used by the financial industry in the model of section 2, the interest-rate spread depends on the level of loans. This technology guarantees that the model has a well defined steady state and no other assumption is required in this regard. Agénor (1997) introduces banks in a model of SOE to study the effect of an increase in the risk premium on international markets induced by a contagion effect. However banks are modelled as a costless technology and the interest-rate margin arises only due to the imposition of reserve requirements.

The rest of this dissertation chapter proceeds with other three sections. Section two presents the model and section three contains its quantitative properties. The model is calibrated to Argentina for the period 1970-1999, in order to evaluate the importance of the banking system and the demand for working capital for the business cycles of that economy. The final section contains concluding remarks.

1.2 The Model

Consider a small open economy with three type of agents: banks, firms and households. They interact in four competitive markets: labor, capital, loans, and goods. The economy grows at a constant and exogenous rate, γ , determined by a standard labor augmenting technological change. Therefore, non-price variables, except labor, are detrended accordingly.

1.2.1 The Household Problem

The representative household (RH) has an infinite life and wants to maximize its objective function

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t^s) \quad (1.1)$$

where c_t represents consumption and n_t^s the labor supplied by the household; the time endowment is normalized to one and household's leisure is the time not spent working, i.e. $1 - n_t^s$. The instantaneous utility function is continuously differentiable and

concave. $\beta \in (0, 1)$, is the intertemporal discount factor and E_0 indicates expectations as of time $t=0$, conditional on the information set Ω_0^h .⁵ The household faces the following flow budget constraint:

$$w_t n_t^s + r_{k,t} k_t^s + \pi_{b,t} + \pi_{y,t} + (1 + \gamma) L_{h,t+1}^d \geq c_t + i_t \left[1 + H \left(\frac{k_{t+1}^s}{k_t^s} \right) \right] + L_{h,t}^d (1 + r_{L,t}) \quad (1.2)$$

The RH's total income in the left hand side of eq. (1.2) is given by the sum of labor and property income. Labor income depends on the wage rate, w_t , and the labor services supplied. Property income has three components. First, the net financial income coming from net interest earnings on household loans, $(1 + r_{L,t}) L_{h,t}^d$. Second, as the RH is the owner of both banks and firms, it receives profits from these two sectors, $\pi_{i,t}$ ($i=b, y$). Third, the RH also counts on income coming from renting capital k_t^s at the rental rate $r_{k,t}$.

The RH uses of income in the right-hand-side of eq. (1.2) are purchases of consumption and investment goods, i_t , including installation costs, $(H(\cdot) i_t)$. Excesses of expenditures over income are covered by increasing the demand for bank loans.

While perfect competition will make firm profits equal to zero, bank profits will fluctuate over the business cycle. This is because banking skills are in fixed supply. Investment (net of adjustment costs) and the law of motion of the household's capital stock are defined by:

$$i_t = (1 + \gamma) k_{t+1}^s - k_t^s (1 - \delta) \quad (1.3)$$

where δ is the depreciation rate. Except at the steady state, adjusting the stock of capital is costly. The convex function $H(\cdot)$ represents the adjustment costs.

To avoid having an infinite level of household debt, the no-Ponzi game condition is imposed:

$$\lim_{t \rightarrow \infty} E_0 \frac{k_t^s - L_{h,t}^d}{\prod_{v=0}^{t-1} (1 + r_{L,v})} \geq 0 \quad (1.4)$$

⁵The true discount factor, B , is different from β since the latter also depends on γ and preference parameters (see section 1.3.1). As usually, $1/B - 1$ is the pure rate of time preference.

The initial conditions for the capital stock and household loans, $L_{h,0}, k_0$, respectively, as well as the initial conditions for the productivity and interest-rate shocks complete the description of the RH's problem

The RH's Optimality Conditions

The RH chooses the contingent sequences $\{c_t, n_t^s, L_{h,t+1}^d, k_{t+1}^s\}_{t=0}^\infty$, so as to maximize eq. (2.10a) subject to eqs. (1.2) to (1.4), and the initial conditions $L_{h,0}, k_0$. The RH's information set at time t is Ω_t^h and includes the historic values of all variables until time $t-1$ and the value of the state variables at time t . The latter are, $L_{h,t}^d, k_t^s$, and also z_t and r_t , which are the economywide productivity shock and international interest rate, respectively.

The following optimality conditions, along with eq. (1.2), characterize the optimal RH's decision process for $t = 0, \dots, \infty$

$$-\frac{u_n(c_t, 1 - n_t^s)}{u_c(c_t, 1 - n_t^s)} = w_t \quad (1.5)$$

$$(1 + \gamma)u_c(c_t, 1 - n_t^s) = \beta E_t \left[u_c(c_{t+1}, 1 - n_{t+1}^s)(1 + r_{L,t+1}) \right] \quad (1.6)$$

$$(1 + \gamma)u_c(c_t, 1 - n_t^s)P_t^q = \beta E_t \left[u_c(c_{t+1}, 1 - n_{t+1}^s)(P_{t+1}^k + r_{k,t+1}) \right] \quad (1.7)$$

where P_t^q and P_t^k are given by:

$$P_t^q = 1 + H_t + H_t' \frac{1}{k_t} i_t \quad (1.8)$$

$$P_t^k = (1 - \delta)(1 + H_t) + (1 + \gamma)H_t' \frac{k_{t+1}^s}{k_t^{s2}} i_t \quad (1.9)$$

P_t^q is the Tobin's Q, and it represents the consumption value-cost of a marginal unit of new capital. P_t^k is the ex-rental value of a marginal unit of installed capital.⁶ The

⁶By construction, either at the steady state or in the absence of capital adjustment costs, $P^q=1$ and $P^k=1-\delta$.

transversality condition indicates that:

$$\lim_{t \rightarrow \infty} \beta^t E_0 [u_c(c_t, n_t^s)(k_t^s - L_{h,t}^d)] = 0$$

Eq. (1.5) equates the marginal rate of substitution of consumption for leisure to the wage rate. Eqs. (1.6) and (1.7) characterize the optimal saving behavior. The former governs the accumulation of bank debt over time. The optimal borrowing behavior indicates that the RH borrows from the bank until the benefit in terms of actual utility, equals the discounted expected cost of borrowing. This cost is the future utility that will be given up to repay the loan.

The left hand side of eq. (1.7) designates the (gross) utility cost of installing a new unit of capital. The right hand side shows the expected discounted benefits, in utility terms, of doing so. These benefits have two components: the future rental income from an extra unit of capital, $(r_{k,t+1})$, and the future (after depreciation) value of that unit of capital, P_{t+1}^k . The latter includes the benefits from future reductions in adjustment costs as it can be seen in eq. (1.9).

1.2.2 Firms

The representative firm (RF) faces an atemporal problem. It wants to maximize its profits choosing a combination of labor, capital, and working capital, given input prices, the final output price (normalized to one), and the interest rate on bank loans. Working capital is required since the RF must pay a ‘fraction’ of the labor services before selling its output. The RF takes supply decisions in the output market and demand decisions in input and loan markets.

The RF’s objective function is:

$$\pi_{y,t} = e^{z_t} f(k_{y,t}^d, n_{y,t}^d) - r_{k,t} k_{y,t}^d - w_t n_{y,t}^d (1 + \varphi r_{L,t}) \quad (1.10)$$

where f is an increasing and concave production function; $k_{y,t}^d$, and $n_{y,t}^d$ are the capital

and labor services demanded by the firm; φ is the fraction of labor costs paid in advance. This specification of the working capital demand allows for $0 \leq \varphi \leq 1$ rather than imposing $\varphi = 1$. When $\varphi = 1$ the amount of working capital demanded would be incompatible with the amount of total credit available in developing economies. For example, when $\varphi = 1$, if the share of income paid to labor is equal to 60% of the output, so is the working capital to output ratio. Working capital loans are, however, only one component of the demand for credit. Adding the households' stock of loans would return the ratio of credit to GDP of a net debtor country as high as 100%, a fact that is not observed in emerging economies.⁷

The term e^{z_t} is a productivity shock with z_t given by:

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t} \quad (1.11)$$

where $\varepsilon_{z,t}$ is a zero mean, i.i.d. process with $\text{Var}[\varepsilon_z] = \sigma_{\varepsilon_z}^2$.

The FOC's of the RF's problem are standard: for each input, the marginal revenue product is equal to its unit cost, which in the case of labor includes *financing* costs:

$$e^{z_t} f_{n_y}(k_{y,t}^d, n_{y,t}^d) = w_t(1 + \varphi r_{L,t}) \quad (1.12)$$

$$e^{z_t} f_{k_y}(k_{y,t}^d, n_{y,t}^d) = r_{k,t} \quad (1.13)$$

In this economy, firms borrow the following amount of working capital:

$$L_{y,t}^d \equiv n_{y,t}^d w_t \varphi \quad (1.14)$$

At each date t , the RF observes prices $r_{L,t}$, w_t , and $r_{k,t}$, and chooses the amount of labor and capital services according to eqs. (1.12) and (1.13). Condition (1.12) illustrates how interest-rate disturbances impact production.⁸ A rise in $r_{L,t}$ depresses

⁷Recall the ratios from Beck et al. (1999) mentioned in the introduction

⁸Although $r_{L,t}$ is the domestic interest rate, it is going to be shown later that $r_{L,t}$ is positively correlated with the international market rate, r_t .

the gross cost of employing labor and induces firms to raise labor demand and production. Since $\varphi=0$ in the standard RBC small-open-economy model, the interest rate has no effect on the demand side of input markets and any variation in the output level arises from changes in the supply side of these markets.

1.2.3 Banks

The representative bank (RB) is the only domestic agent borrowing and lending in international capital markets. Its balance sheet constraint dictates that the RB lends at home what it borrows abroad. On the other hand, a technological constraint arises because the production of loans imposes administrative costs to the banks. These costs are modelled as requirements of capital, labor, and banking skills. A typical commercial bank hires labor (tellers, managers, etc.) and capital (computers, buildings, ATM's, etc.) for their operations. It also employs the “bankers” whose services are supplied inelastically and are invariant over the business cycle.

Distinguishing between financial and administrative costs is useful for understanding the model dynamics. Therefore, the RB's profit maximization program is presented as a two-stage problem. In the first step, the RB solves for a cost function which returns the minimum (administrative) cost per level of loans, L_t . In the second step, observing both the market rate for its bonds and the market loan rate, the bank decides on the optimal supply of financing.

The cost function depends on the production function of loans which is given by:

$$L_t = e^{z_t} g(k_{b,t}^d, n_{b,t}^d, x) \quad (1.15)$$

where e^{z_t} is the economy-wide productivity shock discussed before; $k_{b,t}^d$ and $n_{b,t}^d$ are the capital and labor demanded by banks; and x is the banking specific factor. The financial intermediation technology, described by $g(\cdot)$, is represented by a continuous and concave function. For any L_t , one can solve for the *conditional* factor demands, and from there for the bank cost function. Factor demands are conditional on the

level of financing, returning the optimal $n_{b,t}$ and $k_{b,t}$ given factor prices and L_t . The bank cost function, BCF_t , can be written as:

$$BCF_t = BCF(L_t, w_t, r_{k,t}, r_{L,t}) = \tilde{k}_{b,t}^d r_{k,t} + w_t \tilde{n}_{b,t}^d (1 + \varphi r_{L,t})$$

where a “ $\sim$ ” over a variable denotes the conditional factor demand, i.e. $\tilde{n}_{b,t}^d = n_b(w_t, r_{k,t}, L_t)$ and $\tilde{k}_{b,t}^d = k_b(w_t, r_{k,t}, L_t)$. The cost function incorporates the fact that the bank must also pay labor services in advance. The working capital demanded by banks is:

$$L_{b,t}^d \equiv \tilde{n}_{b,t}^d w_t \varphi \quad (1.16)$$

The RB maximizes the profit function that depends on the interest-rate margin are the inflows of capital intermediated and the and the administrative costs:

$$\pi_{b,t} = (r_{L,t} - r_t)L_t - BCF_t$$

where r_t is the international interest rate, that evolves according to:

$$r_t = \rho_0 + \rho_r r_{t-1} + \varepsilon_{r,t} \quad (1.17)$$

$\varepsilon_{r,t}$ is a zero mean, *i.i.d.* process with $Var(\varepsilon_{r,t}) = \sigma_{\varepsilon_r}^2$.

The optimal level of bank loans is determined by maximizing $\pi_{b,t}$ with respect to L_t . At the optimal L_t , the intermediation spread is equal to the marginal administrative cost,

$$r_{L,t} - r_t = \frac{\partial BCF_t}{\partial L_t} \quad (1.18)$$

For an alternative interpretation of eq. (1.18), notice that the marginal revenue, $r_{L,t}$, is equal to the marginal cost. The marginal cost is the sum of the marginal financial cost, r_t , and the marginal administrative cost, $\frac{\partial BCF_t}{\partial L_t}$.

The optimal L_t determines: the bank's net position in international capital mar-

kets through the balance sheet constraint, $b_t = L_t$; the capital and labor demanded through the conditional factor demands; and the demand for working capital given the optimal amount of labor demanded.

A critical element of this model is that the RB's marginal cost function has a finite elasticity.⁹ Eq. (1.18) shows that the marginal administrative costs imposes a wedge between the domestic and international interest rate. When the marginal administrative costs curve is flat, i.e. $\partial BCF_t / \partial L_t$ is constant, the interest rate spread $r_t - r_{L,t}$ is independent of L_t . The domestic rate, $r_{L,t}$, will rise in exactly the same magnitude as a given rise in r_t .

This is no longer the case when banks operate a decreasing returns technology. When the marginal administrative cost curve has a finite elasticity, i.e. $\partial BCF_t / \partial L_t$ is increasing in L_t , the financial system acts as a buffer when the economy is hit by a world-interest-rate shock. In this case, a rise in the international rate shifts the credit supply curve up and to the left, and for the same demand for loans, the equilibrium domestic rate rises less than its international counterpart.

The described banking technology endows the model with a well defined steady state without requiring other assumptions typically used in the literature for this purpose.¹⁰ Because of the described loan production process, the domestic interest rate is an endogenous variable and the economy always reaches a steady-state which is independent of the initial conditions.¹¹ For example, if the economy starts with an L_0 lower than the steady state value of L , the equilibrium domestic rate is lower than its steady state value. This induces an intertemporal substitution in consumption that raises the demand for loans, which in turns moves the domestic interest rate up towards its *unique* steady state value. A similar reasoning explains why if the economy starts with an L_0 higher than its steady state value, the economy will converge to exactly the same stationary equilibrium.

⁹Discussing the effect of implicit bank bailouts on financial crises, Burnside, Eichenbaum and Rebelo (2001) *assume* an intermediation technology like the one discussed in the text.

¹⁰See the review in Schmitt-Grohé and Uribe (2002).

¹¹This contrasts with the assumptions behind the open economy version of the RBC model where both the interest rate factor $1 + r$ and the discount factor β are given from the standpoint of the small open economy.

1.2.4 The Competitive Equilibrium

The competitive equilibrium of the described economy is: a sequence of state contingent allocations for each household $\{c_t, n_t^s, k_{t+1}^s, L_{h,t}^d\}_{t=0}^\infty$; a sequence of contingent allocations for each firm, $\{n_{y,t}^d, k_{y,t}^d, L_{y,t}^d\}_{t=0}^\infty$; a sequence of contingent allocations for each bank $\{n_{b,t}^d, k_{b,t}^d, L_{b,t}^d, L_t^s\}_{t=0}^\infty$; and a sequence of nonnegative contingent prices $\{r_{L,t}, r_{k,t}, w_t, P_t^q, P_t^k\}_{t=0}^\infty$ such that,

1. The allocation $\{c_t, n_t^s, k_{t+1}^s, L_{h,t+1}^d\}_{t=0}^\infty$ solves the representative household's problem, i.e. it maximizes its expected lifetime utility, eq. (2.10a), subject to: a) the resource and time constraints; b) the no Ponzi scheme condition; and c) the initial conditions for the state variables; d) the fixed factor x and the subsequent bank's profits ; e) the sequence of nonnegative contingent prices.
2. The allocation $\{n_{y,t}^d, k_{y,t}^d, L_{y,t}^d\}_{t=0}^\infty$ gives the maximum firm profits in every period given the sequence of prices.
3. The allocation $\{n_{b,t}^d, k_{b,t}^d, L_{b,t}^d, L_t^s\}_{t=0}^\infty$ gives the bank its maximum profits in every period taken as given the sequence of prices, and observing the balance sheet constraint, $b_t = L_t$.
4. The following four markets clear in every period:

Capital services:

$$k_t^s = k_{y,t}^d + k_{b,t}^d$$

Labor services:

$$n_t^s = n_{y,t}^d + n_{b,t}^d$$

Loans:

$$L_t = L_{y,t}^d + L_{b,t}^d + L_{h,t}^d$$

Final goods:

$$e^{z_t} f(k_{y,t}^d, n_{y,t}^d) + L_{h,t+1}^d - L_{h,t}^d(1 + r_{L,t}) = c_t + (1 + H_t)i_t$$

1.3 Numerical Analysis

The model is first calibrated according to the Argentine national accounts and also using standard parameter values in the literature. It is then log-linearized around its non-stochastic steady state to compute impulse response functions and second moment statistics under different assumptions about the role of the banking system.

The functional forms of the utility index in eq. (2.10a) and the production functions for firms and banks in eqs. (1.10) and (1.15) are as follows.

$$u(c_t, 1 - n_t) = \frac{\left(c_t - \frac{\nu}{\mu} n_t^\mu\right)^{1-\sigma}}{1 - \sigma} \quad (1.19)$$

$$e^{z_t} f(k_{y,t}, n_{y,t}) = e^{z_t} A_y k_{y,t}^\alpha n_{y,t}^{1-\alpha} \quad (1.20)$$

$$L_t = e^{z_t} A_b k_{b,t}^{\alpha\xi} n_{b,t}^{(1-\alpha)\xi} x^{1-\xi} \quad (1.21)$$

The RH has an isoelastic instantaneous utility index and σ is the risk aversion parameter. The argument of the utility index is the one introduced by Greenwood, Hercowitz and Huffman (1988). Thus, employment decisions are taken to be independent of consumption-saving decisions, and labor supply decisions are free of any intertemporal substitution effect. Both production functions are Cobb Douglas and A_y and A_b are scaling factors; α and $\alpha\xi$ are the share of output paid to capital in the good and financial industries, respectively. Considering that x is a specific type of labor, the share of all types of labor is given by $1 - \alpha\xi$.

The cost of adjusting the stock of capital is given by:

$$H\left(\frac{k_{t+1}}{k_t}\right) = h_1 \left\{ \exp \left[h_2(1 + \gamma) \left(\frac{k_{t+1}}{k_t} - 1 \right) \right] + \exp \left[-h_2(1 + \gamma) \left(\frac{k_{t+1}}{k_t} - 1 \right) \right] - 2 \right\}$$

where h_1 and h_2 are parameters defining the size of this cost.

The model is consistent with trend growth under the following circumstances. First, the productivity of the fixed factor x also grows at the rate γ . Second, preferences specified in eq. (1.19) implicitly mean that because of the existence of some

kind of home production, the disutility of working also grows over time. Greenwood, Rogerson and Wright (1995) show that an economy with home production is observationally equivalent to another without home production but with different preferences.

1.3.1 Model Calibration

Because working capital is an intermediate input used in production, the value of final output is different from output itself in the model economy. In the calibrated economy, bank output is equal to 0.9% of national output. Argentinean National Accounts indicate that the financial industry output is equal to 4% of GDP (in the period 1993-1999). However, banks' output in the model comprises just a fraction of the services the financial system provides in the actual economy.

Argentina has grown at 2.6% per year during the last 25 years, so $\gamma=0.026$ on an annual basis. As for the interest rates, r is initially set equal to 6.5% on annual basis. Beck et al. (1999) database defines the interest rate margin as the ratio of net interest income and total assets. They estimate that the Argentinean banks interest margin is equal to 4.25%, and this is taken to be the value of administrative costs at the non-stochastic steady state.

The parameter σ is set equal to 2; μ is set equal to 1.45 as in Mendoza (1991), and it implies that $\nu=6.15$. Eq. (1.6) implies that β is equal to 0.981 on quarterly basis. Therefore, the true subjective discount factor, B , solves $B(1 + \gamma)^{(1-\sigma)} = 0.981$ and implies $B=0.987$. The share of the goods industry paid to labor, including its financial costs, α , is equal to 0.40. The parameter ξ is set equal to 0.50; considering the factor x as specific labor, total labor share in the financial industry then rises to 0.8, which is consistent with the fact that the financial industry is (relatively) more labor intensive than the rest of the economy. Under this assumption, the share of the fixed factor in national income is equal to 0.43%. For simplicity, $x = 1$.

For y denoting national income, Argentinean national accounts indicate that $c/y=0.79$, and $i/y=0.20$. As it is standard in the business cycle literature, it is assumed that 20% of the time is employed in market activities ($n=0.2$). The speci-

fications detailed above and the model equations in steady state have the following implications. The ratio L/y takes different values depending on the value of φ .¹² In the benchmark model φ is set to 0.5, so that 50% of the labor cost must be financed in advance. Under this assumption $L_h/y=0.56$ and $L_y/y=0.35$. The implied annual capital output ratio is $k/y=2.6$ and $\delta=0.052$ on annual basis; 0.4% of the labor and capital are employed in the financial industry; and $r_k=16.33\%$ is the annual rate of return on capital gross off depreciation.

The autocorrelation coefficient of productivity is chosen so that the model reproduces the output persistence of the Argentinean economy. The persistence parameter ρ_r in the international interest-rate process is initially fixed to 0.9. Two standard deviations for the quarterly interest rate are considered, $\sigma_r=1.00\%$ and 3.00% .

1.3.2 Numerical Results

The quantitative exercises to address the following questions. To what extent do working capital needs magnify the real effects of model interest-rate fluctuations? How do these effects depend on the statistical properties of the productivity and interest-rate shocks hitting the economy? And how important is the buffer-effect induced by the operation of a costly banking system?

To see how other models are nested in the discussed one, first notice that as the interest rate margin, $r_{L,t} - r_t$, approaches zero, banks' output becomes negligible and therefore domestic agents obtain financing by placing bonds in the world capital market. On the other hand, when the parameter φ is set equal to zero, there is no need for working capital and the loan market clears when the stock of loans demanded by households equals the banks' credit supply. Therefore, when $r_{L,t} - r_t = \varphi = 0$, the model becomes the basic small-open-economy RBC model.

¹²Compare two similar economies that only differ in the composition of the demand of the credit market. Given the same supply of loans, to clear the loan market at the same domestic interest rate, the economy with a lower φ must have a higher stock of household debt to compensate the lower demand for working capital.

Impulse Response Functions

Interest-Rate Shocks and Capital Flows. The introduction of a banking system into the RBC model of SOE's adds new transmission mechanisms. Consider the effect of a rise in the international interest rate that could follow a policy of tight money in developed economies or a change in the foreigners' perception of the riskiness of domestic businesses. In the standard RBC model of a SOE as well as in the model of this research, the typical income and substitution effects cause a fall in consumption and investment expenditures and a decrease in the current account deficit. The income effect arises because the shock augments the debt burden in a debtor country; the substitution effect follows from a (relatively) more expensive actual consumption which makes the representative household raise its savings. Investment expenditures fall because an optimal portfolio reallocation indicates that it is profitable to reduce the stock of debt and to cut down the stock of physical capital.

As in Neumeyer and Perri (2001), the introduction of working capital adds a demand effect on factor inputs. Since the interest rate is part of the (gross) cost of hiring labor, higher financing costs induce firms to slow down their production, cutting down their demand for labor and their supply of output. Therefore, output falls more in an economy in which firms demand financial services as an intermediate input than in an economy where all transactions take place simultaneously.

Adding a domestic costly-operated banking system can append an attenuating effect to this interest-rate-driven recession. This is shown in Figure 1.4 where solid lines represent the banking economy and dashed lines the economy under direct financing. The impulse response functions follow a one percent rise in the world interest rate and illustrate the buffer effect induced by banks. The interest-rate shock adds to the financial costs of the banking system and banks restrain their supply of credit. The domestic credit market clears at a higher interest rate and at a lower amount of loans. Inasmuch as administrative costs are increasing in the amount of financing, the domestic interest rate rises less than the world interest rate. In other words, higher financing costs are partially compensated with lower administrative costs. While the effect of the shock persists, the quantity and value of resources allocated to the

financial industry are lower than their steady-state counterparts.

The importance of this attenuating effect is going to depend on: a) the interest-rate elasticity of the supply of loans (affected by the value of the parameter ξ); and b) the share of administrative costs in the domestic rate.

To understand the role of the value of ξ , recall that $L_t = e^{z_t} A_b k_{b,t}^{\alpha\xi} n_{b,t}^{(1-\alpha)\xi} x^{1-\xi}$ and consider two special cases, $\xi = 1$ and $\xi = 0$. When $\xi=1$, input x (which has been normalized to one) becomes useless and the technology to produce loans has constant returns. The supply of loans is infinitely elastic at the interest rate that exactly compensates the unitary administrative cost of financing. In this case, a rise in the financial cost of the banks, i.e. a higher international interest rate, shifts up the supply of loans by the magnitude of the interest-rate increase. The opposite case happens when $\xi = 0$ since the supply of loans becomes perfectly inelastic and the domestic interest rate is demand-determined. As the production of loans is proportional to the amount of the fixed factor x , a given amount of the latter implies a unique level of loans which does not depend on the interest rate $r_{L,t}$. On the other hand, for $0 < \xi < 1$, the elasticity of the loans supply is inversely related to the value of ξ as it is shown in Figure 1.3.

As for the share of administrative costs in the domestic rate, other things held constant, the larger the interest-rate margin, the higher the dampening effect shown above. A larger interest-rate margin is equivalent to a larger component of administrative costs in the domestic interest rate. Hence, shocks affecting the financial marginal cost of the banking system have a lower impact on the domestic rate.

When banks intermediate in the loan market, $r_{L,t}$, and not r_t , is the relative price of future consumption. Since r_t rises more than $r_{L,t}$, the economy without banks observes a larger adjustment in the level of consumption and a larger reallocation of savings. The supply of capital and the demand for labor fall more in the non-intermediated economy. Thus the recession following a capital outflow is milder in the banking economy due to the dampening effect performed by bank administrative costs. The fall in consumption and output is approximately twice as large in the economy under direct financing.

Productivity Shocks. Now consider the (one-percent) productivity shock depicted in Figure 1.5, where dashed lines identify the non-intermediated economy. The existence of the banking system breaks the separation between consumption and investment decisions that is present in the standard SOE-RBC model. The shock makes firms demand more capital. The Tobin's q rises above 1. This induces households to accumulate capital. In the standard model, the trade balance becomes countercyclical when the pro-borrowing effect dominates the pro-saving effect.¹³ However, since the interest rate is constant in that model, the price of future consumption does not induce any additional effect on intertemporal decisions.

This is no longer the case in the model of section 2. For instance, investment and consumption decisions place a higher demand for loans and induce an equilibrium rise in the domestic interest rate. This causes other equilibrium adjustments in the banking economy that are similar to those that follow an interest-rate disturbance and which were discussed above: intertemporal consumption substitution; reallocations of assets; adjustment in the level of production.

The importance of the differences between these two economies is going to depend, again, on the slope of the supply of financing and on the share of administrative costs in the domestic interest rate, as well as on the importance of financial costs of the firms. In Figure 1.5 the domestic rate falls by less than 0.1% on impact and the dynamics of the two economies are quite similar, something that is going to be explored quantitatively next.

Quantitative Comparisons

For the quantitative study of the volatilities induced by the above shocks under alternative setups, models' statistics are compared to those of the Argentinean economy. Some actual business cycles statistics are reported in Table 1.1.¹⁴

¹³The pro-borrowing effect arises because the economy wants to produce more when productivity is high and this requires, among other things, more capital. The pro-saving effect is given by the consumption-smoothing behavior: agents are better off if consumption is higher not only when productivity is high, but in all the remaining periods

¹⁴It should be noted that the Argentinean national accounts (NA's) have several deficiencies. One problem is the truncation of the series. Macroeconomics series have small number of observations

The analysis is carried out in three steps. First, banks are ruled out and the non-intermediated economy is tested under two scenarios: a) the economy is only hit by productivity shocks; b) both productivity and interest-rate disturbances drive business cycles. In the second step, the quantitative importance of the banking system is studied under two interest-rate processes. Third, alternative specifications of the production of loans show that to reconcile the model predictions with the actual volatility of loans, the supply of credit should have a higher slope than in the benchmark model. Two additional quantitative exercises are performed next. One studies the effect of introducing capital controls which are modelled as reserve requirements, and the other explores the significance of productivity shocks that only hit the financial sector.

Interest-Rate Shocks under Direct Financing. The simulations reported in Table 1.2 were performed setting the innovations to productivity so that the model reproduces the (average) Argentinean output volatility, computed from the last two editions of the national accounts: i.e. $\sigma_{\varepsilon_z}=1.26$. Similarly, ρ_z is equated to 0.74 to imitate the actual output autocorrelation; the adjustment cost parameters h_1 and h_2 are both equal to 3.4, to reproduce the relative volatility of investment.¹⁵ To make the role of banks negligible, the intermediation margin is reduced (to 0.5%) but not completely eliminated so that the model still has a well defined steady state.

As Table 1.2 shows, moderate interest-rate perturbations do not have a large impact on output fluctuations or on the investment-saving correlation. This is the neutrality of interest-rate shocks discussed by Mendoza (1991), neutrality that seems to remain invariant under the existence of working capital. When the volatility of the interest rate is raised from 1% to 3%, except for investment expenditures and the trade balance which become significantly more volatile, all other variables have

with a maximum of 80, on quarterly basis (20 years). Series bases change very often. The last edition of the NA's measure macroeconomic variables at 1993 prices and its sample goes from 1993.1 to the present. However, this edition has several differences with the NA's at 1986 prices. Moreover, these two editions show differences with the NA's at 1970 prices. Given these deficiencies, it has been opted to report the business cycles statistics that arise from each of these series instead of reporting single figures.

¹⁵See eq. (1.3) and notice that the convexity of the adjustment cost function rises with h_1 and h_2

approximately the same volatility and autocorrelation coefficients. Moreover, the response of hours to interest-rate disturbances seems to indicate that firms' financial needs have a negligible effect on the labor market, where precisely one expects to see the source of additional impact of interest-rate shocks on production.

To explain how the neutrality of interest-rate disturbances is invariant to the introduction of working capital, notice that the firms' gross cost of a unit of labor is equal to $w_t(1 + \varphi r_{L,t})$. For $\hat{z} \equiv d \ln(z)/dt$, the log-linear version of a change in this cost is $\hat{w}_t + \frac{\varphi r_L}{1 + \varphi r_L} \hat{r}_{L,t}$. For $\varphi = 0.5$ and $r_L = 10.75\%$, the coefficient of $\hat{r}_{L,t}$ is equal to 0.05. Thus a 20% rise in $r_{L,t}$ is equivalent to an exogenous 1% rise in the wage rate. Therefore, the negligible relevance of the changes in the cost of financing on the demand side of the labor market is explained by the relatively small coefficient of $\hat{r}_{L,t}$.

A disruptive result of the economy under direct financing is the volatility of household loans. This is the dimension in which the model without banks performs worst and which hints at an alternative setup. The slope of the credit supply is the natural element to consider in this regard.

Banks versus Direct Financing. Table 1.3 compares the intermediated with the non-intermediated economy, the former subject to interest disturbances of different sizes. The results under direct financing are reproduced from Table 1.2. The characteristic of the exogenous stochastic processes and the adjustment cost parameters are the same as above, while administrative costs are equal to 4.25%.

The neutrality of the economy to interest-rate disturbances is not substantially changed when the banking sector is considered explicitly. While output becomes 8.4% more volatile when the standard deviation of interest rate is equal to 1%, the *absolute* volatility of investment and consumption are quite similar. On the other hand, when the standard deviation of interest-rate shocks is raised to 3% in the banking economy, the simulation results are quite similar, except for the domestic interest rate and the investment-saving correlation. This correlation is higher in the intermediated economy because the financial system breaks the standard separation between consumption and investment decisions.

Working Capital and the Supply of Loans. The simulations above show that the introduction of the banking system raises the volatility of output. The next question is then, to what extent do the results depend on the existence of a demand for working capital in the market for loans? This and the importance of the slope of the supply of financing are explored here. The exercise is performed setting $\sigma_{\varepsilon_z}=1.26\%$ and $\rho_z=0.7$ so as to reproduce the actual output behavior when the economy is only hit by productivity disturbances. The value of the adjustment cost parameters are $h_1=h_2=3.2$ to reproduce the actual (average) volatility of investment.

In Table 1.4 ξ is set equal to 0.9 and 0.5, respectively. As shown in Figure 1.3, the lower the value of ξ , the higher the slope of the supply of funds. The model produces similar results in terms of the volatility of its variables for four different values of φ , showing the robustness of the results to alternative values of this parameter. This confirm that working capital needs are unable to enlarge the response of output to interest-rate disturbances. Varying the value of φ demands a re-calibration of the model. Particularly, there are noticeable changes in the credit market.¹⁶ When $\varphi=0.01$ (ruling out the demand for working capital), $\frac{L_h}{y}=1.04$ and $\frac{L_y}{y}=0$; on the other hand, when $\varphi=0.75$, $\frac{L_h}{y}=0.32$ and $\frac{L_y}{y}=0.45$. When the banking system operates a technology that is “close” to a constant return one, i.e. $\xi=0.9$, the differences between alternative values of the parameter φ are less noticeable, except for household loans. As the credit supply becomes flatter (ξ is rising), the fluctuations of the international interest rate are becoming the only source of variation of the domestic rate.

The volatility of household loans which was extremely large under the absence of banks, is now in line with actual data when ξ is set equal to 0.5. This suggests that diminishing returns in the financial industry is important to account for the kind of volatility that financial variables, like the stock of loans and the interest rate, have in practice.

¹⁶See footnote 12.

1.4 Concluding Remarks

The standard RBC model of a SOE predicts that interest-rate shocks are unable to produce significant output variance. Thus, neither country-specific (international risk free rate plus risk premium) interest rates nor their concomitant capital flows would be an important determinant of business cycles in actual economies. However there is a remarkable contrast between this prediction and the recent events in Latin America and Asia. The prediction also contrasts with the emphasis that macroeconomic forecasts for emerging countries put on international financial variables.

The chapter evaluated the extent to which the introduction of working capital and a neoclassical banking system could close the gap between theoretical predictions and the actual developments in developing countries. The premise is that working capital is a short-term loan typically provided by commercial banks, and whose production requires the employment of factor inputs. More generally, this chapter attempts to provide microeconomic fundamentals to the production of financial services carried out by a banking system that borrows from the rest of the world and lends domestically to both households and firms. The intermediation process is subject to a technological and a balance sheet constraint, and banks are modelled following the industrial organization approach, i.e. banks provide a special type of service in the economy.

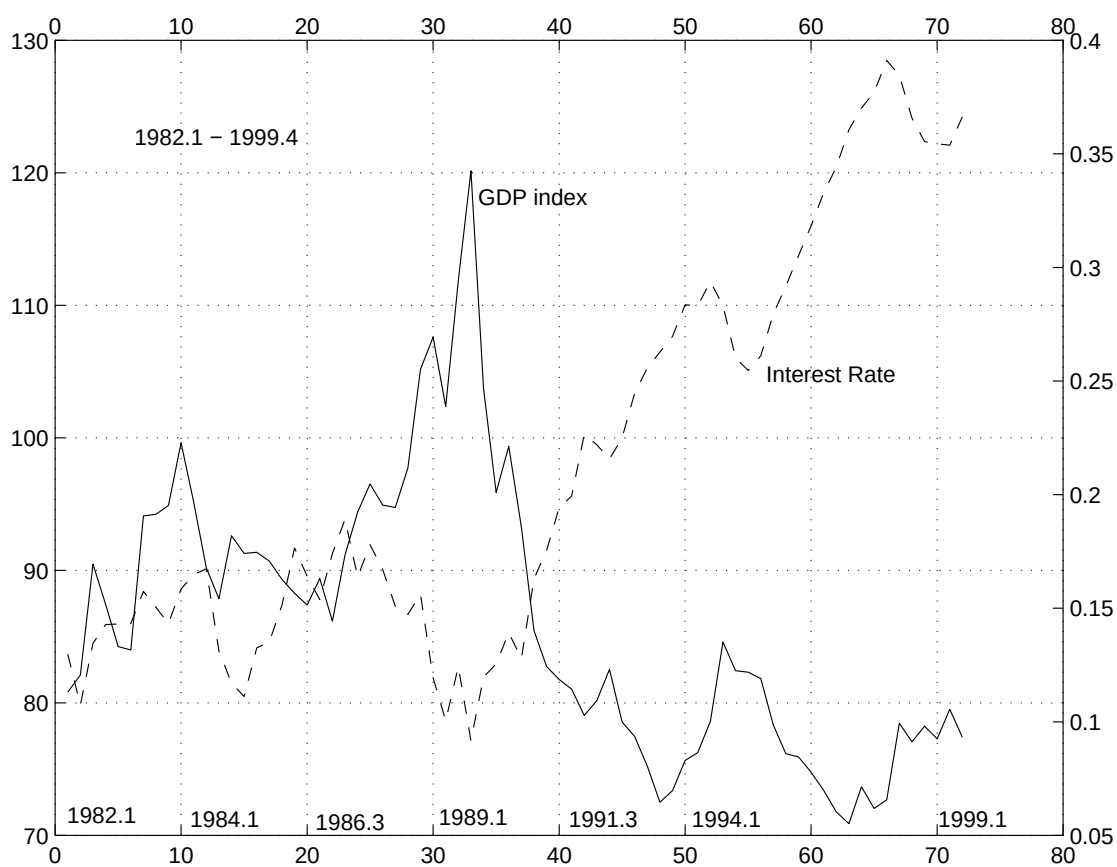
Under the existence of a banking system that operates a decreasing returns technology, the model predicts milder recessions following a decrease in the liquidity of international financial markets than otherwise. The banking technology is the mechanism that produces stationarity in the small-open-economy RBC model. However, it produces a counter-factual behavior of spreads: as the bank intermediates more when the economy faces higher inflows of capital, banking spreads are higher during the booms and lower during the recessions.

The numerical exercises have shown that: first, a demand for working capital needs is not an effective mechanism to align the RBC model's prediction with the actual effect of interest disturbances on domestic output. Second, the importance of including explicitly the role of the financial system depends on how far the technology

is from a constant returns technology. In any case, interest-rate disturbances with a standard deviation of 3% raise output volatility by less than 10%, with respect to the scenario where only productivity shocks drive business cycles. Third, if the supply of domestic financing has not a gentle (or zero) slope, the standard model is unable to reproduce the volatility of loans in actual developing countries.

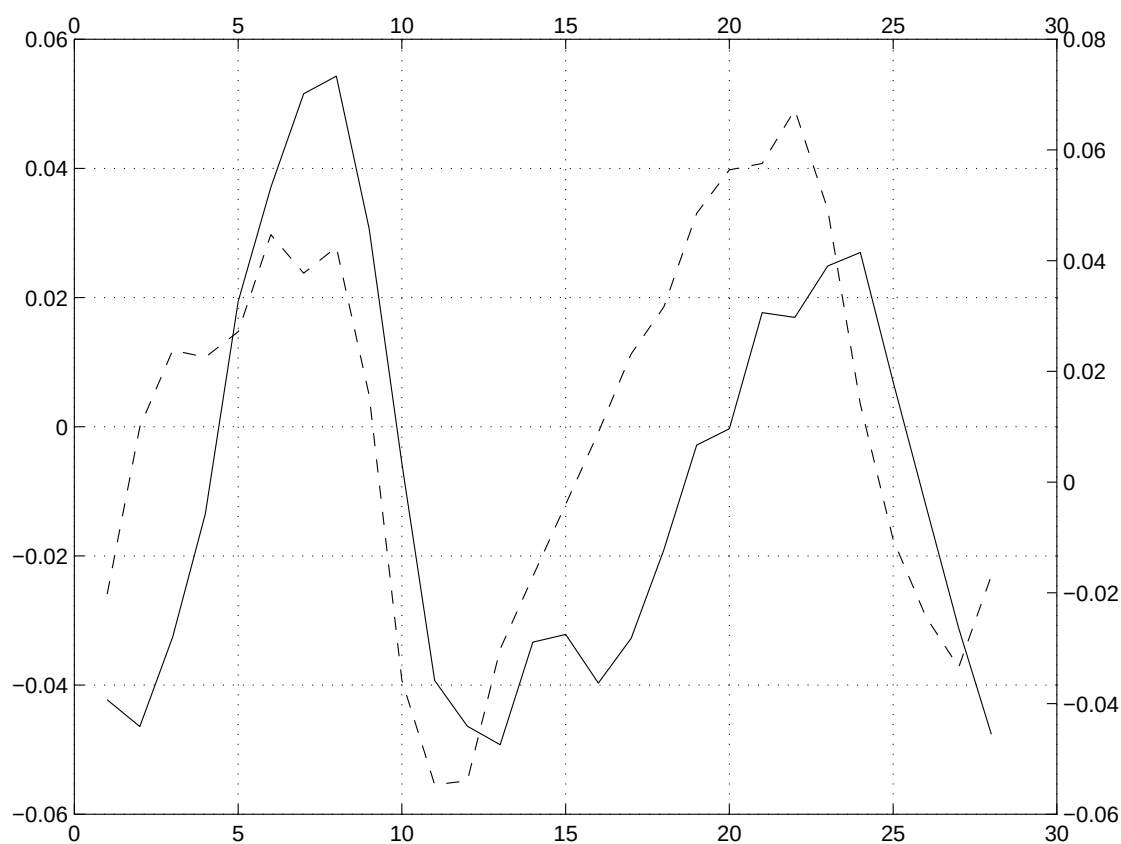
It must be recognized that the characterization made for banks disregards several aspects of the banking system. In particular it does not address risk management in any of its variants (credit risk, liquidity risk), insolvency costs, and equity capital considerations, among other things. It seems that these elements must be part of models intended to account for other links connecting the financial and non-financial sectors in emerging countries, links that cannot be captured by a neoclassical characterization of banks. This hints at models of asymmetric information in banking as a mechanism to explain the macroeconomic volatility induced by capital flows in emerging countries.

Figure 1.1: Argentina GDP index and Country-Specific Interest Rate. 1982-1999



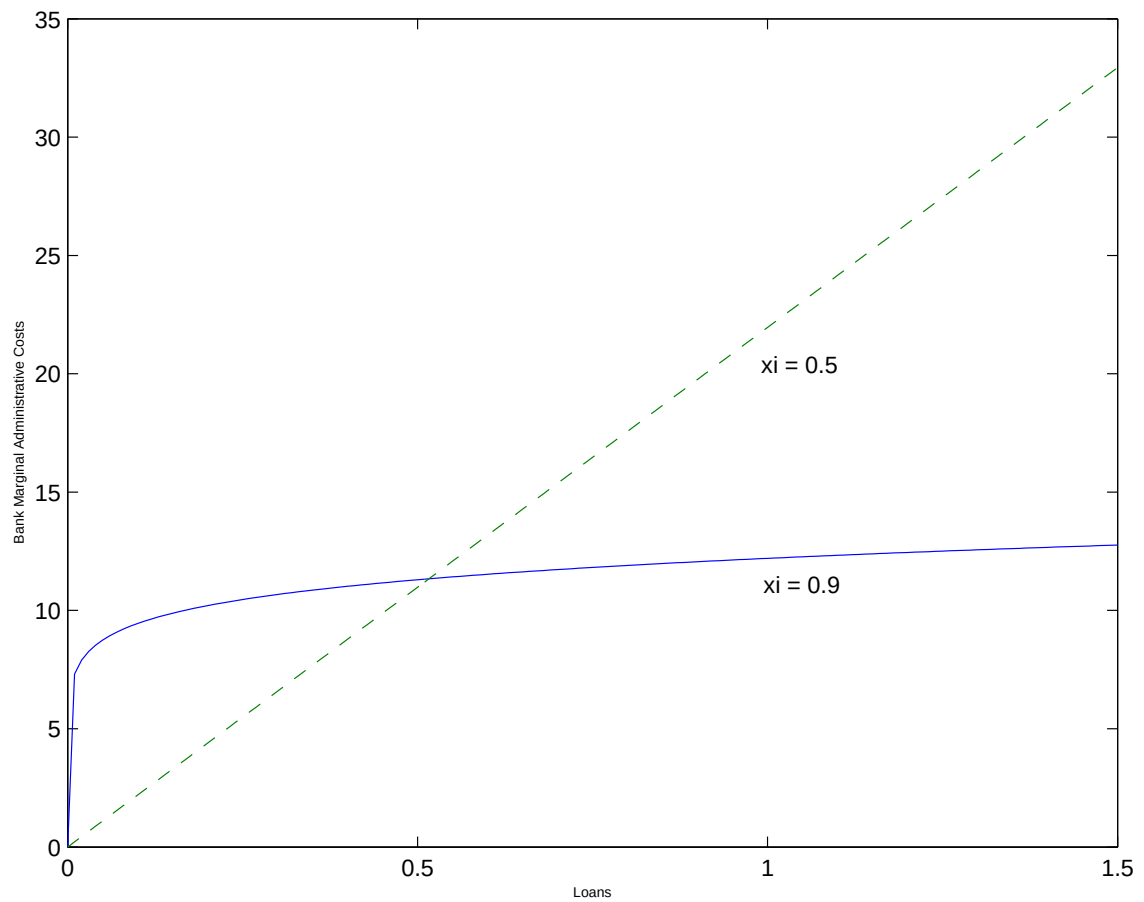
Source: Argentine National Accounts (at 1986 and 1993 prices), Federal Reserve Bank of St. Louis and J.P. Morgan EMBI index.

Figure 1.2: Argentina GDP and Bank Loans, 1993-1999



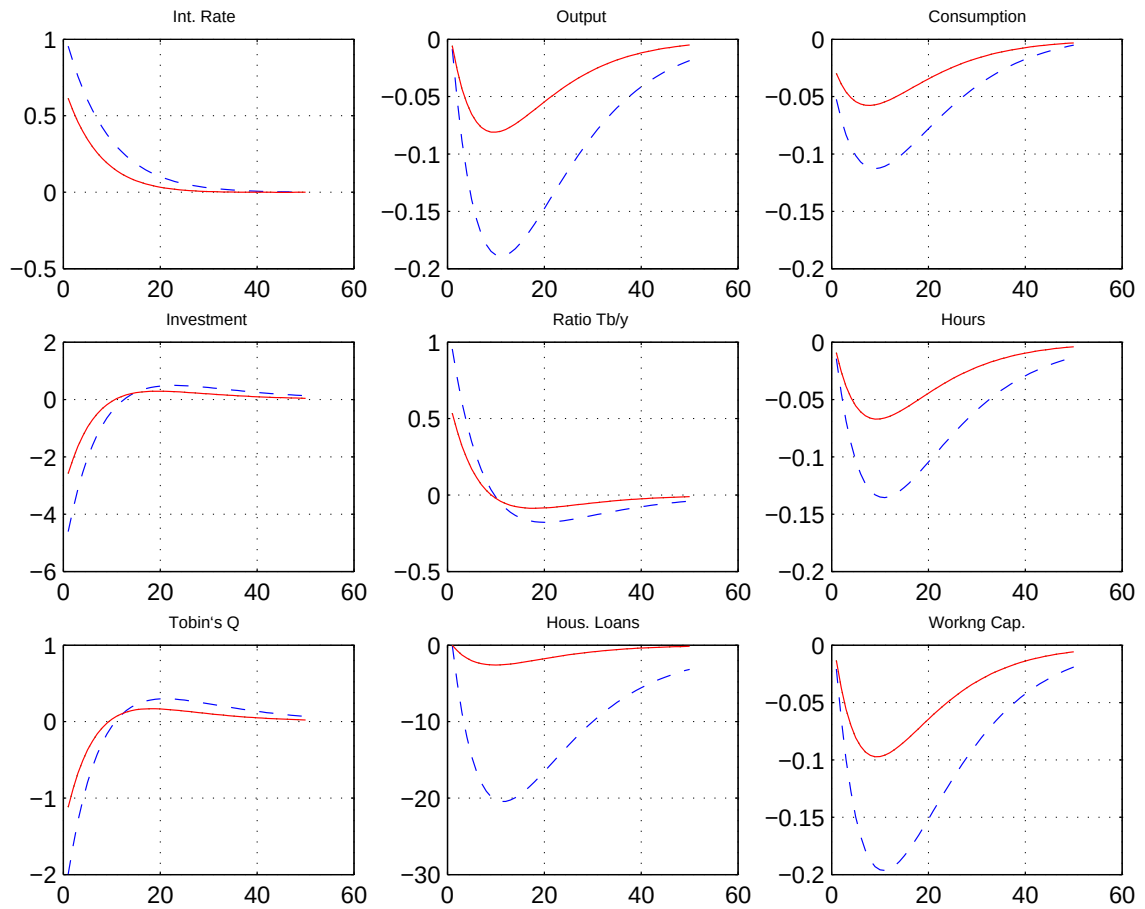
Note: Loans are represented by the dashed line. Source: Argentine National Accounts (at 1993 prices) and Banco Central de la República Argentina.

Figure 1.3: Bank Marginal Operative Costs



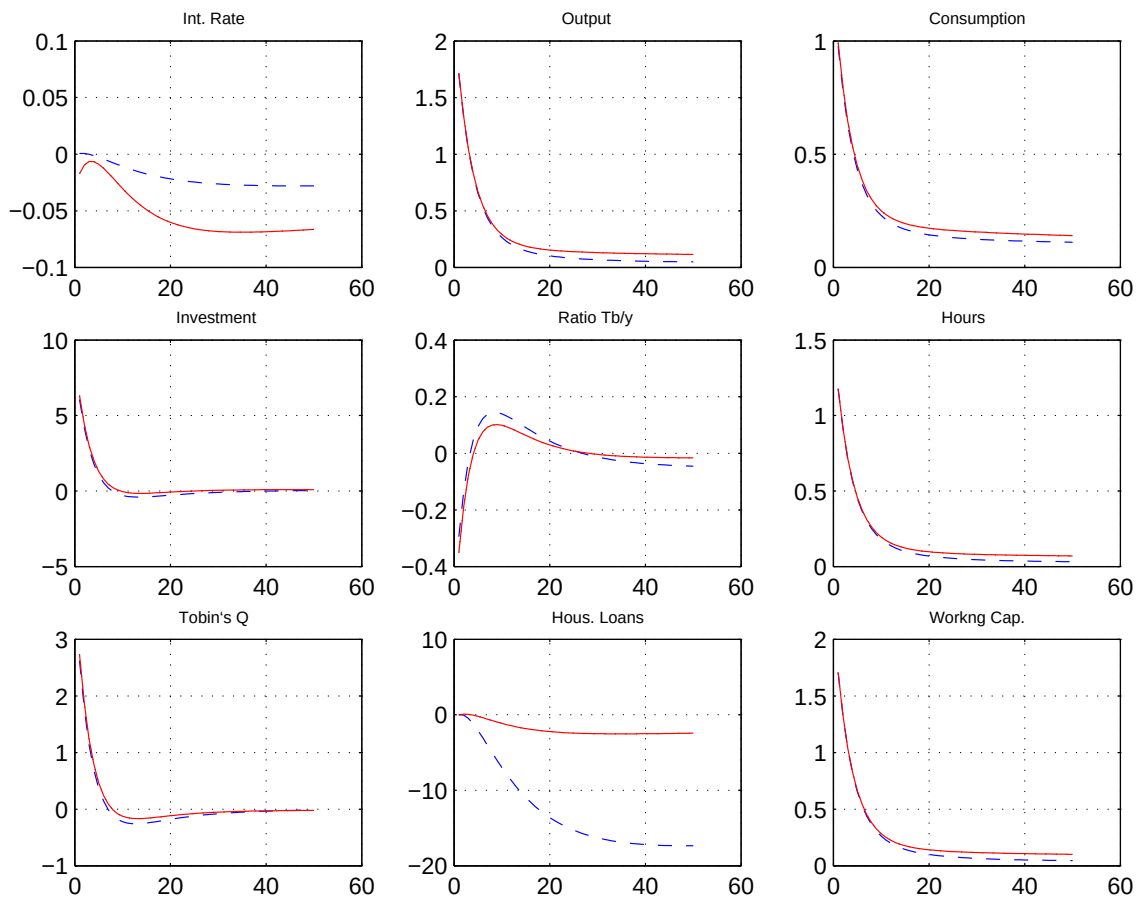
Note: The parameter ξ determines the share of the fixed factor x in the production of loans.

Figure 1.4: Impulse Response Functions following a a 1% Innovation to the International Interest Rate: Banks versus Direct Financing



Note: Dashed lines are used for the non-intermediated economy.

Figure 1.5: Impulse Response Functions Following a 1% Innovation to Productivity Shocks: Banks Versus Direct Financing



Dashed lines are used for the non-intermediated economy.

Table 1.1: Argentine Business Cycles Statistics: National Accounts at 1993, 1986 and 1970 prices

Variable (x)	NA's at 1993 Prices 1993:1-1999:4			NA's at 1986 Prices 1980:1-1996:4			NA's at 1970 Prices 1970:1-1990:4		
	σ_x/σ_y	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}	σ_x/σ_y	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}	σ_x/σ_y	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}
Output (y)	3.002	0.828	1.000	4.343	0.780	1.000	3.121	0.675	1.000
Consumption	1.157	0.826	0.911	1.197	0.794	0.959	3.374	0.716	0.693
Investment	2.673	0.800	0.959	3.031	0.809	0.929	4.954	0.762	0.594
Net Exports.	1.056	0.762	-0.884	2.286	0.836	-0.830	2.922	0.779	-0.649
Fin. System	1.549	0.702	0.809	0.487	0.739	-.—	-.—	-.—	-.—
Int. rate Pesos.	0.681	0.185	-0.235	-.—	-.—	-.—	-.—	-.—	-.—
Int. rate \$	0.473	0.446	-0.235	-.—	-.—	-.—	-.—	-.—	-.—
Tot. Loans	1.638	0.813	0.555	14.636	0.923	-.—	-.—	-.—	-.—
Hours	1.220	0.486	0.521	-.—	-.—	-.—	-.—	-.—	-.—

Notes: The absolute standard deviation of output is reported in the output row. Dashes are used for non available data in the respective edition of the National Accounts (NA's).

Table 1.2: The Effect of Interest-Rate Shocks on Business Cycles under Direct Financing (no Banks)

	Productivity Shocks			Productivity and Interest-Rate Shocks					
	$\sigma_r=0$ $\sigma_y=3.680$			$\sigma_r=1.00$ $\rho_r=0.9$ $\sigma_y=3.710$			$\sigma_r=3.00$ $\rho_r=0.9$ $\sigma_y=3.850$		
Variable (x)	σ_x/σ_y	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}	σ_x/σ_y	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}	σ_x/σ_y	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}
Output (y)	1.000	0.808	1.000	1.000	0.810	1.000	1.000	0.823	1.000
Consumption	0.881	0.921	0.852	0.879	0.921	0.852	0.862	0.923	0.855
Investment	2.850	0.678	0.869	2.996	0.693	0.823	3.895	0.751	0.610
TB/output	0.345	0.933	-0.282	0.398	0.905	-0.238	0.666	0.849	-0.101
Hous. Loans	101.9	1.000	-0.300	102.0	1.000	-0.287	102.8	1.000	-0.191
Work. Cap.	0.995	0.806	1.000	0.996	0.808	1.000	0.999	0.822	1.000
Hours	0.686	0.806	1.000	0.687	0.808	1.000	0.689	0.822	1.000
Capital	0.466	0.536	0.996	0.485	0.538	0.996	0.608	0.568	0.996
$\rho_{S,I}$	0.7947			0.7485			0.531		

Notes: σ_y is the standard deviation of output. σ_r is the percentage standard deviation (volatility) of the international interest rate. ρ_r is the autocorrelation of the interest-rate process. Innovations to productivity are characterized by $\sigma_{\varepsilon_z}=1.26$ and $\rho_z=0.74$. The adjustment cost parameters h_1 and h_2 are both set equal to 3.4.

Table 1.3: The Effect of Interest-Rate and Productivity Shocks on Business Cycles with and without Banks

	Direct Financing			Intermediated Economy					
	$\sigma_r=1.00$ $\rho_r=0.9$ $\sigma_y=3.710$			$\sigma_r=1.00$ $\rho_r=0.9$ $\sigma_y=4.020$			$\sigma_r=3.00$ $\rho_r=0.9$ $\sigma_y=4.030$		
Variable (x)	σ_x/σ_y	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}	σ_x/σ_y	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}	σ_x/σ_y	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}
Output (y)	1.000	0.810	1.000	1.000	0.837	1.000	1.000	0.839	1.000
Consumption	0.879	0.921	0.852	0.786	0.913	0.950	0.785	0.914	0.950
Investment	2.996	0.693	0.823	2.797	0.692	0.864	3.075	0.710	0.782
TB/output	0.398	0.905	-0.238	0.196	0.725	-0.424	0.334	0.769	-0.241
Hous. Loans	102.0	1.000	-0.287	9.358	1.000	-0.466	9.847	0.999	-0.407
Work. Cap.	0.996	0.808	1.000	0.973	0.830	0.999	0.976	0.832	0.999
Int. Rate	0.297	0.924	-0.213	0.286	0.970	-0.450	0.468	0.904	-0.318
Hours	0.687	0.808	1.000	0.671	0.830	0.999	0.673	0.832	0.999
Capital	0.485	0.538	0.996	0.632	0.620	0.998	0.652	0.998	0.622
$\rho_{S,I}$	0.7485			0.9359			0.8358		

Notes: σ_y is the standard deviation of output. σ_r is the percentage standard deviation (volatility) of the international interest rate. ρ_r is the autocorrelation of the interest-rate process. Innovations to productivity are characterized by $\sigma_{\varepsilon_z}=1.26$ and $\rho_z=0.74$. The adjustment cost parameters h_1 and h_2 are both set equal to 3.4.

Table 1.4: The Effect of Working Capital and the Slope of the Supply of Financial Services on Volatility

σ_y	$\sigma_r = 0$	$\sigma_r = 3.00$							
	$\tau=0.01$	$\xi = 0.9$				$\xi = 0.5$			
		$\tau=0.50$	$\tau=0.25$	$\tau=0.50$	$\tau=0.75$	$\tau=0.01$	$\tau=0.25$	$\tau=0.50$	$\tau=0.75$
	3.670	3.710	3.700	3.700	3.700	3.850	3.840	3.820	3.810
Variable (x)	σ_x/σ_y	σ_x/σ_y	σ_x/σ_y	σ_x/σ_y	σ_x/σ_y	σ_x/σ_y	σ_x/σ_y	σ_x/σ_y	σ_x/σ_y
Consumption	0.765	0.772	0.768	0.764	0.760	0.760	0.758	0.755	0.752
Investment	2.854	3.339	3.320	3.296	3.266	3.173	3.109	3.036	2.956
TB/output	0.174	0.407	0.398	0.388	0.377	0.255	0.238	0.219	0.198
Hous. Loans	8.943	5.919	7.215	9.702	15.98	1.060	1.328	1.847	3.176
Work. Cap.	0.975	0.970	0.974	0.979	0.983	0.989	0.993	0.996	0.998
Int. Rate	2.444	0.495	0.493	0.490	0.487	0.473	0.457	0.443	0.435
Hours	0.672	0.669	0.672	0.675	0.678	0.682	0.685	0.687	0.688
Capital	0.597	0.614	0.621	0.628	0.636	0.742	0.740	0.739	0.738
$\rho_{S,I}$	0.9543	0.7817	0.7931	0.8055	0.8187	0.9517	0.9564	0.9616	0.9670

Notes: σ_y is the standard deviation of output. σ_r is the percentage standard deviation (volatility) of the international interest rate. ρ_r is the autocorrelation of the interest-rate process. ξ is the share of the specific factor x used in the production of loans. Innovations to productivity are characterized $\sigma_{\varepsilon_z}=1.26$ and $\rho_z=0.7$. The adjustment cost parameters h_1 and h_2 are both set equal to 3.2.

Chapter 2

Macroeconomic Risk and Banking

Crises in Emerging Market

Countries: Business Fluctuations

with Financial Crashes

2.1 Introduction

Banking crises are not a strange episode in market economies. Indeed, they have been recurrent in emerging market countries during the last 25 years (see ?; and Bordo, Eichengreeb, Klingiegl and Martinez-Peira (2001)). As long as banks are major players in modern economies, a banking crisis sparks off multiple adverse consequences including output losses, monetary instability, and other nonmonetary effects

associated with information losses.¹ The sequels of widespread banking failures are worse in developing countries because their banks own a larger share of total financial assets than the share owned by banks in industrial countries.² Furthermore, the costs of restructuring a bankrupt banking system are inversely related to the degree of poverty of the nations and can be as large as one fourth of the GDP (see Goldstein and Turner (1996) and Hoggart et al. (2001)).

Notwithstanding being costly, to some extent crises are inevitable because every financial system is risky. An examination of the way banks do business reveals that under a reasonably high level of leverage, banks cannot guarantee the claims of investors if large adverse macroeconomic shocks occur Dewatripont and Tirole (1994, chap. 2). The value of the banks' portfolios may fall because of a weak economic performance of the banks' borrowers, specially when some credit risk cannot be diversified. A slowdown of the economy bankrupts a higher proportion of borrowers compared to "normal times", and it is conceivable that the downturn will be severe enough that the banks themselves are in distress and cannot fully repay their creditors. This reasoning along with the greater volatility of emerging market economies can account for the higher vulnerability of these economies to waves of banking failures.³

The causality between macroeconomic conditions and financial instability also goes the other way around because declines in the value of banks' portfolios can weaken the economy. In a world of forward looking economic agents, everybody incorporates the eventual negative effects of a financial crash into his economic decisions. Therefore, not only financial crises but also their likelihood of occurrence affect the economy. Consider the case of international investors which are a large fraction of the banks' creditors in developing countries. As forward-looking agents, these investors demand risk premia and take portfolio decisions that respond to the

¹See Hoggart, Reis and Saporta (2001) for an estimate of the costs arising from generalized banking failures. Bernanke (1983) discusses the nonmonetary effects of banking crises with reference to the US depression; for an international comparison of these effects see Bernanke and James (1991).

²On the importance of banks in the financial system of developing countries, see Beck et al. (1999).

³International business cycles studies indicate that whereas business cycles are qualitatively uniform across countries, macroeconomic volatility is observed to be higher in developing than in industrial countries (see Mendoza (1995) and ?.)

financial fragility of the economy. It is then clear that these investors will not play the passive role suggested by the standard business-cycles model of a small open economy (SOE).⁴

The country-risk premium adds to the interest rate on bank loans to firms. Therefore, a riskier financial system depresses the net rate of return of the productive activities that depend on bank loans. This pushes the demand for inputs down and lowers household employment and income. In sum, on the one hand macroeconomic conditions are an important determinant of the soundness of the financial system and on the other hand the degree of financial fragility pervades all sectors of the economy and affect the real economic activity.

This study builds a model of business cycles in a SOE where aggregate risk produces sporadic bank failures in a world where banks intermediate inflows of capital. Banks issue debt to international investors, fund a large number of firms, and are subject to a capital-adequacy regulation. Firms borrow from banks because they have to pay factors of production before realizing their sales proceeds. As project risk is not completely diversifiable, economic downturns trigger a large ratio of poor project returns depreciating the value of the banks' portfolios. Under some circumstances, recessions are severe enough that they produce the insolvency of the banking system. In this sense, crises are driven by fundamentals and not by a change of expectations.

This chapter of the dissertation abstracts from monetary and liquidity aspects of financial intermediation and stresses the aggregate risk involved in the credit creation process in a SOE. Panics, understood as circumstances where depositors attempt to withdraw their funds simultaneously, play no role in the model as deposits are represented by single-period contracts. A banking crisis is defined here as the inability of the banks to honor their debts.

The probability of widespread banking failures is known in every period and it is part of the information set of all agents in the economy. Consequently, all economic decisions depend on the likelihood of a financial crash due to the eventual crisis sequels. Exogenous capital requirements on banking impose a risk on bankers who

⁴In those models the international-interest rate is given and capital flows are (mainly) demand determined.

can lose part of their wealth during a crisis. International investors face a similar risk insofar as a financial crash affects the repayment capacity of the banks. As long as household income depends on the state of financial markets and is affected by bank failures, the household sector is not immune from financial turmoils either.

The financial frictions in the economy that justify the existence of banks arise from a combination of two related elements. One is the presence of information asymmetries about both firms' output and banks' portfolio returns; the other is the existence of agency costs coming from the employment of a costly-state-verification technology. In this context, financial intermediation resembles delegated-monitoring banking because from time to time international investors have to 'monitor a monitor'. Following Krasa and Villamil (1992a) and Krasa and Villamil (1992b), a two-sided simple-debt contract is the optimal investment mechanism employed to solve the asymmetric information problem.

The contribution of the research consists in addressing the following points using a computable general equilibrium framework. First, macroeconomic conditions and not self-fulfilling expectations trigger banking crises. Second, because the likelihood of a crisis is incorporated into the decisions of every economic agent, both macroeconomic risk and financial fragility affect business cycles. Third, the country-specific interest rate is endogenous because international investors evaluate the fundamentals behind their borrowers' ability to repay.

Following Diamond and Dybvig (1983), most macroeconomic models dealing with banking crises in developing countries emphasize the role of sudden expectation changes in setting off bank runs.⁵ However, the empirical evidence on the determinants of banking crises seems to indicate that pure self-fulfilling expectations are one but not the most important determinant of the crashes. For instance, Kaminsky and Reinhart (1999) point out that "in both [currency and banking] crises we find a multitude of weak and deteriorating economic fundamentals that it would be difficult to characterize them as a self-fulfilling crises."⁶ The two prominent fundamentals that

⁵See for example Chang and Velasco (2000) and Cook and Devereux (2000).

⁶Among other empirical studies that support the view that fundamentals are among the most important factors that tend to breed banking sector problems, the reader can see Gorton (1988), Lindgren, García and Saal (1996), Demirgüç-Kunt and Detragiache (1997), Caprio and Klingebiel

would explain the crises are a weak macroeconomic performance and rising interest rates.

The literature on the effects of credit frictions on macroeconomic fluctuations that emphasizes agency costs follows Bernanke and Gertler (1989). Within the real business cycle paradigm, Fuerst (1995) and Carlstrom and Fuerst (1997) incorporate agency costs into the standard model by assuming informational asymmetries in the production of capital goods. Bernanke and Gilchrist (1999) synthesize this literature and offer a detailed analysis of the so called “financial accelerator”. Both Carlstrom and Fuerst (1998) and Cooley and Nam (1998) compare the consequences of assuming that financial frictions arise in the production of capital goods, vis-à-vis the case where the frictions arise in the production of final output. Despite dealing with credit and financial issues, this literature has not addressed the role of either bank credit risk or aggregate risk. In Bernanke and Gilchrist (1999) banks never default because borrowers offer a state-contingent non-default payment that guarantees the lender a return equal in expected value to the riskless rate. Cooley and Nam (1998) and Carlstrom and Fuerst (1998) take an alternative way out of aggregate risk by assuming that debt contracts are signed after the realization of the stochastic component that explains the macroeconomic uncertainty.

Out of the business-cycle paradigm, several works have studied the relationship between aggregate risk and banking crises by extending the Diamond and Dybvig (1983)’s framework (see Allen and Gale (1998) and the papers cited therein). While this literature shows how business-cycle risk can lie at the root of run-driven banking crises, it is silent about the specific mechanism through which economic fluctuations impact on the bank-portfolio return. Similarly, explaining the way through which the financial fragility exerts influence on business cycles is not the aim of these works either.

Consistent with the empirical evidence, both declining aggregate productivity and rising interest rates are capable of bringing about a banking crisis in the model of this chapter. Particularly, a calibration of the model to emerging market countries shows (1997), and Eichengreen and Rose (2002).

that the probability of a banking crisis goes from 0.5% during a boom to 1% during a deep recession. However, not every recession, no matter how deep, causes a crisis; it certainly does when the downturn is both deep and unexpected. Other results show that the interest rate on the debt issued by banks in international markets as well as the interest rate on debt issued by firms are counter-cyclical and responsive to aggregate risk. Once a crisis is in place, its ability to produce long-lasting effects depends on the existence of explicit crisis-related costs.

The chapter proceeds as follows. The next section describes the model and explains how the probability of a bank failure depends on fundamentals. It also characterizes the financial contracting technology. Section 3 discusses static general equilibrium results, and section 4 discusses the dynamic effects of banking crises and aggregate risk. Section 5 contains concluding remarks and avenues to be explored in future research.

2.2 The Model

This section models a non-monetary open economy where banks intermediate inflows of capital that finance domestic production. Banks borrow from international investors and lend to domestic firms. Firms have no wealth and have to pay for their inputs before getting their sale proceeds. Consequently, firms must borrow working capital from banks to start their production. The outcome of production depends on both an idiosyncratic and an aggregate productivity shock.

All firms in the economy are owned by a mutual fund and every household holds a share of that fund, so that households are isolated from the realization of idiosyncratic productivity shocks. Households do not face financial frictions and have access to international markets where they may borrow and lend at the risk-free rate r_t^* . There is only one good which is freely tradable, and both domestic and international agents own the capital employed domestically.

Bankers and investors are regarded as international agents whose only relationship with the economy is the provision of financial services, i.e. they neither consume nor

invest domestically. This assumption avoids modelling bankers' and investors' economic history. Also, to refrain from dealing with firms credit histories, it is assumed that the only contracting technology available precludes subsequent contracting. Financial markets are competitive so that investors and banks accept any contract giving them profits equal to the opportunity cost of capital.

The economy is subject to hidden information because, at the firm-bank relationship, the bank cannot freely observe the firm's output; and at the bank-investor relationship, international investors cannot assess the return on banks' portfolios without incurring auditing costs. The information asymmetry about the bank-portfolio return arises because the realization of the aggregate productivity shock is non-verifiable straightaway. Therefore, it is impossible to infer both the fraction of firms that has declared bankruptcy and the output that has been produced by these firms. Under these assumptions along with bounded costs of monitoring banks, ? show that two-sided debt contracts dominate any other investment scheme.

The impossibility of observing the value of aggregate productivity shocks provides a rationale for a standard assumption made in the SOE version of RBC models, namely that debt contracts with international investors cannot be made contingent on output Mendoza (1991b). This can be justified by noting that aggregate productivity is a catchall for factors other than labor and capital that affect output. Terms of trade shocks, weather conditions, government intervention in the economy, and political turmoil are a few examples of possible components of aggregate productivity. Furthermore, these components may have uneven consequences across sectors of the economy. Therefore, it is difficult to pick a set of variables whose realizations can be readily observable and reflect the *overall* productivity.

The sequence of events in a given period t can be described in terms of four subperiods, t_1 to t_4 , and it is summarized in Table 2.1. At t_1 , knowing the value of the international risk-free interest rate but ignoring the actual value of firms productivity, firms, banks, and investors sign financial contracts. The contracts establish loan sizes, a bank deposit rate, and a bank loan rate. At t_2 , still ignoring the value of their productivity, firms spend the loan proceeds in hiring labor and capital services and input markets clear. At t_3 , firms start to produce, realize their productivity value

and obtain output. Some firms cannot repay their loans and have to declare their bankruptcy. These firms are audited by the bank. Other firms obtain nonnegative profits which accrue to the household. Under some circumstances the fraction of firms declaring bankruptcy is so high that the value of the bank portfolio falls short of the value of bank liabilities; faced with an insolvent bank, depositors confiscate its portfolio. Before the next subperiod starts, the value of the risk-free international interest rate for $t + 1$ is known. At t_4 , household take consumption and saving decisions; household savings are allocated between physical capital and international risk-free bonds. The amount of capital to be employed in the next period depends on the household allocation of savings and on the international supply of capital. Finally, output market clears in this last subperiod.

Under one of the setups of the model, reorganizing production becomes costly in the aftermath of a banking crisis; specifically, the cost of employing the pre-crisis stock of capital is higher after a crisis has hit the economy than otherwise. This feature recognizes the informational losses derived from a bank failure and shows an additional channel through which a crisis affects macroeconomic aggregates.

2.2.1 Firms

Output is produced by a unit mass of *competitive* firms, with each having access to a decreasing returns technology. Firm i 's output at time t , Υ_{it} , depends on the capital and labor services employed by the firm, k_{it} and n_{it} , respectively and on the productivity shock, z_{it} ; i.e. $\Upsilon_{it} = \Upsilon(z_{it}, k_{it}, n_{it})$. Particularly,

$$\Upsilon(z_{it}, k_{it}, n_{it}) = z_{it} A (k_{it}^\alpha n_{it}^{1-\alpha})^\beta \quad \alpha \in [0, 1]; \quad \beta < 1 \quad (2.1)$$

where the parameter A scales total factor productivity and $\alpha\beta$ is the share of output paid to capital. z_{it} represents the realization of the random variable Z_{it} , which is the sum of other two random variables: a) an idiosyncratic i.i.d. productivity shock, X_{it} ; and b) an economy-wide productivity shock, Y_t , whose dynamics is governed by

a Markov process. That is to say, $Z_{it} = X_{it} + Y_t$.⁷ The realizations of these three random variables are indicated with lowercase letters: z_{it} , x_{it} , and y_t , respectively.

Let $h_X(x_t)$ denote the probability density function of the X_{it} 's; and let $h_Y(y_t|y_{t-1})$ and $h_Z(z_t|y_{t-1})$ denote the conditional probability density function of Y_t , and the Z_{it} 's, respectively, conditional on the realization of the previous period aggregate productivity shock, y_{t-1} . The conditional density function of the firm's productivity shock, $h_Z(z_t|y_{t-1})$, is the convolution between the density functions $h_X(x_t)$ and $h_Y(y_t|y_{t-1})$.⁸ All probability density functions above have a nonnegative support and are positive and differentiable on their respective supports. $H_j(\cdot)$, $j = X, Y, Z$ is the correspondent conditional probability distribution function.

When a firm borrows working capital from a bank to hire inputs, it does not know the realization of its productivity shock, z_{it} . Although firms and banks are fully informed about the conditional distribution of the Z_{it} 's, there exists asymmetric information about the ex-post realizations: while firm i costlessly observes the realization of its own productivity shock, other agents have to employ a costly technology to verify it. Furthermore, the outcome of the verification is only revealed to who monitors the firm.

It is known that under the informational conditions described above, simple-debt contracts (SDCs) dominate any other possible one-sided funding arrangement (see Gale and Hellwing (1985) and Williamson (1986)). Thus, regardless of how the bank funds its loans, a working-capital loan is a SDC. This type of contract is completely characterized by a loan size, m_{it} , and a promised repayment, i.e. a gross interest rate, r_{it}^L . Thereupon, firm i will declare its bankruptcy when it is unable honor that promised repayment. The working-capital loan described here will be part of a two-sided simple debt contract in the recursive competitive equilibrium of the economy

⁷Notice that the random variables Z_{it} are not independent because macroeconomic conditions affect all projects in the same way. This is the source of the non-diversifiable aggregate risk.

⁸Let s and t be two continuous random variables with density functions $\phi_s(s)$ and $\phi_t(t)$ defined for all real numbers, then the *convolution* $\phi_s * \phi_t$ of ϕ_s and ϕ_t is the function given by:

$$\phi_u(u) = (\phi_s * \phi_t)(u) = \int_{-\infty}^{+\infty} \phi_s(u-t)\phi_t(t)dt = \int_{-\infty}^{+\infty} \phi_t(u-s)\phi_s(s)ds$$

to be defined later.

Once firm i has secured a loan of size m_{it} , it seeks to maximize its *expected* profits. Provided these profits are nonnegative, the firm's problem reduces to producing the maximum amount of output subject to the constraint that requires the expenditure on inputs not exceeding m_{it} . Indeed, when the production possibility set posses a convex input structure, it guarantees the existence of the following indirect production function⁹:

$$\Gamma(z_{it}, w_t, r_t^K, m_{it}) \equiv \max_{\{k_{it}, n_{it}\}} \{\Upsilon(z_{it}, k_{it}, n_{it}) : w_t n_{it} + (r_t^K - 1) k_{it} \leq m_{it}\} \quad (2.2)$$

where w_t and r_t^K are the wage rate and the gross rental rate of capital, respectively; Γ expresses output as a function of the productivity shock, parametric input prices, and the loan size. Particularly, for the functional form of Υ_{it} stated in eq. (2.1),

$$\Gamma(z_{it}, w_t, r_t^K, m_{it}) = z_{it} \Lambda_t m_{it}^\beta \quad (2.3)$$

where $\Lambda_t \equiv A \left(\frac{\alpha}{r_t^K - 1} \right)^{\alpha\beta} \left(\frac{1-\alpha}{w_t} \right)^{(1-\alpha)\beta}$ is constant across firms. Furthermore, notice that the duality between $\Upsilon(z_{it}, k_{it}, n_{it})$ and $\Gamma(z_{it}, w_t, r_t^K, m_{it})$ also holds in expectation when z_{it} is replaced by $E[Z_{it}|y_{t-1}]$.

For a given combination of input prices and loan size, i.e. a pair (Λ_t, m_{it}) , it is easy to see from eq. (2.3) that there exists a one-to-one relation between output and the value of z_{it} . Therefore, a value of z_{it} , say $\bar{z}_{it} \in [0, \infty)$, can be found that gives firm i just enough output to repay the loan:

$$r_{it}^L m_{it} = \Gamma(\bar{z}_{it}, w_t, r_t^K, m_{it}) \quad (2.4)$$

Under these conditions the firm is bankrupt when $z_{it} < \bar{z}_{it}$ but it is not when $z_{it} \geq \bar{z}_{it}$.

⁹The convex-input-structure condition is production theory's analogue of convex preferences. It implies that the set of input combinations capable of obtaining or exceeding any given target output level is convex. See Cornes (1992) and Shephard (1970).

In the latter case, the firm repays the loan at the agreed interest rate ($r_{it}^L = \bar{z}_{it}\Lambda_t m_{it}^{\beta-1}$). In the former case, the firm declares its bankruptcy and it is audited and seized by the bank. The nonnegative profits of firm i , i.e. π_{it} , can then be written as

$$\pi_{it} = \begin{cases} \Gamma(z_{it}, w_t, r_t^K, m_{it}) - m_{it}r_{it}^L, & \text{if } z_{it} \geq \bar{z}_{it} \\ 0, & \text{if } z_{it} < \bar{z}_{it} \end{cases} \quad (2.5)$$

The aggregation of profits across firms can be simplified recognizing that all firms are ex-ante identical. Hence, the terms of the SDC as well as the optimal input combination, are exactly the same for every firm.¹⁰ Consequently, aggregate profits, Π_t , are:

$$\Pi_t = \int_{\bar{z}_t}^{\infty} \Gamma(z_{it}, w_t, r_t^K, m_{it}) h_Z(z_t | y_{t-1}) dz_t - m_t r_t^L [1 - H_Z(\bar{z}_t | y_{t-1})] \quad (2.6)$$

where $H_Z(\bar{z}_t | y_{t-1})$ is the measure of firms with a productivity lower than $\bar{z}_t$ and hence bankrupt. The remaining $[1 - H_Z(\bar{z}_t | y_{t-1})]$ firms are solvent and repay their loans at the agreed rate r_t^L .

2.2.2 Banks and the Probability of a Bank Failure

There is a large number of banks (the length of the set is equal to one), each financing a large number of firms through SDCs. The bank's gross income from firm i is given by $\Lambda m^\beta g(\bar{z}_t, x_{it}, y_t)$, where:

$$g(\bar{z}_t, x_{it}, y_t) = \begin{cases} \bar{z}_t, & \text{if } z_{it} \geq \bar{z}_t \\ z_{it} - \mu^f, & \text{if } z_{it} < \bar{z}_t \end{cases}$$

By exploiting the relation between $\bar{z}_t$ and r_t^L stated in eq. (2.4), this expression recognizes the two possible firm outcomes from the bank standpoint: the firm either

¹⁰By dropping the subscript i in all variables but the random variables X_{it} and Z_{it} and their realizations, this allows to simplify the notation hereupon.

repays its loan at the interest rate r_t^L , or surrenders its output to the bank. The latter is costly because monitoring a bankrupt firm costs $\Lambda_t m_t^\beta \mu^f$, a cost that depends on input prices, the loan size, and the parameter μ^f .¹¹

When the bank contracts with N ex-ante identical firms, its average gross income per borrower is equal to $\Lambda_t m_t^\beta G_N(\bar{z}_t, x_{1t}, \dots, x_{Nt}, y_t)$ where G_N involves a simple average:

$$G_N(\bar{z}_t, x_{1t}, \dots, x_{Nt}, y_t) = \frac{1}{N} \sum_{i=1}^N g(\bar{z}_t, x_{it}, y_t)$$

As N becomes infinitely large, the bank diversifies away the idiosyncratic risk and only the realization of Y_t matters for the determination of the average income per borrower. Therefore, the macroeconomic risk remains as a non-diversifiable risk which could cause a bank failure. Absent the macroeconomic risk, the expected and actual income of a bank financing an infinitely large number of firms would be equal.¹²

Once aggregate risk is considered, the average income per borrower, $\Lambda_t m_t^\beta G_N$, is stochastic and converges in probability to another random variable, $\Lambda_t m_t^\beta G(\bar{z}_t, Y_t)$, whose value depends on the realization of the macroeconomic shock. Conditional on the value taken by Y_t , by the strong law of large numbers,

$$\Lambda_t m_t^\beta G_N(\bar{z}_t, X_{1t}, \dots, X_{Nt}, Y_t) \xrightarrow{p} \Lambda_t m_t^\beta G(\bar{z}_t, Y_t)$$

where, for a realization of Y_t such that $0 \leq y_t \leq \bar{z}_t$, the limiting value of the bank gross portfolio return is given by:

$$\Lambda_t m_t^\beta G(\bar{z}_t, y_t) = \Lambda_t m_t^\beta \left\{ \int_0^{\bar{z}_t - y_t} (x_t + y_t) h_X(x_t) dx_t - \mu^f H_X(\bar{z}_t - y_t) + \bar{z}_t [1 - H_X(\bar{z}_t - y_t)] \right\} \quad (2.7)$$

¹¹The constancy of monitoring costs across insolvent firms facilitates aggregation.

¹²The statement is also true when the financial contract is signed after observing the aggregate shock. This is the Carlstrom and Fuerst (1998) and Cooley and Nam (1998)'s way out of aggregate risk. In Bernanke and Gilchrist (1999) the bank-portfolio return is always equal to zero and banks are never insolvent because the borrowers absorb all the risk, either individual or aggregate. They agree to pay a variable interest rate which guarantees that depositors obtain a rate of return equal to the riskless rate.

Here, the first two terms on the right hand side are the bank income coming from bankrupt firms and the third term on that side is the bank income coming from solvent firms. The former is made of surrendered output minus the monitoring costs whereas the latter represents (gross) interest income. Notice that for $y_t > \bar{z}_t$, $G(\bar{z}_t, y_t) = \bar{z}_t$ because the aggregate productivity shock is large enough so that no single firm is bankrupt.

In order to study bank failures, the bank funding mechanism has to be specified: banks issue debt among international investors to finance its portfolio of loans. These investors are risk neutral and of small capacity as compared to the size of the loan, so that each firm would need funds from several investors if it obtained direct financing.¹³ A bank becomes insolvent when its net cash flow turns negative and exceeds its capital. Therefore, the probability of a bank bankruptcy is equal to the probability that the bank cannot repay its depositors.

For κ denoting the regulated bank capital-asset ratio, the bank balance-sheet constraint indicates that the bank must borrow $m_t(1 - \kappa)$. Letting r_t^D be the gross interest rate on bank liabilities, and recalling that the stochastic gross bank-portfolio return is equal to $\Lambda_t m_t^\beta G(\bar{z}_t, Y_t)$, the following are equivalent expressions for the probability of a bank failure:¹⁴

$$\mathbf{P} \left[\Lambda_t m_t^\beta G(\bar{z}_t, Y_t) < r_t^D (1 - \kappa) m_t | y_{t-1} \right] = \mathbf{P} \left[Y_t < G^{-1}(\bar{z}_t, m_t, r_t^D) | y_{t-1} \right]$$

As $H_Y(y_t | y_{t-1})$ is the conditional probability distribution function of Y_t , the probabilities above can be written as:¹⁵

$$H_Y \left[\hat{y}_t(\bar{z}_t, m_t, r_t^D) | y_{t-1} \right] \quad \text{where} \quad \hat{y}_t \equiv G^{-1}(\bar{z}_t, m_t, r_t^D) \quad (2.8)$$

¹³When an investor is able to completely fund a firm, duplicative monitoring costs never arise and delegated monitoring is no longer an efficient investment mechanism.

¹⁴The invertibility of the function $G(\cdot)$ is shown in the Appendix.

¹⁵Notice that the functional form of the threshold $\hat{y}_t$ depends on the distribution function of the idiosyncratic shocks as is clear from eq. (2.7).

The likelihood of a bank collapse converges to the probability that the aggregate productivity shock y_t is lower than a threshold value $\hat{y}_t$, which is implicitly defined by the inverse function G^{-1} . All elements in these two subsections are next joined to characterize the structure of two-sided debt contracts.

2.2.3 The Two-Sided Debt Contract

A two-sided debt contract is the investment mechanism through which banks borrow from international investors and lend to domestic firms. This contract establishes a gross deposit rate r_t^D , a gross loan rate r_t^L (i.e. a value of $\bar{z}_t$), as well as the loan size, m_t . It also specifies two bankruptcy states. One indicates that when the productivity shock is so low that the firm cannot repay the bank, the bank audits and seizes the firm. The other indicates that when the bank portfolio return is so low that the bank cannot repay its depositors, the latter also audit and take possession of the bank portfolio.

Following ?, a two-sided debt contract can be characterized as the problem of choosing triples $(\bar{z}_t, m_t, r_t^D)$ so as to maximize firm expected profits subject to two participation constraints, one for the international investor and another for the banker,

$$\max_{\{\bar{z}_t, m_t, r_t^D\}} \int_{\bar{z}_t}^{\infty} \Gamma(z_t, w_t, r_t^K, m_t) h_Z(z_t | y_{t-1}) dz_t - \Lambda_t m_t^\beta \bar{z}_t [1 - H_Z(\bar{z}_t | y_{t-1})] \quad (2.9a)$$

subject to:¹⁶

$$H_Y(\hat{y}_t | y_{t-1}) E \left\{ \Lambda_t m_t^\beta [G(\bar{z}_t, Y) - \mu^b] | y_{t-1} \right\} + [1 - H_Y(\hat{y}_t | y_{t-1})] r_t^D m_t (1 - \kappa) \geq r_t^* m_t (1 - \kappa) \quad (2.9b)$$

and:

$$[1 - H_Y(\hat{y}_t | y_{t-1})] E \left[\Lambda m_t^\beta G(\bar{z}, Y) - r_t^D m_t (1 - \kappa) | y_{t-1} \right] \geq r^* \kappa m \quad (2.9c)$$

¹⁶An additional constraint guarantees the participation of the firms, i.e. the maximal objective function (2.9a) must be positive.

where the dependency of $\hat{y}$ on the choosing triple has been omitted for notational convenience. The risk-free international interest rate r_t^* represents the opportunity cost of capital for bankers and investors, and its dynamics are governed by a Markov process.

The objective function (2.9a) is the expected value of profits in eq. (2.5) after using the relation between $\bar{z}_t$ and r_t^L implicit in eqs. (2.3) and (2.4). Constraint (2.9b) refers to the bank-investor side of the contract. It states that investors fund risky intermediaries only if the gross expected return from lending to the bank is the same as the return promised by the risk-free asset. Recall that $H_Y(\hat{y}_t|y_{t-1})$ is the conditional probability of a bank failure –which is equivalent to the probability that the aggregate productivity shock be lower than $\hat{y}_t$ – and that banks must fund $m_t(1-\kappa)$ from its creditors. Investors audit and confiscate the portfolio of an insolvent bank. The confiscation proceeds are equal to $\Lambda_t m_t^\beta [G(\bar{z}_t, y_t) - \mu^b]$ where μ^b stands for the costs of auditing the bank. On the other hand, investors earn $r_t^D m_t(1-\kappa)$ when the bank does not fail, something that happens with probability $[1 - H_Y(\hat{y}_t|y_{t-1})]$.

Constraint (2.9c) claims that a competitive bank participates in the contract only if its net expected profits compensate the opportunity cost of the capital. In bankrupt states, the bank loses its capital, κm_t , which is the same as saying that the bank obtains a gross return equal to zero. In solvency states, the bank gets the difference between its gross portfolio return, $\Lambda_t m_t^\beta G(\bar{z}_t, y_t)$, and its dues, $r_t^D m_t(1-\kappa)$, which accrue to investors.

From eq. (2.8) and the characterization of the contract made above, it is clear that the intermediation problem involves choosing a particular probability of bank failure. Consequently, bank failures do not represent a ‘misjudgement’ of the banking risk by any agent or the selection of an inappropriate contract, but the natural consequence of hidden information related to the the bank portfolio return.

A failing bank produces other costs in addition to investors’ auditing costs in actual economies. Reorganizing production in the aftermath of a financial collapse is costly compared to normal times. Transferring the ownership of banks involves significant losses of information that bank-bankruptcy costs do not account for. This

happens because a credit history is built over time and its value depends on the relationship between the bank and the firm. This information is much more valuable in the hands of the originating bank than in the hands of a taking-over bank. However, these information losses do not enter into the contract because they do not affect the return of any of the three agents signing the financial contract.

Whereas several investors have to monitor a single firm under a SDC, under a two-sided simple debt contract investors delegate the verification task to a single bank. This delegation saves on monitoring costs because a failed firm is monitored by just one agent, i.e. the bank. This is true even though sometimes investors have to monitor the bank. The saving of monitoring costs explains why two-sided debt contracts dominate one-sided debt contracts.¹⁷

2.2.4 The Representative Household

The economy is inhabited by a large number of identical and infinitely lived households whose objective is to maximize the utility function

$$U = E_0 \sum_{t=0}^{\infty} \theta^t u(c_t, n_t) \quad (2.10a)$$

$$u(c_t, n_t) = \frac{(c_t - \frac{n_t^\nu}{\nu})^{1-\gamma} - 1}{1-\gamma} \quad (2.10b)$$

subject to the time t ($t = 0, 1, 2, \dots, \infty$) flow-resource constraint

$$c_t + a_{t+1} \leq w_t n_t + \Pi_t + a_t r_t^* \quad (2.11)$$

and the borrowing constraint $a_t > \Delta$, where Δ is an exogenously imposed limit on the household wealth, a_t . c_t is consumption and n_t is the time allocated to work, which

¹⁷For a detailed proof of the argument see Krasa and Villamil (1992a) and Krasa and Villamil (1992b). They also prove that the two-sided contract dominates any other investment mechanism under the hidden information problem discussed in the text.

is constrained to be lower than the time endowment, $\bar{n}$. The rate of time preference implicit in θ is such that $\theta r^* < 1$, which guarantees a stationary and well defined equilibrium (see Lungqvist and Sargent (2000, ch.14). The parameter γ sets the value of the intertemporal elasticity of substitution.

Wealth is composed of the household holdings of physical capital, k_t^H , and international bonds, b_t . As explained below, these two assets are perfect substitutes from the standpoint of the household as well as from the standpoint of international agents, and therefore their rate of return are equal in equilibrium. Notice that inasmuch as foreign residents also supply physical capital, k_t^H constitute only a fraction of the total supply of capital. Household income is composed of labor income, the mutual fund dividends, Π_t , and the gross return on wealth. The household uses its resources for consumption and saving.¹⁸

Equating the marginal rate of substitution of labor for consumption to the wage rate leads to the following labor supply:

$$n_t = w_t^{\frac{1}{\nu-1}} \quad (2.12)$$

The employment of capital in the economy is subject to the following adjustment costs:

$$\Phi(k_t - K_{t-1}) = \frac{\phi}{2}(k_t - K_{t-1})^2$$

where K_t is the per-capita stock of capital employed in the economy at time t . Function Φ indicates that it is costly to employ an amount of capital different from the per-capita amount employed in the previous period.

The supply of capital is derived taking into consideration four elements that specialize the optimal accumulation of wealth. First, the world-capital-market interest rate r_{t+1}^* is known when capital supply decisions are taken at time t ; second, the return on capital at time $t + 1$ depends on the productivity observed at time t because factors of production are paid in advance; third, a financial crisis may enlarge the

¹⁸The assumption that households lend at the rate r_t^* could be justified assuming the existence of a deposit-insurance scheme.

capital adjustment costs shown above because of the nonmonetary sequels that follow a financial crash; fourth, the adjustment costs above. Thus, the supply of capital is described by:

$$r_t^K = r_t^* + \delta + \phi(k_t - (1 - \iota(y_{t-1} < \hat{y}_{t-1})\zeta)K_{t-1}) \quad (2.13)$$

which accounts for the reorganization costs arising from a banking crisis and which do not accrue to bankers and investors. Here δ is the depreciation rate of capital; the function ι is an indicator function returning 1 when the statement in parenthesis is true and zero otherwise; and $0 \leq \zeta \leq 1$ is a parameter that captures the cost of reorganizing production in the aftermath of a banking crisis. During non-crisis time, as well as when ζ is set equal to zero, the arbitrage condition in eq. (2.13) becomes:

$$r_t^K = r_t^* + \delta + \phi(k_t - K_{t-1})$$

When $\zeta \neq 0$ the adjustment costs are larger in a quarter following a crisis. In other words, in a post-crisis period, keeping constant the level of capital is not enough to bring adjustment costs down to zero: it requires reducing the employment of capital in $\zeta \times 100$ percent.

2.2.5 The Recursive Competitive Equilibrium

Equilibrium is defined recursively. The following four variables characterize the state of the economy: $s_t^1 = (r_t^*, y_{t-1}, a_t, K_t, A_t)$; $s_t^2 = (r_{t+1}^*, y_t, a_t, K_t, A_t)$; $S_t^1 = (r_t^*, y_{t-1}, K_t, A_t)$; $S_t^2 = (r_{t+1}^*, y_t, K_t, A_t)$; where A_t denotes the aggregate per-capita level of wealth. Distinguishing between s_t^1 and s_t^2 is relevant because household input-supply and consumption/saving decisions are taken with different information sets (See Table 2.1). S_t^1 and S_t^2 are the corresponding aggregate state variables.

The *recursive competitive equilibrium* of this economy consists of a value function $v(s_t^2)$; a set of financial contract rules, $\bar{z}_t = \bar{z}(S_t^1)$, $m_t = m(S_t^1)$, and $r_t^D = r^D(S_t^1)$; a

labor decision rule $n_t = n(s_t^1)$, and its corresponding aggregate per capita decision, $N_t = N(S_t^1)$; a set of consumption/saving decision rules for the household, $c_t = c(s_t^2)$, $k_{t+1}^H = k^H(s_t^2)$, $b_{t+1} = b(s_t^2)$; a corresponding set of aggregate per capita decision rules $C_t = C(s_t^2)$, $K_{t+1}^H = K^H(S_t^2)$, $B_{t+1} = B(S_t^2)$, plus the international supply of capital $K^W(S_t^2)$, where $K = K^H + K^W$; a set of factor-price functions $w_t = w(S_t^1)$, and $r_t^K = r^K(S_t^1)$; a set of profit functions, $\pi_t = \pi(S_t^2)$, one for each firm; and its corresponding aggregate profit function $\Pi_t = \Pi(S_t^2)$ such that they satisfy:

1. The following household optimality equation:

$$v(s^2) = \max_{\{n, c, k^H, b'\}} \{u(c, n(s^1)) + \beta E[v(s^2)]\} \quad (2.14a)$$

$$\text{s.t. } c + a' \leq w n + r^* a + \Pi \quad (2.14b)$$

$$a = k^H + b \quad (2.14c)$$

$$c \geq 0, \quad n \leq \bar{n}, \quad a \geq \Delta \quad (2.14d)$$

where the symbol $'$ is used to denote the next-period value of a variable.

2. The condition that the financial contract stated in eqs. (2.9) is solved when $\Lambda = A(\frac{\alpha}{r^K - 1})^{\alpha\beta} (\frac{1-\alpha}{w})^{(1-\alpha)\beta}$, so that firms maximize expected profits
3. The consistency of individual and aggregate decisions, that is $c = C$, $a' = A'$, $n = N$, $k^H = K^H$, $b = B$.
4. The aggregate resource constraint:

$$C + A' + \int_{\bar{z}-y}^{\infty} m r^L h_X(x_t) dx_t = \int_{\bar{z}-y}^{\infty} \Gamma(x + y, w, r^K, m) h_X(x) dx + r^* A$$

2.3 Static Equilibrium Results

The equilibrium allocation and prices of the economy of the precedent section are studied first in a static framework to better show the financial structure of the economy. Further specifications are made with respect to the probability distribution functions of aggregate and idiosyncratic shocks and parameter values in the model. Particularly, while the idiosyncratic shocks are assumed to be draws from a normal distribution, i.e. $X_{it} \sim N(\mu_x, \sigma_x^2)$, the law of motion of the aggregate productivity is:

$$y_t = (1 - \rho^y)\bar{y} + \rho^y y_{t-1} + \varepsilon_t^y$$

where $\varepsilon_t^y \sim N(0, \sigma_y^2)$; $E[\varepsilon_t^y, \varepsilon_{t-s}^y] = 0$ for $s = 1, 2, \dots$; $\bar{y}$ is the unconditional mean of the process; and ρ^y is the autocorrelation parameter. As a consequence of this specification, the conditional distribution function of aggregate shocks $h(y_t|y_{t-1})$ is characterized by its first two moments, the first one being equal to:

$$E[Y_t|y_{t-1}] = (1 - \rho^y)\bar{y} + \rho^y y_{t-1}$$

and the second one is σ_y^2 . The dynamics of the world-market interest rate, r_t^* , is also characterized by an autoregressive structure:

$$r_t^* = (1 - \rho^r)\bar{r} + \rho^r r_{t-1}^* + \varepsilon_t^r$$

where ε_{rt} is a zero mean i.i.d. Gaussian process with $\text{Var}[\varepsilon_t^r] = \sigma_r^2$, and $\bar{r}$ and ρ^r parallel $\bar{y}$ and ρ^y , respectively.

Table 2.2 displays the assumed parameter values for the distributions of the productivity shocks. The unconditional mean value of the (net) international interest rate was fixed equal to 6.5% on annual basis. The autocorrelation parameters in the aggregate productivity and interest-rate processes are 0.5 and 0.9 respectively.

As for the remaining parameter values, in the production side of the economy, α is

equal to 0.4 and $\beta=0.95$. The preference parameter ν is set equal to 1.5, so that the elasticity of labor supply is equal to 2. Following Carlstrom and Fuerst (1997), the firm-monitoring cost parameter μ^f is set equal to 0.25. The same value is assumed for μ^b under the (conservative) presumption that restructuring a bank is as costly as restructuring a firm. The capital asset ratio κ is set equal to 0.08 and this value corresponds to the capital adequacy ratio adopted in the 1988 Basle accords, where international negotiations established uniform capital requirements for banks. The parameter ϕ in the adjustment cost function is set equal to 0.008 and on annual basis, $\delta=0.075$.

2.3.1 Macroeconomic Risk, International Liquidity, and Capital Inflows

By fixing the international risk-free interest rate to its mean value, Figure 2.1 maps combinations of aggregate productivity and capital stocks into financial variables and equilibrium input allocations. Notice that the interest-rate spreads (in panels a and b) are invariant to the previous period per-capita stock of capital K_{t-1} . This is consistent with the fact that the cost of borrowing in international capital markets does not change with the size of debtor economies. The internal spread $r^L - r^D$ is also independent of the stock of capital. On the contrary, the loan size is positively related to both aggregate productivity shocks and the previous stock of capital. Thus, larger economies receive more external financing, and capital inflows are procyclical. On the other hand, the interest rate on the debt issued by banks is negatively related to the overall productivity shock. If the country is observing a high productivity, the premium asked by international investors falls because the likelihood of a banking failure is relatively low (see panel d). Conversely, lower values of the aggregate productivity shock hint that a failure is more conceivable and the economy is financed only at an increasing premium.

The relation between aggregate productivity shock and the probability of a banking crash is explored in more detail in Figure 2.2. The first panel of the figure plots

the aggregate productivity shock on which contracts are written, y_{t-1} , against the endogenously-determined crisis value of that variable, $\hat{y}_t$. The dotted line is a 45-degree line. The second panel of Figure 2.2 shows that the difference between the two lines in the first panel is increasing in y_{t-1} , which renders crises as a less likely event at higher levels of the productivity shock.¹⁹ This assertion is verified in the third panel of the figure where the probability of bank bankruptcy is plotted against the value of previous aggregate productivity shock.

Figure 2.3 plots a mapping similar to that of Figure 2.1, but now the stock of capital K_{t-1} is fixed to its unconditional-mean value and the international interest rate is allowed to vary. Given the economy-wide productivity shock, a higher international interest rate raises the difficulties of the firms, and then of the banks, to repay their credits. Therefore, both interest-rate spreads are increasing in the opportunity cost of international capital (see panels a and b). Also, the graph shows that it becomes easier to issue more international debt when world capital markets are relatively more liquid. These results agree with the evidence found by Reinhart and Reinhart (2001) and Calvo, Leiderman and Reinhart (1992) among others, which indicates that contractions in the industrialized world are more prone to make capitals flow into emerging countries than expansions.

2.3.2 Agency Costs, Capital Requirements, and Banking Risk

Completing the study of the financial arrangement, Tables 2.3 to 2.8 show how the value of monitoring-cost parameters, the exogenously fixed bank capital-asset ratio, and the statistical properties of the shocks impact on the financial arrangement. An increment in any of the monitoring costs (Tables 2.3 and 2.4) reduces the inflows of capital (see the loan size m) but takes the probability of bank bankruptcy down. Raising the cost of monitoring the firms (i.e. μ^f) diminishes the external spread $r^D - r^*$, but raises the internal spread $r^L - r^D$. Raising the cost of monitoring the banks (i.e. μ^b) reverses the effect on the spreads: the former rises and the latter falls.

¹⁹Recall that the conditional variance of Y_t is constant. Thereupon, as the two lines are approaching it is more likely that the actual productivity shock be lower than its crisis value.

As expected, raising the required bank capital-asset ratio (Table 2.5) reduces the likelihood of a financial crash and induces larger inflows of capital. The interest rate on bank liabilities falls because the risk premium of a safer bank is lower. However, other things constant, the interest rate on firms debt rises with the capitalization of the banking system. Notice that, other things constant, the bankers losses arising from a bank failure increase with κ . These losses are not compensated by the incremental expected profits arising from the reduced financial costs (i.e. the lower value of r_t^D). Therefore, only by raising the interest rate on their loans, banks find the appropriate incentives to keep intermediating the inflows of capital when the requirements of capital are more strict.

Picking the mean of any of the shocks up produces intuitive results (see Table 2.6)²⁰: capital inflows are larger, and interest rates and the probability of bank bankruptcy are both lower. This is because a higher productivity reduces the likelihood of a negative bank cash flow. This became clear in eq. (2.7) where, in the limit, for some high values of the overall productivity shock, no single firm is bankrupt.

On the other hand, the variance of aggregate productivity shocks is positively related to the probability of a bank failure, while the variance of idiosyncratic shocks is negatively related to it (see Tables 2.7 and 2.8). The former result is in line with the overwhelming empirical evidence that points out that macroeconomic volatility is the main cause of banking sector problems. Furthermore, the result also explains why a change in the perception of the country's macroeconomic risk can alter the amount of capital flowing into a SOE.

The result that indicates that a higher variance of idiosyncratic shocks lowers the probability of a banking failure, which may seem counterintuitive at first sight, is sympathetic with recent remarks made by the Federal Reserve Chairman, Alan Greenspan: "The result of the 1990s of this seeming-heightened instability for individual businesses, somewhat surprisingly, was an apparent *reduction* in the volatility of output and in the frequency and amplitude of business cycles for the macroeconomy".²¹ And he adds "On the same period [second half of the 1990s and into 2000]

²⁰The effect of changes in the mean of the overall productivity shock was studied in Figure 2.2.

²¹Remarks at a symposium sponsored by the Federal Reserve Bank of Kansas City on August 30,

risk spreads on corporate bonds rose markedly on net, implying a rising probability of default”. In the model and from the creditors standpoint, a higher variance of idiosyncratic shocks reduces the importance of the macroeconomic shock on the banks’ portfolio return, and then the probability of a bank failure.

2.4 General Equilibrium Dynamic Results

This section studies the dynamic properties of the model described above. The focus is on the production side of the economy because the model adds nothing to what is already known about the saving decisions of a household that receives a stochastic income flow. In other words, the goal is to understand the macroeconomic consequences of inflows of capital that are driven not by consumption smoothing, but by portfolio and production decisions in the economy. For the sake of completeness, notice that the stochastic household income is given by $w_t n_t + r_t^* a_t + \Pi_t$, and the household knows the return on its savings, i.e. r_{t+1}^* , in advance.

2.4.1 Business Cycles with Banking Crises

The economy described above has two primary sources of fluctuations, namely, the overall productivity shock and the interest rate in world capital markets. The dynamics induced by these two variables are shown in Figures 2.4 and 2.5, under random sequences of shocks for 50 observations. The parameter ζ in eq. (2.13) is set equal to zero so that only the aggregate risk but not crises affect the allocations of domestic agents.

In Figure 2.4, the gross international interest rate, r_t^* , is fixed to its mean (1.065 in annual terms) and the economy is then subject to productivity shocks. The incidence of the overall productivity shock and macroeconomic risk on output fluctuations can be seen in the first row of Figure 2.4. Output swings (in panel 4.c) are the result of two forces: fluctuations in productivity, as in any RBC model; and the amount of

2002, at Hole, Wyoming.

capital flowing into the economy to finance working capital. Capital inflows affect output because larger inflows raise output at the rate β (see eq. (2.3)). This effect is dampened by the lower value of Λ_t arising from the upward pressure on input prices that follows a larger inflow of capital.

For a given value of input prices and a value of β close to one, eq. (2.2) shows that the loan size is almost as important as the productivity shock to explain output dynamics. However, as the loan size (panel f) depends on the expected value of that shock, which depends on its lagged value, the financial structure of the model reinforces the effect of the previous period productivity on the current level of output. The dependency of the size of capital flows on the state of the overall productivity shock can be seen observing the dynamics of the loan size in Figure 2.4.f and the behavior of the overall productivity shock in Figure 2.4.b.

The third row of Figure 2.4 shows how downturns are associated with both a higher probability of a bank failure, and a higher rate of bankruptcy among firms. The annual total spread ($r^L - r^*$) in Figure 2.4.e incorporates these two risks and is counter-cyclical. In linking the state of the macroeconomy to capital inflows and interest rates, a change in the overall productivity shock induces a large adjustment of capital inflows and a small adjustment in the interest rate on bank liabilities. Furthermore, the interest rate on firms' debt is more volatile than the interest rate on banks' debt.

The simulations in Figure 2.4 include a banking crisis in quarter 13 which has been marked in the i-panel of the figure. Section 2 showed that a bank failure happens when the actual overall productivity shock y_t is lower than the threshold value of productivity $\hat{y}_t$. At quarter 13, $y_t=0.3802$ and $\hat{y}_t=0.3821$. Interestingly, not every recession, no matter how deep, is able to produce a crisis; observe for example that output in quarter 45 is lower than in quarter 13. What really triggers a crisis is a *sudden* change in the value of the overall productivity shock. On the other hand, when the economy goes through a period of low productivity shocks, even though a crisis is more likely to happen vis-à-vis a boom (see the static result in Figure 2.2), this is part of the information that agents internalize and incorporate into the contracts. The size of capital inflows along with the interest rates are adjusted accordingly so

that a banking crisis does not necessarily materialize.

The dynamics induced by interest-rate shocks when the overall productivity remains constant are shown in Figure 2.5. There are two interesting results. First, international interest-rate fluctuations are able to drive business cycles. As it was pointed out by Calvo et al. (1992), and more recently by Reinhart and Reinhart (2001), international developments can determine business cycles in emerging countries. However, interest-rate swings induce output changes of second order of magnitude in RBC models of SOE's (see Mendoza (1991b)), and this neutrality to interest-rate shocks is not modified substantially by adding a demand for working capital (see Oviedo (2002)). These results, however, were derived under a constant returns to scale production function and assuming a world of symmetric information which rules out firms and bank bankruptcies. The results in Figure 2.5 show that, under asymmetric information and decreasing returns, the economy is not neutral to disturbances to the interest-rate on world capital markets.

The second result arising from Figure 2.5 is that the likelihood of a banking crisis is invariant to the *level* of the international interest rate, as it has been shown in Figure 2.3. The setup of the model describes a world of fixed-interest-rate debt, which, by definition, prevents an interest-rate change from triggering a bank failure. This is because debt contracts are one-period contracts, and the interest rates are known at the time contracts are signed. In a world of variable interest rates, however, it is easy to see how an interest-rate rise can trigger a banking crisis. The setup of two-sided debt contracts could be changed so that, although r_t^* remains the opportunity cost of capital for bankers and investors, this interest rate is unknown at the time contracts are signed. Therefore, if investors *have* to be compensated at the rate r_t^* , but contracts are written on the expected value of r_t^* , an interest-rate hike raises the value of $\hat{y}_t$. This essentially renders crises more likely under all values of the overall productivity shock.

2.4.2 Reorganization Costs in the Aftermath of a Banking Crisis

Under the model setup discussed above, a banking crisis does not have an effect on equilibrium allocations. While the risk of a crisis is present in the decisions taken by banks, firms, and international investors, and therefore in the model dynamics, the crisis itself is not costly for the economy. This does not imply that a bank failure does not produce wealth losses for bankers and investors. Bankers lose what they have invested in banking, i.e. bank capitals. Also, although investors seize banks' portfolios, the value of these portfolios is lower than the beginning-of-period expected return. However, bankers and investors are not domestic but international agents and the model allocations do not depend on their wealth.

Notice that the economy described in Figure 2.4 recovers completely after the crisis in the 13th quarter. Every bank has rebuilt its bank capital, capital resumes flowing into the economy, and the economy operates exactly the same way it did before the crisis. However, this is not what happens when banking crises hit actual economies. Bank failures destroy the information created through years of bank-client relationships. The informational losses that follow a banking crash make it difficult to produce under the same pre-crisis conditions. To account for the reorganization costs arising from a banking crisis and which do not accrue to bankers and investors, the parameter ζ in eq. (2.13) should be different from zero.

Figure 2.6 compares the effect of assuming two different values for ζ . The sequence of productivity shocks is the same that produced a crisis in the 13th quarter in Figure 2.4. The solid lines in Figure 2.6 are used for the case where $\zeta = 0$ and so the dynamics are the same as those in Figure 2.4. The lines marked with asterisks show the model dynamics when ζ is arbitrarily fixed to 0.25. When it is costly to reorganize production in the aftermath of bank failures, a banking crisis produces long lasting effects. Only by the end of the simulated period, that is approximately after 35 quarters, does the economy recover completely from the crisis. While the probability of another crisis, as well as domestic interest rates, remain constant, now the sequels of a crisis extend

to the labor income of the household (see panels k and l), firms profits, and domestic output.

2.4.3 Agency Costs and Business Cycles

Two-sided agency costs are one of the new features this work proposes to investigate the connection between the state of liquidity of international capital markets, macroeconomic risk, and the business cycles in SOEs. This section shows that changing the value of either the firms' or the banks' bankruptcy cost, does not produce sensible changes in output fluctuations. In Figure 2.7 bank monitoring costs remain constant and the value μ^f is decreased from 0.25 to 0.15. In Figure 2.8 μ^f is equal to 0.25, and μ^b takes three values: 0.25, 0.15, and 0.05. Since debt contracts are single-period loans and credit is extended without any collateral, the persistence of business cycles is independent of the values taken by monitoring costs.

2.5 Concluding Remarks

Banking crises are costly not only because of the costs associated with restructuring a collapsed banking system, but also because they give rise to negative monetary and non-monetary consequences for other sectors. While banking crises are not exclusive to emerging market economies, they are recurrent among these economies. The recurrence of the crises in developing countries is linked to their higher macroeconomic volatility. Furthermore, banking failures have produced devastating consequences in developing countries because their banks play an important role in the credit creation process.

In a world of forward looking economic agents, both macroeconomic risk and the risk of a financial crisis affect the equilibrium prices and allocations of an economy, even during 'normal times'. Therefore, the characterization of business cycles in emerging market economies needs to take into account the likelihood of occurrence of a financial crash. The research here provides a dynamic general equilibrium framework

capable of accounting for two related phenomena. First, banking crises are driven by fundamentals and not by self-fulfilling expectations. Second, financial stability and macroeconomic risk have an important effect on business cycles.

The probability of widespread banking failures is shown to be endogenously determined by the value of the state variables in the economy. Consistent with the empirical literature on banking crises, the numerical results show that crises are more likely to happen during economic downturns and also as a result of interest-rate hikes. The model explains why banking crises are sporadic phenomena and also why not every recession, no matter how deep, is capable of triggering a financial crash. The model also explains how country risk depends on macroeconomic conditions and the strength of the financial system and why country-specific interest rates are counter-cyclical.

It has been shown that the combination of the contracting technology developed by ? along with the assumption of non-verifiable aggregate productivity shocks can provide a rational to one standard assumption in SOE versions of RBC models, namely that debt contracts contingent on productivity shocks are not available. Because aggregate productivity is a non-observable variable, the best investment mechanism involves standard simple-debt-type contracts where, in the absence of bankruptcy, repayments are constant for all levels of productivity.

On the other hand, the combination of a decreasing returns technology in the production of final output and agency costs at the level of firms and banks can successfully explain the mechanism through which domestic fluctuations can be driven by international financial variables. After recognizing that reorganizing the production process in the aftermath of a banking crisis is costly, the simulations also showed that crises have long lasting effects on production and household income.

There are several avenues which could be investigated using the framework developed in this chapter. The agenda for future research includes two policy issues. First, the extent to which tighter capital requirements for banks can help reduce the incidence of macroeconomic risk on domestic fluctuations. Second, given the adverse effects of volatile capital flows on the stability of financial systems, the extent to which restrictions on these flows can reduce their destabilizing effects. An interesting exten-

sion of the model would be to endogeneize the portfolio decisions taken by bankers. By explicitly modelling decisions on how much wealth is allocated to banking, the model would produce a richer interaction between the financial and real sectors of the economy. It would also help to better understand the effect of financial fragility on macroeconomic fluctuations and provide better foundations for the long-lasting effects of financial crises.

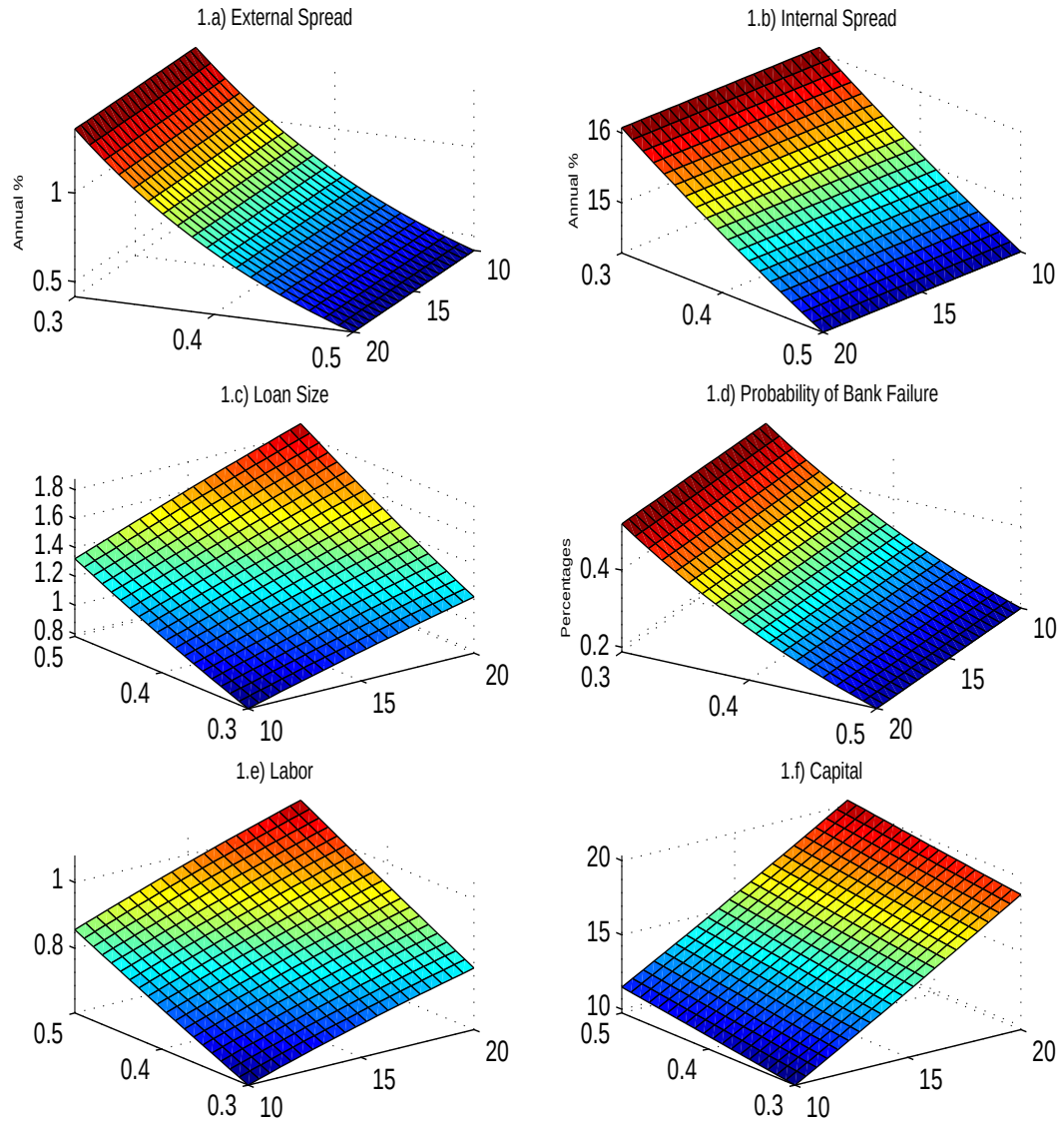


Figure 2.1: One Period Financial Contract with a Fixed International Interest Rate

Notes: Aggregate productivity and capital are plotted on the horizontal axes. The productivity range is $[0.3, 0.5]$ and the capital range is $[10, 20]$. The external (internal) spread is equal to $r^D - r^*$ ($r^L - r^D$), where r^* is the risk-free international interest rate, r^D the interest rate on bank liabilities, and r^L the interest rate on the debt issued by firms.

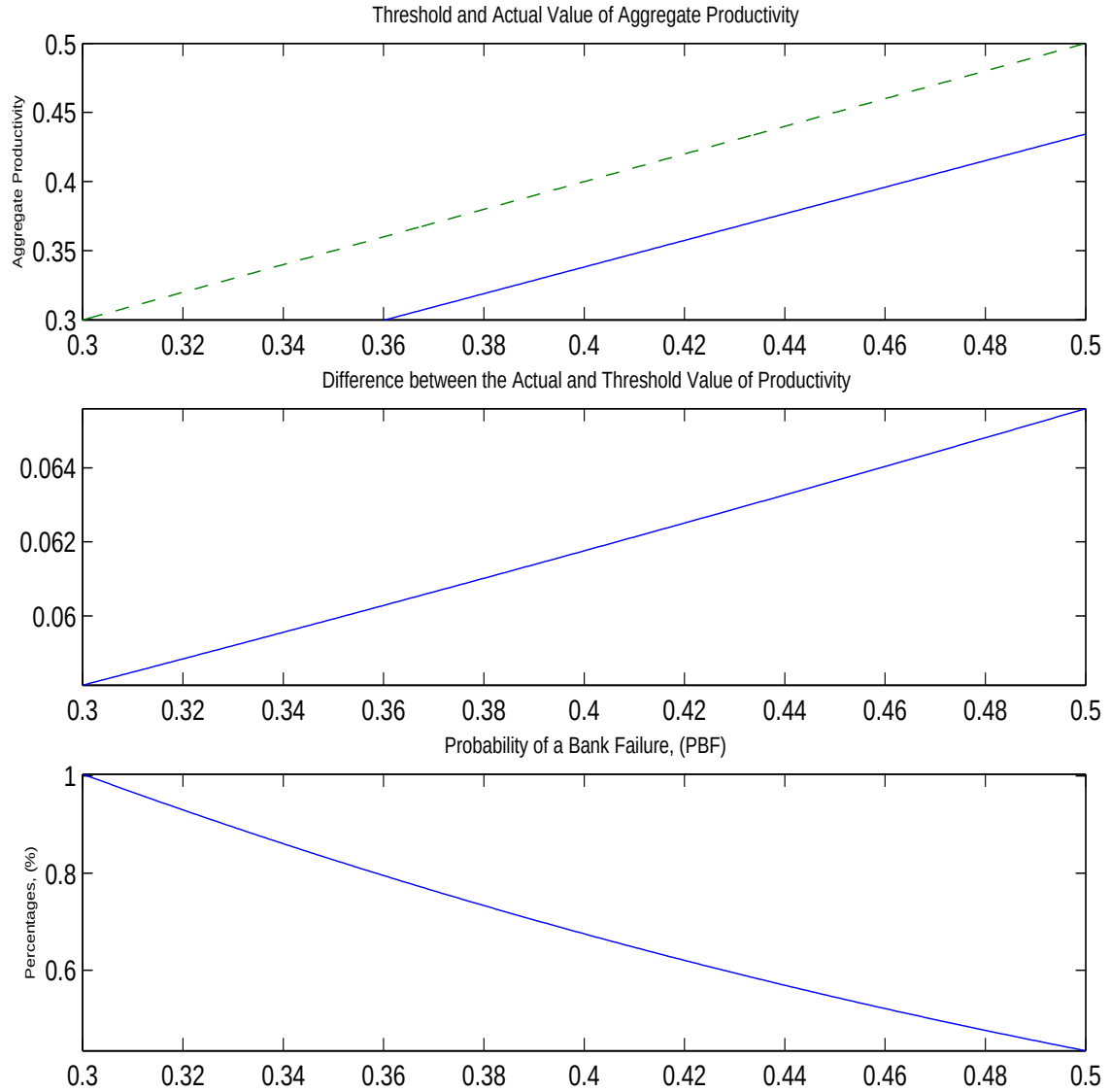


Figure 2.2: Crisis-Threshold-Value of Aggregate Productivity and Probability of a Bank Failure

The top panel of the figure shows the threshold value of aggregate productivity which triggers a banking crash, $\hat{y}$, as a function of actual productivity, y ; and a 45% degrees line (dotted line). Each point of the dotted line can be seen as the mean of (vertically plotted) distributions whose tails go beyond $\hat{y}$. The second panel shows the difference between y and $\hat{y}$ for all levels of y . The third panel of the figure displays the probability of a banking crash at every level of y : the likely of a bank failure falls as productivity rises.

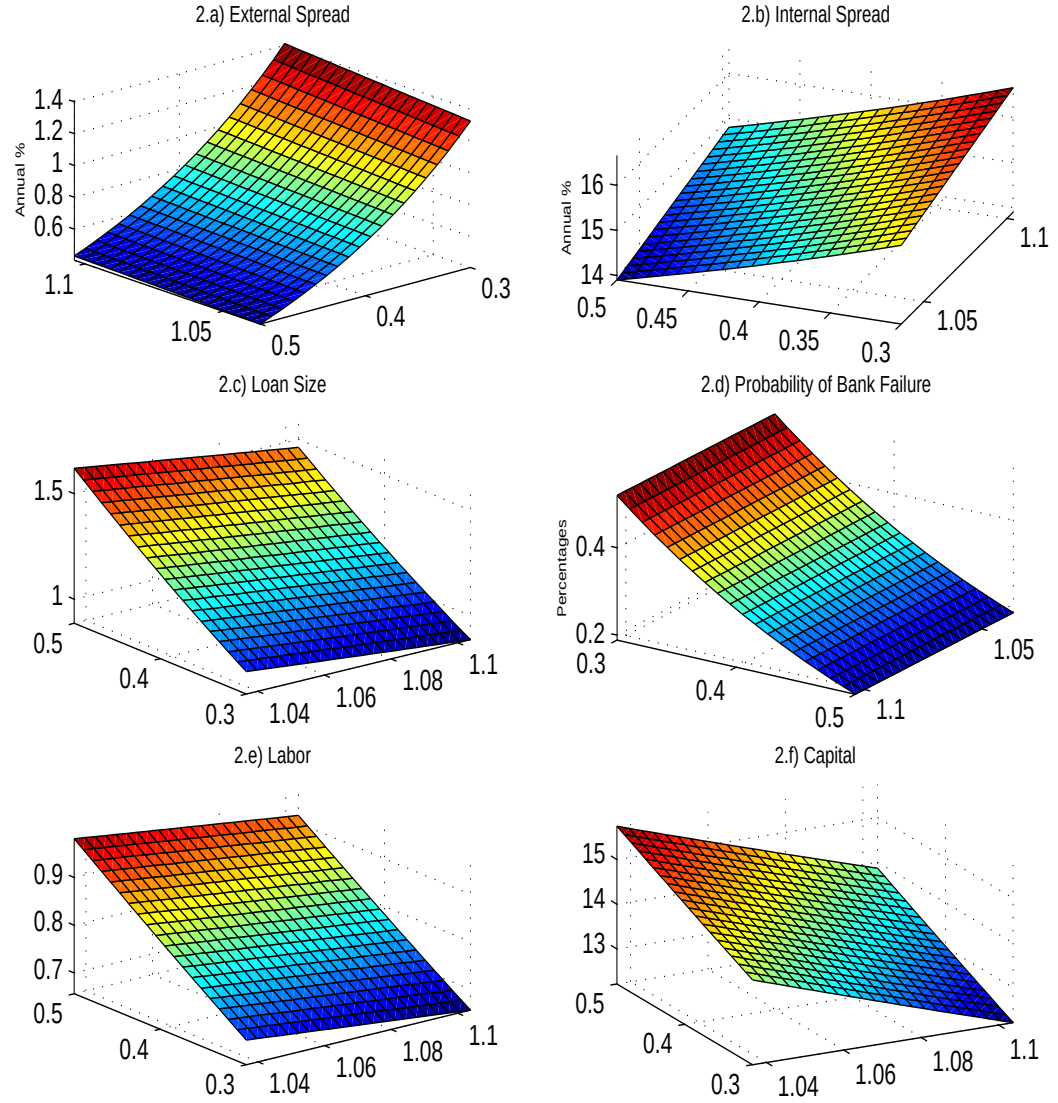


Figure 2.3: One Period Financial Contract with a Fixed Capital Stock

Notes: Aggregate productivity and the international interest rate are plotted on the horizontal axes. The productivity range is $[0.3, 0.5]$ and the risk-free-rate range is $[4\%, 11\%]$, expressed as net annual rates. The external (internal) spread is equal to $r^D - r^*$ ($r^L - r^D$), where r^* is the risk-free international interest rate, r^D the interest rate on bank liabilities, and r^L the interest rate on the debt issued by firms.

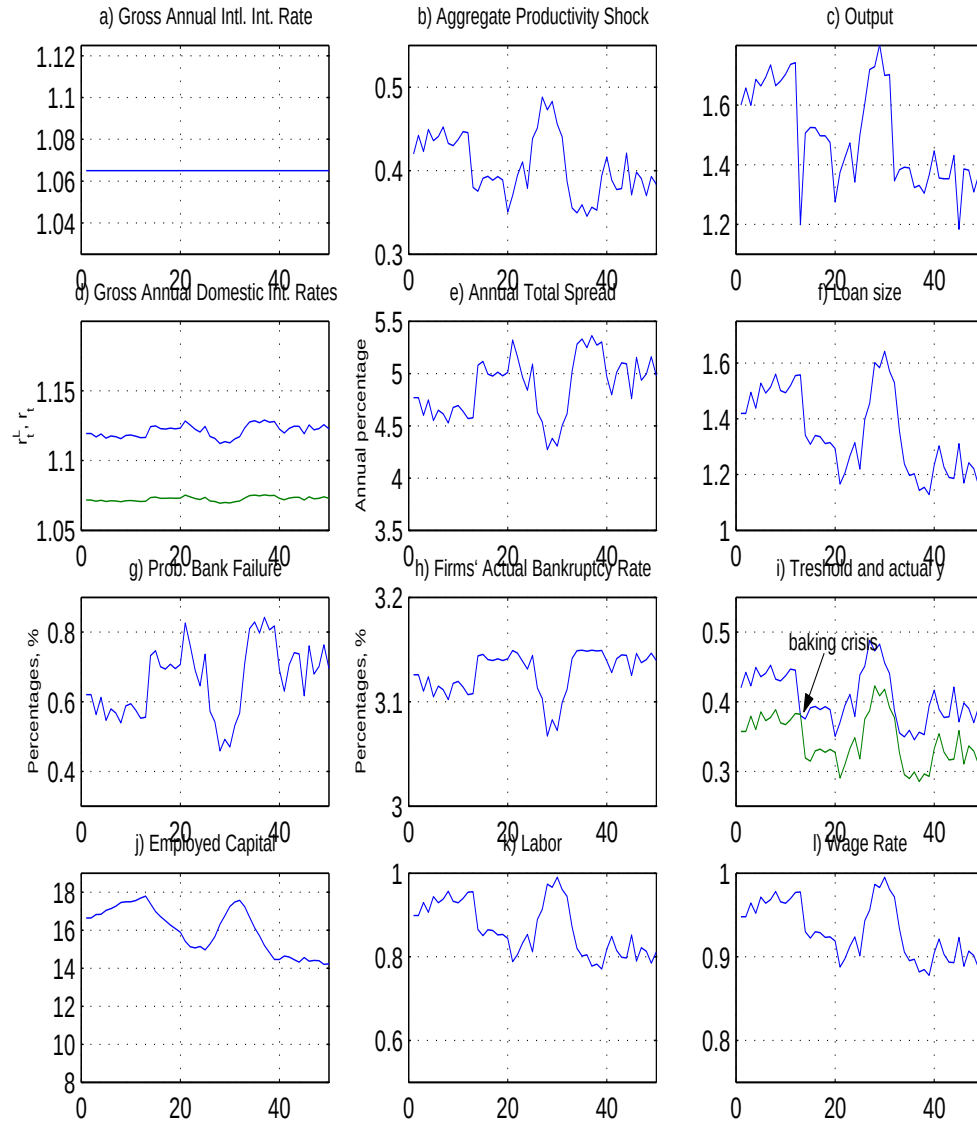


Figure 2.4: Productivity Shocks and Model Dynamics

Notes: The international interest rate r_t^* is fixed at its steady-state value(1.065) and the model is simulated under a random sequence of aggregate productivity shocks that has an autoregressive structure. There is a banking crisis in period 13 that has been marked in panel i.

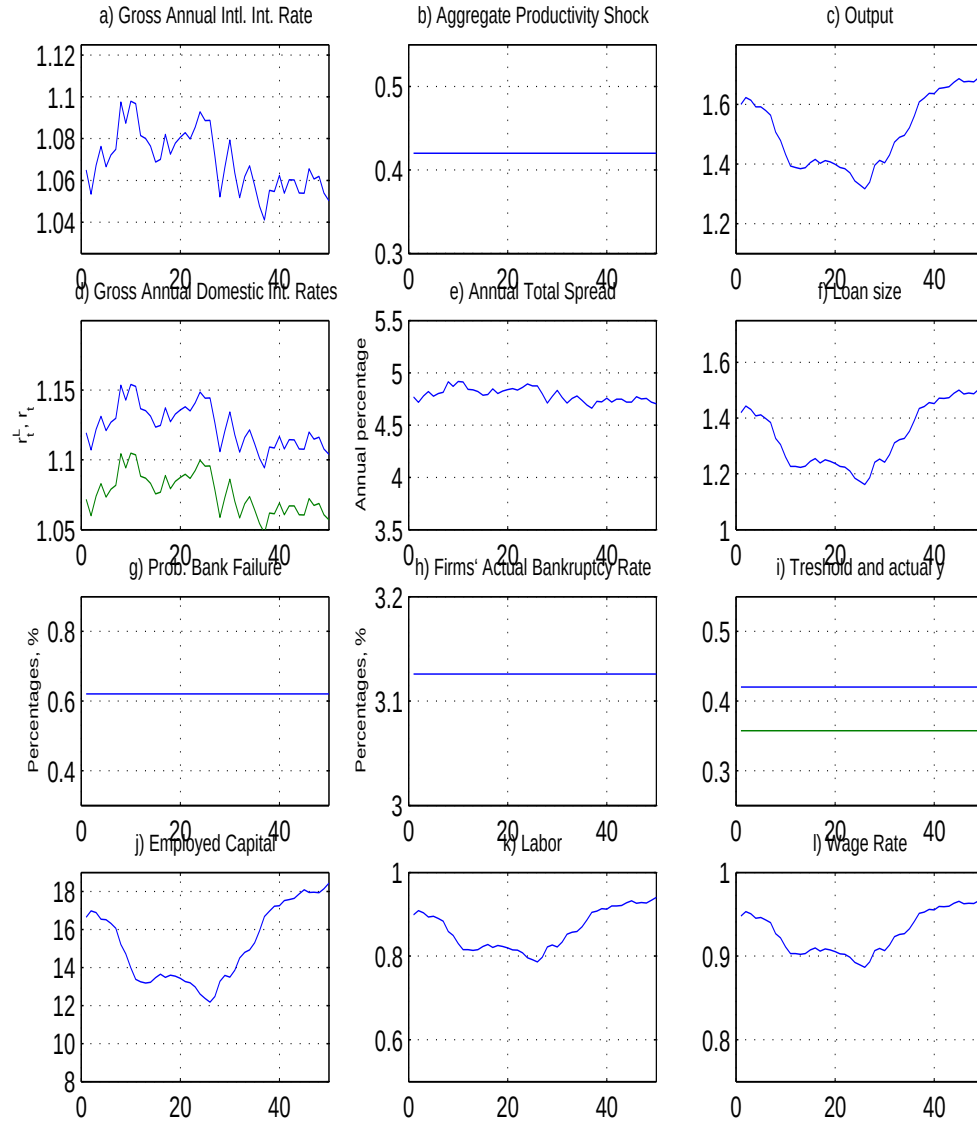


Figure 2.5: Interest-Rate Shocks and Model Dynamics

Notes: The overall productivity shock y_t has been fixed to its mean value (0.42) and the model is simulated under a random sequence of international interest rates r_t^* , which follows an AR(1) process.

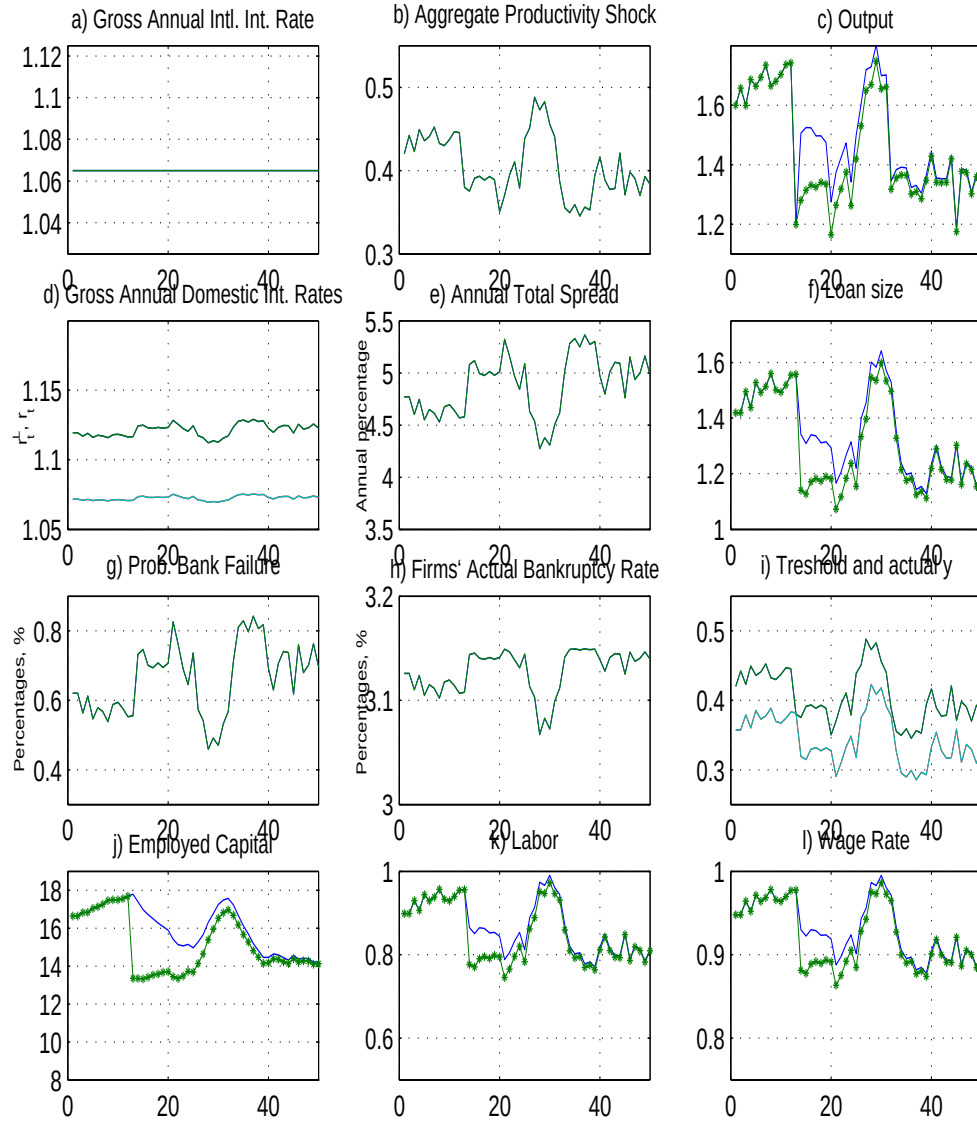


Figure 2.6: The Cost of Reorganizing Production in the Aftermath of a Crisis

Notes: The international interest rate, r_t^* , has been fixed equal to its steady-state value (1.065 in annual terms) and the model is simulated under a random sequence of aggregate productivity shocks that has an AR(1) structure. Asterisks are used for the dynamics generated by the model when it is costly to reorganize the production in the aftermath of a banking crisis. The sequence of overall productivity shocks is the same as in Figure 4, so that a crisis occurs in the 13th quarter.

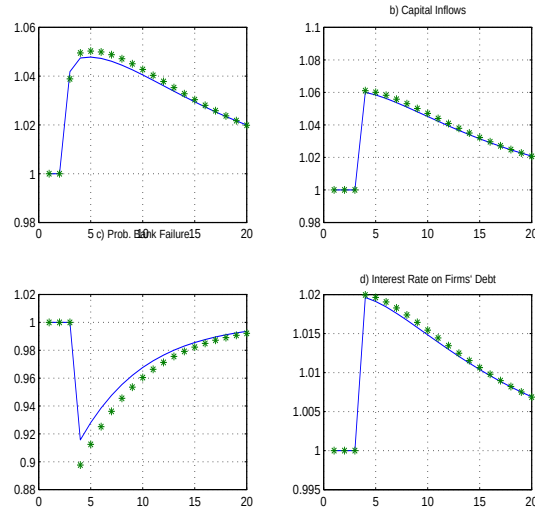


Figure 2.7: Firm Monitoring Costs and Model Dynamics

Notes: Impulse response functions following a 1% deviation in the overall productivity shock in the third quarter, under alternative values of μ^f , i.e. the cost of monitoring the firms. Asterisks are used for the impulse response functions obtained assuming $\mu^f = 0.15$. Solid lines are used for simulations where $\mu^f = 0.25$. All variables have been normalized to be equal to one during the first two quarters.

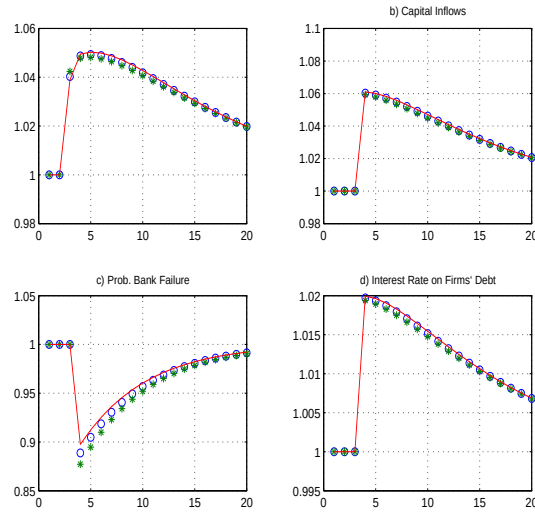


Figure 2.8: Bank Monitoring Costs and Model Dynamics

Notes: Impulse response functions following a 1% deviation in the overall productivity shock in the third quarter, under alternative values of μ^b , i.e. the cost of monitoring the banks. In the economy described by solid lines, $\mu^b = 0.25$; circles are used for the economy where $\mu^b = 0.15$; and asterisks are used for the economy where $\mu^b = 0.05$.

Table 2.1: Sequence of Events in a Given Period

t_1	Information:	y_{t-1} r_t^* K_t a_t	Last period aggregate productivity shock International interest rate Supply of capital Household wealth
	Allocations/ Decisions:	m_{it}	Loan sizes (from financial contracts)
	Prices:	r_t^D r_t^L	Gross bank-deposit rate (from fin. cont.) Gross bank-loan rate (from fin. cont.)
t_2	Allocations:	n_t k_t	Labor employment Capital employment
	Prices	w_t r_t^K	Wage rate Gross rate of return on capital
t_3	New Information:	y_t $x_{1t}, \dots, x_{Nt}$ r_{t+1}^*	Aggregate productivity shock Idiosyncratic productivity shocks Next period international interest rate
	Allocations/ Decisions:	Υ_{it} π_{it}, Π_t	Output Individual and aggregate profits Firm and bank bankruptcy
t_4	Allocations/ Decisions:	c_t b_{t+1} k_{t+1}^H k_{t+1}	Consumption International risk-free asset Household supply of capital Next period total supply of capital

Notes: every period t is divided into four subperiods, t_1 to t_4 . When a variable has two subindexes the first one is used to individualize a firm.

Table 2.2: Mean and Variance of Productivity Shocks

Idiosyncratic shock		Aggregate shock		Total Productivity shock	
μ_X	σ_X	μ_Y	σ_Y	μ_Z	σ_Z
0.470	0.040	0.420	0.025	0.890	0.047

Notes: X and Y are the random idiosyncratic and aggregate productivity shocks respectively.

μ (σ) represents the mean (standard deviation) of the random variable indicated in the subindex.

Table 2.3: Firm-Monitoring-Cost Parameter (μ^f) and the Financial Contract

μ^f	$r^L - 1$ (% annual)	$r^D - 1$ (% annual)	m	$\hat{y}$	PBB (%)
0.150	11.710	7.693	1.658	0.364	1.208
0.200	11.683	7.365	1.509	0.360	0.796
0.250	11.947	7.177	1.420	0.357	0.620
0.300	12.147	7.033	1.340	0.355	0.487
0.350	12.300	6.923	1.270	0.353	0.385

Notes: r^L (r^D) is the gross annual rate on firms (banks) liabilities; m is the loan size; $\hat{y}$ is the threshold value of the aggregate productivity that defines a banking crisis; and PBB is the probability of bank failure.

Table 2.4: Bank-Monitoring-Cost Parameters (μ^b) and the financial contract

μ^b	$r^L - 1$ (% annual)	$r^D - 1$ (% annual)	m	$\hat{y}$	PBB (%)
0.150	12.163	6.871	1.479	0.359	0.739
0.200	12.049	7.037	1.447	0.358	0.675
0.250	11.947	7.177	1.420	0.357	0.620
0.300	11.855	7.296	1.395	0.357	0.575
0.350	11.772	7.399	1.373	0.356	0.535

Notes: r^L (r^D) is the gross annual rate on firms (banks) liabilities; m is the loan size; $\hat{y}$ is the threshold value of the aggregate productivity that defines a banking crisis; and PBB is the probability of bank failure.

Table 2.5: The Capital-Asset Ratio (κ) and the financial contract

κ	$r^L - 1$ (% annual)	$r^D - 1$ (% annual)	m	$\hat{y}$	PBB (%)
0.060	11.749	7.681	1.307	0.362	1.013
0.070	11.845	7.399	1.369	0.360	0.796
0.080	11.947	7.177	1.420	0.357	0.621
0.090	12.048	7.004	1.461	0.355	0.479
0.100	12.142	6.871	1.494	0.353	0.365

Notes: r^L (r^D) is the gross annual rate on firms (banks) liabilities; m is the loan size; $\hat{y}$ is the threshold value of the aggregate productivity that defines a banking crisis; and PBB is the probability of bank failure.

Table 2.6: The Mean of Idiosyncratic Shocks and the Financial Contract

μ_X	$r^L - 1$ (% annual)	$r^D - 1$ (% annual)	m	$\hat{y}$	PBB (%)
0.450	12.186	7.262	0.849	0.358	0.675
0.460	12.065	7.219	1.100	0.358	0.647
0.470	11.947	7.177	1.420	0.357	0.620
0.480	11.832	7.138	1.826	0.357	0.594
0.490	11.720	7.100	2.342	0.357	0.569

Notes: r^L (r^D) is the gross annual rate on firms (banks) liabilities; m is the loan size; $\hat{y}$ is the threshold value of the aggregate productivity that defines a banking crisis; and PBB is the probability of bank failure.

Table 2.7: The Standard Deviation of Idiosyncratic Shocks (σ_X) and the financial contract

σ_X	$r^L - 1$ (% annual)	$r^D - 1$ (% annual)	m	$\hat{y}$	PBB (%)
0.030	10.190	7.250	1.834	0.359	0.699
0.040	11.947	7.177	1.420	0.357	0.620
0.050	14.251	7.114	1.089	0.356	0.552
0.060	17.075	7.067	0.836	0.356	0.502
0.070	20.396	7.037	0.645	0.355	0.467

Notes: r^L (r^D) is the gross annual rate on firms (banks) liabilities; m is the loan size; $\hat{y}$ is the threshold value of the aggregate productivity that defines a banking crisis; and PBB is the probability of bank failure.

Table 2.8: The Standard Deviation of Aggregate Shocks (σ_Y) and the Financial Contract

σ_Y	$r^L - 1$ (% annual)	$r^D - 1$ (% annual)	m	$\hat{y}$	PBB (%)
0.021	11.625	6.802	1.613	0.362	0.280
0.023	11.760	6.975	1.517	0.360	0.438
0.025	11.947	7.177	1.420	0.357	0.621
0.027	12.185	7.401	1.324	0.355	0.822
0.029	12.474	7.644	1.233	0.353	1.038

Notes: r^L (r^D) is the gross annual rate on firms (banks) liabilities; m is the loan size; $\hat{y}$ is the threshold value of the aggregate productivity that defines a banking crisis; and PBB is the probability of bank failure.

Chapter 3

A Toolbox for the Numerical Study of Linear Rational Expectation Models

3.1 Introduction

This chapter explains how to use the author-created toolbox of Matlab functions to study linear difference rational expectation macroeconomic models. Although non macroeconomic models could also be studied using the toolbox, the characteristics of its output make it more suitable for dynamic macroeconomic models. The toolbox is a set of Matlab functions; while some of the functions are model specific, others are applicable to any log-linear model. Having several web sites where written Matlab functions are available to accomplish the same goal as the one in this toolbox, the latter has two distinguishing characteristics.¹ One is the minimization of the compu-

¹Some of the functions in the toolbox were borrow from these other sources although their documentation has been changed here. See for example

tational task required to solve and study the macroeconomic model. This should help a researcher who has almost no experience with computational work to resolve and study his own model. The second distinguishing characteristic of these notes and the associated toolbox of Matlab function is the provision of step-by-step explanations of the procedures a researcher should follow to study his own model.²

Briefly, once the user has outlined his model, the toolbox accompanying this paper helps him to log-linearize the model equations as well as to obtain: a) impulse response functions; b) population moments of all involved variables; c) the behavior of the model when a particular sequence of shocks hits the system; and d) sample moment conditions.

Linear dynamic rational expectation models arise from the linearization of dynamic models around particular stationary points. Commonly, the non-stochastic steady state is the chosen point, and this is also the strategy followed here. There exist several reasons to work with the linear version of a model instead of its original formulation. The most important one is related to the curse of dimensionality to which other solution methods, like value function iteration or those based on a collocation strategy, are subject to. The computer cost of these other methods become significant. Contrarily, linear models can be solved virtually in a couple of seconds. This advantage could be shadowed by a lower accuracy of the results. However, it has been shown that linear models produce very accurate results when the shocks being considered are not too large.³

The organization of the paper is as follows. In the second section we make clear the type of models for which the toolbox is suitable. We do that establishing a convention on the classification of variables and equations. In the third section we develop two examples that will illustrate throughout the paper all steps that a researcher should follow in studying his own model. We discuss the theoretical solution of the model in the fourth section and put the toolbox of Matlab functions to work in the fifth section. As our goal is to offer detailed explanations of these functions, we describe

<http://www.ssc.uwo.ca/economics/faculty/klein/main.htm>

²Both, the notes and the accompanying Matlab functions contain thorough explanations.

³See for example Dotsey and Mao (1992), Danthine and Mehra (1989), and Stephanie and Uribe (2001).

them before their employment in the study of the example models.

3.2 Models to be Considered

We start establishing a convention on how to classify the variables in the models under study. Four types of variables are found in these models. First, predetermined or backward looking variables. Second, nonpredetermined or forward looking variables. Third, innovations to the backward looking variables. Fourth, flow or additional variables. Let us review what we mean by the stated convention.

The value of a predetermined variable at time t is given by the evolution of the system in the previous period and the innovation (third type) that takes place at t . Thus, the value of a predetermined variable at time t is given by the decisions taken at $t - 1$, the evolution of the system between $t - 1$ and t , and the innovations observed at t . Commonly, backward looking variables correspond to the state variables in a dynamic programming framework. Throughout the paper (and also in the accompanying toolbox), we denote predetermined variables at time t as $x_1(t)$.

Within the set of predetermined variables, we distinguish between endogenous and exogenous variables. While the value of an endogenous predetermined variable is affected by economic decisions, exogenous predetermined variables do not depend on any other variable in the model but the innovations to their own processes. The stock of capital and the amount of foreign assets are typical examples of endogenous backward looking variables in an open economy, while a productivity shock is an example of an exogenous backward looking variable. The value of both types of backward looking variables at time t are included in $x_1(t)$. Without loss of generality, we assume that there are n_1 backward looking variables.

A forward looking variable is a variable whose t -value depends on actions taken at time t . We group all forward looking variables in the vector $x_2(t)$ whose size is equal to n_2 . Appealing to the parallelism with the dynamic programming framework, policy variables like consumption or the accumulation of assets are examples of nonpredetermined variables.

Innovations to the backward looking variables typically are iid shocks hitting some of the state variables in x_1 . For example, the international interest rate or the production-function productivity shocks may be modelled as an exogenous autoregressive process subject to period-by-period innovations. Let ϵ denote the vector of innovations to the variables in the model. At a high level of generality, we might consider that all forward and backward looking variables could be subject to innovations. Thus, ϵ is a vector of length equal to $n_1 + n_2$. In most models, however, most of the components of ϵ are equal to zero because some predetermined variables (overall, the endogenous ones) and most, if not all, of the nonpredetermined variables are not subject to any kind of shock.

Finally, the fourth set of variables are called flow or additional variables. They are variables not explicitly included in the model but whose behavior is of interest to the researcher and their dynamics depend on other variables in the model. For example, the trade balance may not be a variable in the model but its dynamic behavior can be computed from those of output, consumption and investment.

With this classification of the variables in hand, we can state the general class of models for which the toolbox accompanying this paper is suitable. The specification of a macroeconomic model usually involves three sets of equations, namely: a set of optimality conditions, a set of market clearing conditions and resource constraints, and a set of equations specifying the evolution of some predetermined variables. Examples of these equations are, respectively, the equalization of the marginal rate of substitution of labor for consumption to the real wage rate; the equalization of output to the sum of investment and consumption; and an autoregressive process describing the productivity-shock dynamics.

After log-linearizing the described sets of equations, the linear version of the model is:

$$AE_t x(t+1) = Bx(t) \quad (3.1)$$

where $x(t)$ is a vector defined by $x(t) \equiv (x_1(t), x_2(t))'$. Inasmuch as the model has n_1 predetermined variables and n_2 non-predetermined variables, the size of $x(t)$ is n , where $n = n_1 + n_2$. A and B are matrices of (scalar) coefficients, both of size $n \times n$.

In most models, the matrix A is singular. This happens when at least one equation in the model does not involve any variable dated at time $t + 1$. An example is the equation describing optimal labor decisions where the marginal rate of substitution of labor for consumption is equal to the wage rate and none of these two involve future dated variables.

The actual evolution of the system is also affected by the innovations to the n variables in x :⁴

$$Ax(t + 1) = Bx(t) + \epsilon(t + 1)$$

When only the backward looking variables are subject to innovations, a partition of the vector ϵ permits writing the system as:

$$A \begin{pmatrix} x_1(t + 1) \\ x_2(t + 1) \end{pmatrix} = B \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} \epsilon_1(t + 1) \\ 0_{n_2 \times 1} \end{pmatrix}$$

And when only the exogenous backward-looking variables are subject to innovations, some components of $\epsilon_1(t + 1)$ are also equal to zero.

Let $x_3(t)$ be the vector of additional or flow variables at time t . Without loss of generality, we are going to assume that there are n_3 flow variables. The value of these variables depends on the value taken by the variables in x_1 and x_2 , both at time t and $t + 1$. After log-linearizing the set of equations defining the flow variables, we have that:

$$x_3(t) = Cx(t + 1) + Dx(t) \tag{3.2}$$

Here, x_3 is an n_3 -vector and C and D are matrices of dimension $n_3 \times n$.

⁴Observe that $Ax(t + 1) = AE_t x(t + 1) + \epsilon(t + 1)$.

3.3 Two Model Examples: an Open and a Closed Economy

This section presents two models that fit into the class of models discussed above. We start with the model of an open economy and then we show how this economy can be “closed” so that we return to the standard neoclassical growth model. After discussing the setup of these models, we are going to discuss their log-linearization; then we are going to show how the components of the linear versions of the models parallels the components of equation (3.1). The actual solution of both models as well as their analysis are conducted in section 3.5.

3.3.1 An Open Economy Facing a Positive Supply of Funds

We consider the problem of a central planner of a small open economy that faces a positively sloped supply of external financing. The function depicting the supply of funds is not derived from an optimal contract but it is an *ad-hoc* one.

The central planner wants to maximize the expected utility of the representative agent,

$$\max_{\{c_t, n_t, k_{t+1}, b_{t+1}\}_{t=0}^{\infty}} \left[E_0 \sum_{t=0}^{\infty} \beta^t \frac{(c_t^\gamma (1 - n_t)^{1-\gamma})^{1-\sigma}}{1 - \sigma} \right] \quad (3.3)$$

subject to the following constraints for $t = 0, \dots, \infty$:

$$e^{z_t} k_t^\alpha n_t^{1-\alpha} + r_t b_t \geq c_t + i_t + b_{t+1} - b_t \quad (3.4a)$$

$$i_t = k_{t+1} - k_t(1 - \delta) \quad (3.4b)$$

$$r_t = r_t^* - \omega(b_t - b) \quad (3.4c)$$

$$z_{t+1} = \rho^z z_t + \epsilon_{t+1}^z \quad (3.4d)$$

$$r_{t+1}^* = (1 - \rho^r)z + \rho^r r_t^* + \epsilon_{t+1}^r \quad (3.4e)$$

$$(b_0, k_0, z_0, r_0^*) \quad \text{given} \quad (3.4f)$$

c_t is consumption and n_t is the labor supplied at time t . The utility index is Cobb-Douglas and σ is the negative of the elasticity of marginal utility.

The flow budget constraint in eq. (3.4a) is standard and says that the sum of consumption, investment, i_t , and the accumulation of international financial assets, $b_{t+1} - b_t$, must not be greater than the sum of production and the net financial income. The production function is Cobb-Douglas and α is the capital share in total output. z_t is the productivity shock. We assume that the country is a net debtor in international financial markets so that b_t is negative. The interest rate on the country's debt is r_t and evolves according to eq. (3.4c). There, r_t^* is the international interest rate on debt issued by countries whose outstanding stock of debt is b ; this is also the steady-state value of the country's debt. When b_t is rising above b , the country risk premium $r_t - r_t^*$ is positive and vice versa.

The investment equation (3.4b) has no adjustment costs and δ is the depreciation rate. Eqs. (3.4d) and (3.4e) are the forcing processes of the productivity shock and the international interest rate. z is the unconditional mean of the productivity shock and the ρ 's are autoregressive parameters.

After substituting i_t and r_t from eqs. (3.4b) and (3.4c) into eq. (3.4a), the first order conditions of the problem, as of time t ($t = 0, \dots, \infty$), are:

$$\frac{1 - \gamma}{\gamma} \frac{c_t}{1 - n_t} = (1 - \alpha) e^{z_t} k_t^\alpha n_t^{1-\alpha} \quad (3.5a)$$

$$MUC_t = \beta E_t [MUC_{t+1} (1 + \alpha e^{z_{t+1}} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} - \delta)] \quad (3.5b)$$

$$MUC_t = \beta E_t [MUC_{t+1} (1 + r_{t+1}^* - \omega(b_{t+1} - b))] \quad (3.5c)$$

$$e^{z_t} k_t^\alpha n_t^{1-\alpha} + [r_t^* - \omega(b_t - b)] b_t = c_t + k_{t+1} - k_t(1 - \delta) + b_{t+1} - b_t \quad (3.5d)$$

where $MUC_t \equiv \gamma c_t^{\gamma(1-\sigma)-1} (1 - n_t)^{(1-\gamma)(1-\sigma)}$ is the marginal utility of consumption

at time t . The interpretation of these conditions is as follows. Eq. (3.5a) makes the marginal rate of substitution of leisure for consumption equal to the marginal productivity of labor. Eq. (3.5b) shows that the economy must accumulate capital until the marginal cost in utility terms (lhs) is equal to the marginal expected benefit, in utility terms, of the future additional consumption (rhs). A similar interpretation fits for eq. (3.5c), but with saving made in international bonds instead of capital goods. Eq. (3.5d) is the budget constraint showed above and the equality arises from no-satiation. An additional limiting condition that must be satisfied by this problem is:

$$\lim_{t \rightarrow \infty} \beta^t E_0 u_{c_t}(k_t + b_t) = 0$$

Four additional or flow variables are considered, namely: output, y_t ; investment, i_t ; the trade balance, tb_t ; and the domestic interest rate, r_t . They are defined by:

$$y_t = e_t^z A k_t^{1-\alpha} n_t^\alpha \quad (3.6a)$$

$$i_t = k_{t+1} - k_t(1 - \delta) \quad (3.6b)$$

$$tb_t = e_t^z k_t^{1-\alpha} n_t^\alpha - c_t - i_t \quad (3.6c)$$

$$r_t = r_t^* - \omega(b_t - b) \quad (3.6d)$$

Thus $x_3(t) = (y_t, i_t, tb_t, r_t)'$.⁵

The solution to this model are infinite sequences for $t = 0$ to ∞ , one for each of the following six variables: c_t , n_t , k_{t+1} , b_{t+1} , r_t^* , and z_t .⁶ We have to use a system of six equations to find the solution. These equations are given by the first order conditions (3.5) and the equations describing the temporal evolution of the interest rates and productivity factor, eqs. (3.4d) and (3.4e). The solution will depend on the initial conditions of the economy, as represented by the initial value of the predetermined

⁵The order of this vector is also important.

⁶If we express this problem in a recursive form, then we could say that for given values of the predetermined variables, we could tell what the solution values for the seven variables above are.

variables.

Categorization of Variables and Equations

Table 3.1 proceeds with the categorization of variables according to the criteria shown in section 3.2.

At time $t = 0$, the economy starts with a stock of capital, a stock of outstanding international assets (see eq. (3.4f)), and initial values of the productivity and interest-rate shocks. These are the four predetermined variables of the model and their value do not depend on any other variable in the model (see eqs. (3.4d) and (3.4e)). At $t = 0$, the central planner decides the optimal value of n_0 and c_0 , as well as the value of the predetermined endogenous variables b_1 and k_1 . Therefore, at $t = 1$ the value of all predetermined variables are given by the decisions taken at $t = 0$ and the innovations to the exogenous predetermined variables.

Eqs. (3.5a) to (3.5c) are the optimality conditions of the model. Eq. (3.5d) is the market clearing condition for goods, and eqs. (3.4d) and (3.4e) are the two forcing processes of the model.

Calibration and Log-Linearization

The calibration of the model to an economy is performed following the statistical information available for the economy. The six equations above are used to solve for a combination of six parameters and variables. Deciding which parameter and variable values are taken from the data and which are arising from the equilibrium conditions of the model depends, among other things, on the available statistical information.

The non-stochastic steady state of the model is characterized by the following six equations⁷:

$$\frac{1 - \gamma}{\gamma} \frac{c}{1 - n} = (1 - \alpha) \frac{y}{n} \quad (3.7a)$$

⁷This is the state where the system would rest in the long run if the innovations to the exogenous backward-looking variables are set equal to zero in every time period. All variables in the system are said to remain at their steady-state value. Therefore, for any variables x_t , $x_t = x_{t+1} = x$, where x is the steady-state value of the variable

$$1 = \beta \left(1 + \alpha \frac{y}{k} - \delta \right) \quad (3.7b)$$

$$1 = \beta (1 + r^*) \quad (3.7c)$$

$$y + r^* b = c + k \delta \quad (3.7d)$$

$$z = \rho^z z \quad (3.7e)$$

$$r^* = (1 - \rho^r) r^* + \rho_z r^* \quad (3.7f)$$

where y is the steady-state value of output (see eq. (3.6a). For $\rho^z \neq 1$, eq. (3.4d) implies that $z = 0$. Eq. (3.7f) is valid for any r^* so its value will be endogenously determined. We then have four equations remaining (3.7a to 3.7d) to solve for a combination of four variables and parameters. Specifically, we choose to solve these equations to find the value of k , b , β , and γ . Setting the value of output equal to one, we start with the following assumptions: $c/y=0.75$; $n=0.2$; $r^*=0.06$; $\alpha=.4$; and $\delta=0.02$. We then obtain that $k=11.5361$; $b=-1.3138$; $\beta=0.9855$; and $\gamma=0.2381$. Plugging these results into the production function, we obtain that: $A=0.9875$.

Since the parameters ω , σ , ρ^z , and ρ^r do not enter into the system of eqs. (3.7a)-(3.7d), we can set their value exogenously. We set $\rho^r = 0.87$, $\rho^z = 0.95$, $\omega=0.0025$, and $\sigma=2$. Furthermore, there are no mathematical constraints on the statistical properties of the innovations to interest rate and productivity shocks so that their mean and standard deviation can be freely chosen. We are going to chose their value to reproduce some statistical properties of output and the interest rate (see section 3.5). Notwithstanding this, we assume that the innovations to both exogenous predetermined variables are zero-mean Gaussian processes.

The toolbox accompanying this paper conducts the log-linearization of the model. However, we are going to illustrate how to do this by paper and pencil using the optimality equation (3.5a). Starting with its *lfh*, totally differentiating and evaluating

the changes around the steady state of the model give:

$$\frac{1-\gamma}{\gamma} \frac{1}{1-n} dc_t + \frac{1-\gamma}{\gamma} \frac{c}{(1-n)^2} dn_t$$

For a variable x , define $\hat{x} = dx/x$; then, we can rewrite the expression above as:

$$\frac{1-\gamma}{\gamma} \frac{c}{1-n} \hat{c}_t + \frac{1-\gamma}{\gamma} \frac{cn}{(1-n)^2} \hat{n}_t$$

Since all variables but the “hat” have numerical values, we can then write the linear version of the eq. (3.5a) as:

$$(3.2)(0.9375)\hat{c}_t + (3.2)(0.2344)\hat{n}_t = 3\hat{c}_t + 0.75\hat{n}_t$$

In order to proceed with the *rhs* of eq. (3.5a), it is convenient to define: $\zeta_t \equiv e^{z_t}$ ⁸. In the first step we totally differentiate to obtain:

$$(1-\alpha)A k^\alpha n^{-\alpha} d\zeta_t + (1-\alpha)\alpha \zeta A k^{\alpha-1} n^{-\alpha} dk_t + (1-\alpha)(-\alpha)\zeta A k^\alpha n^{-\alpha-1} dn_t$$

And converting differentials to relative changes, the second step is:

$$(1-\alpha)\zeta A k^\alpha n^{-\alpha} \hat{\zeta}_t + (1-\alpha)\alpha \zeta A k^{\alpha-1} n^{-\alpha} \hat{k}_t + (1-\alpha)(-\alpha)\zeta A k^\alpha n^{-\alpha} \hat{n}_t$$

And the numerical version of this side of the equation is:

$$1.2\hat{k}_t + 3\hat{\zeta}_t - 1.2\hat{n}_t$$

Combining both sides of the equation the result, which is going to be in the first row

⁸Notice that $d\zeta_t = e^z dz_t$. Also, when $z=0$, $\zeta = 1$ and $d\zeta_t = dz_t$.

of matrix B , is:

$$-1.2\hat{k}_t - 3\hat{\zeta}_t + 3\hat{c}_t + 1.95\hat{n}_t = 0$$

Now using the toolbox to log-linearize the complete model as will be explained in section (3.5), we obtain the following matrices A and B :

$$A = \begin{pmatrix} 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ -0.008 & 0.000 & 0.014 & 0.000 & -0.499 & 0.085 \\ 0.000 & 0.001 & 0.000 & 0.006 & -0.499 & 0.077 \\ 11.536 & -1.314 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \end{pmatrix}$$

$$B = \begin{pmatrix} -1.200 & 0.000 & -3.000 & 0.000 & 3.000 & 1.950 \\ 0.000 & 0.000 & 0.000 & 0.000 & -0.499 & 0.077 \\ 0.000 & 0.001 & 0.000 & 0.000 & -0.499 & 0.077 \\ 11.705 & -1.337 & 1.000 & -0.019 & -0.750 & 0.600 \\ 0.000 & 0.000 & 0.950 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.870 & 0.000 & 0.000 \end{pmatrix}$$

The linearization of the flow variables produces the following two matrices of coeffi-

cients:

$$C = \begin{pmatrix} 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 11.536 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ -11.536 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \end{pmatrix}$$

$$D = \begin{pmatrix} 0.400 & 0.000 & 1.000 & 0.000 & 0.000 & 0.600 \\ -11.305 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 11.705 & 0.000 & 1.000 & 0.000 & -0.750 & 0.600 \\ 0.000 & 0.003 & 0.000 & 0.015 & 0.000 & 0.000 \end{pmatrix}$$

3.3.2 Closing the Open Economy

What distinguishes a closed economy from an open economy is the impossibility of the former to borrow or lend in international financial markets. This is the same as saying that b_t is always equal to zero; alternatively, we can think that $b_{t+1} - b_t = 0$. In the latter case we can redefine the production function as:

$$e^{z_t} \tilde{A} k_t^\alpha n_t^{1-\alpha} \quad (3.8)$$

where $\tilde{A} \equiv A + b r^*$.⁹

The central planner of the closed economy wants to maximize the same expected utility as in eq. (3.3). The maximization is now subject to the following constraints:

$$e^{z_t} \tilde{A} k_t^\alpha n_t^{1-\alpha} \geq c_t + i_t \quad (3.9a)$$

⁹This specification of the production function facilitates the comparison between the models. $b r^*$ is now a parameter.

$$i_t = k_{t+1} - k_t(1 - \delta) \quad (3.9b)$$

$$z_{t+1} = \rho^z z_t + \epsilon_{t+1}^z \quad (3.9c)$$

$$(k_0, z_0) \quad \text{given} \quad (3.9d)$$

Here, the first order conditions are:

$$\frac{1 - \gamma}{\gamma} \frac{c_t}{1 - n_t} = (1 - \alpha) e^{z_t} \tilde{A} k_t^\alpha n_t^{1-\alpha} \quad (3.10a)$$

$$MUC_t = \beta E_t \left[MUC_{t+1} \left(1 + \alpha e^{z_{t+1}} \tilde{A} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} - \delta \right) \right] \quad (3.10b)$$

$$e^{z_t} \tilde{A} k_t^\alpha n_t^{1-\alpha} = c_t + k_{t+1} - k_t(1 - \delta) \quad (3.10c)$$

And the limiting condition is:

$$\lim_{t \rightarrow \infty} \beta^t E_0 MUC_t k_t = 0$$

Categorization of Variables and Equations

The predetermined variables in the closed-economy model are k_t and z_t . c_t and n_t are the nonpredetermined variables and the flow variables are output and investment as they were described in eqs. (3.6a) and (3.6b). Eqs. (3.10a) and (3.10b) are the optimality conditions and eq. (3.10c) is the budget constraint or the good market clearing condition.

Calibration and Log-Linearization

We start assuming again that $c/y=0.75$; $n=0.2$; $\alpha=.4$; and $\delta=0.02$; and we solve the equations characterizing the non-stochastic steady state for k , β , and γ . These

equations are:

$$\frac{1-\gamma}{\gamma} \frac{c}{1-n} = (1-\alpha) \frac{y}{n} \quad (3.11a)$$

$$1 = \beta \left(1 + \alpha \frac{y}{k} - \delta \right) \quad (3.11b)$$

$$y = c + k \delta \quad (3.11c)$$

and they imply the same value of γ , β , and k as in the open-economy model. The production function in eq. (3.8) implies that $\tilde{A}=0.9714 < 0.9875 = A$. We are going to chose the statistical properties of the innovations to productivity shocks to facilitate the comparisons between the closed and open economy. Likewise, $\rho^z=0.95$, and $\sigma=2$, as before.

Using the toolbox to log-linearize the closed-economy model, we obtain the following four matrices of coefficients:

$$A = \begin{pmatrix} 0.000 & 0.000 & 0.000 & 0.000 \\ -0.008 & 0.014 & -0.499 & 0.085 \\ 11.536 & 0.000 & 0.000 & 0.000 \\ 0.000 & 1.000 & 0.000 & 0.000 \end{pmatrix}$$

$$B = \begin{pmatrix} -1.200 & -3.000 & 3.000 & 1.950 \\ 0.000 & 0.000 & -0.499 & 0.077 \\ 11.686 & 1.000 & -0.750 & 0.600 \\ 0.000 & 0.950 & 0.000 & 0.000 \end{pmatrix}$$

$$C = \begin{pmatrix} 0.000 & 0.000 & 0.000 & 0.000 \\ 11.536 & 0.000 & 0.000.0 & 000 \end{pmatrix}$$

$$D = \begin{pmatrix} 0.400 & 1.000 & 0.000 & 0.600 \\ -11.286 & 0.000 & 0.000 & 0.000 \end{pmatrix}$$

3.4 The Theoretical Solution to the Model

A theoretical solution for models like the one specified above may be found applying Klein (1999)'s method. A solution can be stated as a pair of functions, one providing the law of motion of the state (or backward looking) variables and another giving the optimal policy rule (the optimal decision on forward looking variables). The first maps the state space into itself and the second maps the state space into the set of optimal policy functions. Therefore, we are solving for the recursive representation of the stable solution to a system of linear difference equations.

Klein's method is based on the Schur decomposition of the square matrices A and B . This decomposition gives the square complex matrices Q , S , T , and Z such that:

$$A = QSZ^H \quad \text{and} \quad B = QTZ^H$$

where Z^H denotes the transpose of the complex conjugate of Z . Q and Z are unitary matrices, that is $Q^H Q = Z^H Z = I$, and S and T are upper triangular.

Example. If the reader does not feel comfortable with the statement above, notice that it takes just a Matlab call to get the Schur decomposition of two square matrices. Let us

consider the following numerical example. Be

$$A = \begin{pmatrix} 0 & 3 & 2 \\ 5 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 9 & 4 & 0 \\ 3 & 5 & 3 \\ 7 & 7 & 1 \end{pmatrix}$$

Then writing $[S, T, Q, Z] = \text{qz}(A, B)$, Matlab returns the following S , T , Q , and Z matrices.¹⁰

$$S = \begin{pmatrix} 1.3603 & 2.2202 & 3.7181 \\ 0 & 3.6974 & -0.8787 \\ -0.0000 & 0.0000 & 1.9883 \end{pmatrix}; \quad T = \begin{pmatrix} 1.3603 & -0.8365 & 5.3271 \\ 0 & -4.2582 & -5.9526 \\ 0 & 0 & 12.4301 \end{pmatrix};$$

$$Q = \begin{pmatrix} 0.3409 & 0.8863 & -0.3136 \\ -0.9291 & 0.3684 & 0.0313 \\ 0.1433 & 0.2807 & 0.9490 \end{pmatrix}; \quad Z = \begin{pmatrix} 0.2411 & 0.6660 & 0.7059 \\ -0.4266 & -0.5806 & 0.6935 \\ 0.8717 & -0.4683 & 0.1441 \end{pmatrix};$$

The generalized eigenvalues of the matrices A and B are equal to the i th diagonal element of T divided by the i th diagonal element of S . When the matrix A is singular, some of the generalized eigenvalues are infinite. Let us define as stable generalized eigenvalues those that are less than one. The unstable are those larger or equal to one, including infinite values.

The decomposition can be reordered without altering the result. The reordering made here is such that the block of stable generalized eigenvalues come first.

¹⁰In fact, the decomposition that Matlab makes is such that $A = Q^H S T^H$.

Now, define the auxiliary variable y as $y=Z^Hx$, so

$$y(t) = \begin{pmatrix} y^s(t) \\ y^u(t) \end{pmatrix} = Z^H \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \quad (3.12)$$

By virtue of the Schur decomposition, premultiplying both sides of (3.1) by Q^H gives:

$$SE_tZ^Hx(t+1) = TZ^Hx(t)$$

which, employing the auxiliary variables defined in (3.12) becomes:

$$SE_t y(t+1) = Ty(t)$$

A partition of the matrices S and T conformably with y^s and y^u , is:

$$\begin{pmatrix} S_{x_1x_1} & S_{x_1x_2} \\ 0 & S_{x_2x_2} \end{pmatrix} E_t \begin{pmatrix} y^s(t+1) \\ y^u(t+1) \end{pmatrix} = \begin{pmatrix} T_{x_1x_1} & T_{x_1x_2} \\ 0 & T_{x_2x_2} \end{pmatrix} \begin{pmatrix} y^s(t) \\ y^u(t) \end{pmatrix} \quad (3.13)$$

Notice that the second difference equation contains the unstable roots due to the reordering of the eigenvalues mentioned above. Therefore, a stable solution for that equation requires $y^u(t)=0$ for all t . Thus a linear dependency arises among the components of the vector $y^u(t)$. On the other hand, the remaining equations in (3.13) can be written as:

$$E_t y^s(t+1) = S_{x_1x_1}^{-1} T_{x_1x_1} y^s(t) \quad (3.14)$$

where it can be shown that $S_{x_1x_1}^{-1}$ exists. This is because S is a triangular matrix that in turn was reordered in such a way that none of its elements in the diagonal are zero (otherwise a unstable generalized eigenvalue would have arisen and its correspondent block could not be part of the upper right block of S). Hence the determinant of a triangular matrix is equal to the product of the elements in the diagonal, which are all non zero in the matrix S .

When equation (3.12), is premultiplied by Z , we have that $Z Z^H x(t) = x(t) = Z y(t)$ because $Z Z^H = I$. Thus, making a partition of Z conformably with those of x 's and y 's:

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{pmatrix} Z_{x_1 y_s} & Z_{x_1 y_u} \\ Z_{x_2 y_s} & Z_{x_2 y_u} \end{pmatrix} \begin{pmatrix} y^s(t) \\ y^u(t) \end{pmatrix} = \begin{pmatrix} Z_{x_1 y_s} \\ Z_{x_2 y_s} \end{pmatrix} y^s(t) \quad (3.15)$$

The second equality arises because $y^u(t) = 0$. The first of the equations in (3.15) says that $x_1(t) = Z_{x_1 y_s} y^s(t)$. Recalling that the value of $x_1(0)$ is given because it is part of the initial conditions of the problem at hand, one can solve for $y^s(0)$:

$$y^s(0) = Z_{x_1 y_s}^{-1} x_1(0)$$

provided that $Z_{x_1 y_s}$ is invertible. It can be shown that the invertibility is assured when the number of predetermined variables (rows in $Z_{x_1 y_s}$) equals the number of stable roots (columns in $Z_{x_1 y_s}$).

Some of the backward looking variables are affected by innovations in the exogenous process and some others are not. For instance while the next period stock of financial assets may be affected by changes in the interest rate, the stock of capital will be equal to the existent stock plus the net investment undergone this period. Hence we may write $x_1(t+1) = E_t x_1(t+1) + \epsilon_1(t+1)$, with some of the components of $\epsilon_1(t+1)$ equal to zero. On the other hand, after observing that (3.15) implies that:

$$x_1(t) = Z_{x_1 y_s} y^s(t)$$

$x_1(t+1) = E_t x_1(t+1) + \epsilon_1(t+1)$ can be written as:

$$Z_{x_1 y_s} (y^s(t+1) - E_t y^s(t+1)) = \epsilon_1(t+1)$$

So:

$$y^s(t+1) = E_t y^s(t+1) + Z_{x_1 y_s}^{-1} \epsilon_1(t+1)$$

This is not a final solution for $y^s(t+1)$ because its expected value as of time t is also on the right hand side of the equation. But expression (3.14) may then be used to give:

$$y^s(t+1) = S_{x_1 x_1}^{-1} T_{x_1 x_1} y^s(t) + Z_{x_1 y_t^s}^{-1} \epsilon_1(t+1) \quad (3.16)$$

It remains to get the solution in terms of the original variables in $x_1(t)$, something straightforward appealing to the definition of the auxiliary variable $y(t)$ and the partition of Z made above (see equations [3.12] and [3.15]):

$$x_1(t+1) = Z_{x_1 y_s} y^s(t+1) = Z_{x_1 y_s} S_{x_1 x_1}^{-1} T_{x_1 x_1} Z_{x_1 y_s}^{-1} x_1(t) + \epsilon_1(t+1) \quad (3.17)$$

Now, in order to solve for $x_2(t)$, notice that (3.15) implies that:

$$x_2(t) = Z_{x_2 y_s} y^s(t)$$

and again, since $x_1(t) = Z_{x_1 y_s} y^s(t)$,

$$x_2(t) = Z_{x_2 y_s} Z_{x_1 y_s}^{-1} x_1(t) \quad (3.18)$$

To establish a direct link between these results and the matrices contained in the toolbox, define,

$$p \equiv Z_{x_1 y_s} S_{x_1 x_1}^{-1} T_{x_1 x_1} Z_{x_1 y_s}^{-1} \quad (3.19)$$

$$f \equiv Z_{x_2 y_s} Z_{x_1 y_s}^{-1} \quad (3.20)$$

Thus, (3.19) and (3.20) completely describe the evolution of the system once the initial

conditions and the shocks hitting the economy are specified. Particularly, (3.19) is the state transition equation since it governs the evolution of the state variables in the model. Likewise, (3.20) represents the policy function or decision rule and it maps the state of the economy into the forward looking variables. Remember that the initial conditions are specified in $x_1(0)$. Then (3.20) gives the value of $x_2(0)$ and (3.19) gives the value of the backward looking variables at $t + 1$ for a particular shock $\epsilon_1(1)$. Thus, new predetermined values for the variables in x_1 are stated for the next period. At $t=1$ then we can obtain $x_2(1)$ using (3.20) and $x_1(2)$ using (3.19), and so on.

The value of the additional variables is straightforward to get at this point. Making a partition of matrices C and D and the vector x at times t and $t + 1$ in eq. (3.2),

$$x_3(t) = \begin{pmatrix} C_1 C_2 \end{pmatrix} \begin{pmatrix} x_1(t+1) \\ x_2(t+1) \end{pmatrix} + \begin{pmatrix} D_1 D_2 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \quad (3.21)$$

where both C_1 and D_1 are of dimension $n_3 \times n_1$ while both C_2 and D_2 are of dimension $n_3 \times n_2$. Notice that the elements of C_2 are all zeros in most of the cases. Also recalling that $x_1(t+1) = p x_1(t)$ and that $x_2(t) = f x_1(t)$, (3.21) can be re-written as:

$$x_3(t) = (C_1 p + D_1 + D_2 f) x_1(t) + C_1 \epsilon_1(t+1) \quad (3.22)$$

The following notation is used in the toolbox:

$$g \equiv C_1 p + D_1 + D_2 f \quad (3.23)$$

$$h \equiv C_1 p \quad (3.24)$$

3.5 The Toolbox of Matlab Functions

In this section, we explain what each Matlab function in the toolbox does. We recommend reading this section keeping an eye on the Matlab functions mentioned here.¹¹ The Matlab functions in the toolbox can be grouped into three subsets, which are coordinated by the function **Master**. The first subset of functions performs the log-linearization. The second subset provides the solution to the system, that is the two fundamental equations summarized in (3.19) and (3.20). Additionally, the set of coefficients in (3.23) and (3.24) which are used to follow up the evolution of the flow variables are obtained. The functions in this subset are the core of the toolbox. The third subset of Matlab functions, provides the researcher with the tools for plotting impulse response functions, computing business cycles statistics, and simulating the economy under a particular sequence of innovations to the exogenous predetermined variables. Table 3.1 shows the functions included in each of the sets above. The functions that should be modified by the researcher with information on his model are underlined.

In order to run the functions in the toolbox the researcher must save all the files that accompany these notes and whose names coincide with the functions in the Table, in a directory.

In designing the toolbox we preferred to minimize the number of function calls as well as the steps involved in solving a model. Notwithstanding, there are several elements of the toolbox that are model-specific and have to be modified by the researcher according to the specifications of his model. We explain here what he has to modify in the files above. We have limited the number of function calls to just one, the function **MASTER**. There, the user has to indicate details of the model under study and the desired procedures. This minimization of function calls has the advantage of avoiding the researcher to keep track of the order in which several functions should be called.

The researcher should provide the following in-file inputs to **MASTER.m** following

¹¹The functions cab be downloaded from the following website:
www.econ.iastate.edu/faculty/oviedo.

the instructions contained in the function:

1. **Number of each type of variable.** The sorting of variables commented before should be declared here. The researcher should indicate the number of backward looking variables (including both the endogenous and the exogenous), n_1 ; the number of forward looking variables, n_2 ; the number of exogenous variables among the backward looking ones, n_e ; and the number of additional or flow variables, n_3 .
2. **Analysis to be Performed.** *DesProc* is a binary (1×3) vector, with 1's indicating a desired procedure and 0's otherwise. For example, setting *DesProc*=(1,0,0) the toolbox returns the impulse response functions of his model; setting just the second component equal to one the toolbox returns the population moment statistics; and setting just the third component equal to one the toolbox performs the simulation with actual shocks.
3. **Shocks Hitting the Economy.** Vector *shock* indicates what shock or shocks are hitting the economy. Shock's size is $(1 \times n_e)$ and its ordering matters. The first element of *shock* corresponds to the first exogenous state and so on. It is also a binary vector and ones are for variables being shocked.
4. **Statistical Properties of the Shocks.** *StdDev* is the variance covariance matrix of the innovations to the exogenous predetermined variables in the model. It is a square matrix of size n_e . Attention should be paid here to the order in which the standard deviations are entered. Again, the upper-left element of *StdDev* is the variance of the innovations to the first exogenous state variable, and so on.
5. **Location of the Key Variable.** A row vector *kvar* is used to compute the statistics of the economy, either the population moment conditions or the sample moment conditions. *kvar* is a row vector of size 2. It indicates the position of the variable against which the researcher would like to compare the evolution of all variables in his model. In most macroeconomic models, output is the key

variable since the relative volatilities and correlations of all other variables are computed with respect to that of output. The specification is made indicating the type of variable and the position in the respective subset. The first column takes values from 1 to 3; 1 is for backward looking variables, 2 for forward looking variables, and 3 for additional or flow variables. The second column of *kvar* indicates the position of the variable in the respective subset. Thus, if the key variable of a model is the second flow variable, *kvar*=[3,2].

6. **Files.** Finally, **MASTER** asks the user to indicate the name of a directory. This is where results are going to be stored and/or where the functions must read some information.

When simulations with actual shocks are requested, the user should previously provide that sequence of shocks in a **.wk1** file. The spreadsheet in that file has to be filled in the following way. There will be a number of columns equal to the number of exogenous variables and the columns are ordered according to the order these variables have in the log-linearization. The upper-left cell with data is *B2*; the second column of innovations starts at *C2* and so on.

3.5.1 Log-linearizing the Model

Here we explain the steps involved in log-linearizing a dynamic macroeconomic model using the toolbox. Of the three functions in this subset, **st_st.m** and **eqns.m** are mostly model specific and several parts should be rewritten by the researcher according to his own model. **st_st.m** takes the value of parameters and variables that are known and return the value of all parameters and variables in the model. The user may want to check the consistency of the value of parameters and variables. He can write all the equations in his model to see whether or not they are satisfied at the non-stochastic steady state. **st_st.m** is called without any input and its output is a vector *X* which contains the value of the variables at the non-stochastic steady state, and **par**, a vector containing all parameter values.

The actual file `st_st.m` in the toolbox corresponds to the example of the open economy developed above. The file `st_st2.m`, along with all `NameVars2.m`, `eqns2.m`, and `flows.m` are for the closed economy example. Make a permutation of file name to run the example of a closed economy.¹²

The role of the function `call_lin.m` is to call the function `eqns.m` twice, which has the model equations, and twice the function `flows.m`, which has the equations defining the flow variables. `call_lin.m` is not model specific and should not be modified by the user.

Inside `eqns.m` the user has to write the equations of his own model. The inputs are the vectors X and `par` obtained using `st_st.m` and two additional arguments whose functions consist in telling the function whether the matrix A or B is required, whether the log-linearization or a differentiation of the system is required. On the other hand, the function `flows.m` also has to be written for each model. When writing his model equations (in both `eqns.m` and `flows.m`), the user first has to read the parameters and variables in his model where it is specified. Notice that the command `repmat` used in this function has converted the vector X into a matrix where its columns have been repeated. The user should be aware of the places in which the variables enter into vector X because all computations are made assuming the researcher keeps a specific ordering. Concretely, the vectors of variables $x(t)$ and $x(t + 1)$ *must* be ordered such that endogenous states come first, then exogenous states, and finally all the remaining variables. Also, it is convenient to order the model equations in the following way. First, write the dynamic Euler equations; next, other dynamic equations with those governing the dynamics of the exogenous state variables last. And then all the remaining equations.

3.5.2 The Core Functions

The function `MASTER` thus receives all the information about the model and the desired procedures. `MASTER` will call, first, the files necessary to compute p and f [see

¹²The permutation of names is such that when the programs call the function “eqns.m” for example, it should be calling the file that corresponds to the example under study.

eqs. (3.19) and (3.20)] and also g and h [see eq. (3.23) and (3.24)]; then it calls the functions necessary to perform the requested procedures.

The former step is completed calling the functions `rec_repres`, which in turn calls `reorder`, `qzswitch` and the Matlab built-in function `qz`.¹³ `rec_repres`'s output are the functions p to h . `reorder` checks, at pairs and starting from the top, whether or not the generalized eigenvalues are arranged in an ascending order. If they are, then it checks the next pair and so on. If they are not, it then calls `qzswitch`, which in turn completes the desired ordering.

3.5.3 The Procedure Functions

Procedures is a function that reads what the user wants to do once the model has been solved and then it calls the functions required accordingly.

NameVars. A file of the toolbox is called `NameVars` and it must be modified for each model. Following the instructions there, the user must write a name for each of the variables in his model. These names are employed as headings of graphs and tables. The order in which the variable names are sorted should be the same used in the log-linearization. Then, start writing the name of all backward looking variables, then those of all forward looking variables and finally those of the flow or additional variables.

After computing the matrices p , f , g , and h `MASTER.m` can carry out any of the three computation, namely the impulse response functions (calling `impres.m`), obtain the population moment conditions (calling `moments1.m`) and the dynamic exercise under actual shocks (calling `actual_shocks.m`). The main input of these three functions are the matrices p to h . After executing `actual_shocks`, `moments2` compute the sample moment conditions of the variables. Next, we review what each of the mentioned functions does.

The toolbox employ the functions `rec_repres.m`, `qzswitch` and `reorder` to obtain these four matrices. They are then passed to the function `procedures`, which is

¹³`qzswitch` is a function written by Christopher Sims and the toolbox's version of it has minor changes in its notation and documentation.

going to control the output desired by the researcher.

impres.m computes the impulse response functions arising from shocking one or more of the exogenous variables in the system. It is assumed that the impulse is equal to 1% with respect to the steady state value of the variable being shocked. Briefly, what **impres** does is to iterate $x_1(t+1) = px_1(t)$ for the state variables, $x_2(t) = fx_1(t)$ for the control variables, and $x_3(t) = gx_1(t)$ for the additional variables.

Fixing $DesProc = [100]$ in the function **Master**, the researcher obtains a matrix X of impulse response functions. There is a column for each variable and a row per time period. The order of the columns of X have first the backward looking variables, then the forward looking variables, and last the flow variables. A graph per each of the variables of the model is produced and saved in the file indicated in the function **graph** (see line 17).

An important note follows here. In order for the user to save the files in the indicated directory, the utility **PRTFIGS** in Fackler and Miranda (2001) should be in your directory. As it is explained there you must look for a Matlab file called **startupsav.m** and add the following line there: `set(0,'DefaultAxesFontName','B')`. Without this modification the user may obtain an error message and not get the figures.

moments1.m is the function used to calculate the population moment conditions. It reads the variance covariance matrix created in **MASTER** along with the functions p, f, g , and h and gives the standard deviation (or the absolute volatility) of the variables, along with the relative volatility (measured with respect to that of the key variable) and several correlations. The result is a table that will be saved in your directory called **TableBCstats.out**. Each time the researcher requests population moment conditions or to simulate a specific sequence of shocks, the mentioned file is overwritten. The file **TableBCstats.out** can be opened with the Matlab editor.

Setting $DesProc = [0, 1, 0]$, the toolbox calculates the population moment conditions of the variables. Now X is a structure containing those moment conditions. A file containing a table of results is saved in the file specified in **tableBCstats** (see

line 22). To obtain the results in numerical form, write `X.moment`, where `moment` can be any of the following arguments. `SD`: standard deviations; `RV`: ratio of the standard deviation of each of the variables to the standard deviation of the key variable; `Corr`: correlation between each of the variables and the key variable; `Autocorr`: correlation between each of the variables dated at $t + 1$ and the key variable at t ; `AutoCorr`: autocorrelation of each variable.

actual shocks reads the shocks from the directory and then computes the same statistics as with `moments1.m`, but now they are moments of a sample and not population moments. The statistics are also reported in the file `TableBCstats.out` and remember that the file is modified each time the user run either `actual_shocks` or `moments`. User provided shocks should be saved in a file with extension `.wk1` and named `shocks`.

When the researcher sets $DesProc = [001]$ the researcher obtains simulations and model sample moments arising from the sequence of actual shocks provided by him. The result X returned by the function `Master` has two components. One, $X.Data$, is a matrix similar to the matrix of impulse response described above. The other, $X.Moments$, corresponds to the numerical values of the sample moments conditions. $X.Data$ has a column per variable and row per observation. $X.Moment$ contains the same moments as above and the calling is now $X.Moments.moment$ where `moment` is one of the arguments described in the precedent paragraph.

3.6 The Toolbox at Work

In this section, we show how the toolbox can be used to solve and study a dynamic macroeconomic model of the type described in section 3.2. We do so using the models of section 3.3. Since the log-linearization of these models was shown in subsections 3.3.1 and 3.3.2, we concentrate here on the utility of the Core and Output Functions (see Table).

With the information provided by the researcher, the file `Master` calls the function

`rec_repres` and four matrices are returned: a) The transition function summarized by matrix p which gives the expected value of the predetermined variables one period forward (see eq. (3.19); b) The policy function that prescribes the optimal decisions regarding the forward looking variables. This function is summarized by matrix f (see eq. (3.20); c) The two matrices g and h in eqs. (3.23) and (3.24) track the flow variables' dynamics.

In the open economy example, these four matrices are:

$$p = \begin{pmatrix} 0.654 & 0.009 & 1.226 & -0.266 \\ -3.199 & 1.079 & 9.992 & -2.407 \\ 0.000 & 0.000 & 0.950 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.870 \end{pmatrix}$$

$$f = \begin{pmatrix} 0.198 & -0.011 & 0.542 & -0.068 \\ 0.311 & 0.017 & 0.705 & 0.105 \end{pmatrix}$$

$$g = \begin{pmatrix} 0.586 & 0.010 & 1.423 & 0.063 \\ -3.765 & 0.099 & 14.143 & -3.068 \\ 4.203 & -0.080 & -13.127 & 3.182 \\ 0.000 & 0.003 & 0.000 & 0.015 \end{pmatrix}$$

$$h = \begin{pmatrix} 0.000 & 0.000 & 0.000 & 0.000 \\ 7.540 & 0.099 & 14.143 & -3.068 \\ -7.540 & -0.099 & -14.143 & 3.068 \\ 0.000 & 0.000 & 0.000 & 0.000 \end{pmatrix}$$

In the closed economy example, the matrices f , p , g , and h are the following.

$$f = \begin{pmatrix} 0.544 & 0.491 \\ -0.221 & 0.783 \end{pmatrix} \quad p = \begin{pmatrix} 0.966 & 0.096 \\ 0.000 & 0.950 \end{pmatrix}$$

$$g = \begin{pmatrix} 0.267 & 1.470 \\ -0.141 & 1.102 \end{pmatrix} h = \begin{pmatrix} 0.000 & 0.000 \\ 11.536 & 0.000 \end{pmatrix}$$

Before showing the results of the procedures performed with the functions in the toolbox, we are going to discuss the ordering of equations and variables. Equations in both `eqns.m` and `flows.m` were ordered as they were introduced in the text. In our `eqns.m` of the open economy example, we wrote first the marginal rate of substitution of labor for consumption, then the optimal condition for the accumulation of capital; the the optimal condition for international assets accumulation; and finally the budget constraint. The ordering of the variables in the open economy example is: k , b , ϵ^z , ϵ^r , r , c , n , y , i , tb , and r .

3.6.1 Business Cycles in the The Open and Closed Economy

Figures 3.1 to 3.9 show the impulse response functions that follow a 1% increase in the domestic productivity shock in the open economy.

To calculate the population moment conditions we have first set the standard deviation of the interest rate equal to zero and the standard deviation of the innovations to productivity shocks so that the standard deviation of output is equal to 4%. Under this setting, the population moment conditions are described in Table 3.3. We then add a shock to the international interest rate. We assume that the international interest rate has a standard deviation equal to 2%. The results are contained in Table 3.4.

To compare the previous results with those of a closed economy, we we compute the business cycle statistics of the closed economy under the same type of productivity shocks that hit the open economy. The results are in Table 3.5.

Figure 3.1: Impulse Response Function of the Productivity Shock in the Open Economy

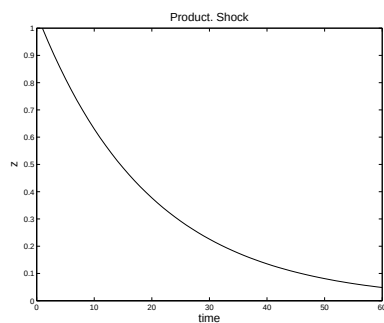


Figure 3.2: Impulse Response Function of the Domestic Interest Rate Following a Productivity Shock in the Open Economy

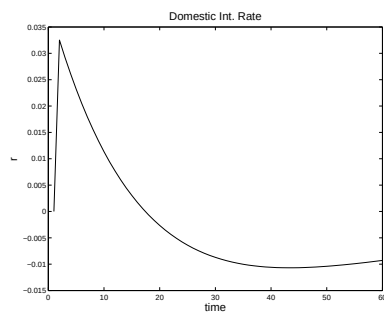


Figure 3.3: Impulse Response Function of the Stock of Capital Following a Productivity Shock in the Open Economy

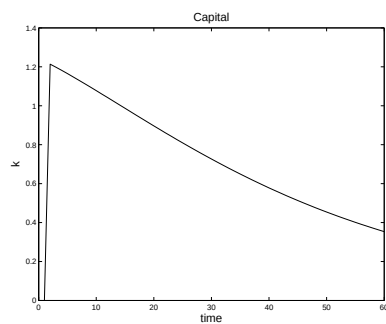


Figure 3.4: Impulse Response Function of the Trade Balance Following a Productivity Shock in the Open Economy

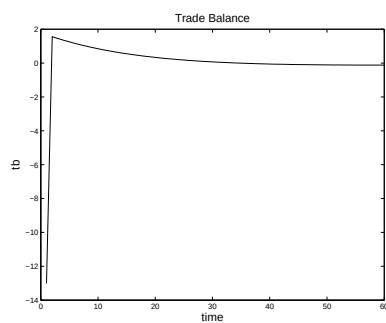


Figure 3.5: Impulse Response Function of International Financial Assets Following a Productivity Shock in the Open Economy

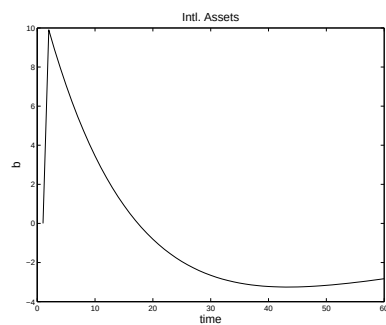


Figure 3.6: Impulse Response Function of Consumption Following a Productivity Shock in the Open Economy

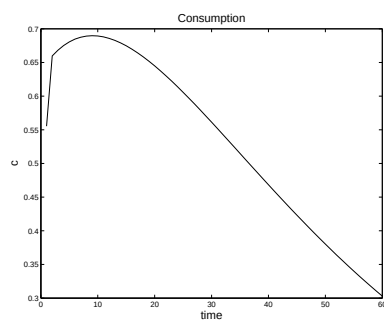


Figure 3.7: Impulse Response Function of Labor Following a Productivity Shock in the Open Economy

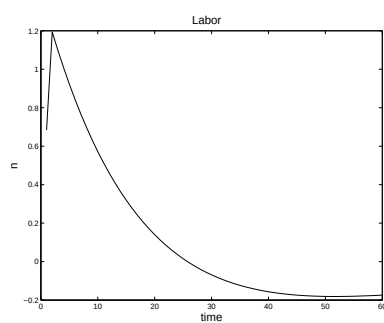


Figure 3.8: Impulse Response Function of Output Following a Productivity Shock in the Open Economy

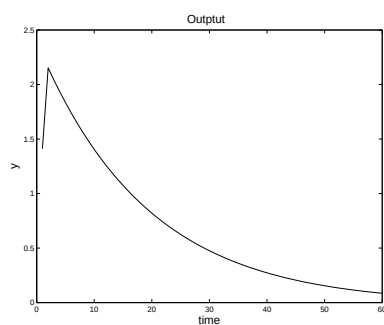


Figure 3.9: Impulse Response Function of Investment Following a Productivity Shock in the Open Economy

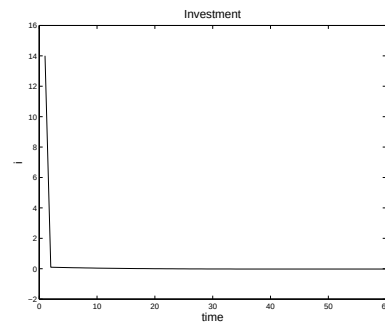


Table 3.1: Categorization of Variables and Equations in LDRE Models

Predetermined Variables		Non-Predetermined Variables	Flow Variables
Endogenous	Exogenous		
k_t, b_t	z_t, r_t^*	c_t, n_t	y_t, i_t, tb_t, r_t

Table 3.2: Matlab Functions to the Study of LDRE Models

Log Linearization	Core Functions	Output Functions
st_st.m call_lin.m eqns.m flows.m	rec_repres.m qzswitch.m reorder.m	procedures.m NameVars.m impres.m actual_shocks.m tableBCstats.m graph.m moments1.m moments2.m prtfigs.m shokcs.wk1

Table 3.3: Population Moment Conditions: Productivity Shocks in the Open Economy

Variable	St.Dev. (percent)	Relative Volatil.	Correlation key(t) with x()				
			t	t+1	t+2	t+3	t+4
k	3.604	0.901	0.882	0.907	0.891	0.875	0.859
b	17.92	4.480	0.385	0.365	0.279	0.200	0.126
z	1.859	0.465	0.993	0.944	0.896	0.852	0.809
rs	0.000	0.000	0.665	0.697	0.716	0.719	0.708
c	2.588	0.647	0.836	0.837	0.833	0.828	0.822
n	1.857	0.464	0.791	0.741	0.658	0.581	0.508
y	4.000	1.000	1.000	0.972	0.921	0.873	0.827
i	8.130	2.032	0.216	0.011	0.009	0.008	0.006
tb	7.925	1.981	0.079	0.274	0.251	0.230	0.210
r	0.059	0.015	0.385	0.365	0.279	0.200	0.126

Table 3.4: Population Moment Conditions: Productivity and Interest-Rate Shocks in the Open Economy

Variable	St.Dev. (percent)	Relative Volatil.	Correlation key(t) with x()				
			t	t+1	t+2	t+3	t+4
k	3.657	0.910	0.881	0.903	0.886	0.869	0.852
b	18.66	4.646	0.394	0.371	0.286	0.207	0.134
z	1.859	0.463	0.989	0.940	0.893	0.848	0.806
rs	2.000	0.498	-0.078	-0.068	-0.059	-0.051	-0.045
c	2.591	0.645	0.835	0.836	0.832	0.826	0.820
n	1.870	0.466	0.793	0.742	0.658	0.581	0.508
y	4.018	1.000	1.000	0.971	0.919	0.871	0.825
i	8.860	2.205	0.187	0.003	0.003	0.002	0.001
tb	8.720	2.170	0.085	0.257	0.235	0.215	0.196
r	0.061	0.015	0.357	0.339	0.258	0.183	0.113

Table 3.5: Population Moment Conditions: Productivity Shocks in the Closed Economy

Variable	St.Dev. (percent)	Relative Volatil.	Correlation key(t) with x()				
			t	t+1	t+2	t+3	t+4
k	3.324	0.991	0.768	0.794	0.817	0.836	0.853
z	1.859	0.554	0.978	0.929	0.883	0.839	0.797
c	2.478	0.739	0.920	0.922	0.921	0.919	0.916
n	1.157	0.345	0.743	0.665	0.592	0.524	0.460
y	3.355	1.000	1.000	0.967	0.935	0.904	0.875
i	1.797	0.536	0.915	0.852	0.793	0.738	0.686

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