ABSTRACT

MERRITT, RONALD L., JR. The Effect of Concept Mapping on Community College Precalculus Students' Conceptual Understanding of Inverse Functions. (Under the direction of William M. Waters, Jr.)

The purpose of this study was to investigate the efficacy of concept mapping on community college precalculus students' conceptual understanding of inverse functions. This study employed a quasi-experimental nonequivalent control group design in which a single instructor taught one experimental precalculus algebra class and one control precalculus algebra class. Students in the experimental group (**n** = 15) participated in one collaborative "System of Equations" concept mapping exercise. These students also individually constructed maps given the seed concepts "Inverse" and "Functional Inverse." Other than the concept mapping treatment, all assignments, assessments and instruction were equivalent for the experimental and control groups (**n** = 21). The duration of the experiment was about 12 weeks.

Three veteran mathematics community college instructors and two professors of mathematics education from a local university collaborated to create criterion maps for this study. The Markham, Mintzes and Jones' rubric for scoring science-oriented concept maps and these

criterion maps were used to quantify students' individual maps. Quantification of the maps relied on seven components: concept, link, hierarchy, initial branching, successive branching, crosslink, and example. Other data collected for analysis in this experiment includes pretest diagnostic scores, unit test scores and selected subscores, a routine writing assignment score, final examination subscore, and a variety of demographic data. ANOVA and a Backward Elimination model ($\alpha = .05$) revealed that the inverse function map score is significant and contributes to significant variation in the final course grade. However, distribution-free and independent non-equivalent t-tests disclose very few significant differences between the two groups for the duration of the course. Qualitative analyses of the (1) mathematics instructors and professors surveys on concept mapping usefulness, (2) system of equations and inverse function maps, and (3) the follow-up survey provided further evidence that concept mapping supports the *NCTM* and *AMATYC Standards*.

THE EFFECT OF CONCEPT MAPPING ON COMMUNITY COLLEGE PRECALCULUS STUDENTS' CONCEPTUAL UNDERSTANDING OF INVERSE FUNCTIONS

by

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APPROVED BY:

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DEDICATION

Mr. Ronald L. Merritt, Sr.

In Memory of
Mrs. Beverly M. Merritt, Mrs. Addie M. Merritt,
and Mrs. Ann W. Jones

To the valiant deceased of September 11, 2001...
”One nation under God”

BIOGRAPHY

Ronald Lee Merritt, Jr. (Ronnie) was born to Ronald Lee Merritt, Sr. and Beverly Brownstein Merritt in Los Angeles, California on May 29, 1968. He received a Bachelor of Science Degree in Mathematics and Secondary Mathematics Education at Greensboro College in May of 1989 and a Master of Science Degree in Mathematics at North Carolina Central University in August of 1996. Ronald Merritt entered the Ph.D. program in Mathematics Education at North Carolina State University in 1997, where he focused on heuristics that strengthen mathematics conceptual understanding.

Ronald Merritt has served as an adjunct and full-time faculty member at Durham Technical Community College for 8 years in the departments of mathematics and developmental education. While there, the faculty and staff of the college recognized him with the coveted Excellence in Teaching Award for 2001-2002. In August 2002, he will begin serving the students at Lee University as Assistant Professor of Mathematics.

Ronnie serves the Lord, and thus, he participates in the music department as church pianist at the Hillsborough Church of God. He currently resides in Raleigh, North Carolina.

ACKNOWLEDGEMENTS

"The race is not given to the swift, but to him who endures to the end" (Eccl. 9:11, Matt. 10:22). This statement perfectly describes the dissertation creation exercise. Others' faith in me and my endurance caused this work to coalesce.

First, I offer my sincerest appreciation to the members of my dissertation advisory committee. I am honored to be the last doctoral student for whom Dr. William M. Waters, Jr. will serve as chairman of a doctoral advisory committee. I thank him for his insatiable sense of humor and his use of psychology that urged me to complete this work. Likewise, I extend thanks for the time, education and effort extended by Drs. Glenda S. Carter, Lee V. Stiff, and Ernest Stitzinger.

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Thirdly, I acknowledge my family members who endured my absence in many family activities during the last few years, so that I may complete this work. To my father, Ron; brothers, Clint and Jason; sisters-in-law, Melissa and Heather; nieces, Samantha and Meghan; and nephew, Scott, I offer thanks. Perhaps I will get to know the nieces and nephew better now.

Finally, I offer my highest praise to Mrs. Ann W. Jones (with the Lord), Mr. and Mrs. Joseph and Teresa Scott and the Glenwood-Pinecroft Church of God family, Rev. James Boyd and the Hillsborough Church of God family for their faith and encouragement during my doctoral studies. None of my college work would have been successful without the assistance of my grandma, Mrs. A. Mae Merritt, who helped me move into college in 1986. Yet, with all of these acknowledgments, none of this would have been possible without my Lord, Jehovah Jireh, The Lord our Provider.

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CHAPTER 1

Introduction

The Community College Student

Wood (2001) and Carlon and Byxbe (2000) reveal an astounding statistic of the 1996 report of the National Center for Educational Statistics: nearly 50% of the total undergraduate population in the United States enroll in credit courses at the community college. Fredrickson (1998), an administrative official with the North Carolina Community College System, describes the typical transfer student as a 26-year-old white woman who holds a part-time job and enrolls in college part-time. She also declares that 58.2% of all college transfer students attend community colleges part-time.

Wood (2001) reports that the National Science Foundation (NSF) estimates that during the 1990's, "Thirty-four percent of all course enrollments in science, mathematics, engineering and technology were found in the two-year colleges" (p. 363). Furthermore, Wood claims two-year colleges increasingly impact teacher preparation, especially relative to elementary and middle grades teachers. The community college at which this author teaches will implement mathematics pedagogy courses for the elementary grades starting in the fall of 2003.

Community College Precalculus Transfer Students

Students enrolled in a precalculus course at the community college are traditionally transfer students who require precalculus as a prerequisite for some higher level mathematics course; typically, an algebra-based calculus or a transcendental-based calculus course. McIntyre (1987) asserts that transfer education supersedes many of the functions of the community college, although its assessment remains difficult. Typical California transfer students take more than the traditional two years of lower division work prior to transferring. Yet, it has been the experience of this researcher that transfer students at his community college (in a central North Carolina urban area) transfer to senior area universities prior to completing a full two-year associate in science or associate in arts degree.

One subset of precalculus students consists of reverse transfer students. McIntyre defines a reverse transfer student as a student attending a community college who attended a four-year academic institution for at least one semester. A second subset of traditional precalculus students at the community college is the developmental mathematics graduates. These students take courses approximating the first two years of high school algebra. These courses are taken normally over a period of two semesters. A third subset of the traditional precalculus students at the community college consists of those who actually place into the course based on college placement testing.

Unfortunately, senior university educators criticize students who complete courses, such as precalculus or calculus at the community college (Carlon & Byxbe 2000). Carlon and Byxbe (2000) briefly discuss two major criticisms of lack of preparedness of these students at the university: lower aptitude and over-nurturing. The researchers cite opposing research dispelling these common myths. However, Carlon and Byxbe found that "transfer students who earn good grades at the community college appear to be well-prepared for senior college endeavors, except for those students entering business and science disciplines" (p. 33).

Crossroads in Mathematics (Cohen, et al 1995) identifies the categories from which the typical community college student emanates. The traditional student is older, is employed, returns to college after an interruption in education, intends on transferring to a four-year college or university, needs formal developmental work in one or more disciplines, studies part-time, or has no family history in post-secondary education (p. 4, Cohen, et al 1995). Clearly, the community college student's background differs from that of the traditional university freshman student's.

AMATYC Standards

However, standards for intellectual development, mathematical content, and mathematical pedagogy are quite commensurate with those of the traditional four-year college or university. The American Mathematical Association of Two-Year Colleges (AMATYC) Standards for Instructional Development promote making connections; communication through reading, writing, and listening to mathematics; and development of mathematical power by engaging students in nontrivial explorations in mathematics. Of the seven major standards for content, this report requires close scrutiny of standard 4. Standard 4 of the Standards for Mathematical Content states, "Students will demonstrate understanding of the concept of function by several means (verbally, numerically, graphically, and symbolically) and incorporate it as a central theme into their use of mathematics" (p. 13, Cohen et al 1995). Crossroads specifically catalogs the topics students study during the course of a two-year college precalculus curriculum:

1. Functions

   Linear, power, polynomial, rational, algebraic, logarithmic, trigonometric, and inverse trigonometric

2. Inverse and relationship to function
3. Parametric, polar, rectangular coordinate systems
4. Categorization of functions (families) and exploration of their properties
5. Analysis of functions: zeros, intervals over which functions increase or decrease, approximate extrema, describe end behavior
6. Manipulate and apply matrices, store data, represent graphs, solve linear systems of equations.

Recommendations for Precalculus Reform

Beginning in the late 1980's the Mathematical Association of America (MAA) encouraged calculus reform. Subsequently, the College Board reinforced the calculus reform by considering ways in which post-secondary precalculus might be reformed (Haruta, Tarpon & McGivney 1998). The College Board supports increased emphasis on realistic applications based on real data sets, the use of graphing utilities, collaborative and exploratory activities, and "extensive oral and written reporting about key concepts" (p.27). The author of this report complies with these four objectives within the framework of his precalculus and calculus instruction.

Research on Teaching and Learning Functions

In an effort the provide insights into how "high performing" precalculus students develop an understanding of the major aspects of the function concept, Carlson (1997) studied 30 students who had just completed a function-integrated college algebra course with a grade of A. Each took a 25-item exam developed to measure the students' conceptions of the roles of functions. Carlson discovered these students misunderstood functions, inaccurately interpreted graphs, and misapplied or failed to use function notation in problems with real-world relationships. In addition, these students conceived functions as a "sequence of memorized operations to be carried out" (p. 48). As a result of Carlson's research, she recommended precalculus instructors engage students in activities which

1. Develop the vocabulary for referencing and constructing aspects of both algebraic and graphic function representations,

2. Develop a view of functions as a process which accepts input and produces output (p. 57),
3. Extend work with new concepts to replace memorization with understanding,
4. Probe students' conceptions to make necessary adjustments in teaching.

The NSF supports a joint effort of two Virginia universities and two community colleges in their course revisions in mathematics and science. Among many objectives of the reformed courses, these mathematics and science courses

1. incorporate current teaching technology that enhances active student learning,
2. "create a sense of intellectual community fostered by small group collaborations and electronic means",
3. nurture student awareness of establishing connections (p. 364).

Since the mid 1990's, the NSF has been investing in the two-year colleges' involvement in the mathematics preparation of prospective teachers. Since 1995, the American Mathematical Association of Two-Year Colleges has pursued teacher preparation initiatives through discussions, workshops, sessions and teacher preparation symposia (Wood 2001).

Research on Teaching and Learning Inverse Functions

Mathematics education researchers devote very little to the concept of functional inverse. Vidakovic (1996) shares a limited number of conclusions gleaned form three researchers: Snapper, Flores and Zazkis. Snapper suggests explaining the concept of inverse function via interchanging the domain and range—hence, interchanging the x- and y-axes. Flores relies on obtaining an inverse function through a series of reversing operations. He claims students acquire an informal understanding of functional inverse by perfecting this reversal process. Zazkis reports that

1. Students fail to acknowledge their previous knowledge about inverses (of course, this is common across many topics and disciplines)
2. Students discover through trial-and-error that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

The second discovery, if made by precalculus students, would be marvelous.

Perhaps, some mathematics misconceptions transfer from preservice and veteran mathematics teachers. Even's paper (1992) on the prospective teacher's knowledge and comprehension of inverse functions reveals some of these misconceptions held by prospective teachers. Many prospective teachers habitually perform unnecessary calculations when responding to an item, such as

> "Given $f(x) = 2x + 10$ and $f^{-1}(x) = \frac{x+10}{2}$, find $\left(f^{-1} \circ f\right)(512.5)$. Explain."

(p. 558). Some preservice teachers responded correctly to the item, invoking both inverse properties and calculations. Few of the participants (approximately 20%) responded correctly using solely the inverse property. Unfortunately 53 of 152 either provided no solution or an incorrect solution. This reflects a worse problem than merely examining the participants' strengths in mathematics cognition relative to inverse functions. Clearly, these preservice teachers avoid properties they've learned and regress from the notion of "undoing".

Nevertheless, Even asserts that the notion of "undoing" is insufficient, especially with regard to exponential and power functions. For example, $f(x) = 4^x$ is an elementary exponential function whose inverse is $g(x) = \log_4 x$. A misconception students have is that $g_1(x) = x^{\frac{1}{4}}$ is the inverse function of f. Clearly,

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(\log_4 x) \\ &= 4^{\log_4 x} \\ &= x\end{aligned}$$

and

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(4^x) \\ &= \log_4 4^x \\ &= x \log_4 4 \\ &= x,\end{aligned}$$

yet,

$$\begin{aligned}(f \circ g_1)(x) &= f(g_1(x)) \\ &= f(x^{\frac{1}{4}}) \\ &= 4^{x^{\frac{1}{4}}} \\ &\neq x\end{aligned}$$

Obviously, if $x^{\frac{1}{4}}$ (rational power function), and x^4 (power function), and 4^x (exponential function) cause confusion among preservice mathematics teachers, then it is quite likely that the transfer of this confusion takes place between inservice teachers and students. Even concludes that prospective secondary teachers "did not understand the difference between exponential and power functions and thought that taking the logarithm and taking the root were the same thing" (Even 1992, p. 562). Furthermore, Even advocates that mathematics educators and mathematics teachers make better connections between procedural and conceptual knowledge.

CHAPTER 2

Concepts as a Foundation of Learning

Critics of mathematics and science educators claim students graduate from various public schools and institutions of higher learning seriously lacking critical thinking skills requisite for technical vocations. Fortunately, mathematics educators challenge these critics with appropriate proof. During the last half of the twentieth century, researchers such as the psychologist, David Ausubel, and biologist, Joseph Novak, investigated how students develop concepts and how they undergo conceptual change. Effective teachers depend on understanding their students' thinking processes. Their analyses of these processes ultimately lead to effective instructional techniques. However, without concrete, appropriate and reliable methods of tracking students' conceptual development, criticism of mathematics and science educators regarding the lack of students' critical thinking skills might be valid.

Meaningful Teaching

First, successful mathematics teachers understand the definition of concept. Secondly, they concretely follow the development of students' conceptions of mathematical ideas or objects (Clark 1997). Solely teaching procedures limits mathematics students potential to make connections or solve non-routine real-world problems. Procedural pedagogical style inhibits meaningful learning and impedes advantageous conceptual change (Cohen 1995). According to the coincidental opinions of Joseph Novak and the 1961 Educational Policies Commission, the primary objective of education is to facilitate the ability of students to think (Novak 1998). Novak (1998a) claims that "the central purpose [of education] is to empower learners to take charge of their own meaning making" (p. 9). Furthermore, Edmondson supports Gowin's claim that the development of meaning is a shared experience (Mintzes, Wandersee & Novak 2000). Teachers and students "work together to construct knowledge and negotiate meaning" (Novak, Wandersee & Mintzes 2000, p. 15). Teachers initiate this collaboration through concept mapping. Concept maps are graphical representations of students' comprehension of concepts. Before we investigate the history and function of concept maps, we examine the meaning of concept, the epistemological view of constructivism and conceptual change.

Definitions of Concept

Novak defines a concept as "a perceived regularity in events or objects, or records of events or objects, designated by a label" (Novak 1998a, Novak 1990). However, perceptions depend on personal experiences. Ausubel (1968) proposed that cognitive framework (thought), actions and emotions form meanings of objects or events within an individual. Malone and Dekkers (1996) record Ausubel's definition of concept as "…any objects, events, situations or properties that possess common critical attributes and are designated in any given culture by some accepted sign or symbol." Ultimately, concepts comprise the individuals cognitive framework (Ausubel 1968). Propositions consist of a statement composed of concepts. Novak (1998b) refers to propositions as semantic units, and logicians call them declarative statements. One assigns truth to propositions. The connections between propositions form meaning. So, concepts are neither true nor false, yet propositions can be valid or invalid. This is a common misconception among teachers. Novak (1990, 2000) clearly defines meaning as the representation of all propositional linkages an individual could construct that include a given concept.

Vinner augments the definition of concept with concept name and concept image (Tall 1991). Concept names evoke images associated with this name. Vinner reasonably claims students evoke concept images, a non-verbal or visual representation of the concept, before they translate the concept into a verbal form. Therefore, researchers of mathematics pedagogy must investigate how students think they acquire mathematical concepts and how they actually acquire them.

Furthermore, Vinner asserts knowledge of a concept definition does not imply conceptual understanding. Any competent mathematics teacher concurs with this view. An excellent illustration of this is the empty set. First, the mathematics teacher adequately introduces sets relying on examples and counterexamples of set, thus invoking the formation of a proper concept image. Secondly, the teacher solicits suggestions for the definition of the empty set from the students or provides an acceptable definition, such as a set that contains no elements. Quite often, students have difficulty visually differentiating between a set containing nothing and a set containing the number zero. The concept image that appears in this researcher's mind is an empty box. Of course, the box contains nothing, not even the number zero.

Conceptual Change

Ausubel's profound statement about assimilation theory of cognitive learning supports the constructivist epistemology. In 1968, Ausubel exclaimed, "If I had to reduce all of educational psychology to just one principle, I would say this: The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly" (Novak 1990). Edmondson (Mintzes, Wandersee & Novak 2000) asserts shared meaning, as identified by Gowin, leads to meaningful learning. Students build upon previously acquired knowledge and assimilate the new knowledge with what they already know. Beamans and Simons (Wolfgang, Vosniadou & Carretero 1999) describe previously acquired knowledge (prior knowledge) as "all the knowledge that the learners have when entering a learning environment, which is potentially relevant for constructing new knowledge." Prior knowledge is part of the basis of the epistemological view of constructivism. Edmondson distinguishes the two essential parts of constructivism: that knowledge is built upon prior knowledge, and the nature of this knowledge is symbiotic with the human experience (Novak 1998b). Hence, this is the basis of Ausubel's cognitive theory of assimilation. The construction of one's knowledge base could be faulty if the prior knowledge is invalid, the perceived meanings are not the ones the teacher wishes students to perceive, or there is some form of complication in the student's conceptual change.

Biemans and Simons delineate conceptual change as "partial or radical change of existing connections, as opposed to integration of new information into preconceptions without really changing these ideals" (p. 249, Wolfgang, Vosniadou & Carretero 1999).

Obviously, rote learning is antithetical to meaningful learning, and meaningful learning is a component of constructivist epistemology. Novak (1998) states that a student who learns meaningfully possesses prior knowledge with which to relate it to the new information, realizes the relevancy to other knowledge and deliberately relates new knowledge and prior knowledge in some nontrivial way. Malone and Dekkers (1984) concur with Novak's view, and they report meaningful learning relates new knowledge to prior knowledge via concepts.

Meaningful Learning

Wells (1999) and Clarke (1997) observe the teacher's role as facilitator of the process of meaningful learning development. Wills clearly defines the role of the teacher with respect to two levels: the macro level and the micro level. At the macro level, the mathematics teacher selects themes for curricular units and how students address these themes. Furthermore, the mathematics teacher presents students with clear expectations and evaluates and reflects upon processes and outcomes. At the micro level, the mathematics teacher observes how students react to the selection of units, teacher expectations, teacher evaluations of their performance and peer reflections and evaluations. Vygotsky characterized this type of teaching Wells analyzes as "working in the student's zone of proximal development" (p.243, Wells 1999). Within the zone of proximal development, "learning awakens a variety of internal development processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers" (p. 25, Wells 1999).

Clarke (1997) suggests maximization of the zone of proximal development occurs (which subsequently maximizes meaningful learning) when mathematics teachers

1. adapt materials and instruction "according to local contexts and teachers' knowledge of students and [their] needs"
2. use varied learning communities, such as collaborative or cooperative learning groups

3. develop a mathematical discourse community wherein the teacher appreciates and elaborates on students' solutions and methods
4. identify and focus on principles of mathematics
5. asses students' performances using alternate methods for making instructional decisions.

Possession of these components provides teachers "a window into their [students'] thinking that can be used to plan further instruction", and students with an "atmosphere of conjecture and justification of mathematical ideas…[that] provide the basis for discussion of problems" (p. 278, Clarke 1997).

Graphic Organizers

Trowbridge and Wandersee define graphic organizers as "visual representations that are added to instructional materials to communicate the logical structure of the instructional material" (p. 96, Mintzes, Wandersee & Novak 1998). Another term used in the literature for graphic organizers is representational diagram systems (Stensfold and Wilson 1990). Originally, reading researchers intended specialists to construct the graphic organizers to assist the learner. The constructions of these early graphic organizers consisted of a hierarchy of boxed concepts connected with unlabelled segments. Trowbridge and

Wandersee report the taxonomy of graphic organizers as presented by Readance, Been and Baldwin. The basic taxonomy of visual organizational patterns include

1. Cause/Effect: the graphic organizer connects reasons with results
2. Comparison/Contrast: the graphic organizer highlights apparent likenesses and differences between objects or events
3. Time Order: the graphic organizer chronologically sequences objects or events
4. Simple Listing: the graphic organizer groups related items
5. Problem/Solution: the graphic organizer shows how a question can be answered

(p. 98, Mintzes, Wandersee & Novak 1998).

Concept Circles

Among the variety of graphic organizers Trowbridge and Wandersee discuss, they focus their discussion on three Ausubelian tools for science teaching: Wandersee's Concept Circle, Gowin's Vee Diagram and Novak's Concept Map. Wandersee's Concept Circle resembles Euler's Circles. These graphic organizers represent the judgment of the circle constructor, and they reveal relationships. Such relationships include class exclusion, class equality, class product, and class sum (Mintzes, Wandersee & Novak 1998). Nobles, Konopak and Nicols research reveals the usefulness of concept circles as a diagnostic tool for assessing prior knowledge and as a strategy for peer evaluation. An example of concept circle appears in Figure 1.

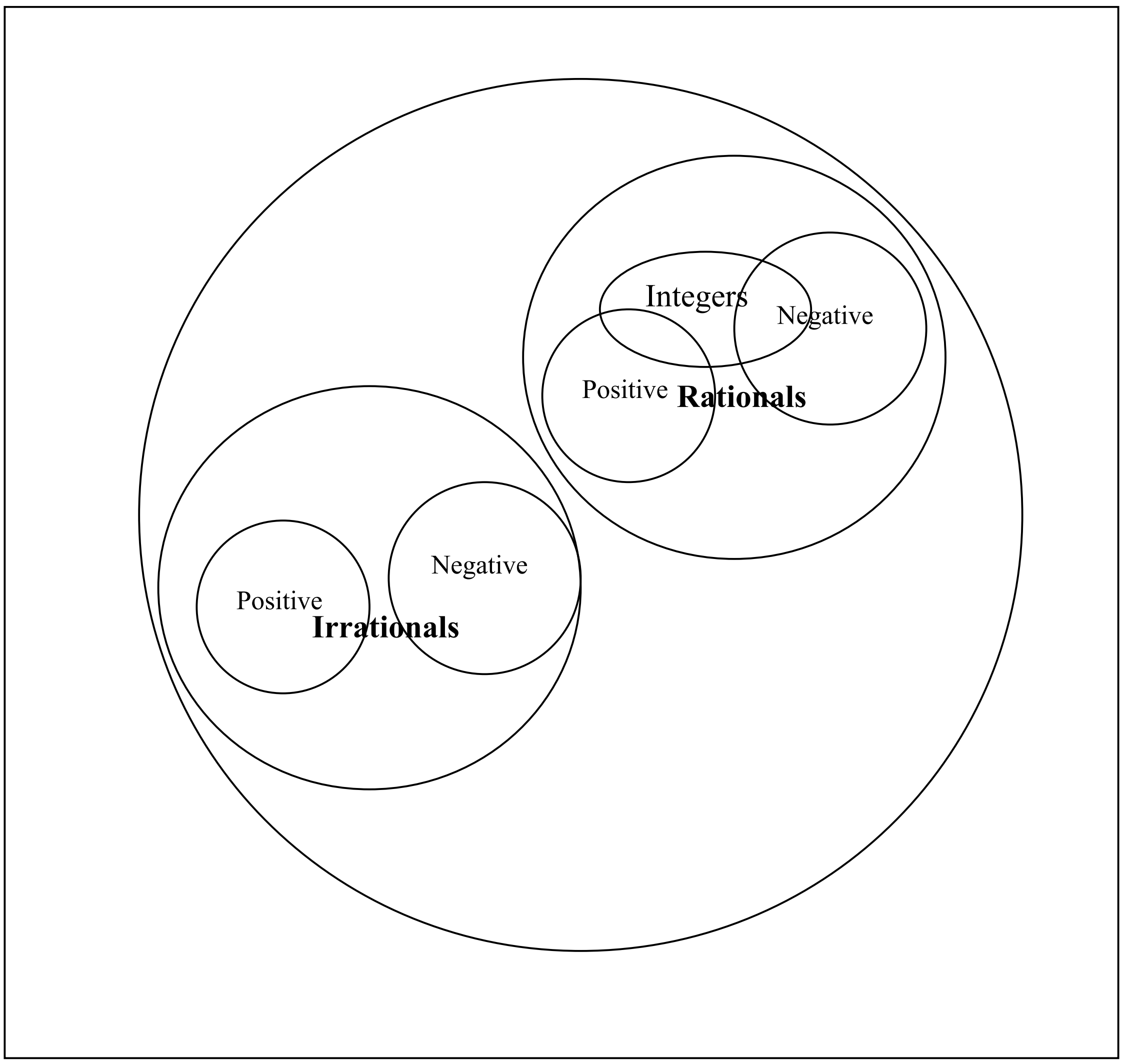


Figure 1. Concept Circle: Elements of the Set of Real Numbers.

Vee Diagrams

Gowin's epistemological Vee displays a large V with a focus question for scientific investigation in the middle of the graphic organizer, a methodological side and an epistemological side. On the methodological side, the constructor of the Vee records the experiment, data collected transformations (graphs), knowledge claims extracted from the transformations, and value claims relative to solving current problems. The constructor records concepts, principles, theory and philosophy necessary for thoroughly examining the focus question.

Concept Maps

Stensfold and Wilson (1990) classify the third type of Ausubelian graphic organizer as concept. The two most commonly used representational diagram systems are Gowin's Vee Diagram and Novak's Concept Map (Novak & Gowin 1984). Novak (1990, 1998, 2000) claims the representational diagram system of concept mapping emphasizes meaningful learning as previously defined in this paper. Other terms, such as webbing, identify concept mapping within the context of social studies and language arts pedagogy (Wilcox & Lanier 2000). In 1972, a research group at Cornell University, directed by Joseph Novak and Bob Gowin, developed concept maps to better represent meaning for a specific domain of knowledge, and to specifically understand change in children's knowledge of science (Novak 1990). Initially, concept maps represented students' knowledge structures before and after instruction. However, researchers and teachers use concept maps as research or pedagogical tools that superseded the general purpose as demonstrated by the Cornell team. However, Edmondson (Mintzes, Wandersee & Novak, 1998) exclaims, "The benefits of using concept maps seem to lie more in improving the process of learning and in providing formative feedback than in providing additional quantitative information that may be translated into a grade" (p. 28). Furthermore, Willerman and MacHarg (1991) proclaim that students' concept maps reveal their teacher's conceptions of a specific area of instruction, such as the areas in a unit that received more attention relative to other areas.

Concept Map Structure

Prior to the construction of a concept map, teachers provide an event or topic they wish students to understand (Roberts 1999). Sometimes teachers offer a list of words among which students subsequently make connections; although, allowing students to generate their list reinforces empowerment as recommended by the *NCTM Standards*

2000. The traditional map consists of a "most inclusive, most general" concept at the top of the map (p. 19, Novak 1998b) enclosed by a rectangle.

The simplest concept map consists of two concepts linked by a segment (Malone & Dekkers 1984). An elementary example follows in Figure 2.

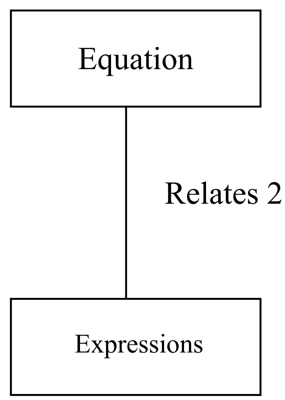


Figure 2. Example of elementary components of concept map.

Adjacent to the segment is the proposition (Novak 1998b, Malone & Dekkers 1984). A proposition is a description of the relationship between concepts (Novak 1998). Next, students augment their maps by relating terms to the placed concepts in a similar fashion. They link these subconcepts, the more specific concepts, with the more generalized concepts in a hierarchical manner. Subsequently, students create more links between the lower level terms.

Concept maps are hierarchical diagrams. McClure, Sonak and Suen (1999) define hierarchies as “branching structures that show superordinate-subordinate categorical relationships among concepts. Crosslinks are relationships identified between concepts located in different hierarchical branches” (p. 484).

Research Inquiries

This report addresses the following questions:

1. Is there a significant difference in achievement between the experimental group and control group as measured on end-of-unit items relevant to the concept of inverse?
2. Within the experimental group, is there increased confidence in mathematical ability within and between genders as a result of the treatment of concept mapping?
3. Is there a significant difference between the experimental and control groups’ genders with respect to the treatment of concept mapping?
4. What are the subjects and assessors views of the usefulness of concept maps on learning and teaching mathematics?

CHAPTER 3

Literature Review

Variations in Concept Map Structure

Malone and Dekkers (1984) detailed a procedure for constructing concept maps. They suggest the teacher

1. select a “stand-alone topic”
2. encourage students to read their text, and subsequently, devise a list of concepts related to the topic or brainstorm the subject
3. direct his students to rank their concepts from most inclusive to most exclusive
4. instruct his students to prepare the map.

Relative to the second part of their paradigm was classification of the essential concepts into one of three broad categories. These categories are entity concepts (objects), relational concepts (such as equal, greater than, proportional to, etc.) and qualitative concepts (such as area, density, volume, motion, etc.). Variations on concept maps include directed or non-directed graphical representations. Directed maps usually include cause and effect relationships or inclusive relationships. Figure 3 displays an example of a portion of a directed concept map on polygons as given by Malone and Dekkers (1984).

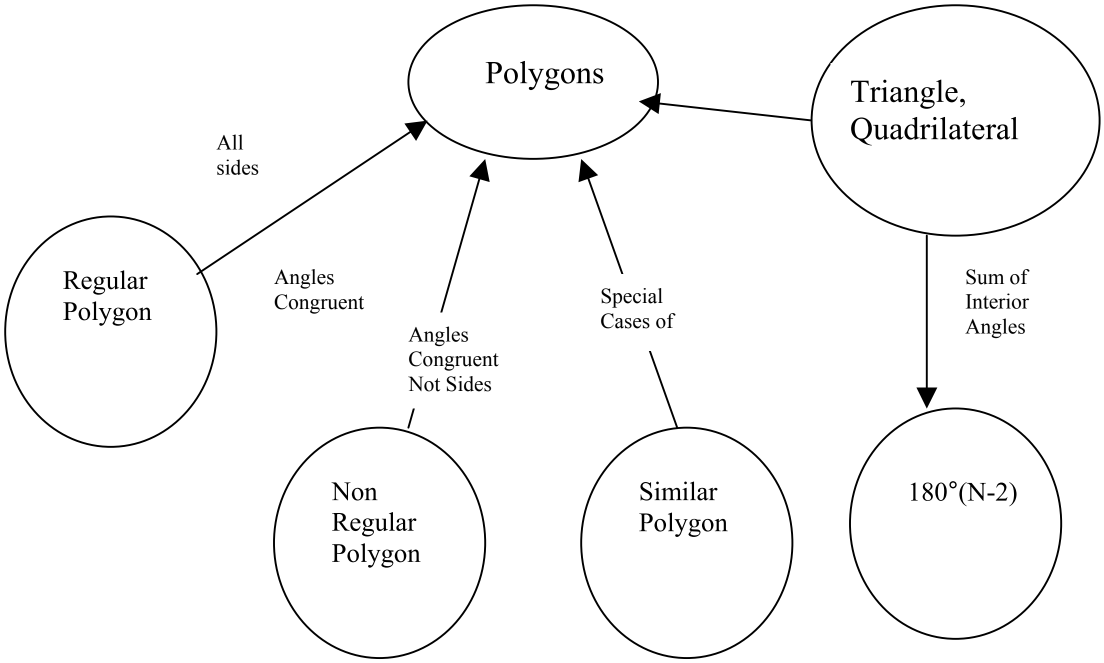


Figure 3. Directed concept map on polygons.

Yet, most researchers and teachers avoid directed maps, simply because the proposition inherently identifies the direction. Figure 4 shows Novak's concept map of a concept map (Novak 1998, p. 32). To this figure, this author applies the term meta-cartography, mapping about maps.

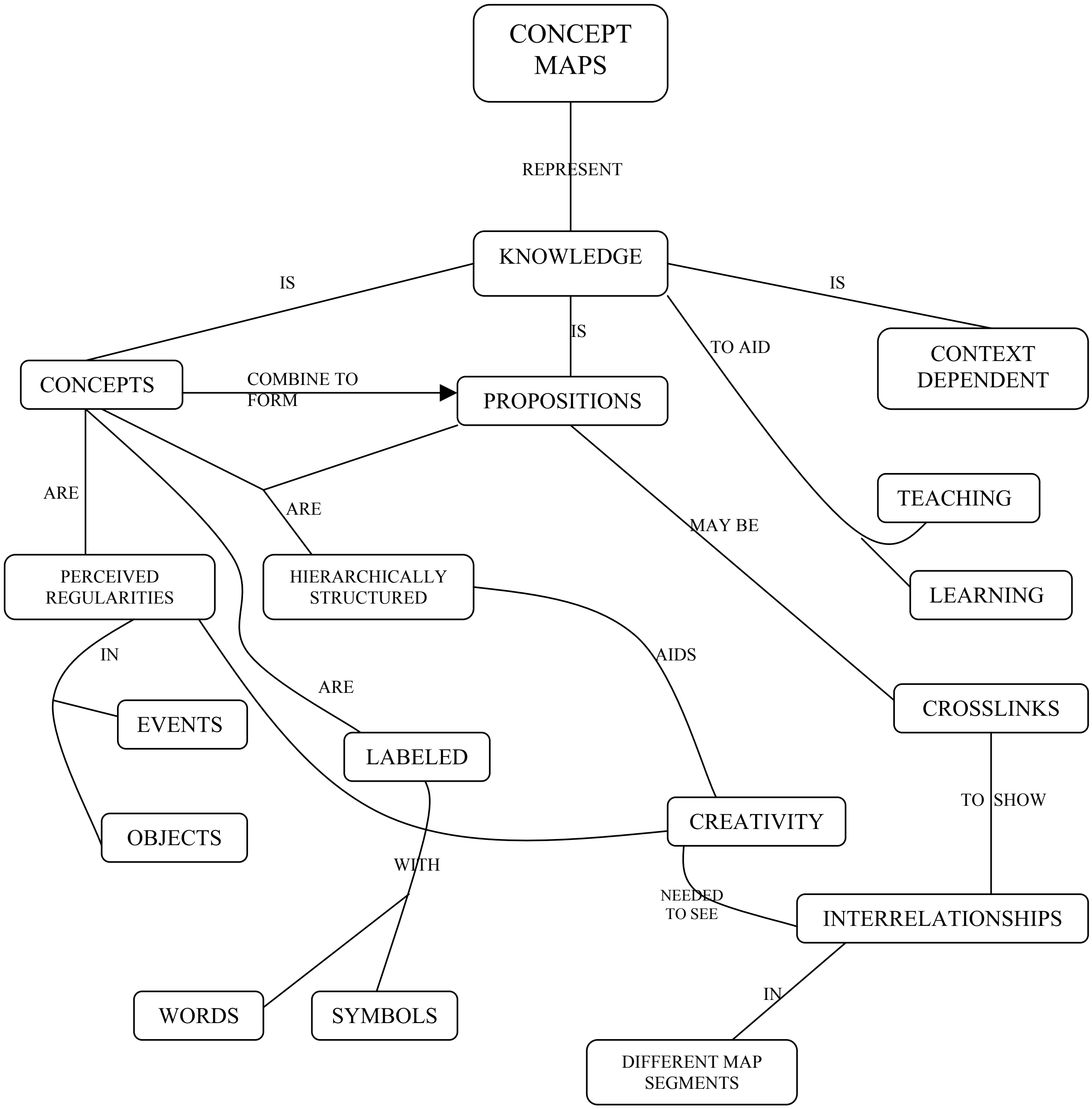


Figure 4. Novak's Concept Map of a Good Concept Map.

Novak suggests constructing a preliminary map from a clearly defined list of related concepts. Students relocate the concepts through self or group negotiations. Novak suggests software that assists the user in the generation of such representational diagrams. Furthermore, he insists that students revise concept maps at least three or more times (Novak 1998b). Since they concretely reflect the cognitive structure of a student's knowledge of an object or event, the total of all possible concept maps represent the student's full perception of the event or object.

Table 1 enumerates the steps for constructing an acceptable concept map as proposed by Trowbridge and Wandersee (p. 119, Mintzes, Wandersee & Novak 1998). The table has been abridged for mathematics and appropriate pedagogical technology.

Table 1

Trowbridge and Wandersee's Steps for

Constructing a Concept Map

1.	Select from 1 to 12 concepts from the mathematics content material being considered (e.g., lecture notes, videotape, journal article, textbook, website, compact disc).
2.	Write each concept on a separate note card. Lay these on a large sheet of paper. Use a marker board with colored dry erase markers, if available.
3.	Select a superordinate concept to be placed at the top of the map. This is the organizing concept for the map.
4.	Arrange the other concepts in a distinct hierarchy under the superordinate concept. The concepts should be arranged from general to specific, in levels from top to bottom on the map.
5.	Once the concepts have been arranged, construct segments between related concepts and label each linking segment with words that characterize the relationship between these concepts.
6.	Create cross-links between two subordinate concepts in different branches of the map by using a dashed segment and label their relationship by writing adjacent to the dashed segment.
7.	Provide examples of certain concepts enclosed within broken ovals. Connect these to their source concepts with a solid segment.
8.	Review and reflect. Upon satisfaction with the map's revised arrangement, redraw the map in final form.

Wandersee (Mintzes, Wandersee & Novak 1998) proclaims the cognitive power of the revision and reflection step as described in Table 1. The concept mapper reflects on the map's mathematical validity in terms of conceptual linkages, the suggested propositions between concepts, the hierarchy or prioritization of the concepts, and the relevancy of examples. Obviously, the mapper spends more time working with concepts as opposed to reviewing lecture notes or text materials alone.

In addition to suggesting methods of construction of concept maps, Trowbridge and Wandersee (Mintzes, Wandersee & Novak 1998) suggest incorporating micromaps, macromaps and progressive maps. They define a micropmap as a concept map that limits the construction to 10-15 elements. Since the mapper minimizes the concept map, "the map does not become too large or cluttered, the mapper must prioritize elements, it takes less time, and it is easier to evaluate" (p. 120). The authors recommend micromapping at least ten maps to obtain proficiency in concept mapping. The macromap consists of a composite of micromaps relating the macromap's concepts to those of the upper level concepts of the micromaps. Progressive maps, also known as time series maps, are macromaps that document changes in map constructions over time. Subsequently, evaluators reconstruct the process of conceptual change. This is likely the most valuable application of concept maps to mathematics education researchers with interests in mathematics conceptual development.

Functions of Concept Mapping

General Purposes

Concept maps serve a variety of purposes. White and Gunstone (1992) cite numerous aims for concept maps within the context of the classroom environment. Students explore their understanding of a limited aspect of a topic, determine relationships between distinct topics and engage in meaningful discussion by using concept maps. Using Assessment in Mathematics Teaching (2000) describes three aspects of the potential power of concept maps for students:

1. They are tools for student self-assessment
2. They empower students
3. They are a means by which to communicate students' mathematical accomplishments to each other.

In addition to providing help for students, concept maps reflect students' understanding of a given topic. This enlightens teachers about students' strengths and weaknesses and how to direct (or redirect) future instruction on the topic (Wilcox & Sahloff 1998, Wilcox 1996). Hence, the concept map becomes an additional form of assessment. Furthermore, they offer "a way for students to show growth in understanding over time" (Wilcox 1998).

Assessment

Leach, Neutze and Zepke (Brown, Armstrong & Thompson 1998) concur that assessment metaphorically loiters outside the classroom in anticipation of entering the room to serve the learner. Can assessment improve meaningful learning? Leach, Neutze and Zepke claim motivation for learning in adult students increases when instructors integrate assessment so that assessment appears to be part of learning, and when they relinquish more control of the learning process and assessment to their students. The AMATYC Standards of Assessment confirm this view.

Furthermore, the fundamental goal of assessment delineated in the NCTM Assessment Standards for School Mathematics (1995) coincides with Leach, Neutze and Zepke's views of assessment. The learning standard of assessment clearly defines assessment as "a communication process in which assessors—whether students themselves, teachers, or others—learn something about what students know and can do and in which students learn something about what assessors value" (Brown, Armstrong & Thompson 1998). The document narrows this process to the mathematics teacher gathering data about the students' knowledge, application of, and attitudes toward mathematics. Subsequently, the mathematics teacher draws inferences from these data for evaluation. <u>Crossroads</u> states assessments measure students' knowledge of mathematics content, their ability to solve problems, to communicate, to work in groups, and to read technical material (Cohen et al 1995). Moreover, the <u>NCTM Assessment Standards</u> (1995) indicate records assessment fosters growth toward high expectations. Mathematics teachers with the vision of reform employ multiple sources of assessment that measure mathematics knowledge and its connections. Student concept maps provide many opportunities for this type of perceived assessment.

In addition to concisely defining appropriate assessment, the <u>NCTM Assessment Standards</u> states, "Many products of classroom activity are indicators of mathematics learning: oral comments, written papers, journal entries, drawings, computer-generated models, and other means of representing knowledge" (p. 2 of 3 standards.nctm.org/--/Leasstds.htm). Although not explicitly stated, concept maps belong conveniently under the category of "other means of representing knowledge."

<u>Concept Map as Advance Organizers</u>

Ausubel suggested the use of advance organizers to bridge the gap between prior knowledge and knowledge to be assimilated into a students existing cognitive structures.

An advance organizer is “a small segment of instruction offered prior to a larger instructional unit that is general and more abstract than material in the larger unit” (Novak 1998). Effective employment of advance organizers requires the identification of the learner’s conceptual and propositional knowledge and appropriate organization and sequencing of new knowledge that relates well to existing concepts and propositions already held (Ausubel 1968).

Joseph Cliburn delineates an advance organizer as “a preinstructional strategy that presents the major background concepts for a subsequent unit of study” (Cliburn 1990). Cliburn, a biology instructor at a Missouri community college, constructs a concept map for an entire unit of study at the beginning of a unit. He presents the general map on the overhead projector (much more convenient and visually pleasing using PowerPoint), he distributes copies of the same map to the students, and he explains them to the students. At appropriate times during the exposition of the unity of study, Cliburn refers to the general concept map—the advance organizer—and relates it to a new concept map on a subtheme of the unit.

Willerman and MacHarg (1991) researched the application of concept maps as advance organizers in eighth grade science classes. Their sample consisted of 42 students in the control group and 40 students in the experimental group taken from four physical science classes. The students' SES ranged from the poverty level to upper class. Each teacher in the experimental sections presented a completed concept map including propositions. Each experimental subject received a blank concept map to be completed as the teacher explained her map within the context of an advance organizer. Teachers instructed their students that they could modify their concept maps during the exercise as needed. Willerman and MacHarg reported a positive significant difference of achievement in favor of the experimental group based on a teacher-constructed objective test administered at the end of a 2-week unit on elements and compounds. They claimed (1) the organizational and visual relationships of the concept map introduced the topic better than prose or written exposition, (2) the complete and accurate teacher-constructed map represented the new information better than the students might construct, and (3) the facts from the teacher-constructed map overlapped with the teacher-constructed test (1991). Unfortunately, Novak's perspective on the use of concept maps vastly differs from that of Willerman and MacHarg, yet aligns with that of Cliburn. Willerman and MacHarg fail to record any perpetuated student or teacher use of concept maps following the initial day of instruction. Recall Novak insists student-constructed maps are more valuable as a learning tool than teacher-constructed maps.

Malone and Dekkers (1984) enumerate six examples of situations in which teachers use concept maps. Teachers

1. more effectively organize information on a topic
2. motivate the study of a new topic (Ausubel's advance organizer)
3. review a topic
4. generate discussion on a topic
5. rank important ideas on a topic from most exclusive to most inclusive
6. demonstrate interrelationships between ideas.

McClure, Sonak and Suen (1999) recapitulate Novak's remarks from the December 1990 issue of Journal for Research in Science Teaching. They report that concept mapping is a learning strategy used for improving science education, an instructional strategy, a strategy for planning curriculum, and a means for assessing students' understanding of science concepts. In addition to reporting Novak's assertions, McClure, Sonak and Suen state that teachers facilitate diagnoses of misunderstandings easier through students' graphical distortions, intrusions or omissions of content (1999). In comparison to traditional subjective assessments, the production time for developing concept maps is relatively short. Shaka and Bitner (1996) confirm math and science educators use concept maps as instructional tools, assessments of processes and products, and as a heuristic for developing science curricula.

Application Concept Maps to Academic Disciplines

Early History of Concept Maps

Educators use concept mapping within a variety of academic disciplines. Among these are social sciences, physical sciences, mathematics, statistics and elementary teacher education. In 1975, Cardemone prepared a master concept map for the topic of ratio and proportion and distributed it for student use (Novak 1990). Bogden (1977) reported advantageous results when students learned about genetics in small groups. However, in both studies, the research designers or professor constructed their concept maps (and without propositions). The participants who gained the most benefit from concept mapping were not the students—in both cases students reported confusion; yet, the instructors informally claimed a positive correlation (1) between concept map design and course examination design and (2) between concept map design and interpretation of answers for these examinations (Novak

1990). Therefore, Novak concluded the persons who construct the representational designs benefit the most.

Applications of Concept Mapping to Mathematics

Park (1993) studies the comparison of a traditional calculus course to a course that combines calculus with a Mathematica lab component. Mathematica is a computer algebra system. Universities and community colleges now implement a computer algebra as a component of the calculus curriculum (e.g., North Carolina State University, Durham Technical Community College). As a part of Park's research analysis, Park focuses on conceptual understanding by using what he labels "a new instrument", the concept map. Based on two analysis methods for comparing students' concept maps, Park reveals favorable results regarding conceptual understanding for the Calculus & Mathematica experimental over the traditional group.

Corporations frequently use organizational software for a variety of problem solving activities (Novak, 1998a). Edwards (1993) examines whether or not organization software positively effects students' concept mapping ability. Sixty-four Midwestern high school geometry students participated in the study, 44 of whom used the organizational software, Org Plus, for use on Macintosh computers to produce concept maps. First, Edwards discovers students using the organizational software produced significantly more branches and levels of hierarchies than the students constructing concept maps by pencil and paper. Secondly, he finds that females produced significantly more complex maps than males. Finally, Edwards concludes that students using the organizational software have a more positive attitude toward concept mapping than students using pencil and paper.

Tsao (1995) appropriately claims, "Conceptual understanding and skill acquisition seem to be like two extremes in the spectrum of learning mathematics." His study examines how to construct connections between conceptual understanding and skill acquisition for two students within the context of solving simple linear equations in one variable. Two subjects, a sixth grader and a seventh grader, participated in the study. The subjects underwent eight ninety-minute tutoring sessions during which Tsao "covered" algorithms of integers, creating equations, and five methods for solving linear equations. Tsao reported the subjects discovered connections among the concepts of variable, equality, solution and equivalent equations. Figure 5 shows one of the concept maps created by a middle grades mathematics student named Albert (p. 227, Tsao, 1995).

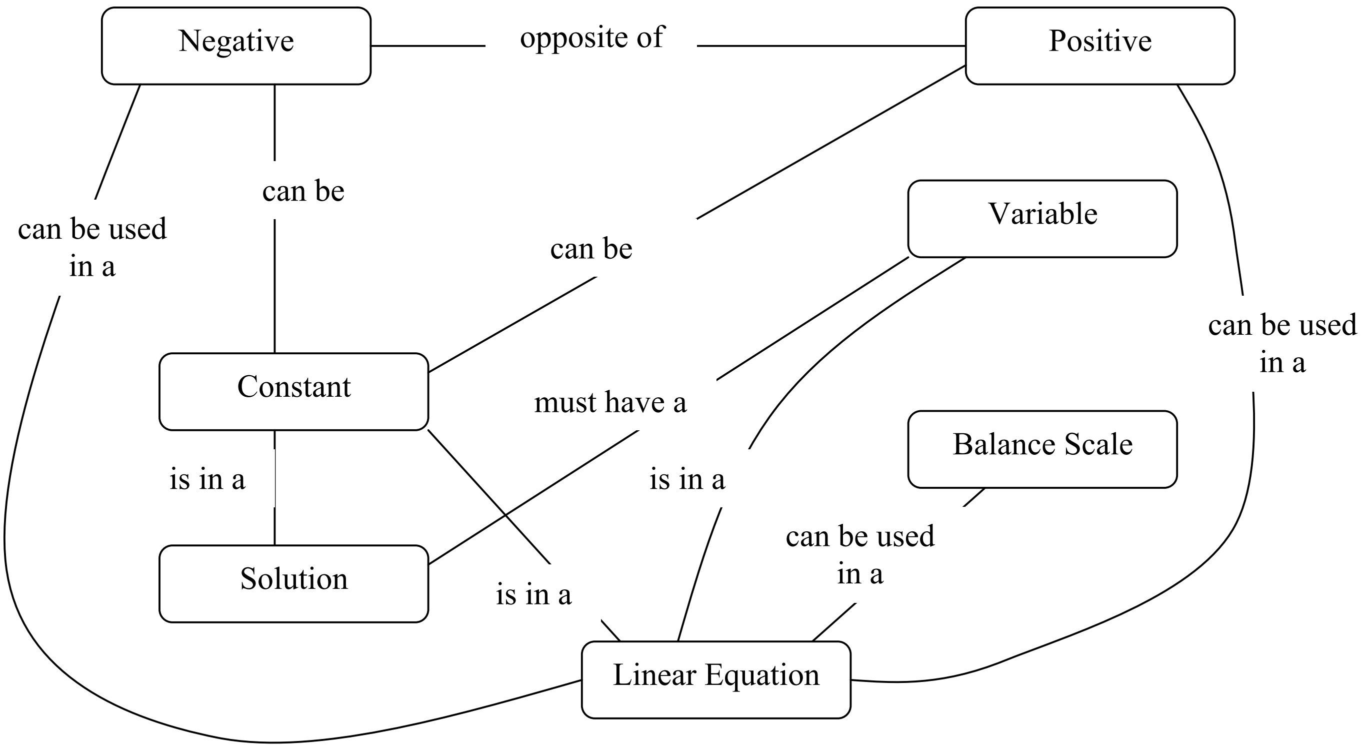


Figure 5. Albert's Concept Map 14.

Wilcox and Sahloff (1998) implemented concept mapping as a strategy to determine how grade 8 algebra students' knowledge of content domain increased over time. Sahloff, a support teacher for mathematics in the junior high school, implemented the experiment in the classroom setting. These students already had some experiences with concept mapping under the guise of clustering in their language arts classes (Wilcox & Lanier 2000). She began teaching her students with non-mathematical ideas, such as "rooms", followed by mathematical ideas such as "integers", "operations" and "statistics". The study began as Sahloff instructed, "Tell me anything you can about what you know about...analytic geometry" (p. 466, Wilcox & Sahloff 1988, p. 132, Wilcox & Lanier 2000). Students' constructions of these pre-unit maps were primitive as expected. After five weeks of instruction supplemented by extensive use of the graphing utility, Sahloff returned the pre-unit maps and instructed her students to construct what they know about analytic geometry. Sahloff refused her students access to their textbooks or notes during this post-unit assessment.

The researchers for this study noted similarities across maps. Every map had hierarchies. Higher level concepts, such as equations, systems of equations, variation, and grids were common. More specific terms at lower levels also appeared strikingly similar (e.g., degree, standard forms, number of solutions, etc.). Finally, some maps displayed mathematical conventions, such as the Cartesian plane. Unfortunately, some of the maps resembled vocabulary lists, and none displayed propositions. An example of one of Sahloff's students' pre-unit and post-unit maps appears in Figure 6 and Figure 7, respectively.

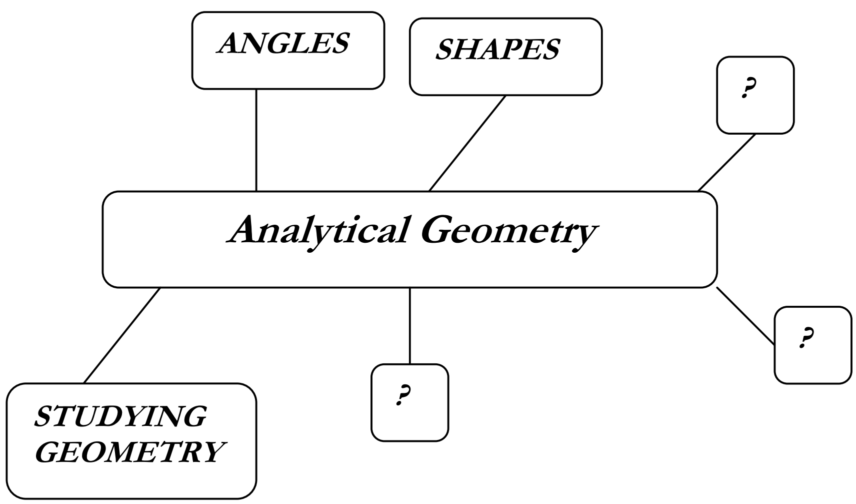


Figure 6. Greg's pre-unit concept map for analytic geometry.

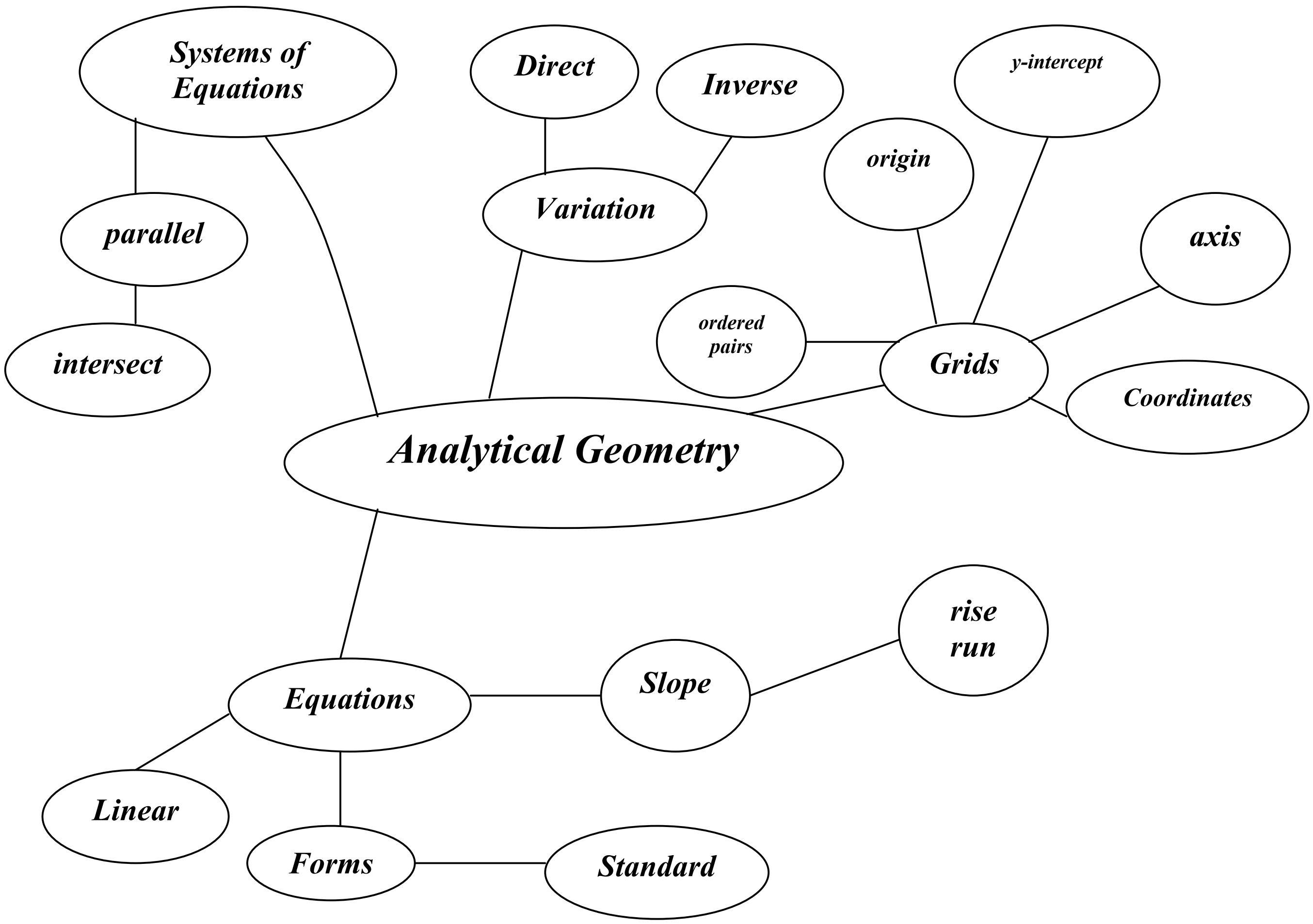


Figure 7. Greg's post-unit map for analytic geometry.

Sahloff extended the concept construction experiment. She asked her students to respond to the following two items:

1. Write about something from your concept map that you know a lot about, and tell everything you can.

2. Write about something from your concept map that you do not know much about, and tell why you think you are having trouble with this idea

(p. 466, Wilcox & Sahloff 1998). The students' responses to these items indicated what Sahloff might do next relative to instruction.

Chilcoat (1998) researches the use of a teacher-developed concept map within a College Algebra curriculum. The curriculum includes writing and graphing calculators in conjunction with the teacher-developed concept map. Chilcoat examines the impact of these three techniques on students' conceptual understanding in the College Algebra curriculum. One hundred thirty students from four sections of College Algebra classes, taught by two instructors, participated in the study. Chilcoat reports the mean for the conceptual posttest of the experimental groups was practically greater, but not significantly greater, than the mean for the conceptual posttest of the control groups. Yet, he also reports insignificant differences in procedural scores and attitudes/beliefs due to treatment. Again, this study exemplifies only the use of teacher-generated map, and it fails to report how students used these maps. An example of one of Chilcoat's teacher-generated maps is located in Figure 8 (p. 136).

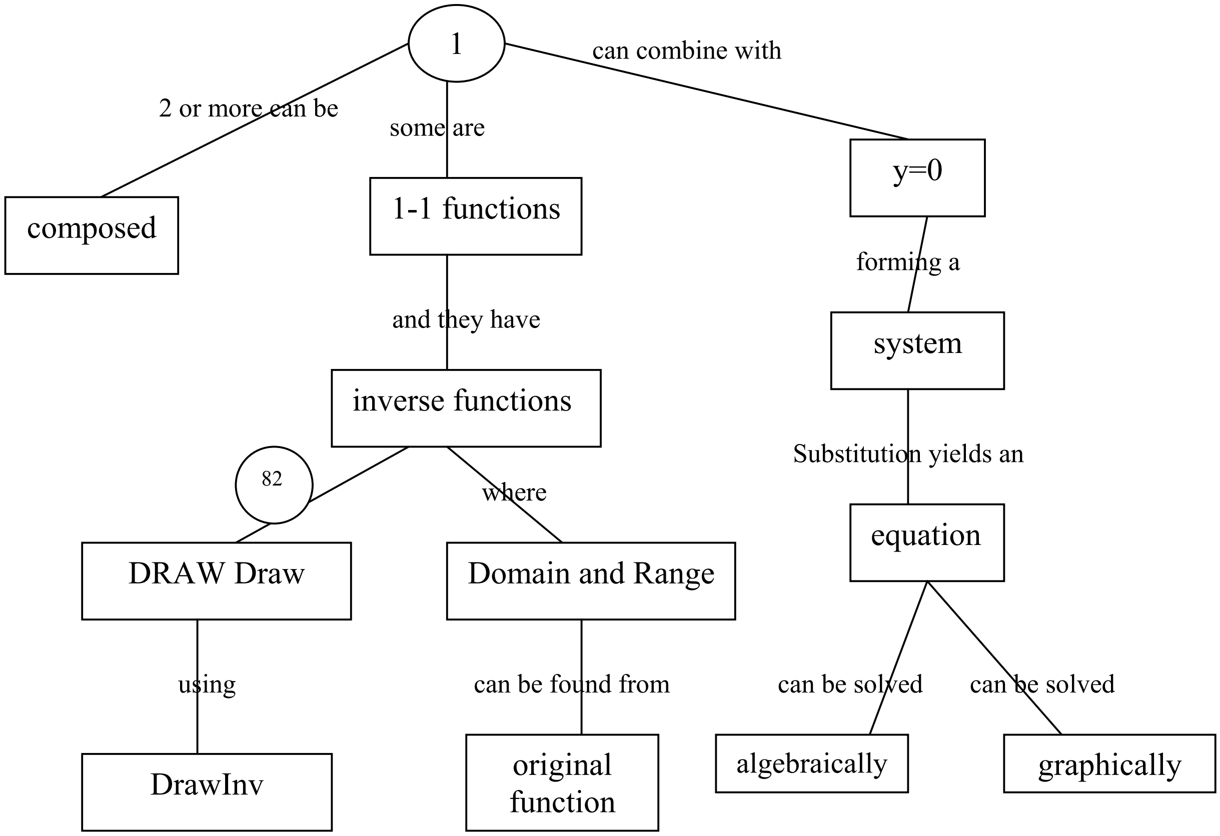


Figure 8. Chilcoat's example of a teacher-generated concept map.

Bolte (1998) incorporates concept mapping as a means for students to clarify their mathematical thinking processes. Furthermore, she uses concept maps as a vehicle for her students to elaborate on their ideas in the form of "interpretive essays" (Bolte, p.29). At the beginning of the Calculus I course, she directs her students to construct concept maps incorporating a review of precalculus terms, such as "mapping, graph, domain, range, inverse and onto" (p. 29). Finally, at the end of the course her students construct a summative concept map including terms such as limit, continuous, and optimization. Figure 9 illustrates one of Bolte's student's concept of differential and integral calculus (p. 31).

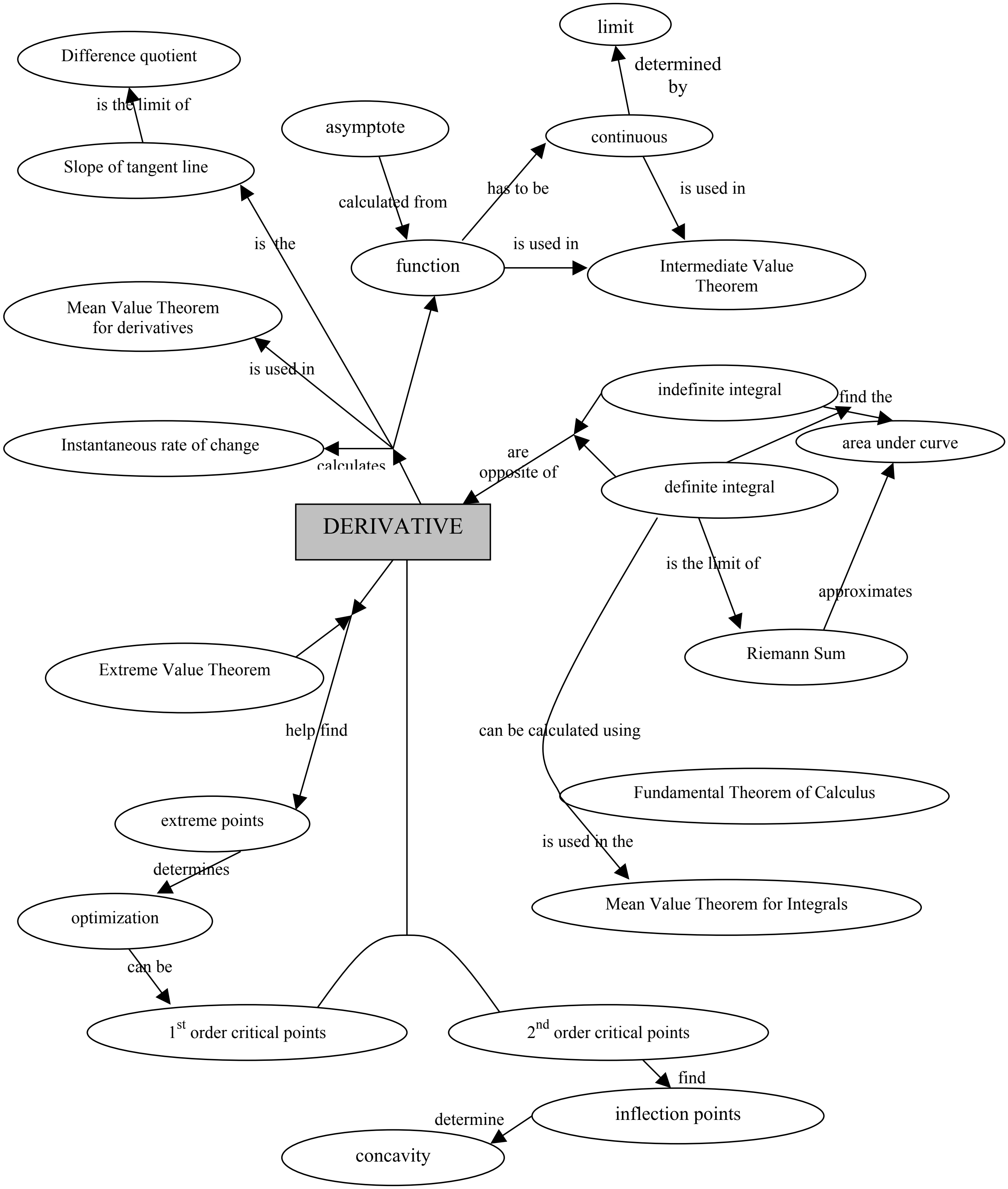


Figure 9. Student concept map for differential and integral calculus concepts

Bolte offers instruction on how to construct a concept map during the course of a regular class meeting at the beginning of the academic quarter. She advocates individual construction of concept maps given a list of key terms. However, since developing propositions seems to be a difficult task for her students (corroborated by previously mentioned studies), she solicits the entire class for appropriate linking terms.

Ultimately, Bolte identifies four advantages of concept mapping for students in the Calculus I course. Students

1. reflect on their work, because the mathematical connections are explicitly depicted,
2. modify and extend their knowledge while constructing the map,
3. experience mathematics as a creative activity,
4. create a useful device as a foundation for developing an interpretive essay (p. 33).

Finally, Bolte claims that this alternate means of assessment helps students realize their strengths and weaknesses, and it encourages written and oral mathematics communication. Hence, concept mapping in the Calculus I class supports both the NCTM and AMATYC standards of collaborative learning, communications and making connections.

Roberts' (1999) study examines the efficacy of concept mapping within the context of two aspects of statistics, problem definition and statistical inference. Roberts' study takes the form of action research. Nineteen students enrolled in a third semester course in statistics. Each student already successfully completed a course in elementary inference and a course on a particular statistical software package. The instructor lectured on concept mapping during the first week of the course. For the first assignment, students drew concept maps using a list of terms procured from the first chapters of their statistics texts. Roberts collected the initial maps and subsequently returned them for group discussion. She displayed unidentified student models as demonstrations of good concept maps and maps with some misconceptions.

After two weeks passed, intended for reflection, Roberts directed students to construct a second map. The students used a list of terms associated with statistical inference and constructed concept maps linking different statistical tests with concepts of hypothesis testing. Roberts provided class time for informal small-group discussion, yet she requested each student to prepare his or her own map. Figure 10 illustrates a student's concept map used to measure statistical understanding.

At the end of the thirteen-week course, the researcher asked the students to construct two more concept maps under parallel instructions given for the first two maps. However, many students failed to perceive the relevance in repeating the exercise, so not all students produced the second set of maps. Therefore, Roberts could not easily track conceptual change at any rudimentary level. Roberts claims concept map scores (as determined by a scoring rubric) correlated highly positively with corresponding formal assessments.

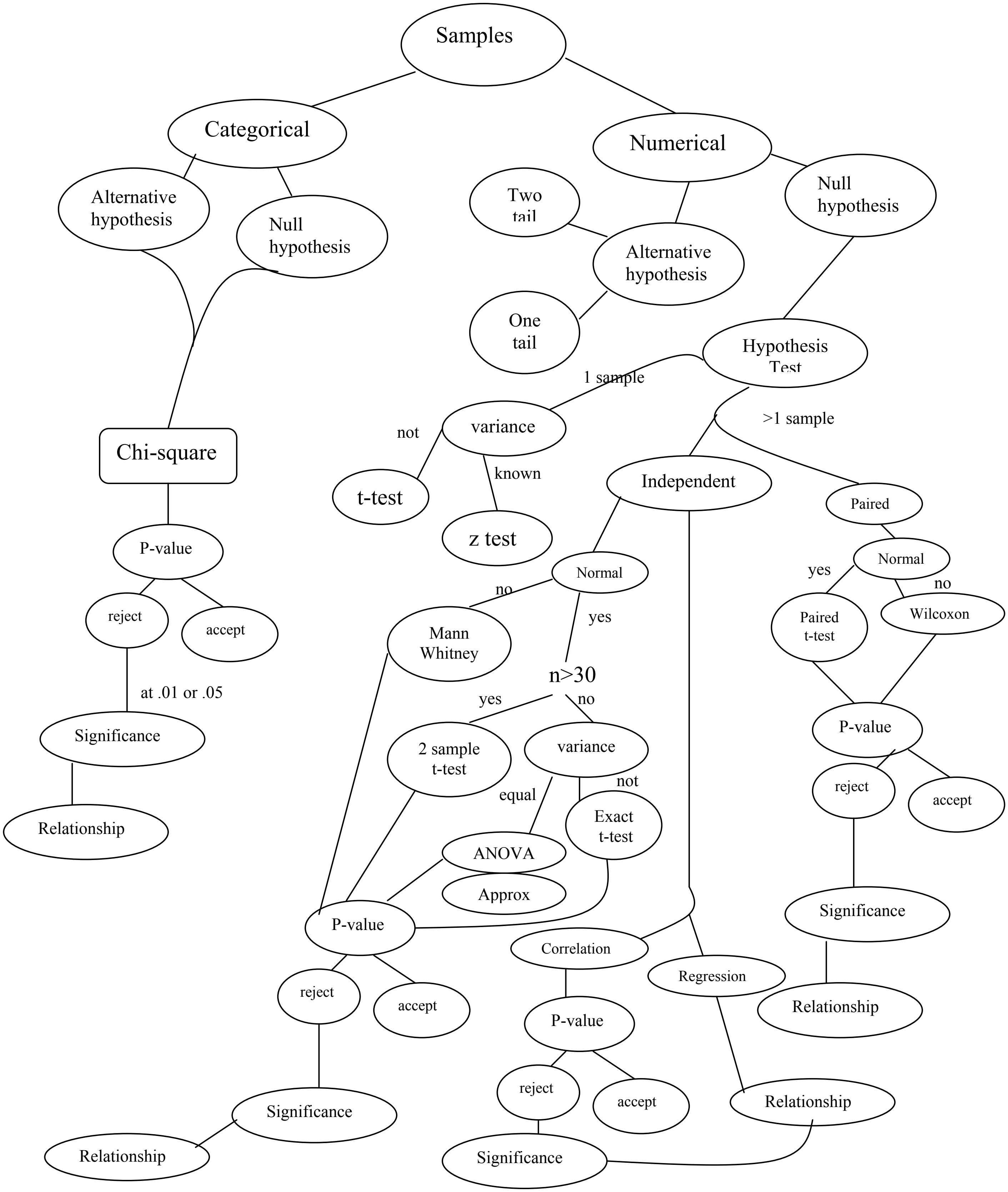


Figure 10. Diane's map for statistical hypotheses testing.

Applications of Concept Mapping to Mathematics Education

An early use of concept mapping within the context of mathematics education occurs at the elementary preservice level. Merrill (1987) studies preservice elementary teachers and their conceptual understanding of division. The researcher divided the elementary into high and low mathematics achieving groups. Each group underwent concept map instruction during the first of two treatment sessions. Each student admitted into the second session completed the first concept mapping activity at or above the 75% level of accuracy. All students mapped the division concept during the second treatment. Merrill finds that

1. no subject mapped the division concept at or above the 75% level of accuracy;
2. the high achieving group maps significantly better than the low achieving group;
3. hierarchy, grouping and branching component scores predict mathematics achievement.

Subsequently, Merrill recommends that preservice elementary teachers

1. be taught division in a mode that promotes the conceptual understanding of division;
2. be taught concept mapping to perpetuate material organization and to test understanding; and

3. develop classification skills.

Figure 11 displays one of the preservice elementary students' high-scoring maps (p. 87, Merrill 1987). The map demonstrates complex groupings.

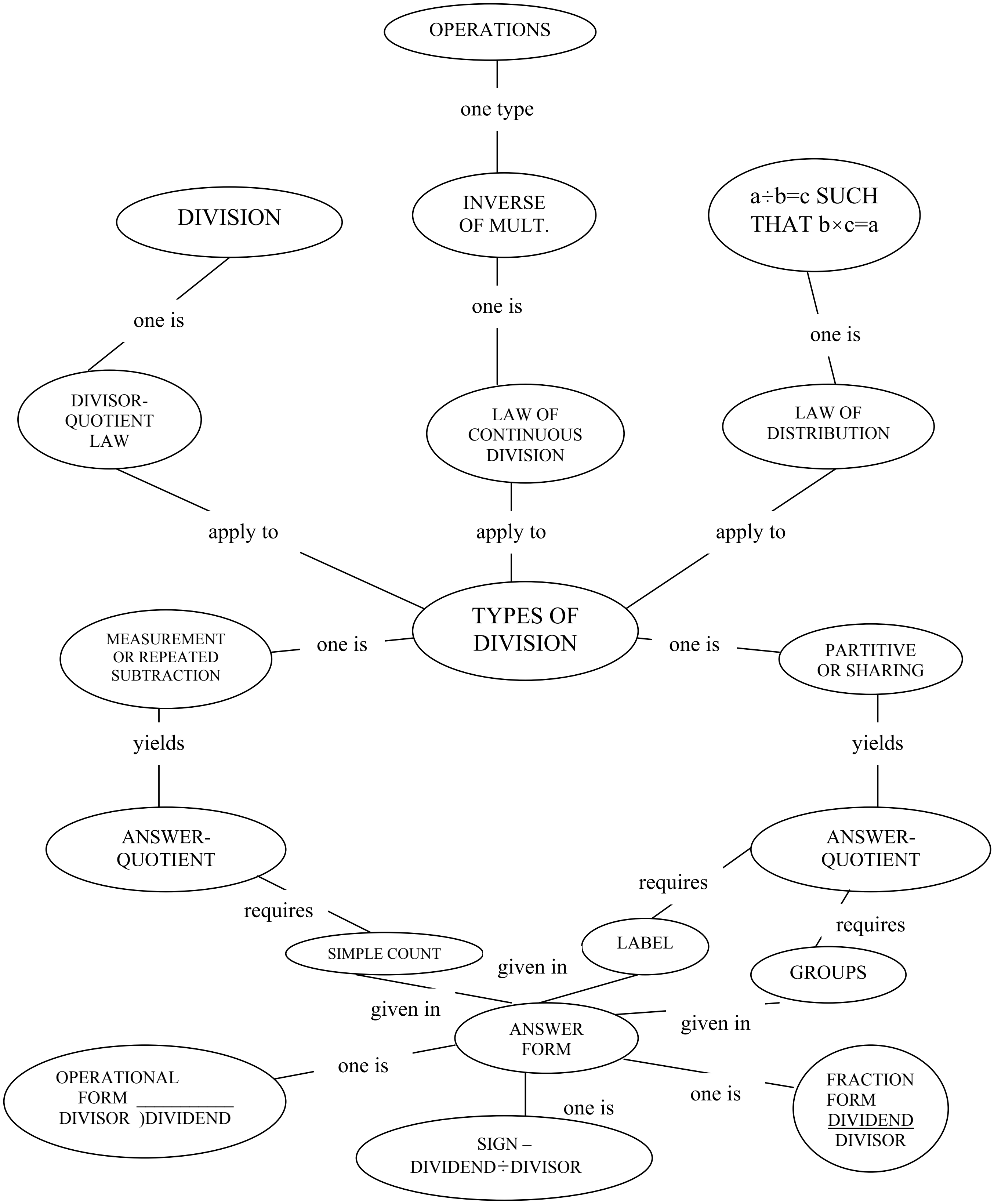


Figure 11. High scoring map which demonstrates the complex groupings used in higher scoring maps

Wallace (1990) investigates the dimensions of the subject-matter and pedagogical content knowledge of four geometry teachers. Wallace observed each teacher for a period of two weeks, and each teacher responded to interviews and a variety of tasks designed "to elicit information about the content and organization of their subject-matter and pedagogical content knowledge in geometry." Each geometry teacher constructed a concept map, planned simulated teaching unit, and sorted geometry problems according to perceived student difficulty and to connections. Wallace determined each teacher had learned pedagogical knowledge from high school or college mathematics teachers, but they learned more about geometry from colleagues, workshops and other inservice activities. However, Wallace reports none of the participants ascribe their pedagogical knowledge to undergraduate mathematics methods courses.

The concept maps Wallace requests her subjects construct follow a different construction procedure than normally accepted. She offers the geometry teachers a list of 24 labels, and subsequently asks them to position the labels on a sheet of paper. Her instructions follow:

1. If you find two concepts that are identical, put them next to each other and draw a box around them.

2. When you see two concepts that are related place them in such as way that you can eventually draw lines connecting them.
3. If you find a term that is not connected to any other, isolate it.
4. If you find a term that is unfamiliar to you, isolate it and cross it out.
5. When you are satisfied with your arrangement, attach the labels to the paper and draw the connecting lines to make a conceptual map.
6. Now place a number on each line and write sentences explaining the relationship you have indicated. Try to reconstruct for me your thinking in making this map.

(p. 233). An example of one of the geometry teachers' concept maps is located in Figure 12.

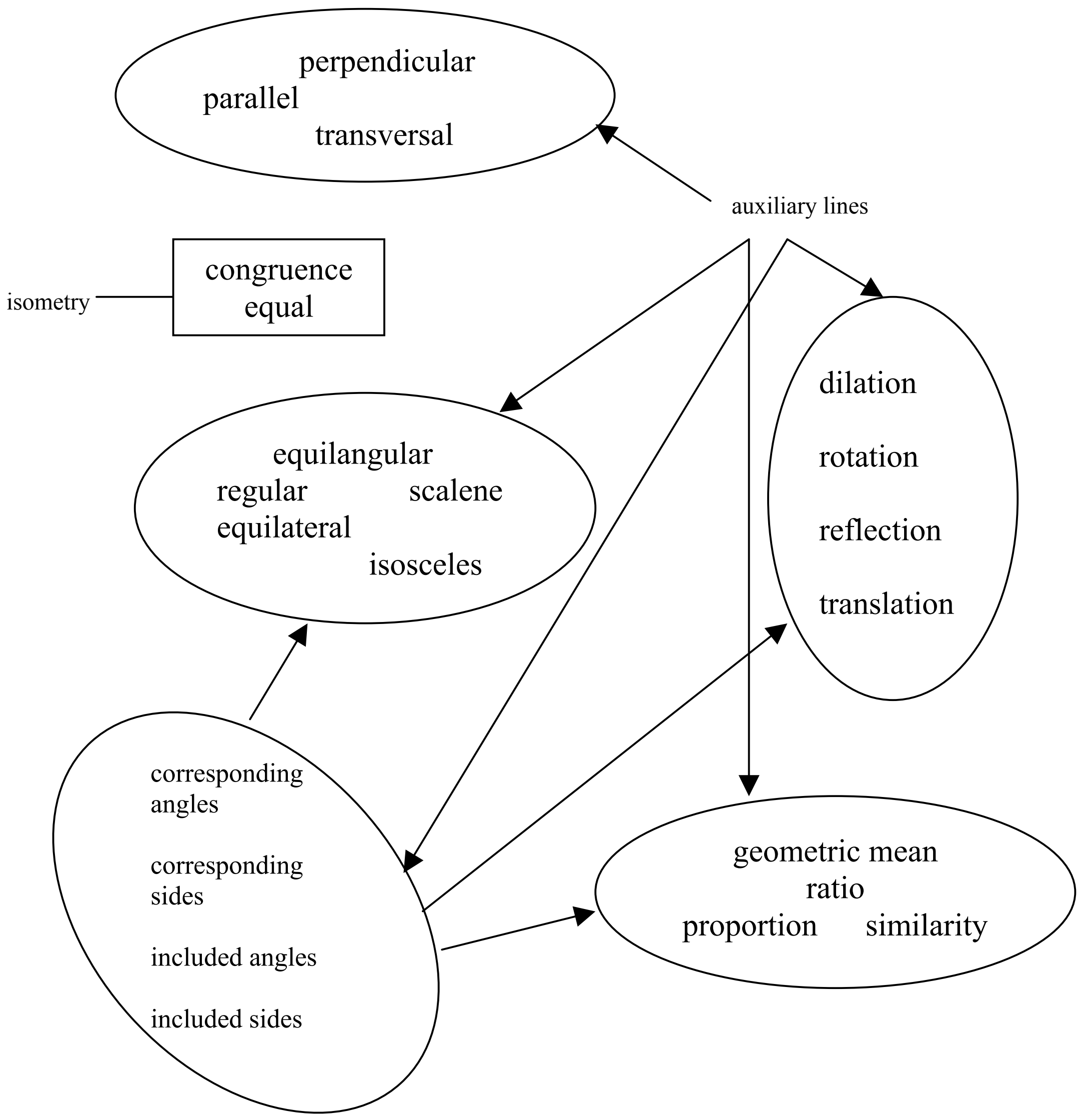


Figure 12. Chris' concept map for Wallace geometry teacher study

North Carolina aligns itself with many other states with the edict that all students graduate from high school with the equivalent of one year of Algebra I. Sjostrom (2000) examines the reasons for students' failure rates in Algebra I within a state requiring Algebra I for graduation. She focuses on

1. teachers' beliefs about the nature of algebra
2. teachers' attributions for student failure
3. teaching efficacy relative to student failure
4. means by which teachers' attributions influence their practices.

Sjostrom's qualitative study participants consisted of four metropolitan high school algebra teachers. The student population of the high school in which she executes the study has a "diverse student population." Sjostrom develops a concept map that provides a framework for analyzing the teacher attributions data. Her concept map implies that the teachers attribute their students' failures to the students; taking little or no responsibility. However, Sjostrom noted the "four teachers had low general teaching efficacy." Figure 13 shows the concept map she created (p. 88, Sjostrom, 2000).

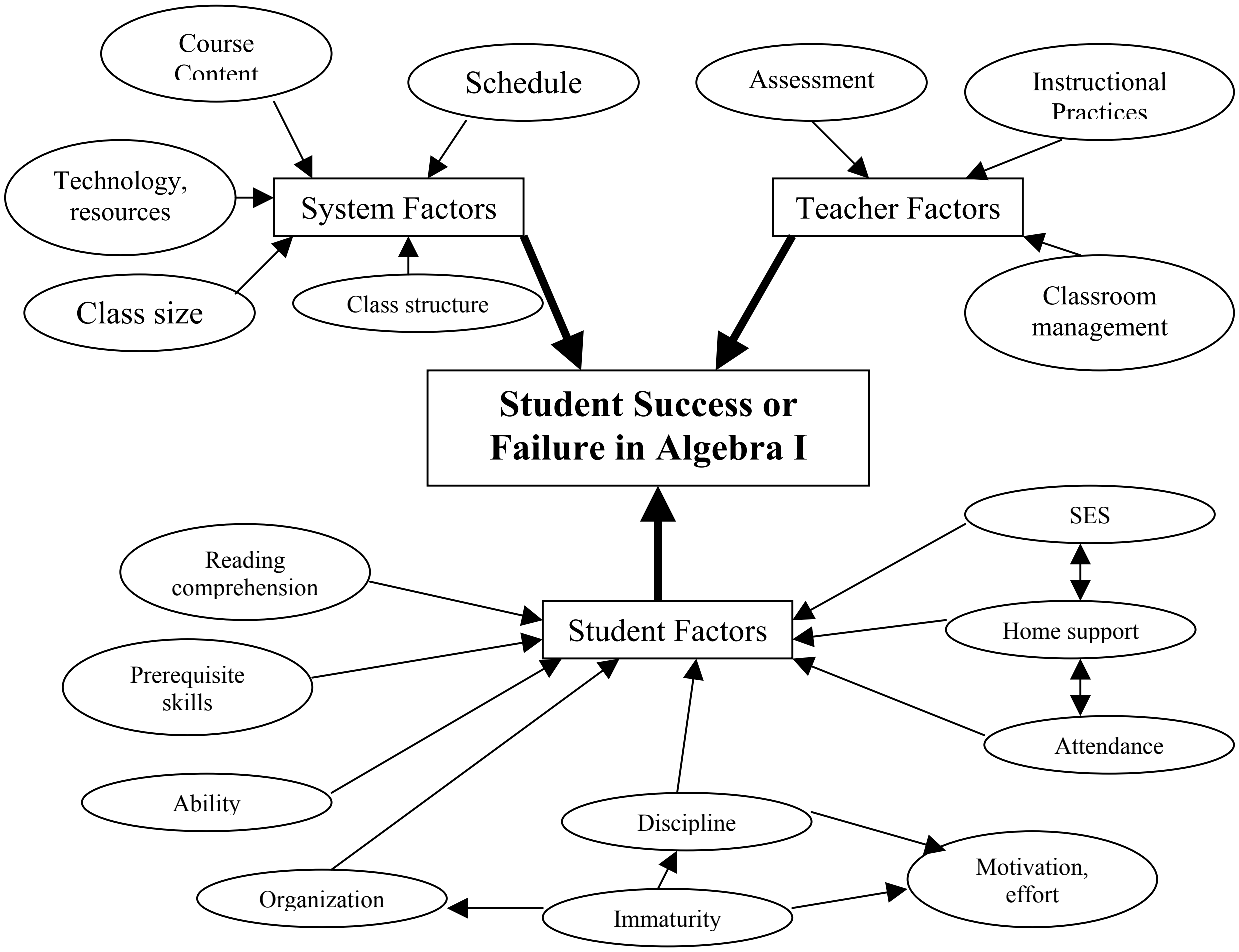


Figure 13. Sjostrom's concept map that classifies reasons for student failures in Algebra I

Baroody and Bartels (2000) recommend implementation of concept mapping as a teaching tool in secondary and post-secondary mathematics curricula, preservice mathematics education courses and in-service teacher education. Figures 14 and 15, respectively, illustrate a fourth graders concept of a geometric figure and a group of preservice teachers' concept of the real number system (p. 606 & p. 608).

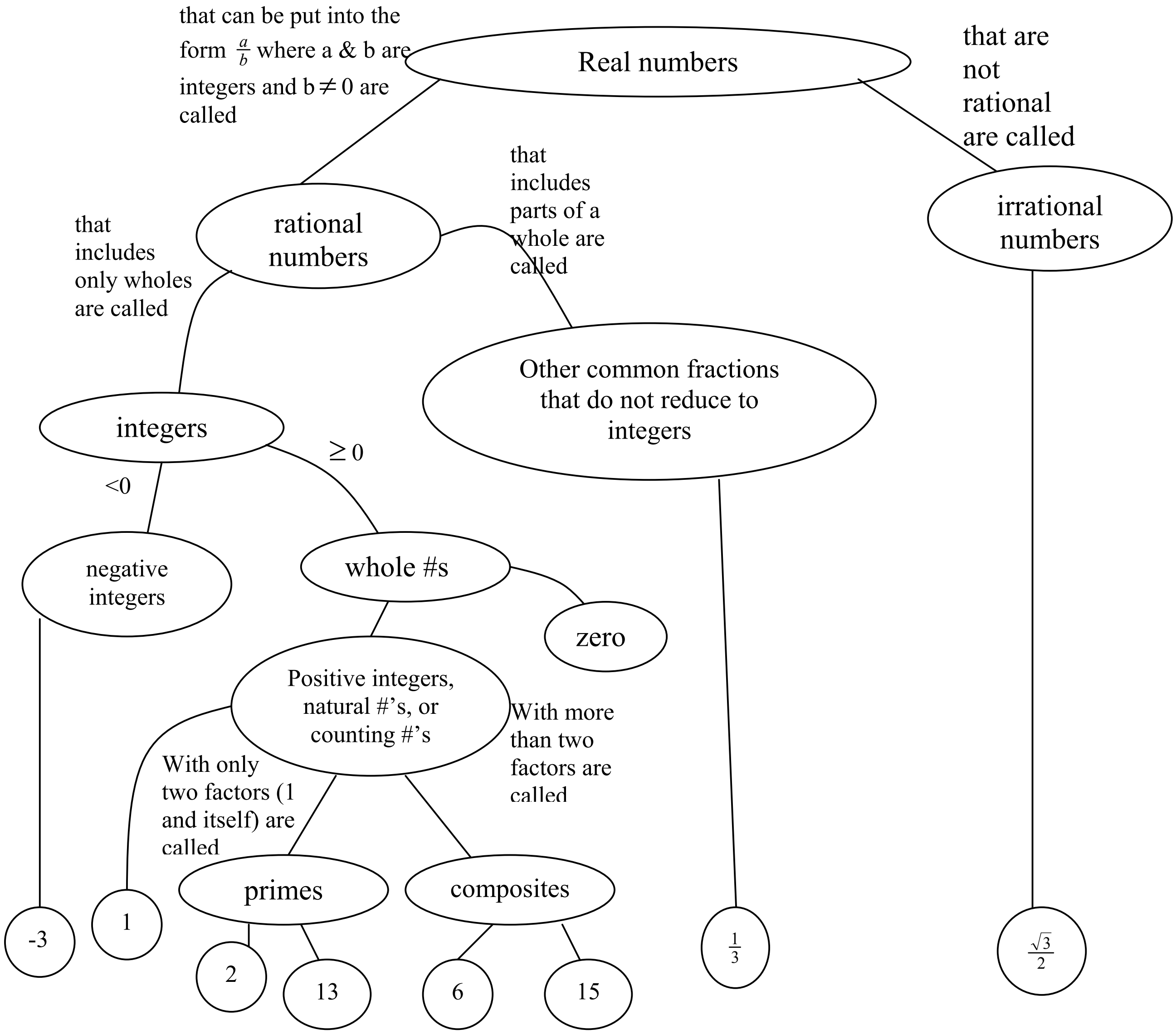


Figure 14. A hierarchical concept map of the real-number system drawn by preservice teachers.

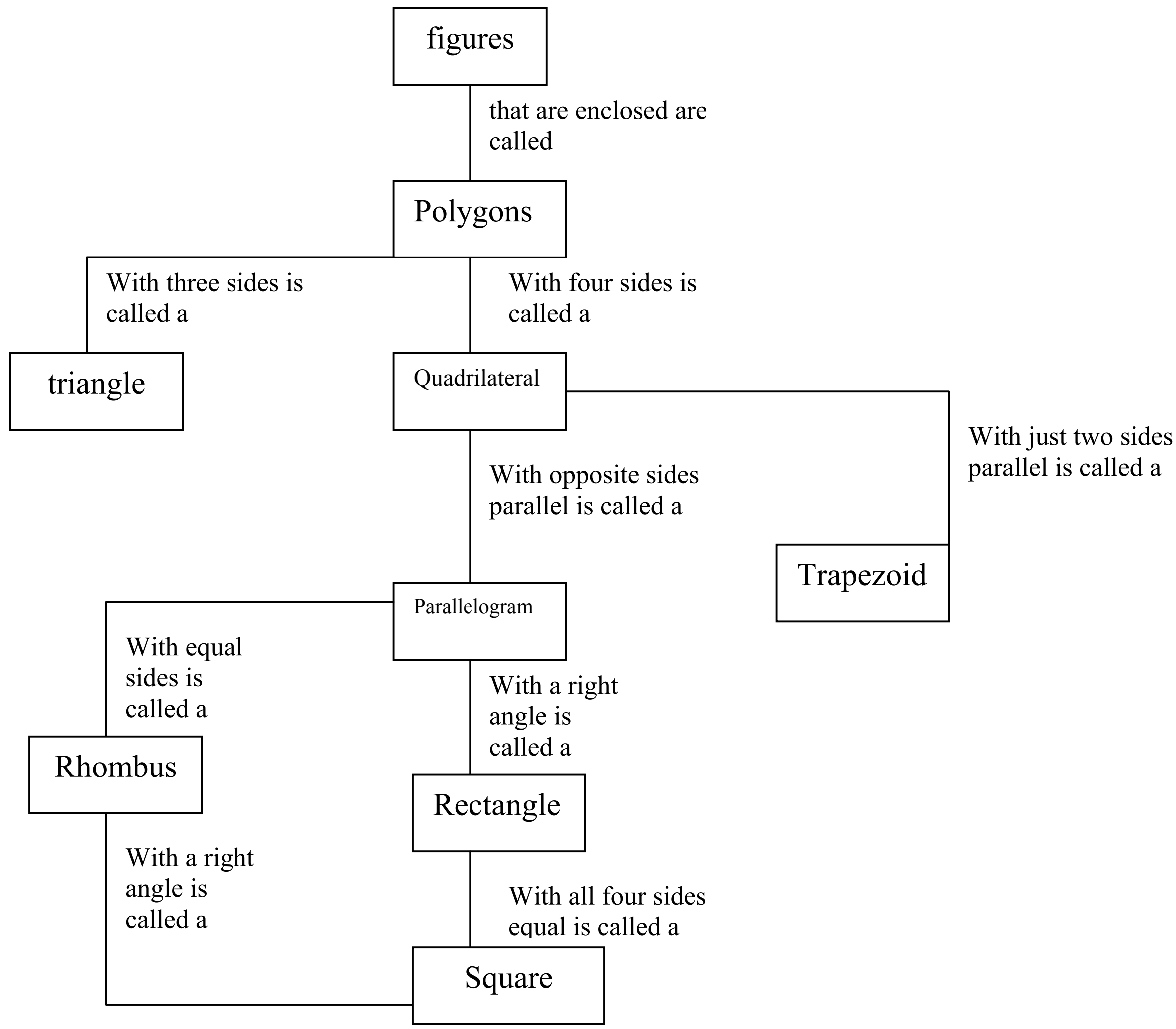


Figure 15. A fourth grader's concept map of some geometric concepts.

Baroody and Bartels claim mapping promotes inquiry-based learning in 10 ways. Concept mapping

1. serves as an excellent advance organizer (Ausubel)
2. encourages conscious construction of concepts (Bruner)
3. fosters metacognitive knowledge and autonomy (Piaget)
4. motivates conjecture making and testing (Polya)
5. underscores personal interpretation (Schoenfeld)
6. encourages engagement in logical reasoning
7. prompts problem solving (Polya/Schoenfeld)
8. incites dialogue and a perception of the construction of mathematical knowledge as a social process (Vygotsky)
9. promotes a view that knowledge is fluid
10. and necessitates introduction and practice of algebraic notation.

This author provides the associated researchers whose areas of expertise Baroody and Bartels mentions relative to the 10 areas concept mapping promotes.

Applications of Concept Mapping to Science

Novak conducted Cornell’s first comprehensive study of concept maps with junior high school students in 1983. The researcher’s primary goal concerned the process of seventh and eighth grade students in acquiring concept mapping skills. They found

1. Concept mapping improved over the span of the academic year
2. Students who used concept mapping vastly outperformed their counterparts on a test of novel problem solving
3. It took nearly two years for the seventh grade group to adapt to meaningful learning strategies.

In 1985, Basconas and Novak (Novak 1990) validated their claim that concept mapping in high school physics conjoined with "other educational strategies" (p. 42) led to superior achievement compared to students matriculating in a typical physics curriculum. Sherris and Kahle (1984), Okebukala (1990), and Pankratias and Keith (1987) produced significant results in high school biology studies and a ninth grade general science study, respectively (Novak 1990).

As previously mentioned, Cliburn uses concept maps as advance organizers. During a given unit of study, Cliburn makes "color-coded composite maps" (p. 215) that joins all the individual subtopic maps. For example, a concept colored green on the first general concept map consistently appears hilighted in green on subsequent, more specific maps. The color-coding emphasizes that learning depends on prior knowledge, and learning depends on the appropriate integration of such knowledge with newly acquired knowledge. Cliburn cautions students not to memorize maps, and furthermore, to develop their own. Although Cliburn reports students' positive reception of his concept maps, he asserts, "Students who produce original maps should be encouraged, and those originals should be evaluated" (p. 215). Recall that Novak commented the developer of the concept map benefits the most from its construction. Nevertheless, Cliburn discovered the maps generated more discussion in the experimental group, and the experimental group performed significantly better on his objective test of the skeletal system (Cliburn 1990).

Heinze-Fry and Novak (1990) investigated the use of concept mapping as an enhancement tool to promote meaningful learning in college autotutorial biology students. The biology course was self-paced, emphasizing conceptual development of students. They used a traditional biology text and a locally prepared study guide. There were 10 units of study, supplemented by the following: a list of readings, objectives requiring responses, and attendance to demonstrations in the Biology Study Center. Teaching assistants answered questions, created the tests, and gave feedback on areas of weakness at the end of the tests. Teaching assistants allowed retesting until students passed.

Twenty students enrolled in the self-paced course volunteered to try the concept mapping strategy. These students received handouts on concept mapping as a learning strategy, characteristics of concept maps, examples of good and bad maps, and directions on how to construct a concept map. The experimental group employed the strategy for the biological units of nutrition, gas exchange and transport. Only comparisons between the experimental and control groups were made for the third unit of study. The control group of 20 volunteers consisted of self-paced students whose SAT scores closely matched each of the 20 experimental subjects' scores.

Although Heinze-Fry and Novak reported no statistically significant differences between the two groups, they claim high-SAT and low-SAT mappers' retention and learning efficiencies benefited. According to two error analyses and student interview results, mapping enhanced clarity of learning, integration and retention of knowledge and transferability of knowledge to new situations.

Roth and Roychoudhury (1993) examined 27 students enrolled in a physics course for elementary education majors. Most of the students had been teaching or had a baccalaureate degree in another major area. Throughout the course, students participated in collaborative discussion groups of two to four students. The instructor used concept mapping to summarize main ideas for each of the assigned readings of the course text. Students learned how to construct concept maps according to the guidelines recommended by Novak and Gowin for students at the college level. During the course of the semester, the students collaborated on concept map construction, which summarized assigned chapter readings, expressions of theoretical backgrounds of experiments, and representations of their learning during these laboratory experiments as part of a Vee diagram (p. 238).

Roth and Roychoudhury discovered the number of background concepts nearly doubled. As time progressed, not only did quantity increase, the researchers noted significant increases in the quality of the propositions. The propositions were increasingly complete and meaningful. Furthermore, the researchers identified 81% of the students as having used their graphic representations as a tool for negotiating meaning within the collaborative context. One subject wrote, "...plus if you can explain a concept to someone else and they understand it, then you know that you really do understand it well" (p. 241). Finally, Roth and Roychoudhury reported that concept maps incited the preservice elementary teachers to reflect on the process of learning and teaching. They reiterate Tilgner's advice "that preservice teachers be taught in the same way as one expects them to teach in their own classes" (p. 243). Hence, we must practice what and how we teach.

Roth and Roychoudhury (1994) collected data over a two-year period at a private college preparatory school. The subjects ($N < 150$) attended either a junior introductory or senior second-year physics course. The researchers taught the physics courses during the progress of the study. They concluded, "The concept map has become a conscription device which engages students and teachers alike in an extended discourse toward the integration or reconstruction of knowledge" (p. 13). Essentially, collaborating members develop a source to which everyone refers during their dialogue. Hence, this research reinforces NCTM and AMATYC standards relative to communication and reasoning.

Sizmur and Osborne's (1997) sample consisted of children in Years 5 and 6 in England. Years 5 and 6 correspond to ages 9-11 year olds. Eighty-four students of a variety of SES participated in the study. The researchers selected classes whose teachers had already implemented concept mapping as a part of their science instruction. Sizmur and Osborne investigated the group interaction of students while they constructed concept maps for the topics: habits, earth in space, and sound and hearing. They found the dialogue produced by the students within the collaborative groups significantly differed from the more traditional triadic teaching exchanges. A triadic teaching exchange means the teacher questions, the students respond, and subsequently, the teacher evaluates. The most striking discovery Sizmur and Osborne (1997) made was the phenomenon of children building on each other's contributions. This is exactly the behavior of research community participants, and similar to the mathematics community as advocated by Alan Schoenfeld. Consequently, it appears the combination of collaborative learning and concept mapping is quite dynamic within the context of intermediate grades science.

Concept Map Assessment

Although the volume of research on concept mapping within the mathematics and science classroom environment increases over time, little of it focuses on concept map assessment. However, the research does not completely lack some notions of formal assessment.

Ruiz-Primo and Shavelson (1996) classify concept map scoring into three categories: (1) component scoring, (2) criterion map comparison and (3) a combination of component scoring and criterion map comparison. Novak and Gowin's (1984) scoring scheme, a type of component scoring procedure, accounts for the existence of propositions, hierarchy, crosslinks, and examples. They assign 1 point for each valid proposition, 5 points for each valid level of hierarchy, and 10 points for each valid and significant crosslink. Yet, they only award 2 points for a valid crosslink that lacks sufficient description of the relationship between the two segments of hierarchy. Novak and Gowin assign 1 point for each example. Several studies in concept mapping present scoring rubrics that modify the Novak and Gowin scoring scheme.

Malone and Dekkers (1984) offer a rubric for assessing concept maps containing five components. The five parts of their assessment rubric are concept recognition, grouping, hierarchy, branching and proposition. Malone and Dekkers provide a definition and corresponding point values for each of these five components.

First, they define concepts almost exactly as Novak defines the term, "Concepts are objects, events, situations or properties of things that are designated by a label or symbol" (p. 277). The assessor assigns one point for each concept connected to another concept by a proposition.

Secondly, Malone and Dekkers delineate groups based on how students link concepts. Furthermore, they subdivide grouping into point grouping, open grouping and closed grouping. A point group is a subset of a concept map that consists of a single concept linked to more than one subconcept. Figure 16 offers a simple example.

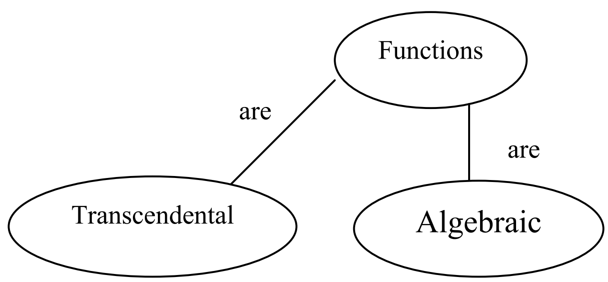


Figure 16. Simple Example of Point Group

An open group is a subset of a concept maps that contains three or more concepts linked in a single chain. Figure 17 displays an example of this type of grouping.

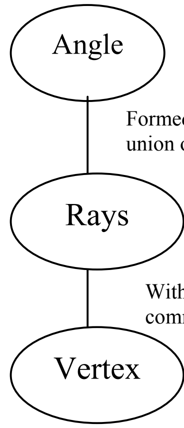


Figure 17. Elementary Example of Open Group

Malone and Dekkers circularly define a closed group. However, this author perceives a closed group as a subset of a concept map that contains three or more interrelated concepts. The set of emboldened ovals exemplifies a closed group in the elementary concept map located in Figure 18.

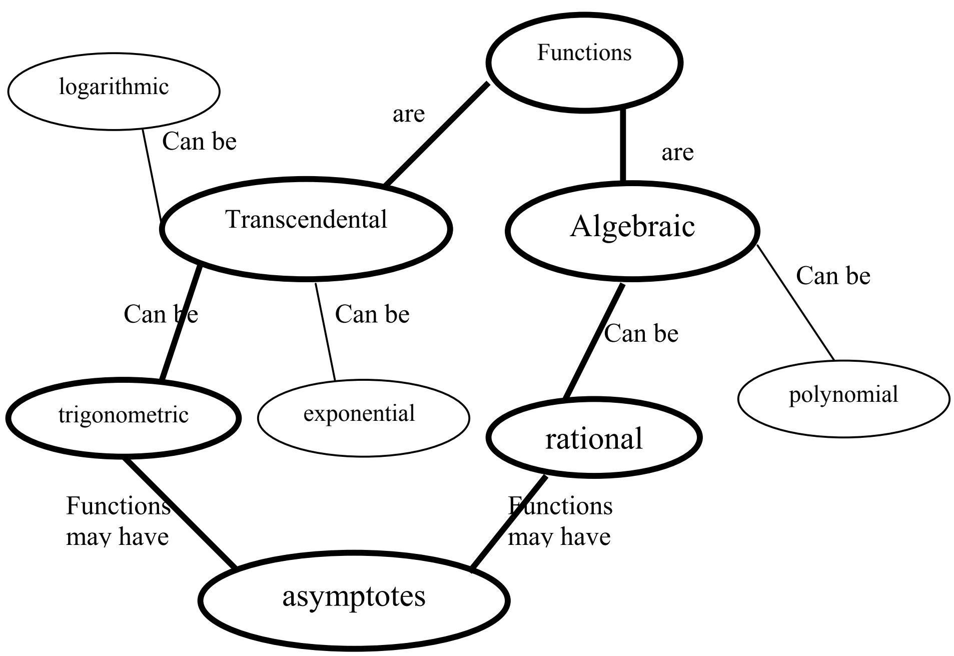


Figure 18. Closed-grouping for types of functions studied in precalculus.

The assessor of a concept map assigns 1 point for each concept contained within a point grouping, 2 points for each concept contained within an open grouping and 3 points for each concept contained within a closed grouping (Malone & Dekkers, 1984).

Thirdly, Malone and Dekkers define hierarchy as "a structure in which the more general, more inclusive concepts are at the top of the map; the more specific and exclusive concepts are at the lower end of the map" (p. 227). For the Malone and Dekkers scoring scheme, the authors suggest teachers give a list of selected terms and identify how many levels of hierarchy student maps should have. The assessor awards 4 points for each concept assigned to a level, 2 points for each concept on a level removed from an assigned level, and a score of 0 for concepts two levels removed.

Malone and Dekkers describe a proposition as a connecting word or connecting words or phrases written on a segment joining any two concepts. The authors partition proposition into two types: simple propositions and scientific propositions. A simple proposition is an English word or phrase, and a scientific proposition is a "phrase or statement composed of technical or scientific words” (p. 228). The assessor assigns one point for each new simple proposition viewed and one-half point for any subsequent use. The assessor doubles these point values for scientific propositions (i.e., 2 points for new scientific proposition and 1 point for any subsequent use). The notion of scientific proposition can be equivalently applied to mathematics vocabulary. Instead of naming the proposition “scientific”, one would call it “mathematical” within the context of a mathematics concept map.

Finally, Malone and Dekkers delineate branching as the extent to which a map connects more general concepts to more specific concepts. The assessor awards one point for each concept with two or more distinct links with different propositions.

Stensvold and Wilson’s (1990) rubric for scoring concept maps is elementary compared to that of Malone and Dekkers'. Their assessors count the number of concept words and the number of researcher-determined segments between the concepts (otherwise known as propositions) for each map. Stensvold and Wilson fail to distinguish between a concept consisting of one word and “concept words.” This author assumes “concept word” means either a one-word concept or a concept represented by more than one word.

The quotient of the number of concepts and appropriate links determines the Stensvold-Wilson map score.

Markham, Mintzes and Jones (1994) recommend a modified version of Novak’s scoring rubric. Table 2 displays the scoring scheme these researchers propose.

Table 2

Markham, Mintzes and Jones Scoring Scheme

Map Component	**Point Value of Component**
Valid Concept	1
Valid Relationship	1
First Branching	1
Successive Branching	3
Hierarchy	5
Crosslink	10
Example	1

A student’s branching ability reflects the student’s breadth of knowledge, commonly known as progressive differentiation (Markham, Mintzes & Jones 1994). A student’s ability to establish a hierarchy of concepts and subconcepts reflects the student’s depth of knowledge. Novak (1998a) terms this hierarchy of knowledge as subsumption. The existence of crosslinks in a concept map reveals the student’s ability to integrate knowledge, and examples show the student’s ability to be specific. Figure 16 displays a concept map constructed by Kelly, a freshman non-science major (p. 96, Markham, Mintzes, & Jones 1994). The coded map illustrates how a rater scores

Kelly's map. She receives 13 concept points (1 point for each of the 13 ovals), 12 relationship points (1 point for each of the link), 4 branching points (1 point for the first branching from MAMMAL and 3 points for the successive branching from BODY), 15 hierarchy points (5 points for each hierarchy: MAMMALS to BODY, BODY to MAMMARY GLAND, MAMMARY GLAND to MILK as indicated by the dashed segments), 0 crosslink points and 3 example points (BATS, WHALES, and PEOPLE). This yields a composite score of 47 for Kelly's map as shown in Figure 19.

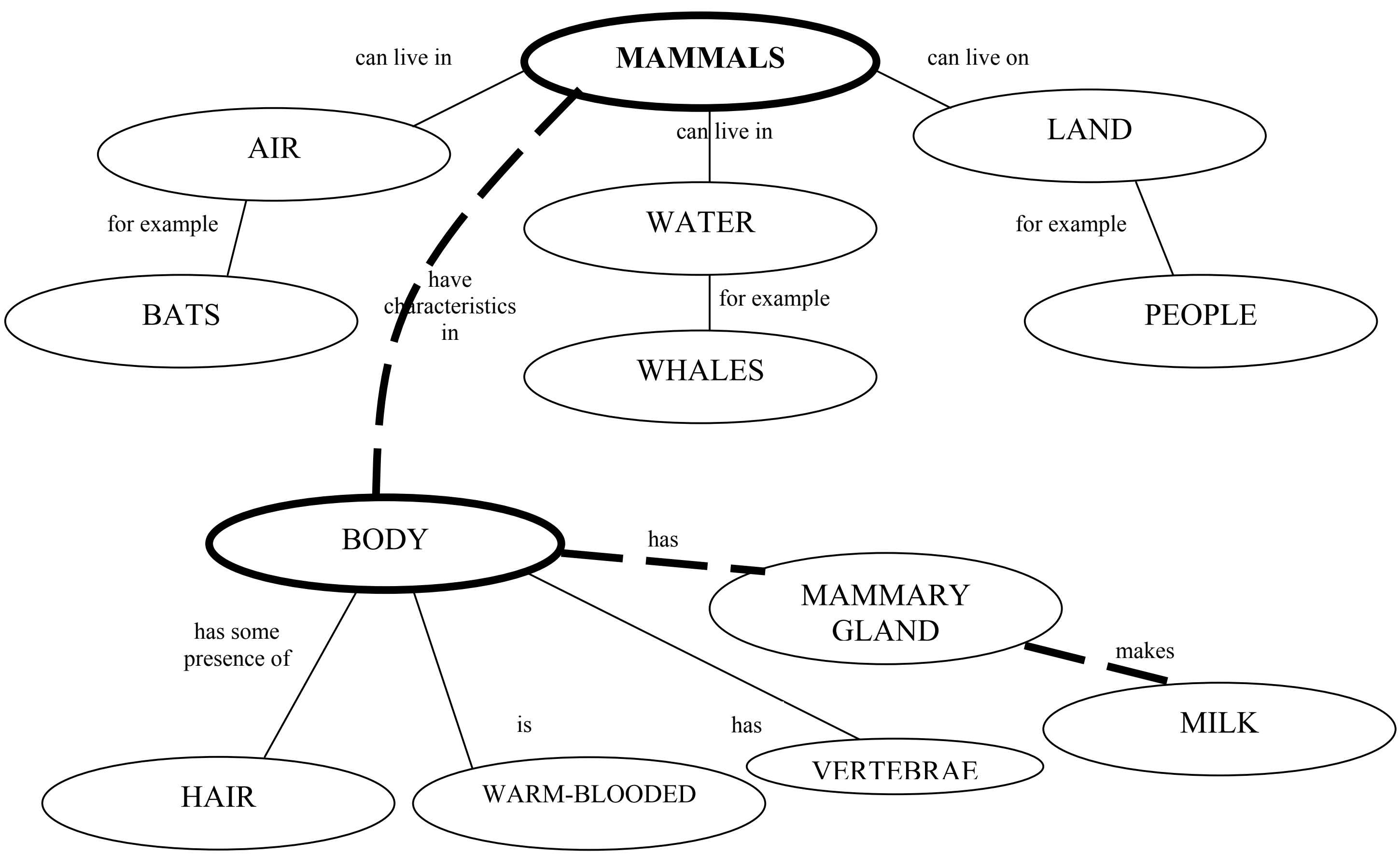
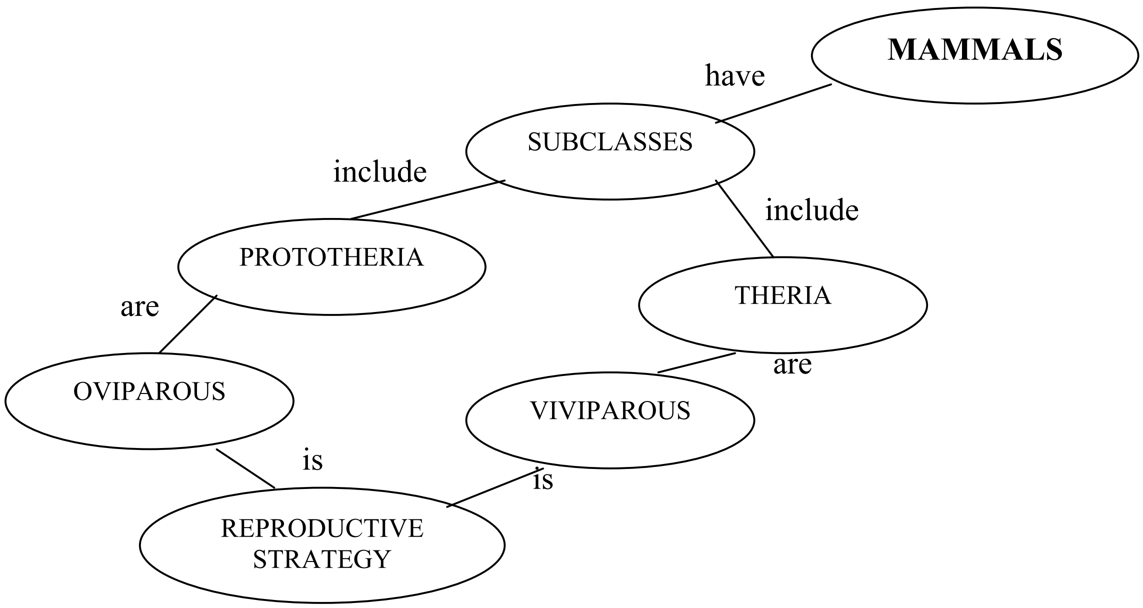


Figure 19. Concept Map: Kelly, a freshman non-science major.

Figure 20 displays a crosslink found in a graduate biology student's concept map (p. 96, Markham, Mintzes, & Jones 1994).



<u>Figure 20.</u> Graduate Biology Student's (Kevin) Concept Map Excerpt of a Crosslink

Ruiz-Primo and Shavelson (1996) recommend criterion map comparison as a scoring scheme above that of component scoring. Usually, concept map assessors rely on one or more discipline experts, one or more teachers, or maps from one or more top students to develop a criterion map. The Lomask Scale (1992) is a scoring rubric that relies on a criterion map. This rubric creates two scores based on the following formulae:

$$\text{size quotient} = \frac{\text{number of concepts on student map matching criterion map}}{\text{number of concepts on criterion map}}$$

$$\text{strength quotient} = \frac{\text{number of necessary and accurate connections}}{\text{number of map connections on criterion map}}$$

Subsequently, the raters compare these quotients to the Lomask matrix of size and strength to assign a composite score to each map. Table 3 displays the Lomask matrix (p. 583, Ruiz-Primo & Shavelson, 1996).

Table 3

Lomask Matrix of Composite Scores Based on Combinations of Size and Strength of Students' Concept Maps

		Strength			
Size	Size Percent	Strong (100%)	Medium (50-99%)	Weak (1-49%)	None (0%)
Complete	100	5	4	3	2
Substantial	67-99	4	3	2	1
Partial	33-66	3	2	1	1
Small	1-32	2	1	1	1
None/ irrelevant	0	1	1	1	1

Suppose a concept map has 75% as a size percent score and 50% as a strength percent score. The rater assigns a composite score of 3 to the map.

McClure and Bell's (1990) technique of relational scoring is based on the accuracy of concept map propositions. Relational scoring requires arrows indicating the relationship between concepts (e.g., cause and effect). Given a proposition, if there is no relationship between the concepts of the proposition, then the rater assigns 0 points to the link. If there is a label for the relationship, but it is inaccurate, then the rater assigns 1 point to the link. If the label is correct, but the proposition omits the arrow or the arrow fails to accurately indicate a hierarchical, causal or sequential relationship between the concepts of the proposition compatible with the label, then the rater assigns 2 points to the proposition. The rater assigns 3 points to a proposition with a valid relationship, label and direction. The composite score consists of the sum of all the separate propositional scores. An example of a directed map can be found in Figure 9, the calculus student's map for differential and integral calculus.

Mintzes, Wandersee and Novak (1997) recommend using Wandersee and Trowbridge's Standard Concept Map Checklist prior to or in lieu of a quantitative assessment. This assessment is easier to read and requires very little interpretation. Table 4 enumerates the items on the

Wandersee and Trowbridge Checklist (Mintzes, Wandersee and Novak 1997, p. 122).

Table 4

Wandersee and Trowbridge Standard Concept Map Checklist

Items to consider when evaluating a concept map (micromap)
1. Does the map contain five seed concepts provided by the instructor?
2. Are all the links between the concepts precisely linked?
3. Does the map have any labeled cross-links?
4. Does the map also contain examples (preferably novel examples)?
5. Is the map treelike (dendritic) instead of stringy (linear)?
6. Is the superordinate concept the best choice, given the way the rest of the concepts [are] arranged?
7. Are the examples included appropriate?
8. Is the map of acceptable scientific quality?
9. Has the mapper used the proper map symbols and followed standard mapping conventions?
10. Is the map limited to approximately 12 elements?

Reliability and Validity

Reliability of Concept Mapping

Ruiz-Primo and Shavelson (1996) define reliability as the consistency of scores assigned to students' concept maps. Few concept map researchers devote much time or resources to studying the reliability of concept mapping. However, Ruiz-Primo and Shavelson report reliability results for some studies executed within the last decade.

Although they report reliability results of some studies, the procedures used to establish reliability remain virtually unreported.

Researchers who reported reliability results focused on interrater reliability. Berenholz and Tamir (1992) reported a percentage of agreement exceeding 80%, yet they failed to give a procedure used to estimate this figure. In the Lomask study (1992), the authors computed three reliability coefficients: r = 0.87 for the number of concepts, r = 0.81 for correct connections, and r = 0.72 for expected connections of the map. McClure, Sonak and Suen (1999) report a g-coefficient reliability score of r = 0.76 for relational scoring with a master map. Relational scoring is a procedure for rating propositional relationships of a concept map, similar to McClure and Bell's technique.

Shaka and Binter (1996) provide one of the most extensive discussions of a procedure for the establishment of interrater reliability. Six raters who participated in the reliability study scored concept maps created by elementary science methods students. Shaka and Bitner (university professors), two secondary science preservice teachers, an elementary school teacher and a middle grades teacher scored 10 maps randomly selected from the larger set of concept maps. The researchers used the Student SYSTAT Program to compute the correlation between scores for seven concept map attributes and total map score. The seven attributes are enumerated in Table 5 (Shaka & Bitner 1996, p. 13).

Table 5

Seven Concept Map Attributes for Scoring Concept Maps

1. Propositions
2. Hierarchy
3. Branches
4. Crosslinks
5. Examples
6. Degree of conceptualization (indicates how well the superordinate concept and its connected subordinate concepts are understood)
7. Differentiation of concepts (elaboration of subconcepts or subordinates within each branch)

Each of the 6 raters used a scale of 0 to 4 points from which to assign a score for each of the 7 attributes for each of the 10 concept maps. In addition, they computed a Pearson correlation matrix for each of the seven attributes. Table 6 displays the correlation matrix for total scores of the two researchers (university professors) and the two preservice secondary science majors (Shaka & Bitner 1996, p. 13).

Table 6

Interrater correlations for

Science Concept Map Scores

	Rater 1	Rater 2	Rater 3	Rater 4
Rater 1	1.00	.76**	.84***	.82***
Rater 2		1.00	.82***	.61*
Rater 3			1.00	.85***
Rater 4				1.00

It is worth noting that many correlations between the secondary science majors and the two schoolteachers were either deemed insignificant or even slightly negatively correlated.

Validity

Ruiz-Primo and Shavelson (1996) define validity as "the extent to which inferences to students' cognitive structures, on the basis of their concept map scores, can be supported logically or empirically" (p. 592). For most concept map studies, independent discipline experts or teachers establish content validity for concept mapping as appropriate to the respective content area. Researchers establish concurrent validity by correlating map scores with achievement measures, examination scores, or assessment scores for logical thinking. Correlations between concept map scores and school achievement measures range from r = 0.49 for a standardized science assessment to $r = 0.74$ for the Otis-Lennon (Anderson & Huang 1989) and from $r = -0.02$ for SAT Reading to $r = 0.34$ SCAT verbal (Novak 1983).

CHAPTER 4

Methodology

Research Hypotheses

As long ago as 1998 (Chilcoat, Wilcox & Saloff), there have been calls for more research on the use of mathematics student-generated concept maps for constructivist learning, as opposed to the use of teacher-generated concept maps. Hence, this study attempts to respond to this call, and thus, addresses the following four questions:

1. Does concept mapping improve conceptual understanding of community college precalculus students relative to inverse functions?
2. Do mapping, pretest performance, inverse function items on assessments, and demographic variables, such as employment status, language and gender affect student achievement?
3. Does concept mapping improve community college precalculus students' attitudes and beliefs about mathematics?

This research addresses the following null hypotheses relative community college precalculus students.

H_{0_1}: After treatment, no significant differences exist between the experimental and control groups as measured by selected conceptual inverse function items on Unit #4 test and components of a routinely assigned writing exercise.

H_{0_2}: After treatment the general linear model will have coefficients $\beta_i = 0, 1 \le i \le 7$, where X_1 = Inverse Function Concept Map Score, X_2 = Test #4 inverse function items subscore, X_3 = writing assignment score, X_4 = final examination inverse function items subscore, X_5 = employment status, X_6 = language and X_7 = gender.

H_{0_3}: After treatment, no significant differences exist in student beliefs about mathematics for the experimental group as measured by the Aiken Revised Attitude Mathematics Scale.

Variables

This quasi-experimental study focuses on the use of student-generated concept maps. The immediate goal of this study is to determine whether or not implementation of concept mapping affects students' conceptual understanding of inverse functions. Therefore, the dependent variables consist of two map scores: a first complete individual concept map score on "Inverse", and a second complete individual concept map score on "Functional Inverse."

Other variables, such as age, gender, employment status, attitude toward mathematics, precalculus course pretest scores, and first three course tests might impact on the results of this study. Other dependent variables in this experiment consists of subscores for inverse items on Unit Test #4, inverse items on the final examination and scores from the writing exercise on inverse functions.

Research Design

Since random assignment of subjects to treatment groups could not be accomplished, due to the intact nature of the two community college precalculus classes, the researcher employs a modified pretest-posttest control group design as recommended by Campbell and Stanley (1963). The model for this design is

O_1	O_2	O_3	X_1	O_4	O_5	O_6	O_7	O_8
O_1	O_2	O_3					O_7	O_8

O_1: Pre-Mathematics Attitude Survey

O_2: Precalculus Pretest Diagnostic Score

O_3: Demographic Observations

X_1: Learning Concept Mapping

O_4: Group Concept Map Score

O_5: First Individual Concept Map Score

O_6: Second Individual Concept Map Score

O_7: Written Assignment Score

O_8: Unit Test Functional Inverse Subscores

O_9: Final Examination Functional Inverse Subscores

Instrumentation

The researcher employed a variety of instruments to collect valuable data relevant to this study. On the first day of class, all precalculus students took a diagnostic assessment as an informal predictor of their success in the course (Appendix C). At the end of the first class meeting, the instructor of the precalculus class distributed an information data sheet entitled, "Who Am I?" Near the end of the course, students completed a follow-up demographic questionnaire to update the data from "Who Am I?" The students completed the data sheets and returned them to the instructor at the beginning of the next class meeting. University Transfer Department guidelines strongly suggest that instructors issue the diagnostic assessment and the student information sheet to all students on the first day of class.

Prior to beginning the experiment, each student enrolled in experimental sections complete the Aiken Revised Math Attitude Scale (Appendix B). Each subject from the experimental group completed the assessment a second time, following the fourth unit test. This reliable and valid assessment consists of 24 statements about attitudes and beliefs relative to studying mathematics. Each item offered 5 consistent choices among which the subject selected only one. These selections are strongly agree, agree, undecided, disagree and strongly disagree. The maximum score is 48 and the minimum score is −48.

During the experimental period, experimental subjects learned about concept mapping. On the day in which concept the instructor introduced concept mapping, the instructor provided the subjects handouts entitled, "Concept Map Construction", "A Hierarchical Concept Map of the Real Number System", "Scoring Rubric for Your Concept Maps", and "Concept Map Requirements" (Appendices E-H). The first handout defined the parts of a concept map. The second handout provided subjects an example of other undergraduates' group concept map. The third one informed subjects of how their instructor would score their group and individual maps. Finally, the last handout detailed the order in which subjects submit concept mapping related assignments to their instructor.

The final four instruments consisted of a writing assignment on functions completed by both groups by virtue of being in the precalculus class, selected items from the fourth unit test, a follow-up questionnaire on concept mapping and inverse functions, and selected items from the final examination. Students had 2 ½ weeks to complete the writing assignment. They took Unit Test #4 during the first week of May, completed the follow-up survey on concept mapping at the beginning of the second week of May, and they took their final examination at the end of the week.

The qualitative portion of this study required data solicited by three instruments: the concept mapping criterion construction group (expert group), group concept maps on systems of equations with subjects' supporting documentation, and subjects' responses to the concept mapping follow-up survey. Although the researcher intended to conduct interviews, the lack of time at the end of the semester preempted them. All items from the intended interviews were distributed in the form of the survey, and the instructor provided ample time for experimental subjects to respond. The follow-up survey is located in Appendix D.

Population and Study Environment

The experimental group in this study consisted of students (**N** = 36) enrolled in the first semester of a sequence of two precalculus courses. Fifteen subjects included in this study who attended the experimental section completed all concept-mapping activities and finished the course. The remaining 21 subjects were enrolled in the control section. The course was named MAT 171, Precalculus Algebra. Precalculus Algebra is a 5-contact hour course, offered at many of the 58 community colleges of the North Carolina System of Community Colleges. The class met for 16 weeks, yielding a total of 80 contact hours. The exact course description is given below:

> This is the first of two courses designed to emphasize topics that are fundamental to the study of calculus. Emphasis is on equations and inequalities; functions (linear, polynomial, and rational); and systems of equations and inequalities. Upon completion, students should be able to solve practical problems and use appropriate models for analysis and predictions. Additional topics include, but are not limited to, exponential and logarithmic functions and their applications... Upon completion, students should be able to solve problems, apply critical thinking, work in teams, and communicate effectively. This course has been approved to satisfy the Comprehensive Articulation Agreement [CAA] for the general education core requirement in natural sciences and mathematics.

This course is a transferable course per the North Carolina University and Community College Articulation Agreement. However, a few major institutions only grant elective credit for this course. Although the CAA addresses the transfer of credits among the North Carolina Community

College System or constituent institutions of the University of North Carolina, the CAA does not address admissions policies of the constituent institutions of the University of North Carolina.

General performance measures for Fall 1998 (1) transfers from North Carolina community colleges to the constituent institutions of the University of North Carolina and (2) transfers from the community college (where this research took place) to constituent institutions of the University of North Carolina are located in Appendix A. Most of the students who transfer from the community college in which this researcher selected a sample of mathematics students ultimately transfer to North Carolina Central University, North Carolina State University or the University of North Carolina at Chapel Hill. This set of performance measures includes the number of transfers and the mean grade point averages for math and science courses combined for each constituent university. According to the data, 65% of the fall 1998 transfers from the community college in this study to North Carolina State University took a math or science course and the composite grade point average was 2.35. Although this information might be helpful, the performance measures fail to distinguish between

1. math and science performance
2. the success of transfers who had completed their mathematics requirements at the community college rather than at the university.

Of interest to this study would have been the number of students accepted for transfer into a constituent university with just having completed one or both of the precalculus courses prior to transfer. However, the General Administration of the University of North Carolina does not allow data with this much specificity available for public scrutiny. Most students enrolled in the precalculus sequence are enrolled as associate in science degree students, and they intend on majoring in areas such as computer science or many of the varied engineering fields upon transferring into a senior university. Therefore, these students must take at least one year of calculus following this precalculus sequence.

The performance measure of mean grade point average for math and science courses of transfers from the community college in this study to UNC-Chapel Hill is abysmal at best. Yet, a larger proportion of students who transfer to UNC-Chapel Hill are associate in arts students; unlike those who transfer to North Carolina State University. The level of mathematics courses required for graduation requires a prerequisite level of high school algebra I. Yet, only 26% of the transfers took a mathematics or science course. Many of the students with high mathematics anxiety wait until after transferring to this liberal arts institution to take them. These performance measures on transfer students in conjunction with demographic data are crucial to understanding the precalculus student population.

The control group consisted of a set of transfer students attending a community college located in an urban area of central North Carolina with a population of approximately 175,000 people. Experimental sample participants attended their precalculus classes on Mondays, Wednesdays and Fridays from 8:00 a.m.-9:25 a.m., while control sample participants attended their classes on Tuesday and Thursday afternoons from 3:30 p.m.-5:40 p.m. Per the requirement of the president of the college, the researcher informed all participants of their participation in this study. The instructor of both groups distributed a participation consent form to all subjects and allowed any student to decline participation.

Procedure and Data Collection

Each group of subjects took a preliminary diagnostic test on the first class day. The results from this test were used to detect any significant differences in initial diagnostic means differ. Pretest sample items are located in Appendix C. On the same day the instructor distributed an information survey on which subjects from both groups provided their names, gender, last two mathematics courses taken (and grades received) prior to enrolling in the precalculus courses, GPA, last two English courses taken (and grades received), and any other pertinent information. Following the diagnostic test, the instructor distributed the Aiken Revised Attitude to Mathematics Scale as used in Taylor’s study (1997), which is a reliable and valid assessment that focuses on identifying attitudes toward mathematics. Subjects returned the attitude survey upon completion.

Before beginning the study, the instructor distributed a form that allowed the researcher to use any data related to this experiment for a doctoral dissertation. No subject declined participation in the study. If any subject had refused, then the alternate activity would have been a linear regression activity routinely assigned by the precalculus instructors.

The experimental section employed the concept mapping heuristic. The instructor provided instruction on the parts of a concept map, how to construct a concept map, the function of a concept map as a study tool, and the importance of the revision of the map during a particular unit of study.

Initially, the instructor offered an example of concept mapping during a period of expository teaching. The instructor was cautious to involve students in the development of the first concept map. Therefore, the instructor solicits a variety of mathematical concepts studied during the first part of the course. The instructor began a concept map by using the "Draw" functions of Microsoft Word. Furthermore, the instructor persistently solicited more subconcepts and writes them on the board. Next, he asked the subjects to arrange them underneath the main concept. Subsequently, the subjects offered linking phrases that created propositions relating the concepts.

After the structured classroom exercise, the instructor clearly explained the second handout, "A Hierarchical Concept Map of the Real Number System." He related each of the essential concept mapping components to the map. Next, he identified how the researcher would score their concept maps by referring to the handout, "Scoring Rubric for Your Concept Maps". As a part of the first assignment for concept mapping, the instructor directed the experimental subjects to use the given scoring rubric for scoring the Real Number Concept Map.

At the end of the introductory session on concept mapping, the instructor grouped students into trios to prepare the first concept map for submission. Prior to the class meeting, the instructor partitioned the experimental group into 3 subgroups based on the Unit #1 and Quiz #1 scores. Subgroup 1 consisted of the top third, subgroup 2 consisted of the middle third, and subgroup 3 consisted of the lower third. The instructor used the randint(lowerbound,upperbound) function on the TI-83+ graphing utility to randomly assign members to one of 8 groups of 3. Following the assignment of these groups, the instructor notified students to produce a concept map of "Systems of Equations" (specifically not mentioning the term *linear*). The instructor provided a seed list of five words or phrases for their maps on the handout entitled, "Concept Map Requirements."

During the subsequent class period, the instructor allowed subjects time to meet with respective group members for a brief comparison of their "Systems of Equations" concept maps. The instructor informed the groups of three to collaborate further outside of class time and to prepare a single group concept map for “Systems of Equations.” The groups submitted this map one week later.

During the normal course of the semester, students study at least two types of inverses, but this study shall be limited to inverse functions. The instructor did not provide lists of words during the course of the experiment. Instead, he expected subjects to follow the methods of construction as presented on the day the instructor introduced concept mapping to the experimental subjects. Two more maps were required of the subjects: an individual map on “Inverse” and an individual map on “Functional Inverse.” Subjects submitted the “Inverse” concept map approximately one week after the group assignment was due. The final map was submitted a week following instruction on inverse functions.

A team of three mathematics instructors from the community college at which this researcher conducted this study and two mathematics educators from an area university created a criterion concept map for “System of Equations” “Functional Inverse.” One of the two mathematics educators had some knowledge of concept mapping. Nevertheless, the researcher offered the same instruction to this expert group on concept mapping provided to the subjects in the experimental group. The criterion maps guided the researcher for scoring subjects' maps based on the Markham, Mintzes and Jones' Rubric for scoring concept maps.

Finally, the researcher collected data from Unit Test #4, the selected writing exercise on function behavior (Appendix K), selected final examination items (Appendix L), follow-up survey (Appendix D), expert group questionnaire (Appendix I), and the follow-up demographics instrument (Appendix M). The researcher collected data from graded assessments relevant to the topic of inverse functions only.

Data Analysis

For all variables in question, the researcher supplied pertinent descriptive statistics. Since it is possible that variables may not emanate from normally distributed populations, it was preferable to use distribution-free statistical tests (formerly known as nonparametric tests) that preempt false assumptions. A Chi-square, Anderson-Darling, and Carmer-von Mises Tests for normality were used to determine whether or not concept map scores and appropriate assessment scores were from normally distributed populations. If not, then the preferred statistical procedures needed to be distribution-free. Assuming normality (and other assumptions that shall be appropriately noted),

1. Chi-square procedures will identify any differences in demographic data, such as gender, language, employment status, and mathematics preparation prior to enrolling in the course,
2. Pairwise and unpaired t-tests or Kruskal-Wallis procedure identified differences between means for pertinent variables,
3. Multiple Linear Regression and Backward Elimination Procedures to determine which variables (gender, language, level of mathematics preparation, inverse concept map score, inverse function items subscore from Unit Test #4, and inverse function items subscore from the final examination) contribute to the general linear model for final course grade, and
4. ANOVA or Kruskal-Wallis and Ansari-Bradley procedures identified any significant differences in concept mapping scores based on subgroup comparisons.

Finally, the researcher coded the follow-up responses, selected items from Unit Test #4, and the expert group survey responses. The researcher reported any qualitative data that clearly supported or refuted the research hypotheses.

CHAPTER 5

Results

Development of the Criteria Maps

Three community college mathematics instructors, one mathematics educator responsible for mathematics teacher education at a centrally located university in North Carolina, and a college dean from the same university (formerly responsible for mathematics teacher education at the same university) collaborated to create two criterion concept maps: Systems of Equations and Inverse. Although the three mathematics instructors and one of the mathematics education professors had very limited exposure to concept mapping prior to the 2-hour session on concept mapping, one mathematics professor had modest professional exposure to concept mapping. However, none of these professionals ever implemented concept mapping in the classroom environment. During the 2-hour session, this researcher followed the same introductory lesson plan on concept map development that the experimental group underwent. The researcher distributed handouts delineating relevant terms that the researcher requested the five participants to use during the session.

First, the researcher guided the participants through their construction of a “tree” concept map. Following this exercise, the researcher instructed the group to create their first map employing the seed concept of "System of Equations." The participants used a large marker board on which to record their first map. One of the community college instructors agreed to serve as the scribe for both of their concept maps. During the construction of the first map, the participants politely argued about how to advance beyond the first level of hierarchy, yet they finally decided how to proceed with its construction. After many revisions, the group finally decided on the map in Figure 21. Subsequently, the researcher instructed the group to construct an "Inverse" concept map. The newly formed expert group needed no guidance regarding which kinds of inverses to include on their map, since all of them had taught a precalculus algebra or college algebra course, and they were cognizant of the traditional curriculum includes inverse functions. The map the participants constructed is located in Figure 24. However, in addition to these concept maps, this researcher provided alternate criteria maps for "Systems of Equations" (Figures 22 & 23) and for "Inverse" (Figure 24). Figure 25 shows a researcher-constructed concept map for Functional Inverse.

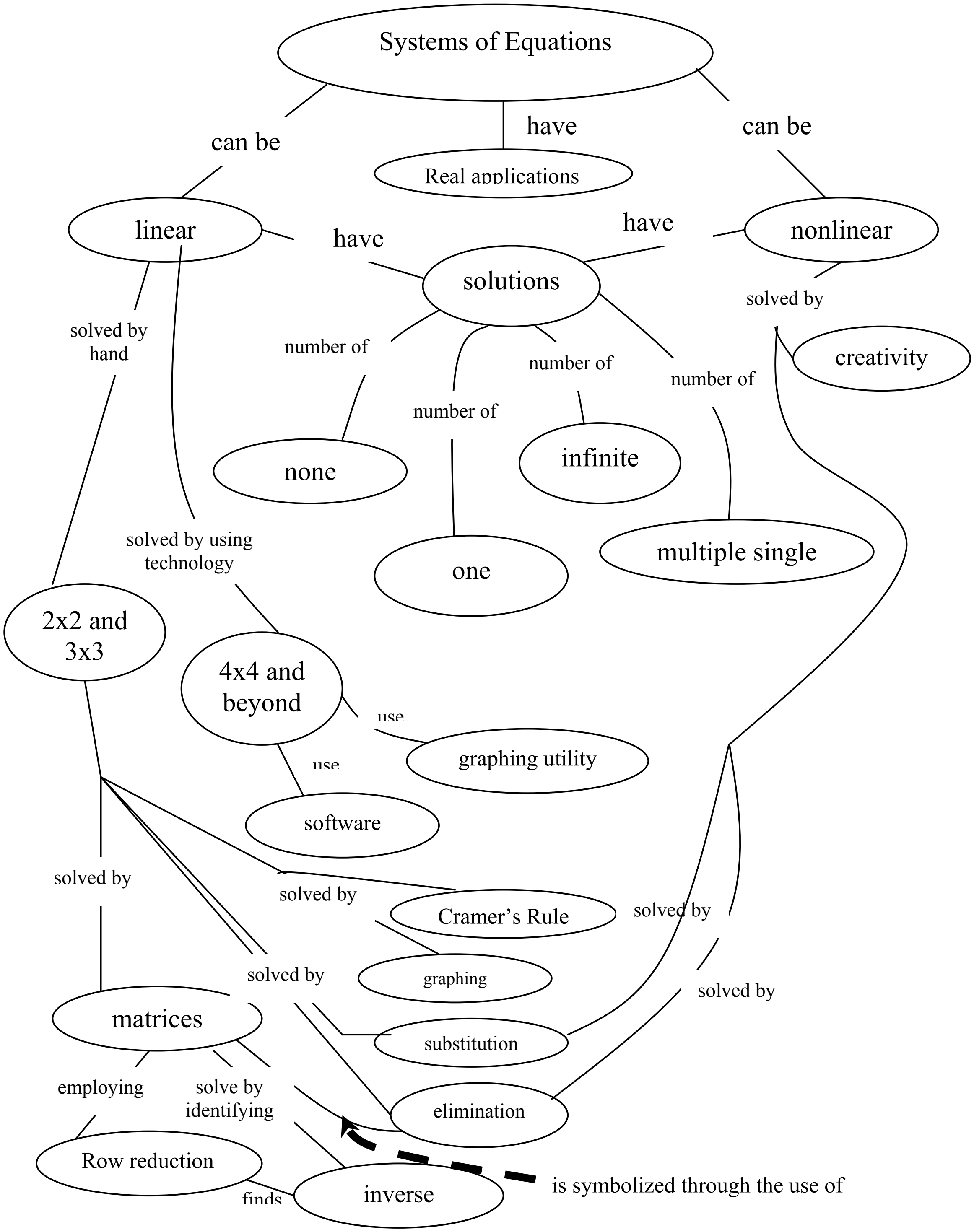
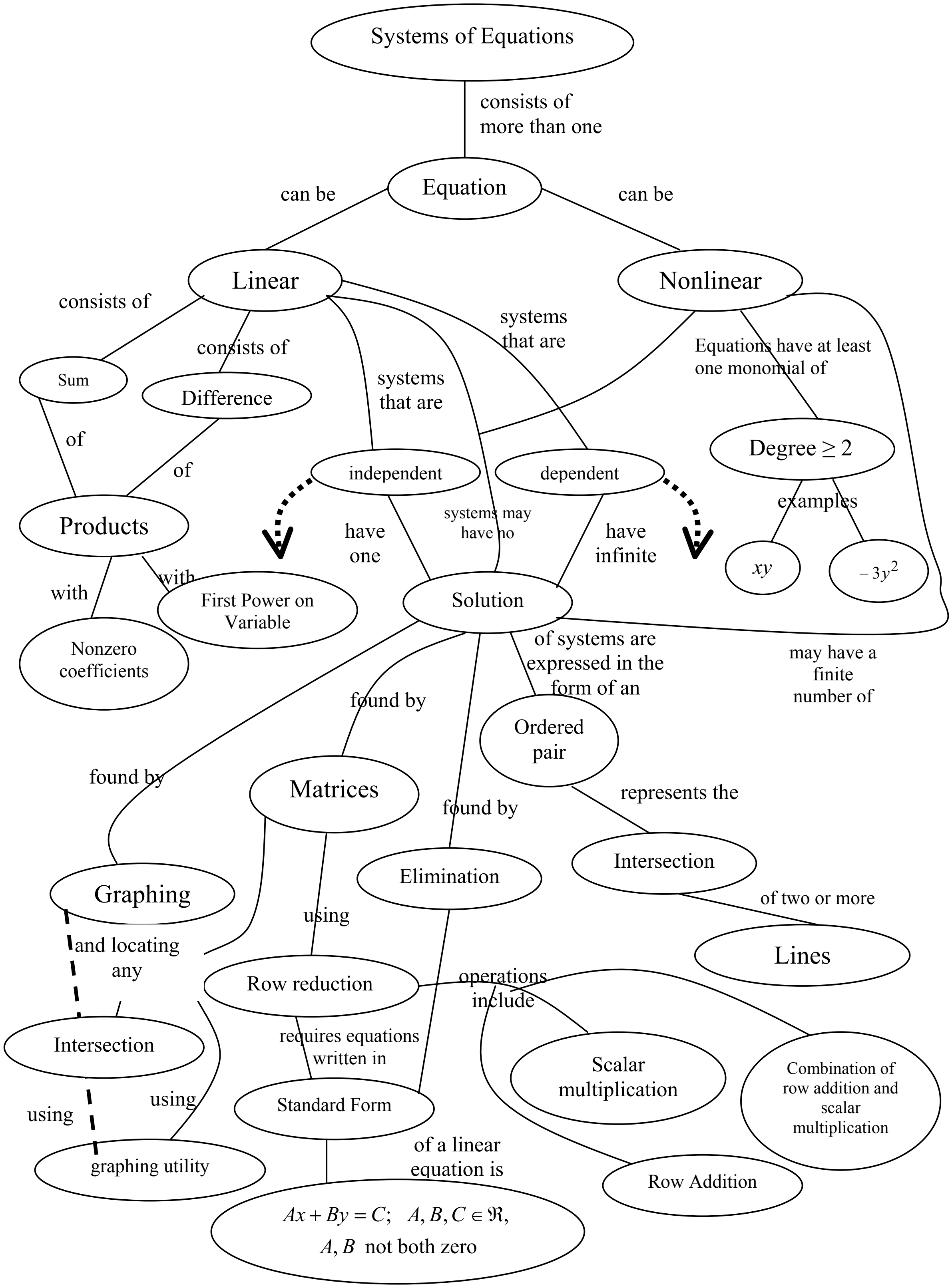
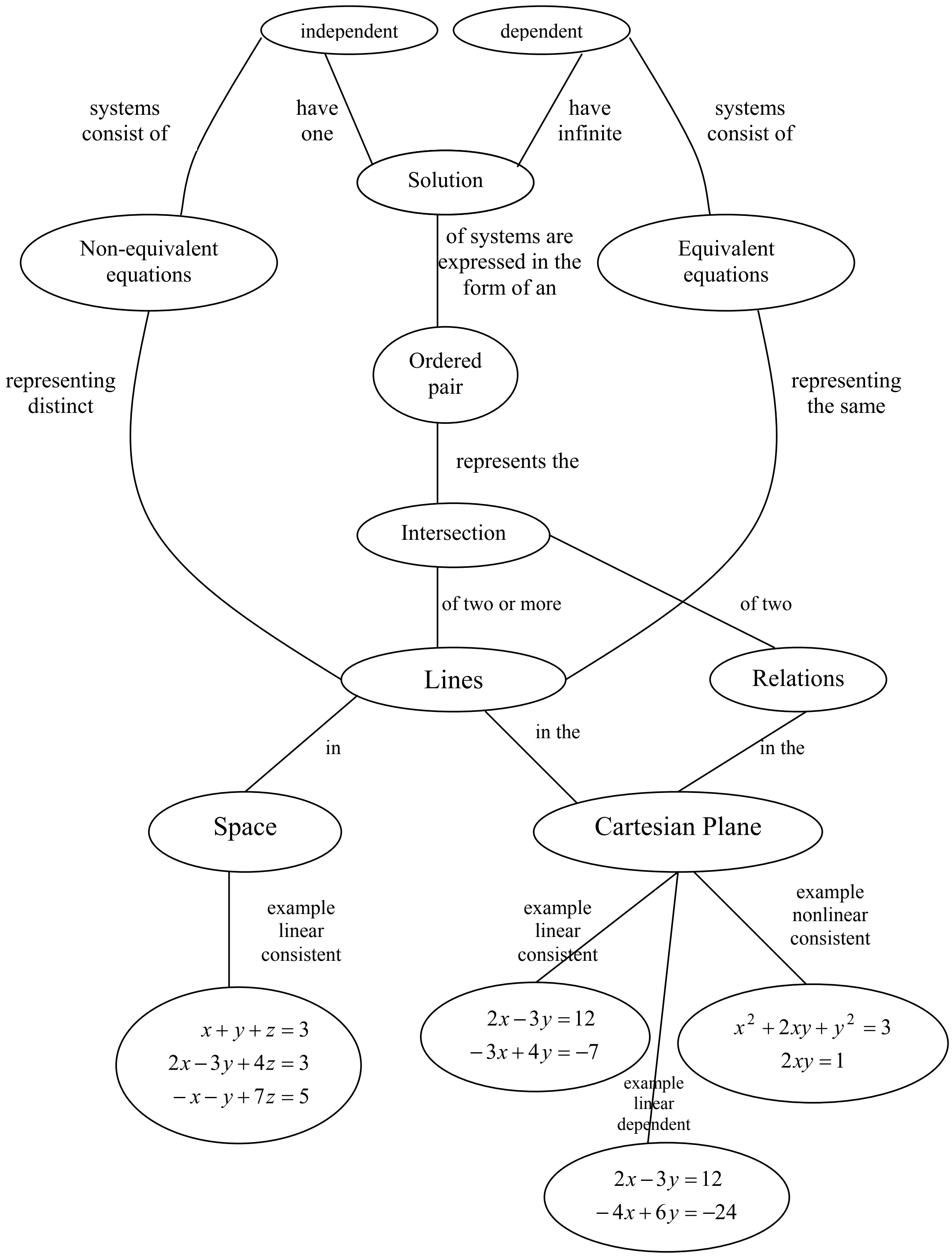


Figure 21. Criterion map for "Systems of Equations"



<u>Figure 22.</u> Researcher-Constructed Concept Map for "Systems of Equations"



<u>Figure 23.</u> Researcher-constructed concept map for "Systems of Equations" (Continued)

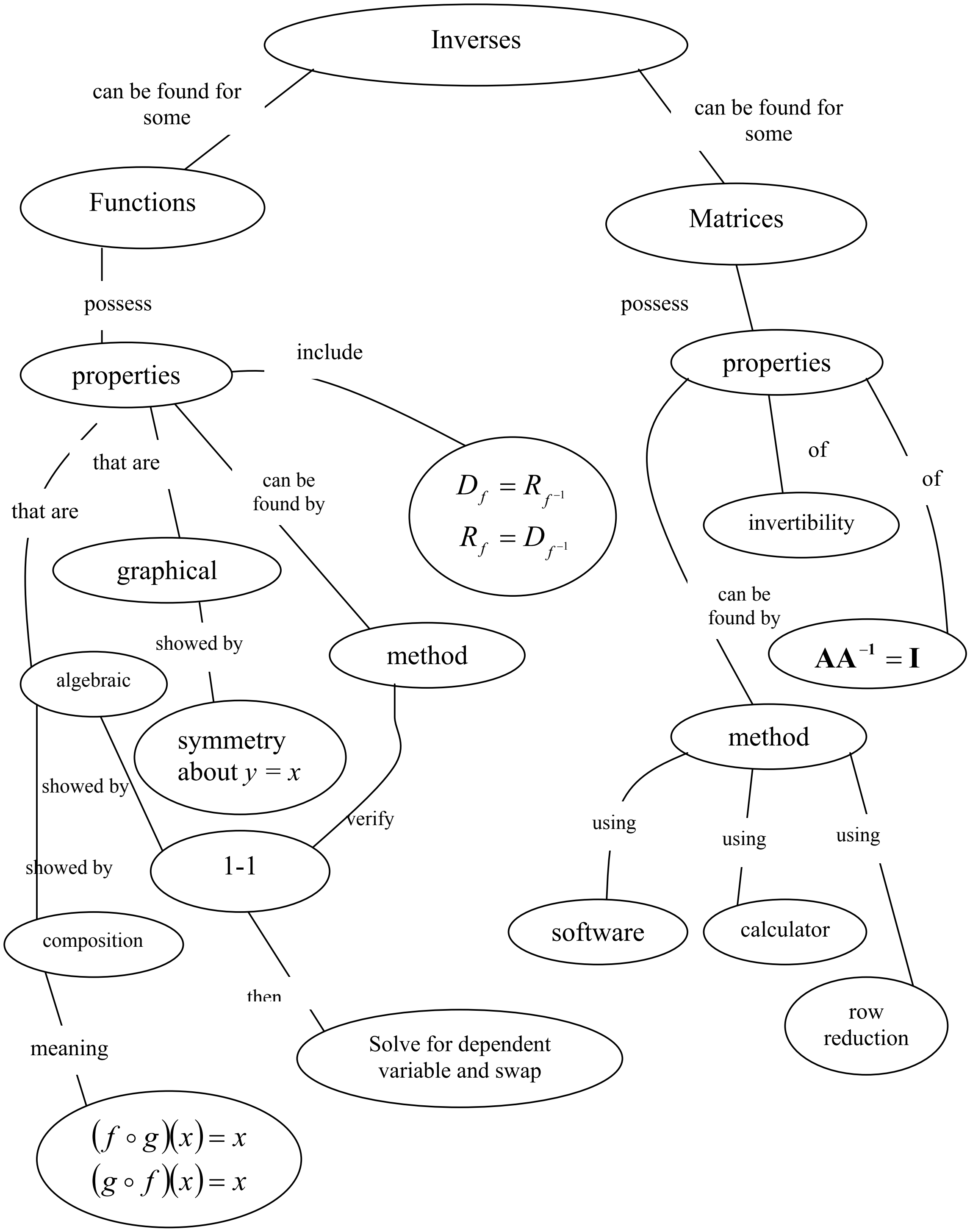


Figure 24. Criterion map for "Inverse"

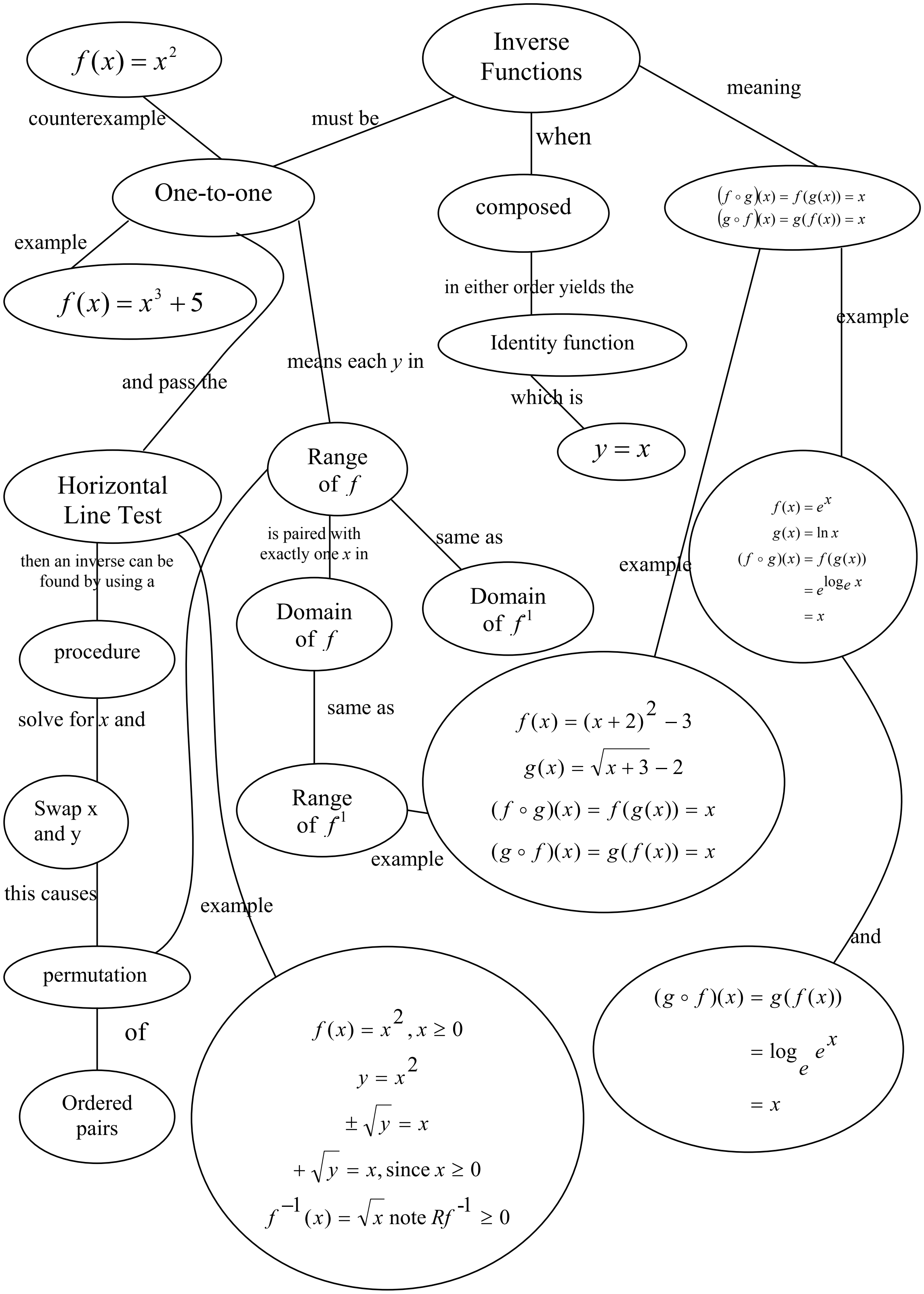


Figure 25. Criterion map for "Inverse Function"

Expert Group Survey Analysis

Following the concept mapping session, this researcher requested the expert group to respond to a questionnaire (Appendix I) regarding their teaching experience and their perspectives on the utility of concept maps in the precalculus algebra setting. The 5-item questionnaire solicited from the mathematics instructors produced rich responses from the expert group

Item 1 of the survey inquires:

> **Approximately how many years have you been teaching mathematics?**

The years of mathematics experience ranges from 10 to 40. The mean and median number of years of teaching experience are 26.2 years and 24 years, respectively. The only male participant in the expert group was the participant with the least number of years of teaching experience.

Item 2 requests:

> **In which level of mathematics teaching have you had experience?**

The responses included middle school, high school, a variety of levels of university mathematics, and secondary mathematics teacher education courses. This variety of teaching experience represented 131 years of service in secondary and post-secondary education.

Item 3 is an open-response item:

> **How might you find concept mapping useful within the mathematics courses you teach?**

The members of the expert group responded that concept mapping

1. Stimulates discussion among knowledgeable group students
2. Organizes thoughts and thinking processes
3. Illustrates connections among concepts, ideas, properties and procedures.

One of the participants, the youngest in fact, wrote,

> If you let the students create without your input you can get a good idea of where they are at and what they know—especially useful early in the semester to check their prerequisite knowledge.

Discussion, organization, and making connections are three essential components that comprise a portion of the *NCTM* and *AMATYC Standards*.

Item 4 states:

> **Explain whether or not you think concept mapping would be appropriate for MAT 171, Precalculus Algebra.**

Of the five participants, two respondents were undecided, two were positive and one was negative. An undecided member of the group wrote, "Probably not...It would take up too much class time, but I will think about it and prefer it." A member who responded affirmatively declared, "Yes, especially for the purposes mentioned in #3", [review of prerequisite knowledge]. The lone member who responded clearly negatively argued, "It is a very long, complex procedure. I would much consider this time being spent in problem-solving." However, this researcher declares that the group construction of a concept map invokes the dynamic of a mathematics community. Thus, one may argue that the process of constructing a concept map is within itself a problem-solving activity. From this perspective, concept mapping meets the NCTM and AMATYC problem-solving standards.

Item 5 asks:

> **What did you find difficult about the concept mapping activity?**

The participants responded with

1. Remembering to put in links to make propositions
2. Organization
3. Logically organizing one's thoughts
4. Getting ideas organized
5. Agreement among the members of the group.

One might argue that this type of negotiation and organization reaches or exceeds the Generalization Level of Avital and Shettleworth's Taxonomy of Mathematics Learning

(Avital & Shettleworth, 1968). Although concept mapping is not uniquely a mathematics exercises, the Open Search Level of the Avital and Shettleworth (A & S) Taxonomy states a student should be able to "apply higher mental processes to mathematics through open search in order to solve problems that are remote from the student's experience." Concept mapping requires analysis, a breakdown of the superordinate concept; and synthesis, a reorganization of subordinate concepts. These levels (4 and 5) of Bloom's Taxonomy (1956) relate directly to the Open Search Level of the A & S Taxonomy.

Qualitative Analysis of Subjects' Concept Maps for 'System of Equations' and Associated Work

During the course of the experiment, the researcher directed subjects to construct three maps: system of equations, inverse and functional inverse. Collaborative groups constructed the system map, and each subject composed the last two maps individually. Initially, the researcher stratified 8 groups of 3 students based on measures from the first unit test and quiz. Only one group member withdrew from the course during this preliminary part of the experiment leaving **N** = 23 subjects. Therefore, Group 2 was disbanded and the researcher reassigned the remaining group members to other groups via the randint function on the TI-83^+ graphing utility.

Each student constructed a preliminary individual map for "System of Equations." Following this construction, groups convened to construct a map based on the individual maps each member composed. Subjects discussed their maps and began to coalesce their creations into group maps during class time provided by the instructor. Subsequently, the instructor allowed two weeks for the groups to comprise final maps and written reports detailing the development of the group concept maps. The requirements for the written report are provided in Appendix H, Item 3. Each group submitted a concept map and a written report, and the researcher cited important statements relating to the group process and commented on the reports and concept maps. In addition, the researcher tailored questions for each group about their respective maps to which each group would respond.

The groups' reports on the constructions of their maps centered about 6 common themes:

1. Communication and collaboration
2. Procedure for solving a system of equations
3. Introduction of concepts not yet broached in class

4. Employment of a variety of subconcepts
5. Resubmission of a revised group concept map with the responses to questions posed by the researcher about the group concept map
6. Use of computer.

Six of the 7 groups explicitly mentioned the positive nature of the impact of collaboration on developing a concept map. Group 1 reported,

> Communication was the key word here. We took what we wanted from each [map]...we added all of those [linking phrases] during our meeting and some were done away with due to their lack of clarity. It seems hard to turn this map in due to the many concepts that could be on the map, but this is what we all decided.

Group 3 declared,

> ...we finally came to realized [sic] this map may not be perfect, but by working together we were able to accomplish much more by working as a team.

Group 4 corroborates with Group 1,

> Having many subtopics, our group found that there were numerous ways to construct a map of this particular superordinate...group's open communication...Each person's individual maps and many ideas contributed to an excellent final concept map.

Group 5 considered the perspective of an external observer,

> It was interesting to see how our ideas unfolded and changed through the construction of the map...We decided that a map which was easily understandable by its viewer would be most helpful in explaining a concept as broad

as systems of equations.

Additionally, Group 6 reported, "A peaceful debate transpired," and Group 8 admitted, "Our map went through many changes." Although, one group, Group 7, concluded, "...this project required many hours and most importantly teamwork," members of Group 7 complained of conflicts among group members, but the instructor deemed them as superfluous personality conflicts.

Groups 1, 3, 4, 5, 6, and 8 indicated the importance of the selection process and deliberation. Furthermore, the deliberation implies the groups reached the fourth and fifth levels of Bloom's Taxonomy (1956): analysis—selection of appropriate related subordinate concepts beyond the 5 provided in the instructions; and synthesis—connection of the subordinate concepts with appropriate links. Secondly, every map and associated report included evidence of procedures to solve a system of *linear* equations. This likely indicates the importance the instructor stresses on systems of linear equations. At the time the instructor assigned this group exercise, the class had not yet discussed any other types of systems.

Thirdly, 4 of the 7 groups introduced concepts on their maps not yet broached by the instructor. Groups 4, 5, 6 and 7 included the subordinate concept of nonlinear, although the branch(es) under the nonlinear concept were not necessarily correct. See Figure 26. Members of Group 5 placed the concept of "linear inequalities" on their map. Group 7 uniquely related matrices to nonlinear systems of equations. The group indicated matrices can be used to solve a nonlinear system; however, the researcher cautioned Group 7 against generalizing this over all nonlinear systems.

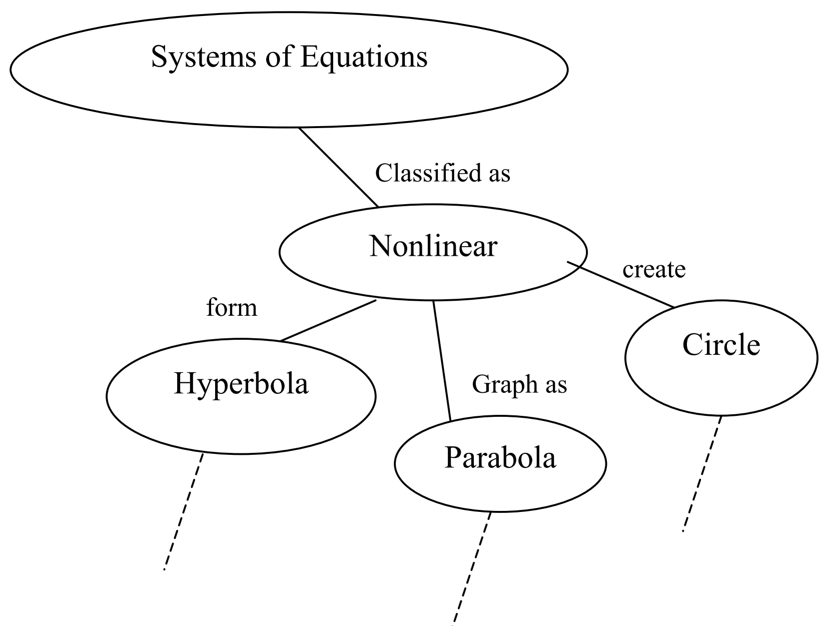


Figure 26. Excerpt from Group 4 System of Equations Concept Map

Next, given only the five concepts, solutions, standard form, row reduction, intersection and linear, many groups provided a plethora of subconcepts, such as inconsistent, augmented, dependent, identity, ordered pair, linear, nonlinear, inequalities, graphing utility, trace function, intersection function, substitution, elimination, etc. In addition to excellent employment of subconcepts, five groups (1, 3, 5, 6, 8) used the computer to construct concept maps. The researcher provided a short tutorial on using Microsoft WORD tools to create a concept map on the computer. This tutorial transpired at the end of the lesson on concept map construction, but he left it to the discretion of the groups to determine whether or not their groups should use the computer for the construction. Four of the 5 groups that used the computer to construct their maps provided revisions of their group maps along with their responses to the questions tailored for these groups.

Close examination of the group concept maps revealed common types of errors in subconcept names, interpretations of subconcepts through vague links, and in the interpretation of the term, “concept.” The groups invoked terms such as “consistents” for “coefficients”; “determinates” for “determinants”; “coordinate plain” for “coordinate plane”; and “intersecting point.” Group 1 submitted a response informing the researcher of the "consistent" error. Although the instructor of the course avoided using determinant methods for solving linear systems of equations, the researcher informed members of Group 5 of this valiant attempt to research beyond the timeline of study for the course. Group 8 provided a revised group map correcting the spelling of "coordinate plain," and in the group's response to the researcher questions tailored for the group's map, the group reported that "intersecting point" meant "point of intersection." The researcher inquired about this since one might interpret "intersecting point" as a point that intersects, and points fail to intersect, whereas lines might.

The groups' concept maps for system of equations revealed errors in conceptual interpretations. Group 1 gave the example of the linear equation, $y = 2x + 4$, as a "dependent solution." The researcher probed Group 1 about this. The group reported that a system is dependent if two linear equations reduced to one. Group 1 never noted the solution to a dependent system consists of an infinite number of ordered pairs. Group 4 recorded a similar misconception about "solutions": the intersection is a real solution as opposed to an ordered pair of real numbers. Group 5 suggested "$ax + by = c$" is an example of a linear system. In the response to the researcher question, Group 5 amended, "As for an example of a linear system $ax + by = c$ and $dx + ey = f$ would have constituted a system rather than simply $ax + by = c$." In Figure 27, Group 6 relates matrices and row reduction with the linking phrase, "to solve consistent." After questioning the meaning of this portion of the map, Group 6 responded,

> The phrase, 'to solve consistent,' between matrices and row reduction was meant to imply that if we have a matrix that is augmented we can use row reduction to solve it. We as a group decided to change the phrase to 'use' between matrices and row reduction. We figure this would be easier to understand.

An excerpt of Group 6's map appears in Figure 27 below.

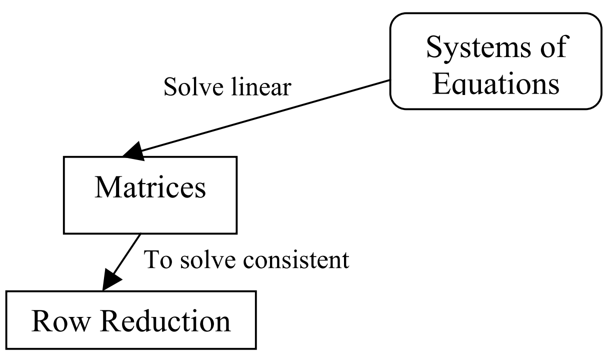


Figure 27. An excerpt of Group 6 concept map.

Group 7 indicated "inequalities" as a type of an equation.

Finally, some of the group maps submitted placed too many concepts within a concept bubble or completely outside of one. For instance, two groups tended to enclose entire definitions within bubbles as opposed to a single term or a short phrase. However, subsequent individual maps revealed better decomposition of definitions into multiple subordinate concepts and links.

Brief Analysis of Inverse Maps

Subjects individually completed the second of three concept maps for the seed concept, inverse. The researcher insisted on using the general term, inverse, as opposed to a more specific type of inverse, in anticipation of their discoveries of inverse not yet discussed in class. The instructor had recently completed a mini-unit on matrices, which included an introduction to 2-dimensional and 3-dimensional matrices. However, the instructor made no references to inverse functions and motivated the topic of matrix inverses using an advance organizer involving real number examples.

The researcher provided no subordinate concepts for the seed concept. Of the 15 completed inverse maps, all 15 made references to matrices, 7 of them ascribed to either real additive or multiplicative inverses (or both), and 7 of the maps referred to inverse functions. Over half of the maps inferred subjects investigated beyond expectation, given the class timeline. This is an excellent finding, given that only 1 of these 15 students reported having taken high school precalculus; 2 of them had unsuccessfully completed MAT 171 with a grade of W, D or F; 8 of them completed either Intermediate Algebra, MAT 080, or high school Algebra II; and 3 of them had not had any mathematics for at least one year. Not only did the maps reflect investigation in a topic yet to be discussed, the maps indicated the subjects' revision of what they had recently studied. Hence, the community college mathematics instructor who suggested concept maps serve best as an assessment for prerequisite knowledge confirms this discovery.

Brief Analysis of Functional Inverse Map

The classroom instructor assigned the third concept mapping exercise immediately after teaching the section on inverse functions. This instruction ensued approximately 12 weeks into the course. The instructor allowed one week for completion of this second individually constructed map. Again, the researcher supplied only the seed concept yielding to the subjects' creativity. Table 7 recapitulates the more common concepts that appeared on the functional inverse maps.

Table 7

Common Concepts on Functional Inverse Maps

Concept	**Appearance**
Seed Concept of Inverse Function	15
One-to-One	15
Horizontal Line Test	14
Identity function or y = x	10
Symmetry	8
Domain	7
Range	7
Strictly Increasing	6
Strictly Decreasing	6
Exponential	6
Logarithmic	6
Permute/Switch	5
$f(f^{-1}(x)) = x = f^{-1}(f(x))$	5
Graph	4
Counterexamples	4
Examples	4
Domain of f = Range of f^{-1}	3
Range of f = Domain of f^{-1}	3
Domain of f	2
Range of f^{-1}	2
Range of f	2
Domain of f^{-1}	2
Ordered Pairs	2
Even/Odd	2

There are two noteworthy observations. First, two subjects' maps displayed concepts that appear as links in other subjects' maps, and conversely. These "concepts as links" are symmetry and permute (or switch). Secondly, the maps clearly indicated a departure from a procedure-generated map to a more valid concept map. This researcher defines a procedure-generated map as one containing mostly algorithms that demonstrate how to solve rudimentary mechanical problems, such as solving a linear system of equations using substitution, graphing, elimination, or row reduction. Some system maps were procedure-generated maps, but they were not entirely flowcharts, since subjects implemented valid concepts.

Comparison of Concept Maps with
General Item 1 (c), Page 3, Test #4

The 4-part item appears exactly as follows on Test #4. Values enclosed within brackets indicate the point value of 38 possible points. Subjects' responses to Item 1 (c) and excerpts of their Functional Inverse Concept Maps follow. These maps were turned in and reviewed by the researcher before the subjects' took Test #4.

1. Use the function $f(x) = e^x$ to respond to each of the following items.

   (a) As $x \to -\infty$, which value of y does the function approach? [1]

   (b) What is the inverse function? [1]

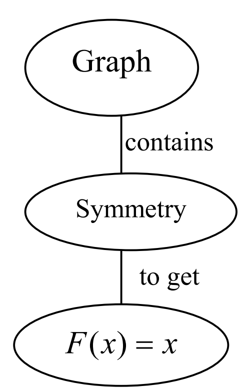
   (c) Show or describe why the inverse you selected for the function $f(x) = e^x$ is the inverse function. You should have at least three different detailed statements to defend your choice of inverse. [3]

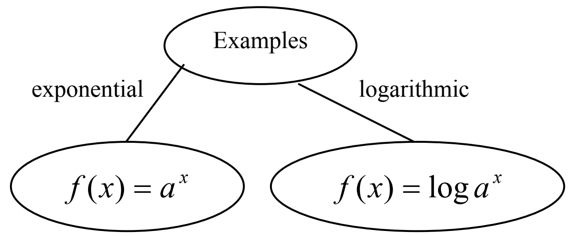
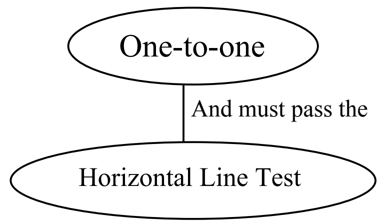
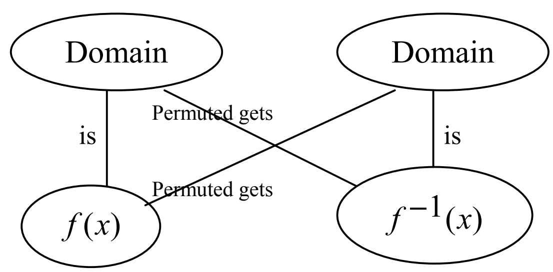
   (d) As $x \to 0$ for the inverse you gave in part (b), which value of y does the function approach? [1]

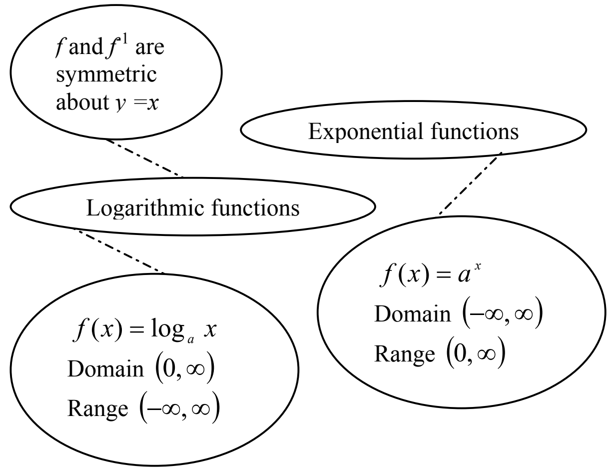
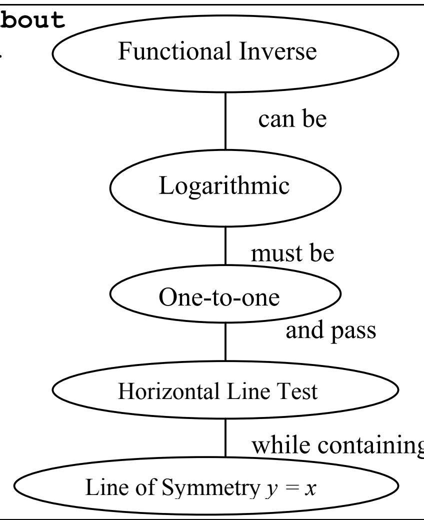
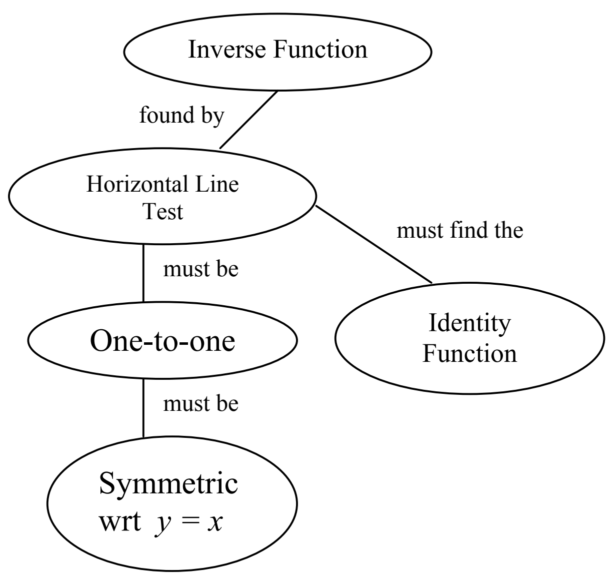
Table 8 indicates the portion of students' concept maps that directly relates to the response given by the students to this valid test item.

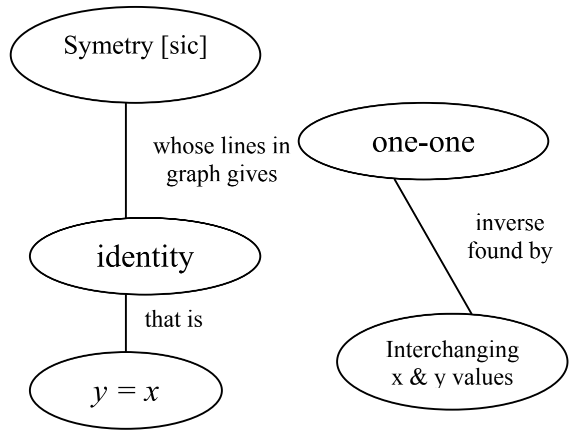
Table 8

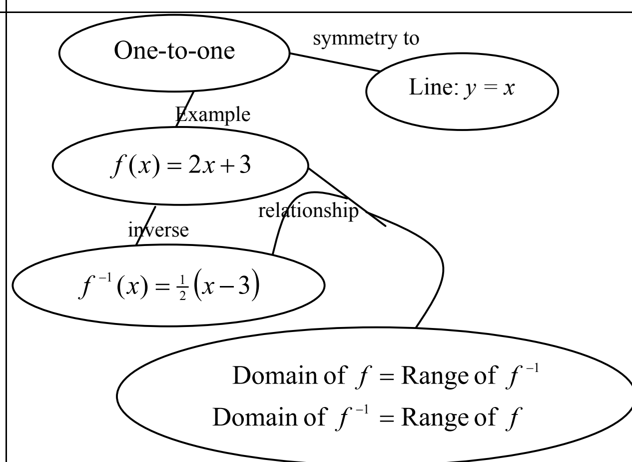
Relationship between Student Responses on Test #4 with Concept Map

Name	**Item (c) Responses**	**Concept Map Excerpt**
Mark	Graph Symmetric about y = x	

Gabriel	Passes Horizontal Line Test; graph symmetric with respect to (wrt) y = x; $\log_e x$ is the reflection of $y = e^x$	 **(Incorrect—should be** $f(x) = \log_a x$**)** ***[and]*** 
Ezra	$f(f^{-1}(x)) = f^{-1}(f(x))$	No corresponding part of the Inverse Function Map to this composition statement.
Deborah	Verified correctly $(f \circ f^{-1})(x) = x$ $(f^{-1} \circ f)(x) = x$ The coordinates have been permuted; graphs of both functions provided wrt y = x; small tables for e^x and $\ln x$ provided	See entire map in Figure 28
Joshua	Reflected about Y = x; ordered pairs are permuted; Domain of e^x = Range of $\ln x = (-\infty, \infty)$	 **(nothing on map about reflection or ordered pair)**
Abraham	Incomprehensible response	Has a good map, but responses to test items made no sense

Hannah	Symmetry wrt y = x; domain and range are switched; Domain $x > 0$ and range $(-\infty, \infty)$ for e^x; Range $(-\infty, \infty)$ and domain $(0, \infty)$ for $\ln x$	
David	Inverse asymptote x = 0; Line of symmetry y = x; range of f^{-1} is the domain of f; domain of f^{-1} is range of f	**(nothing about domain and range)** 
Jacob	Graph provided with y = x; the $\ln x$ and e^x both create the identity function	

<table>
<tr><th></th><th></th><th></th></tr>
<tr><td>Aaron</td><td>The graph of $y = \ln x$ reflects $y = e^x$ wrt the line $y = x$; all the values of y and x in $y = e^x$ have been interchanged; algebraically solving for x and interchanging x and y</td><td>


</td></tr>
<tr><td>Matthew</td><td>$f(g(x)) =$ identity function; permuted domain and range; solve for x (procedure)</td><td>See entire map in Figure 29</td></tr>
<tr><td>Luke</td><td>The x and y values are permuted. It reflects across y = x. It passes HLT</td><td>Only Horizontal Line Test is displayed in the map.</td></tr>
<tr><td>Isaac</td><td>Graph provided and y = x; symmetry wrt y = x; Domain f(x) = Range g(x); Range f(x) = Domain g(x)</td><td>


</td></tr>
<tr><td>Philip</td><td>If you permute the x and y coordinates for $f(x) = e^x$ you get the inverse</td><td>Answer does not compare with map.</td></tr>
<tr><td>Elizabeth</td><td>Graph of $y = x$ and functions provided; $\log_e x$ and e^x are reflected about the line $y = x$; permutation of all ordered pairs</td><td>See entire map in Figure 30.</td></tr>
</table>

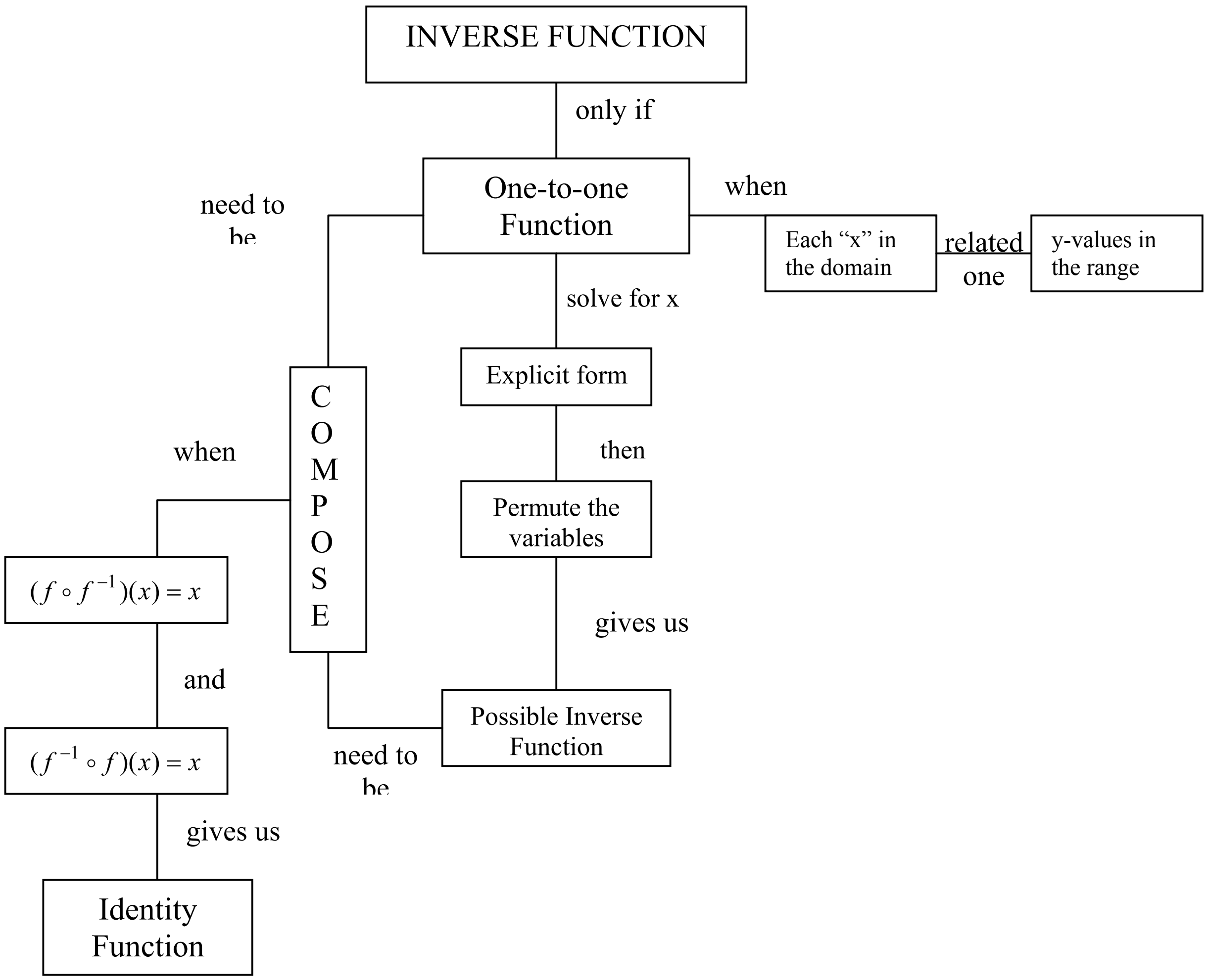


Figure 28. Deborah’s functional inverse concept map.

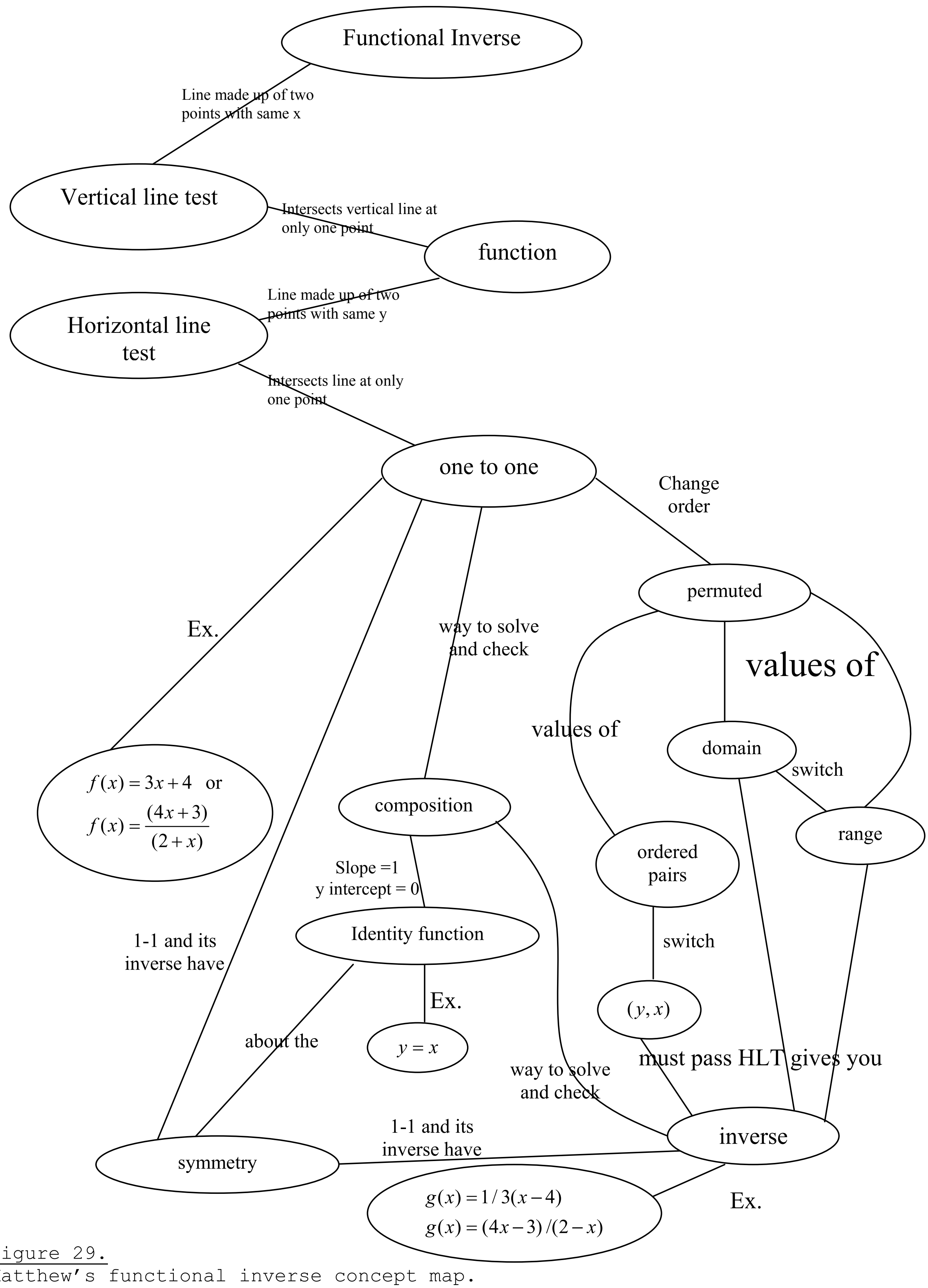


Figure 29.
Matthew's functional inverse concept map.

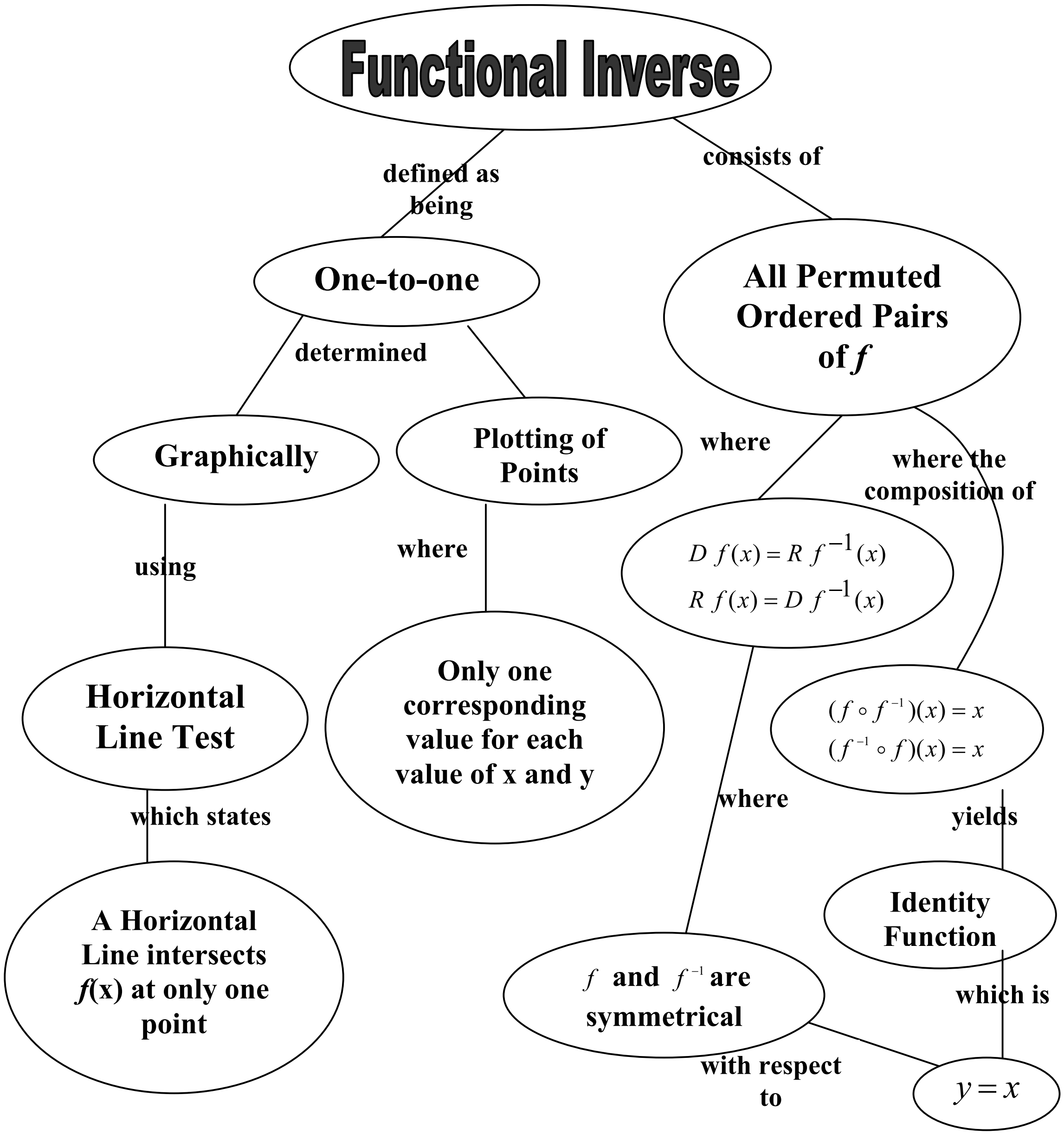


Figure 30. Elizabeth's functional inverse concept map.

Quantitative Analysis

Analysis of Demographic Data

Although studies of undergraduate level students include SAT-Math and SAT-Verbal scores, the institution at which the students attended who participated in this experiment did not require SAT scores for entrance. General demographic characteristics of concern are age, gender, number of hours employed per week, reported amount of study hours per week, mathematics preparation prior to taking the course and primary language. Since the community college routinely enrolls more international students than most of the other 57 community colleges in the state of North Carolina, consideration of language is importance. Furthermore, the examination of the other parameters is necessary, since these typically differ from counterparts at the traditional four-year college or university.

The Pearson Chi-Square test reveals no statistical significant differences between the experimental and control groups ($\chi^2 = 0.7754, p = 0.3786$) relative to gender. Since the experimental section had fewer than 5 females, this necessitates Fisher's Exact Test. This test reveals no further evidence that the groups are statistically different based on gender ($p = 0.3109$).

Table 9

Gender Percentages

Section	Male	Female	Sample Size
Experimental	80.0%	20.0%	15
Control	61.9%	38.1%	21

With regard to language, the data clearly indicates no need for a test. Remarkably, $p_E - p_C = 0$ for either group whose language is English versus subjects whose primary language is other than English.

Table 10

Language Percentages

Section	English	Non-English	Sample Size
Experimental	67.7% $\underline{\mathrm{n}}_E = 10$	33.3% $\underline{\mathrm{n}}_I = 5$	15
Control	67.7% $\underline{\mathrm{n}}_E = 14$	33.3% $\underline{\mathrm{n}}_I = 7$	21

A Two-Sample T-Test for equal means strongly verifies no significant difference between mean ages for both samples $(t = 0.430, p = 0.671)$. Table 11 displays means of ages and corresponding standard deviations.

Table 11

Simple Age Statistics

Group	Mean Age	Standard Deviation
Experimental	24.267	6.984
Control	23.333	5.526

The Kolmogorov-Smirnov Test for Normality indicates no significant departures from normality for both the experimental group $(D = 0.169, \mathrm{p} > 0.150)$ and the control group $(D = 0.133, \mathrm{p} > 0.150)$. Furthermore, the kurtosis and skewness sample statistics located in Table 12 support this evidence. Both groups indicate virtually no skew, and both distributions are slightly platykurtic.

Table 12

Essential Normality Statistics

Section	Kurtosis	Skewness	Mean	Standard Deviation
Experimental	-0.765	0.092	73.667	10.601
Control	-0.949	-0.077	66.429	16.136

Now, since normality of the population is confirmed with a reasonable error rate, the two-sample test for equal variance indicates a significant difference between the two group variances for $p > 0.112$ with $F = 0.43\,(df_{num} = 14, df_{denom} = 20)$.

Hence, with concern for the critical nature of this p-value, assuming the population variances are equivalent, the two-sample independent t-test indicates no difference between the pretest score means $(t = 1.516, p = 0.139)$. Again, the p-value is rather marginal, but outside the traditionally accepted experiment-wise error of $\alpha = 0.05$.

An ANOVA for 6 levels of pretest scores and the Sheffé Procedure yields a difference in means for subjects who completed the course in the experimental section and the mean for subjects who withdrew from the control section $(F = 2.74, p = 0.022)$. Table 13 displays the 6 levels of scores and their corresponding means and standard deviations.

Table 13

Simple Statistics for Six Pretest Subgroups

Level of Pretest	**N**	**Mean**	**Standard Deviation**
Completed Experimental	15	73.667	10.601
Completed Control	21	66.429	16.136
Withdrew/Drop Experimental	13	58.077	17.385
Withdrew/Drop Control	11	51.364	22.593
All Subjects taking pretest Experimental	28	66.429	15.978
All Subjects taking pretest Control	32	61.250	19.634

Experimental Results

Each student completed a supplementary data sheet that included some items that appeared on the original data sheet completed by all students on the first day of class. Other than age and gender, the items requested the number of hours employed per week, the number of study time hours per week and a more detailed item about mathematics preparation prior to enrolling in the course. Tables 14, 15, and 16 summarize these data. The more conservative Mantel-Haenszel Chi-Square Test of regression on $\mathbf{r} \times \mathbf{2}$ categorical (or ordinal) data suggests either no significant differences or marginal differences between the experimental and control groups with respect to hours employed per week ($\chi_1^2 = 0.1337, p = 0.7146$).

Table 14

Hours Employed per Week

Number of Hours	**Experimental**	**Control**
<10	20% (3)	23.81% (5)
10-19	26.67% (4)	14.29% (3)
20-29	13.33% (2)	28.57% (6)
>30	40% (6)	33.33% (7)

Similarly, the same test intimated no difference among categorized reported hours of study of precalculus per week ($\chi_1^2 = 2.1921, p = 0.1387$).

Table 15

Hours Studied Mathematics per Week

Number of Hours	**Experimental**	**Control**
≤ 1	0% (0)	4.76% (1)
2-4	33.33% (5)	52.38% (11)
5-6	40% (6)	28.57% (6)
>6	26.67% (4)	14.29% (3)

Finally, no differences among categories of mathematics preparation prior to entering the course exist ($\chi_1^2 = 0.5265, p = 0.4681$). Notice that the row for 2-4 hours of study time contributes most to the Mantel-Haenszel Chi-square statistic.

Table 16

Most Recent Mathematics Preparation Prior to Course

Mathematics Preparation	Experimental	Control
MAT 070 (Introductory Algebra) or High School Algebra I	6.67% (1)	0% (0)
MAT 080 (Intermediate Algebra) or High School Algebra II	53.33% (8)	66.67% (14)
MAT 171 (Precalculus Algebra) Grade of W,D or F	13.33% (2)	9.52% (2)
>1 year since last mathematics course	20.00% (3)	23.81% (5)
High School Precalculus	6.67% (1)	0% (0)

A One-way ANOVA for the first three tests fails to show any significant differences among first three unit test means $(F = 0.85, p = 0.520)$. However, the mean test scores for the experimental group consistently place lower than the mean test scores for the control group. The Scheffé Test for Mean Comparisons confirms this. Table 17 shows the means for each test taken before Test #4, the test that assesses inverse functions.

Table 17

Mean Scores for First Three Tests

Test	Mean Score Experimental (Standard Deviation)	Mean Score Control (Standard Deviation)	Mean Differences
1	73.60 (13.16)	80.57 (15.71)	**-6.97**
2	72.07 (11.68)	76.57 (18.27)	**-4.50**
3	76.67 (12.64)	80.52 (19.57)	**-3.85**

Two weeks into the semester and 4 days before the experimental subjects took their final examination, the experimental subjects completed the Aiken Revised Mathematics Attitude Scale consisting of 24 items. Cramer-von Mises and Anderson-Darling statistics reveal significant departures from normality for both groups as displayed in Table 18.

Table 18

Important Aiken Survey Statistics

Aiken Trial	Mean Score	Standard Deviation	Test	Statistic	P-value
1	12.733	2.4348	Cramer-von Mises	$W^2 = 0.1153$	$p = 0.066$
			Anderson-Darling	$A^2 = 0.7163$	$p = 0.048$
2	14.133	3.1242	Cramer-von Mises	$W^2 = 0.2005$	$p < 0.005$
			Anderson-Darling	$A^2 = 1.0667$	$p = 0.006$

Since normality cannot be assumed about the population from which the samples emanate, the Kruskal-Wallis test for equivalent medians is invoked. The test reveals no significant differences between medians $(W = 223, p = 0.6928)$, and furthermore, The Ansari-Bradley test for dispersion differences indicates no significant differences $(C = 116.667, p = 0.7797)$.

Analysis of Inverse and Functional Inverse Concept Maps

The researcher scored the individual concept maps based on the Markham, Mintzes and Jones rubric. The distribution of scores for "Inverse" concept maps $(D = 0.156, p > 0.150)$ and "Functional Inverse" concept maps $(D = 0.172, p > 0.150)$ have no statistically significant departures from normality according to the Kolmogorov-Smirnov Goodness-of-Fit Test for Normal Distribution. However, according to the Anderson-Darling $(A^2 = 0.600, p = 0.098)$ and the Cramer-von Mises $(W^2 = 0.088, p = 0.147)$ Tests for Normality, the distribution of scores for the Functional Inverse concept maps might not emanate from a normal population. Furthermore, the distribution of Functional Inverse scores could be positively skewed (1.414) and leptokurtic (3.975). Table 19 records simple statistics for both map composite scores.

Table 19

Statistics for Map Distributions

Map	**N**	**Mean**	**Median**	**Standard Deviation**	**Range of Scores**
Inverse	15	46.33	53.00	21.303	13-83
Functional Inverse	15	54.67	53.00	19.412	23-109

According to the Two-sample Paired T-test for means, the data suggests no strong statistical difference $(t = -1.614, p = 0.129)$ between the concept map mean scores, although the *p*-value appears marginal in favor of the "Functional Inverse" map. Interestingly, this map had a more specific, targeted seed concept, and still there was no statistical difference.

The Wilcoxon Two-Sample test was used to examine median differences in Inverse Function Concept map scores between males and females and between English-speaking subjects and subjects whose primary language is other than English. The Wilcoxon $(W = 29.50, p = 0.4747)$ and Ansari-Bradley $(S = 12.50, p = 0.4655)$ tests suggest little statistical difference in median scores and dispersion of the distribution of map scores with respect to gender. In addition, the Wilcoxon Test $(W = 31.00, p = 0.1441)$ suggests marginal differences between median scores with respect to language. However, it should be noted that there were only 3 female participants in the experimental group, and Table 20 displays some of the statistics comparing genders. Since there were only 3 female participants, the correlation matrix yields mostly nonsense; although the correlation coefficients tended to be highly significant.

Table 20

Simple Statistics for Male and Female Experimental Subjects

Female (n = 3)			**Male (n = 12)**		
Mean Score	Median	Standard Deviation	Mean Score	Median	Standard Deviation
70	55	34.073	50.833	52.50	13.723

The Ansari-Bradley $(S = 21.00, p = 0.4674)$ test indicates little significant difference in dispersion of the distribution of map scores with respect to language.

However, the Kruskal-Wallis Test for differences in median scores relative to the most recent mathematics preparation indicates some differences. The four levels of mathematics preparation prior to enrolling in MAT 171 are located in Appendix N. The Kruskal-Wallis Test indicates a significant difference among the preparation level of the four groups $(\chi_3^2 = 5.9943, p = 0.0867)$. Furthermore, the group that differs consists only of 2 subjects, and these subjects indicated not having taken the prerequisite course for MAT 171.

Table 21 displays some simple statistics for the components of each concept map.

Table 21

Simple Statistics for Individually Constructed Maps

Component	Mean Map 1	Standard Deviation Map 1	Mean Map 2	Standard Deviation Map 2
Concept	11.000	5.976	12.667	3.474
Linking Phrase	7.267	5.391	9.467	6.844
Initial Branching	1.533	0.915	1.000	0.000
Successive Branching	4.067	4.440	3.668	6.200
Hierarchy	19.333	7.988	20.333	6.114
Crosslink	0	0	2.000	4.140
Example	2.867	2.293	3.400	3.738
Composite	46.333	21.303	54.667	19.412

Upon examination of the kurtosis and skewness statistics, this researcher assumes non-normality for most of the component distributions for both maps. So, the Ansari-Bradley Two-Sample Nonparametric Test reveals differences between median scores for linking phrases $(\chi_1^2 = 2.2914, p = 0.065)$, initial number of branches $(\chi_1^2 = 4.4615, p = 0.035)$, successive branches $(\chi_1^2 = 3.8238, p = 0.051)$, crosslinks $(\chi_1^2 = 3.222, p = 0.073)$, and examples $(\chi_1^2 = 5.6988, p = 0.017)$. Therefore, Kendall's τ_b is used to discover statistically significant correlation coefficients for the Inverse Map (Table 22) and Functional

Inverse Map (Table 23). Cells for which there is no report of a coefficient have sample standard deviations of $s = 0$.

Table 22

Kendall's τ_b for Inverse Map

Component/ Component	Concept	Linking Phrase	Initial Branch	Successive Branch	Hierarchy	Example
Concept	1.0000	0.8200*	0.4697***	0.4552**	0.5344**	0.6427*
Linking Phrase		1.0000	0.4200	0.3755	0.6951**	0.4824***
Initial Branch			1.0000	0.6177***	0.2829	0.4820***
Successive Branch				1.0000	0.2024	0.4552***
Hierarchy					1.0000	0.5162***
Example						1.0000

$* \ p < 0.001, ** \ p < 0.01, *** p < 0.05$

Table 23

Kendall's τ_b for Functional Inverse Map

Component/ Component	Concept	Linking Phrase	Initial Branch	Successive Branch	Hierarchy	Crosslink	Example
Concept	1.0000	0.3031 $p = 0.1313$	----	0.4091***	0.0551	0.1684 $p = 0.1680$	0.3231
Linking Phrase		1.0000	----	0.1125	0.4146***	0.3167	0.2132
Initial Branch			----	----	----	----	----
Successive Branch				1.0000	0.2087	0.0375	0.4078
Hierarchy					1.0000	-0.2728	0.1163
Crosslink						1.0000	-0.2308
Example							1.0000

$p < 0.001*, p < 0.01, ** \ p < 0.05 ***$

A backward elimination linear regression model over the dependent variable, course grade, employs several important dependent variables: Inverse Concept Map, Inverse Function Concept Map score, Test #4 inverse function items, Function Project, Final Examination inverse items, mathematics preparation prior to taking the course, language, and gender. The independent variables, Inverse Function Concept Map score and Test #4 inverse function items, remain in the model, and this reduced model is significant at the 0.05 level. The final ANOVA and model appears in Table 24:

Table 24

ANOVA and Backward Selection Model

R-Square = 0.7133

Analysis of Variance

Source	**Degrees of Freedom**	**Sum of Squares**	**Mean Square**	**F Value**	**p-value**
Model	2	850.86794	425.43397	14.93	0.0006
Error	12	341.92939	28.49412		
Corrected Total	14	1192.79733			

Variable	Parameter Estimate	Standard Error	Type II Sum of Squares	F Value	p-value
Intercept	49.23687	4.26743	2489.65527	87.37	<0.0001
Map Score	0.25039	0.07382	324.87050	11.51	0.0053
Test #4 Inverse Items	3.23166	0.81823	444.48907	15.60	0.0019

The model with all variables included has $R^2 = 0.7710$. Although the amount of variation explained by the model is not very good, the reduction in variation is quite modest, $d = 0.0577$.

Table 25

Partial Table of Kendall's τ_b for

Functional Inverse Map and Other Variables

	Composite Score	Inverse Project	Crosslink	Number of Concepts	Test #4 Inverse Items	Final Exam Inverse Items	Final Grade
Composite Score (CS)	1.0000					$0.4649^{\dagger}$	$0.3817^{\dagger\#}$
Inverse Project		1.0000	0.5196***				
Crosslink			1.0000				
Number of Concepts				1.0000		0.6855**	0.4450***
Test #4 Inverse Items					1.0000	$04843^{\dagger}$	0.5477**
Final Exam Inverse Items						1.0000	0.6728**
Final Grade							1.0000

$p < 0.001^{*}, p < 0.01,^{**}\ p < 0.05^{***}, p < 0.1^{\dagger}$

#much better results with Pearson

A simple One-Way ANOVA illustrates no significant differences in function writing assignment, Unit Test #4 inverse function items, and final examination inverse function items. However, analysis of variance and Levene's Test for Homogeneity of Variance detects a somewhat appreciable difference between the two group means and variances relative to these variables. Table 26 displays these dependent variable statistics.

Table 26

Dependent Variable Statistics

Variable	F Value	p-value	Levene's Test of Homogeneity	p-value
Function Writing Assignment	0.24	0.6249	1.49	0.2307
Test #4 Inverse Function Items	3.47	0.0712	3.13	0.0859
Final Exam Inverse Items	1.29	0.2639	2.56	0.1190

Finally, there are no appreciable significant differences between final course grades for the experimental and control groups. Since the final course grade distributions do not show any significant departures from normality, a Two-Sample t-test for the course grade means indicates no statistical difference $(t = 0.907, p = 0.3706)$.

Table 27

Sample Statistics for Course Grade

Group	N	Mean	Standard Deviation
Experimental	15	76.713	2.3833
Control	21	80.376	2.9500

Subjects' Responses to Follow-up Questions

Approximately one week prior to the end of the course, the experimental section completed a second Aiken Revised Mathematics Attitude Scale (ARMAS) and an open-ended survey relative to the concept mapping exercises. The instructor did not inform the subjects they would be completing an open-ended survey nor a second ARMAS. The analysis of the ARMAS data is located in the quantitative section of this work.

Fifteen (**N** = 15) students responded to the Follow-up survey. For the sake of anonymity, the following pseudo names shall be employed in lieu of real subject names: Hannah, Joshua, Deborah, Jacob, Gabriel, Ezra, Luke, Abraham, Philip, Elizabeth, Aaron, Matthew and David (as were used in the analysis of inverse function maps). The instrument consists of seven items, four of which refer to concept mapping and the remaining three focus on inverse functions.

Item #1 asks,

What did you find easy about constructing your concept maps?

The more popular responses to this item were

1. Sources that made the process easier, such as notes, lecture, text, and feedback on group concept maps
2. Finding valid concepts—collecting data for the maps
3. Constructing the initial branches of the map
4. Creating examples
5. Using the computer
6. Determining valid relationships.

Item #2 requests that the subjects,

Describe what was most difficult about constructing your concept maps.

Forty percent (**n** = 6) of the subjects agreed creating appropriate propositions was a source of trouble. A third of the sample reported greatest difficulty with constructing crosslinks. Deborah wrote, "...hard time trying to find the perfect link word or phrase to connect two ideas together." Mark, an international student, concurs, "Try to come a word which related to two part of the map." Other relevant responses included computer construction, hierarchy, deciding relevant concepts and concept organization. Two subjects explicitly stated that concept organization was quite difficult, and the members of the criterion map expert group reported the same challenge. Of course, problem solving by its nature typically presents a challenge. Gabriel indicated that the arrangement of concepts in some meaningful order, easily understood by a mathematics reader, presented difficulty when constructing the maps.

Item #3 asks,

> **Was the classroom explanation about how to construct a concept map clear? If any part of the explanation was unclear, please elaborate. How could it be better?**

Subjects positively declared, “The explanations in class were always good...I did understand well...Seeing it done and doing it yourself are two different things.” However, some subjects requested better explanations on how to create crosslinks.

Item #4 commands,

> **Explain whether or not the collaborative experience (group work) with systems of equations helped you construct better maps for inverse and inverse function.**

Thirteen subjects responded positively and two subjects, who belonged to disparate groups, responded negatively. Joshua stated, “You’re able to feed off of each others’ ideas, and it helps you think more independently for the ones you do on your own.” Ezra affirms that the collaborative experience “totally helped me on my following maps...ideas to look up in the back of the book...talking to people really helped me understand the math better." Elizabeth concurs, "Absolutely because we were able to bounce ideas off one another and ask questions within the group...some concepts I didn't think to break down." Luke stated that there were "more concepts to talk about and ways to solve them." Obviously, Luke failed to clearly understand what concept means, since one cannot "solve" a concept. Finally, Hannah retorts negatively to this item, although her answer indicates a part of the problem-solving process occurs within a mathematical community. She proclaimed, "It was very frustrating, because I had to agree with decisions that I did not think were properly justified."

Item #5 poses,

> **What does it mean for a function to have an inverse?**

The subjects' responses included the following descriptions.

1. The Horizontal Line Test (**n** = 6)
2. One-to-one (**n** = 7)
3. Horizontal Line Test and One-to-one (**n** = 5)
4. Identity function (**n** = 5)
5. Symmetric with respect to the identity function (**n** = 5)

6. Permuted, changed or switched ordered pairs (**n** = 5)

7. Exactly three valid descriptions (**n** = 2)

8. At least two valid descriptions (**n** = 10)

9. Exactly one valid description (**n** = 2)

10. Completely invalid (**n** = 3)

Most of the responses to the item were graphical in nature, and only two subjects, Ezra and Jacob, referred to either "put[ting] them together and arrive at the identity function," or, "produces the identity function." Unfortunately, no answer directly included references to $f(g(x)) = x = g(f(x))$, meaning that f and g are inverses of each other, although 5 inverse function concept maps included this.

Item #6 states,

> **Provide an example of a function that has an inverse and give the explicit forms for each: $f(x) = ...$ and $f^{-1}(x) = ...$Defend your choice.**

Most responses revealed a lack of concern for domain and range; however, many responses reveal other important results. Of the 15 replies, 7 pairs of submitted functions were inverses, 3 pairs were "almost" correct, 2 pairs were incomplete (meaning an invertible function is given, but not its inverse), and 3 are incorrect. Table 28 displays correct answers by seven subjects.

Table 28

Table of Correct Answers to Item #6

Subject Name	Function $f(x)$	Inverse Function $f^{-1}(x)$	Defense	Concept Map As Defense Relates
Hannah	x	x	Domain & Range are switched; symmetric with respect to $y = x$	Both appear
Joshua	e^x	$\ln x$	Sketch provided; symmetric with respect to $y = x$	Sketches of graphs appear as examples; symmetry unmentioned
Isaac	x^3	$\sqrt[3]{x}$	None	N/A
Ezra	e^x	$\ln x$	Showed that $f(f^{-1}(x))$ $= x$ $= f^{-1}(f(x))$	Does not appear on map; although exponential and logarithmic do
Philip	x^3	$\sqrt[3]{x}$	Showed that $f(f^{-1}(x))$ $= x$ $= f^{-1}(f(x))$	$f(f^{-1}(x)) = x$ appears on the map
Matthew	$x^2, x \geq 0$	$\sqrt{x}$	None, yet took care for the restriction on the original function to make it one-to-one	N/A
David	$2x+5$	$\frac{x-5}{2}$	Only showed that $f(f^{-1}(x)) = x$	Notation appears, but not $f(f^{-1}(x)) = x$

The table indicates that 5 of the 7 students' defenses who responded have direct connections to their Functional Inverse Concept Maps. Table 29 verifies that Mark, Luke and Aaron neither graphically nor algebraically verify "one-to-one-ness" of their choices for functions:

Table 29

Table of "Almost" Correct Answers to Item #6

Subject Name	**Function** $f(x)$	**Inverse Function** $f^{-1}(x)$	**One-to-one appear on map?**
Mark	x^2+2	$\sqrt{x-2}$	Yes
Luke	x^2	$\sqrt{x}$	No, but Horizontal Line Test does
Aaron	x^2+2	$\sqrt{x-2}$	Yes

These students are obviously unaware that even functions are not invertible. Common problems occur when solving for x:

$$\begin{aligned} y &= x^2+2 \\ y-2 &= x^2 \\ x &= \pm\sqrt{y-2} \end{aligned} \quad .$$

Students making this error typically record $x=\sqrt{y-2}$ before permuting x and y, paying little or no attention to the result as not being a function, much less one-to-one.

Deborah answered with a pair of functions that are multiplicative inverses: $f(x)=3^x$ and $f^{-1}(x)=(\frac{1}{3})^x$. Vidakovic reports this error is quite common, yet it is not the case in this experiment. Jacob fails to provide a function in explicit form, but he does construct two tables:

$f(x)$		$f^{-1}(x)$	
x	y	x	y
-1	1	1	-1
0	2	2	0
1	3	3	1

However, if Jacob were to have explicitly stated that $f = \{(-1,2);(0,2);(1,3)\}$ were finite, then the inverse function would have been correct. Although Elizabeth provided an incomplete solution, she gave $f(x) = x^3$ and an excerpt of the tables for f and f^{-1}.

$f(x)$		$f^{-1}(x)$	
x	y	x	y
1	1	1	1
2	8	8	2
3	27	27	3

Finally, item #7 requests

> **What are some inverses other than functional inverses? How do they relate to the behavior of functional inverses?**

Eight subjects responded with matrices, 2 with additive real, 3 with multiplicative real, 2 with function (both subjects who answered with this were international students), and 3 subjects either did not know or left the item blank. No subject related their response to the behavior of functional inverses.

Chapter 6

Summary, Conclusions, and Recommendations

Summary

The purpose of this study was to determine if concept mapping augments community college precalculus students' understanding of inverse functions. Of the two selected groups, only the treatment of concept mapping differentiated them. There was no difference in instructional delivery to either group. Analysis of nominal and ratio data was useful in this investigation. The necessity for diverse methods for understanding mathematics concepts that incite making connections, drawing conclusions, solving problems and working within a mathematics community at the college level inspired this study.

The experiment consisted of two precalculus algebra classes taught at a central North Carolina community college. Random assignment of the instructors to the precalculus classes is not how the mathematics discipline chair allocates class assignments. Since there were only three precalculus algebra classes available for this research at the college, and anecdotal evidence of the third section indicated performance was likely different from the two classes selected, the discipline chair, precalculus instructors and this researcher agreed the researcher should use the two classes that were "similar." Furthermore, since this researcher had ten year of experience teaching the course (or some variation of it), the mathematics chair strongly urged this researcher to teach both the experimental and control sections.

The instructor informed the experimental and control sections that subjects' grades and demographic data would be used for a study in mathematics education after the drop/add period ended for the semester. However, the researcher enumerated the sample size based on the second day of attendance for both classes, $\underline{\mathbf{N}} = 60$; yet, only those subjects who completed the course (including having taken the final examination) were included in the study. Ultimately, the adjusted sample size became, $\underline{\mathbf{N}} = 36$, which consisted of 15 experimental subjects and 21 control subjects.

At the beginning of the semester, students to a routine precalculus pretest and completed an information data sheet. Only the experimental subjects complete a mathematics attitude survey (ARMAS). The data from these instruments yielded no significant differences in pretest score means between the control and experimental subjects who endured or between control and experimental subjects who withdrew. The only significant demographic difference occurred relative to gender. There was a significant difference between the proportion of females and males in the experimental section and the proportion of females in the experimental section to males or females of the control section.

During the course of the semester, instruction remained equivalent, except for concept mapping. Every assessment was exactly the same, including all 4 of the unit tests, quizzes, writing assignments or projects, homework assignments and the final examination. Occasionally, the instructor prefers to administer different test forms, but for the sake of internal validity, he belayed this practice for the semester. The control subjects completed a second project during the first half of the semester in lieu of the concept mapping activities. During the early part of the last week of classes, experimental subjects completed a second ARMAS and a follow-up survey on concept mapping and inverse functions.

Conclusions

Research question 1 asks, "Does concept mapping improve conceptual understanding of community college precalculus students relative to inverse functions?"

Results for Hypothesis 1 respond to this question. Recall that Hypothesis 1 states, "After treatment, no significant differences exist between the experimental and control groups as measured by selected conceptual inverse function items on Unit Test #4, components of a routinely assigned writing exercises, and inverse function final examination items."

Mean scores of the control and experimental groups did not significantly differ for the inverse function items in one of the course writing assignments, Test #4 inverse function items or final examination inverse function items. However, there is an accepted difference in means for Test #4 inverse function items for $\underline{\mathbf{p}} > 0\ 0712$ ($\underline{\mathbf{F}} = 3.47$). In addition, and independent sample t-test yielded no significant difference between group's course grade means.

Almost every subjects' response to Test #4 inverse items were referenced on their concept maps. Most students mentioned higher order concepts, such as symmetry, Horizontal Line Test, composition (either explicitly or symbolically), domain, range and permute. Likewise, subjects with higher inverse function map scores responded correctly to Item #6 of the follow-up survey.

Question 2 poses, "Do mapping, pretest performance, inverse function items on assessments, and demographic variables, such as employment status, language and gender affect student achievement?" The corresponding hypothesis states, "After treatment the general linear model will have coefficients $\beta_i = 0, 1 \le i \le 7$, where X_1 = Inverse Function Concept Map Score, X_2 = Test #4 inverse function items subscore, X_3 = writing assignment score, X_4 = final examination inverse function items subscore, X_5 = employment status, X_6 = language and X_7 = gender." The backwards elimination regression model revealed quite an interesting result. The model assigned $\beta_i = 0$ for all variables except for the Inverse Function Concept Map Score and Test #4 inverse function items. The model was significant with a p-value of $p = 0.0006$, and these two independent variables were significant at $p \ge 0.006$. This clearly gives credence to concept mapping as a significant means of assessment affecting mathematics achievement. However, it should be noted that Kendall's τ_β for the map score and final grade is $r = 0.3817$ ($p < 0.1$) and Test #4 inverse function items is $r = 0.5477$ ($p < 0.01$).

Research question 3 asks, "Does concept mapping improve community college precalculus students' attitudes and beliefs about mathematics?" The corresponding hypothesis states, "After treatment, no insignificant differences exist in student beliefs about mathematics for the experimental group as measured by the Aiken Revised Mathematics Attitude Scale." The ARMAS was administered twice, before the mapping treatment and following the close of all treatment exercises. Although the mean score was approximately 1.5 points higher for the second administration there was no significant difference between mean scores according to the two-sample t-test for paired replicates. This is a positive revelation in that concept mapping did not appear to reduce overall interest in mathematics and its relevance.

Limitations

Unfortunately, true randomization of subjects was impossible. Many of the subjects chose their precalculus section based on employment schedules, family obligations or simply time preference. However, no subject knew about the research experiment until after the drop/add period, as previously mentioned. Only students who wish to pursue an Associate in Science degree mush complete each of two precalculus courses in the sequence with a 'C' or better. An analogous, but less rigorous, course called College Algebra, MAT 161, is available for Associate in Arts students. So, not just any community college student takes Precalculus Algebra, MAT 171.

Although a sample size of $\underline{\mathbf{N}}$ = 36 is acceptable, it is insufficient. Subjects from the third precalculus class would have augmented the data set and could have influenced the statistical analyses. Perhaps data from the College Algebra sections may have influenced the outcome of the study.

Few, if any, issues threatened the internal validity of this study. Subjects who remained in the study did not undergo any known biological or psychological changes. Since experimental subjects completed at most three concept maps during the course of the semester, neither testing replication nor statistical regression influenced the results. Furthermore, the treatment instrument remained constant. Since the same instructor taught the control and experimental sections, and the instructor clearly understood the gravity of the study, the instructor minimized compensatory equalization of the experimental group and demoralization of the control group. The initial and final statistics revealed very little difference between the two groups; so, there was no evidence of differential selection. Since the researchers informed control subjects that only some of their scores from graded course material would be used, it seemed unlikely the control members put any extra effort in their work beyond what would be considered normal for the population.

Although most of these threats never surfaced during the experiment, subject attrition could have altered the outcome of the study. Forty-six percent of the experimental subjects either dropped the course during the first week or withdrew because of employment, family obligations, or inability of performing adequately due to inadequate prerequisites (8 of 13 subjects who withdrew did not have appropriate prerequisites). Thirty-five percent of the subjects withdrew because of employment or family obligations. Only 3 of the 11 control subjects who withdrew from the control group had inadequate prerequisites. The community college administration refuses to give authority to instructors for initiating forced withdrawals of students who fail to have adequately completed appropriate prerequisite courses. It is unknown whether or not the interaction between selection and attrition influenced the validity of this experiment.

Likewise, generalization of the results of this experiment to the larger population of community college precalculus students does not appear impaired. First, the researcher avoided the Hawthorne Effect by not having the course instructor refer to the concept mapping exercises as being part of the study. Secondly, the sample of precalculus students well represented the general population. Thirdly, although subjects in the experimental section had little or no prior experience with the treatment or any variation of it, they did not know that other classes did not participate in concept mapping activities; thus, avoiding the Novelty Effect. Finally, since the treatment of concept mapping was the sole difference between the two groups, multiple treatment did not apply.

Recommendations

Numerous components of this study should be expanded. Current research in the application of concept mapping in the post secondary mathematics classroom remains quite limited. No research exists on the use of concept mapping in the developmental mathematics environment at the community college. Moreover, augmentation of this investigation to include another semester of mathematics prior to precalculus algebra should reveal ameliorated results. The natural course to include in future research would be Intermediate Algebra, MAT 080 (North Carolina Community College System). However, since courses that follow prerequisites rarely have exactly the same students, this could present logistical and administrative challenges.

Since time limited the results of this study, this researcher recommends further replications for Precalculus Algebra or College Algebra. More time allows for better and multiple subject interviews, and subsequently, ameliorated qualitative analysis. These replications of the concept mapping exercises should strengthen results submitted in this report. Comparisons among community colleges, university and secondary levels of precalculus groups might uncover interesting findings. Questions worth investigating include:

1. Is there a significant difference in achievement among community college, university and high school students in precalculus groups, given the concept mapping treatment?
2. Does concept mapping influence students' mathematics achievement after at least one full semester of experience with concept mapping?
3. Does concept mapping affect higher level mathematics proving ability?
4. Does concept mapping improve student achievement in groups that have little or no access to graphing utilities or software?

5. Is there a significant interaction between the use of a graphing utility and concept mapping?

Although the mathematics instructor in this study claimed no difference in instruction, employment of a variety of instructors in replications of this study should increase this study's validity.

Another issue of concern is concept map scoring reliability. Whereas the researcher employed a reliable scoring rubric, it is likely that subjects' scores would likely vary from rater to rater. A logical extension of this study would include a replication with at least 3 raters using the same scoring rubric.

Unfortunately, the experimental subset of the sample in this study had a disproportionate number of males versus females. More replications with statistically significant differences relative to gender would yield more valid and reliable results when analyzing the differences between males and females. Furthermore, the experimental subset had only 3 female subjects; this impeded the use of many standard statistical tests used for quantitative analyses.

Finally, the experts who produced the criterion maps and the experimental subjects agreed that the mapping exercises stimulate discussion and deliberation, promote organization and illustrate connections. Moreover, concept mapping prompts independent, logical and creative thinking. The nature of the instructions provided for concept mapping in this study encouraged creative thinking as shown in the development of the system of equations map and inverse function map. Ultimately, Bruner would have been pleased that concept mapping promotes discovery learning.

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Appendix A

Performance Measures for 1998-1999 Transfers from 005448

Durham CC to UNC Institutions

PERFORMANCE MEASURES FOR 1999 - 2000 TRANSFERS FROM 005448 Durham CC TO UNC INSTITUTIONS

STUDENT SELECTION CRITERIA: RACE= ALL SEX= ALL CLASS LEVEL= ALL TYPE= FULL & PART-TIME

UNC INSTITUTIONS	No. of Trans-fers	FALL TERM MEASURES: Trans-fers Mean GPA	Mean Letter Grade Hours EARNED	Mean Hrs. of 'I','W' 'Pass/Fail'	English N	English Mean GPA	Math/Science N	Math/Science Mean GPA	Social Science N	Social Science Mean GPA	END OF YEAR MEASURES (SUMMER 1999 - SPRING 2000): Trans-fers Mean GPA	Academic Standing (% of ransfers): Good Stand-ing	Pro-bation	Sus-pended	With-drew	Grad-uated
Appalachian	4	3.27	12.0	0.0	2	3.00	1	3.00	4	3.28	3.08	100.0	N/A	N/A	N/A	N/A
East Carolina	7	2.47	12.9	0.6	1	4.00	4	2.00	6	1.22	2.54	71.4	N/A	N/A	N/A	N/A
Elizabeth City	0	****	****	****	****	****	****	****	****	****	****	****	N/A	N/A	N/A	N/A
Fayetteville	0	****	****	****	****	****	****	****	****	****	****	****	N/A	N/A	N/A	N/A
N.C. A and T	3	1.71	9.3	0.0	2	2.00	2	1.33	0	0.00	1.57	0.0	N/A	N/A	N/A	N/A
N.C. Central	45	2.96	9.8	1.4	15	3.04	18	2.68	24	2.88	2.89	73.3	N/A	N/A	N/A	N/A
N.C. School of Arts	1	****	****	****	****	****	****	****	****	****	****	****	N/A	N/A	N/A	N/A
N.C. State	26	2.68	12.0	0.2	3	2.44	22	2.59	17	2.44	2.54	69.2	N/A	N/A	N/A	N/A
UNC-Asheville	2	****	****	****	****	****	****	****	****	****	****	****	N/A	N/A	N/A	N/A
UNC-Chapel Hill	41	2.81	13.7	0.3	4	2.93	16	2.69	21	2.79	2.90	78.0	N/A	N/A	N/A	N/A
UNC-Charlotte	4	2.40	8.8	2.8	2	2.50	1	3.00	0	0.00	2.48	50.0	N/A	N/A	N/A	N/A
UNC-Greensboro	12	3.14	11.1	5.3	1	2.00	7	2.22	4	3.17	3.13	66.7	N/A	N/A	N/A	N/A
UNC-Pembroke	1	****	****	****	****	****	****	****	****	****	****	****	N/A	N/A	N/A	N/A
UNC-Wilmington	3	2.32	12.0	3.7	2	1.92	0	0.00	2	2.56	2.25	66.7	N/A	N/A	N/A	N/A
Western Carolina	1	****	****	****	****	****	****	****	****	****	****	****	N/A	N/A	N/A	N/A
Winston-Salem	0	****	****	****	****	****	****	****	****	****	****	****	N/A	N/A	N/A	N/A
UNC TOTAL	150	2.81	11.5	1.2	34	2.79	71	2.53	81	2.67	2.79	71.3	N/A	N/A	N/A	N/A

Definitions:

1) A full-time transfer student is one who completed 12 hours or more of letter-graded coursework in the fall semester and in the spring semester.
2) Class level is based on the number of hours 'accepted' for transfer from all previously attended institutions (0<30 = Freshman, 30<60 = Sophomore, 60+ = Upper Division).
1) Letter Grade courses include those which are graded 'A', 'B', 'C', 'D', 'F' or 'WF' or the equivalent thereof. They Exclude Advanced Placement, CLEP and regular courses with grades of 'I', 'W' or 'Pass/Fail'. The only exception to this definition occurs at UNC-CH, where a grade of 'I' is treated as an 'F' and is included in Letter Grade Hours Earned and in the Mean GPA.
2) For 1996-97 and later, Good Standing means completing the first year with GPA >= 2.0. Information about Probation, Suspension, Withdrawal, and Graduation is not available.

UNC-GA ProgAssess/TSP.PR001C/04FEB02

Performance Measures for 1998-1999 from

Community Colleges to UNC Institutions

PERFORMANCE MEASURES FOR 1999 - 2000 TRANSFERS FROM COMMUNITY COLLEGES TO UNC INSTITUTIONS

STUDENT SELECTION CRITERIA: RACE= ALL SEX= ALL CLASS LEVEL= ALL TYPE= FULL & PART-TIME

UNC INSTITUTIONS	No. of Trans-fers	FALL TERM MEASURES: Trans-fers Mean GPA	Mean Letter Grade Hours EARNED	Mean Hrs. of 'I','W' 'Pass/ Fail'	English N	English Mean GPA	Math/ Science N	Math/ Science Mean GPA	Social Science N	Social Science Mean GPA	END OF YEAR MEASURES (SUMMER 1999 - SPRING 2000): Trans-fers Mean GPA	Academic Standing (% of Transfers): Good Stand-ing	Pro-bation	Sus-pended	With-drew	Grad-uated
Appalachian	336	2.75	12.4	0.6	69	2.65	101	2.10	177	2.66	2.76	76.8	N/A	N/A	N/A	N/A
East Carolina	533	2.62	12.0	1.3	141	3.11	283	2.15	339	2.37	2.65	67.0	N/A	N/A	N/A	N/A
Elizabeth City	46	3.09	13.2	0.8	28	3.01	22	2.65	45	3.26	3.19	82.6	N/A	N/A	N/A	N/A
Fayetteville	134	2.84	9.8	2.0	48	2.77	66	2.52	58	2.90	2.93	70.1	N/A	N/A	N/A	N/A
N.C. A and T	100	2.28	11.6	0.0	40	2.44	65	1.85	21	2.05	2.31	65.0	N/A	N/A	N/A	N/A
N.C. Central	109	2.76	10.5	1.4	33	2.86	49	2.31	48	2.56	2.75	67.9	N/A	N/A	N/A	N/A
N.C. School of Arts	7	3.50	10.7	4.6	0	0.00	0	0.00	1	3.00	3.45	71.4	N/A	N/A	N/A	N/A
N.C. State	400	2.58	11.8	0.5	58	2.88	291	2.41	260	2.55	2.59	68.0	N/A	N/A	N/A	N/A
UNC-Asheville	113	2.61	10.7	1.2	26	3.00	56	2.28	44	2.77	2.67	66.4	N/A	N/A	N/A	N/A
UNC-Chapel Hill	174	2.73	12.0	0.5	20	2.63	61	2.33	94	2.58	2.77	66.1	N/A	N/A	N/A	N/A
UNC-Charlotte	632	2.51	10.5	1.3	99	2.74	322	2.08	276	2.45	2.56	61.6	N/A	N/A	N/A	N/A
UNC-Greensboro	378	2.72	10.5	2.2	91	2.83	128	2.28	166	2.54	2.78	69.0	N/A	N/A	N/A	N/A
UNC-Pembroke	180	2.85	10.3	2.3	36	2.87	91	2.43	93	2.66	2.90	64.4	N/A	N/A	N/A	N/A
UNC-Wilmington	443	2.59	11.6	1.1	84	2.79	207	2.12	204	2.59	2.63	64.1	N/A	N/A	N/A	N/A
Western Carolina	241	2.50	12.2	0.1	89	2.06	125	2.06	90	2.56	2.58	72.2	N/A	N/A	N/A	N/A
Winston-Salem	70	2.48	10.5	1.0	24	2.00	39	1.71	31	2.50	2.44	54.3	N/A	N/A	N/A	N/A
UNC TOTAL	3,896	2.63	11.3	1.1	886	2.74	1,906	2.20	1,947	2.56	2.67	67.1	N/A	N/A	N/A	N/A

Definitions:

1) A full-time transfer student is one who completed 12 hours or more of letter-graded coursework in the fall semester and in the spring semester.
2) Class level is based on the number of hours 'accepted' for transfer from all previously attended institutions (0<30 = Freshman, 30<60 = Sophomore, 60+ = Upper Division).
3) Letter Grade courses include those which are graded 'A', 'B', 'C', 'D', 'F' or 'WF' or the equivalent thereof. They exclude Advanced Placement, CLEP and regular courses with grades of 'I', 'W' or 'Pass/Fail'. The only exception to this definition occurs at UNC-CH, where a grade of 'I' is treated as an 'F' and is included in Letter Grade Hours Earned and in the Mean GPA.
4) For 1996-97 and later, Good Standing means completing the first year with GPA >= 2.0. Information about Probation, Suspension, Withdrawal, and Graduation is not available.

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Appendix B

Aiken Revised Attitude to Mathematics Scale (1979)

For each item completely fill in one bubble corresponding to a single choice from the following scale:

A = Strongly Agree
B = Agree
C = Undecided
D = Disagree
E = Strongly Disagree

1. Mathematics is not a very interesting subject.
2. I want to develop my mathematics skill and study this subject more.
3. Mathematics is a very worthwhile and necessary subject.
4. Mathematics makes me feel nervous and uncomfortable.
5. I have usually enjoyed studying mathematics at school.
6. I don't want to take any more mathematics than I absolutely have to.
7. Other subjects are more important to people than mathematics.
8. I am very calm and unafraid when studying mathematics.
9. I have seldom liked studying mathematics.
10. I am interested in acquiring further knowledge of mathematics.
11. Mathematics helps to develop the mind and teaches a person to think.
12. Mathematics makes me feel uneasy and confused.
13. Mathematics is enjoyable and stimulating to me.
14. I am not willing to take more than the required amount of mathematics.

Aiken Scale Page 2

15. Mathematics is not especially important to everyday life.

16. Trying to understand mathematics doesn’t make me anxious.

17. Mathematics is dull and boring.

18. I plan to take as much mathematics as I possibly can during my education.

19. Mathematics has contributed greatly to the progress of civilization.

20. Mathematics is one of my most dreaded subjects.

21. I like trying to solve new problems in mathematics.

22. I am not motivated to work very hard on mathematics problems.

23. Mathematics is not one of the most important subjects for people to study.

24. I don’t get upset when trying to work mathematics problems.

Appendix C

The MAT 171 Selected Diagnostic Pretest Items

4. Multiply $(3-y)(2+4y-y^2)$
   A. $-y^3+10y^2+2y+6$
   B. y^3-5y^2+2y+6
   C. y^3-6y^2+3y+6
   D. $y^3-7y^2+10y+6$

9. Factor $27a^3-8$ completely.
   A. $(9a^2-4)(3a+2)$
   B. $(3a-2)(9a^2+6a+4)$
   C. $(3a-2)^3$
   D. $(3a-2)(3a^2+4)$

13. Simplify $13\sqrt{32}-(15\sqrt{18}-2\sqrt{8})$.
   A. $-11\sqrt{2}$
   B. $-4\sqrt{2}$
   C. $11\sqrt{2}$
   D. $4\sqrt{2}$

14. The graph of $3x+2y=-6$ has _______ and ________ as intercept points.
   A. $(-2,0)$ and $(0,-3)$
   B. $(2,0)$ and $(0,-3)$
   C. $(3,0)$ and $(0,2)$
   D. $(-3,0)$ and $(0,2)$

19. Solve $x^2+6x=16$ for x.
   A. $x=-8$ or $x=2$
   B. $x=0$ or $x=6$
   C. $x=6$
   E. $x=-2$ or $x=8$

Appendix D

Follow-up Questions
Concept Mapping and Inverse Functions

Please respond completely as possible to each of the following items.

1. What did you find easy about constructing your concept maps?

2. Describe what was most difficult about constructing your concept maps.

3. Was the classroom explanation about how to construct a concept map clear? If any part of the explanation was unclear, please elaborate. How could it be better?

4. Explain whether or not the collaborative experience (group work) with systems of equations helped you construct better maps for inverse and inverse function.

5. What does it mean for a function to have an inverse?

6. Provide an example of a function that has an inverse and give the explicit forms for each: $f(x) = \ldots$ and $f^{-1}(x) = \ldots$ Defend your choice.

7. What are some inverses other than functional inverses? How do they relate to the behavior of functional inverses?

Appendix E

Concept Map Construction

Effects of Concept Mapping on
Community College Precalculus Students' Understanding of Functional Inverse

What is a concept map?

A **concept map** consists of two or more ideas showing how these ideas relate to each other. If two concepts are related to each other, then a segment or curve connects the two concepts. Adjacent to the segment is a word or small phrase that elaborates the relationship between the two concepts. The segment is called a **link**, and the words or small phrases are called **propositions**. Ovals or rectangles enclose each concept on the map.

An example of a concept map appears on the next page. This map reviews the concept of real numbers. Notice the central concept is "Real Numbers." There are numerous sub-concepts, some of which include "rational numbers", "irrational numbers", and "integers." The phrase, "that includes only wholes and their opposites," is the proposition that links the two concepts "rational numbers" and "integers" together. **Every link** should have a propositional label.

The hierarchy of a concept map is quite important. The **hierarchy** refers to the branch of the map with the most linear segments connecting concepts in a top-down fashion. The bold dashed segments illustrate how to determine the hierarchy of the concept map of real numbers on the next page. The map has a hierarchy of 5.

The four emboldened sub-concepts, "integers", "whole #s", "negative integers" and "positive integers…" form a closed group. This closed group also refers to what is called a crosslink. A **crosslink** simply connects two separate branches of the concept map together with a link. Finally, concept maps should also include **examples**. However, examples should be located at the outer fringes of the map.

Concept Map Construction
Page 2

Keep in mind the following questions when constructing your concept maps.

1. Does the map contain concepts appropriate to the superordinate concept provided by the instructor?

2. Are all the links between the concepts precisely linked?

3. Does the map have any labeled cross-links?

4. Does the map also contain examples (preferably novel examples)?

5. Is the map treelike (dendritic) instead of stringy (linear)?

6. Is the superordinate concept the best choice, given the way the rest of the concepts [are] arranged?

7. Are the examples included appropriate?

8. Is the map of acceptable scientific quality?

9. Has the mapper used the proper map symbols and followed standard mapping conventions?

Mintzes, Wandersee, & Novak. (2000). Assessing science understanding: A human constructivist view. San Diego, CA: Academic Press.

Appendix F

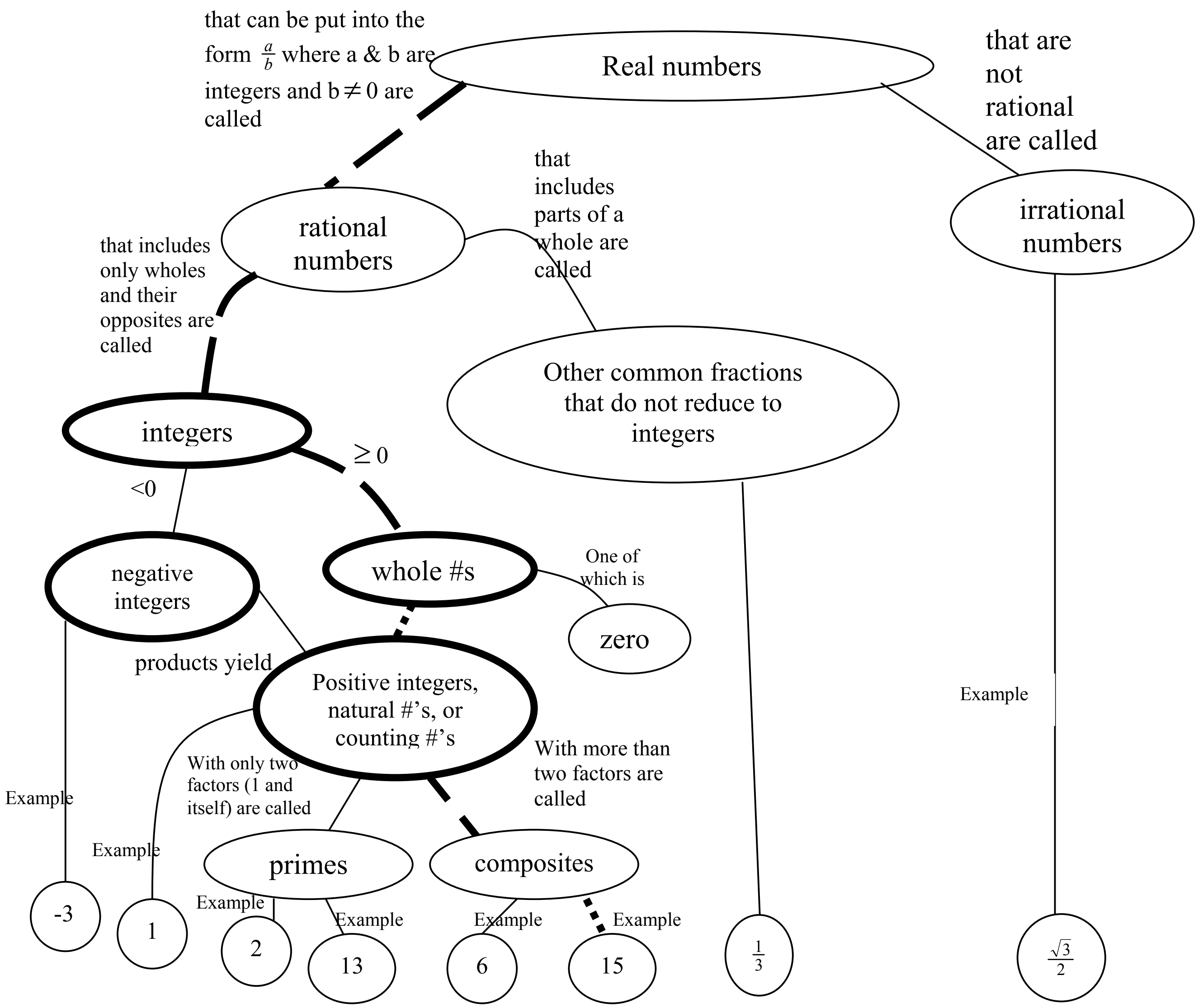
Scoring Rubric for Your Concept Maps

Map Component	Point Value of Component
Valid Concept	1
Valid Relationship	1
First Branching	1
Successive Branching	3
Hierarchy	5
Crosslink	10
Example	1

Markham, K. M., Mintzes, J. J., & Jones, M. G. (1994). The concept map as a research and evaluation tool: Further evidence of validity. Journal of Research in Science Teaching, 31, 91-101.

Appendix G

A hierarchical concept map of the real-number system drawn by preservice teachers



Baroody and Bartels (2000), Using concept maps to link mathematical ideas. Mathematics Teaching in the Middle School, 5(9), page 606. Parts augmented by R. L. Merritt, Jr. (2002).

Appendix H

<u>MAT 171</u>
Concept Map Requirements

Each of you has been randomly assigned to a group of 3 members. The initial meeting takes place during today's class period. During the time allotted, please begin item #1 from the list of project requirements below. You will be given information sheets to complete that you will share with your group members so that you might complete the group component of this project. Your project consists of several parts:

1. **Initial group concept map based on "Systems of Equations", given a list of seed concepts**
2. **Refined "Systems" concept map**
3. **Written report of the development of the "Systems of Equations" concept map**
   This report should include
   a. Deletions from the map at any given time (in class and outside of class).
   b. Additions of concepts to the list of seed concepts
   c. Why these changes were made
   d. Include interactions by **<u>each</u>** group member.
   This report should be typed and should comply with the guidelines for appropriate college writing as enumerated in the ***Dot System*** outline or ***Guidelines for Writing an Acceptable Lab Report.***
4. **Individual concept map based on "Inverse"**
5. **Individual concept map based on "Functional Inverse"**
6. **Written report on a specified function, tracing the development of its functional inverse and revealing the behavior of the functional inverse.** In addition, your report should refer to your concept map. This report should follow the guidelines for appropriate college writing as enumerated in the ***Dot System*** outline or ***Guidelines for Writing an Acceptable Lab Report.*** The function for this exercise will be given during a regular class period. Finally, only typed reports shall be accepted. The reports from items 3 and 6 shall be graded.

The seed concepts for "Systems of Equations" are

Solutions
Standard Form
Row Reduction
Intersection
Linear

Appendix I

Brief Questionnaire of Criterion Concept Map Participants

1. Approximately how many years have you been teaching mathematics?

2. In which levels of mathematics teaching have you had experience (e.g., middle school, university, etc.)?

3. How might you find concept mapping useful within the mathematics courses that you teach?

4. Explain whether or not you think concept mapping would be appropriate for MAT 171, Precalculus Algebra.

5. What did you find difficult about the criterion concept mapping exercise?

Appendix J

Scoring for Inverse Map

Name: ____________________________

Map Component	**Point Value of Component**	**Total Correct Map Components Observed**	**Total Observed Per Component**
Valid Concept	1		
Valid Relationship	1		
First Branching	1		
Successive Branching	3		
Hierarchy	5		
Crosslink	10		
Example	1		
		Composite Score	

Appendix K

MAT 171 Name ___________________
Mini-Project: Functions Date ___________________

Honor Statement

Sign the following pledge upon completion of this take-home assignment.
I have neither offered nor received any assistance from any person while completing this assignment.

____
Signature

Respond to the following items according to the guidelines given for writing assignments in your syllabus. These guidelines are located on the Dot System page of your syllabus. If you need a copy of the set of guidelines, then inform the instructor. You may use your text, notes, quizzes, test or other paperwork to assist you. Document the source material you used (if any) at the beginning of each item. Attach this cover sheet to the front of your work.

1. Analyze the behavior of the function $f(x) = 3x^4 - 20x^3 + 5x^2 - 80x - 28$.

2. Does the function in item #1 have an inverse function? Defend your response.

3. Analyze the behavior of the function $h(x) = \dfrac{2x^2 - x - 1}{x + 2}$.

4. Why does the function $g(x) = x^2 - 5x + 6$ not have an inverse function? How would you make changes to this function so that it is invertible?

5. Find the inverse for each of the following functions, if it exists. If the inverse does not exist, then state why the function does not have an inverse. If the function does have an inverse, show how you would substantiate your choice.

   a. $f(x) = 4 - x^2, x \geq 0$

   b. $f(x) = 4 - x^2, -2 \leq x \leq 2$

   c. $f(x) = \dfrac{1}{1 - x}$

   d. $f(x) = 5^x$

Appendix L

Inverse function items from MAT 171 Final Exam:

10. Consider the function $f(x) = \ln x + 2$. Which of the following is true?

    I. $f^{-1}(x) = e^{x-2}$

    II. The domain of f is $(0, \infty)$.

    III. The x-intercept of f is $(0.135, 0)$.

    (a) I only (b) I and II only (c) II and III only (d) I, II and III

31. The functional inverse of $f(x) = \sqrt[3]{2x+1}$ is

    (a) $f^{-1}(x) = \sqrt[3]{2y+1}$
    (b) $f^{-1}(x) = \frac{1}{2}(x^3 - 1)$
    (c) $f^{-1}(x) = \frac{1}{2}(y^3 + 1)$
    (d) $f^{-1}(x) = \dfrac{1}{\sqrt[3]{2x+1}}$

32. Which of the following statements is true about the relationship between $f_1(x) = \ln x$ and $f_2(x) = e^x$?

    (a) They are functional inverses of each other.
    (b) They are symmetric about the line $y = x$.
    (c) The domain of $f_1(x) = \ln x$ is exactly the range of $f_2(x) = e^x$.
    (d) All of the above statements are true.

Appendix M

(Reduced Print)

MAT 171
Personal Information Update
This information will be kept confidential. Thank you for your valuable input.

Course Section ___________________________

Name __________________________________

Circle appropriate gender: Female Male

Check the average range of hours per week you have been employed this semester.

________ Less than 10 hours

________ 10 – 19 hours

________ 20 – 29 hours

________ 30 or more hours

Check the average range of hours per week you have studied for this course. Include completing project(s), homework, or other take home exercises. Do not include time spent in class. **YOUR END OF COURSE GRADE WILL NOT BE AFFECTED BY YOUR ANSWER**. So, please be honest.

_________ 1 hour or less

_________ 2 – 4 hours

_________ 5 – 6 hours

_________ More than 6 hours

Check the appropriate blank corresponding to your mathematics preparation prior to taking this course. If it has been a year or more since you have taken your last math course, then check only that blank.

_________ Less than two years of high school algebra or completion of MAT 070, Introductory Algebra only.

_________ Completion of high school algebra II or completion of MAT 080, Intermediate Algebra

_________ Unsuccessful completion of MAT 171, Precalculus Algebra, in a prior semester. This includes D, F, or W.

_________ It has been more than one year since I had taken a math course anywhere prior to completing this one.

What is your primary or native language? ____________________________

Appendix N

Composite Scores for Map I and Map II

Name	Map I	Map II
Mark	18	40
Gabriel	54	53
Ezra	67	73
Deborah	65	46
Joshua	59	52
Abraham	13	61
Hannah	83	109
David	53	61
Jacob	33	23
Aaron	69	55
Matthew	32	65
Luke	60	42
Isaac	27	38
Philip	23	47
Elizabeth	39	55