ABSTRACT

DAVIS, CLARENCE EDWARD “C.E.” Prospective Teachers’ Subject Matter Knowledge of Similarity. (Under the direction of Sarah B. Berenson and Ron Tzur.)

Teachers’ knowledge of the subject matter needed for teaching is seen as diverse, multidimensional, and vital to a teachers’ knowledge base for teaching (Ball, Lubienski, & Mewborn, 2001; Cooney & Wilson, 1995; Even, 1993; Grossman, Wilson, & Shulman, 1989; Ma, 1999; Shulman, 1986; Thompson, 1992; Wilson, Shulman, & Richert, 1987). Understandings of subject matter knowledge become important when researching students’ conceptual understandings because teachers’ knowledge of organization, connections among ideas, ways of proof and inquiry, and knowledge growth within discipline are important factors needed to teach the subject.

Some research has been done on secondary mathematics teachers’ subject matter knowledge (e.g., Cooney, Shealy, & Arvold, 1998; Cooney & Wilson, 1995; Even, 1993; Leinhardt, 1989; Leinhardt & Smith, 1985), however, today’s studies tend to report more on elementary prospective and inservice teachers’ subject matter knowledge rather than the knowledge of middle grades and high school prospective and inservice teachers. Therefore, the focus of the research is in reaction to researchers’ statements about research and teachers’ subject matter knowledge (e.g. Ball et al., 2001, Borko & Putnam, 1996; Schoenfeld, 1999) and is driven by two interrelated questions: (1) What is the nature of the subject matter knowledge that prospective teachers rely on when planning a lesson to introduce the concept of similarity, and (2) What growth in the subject matter

knowledge of similarity is revealed as prospective teachers plan a lesson to introduce the topic.

To analyze the subject matter knowledge of similarity that prospective teachers rely on when planning a lesson to introduce the topic of similarity, the transcripts of the interviews, the group presentation, and the written artifacts from the final individual lessons were coded using Even's (1990) seven aspects of subject matter knowledge and Shulman's (1986) three forms of teacher knowledge. The growth in subject matter knowledge of similarity was assessed within the Berenson, Cavey, Clark and Staley (2001) adaptation of the Pirie-Kieren (1994) model noting instances of folding back, collecting, and thickening. Within each level of the Berenson et al. model, Even's (1990) aspects of subject matter knowledge were looked at for potential growth in the context of *what* and *how* to teach.

Results from the study show that the prospective teachers' ideas of *what* and *how* to teach focused on procedural generalizations conveying meanings. In some instances the prospective teachers used one or two examples of procedures and expected students to make generalizations from these examples. Another result from the study was that the prospective teachers' images of *what* and *how* to teach had to contend with their belief structures about teaching and learning. The prospective teachers in the lesson plan study relied heavily on their belief structures about teaching and learning and changed them minimally. Another result from the study is that the prospective teachers' starting place for growth in the subject matter knowledge is determined by their existing knowledge. The implications of this conjecture are that teacher education programs need to find ways to benefit from the vast knowledge and backgrounds of all their prospective students.

Lastly, prospective teachers were limited in their substantive and syntactic knowledge of the concept of similarity. The prospective teachers seemed to fixate on particular aspects of similarity; however, the connections between these facts and the proof they established in their lessons were in need of reinforcement.

PROSPECTIVE TEACHERS' SUBJECT MATTER KNOWLEDGE OF SIMILARITY

by
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DEDICATION

I would like to dedicate my work to the memory of my grandmother, Doris McCraw. Without her support early in my life I do not feel as if I would have achieved all that I have. Her strength, kindness, and love were immeasurable.

Granny, I will always love you.

PERSONAL BIOGRAPHY

Clarence “C.E.” Davis was born August 21, 1970 in Lynchburg, Virginia. In 1988, he graduated from Rustburg High School in Rustburg, V.A. After receiving an Associates Degree in Education in 1992 from Central Virginia Community College he transferred to Longwood College in Farmville, V.A. In 1995, C.E. graduated from Longwood College with a B.S. in Pure and Applied Mathematics and a Virginia teaching certificate. In the fall of 1995, he attended Eastern Kentucky University and graduated from EKU with a M.S. Degree in Mathematical Sciences in 1997. It was during this time that he met and married his wife Qiaofang Ding Davis. After graduating from EKU, he worked as an eighth grade mathematics teacher at Caswell County Schools in North Carolina. In 1998, C.E. moved to Oxford, N.C. and began working at South Granville High School as a high school math teacher. It was during this time that C.E. began his Ph.D. studies in the Department of Mathematics, Science, and Technology Education at North Carolina State University. In 2002, C.E. and Qiaofang welcomed the birth of their daughter, Eylora Dorothy Lan-Lan Davis-Ding, who has become the joy in both of their lives.

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INTRODUCTION

> Why research has focused on elementary teachers reflects a continuing assumption that content knowledge is not a problem for secondary teachers, who, by virtue of specialized study in mathematics, know their subjects. However, research on secondary teachers repeatedly reveals the fallacy of this assumption (Ball, Lubienski, & Mewborn, 2001, p.444).

This introduction serves several purposes for the research in question. The first purpose is to give an overview of past research on subject matter knowledge (SMK) and explain how the research on SMK has changed over the course of the last century. The second purpose of this introduction is to define SMK, since many researchers see it differently, and to stress the significant role it plays in teaching. The third purpose is to discuss some aspects of SMK needed in teaching similarity. Lastly, this introduction discusses the focus of the research and how it is concerned with prospective teachers' SMK of similarity.

Historical Aspects of Subject Matter Knowledge

Over the course of the last century several researchers have made strong statements about teachers' SMK. In 1902, Dewey stated that when teachers study a subject they extract a quality from their studies unlike most scientists, requiring the teacher to think of the subject matter as a factor in a child's growth. During that time, some of the earliest known research on SMK in mathematics and education was conducted (Archibald, 1918; Kandel, 1915; Rice, 1902; Rice, 1903; Thorndike, 1922;

Thorndike, 1923). In one of the first studies on teacher preparation, Archibald (1918) stated, “if a high minimum standard were [sic] required on the part of each teacher, and the position of the teacher were [sic] made such as to attract in sufficient numbers the best talent in the country, other difficulties would disappear (p. 4).” The difficulties that Archibald referred to were the problems that teachers had teaching particular topics, the amount of stress placed on algebraic manipulation, and the importance of solving applications. Also during that time, teachers published writings on their classroom teaching (Kaput, 1992). However, these writings did not attempt to find out why a phenomenon happened or to try and explain the phenomena, and only presented lessons and activities that other teachers might like to try. These early writings began the process of inquiry in the field of mathematics education.

> Through their own activities and those of their students, they [teachers of the early 1900’s] laid the foundations for research that would be done neither out of the mathematician’s passing curiosity nor to serve the psychologist’s need for subject matter but in response to a profession’s questions about its practice (Kaput, 1992, p. 12).

Around the middle of the 20th century, debate began among mathematics educators as to the direction of the discipline. Researchers were worried that mathematics education was seen as a strict science unconcerned about knowledge and learning while psychology was devoid of the subject matter content needed to make progress in the field (Brown, 1952; Brownell & Moser, 1949). Therefore, the aim of these mathematics education researchers was to incorporate both mathematics and psychology to implement new directions in research. As the discipline of mathematics education began to define itself, several theorists wrote papers that would begin the foundations for the study of SMK of teachers in mathematics education. For example, Brownell and Moser's (1949) "meaningful arithmetic" in Duke University Research Studies in Education explored the subject matter learning of arithmetic and numeration. Around that time, Brown's (1952) *The Teaching of Secondary Mathematics*, stated that teaching programs that emphasize only theoretical uses or only the applications in mathematics are not capable of meeting the needs of students and that there needs to be more integration between the philosophies of life, education, and mathematics.

Thirty years ago, most of the research on teachers' SMK was directed towards understanding their knowledge of the content they were teaching (Ball, 1991; Borko & Putnam, 1996; Wilson et al., 1987). For example, some researchers looked at the number of university courses a teacher took (Begle, 1972; Begle, 1979; Begle & Geeslin, 1972; Druva & Anderson, 1983; Eisenberg, 1977; Everston, Hawley, & Zlotnik, 1985) or teachers' test scores and grade point average in their university studies (Andrews, Blackmon, & Mackey, 1980; Ayers & Qualls, 1979; Ducharme, 1970; Guyton & Farokhi, 1987). Although sometimes given to criticism, these studies give insight into the directions that mathematics education is currently heading. Teachers' knowledge of the subject matter needed for teaching is seen as diverse and multidimensional (Grossman et al., 1989; Wilson et al., 1987). Research during this time began to study many of the areas of teachers' knowledge, including substantive structures of content, beliefs, and the modes of instruction used in classrooms (Begle, 1972; Begle, 1979; Begle & Geeslin, 1972; Eisenberg, 1977).

Within the last fifteen years, many researchers have placed considerable emphasis on teacher preparation, including SMK and pedagogical content knowledge (Ball et al., 2001; Berenson, Van Der Valk, Oldham, Runesson, Moreira, & Broekman, 1997; Cooney & Wilson, 1995; Eisenhart, Borko, Underhill, Brown, Jones, & Agard, 1993; Even, 1990; Even, 1993; Grossman et al., 1989; Leinhardt, & Smith, 1985; Ma, 1999; Schram, Wilcox, Lanier, & Lappan, 1988; Shulman, 1986; Simon, 1995; Simon & Blume, 1994; Thompson, 1992; Towers, 2001; Wilson et al., 1987). Some research has specifically been done on secondary mathematics teachers' SMK (e.g., Cooney, Shealy, & Arvold, 1998; Cooney & Wilson, 1995; Even, 1993; Leinhardt, 1989; Leinhardt & Smith, 1985). However, most studies still report on elementary preservice and inservice teachers' SMK than on the knowledge of middle grades and high school preservice and inservice teachers (e.g., Ball, 1991; Ball & McDiarmid, 1990; Ball & Wilson, 1990; Ma, 1999; McDiarmid, Ball & Anderson, 1989; Simon & Blume, 1994).

Over the course of the years, teacher education programs took on the responsibility for much of the research, training, and support of prospective teachers' SMK. Given the possible repercussion that teachers' knowledge of subject matter has on their pedagogy, many teacher educators considered ways to incorporate discussions of subject matter into teacher education programs (Grossman et al., 1989). Ma (1999) stated that it is during teacher education programs that prospective teachers have one of three opportunities to cultivate their SMK of school mathematics and that "their mathematical competence starts to be connected to a primary concern about teaching and learning school mathematics" (p.145). Several researchers worked with prospective teachers to advance teacher education programs and develop courses that promote teachers' SMK and pedagogical content knowledge (Ball, 1988; Ball & McDiarmid, 1990; Baturo & Nason, 1996; Lappan & Even, 1989; Schram et al., 1988). However, some researchers question the effect of too few university courses on teachers' knowledge when these courses are intended to replace some of the instilled beliefs about teaching and learning that prospective teachers developed as students in their K-12 mathematics classes (Ball et al., 2001; Zeichner & Tabachnick, 1981).

Subject Matter Knowledge

An essential part of teachers' knowledge that goes beyond specific topics within a curriculum is the subject matter that is to be taught. The subject matter of any area of study, in very broad terms, includes the topics, facts, definitions, procedures or algorithms, concepts, organizing structures, representations, influences, reasons, truths and connections within the area of study and the connections outside the area of study to other areas. Leinhardt and Smith (1985) defined mathematical SMK as the knowledge of "concepts, algorithmic operations, the connections among different algorithmic procedures, the subset of the number system being drawn upon, the understanding of classes of student errors, and curricular presentations" (p.247). This definition suggests that SMK has several influences that shape the learning and teaching of prospective teachers. Ma (1999) found several parallels between elementary teachers' SMK and the ability to function as an effective teacher within the classroom. A teachers' SMK influences both their actions in the classroom and their interactions with students.

Grossman, Wilson, and Shulman (1989) defined the dimensions of SMK as the ***content knowledge***, ***substantive knowledge***, and ***syntactic knowledge*** of teaching, along with the ***teachers' beliefs*** about the subject matter (see Figure 1).

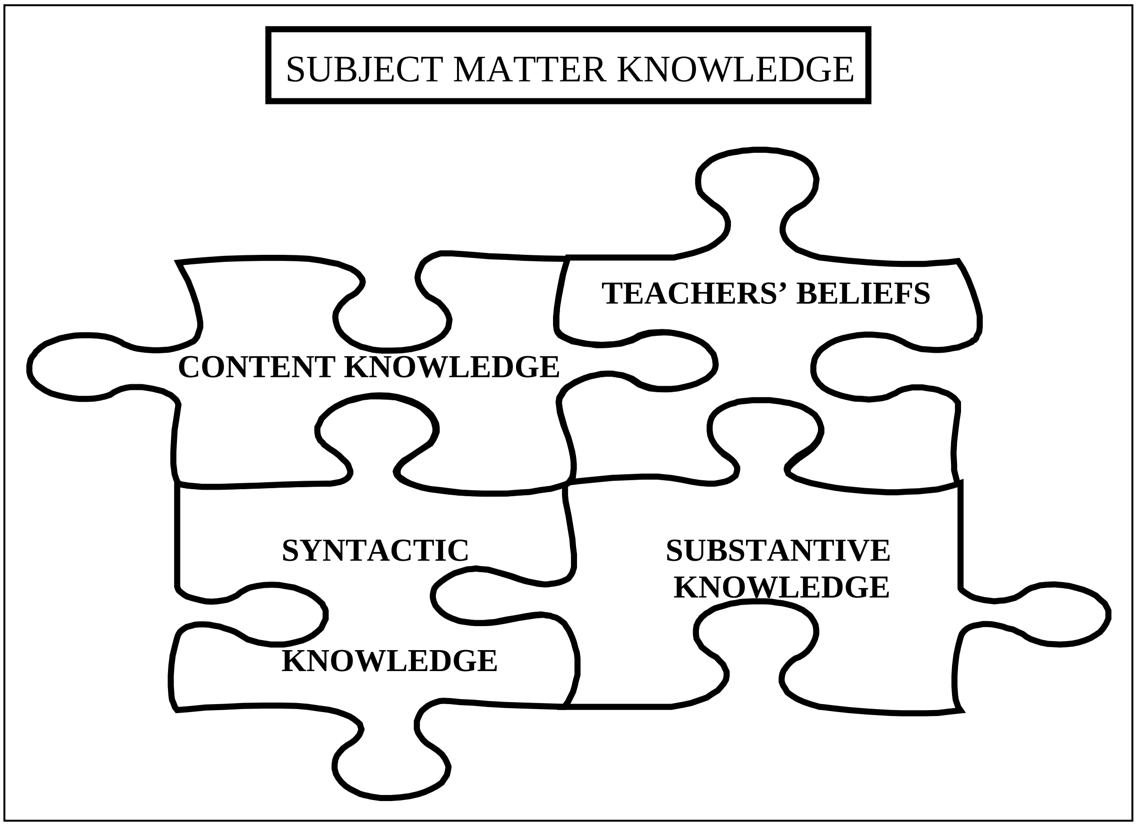


Figure 1. The dimensions of subject matter knowledge for teaching.

Teacher knowledge is diversified (Ball, 1991; Borko & Putnam, 1996; Leinhardt & Smith, 1985; Grossman et al., 1989; Shulman, 1986; Shulman, 1987; Wilson et al., 1987). The components of SMK can be seen as just as diverse, however, the dimensions of SMK are interrelated and affect each other, as the puzzle piece above suggests. Borko and Putnam (1996) pointed out that there are multiple definitions for what is deemed

SMK and it is essential to recognize "that teachers need to know more than just the facts, terms, and concepts of a discipline" (p. 676).

The National Council of Teachers of Mathematics' (NCTM) *Professional Standards for Teaching Mathematics* (1991) stated:

> Knowing mathematics includes understanding specific concepts and procedures as well as the process of doing mathematics. Mathematics involves the study of concepts and properties of numbers, geometric objects, functions, and their uses–identifying, counting, measuring, comparing, locating, describing, constructing, transforming, and modeling…. At any level of mathematical study, there are appropriate concepts and procedures to be studied (p.132).

This statement gives educators an insight into the dimensions of SMK and shows its importance to teaching mathematics. Numerous teacher educators argued that teaching in accord with current reform efforts (NCTM, 1989; NCTM, 1991; NCTM 2000) emphasizes that teachers must have rich, flexible SMK (Ball et al., 2001; Berenson et al., 1997; Borko & Putnam, 1996; Cooney et al., 1998; Cooney & Wilson, 1995; Garofalo, Drier, Harper & Timmerman, 2000; Eisenhart et al., 1993; Even, 1990; Even, 1993; Lappan & Even, 1989; Leinhardt, & Smith, 1985; Ma, 1999; Manoucherhri, 1999; Schram et al., 1988; Shifter, 1996; Shulman, 1986; Simon & Blume, 1994). Knowledge of subject matter influences the representations and approaches that teachers choose in teaching concepts in mathematics (Ball et al., 2001; Berenson et al., 1997, Garofalo et al., 2000; Eisenhart et al, 1993; Even, 1990; Even, 1993; Lappan & Even, 1989; Leinhardt & Smith, 1985; Ma, 1999; Manoucherhri, 1999). Subject matter knowledge goes beyond knowledge of content and procedures, to including the processes of doing mathematics, establishing inquiry and discourse in teaching mathematics, and reflecting teachers' beliefs about mathematics (Ball et al., 2001; Borko & Putnam, 1996; Cooney et al., 1998; Cooney & Wilson, 1995; Eisenhart et al., 1993; Even, 1990; Even, 1993; Ma, 1999; Schram et al., 1988; Schifter, 1996; Shulman, 1986; Simon & Blume, 1994). The dimensions of mathematical SMK are discussed below.

The Mathematical Content Knowledge of Teaching

The ***content knowledge*** of teaching is the most widely researched area in SMK. In many cases content knowledge is referred to as the knowledge of the subject. It is the factual information, organizing principles, and central concepts of a discipline. Mathematical content knowledge includes "specific concepts, definitions, conventions, and procedures" (Ball et al., 2001, p. 440). The factual information, organizing principles, and central concepts involved in the mathematical knowledge acquired by prospective secondary mathematics teachers come mainly from their K-12 experience along with the courses required from their discipline major at the university level.

Teachers who lack a distinct understanding of content knowledge have troubles in other areas of teaching (Grossman et al., 1989; Ma, 1999; McDiarmid et al., 1989). Content knowledge is seen as an integral part of teaching and is needed to make effective instruction possible (Ball, 1991; Ball et al., 2001; Ball & McDiarmid, 1990; Ball & Wilson, 1990; Brown & Baird, 1993; Cooney, 1994; Even, 1993; Fennema & Franke, 1992; Garofalo et al., 2000; Graeber, 1999; Grossman et al., 1989; Leinhardt & Smith, 1985; Ma, 1999; NCTM, 1991; Wilson et al., 1987). For example, the topic of similarity is one that impacts several different curricula (e.g., Pre-Algebra, Algebra, Geometry and Trigonometry) and understandings of this topic could be beneficial for prospective teachers. The NCTM (1991) stated that one of the necessary standards for the professional development of teachers of mathematics is “knowing mathematics and school mathematics” (p. 6). Wilson, Shulman, and Richert (1987), in exploring SMK, pointed to the intimate relationship between content and pedagogy.

> In order to transform or psychologize the subject matter, teachers must have a knowledge of the subject matter that includes a personal understanding of the content as well as knowledge of ways to communicate that understanding, to foster the development of subject matter knowledge in the minds of students (Wilson et al., 1987, p. 110).

Ma (1999) pointed to the relationship between content and pedagogy several times when addressing areas of elementary teachers content knowledge and the representations they use while teaching. Research stated that expert teachers use their content knowledge in ways that promote learning (Leinhardt, 1989; Leinhardt & Smith, 1985; Ma, 1999). However, Grossman, Wilson, and Shulman (1989) pointed out that having knowledge of only the content is not sufficient. For example, a prospective teacher with a strong content knowledge may not be able to make connections and illustrate relationships between mathematical topics and may teach them as if they are fragmented pieces of information. Thus, disciplinary knowledge must also include knowledge of the underlying structures (substantive knowledge) and knowledge of how to conduct inquiry (syntactic knowledge).

<u>The Substantive Knowledge of Mathematics</u>

Proficient mathematics teachers understand not only the content knowledge of what they teach, but also how ideas within the discipline are related – in other words, the *substantive knowledge* of mathematics. A general definition for substantive knowledge of teaching refers to understandings of particular topics within a discipline, procedures, concepts, and their relationships to each other. Wilson, Shulman, and Richert (1987) in their definition of substantive knowledge, included "the ways in which the fundamental principles of a discipline are organized" (p.113). Ball (1996) stated that mathematical substantive knowledge is the knowledge *of* mathematics and includes an understanding of particular mathematical topics, procedures, concepts, and the connections and organizing structures within mathematics. Ball's definition is a combination of what was previously defined as content knowledge and substantive knowledge. Substantive knowledge, including content knowledge, is often assumed to be SMK by many researchers.

Grossman, Wilson, and Shulman (1989) stated that substantive knowledge structures of teachers most likely develop during the discipline coursework that they complete in the departments of arts and sciences at their universities. However, research shows that the underlying structures of these disciplines are not discussed or explored in lower level undergraduate courses. Ball (2002) stressed that teacher educators and researchers look for more effective ways to support good teaching in teacher programs, because good teaching "can not only help teachers learn teaching mathematics, but it will also help them learn about mathematics" (p. 6).

Teachers' substantive knowledge of discipline structures has strong implications for what and how teachers choose to teach. To learn mathematics with understanding, students must be exposed to relevant mathematical relationships and connections in their mathematics courses (Ball, 1988; Borko et al., 1992; Cooney & Wilson, 1995; Eisenhart et al., 1993; Garofalo et al., 2000; NCTM, 2000; Simon, Tzur, Heinz & Kinzel, 2000). One of the implications is the influence teachers' substantive knowledge can have on curricular decisions. Teachers with a strong knowledge *of* mathematics make connections among relationships within topics that promote students' conceptual understandings (Ball, 1991; Borko & Putnam, 1996; Eisenhart et al., 1993; Even, 1993; Leinhardt, 1989; Leinhardt & Smith, 1985; Ma, 1999; McDiarmid et al., 1989; Simon et al., 2000; Simon & Blume, 1994; Stigler & Hiebert, 1999). "Given the potential impact teachers' knowledge of substantive structures may have on their pedagogy, teacher educators need to consider ways to incorporate discussions of substantive structures into programs of teacher education" (Grossman et al., 1989, p.29).

The Syntactic Knowledge of Mathematics

The syntactic knowledge of teaching is the evidence and proof that guide inquiry within the discipline. It focuses on where the discipline comes from, how it changes, and how truth is established within the discipline. Ball (1991) emphasized that syntactic knowledge of mathematics is knowledge *about* mathematics. Syntactic knowledge is seen as the nature of the knowledge in a field of study. "Knowledge *about* mathematics also includes what it means to 'know' and 'do' mathematics, the relative centrality of different ideas, as well as what is necessary or logical, and a sense of philosophical debate within the discipline" (Ball, 1991, p. 7).

Like the other dimensions of SMK, teachers' knowledge of syntactic structures has many different components. These components are concerned with establishing truth within a discipline. Truth within a discipline comes from its preserved foundations and new evidence or inquiry that gives rise to debate (Clements & Battista, 1992; Lakatos, 1976). For example, Schoenfeld (1999) asked researchers and professionals within the discipline of education to "characterize fundamentally important educational arenas for investigation, in which theoretical and practical progress can be made over the century to come" (p. 4). Since this challenge for debate and inquiry, research has focused and made some improvement in many of the areas Schoenfeld called "sites for progress."

Grossman, Wilson and Shulman (1989) stressed that teachers' lack of syntactic knowledge can limit their abilities to learn new information in their fields. Teachers with limited syntactic knowledge may not be able to distinguish between more or less legitimate claims within a discipline. Furthermore, a lack of syntactic knowledge may also cause teachers to misrepresent the mathematics they are teaching. For example, the substantive structures of similarity are heavily influenced by the relationship between angle measurement and proportional sides. Teachers with a limited syntactic knowledge of similarity may be unable to sufficiently explain this relationship or engage in discourse to allow their students to explore angle measurement and corresponding side lengths. Teachers' perspectives of their discipline influence their views of the roles of factual knowledge, evidence and inquiry. The syntactic knowledge of teachers is instrumental in determining the classroom environment that they nurture. Teachers with a strong sense of mathematical syntactic structures are more likely to have classrooms that include discussions and activities aimed at developing their students' awareness and understanding because they go beyond memorization and regurgitation of facts. For example, Even (1993) suggested that teachers' SMK is reflected by the questions they ask and activities they design. Also, teachers with an immature conception of mathematics, see it as mechanical, abstract, and consisting of meaningless facts and rules to be memorized and their teaching style reflects this particularly when they are unsure of the situation they are in.

Teachers' Beliefs about Subject Matter

Teachers' beliefs about subject matter are the ways they think about the nature of mathematics in relation to how it is learned and how to facilitate students' learning of content. Similar to the syntactic structures of teachers' knowledge, teachers' discipline beliefs and experiences play an important role in their teaching. Teachers' ideas or beliefs about mathematics do not exist separately from the other dimensions of SMK (Ball, 1991; Cooney, 1994; Cooney & Wilson, 1995; Even, 1993; Graeber, 1999; Grossman et al., 1989; Ma, 1999; McDiarmid et al., 1989; Simon & Blume; 1994; Thompson, 1992). Borko and Putnam (1996) stated, "contemporary cognitive theories view learning as an active, constructive process that is heavily influenced by an individual's existing knowledge and beliefs" (p. 674). Just as several things including students' beliefs, influence their learning, teachers' beliefs and the other dimensions of their SMK influence teaching. For example, beliefs that mathematics is only good for algebraic manipulations may cause teachers not to admire the strength of a concept such as similarity, and not relate it to the building of scale models, or right triangle trigonometry. Research on learning to teach demonstrates that teachers' existing knowledge and beliefs are critical in shaping what and how they learn from experiences, how they think about teaching, and how they conduct themselves in classrooms (Ball, 1991; Ball & McDiarmid, 1990; Borko et al., 1992; Borko & Putnam, 1996; Cooney & Wilson, 1995; Eisenhart et al., 1993; Even, 1993; Garofalo et al., 2000; Graeber, 1999; Leinhardt, 1989; Leinhardt & Smith, 1985; Ma, 1999; McDiarmid et al., 1989; Simon et al., 2000; Simon & Blume, 1994; Stigler & Hiebert, 1999; Thompson, 1992). Thompson (1992) stated that if teachers and teacher educators want to cultivate positive beliefs and attitudes in students, "then we must examine the extent to which the structure and conduct of our mathematics classes are conducive to cultivating them" (p. 142).

Grossman, Wilson, and Shulman (1989) distinguished between knowledge and beliefs by stating that unlike knowledge, beliefs are subjective and rely heavily on significant and personal reflections and evaluations. Ball (1991) concluded from research with preservice teachers that, "understanding mathematics is colored by one's emotional response and sense of self in relation to it" (p. 7). Beliefs about a discipline do not need to meet the syntactic requirements, previously mentioned, that must be met by new knowledge being introduced within a discipline. McDiarmid, Ball, and Anderson (1989) stressed that since teachers' direct experience may also suggest ways of representing subject matter that they need to develop standards to judge the validity and usefulness of these representations. Cooney and Wilson (1995) stressed that teachers' beliefs have an affect on their ability to teach disciplines effectively. The Third International Mathematics and Science Study (TIMSS) found that "teachers' beliefs about mathematics learning and instruction are to some degree related to their preparation" (Mullis et al., 2000, p. 191). TIMSS (Mullis et al., 2000) suggested that teachers with mature beliefs about teaching and student learning spend more time developing lessons that engage students.

Teachers' beliefs influence the choice of disciplines in which they teach. Other people or circumstantial events in their lives can influence these beliefs. Generally, positive attitudes and a sense of accomplishment within a certain discipline influence individuals to become teachers in those disciplines. Also, Cooney (1996) stated that most mathematics teachers "enjoy doing mathematics at some level and have a positive attitude towards mathematics"(p. 5), implying that prospective teachers that have chosen mathematics as their discipline of instruction have been successful in it as students and have a positive attitude towards it.

Importance of Subject Matter Knowledge

Understandings of SMK become important when researching students' conceptual understandings because teachers' knowledge of organization, connections among ideas, ways of proof and inquiry, and knowledge growth within discipline are important factors needed to teach the subject. The limitations in teachers' SMK may cause them to focus on procedural understandings of mathematics (Borko et al., 1992; Eisenhart et al., 1993; Even, 1990; Ma, 1999). Procedural knowledge consists of the mastery of computational skills and procedures for identifying mathematical components (Eisenhart et al., 1993). Therefore, a teacher that teaches for procedural knowledge may not focus much attention on the substantive and syntactic structures of a concept.

In her research of American and Chinese elementary teachers, Ma (1999) reported that SMK plays several important roles in teaching. For example, a limited knowledge of a topic can cause a teacher to be unable to define a student's mistake and resolve this problem. Limited knowledge in the subject matter can cause teachers to be uncomfortable when using manipulatives or other materials, or to use these tools in a way that can cause students to have incorrect ideas about the topic. Ma also stated that pedagogical knowledge does not make up for limited SMK. For example, teachers with limited SMK may be unable to address students' non-routine questions, or plan exploratory tasks that connect substantive structures in mathematics.

Subject matter knowledge, pedagogical knowledge, and curricular knowledge are seen as domains central to teachers' knowledge base. Shulman (1987) lists the components of SMK as part of the knowledge base that underlies teachers' understanding needed to promote comprehension among students. Again, teachers' SMK is shown to be essential in helping students to understand. TIMSS (Mullis et al., 2000) reported that "average mathematics achievement [of students] is related to how well teachers felt they were prepared to teach mathematics, with higher achievement related to higher levels of teaching confidence in their preparation [to teach]" (p. 190). To further emphasize the importance of SMK, Shulman (1986) stated that a "teacher need not only understand *that* something is so; the teacher must further understand *why* it is so, on what grounds its warrant can be asserted, and under what circumstances our beliefs in its justification can be weakened and even denied" (p. 9).

The present study focuses on the mathematical content knowledge, substantive structures and syntactic knowledge as they relate to prospective teachers' SMK of similarity. In looking at the mathematical content knowledge, this researcher looked at the factual information, organizing principles, and central concepts of similarity used in explaining definitions, examples, and problems as they pertain to prospective teachers' SMK of similarity. Mathematical substantive knowledge of similarity includes an understanding of particular mathematical topics, such as corresponding sides and congruent angles, procedures, representations, concepts, and the connections and organizing structures of similarity. A prospective teacher's substantive knowledge of similarity may contain procedures as how to find the measurement of the missing side or angle using indirect measurement, or the connection between similarity and the trigonometric identities.

The Subject Matter Knowledge of Similarity

"Similarity is not an easy concept" (Lappan & Even, 1988, p.33). Similarity is an important spatial-sense, mathematical concept that can facilitate students' understanding of indirect measurement and proportional reasoning. Woodward, Gibbs, and Shoulders (1992), in their fifth-grade unit on similarity, pointed out that "numerous everyday applications of similarity involve such visual material as blueprints, maps, graphs, models, and photographs" (p. 22). Similarity is a topic in mathematics that can help represent concepts and relations in other branches of mathematics, such as right triangle trigonometry (Chazan, 1988). Senk and Hirschhorn (1990) lend support to this statement and reported that geometry can be used as a "vehicle for making connections among various mathematical subjects or between mathematics and other subjects" (p. 274). Friedlander and Lappan (1987) illustrated that similarity is important to teaching because the acquisition of this concept in geometry is related to several ideas in a child's geometrical understanding. The National Assessment Governing Board (2001) in their draft for public review of the *Mathematics Framework for the 2004 National Assessment of Educational Progress* (NAEP) emphasized that "geometry provides an especially appropriate and rich context for the development of reasoning skills, including making conjectures and validating them formally and informally" (p.16).

Although many see geometry as a significant subject in mathematics and similarity is seen as a key concept within geometry, there is very little research on prospective teachers' understanding of the concept of similarity. Many of the research studies that investigated learning similarity focused on school age children (Chazan, 1988; Friedlaner & Lappan, 1987; Lappan & Even, 1988; Ren, 1995; Senk & Hirschborn, 1990; Woodward et al., 1992). The studies that involved prospective teachers and geometry related topics focused on topics that are closely related to the substantive structures of similarity, but not exactly similarity (Baturo & Nason, 1996; Berenson et al., 1997; Lappan & Even, 1988; Simon & Blume, 1994; Swafford, Jones, & Thornton, 1997). These studies will be elaborated on in Chapter 2.

TIMSS (Mullis et al., 2000) surprisingly reported that only 75% of the teachers sampled for the United States feel well prepared to teach "geometric figures – symmetry motions and transformations, congruence and similarity" (p. 284). With the emphasis that is placed on geometry and geometry related concepts, like similarity, throughout all of mathematics it is necessary that teachers feel adequately trained to teach these topics (Chazan, 1988; NCTM, 1989; NCTM, 1991; NCTM, 2000; Woodward et al., 1992).

One must answer several questions to understand the four dimensions of prospective teachers' SMK *of* and *about* similarity. *What is the prospective teachers' content knowledge of similarity? What are the prospective teachers' substantive* *structures of similarity? What are the prospective teachers' syntactic structures of similarity? What are teachers' beliefs of similarity?*

The content knowledge of similarity at first may seem simplistic. However, when looking at the fundamental theorem of similarity it illuminates the complexity in defining the content knowledge, substantive structures, and syntactic structures of similarity. The fundamental theorem of similarity states that if two figures are similar with ratio of similitude c, then corresponding angle measures are equal; corresponding lengths and perimeters are in the ratio c; corresponding areas are in the ratio c^2; and corresponding volumes and weights are in the ratio c^3. Along with the fundamental theorem of similarity there are other topics throughout mathematics that contribute to the content knowledge of similarity. For example, there are angle and segment measures, proportions, ratios, scale drawing, equivalent fractions, and multiplicative relationships, just to name a few. All of these topics lend support and are needed to build strong substantive structures for similarity (see Figure 2).

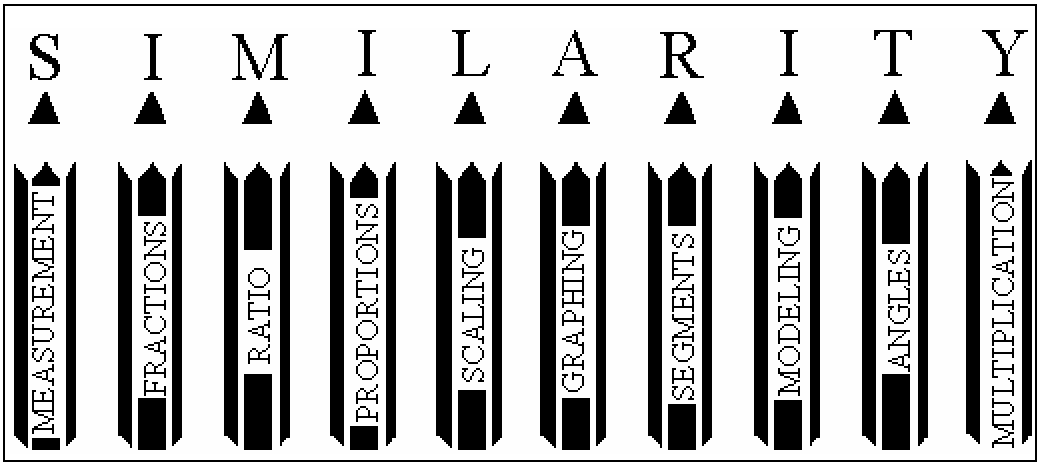


Figure 2. Content knowledge that contributes to the substantive structures of similarity.

The substantive structures of similarity include the properties and relationships between the angles and sides of similar triangles and are reflected by indirect measurement, proportional reasoning, scale drawing, and modeling. Indirect measurement is a tool used to find measures of corresponding sides, and equivalent corresponding angles. Scale drawing and modeling are seen as visual applications that rely heavily on similarity. The underlying connections of indirect measurement and the visual representations in teachers' substantive structures can give them opportunities to show their students multiple representations of similarity. These experiences with drawing and visualizations can lay the foundations for students to later understand the theoretical results of similarity. Since similarity has close connections with proportional reasoning skills, prospective teachers' can use these representations to support their students' proportional reasoning abilities.

The syntactic structures of similarity that prospective teachers have will rely on their experiences in high school and university mathematics courses. Geometry and other areas of mathematics use empirical and deductive methods to interact and reinforce each other to help establish mathematical truth (Clement & Battista, 1992). For example, it may be possible for a prospective teacher to rely heavily on two-column or other types of proofs in establishing justifications within geometry. What the prospective teachers see as sufficient evidence to accept or decline new information to their knowledge structure is dependent on their experiences in mathematics courses. Similarly, prospective teachers' beliefs about similarity and teaching will vary because their beliefs will be dependent on experiences as prospective teachers, as well as when they were secondary school students.

Research Focus

The focus of this research is in reaction to statements about teachers' SMK. Ball, Lubienski, and Mewborn (2001) stated that research on secondary teachers' SMK has been overlooked. Borko and Putnam (1996) suggested that learning opportunities for teachers be grounded in the teaching of subject matter and "provide opportunities for teachers to enhance their own subject matter knowledge and beliefs" (p. 702). In response to these two assertions and Schoenfeld's (1999) call for theoretical research that is focused on practical and relevant applications throughout education, the focus of this research is driven by two interrelated questions:

1. What is the nature of the subject matter knowledge that prospective teachers rely on when planning a lesson to introduce the concept of similarity?
2. What growth in the subject matter knowledge of similarity is revealed as prospective teachers plan a lesson to introduce the topic?

LITERATURE REVIEW

Research reports on teaching the content and substantive structures of similarity emphasize its importance in geometry and other areas of mathematics. The importance that SMK has for prospective teachers' teaching knowledge has been exemplified through numerous research studies of elementary prospective teachers. The need and focus of the research takes both topics into consideration and looks at middle and secondary prospective teachers' SMK of similarity. To review the SMK of the prospective teachers, conceptual frameworks for analyzing SMK were developed.

Conceptual Frameworks

The purpose of this section is to discuss the importance of conceptual frameworks and the conceptual frameworks used in this research. The conceptual frameworks used in this research help explain the growth of the prospective teachers' SMK in the concept of similarity and the nature of the subject matter they rely on when planning to teach a lesson on the topic. The first conceptual framework addresses the nature of the SMK that prospective teachers rely on when planning a lesson to introduce the concept of similarity to a high school geometry class. This framework uses components of Shulman's (1986) knowledge base in teaching along with Even's (1990) analytic framework of SMK for teaching a specific topic in mathematics. The second conceptual framework incorporates Even's (1990) aspects of SMK with Pirie-Kieren's (1994) model for growth in mathematical understanding that was adapted to teacher preparation by Berenson, Cavey, Clark and Staley (2001). The second conceptual framework includes Pirie and Martin's

(2000) feature of collecting and Martin's (1999) feature of thickening to explain the growth in the SMK of similarity.

Maxwell (1996) stated that a key part to the design of any research is the conceptual context or conceptual framework. A conceptual framework is an outline for the system of concepts, assumptions, expectations, beliefs, and theories that support and inform the research. Schram (2002) asserted that conceptual frameworks determine when and how theory plays a role in research. There are several benefits to using a conceptual framework over both a theoretical or theory grounded in practice framework. One such benefit is that conceptual frameworks are less rigid then theoretical and practical frameworks and highlight what is relevant to the particular study. Eisenhart (1991) also noted that conceptual frameworks incorporate ways of investigating a research problem without drawing unwarranted conclusions or unsupported explanations as many have done in using theoretical and practical frameworks.

Shulman's Knowledge Growth in Teaching.

Shulman (1986), one of the leading researchers in the program "Knowledge Growth in Teaching", wrote a landmark article that has been critical in several research studies over the last sixteen years. The research was based on observation, interviews, and longitudinal studies of participants from several different fields of study. The article challenges the ways in which teachers' knowledge is tested and researched. He noted that the standards for testing teachers have shifted and more emphasis is placed in the procedures of teaching instead of the knowledge of the teacher. He noted: "The person who presumes to teach subject matter to children must demonstrate knowledge of that subject matter as a prerequisite to teaching" (Shulman, 1986, p. 5). Shulman calls the absence of subject matter in educational research, the "missing paradigm" because the content of what is being taught seems to be ignored, as if it were inconsequential. Within the last fifteen years, Shulman's article had a profound effect on much of the SMK and pedagogical content knowledge research (Ball, 1991; Ball et al., 2001; Even, 1993; Ma, 1999). He tried to refocus research on teaching by articulating his perspective on teacher knowledge and the forms of teacher knowledge. His ideas come from trying to define prospective teachers and in-service teachers comprehension in understandings, conceptions, and orientations of the subjects they teach. Shulman's perspective on teacher knowledge is directed towards their content knowledge and how it grows.

Shulman described three categories of content knowledge: subject matter content knowledge, pedagogical content knowledge, and curricular knowledge. Content knowledge "refers to the amount of knowledge per se in the mind of the teacher" (p. 9). Subject matter content knowledge is seen as more than the knowledge of facts and concepts of a subject, but also includes understanding of the substantive and syntactic structures (Grossman et al., 1989; Shulman, 1986; Wilson et al., 1987). Pedagogical content knowledge goes beyond SMK for content to the SMK for teaching. Pedagogical content knowledge is a type of SMK that is improved and embellished by knowledge of the learners, knowledge of the curriculum, and knowledge of context and pedagogy (Wilson et al., 1987). Curriculum knowledge is seen as the knowledge of the range in programs, the materials available, and characteristics of a curriculum, at any grade level.

Lastly, Shulman described three forms of knowledge that teachers use in their practice: propositional knowledge, case knowledge, and strategic knowledge.

Propositional knowledge is knowledge from examples of literature that contain useful principles about teaching. Propositional knowledge is also the wisdom of practice, empirical principles, norms, values, and the ideological and philosophical principles of teaching. Case knowledge is specific, well-documented, descriptive events of propositional knowledge. Cases are specific instances in practice that are detailed and complete descriptions that exemplify theoretical claims and communicate principles of practice and norms. Strategic knowledge is used when a teacher cannot rely on propositional or case knowledge, and must formulate answers when no simple solution seems possible. These strategies about teaching go beyond principles or specific experiences and are formulated using alternate approaches.

Shulman's forms of knowledge, propositional, case, and strategic, will be used in conjunction with Even's (1990) aspects in examining the prospective teachers' SMK. The prospective teachers' examples, definitions, activities, and explanations are looked at to see if they are using propositional, case, or strategic knowledge. If the prospective teachers use approaches that are grounded in theory, such as cooperative learning, then it may be possible they are accessing their propositional knowledge. When the prospective teachers have been previously exposed to an approach, definition, or example and use it in their explanations, examples, or activity, they may be accessing their case knowledge. The prospective teachers' strategic knowledge may be looked at if they seem to encounter a situation that they are unfamiliar with and resolve it with a new approach that they had not seen or used before. It is important to note that the learners, as well as the context, determine the type of knowledge being used.

Even's Aspects of Subject Matter Knowledge

Even (1990) established a framework for analyzing teachers' SMK about a specific concept in mathematics. She stated:

> The analysis of teachers' subject matter knowledge about a specific piece of mathematics should integrate several bodies of knowledge; the role and importance of the topic in mathematics and in the mathematics curriculum; research and theoretical work on teachers' subject matter knowledge and its role in teaching (p. 523).

She stated that the seven aspects that form teachers' SMK about a specific mathematical concept are:

1. Essential Features
2. Different Representations
3. Alternative Ways of Approaching
4. The Strength of the Concept
5. Basic Repertoire
6. Knowledge and Understanding of a Concept
7. Knowledge about Mathematics

Each of these seven aspects is further explained below.

The first aspect addressed by Even is *essential features*, or concept image. The *essential features* of a mathematical topic are the mental ideas or concept images that the teacher has about the topic and the set of properties that are associated with the topic. She stated that good teachers have a "match between their understanding of a specific mathematical concept they teach and the 'correct' mathematical concept" (p. 523). To obtain the “correct” mathematical concept, teachers choose the definitions and common examples associated with the topic. A common definition of similarity is, “Two polygons are similar if and only if their corresponding angles are congruent and the measures of their corresponding sides are proportional” (Boyd et al., 1998). This implies that the essential features of similarity are corresponding congruent angles, proportional corresponding side lengths, thus, the relationship between these are seen as necessary in developing the correct mathematical concept of similarity. Teachers’ pedagogical and cultural decisions are instrumental in correctly identifying concept images. “Mathematics teachers who are constrained by their limited and underdeveloped concept image may also be deprived from understanding current mathematics…”(Even, 1990, p. 524).

The second aspect of Even’s framework involves the *different representations* of the concept. Different representations give more insight into the topic and allow for a deeper understanding of a concept. Teachers who can represent concepts in different ways are able to use these representations in their teaching. When teachers use only definitions and common examples, they run the risk of ignoring relevant concepts by being limited in their representations. By using more than one representation, one can abstract and grasp properties, concepts, and the common qualities of a topic (Even, 1990). Examples of representations that could be used for similarity, range from simplistic identifications to how similarity is related to right triangle trigonometry, or possibly tiling.

The third aspect in Even’s framework, stresses the need for *alternative ways of approaching* mathematical concepts. It is helpful to see a concept in various forms: in different divisions of mathematics and other disciplines, or everyday life. For example, Chazan (1988) in his research on secondary students interpretations of similarity used the *Geometric Supposer* to help students in their understanding of the concept. Using the most appropriate way to approach the concept infers that teachers have the necessary knowledge to make the correct choices in determining the most effective method. “Teachers should be familiar with the main alternative approaches and their uses” (p. 525). Using their substantive and syntactic knowledge, many of the prospective teachers chose approaches that have influenced their SMK and pedagogical content knowledge.

The fourth aspect of Even’s framework is the *strength of the concept*. The strength of a concept is seen as the new opportunities that can be explored or addressed by it. Teachers with a good understanding of the powerful characteristics of a concept, show good knowledge of the concept. The use of multiple representations is in an effort to help students understand and appreciate the usefulness and strength of the concept. To understand the strength of a concept requires the general knowledge of the concept, as well as the more formal definitions and examples for sophisticated mathematical knowledge, thus, the strength of any concept is seen as having important sub-topics and sub-concepts. The strength in a topic like similarity is that it can be applied to everyday examples such as models and maps, along with other areas of mathematics including right triangle trigonometry and fractal geometry.

The fifth aspect Even described is the basic *repertoire* of examples that teachers access. A seasoned basic repertoire contains easily accessible powerful examples that illustrate important principles, properties, theorems, and definitions. These examples should not be memorized or used without a basic understanding of the concept. Having a mature basic repertoire allows insight and understanding of general and more complicated knowledge. Also, a mature basic repertoire can be used as a reference to monitor ways of thinking and acting. For example, a teacher with a good basic repertoire of similarity would have knowledge of representations that are efficient and powerful to explain the importance of the proportional sides and the congruent angles of similar figures.

The sixth aspect is *knowledge and understanding of a concept* and it involves both procedural and conceptual knowledge and the relationships between them. Procedural knowledge is made up of algorithms, definitions, and common examples. Conceptual knowledge contains relationships and connections among the ideas of a concept. In having a meaningful understanding of a concept, teachers possess a basic repertoire of procedural and conceptual principles and properties. If someone has knowledge and understanding of a concept they have a network of concepts and relationships at their disposal. “When concepts and procedures are not connected, people may have a good intuitive feel for mathematics but not be able to solve problems, or they may generate answers but not understand what they are doing” (p. 527). Teachers with a knowledge and understanding of similarity will include examples that allow students to develop a conceptual idea of the relationship between the corresponding congruent angles and proportional corresponding side lengths, while giving them the opportunity to reinforce their ideas through multiple representations.

The seventh aspect Even described is *knowledge about mathematics*. Knowledge about mathematics is the more general knowledge that gives inquiry, truth, and the construction and use of knowledge and understanding within mathematics. This is consistent with what Grossman, Wilson and Shulman (1989) call syntactic structures and what Ball (1991) calls knowledge *about* mathematics. The knowledge about mathematics goes beyond conceptual and procedural understandings, and is seen as the nature of the knowledge in mathematics. The knowledge about mathematics is concerned with preserving the foundations of mathematics and establishing truth within new evidence or inquiry that gives rise to debate (Ball, 1991; Grossman et al., 1989; Shulman, 1987; Wilson et al., 1987).

Even's framework for analyzing SMK is related to the four dimensions discussed in Chapter 1. Because of the way Even defines her framework, aspects such as different representations, ways to approach, along with understanding and knowledge of the concept are in more than one dimension. The dimension of mathematical content knowledge includes the aspects of essential features, basic repertoire, representations, and understanding and knowledge of a concept. However, the essential features and representations are unconnected facts and examples, as they relate to content knowledge. Substantive knowledge contains the underlying connections in mathematical content knowledge and includes the different representations and strength of the concept the prospective teachers apply. The dimensions of syntactic knowledge include the aspects of different representations, ways of approaching, and knowledge about mathematics that the prospective teachers have learned. The beliefs of prospective teachers are illustrated by the representations and ways of approaching the topic they choose during the lesson plan study.

The Conceptual Framework for Relying on Subject Matter Knowledge.

The nature of the SMK that prospective teachers rely on when planning a lesson on similarity is analyzed using Even's (1990) analytic framework of SMK for teaching specific topics and consolidating ideas from Shulman's (1986) knowledge base for teaching. Shulman's three forms of teacher knowledge are used as they relate to SMK. Since Shulman (1986) defines these three forms of knowledge for content knowledge and states that subject matter content knowledge is a category of content knowledge, then this definition can be directly applied to Even's framework. The propositional knowledge, case knowledge, and strategic knowledge that prospective teachers rely on when engaged in planning a lesson are examined as they relate the Even's (1990) framework (see Figure 3).

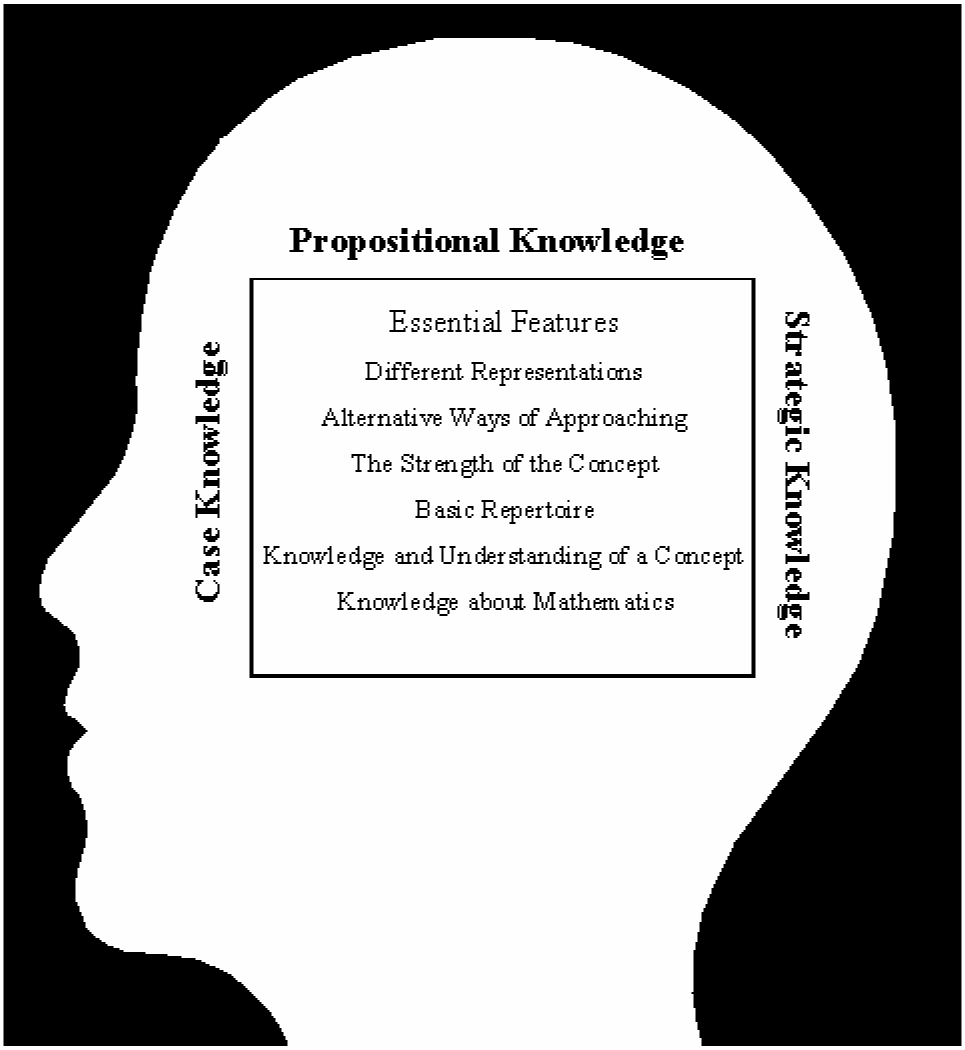


Figure 3. Relating Shulman's (1986) forms of knowledge to Even's (1990) framework.

Shulman's (1986) three forms of knowledge are exemplified through the prospective teachers choice of representations, definitions, explanations, and approaches they use during the study. In looking at the research data, Even's (1990) aspects of SMK are instrumental in framing and categorizing the SMK prospective teachers rely on, while reflecting on the forms of knowledge they use.

Mathematical Understanding

The framework used to discuss the growth of SMK in similarity is based upon Pirie-Kieren's (1994) model for growth in mathematical understanding that was adapted to teacher preparation by Berenson, Cavey, Clark and Staley (2001). It is necessary to discuss mathematical understanding as it relates to these frameworks, in order to address how understanding is looked at during this study. The theories used in these frameworks to determine understanding draw from Ball (1990), Piaget (2001), Skemp (1976), and Glasersfeld (1987). The constructivist definition of understanding (von Glasersfeld, 1987) is used to exemplify Pirie and Kieren's (1994) theory of growth of mathematical understanding "as a whole, dynamic, leveled but non-linear transcendently recursive process" (p. 166). The constructivist theory sees understanding as a process of continually organizing one's own knowledge structures (Simon, 1995; von Glasersfeld, 1987). To explain the relational and conceptual understandings of what and how to teach, Berenson, Cavey, Clark, and Staley (2001) incorporated the theories of Ball (1990) and Skemp (1976). These theories of understanding will also be used in examining the understanding of the prospective teachers' growth of SMK in similarity.

The Teacher Preparation Model along with the Features of Collecting and Thickening.

The Berenson, Cavey, Clark and Staley (2001) teacher preparation model consists of five levels of understanding and the feature of folding back that were adapted from the Pirie-Kieren (1994) model of growth in mathematical understanding. The teacher preparation is a conceptual framework that is designed to capture the process of learning as prospective teachers come to an understanding of both *what* and *how* to teach in high school mathematics. The “what” to teach refers to the school mathematics that prospective teachers will teach after their teacher education training. The “how” to teach is the teaching strategies that are used by the prospective teachers. Modifications were made to the Pirie-Kieren model for growth in mathematical understanding to accommodate tasks of teacher preparation. The teacher preparation model relies heavily on the integrity of the first five levels of the Pirie-Kieren model for growth in mathematical understanding, along with the feature of folding back. The five levels of the teacher preparation model are called primitive knowledge, making an image, having an image, noticing properties, and formalizing (see Figure 4). The five levels of the teacher preparation model reflect the same first five levels of the Pirie-Kieren model. The feature of folding back in the Pirie-Kieren model is seen as essential and was used in the teacher preparation model. The features of collecting (Pirie & Martin, 2000) and thickening (Martin, 1999) that evolved after the publication of the Pirie-Kieren model will be discussed as they pertain to teacher preparation.

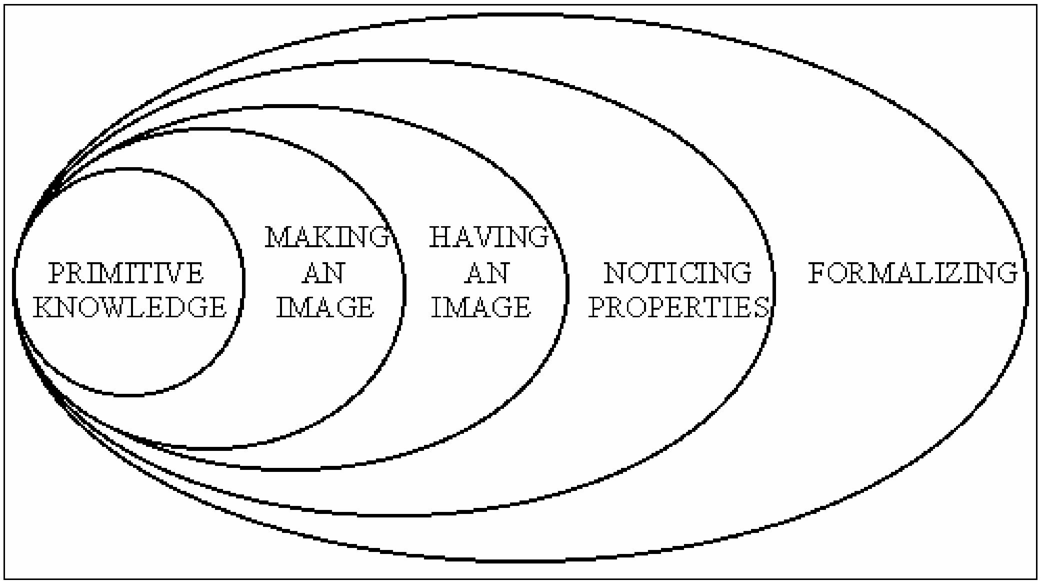


Figure 4. Levels of the teacher preparation model (Berenson et al., 2001).

The first level of the teacher preparation model is called *primitive knowledge*. The instructor or observer sees primitive knowledge as something that is assumed that the student or participant knows mathematically, before coming to the new task. Berenson, Cavey, Clark, and Staley (2001) list several aspects of primitive knowledge that are assessed during the prospective teachers' interviews and activities, including: knowledge of college mathematics, knowledge physics and chemistry, knowledge of school mathematics, and knowledge of teaching strategies. Pirie and Kieren (1994) point out that primitive knowing does not imply a low level of mathematics, but a starting place for growth. For example, a prospective teacher's primitive knowledge of mathematics may contain the ideas of congruent corresponding angles, without relating the proportional corresponding side lengths. Within a prospective teachers primitive knowledge of teaching strategies, there may be a strong desire and knowledge of using a lecture-style teaching strategy. It is important to note that, while one can never establish the extent of the prospective teacher's primitive knowledge, one however can make inferences based on aspects from the learning activity.

Making an image is when participants or students use their primitive knowledge in new ways. When using their knowledge in new ways, participants make distinctions in what they already know and apply it to a new situation that builds directly on their previous knowing (Piaget, 2001). A possible example of making an image of *what* to teach may be indicated when prospective teachers make images and concludes that all the corresponding side lengths of similar triangles can be expressed as the same ratio in their lesson. And an image of *how* to teach could be indicated when a prospective teacher makes images of an approach for their lesson on similarity.

When *having an image*, a new image is formed as a result of the activities in making an image. When participants are having an image, they can bring meaning to their image making activities. Then they can mentally manipulate ideas and consider different aspects of the task without having to repeat the activities that brought forth the new meaning. Using the last example of *what* to teach, prospective teachers may have an image when they realizing that the Side-Side-Side similarity postulate can be used as a way to illustrate if triangles have proportional corresponding side lengths. An example of having an image of *how* to teach, could be indicated when prospective teachers are clear about possible benefits of a new strategy. In both of the previous examples, the level of having an image allows the prospective teachers to manipulate the image they made, and bring new meaning to that image.

When *noticing properties* of a task, the prospective teachers manipulate and combine images that they made about their lessons. Prospective teachers may be noticing properties of *what* to teach when they combine their images of the relationship between the corresponding congruent angles and the proportional corresponding sides and incorporate these ideas in their lesson plan. Also, when noticing properties of *how* to teach, prospective teachers may begin to devise ideas on how effective the approach they choose may be to their students' learning.

When *formalizing* prospective teachers can identify common features, patterns, algorithms, or formulas of the image they made while noticing properties. Prospective teachers may be formalizing *what* to teach when they realize that both corresponding proportional sides and congruent angles are important properties of similarity and are needed in using the concept. When formalizing ideas of *how* to teach, prospective teachers could establish a pattern in their teaching strategies such as a conceptual activity to develop the concept image, followed by several procedural activities for reinforcement. Pirie and Kieren (1994) stated, "anyone formalising, would be ready for, and capable of enunciating and appreciating a formal mathematical definition or algorithm" (p. 171).

The images of *what* and *how* to teach reflect the primitive knowledge inferences made by the prospective teachers. These images build on formalized knowledge of material related to similarity from mathematics classes and that it was a concept that either they understood or as one participant said, they needed to "relearn" all the geometry that they need to teach. The images made of *what* and *how* to teach will have an effect on the determination of the outer levels of the teacher preparation model.

The notion of *folding back* is essential to the growth in understanding and reveals the non-linear and recursive nature of coming to understand mathematics (Berenson et al., 2001; Pirie & Kieren, 1994). Pirie and Kieren's (1994) feature of *folding back*, which is critical to mathematical understanding, is built on Piaget's (2001) concept of reflective abstraction. Reflective abstraction is when students abstract a property of understanding from coordinating their actions. Folding back is seen as necessary to build an in-depth understanding and occurs when a learner is faced with a new issue or problem that is not immediately solvable with his or her current knowledge. Folding back is a recursive process in which the participant returns to an inner level of understanding to extend her current understanding. Martin (1999) suggested that the effectiveness of the activity of folding back is dependent on the learner and learning environment.

The process of *folding back* involves more than just borrowing from an inner level to construct understanding of a concept on an outer level. Two aspects of folding back are called *thickening* (Martin, 1999) and *collecting* (Pirie & Martin, 2000). In the process of *folding back*, if students become aware of their understanding and realize that an inner level activity cannot be identical to what was performed previously, they build a *thicker* understanding at the inner level to support and extend their understanding at the outer level in which they return (Martin, 1999). When folding back, *collecting* occurs when students are aware that their understanding is lacking and they need to recollect some inner understanding and use it at an outer level to get a better understanding of the concept in question (Pirie & Martin, 2000).

<u>The Conceptual Framework for the Growth of Subject Matter Knowledge in Similarity.</u>

The Pirie and Kieren (1994) model for growth in mathematical understanding was adapted by Berenson, Cavey, Clark and Staley (2001) creating a model for studying prospective teachers' understanding of *what* and *how* to teach high school mathematics.

Incorporated into the teacher preparation model are the features of collecting (Pirie & Martin, 2000) and thickening (Martin, 1999). Using the teacher preparation model in conjunction with Even's (1990) aspects of SMK, helps to define the *what* and *how* of the prospective teachers' teaching. Even's (1990) framework is mapped onto the teacher preparation model of understanding of *what* and *how* to teach, and grows via the levels of the teacher preparation model (see Figure 5).

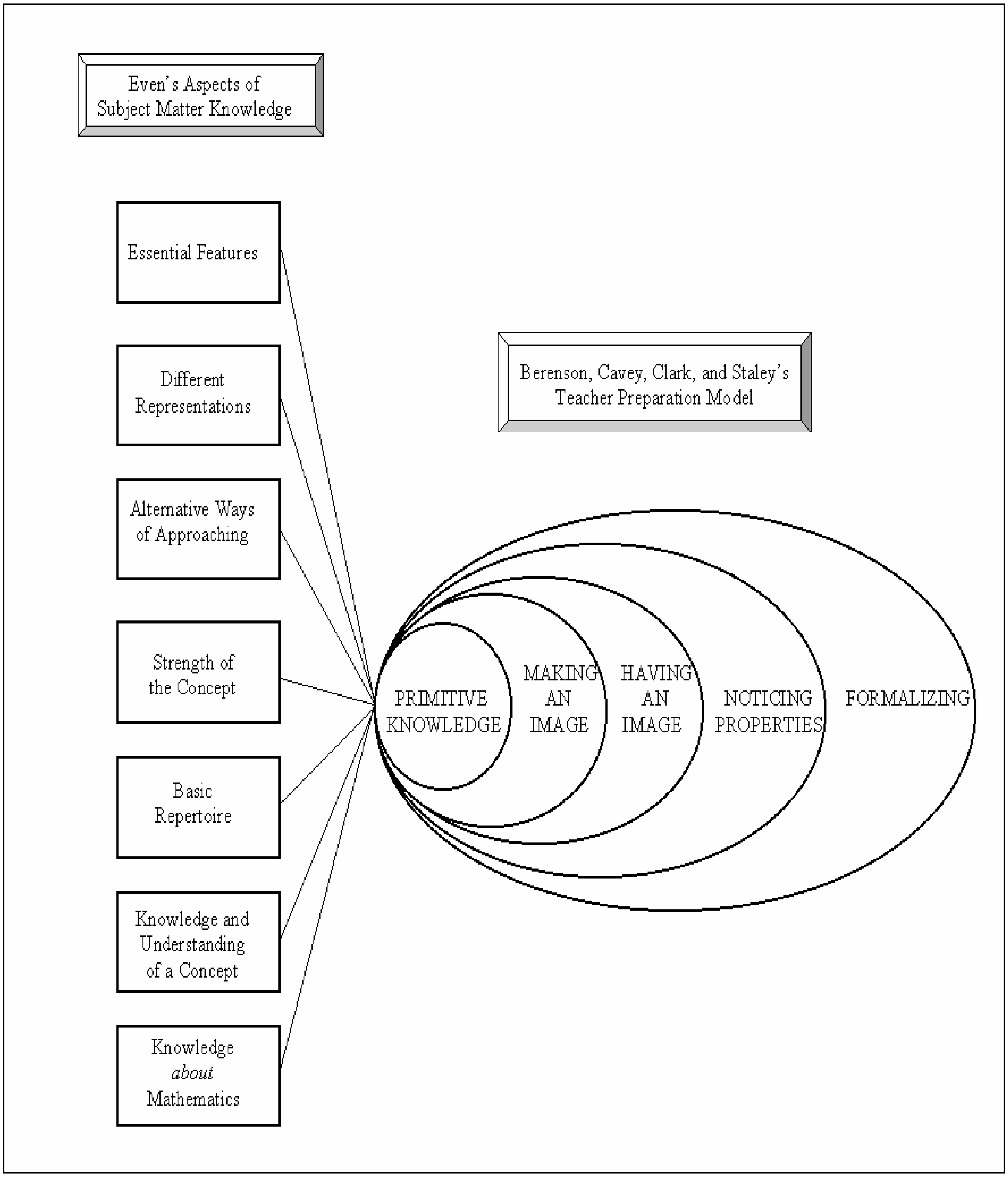


Figure 5. The conceptual framework for growth in subject matter knowledge.

Since each level of Berenson et al.'s model considers the *what* and *how* of teaching high school mathematics, each of Even's (1990) aspects of SMK will be included at each level. It is within the discussions, planning, and presentations of a lesson to introduce the topic of similarity, that the primitive knowledge and growth of prospective teachers' SMK of similarity can be explored.

Review of Literature on Similarity and Subject Matter Knowledge

The purpose of this section is to illuminate research on similarity, prospective teachers' SMK of concepts related to similarity, and the implications of SMK to teacher preparation. Similarity is seen as a very important concept that is related to other topics within mathematics. It has everyday uses that can be seen throughout the natural and social sciences. Concerns have been expressed in research about prospective and inservice teachers' SMK of similarity related concepts. The SMK of teachers has been shown to go beyond any particular topic or discipline to several areas of teachers' knowledge. Furthermore, much of the research available on teacher SMK focuses on what they do not have, instead of what they can do. It has been shown that teachers need to know and experience a deep, flexible understanding of what mathematics is and what it means to do mathematics. Lastly, the need and focus of the research is in reaction to research on similarity and the SMK preparation of prospective teachers.

The Knowledge of Similarity and its Related Concepts

Similarity and its related concepts are central components of geometry (Chazan, 1988; Friedlaner & Lappan, 1987; Lappan & Even, 1988; Ren, 1995; Senk & Hirschborn, 1990; Woodward et al., 1992). Similarity is a visual representation of many topics throughout mathematics involving proportional reasoning (Friedlaner & Lappan, 1987; Lappan & Even, 1988). Similarity has been shown by researchers not to be an isolated simplistic topic without relevant applications throughout mathematics and other content areas (Chazan, 1988; Friedlaner & Lappan, 1987; Lappan & Even, 1988; Ren, 1995; Senk & Hirschborn, 1990; Slovin, 2000; Woodward et al., 1992). It has everyday uses such as maps, models, tiling, blueprints, and photographs and can be seen throughout physics and other sciences. "Phenomena that require familiarity with enlargement, scale factor, projection, area growth, indirect measurement, and other similarity related concepts are frequently encountered by children in their immediate environment and in their studies of natural and social sciences" (Friedlaner & Lappan, 1987, p.36). Though its implications throughout prospective teachers' SMK seem to be substantial, much of the research on similarity has been directed towards the teaching of similarity and its related concepts to school children. However, studies show that prospective teachers' and inservice teachers' SMK of similarity related concepts are of a concern and that more research needs to be focused in these areas (Baturo & Nason, 1996; Berenson et al., 1997; Lappan & Even, 1988; Simon & Blume, 1994; Swafford et al., 1997).

Research directed towards teaching similarity stresses several important reasons for the study of this topic. Clements and Battista (1992) stressed, "we need teaching/learning research that leads students to construct robust concepts through meaningful synthesis of diagrams and visual images on the one hand, and through verbal definitions and analyses on the other" (p. 457). There are several unifying themes throughout much of the research on teaching similarity and its related concepts, such as the complexities involved in teaching similarity and its related concepts, including reasons for studying similarity and the substantive structures of similarity.

In their work on the Middle Grades Mathematics Project Series, Friedlaner and Lappan (1987) and Lappan and Even (1988) discussed several reasons for the study of similarity in the middle grades. Similar geometric shapes provide a helpful mental image of ratios, equivalent fractions, proportional reasoning, and some models for rational number concepts. The NCTM (1989) supports these claims and stated that learning similarity concepts could facilitate children's understanding of proportional reasoning. Tourniaire and Polus (1985) suggested that the use of task-centered approaches to proportional reasoning in which the context variables are familiar to the students would foster their performance in related tasks. Woodward, Gibbs and Shoulders (1992) included that omitting the topic of similarity is "inappropriate because students will not gain the prerequisite informal geometry experiences for further study" (p. 22). For example, in their study on the emergence of similarity, Lehrer, Strom, and Confrey (2002) found that children in the third grade were able to connect the algebraic concept of ratios with similar rectangles. Senk and Hirschborn (1990) extended the importance of similarity to right triangle trigonometry and the concept of self-similarity, which is characteristic of fractals and fractal geometry. "In the traditional curriculum similar triangles are studied synthetically in both middle and high school, with applications to indirect measurement in the former and to right-triangle trigonometry in the latter" (Senk & Hirschborn, 1990, p.274).

Similarity is a complex and demanding topic to teach. Because of the substantive structures and possible approaches between these structures, the SMK and pedagogical content knowledge required of teachers must be flexible and mature (Friedlaner & Lappan, 1987; Lappan & Even, 1988; Senk & Hirschborn, 1990; Thompson & Bush, 2003). The ideas of Senk and Hirschborn (1990) included the need for teachers to have a strong SMK and pedagogical content knowledge for making connections between algebra and geometry, while making applications to other disciplines and other branches of mathematics. In alluding to SMK, Friedlaner and Lappan (1987) stated that, “our task as teachers of geometry is to provide the kinds of experience for students that will enhance their understanding of the space about them” (p. 130). Lappan and Even (1988) suggested that many teachers might not have the SMK or pedagogical content knowledge to teach mathematics in order for their students to develop a conceptual understanding of similarity. In looking at middle school teachers’ proportional reasoning, a substantive structure in similarity, Thompson and Bush (2003) stressed that for teachers to help students develop proportional reasoning skills, they must realize that it is developmental and emerges gradually over the course of several years.

Research studies demonstrate that prospective teachers’ and inservice teachers’ SMK of similarity related concepts are of a concern and that more research needs to be focused in these areas (Baturo & Nason, 1996; Berenson et al., 1997; Lappan & Even, 1988; Simon & Blume, 1994; Swafford et al., 1997). Many of the concerns brought up by these researchers are similar to those brought up by researchers studying how to teach similarity related concepts to school age children (Chazan, 1988; Friedlaner & Lappan, 1987; Ren, 1995; Senk & Hirschborn, 1990; Thompson & Bush, 2003; Woodward et al., 1992). Heaton (1992) expressed concern about the role of mathematical subject matter and the responsibility of teachers and inservice programs in reforming mathematics teaching and learning. She especially targeted programs that promote meaningful mathematics but are devoid of mathematical content. Similar concerns have been expressed by several researchers, not just in inservice development programs, but also in the subject matter reinforcement in teacher education preparations (Battista, Wheatley, & Talsma, 1989; Baturo & Nason, 1996; Berenson et al, 1997; Shulman, 1986; Swafford et al., 1997).

Swafford, Jones, and Thornton (1997) examined the effects of an intervention program that was designed to improve upon 49 middle grade teachers' knowledge of geometry and student cognition. The intervention program was a content course in geometry and seminar on the van Hiele theory (1959). They asserted the relationship between student achievement and instructional practice and recalled the old adage "the more a teacher knows about a subject and the way students learn, the more effective that individual will be in nurturing mathematical understanding" (p. 467). They found that the intervention program did show improvement in the teachers' SMK, particularly on instructional practice and student cognition. Teachers in their study "not only spent more time teaching geometry but devoted more quality time to instruction in geometry" (p. 480). The researchers found that, not only did students' cognition of geometry related concepts improve, but also the participants [teachers] extended their own knowledge of concepts that they were expected to teach.

Baturo and Nason (1996) in a study of 13 first-year education students examined their SMK of the topic of area measurement. The researchers interviewed the prospective teachers using eight area measurement tasks to explore their SMK, particularly their "substantive knowledge about area measurement, their knowledge about the nature and discourse of mathematics in the domain of area measurement, their knowledge about area measurement in society and their dispositions towards area measurement" (p. 240-241).

They stated that the area measurement SMK of the subjects was weak. The subjects' substantive structures, knowledge about the nature and discourse of area measurement, and their dispositions towards area measurement were limited. They stated that the, "impoverished nature of the students' area measurement subject matter knowledge would extremely limit their ability to help their learners develop integrated and meaningful understandings of mathematical concepts and processes" (p. 263). The researchers felt that the most telling indictment discovered in their study was the prospective teachers' negative beliefs about their abilities and how that will hinder their effectiveness in teaching mathematics.

Berenson, Valk, Oldham, Runesson, Moreira, and Broekman (1997) in an international study used a lesson planning activity on area to explore prospective elementary and secondary teachers' mathematical content knowledge and pedagogical content knowledge. The choice of the area concept was seen as beneficial in many different respects. It allowed teachers from all grade levels to participate, it was seen to encourage many different approaches to planning the lesson, and it entertained a variety of prospective teachers' mathematical backgrounds. The researchers found that the prospective teachers in the study had varying approaches to teaching the concept of area. The approaches of these participants were put into three different categories: concept–centered with some procedure knowledge, procedure–centered with some knowledge of concepts, and procedure only. They suggested that teacher education programs should address all three varieties of prospective teachers and help them to develop understandings of the topics that they plan to teach in the future and to encourage them to incorporate these understandings in their lesson plans and their teaching. Teacher educators need to look for “situations that make it possible for them [prospective teachers] to discover the different aspects of the topics they will teach, and also facilitate them in doing the same for their students” (Berenson et al., 1997, p. 148).

Simon and Blume (1994) used a constructivist teaching experiment to look at prospective elementary school teachers’ nature of mathematical understanding and reasoning processes of the area of a rectangular region as a multiplicative relationship between the side lengths. The study was part of a larger study in the development of prospective elementary teachers’ abilities to distinguish between additive and multiplicative situations. They found that the prospective elementary teachers had a lack of conceptual knowledge of the area of a rectangle with regards to the multiplicative relationship between length and width of its sides. In looking at the lack of conceptual development of the participants, they suggested that it might reflect a more widespread problem. “As teacher educators, we must question the extent to which we are preparing teachers for this important aspect of their teaching practice” (p. 493).

Battista, Wheatley, and Talsma (1989) looked at a substantive structure in similarity by focusing on visualization, reasoning, and geometric problem-solving strategies of prospective teachers. These strategies were seen as underlying structures throughout all the concepts in geometry. They believed that teacher education programs should focus on problem solving strategies and abilities. Further, they looked at five different sections of a geometry course for prospective elementary teachers. The participants were involved in quantitative research that tested their abilities at the beginning of the class and then tested their abilities after focusing on spatial visualizations, formal reasoning, and geometric problem solving. They suggested that teacher educators give prospective teachers several geometric and spatial problems to solve and discuss the various types of strategies used in solving them. They also encouraged teacher educators to find ways to motivate prospective teachers to think about and monitor the use of their strategy selections.

Similarity and its related concepts are complex structures in geometry (Chazan, 1988; Friedlaner & Lappan, 1987; Lappan & Even, 1988; Ren, 1995; Senk & Hirschborn, 1990; Woodward et al., 1992). Similarity is seen as a substantial part of mathematics with connections to other content areas. Research suggests that to teach similarity and its related concepts require knowledge of a mature subject matter repertoire (Friedlaner & Lappan, 1987; Lappan & Even, 1988; Senk & Hirschborn, 1990; Thompson & Bush, 2003). In looking at prospective teachers' and inservice teachers' SMK of similarity-related concepts, research studies reported that there is need for concern and that more research needs to be focused in these areas (Baturo & Nason, 1996; Battista et al., 1989; Berenson et al., 1997; Lappan & Even, 1988; Simon & Blume, 1994; Swafford et al., 1997). However, there does seem to be promising research in techniques that may encourage development in the SMK of similarity and its related concepts (Battista et al., 1989; Berenson et al, 1997; Simon & Blume, 1994; Swafford et al., 1997).

The Impact of Subject Matter Knowledge to Teacher Preparation

The implications of SMK to teacher preparation go well beyond that of similarity or any particular mathematics course, to multiple areas of teacher knowledge (Ball, 1988; Ball & Wilson, 1990; Borko et al., 1992; Borko & Putnam, 1996; Cooney & Wilson, 1995; Cooney et al., 1998; Eisenhart et al., 1993; Even, 1993; Fennema & Franke, 1992;

Graeber, 1999; Grossman et al., 1989; Lappan & Even, 1989; Ma, 1999; Shram et al., 1988; Shulman, 1986; Shulman, 1987; Thompson, 1992; Van Dooren et al., 2002; Wilson et al., 1987). Teachers' SMK is also reflected by their mathematical abilities and their beliefs (Ball, 1988; Ball & Wilson, 1990; Borko et al., 1992; Borko & Putnam, 1996; Cooney & Wilson, 1995; Cooney et al., 1998; Eisenhart et al., 1993; Even, 1993; Graeber, 1999; Lappan & Even, 1989; Ma, 1999; Shram et al., 1988; Simon & Blume, 1994; Van Dooren et al., 2002). Unless teacher education programs undertake an active role in determining the limitations of prospective teachers' SMK, research suggested these limitations might never be addressed (Borko et al., 1992; Borko & Putnam, 1996; Cooney & Wilson, 1995; Eisenhart et al., 1993; Graeber, 1999; Lappan & Even, 1989; Ma, 1999; Shram et al., 1988). In their examination of teacher's learning, Borko and Putnam (1996) stated:

> Subject matter knowledge of teachers does make a difference in how they teach, and that novice and experienced teachers alike often lack the rich and flexible understanding of the subject matter they need in order to teach in ways that are responsive to students' thinking and that foster learning with understanding (p. 690).

In looking at the mathematical abilities of prospective teachers' SMK, researchers have noted several areas of concerns (Ball, 1988; Ball & Wilson, 1990; Borko et al., 1992; Borko & Putnam, 1996;Eisenhart et al., 1993; Even, 1993; Lappan & Even, 1989; Ma, 1999; Schram et al., 1988; Simon & Blume, 1994; Van Dooren et al., 2002). Notwithstanding the findings discussed earlier in studying the concepts of similarity and geometry, researchers have found other areas of concern in looking at the SMK and mathematical content of prospective and inservice teachers (Ball, 1988; Ball & Wilson, 1990; Borko et al., 1992; Borko & Putnam, 1996; Cooney & Wilson, 1995; Cooney et al., 1998; Eisenhart et al., 1993; Graeber, 1999; Ma, 1999). For example, Ma (1999) stated: “Two main factors may have precluded the U.S. teachers from a successful mathematical investigation – their lack of computational proficiency and their layperson-like attitude toward mathematics” (p. 104).

In a research study, Van Dooren, Verschaffel, and Onghena (2002) studied the problem-solving strategies and skills of 97 prospective primary and secondary school teachers. They reported on the relationship between the future teachers’ problem-solving skills and strategies for solving arithmetic and algebra word problems. They were also concerned with the prospective teachers’ abilities to evaluate student’s work on some of these problems. The study showed that many of the primary school prospective teachers did not “spontaneously use algebraic strategies when necessary, because they had not mastered algebra and did not understand it sufficiently or because it was perceived as mindless manipulating of meaningless mathematical symbols” (p. 345). More surprisingly it found that the secondary prospective teachers’ mathematical abilities were focused only on the algebraic method of solving problems, even when these strategies were inefficient. They stressed concern about the attitudes that secondary prospective teachers had towards solving problems and their pedagogy. Lastly, the authors felt as if there are a substantial number of primary and secondary prospective teachers who currently do not appreciate arithmetical as well as algebraic problem-solving skills.

In a study of 162 prospective secondary teachers’ SMK of functions, Even (1993) investigated the relationship between teachers’ SMK and pedagogical content knowledge.

She stressed that when a prospective teachers' mathematical concept image of a particular topic is limited, they may be prevented from understanding current mathematics that is based on modern concepts. She suggested two important implications from her study. First, prospective teachers should have access to better subject matter preparation; however, this does not necessarily mean changing the number of mathematics courses they take. Secondly, prospective teachers need to have learning environments that nurture powerful constructions to help in the development of strong teaching repertoires of teaching skills for mathematical concepts.

Other researchers and organizations lend support to Even's (1993) suggestions and implications (Ball, 1988; Borko et al., 1992; Borko & Putnam, 1996; Eisenhart et al., 1993; Lappan & Even, 1989; Ma, 1999; NCTM, 2000; Schram et al., 1988; Simon & Blume, 1994). Several professional teaching organizations, including the Conference Board of the Mathematical Sciences (CBMS, 2001) have stressed the need for developing the mathematical maturity of prospective teachers. The CBMS (2001) stressed the need for secondary teacher preparation programs to focus on the connections for algebra, geometry, functions, data analysis and discrete mathematics. To meet these needs they suggested that the education of prospective high school mathematics teachers should develop:

- Deep understanding of the fundamental mathematical ideas in grades 9-12 curricula and strong technical skills for application of those ideas.
- Knowledge of the mathematical understanding and skills that students acquire in their elementary and middle school experiences, and how they affect learning in high school.

- Knowledge of the mathematics that students are likely to encounter when they leave high school for collegiate study, vocational training or employment.
- Mathematical maturity and attitudes that will enable and encourage continued growth of knowledge in the subject and its teaching (p. 39).

The CBMS (2001) suggested that to attain these goals, mathematics courses for prospective secondary teachers should be redesigned to make insightful connections between secondary mathematics and the advanced mathematics they are required to take. It is also suggested that these mathematics courses focus on the conceptual difficulties, fundamental ideas, and techniques of high school mathematics from an advanced standpoint. These sentiments were echoed throughout many research studies that found deficiencies in prospective teachers' SMK of mathematical content (Ball, 1988; Ball & Wilson, 1990; Baturo & Nason, 1996; Battista et al., 1989; Berenson et al., 1997; Borko et al., 1992; Borko & Putnam, 1996; Cooney & Wilson, 1995; Cooney et al., 1998; Eisenhart et al., 1993; Even, 1993; Graeber, 1999; Lappan & Even, 1989; Ma, 1999; Shram et al., 1988; Simon & Blume, 1994; Swafford et al., 1997; Van Dooren et al., 2002).

Similar to these suggestions, Schram, Wilcox, Lanier and Lappan (1988) and Lappan and Even (1989) examined a new sequence of mathematics courses for undergraduate majors that "emphasize the conceptual foundations of mathematics and actively engage prospective elementary teachers in making sense of mathematical situations" (Schram et al., 1988, p. 4). The purpose for these new courses were that mathematics was seen as an area in the elementary curriculum that was weak and that children were not learning much mathematics. Schram, Wilcox, Lanier and Lappan (1988) addressed the discrepancy between the conceptions of mathematics teaching and learning that is derived from research with how mathematics is taught and learned by prospective teachers in their teacher training courses. Results from the study showed that the majority of students in these new courses began to question the view of mathematics they brought to the class, how they were thinking about mathematics for themselves, and how they began to appreciate the use of problem situations to explore ideas (Schram et al., 1988). The study did have an effect on the prospective teachers' SMK. "They spoke of connections between ideas that they had never encountered before. Many for the first time came to understand why rules and procedures they had memorized years ago really worked" (Schram et al., 1988, p. 25). The effects on the prospective teachers' SMK, the authors suggested, were because the class focused on patterns, relationships within mathematics, multiple representations, and solving rich problems in a challenging problem-solving situation that forced them to solve them using new strategies. Lappan and Even (1989) stated:

> If students in elementary/middle school are to learn in environments that support the development of mathematical power… teachers themselves need to know mathematics and experience learning in ways that build a deep and flexible understanding of what mathematics is and what it means to do mathematics (p.22).

Another implication of SMK is how prospective teachers' beliefs largely reflect and sometimes contradict their SMK. Teachers' beliefs are guided by their SMK (Ball, 1988; Borko et al., 1992; Cooney & Wilson, 1995; Cooney et al., 1998; Eisenhart et al., 1993; Ma, 1999; Van Dooren et al., 2002). Some of the prospective teachers' beliefs that are a part of their SMK, effect the representations they choose, their approaches to teaching, and their ideas of student learning (Ball, 1988; Ball & Wilson, 1990; Baturo & Nason, 1996; Battista et al., 1989; Berenson et al., 1997; Borko et al., 1992; Eisenhart et al., 1993; Even, 1993; Graeber, 1999; Lappan & Even, 1989; Ma, 1999; Shram et al., 1988; Simon & Blume, 1994; Van Dooren et al., 2002). One flaw in many studies on teachers' beliefs is their disconnection with teachers' SMK. Thompson (1992) stated "to look at research on mathematics teachers' beliefs and conceptions in isolation from research on mathematics teachers' knowledge will necessarily result in an incomplete picture" (p. 131).

Cooney and Wilson (1995) investigated the abilities and confidence of two secondary teachers as they advanced through their mathematics education courses. They focused their efforts in examining three aspects of knowing: what the teachers seem to be able to do mathematically, what beliefs they seem to hold about mathematics and how those beliefs are structured, and the implications of their knowledge and beliefs about mathematics for the teaching of mathematics. The prospective secondary teachers held different views about the nature of mathematics and understood mathematics differently: one student enjoyed mathematics because it was either right or wrong and saw the teacher as the purveyor of answers, whereas the other had similar views but extended ideas to ones that were mathematically rich and profitable. In looking at the belief structures of the prospective teachers, the researchers found that one of the prospective teacher's beliefs structures were more permeable than the other's. The researchers noted this revelation because they felt that if a prospective teacher's belief structures were able to adapt to different ideas, then they would be able to reform their teaching over time.

In examining a middle school student teachers' knowledge and beliefs related to division of fractions, Borko, Eisenhart, Brown, Underhill, Jones, and Agard (1992) and Eisenhart, Borko, Underhill, Brown, Jones and Agard (1993) examined how the topic of division of fractions was handled in a mathematics methods course. The researchers found that in spite of advanced mathematics courses that they take, some student teachers have only a minimal understanding of topics in the middle grades curriculum. The research studies suggested that prospective teachers' beliefs about good mathematics teaching could not be implemented because they do not have a conceptual understanding of the topic. It seemed as if prospective teachers had a limited repertoire of instructional representations and limited knowledge of what students understand about mathematical topics. Lastly, the researchers suggested that it might be possible for prospective teachers to enter their student teaching year with only rote knowledge of the topics they need to teach and no knowledge of representations that might enable them to teach the topic except by demonstration of algorithms.

Ball (1988) examined the knowledge and beliefs of prospective teachers. She conducted interviews with 19 prospective teachers – 10 elementary and 9 secondary – examining their knowledge and beliefs of division in order to assess what teacher candidates bring to their formal preparation in becoming mathematics teachers. Her research challenged common assumptions about SMK and the preparation of teachers, while trying to provide a synthesis of the substantive knowledge of division held by these prospective teachers and to "illustrate an approach to the examination and analysis of teachers' substantive knowledge of mathematics" (p. 4). Her research looked at the prospective teachers' substantive structures for: truth-value, legitimacy, and connectedness. She found that many prospective teachers' image of truth-value involves simply restating rules. She also stated that the prospective teachers were able to give more legitimate answers, than explanations of these answers. She found that prospective teachers focused on the rules instead of the underlying meanings of the problems presented. Ball and Wilson (1990) suspected that the ideas that the prospective teachers have are brought from much of their experiences as students to their teaching of mathematics. These are the experiences that teacher education programs must address, in their preparation, or the knowledge and beliefs that the prospective teachers already have, from their secondary school experiences, will 'wash out' any training they receive (Ziechner & Tabachnick, 1981).

Similar to these studies, Graeber (1999) investigated ideas about forms of knowing mathematics and what should be included in mathematics methods courses for prospective teachers. She found several ideas that reemphasize what has been said about SMK, beliefs, and prospective teacher education. She found that if prospective teachers are not familiar with the advantages of instructional principles that require children to explain and justify their answers, then they are likely to implore the strategy of teach by telling, as in many mathematics classes. She stressed that prospective teachers need both time and experience as learners and designers of lessons that incorporate the notion of justification for their students, and that by doing these activities the prospective teachers are building stronger connections between their pedagogical content knowledge and SMK. "Clearly knowing different models and various approaches to topics places demands on the prospective teachers mathematical knowledge as well as on their attitude" (p. 203). Graeber (1998) stressed that the prospective teachers need these experiences of multiple approaches or they will likely recall and implement the old way of homework review, lecture, and practice, because that worked for them. In support of this statement, Ball (1988) stressed that the understandings of mathematics that prospective teachers bring with them suggests that teacher education programs cannot assume that their subject matter preparation will happen elsewhere and that it needs to become a central focus in teacher education.

In summary, teachers' insufficient SMK impedes their abilities to use curriculum materials effectively, interpret and respond to students' work, and choose correct representations, tools, and reinforcement activities within a lesson. "Observers note that interpreting reform ideas, managing the challenges of change, using new curriculum materials, enacting new practices, and teaching new content all depend on teachers' knowledge of mathematics" (Ball et al., 2001, p.437). In support of this idea, Borko and Putnam (1996) assert in their discussion of SMK, "teacher educators have argued that teaching in accord with current reform efforts emphasis on understanding and use of knowledge, requires teachers to have rich and flexible knowledge of the subjects they teach" (p.685).

The Need and Focus

Similarity is seen as a very important concept that is related to other topics throughout mathematics. Also, the SMK of teachers is seen as a vital component in teachers' knowledge base for teaching. Researchers advocate subject matter training for prospective teachers in concepts like similarity. They see that the current structures of mathematics courses are deficient in teaching prospective teachers the underlying structures of mathematics. "Evidence is mounting that students can do satisfactory work in mathematics courses without developing a conceptual understanding of subject matter knowledge" (Borko et al., 1992, p. 219). Therefore, it is the intent of this research to use a conceptual framework that is structured using aspects of SMK to answer questions that are lacking in research on prospective teachers' SMK of similarity:

1. What is the nature of the subject matter knowledge that prospective teachers rely on when planning a lesson to introduce the concept of similarity?
2. What growth in subject matter knowledge is revealed as prospective teachers plan a lesson to introduce the concept of similarity?

METHODOLOGY

This chapter discusses the methodology of lesson plan study (LPS) that was used for the research. Lesson plan study is a hybrid teaching experiment that was used to investigate prospective teachers' SMK of similarity. Lesson plan study was adapted from Japanese lesson study incorporating some of its aspects including: peer participation, collaborative efforts in planning and revising a lesson, and live observations. The prospective teachers involved in the LPS characterize the uniqueness and vast differences among prospective teachers' primitive knowledge. The sources of data collected and analyzed, from the LPS, included videotaped interviews, transcripts, and other written artifacts. The data was analyzed using the conceptual frameworks discussed earlier in Chapter 2. Lastly, this chapter discusses issues of data credibility.

The Investigation

The investigation of prospective teachers' SMK of similarity was part of a larger study that investigated prospective teachers' knowledge of *what* and *how* to teach concepts dealing with proportional reasoning, while engaged in LPS. The larger study was concerned with developing prospective teacher knowledge in the concepts of ratio, proportions, rate of change, similarity, and right triangle trigonometry. The studies were done over a three-year period, involving several different classes of prospective teachers.

The LPS involved a combination of teaching and research methods. The research method was based on the idea of Japanese lesson study and was looked at ways prospective teachers developed an introductory lesson on a topic concerning proportional reasoning. However, since the lesson plan was seen as part of their methods course, it was imperative that the prospective teachers had an idea of some of the content and substantive structures involved in the proportional reasoning topic. In some instances it was necessary for the researchers to guide the participants towards these ideas. The researchers were a team of graduate students and faculty members that conducted interviews and were participant observers in various phases of the studies.

Japanese Lesson Study

In Japan, the impact of research lessons called "*kenkyuu jugyou*" help to foster a shared professional community and is not just a professional development activity. The essential features of Japanese lesson study are that teachers agree upon a shared goal, there is a focus on a particular content area and student learning and development, and there are live observations (Lewis, 2000; Lewis, 2002; Lewis & Tsuchida, 1998; Watanabe, 2002; Yoshida, 1999). When focusing on content areas, teachers look for things such as difficulty in teaching a topic, a new subject added to the curriculum, or time management in covering a lesson sufficiently. When focusing on students, the observers note if topics are difficult for students to learn, student engagement, and other student activity. Lastly, live research lessons are the heart of lesson study. Ball (2002) pointed out that lesson study engages teachers in learning mathematics in ways connected with practice, while also focusing their attention on student thinking and to the integrity of the mathematical ideas. "Japanese teachers also mentioned that research lessons provide a good way to deepen one's knowledge of subject matter – particularly for topics newly added to the curriculum" (Lewis, 2002, p.15).

In Yoshida's (1999), ***Lesson Study in Elementary School Mathematics in Japan: A*** *Case Study*, he described a typical cycle of a lesson study (see Figure 6). There are five steps in the lesson cycle. Within each cycle are a set number of tasks that need to be completed in order for the teachers to accomplish the maximum intended effect. In the first step, the preparation for the lesson study begins. The preparation includes a group meeting in which the teachers incorporate ideas for the pending lesson. The second step is the implementation of the prepared lesson. Step three usually consists of group meetings that are held to reflect on the lesson and make improvements. Implementation of the improved study lesson is the fourth step. Further reflection and group discussion are summarized and placed in a booklet, which is then placed on a bookshelf in the staff room.

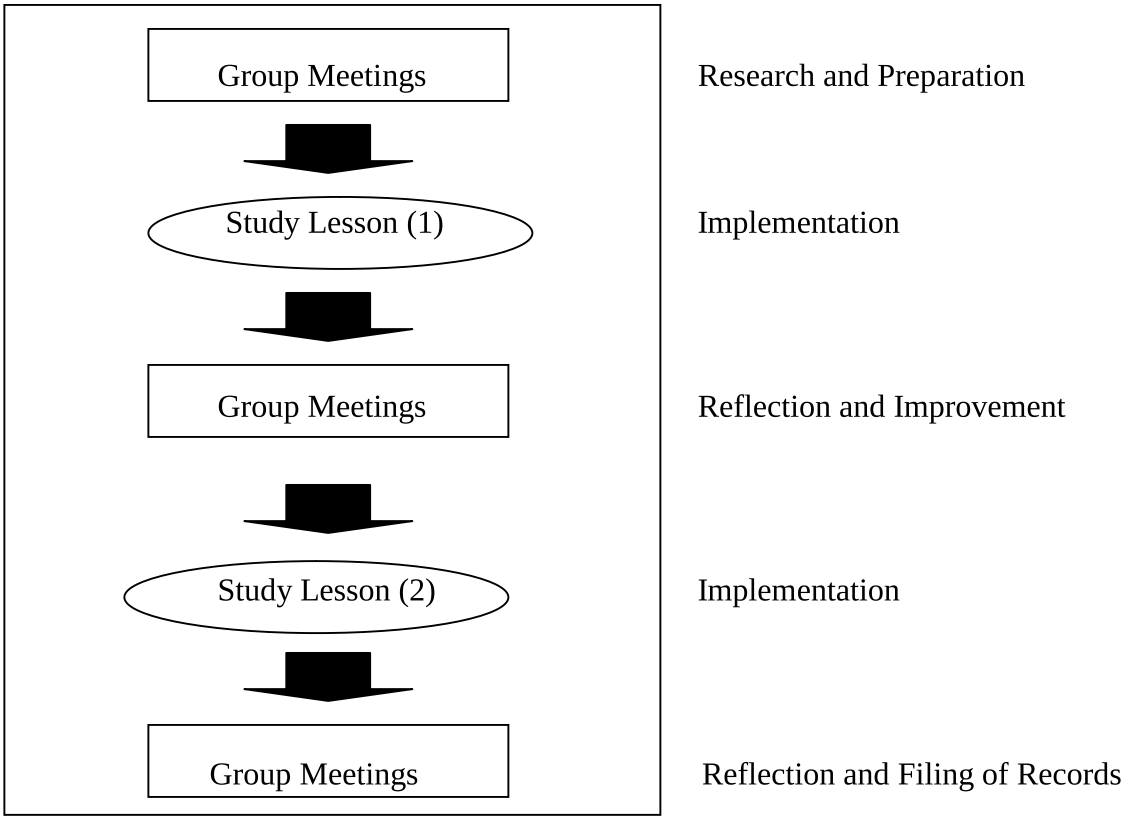


Figure 6. A typical lesson study cycle (Yoshida, 1999).

From the United States, examples of lesson study have focused on goals of academic content areas, testing, and other goals set out by lesson study leaders (Lewis, 2002). More research is needed to understand which adaptations of Japanese lesson study are successful and which are not in the United States (U.S. Department of Education, 2000; Watanabe, 2002). Japanese educational researchers want to collaborate on this research. Preliminary evidence suggests that, at least at some sites, U.S. teachers have found lesson study useful (Lewis & Tsuchida, 1998). Ball (2002) further supported this research by stating that, “probing the surfaces of the practices involved in lesson study suggests that there are important aspects of this work that offer promise for teachers to learn in and from practice, in the company of other professionals” (p. 11).

Data Collection

The structure for the LPS for this research was modeled using aspects that are similar to those described by researchers of Japanese lesson study. Lesson plan study can be seen as a hybrid of two types of teaching experiments: classroom experiments (Borba & Confrey, 1993; Cobb & Steffe, 1983; Steffe, 1991) and preservice teacher development (Simon, 1995). Lesson plan study is a hybrid teaching experiment because the research team collaborated with the teacher of the methods course and organized and developed experiments (e.g. the lesson planning cycles) for the prospective teachers, and assumed responsibility for the instruction, if necessary, on similarity and the other proportional reasoning activities during the study (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003). The activities for the LPS have been modified throughout the larger study in order to find better ways to study prospective teachers' proportional reasoning abilities. For example, in this particular study the topic of similarity was used to create a bridge in the prospective teachers proportional reasoning abilities between ratio/proportion and right triangle trigonometry.

During a semester a LPS may consist of two or three different cycles, with each cycle focusing on a proportional reasoning concept (e.g. rate of change, similarity, or right triangle trigonometry). The LPS on similarity was the first of two cycles that took place during the sixteen-week methods course. Data from the similarity LPS cycle was collected between the second and seventh week of the semester. Each LPS cycle consisted of four parts: *Individual Interview*, *Group Interview*, *Group Presentation*, and *Individual Lesson* (see Figure 7).

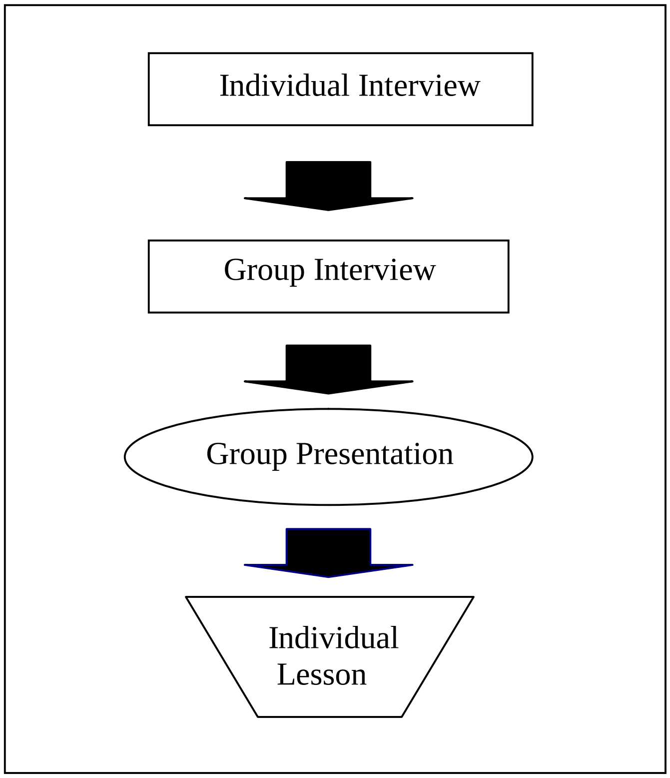


Figure 7. The structure of the lesson plan study cycle.

The schedule of interviews and group placements were done to accommodate the prospective teachers' schedules in the blocks of time that the researchers set aside. Two weeks were needed to conduct all of the individual interviews, while four times were set aside in a one week period to conduct the group interviews. Two class periods were used for the group presentations, and two weeks after the group presentation the participants were expected to turn in an individual lesson plan on introducing the topic of similarity.

Step 1: The Individual Interview

During the individual interview, the researchers tried to get an understanding of what the research participants knew about similarity and what their initial understandings were of how to teach similarity. The protocol for the individual interview consisted of three major components: pre-interview, lesson planning, and post-interview (see Appendix B). During the pre-interview stage, participants discussed what they recalled from previous high school and college classes about similarity and the methods used to teach them. The lesson planning stage of the individual interview was 45-mintues long and during this time the prospective teachers devised their first lesson plan on similarity. During the lesson planning stage, the participants had a variety of materials to use or incorporate within their lesson plan. The materials that they had to choose from were a geometry textbook, the North Carolina Standard Course of Study for Geometry, an article that they were reading in class on ratio and proportion, geoboards, plastic and wooden triangles, protractors, rulers, compasses, a graphing calculator, rubber bands, transparencies, paper, scissors, pens and pencils. After the 45-minutes of lesson planning, the prospective teachers were asked to explain their lesson and were further probed about their lesson and their understanding of similarity.

The purpose of the individual interview was to ascertain what the prospective teachers' initial subject matter understanding of similarity was and how they might teach similarity. This allowed the researcher to begin to infer their initial images of similarity, before any influences took place by group members or the instructor during the group presentation. The prospective teachers primitive knowledge of material related to similarity is reflected in their images of *what* and *how* to teach similarity. What is inferred as their primitive knowledge and initial images of *what* and *how* to teach will have an effect on the determination of the outer levels of the teacher preparation model. For example, if a participant were to relate an idea at the beginning of the study that they use consistently throughout the remainder of the LPS cycle, then there may be strong evidence that this initial image is a part of that participant's formalized knowledge of *what* and *how* to teach

Step 2: The Group Interview

During the second stage of the LPS, groups were formed so that the participants could construct a lesson on similarity and present it one week later to their classmates in their methods course. The participants planned times that were convenient for them to come in for the group interview. Each of the groups consisted of five members. The purpose of this interview protocol was twofold (see Appendix C). In the first part of the interview, the participants were left alone for an hour, with the same materials that they had in the individual interview, and allowed to discuss ideas about the presentation without any observation other than a video camera. It was during this time that the group began to agree about the main ideas needed in teaching similarity. The second part of the group interview occurred immediately after the one-hour discussion, when the interviewer checked on their progress and asked them questions about the group's ideas to probe their development and the progress made on the similarity presentation.

The purpose of the group interview was to allow time for the participants to share their SMK and ideas about teaching similarity. It was during this time that the prospective teachers' ideas of *what* and *how* to teach became clearer and some began to make and have different images about teaching similarity. For example, in discussing what they did in the individual interview, they may come to realize other ways to teach similarity and develop and keep these alternative ways of approaching the topic for themselves.

Step 3: The Group Presentation

The third stage of the LPS was the groups' presentations of their ideas on what should be in a lesson introducing similarity. The presentation was for their methods course classmates and occurred one week after the group interviews. After the groups presented their lessons, they received feedback from the instructors and their classmates. The presentations were scheduled to take 20 minutes with an additional 10 minutes allowed for feedback from other students and the instructors. Since the group interviews were only an hour, the presentation helped to reinforce any idea that they had and any new idea that they came up with later on.

Step 4: The Individual Lesson

In the last stage of the LPS, participants constructed another individual lesson plan on similarity. The prospective teachers were given expectations for this lesson planning activity (see Appendix D). Other than connecting ratio and proportion and including a hands-on activity, the prospective teachers were given ample opportunity to develop the lesson as they saw fit. The written lesson plan was due two weeks after the group presentations and it was taken for a grade in their methods course. Given more time to develop the lesson allowed the prospective teachers the chance to express their SMK in similarity. Because this part of the LPS was being graded there was more pressure on having accurate representations and examples.

Participants

The prospective high school teachers in the larger study were sophomores or juniors, had Grade-Point-Averages of 2.5 or higher, and at the time of the study were enrolled in their first of four methods courses. All the prospective teachers in the class were invited to participate and have their data used for the research study. The participants signed an informed consent form in order to have their data used for publication purposes (see Appendix A).

For the present research, all the members of the methods course volunteered for the study. Five participants were chosen from the Spring 2002 study for an in-depth analysis of prospective teacher SMK of similarity. The participants were all female and from the same group, known as Group C. The names (pseudonyms) of the participants in Group C are Alice, Anne, Ava, Rose, and Mary. The participants from Group C were chosen as the subjects for this study because of the unique dynamics that were contained within the group. The dynamics of this group that interested the researcher were their cultural diversity and backgrounds. These dynamics will not be the focus of this research, but are interesting to note and could be a topic for further research.

Alice was a junior during the present study. Her desire to become a teacher was something that she recalled since the age of six, when she was inspired by her kindergarten teacher's ability to teach and help her. As she grew older, her interest in careers changed from wanting to be a teacher or a lawyer. She entered college as an English major and quickly changed her major to Computer Science. She eventually changed her major again to one that would make her happy, teaching mathematics. Alice said in her autobiography, "Teaching and math are my heart and my dream and my goal."

Anne, at the time of the present study, was 19 years old and a sophomore. Her plans were to graduate with a degree in Middle School Math Education. She has had previous experience working with children at a day camp in her church. In her search for excellence she viewed becoming a teacher as an ongoing development of becoming the best that she could. Expressing her goal to become a good teacher, Anne asked, "Children are our future, if they are not taught right what will the world look like in the years to come?"

Ava, like Alice, felt as if she was influenced early in life to become a teacher. During the time of the study, Ava was a junior majoring in Mathematics Education. She recalled volunteering to teach her first grade class when her teacher lost her voice. While in elementary school she undertook many supportive roles as a hall monitor and a stair monitor. In middle school, she continued to work in supportive lead roles, such as a paper grader and a group leader. It was during her middles grades that she felt as if one of the greatest influences on her decision to become a teacher was a math teacher who showed her genuine concern and openness. She worked as a teacher for an after-school program teaching seventh graders. Ava said, "Each child is free to choose to stay on the path or go astray, but it is the role of the teacher to make the light visible to each child, to save them from blindness."

Mary, who was the oldest member in the group, was 28 years old during the present study. She originally grew up in Europe before coming to the United States after her high school graduation. She attended a community college before transferring in 2002 as a double major in Mathematics and Mathematics Education, with plans of getting a coaching minor. Like Anne and Ava, Mary had opportunities to work with children and students as a basketball coach and as an au pair. Mary felt that, “Teachers have a big responsibility, and they have to do their very best to teach students to their best ability.”

Rose was a junior double majoring in Mathematics and Mathematics Education during the present study. Crediting wanting to become a math teacher from experiences that she had in school, she recalled that her math teacher once grounded her until she began to make better grades. The teacher then devoted time between classes and after school to help Rose raise her grades. She has seen mathematics teachers as positive influences in her life and went to them for help with academic problems outside of mathematics. Like Alice, Rose was unsure of her major but decided on Mathematics and Mathematics Education because she felt she had a strong mathematics background. Rose stated, “In ten years, I hope to be a successful teacher that any student can come to with any problems they may have.”

The participants in Group C had diverse backgrounds. However, their ideas and experiences of teaching mathematics were seen as more important because this will reflect their knowledge base for teaching, SMK, and their growth in SMK of similarity. Lastly, many teacher education programs throughout the world are challenged with similar issues of diversity in knowledge and experience in their prospective teachers (Berenson et al., 1997; Cooney & Wilson, 1995; Van Dooren et al., 2002).

Sources of Data

The data sources included videotapes of the individual interviews, group interviews, and the group presentations and transcripts of these videotapes. Written artifacts such as individual lessons, group lessons, and any written materials or manipulatives were also analyzed. The instructors' feedback was reviewed as part of the data during the group presentations and on the final individual lesson plan. The participants' autobiographies and teaching philosophies were also examined for any background information pertinent to the research study.

Case Study Research

The history of case study research is marked by periods in which its use fluctuates (Bogden & Bilken, 1998). The earliest form of case study research can be traced throughout Europe, but it was predominantly used in France. Case study research is said to have gotten its name from either the fields of Law or Medicine, where "cases" make up a large body of the work. There are some areas that have used case study techniques extensively, particularly in government and in evaluative situations. These types of studies are carried out to determine whether particular programs were efficient or if the goals were met.

As defined by Bogden and Bilken (1998), a case study is a "detailed examination of one setting, or a single subject, a single depository of documents, or a particular event" (p. 54). There are several types of case studies that can be researched. For example, historical organizational, observational, life history, and documents are all different types of case studies that are well respected in qualitative research. Creswell (1998) stated that a case study is "an exploration of a 'bounded system' or a case (or multiple cases) overtime through detailed, in-depth data collection involving multiple sources of information rich in context" (p. 61).

The participants are seen as a case of prospective teachers' SMK of similarity. Prospective teachers that enter many university education programs have a wide variety of backgrounds, experience, and range of knowledge (Berenson et al., 1997; Cooney & Wilson, 1995; Van Dooren et al., 2002). Many teacher educators are faced with many of the diverse characteristics of this group of five individuals. Case study is appropriate because the diversity captured by the members of Group C range from their educational backgrounds, race, age, cultural diversity, professional experiences, influences on their lives, and ideas about teaching. To give insight into prospective teachers' SMK of similarity and possible growth in their SMK, ideas of Pirie and Kieren (1994), Berenson, Cavey, Clark, and Staley (2001), Even (1990), Martin (1999), Pirie and Martin (2000), and Shulman (1986) are used to frame and guide the data analysis and discussion for this research.

Strategy for Data Analysis

The conceptual frameworks used were instrumental in the analysis of the data. Analysis of the data for the research involved coding, sorting into categories of significance, and establishing patterns among the categories. Data from interviews and the lesson plans from Group C were coded and evidence of SMK was extracted and examined.

To analyze the SMK of similarity that prospective teachers rely on when planning a lesson to introduce the topic of similarity, the transcripts of the interviews, the group presentation, and the written artifacts from the final individual lessons were coded using Even's (1990) seven aspects of SMK (*essential features, different representations, alternative ways of approaching, strength of similarity, basic repertoire, knowledge and* *understanding of similarity, and knowledge about mathematics*) and Shulman's (1986) three forms of teacher knowledge (*propositional knowledge, case knowledge, and strategic knowledge*). Any themes found, were looked at for consistency throughout the LPS.

The growth in SMK of similarity was assessed within the Pirie-Kieren (1994) model of growth in understanding as adapted by Berenson, Cavey, Clark and Staley (2001) to teacher preparation, while noting instances of folding back, collecting, and thickening. Within each level of the teacher preparation model, Even's (1990) aspects of SMK were looked at for potential growth in the context of *what* and *how* to teach. The transcripts and artifacts were examined for instances of *primitive knowledge*, *making an image*, *having an image*, *noticing properties*, and *formalizing*, while also looking for occurrences of *folding back*, *collecting*, and *thickening*. The coding of the data used the categories from the Berenson, Cavey, Clark and Staley (2001) model for teacher preparation along with Even's (1990) aspects of SMK.

Credibility

The issues of credibility are of utmost concern, and steps were taken to assure that the findings are accurate and credible. Wood (2000) pointed to different techniques that help in assessing the credibility of qualitative research. Four such techniques are integrity of the observations, peer debriefing, member checks, and theoretical validity.

In looking at the credibility of the observations, several precautions were taken in dealing with the observed data. This researcher tried to separate himself from the collection of the data as much as possible, to countermand any influences on the data.

However, it cannot be said that one can completely separate themselves from the data. There were instances in which this researcher had some interaction with participants of the study, as a cameraman and participant observer in the group presentations. All the observations were recorded on videotape and the data from each episode was compared to previous episodes when analyzing the data.

To help in the accuracy of the interpretations of the data, this researcher used a triangulation protocol. Stake (1995) stated that, "to gain needed confirmation, to increase credence in the interpretation, to demonstrate commonality of an assertion, the researcher can use any of several [triangulation] protocols" (p. 112). Data source triangulation was used to see if the phenomenon or case remains as persons interact differently. It is important that data source triangulation be used to help support the observations made, to make sure they carry the same meaning when found under different circumstances. The triangulation protocol used data sources from all three video transcripts, written artifacts, and instructor feedback to make interpretations about the prospective teachers' SMK and growth in SMK.

By employing the use of peer debriefing, this researcher asked an outside, uninterested third party to review the present research report. This third party was given the opportunity to review the materials and to identify strengths and weaknesses in the report. While not uninterested, the research interviewers and instructor of the methods course were given the opportunity to review the report. These opportunities lend credibility and support to the conclusions drawn.

By the use of a member check, this researcher is gave the participants of the present study the opportunity to review any materials, and this report on their SMK of similarity (see Appendix E). Their input as to how the data was looked helps in establishing a greater credibility to the conclusions. To lend credibility to the findings, it is important that the participants be given the opportunity to help in the reconstruction of the events that took place during the LPS.

Theoretical validity is the degree in which a theory or theoretical explanation developed from a research study fits the data and is credible and defensible (Johnson, 1997). The theories chosen in the conceptual framework lend credibility to the research on the growth of prospective teachers' SMK and what aspects of SMK they rely on when planning a lesson to introduce the topic of similarity. The conceptual frameworks for this research study incorporate Even's (1990) aspects of SMK for teaching a particular topic. Also, the protocols were designed to investigate the *what* and *how* of teaching a proportional reasoning topic (see Appendix B and Appendix C). The protocols have been modified during the course of the larger three-year study to help in attaining more data from the participants. For example, the protocols are seen as semi-structured interviews in which if it becomes necessary for the interviewer to deviate from the protocol to probe a participants understanding deeper, then they can do so while keeping intact the focus of the LPS. The protocols for this LPS cycle were focused on the *what* and *how* to introduce the topic of similarity to a high school geometry class.

It needs to be pointed out, that as a doctoral student who has worked closely with prospective teachers and in-service teachers throughout the state, this researcher looks at better ways for teachers to learn and understand the content they are expected to teach. This could have an affect on the study. This researcher has also used other data from previous lesson plan studies in research projects for conferences (Davis & Staley, 2002; Staley & Davis, 2001).

Some may question if the present study was completely voluntary. Students may have felt compelled to be in the study because they were members of the class, however students were informed that there would be no consequences if they declined to be a part of the study. Some aspects of the LPS were required of all students whether they chose to have their data used in the research or not. For example, the individual lesson plan that was due at the end of the cycle was required of everyone in the class, regardless of their participation in the study. The last individual lesson plan was turned in for a grade and was reflected in the student's overall grade for the course. While this pressure is uncharacteristic of Japanese lesson study, it could have an affect on the data for the LPS. However, this researcher had no input in determining the grades for any of the participants at any time before, during, or after the studies.

PRESENTATION OF FINDINGS

The purpose of this chapter is to report the findings related to the prospective teachers' SMK of similarity during a LPS cycle. The chapter is divided into two sections: the nature of SMK and growth in the SMK of similarity. The nature of the SMK that prospective teachers rely on when planning a lesson on similarity is analyzed using Even's (1990) analytic framework of SMK for teaching specific topics and Shulman's (1986) three forms of teacher knowledge within the knowledge base for teaching. The growth in the SMK of similarity during the planning of the lesson is analyzed using Berenson, Cavey, Clark and Staley's (2001) adaptation to Pirie and Kieren's (1994) model of growth in mathematical understanding to teacher preparation along with Even (1990) and the two features of thickening (Martin, 1999) and collecting (Pirie & Martin, 2000).

The Nature of Subject Matter Knowledge

The SMK for teaching specific topics is formed by seven aspects: essential features, different representations, alternative ways of approaching, strength of the concept, basic repertoire, knowledge and understanding of a concept, and knowledge *about* mathematics (Even, 1990). These aspects are used to frame the results reported on the nature of the SMK that prospective teachers rely on planning a lesson on the topic of similarity. Shulman's (1986) three forms of teachers' knowledge will be incorporated to lend support throughout Even's analytical framework. For each feature of Even's framework of SMK a brief description will be given along with the interpretations of the data.

Essential Features

Even (1990) described essential features as the image of the concept that pays attention to its essence. The essential features that the prospective teachers use in recalling and working with similarity will be their mental pictures of the topic and the properties that they associate with it. Concept images are usually different for different people and are influenced by their analytical judgments and the prototypical examples they use. Two major themes about the essential features of similarity seemed evident in the data collected on the prospective teachers: *Same shape but different sizes* and *the role of side lengths compared to angle measures.*

Same Shape but Different Sizes:

The first theme deals with the idea of similar triangles having the same shape but not necessarily the same size. This feature became evident as the prospective teachers began to discuss their initial ideas about similarity along with what they incorporated within their lessons or lesson outlines.

Rose: Okay, if I am going to start out I would state that the definition between similar triangles which would be that they are the same shape but not exactly the same size.

Anne: I told them that similar figures have the same shape but not necessarily the same size; they are either enlargements or reductions of the original object. And I gave them the original triangle, and the original triangle can be

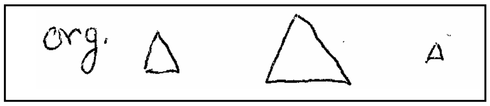


Figure 8. Anne's enlargements or reductions of the original object.

> bigger than what it was or smaller than what it was and be similar, they are similar because they are the same shape triangles but different sizes.

The idea that similarity meant that the triangles have "the same shape but different sizes" was discussed, in some part, by all the prospective teachers. Building on this idea along with what they recalled about similarity shows what the prospective teachers meant by "the same." The transitions that the prospective teachers made from generalizations about "the same" when discussing similar triangles, to the exactness of what they meant by "the same" led to the other major theme within the essential features.

The Role of Side Lengths Compared to Angle Measures:

The second theme in prospective teachers' essential features of similarity deals with the role of side lengths compared to the angle measures. In many of the first examples discussed by the prospective teachers, side lengths were dominant in their prototypical examples of similarity. In discussing what she recalled about what her teacher did with similarity Rose stated:

> "If we are doing similarity, she would put the basic triangle and she would put this one

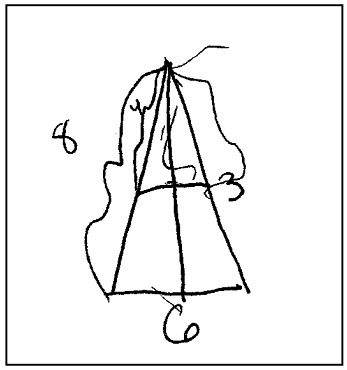


Figure 9. Rose's "basic" triangle.

say, okay, if those one has length 6, and this one would be half of that and this one would be 4, or length 3, I mean. Then she'll do sides, she'll do the proportions and ratios that we were doing in class. Like if that one had an 8 or something and this one would be half of that a 4, and the same on the other side. She just drew diagrams like that when doing similarity."

When asked what she recalled from similarity Alice used ideas that matched Rose and others:

"Similar triangles **[begins to point at the table]**. Similar triangles **[starts making shapes with her fingers]** their sides are…their sides are proportional to each other. Umm… let's see, you have a little one and a big one say,

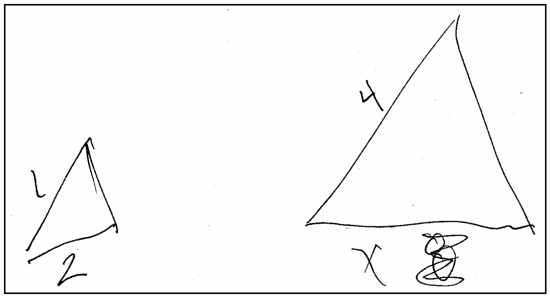


Figure 10. Alice's recollection of the proportional sides.

that this side is 1 and this side is 4 and this side is 2, and this side would be 8. I think that's right, yeah."

In both of these examples, the prospective teachers connected the idea of similarity with proportional sides. When asked, "what do you recall about similarity," these responses focused only on the side lengths as the indicator for similarity. In the beginning, the ideas about the importance that the prospective teachers placed on angle measures were at times an afterthought or not even recognized.

Int.: The proportionality of the side lengths, and you also mentioned the congruency of angles…

Ava: Umm-hmm **[nods head]**.

Int.: Do you think of both of those when you think of similarity?

Ava: Umm…yeah, but I think probably because it was introduced to me first, that I look at the side lengths first. When I think of similarity I think that these sides are multiples of some number to get all the rest of them, then it clicks with me that they must have all the same angles… It's like a second, it follows right after.

Here, Ava considered angle measures a part of similarity, but only as "a second" or afterthought. In the example below, Anne knew that angles were mentioned when she learned about similarity in high school, but could not remember their importance:

Int.: So we got this correspondence **[points to a proportion Anne set up earlier]**, this one goes with this one and this one goes with this one.

Anne: And that's what mostly I remember and something about the angles that I can't remember.

As the participants were further exposed to the LPS on similarity, they began to place more emphasis on angle measurements. In their individual lesson plans, at the end of the study, all of the prospective teachers brought up the importance of angle measurements in their hands-on activities and incorporated this idea in discussing the similarity postulates, used in their lessons. In the example below, Ava is discussing with the group the student activity she made for understanding similarity in her first individual lesson plan, and what she felt was important:

> Then the teacher facilitates a class discussion toward the mathematical definition of similarity, and then some of the rules for similarity, like they are the same shape but not necessarily the same size, corresponding angles must be equal, corresponding sides must be proportional. And then this is where I took over as a teacher and the lecture style and started talking about Side-Side-Side, Angle-Angle, and Side-Angle-Side.

As Ava explained her first individual lesson plan, it is clear that she changed her ideas about similarity, and she now placed more emphasis on angle measurements. Hence, the importance of the same shape but different sizes and the role of the sides compared to the angle measures that the prospective teachers first discussed were major themes that they all commonly had within their essential features, and influenced many of their images and discussions as they progressed throughout the LPS.

These ideas, same shape but different sizes and the role of the sides compared to angles, are two important themes in the prospective teachers' essential features and begin to illuminate their substantive structures of similarity. The prospective teachers' knowledge of substantive and syntactic structures about similarity is largely influenced by a combination of their propositional and case knowledge of ratio and proportions, along with enlargements and reductions. These structures are influenced by how the prospective teachers have learned similarity and the importance they place on these features within similarity. Their substantive structures seem to connect ratio and proportion to similarity with the idea that these two topics, at first, are more important to them than angle measurement. Through their experiences they have come to see that proportional corresponding side lengths are important in proving two triangles similar. The syntactic structures within the essential features are how the prospective teachers use examples of ratio and proportion to *prove* two triangles similar.

This type of fragmented understanding in prospective teachers substantive and syntactic structures has been discussed by other research studies concerned with prospective elementary teachers (Ball, 1988; Ball, 1990; Cooney et al., 1998; Ma, 1999; Simon & Blume, 1994; Van Dooren et al., 2002). In looking at prospective elementary teachers' understanding of division Ball (1988) noted, "prospective [elementary] teachers' focus on the surface differences… suggests their understanding is comprised of remembering the rules for specific cases, not a web of interconnected ideas" (p. 22). It seems evident that the prospective secondary teachers in the LPS, at first, may not have "a web of interconnected ideas" when it comes to the concept of similarity. They seem to focus on a generalization that similar triangles are the same shape but different sizes and on the aspect that similar triangles have proportional sides. However, as they progressed in the LPS, they began to construct images and connections between these initial ideas.

Different Representations

Even (1990) discussed the importance of using different representations when discussing a topic. Different representations give more insight into the topic and allow for a deeper understanding of a concept. By using more than one representation, one can abstract and grasp properties, concepts, and the common qualities of a topic.

The representations that the prospective teachers used in explaining similarity involved looking at combinations of triangles. In the student activity part of their lessons, the prospective teachers devised activities that used triangles and asked to categorize them or deduce why they are considered similar. Each of the prospective teachers used different approaches to their discussion of similar triangles, but the representations that they used were focused on the examples of right triangles, isosceles triangles, and whether or not they are similar.

This is important because if the prospective teachers use only right and isosceles triangles, their students may not be able to grasp properties, concepts, and common qualities of similar triangles. It may be possible for students to construct images as Rose had done in her initial interview:

Int.: Yeah, **[has a piece of paper off camera]** I tried to draw a couple of isosceles triangles there; do you see those as being similar?

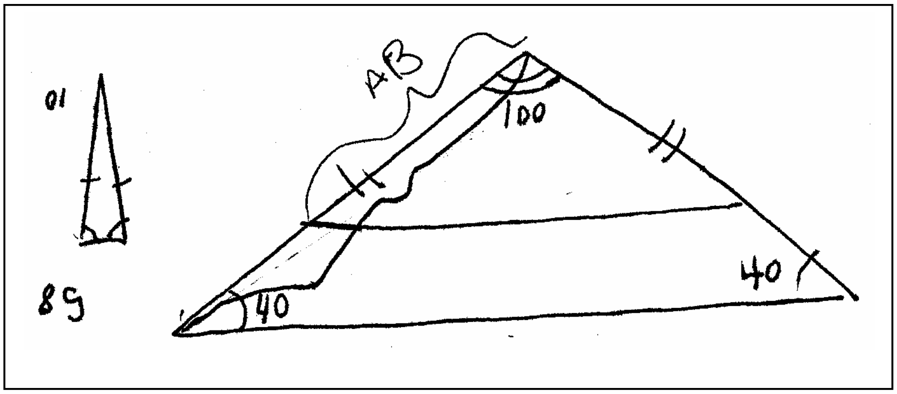


Figure 11. Interviewer draws two isosceles triangles.

Rose: Yeah, **[pointing at the interviewers paper, off camera]** even though their angle was different they are based on that same concept that both sides are the same… that… I don't know. I would still think that they are similar because they were isosceles.

This example clearly illustrates that it may be possible for students to confuse aspects of mathematical similarity with the everyday usage of the word similar, like Rose did. Since angles and sides are used to classify triangles, using examples that are concentrated on particular types of triangles can cause students to connect these triangles, like Rose, with the concept of similarity. Therefore, it is important that the prospective teachers use more than one or two types of triangles in their examples, so that their students can see the properties of similarity.

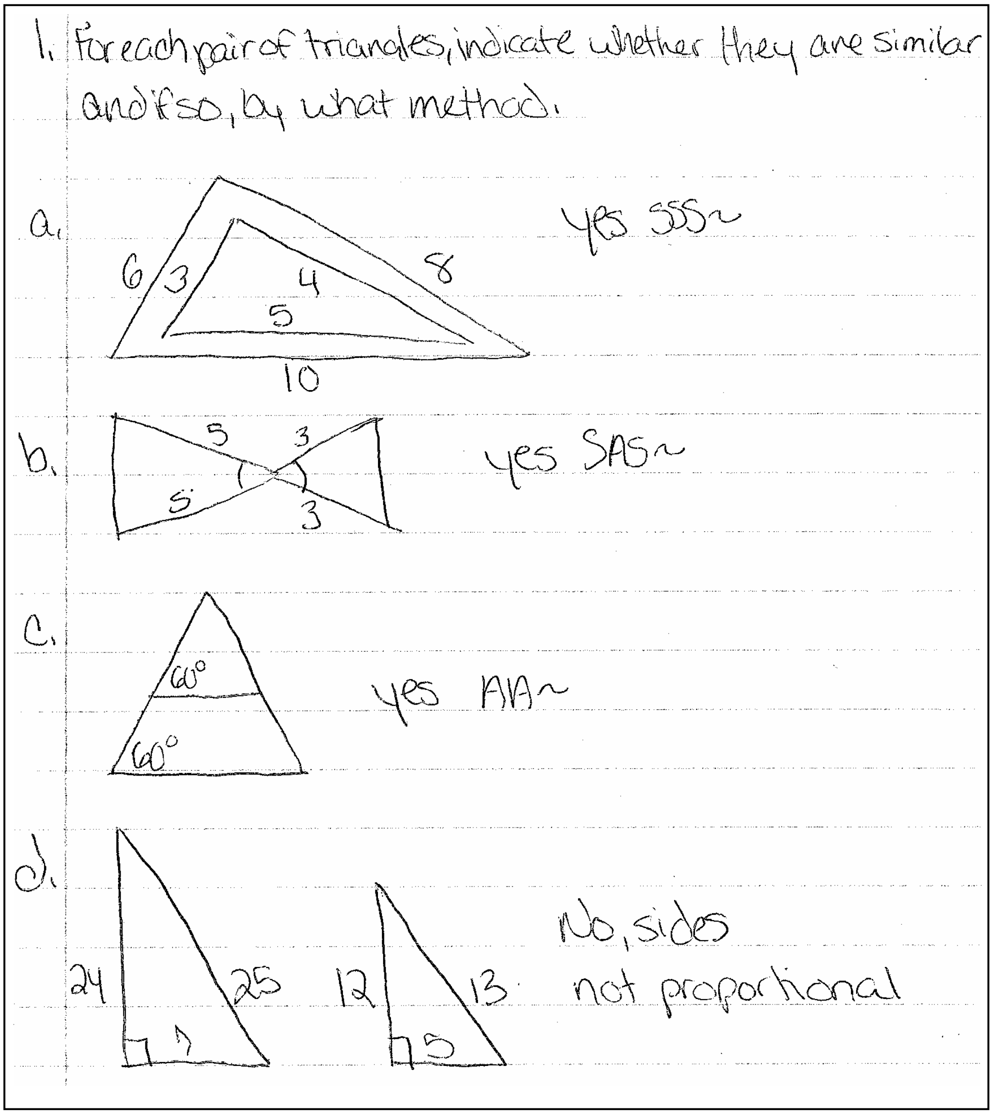


Figure 12. Problems that Rose used to help reinforce similarity.

The representations chosen by Rose, in the figure above, are designed so that her students can reinforce their ideas of what makes two triangles similar and the similarity postulates. All of her examples on this page reflect many of the right triangle and isosceles triangle representations used by the other prospective teachers. Over half of the representations used by the five prospective teachers involved either right triangles or isosceles triangles.

With the focus being on these two types of triangles, the prospective teachers are using approaches that they feel comfortable with, but may not be the "best" for their students.

The substantive structures of what makes two triangles similar, and examples to show these structures were the focus of the representations that the prospective teachers used. Many of these examples were representations of proportional sides that heavily influenced the prospective teachers' discussions of similarity. Also, many of these examples were either isosceles or right triangles, which may lead to problems in their students understanding of similarity. The prospective teachers seemed to use their case knowledge of how to represent and approach the topics. Rose, Ava, and Anne mentioned either the influences of other teachers or opportunities for teaching on the representations and approaches that they used.

Int.: Yes. **[talking about Rose's triangle that she drew]** So the smaller triangle is similar to the larger triangle in that …

Rose: In that they are both isosceles and that the parts are similar. That's what I would draw.

Int.: Any reason why you drew isosceles triangles?

Rose: No, I just… **[shaking head]** No, you could have drawn a scalene one, but that is the one I grew up with **[laughs]**.

Int.: Really, so you think that is pretty close to what your teacher drew.

Rose: Yeah. Most teachers use isosceles triangles when doing anything.

These influences are similar to those mentioned by researchers (Ball, 1988; Borko et al., 1992; Cooney et al., 1998). It is evident in Rose's case that she is influenced by examples that her former geometry teacher may have used in class. These experiences with mathematics influence prospective teachers long before they enter the formal world of mathematics education (Cooney et al., 1988). These experiences heavily influence students in the secondary school mathematics courses and are what teacher educators having to deal with as prospective teachers enter teacher education programs (Zeichner & Tabachnick, 1981).

Alternative Ways of Approaching

Even (1990) stressed the need for alternative ways of approaching a concept. It is very helpful to see a concept in various forms: in different divisions of mathematics and other disciplines, or everyday life. Using their substantive and syntactic knowledge, many of the prospective teachers chose an approach that has influenced their SMK and pedagogical content knowledge. Many of the prospective teachers devised a student activity that would help their students to see patterns and relationships that they could then focus on and come up with what it meant for two triangles to be similar. In the analyzing the prospective teachers' approaches, two themes were noticed: *The use of one example to make generalizations* and *the ability to anticipate students' responses*.

One Example in Order to Make Generalizations:

Alice, Rose and Anne, in their first individual lesson, used one example to show important concepts in similarity. In the example below from Anne's individual interview, she devised an introductory activity for the concept of similarity. It is evident that she felt as if one example was enough for students to abstract the properties of similarity.

Anne: My other triangle, I didn't do too well, but I told them to get into groups and measure the angles and sides and to fill in the answers and see what patterns they notice. They'll label the M, L, N, the Q, R, and S,

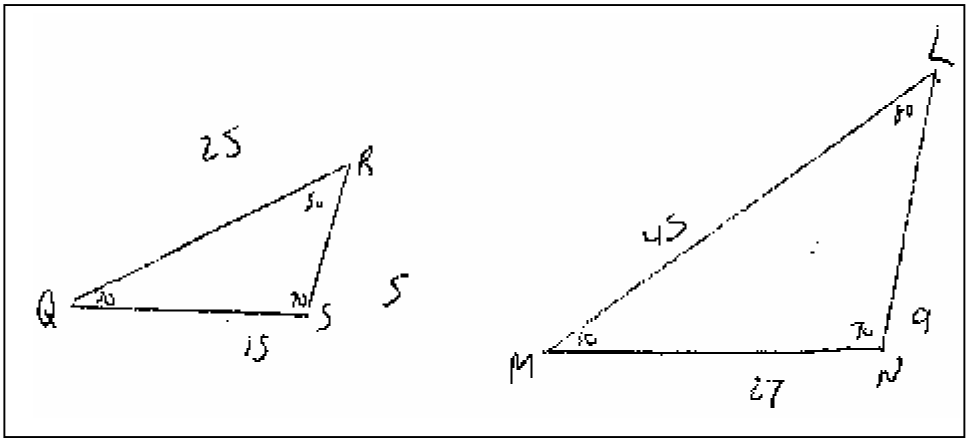


Figure 13. Anne's group work using measurement.

so they just have to say that the triangles are similar. And after they finish all of that, I am going to ask for volunteers of what they found or what they deduced. Then we will talk about did they notice that the angles match. I am sure that somebody will get that and that all the sides reduce down to 9 over 5.

In her activity, Anne assumed that by measuring the sides and angles of the triangles that it would be enough for her students to realize the properties of similar triangles. She predicted by doing this activity that volunteers would be able to "notice that the angles match" and "the sides reduce to 9 over 5." This type of thinking exemplifies prospective teachers need to be exposed to different activities that bring the activities of teaching into context.

Anticipating Student Responses:

Mary and Ava used only one type of activity to introduce the concept of similarity in their lesson outlines and predicted their students' responses during the activities. However, Mary and Ava used more than one example in their introductory activity.

Mary: Oh, okay. I thought I would start with this triangle **[holds up transparency with triangle]**. It has equal sides… **[holds up red plastic triangle]** I just used this one… and I put it on the overhead, and I wrote down how many centimeters there are.

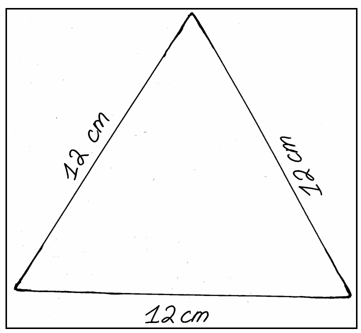


Figure 14. Mary's first transparency of an equilateral triangle.

Then 1 to 3 students, depending on the number, will come up and measure the sides on the overhead projector, to find out if they are the same sides – if that's true - and then, of course they will come up with that it is longer, but then I wanted to see what they conclude from that. Then I was thinking well they might conclude that you add to each side in order to make it bigger or longer, so I made another triangle **[points to transparency with triangle]** where the sides aren't equally, and do just the same thing so they see that it is not through addition that makes it similar.

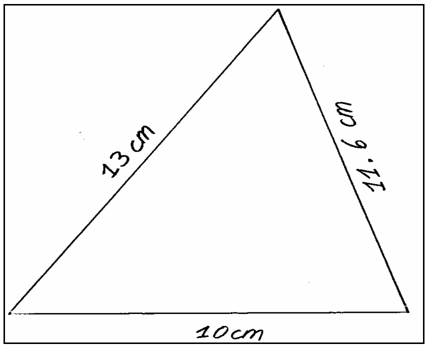


Figure 15. Mary's second triangle used to de-emphasize additive lengths.

Then I'll take this off after they make their conclusion and then I will put this on, and then do the same thing where they measure again the sides, and then see is it again that you add a certain amount to each side. I guess that then they will realize that conclusion is wrong and just kinda see if they can come up with a conclusion.

In Mary's activity, she felt that it was necessary to use more than one example to establish a pattern and allow her students to abstract the qualities of the concept of similarity. Mary also anticipated students' additive thinking and included another example to try and avoid this type of thinking. It is interesting to note that she thought that one counterexample would be enough to show her students the correct way of reasoning (multiplicative) and she thought it was powerful enough to persuade her students from thinking about proportions incorrectly (additively). This view is compatible with Simon et al.'s (2000) notion of perception-based perspective.

In Ava's activity, she used several types of triangles in her activity to help her students in defining similarity.

> Ava: I gave them a packet of 12 triangles that were sorted by color and different sizes, and different angle measurements, but there are some of each that are represented, and so that each group must find several ways to categorize that triangles and they share ideas with the rest of the class and write them up and they have rulers and compasses and they can measure different things, if they want to.

In her previous teaching experiences, Ava was exposed to the multiple learning styles. The activity that she devised was directed towards multiple learning styles including visual, hands-on, and collaborative learning. She felt that her activity would allow her to teach students who were better with visual manipulatives, with hands-on activities, and with learners who worked best in groups and could share their ideas.

The idea that the prospective teachers established in their lessons is largely through their use of prototypical examples to establish patterns. This indicates that the prospective teachers are trying to "create" new mathematics by their students' deductions (Battista & Clements, 1995). To allow students to derive their own definitions and meanings of the concept of similarity addresses their propositional knowledge and many of the ideas explored in their university education courses. The syntactic structures of the prospective teachers seem to be rooted in using examples to prove or disprove an idea. Also, their use of examples to establish the meaning of similarity addresses their need for stronger substantive structures on this topic in order to further their knowledge of similarity.

The prospective teachers' perceptions of how their students would react to the activities they designed were important in their planning of what to teach (Ball & Wilcox, 1989; Berenson et al, 2001; Simon, 1995; Simon et al., 2000; Thompson & Bush, 2003). Ball and Wilcox (1989) stated "teaching involves weaving together various kinds of knowledge and skill with considerations of context" (p. 21). Thompson and Bush (2003) suggested that images like Mary's about students connecting an additive relationship with proportional reasoning are important because only 52 percent of eighth graders are able to use multiplicative reasoning as opposed to additive reasoning in solving proportions.

The Strength of Similarity

Even (1990) saw the strength of a concept as the new opportunities that can be explored or addressed by it. Also, the strength of any concept is seen as having important sub-topics and sub-concepts. In the beginning, Mary was the only prospective teacher that went beyond using ratios and proportions, in discussing the sub-topics or sub-concepts of similarity. In the explanation of her first individual lesson plan, Mary related similarity to models and scaling, and indirect measurement to find the height of objects:

> "Then I would ask why is this idea important or what we need this for with the ratios? And then I was just saying a lot of models are being used like in architecture or if you build an airplane, you first have your little model and just go from there."

During the group interview Mary emphasized the importance of using similarity for indirect measurement to find the height of objects that cannot be easily measured. It was during this moment that Mary impressed this idea on the other group members, and many of them incorporated this idea in their lesson plans. For example, Ava in her final lesson plan devised a problem to find the height of a lighthouse (see Figure 16).

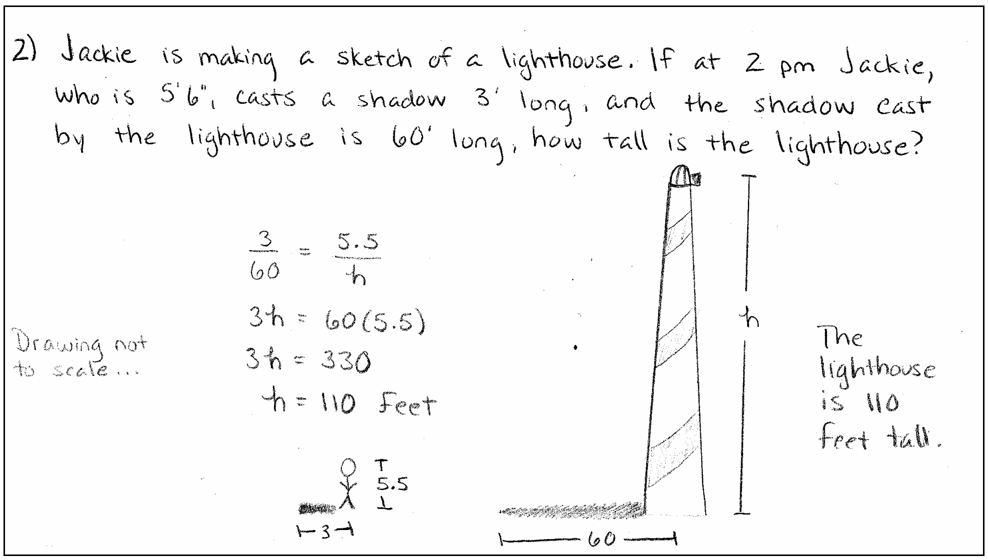


Figure 16. Ava's indirect measurement problem from her individual lesson.

In this example, Ava used a "real-life" example of similarity in setting up and solving a proportion to help in finding the height of the lighthouse. None of the prospective teachers had initially included this idea in their lesson plans, and since all of them used this idea in their final lesson plans, it is obvious that they had been exposed to this important application of similarity. The knowledge that the prospective teachers seem to access is their propositional and case knowledge for finding an application of similarity.

For the most part, the prospective teachers used their propositional knowledge of ratios and proportions in finding the missing sides of triangles. However, when Mary emphasized her case knowledge about finding the length or height of something real, the group used cases that they were familiar with and incorporated this approach in their lessons to emphasize the importance of similarity. This analysis, again, suggests that the prospective teachers were limited in their substantive and syntactic knowledge of similarity. At first, many of the prospective teachers made connections that were procedural and involved two explicit triangles with sides and angles, and at many times the same orientation. As for a reason why many of the prospective teachers could not initially see the strength of similarity, Borko, Eisenhart, Brown, Underhill, Jones, and Agard (1992) suggested that prospective teachers take courses, during their university study, that do not stress conceptual topics or treat topics at high levels of abstraction encouraging rigorous proof.

Basic Repertoire

Even (1990) described the basic repertoire that teachers have as easily accessible powerful examples that illustrate important properties. These examples should not be memorized or used without a basic understanding of the concept. Having a mature basic repertoire allows insight and understanding of general and more complicated knowledge. A mature basic repertoire can be used as a reference to monitor ways of thinking and acting. Two themes were evident in the prospective teachers' basic repertoire: *Daily language versus mathematical language* and *procedural aspects to make generalizations*.

Daily Language versus Mathematical Language

The prospective teachers' basic repertoire concentrated heavily on the daily language use of the term similar as a frame of reference for their students.

> Ava: I think to always start out on something that they feel more comfortable with; they know that a plant…a tree, bush and plant might be similar, but they are still different in ways. Then why are they similar and not different, you know, those characteristics. And then kind of broaden that view of similarity and then bring it back down too... but in a mathematical sense similarity means this definition.

The prospective teachers when either trying to discuss their initial ideas of similarity or when they were asked to again explain similarity to someone who did not understand their lesson used this frame of reference. They saw that it was important to make use of students' existing knowledge of the word similar and its everyday use to develop the mathematical definition of similarity. However, for some of the prospective teachers involved in the study, it was this use of the everyday meaning of similar that clouded their initial understanding of mathematical similarity, when asked what they recalled about similar triangles.

> Int.: Do you associate area with similarity?
>
> Rose: If you are finding out the area of two things…if I am doing anything with similarity, I'll find some base concept like area, or perimeter or something. Say well okay, this is similar to this maybe by half or by twice or whatever, that's how I do anything that is similar…

It was during her individual interview that Rose associated the use of the everyday meaning of the word similar with mathematical similarity. This association, at first, seemed to impede her understanding of mathematical similarity. She considered two triangles to be similar if they had anything in common. Two triangles were similar if they had the same area, perimeter, length of base, or were the same type of triangle. This connection between similar and mathematical similarity dominated the first portion of Rose's individual interview and it was not until she brought forth her understanding of congruent triangles, that her knowledge of mathematically similar triangles emerged.

Procedural Aspects to Make Generalizations

The second theme that became evident in the prospective teachers' basic repertoire was the use of the Side-Side-Side, Side-Angle-Side, and Angle-Angle similarity postulates. Ava first discussed these ideas in her individual interview as a way for students to check whether or not two triangles were similar. During the group planning stage of the LPS, Ava brought up these "short-cuts," as she called them, to the group. The "short-cuts" became a large part of the group's presentation (see Figure 17) and their final individual lesson plans.

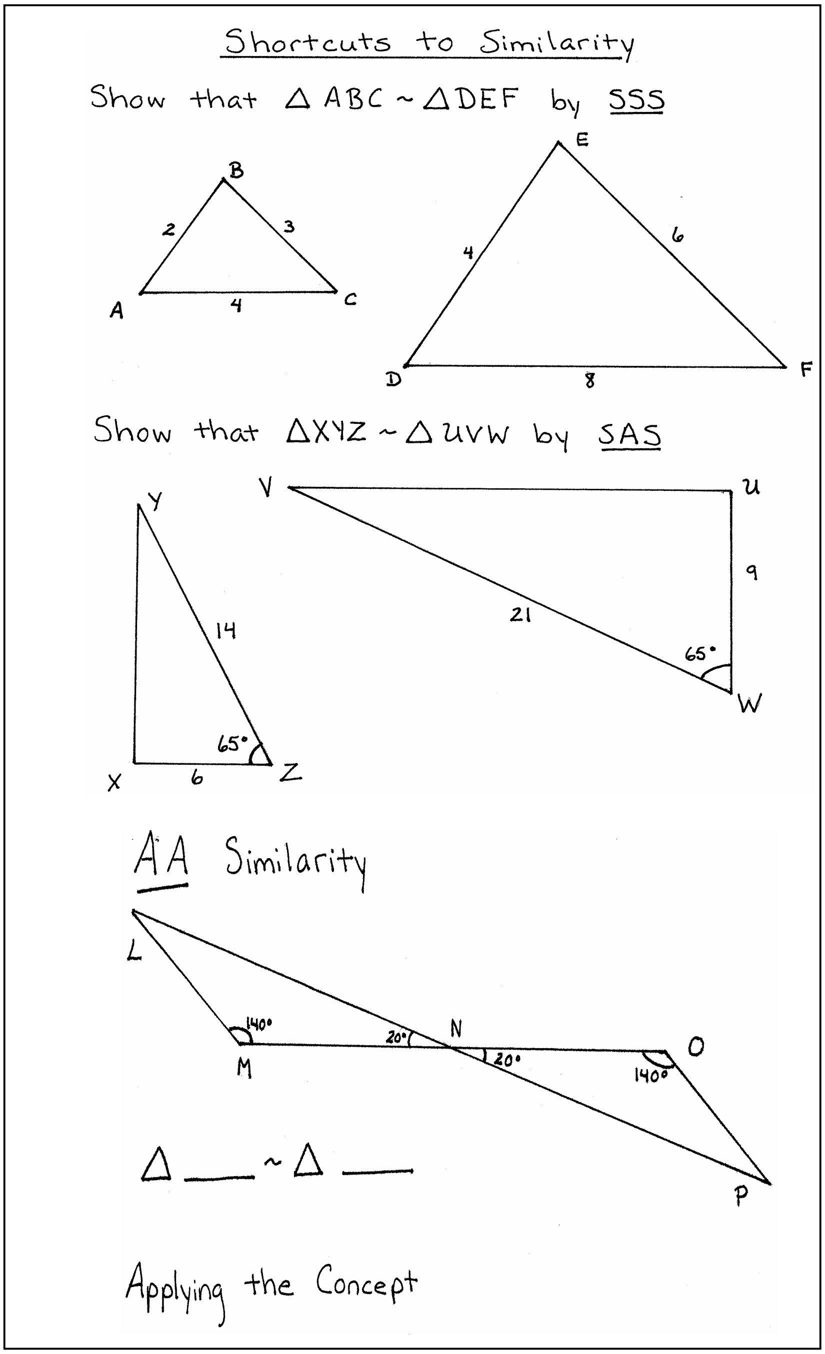


Figure 17. Ava's short cuts discussed during group presentation.

Like the prospective teachers' initial ideas of side lengths compared to angle measurements, the similarity postulates became a procedure for checking whether or not two triangles were similar. The prospective teachers used the similarity postulates as a check-off list for finding similar triangles. In their development of the similarity postulates, the prospective teachers used examples of how they worked, as opposed to why they work. In the lessons they developed for their presentation and the final individual lessons, the prospective teachers focused on the procedural aspects of the similarity postulates without explaining how they worked. Even (1993) stressed similar results when looking at prospective teachers' knowledge of functions, they seemed to know *how* something works without knowing *why* something works.

Knowledge and Understanding of Similarity

Even (1990) stated that knowledge and understanding of a concept involves both procedural and conceptual knowledge and the relationships between them. If someone has knowledge and understanding of a concept then they have a network of concepts and relationships at their disposal. Also, a good intuitive feel for mathematics - to solve problems, generate answers, and understand what they are doing - is involved in the understanding of a concept.

Many of the prospective teachers sought to teach for understanding through activities and examples that helped their students develop their own understanding of the concept of similarity for themselves. When recalling what they remembered about similarity, most of the prospective teachers' responses were immature and of a procedural nature. However, as they planned the lessons their ideas became more concrete and confident. In planning their lessons, it seems as if they were searching for more than definitions and examples, but connections between ratio, proportion, and what similarity actually means. Teaching for procedural knowledge was still a large part of some of their lessons, but many did try incorporating a conceptual aspect.

In their lessons Ava, Alice, Anne, and Mary had intended to use conceptually centered activities for their students to discover the properties of similarity. Ava had a set of 12 triangles in which students categorized them and hopefully came up with what similar triangles meant mathematically. Alice began her lesson with a series of geometric figures in which her students knew they were similar and expected them to find a pattern of properties that made them so. Anne, in her lesson, allowed students to measure angles and sides of two similar triangles to explain why they are considered similar. In her lesson, Mary used an overhead projector and projected the picture of a triangle on the overhead screen. Mary then expected her students to measure and deduce why the triangles on the overhead transparency were considered similar to those on the overhead screen. All of these activities can be considered conceptually oriented, because they tried to focus on “the relationships and interconnections of ideas that explain and give meaning to mathematical procedures” (Eisenhart et al., 1993. p.9). Although limited by their lack of substantive and syntactic structures, the prospective teachers developed activities that made connections between the side lengths and angle measurements in defining similarity. Because of this lack in their SMK, the activities the prospective teachers’ constructed were actually more procedural in nature.

The procedural knowledge that the prospective teachers incorporated within their lessons was other examples of whether two triangles were similar. Also, many prospective teachers did problems that used the proportional sides of two similar triangles to do indirect measurement.

Rose: The last page that I finally came to is that I was trying to show them an example of proportions? Yeah, and I said okay, you have two similar triangles here.

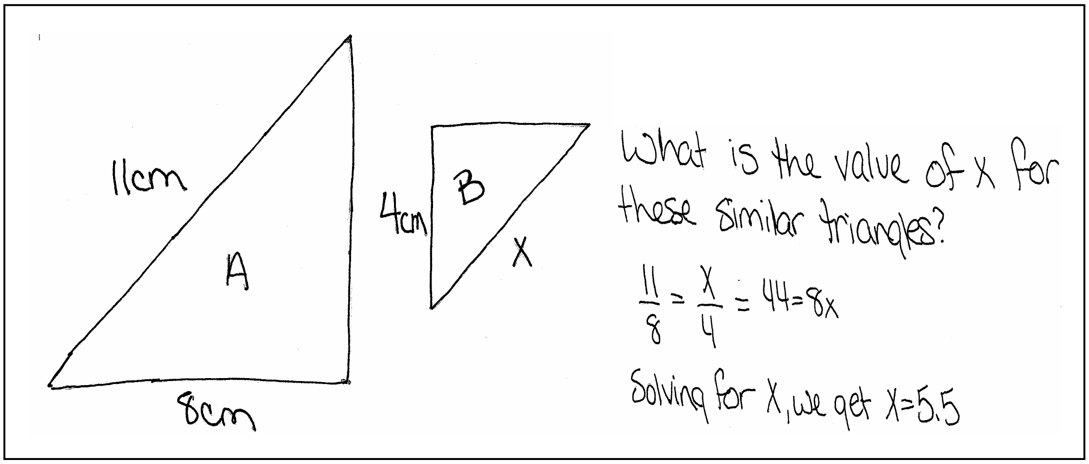


Figure 18. Rose's indirect measurement in her initial interview.

I mixed them so they wouldn't have to turn them around in their head to figure it out. Okay that hypotenuse is 11 to 8, so how would you find out that hypotenuse based on that one and solving the… using that ratios.

In trying to stress the importance of corresponding proportional sides, Rose used two triangles where one was oriented differently than the other. During her initial interview, this was the only time that she anticipated that her students might have difficulties with corresponding sides in similarity. Again, this anticipation was limited to her students incorrectly setting up a procedure that she had previously explained. This illustrates Rose's limited knowledge and understanding of similarity along with her images of teaching, at the beginning of the LPS.

A major concern that goes beyond the prospective teachers' knowledge of similarity is their knowledge and understanding of mathematics and how it affects the representations and examples that they used. The prospective teachers' limited knowledge of substantive structures in their network of concepts and the relationships caused them to make mathematical mistakes. The concern is not just that some examples were mathematically incorrect, but they were also visually confusing. Anne's example of getting her students to fill-in the measurements of two similar triangles contradicts the Triangle Inequality Theorem (see Figure 19).

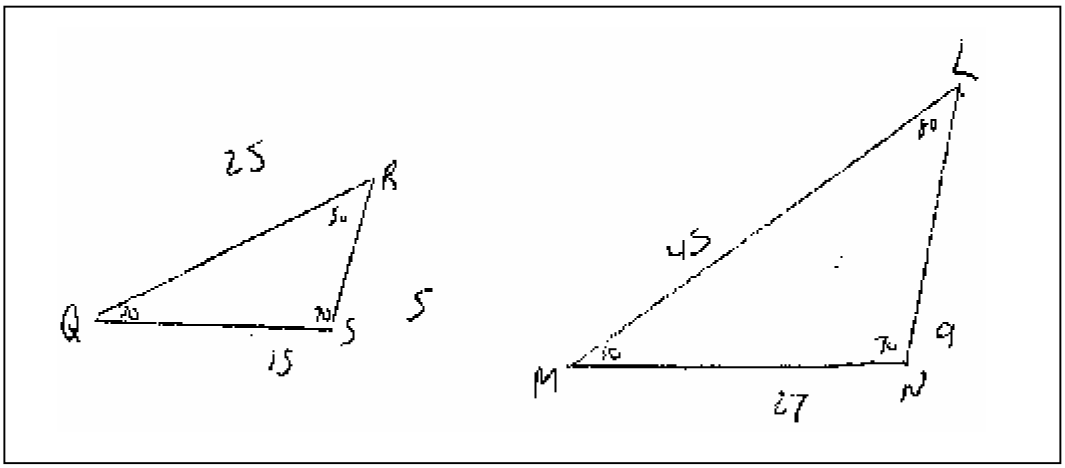


Figure 19. Anne's group work using measurement.

The Triangle Inequality theorem states, "The sum of the lengths of any two sides of a triangle is greater than the length of the third side" (Boyd, et. al., 1998). In both triangles the sums of the lengths of the smaller sides are less than the length of the third side.

An open ended problem that Rose planned on asking her students did not contain enough information for the students to give one correct answer. Rose, in an endeavor to include more open-ended responses, included the following problem in her final lesson plan:

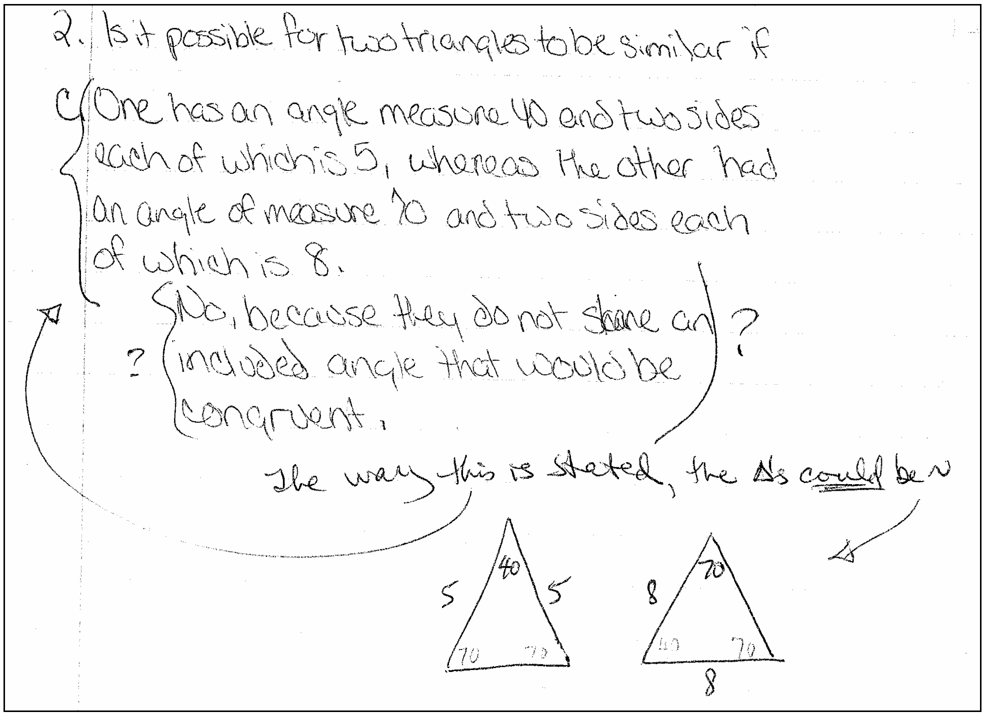


Figure 20. Rose's open-ended problem with instructor feedback.

In order to get the answer that she had hoped, Rose needed to mention that the vertex angle in the isosceles triangles were 40 and 70. The instructor feedback points to the fact that her wording was a little unclear.

In her first attempt to illustrate the importance of the relationship between angle measurement and proportional sides in mathematical similarity, Alice was unaware that the representation that she used to show that two triangles were not similar is a mathematical paradox.

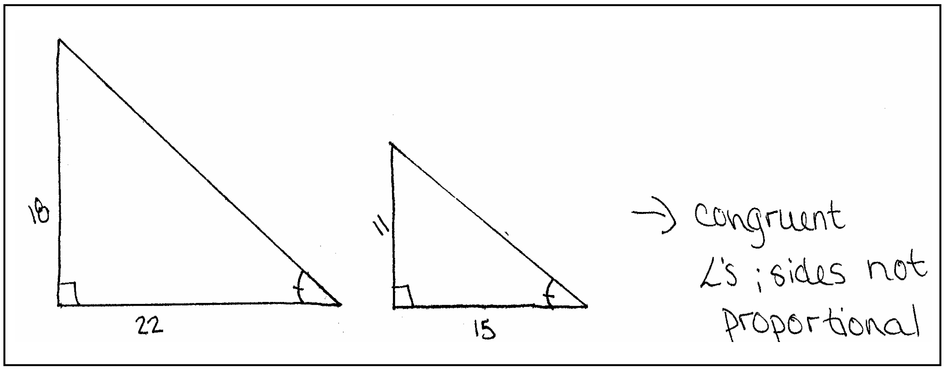


Figure 21. Alice's mathematical paradox.

The fallacy in her argument about the triangles is that she has shown two triangles that are similar by the Angle-Angle Similarity Postulate. The Angle-Angle Similarity Postulate states, "If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar" (Boyd, et. al., 1998).

Lastly, the individual lesson plans that many of the prospective teachers turned in contained images that could visually confuse their students. In some instances the side lengths given for the triangle were those of right triangles and the figures drawn were not recognizable as right triangles. The examples below support the theme brought up earlier in the prospective teachers essential features that they were focused on the ideas of same shape but different sizes.

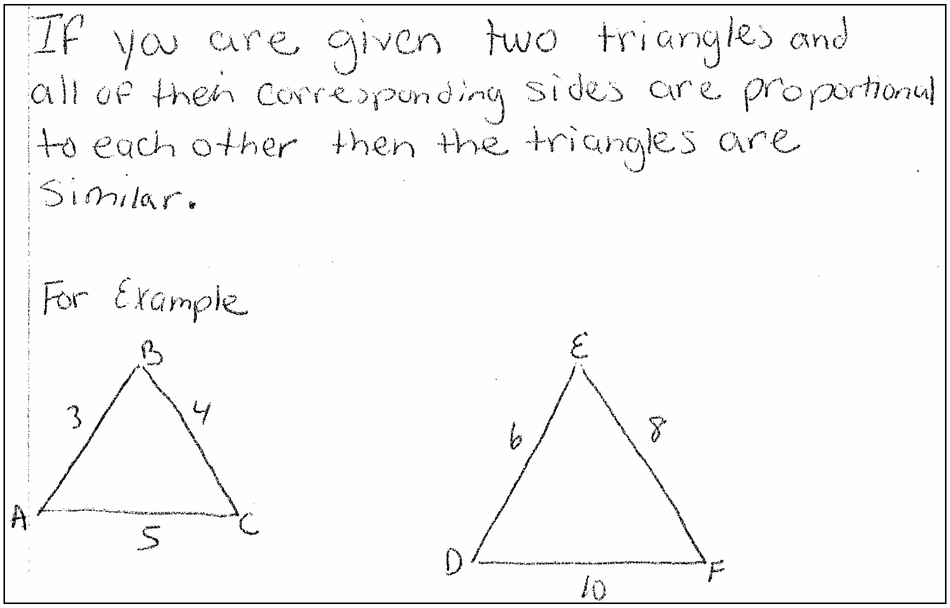


Figure 22. Anne’s side lengths and visual picture are confusing.

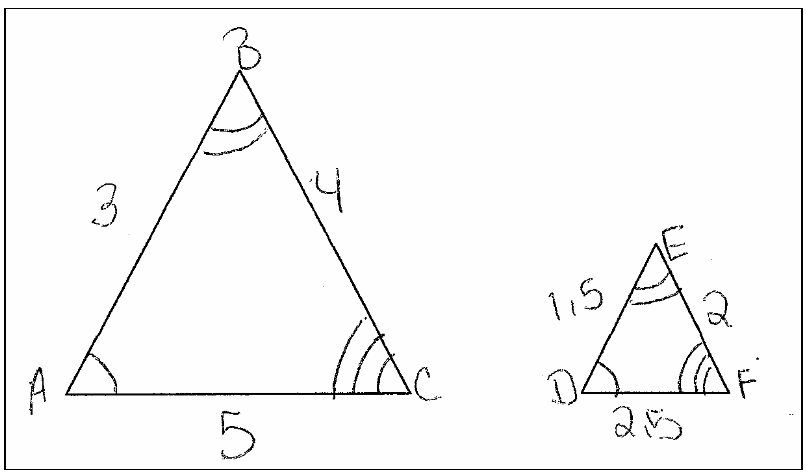


Figure 23. Rose’s side lengths and visual picture are confusing.

Since similarity has been shown to be a complex concept that has implications throughout mathematics, it is important to consider prospective teachers’ knowledge of mathematics. Prospective teachers’ content knowledge, substantive knowledge, and syntactic knowledge of other concepts within mathematics have an effect on the representations they use to illustrate important ideas in the teaching of similarity. The examples and illustrations used by prospective teachers must be mathematically and visually accurate to support their students' understanding.

Knowledge *about* Mathematics

Even (1990) described knowledge ***about*** mathematics as the knowledge that gives inquiry, truth, and the construction and use of knowledge and understanding within mathematics. Knowledge about mathematics is seen to include the ways, means, and processes by which truths are established as well as different ideas (Ball, 1991; Even, 1990; Shulman, 1986).

The prospective teachers' ideas about establishing truth in geometry were vague. In some of their lesson plans, the prospective teachers did allow students to harvest their own knowledge, but as to how this was handled afterwards varied. Usually after the prospective teachers expected the students to derive the meaning of similarity, they told the students the definition. The instances in lesson plans where definitions came before examples and then more definitions and examples were limited. However, the teacher still seemed to be the authority in establishing truth in the classroom.

Rose: And then I will put the way to define if two triangles are similar is to put them in proportion to each other, **[points to paper]** which I think I got that one wrong because I don't think its 5.8, but still. And so I did a nice little example for that

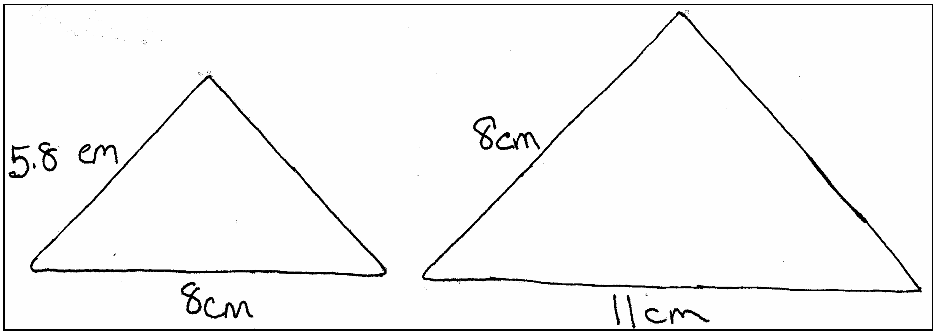


Figure 24. Rose’s example of proportional sides.

> saying that for every 5.8 is to 8 for the left side and 8 to 11 for the base of the triangles.

There were instances in which the prospective teachers sought out the expertise of knowledge from the textbook. In this case, the textbook in any classroom would be its prized resource. When faced with a conflict in whether or not all right triangles were similar, Rose used this resource as her guide:

> Okay you would say that 1 is to *x*, which is equal to 10 to 20… Okay that would be wrong. Hmm.

$$\frac{1}{x} = \frac{10}{20}$$

$$20 = 10x$$
$$x = 2$$

Figure 25. Rose’s contradiction that all right triangles are similar.

> **[Grabs the textbook and starts turning pages]** The way that I was reading it in the book… It said that they would still have similar, that they would still be similar, no matter… **[still turning pages]** If this book had answers in the back it would be more helpful. **[stops turning pages and has book open]** Like right here is this what you are talking about? **[points at book and turns it to interviewer]** It's similar because they are both right angles, but they don't have any kind of answers in the back of the book.

The instances where the prospective teachers told the definitions and correct answers, reflect the idea that, "teachers are inclined to tell and show students how to do mathematics instead of creating activities that help students construct understanding of content" (Ball & Mosenthal, 1990, p. 3). Also, Even (1993) suggested that the instances of relying on the textbook, when faced with contradictions of their own understanding and situations of unfamiliarity, strongly reflect the prospective teachers' limited SMK.

Summary of the Nature of Subject Matter Knowledge

The subject matter preparation of teachers has as a very important role in the development of prospective teachers. Topics such as similarity, which are powerful and useful, have many sub-topics and sub-concepts that need to be explored by prospective teachers. In the previous analysis it seems that the prospective teachers in question had a grasp of ratio and proportion, and some good ideas about teaching students. However, they seemed to lack exactness in their visual representations and strength in the concept of similarity for any topic other than indirect measurement. The knowledge base that the prospective teachers used varied from their propositional knowledge, in which the norms that they have experienced guided them, to instances where their case knowledge as students and working as teachers yielded different practices. Within these interviews it is easy to see that the prospective teachers began to change their ideas about similarity, thus causing them to rely more heavily on their SMK to explain examples, definitions, student responses, and activities.

Growth in the Subject Matter Knowledge of Similarity

The growth in the SMK of similarity during the planning of the lesson is analyzed using the Berenson, Cavey, Clark and Staley (2001) adaptation to the Pirie-Kieren (1994) model of mathematical understanding to teacher preparation, along with Even (1990) and the features of collecting (Pirie & Martin, 2000) and thickening (Martin, 1999). The teacher preparation model consists of five levels of understanding and the feature of folding back that were adapted from the Pirie-Kieren model of mathematical understanding: primitive knowledge, making an image, having an image, noticing properties, and formalizing, imitate the first five levels of the Pirie-Kieren model, primitive knowing, image making, image having, property noticing, and formalising. The feature of folding back in the Pirie-Kieren model is seen as essential and was absorbed into the teacher preparation model. The features of collecting (Pirie & Martin, 2000) and thickening (Martin, 1999) that evolved after the publication of the Pirie-Kieren model will be discussed as they pertain to teacher preparation.

The teacher preparation model is a framework for studying prospective teachers' understanding of *what* and *how* to teach high school mathematics. Using the teacher preparation model in conjunction with Even's (1990) aspects of SMK, helps to define the *what* and *how* of teaching. The *what* of teaching includes the essential features of the concept, representations, strength of the concept, and the knowledge and understanding of the concept to be taught. The *how* of teaching incorporates the ways of approaching the topic, the basic repertoire in teaching the topic, and the prospective teachers' knowledge about mathematics. It is within the discussions, planning, and presentations of a lesson to introduce the topic of similarity, that the primitive knowledge and growth of prospective teachers' SMK of similarity can be explored.

The first level of the teacher preparation model is called *primitive knowledge*. The instructor or observer sees primitive knowledge as something that is assumed that the student or participant knows mathematically, before coming to the new task. Berenson, Cavey, Clark, and Staley (2001) list several aspects of primitive knowledge that are assessed during the prospective teachers' interviews and activities, including: knowledge of college mathematics, knowledge of physics and chemistry, knowledge of school mathematics, and knowledge of teaching strategies. Pirie and Kieren (1994) pointed out that primitive knowing does not imply a low level of mathematics, but a starting place for growth. *Making an image* is when participants or students use their primitive knowledge in new ways. When using their knowledge in new ways, participants make distinctions in what they already know and apply it to a new situation that builds directly on their previous knowing (Piaget, 2001). During the LPS, the images the prospective teachers make of *what* and *how* to teach reflect their primitive knowledge of mathematics, along with various other aspects of their SMK backgrounds. While *having an image*, a new image is formed as a result of the activities in making an image. If participants are having an image, they can bring themselves meaning of their image making activities.

Once they have brought themselves meaning, they can mentally manipulate ideas and consider different aspects of the task without having to repeat the activities that brought forth the new meaning. When *noticing properties* of a task, the prospective teachers' can manipulate and combine images that they made about their lessons. During the LPS, noticing properties becomes evident by the consistent images of topics and teaching strategies that the prospective teachers use while discussing their lessons. While *formalizing*, prospective teachers can identify common features, patterns, algorithms, or formulas of the image they made while noticing properties. Within the LPS, *formalizing* is seen as the knowledge of common features and patterns that the prospective teachers would consistently incorporate in their lessons on *what* and *how* to teach similarity. Pirie and Kieren (1994) stated, "anyone formalising, would be ready for, and capable of enunciating and appreciating a formal mathematical definition or algorithm" (p. 171).

While one can never establish the extent of the prospective teachers' primitive knowledge, one however can make inferences based on aspects from the learning activity. In the case of the LPS, participants' interviews, their lesson plans, and the themes that were evident throughout the study were instrumental in helping the researcher come to conclusions about changes in their SMK of similarity. The images of *what* and *how* to teach reflect the primitive knowledge and initial images inferences made by the prospective teachers. These images build on formalized knowledge of material related to similarity from mathematics classes and that it was a concept that either they understood or as one participant said, they needed to "relearn" all the geometry that they need to teach. The images made of *what* and *how* to teach will have an effect on the determination of the outer levels of the teacher preparation model.

The notion of *folding back* is essential to the growth in understanding and reveals the non-linear and recursive nature of coming to understand mathematics (Berenson et al., 2001; Pirie & Kieren, 1994). Folding back is seen as necessary to build an in-depth understanding and occurs when a learner is faced with a new issue or problem that is not immediately solvable with his or her current knowledge. Folding back is a recursive process in which the participant returns to an inner level of understanding to extend her current understanding. Martin (1999) suggests that the effectiveness of the activity of folding back is dependent on the learner and learning environment.

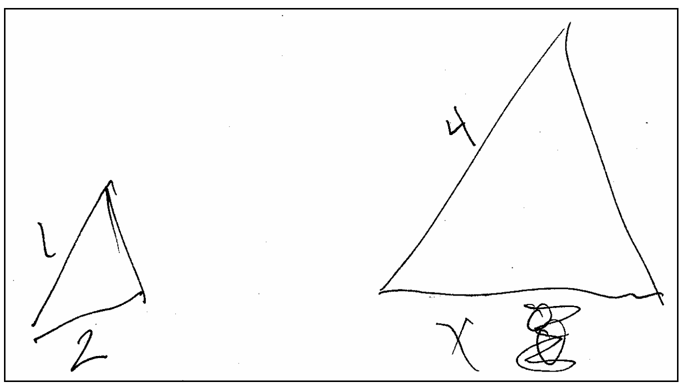
The process of *folding back* involves more than just borrowing from an inner level to construct understanding of a concept on an outer level. Two aspects of folding back are called *thickening* (Martin, 1999) and *collecting* (Pirie & Martin, 2000). In the process of folding back, if students become aware of their understanding and realize that an inner level activity cannot be identical to what was performed previously, they build a *thicker* understanding at the inner level to support and extend their understanding at the outer level in which they return (Martin, 1999). When folding back, *collecting* occurs when students are aware that their understanding is lacking and they need to recollect some inner understanding and use it at an outer level to get a better understanding of the concept in question (Pirie & Martin, 2000).

The conceptual framework incorporating Berenson et al., (2001), Even (1990), Martin (1999), and Pirie and Martin (2000) will be used to analyze each participant's growth of SMK in similarity. The growth will be noted from the beginning of their individual interview through their final individual lesson that they turned in at the end of the LPS cycle.

<u>The Case of Alice.</u>

Alice's images of *what* to teach were focused on the essential features of the proportional sides and congruent angles of similar triangles. Although both concepts were discussed, she put more emphasis on the proportional sides of similar triangles in her first interview. In the example below, she answers the interviewers (Int.) question about what she recalled from her geometry class about similarity:

Alice: **[begins to point at the table]**. Similar triangles **[starts making shapes with her fingers]** their sides are… proportional to each other. **[Begins to draw figure]** Umm… let's see, you have a little one and a big one



<u>Figure 26.</u> <u>Alice's initial conversation about similar triangles.</u>

say that this side is 1 and this side is 4 and this side is 2, and this side would be 8. I think that's right, yeah.

Alice: I remember about the problem about setting up the ratio.

Int.: Okay, can you do it now?

Alice: Umm-hmm **[nods head]**.

Int.: Can you go ahead and set a ratio for me then?

Alice: Umm…let's take that **[Scratches out the 8 on the picture and writes in *x*]** and put the *x* **[She then sets up the following proportion and immediately starts to solve it]**

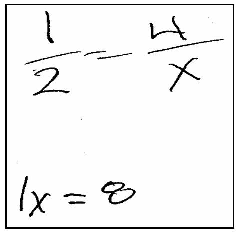


Figure 27. Alice solves for a missing side.

…cross multiply… That was easy **[giggles]**.

In her initial images of similarity, Alice placed considerable emphasis on her knowledge of mathematics. Looking at the above data, Alice seemed to have understanding of corresponding sides, and setting up a ratio between the two similar triangles. She also had knowledge of finding the length of a missing side in a triangle that is similar to another. There seemed to be some confidence in her abilities to set up and solve proportions.

At first, in her discussions on angles, Alice recalled that, "Some of the angles have to be the same measurement," and she later explained her images of angles and their importance in *what* to teach. It was this aspect of her primitive knowledge of *what* to teach that helped her to make initial images in her lesson plan (see Figure 28).

Similar Triangles

- regular everyday definition of similar
  - examples
- mathematical definition of similar
  - angles are same or congruent (corresponding)

    +
  - sides are proportional

~~make sure everyone knows what proportional means.~~

- examples of triangles w/ congruent corresponding angles and un-proportional sides
- examples of triangles w/ non-congruent corresponding angles and proportional sides
- examples of similar triangles
- let them practice drawing similar triangles

Figure 28. Alice's outline of her initial images of what and how to teach.

Other aspects of Alice's ideas of *what* to teach, involved similar figures such as squares, rectangles, and pentagonal shapes. In looking above at her outline, it is evident that procedural aspects of the concept of similarity influence her knowledge of *what* and *how* to teach.

In looking at *how* to teach, Alice's image was focused on using patterns and finding relationships.

> Alice: So like what conclusions can you draw from what similar means from those shapes since, this is a square and rectangle. Then add a shape that's like neither one of them, and see if that same definition holds, and then why it does or why it doesn't… and then move on.

In her final lesson plan, Alice's hands-on activity formalized an approach that mirrored this original image:

> To introduce the topic of similarity and similar triangles, the class will be split into groups of three or four. Each group will be given a copy of a worksheet that contains two squares, two rectangles. They will be told that the squares are similar to each other, the rectangles are similar to each other. They will be asked to discuss and try and reach a consensus on the definition of similar. I will then pull them together long enough to get what each group came up with a definition of. They will then receive two other polygons that are similar to each other. Also, handed out at this time will be two similar triangles. They will be asked to reevaluate the definition that they have of similar and see if it still applies.

Figure 29. Alice's hands-on activity for her final lesson plan.

These formalized images of *how* to teach grew from initial images in her first individual lesson, into images that she built on in the course of the LPS. Her formalized images of *how* to teach used only one approach, the use of finding patterns from three sets of similar figures, and expects students to be able to make generalizations about similarity. Like many of the other prospective teachers, Alice felt that it was necessary for her students to be told the definition of similarity after the activity.

In looking at other approaches and representations of *how* to teach, Alice had images of examples and teacher definitions to reinforce her students' knowledge about similarity. In looking below at a section of her final lesson plan, it is evident the importance that Alice placed on teacher intervention in her lesson.

> After this, I will formally state the mathematical definition of similar and the components necessary. I will also tell them that both of these components, the angle congruence and the proportionality relationship between the sides are necessary.

Figure 30. Alice reinforcing the definition of similarity.

She also had images in the context of everyday similarity in trying to connect ideas that students may have with mathematical similarity. It was this use of daily language versus mathematical language that was used by many of the prospective teachers. The images used by the prospective teachers connected mathematical similarity with their understanding of the everyday use of the word similar.

Alice: Okay, So I start out with the regular everyday definition of similar, just like what types of things people think are similar.

Int.: Like what?

Alice: Umm, more like real world situations like, “Would you say two cars are similar and why would you say?” Or just something like that to start off with…

It was during her initial interview that Alice had an image about *how* to teach the importance of both the proportional sides and the angle measurements.

Alice: Then set the definition of similar by congruent angles and proportional sides and then go on to triangles with what I have. I have examples of triangles with congruent angles but the sides aren’t proportional, and proportional sides but the angles aren’t proportional (see Figure 31).

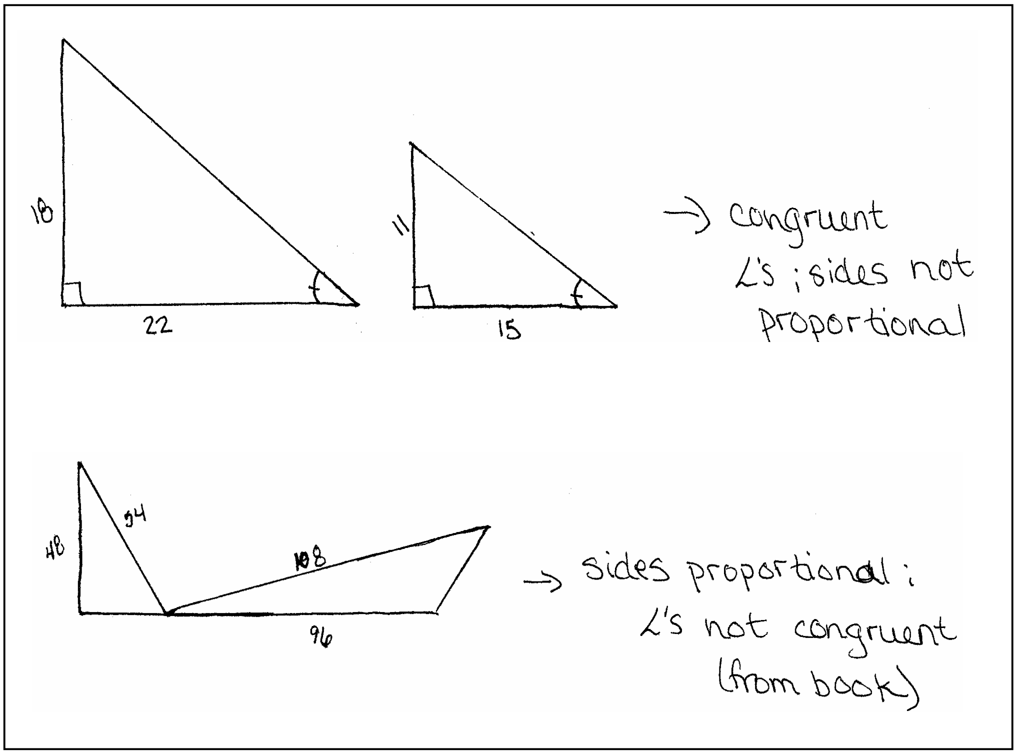


Figure 31. Alice's image about how to teach the importance of sides and angles.

Although her initial image was mathematically inaccurate, it was one that eventually supported her in having other images of incorporating ideas of indirect measurement and the similarity postulates within her final lesson. Since Alice noticed the importance on these two properties within ***what*** to teach in similar triangles and had images of *how* to teach them, when the discussion of the similarity postulates came up during the group interview and presentation, she was able to include these "short-cuts" of finding out if two triangles are similar into her individual lesson plan (see Figure 32). Alice folded back to her primitive knowledge of mathematics and thickened the images she had made using her mathematical primitive knowledge of indirect measurement and the similarity postulates, and incorporated these ideas in her lesson plan. This illustrates how her ideas and understanding of similarity changed during the course of the LPS.

> From this point in the lesson, I will go into triangle similarity shortcuts. What I mean by this is that I will tell class there are shortcuts for determining whether two triangles are similar or not. I don't think that there will be enough time to go through all of the shortcuts, so I will do SSS in class. I will then pass out a worksheet that covers AA and SAS similarity shortcuts.

Figure 32. Alice's image about the similarity postulates.

Alice's interaction with the group was limited. She was not vocal during the group planning stage of the LPS, and during the presentation her comments were scripted. The properties of *what* and *how* to teach that Alice eventually noticed and formalized in her final lesson plan incorporated the use of the everyday meaning of the word similar, getting her students to derive the mathematical definition of similarity using patterns, and the use of corresponding proportional sides along with congruent angles. She also formalized an understanding that it was the teacher's role to tell students the correct definitions and answers in problems. Alice had images of using indirect measurement to illustrate the strength of similarity, and using the similarity postulates to devise "short-cuts" and incorporated these ideas within her final lesson. Alice's initial images of *what* and *how* to teach the topic of similarity shifted from focusing on algebraic manipulations using proportional sides and congruent angles to applications of indirect measurement and the similarity postulates.

The Case of Anne.

Throughout the LPS, Anne's images of similarity and similarity related concepts were largely influenced by her formalized proportional reasoning abilities. Her images of *what* to teach were focused on solving for missing side lengths of similar triangles and did not initially relate angle measures with similarity. Her ideas in solving these types of problems were confident throughout the LPS.

Int.: Can you draw **[points to paper]** two similar triangles and show me?
I just want to make sure that you have a good understanding before
I turn you loose to uh… plan.

Anne: I'm not a very good drawer

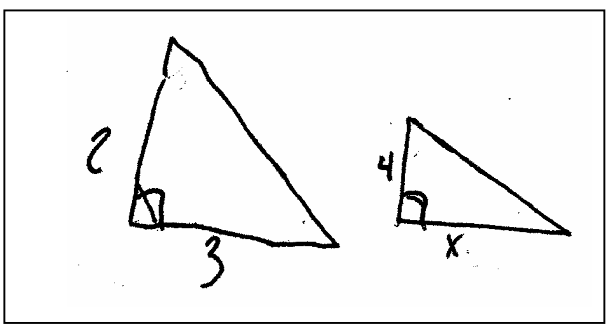


Figure 33. Anne's drawing of two similar triangles.

And then you solve that, 2 over 3 equals 4 over x.

$$\frac{2}{3} = \frac{4}{x}$$

Figure 34. Anne solving for the missing side.

Int.: **[points to proportion]** And this is your proportion?

Anne: Umm-hmm **[nods head]**.

Int.: And you remember solving for here the missing side, side *x*. Is there any information if you gave this to students, any information that they would need ahead of time?

Anne: Umm… I would have to make sure that they understood proportions and ratios first, before we went into this. And I would explain to them…I guess, the relationships of the sides. How similar triangles relate, I guess.

Int.: Okay, they would need – in order to solve for this, would they need to know that these are two similar triangles?

Anne: They would need to know that first and even how to set up the equation. Because they might mix them up and put 3 over 4, thinking that you use any numbers.

Int.: Ah… because if they set it up 3 over 4 and 2 over *x*.

Anne: That wouldn't work.

Anne's images of *what* to teach about ratio and proportion were consistent throughout the LPS. Looking at a portion of her first lesson plan compared to her last lesson plan, it is evident that Anne had images and noticed the importance of these properties in her lesson.

Anne: First I would review ratio and proportion just in case we had done it a while back. Give the definition of a ratio, and put like just a small example to refresh the memory.

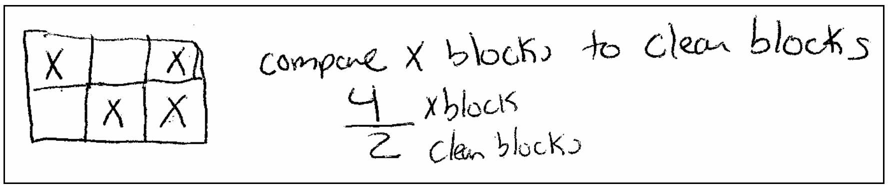


Figure 35. Anne's first example of ratio using blocks.

The *x* blocks compared to the clear blocks. Then the definition of proportion, an example right there.

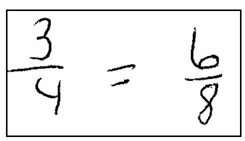


Figure 36. Anne's first example of proportion.

And tell them that we are going to take what we have learned and apply it to our geometry.

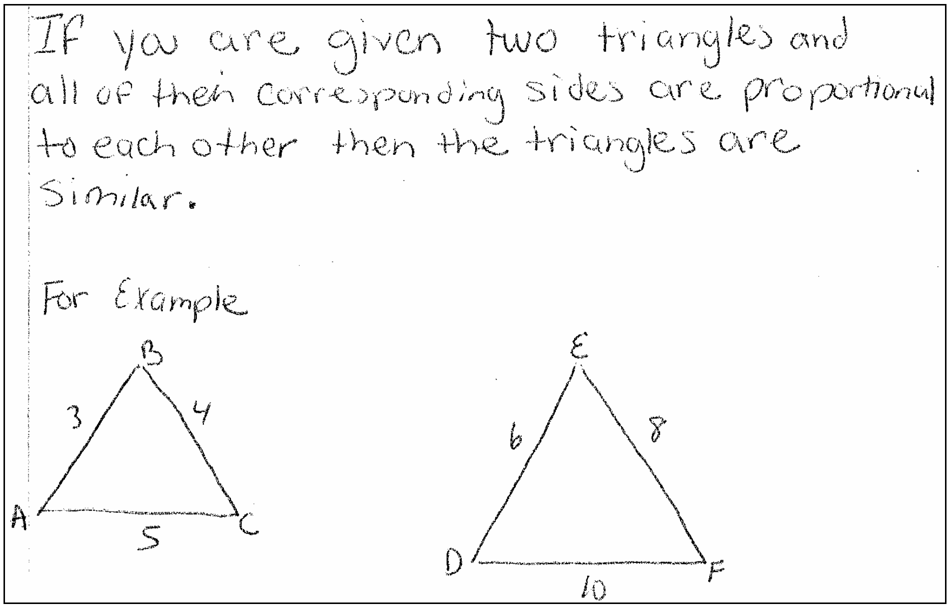


Figure 37. Anne's use of proportion in her final lesson plan.

In these examples of similarity from her first lesson plan and her last, it is evident that Anne placed little importance on the angle measurements of similar triangles. She made an image about angle measures in her individual interview, but this image was overshadowed by another image about the shape and size of similar triangles.

Anne made an image about *what* and *how* to teach that was evident not only throughout her discussions about similarity, but among other participants, the idea was that, "similar figures have the same shape but not necessary the same size."

Anne: I told them that similar figures have the same shape but not necessarily the same size; they are either enlargements or reductions of the original object. And I gave them the original triangle,

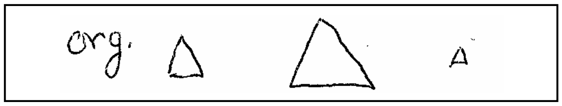


Figure 38. Anne’s reduction example for same shape, not same size.

and the original triangle can be bigger than what it was or smaller than what it was and be similar, they are similar because they are the same shape triangles but different sizes.

It is evident from this example, and the examples to follow, that Anne and the other prospective teachers used the everyday meaning of the word similar to make a connection for students. The prospective teachers saw this as an important image in *how* to teach.

Anne’s initial images about *how* to teach were at first guided by teaching strategies of pattern recognition and discourse, along with the everyday use of the word similar.

Anne: Like **[turns to one of her pages]** different shapes, like if I say these shapes are similar, and these shapes are similar. So like what conclusions can you draw from what similar means from those shapes since, this is a square and rectangle. Then add a shape that’s like neither one of them

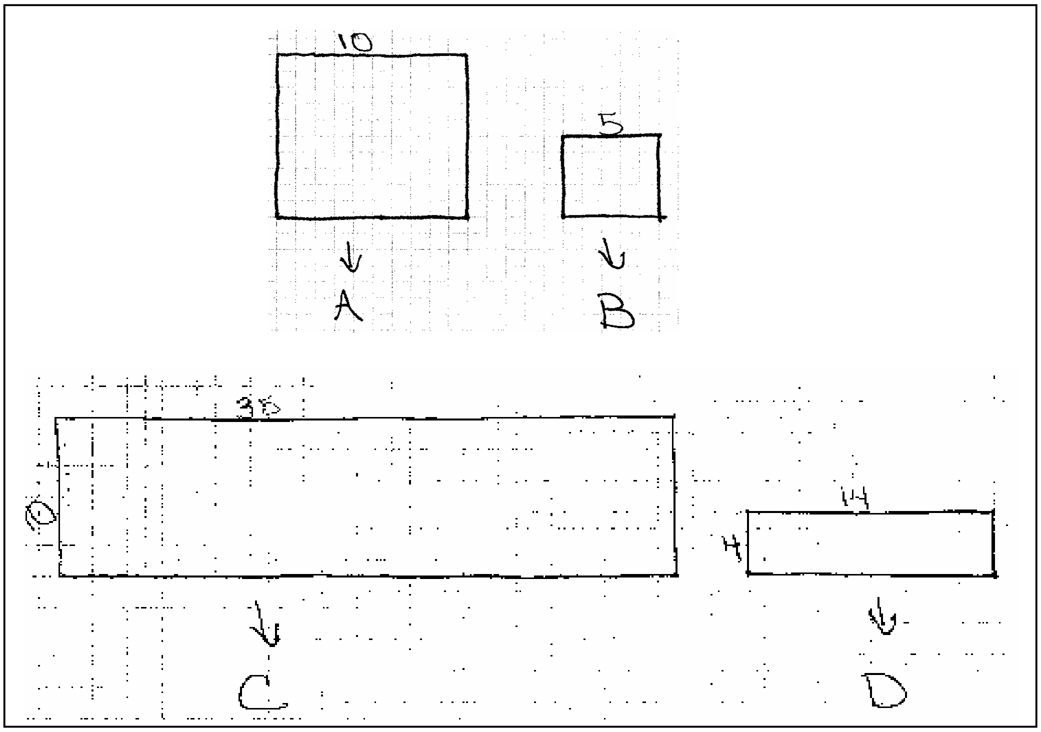


Figure 39. Anne's pattern approach with squares and rectangles.

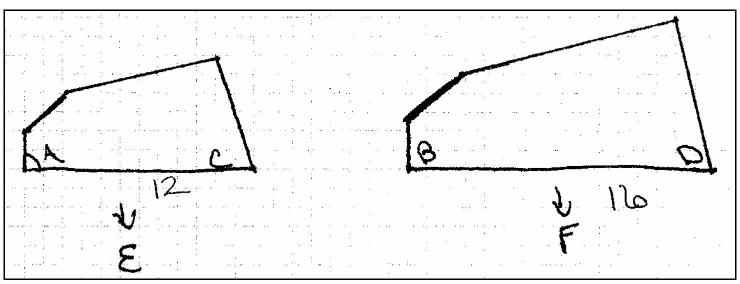


Figure 40. Anne's pattern approach using pentagonal shapes.

and see if that same definition holds, and then why it does or why it doesn't and then move on. Then set the definition of similar by congruent angles and proportional sides and then go on to triangles with what I have.

These images of discourse included the students establishing what similar means mathematically and what patterns they found in doing Anne's exercises.

Anne: Then I ask the question, "Who can define what the word similar means or give me examples of something that would be similar?" Cause I would have them talking, and I would write down whatever they said.

Int.: Just in sort of everyday terms, like a tree is similar to a flower because they are both plants or something…

Anne: Yeah…

Int.: Kind of everyday similar.

Anne: And I got their list of whatever they said, and then like at the end give them a simple example, just different things, and my input.

However, Anne's images of discourse quickly shifted into a teacher-led lecture. This is evident by her lecture of the similarity postulates in her final individual lesson plan.

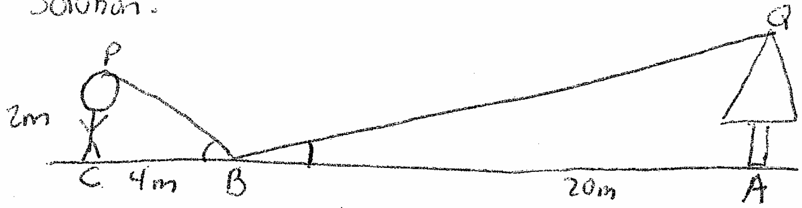
> I will explain to the class that we have short cuts to find out if two triangles are similar or not. If it would take to long to see if all the angles are congruent or if the sides are proportional. What would we do if we were missing some information from the triangles? That is why we have conjectures.

Figure 41. Anne's lecture on the similarity postulates.

As Anne progressed through the study, she folded back and thickened her images of *what* to teach and had other images of indirect measurement and the similarity postulates.

I would not grade this homework for right or wrong, but for effort. We would start our class off tomorrow discussing this problem

Solution:



You assume the man is standing straight and the ground is flat. So $\angle PBC = \angle QBA$

$$\frac{|QA|}{|PC|} = \frac{|AB|}{|CB|}$$

$$= \frac{|QA|}{2} = \frac{20}{4}$$

$10 = |QA|$

The height of the tree is 10 meters.

Figure 42. Anne's images of indirect measurement.

While discussing approaches for teaching the similarity postulates, the group entertained the idea of doing all the postulates first, then doing examples to explain them. However, Anne folded back to images that she had and made an image during the individual interview about *how* to teach and had an image about the approach that changed the entire discussion and had an impact on the presentation.

Anne: **[talking to Ava]** I have a question. What if we were to start off with the whole triangle, all the sides and all the angles, and then break it down to Side-Angle-Side? Start off with everything so that they can see what the whole similar triangles look like and then go from there and break it down in all these, so they can see…

Anne's idea to do a postulate then an example was received as a way to go "step-by-step" by the other members of the group. In this example, Anne anticipated how students would react to a series of definitions followed by a series of examples. She had an image based on images made about previous discussions concerning the group presentation. This image of a step-by-step approach eventually turned into a teacher-led discussion that included the definitions of the similarity postulates and examples that followed.

There was an instance in which Anne folded back during the group interview and collected images about the correct way to set up proportions.

Anne: And like doing ratios is two, like comparing two things. Cause I remember when I was in there last week **[points to paper, softly]** you can have these set up wrong.

Ava: And the corresponding sides need to be proportional…

Anne: Yeah.

By folding back on her knowledge of mathematics about the correct way to set up a proportion, this gave members of the group images about ways to assess the students understanding of corresponding sides. Anne folded back on an image that she made about setting up proportions correctly, anticipating that this would be an area of concern in teaching similarity, the group then decided to do examples in which the students would have to identify the corresponding sides of the similar triangles in order to set up the correct proportions.

Anne made considerable changes in her initial images during the LPS. In her last lesson plan, Anne formalized ideas that resembled many of the major ideas of the other prospective teachers' lesson plans. She included the everyday meaning of the word similar as an image for students to begin to develop the idea of similarity. She made and had images of *what* and *how* to teach that anticipated potential problems for her students. Anne formalized ideas of using patterns to help her students derive the mathematical definition of similarity, although these became instances of a teacher led lecture, and she noticed the importance of indirect measurement to illustrate the strength of similarity. Anne also noticed the importance of using the similarity postulates to devise "short-cuts" for her students in *what* to teach. Similar to Alice, Anne's initial images of *what* and *how* to teach the topic of similarity shifted from focusing on algebraic manipulations using proportional sides and congruent angles to applications of indirect measurement and the similarity postulates.

<u>The Case of Ava</u>

Like Anne's initial images of teaching similarity, Ava's knowledge of *what* to teach was largely influenced by her proportional reasoning abilities. From the beginning, Ava related her ideas of similarity with the similarity postulates.

> Ava: I remember talking about proportional lines and definitely in my geometry, in 9^{th} grade learning my ASA and SAS rules for proving that two triangles were similar.

In the beginning of the LPS, Ava's images about ratios and proportions were important to her beliefs about similarity. She openly admitted that the importance of angle measurement was secondary.

Int. The proportionality of the side lengths, and you also mentioned the congruency of angles…

Ava: Umm-hmm **[nods head]**.

Int.: Do you think of both of those when you think of similarity?

Ava: Umm…yeah, but I think probably because it was introduced to me first, that I look at the side lengths first. When I think of similarity I think that these sides are multiples of some number to get all the rest of them, then it clicks with me that they must have all the same angles…

Int.: Oh, okay…

Ava: It's like a second, it follows right after.

As the LPS progressed, Ava had an image of a stacking method with triangles she had created, incorporating her formalized knowledge of angle measurements to show its importance in similarity. In the example below, it becomes evident that Ava was making images that caused her to consider the importance of angle measurements in similarity.

Ava: It might be easier for them in some cases, especially visual learners, is like a stacking process **[holds two paper cut out triangles so they overlap]** if they put them all together they see that this is the same angle because they all match up, and they have the different sides because they are obviously proportional because this band width is all away across, and that helps if people are like, "What does it mean that the angles are the same?" It means that you can stack them together and it is not lopsided one way or the other. And then the teacher will facilitate a discussion with the class about a true definition of some of the similar rules, what similar means mathematically

In explaining her lesson, Ava began to place more emphasis on the importance of angle measurements. As she explained images about *what* and *how* to teach she began to anticipate important attributes that her students may focus on in her activity. The shift in her thinking was brought about because she realized the importance of her manipulatives and had images of what students might do while they are discovering similarity.

Similar to Alice, Ava made images of *how* to teach that included ideas of counter examples. She made images of open-ended questions that challenged the students' ideas about what they had learned.

Ava: As far as homework or a second assessment would be to go is ask them, after we have covered some of the short-cuts, "Is SSA a shortcut, can you prove it or can you disprove it?" You know, try and come up with triangles that meet that the SSA but to show that they are not similar, and that would be something for them to go home and look at something we have already done and try to work through it on their own and let them try and work through it, kind of experience what other mathematicians have done in coming up with these conjectures themselves, so that is basically how I would teach that.

Ava's initial images of *how* to teach similarity were largely based on discovery methods and discourse that were focused on the conceptual learning of similarity. Like other prospective teachers, she used only one approach in initially introducing the concept of similarity.

Ava: I gave them a packet of 12 triangles that were sorted by color and different sizes, and different angle measurements, but there are some of each that are represented, and so that each group must find several ways to categorize that triangles and they share ideas with the rest of the class and write them up and they have rulers and compasses and they can measure different things, if they want to. Then as a class we show which the most appropriate way to show the mathematical similarity of triangles. You got color is irrelevant…

Int.: When you say similar, in this case similar means how they can be alike…

Ava: Right, right, how they can be alike but then you move it on to mathematically, how they – what similar really means.

These ideas were consistent throughout all of her planning. In the group interview, it was Ava's discovery method of teaching using the twelve triangles became the hands-on activity for the group presentation and her initial ideas of the "short-cuts" became predominant in the other group members lesson plans.

Ava: Then that is when I would get into the conjectures of Side-Side-Side, how do you prove it.

Rose: See, I was going to put that in mine. I was thinking about that but I didn't have enough time. I don't want to go back and review that stuff anyway. But I do think that would be an easier way.

Ava: There is discovery ways for them to do it, to think about what to actually make something similar.

Ava encountered situations that caused her to have new images as to her approaches and knowledge about mathematics. The first occurred in the group lesson planning stage of the LPS.

Ava: And that is a really good way to explain it to them as far proportions go, because I know they understand ratios and proportions, but to say that it is an enlargement or a reduction. Putting it in terms that they might be more comfortable with even though they understand proportions.

In this situation, Ava used Anne's idea of reduction and enlargement to help explain the idea of similarity. This allowed her to incorporate a different approach and consider more procedural aspects of her lesson.

Ava had another image related to using procedural examples in her lesson. In the example below, Ava used her previous images about ratio and proportion and anticipated the need to guide students more than she had previously considered.

Ava: Okay, so you just want to do the review of the proportion, give them two triangles, but have…

Rose: Everything.

Ava: everything labeled.

Rose: Or you could just take those two triangles and break it down like in parts.

Ava: I think if then we could talk about certain things that need to be there before Side-Side-Side, then you go on to conjectures. How do you want to do that from here?

In this example, Ava shifted her thinking about how her students will understand her discovery activity and tried to lead them with examples and lecture. She had these new images because her style of teaching was not focused on teacher led examples and lecture, but by inquiry and discovery. She began to anticipate that not all of her students would be appreciative of this style of teaching and tried to incorporate more strategies in her teaching, while still addressing the different types of learners.

A third image that Ava had was related to incorporating real-life examples in her final individual lesson. This was an idea that she did not incorporate in her previous lesson, but because of a discussion with Mary and Rose, she was impacted throughout the remainder of the LPS.

Mary: I think we should, if we have time and I think we still have time. Some way relate everyday life, not just triangles.

Rose: I got that on here **[pushes paper to Mary]**.

Mary: I don't mean just talking about it, for example have something that you want to measure and it's to high and you can't, but you find something else that you can measure.

Ava: I think we should definitely do that at the end.

Mary: Yeah, at the end so you show where it's…used.

This image allowed Ava to have an image in her last lesson plan that incorporated some application uses for similarity. Ava, while using methods of inquiry and discovery to examine the properties and postulates of similarity, did not go beyond these topics in her initial interview. It was in the episode above that Ava had images of important applications of similarity. She later used these images in her final lesson plan.

Ava was very active in leading the group through the remainder of the LPS; Mary, Alice, Anne, and Rose all incorporated her initial ideas of using the postulates of similarity in their lessons. The group experience seemed to shift Ava's approaches in teaching from an open-ended conceptual exploration to including some procedural aspects where she lectured the students. She also began to anticipate student responses to her lessons and tried to include teaching strategies that allowed her to help more learners. Her initial images of *what* to teach shifted from proportion-centered to the incorporation of the importance of angles. Her formalized images of *what* and *how* to teach included her conceptual images of allowing students to derive the definition of mathematical similarity and using the similarity postulates to create "short-cuts" to determine if two triangles were similar. Ava noticed the importance of indirect measurement to illustrate the strength and importance of the topic.

The Case of Mary

During the first part, and throughout the entire LPS, Mary's images of *what* to teach were directed towards ratios and proportions. Similar to the other prospective teachers in the study, Mary's initial images focused on side lengths and slowly shifted towards angle measurements.

Mary: **[pointing at the geoboard]** I mean they're similar triangles, and this side is to this side, as this side as to this. So there's where you get the ratio.

Mary: Yeah, but these two are proportional **[pointing at the geoboard]** or this is a part of this and this is a part of this

Mary made images of *what* to teach that included the importance of angles and proportional sides. She included this idea in her first lesson plan:

Mary: And then I was just thinking on the board to draw two different sizes of triangles with equal angles

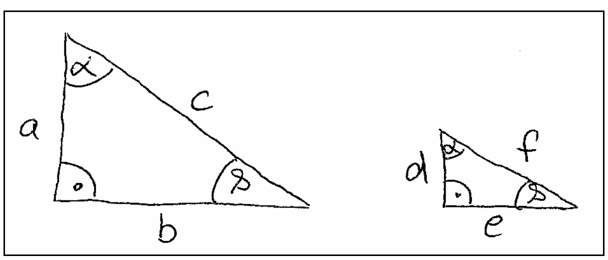


Figure 43. Mary's picture to exemplify two similar triangles.

[shows interviewer her notes] and then just…

Int.: Like these two here? **[points at Figure 43]**

Mary: Yeah, and then just show. I figured after this one **[points to one of the transparencies]** when we talk about it, that we would come up with these ratios and that they would be equal **[pointing at paper]**

show ratios $\frac{a}{d} = \frac{c}{f} = \frac{b}{e} \Rightarrow$ if that is true $\Rightarrow$ similar triangle

Figure 44. Mary's ratio examples that were abstracted from a picture.

and then to clarify, just one more time, I would draw it on the board to show it again. Then I would draw them two triangles that have equal angles – it doesn't have to be 90 degrees, but just equal the same angles on each triangle and then ask if the opposite would be true too. If *d* over *a* equals *f* over *c* equals *e* over *b*, if you can switch it

Mary incorporated the use of repetition in her lesson and anticipated students' responses throughout the lesson study. Her initial images continued and became more developed as the LPS progressed. In the following example, from her last lesson plan, she used examples of ratio and proportion in conjunction with the similarity postulates to make a worksheet for her students to use.

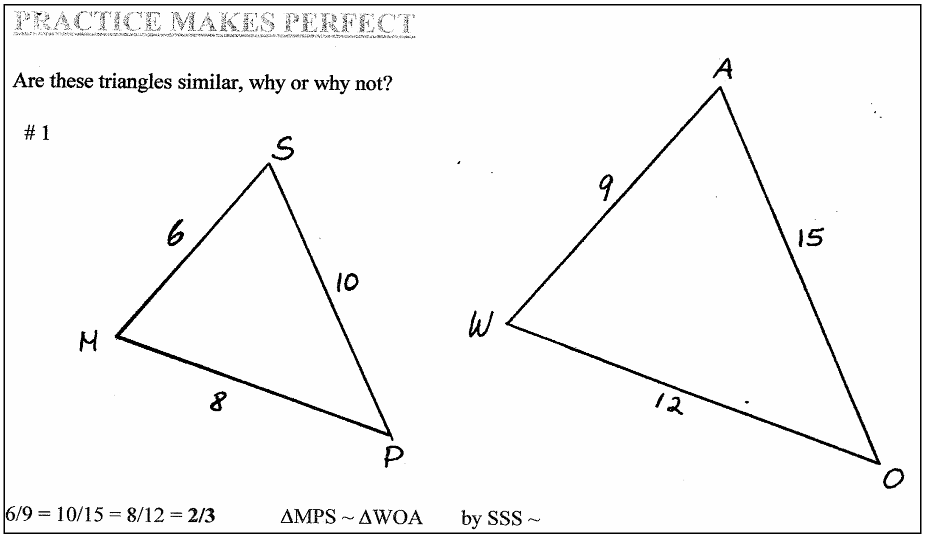


Figure 45. Mary's continued use of ratios and proportions in her lessons.

Mary apparently made an image of using the similarity postulates that was influenced by her interactions with the group. This image apparently came about during the group planning activity. Many of the group members had similar images of using Ava's ideas about the similarity postulates. Mary shifted her images of *what* to teach to include the similarity postulates as "short-cuts" to help students in determining if two triangles were similar.

Mary also made images of indirect measurements and applications in her discussions of *what* to teach. She was the first in the group to mention other strengths of similarity.

Mary: Then I would ask why is this idea important or what we need this for with the ratios? And then I was just saying a lot of models are being used like in architecture or if you build an airplane, you first have your little model and just go from there.

Out of the five prospective teachers in the lesson study, Mary was the only one in the group to use indirect measurement to find the height of objects in her first lesson plan. During the group planning, when the other prospective teachers were trying to relate similarity to everyday life by use of daily language, it was Mary who suggested relating similarity to applications that involved finding the height of something.

Mary folded back and collected previous mathematical knowledge of *what* to teach in her individual interview and made an image. She made an image based on a boat mast problem that she recalled from school. With this folding back and collecting on her mathematical knowledge, she made an image that this type of problem of indirect measurement was important in learning about similarity.

Mary: And then so because I remembered that boat example, I was thinking that we could go outside – and I have made a little note to that – if we could go outside **[points to paper]**

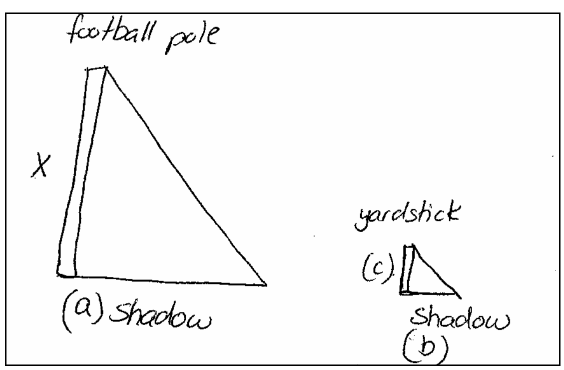


Figure 46. Mary's image of how to use her idea of "the boat example."

and measure a tall thing, like a football goal, most high schools have that. That football goal…

Int.: The football post?

Mary: Yeah… Well since you can't measure how long it is by putting a yardstick to it. Just do it **[points at Figure 46]** with the shadow that's cast, how long the shadow is and then use a yard stick and do the same thing and then you have the same proportions again and you can figure out how long the football pole is.

It was this process of folding back in her individual interview that had a huge affect on the remainder of the lesson study. Her image of a boat mast example that had caused her problems when she was studying similarity in school was so important to her that she folded back and made an image of a similar problem to help her students see the importance of similarity. During this episode, she anticipated problems of her students by using her own difficulties in studying similarity. There were other episodes where Mary thought about students' reactions to activities. During the group planning stage, she had an image about the activity they were planning and the students' reaction.

Mary: See what I was thinking is somehow, **[placing plastic triangles in some order]** let's pretend that the yellow one is bigger **[has three plastic triangles]** and these are my three 45-45-90 problems.
Then after we have come together and seen what they've found out, then they go back in groups and say take these three and see what you find out how they can be grouped. What is the special thing that I would put these three in a group?

These images that she had caused the group to consider their actions from the students point of view. Some of the prospective teachers in the lesson study had already anticipated students' responses and this allowed those who had not, to see the benefit to this anticipation.

Mary's initial images of *how* to teach involved discovery lessons and applications. The discovery lesson that Mary invented used the overhead projector to enlarge a picture of a triangle.

Mary: Oh, okay. I thought I would start with this triangle **[holds up transparency with triangle (see Figure 47)]**.

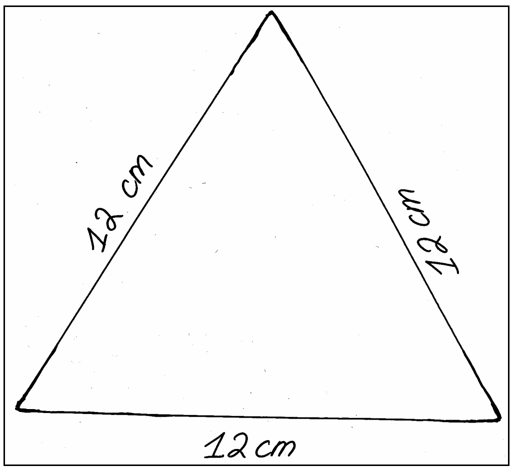


Figure 47. Mary's transparency to project on the overhead.

It has equal sides… **[holds up red plastic triangle]** I just used this one… and I put it on the overhead, and I wrote down how many centimeters there are. Then 1 to 3 students, depending on the number, will come up and measure the sides on the overhead projector, to find out if they are the same sides – if that's true - and then, of course they will come up with that it is longer, but then I wanted to see what they conclude from that.

In her first lesson plan, she included the "football goal" problem and finding the height of a plant, as applications of the strength of similarity. These two ideas developed into several other images using indirect measurement on figures other than triangles.

"Activate your Brain" Transparency

"You are in the Outer Banks with a big group of friends enjoying summer break at camp. Since lying in the sun all day gets boring quickly, your camp leader Sam made up a fun scavenger hunt. You are in groups of four, running through town trying to get as many points as quickly as possible. Your next task is to find the height of the lighthouse at the beach. How will you solve this problem?"

Figure 48. Example from Mary's final lesson involving indirect measurement.

She also made an image of a project in which the students used what they had learned (see Figure 49).

NOTHING IS TOO HIGH FOR US (due 1 week from today)

Setting the stage:

Oh no, another hurricane is coming. Your mother is worried about the tall tree in your yard falling on your house. She wants to know the height of the tree to see if it would hit the house when it fell. She is very worried and does not know how to figure it out, since she does not have any tools for measuring tall objects. You come home from school and tell your mom that you just learned in school how to solve that problem. She is stunned and just watches you proudly as you go to work.

Figure 49. Mary's out of class project to help students.

Mary's ideas and suggestions of incorporating ideas that related mathematical similarity to "real-life" influenced many members of her group. The members of the group were able to make images and begin to see the strength of similarity, in something other than finding missing sides of triangles. None of the other participants in Group C, until they encountered Mary's formalized ideas of scale models and indirect measurement to find the height of an object, had incorporated these images from their mathematical primitive knowledge into their images of *what* or *how* to teach. Mary's ideas of *what* and *how* to teach also changed and she incorporated other group member's images within her final lesson, and anticipated students' responses to these activities. The property that Mary eventually had and noticed as being important in *what* to teach about similarity included Ava's use of the similarity postulates. Mary's formalized ideas of *what* to teach included congruent angles, corresponding proportional sides, along with modeling and indirect measurement.

<u>The Case of Rose</u>

Rose began her discussion about similarity using an everyday definition of what similar means. When the interviewer tried to probe her understanding of congruence it became evident that this was her initial idea of similarity.

> Int.: Do you associate, do you associate area with similarity?
>
> Rose: If you are finding out the area of two things…if I am doing anything with similarity, I'll find some base concept like area, or perimeter or something. Say well okay, this is similar to this maybe by half or by twice or whatever, that's how I do anything that is similar… **[Interviewer begins to speak]** like people, go ahead.

Int.: Umm… how you do anything that is similar, you try and find relationships like area and perimeter?

Rose: Yeah, like if I am doing a person I will be like this person is similar to this person, like likes and dislikes, or something. I'll have to find some root thing.

Int.: So if umm… this person likes pepperoni and his pizza and this person likes pepperoni and his pizza, then those two people are similar in …

Rose: Yeah, in that scenario.

Rose's idea of the use of everyday similarity caused confusion during her individual interview. She eventually replaced this image with one that reflected mathematical similarity, but it was not until she after she had expressed other visual representations, she felt exemplified similarity. Rose had a formalized illustration that she associated with *what* to teach about similarity.

Rose: Lets just use 3, 4, 5, and then if you had a littler triangle,

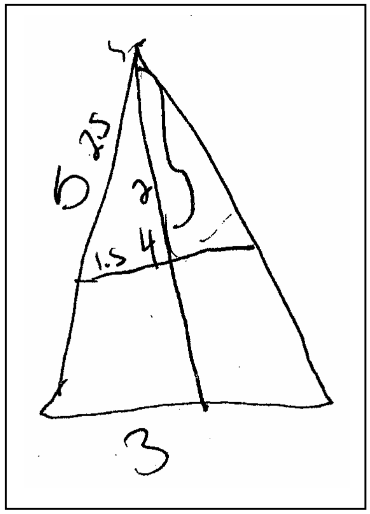


Figure 50. Figure that Rose associated with similarity.

and this was half of that than this would be 2.5 and then 2, and then 1.5. Then find out the area of that one, and those would be similar because the area is twice the other one.

This illustration with the parallel bases through the midpoints of two remaining sides was seen as necessary for Rose to fold back and collect in order to do similar triangles using ratio and proportion. However, this image was replaced when she was realized that her understanding of mathematically similar triangles had been confused with her the understanding congruent triangles.

Rose: Because I don't really remember similarity but I would do congruence and things like that…

Int.: Well, let's start off with that, what does congruence mean to you?

Rose: Isn't that what this is kind of? Like, congruence is kind of like equal, in away.

Int.: Equal in what way?

Rose: Like one triangle would be… **[points to a set of triangles on a page]** Like this one, this triangle is congruent to this one because they both have like the same shape and almost the same size.

Int.: What are you talking about? **[points to the set of triangles]** This big one congruent to…

Rose: Yeah, well like if you had this triangle

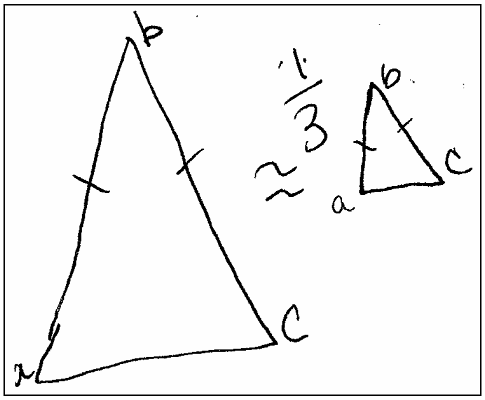


Figure 51. Rose draws her idea of congruent triangles.

say *a*, *b*, *c*, would it kind of be congruent to this smaller version of it. Considering they would have the same lengths and stuff, well not the same lengths. They would both be isosceles, except maybe this one would have like 3 and this length would be a third smaller.

As the LPS proceeded, Rose made more images of the substantive structures in *what* to teach, such as indirect measurement and the similarity postulates.

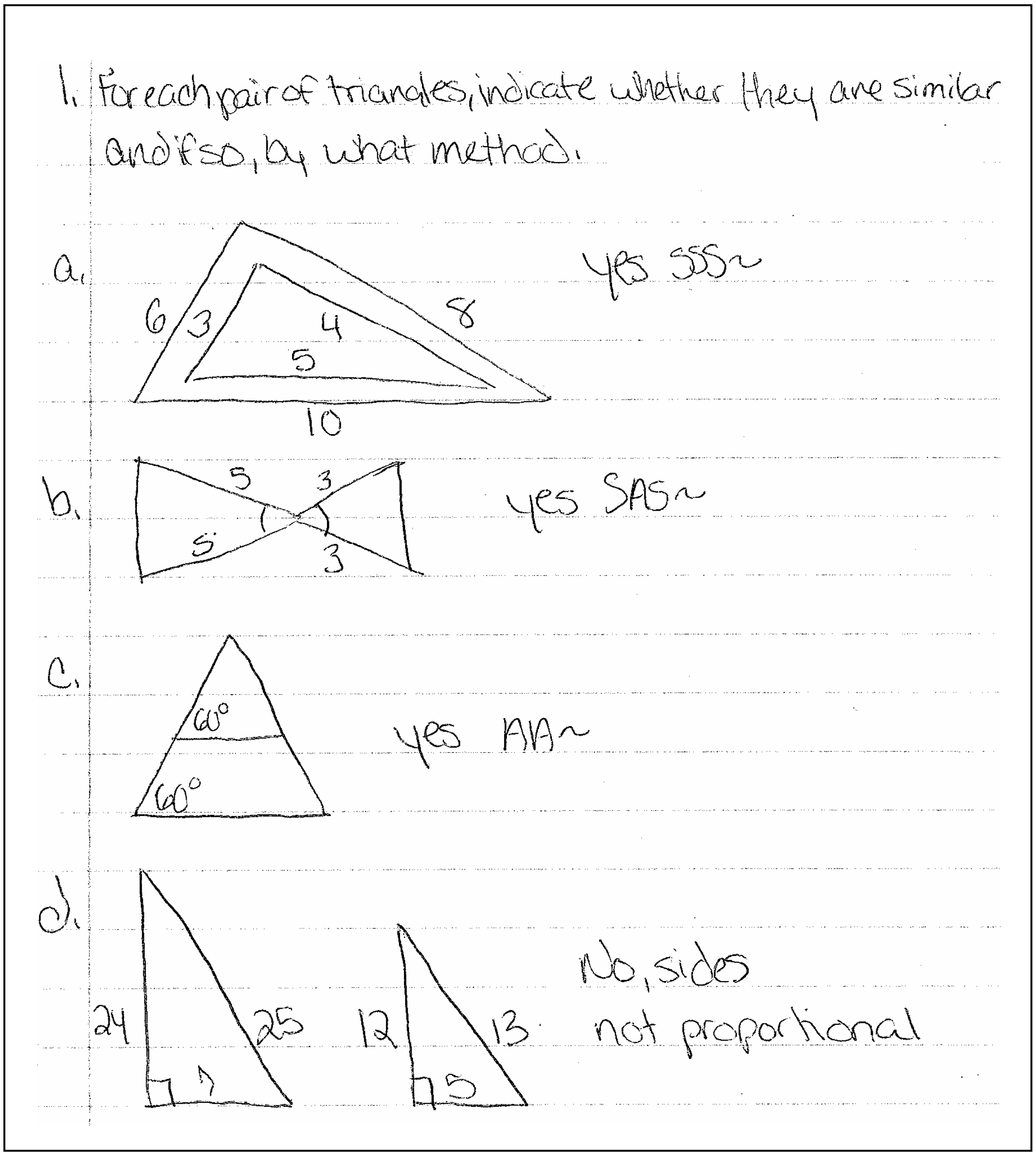


Figure 52. Problems that Rose used to help reinforce similarity.

Throughout the LPS, Rose made images of similarity that reflected either isosceles or right triangles. She focused on these two types of triangles because, as she stated, “teachers usually use isosceles triangles.” She seldom included any representations other than right or isosceles triangles.

In trying to help herself come to a more correct meaning of *what* to teach about mathematical similarity, Rose made an image during the interviewers discussion about an a figure he drew. Rose was able to relate to this figure and she began to change her images of similarity.

Rose: **[pointing at a figure of a large triangle]** But that would still be 40 right there, and that would still be 40 and so they would have like similar… the angles would be the same, only the side lengths changes. Oh, yeah we do angles too.

Int.: When we drew a smaller triangle that was similar and you said that the angles would be the same, it's just the side lengths are different.

Rose: Yeah.

Int.: In your mind how is similarity related to ratios now?

Rose: Well because… like… well the only way that I can think of it being related is because of this example. This example is like if you are comparing one thing to another and if they are both similar maybe then they have the same kind of angles. Then the sides of it are going to have to be in some kind of… You can always say that, like, this side is twice this side or something like that. You can always do some kind of ratio to figure out.

In this example, Rose shifted her images of *what* to teach in similarity from only proportional sides to including angle measurements. Until this point she had not considered angle measurements and only used proportional sides to determine whether two triangles were similar. After this episode, Rose began to incorporate the use of angle measurements, however, she still focused on proportional sides.

In discussing ideas about right triangles, Rose folded back and thickened her understanding about angles and made new images. She extended these images about right triangles to particular types of right triangles.

Int.: So a 30-60-90 triangle is similar to another 30-60-90 triangle.

Rose: Yeah.

Int.: Well what about a 30-60-90 and a 45-45-90?

Rose: They can't be because your angles would make them have different lengths on the sides.

The previous example illustrates that Rose made images that the corresponding angles of two similar triangles needed to be congruent. It also illustrates that she shifted from looking for aspects such as "area, perimeter, or some basic concept that makes them similar" of *what* to teach to a more mathematical understanding of similarity.

After extending her images to right triangles, she began to extend her ideas and she had an image about other triangles.

Rose: And then I drew two different triangles

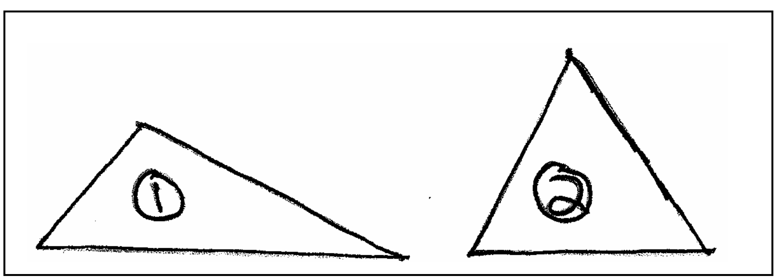


Figure 53. Two triangles that are not similar because they are different types.

and I'll say that those two couldn't be similar because one is scalene and one is equilateral.

Rose's formalized knowledge of *how* to teach involved a teacher-led lecture. In discussing the method of instruction her teacher used, Rose commented that this is a method that she will employ. These same sentiments reflect her teaching strategies in her lessons throughout the LPS. She addressed the use of these formalized images in her teaching philosophies:

> "I believe that the teacher is the core of the classroom."
>
> "During instruction, I believe that the classroom needs to be of lecture style"
>
> "The teacher is the backbone of the classroom and what holds it together."

While Rose felt it was important for the students to learn, she also saw the textbook, teachers, and examples as the ways of establishing knowledge about mathematics in *how* to teach.

> Rose: Okay that would be wrong. Hmm. **[Grabs the textbook and starts turning pages]** The way that I was reading it in the book…
>
> Int.: Umm-hmm.
>
> Rose: it said that they would still have similar, that they would still be similar, no matter…
>
> Int.: Show me what you were looking at earlier.
>
> Rose: **[still turning pages]** If this book had answers in the back it would be more helpful.

When faced with this contradiction in her understanding of *what* to teach about right triangles, she immediately used the textbook as a support reference. This same support reference was instrumental in her images of *how* to teach. Her need for the teacher to be an authority in the classroom and the use of the textbook exemplify her limited SMK.

During the group lesson-planning stage of the LPS, Rose made an image about *how* to teach using the hands-on activity that Ava had used in her first lesson.

Anne: What should be our hands-on activity?

Rose: I like her idea **[points to Ava, Anne nods]**, I really do…

Anne: Yeah.

Alice: Me too.

Rose: Cause hers would take away my need to go, say okay well, “If you have two triangles and try and figure out if they are similar or not, and that actually teaches them.

Another image that Rose made, dealt with how to relate the similarity postulates with indirect measurement.

Anne: Are you sure that you want to do the Side-Side-Side, and just do the basic concept?

Rose: Yeah, that is what I would do. If you gave them two triangles, because they know what similar is, and we’ve already talked about similarity. Okay, if you can find the length of one side based on the other triangle if you know the two sides of that and all the angles. It’s kind of like yours **[points to Ava]** but not exactly the same, like here is Side-Side-Side.

Ava: I think that is a good idea.

The two previous episodes show that while Rose may have appreciated other teaching strategies, she has to overcome her limited SMK and belief structures that prevent her from employing these teaching strategies.

While wrestling with the image of how to write a scale factor in discussing ***what*** to teach, Rose folded back and thickened previous images that she had from classroom lectures. She folded back to an example that had happened earlier in the methods class and thickened on information to help her make an image how to write a scale factor.

Rose: Yeah. Well I just went through and said that if you had that… that for every… what is it, 5.5…or something. I don't know what I did, then the lengths are twice. That for every two, that for two… Ahhh.

Int.: Well, all these side lengths are twice as long as the corresponding side lengths in that triangle.

Rose: **[writes on paper]** Yeah but it would be *A* is equal to 2*B*. Instead of 2*A* is equal to *B*, I messed up there, yeah. And for every *A* **[writes on paper]** you would have 2*B* or 1 to 2. **[points to paper]** That's what you were doing, when you were doing this in class, that whole little rock salt thing. This is what I'm saying, because if you are going to ask us a question and say well you need to know the proportion of rock to ice then you can't switch them back, because the numbers can't actually like, like the numbers need to stay in the same order as what you are saying. That is what I was talking about in class and that would mess me up if they were doing ratios on these two triangles, and they said instead of … If I am asking them like from *A* to *B*, like I did right here *A* is to *B*, they would have to do that right here instead of switching it around.

Int.: Right, because we talked about it in class having 1-7 versus 7-1…

Rose: 7-1. And then if you like have the 1 over 7 or even the 1 over *A*, I wouldn't even put 1 over *A* because we don't do stuff like that. But if umm, you were doing it that way, then they could switch that and say originally it was supposed to be 7 to 1, because it was supposed to be ice to rock. I wouldn't use a fraction that way because they would disregard the 1 and just use the 7.

Int.: Yeah, you could just say 7 is the ratio of scoops of ice to scoops of rock salt. You got 7 times, no matter how many scoops of rock salt you put in there you got 7 times the ice.

After having this image Rose was forced to fold back and collect this image during her presentation to defend a definition that her group had made. When folding back on the image, Rose was able to explain properties that she had noticed to Student 1 (S1) about how to write the definition of ratio.

S1: Can I make a comment real quick? Just the math on that one, where it says, "The ratio is…", you can put it back up.

[Alice puts her transparency back on the overhead]

It says, "division." Alright? It says, "that compares two quantities by division." That's not necessarily true right? Because if you have the ratio of, out of 9 starting baseball players on a team the ratio of millionaires to non-millionaires, right? That would be 9 to 0. In most cases, right? You couldn't express that in division. You couldn't have 9 over 0. So…

Rose: Well, with that example you could probably still do it, if like you are, because I remember that we were doing the rock salt example where you have the, uh…What am I trying to say? You have to have the wording exactly right, but you could still do that problem using division because if you have 0 over 9, your answer would still be zero.

S1: Yes, but a ratio is the expression of two ordered quantities in a fixed multiplicative relationship, not a division relationship.

Rose: Well I am showing you how it is a division relationship, using your example. If you had 0 over 9, yes that would be a division problem and you would get an answer out of it.

S1: But that is not the same ratio though. That's the ratio of non-millionaires to millionaires, but what I am talking about are the millionaires to non-millionaires.

Rose: Yeah, but you could still switch the order around. **[loudly]** I am not trying to argue. I am not trying to argue the point with you. I am just trying to say, yes you are right if you are wording it that way. But you can still change the way you are stating it to be part of a division problem so that it would fit the answer.

S1: The article that we read for class at least made a distinction…

Rose: Yeah.

S1: Ratios are not a division relationship because of that fact, because you can't have 0 as…

Rose: Yeah, you can't have 0 in the denominator in a division relationship.

S1: Yeah.

Rose: I understand what you are saying.

Rose was an active participant in the similarity LPS and her vocalizations helped her to make and have images. Rose's images at the beginning of the LPS were eventually replaced as she folded back to collect and thicken her understanding. Rose's mathematical knowledge of similarity was fragmented and effected her images of *what* to teach. Her initial images of *what* and *how* to teach changed during the LPS and apparently she noticed properties about the importance of indirect measurement, the similarity postulates, and the use of a conceptual based activity in helping her students understand mathematical similarity. Rose formalized her images of proportional sides, congruent angles, and the use of teacher-led lecture as a teaching strategy. There is evidence to suggest that Rose did not anticipate the students' responses during her lesson because of her beliefs in the authoritarian role of the teacher.

Summary of the Growth in Subject Matter Knowledge of Similarity

The prospective teachers' SMK changed while involved in the LPS. Their images of *what* and *how* to teach became more evident as they progressed in the study. At first, the prospective teachers related images of similarity with proportional sides and congruent angles. During the LPS, some of the prospective teachers' approaches to teaching were affected by the interactions and opportunities to reflect and rewrite their lesson plans. For example, Rose moved from a procedure-only approach of teaching to having an image about the importance of a conceptual activity. While she did notice the importance of this activity, she never formalized this teaching strategy within her knowledge of how to teach and still believed that the best approach was a teacher-led lecture. Most of the prospective teachers made images of activities that allowed their students to derive mathematical similarity. During the LPS, most of the prospective teachers anticipated their students' responses to their activities. The group planning allowed the participants to make new images of *what* and *how* to teach by listening, discussing, and reflecting on their ideas. After the group planning stage of the LPS, everyone collected Ava's images of the similarity postulates and Mary's images of modeling and indirect measurement. Instances of folding back on various aspects of their primitive knowledge or images helped the prospective teachers to understand the importance of the concepts and approaches they used. The features of folding back, collecting, and thickening were used in the LPS by the prospective teachers in order move to outer levels of understanding.

DISCUSSION AND CONCLUSION

The purpose of this chapter is to summarize and discuss the findings related to the prospective teachers' SMK of similarity during a lesson planning cycle. The chapter's findings are divided into two sections: the SMK of similarity relied on and the growth in the SMK of similarity. The last part of this chapter discusses the implications that these findings have for teacher preparation.

The conceptual frameworks used to analyze the data help explain growth of the prospective teachers SMK of similarity and the nature of the SMK they rely on when planning to teach a lesson on the topic. Both frameworks for the study incorporated Even's (1990) analytic framework of SMK for teaching a specific topic in mathematics. In focusing on the first research question, Even (1990) is used in conjunction with Shulman's (1986) three forms of teacher knowledge for looking at the SMK the prospective teachers rely on when planning a lesson to introduce the topic of similarity. The second conceptual framework uses Even (1990) with Pirie and Kieren's (1994) model of mathematical understanding that was adapted by Berenson, Cavey, Clark, and Staley (2001) to teacher preparation along with the features of collecting (Pirie & Martin, 2000) and thickening (Martin, 1999). The second framework analyzed the growth in SMK of similarity as the prospective teachers were engaged in LPS.

The Subject Matter Knowledge of Similarity Relied on

In looking at the essential features that the prospective teachers relied on when planning the lesson, two themes were evident: same shape but different sizes and the role of the side lengths compared to angle measures. The first theme dealt with the idea of similar triangles having the same shape, but not necessarily the same size. The prospective teachers took this generalization of similarity and began to develop their lessons. Eventually they came up with a particular definition for similarity, defining their ideas of what “the same” meant. The second theme in the prospective teachers’ essential features was the roles of the side lengths compared to the angle measures of two similar triangles. Initially, angles were ignored, but as the LPS progressed, angle measurements became more important within the prospective teachers’ lessons, however, they still placed more emphasis on side lengths.

Researchers’ discussed the fragmented understandings of prospective elementary teachers (Ball, 1988; Ball, 1990; Baturo & Nason, 1996; Cooney et al., 1998; Ma, 1999; Simon & Blume, 1994; Van Dooren et al., 2002). These studies, while focusing on prospective elementary teachers, lend support to identifying that prospective secondary teachers may also suffer from not having a “web of interconnected ideas” (Ball, 1988, p. 22). It is evident by their generalizations of essential features such as “same shape but different sizes” and the role the side lengths play in determining whether two triangles are similar, that the prospective teachers do not have strong substantive and syntactic structures. Ma (1999), in looking at U.S. elementary teachers teaching approaches, referred to this as “splintered teaching” (p. 53). By focusing on one aspect of similarity, such as side lengths, the prospective teachers establish their algebraic prowess and use this strength in developing their lesson. However, since they devoted more emphasis to side lengths, they downplay the importance that angle measures have in the concept of similarity. The idea that the prospective secondary teachers are guided by their content knowledge and beliefs were reflective of research done on elementary school teachers

(Baturo & Nason, 1996; Cooney et al., 1998; Ma, 1999; Van Dooren et al., 2002). Baturo and Nason (1996) suggested that prospective teachers' lack of SMK would limit their ability to help their learners develop integrated and meaningful understandings of mathematical concepts. Therefore, it is suggested that when the secondary prospective teachers fixate on generalizations and a single aspect of a complex concept such as similarity, they too will hinder their effectiveness in teaching mathematics.

The different representations that the prospective teachers used when planning the lesson reflected the ideas that the prospective teachers felt most comfortable in using. Over half of the representations used by the prospective teachers involved either an isosceles or right triangle. Because the prospective teachers are in their first methods course, it is not surprising that they have a limited repertoire of instructional representations for mathematical concepts (Ball, 1990; Borko et al., 1992; Eisenhart et al., 1993). Prospective teachers are influenced by every aspect of their SMK while choosing appropriate representations for their teaching, especially their belief structures (Cooney & Wilson, 1995). Cooney and Wilson (1995) suggest the prospective teachers' belief structures are influential in the representations and practices that employ in the classroom. The limitations in the prospective teachers' representations imply that their students might not get a robust understanding of similarity.

The alternate ways to approach the topic of similarity relied heavily on pattern recognition. Many of the prospective teachers devised a single activity that would help their students to see patterns, and relationships that they could then use to come up with what it meant for two triangles to be similar. The prospective teachers, in discussing their activities, also began to anticipate their students' responses and reactions to their lessons. For example, Alice, Anne, and Rose's limited SMK caused them to make unrealistic assumptions about their activities. They used activities with only a few examples that fixated on procedures and expected their students to make generalizations from these examples. Eisenhart et al. (1993) pointed out that these types of examples are not conducive for students' conceptual understanding of topics.

Cooney, Davis, and Henderson (1975) pointed out that understanding a mathematical concept is different from understanding a mathematical procedure. Lakatos (1976) suggested that this type of teaching strategy was contradictive of the principles that established proof in mathematics. Ma (1999) suggested that the prospective teachers' inability to devise meaningful activities reflected their limited substantive and syntactic structures. Mary and Ava, while having only one approach, imagined their students' responses and devised their activities with these images in mind. Mary used only two triangles in her approach but she specifically chose the second one to discourage students from thinking about proportional sides additively. Research suggests that this type of additive thinking is common among students (Simon & Blume, 1994; Thompson & Bush, 2003; Tourniaire & Pulos, 1985), and thus a teachers' expectation that a single example is sufficient would most likely not bring about the target learning in students.

Ava decided to use several representations of similar triangles in her activities. She too was guided by her students' actions while engaged in her activity. Ava felt that by having multiple representations, students would direct their thinking to side lengths and angle measurements. Mary and Ava's ability to project themselves in the role of their students exemplify their substantive structures of similarity. They used their connections between angles, sides, and proportions to experience what their students might encounter.

The strength in the concept of similarity was largely focused on its application of ratio and proportions to indirect measurement. In the beginning, all the prospective teachers were able to relate similarity with a visual representation of ratio and proportion, and indirect measurement. However, it took Mary's intervention in the group interview for the others to realize that similarity had other uses such as modeling, and scaling, and indirect measurement on things other than triangles. The prospective teachers' limited knowledge of strength of similarity caused them to ignore connections to topics that their students might encounter in other branches of mathematics and science. Ma (1999) suggested that teachers' ability to recognize the strength in a concept was reflective of their basic mathematical content knowledge. Even (1993) suggested that secondary teachers' images of strengths in concepts were limited by their SMK. Mary's ability to recognize strength in similarity shifted the group's "lay-person attitude" (Ma, 1999, p. 104) from everyday uses of the word similar (e.g. plants and trees) to more meaningful applications of similarity in mathematics (e.g. finding the height of structures).

Two major themes that were evident in the prospective teachers' basic repertoire of similarity were: the recognition of differences between daily language versus mathematical language and the attempt to use procedural aspects as a means to foster generalizations. The first theme that was evident in the prospective teachers' basic repertoire was the everyday usage of the word similar as a frame of reference. The prospective teachers considered their students' existing knowledge of the word similar in its everyday use to develop the definition of mathematical similarity. Ma (1999) suggested that this type of attitude towards mathematics might hinder the prospective teachers from having successful mathematical investigations. Looking back on the episode with Rose and how she had originally thought of mathematical similarity in the context of everyday language, it is clear that her ability to think about mathematical similarity was limited. However, with constant probing and interviewer intervention, Rose began to associate, mathematically, the ideas of similarity.

The second theme that became evident during the LPS was the use of procedural aspects of similarity, such as the Side-Side-Side, Side-Angle-Side, and Angle-Angle similarity postulates, to make generalizations. Ava first discussed these ideas in her individual interview and in the group planning stage of the LPS. These "short-cuts" as they were called in the group discussions became a large part of the group's presentation and their final individual lesson plans. However, the approach that the prospective teachers used was to give a similarity postulate and an example of how it worked without any connections to the proportional sides or congruent angles. Shulman (1986) suggested that such use of "short-cuts" de-emphasizes the importance of similarity in the lesson and that it is more important to understand why something is so, as opposed to how it is used. In their teaching of the similarity postulates, the secondary teachers exemplify many of the procedural approaches discussed by other researchers (Ball, 1988; Ball, 1990; Borko et al., 1992; Eisenhart et al., 1993; Even, 1993; Ma, 1999). The prospective teachers rely on the rote understandings of the similarity postulates to help their students' understanding of similarity. By relying on these rote understandings, the prospective teachers might influence their students to do the same, which could overshadow the definition of similarity and the importance of both side lengths and angle measurements.

The prospective teachers' knowledge and understanding of similarity were reflected in the activities and examples that they chose for helping their students develop an understanding of the concept of similarity. When recalling what they remembered about similarity, most of the prospective teachers' responses were immature and of a procedural nature. However, as they planned their lessons their ideas became more concrete and robust. Their procedural knowledge still played a large role in some of their lessons, but many did try incorporating a conceptual aspect within their lessons. However, the prospective teachers' activities and teaching style mirrored those reported by Borko, Eisenhart, Brown, Underhill and Agard (1992) in that they eventually focused on teacher-led lecture and algorithmic procedures. Cooney, Shealy, and Arvold (1998) suggested that prospective teachers' belief structures along with their limited substantive structures influenced their discussions about how to teach similarity. The prospective teachers might not be able to go beyond teaching procedures and algorithms because of their lack of a conceptual understanding of similarity (Borko et al., 1992; Eisenhart et al., 1993).

In looking at the prospective teachers' representations, there seems to be a lack of mathematical and visual correctness in their examples. Attempts to show particular aspects of similarity or give different approaches to looking at similarity resulted in the prospective teachers making mathematical mistakes in their visual representations. It is important to point out that to have knowledge about similarity, teachers should have knowledge in other areas of mathematics. Research has shown that teachers' explanations and answers to problems are reflective of what they know and, hence, of what their students learn (Ball, 1990; Berenson et al., 1997; Borko et al., 1992; Borko & Putnam, 1996; Cooney et al., 1998; Cooney & Wilson, 1995;Eisenhart et al., 1992; Ma, 1999; Rider, 2002). Graeber (1999) suggests that in order to build the connections to help the prospective teachers in constructing knowledge and understanding of concepts like similarity, teacher education programs need to reemphasize SMK, beliefs, and teacher education in mathematics methods courses. “Without a solid knowledge of what to represent, no matter how rich one’s knowledge of students’ lives, no matter how much one is motivated to connect mathematics with students’ lives, one still cannot produce a conceptually correct representation” (Ma, 1999, p. 82).

The prospective teachers’ knowledge about mathematics and establishing proof in geometry were vague. In some of their lesson plans, the prospective teachers did allow students to harvest their own knowledge, but as to how this was handled varied greatly. There were instances in which, after students derived the meaning of similarity, they were then told the definition by the teacher. Repetition of such instances in classroom teaching might cause students to wait for the correct answer, instead of engaging in an exploration for the answer. Ma (1999) suggested that the teacher’s own knowledge of mathematical inquiry guides their images of mathematical learning. The prospective teachers, while trying to use teaching strategies that they have been taught were “good” (e.g. inquiry, discovery, etc.) still regarded the teacher and textbooks as authorities in establishing truth in the classroom. These actions reflect largely what Ball and Feiman-Nemser (1986) addressed with prospective elementary teacher programs. Research suggests that these issues of proof and inquiry in the classroom are instilled in the prospective teachers’ belief structures (Ball, 1991; Cooney, 1996; Cooney et al., 1998, Cooney & Wilson, 1995; Mullis et al., 2000).

There were several common themes among the prospective teachers' SMK. They all worked with their SMK in ways that many of them had not worked with before, by reflecting on their beliefs, content knowledge, substantive knowledge, and syntactic knowledge of similarity and development of lesson plans. In planning their lessons, the amount of SMK that they relied on increased and varied as the LPS progressed. It seemed that the prospective teachers in question had a grasp of ratio and proportion, and some good ideas about teaching students. However, they seemed to lack exactness in their visual representations and strength in the concept of similarity for any topic other than ratios, proportions, and indirect measurement. The knowledge base that the prospective teachers used varied from their propositional knowledge to instances where their case knowledge yielded different practices. In using their propositional knowledge, the prospective teachers looked at their experiences in their teacher education program or other professional experiences. Their case knowledge was heavily influenced by their experiences as mathematics students, in looking at the ways that they had learned or been taught, and what was appropriate for the LPS. Within these interviews it is easy to see that the prospective teachers began to change their ideas about similarity, thus causing them to rely more heavily on their SMK to generate and explain examples, definitions, and activities.

The Growth in Subject Matter Knowledge of Similarity

The starting place for growth of *what* and *how* to teach for all five participants was different. Alice's images of *what* and *how* to teach consisted of patterns and finding relationships while focusing on the essential features of proportional sides and congruent angles of similar triangles. Anne's teaching strategies of pattern recognition and discourse conveyed her primitive knowledge of mathematics and similarity with her ideas of "same shape but different sizes." Anne focused her proportional reasoning skills on solving for missing side lengths in similar triangles. Ava's images of *what* to teach relied on her proportional reasoning skills and her knowledge of the similarity postulates. Ava's teaching strategies were largely based on discovery methods and discourse that was focused on the conceptual learning of similarity. Mary's images of *what* to teach included examples of ratios, proportions, congruent angles, and indirect measurement. The teaching strategies that she incorporated involved exploratory examples and teacher led lecture, with relevant applications involving indirect measurement. Rose's images of *what* to teach began with the everyday meaning of the word similar, and later related ratios and proportions. Rose used a teacher-led lecture as her teaching strategy that followed the pattern of definition, examples, and more definitions. It is important to note that since their starting places for growth were different, this had an effect on the outer levels of the teacher preparation model. For example, Rose came into the LPS with formalized beliefs of *how* to teach and while she was able to appreciate approaches such as the one Ava used, she did not change her initial image.

Research suggests that many of the first images made by the prospective teachers' were reflected on their content and beliefs (Ball & Mosenthal, 1990; Cavey, 2002; Cooney et al., 1998, Cooney & Wilson, 1995; Graber, 1999; Ma, 1999). In many instances the prospective teachers relied heavily on their primitive knowledge in making images of *what* and *how* to teach. The prospective teachers' SMK ranges from insufficient to rich and flexible (Ball & Mosenthal, 1990; Cavey, 2002; Clark, 2001;

Even, 1990; Even, 1993; Ma, 1999). Some of the prospective teachers in the study had direct teaching experiences that helped them in the LPS. Ma (1999) suggested that in order for teachers to develop their SMK, they must be given opportunities that stress its role in teaching. For example, Ava's ideas of directing her teaching style towards different learners came from teaching in an after-school program. Consequently, she created a lesson that included more than one example, a strength in terms of students' likelihood for learning. Mary and Ava were the two prospective teachers who had images of anticipated student responses to their activities and adjusted them accordingly. Mary saw that some students could be led to believe that proportional sides were found additively and used another example to discourage that type of thinking. Ava actually went through "stacking," a concept that she thought her students might try in order to come to an understanding of congruent angles. Rose, Anne, and Alice, in the data, did not anticipate students' responses to the degree of Mary and Ava. Therefore, during the course of teaching, their beliefs about mathematics teaching and learning, illustrated in their lesson plans, reflect this lack of anticipation (Mullis et al., 2000).

Since the outer levels of the teacher preparation models were determined by what was inferred to be the prospective teachers' initial images of *what* and *how* to teach, each prospective teacher changed their images differently. For example, it was possible for Ava to make images of an activity and then for Rose to have an image about the activity during the group's planning stage. Research indicates that Rose may not have formalized Ava's image because it was not part of her belief structures (Ball, 1990; Clark, 2001; Cooney et al., 1998; Cooney & Wilson, 1995). All of the prospective teachers made images during the LPS about *how* to teach similarity. Alice made an image about how to show the importance of the relationship between the proportional sides and angles measurements. Anne made an image, during the group planning stage, about doing a postulate then an example, instead of doing all the postulates then example in their presentation that changed the approach of the presentation. Ava's image that she made moved her from a conceptual approach with generalizations to one that included more procedures and algorithms. Mary's folding back helped her to have an image of an indirect measurement problem similar to a boat mast problem that she remembered from secondary school mathematics. One of Rose's images, as previously mentioned, involved Ava's hands-on activity and how allowing students to deduce what similarity was, meant that she did not need to "teach" it because they would learn it on their own. Ball and Mosenthal (1990) suggested that prospective teachers' ability to change their images of *what* and *how* to teach depended on their own understanding of the subject matter.

Not all the prospective teachers progressed to the exact same level of understandings, or in the exact same manner, within the teacher preparation model. Their levels of understanding depended on their participation within the LPS. For example, the outer levels of understanding and the other features are hard to determine for Alice because she was not as active as other members of the group. Alice's participation in the group was not as vocal as some, and it makes it more difficult, after the individual interview, to determine the images she had and other levels of understanding she reached. In looking at the data, everyone seemed to fold back to their knowledge of mathematics within the LPS to make or have images about aspects of the lesson plan that they had not originally considered in *what* to teach. For example, Alice, Anne, Mary and Rose all collected images of Ava's ideas of the similarity postulates. Ava made images of *how* to teach and incorporated more procedural aspects from members in the group. In each case of folding back, the participants used their primitive knowledge or previous images to help them reach an outer level of understanding.

The prospective teachers engaged in the LPS to introduce the topic of similarity changed their initial concept images of what and how to teach the topic. The data collected showed that all the participants gained some benefit from the LPS. The LPS allowed the prospective teachers to makes shifts in their understanding by discussing mathematics and how to teach mathematics with interviewers and their peers. All the participants made images to help in their understanding and went beyond image making to actually noticing properties in the substantive structures of similarity. Again, it is important to note that all the outer levels and the folding back features were contingent on the assumption made about the prospective teachers' primitive knowledge and initial images in the SMK of similarity. Research has shown the more teachers know about subject matter, the more likely they are to have aspects in their teaching that address meaningful teaching and student learning (Borko & Putnam, 1996; Borko et al., 1992; Eisenhart, 1993; Even, 1990; Even, 1993; Ma, 1999)

Implications for Mathematics Teacher Preparation

The present research has four major implications for mathematics teacher preparation. The first is reflective of the prospective teachers' generalizations and representations in their lesson plans. The second addresses the need for teacher preparation activities to be grounded in various contexts, such as those in LPS. The third addresses the backgrounds of the prospective teachers and their ideas of *what* and *how* to teach. The last looks at the prospective teachers' substantive and syntactic structures of the concept image of similarity. Each of these implications is addressed below.

Conclusion 1: Prospective teachers' ideas of what and how to teach similarity focus on procedural generalizations conveying meanings. During the LPS, the prospective teachers focused on generalizations to define the concept of similarity. They used one or two examples and expected students to make generalizations about similarity based on these examples. This type of teaching strategy contradicts the principles of proof and inquiry in mathematics (Ball, 1990; Clements & Battista, 1992; Lakatos, 1976). The prospective teachers also focused their lessons on developing the similarity postulates as "short-cuts" to aid in determining whether or not two triangles are similar. It has been suggested that their mathematics courses at the university level play an important role in these generalizations. It is suggested that university mathematics courses do not stress conceptual connections, or abstractions that encourage rigorous proof (CBMS, 2001; Borko et al., 1992; Eisenhart et al., 1993). The CBMS (2001) suggested that university mathematics courses be redesigned to promote the previous ideas. It seems as if prospective teachers are entering teacher education programs with knowledge of short-cuts, tricks, and how things work in mathematics, as opposed to why they work. In support of the CBMS (2001) it is important to point out that Rose, who was double majoring in Mathematics and Mathematics Education, had a limited knowledge of mathematics and employed procedural representations. Ma (1999) suggested that with this knowledge, prospective teachers could only bring students to a level of understanding that they themselves have.

Conclusion 2: Lesson plan study gives prospective teachers opportunities to change their images of what and how to teach. Lesson plan study may be a method that teacher education programs could use to find ways to address prospective teachers' SMK needs. The prospective teachers in the LPS extracted different images about *what* or *how* to teach similarity. Teacher education programs need to find ways to benefit from the vast knowledge and backgrounds of all their prospective students. In using a method like LPS, teacher education programs can incorporate various mathematical topics, contexts, and foci to be central issues for the prospective teachers thus allowing all prospective teachers to begin the study with some knowledge of the activity (Berenson et al., 1997; Cavey, 2002; Cavey, Berenson, Clark & Staley, 2001). It affords prospective teachers the opportunity to review their mathematical content knowledge, substantive structures of mathematics, syntactic structures of mathematics, and illuminates their personal beliefs about mathematics. It also allows them to work collaboratively with their peers and share images of *what* and *how* to teach. It is important to point out that if prospective teachers are not given these opportunities to address their SMK deficiencies in teacher education programs, then it is unlikely that they will address them when they become teachers (Borko et al., 1992; Borko & Putnam, 1996; CBMS, 2001; Cooney & Wilson, 1995; Eisenhart et al., 1993; Graeber, 1999; Lappan & Even, 1989; Ma, 1999; Shram et al., 1988).

Conclusion 3: During the lesson plan study, images of what and how to teach similarity have to contend with prospective teachers' backgrounds and belief structures. The prospective teachers came into the LPS with approximately 15 years of beliefs about education and teaching (Zeichner & Tabachnick, 1981). Some of these beliefs were positive and others were not. The prospective teachers in the LPS relied heavily on these beliefs during the study and changed them minimally. For example, Rose considered using an activity in her final lesson plan, but she made it apparent in her teaching philosophies at the end of the methods course, that the teacher was a leader and the "back bone" of the classroom. Teachers that are inclined to this authoritarian role in the classroom are more likely to tell students answers, show them how to do mathematics, and be less frequent in devising activities that help students construct meaningful mathematics (Ball, 1990; Ball & Mosenthal, 1990; Cooney et al., 1998; Borko et al., 1992; Eisenhart et al., 1993; Even, 1993; Ma, 1999). Rose was unable to overcome her preconceptions about how students learn and the role of a classroom teacher, however, through LPS she was exposed to different teaching strategies and approaches.

Conclusion 4: The substantive knowledge and syntactic knowledge of prospective teachers are limited and fragmented. At the beginning of the LPS the prospective teachers did not relate the proportional sides of two similar triangles with congruent angles. If they did it was considered an afterthought, when the sides had been proven proportional. However, during the course of the LPS, the prospective teachers shifted their knowledge of this mathematical concept to include angle measures. The shifts in their images of similarity suggest that the prospective teachers had many of the necessary components of the mathematical content knowledge needed to teach similarity (e.g. ratio/proportion, segment measurement, angle measurement, indirect measurement, and the similarity postulates), but were constricted by their fragmented understandings of the substantive structures. The prospective teachers' understanding of similarity included the use of the everyday meaning of the word similar. Research studies have shown these styles of thinking are also evident in prospective elementary teachers (Ball, 1988; Ball, 1990; Cooney et al., 1998; Ma, 1999; Simon & Blume, 1994; Van Dooren et al., 2002). It has been suggested that prospective teachers with limited and fragmented substantive and syntactic structures focus on surface differences, which suggests that their understanding is rote and algorithmic (Ball, 1988; Borko et al., 1992; Eisenhart et al., 1993; Ma, 1999). It has been shown in this study (e.g. Rose) that this type of understanding may lead to confusion and inappropriately connecting mathematical concepts by their uses in everyday language.

Conclusion

Like many of the prospective teachers that enter teacher education programs, Alice, Anne, Ava, Mary, and Rose each possess different SMK in mathematics. Their mathematical content knowledge, substantive knowledge, syntactic knowledge, and beliefs have developed throughout years of mathematics courses and having personal experiences inside and outside of the classroom. Looking at a topic as important as similarity has shown that we cannot assume that secondary prospective teachers have the necessary SMK needed to benefit all of their future students. Hopefully this research has shown that there are areas of promise in prospective teachers' SMK of similarity and areas that need attention.

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APPENDICES

APPENDIX A

Informed Consent Form

Pre-service Teachers

Learning What and How to Teach: Knowledge Base of Teaching Secondary Mathematics
Dr. Sarah B. Berenson, Principle Investigator

You are invited to participate in a teaching experiment. The purpose of this experiment is to understand how preservice teachers learn to teach secondary mathematics. In doing regular class assignments, you will contribute to this experiment by:

1. Individually designing three lessons and participating in follow up interviews
2. Collaborating with a small-group on lesson designs and participating in follow-up interviews
3. Presenting group plans to the whole class
4. Individually revising group plans for anonymous web publication

All interviews and class presentations will be videotaped. The planning sessions and follow up interviews will be scheduled outside of regular class time, but are required as class assignments. Each planning session and follow up interview will take approximately 1 1/4 hours to complete.

There will be no risk associated with your participation in the experiment and previous participants' comments indicate that these are meaningful learning experiences. The knowledge we gain from your ideas will add to the knowledge base in mathematics education, especially with regard to teaching secondary mathematics.

The information we derive from the class activities and assignments will be kept strictly confidential. It will be stored securely in a locked file and will be made available only to the researchers unless you specifically give permission in writing to do so. No reference will be made to your name either in oral or written reports and transcripts that could link you individually to the study.

You are free to withdraw from the study at any time; however, you will still participate in all of the activities, including the individual and group interviews, since they are class requirements. If you have any questions at any time, you may contact Dr. Sarah Berenson at 515-2013. Her address is 315 Poe Hall, NC State University. If you feel you have not been treated according to the descriptions in this form, or your rights as a participant in research have been violated during the course of this project, you may contact Dr. Matthew Zingraff, Chairperson of the NCSU Human Subjects Committee, Box 8101, NCSU Campus.

CONSENT

I have read and understood the above information. I have received a copy of this form. I agree to participate in this study.

Participant's signature ___________________________________ Date________________

Investigator's signature _________________________________ Date________________

APPENDIX B

Interview Protocol for Spring 2002 EMS 203 Similarity Lesson-Planning Activity (Individual Task)

Name(s) of Interviewer(s):
Name of Participant:
Date:

I. Pre-Task Interview

1. Explain the purpose of the study

   - To understand how pre-service teachers would introduce the topic of similar triangles to a geometry class

   - To understand how pre-service teachers learn about geometry-related concepts and how they learn about teaching geometry-related concepts

2. Explain the process for this part of the study.

   - Pre-task interview

   - Lesson-planning activity

   - Post-task interview

3. *What do you recall from school about similarity? Similar triangles?*

4. *What do you remember about what your teacher showed you on paper, on the board, or on the overhead about similar triangles?* Ask the student to sketch this on paper. Probe if nothing about the notation:

5. *What problems, if any, did you have learning about similarity in school?* Ask the student to sketch this on this on paper.

6. *In your mind, how is similarity related to ratios?*

7. Explain to the student where the student will plan a lesson. Give them the lesson plan Task and explain as needed. Explain what materials will be available to help the student plan the lesson. Ask the student to make notes about the lesson so that the student will be able to explain it in the post-task interview and then be able to use those notes for the group activity later.

8. *Do you have any questions you want to ask me?*

II. Lesson-Planning Activity

Limit this to 45 minutes

Instructions for the Individual Lesson-Planning Activity

Plan a lesson that **introduces the concept** of similar triangles to a geometry class of eighth, ninth, or tenth graders. In the lesson connect the concept of similarity to ratio and proportion.

Use any of the materials that are on the table to help you plan your lesson. Please outline your lesson plan on paper so that you can refer to it later in the interview.

You can assume that the students in the class understand ratio and proportion and know how to use a compass and a protractor.

III. Post-Task Interview

1. *Explain your lesson plan.* (Or, *So what did you come up with?)* Probe for explanations, rich in detail, concerning the representations they would use and their justification for the lesson plan.

2. *How would you describe similarity to a geometry student*?

3. *How would you describe the relationship between similarity and congruence*?

4. *What helps you learn about concepts in geometry*?

5. *What helps you learn about teaching concepts in geometry*?

APPENDIX C

REMINDERS FOR BEFORE INTERVIEW:

- Photocopy the lesson notes for the individuals who are in the group and have them ready to pass out when the students arrive.
- Set up the flip chart.
- Get two tapes ready.
- Put Do Not Disturb sign on door.

Pre-Task: Go over the instructions and let them ask questions. Turn on the camera right before you leave.

After the one-hour planning session, put in a new tape.

Protocol

1. So what did you come up with?
2. How do you plan to involve your students? What are their roles?
3. How have you changed the way you think about similarity? (Ask everyone individually.)
4. How have you changed the way you think about ratios as they relate to similarity? (Ask everyone individually.)
5. At this point, what is the most important thing you need to learn to become a good teacher of geometry-related topics?

Instructions for Group Lesson-Planning Activity

Your task is to plan, as a group, a lesson that begins with a hands-on student activity to introduce the topic of similar triangles to a high-school geometry class. In the lesson, connect the concept of similarity to ratio and proportion. You can use your notes from your individual plans as a starting point for your discussion, but you are not restricted to those ideas. You can assume that the class period for the lesson is about an hour, that the students understand ratio and proportion, and that the students know how to use a compass and protractor.

You will have one hour today for planning, and you may want to meet later on your won to prepare for you EMS 203 class presentation. You can assign group roles (for example, moderator, secretary, materials coordinator, etc.) for the planning period if you want to. You can make notes on paper, overhead transparencies, and flip-chart pages and take them with you.

During the planning period

- You should focus on how you want to teach and organize the lesson, discussing how the mathematics relates to what the students and teacher would be doing, but you should not get bogged down in coming up with specific numbers for examples. You can add details later.
- Everyone should participate in the discussion and ask questions if you do not understand an activity or a problem that someone is describing.
- At the end everyone should volunteer to complete some part of the lesson that you as a group have agreed on or volunteer to prepare something for the EMS 203 class presentation.

APPENDIX D

Expectations of your first individual lesson plan

Your task is to plan (individually) a lesson that begins with a hands-on activity to introduce the topic of similar triangles to a high-school geometry class. In the lesson, connect the concept of similarity to ratio and proportion. You may assume that the class period for the lesson is about an hour, that the students understand ratio and proportion, and that the students know how to use a compass and a protractor.

You may either hand write or type your lesson plan, however, it must be neat, legible, and professionally presented (yes, grammar and spelling matter!), as well as being mathematically accurate and correct.

Develop your lesson, including the specific activities, examples, exercises and/or problems you'll use and/or assign (including their directions), as well as anticipated results, solutions, etc. (with work shown and explanations given). Be sure the activities, examples, exercises and/or problems match the objective(s) of the lesson and put them in a context whenever possible.

<u>Be sure to include the following:</u>

- ♦ Statement of the objective(s) of the lesson
- ♦ Rationale for the lesson
- ♦ List of materials needed
- ♦ Hands-on opening activity to introduce the topic
- ♦ Description of ALL activities, including the homework to be assigned (e.g. specific problems and their solutions), and a description of what the students will be doing and what the teacher will be doing during each activity/part of the lesson.
- ♦ Questions you plan to ask and anticipated responses
- ♦ Clear explanations (of activities, of problems, of examples, of mathematics, etc.)
- ♦ Logical sequence of events (e.g. progressing from students' real world notion of similarity to a precise mathematical definition of it, or from easier to more difficult)
- ♦ Student involvement (actively engage the students as much as possible)
- ♦ Strong verbal communication of mathematics: appropriate terminology, mathematically accurate statements
- ♦ Accurate representations
- ♦ Work out ALL examples, exercises, problems (including homework).
- ♦ Tell how you will "close" the lesson

APPENDIX E

Report and Data Review

Former EMS 203 Research Participant,

I would like to invite you to review some of the material that was gathered during your participation in the proportional reasoning lesson planning activities in the Spring of 2002. This material consists of transcripts from your videotaped interviews and written artifacts gathered from the interviews and classroom exercises. Also, there is a summary of the findings and other information pertaining to the data that was collected.

I would like to assure you that information that was derived from class activities and assignments are kept strictly confidential. Pseudonyms, or aliases, have been used to protect your identities throughout the research process and no reference to your name has been given in any oral or written reports.

Personally, I would like to thank you for your participation and your contribution to the research efforts at North Carolina State University. Your participation and input have been vital in my research and I would like to give you this opportunity to review what has been written. Please respond no later than June 13, 2003 if you would like to review the data and report.

Thank you very much,

Clarence E. Davis