

ABSTRACT

AL-IBRAHIM, ANWAR A. Optimizing Roof Maintenance and Replacement Decisions. (Under the direction of Dr. David W. Johnston.)

The objective of this research is to develop a Decision Support System that helps allocation of available funds to optimize roof maintenance strategies. The Decision Support System analyzes different maintenance alternatives available for each roof element and chooses the alternative that maximizes the benefits due to savings resulting from postponing element replacement.

The analysis uses the Life-Cycle Cost to calculate the Reduction in Uniform Annual Cost (RUAC) that is used as the economical decision criterion to select the most economical maintenance alternative for each roof element at different condition states. The analysis utilizes integer linear programming to optimize the selection of maintenance actions for the different elements by maximizing the RUAC under budgetary constraints. Excel and AMPL software are used to implement the analysis.

This study uses the roofing systems of 28 buildings on the North Carolina State University campus as a model to develop the Decision Support System. Parameters necessary to develop the model are estimated. The analysis is tested on the available data base.

OPTIMIZING ROOF MAINTENANCE AND REPLACEMENT DECISIONS

by

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This work is dedicated to my dearly loved parents Adel Al-ibrahim and Maria Gallerani who supported me spiritually and financially throughout my journey.

BIOGRAPHY

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LIST OF NOTATION

ACSSL	= Average Condition State Service Life of
AMPL	= A Mathematical Programming Language
ASL	= Additional Service Life
AUCST	= Annual User Cost
c	= Element condition state before maintenance
CS	= Condition State
DM&R	= Department of Maintenance and Repair
DSS	= Decision Support System
ECSDF	= Element Condition State Damage Factor
EL	= Expected Life
EUAC	= Equivalent Uniform Annual Cost
EX	= Extension in element service life
f	= Inflation rate
FCI	= Flashing Condition Index
i	= Building number
ICI	= Insulation Condition Index
INCBEN	= Incremental Benefit-Cost
IRC	= Initial Replacement Cost
j	= Roof section
k	= Element condition state after maintenance
l	= Maintenance element

LCC	= Life-Cycle Cost
MA	= Maintenance Action
MAUCST	= Maintenance Action Unit Cost
MCST	= Maintenance Cost
MCI	= Membrane Condition Index
MMC	= Major Maintenance Cost
OPBRIDGE	= Optimum Bridge
P_{cc}	= Probability of a condition state c quantity staying in state c after one year
Qty	= Quantity
r	= Nominal rate of return
RAMP	= Roof Asset Management Program
RCC	= Replacement-Cycle Cost
RCI	= Roof Condition Index
$RCI_{improved}$	= Improved Roof Condition Index
RCST	= Re-roofing Cost
REUAC	= Reduction in Equivalent Uniform Annual Cost
RFUAC	= Re-roofing Uniform Annual Cost
RM CST	= Regular Maintenance Cost
RRFUAC	= Reduction in Re-roofing Uniform Annual Cost
RRR	= Real Rate of Return
RSL	= Remaining Service Life
s	= Roof section
SL	= Service Life of element

VB = Visual Basic program

$X_{(i,j,l,c,k)}$ = Decision variable for building i, roof section j, element l, initial condition state c,
and final condition state k

$X(t)$ = Vector of the CS of the element at the start of year t

μ_c = Average years an element stays at a condition state before deteriorating to a
lower condition state

1. INTRODUCTION

The condition and quality of buildings mirror the quality of life and the owner's awareness of building maintenance [1]. Each building element has an approximate life span. Experience has shown that proper maintenance can not only ensure a normal life span but can add additional life to the span expected. Proper planning and allocating the right resources for maintenance - workers, materials, equipment, and funds - will help ensure the availability of resources when a component is to be maintained, rehabilitated or replaced [2].

During a building's life there are many elements to be maintained. These elements can fall under the following categories: 1) General construction (roof maintenance, window caulking, carpet replacement, repainting, etc.), 2) Mechanical construction (HVAC, boilers, electrical, plumbing, etc.), and 3) Custodial aspects (routine cleaning services).

1.1 Problem Statement

Roof Maintenance Management is typically a major concern for the owners of facilities. Decisions are too often made only in reaction to a problem, such as a leak. The use of computer programs is limited to information systems that tabulate needs from inspections and tabulate needs from maintenance schedules. When resources are limited, a tradeoff between maintained elements will occur. The tradeoff is to give some elements maintenance priority over other elements to stop the deterioration beyond acceptable users' standard. Currently the tradeoff is based on expert opinion.

Roof Maintenance Management should be aided by a Decision Support System (DSS) which has the capability of optimizing the work performed on each element to reduce the elements' maintenance life cycle costs over the long run. The tradeoff will then be based on expected costs and benefits associated with the work performed on each element for it to be maintained at an optimal, and no less than minimum acceptable, condition state.

1.2 Research Objectives

The objective of this research is to develop a DSS that optimizes roof element maintenance Condition State (CS). Concepts of economic analysis will be used to evaluate the different maintenance options. In addition, a methodology for analyzing maintenance cost and deterioration rate data will be implemented to optimize element condition state alternatives.

This study will be directed toward the proper allocation of funds for building roof maintenance work for North Carolina State University buildings by using a DSS. The first step is to identify the availability of data and information required for developing the DSS. The DSS is then developed to optimize the allocation of available funds for proper roof maintenance and replacement work. The system could also provide information on the action required on a roof element depending on its status. The action suggested might be routine maintenance or replacement, based on the level of improvement required.

1.3 Overview of Proposed Model

This study will use the roofing systems of 28 buildings on the North Carolina State University campus as an inventory to develop the DSS model. The campus has many more buildings but additional data will be needed to implement the DDS on the entire building inventory.

The proposed model will analyze maintenance costs and deterioration rates as obtained from the North Carolina State University Department of Maintenance and Repair (DM&R) database or as provided by personnel working in the department. The analysis will compare the different maintenance activities based on their costs and expected benefits to determine the most economical maintenance action for each roof element.

The model then takes the available budget of the DM&R into consideration and tries to optimize the choices of different maintenance actions taking into account the available funds, and the number and quantity of roofing elements to be maintained.

2. LITERATURE REVIEW

2.1 Related Literature in Roof Management

Roof Management is currently performed by using computer programs that tabulate the available resources and provide the user with a preventive maintenance program that reduces maintenance cost. Examples of these software programs are summarized in this section.

2.1.1 RoofManager

RoofManager [3] is a program that facilitates creation of a database of roof information for multiple buildings at multiple sites for a single client. Information about each roof sector is entered. Examples of the information entered are: roof area, roof age, construction and historical data including drawings, roof designer and contractor that installed the roof, contractors that performed any repair work on the roof, repairs performed, unit prices, and quantities.

This software is a helpful tool used to track and manage multiple sites and buildings and their related repair budgets, deficiencies and repair orders. In addition the database can incorporate site plans, drawings, and digital photos. Also, this software provides the user with reporting capabilities by providing summaries and detailed roof information to support roofing expenditures and budgetary plans.

2.1.2 Garland Roof Asset Management Program (RAMP) Computer Software

This computer program creates a data base that includes construction details, inspection reports, roof maintenance histories and digital documentation of problem areas [4]. RAMP computer software will then analyze the data and propose recommendations based on specific user needs, expectations, and budget. The software provides different repair and replacement solutions. The user is then to choose which solution best fits the available budget.

No work has been done toward maximizing the benefits gained from extending the roof life through maintenance while working within the available budget by using an optimization algorithmic. Parallel work is found in bridge management where research has been done to provide systems to optimize the allocation of limited budgets to maintenance work while maximizing the benefits resulting from the investment.

2.2 Related Literature in Roof Maintenance

2.2.1 Roofer

Roofer is a roofing maintenance management system developed by the U.S. Army Construction Engineering Research Laboratory with the assistance of the U.S Army Cold Regions Research and Engineering Laboratory and the U.S. Army Engineering and Housing Support Center [5]. Its purpose is to provide engineers with data and procedures to aid in practical decision making for cost-effective maintenance and repair of building roofs. Roofer provides the user with a practical tool for evaluating roofs, determining maintenance priorities, and selecting repair strategies that ensure the maximum return on investments.

Roofer uses a Roof Condition Index (RCI) to evaluate the overall condition of a roof section and to compare between different roof sections. The RCI is calculated by combining the membrane, flashing and insulation indexes, MCI, FCI and ICI respectively. Each of the three indexes (MCI, FCI and ICI) provides a measure of the component's ability to perform its function, the needed level of repair and the potential for leaks. The three component indexes have a direct relationship in determining the needs for maintenance and repair of a roof section, where the RCI provides an overall indication of the maintenance and repair needs. The RCI is based on a scale from 0 to 100 with 100 indicating that only routine maintenance is needed. The three component Indexes also range numerically from 0 to 100, where 100 represents excellent condition. The MCI and FCI [6] are determined by visual inspection, while the ICI is determined by evaluation techniques such as infrared, electrical capacitance or nuclear moisture measurement.

RCI is calculated using Eq. 2.1 where 70% of the total RCI weight is given to the lowest of the three indexes (MCI, FCI and ICI), while each of the other two indexes contribute 15% of the weighting.

$$\text{RCI} = (0.7 \times \text{Lowest condition index}) + (0.15 \times \text{Sum of remaining condition indexes}) \quad (2.1)$$

Roofer uses the RCI to describe the overall condition of a roof section and by combining the value of the three indexes (MCI, FCI and ICI) it gives the user an indication of the level of repair needed Table 2.1

Table 2.1 Maintenance, repair and replacement recommendations

RCI	Corrective Action
86 -- 100	Routine Maintenance
71 -- 85	Minor Repairs Needed
56 -- 70	Moderate Repairs Needed
41 -- 55	Major Repairs Needed
26 -- 40	Replacement Probable
11 -- 25	Replacement Needed
1 -- 10	Replacement Critical

The RCI and the current age of a roof section are then used to determine the roof section expected life (EL) from graphs of appropriate deterioration curves. The graphs present a series of curves representing possible roof section deterioration. The rates can differ from the curve defined by Roofer as the normal deterioration rate of a roof section. Roofer considers a roof of 20 years expected life, which corresponds to RCI=33, to be theoretically defined as "normal". After 20 years, the RCI will be in the "Replacement Probable" stage. The curves falling below the "normal" curve represent roof sections deteriorating at a faster rate and predicted to fail before 20 years, while the curves above the "normal" curve are performing better than a 20-year roof. The expected life is defined as the time from construction to the time at which the roof is expected to reach an RCI of 33. The remaining service life of a roof section is defined as the time remaining until the end of service life is reached (RCI=33) and is determined by the following equation:

$$RSL = EL - Age \quad (2.2)$$

For Roofer to determine which maintenance alternatives to perform (major maintenance or roof replacement), the additional service life (ASL) of the roof section that

will result from performing a major maintenance action is calculated using the following equation:

$$ASL = RSL' - RSL \quad (2.3)$$

Where RSL' is determined from curve that requires an RCI_{improved} which is the recalculated RCI for the repaired roof section

For the optimum maintenance alternative to be selected, a cost analysis should be made to determine if the selected alternative is cost effective. The cost to repair per year of ASL is compared to the cost per year of service life to replace the roof section by calculating a cost ratio.

$$\text{Cost of major repair/year} = \frac{\text{total repair cost}}{ASL} \quad (2.4)$$

$$\text{Cost of replacement/year} = \frac{\text{total replacement cost}}{\text{service life (20 years)}} \quad (2.5)$$

$$\text{Cost Ratio} = \frac{\text{cost of major repair/year}}{\text{cost of replacement/year}} \quad (2.6)$$

It is assumed that when the cost ratio exceeds 1.0 for roofs at an early age, then replacement is justified. However, as a roof ages, it eventually reaches a state where it will wear out because of physical changes to the material. To compensate for this, an aging factor is used to adjust the cost ratio as shown in Eq. 2.7. If the adjusted cost ratio is less than 0.8, it is best to repair. If the ratio is greater than 1.2, replacement is the preferable alternative. When the adjusted ratio falls within 0.8-1.2 the roof engineer has the option to replace or repair.

$$\text{Adjusted Cost Ratio} = \text{Cost Ratio} + (0.01 \times \text{Age}) \quad (2.7)$$

Roofer is used to provide engineers with data and procedures to aid in practical decision making for cost-effective maintenance and repair of building roofs. This system has the following limitations:

- 1- The procedures used to calculate MCI, FCI and ICI are lengthy and time consuming,
- 2- The cost analysis is a simplified approach and does not take into account the cost of money including inflation and discount rates,
- 3- The system does not allow any tracking of the deterioration of roof section elements.
- 4- The analysis is done on a yearly basis. There is no forecasting for maintenance work required in the future,
- 5- The optional range of the adjusted cost ratio from 0.8 to 1.2 is very large. This large range may result in a non-optimum maintenance alternative selection because it is based only on the roof engineers' subjective opinion.

2.3 Related Literature in Bridge Management

2.3.1 INCBEN

Farid, Johnston, Chen, Laverde and Rihani [7] investigated the applicability of the Incremental Benefit-Cost (INCBEN) Technique in allocating limited budgets to bridge improvement alternatives at the system level. A sample of 25 in-service bridges in North Carolina with varying degrees of structural or functional deficiencies were investigated. The report concluded that:

1. INCBEN is superior to empirical priority ranking methods used as guidelines for knowledgeable, responsible bridge managers who select bridge improvement alternatives under budget constraints.
2. Under budget constraints INCBEN selects near optimal sets of improvement alternative, while it selects optimal alternative if the budget restriction is omitted.
3. The report overcame the “do-nothing” alternative by ensuring that the least-cost alternative is funded first. The budget balance is then allocated in such a way as to increase the level of improvement.

The limitations of INCBEN application in a Roof Management System are similar to those in Bridge Management Systems.

1. INCBEN does not maximize the benefits expected from improvement alternatives selected under a limited budget.
2. INCBEN uses the total benefits (Net Benefit + Initial Cost) while the algorithm objective is to maximize net benefits subjected to a limited budget constraint.
3. The “do-nothing” alternative should be considered when using INCBEN in a Roof Management system for two reasons. First, it may leave out a roof that is deficient which might cause leak and result in losses to the building users. Second, INCBEN disregards the results of leaving the roof with no improvement because it assumes that the benefit of such alternative is equal to zero.
4. One year is usually the horizon for a limited budget allocation produced by INCBEN. This is considered a limited or short period. In addition to the INCBEN possible selection of sub-optimal alternative set that has a small initial cost but very high future cost due to the roof deterioration. This results because

the denominator of the incremental benefit-cost ratio only includes the first cost for every improvement alternative.

2.3.2 OPBRIDGE

Al-Subhi, Johnston and Farid [8] developed a system which forecasts and allocates an optimum budget for bridge maintenance and replacement (OPBRIDGE) considering both the owner and user costs. The system determines the optimum improvement action and time for each individual bridge in the system under various levels of service goals and funding constraints over an analysis horizon. OPBRIDGE optimizes decisions for every year in the analysis horizon using a 0-1 integer-linear programming formulation. At the end of every year, OPBRIDGE ages bridges one year and predicts the required information to allow the system to continue the analysis for the next year. The output of OPBRIDGE is a detailed bridge by bridge recommended current and future major action. It also produces a county by county output showing costs of major actions and the budget required for each county. OPBRIDGE tabular and graphical outputs show the future performance level of the bridge system over the specified study horizon.

2.3.3 Bridge Maintenance Level of Service Optimization Based on an Economical Analysis Approach

Jabreen [9] adopted an economical analysis and decision making approach in optimizing bridge routine and preventive maintenance levels of service. Level of service is a trigger condition state that signals a need for maintenance effort to be applied to the bridge

element. In this approach, different maintenance alternatives are analyzed to select the best levels of service that will maximize the benefits under a budget limitation.

A computer program in FORTRAN-77 was developed to implement the economical analysis using Benefit-to-Cost Ratio and Life-Cycle Cost methods through an integer programming algorithm to optimize the selection of levels of service for different elements. The program determines the best economical maintenance alternative for each bridge element at different condition states. The benefits and costs of the various levels of service are determined through maintenance costs, replacement costs and element deterioration rates. The parameters for estimating benefits and costs of various maintenance alternatives are estimated.

2.4 Economic Analysis Decision Criteria

Four basic decision criteria are often used for evaluating highway improvements [7]. These criteria can also be used to evaluate roof improvement alternatives.

1. First-Cost Analysis
2. Life-Cycle Cost Analysis
3. Simple Benefit-Cost Analysis
4. Incremental Benefit-Cost Analysis

The selected criterion should provide the analyst with correct and consistent results in attempting to:

1. Determine economic justification of proposed improvement alternatives. A project is deemed desirable if the present value of all project benefits is equal to or greater than the present values of all project costs

2. Compare merits of a group of mutually-exclusive option alternatives for selecting the most desirable alternative.

2.4.1 First-Cost Analysis

First-cost analysis is the simplest of the four economic evaluation techniques mentioned earlier. In this technique, alternatives are compared based on the first expenditure on roof element improvements. The alternative with the lowest first-cost is selected as the most economical disregarding any future costs.

In spite of being the simplest evaluation technique, first-cost analysis has the following disadvantages:

1. Only alternatives with identical life span, performance, and maintenance needs can be evaluated.
2. Different types of roof improvements- replacement, rehabilitation, and maintenance- cannot be compared.

For this technique to be meaningful when comparing alternatives with different types of roof improvements, the present value of future expenditures for each alternative must be determined.

2.4.2 Life-Cycle Cost Analysis

Life-cycle cost (LCC) analysis combines initial and future costs of an improvement alternative into a single present value, LCC. Future costs may include maintenance costs, rehabilitation costs, and replacement costs minus salvage value at the end of the service life.

The alternative with the lowest LCC is selected as the most economical alternative. The disadvantages of this technique are:

1. This technique cannot establish economic desirability, which is determined by comparing benefits to costs.
2. Various alternatives must be compared over the same economic horizon (time period), otherwise the analysis may be biased towards the shortest time horizon.
3. This technique should only be applied to improvement alternatives which have equal expected user benefits. Thus, life-cycle cost analysis requires that improvement alternatives be compared under similar levels of services

2.4.3 Simple Benefit-Cost Analysis

This technique compares the benefits of improvement alternatives with their costs.

The advantages of this technique when compared to the previous two techniques are:

1. It can be applied to roofs with different levels of service because it considers user costs as well as agency costs.
2. The analysis can be conducted at both roof and system levels.

The problem with this technique is estimating user and agency benefits in monetary terms.

2.4.4 Incremental Benefit-Cost Analysis

Incremental benefit-cost ratio is defined [7] as “the ratio of the extra benefits of advancing from one improvement level to the next, divided by the corresponding extra cost.” For the incremental increase in benefits to be economically justified the ratio should be greater or equal to one.

3. ROOF COMPONENTS

The primary purpose of a roof system is to protect the interior of the building from moisture and to prevent excessive heat loss. The roof structure also serves as an integral part of the structural frame and must be designed to sustain loads due to wind, snow, and rain, as well as to contribute to the building's exterior appearance. Building roofs are generally divided into two categories. Low-slope roofs with a slope of 3:12 or less, and steep-slope roofs with a slope of more than 3:12. This chapter will cover these two types of roofs and the roofing system components characterizing each type.

3.1 Low-Slope Roofs

A Low-slope roof is composed of several interrelated parts, and collectively these parts are referred to as either the roof assembly or the roof system [10]. A roof assembly includes the roof deck, vapor retarder (if present), roof insulation, roof membrane and roof covering or surfacing. The roof system includes all the components of a roof assembly excluding the roof deck.

3.1.1 Roof Decks

The roof deck serves as the structural foundation onto which a roof system is applied and may provide slope. A roof deck maybe made of wood, concrete, metal, gypsum, or lightweight cellular concrete.

3.1.2 Vapor Retarder

Vapor retarders are used to prevent moisture vapor within the building from infiltrating the insulation and condensing, thereby destroying the insulating value, and eventually causing blistering or other damage to the roof covering, Figure 3.1. The vapor retarder may be located at various points in the roof assembly depending on the type of construction. The most common position for vapor retarder is over the roof deck, just under the insulation, Figure 3.2.

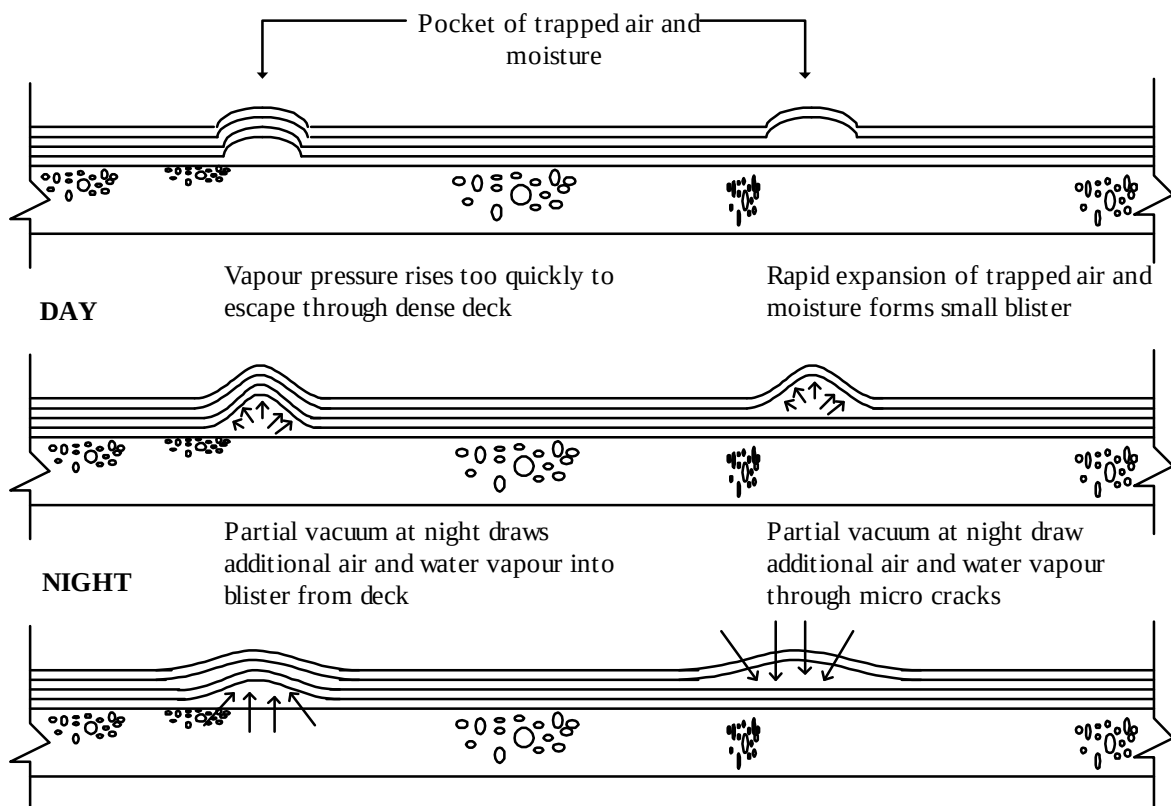


Figure 3.1 Formation of blister in roof membrane [11]



Figure 3.2 Vapor retarder [12]

3.1.3 Roof Insulation

Roof insulation provides the building with thermal resistance which will reduce the energy used by the air-conditioning and heating systems and will result in energy cost saving. Roof insulation is also used as a base for most membranes over any structural deck and to create slopes on flat roofs to allow for drainage and to help control ponding during heavy storms. Insulation is applied over the vapor barrier or directly over the roof deck. There are four types of roof insulation:

1. Rigid board - perlite, wood fiber or polyisocyanurate foam, Figure 3.3,
2. Composite board,
3. Cellular Glass Roof Insulation, and
4. Glass Fiber Insulation boards.



Figure 3.3 Rigid board insulation [12]

3.1.4 Roof Membrane

Roof membranes serve as the weatherproofing component of the low-slope roof. The membrane is installed over the insulation or directly over the roof deck. Common roof membrane types are:

1. Built-up Roofing Membrane, Figure 3.4,
2. Thermoplastic Single-Ply Membrane,
3. Thermoset Single-Ply Roofing Membrane, and
4. Modified Bitumen Roofing Membrane.



Figure 3.4 Built-up roofing membrane [12]

3.1.5 Roof Covering or Surfacing

Roof coverings are used to protect the roof membrane from direct sunlight and to reflect solar radiation [13]. It thus slows down weathering of the roofing membrane. Roof surfacing also protects the membrane against casual foot traffic. Roof surfacing may include:

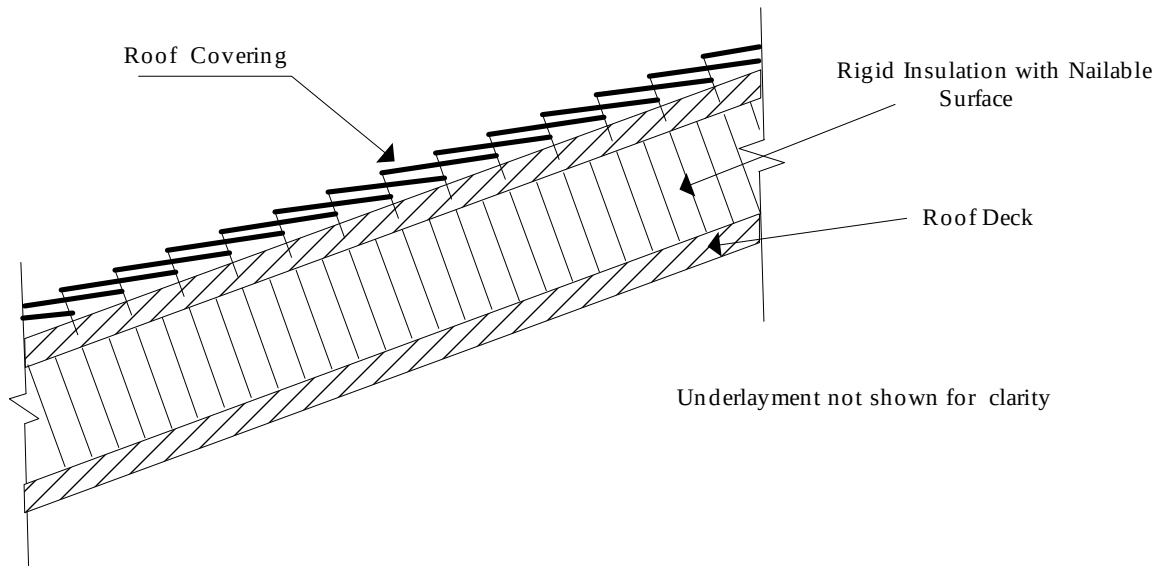
1. Aggregate, Figure 3.5,
2. Cap sheets, and
3. Smooth-surfaced Aggregate.



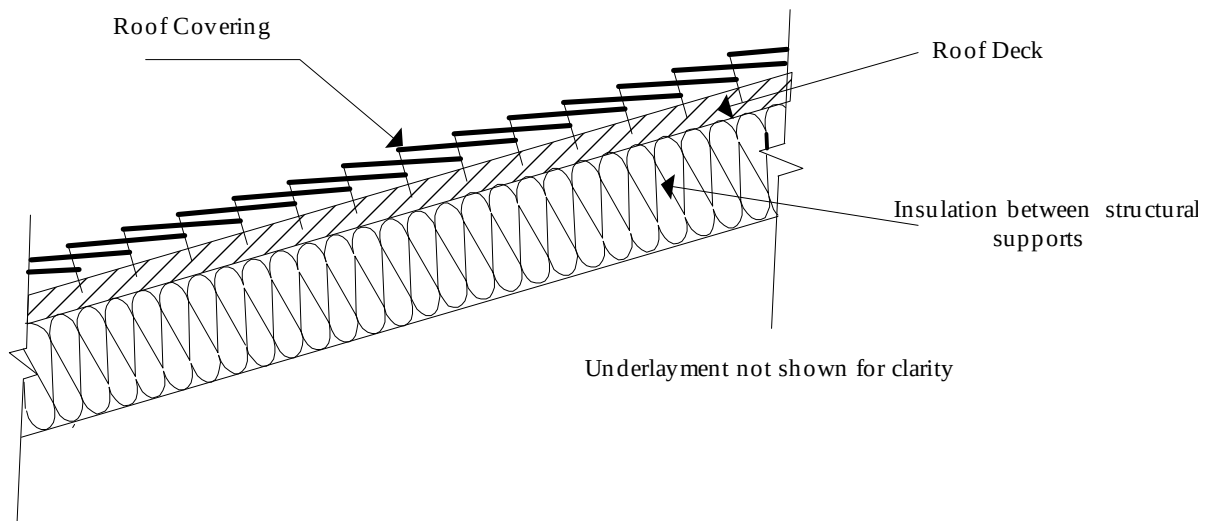
Figure 3.5 Aggregate covering [12]

3.2 Steep-Slope Roofs

Steep-slope roofs are water shedding roof systems that function with gravity to shed water from one course to the next, thereby draining roof surfaces. Steep-slope roofs are composed of a roof covering, underlayment, insulation (covered by a nailable substrate if installed on top of a roof deck) and roof deck. Steep-slope roofs are divided into two categories. Compact, or warm, roof assemblies incorporate insulation directly above or below roof decks as shown in Figure 3.6. Ventilated, or cold, roof assemblies have ventilation cavities beneath roof decks. A ventilation cavity may be an attic or a vented space just below the nailable surface of a roof covering as shown in Figure 3.7.

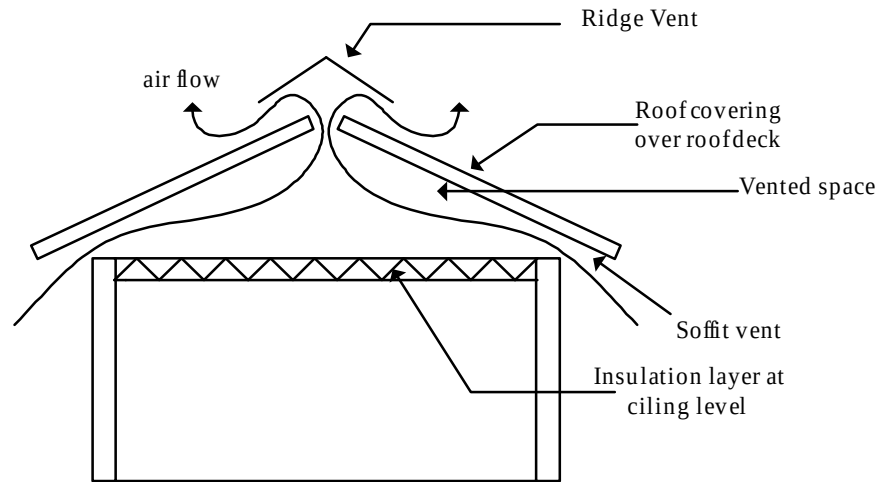


a) Insulation above roof deck

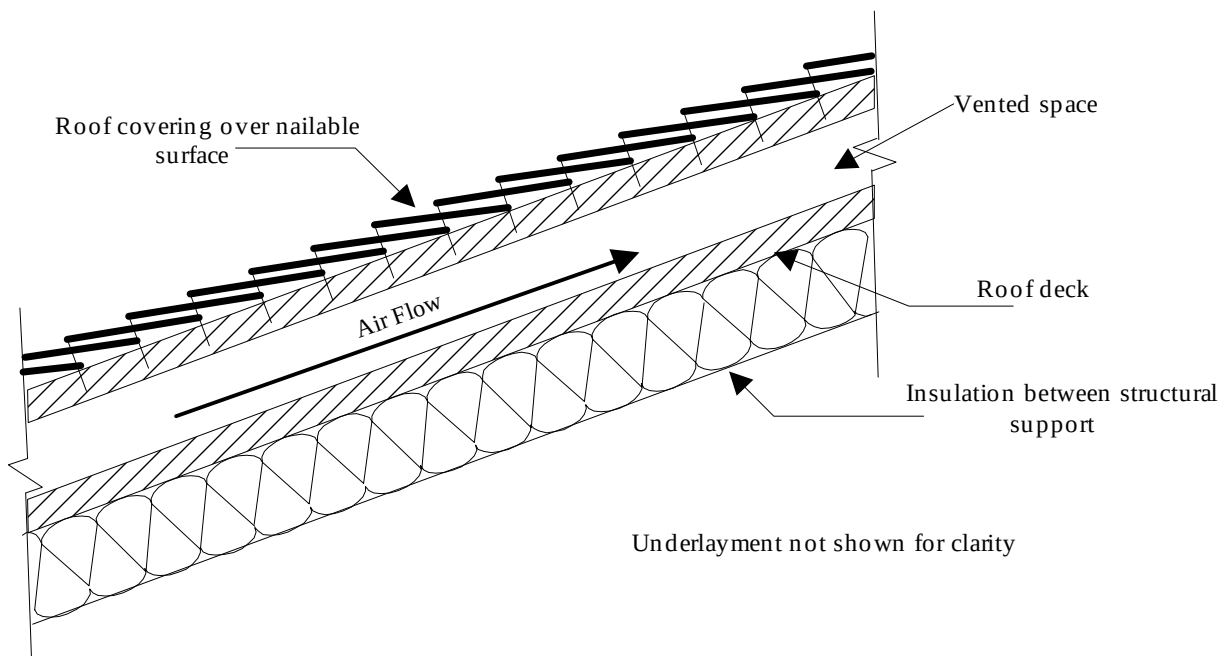


b) Insulation below roof deck

Figure 3.6 Compact or warm roof assembly [14]



a) Vented attic



b) Vented compact roof

Figure 3.7 Vented or cold roof assembly [14]

3.2.1 Roof Covering

Roof coverings in steep-slope roofs are composed of slates, tiles, asphalt shingles, thatch, wood shingles or metal roofing. These materials are water shedders and not water barriers and cannot waterproof a structure where water can gather and stand on the roof surface.

3.2.2 Underlayment

Underlayments are used in steep-slope roofs as the waterproofing component. Organic felts are most commonly used as waterproof underlayment. Subsequently organic felts were replaced by fiberglass felts, which is an inorganic material. Underlayments are usually placed below the roof covering.

3.2.3 Insulation

The insulation used in steep-slope roofing may be made of polyisocyanurate, glass fiber, composites and wood fiberboard. Insulation is used to reduce heating and cooling costs by providing thermal resistance.

3.2.4 Roof Deck

Roof decks are boards or sheet materials that are fastened to the roof structure to cover a house or building. The roof deck also serves as the structural foundation onto which a roof system is applied. In steep slope roofs the decks are either wood or metal.

3.3 Other Roofing Components

For a building to be water tight, its roof should include components other than just the roofing material to direct water away from, or to keep water out of, buildings. The drainage

system can be considered as a first line of defense against water penetration followed by roof flashing. The following sections will describe the drainage system and roof flashing in more detail.

3.3.1 Drainage Systems

Drainage systems are used as the first line of defense against moisture by directing water away from the roof and eliminating water ponding on the roof surface. Drainage systems used in low-slope roofs differ than the drainage systems used in steep-slope roofs. Low-slope roof drainage systems are in the interior of the roof surface with the scupper built into the roof and the drain strainer projecting above the roof surface, Figure 3.8. The drainage system of the steep-slope roof consists of the gutter that runs around the parameter of the roof collecting the water moving downward due to gravity, and the downspout that is attached to the gutter to take the water out of the gutter and down to the drainage pipe or splash block on the ground.

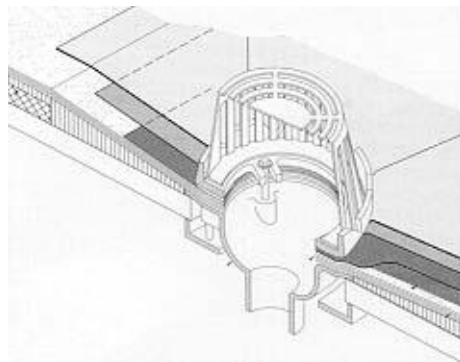


Figure 3.8 Built-in scupper and drain strainer [12]

3.3.2 Roof Flashing

Roof flashings are used in both low-slope and steep-slope roofs wherever there is an interruption in the roof surface. Flashing is used to seal and protect joints in a building from water penetration. The joints created by the intersection of the roof and roof rising structures and projections, such as parapets, hatches, skylights, chimneys, vent stacks, or towers, are among the most vulnerable areas of roofing systems. They constantly expand and contract in response to changes in humidity and temperature. The greater the number of such projections, the greater the potential for serious leaks. Flashing is used at these intersections to keep rainwater from leaking into the building. It makes joints at these junctions watertight, while at the same time allowing the natural expansion and contraction of materials to continue. Flashing materials are overlapped when installed in such a way as to discourage water entrapment. Roof flashing can be divided into two groups: Vertical Flashing and Horizontal Flashing [15].

1. Vertical Flashing: Vertical flashings are installed at upturned edges, such as walls.

Vertical Flashings are divide into three parts:

- Cant: A cant is a triangular-shaped strip made of perlite, wood fiberboard or wood. Cants are used in corners so that flashing materials do not have to bend so much as to cause a tear. A cant is adhered or mechanically fastened.
- Base Flashing: A base flashing may be an extension of the roof membrane running up to the vertical parapet wall or other projecting element or may be of a different waterproof material that is compatible with the roofing materials used. Base flashings are used to exclude moisture from

interruptions to the roof surface. Depending on the material selected, the base flashing may be fully adhered or mechanically fastened to the vertical side face of the parapet. Most base flashings are made of sheet metal (copper, lead, aluminum, or galvanized iron), fabric saturated with asphalt, or pliable synthetic material, Figure 3.9,

- Counter Flashing: Counter flashing is usually made of metal and is used to shield the top edges and joints of the base flashing, Figure 3.9,

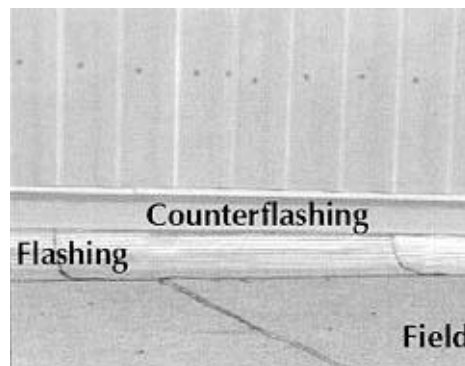


Figure 3.9 Counter flashing and base flashing [12]

2. Horizontal Flashing: Horizontal flashing is installed at drains, roof edges and vents. Horizontal flashing is installed around objects on the flat surface of the roof. The horizontal flashings are made of felts, asphalt, flashing cement, lead and other metals.

4. METHODOLOGY

4.1 Identification and Selection of Maintenance Elements

The DM&R at North Carolina State University funded a roof management survey carried out by Stafford Consulting Engineers. The survey results are stored in RoofManager and provide a data base for multiple building roofs at different sites belonging to the university. RoofManager also lists re-roofing costs and repair costs by item to provide the user with additional insight into the decision of re-roof versus repair. This research will adopt the information provided by the RoofManager database for each roof element. Missing information is provided by the roofing expertise of the staff of the DM&R.

RoofManager also provides a list of recommended repair methods. Each group of repair methods relates to a certain maintenance element in the roof. A maintenance element is a physical part of a roof that requires a substantial maintenance effort [9]. The major maintenance elements identified by the DM&R staff and provided by the RoofManager database are used in this research. For the purpose of this research, each element is assigned a number which is used as a reference in the optimization model developed. For each element in the system, certain parameters are needed by the optimization which were not available in RoofManager, particularly the element's expected rate of deterioration and the element's expected service life.

4.1.1 Element Rate of Deterioration

Roof elements deteriorate with time. The decline in condition can be described as a series of condition states such as those described in Table 4.1 which were developed during this research effort. Each element will have a condition state service life at each condition state (CS). The condition state service life is associated with the average time (in years) that an element will stay at a condition state before declining to a lower condition state. The average condition state service lives presented in Table 4.1 were estimated by the DM&R roofing staff personnel. The element average condition state service life at a condition state will be characterized by $ACSSL_{(e,c)}$ where (e) represents the element and (c) is the condition state before maintenance.

4.1.2 Element Service Life

Service life is another important parameter of an element. Element service life is the time span that a roof element can serve its purpose before it needs replacement [9]. The element service life can be calculated from:

$$SL_{(e)} = \sum_c^5 ACSSL_{(e,c)} + EX_{(e,c,k)} \quad (4.1)$$

Where,

$SL_{(e)}$ = Service life of element (e) starting new,

$ACSSL_{(e,c)}$ = Average Condition State Service Life of element (e) at condition state (c) during first cycle of deterioration,

$EX_{(c,k)}$ = Extension in element (ϕ) service life when the condition state is increased from condition state (c) to condition state (k) after first cycle of deterioration.

Table 4.1 Element condition states and average condition state service life

No.	Element	CS	Condition State Description	ACSSL
1	Single-ply membrane	5	New/excellent Membrane- New condition	13.00
		4	Loose/dry Membrane Lap or wrinkled/ridging Membrane/ slightly deteriorated- No deficiency	5.00
		3	Infiltration of water in roof system/ partially deteriorated- Deficient with no damage	2.00
		2	Infiltration of water in building- Damaged	1.00
		1	Deteriorated/damaged Membrane- Useless/ non-functional	0.00
2	Multiple-ply membrane	5	New/excellent Membrane- New condition	18.00
		4	Loose/dry Membrane Lap or wrinkled/ridging Membrane/ slightly deteriorated- No deficiency	6.00
		3	Infiltration of water in roof system/ partially deteriorated- Deficient with no damage	1.50
		2	Infiltration of water in building- Damaged	0.75
		1	Deteriorated/damaged Membrane- Useless/ non-functional	0.00
3	Insulation	5	New/excellent Insulation- New condition	15.50
		4	Shuffled/loose Insulation, loose Insulation fastener/ slightly damaged- No deficiency	4.50
		3	Small Area of saturation/ partially damaged- Deficient with no damage	2.00
		2	Large area of saturation- Damaged	0.75
		1	Collapsed Insulation/ fully saturated- Useless/ non-functional	0.00
4	Base Flashing Single-ply	5	New/excellent Base Flashing- New condition	10.00
		4	Small hair-line cracks- Deterioration with no deficiency	5.00
		3	Wide spread cracks- Deficiency with no damage	2.00
		2	Partially deteriorated /loose Base Flashing- Damaged	1.25
		1	Dried/deteriorated Base Flashing- Useless/ non-functional	0.00
5	Base Flashing Multiple-ply	5	New/excellent Base Flashing- New condition	12.50
		4	Small hair-line cracks- Deterioration with no deficiency	6.00
		3	Wide spread cracks- Deficiency with no damage	3.00
		2	Partially deteriorated /loose Base Flashing- Damaged	2.00
		1	Dried/deteriorated Base Flashing- Useless/ non-functional	0.00
6	Counter Flashing	5	New/excellent Counter Flashing condition	5.50
		4	Slightly deteriorated/damaged/rusted Counter Flashing- No deficiency	4.00
		3	Partially deteriorated/damaged Counter Flashing- Deficiency with no damage	3.50
		2	Loose Counter Flashing/severely damaged- Damaged	2.00
		1	Missing Counter Flashing- Useless/ non-functional	0.00
7	Gutter	5	New/excellent Gutter- New condition	5.00
		4	Slightly deteriorated/loose Gutter- No deficiency	3.00
		3	Partially disconnected & leaking Gutter/partially damaged- Deficient with no damage	2.00
		2	Substantially disconnected & leaking Gutter/ severely damaged- Damaged	0.75
		1	Missing or ineffective Gutter/ useless/ non-functional	0.00

Table 4.1 Continued

8	Downspout	5	New/excellent Downspout- New condition	5.00
		4	Slightly deteriorated/ clogged/loose Downspout- No deficiency	3.00
		3	Partially disconnected/ clogged & leaking Downspout/ partially damaged- Deficient with no damage	2.00
		2	Substantially disconnected/ clogged & leaking Downspout/ severely damaged- Damaged	0.75
		1	Missing or ineffective Downspout- Useless/ non-functional	0.00
9	Drain Strainer	5	New/excellent Drain Strainer- New condition	7.00
		4	Partially clogged Drain Strainer/ slightly deteriorated/damaged- No deficiency	5.00
		3	Fully clogged Drain Strainer/ partially deteriorated- Deficient with no damage	5.00
		2	Deteriorated/damaged Drain Strainer- Damaged	2.00
		1	Missing Drain Strainer- Useless/ non-functional	0.00
10	Coping Cap	5	New/excellent Copping Cap- New condition	6.00
		4	Failure of Copping Cap Head Joint- Deterioration with no deficiency	4.00
		3	Failure of Copping Cap Bed Joint- Deficiency with no damage	2.50
		2	Masonry saturation & efflorescence- Damaged	1.00
		1	Unleveled Copping Cap Units-missing- Useless/ non-functional	0.00
11	Termination Bar	5	New/excellent Termination Bar- New condition	5.50
		4	Slightly loose- No deficiency	3.50
		3	Partially loose but not damaged Termination Bar- Deficient with no damage	2.50
		2	Partially loose/ damaged Termination Bar- Damaged	5.00
		1	Missing Termination Bar- Useless/ non-functional	0.00
12	Edge Metal	5	New/excellent Counter Flashing condition	7.50
		4	Slightly deteriorated/damaged/rusted Counter Flashing- No deficiency	3.00
		3	Partially deteriorated/damaged Counter Flashing- Deficiency with no damage	1.75
		2	Loose Counter Flashing/severely damaged- Damaged	0.75
		1	Missing Counter Flashing- Useless/ non-functional	0.00
13	Shingles	5	New/excellent Shingles- New condition	15.00
		4	Loose Shingle tabs/ buckled Shingles- No deficiency	6.00
		3	Partially deteriorated/damaged Shingles- Deficient with no damage	2.00
		2	Loose/split Shingles- Damaged	1.00
		1	Missing Shingles- Useless/non-functional	0.00
14	Slate	5	New/excellent Slate- New condition	30.00
		4	Slightly worn/aged Slate- No deficiency	25.00
		3	Loose Slate/ partially damaged- Deficient with no damage	15.00
		2	Broken Slate- Damaged	3.00
		1	Missing Slate- Useless/ non-functional	0.00
15	Hip/Ridge Cap	5	New/excellent Hip/Ridge Cap - New condition	15.00
		4	Worn/aged/rusted Hip/Ridge Cap - Deterioration with no deficiency	7.00
		3	Slightly deteriorated Hip/Ridge Cap- Deficiency with no damage	3.00
		2	Deteriorated/damaged Hip/Ridge Cap- Damaged	1.50
		1	Missing Hip/Ridge Cap- Useless/ non-functional	0.00

4.2 Element Condition State

Each element has a rate at which it deteriorates from a better Condition State (CS) to a worse condition state if no Maintenance Action (MA) is performed on the element. For the purpose of this research, each element will have five condition states describing the element deterioration from the new condition, represented by 5, to a condition where the element fails, represented by 1. The five condition states were developed with the help of personnel at the DM&R by defining the state of each element if only regular roof maintenance work is performed during its life time. Through an inspection process, each element in each roof section will be described by its condition state. The five condition states are shown in Table 4.2.

Table 4.2 Description of condition state

State	Condition State Description
5	New/ excellent condition
4	Slightly deteriorated but no deficiency
3	Deficient but no damage
2	Damaged
1	Non-functional

Figure 4.1, shows a typical diagram of element deterioration with time where the condition state drops one step every few years. The time between two consecutive steps is the average time an element stays at a condition state (ACSSL).

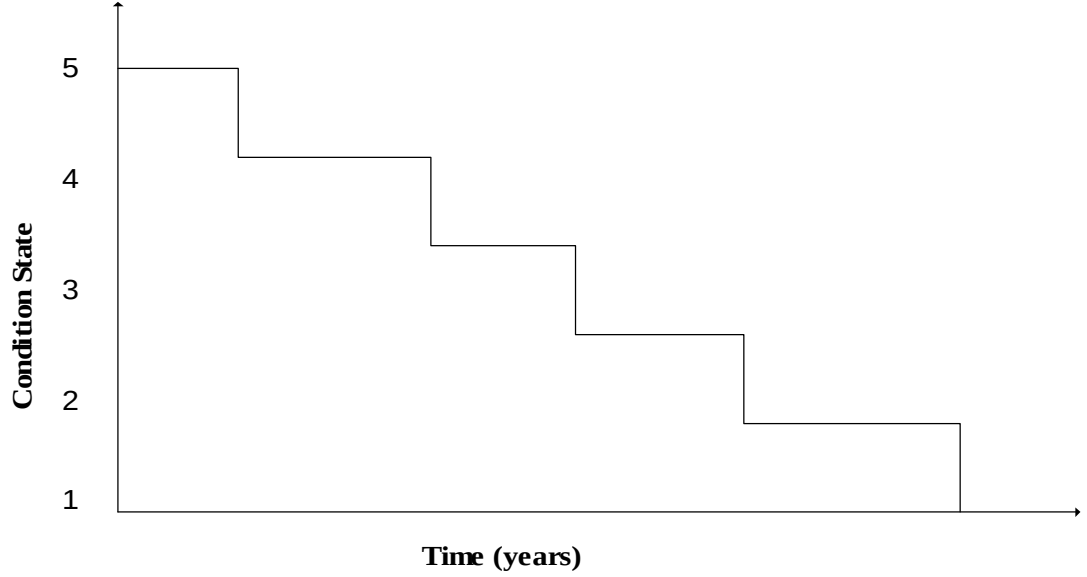


Figure 4.1 Element deterioration with time

4.3 Element Maintenance Action

Each element will have one or more maintenance action (MA) alternatives at each condition state. The higher the element condition state the lower the number of maintenance action alternatives that will be available for the element. Each maintenance action will require the following information:

1. The unit cost of performing the maintenance action.

Maintenance costs were usually provided from one of two sources: North Carolina State University historical cost data or the Means Building Construction Cost Data. RoofManager has work function codes for the types of maintenance and repair performed on roofs. There are 162 function codes, each describing a certain type of maintenance work item. Each group of maintenance work items generally relates to a particular roof element. Unit costs for maintenance work function codes will be denoted in this research by $MAUCST_{(e,c,k)}$ where (e) is the element, (c) is the

condition state before maintenance and k is the condition state after maintenance. Figure 4.2 shows the maintenance action unit cost matrix representing the unit cost of improving an element from a condition state (c) to a condition state (k). These unit costs were partially available from Roofmanager and partially developed in this research.

2. The effect of maintenance action on the condition of element.

It is assumed that each maintenance action will improve the state of the element, which will lengthen its service life and postpone the need for replacement. This improvement can be determined as the increase in the element condition state after maintenance. The higher the increase in the element condition state, the higher the cost of maintenance. For routine maintenance performed annually on each roof section, the element condition state will not increase, but will help in slowing the element deterioration.

Figure 4.3 shows the effect of increasing the element condition state to a higher state that results in the extension of the element service life.

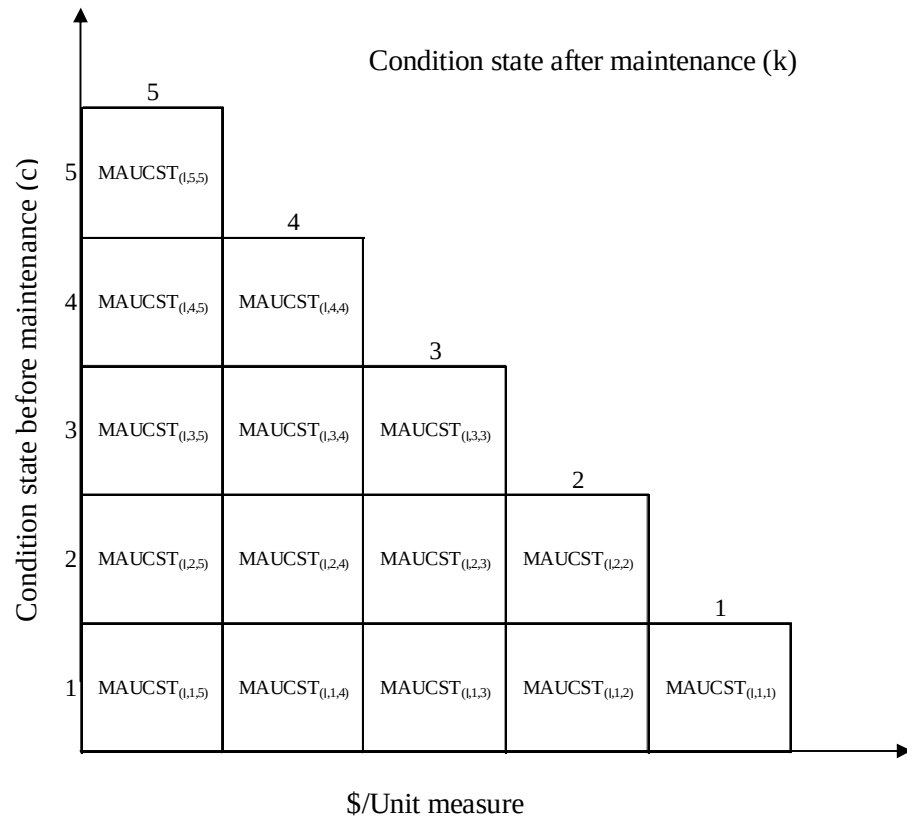


Figure 4.2 Maintenance action unit cost matrix

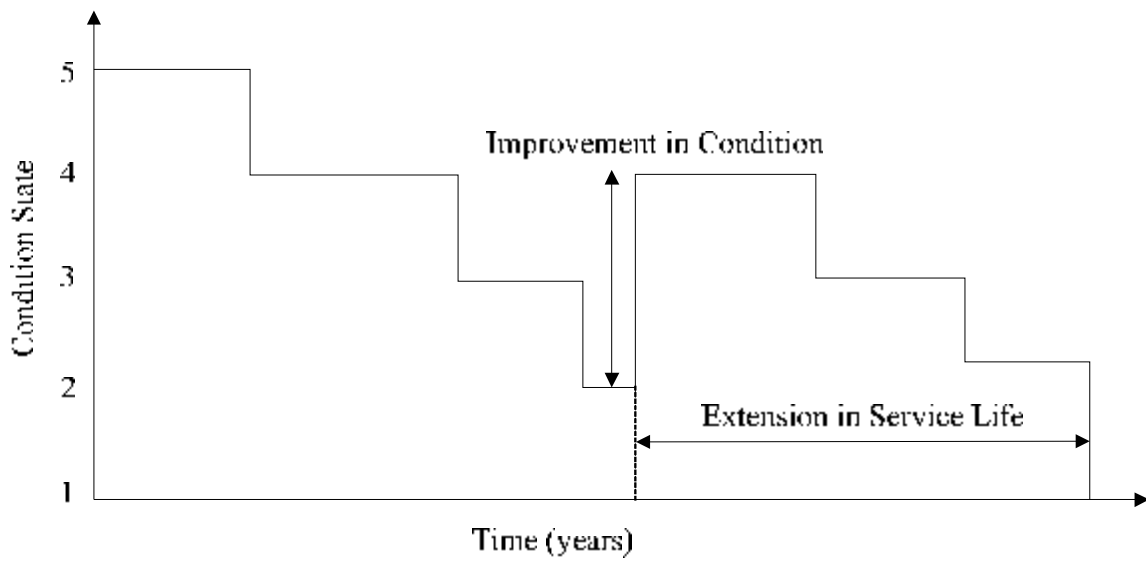


Figure 4.3 Extension in service life due to improvement in condition state

4.4 Annual User Cost

Annual user cost is the cost incurred by a building user if damage occurs due to water infiltration into the building interior. It can also be defined as the penalty paid by the agency as result of selecting one maintenance alternative instead of the other.

Damage to a building interior is calculated by multiplying the cost of repairing a unit area of the building interior by an Element Condition State Damage Factor. The annual user cost is calculated as follows:

$$AUCST_{(l,c)} = \text{Damage Cost} * ECSDF_{(l,c)} \quad (4.2)$$

Where,

$AUCST_{(l,c)}$ = Annual User Cost per unit measure of element (l) at initial condition state (c) (\$/unit measure of element),

Damage Cost = Cost of repairing a square foot of the building interior (\$/SF),

$ECSDF_{(l,c)}$ = Element Condition State Damage Factor of element (l) at initial condition state (c) (unit area of interior damage/ unit measure of element).

4.4.1 Cost of Damage to Building Interior

The cost of damage to a building interior resulting from a roof leak was estimated by the DM&R based on their records, experience, and consideration of standard repair costs [16]. Table 4.3 shows a breakdown of the total cost of repair to a building interior. The total unit cost of repair does not include the value of research, business continuation, etc.

Table 4.3 Total cost of repair to building interior

Item Description	Cost (\$/SF)
Demolition	
Clean-up	2.50
Demolition	5.00
Finishes	
Partitions, wall finishes	12.00
Ceilings	5.50
Flooring	7.00
Systems	
Lighting	8.00
HVAC	5.00
Electrical	6.00
Equipment and furnishings	
Furniture	20.00
Computers/Electronics	25.00
Miscellaneous furnishings	7.50
Total	103.50

4.4.2 Element Condition State Damage Factor (ECSDF)

The $ECSDF_{(c)}$ was estimated with the help of the DM&R expert personnel. To estimate the factor the following considerations were accounted for,

1. The probable number of leaks per year resulting from having a specific element quantity at certain condition state were estimated by the DM&R expert staff. For example, a 1000 sq.ft. single-ply membrane at a condition state 3 will probably result in 4 leaks/year.
2. The number of leaks will be greater for a lower condition state. For example, an element at condition state 1 will result in more damage to a building interior than the same quantity of the element at condition state 3 due to the greater element deterioration at condition state 1.

3. The area, in square feet of roof, affected by the leak resulting from a specific element quantity at certain condition state was also an approximate estimate provided by the experts working at the DM&R. For example, 1000 sq.ft. of single-ply membrane at a condition state 3 will have about 4 leaks/year that will damage a total area of about 7 sq.ft. of building interior in each year.

The $ECSDF_{(c)}$ of an element (ϕ) at an initial condition state (c) is the ratio of the probable damaged area of the building interior to the quantity of the element at the condition state.

$$ECSDF_{(c)} = \frac{\text{Area of damaged building interior}}{\text{Quantity of element unit measure causing the damage}} \quad (4.3)$$

Table 4.4 shows the element condition state, element unit measure causing the leak, number of leaks caused by having the element unit measure at a certain condition state, and the corresponding ECSDF of each element at each condition state.

4.5 Equivalent, Uniform Annual Cost Calculation for Maintenance Action Alternatives

Three types of maintenance action alternatives are available for an element: do-nothing, major maintenance, or replacement. This section describes the method used to estimate the Equivalent, Uniform Annual Costs ($EUAC_{(c,k)}$) of the alternatives. The reductions in $EUAC_{(c,k)}$ that are associated with the optimum set of improvement

alternatives will be the objective function value. In addition, some minor routine maintenance is performed for each element on an annual basis.

4.5.1 Element Replacement Alternative

The replacement of an existing element with a new one improves the element condition state to level 5. At condition state 5 it is assumed that no user costs result due to water infiltration into the building interior. Figure 4.4 shows the cost profile for one replacement cycle of a new element.

With time the new element will deteriorate. Due to element deterioration user costs are generated as soon as the element condition state drops below condition state 4, even though routine maintenance is performed. To reduce deterioration and decrease user cost, one or more major maintenance actions would be performed to maintain the condition state of the element at an acceptable level. The preferred lowest acceptable condition state for all elements is condition state 3. As a result of major maintenance, the element service life can be extended for a few more years during which routine maintenance continues.

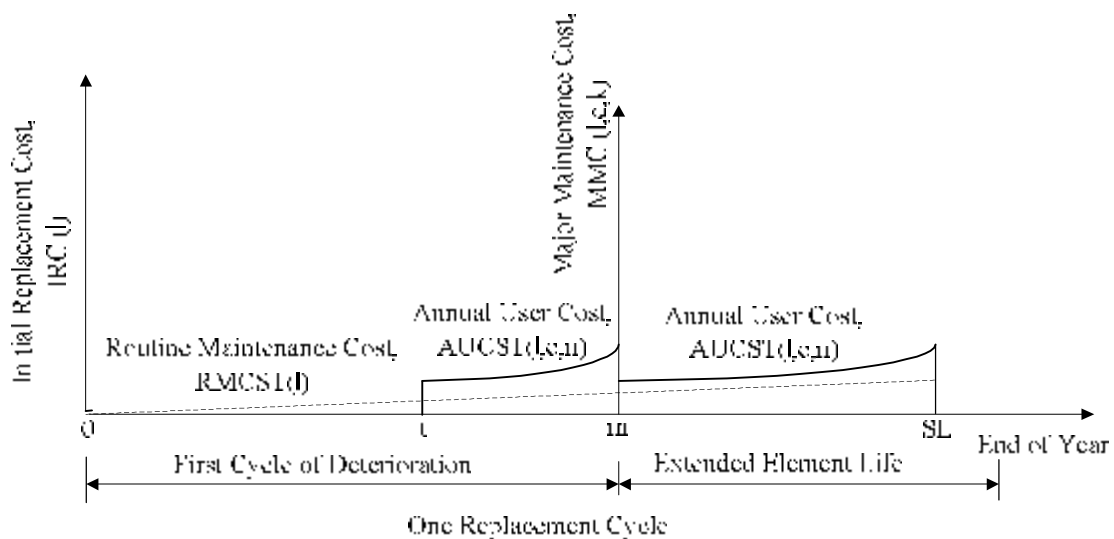


Figure 4.4 Element replacement cycle profile

Table 4.4 ECSDF calculation

Element	Condition State Description	CS	Element unit measure causing leak	# Leak/year	Damage building interior (SF)	ECSDF
Single-ply membrane (SF)	Deteriorated/damaged Membrane- Useless/ non-functional	1	1000	13	39	0.04
	Infiltration of water in building- Damaged	2	1000	9	27	0.03
	Infiltration of water in roof system/ partially deteriorated- Deficient with no damage	3	1000	4	7	0.01
	Loose/dry Membrane Lap or wrinkled/ridging Membrane/ slightly deteriorated- No deficiency	4	1000	0	0	0.00
	New/excellent Membrane- New condition	5	1000	0	0	0.00
Multiple-ply membrane (SF)	Deteriorated/damaged Membrane- Useless/ non-functional	1	1000	13	39	0.04
	Infiltration of water in building- Damaged	2	1000	9	22	0.02
	Infiltration of water in roof system/ partially deteriorated- Deficient with no damage	3	1000	4	5	0.01
	Loose/dry Membrane Lap or wrinkled/ridging Membrane/ slightly deteriorated- No deficiency	4	1000	0	0	0.00
	New/excellent Membrane- New condition	5	1000	0	0	0.00
Insulation (SF)	Collapsed Insulation/ fully saturated- Useless/ non-functional	1	1000	0	3	0.00
	Large area of saturation- Damaged	2	1000	0	2	0.00
	Small Area of saturation/ partially damaged- Deficient with no damage	3	1000	0	1	0.00
	Shuffled/loose Insulation, loose Insulation fastener/ slightly damaged- No deficiency	4	1000	0	0	0.00
	New/excellent Insulation- New condition	5	1000	0	0	0.00
Single-ply base flashing (LF)	Dried/deteriorated Base Flashing- Useless/ non-functional	1	100	9	13	0.13
	Partially deteriorated /loose Base Flashing- Damaged	2	100	6	5	0.05
	Wide spread cracks- Deficiency with no damage	3	100	3	3	0.03
	Small hair-line cracks- Deterioration with no deficiency	4	100	0	0	0.00
	New/excellent Base Flashing- New condition	5	100	0	0	0.00

Table 4.4 Continued

Multiple-ply base flashing (LF)	Dried/deteriorated Base Flashing- Useless/ non-functional	1	100	9	10	0.10
	Partially deteriorated /loose Base Flashing- Damaged	2	100	3	6	0.06
	Wide spread cracks- Deficiency with no damage	3	100	3	3	0.03
	Small hair-line cracks- Deterioration with no deficiency	4	100	0	0	0.00
	New/excellent Base Flashing- New condition	5	100	0	0	0.00
Counter flashing (LF)	Missing Counter Flashing- Useless/ non-functional	1	100	9	13	0.13
	Loose Counter Flashing/severely damaged- Damaged	2	100	4	5	0.05
	Partially deteriorated/damaged Counter Flashing- Deficiency with no damage	3	100	2	3	0.03
	Slightly deteriorated/damaged/rusted Counter Flashing- No deficiency	4	100	0	0	0.00
	New/excellent Counter Flashing condition	5	100	0	0	0.00
Gutter (LF)	Missing or ineffective Gutter/ useless/ non-functional	1	100	0	3	0.03
	Substantially disconnected & leaking Gutter/ severely damaged- Damaged	2	100	0	2	0.02
	Partially disconnected & leaking Gutter/partially damaged- Deficient with no damage	3	100	0	2	0.02
	Slightly deteriorated/loose Gutter- No deficiency	4	100	0	0	0.00
	New/excellent Gutter- New condition	5	100	0	0	0.00
Downspout (LF)	Missing or ineffective Downspout- Useless/ non-functional	1	12	0	3	0.25
	Substantially disconnected/ clogged & leaking Downspout/ severely damaged- Damaged	2	12	0	2	0.17
	Partially disconnected/ clogged & leaking Downspout/ partially damaged- Deficient with no damage	3	12	0	1	0.08
	Slightly deteriorated/ clogged/loose Downspout- No deficiency	4	12	0	0	0.00
	New/excellent Downspout- New condition	5	12	0	0	0.00
Drain strainer (EA)	Missing Drain Strainer- Useless/ non-functional	1	2	0	3	1.50
	Deteriorated/damaged Drain Strainer- Damaged	2	2	0	2	1.00
	Fully clogged Drain Strainer/ partially deteriorated- Deficient with no damage	3	2	0	1	0.50
	Partially clogged Drain Strainer/ slightly deteriorated/damaged- No deficiency	4	2	0	0	0.00
	New/excellent Drain Strainer- New condition	5	2	0	0	0.00

Table 4.4 Continued

Coping cap (LF)	Unleveled Copping Cap Units-missing- Useless/ non-functional	1	100	9	7	0.07
	Masonry saturation & efflorescence- Damaged	2	100	4	5	0.05
	Failure of Copping Cap Bed Joint- Deficiency with no damage	3	100	2	4	0.04
	Failure of Copping Cap Head Joint- Deterioration with no deficiency	4	100	0	0	0.00
	New/excellent Copping Cap- New condition	5	100	0	0	0.00
Termination bar (LF)	Missing Termination Bar- Useless/ non-functional	1	100	9	7	0.07
	Partially loose/ damaged Termination Bar- Damaged	2	100	4	3	0.03
	Partially loose but not damaged Termination Bar- Deficient with no damage	3	100	2	1	0.01
	Slightly loose- No deficiency	4	100	0	0	0.00
	New/excellent Termination Bar- New condition	5	100	0	0	0.00
Edge metal (LF)	Missing Edge Metal	1	100	10	20	0.20
	Deteriorated/damaged Edge Metal	2	100	6	11	0.11
	Loose Edge Metal- Deficiency with no damage	3	100	3	3	0.03
	Slightly deteriorated Edge Metal- No deficiency	4	100	0	0	0.00
	New/excellent Edge Metal condition	5	100	0	0	0.00
Shingle (SF) (Asphalt)	Missing Shingles- Useless/non-functional	1	100	4	3	0.03
	Loose/split Shingles- Damaged	2	100	2	2	0.02
	Partially deteriorated/damaged Shingles- Deficient with no damage	3	100	1	1	0.01
	Loose Shingle tabs/ buckled Shingles- No deficiency	4	100	0	0	0.00
	New/excellent Shingles- New condition	5	100	0	0	0.00
Slate (SF)	Missing Slate- Useless/ non-functional	1	100	8	3	0.03
	Broken Slate- Damaged	2	100	4	2	0.02
	Loose Slate/ partially damaged- Deficient with no damage	3	100	2	1	0.01
	Slightly worn/aged Slate- No deficiency	4	100	0	0	0.00
	New/excellent Slate- New condition	5	100	0	0	0.00
Hip/Ridge cap (LF)	Missing Hip/Ridge Cap- Useless/ non-functional	1	100	1	3	0.03
	Deteriorated/damaged Hip/Ridge Cap- Damaged	2	100	1	2	0.02
	Slightly deteriorated Hip/Ridge Cap- Deficiency with no damage	3	100	1	1	0.01
	Worn/aged/rusted Hip/Ridge Cap - Deterioration with no deficiency	4	100	0	0	0.00
	New/excellent Hip/Ridge Cap - New condition	5	100	0	0	0.00

The Replacement-Cycle Cost, $RCC_{(i,c,k)}$, of element (i) at a beginning condition state (c) and final condition state k can be expressed as:

$$\begin{aligned}
 RCC_{(i,c,5)} = & IRC_{(i)} + \sum_{n=0}^{SL} RMCST_{(i,n)} * (P / F, r, n) + \sum_{n=t}^{SL} AUCST_{(i,c,n)} * (P / F, r, n) \\
 & + MMC_{(i,c,k)} * (P / F, r, m) + MMC_{(i,c,k)} * (P / F, r, m + z) \\
 & + MMC_{(i,c,k)} * (P / F, r, m + 2z) + \dots \dots \dots
 \end{aligned} \tag{4.4}$$

where,

- $IRC_{(i)}$ = Initial Replacement Cost of element (i),
- $RMCST_{(i,n)}$ = Regular Maintenance Cost of element (i) at year (n),
- $MMC_{(i,c,k)}$ = Major Maintenance Cost of element (i) at beginning condition state (c) improved to final condition state (k),
- $AUCST_{(i,c,n)}$ = Annual User Cost of element (i) at initial condition state (c) at year (n),
- SL = Expected Service Life of the element,
- $(P/F, r, n)$ = Single-payment present value factor; and
- r = Nominal rate of return
- z = m-t, from Figure 4.4

It is assumed that an element will always be replaced at the end of its service life, because the element is assumed to be always required on the roof. The cost profile for

repeated replacement cycles in perpetuity, i.e., forever repeated at SL intervals is shown in Figure 4.5. The Life-Cycle Cost, $LCC_{(i,c,5)}$, of a replacement alternative in perpetuity for element (i) at beginning condition state (c) and final condition state 5 is expressed as:

$$LCC_{(i,c,5)} = \frac{RCC_{(i,c,5)}}{1 - (1 + r)^{-SL}} \quad (4.5)$$

The Equivalent, Uniform Annual Cost, $EUAC_{(i,c,5)}$, of the replacement alternative for element (i) at initial condition state (c) and final condition state 5 is computed as follows:

$$EUAC_{(i,c,5)} = LCC_{(i,c,5)} * r \quad (4.6)$$

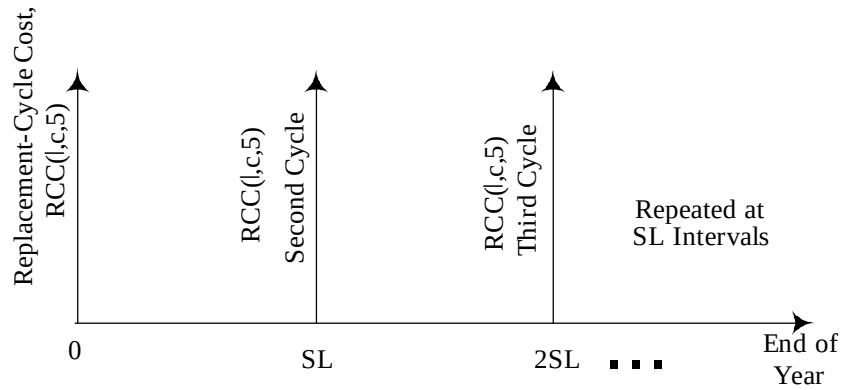


Figure 4.5 Perpetual replacement cycle

4.5.2 Major Maintenance Alternative

The second improvement alternative is to perform major maintenance on the element. Performing major maintenance on an element will extend its life by a few years. By performing major maintenance on an element, the condition state of the element will be

raised to a higher condition state. Thus, the extension in the element life will be equal to the number of years an element stays at the new condition state before dropping to a lower condition state. The number of years which an element stays in a condition state before dropping to a lower condition state was defined in Section 4.1.1 as the average condition state service life. At the end of the extended life, the element will be replaced by a new one. The cost profile for a major maintenance cycle of an existing element followed by replacement is shown in Figure 4.6.

The Life-Cycle Cost, $LCC_{(i,c,k)}$, of a major maintenance alternative in perpetuity for element (i) at an initial condition state (c), and final condition state (k) is expressed as:

$$LCC_{(i,c,k)} = MMC_{(i,c,k)} + \sum_{n=0}^{Ex} RMCST_{(i,n)} * (P / F, r, n) + \sum_{n=0}^{Ex} AUCST_{(i,c,n)} * (P / F, r, n) + LCC_{(i,c,5)} * (P / F, r, Ex) \quad (4.7)$$

where,

$MMC_{(i,c,k)}$ = Major Maintenance Cost for element (i) at initial condition state (c) and final condition state (k); and

Ex = Extended element life of element (i) due to major maintenance.

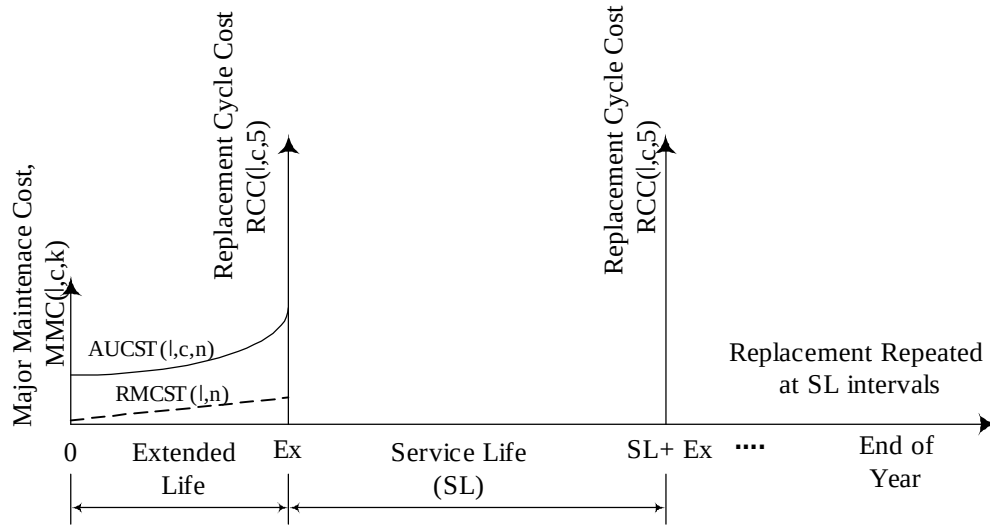


Figure 4.6 Major maintenance cycle profile

The Equivalent, Uniform Annual Cost of a major maintenance alternative, $EUAC_{(i,c,k)}$, for element (i) at initial condition state (c) and final condition state (k) can be computed as:

$$EUAC_{(i,c,k)} = LCC_{(i,c,k)} * r \quad (4.8)$$

4.5.3 Do-Nothing Alternative

The last improvement alternative is the Do-Nothing alternative. This alternative does not improve the element condition state. The element will remain in its initial condition state for several years before dropping to a lower condition state. The number of years that an element remains in its present condition state before dropping to a lower condition state is equal to the $ACSSL_{(i,c)}$ of element (i) at an initial condition state (c). At the end of the remaining life of the element, it will be replaced by a new one. The cost profile for the do-nothing cycle of an existing element is shown in Figure 4.7.

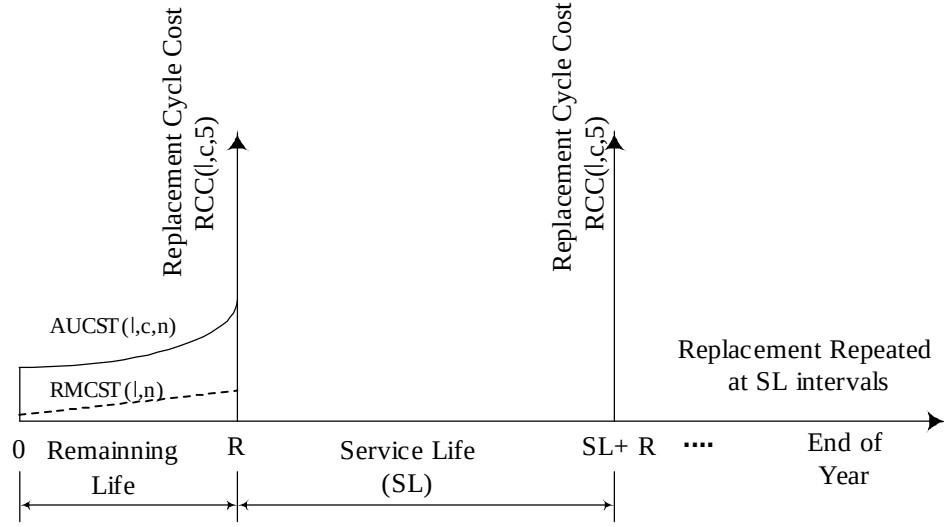


Figure 4.7 Do-nothing cycle profile

The Life-Cycle Cost, $LCC_{(l,c,c-1)}$, of a major maintenance alternative in perpetuity for element (l) at an initial condition state (c), and final condition state ($c-1$) is expressed as:

$$LCC_{(l,c,c-1)} = \sum_{n=0}^R RMCST_{(l,n)} * (P/F, r, n) + \sum_{n=0}^R AUCST_{(l,c,n)} * (P/F, r, n) + LCC_{(l,c,5)} * (P/F, r, R) \quad (4.9)$$

The Equivalent, Uniform Annual Cost of a do-nothing alternative, $EUAC_{(l,c,c-1)}$, for element (l) at initial condition state (c) and final condition state ($c-1$) can be computed as:

$$EUAC_{(l,c,c-1)} = LCC_{(l,c,c-1)} * r \quad (4.10)$$

4.6 Reduction in Equivalent Uniform Annual Costs

Maximizing the total amount of reductions in annual cost will assure the optimal maintenance solution for all roof elements available under budgetary constraint.

To optimize roof maintenance, the Reduction in Equivalent Uniform Annual Costs, $REUAC_{(i,j,\ell,c,k)}$ for all maintenance alternatives that are possible for each roof element (ℓ) at initial condition state (c) and final condition state (k) has to be calculated. $REUAC_{(i,j,\ell,c,k)}$ calculates the saving in annual cost associated with a specific alternative.

The $REUAC_{(i,j,\ell,c,k)}$ for building (i), section (j), element (ℓ) at initial condition state (c) and final condition state (k) is calculated as follows:

$$REUAC_{(i,j,\ell,c,k)} = AUCST_{(\ell,c)} - EUAC_{(i,j,\ell,c,k)}$$

(4.11)

As an example, assume an element is at condition state 2 and the possible maintenance alternatives are: do-nothing or replace the element with a new one. The do-nothing alternative is to leave the element at its current condition state, CS= 2, while the replacement alternative is to improve the element condition state to CS= 5 by replacing the element with a new one. The following hypothetical values are available as an example:

AUCST(l,c) (\$)	Alternative	EUAC(i,j,l,c,k) (\$)	REUAC(i,j,l,c,k) (\$)
279.45	Do-nothing	55.95	223.50
	Replacement	39.99	239.46

To maximize the saving in annual cost, it is necessary to maximize the total amount of reductions in equivalent uniform annual cost. For the previous example, the replacement alternative would be chosen under budgetary constraint. Minimizing $EUAC_{(i,j,l,c,k)}$ or $AUCST_{(c)}$ in isolation will not produce the optimal solution. The reason for including $AUCST_{(c)}$ in the calculation of the $REUAC_{(i,j,c,k)}$ is because building users are the ultimate owners of the buildings.

4.7 Reduction in Re-roofing Uniform Annual Costs

Maximizing the total amount of reductions in re-roofing annual cost will ensure the optimal maintenance solution for all roof sections under budgetary constraint. To optimize roof maintenance, the Re-roofing Uniform Annual Cost, $RFUAC_{(s)}$ for each section (s) has to be calculated. $RRFUAC_{(s)}$ is the saving in annual cost due to choosing re-roofing the roof section. The $RRFUAC_{(s)}$ for section (s) is calculated as follows:

$$RRFUAC_{(s)} = MCST_{(s)} - RFUAC_{(s)} \quad (4.12)$$

where,

$MCST_{(s)}$ = Total maintenance cost of section (s) resulting from maximizing

$REUAC_{(i,j,c,k)}$

$RFUAC_{(s)}$ = Re-roofing Uniform Annual Cost of section (s) calculated using the following equation,

$$RFUAC_{(s)} = RCST_{(s)} \frac{r}{1 - (1 + r)^{-SL_{(s)}}} \quad (4.13)$$

where,

$$RCST_{(s)} = \text{Re-roofing cost of section (s)}$$

As an example, assume that total maintenance cost of section (1) equals \$ 300, while the total maintenance cost of section (2) equals \$2500. If the RFUAC for both sections (1) and (2) equals \$500, then the RRFUAC equals -\$200 for section (1) and \$2000 for section (2). The model will choose section (2) for re-roofing while maintenance work is performed on roof section (1). This is all performed under a budget restriction.

4.8 Projection of Future Element Condition State

Element condition deteriorates with time. The finite Markov Chain class will be used to project the quantity of each element at each condition state for the subsequent analysis year. A Markov chain is a discrete-state random process in which the evolution of the state of the process beginning at a time (t) depends only on the current state $X(t)$, and not how the chain reached its current state or how long it has been in that state. Therefore, a finite Markov chain will be used since the condition states that a roof element may be in are considered to be discrete and countable and the probability that an element will transition from condition state (c) to condition state (k) does not depend on how the (c) state was arrived at. This means that the current state will always be considered as the starting point because the time between state changes is a random variable with a memory-less distribution. The construction of a Markov chain requires two basic parts, an initial distribution and a transition matrix.

4.8.1 Initial Distribution

Each element is given an ACSSL at each CS. Assuming that the life of an element at any CS is exponentially distributed, knowledge of the ACSSL identifies the distribution of the life of the element, which shall be denoted by the random variable Y_c , in state (c) as:

$$f_{Y_c}(t) = \frac{1}{m_c} e^{-\frac{t}{m_c}} \quad (4.14)$$

Assuming that the deterioration in the element CS leads to the next lower CS (and no to other state) then the probability of the element remaining at the same CS for one year is given by the probability that it will survive longer than one year in that state,

$$p_{cc} = \Pr(Y_c \geq 1) = \int_{t=1}^{\infty} \frac{1}{m_c} e^{-\frac{t}{m_c}} dt = e^{-\frac{1}{m_c}} \quad (4.15)$$

and the probability that one element will deteriorate to CS (c-1) is

$$p_{cc-1} = 1 - p_{cc}, \quad c = 5, 4, 3, 2 \quad (4.16)$$

while,

$$p_{11} = 1 \quad (4.17)$$

This means that an element reaching a failure condition state, CS=1, will remain in that condition state forever if no maintenance action is performed.

4.8.2 Transition Matrix

Based on the ACSSL defined in Table 4.1 for each element at each CS, and the assumption that the deterioration in an element CS leads to the next lower CS (and no to other state), a one year transition probability matrix P is constructed as follows:

$$P = \begin{bmatrix} P_{55} & 1 - P_{55} & 0 & 0 & 0 \\ 0 & P_{44} & 1 - P_{44} & 0 & 0 \\ 0 & 0 & P_{33} & 1 - P_{33} & 0 \\ 0 & 0 & 0 & P_{22} & 1 - P_{22} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where,

P_{cc} = Probability of a condition state c quantity staying in state c after one year,

$c = 2, 3, 4, 5$, and

$P_{11} = 1$, because we have only 5 condition states; therefore, an element deteriorates to condition state 1 and remains at condition state 1 with no further deterioration.

The probabilities (P_{cc}) of all elements remaining at their initial condition state (c) more than one year are given in Table 4.5. It should be noted that, for the sake of brevity, the probabilities in the table are written for only two significant figures. However, the full accuracy of the exponentiation operation was retained throughout the calculations.

As an example consider the do-nothing maintenance action. The element will continuously deteriorate year after year until it reaches CS = 1, where it stays forever. Let $X(t)$ denote the vector of the CS of the element at the start of year t ; given in its expanded form as

$$X(t) = (x_5(t), x_4(t), x_3(t), x_2(t), x_1(t)) \quad , \quad t = 1, 2, 3, \dots$$

The starting CS in the first year is denoted by $X(0)$ and is given by

$$X(0) = (1, 0, 0, 0, 0),$$

This means that the entire element quantity is at CS = 5. Clearly,

$$X(1) = X(0) P,$$

which can be written in an expanded form as follows:

$$\begin{aligned} & (x_5(1), x_4(1), x_3(1), x_2(1), x_1(1)) \\ &= (x_5(0), x_4(0), x_3(0), x_2(0), x_1(0)) \begin{bmatrix} P_{55} & 1-P_{55} & 0 & 0 & 0 \\ 0 & P_{44} & 1-P_{44} & 0 & 0 \\ 0 & 0 & P_{33} & 1-P_{33} & 0 \\ 0 & 0 & 0 & P_{22} & 1-P_{22} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\ &= (P_{55}, 1-P_{55}, 0, 0, 0) \end{aligned}$$

So generally,

$$X(t) = X(t-1) P$$

Substituting for $X(t-1) = X(t-2) P$ one gets

$$X(t) = [X(t-1) P] P = X(t-1) P^2$$

Continuing recursively all the way we secure,

$$X(t) = X(0) P^t$$

So to calculate the distribution of an element after year t , the transition matrix P is raised to the t^{th} power and pre-multiplied by the original state $X(0)$.

Table 4.5 Transition probability for maintenance element

No.	Element	CS	ACSSL	Pck
1	Single-ply membrane	5	13	0.93
		4	5	0.82
		3	2	0.61
		2	1	0.37
		1	0	1.00
2	Multiple-ply membrane	5	18	0.95
		4	6	0.85
		3	1.5	0.51
		2	0.75	0.26
		1	0	1.00
3	Insulation	5	15.5	0.94
		4	4.5	0.80
		3	2	0.61
		2	0.75	0.26
		1	0	1.00
4	Base Flashing Single-ply	5	10	0.90
		4	5	0.82
		3	2	0.61
		2	1.25	0.45
		1	0	1.00
5	Base Flashing Multiple-ply	5	12.5	0.92
		4	6	0.85
		3	3	0.72
		2	2	0.61
		1	0	1.00
6	Counter Flashing	5	5.5	0.83
		4	4	0.7788
		3	3.5	0.75
		2	2	0.6065
		1	0	1.00
7	Gutter	5	5	0.82
		4	3	0.72
		3	2	0.61
		2	0.75	0.26
		1	0	1.00

Table 4.5 Continued

8	Downspout	5	5	0.82
		4	3	0.72
		3	2	0.61
		2	0.75	0.26
		1	0	1.00
9	Drain Strainer	5	7	0.87
		4	5	0.82
		3	5	0.82
		2	2	0.61
		1	0	1.00
10	Coping Cap	5	6	0.85
		4	4	0.78
		3	2.5	0.67
		2	1	0.37
		1	0	1.00
11	Termination Bar	5	5.5	0.83
		4	3.5	0.75
		3	2.5	0.67
		2	5	0.82
		1	0	1.00
12	Edge Metal	5	7.5	0.88
		4	3	0.72
		3	1.75	0.56
		2	0.75	0.26
		1	0	1.00
13	Shingles	5	15	0.94
		4	6	0.85
		3	2	0.61
		2	1	0.37
		1	0	1.00
14	Slate	5	30	0.97
		4	25	0.96
		3	15	0.94
		2	3	0.72
		1	0	1.00
15	Hip/Ridge Cap	5	15	0.94
		4	7	0.87
		3	3	0.72
		2	1.5	0.51
		1	0	1.00

4.9 Optimization Model

Two optimization models will be used in this study. The first optimization maximizes the saving in annual cost by maximizing the total amount of reduction in equivalent uniform annual cost. The second optimization also maximizes the saving in annual cost by maximizing the total amount of reduction in annual re-roofing cost.

4.9.1 Maximizing Total Amount of Reduction in EUAC

After quantifying the REUAC and MAUCST for each maintenance alternative of each roof element, a binary linear programming approach was used to optimize the selection of maintenance actions to perform on each roof element under a limited budget. The following model was used:

Objective function:

$$\text{Max} \sum_{i=1}^{28} \sum_{j=1}^{12} \sum_{l=1}^{15} \sum_{c=1}^5 \sum_{k=c-1}^5 X_{(i,j,l,c,k)} * Qty_{(i,j,l,c,k)} * REUAC_{(i,j,l,c,k)} \quad (4.18)$$

Subject to:

Constraint 1:

$$\sum_{i=1}^{28} \sum_{j=1}^{12} \sum_{l=1}^{15} \sum_{c=1}^5 \sum_{k=c-1}^5 X_{(i,j,l,c,k)} * Qty_{(i,j,l,c,k)} * MAUCST_{(i,j,l,c,k)} \leq BUDGET \quad (4.19)$$

Constraint 2:

$$\sum_{k=c-1}^5 X_{(i,j,l,c,k)} = 1, \quad \forall i,j,l \text{ values}, c = 1, 2, \dots, 5 \quad (4.20)$$

Constraint 3:

$$X_{(i,j,\ell,c,k)} = 0 \text{ or } 1 \quad (4.21)$$

where,

BUDGET = Available maintenance budget

i = Building number

j = Roof section

ℓ = Maintenance element

c = Element condition state before maintenance

k = Element condition state after maintenance

$X_{(i,j,\ell,c,k)}$ = Decision variable for building (i), roof section (j), element (ℓ), initial condition state (c), and final condition state (k)

The model will compute the maintenance actions performed on each roof element that will maximize the total savings in annual cost within the available budget. For each element on each roof section of each building, there is only one decision variable $X_{(i,j,\ell,c,k)}$ which will have a value of 1. This means that for each element (ℓ) the model will select one maintenance action that will improve the element from condition state (c) to condition state (k) and maximize the total savings.

The benefits are calculated as the saving in future replacement costs due to applying the maintenance action on the element. After obtaining the recommended maintenance action on each element, the condition state of the elements is updated. The quantity of elements improving to a better condition state and the quantity of elements deteriorating to a worst

condition state will be calculated and the resulting element condition state will help in creating the inputs for the following year analysis.

4.9.2 Maximizing Total Amount of Reduction in Re-roofing Costs

After maximizing the total reduction in EUAC, another optimization is performed to ensure that the total cost of maintenance actions performed on a roof section does not exceed the cost of re-roofing the section. The 28 buildings available for this study include a total of 116 roof sections. A binary linear programming approach was used to optimize the decision of maintaining or re-roofing a roof section. The following model was used:

Objective function:

$$\text{Max} \sum_{s=1}^{116} X_{(s)} * RRFUAC_{(s)} \quad (4.22)$$

Subject to:

Constraint 1:

$$\sum_{s=1}^{116} X_{(s)} * RCST_{(s)} \leq RBUDGET \quad (4.23)$$

Constraint 2:

$$X_{(s)} = 0 \text{ or } 1 \quad (4.24)$$

where,

RBUDGET = Available re-roofing budget

s = Roof section

5. INVENTORY, CONDITION AND COST DATA

This chapter presents input data required to run the optimization model. The data was obtained from or developed with the assistance of the Department of Maintenance and Repair (DM&R) at North Carolina State University.

5.1 Number of Buildings and Buildings Roof Sections

The North Carolina State University campus has an inventory of the 128 buildings. Inspections of these buildings are performed biennially and the repairs performed on each building are stored in the department data base.

For the purpose of this model application, only 28 buildings were chosen because of the availability of data on their roof elements. Each building roof is divided into one or more sections. The division of roof buildings into sections is based on the variation in roof elevation or on the use of different roofing material, and in some cases, it is based on both. Table 5.1 lists the 28 buildings and the number of sections associated with each building.

5.2 Number of Maintenance Elements

DM&R defines 15 roof elements in its database. The following is a list of these maintenance elements:

- 1- Hip/Ridge Cap
- 2- Coving Cap
- 3- Base Flashing
- 4- Edge Metal
- 5- Counter Flashing
- 6- Shingle

- 7- Slate
- 8- Gutter
- 9- Downspout
- 10- Drain Strainer
- 11- Membrane
- 12- Insulation
- 13- Termination Bar
- 14- Scupper
- 15- Penetration Flashing

Table 5.1 Building name and number of sections associated

Building Name	Number of Sections
Alumini Memorial Building	3
D.H.Hill Library	4
Harris Hall	1
Pullen Hall	1
West Dunn Building	2
Winston Hall	6
Head House Unit 2	1
Hillsborough Building	2
Hodges Wood Products Lab	2
Jordan Hall	10
Page Hall	1
Peele Hall	1
Phytotron	5
Pulp and Paper Labs	7
Thompkins Hall	3
Williams Hall	12
Withers Hall	3
Administrative Services	2
Avent Ferry Convention Ctr	5
Biltmore Hall	4
Bostian Hall	6
Bureau of Mines	1
Caldwell Hall	5
Cates Avenue Steam Plant	5
Daniels Hall	8
Don E. Ellis Lab	2
Gardner Hall	12
Harrelson Hall	2

For the purpose of this model application, two elements were removed from the previous list. One removed element is the scupper; it was removed because no maintenance action is performed on it and it does not affect the roof maintenance cost. Penetration flashing was also removed because there were no available data in the DM&R database. Because of the important role penetration flashing plays in preventing water infiltration at roof projections, an approximate quantity was calculated and added to the base flashing quantity of each roof section.

Another modification was done to the maintenance element list. The membrane and base flashing were subdivided into single-ply and multiple-ply types. This subdivision allowed deterioration rates and maintenance costs of these materials to be better represented. No subdivision was needed for the rest of the elements due to the similarity in the average deterioration rates and maintenance costs for the material types encountered within each.

When the optimization model was first executed it was noted that the condition state weighted average of insulation sometimes improved when the membrane did not. This was logically impossible because the insulation can not be replaced or fixed without replacing the membrane. To solve this problem, the insulation was left in the model but was given zero quantities and half of the insulation maintenance cost was added to the cost of membrane maintenance. This assures that the insulation would be repaired or replaced over 50% of the area of membrane maintenance or replacement. Table 5.2 shows the modified list of the maintenance elements.

Table 5.2 Maintenance element and corresponding unit measure

Maintenance element	Unit measure
Single-ply membrane	Sq. Ft.
Multiple-ply membrane	Sq. Ft.
Insulation	Sq. Ft.
Single-ply base flashing	Linear Ft.
Multiple-ply base flashing	Linear Ft.
Counter flashing	Linear Ft.
Gutter	Linear Ft.
Downspout	Linear Ft.
Drain strainer	Each
Coping cap	Linear Ft.
Termination bar	Linear Ft.
Edge metal	Linear Ft.
Shingle	Sq. Ft.
Slate	Sq. Ft.
Hip/Ridge cap	Linear Ft.

5.3 **Quantity of Maintenance Elements at Each Condition State**

The quantity of maintenance elements and their distribution among different condition states are important factors in determining the annual cost of performing maintenance. They also have an important role in the optimization model because costs and benefits of the decisions increase and decrease with the increase or decrease in the quantity and present condition of units maintained.

The records in the DM&R database are kept for each roof section. The data available of interest to the research consists of the total square feet area and the perimeter length of each roof section. The types of repairs to be performed on each element and the quantity of element to be repaired are also available in the DM&R database. The availability of this information helped in developing the allocation of element quantities among the different

condition states. Table 5.3 shows Harris Hall, Pullen Hall, and West Dunn building element quantities distributed over their condition states. A table showing all buildings element initial quantities distributed over their condition states is available in Appendix A3.

5.4 **Real Rate of Return**

The interest rate should reflect the time value of money. It is used to convert costs occurring at different future times to equivalent costs at a common point in time. There are two main types of Interest rates; nominal and real. The nominal interest rate includes general price inflation, while the real interest rate, called also the Real Rate of Return (RRR), the net of price inflation. The nominal rate of return is calculated as follows,

$$\text{Nominal rate of return} = \text{RRR} + (1 + \text{RRR}) f \quad (5.1)$$

where,

$$f = \text{Inflation rate}$$

For computational simplicity, inflation will be set to zero and the nominal rate of return will equal the RRR. Thus, for this analysis, the budgets are estimated in today's dollar.

Table 5.3 Examples of element quantity distribution over different condition states

Building	Section	CS	Maintenance Elements														
			Single-ply membrane (SF)	Multiple-ply membrane (SF)	Insulation (SF)	Single-ply base flashing (LF)	Multiple-ply base flashing (LF)	Counter flashing (LF)	Gutter (LF)	Downspout (LF)	Drain strainer (EA)	Coping cap (LF)	Termination bar (LF)	Edge metal (LF)	Shingle (SF)	Slate (SF)	Hip/Ridge cap (LF)
Alumini Memorial Building	A	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		5	0.0	0.0	0.0	239.7	0.0	239.7	239.7	0.0	0.0	0.0	0.0	239.7	4871.0	0.0	0.0
	B	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		4	0.0	0.0	0.0	0.0	0.0	77.3	0.0	0.0	1.0	0.0	0.0	20.0	0.0	0.0	0.0
		5	2207.0	0.0	0.0	154.7	0.0	77.3	0.0	0.0	3.0	0.0	0.0	134.7	0.0	0.0	0.0
	C	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		3	0.0	0.0	0.0	0.0	0.0	4.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		4	0.0	0.0	0.0	0.0	0.0	23.9	0.0	36.0	0.0	15.0	56.9	0.0	0.0	0.0	0.0
		5	368.0	0.0	0.0	75.8	0.0	47.9	0.0	0.0	0.0	60.8	19.0	0.0	0.0	0.0	0.0
D.H.Hill Library	A1	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	98.6	0.0	0.0	0.0
		4	429.0	0.0	0.0	0.0	0.0	0.0	98.6	24.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	A2	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	191.0	0.0	0.0	0.0
		4	3129.0	0.0	0.0	235.3	0.0	235.3	235.3	24.0	0.0	0.0	0.0	44.3	0.0	0.0	0.0
		5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	A3	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	573.0	0.0	0.0	0.0
		4	15876.0	0.0	0.0	573.0	0.0	573.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	A4	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		4	260.0	0.0	0.0	577.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	577.6	0.0	0.0	0.0
		5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

5.5 Average Condition State Service Life

The average condition state service life is an important factor in determining its service life and in calculating its life-cycle cost. In addition, the average condition state service life determines the frequency in which major maintenance is performed on different elements. The average condition state service life for each element at each condition state was estimated by the roof maintenance personnel at the DM&R. For the elements used in this study, Table 5.4 provides the average condition state service life values which are the average times the elements will remain at a condition state without maintenance before deteriorating to a lower condition state.

Table 5.4 Average condition state service life for each element at each condition state

Maintenance Element	Average condition state service life (years)				
	5	4	3	2	1
Single-ply membrane	13.00	5.00	2.00	1.00	-
Multiple-ply membrane	18.00	6.00	1.50	0.75	-
Insulation	15.50	4.50	2.00	0.75	-
Single-ply base flashing	10.00	5.00	2.00	1.25	-
Multiple-ply base flashing	12.50	6.00	3.00	2.00	-
Counter flashing	5.50	4.00	3.50	2.00	-
Gutter	5.00	3.00	2.00	0.75	-
Downspout	5.00	3.00	2.00	0.75	-
Drain strainer	7.00	5.00	5.00	2.00	-
Coping cap	6.00	4.00	2.50	1.00	-
Termination bar	5.50	3.50	2.50	5.00	-
Edge metal	7.50	3.00	1.75	0.75	-
Shingle	15.00	6.00	2.00	1.00	-
Slate	30.00	25.00	15.00	3.00	-
Hip/Ridge cap	15.00	7.00	3.00	1.50	-

5.6 Unit Cost of Maintenance Actions

The unit cost of maintenance work performed on each roof element can be obtained from North Carolina State University historical cost data or the Means Building Construction Cost Data [16]. Since the maintenance costs required in this study are those incurred to improve the roof element from a lower condition state to a higher one, a group of experts at DM&R were asked to give their estimate of maintenance cost at each condition state for all elements and the corresponding improvement resulting from the performed maintenance. The group used their knowledge and the available data in the database. The individuals in the group were allowed to discuss their estimates to reach an agreement on the proper estimate for each element at each condition state. The results of this process are presented in Appendix A1.

5.7 Equivalent Uniform Annual Cost

The Equivalent Uniform Annual Cost is calculated to estimate the savings in annual costs resulting from performing a maintenance action. The EUAC is calculated for each maintenance action performed on each maintenance element at each condition state. The calculation of EUAC is discussed in detail in section 4.5. The values of all EUAC maintenance action alternatives calculated for all elements are available in Appendix A2.

6. MODEL DEVELOPMENT

Two software programs, Excel and AMPL, were used for the optimization analysis. Microsoft Excel is a spreadsheet program, while AMPL is an optimization program that uses algebraic modeling language to solve linear, nonlinear and integer programming problems. Excel operates on a MS Windows PC and AMPL operates on a SUN microcomputer platform.

This chapter will present the use of the two programs to perform the optimization and analysis. The chapter will also present two different approaches for optimizing roof maintenance and replacement.

6.1 Optimizing Roof Maintenance Using Separated and Combined Budgets

Currently, the DM&R has two budgets. The first budget is used to perform maintenance activity on university building roof elements. Maintenance work is performed by department maintenance personnel, and consists of repairing or replacing roof elements. The second budget available to the department is a capital budget which is used to perform re-roofing of a building roof section. Re-roofing replaces all roof section elements. Contractors usually perform the re-roofing work. If these two total budgets for all university building roofs are apportioned on the basis of sq.ft. of roof, the amounts for the 28 buildings in this study are \$54,000 for maintenance activity and \$246,000 for the re-roofing capital budget. Two approaches will be considered, one based on separated budget amounts and a second based on the combined budget amount.

In the separated budget approach, two optimizations are performed using the two separate budgets. First optimization is performed to select the optimum maintenance action on each roof element using the \$54,000 budget. The total maintenance costs for each roof section resulting from the first optimization are added and compared and the cost of performing re-roofing on the section. Second, optimization is performed by maximizing the difference between the total costs of performing maintenance work and the cost of re-roofing the roof section using capital budget.

In the combined budget approach, the two budgets available are combined and used to perform one optimization. The optimization will select the optimum maintenance or replacement action on each roof element using the combined budget of \$300,000. For both approaches, separated and combined, Excel is used for data entry and performing certain analysis, while AMPL performs the optimization.

6.2 Excel Operations

Excel was used to tabulate the input data, create the optimization model in algebraic language, save the created model in a text file readable to AMPL, and use the optimized results generated by AMPL to complete the analysis. An Excel file containing multiple sheets was developed for this purpose.

The initial quantities obtained from the DM&R database are entered in the “Initial Quantity” data sheet. Each element quantity entered at a specific condition state in the “Initial Quantity” sheet, Table 6.1, will then be displayed in the fifth column of the “Initial Building Data” sheet, Table 6.2. The entered quantity in “Initial Quantity” sheet will appear in more than one cell in the “Initial Building Data” sheet depending on the maintenance

actions available for the specific condition state. If, for example, 15 LF of coping cap at condition state 4 is entered for Alumini Memorial Building roof section C, Table 6.1, the quantity will appear in the “Initial Building Data” sheet at condition state 43, 44, and 45, Table 6.2. This means that the possible maintenance action for the 15 LF coping cap at condition 4 are do nothing, 43 and 44, or replace the element, 45.

Other data entered into the “Initial Building Data” sheet are EUAC, ECSDF, Damage cost, and MAUCST. The values displayed in these columns are obtained from different sheets used to perform the calculations.

The “Initial Building Data” sheet will also be used to create the optimization model only for the first year of the analysis horizon that has a total of 15 years. Table 6.2 shows a sample of the data entered into the “Initial Building Data” sheet for building 1, section 3 and elements 10 and 11. Table 6.3 describes the content of each column in Table 6.2.

Table 6.1 "Initial Quantity" data sheet

Building	Section	CS	Maintenance Elements														
			Single-ply membrane (SF)	Multiple-ply membrane (SF)	Insulation (SF)	Single-ply base flashing (LF)	Multiple-ply base flashing (LF)	Counter flashing (LF)	Gutter (LF)	Downspout (LF)	Drain strainer (EA)	Coping cap (LF)	Termination bar (LF)	Edge metal (LF)	Shingle (SF)	Slate (SF)	Hip/Ridge cap (LF)
Alumini Memorial Building	A	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		5	0.0	0.0	0.0	239.7	0.0	239.7	239.7	0.0	0.0	0.0	0.0	239.7	4871.0	0.0	0.0
	B	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		4	0.0	0.0	0.0	0.0	0.0	77.3	0.0	0.0	1.0	0.0	0.0	20.0	0.0	0.0	0.0
		5	2207.0	0.0	0.0	154.7	0.0	77.3	0.0	0.0	3.0	0.0	0.0	134.7	0.0	0.0	0.0
	C	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		3	0.0	0.0	0.0	0.0	0.0	4.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		4	0.0	0.0	0.0	0.0	0.0	23.9	0.0	36.0	0.0	15.0	56.9	0.0	0.0	0.0	0.0
		5	368.0	0.0	0.0	75.8	0.0	47.9	0.0	0.0	0.0	60.8	19.0	0.0	0.0	0.0	0.0
D.H.Hill Library	A1	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	98.6	0.0	0.0	0.0
		4	429.0	0.0	0.0	0.0	0.0	0.0	98.6	24.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	A2	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	191.0	0.0	0.0	0.0
		4	3129.0	0.0	0.0	235.3	0.0	235.3	235.3	24.0	0.0	0.0	0.0	44.3	0.0	0.0	0.0
		5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	A3	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	573.0	0.0	0.0	0.0
		4	15876.0	0.0	0.0	573.0	0.0	573.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	A4	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		4	260.0	0.0	0.0	577.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	577.6	0.0	0.0	0.0
		5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 6.2 “Initial Building Data” sheet

Building	Section	Element	CS	Qty-1	EUAC	ECSDf	Damage cost	MAUCST	Obj. Fun. Coeff.	Budget Coeff.
1	3	10	10	0.0	0.00	0.00	103.50	0.00	0.00	0.00
			11	0.0	3.50	0.07	103.50	0.00	0.00	0.00
			12	0.0	1000.00	0.07	103.50	100.00	0.00	0.00
			13	0.0	1000.00	0.07	103.50	100.00	0.00	0.00
			14	0.0	1000.00	0.07	103.50	100.00	0.00	0.00
			15	0.0	3.28	0.07	103.50	25.00	0.00	0.00
			21	0.0	0.00	0.00	103.50	0.00	0.00	0.00
			22	0.0	3.49	0.05	103.50	0.00	0.00	0.00
			23	0.0	1000.00	0.05	103.50	100.00	0.00	0.00
			24	0.0	1000.00	0.05	103.50	100.00	0.00	0.00
			25	0.0	3.28	0.05	103.50	25.00	0.00	0.00
			32	0.0	0.00	0.00	103.50	0.00	0.00	0.00
			33	0.0	3.53	0.04	103.50	0.00	0.00	0.00
			34	0.0	3.22	0.04	103.50	10.00	0.00	0.00
			35	0.0	3.28	0.04	103.50	25.00	0.00	0.00
			43	15.0	0.00	0.00	103.50	0.00	0.00	0.00
			44	15.0	2.92	0.00	103.50	0.00	-43.74	0.00
			45	15.0	3.28	0.00	103.50	25.00	-49.20	375.00
			54	60.8	0.00	0.00	103.50	0.00	0.01	0.00
			55	60.8	2.75	0.00	103.50	0.00	-167.24	0.00
		11	10	0.0	0.00	0.00	103.50	0.00	0.00	0.00
			11	0.0	0.59	0.07	103.50	0.00	0.00	0.00
			12	0.0	1000.00	0.07	103.50	100.00	0.00	0.00
			13	0.0	1000.00	0.07	103.50	100.00	0.00	0.00
			14	0.0	1000.00	0.07	103.50	100.00	0.00	0.00
			15	0.0	0.37	0.07	103.50	3.00	0.00	0.00
			21	0.0	0.00	0.00	103.50	0.00	0.00	0.00
			22	0.0	0.84	0.03	103.50	0.00	0.00	0.00
			23	0.0	1000.00	0.03	103.50	100.00	0.00	0.00
			24	0.0	1000.00	0.03	103.50	100.00	0.00	0.00
			25	0.0	0.37	0.03	103.50	3.00	0.00	0.00
			32	0.0	0.00	0.00	103.50	0.00	0.00	0.00
			33	0.0	0.46	0.01	103.50	0.00	0.00	0.00
			34	0.0	0.36	0.01	103.50	1.00	0.00	0.00
			35	0.0	0.37	0.01	103.50	3.00	0.00	0.00
			43	56.9	0.00	0.00	103.50	0.00	0.01	0.00
			44	56.9	0.33	0.00	103.50	0.00	-18.94	0.00
			45	56.9	0.37	0.00	103.50	3.00	-20.98	170.62
			54	19.0	0.00	0.00	103.50	0.00	0.00	0.00
			55	19.0	0.31	0.00	103.50	0.00	-5.95	0.00

Table 6.3 Column content description for the “Initial Building Data” sheet

Column No.	Column Content Description
1	Building number
2	Section number
3	Element number
4	Element initial condition state and suggested final condition state
5	Quantity of element at initial condition state
6	Equivalent, uniform annual cost for improving the element from the initial condition state to the final condition state
7	Element Condition State Damage Factor of the element at the initial condition state
8	Damage cost
9	Maintenance action unit cost of maintaining, improving or replacing an element
10	Objective function coefficient, calculated as: $Qty * (ECSDF * Damage\ cost - EUAC)$
11	Budget constraint coefficient, calculated as: $Qty * MAUCST$

The “Building Data” sheet was used to enter the data and perform the analysis for the remaining 14 years in the analysis horizon. The “Building Quantity” sheet is similar to the “Initial Quantity” sheet but it is linked to column 14 of “Building Data” sheet and is used to display the condition state and quantity of each roof element after suggested maintenance is applied for the analyzed year. Table 6.4 shows a sample of the data entered into the “Building Data” sheet for building 1, section 3 and elements 10 and 11. Table 6.5 describes the content of each column in Table 6.4.

The “Decision” sheet was used to perform the re-roofing optimization. The resulting decision of whether to perform maintenance or re-roofing will appear in column 12 of the “Building data” sheet, Table 6.4.

Table 6.4 “Building Data” sheet entry

Building	Section	Element	CS	Qty-1	EUAC	ECSDF	Damage cost	MAUCST	Ample output	MUCST	Maintenance Decision	New Qty-1	Qty-2	Aged Qty-2	Obj. Fun. Coeff.	Budget Coeff.
1	3	10	10	0.0	0.00	0.00	103.50	0.00	1	0.00	Maintenance	0.0	0.0	0.0	0.00	0.00
			11	0.0	3.50	0.07	103.50	0.00	0	0.00	Maintenance	0.0	0.0	0.0	0.00	0.00
			12	0.0	1000.00	0.07	103.50	100.00	0	0.00	Maintenance	0.0	0.0	0.0	0.00	0.00
			13	0.0	1000.00	0.07	103.50	100.00	0	0.00	Maintenance	0.0	0.0	0.0	0.00	0.00
			14	0.0	1000.00	0.07	103.50	100.00	0	0.00	Maintenance	0.0	0.0	0.0	0.00	0.00
			15	0.0	3.28	0.07	103.50	25.00	0	0.00	Maintenance	0.0	0.0	0.0	0.00	0.00
			21	0.0	0.00	0.00	103.50	0.00	1	0.00	Maintenance	0.0	0.0	0.0	0.00	0.00
			22	0.0	3.49	0.05	103.50	0.00	0	0.00	Maintenance	0.0	0.0	0.0	0.00	0.00
			23	0.0	1000.00	0.05	103.50	100.00	0	0.00	Maintenance	0.0	0.0	0.0	0.00	0.00
			24	0.0	1000.00	0.05	103.50	100.00	0	0.00	Maintenance	0.0	0.0	0.0	0.00	0.00
			25	0.0	3.28	0.05	103.50	25.00	0	0.00	Maintenance	0.0	0.0	0.0	0.00	0.00
			32	13.2	0.00	0.00	103.50	0.00	0	0.00	Maintenance	0.0	0.0	13.8	0.00	0.00
			33	13.2	3.53	0.04	103.50	0.00	0	0.00	Maintenance	0.0	0.0	13.8	8.47	0.00
			34	13.2	3.22	0.04	103.50	10.00	1	187.03	Maintenance	13.2	0.0	13.8	12.72	137.71
			35	13.2	3.28	0.04	103.50	25.00	0	0.00	Maintenance	0.0	0.0	13.8	11.84	344.29
			43	49.0	0.00	0.00	103.50	0.00	1	0.00	Maintenance	49.0	62.3	50.6	0.01	0.00
			44	49.0	2.92	0.00	103.50	0.00	0	0.00	Maintenance	0.0	62.3	50.6	-147.48	0.00
			45	49.0	3.28	0.00	103.50	25.00	0	0.00	Maintenance	0.0	62.3	50.6	-165.88	1264.26
			54	13.6	0.00	0.00	103.50	0.00	1	0.00	Maintenance	13.6	13.6	11.5	0.00	0.00
			55	13.6	2.75	0.00	103.50	0.00	0	0.00	Maintenance	0.0	13.6	11.5	-31.59	0.00
		11	10	0.0	0.00	0.00	103.50	0.00	1	0.00	Maintenance	0.0	0.0	0.0	0.00	0.00
			11	0.0	0.59	0.07	103.50	0.00	0	0.00	Maintenance	0.0	0.0	0.0	0.00	0.00
			12	0.0	1000.00	0.07	103.50	100.00	0	0.00	Maintenance	0.0	0.0	0.0	0.00	0.00
			13	0.0	1000.00	0.07	103.50	100.00	0	0.00	Maintenance	0.0	0.0	0.0	0.00	0.00
			14	0.0	1000.00	0.07	103.50	100.00	0	0.00	Maintenance	0.0	0.0	0.0	0.00	0.00
			15	0.0	0.37	0.07	103.50	3.00	0	0.00	Maintenance	0.0	0.0	0.0	0.00	0.00
			21	0.0	0.00	0.00	103.50	0.00	1	0.00	Maintenance	0.0	0.0	0.0	0.00	0.00
			22	0.0	0.84	0.03	103.50	0.00	0	0.00	Maintenance	0.0	0.0	0.0	0.00	0.00
			23	0.0	1000.00	0.03	103.50	100.00	0	0.00	Maintenance	0.0	0.0	0.0	0.00	0.00
			24	0.0	1000.00	0.03	103.50	100.00	0	0.00	Maintenance	0.0	0.0	0.0	0.00	0.00
			25	0.0	0.37	0.03	103.50	3.00	0	0.00	Maintenance	0.0	0.0	0.0	0.00	0.00
			32	17.7	0.00	0.00	103.50	0.00	0	0.00	Maintenance	0.0	0.0	17.9	0.00	0.00
			33	17.7	0.46	0.01	103.50	0.00	0	0.00	Maintenance	0.0	0.0	17.9	10.27	0.00
			34	17.7	0.36	0.01	103.50	1.00	1	36.11	Maintenance	17.7	0.0	17.9	12.05	17.93
			35	17.7	0.37	0.01	103.50	3.00	0	0.00	Maintenance	0.0	0.0	17.9	11.94	53.79
			43	54.4	0.00	0.00	103.50	0.00	1	0.00	Maintenance	54.4	72.1	54.8	0.01	0.00
			44	54.4	0.33	0.00	103.50	0.00	0	0.00	Maintenance	0.0	72.1	54.8	-18.25	0.00
			45	54.4	0.37	0.00	103.50	3.00	0	0.00	Maintenance	0.0	72.1	54.8	-20.22	164.48
			54	3.7	0.00	0.00	103.50	0.00	1	0.00	Maintenance	3.7	3.7	3.1	0.00	0.00
			55	3.7	0.31	0.00	103.50	0.00	0	0.00	Maintenance	0.0	3.7	3.1	-0.97	0.00

Table 6.5 Column content description for the “Building Data” sheet

Column Number	Column Content Description
1	Building number
2	Section number
3	Element number
4	Element initial condition state and suggested final condition state
5	Quantity of element at initial condition state
6	Equivalent, uniform annual cost for improving the element from the initial condition state to the final condition state
7	Element Condition State Damage factor at initial condition state
8	Damage cost
9	Maintenance action unit cost of maintaining, improving or replacing an element
10	Ampl output, the optimum value of the decision variables obtained from AMPL
11	Maintenance and user cost resulting from implementing the selected maintenance action
12	Maintenance or Re-roof decision when MUCST is compared to the equivalent uniform annual re-roof cost
13	Element quantity available at each condition state based on the maintenance or re-roof decision
14	Addition of all element quantity improved to or remained at each condition state
15	Element quantities aged using Markov Chain. The aged quantities will serve as element initial quantities for next year analysis
16	Objective function coefficient, calculated as: $\text{Aged Qty}_2 * (\text{ECSDF} * \text{Damage cost} - \text{EUAC})$
17	Budget constraint coefficient, calculated as: $\text{Aged Qty}_2 * \text{MAUCST}$

6.3 AMPL Operations

AMPL is the acronym for A Mathematical Programming Language. AMPL is a comprehensive, powerful and flexible algebraic modeling language for the linear, nonlinear and integer programming problems often encountered in optimization. It uses familiar algebraic notation to express models concisely, while allowing the separation of model from the data file. This feature helps in modifying and maintaining the model by allowing sets,

variables and constraints to be added, changed or deleted. The separation feature helps also in updating the data file quickly. AMPL is the software program used in the research to perform the optimization task. AMPL uses the branch and bound algorithm and CPLEX for its solver.

The model files required by AMPL are generated using Excel. A visual basic program, VB, generates the model file required for the first year analysis, while the visual basic program VB1 generates the maintenance model file required for the remaining years of the analysis horizon. VB3 is a third visual basic program used to generate the re-roofing model file required for all years in the analysis horizon when using the separated budget approach.

The data file, containing the parameter values required to run the optimization model, is created in advance and stored in a path retrievable by AMPL. There are two data files. The first data file, “data.dat”, contains the data required to run the maintenance model generated by both VB and VB1. The second data file, “r_data.dat”, is used to run the re-roofing model generated by VB3. Both data files can be found in Appendix B3.

The following commands are used with AMPL to obtain the optimal value of the decision variables for each year in the analysis horizon for the maintenance part of the analysis when performing the separated budget approach analysis (i represents the analyzed year):

```
ampl: Reset;  
ampl: model"/afs/unity.ncsu.edu/users/a/aaalibra/outputs/model_year i.mod";  
ampl: data"/afs/unity.ncsu.edu/users/a/aaalibra/data.dat";  
ampl: solve;  
ampl: print {i in I1, j in J1, l in L1, k in K1} x [i, j, l, k]>output;  
ampl: close output;
```

The following commands are fed into AMPL to obtain the optimal value of the decision

variables for each year in the analysis horizon. These commands are only used for the re-roofing part of the analysis when performing the separated budget analysis approach (i represents the analysis year):

```
ampl: Reset;  
ampl: model"/afs/unity.ncsu.edu/users/a/aaalibra/outputs/reroof_year i.mod";  
ampl: data"/afs/unity.ncsu.edu/users/a/aaalibra/r_data.dat";  
ampl: solve;  
ampl: print {n in N1} x [n]>output;  
ampl: close output;
```

6.4 Separated Budget Optimization

This optimization is called “separated budget” because two budgets are used in performing separate optimizations. The first budget, \$54,000, is used to optimize maintenance work. Maintenance work includes repairing or replacement of roof elements only. With this budget no re-roofing is performed. The second budget is the capital budget, \$246,000, and is used to perform re-roofing work. Re-roofing includes replacing all elements of a roof section. The steps followed to perform the analysis are shown in Figure 6.1.

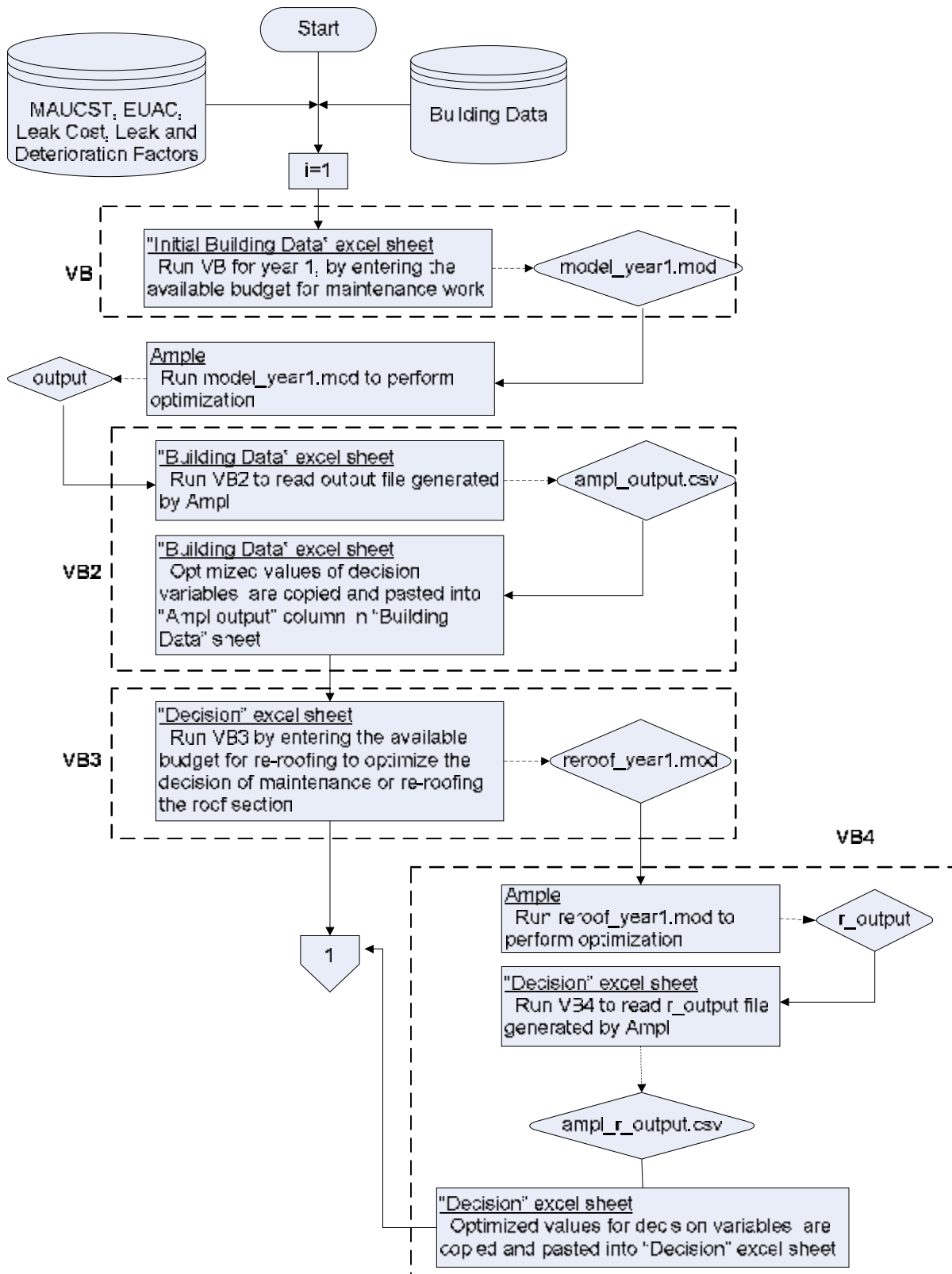


Figure 6.1 Flowchart for use of Excel and Ampl to optimize roof maintenance using the separated budget approach

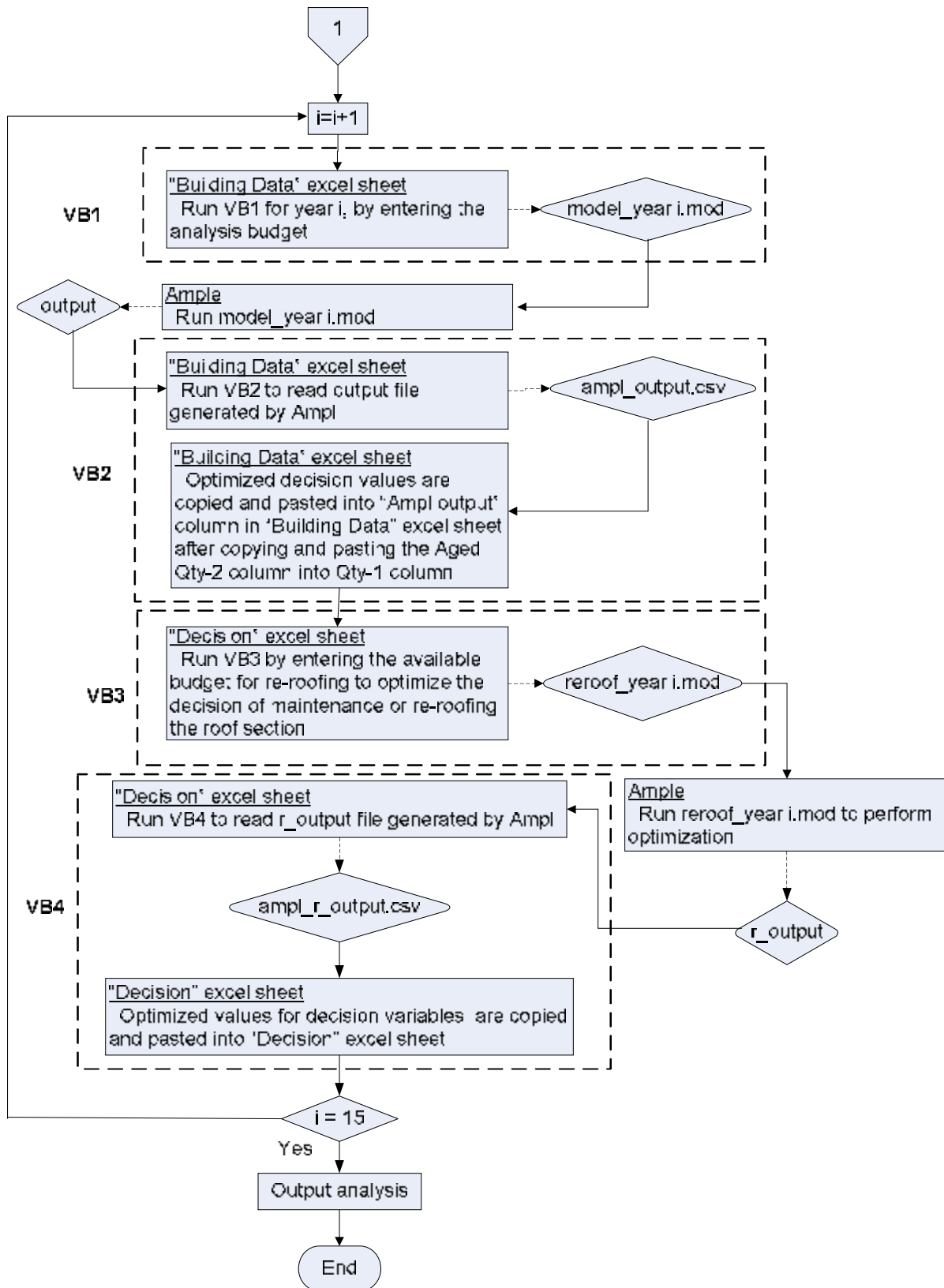


Figure 6.1 Continued

6.4.1 First Year Analysis

The input data are entered in “Initial Quantity” sheet. The data is then displayed in column 5 of “Initial Building Data” sheet as explained earlier in the chapter. Before starting the optimization, the initial element quantities displayed in column 5 in “Initial Building Data” sheet are copied from and pasted in column 5 of “Building Data” sheet. The Visual Basic program VB is then run from the “Initial Building Data” sheet (see figure 6.1). A dialog box will appear asking the user to enter the maintenance budget for the analysis. By entering the budget a binary integer linear program text file is created in an algebraic modeling language readable by AMPL. The output file generated by VB is named “model_year1.mod” and is stored in a path retrievable by AMPL. The VB code used to generate “model_year1.mod” text file can be found in Appendix B1.1. A sample generated “model_year1.mod” file can be found in Appendix B2.1.

After obtaining the output file resulting from performing the optimization using AMPL, a second Visual Basic program, VB2, is run from the “Building Data” sheet. VB2 reads the output file generated by AMPL containing the optimized values of the decision variables. By executing VB2, a message box appears notifying the user that a file called “ampl_output.csv” is created. The created file is an Excel file containing the optimized values of the maintenance decision variables generated by AMPL but translated by Excel from binary format to ascii or text format. The binary decision variables will be displayed column wise. The contents of “ampl_output.csv” are then copied and pasted in column 10 of “Building Data” sheet. The VB2 code used for this step can be found in Appendix B1.3.

A third Visual Basic program, VB3, will be run from the “Decision” sheet. A dialog box appears asking the user to enter the budget available for re-roofing. By entering the

budget, a file named “reroof_year.mod” is created in an algebraic modeling language readable by AMPL, and stored in a path retrievable by AMPL. The VB3 code used to generate “reroof_year.mod” can be found in Appendix B1.4. An example of the file “reroof_year.mod” generated by VB3 can be found in Appendix B2.2.

After obtaining the optimization results from AMPL, a forth Visual Basic program, VB4, is run. VB4 reads the output file generated by AMPL containing the optimized values of the re-roofing decision variables. By executing VB4, a message box appears notifying the user that a file called “ampl_r_output.csv” is created. The created file is an Excel file containing the optimized values of the re-roofing decision variables generated by AMPL but translated by Excel from binary format to ascii or text format. The binary decision variables are displayed column wise. The values of the later file are copied and pasted into the “Ampl Output” column in the “Decision” sheet. The VB4 code used for this step can be found in Appendix B1.5. At this point, the analysis for the first year is completed.

6.4.2 Remaining Years Analysis

Before starting the analysis of the next year, the aged element quantities in column 15 of “Building data” sheet are copied and pasted in column 5 of the same sheet. To start the next year analysis, VB1 (see Figure 6.1) is run. A dialog box appears asking the user to enter the maintenance budget for the analysis. By entering the budget a message box will appear telling the user that a file called “model_year.mod” is created. The created file is in an algebraic modeling language readable by AMPL and will be stored in a path retrievable by AMPL. The VB1 code used to generate the binary linear program text file can be found in Appendix B1.2. An example of the file “model_year.mod” generated by VB1 can be found

in Appendix B2.1. After obtaining the output file resulting from performing the optimization using AMPL, VB2 is run from “Building Data” sheet and the decision variable values in “ampl_output.csv” are copied into column 10 as explained in the first year analysis. VB3 and VB4 will also be run from “Decision” sheet as explained in the previous section.

The steps in this section are repeated for the number of years remaining in the analysis horizon. The steps followed to perform the analysis for the separated budget approach are shown in Figure 6.1.

6.5 Combined Budget Optimization

This step is used to assess the total budget available to the DM&R. The hypothesis is that a combined budget will show better future savings when compared to the analysis approach results of the separated budgets discussed earlier.

In the combined budget approach, both the available budget for maintenance and the available budget for re-roofing are combined and considered to be the total budget used to optimize roof maintenance.

The Excel file used in the previous approach is also be used for this approach. All sheets are used except for the “Decision” sheet. Since the maintenance and re-roofing budgets are combined into one budget, this approach will perform only the first optimization of the previous approach. For this reason VB3 and VB4 will not be used and AMPL will only be used to run the first optimization of the separated budget approach using the same commands.

The steps followed to perform one budget analysis for the combined budget approach are similar to the steps followed to perform one budget analysis for the separated budget

analysis except for the re-roofing part of the analysis that will not be executed. The steps followed to perform one year analysis of the combined budgets are explained below and shown in Figure 6.2.

6.5.1 First Year Analysis

The input data are entered in “Initial Quantity” sheet. The data is then displayed in column 5 of “Initial Building Data” sheet as explained earlier in the chapter. Before starting the optimization, the initial element quantities displayed in column 5 in “Initial Building Data” sheet are copied from and pasted in column 5 of “Building Data” sheet. A Visual Basic program called VB is then run from “Initial Building Data” sheet (see Figure 6.2). A dialog box will appear asking the user to enter the maintenance budget for the analysis. By entering the budget, a binary integer linear program text file is created in an algebraic modeling language readable by AMPL. The output file generated by VB will be called “model_year1.mod” and will be stored in path retrievable by AMPL. The VB code used to generate “model_year1.mod” text file can be found in Appendix B1.1. A sample generated “model_year1.mod” file can be found in Appendix B2.1.

After obtaining the output file resulting from performing the optimization using AMPL, a second Visual Basic program, VB2, is run from “Building Data” sheet. VB2 will read the output file generated by AMPL containing the optimized values of the decision variables. By executing VB2, a message box will appear notifying the user that a file called “ampl_output.csv” is created. The created file is an Excel file containing the optimized values of the maintenance decision variables generated by AMPL but translated by Excel from binary format to ascii or text format. The binary decision variables will be displayed

column wise. The content of “ampl_output.csv” are then copied and pasted in column 10 of “Building Data” sheet. The VB2 code used for this step can be found in Appendix B1.3. Upon reaching this step, the analysis of the first year is completed.

6.5.2 Remaining Years Analysis

Before starting the analysis of the next year, the aged element quantities in column 15 of “Building data” sheet are copied and pasted in column 5 of the same sheet. To start the next year analysis, VB1 (see Figure 6.2), is run. A dialog box will appear asking the user to enter the maintenance budget for the analysis. By entering the budget a message box will appear telling the user that a file called “model_year.mod” is created. The created file is in an algebraic modeling language readable by AMPL and will be stored in a path retrievable by AMPL. The VB1 code used to generate the binary linear program text file can be found in Appendix B1.2. An example of the file “model_year.mod” generated by VB1 can be found in Appendix B2.1. After obtaining the outputs file resulting from performing the optimization using AMPL, VB2 is run from “Building Data” sheet and the decision variable values in “ampl_output.csv” are copied into column 10 as explained in the first year analysis.

The steps in this section are repeated for the number of years remaining in the analysis horizon. The steps followed to perform the analysis for the combined budget approach are shown in Figure 6.2.

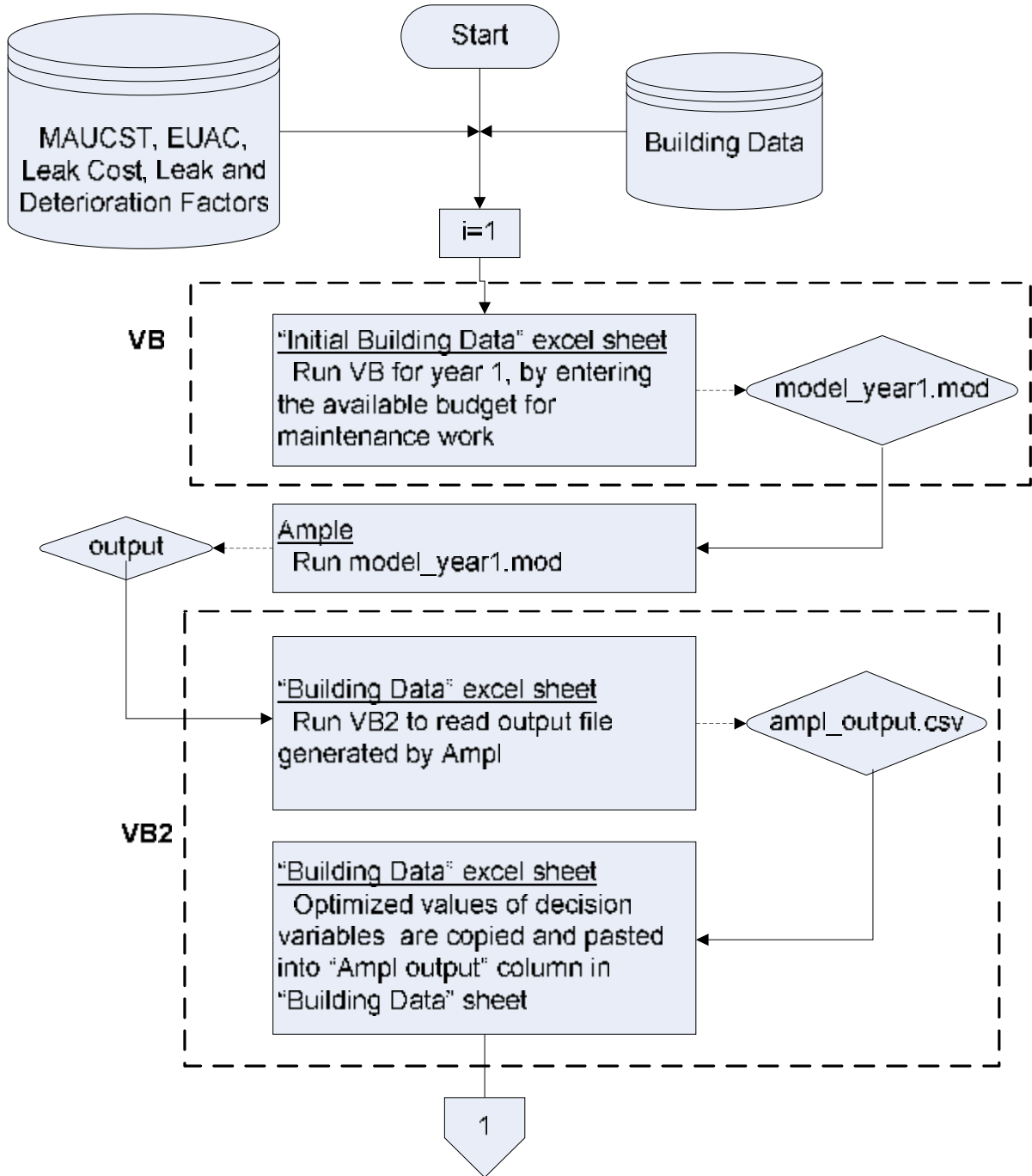


Figure 6.2 Flowchart for use of Excel and AMPL to optimize roof maintenance using combined budget approach

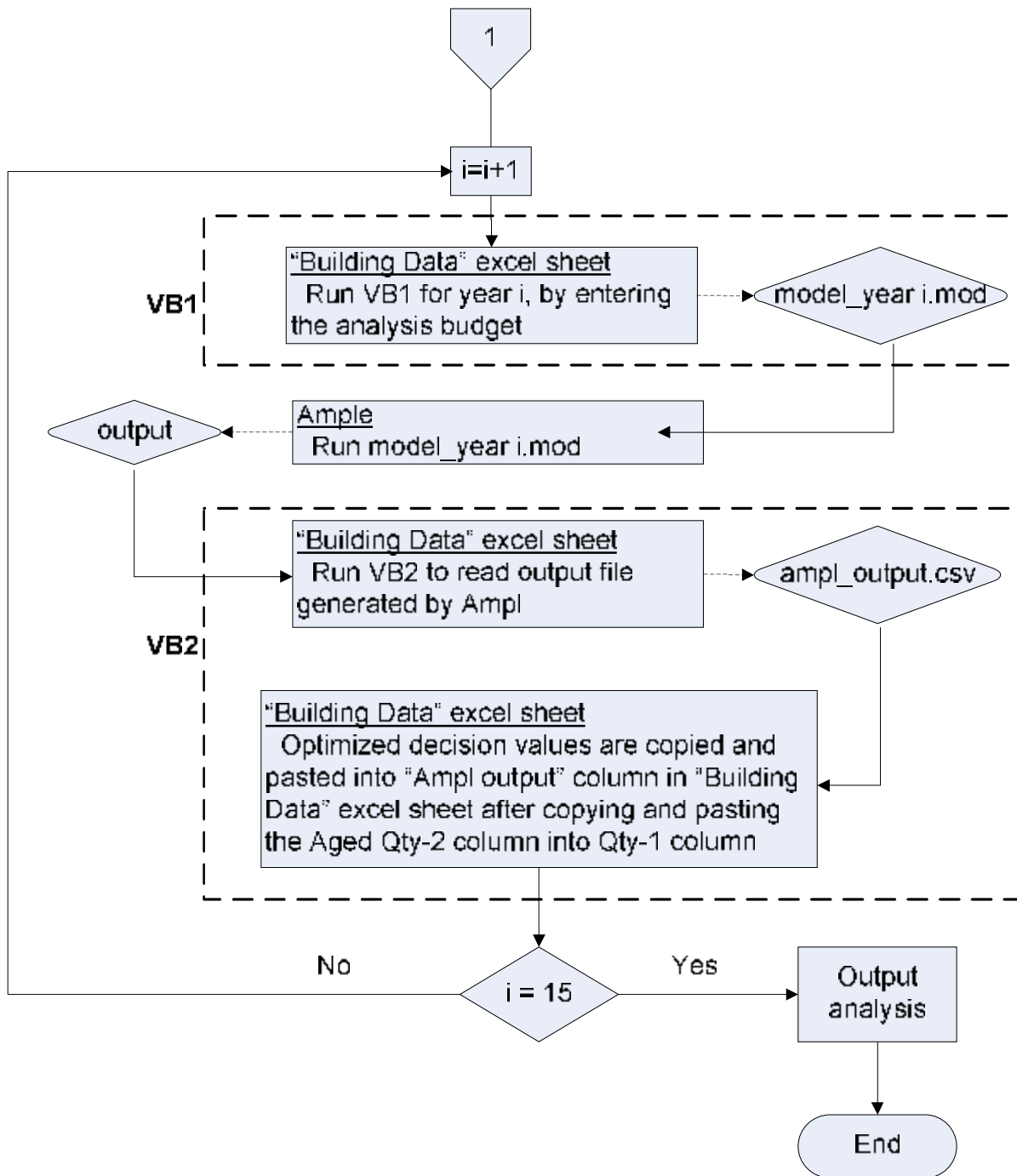


Figure 6.2 Continued

7. ANALYSIS AND RESULTS

This chapter presents an evaluation of the impact of different budget levels and real rates of return on the model outputs. The results from the analysis are the user costs, total costs and the weighted average condition state of each roof element. The user costs and total costs will be used as indicators of cost effects the DM&R may encounter using the Decision Support System. Better maintenance of roofs results in a higher condition state of the roof elements and a reduction in the user costs. However, better maintenance results in increases in maintenance costs offsetting some of the user cost savings.

The analyses conducted are divided into three groups to evaluate the optimization model. The first group examines the effect of using a combined budget approach vs. a separated budget approach. The second group examines the sensitivity of the model to budget reductions and increases. The last group examines the sensitivity of the model to variations in the required real rates of return. The three groups are shown in Table 7.1.

7.1 **Separated Vs. Combined Budgets**

As mentioned in the previous chapter, the DM&R has two budgets: the maintenance budget, which is available for all maintenance work including replacement of individual roof elements, and the capital budget, available to the department for re-roofing purposes. This section will present the optimization results from using the two budgets separately, \$54,000 for maintenance work and \$246,000 for re-roofing work, versus combining the two

budgets and considering it to be the only budget available and used for maintenance work or re-roofing.

Table 7.1 Analysis groups

Group		Total budget	Real Rate of Return
1	Separated	\$54,000 maintenance + \$246,000 re-roofing	3.0%
	Combined	\$300,000	3.0%
2	Budget 1	\$200,000	3.0%
	Budget 2	\$250,000	3.0%
	Budget 3	\$300,000	3.0%
	Budget 4	\$350,000	3.0%
	Budget 5	\$400,000	3.0%
3	RRR 1	\$300,000	1.5%
	RRR 2	\$300,000	3.0%
	RRR 3	\$300,000	4.5%
	RRR 4	\$300,000	6.0%
	RRR 5	\$300,000	7.5%

The User costs resulting from the separated budget maintenance optimization and combined budget optimization are shown in Table 7.2 and Figure 7.1.

Table 7.2 User costs for the separated and combined budget cases

Year	User Cost (\$)		Difference (\$)
	54K & 246K	300K	54K-300K
1	130,912	109,069	21,842
2	282,068	185,676	96,392
3	438,096	271,675	166,421
4	518,957	334,341	184,616
5	558,334	368,194	190,139
6	559,835	376,320	183,515
7	552,484	356,988	195,495
8	540,056	314,721	225,335
9	523,633	263,808	259,825
10	488,968	205,652	283,315
11	445,650	138,441	307,209
12	399,281	79,230	320,051
13	349,705	33,644	316,061
14	301,206	19,001	282,205
15	270,692	6,600	264,092
Sum	6,359,877	3,063,362	3,296,515

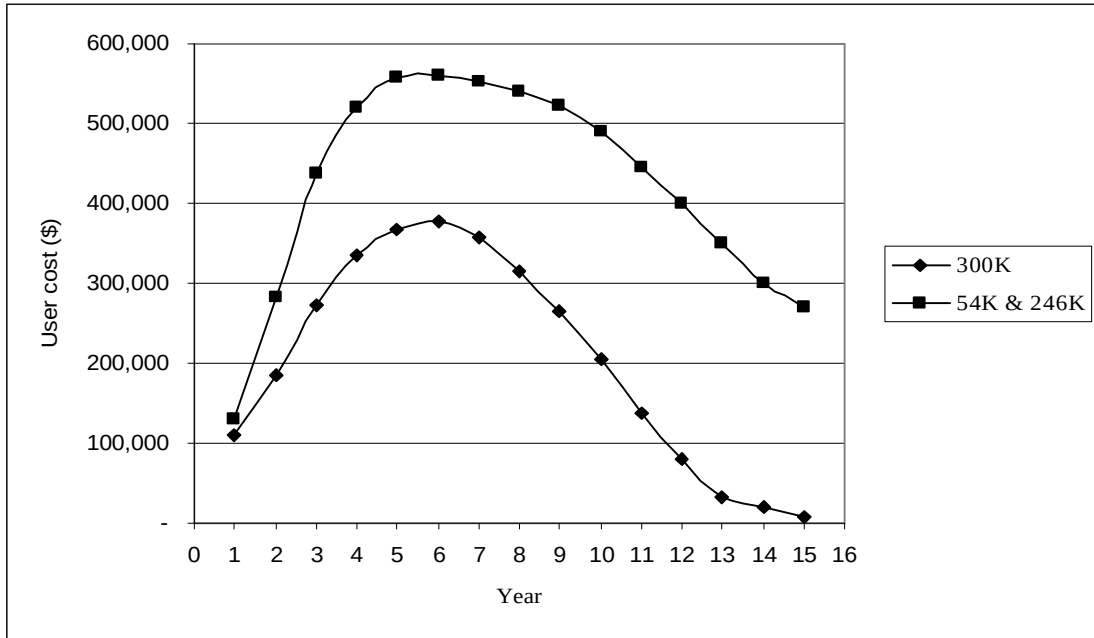


Figure 7.1 User costs for the separated and combined budget cases

From the results of this analysis, the user costs of the combined budget are always lower than the user costs when the budget is separated. Table 7.2 and Figure 7.2 show the difference between the resulting user costs for the two budget approaches. It is also shown that the difference between the two user costs reaches a peak of about \$320,000.

The results of this analysis indicate that it is better to utilize the total combined budget available to perform maintenance and re-roofing rather than constrain the budget in fixed categories. Based on these results, the combined budget approach will be used when investigating the sensitivity of user costs and weighted average condition state to different budget levels and real rates of return.

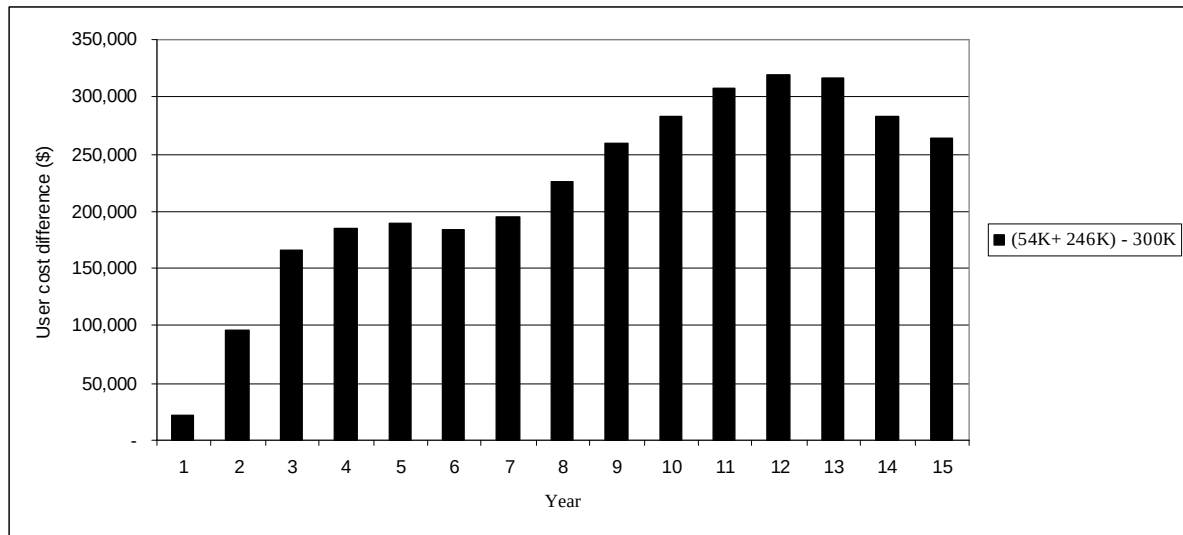


Figure 7.2 User cost difference between the separated and combined budgets

7.2 Sensitivity of User Costs and Element Weighted Average Condition State to Budget Reduction and Increase

Five budget levels were used to analyze the effect of budget reductions and increases on the user costs and weighted average element condition state. The five different budgets used for the analysis are shown in Table 7.1. Tables and graphs showing the response of user costs and element weighted average condition state to each budget level are provided in Appendix C1 and C2 respectively.

7.2.1 User Costs Sensitivity to Budget Levels

Results in Table 7.3 and Figure 7.3 indicate user cost is significantly sensitive to changes in budget available for maintenance and re-roofing work. As the available budget increases, the user costs decreases. With higher budgets, more funds are available to repair or replace deteriorated elements. This increases roof elements to higher condition state

levels. By keeping elements at higher condition states the user costs decrease by reducing or eliminating leaks which cause internal damage to the building.

Table 7.3 User cost sensitivity to budget levels

Year	User Cost (\$)				
	200K	250K	300K	350K	400K
1	122,092	115,462	109,069	102,388	95,693
2	264,867	238,235	211,414	184,618	158,118
3	450,840	386,149	327,545	269,308	211,600
4	592,859	488,561	391,776	307,086	216,833
5	689,578	547,613	423,910	301,030	160,825
6	751,407	575,171	422,622	253,603	79,476
7	787,391	581,345	386,529	190,704	38,401
8	804,597	566,147	339,269	117,224	12,327
9	807,275	538,245	286,247	41,684	-
10	793,970	501,128	225,688	20,458	-
11	774,870	462,605	159,477	1,961	-
12	752,278	419,951	88,884	-	-
13	711,696	375,635	34,709	-	-
14	691,896	330,351	19,297	-	-
15	666,189	284,779	4,791	-	-
Sum	9,661,805	6,411,375	3,431,228	1,790,065	973,273

Table 7.3 and Figure 7.3 show that for the current \$300,000 budget available to the DM&R, the maximum user cost reached was \$423,910 at year 5 and by year 15 the user cost reaches its' lowest value of \$4,791. An increase in the budget to \$350,000 will reduce the maximum user cost reached during the 15 year horizon to \$307,086. A zero user cost is reached by year 12 and the whole roof maintenance cost will be less than the available budget, which means reduction in the fund expenditure, Figure 7.4 and Table 7.4. If the budget is increased to \$400,000, then the maximum user cost reduces to \$216,833. The university will then start reducing its expenditure by year 9, Figure 7.4, because the user cost at that year will equal zero. If the available budget decreases to \$250,000, the maximum user cost reached during the analysis horizon will be \$581,345. If the budget reduces to

\$200,000, then the maximum user cost will increase to \$807,275. In both the \$250,000 and \$200,000 budget, the fund expenditure will remain constant and equal to the available budget, Figure 7.4.

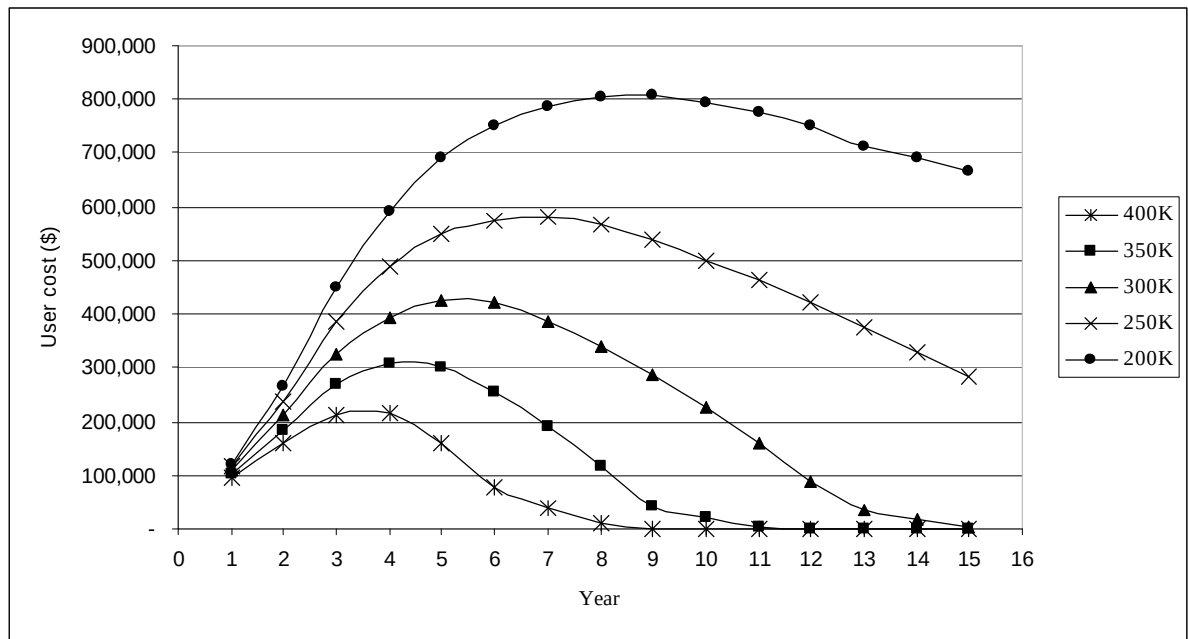


Figure 7.3 User cost sensitivity to budget levels

The total cost to the university is the sum of the actual budget used plus the user cost resulting from deteriorated roof elements. Figure 7.5 and Table 7.5 show that the total cost decreases with the increase of the budget for element maintenance and replacement. It also shows that before year 3, the higher the budget, the higher the total cost. This changes to the opposite beyond year three and continues throughout the analysis horizon. Figure 7.5 and Table 7.5 show that the lowest budget needed to maintain roof elements at an acceptable levels (CS 4 and 5) is around \$200,000. This is only true after bringing all deteriorated

elements to condition state 4 or 5 by maintenance or replacement with higher initial budget levels.

Figure 7.4 and Table 7.4 show the actual fund expenditure for different budget levels. In the long run, the higher the budget available, the lower the total cost to university even if the total funds expended increase initially with the increase of available budget.

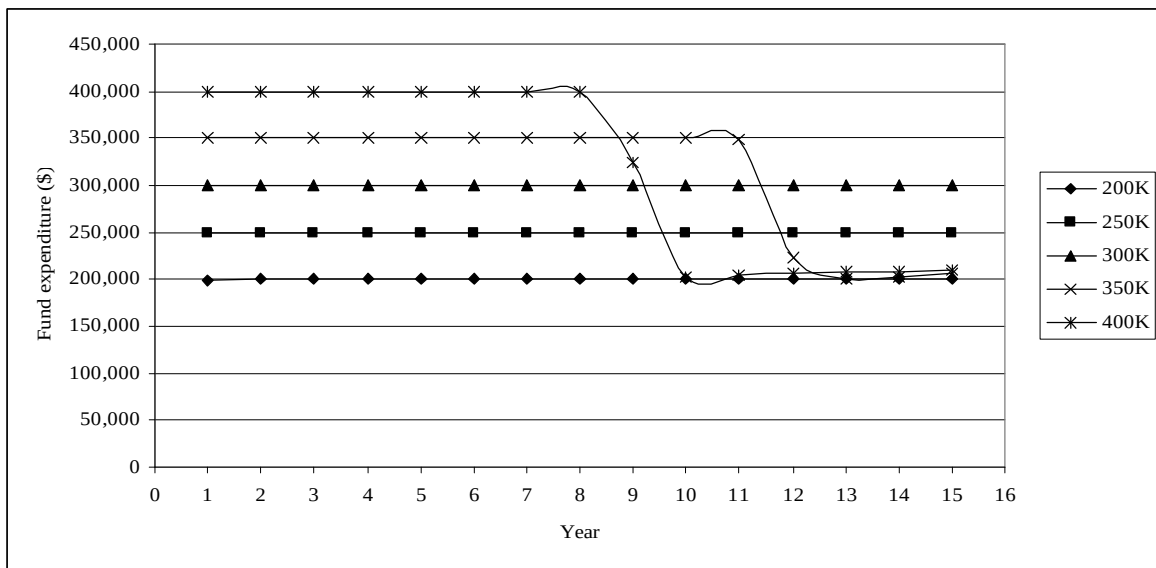


Figure 7.4 Funds expended for different budget levels

Table 7.4 Funds expended for different budget levels

Year	Budget Used (\$)				
	200K	250K	300K	350K	400K
1	199,498	249,828	299,945	349,777	399,915
2	199,954	249,983	299,993	349,956	399,953
3	199,997	249,961	299,997	349,997	400,000
4	199,998	249,974	299,968	349,998	399,995
5	199,993	249,999	299,989	349,999	399,989
6	199,988	249,999	299,998	349,999	399,994
7	199,998	249,994	299,989	349,993	399,991
8	199,999	249,996	299,998	349,996	399,997
9	199,999	249,998	299,996	349,988	324,061
10	199,999	249,997	299,999	349,996	203,139
11	199,999	249,998	299,996	349,518	205,079
12	199,999	250,000	299,999	223,308	206,655
13	200,000	250,000	299,986	200,215	207,934
14	200,000	249,993	299,970	203,093	208,971
15	200,000	250,000	299,980	205,331	209,813
Sum	2,999,421	3,749,720	4,499,805	4,681,164	4,765,485

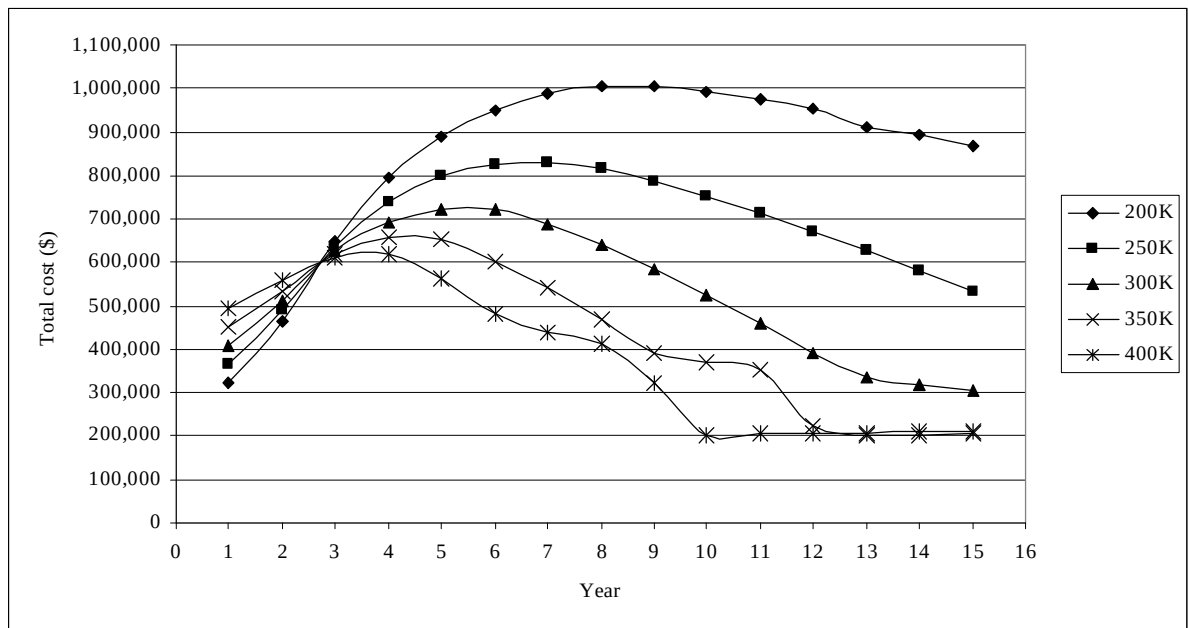


Figure 7.5 Total cost for different budget levels

Table 7.5 Total cost for different budget levels

Year	Total Cost (\$)				
	200K	250K	300K	350K	400K
1	321,590	365,290	409,014	452,165	495,608
2	464,821	488,218	511,407	534,575	558,071
3	650,837	636,110	627,542	619,305	611,600
4	792,857	738,534	691,744	657,085	616,828
5	889,571	797,613	723,899	651,029	560,814
6	951,394	825,169	722,620	603,601	479,471
7	987,389	831,338	686,518	540,696	438,391
8	1,004,596	816,143	639,267	467,220	412,324
9	1,007,275	788,244	586,243	391,672	324,061
10	993,969	751,125	525,688	370,455	203,139
11	974,869	712,603	459,474	351,479	205,079
12	952,277	669,951	388,883	223,308	206,655
13	911,696	625,635	334,695	200,215	207,934
14	891,896	580,344	319,268	203,093	208,971
15	866,189	534,779	304,771	205,331	209,813
Sum	12,661,226	10,161,095	7,931,033	6,471,229	5,738,757

7.2.2 Element Weighted Average Condition State Sensitivity to Budget Levels

From Tables and Figures in Appendix C2 showing the sensitivity of each roof element weighted average condition state to budget changes, it is found that only three roof elements are significantly sensitive to budget changes. Figures 7.6, 7.7 and 7.8 show the sensitivity of single-ply membrane, multiple-ply membrane and shingles respectively to budget changes.

Figure 7.6 shows that the lower the budget the later the single-ply membrane element reaches a higher condition state. It is shown from the sensitivity analysis also that the element reaches condition state 4 from the first analysis year except for budgets of \$200,000 and \$250,000 where the element reaches condition state 4 at year 8 and 3 respectively.

Figure 7.6 and Table 7.6 show that the single-ply membrane has a relatively low sensitivity to changes in budget levels since the changes in its condition state ranges between 3.82 and 4.74.

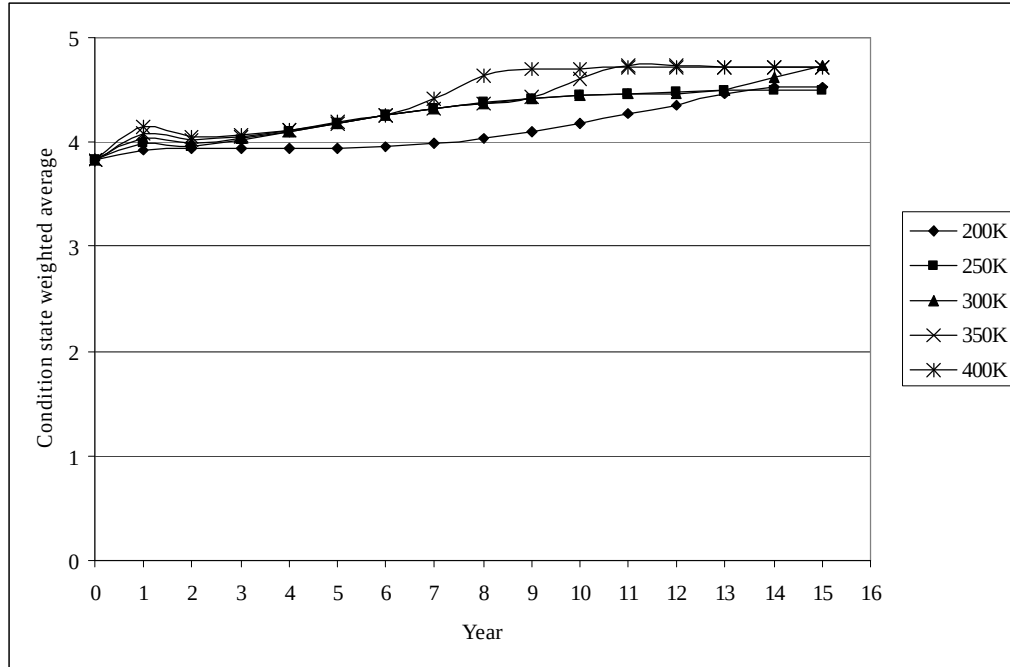


Figure 7.6 Single-ply membrane condition sensitivity to budget level

Table 7.6 Single-ply membrane condition sensitivity to budget level

Year	Single-ply membrane condition state weighted average				
	200K	250K	300K	350K	400K
0	3.82	3.82	3.82	3.82	3.82
1	3.93	3.98	4.04	4.09	4.14
2	3.93	3.96	3.99	4.02	4.05
3	3.94	4.02	4.03	4.04	4.06
4	3.94	4.10	4.10	4.11	4.12
5	3.94	4.18	4.18	4.18	4.19
6	3.95	4.26	4.26	4.26	4.26
7	3.98	4.33	4.32	4.32	4.42
8	4.03	4.38	4.37	4.37	4.64
9	4.09	4.42	4.41	4.42	4.71
10	4.18	4.44	4.44	4.60	4.71
11	4.27	4.46	4.46	4.73	4.71
12	4.34	4.48	4.46	4.72	4.71
13	4.47	4.49	4.50	4.72	4.71
14	4.53	4.50	4.63	4.72	4.71
15	4.53	4.50	4.74	4.72	4.71

With the current budget available to the DM&R the single-ply membrane element improves to the highest condition state reached by the element at year 15, as shown in Table 7.6. An increase in the budget to \$350,000 allows the element to reach its highest condition state at year 11. An increase in the budget to \$400,000 allows the element to reach its highest condition state at year 9. Reducing the available budget to \$250,000 or \$200,000 will cause the element to reach its highest condition state at a year out side our analysis horizon.

Figure 7.7 shows the sensitivity of the multiple-ply membrane to changes in budget levels. As the available budget for maintenance decreases the element reaches a higher condition states at later years when compared to the same element but with a higher maintenance budget. The multiple-ply membrane has a relatively high sensitivity to budget changes since its condition state ranges between 1.29 and 4.81, as shown in Figure 7.7 and Table 7.7.

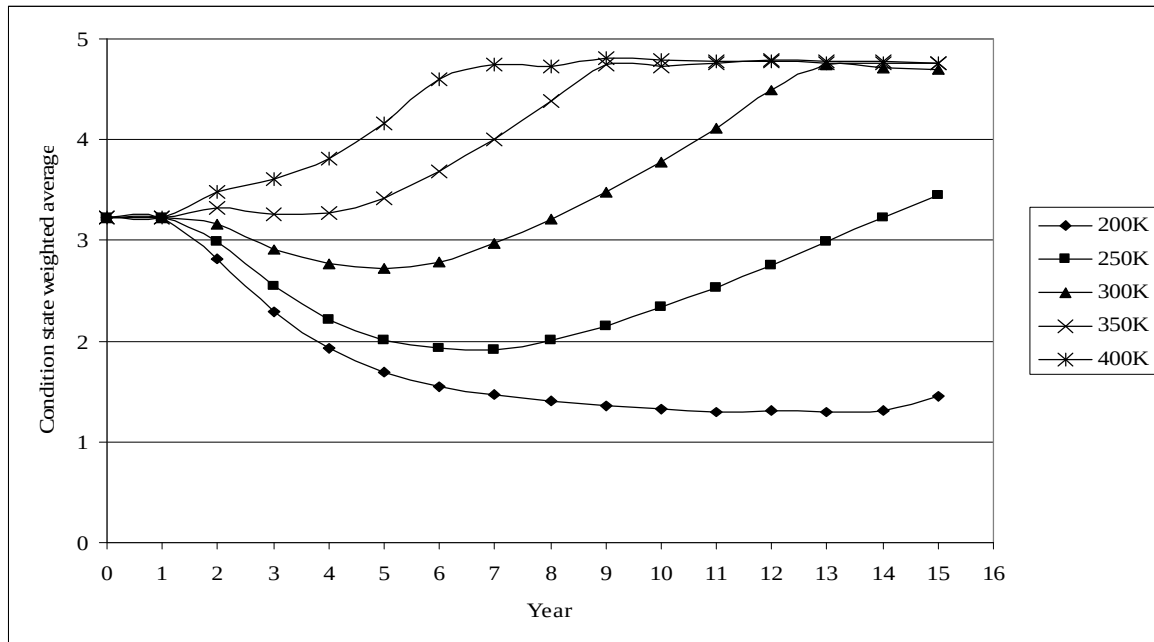


Figure 7.7 Multiple-ply membrane condition sensitivity to budget level

Table 7.7 Multiple-ply membrane condition sensitivity to budget level

Year	Multiple-ply membrane condition state weighted average				
	200K	250K	300K	350K	400K
0	3.23	3.23	3.23	3.23	3.23
1	3.23	3.23	3.23	3.23	3.23
2	2.82	2.99	3.16	3.33	3.49
3	2.30	2.55	2.91	3.26	3.61
4	1.93	2.22	2.77	3.28	3.81
5	1.70	2.02	2.72	3.42	4.17
6	1.56	1.93	2.78	3.68	4.60
7	1.47	1.92	2.97	4.01	4.74
8	1.41	2.00	3.21	4.38	4.72
9	1.36	2.15	3.48	4.75	4.81
10	1.33	2.33	3.79	4.73	4.80
11	1.30	2.54	4.12	4.77	4.78
12	1.31	2.76	4.50	4.80	4.78
13	1.29	2.99	4.75	4.78	4.77
14	1.31	3.22	4.72	4.77	4.76
15	1.45	3.45	4.70	4.77	4.76

For the current budget available, the multiple-ply membrane condition state drops to its lowest value, 2.72, at year 5 and increases each year after reaching its highest at year 13, Table 7.7. An increase in the budget to \$350,000 or \$400,000 will cause the element to increase its condition state gradually reaching their highest condition state at year 11 and 9 respectively. A decrease in the available budget to \$250,000 or \$200,000 will cause the element condition state to reach low condition states of 1.27 and 2.08 respectively. After reaching their lowest condition state, the \$200,000 and \$250,000 budget curve will move upward reaching probably their highest at years beyond the analysis horizon.

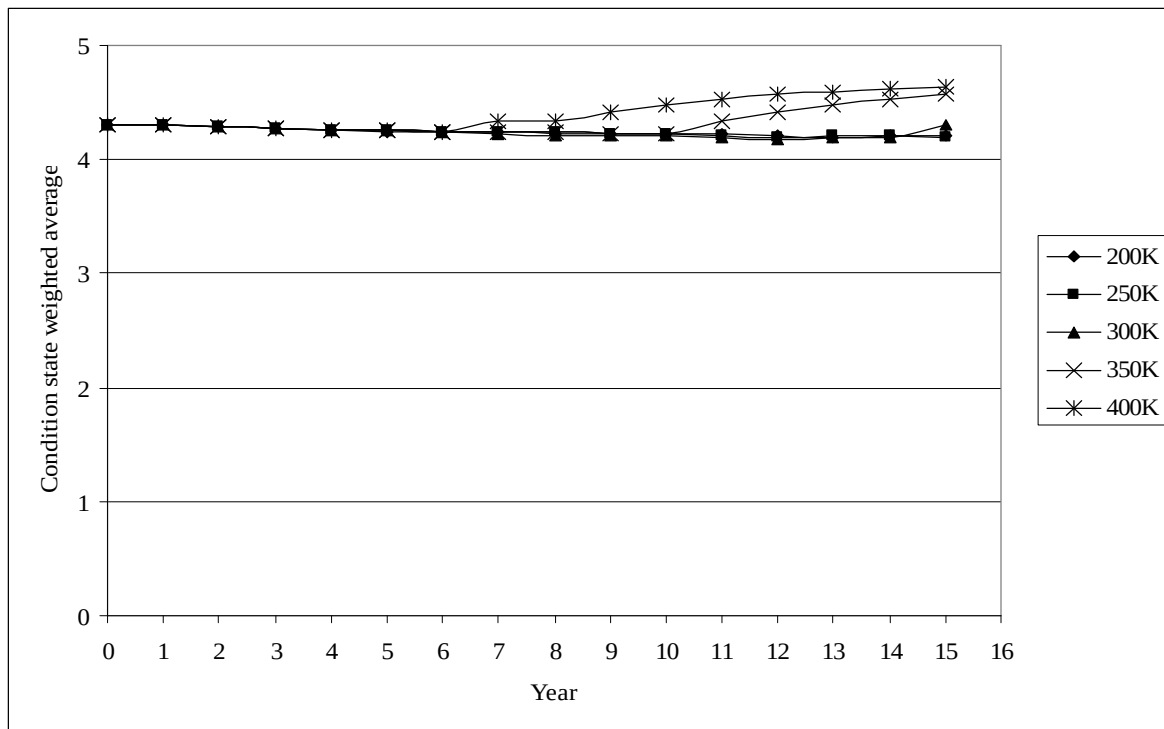


Figure 7.8 Shingle condition sensitivity to budget level

Table 7.8 Shingle condition sensitivity to budget level

Year	Shingle condition state weighted average				
	200K	250K	300K	350K	400K
0	4.31	4.31	4.31	4.31	4.31
1	4.31	4.31	4.31	4.31	4.31
2	4.29	4.29	4.29	4.29	4.29
3	4.27	4.27	4.27	4.27	4.27
4	4.25	4.26	4.26	4.26	4.26
5	4.25	4.25	4.25	4.25	4.25
6	4.24	4.25	4.25	4.25	4.24
7	4.24	4.24	4.23	4.24	4.34
8	4.23	4.23	4.21	4.23	4.33
9	4.23	4.23	4.22	4.23	4.41
10	4.23	4.23	4.20	4.23	4.48
11	4.22	4.21	4.19	4.33	4.53
12	4.21	4.20	4.18	4.41	4.56
13	4.20	4.20	4.19	4.48	4.60
14	4.20	4.21	4.19	4.53	4.62
15	4.21	4.19	4.30	4.57	4.64

Figure 7.8 shows the sensitivity of shingles to changes in budget levels. Shingles show no significant sensitivity to changes in budget level through year 6. Years following year 6 show a relatively low sensitivity to changes in budget levels where condition states range between 4.19 and 4.64.

Table 7.8 shows that for the \$300,000 budget the element condition state decreases to its lowest condition state of 4.18 at year 12 and then the condition state increases reaching 4.30, which is the highest value for this budget at year 15. Increasing the available budget to \$350,000 or \$400,000 will cause the element to reach its lowest condition state of 4.23 at year 8 and 4.24 at year 6 respectively.

The highest condition state reached by increasing the available budget to \$350,000 is 4.57, while the highest condition state reached by the element due to increasing the budget to \$400,000 is 4.64. A decrease in the available budget to \$250,000 will cause a decrease in the element condition state through the whole analysis horizon reaching its lowest condition state of 4.19 at year 15. A decrease in the budget to \$200,000 will cause a decrease in the element condition state through the whole analysis horizon. The lowest condition state reached under the \$200,000 budget is 4.20 at years 13 and 14.

7.3 Sensitivity of the User Costs and Element Weighted Average Condition State to Real Rate of Return

Five Real Rate of Return levels were used to analyze the effect on the user costs and element weighted average condition state. The five levels used for the analysis are shown in Table 7.1. Tables and graphs showing the response of user costs and element weighted average condition state to each Real Rate of Return level are available in Appendix C1 and C2 respectively.

7.3.1 User Cost Sensitivity to Real Rate of Return Changes

Table 7.9 and Figure 7.9 indicate that user cost is not very sensitivity to changes in Real Rate of Return levels considered in this analysis. It might be expected that as the Real Rate of Return increases, the curve of the user cost would move upward. This expectation would be correct if the Real Rate of Return increase causes the EUAC values to be greater than the user costs for all elements at all condition states. If this happens the objective function coefficient would be negative and would not be picked by the model. This would

cause the elements to deteriorate causing a higher user cost, which in result would shift the user cost curve upward.

Table 7.9 User cost sensitivity to Real Rate of Return

Year	User Cost				
	1.5%	3.0%	4.5%	6.0%	7.5%
1	108,856	109,069	108,853	108,919	108,875
2	212,040	211,414	211,381	211,430	211,230
3	329,285	327,545	327,585	327,779	305,995
4	400,614	391,776	397,919	398,041	375,936
5	430,144	423,910	425,909	426,138	403,818
6	429,231	422,622	422,751	422,324	399,045
7	394,051	386,529	386,987	386,686	371,808
8	348,295	339,269	340,036	339,108	329,492
9	292,318	286,247	285,844	285,398	276,891
10	229,711	225,688	225,435	225,553	217,987
11	165,029	159,477	162,824	162,055	154,603
12	92,172	88,884	90,491	94,985	83,967
13	34,991	34,709	34,823	34,849	35,739
14	19,532	19,297	19,286	19,416	20,076
15	4,618	4,791	4,787	4,906	4,947

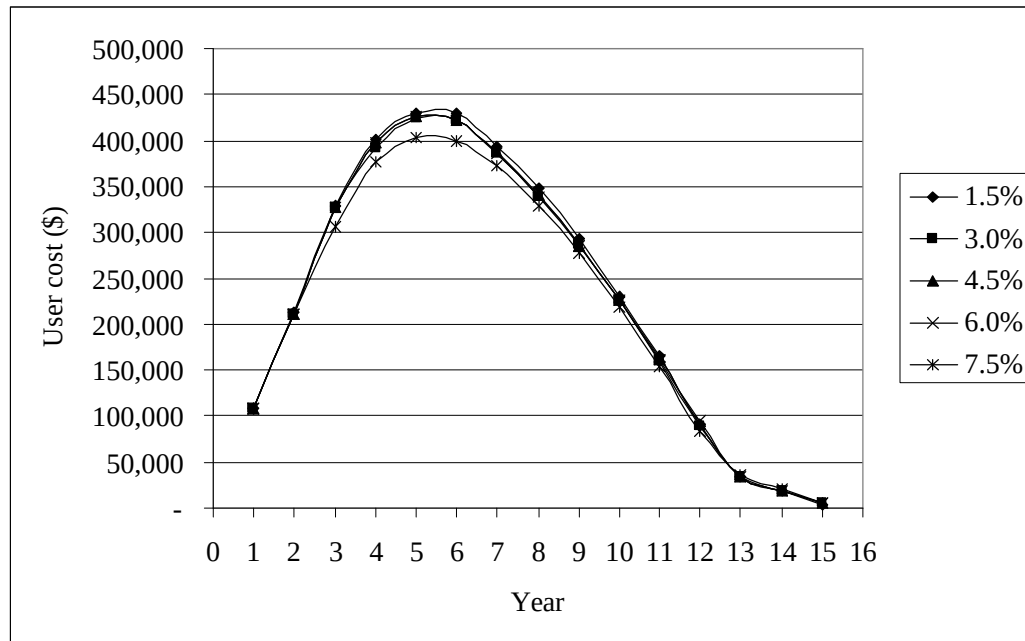


Figure 7.9 User cost sensitivity to Real Rate of Return

This was not the case for the Real Rates of Return used in this study. For the analyzed Real Rates of Return, as the Real Rate of Return increases the EUAC for each roof element increases too, but the increase never exceeded the user cost. The increase of the EUAC caused some elements to have an objective function more attractive to be picked by the optimization model than other elements.

For the 1.5%, 3%, 4.5%, and 6% Real Rates of Return, the increase of the EUAC was not sufficient to change the model optimized decision. This resulted in having nearly the same user cost for all four Real Rates of return and their curves are drawn on top of each other, Figure 7.9, indicating low user cost sensitivity to changes in the Real Rate of Return.

The increase of the EUAC due to the 7.5% Real rate of Return caused the model to change its optimizing decisions. For this Real Rate of Return, it can be shown from the analysis results in Appendix C1 that it was more attractive to the model to use more of the budget to maintain the multiple-ply membrane. Since the single-ply and multiple-ply membrane are the elements that result in highest user cost if damage occurs to them, any maintenance or replacement will drop the user costs significantly in comparison to other roof elements. For this reason the 7.5% resulted in slightly lower total user cost - when compared to the other analyzed Real Rates of Return - which shifted its user cost curve downward.

7.3.2 Element Weighted Average Condition State Sensitivity to Real Rate of Return Changes

From Tables and Figures in Appendix C2 showing the sensitivity of each roof element weighted average condition state to Real Rate of Return changes, it is found that roof

element condition state have no significant sensitivity to changes in Real Rates of Return changes except for multiple-ply membrane, which shows a low sensitivity to Real Rate of Return changes. Figure 7.10 shows the sensitivity of multiple-ply membrane to changes in Real Rate of Return.

Table 7.10 and Figure 7.10 show that multiple-ply membrane weighted average condition state has a very low sensitivity to changes in Real Rates of Return analyzed in this study except for the 7.5% level. By increasing the Real rate of Return to 7.5% the curve in Figure 7.10 moves upward. The upward movement of the curve indicates a better element weighted average condition state compared to the other Real Rate of Return weighted average. However, this appears to be due to minor switches in actions selected rather than a general trend.

Table 7.10 Multiple-ply membrane weighted average condition state sensitivity to Real Rate of Return

Year	Multiple-ply membrane condition state weighted average				
	1.5%	3.0%	4.5%	6.0%	7.5%
0	3.23	3.23	3.23	3.23	3.23
1	3.23	3.23	3.23	3.23	3.23
2	3.15	3.16	3.16	3.16	3.16
3	2.89	2.91	2.91	2.91	2.98
4	2.72	2.77	2.75	2.75	2.82
5	2.68	2.72	2.71	2.71	2.78
6	2.74	2.78	2.78	2.78	2.86
7	2.92	2.97	2.96	2.97	3.01
8	3.15	3.21	3.20	3.21	3.24
9	3.44	3.48	3.48	3.48	3.51
10	3.76	3.79	3.78	3.79	3.81
11	4.09	4.12	4.10	4.11	4.13
12	4.48	4.50	4.49	4.45	4.56
13	4.75	4.75	4.75	4.75	4.75
14	4.72	4.72	4.72	4.72	4.72
15	4.70	4.70	4.70	4.70	4.70

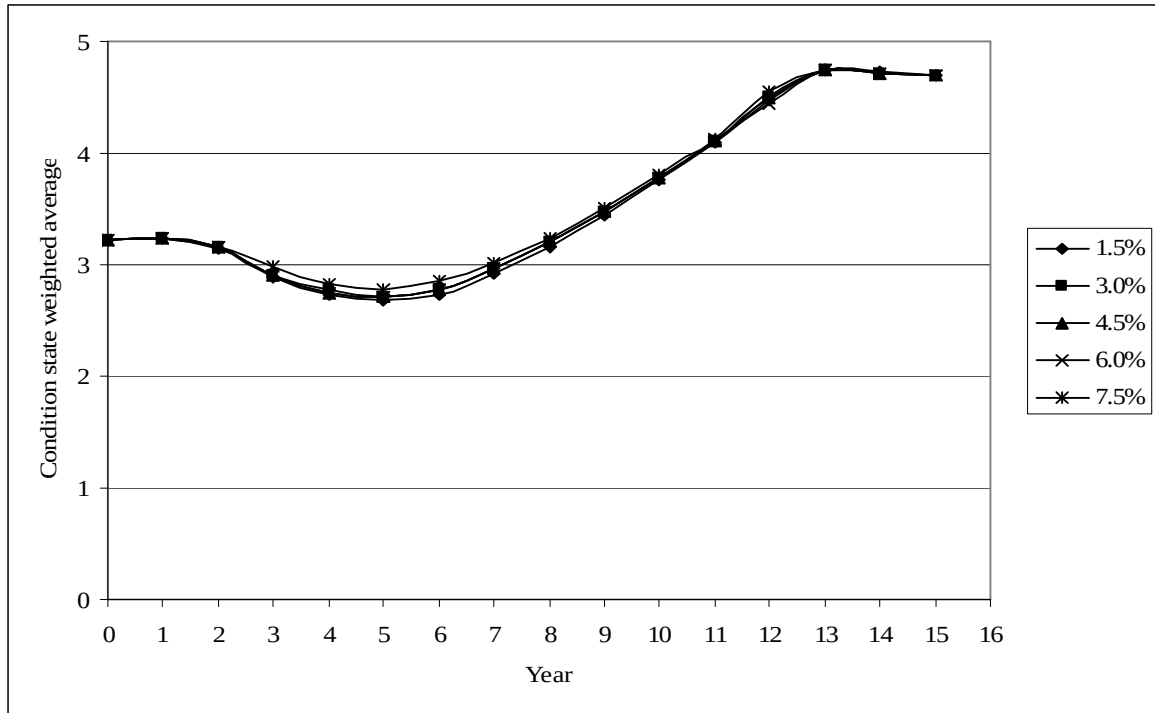


Figure 7.10 Multiple-ply membrane weighted average condition state sensitivity to Real Rate of Return

8. CONCLUSIONS AND RECOMMENDATIONS

The main objective of the research was to develop a Decision Support System that optimizes the allocation of available funds to the DM&R by maximizing the saving in annual cost through maximizing the total amount of reductions in equivalent uniform annual cost. Based on the developed methodology and data available or developed for the analysis, this chapter lists the conclusions of this study and recommendations for future research and development.

8.1 Conclusions

1. The Decision Support System developed in this study provides an allocation of the available funds to the Department of Maintenance and Repair that results in optimal sets of roof element improvement.
2. The combined budget approach produces lower total cost in comparison to the separated budget approach. This is due to non-optimal restriction of the available budget into fixed categories in the separated budget approach.
3. For the combined budget approach, the higher the available budget, the lower the resulting user cost. The analysis shows that the user cost is sensitive to budget levels. Increasing the funding for roof maintenance and replacement would reduce total costs by reducing user costs (building damage).
4. For the developed model and data available, the multiple-ply membrane has a high weighted average condition state sensitivity to budget levels. The single-ply membrane

and shingle elements show less condition sensitivity to budget levels in comparison to multiple-ply membrane. All other elements have shown low condition sensitivity to budget changes.

5. For the developed model and data available, the user costs show very low sensitivity to changes in Real Rate of Return.
6. No significant condition sensitivity is shown by any roof elements to changes in Real Rate of Return except for the multiple-ply membrane where it shows a slight sensitivity.

8.2 Recommendations

1. Expand the study to include the remaining university buildings when the data is available. To do that, another computer program has to be used due to Excel capacity limitation in data entry.
2. The values of ECSDF and ACSSL used in this study were estimated with the help of experts at the Department of Maintenance and Repair. More research would be desirable to improve the accuracy and precision of this data.
3. Remove the exponential distribution assumption and empirically determine the values of ACSSL based on historical data to obviate the need to assume any real life distribution, which is typically extremely difficult to assess.
4. Expand the analysis horizon to test the model adequacy beyond the 15 years.
5. Modify the model developed in this study to incorporate the dependence relationship between membrane and insulation maintenance and replacement.
6. Allow the element quantity to deteriorate to any condition state in one year in case of natural disaster or poor element installment.

7. The DSS model analysis system needs to be automated so that it becomes user friendly for optimizing roof maintenance.

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APPENDIX A

APPENDIX A1

This appendix includes the MAUCST matrices of all maintenance elements.

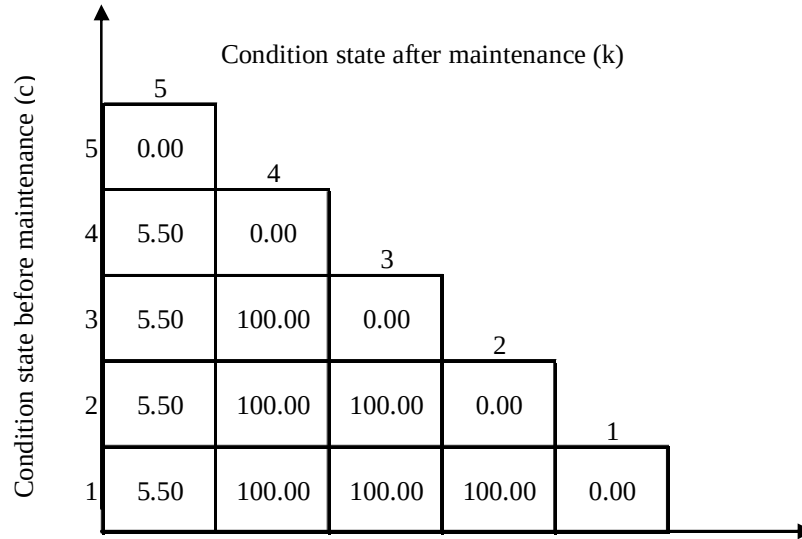


Figure A1.1 Single-ply membrane MAUCST (\$/SF)

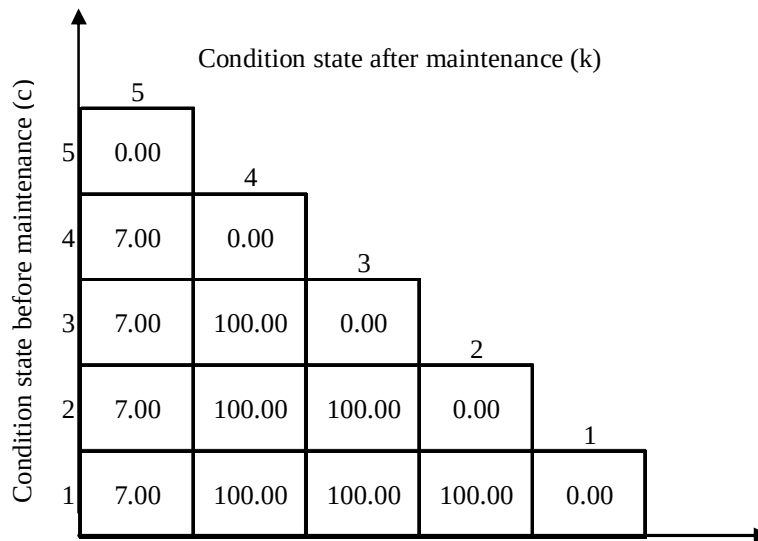


Figure A1.2 Multiple-ply membrane MAUCST (\$/SF)

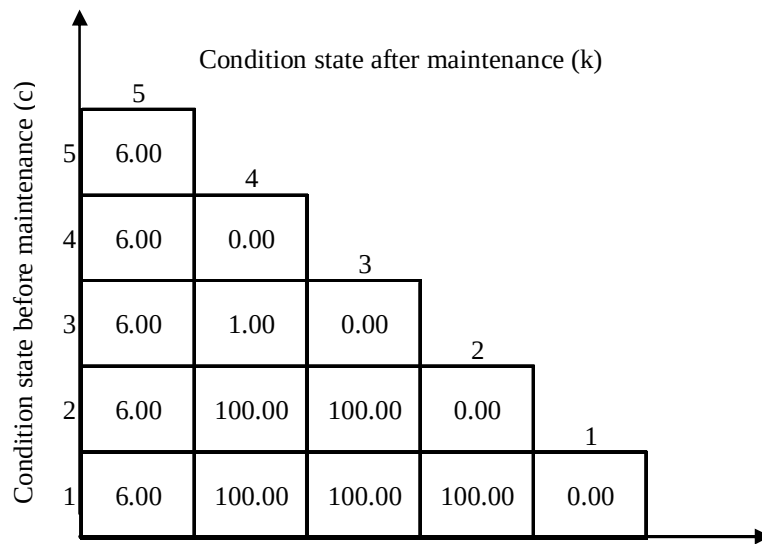


Figure A1.3 Single-ply base flashing MAUCST (\$/LF)

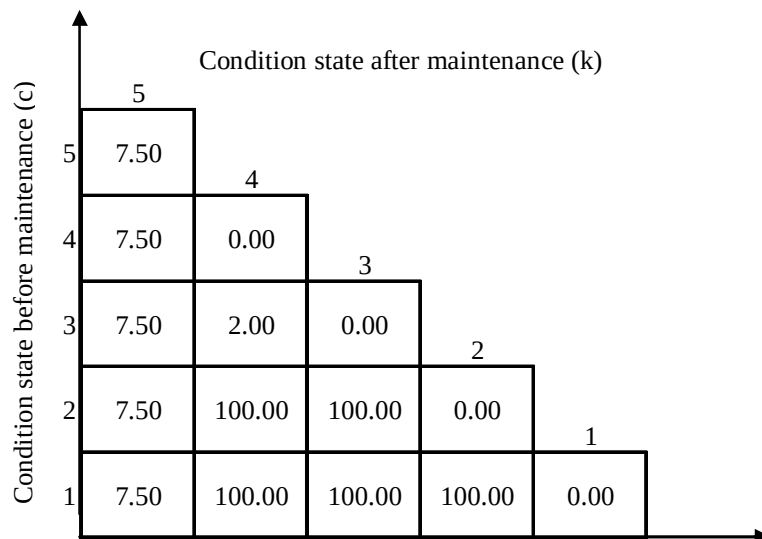


Figure A1.4 Multiple-ply base flashing MAUCST (\$/LF)

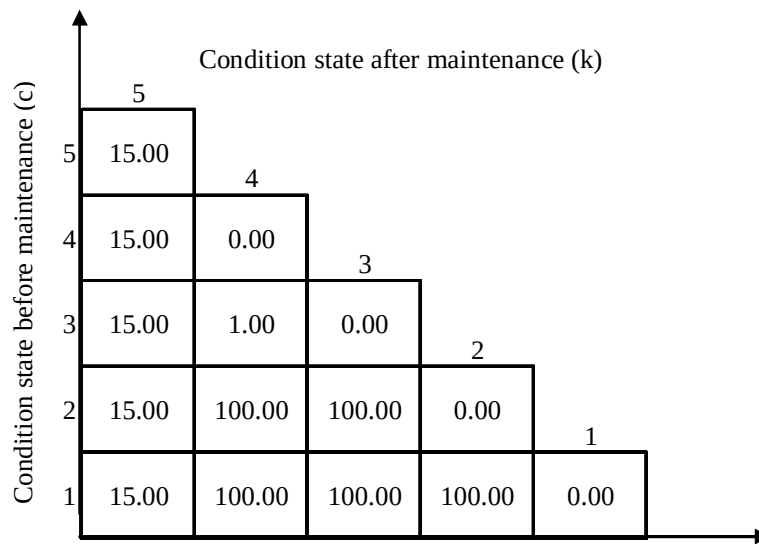


Figure A1.5 Counter flashing MAUCST (\$/LF)

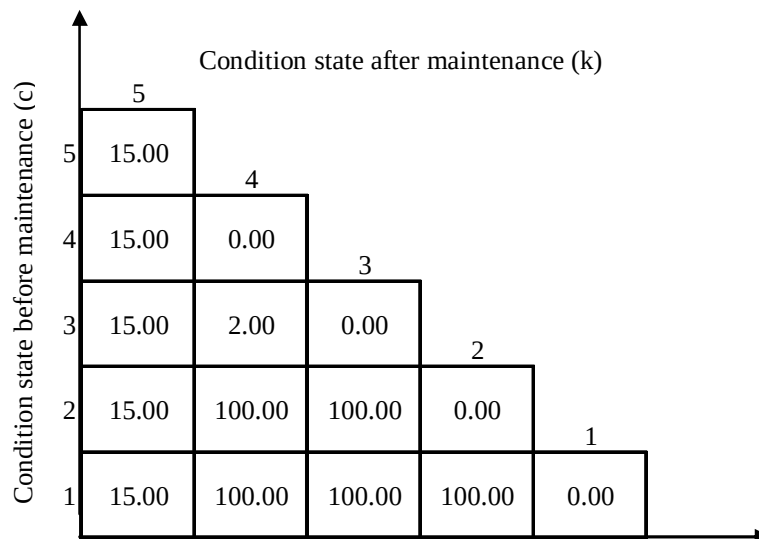


Figure A1.6 Gutter MAUCST (\$/LF)

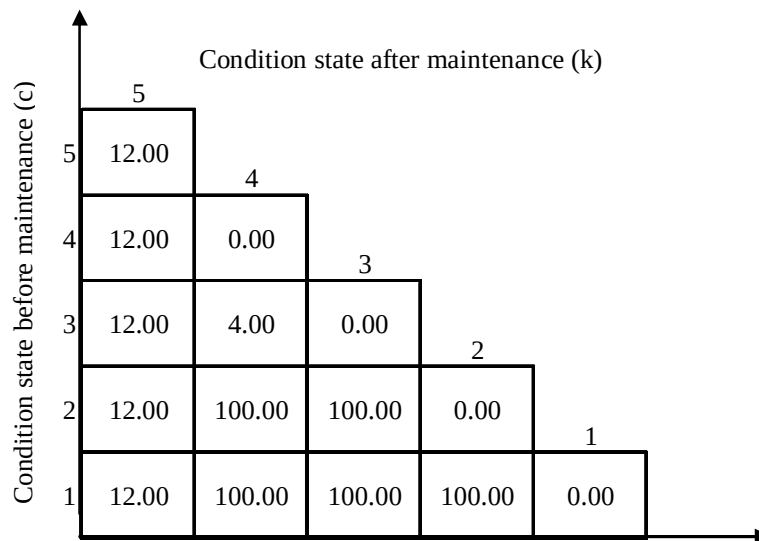


Figure A1.7 Downspout MAUCST (\$/LF)

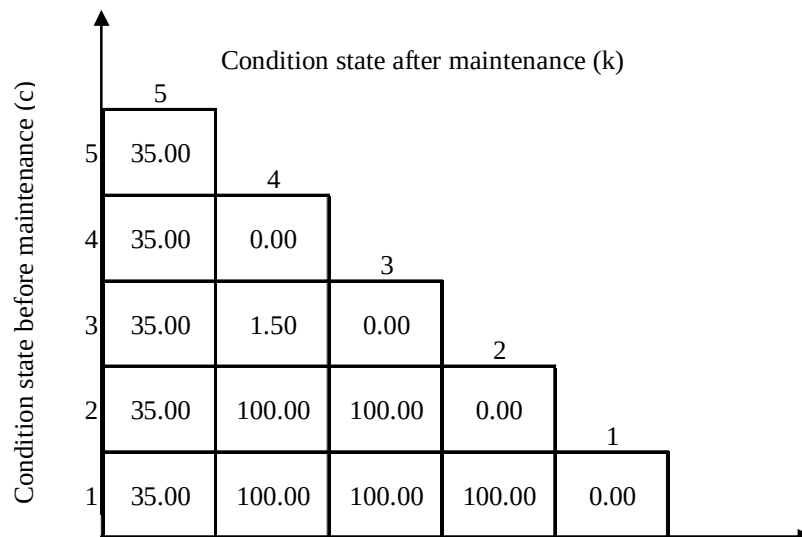


Figure A1.8 Drain strainer MAUCST (\$/EA)

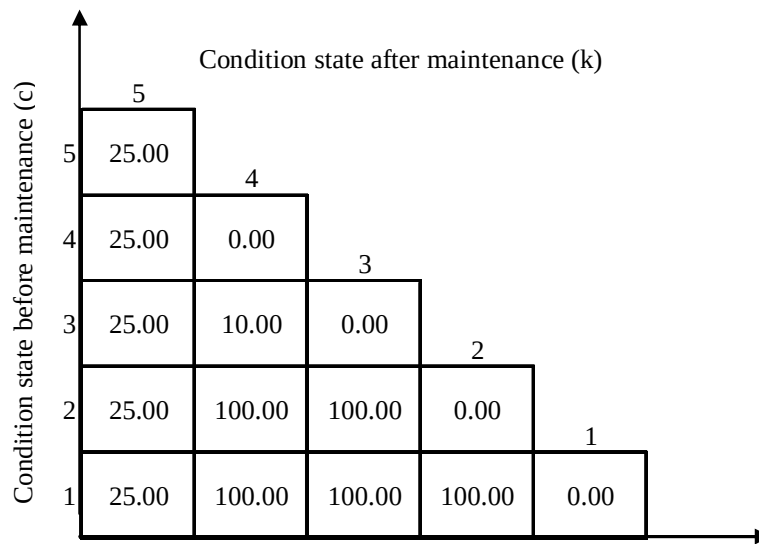


Figure A1.9 Coping cap MAUCST (\$/EA)

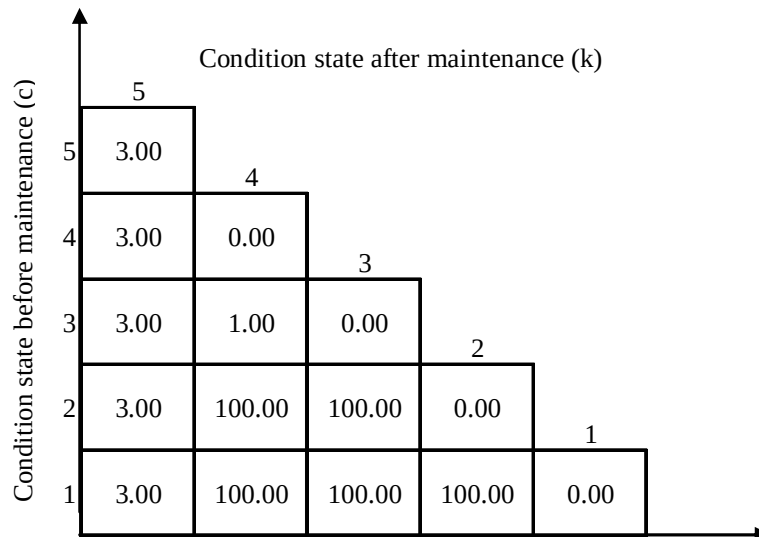


Figure A1.10 Termination bar MAUCST (\$/LF)

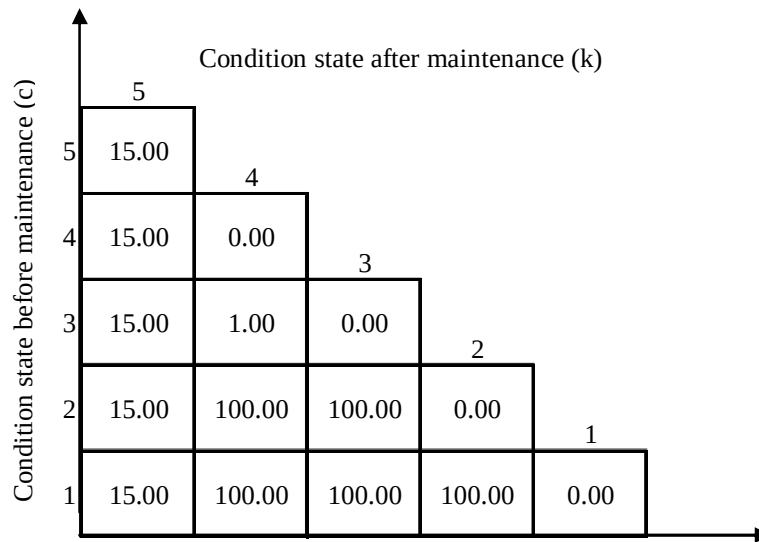


Figure A1.11 Edge metal MAUCST (\$/LF)

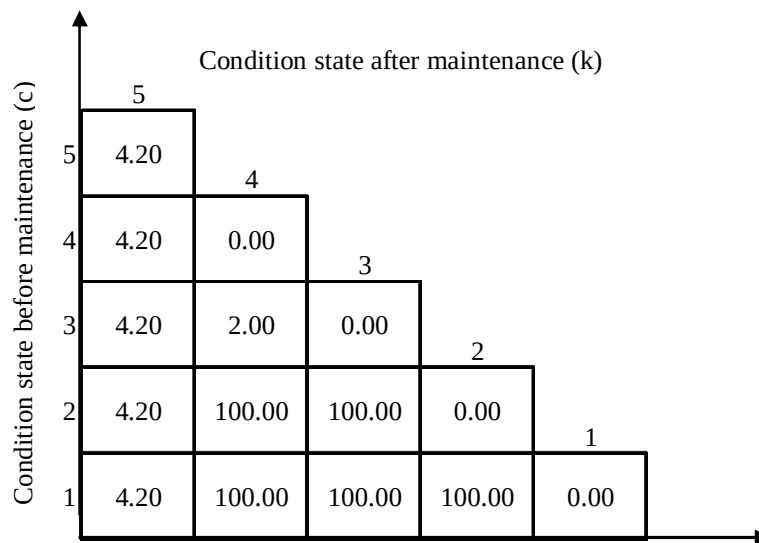


Figure A1.12 Shingle MAUCST (\$/SF)

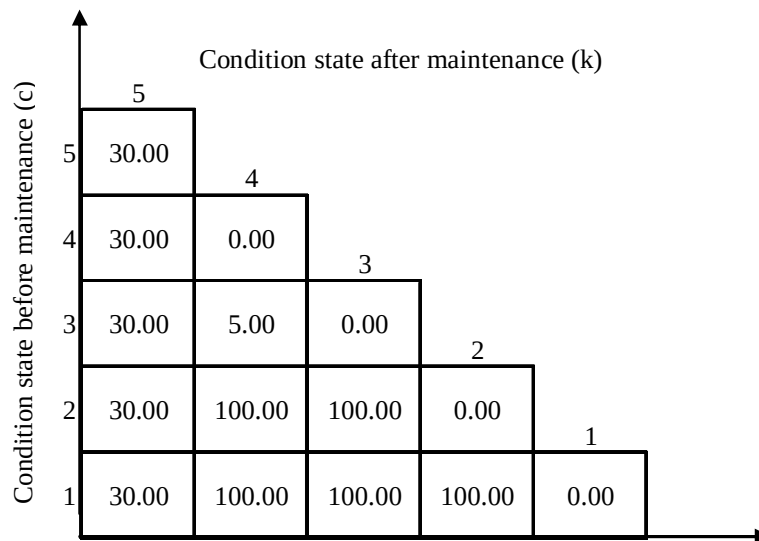


Figure A1.13 Slate MAUCST (\$/SF)

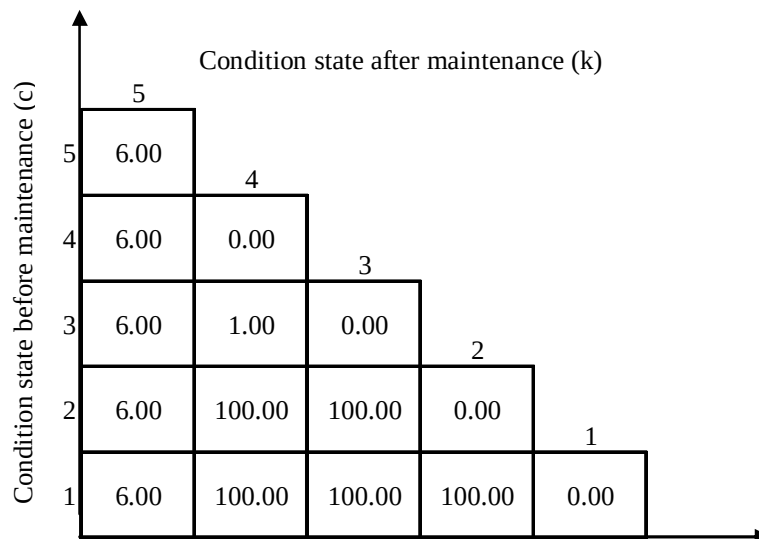


Figure A1.14 Hip/Ridge cap MAUCST (\$/LF)

APPENDIX A2

This appendix includes the EUAC matrices of all maintenance elements.

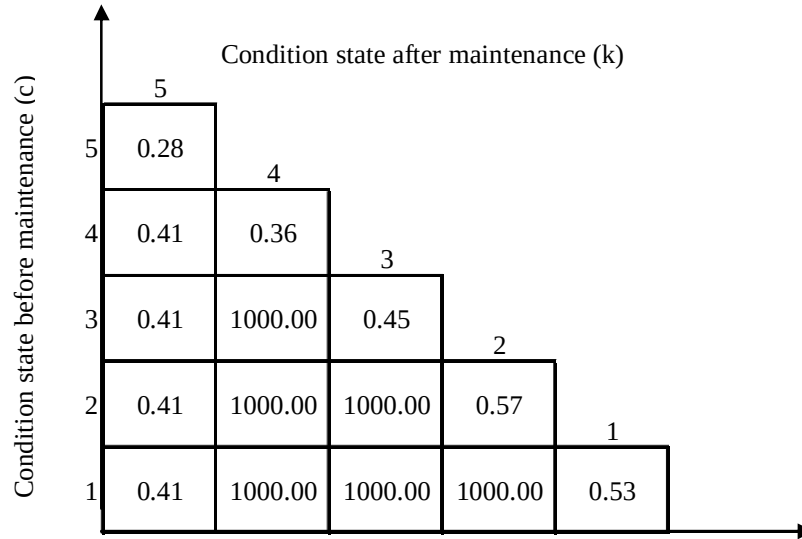


Figure A2.1 Single-ply membrane EUAC (\$/SF)

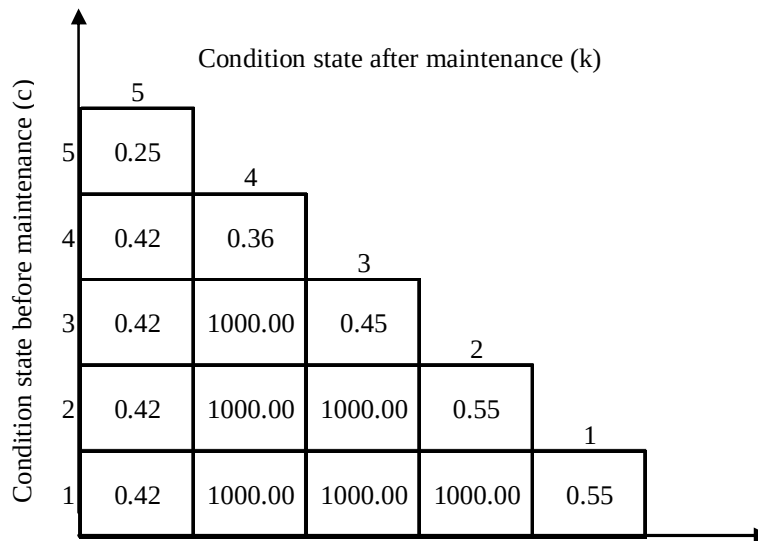


Figure A2.2 Multiple-ply membrane EUAC (\$/SF)

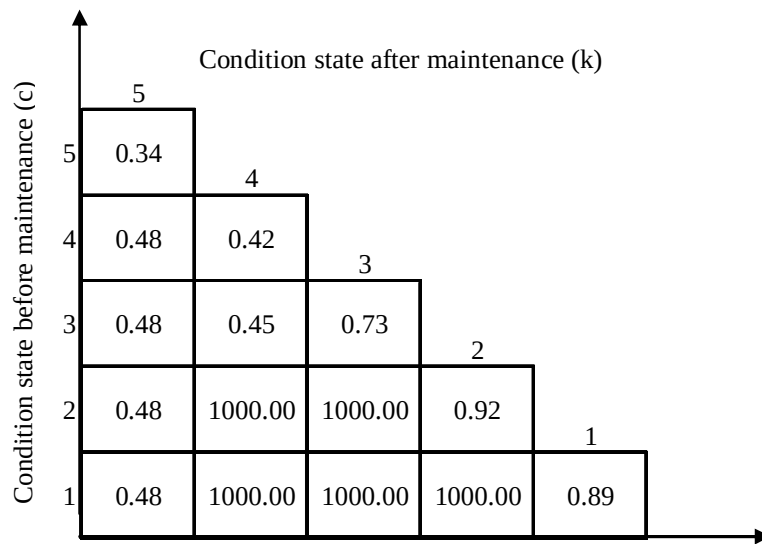


Figure A2.3 Single-ply base flashing EUAC (\$/LF)

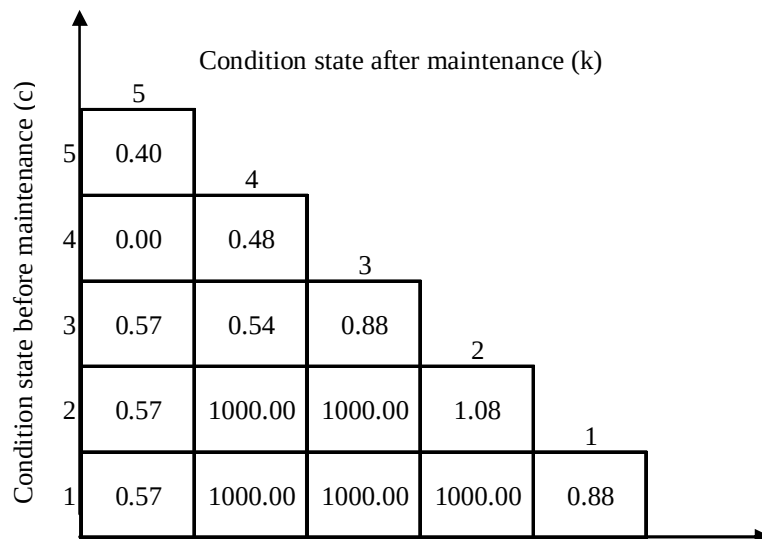


Figure A2.4 Multiple-ply base flashing EUAC (\$/LF)

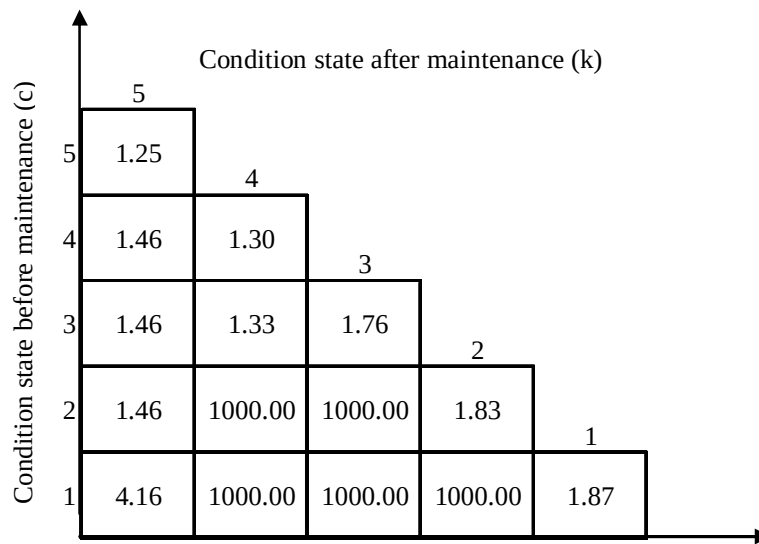


Figure A2.5 Counter flashing EUAC (\$/LF)

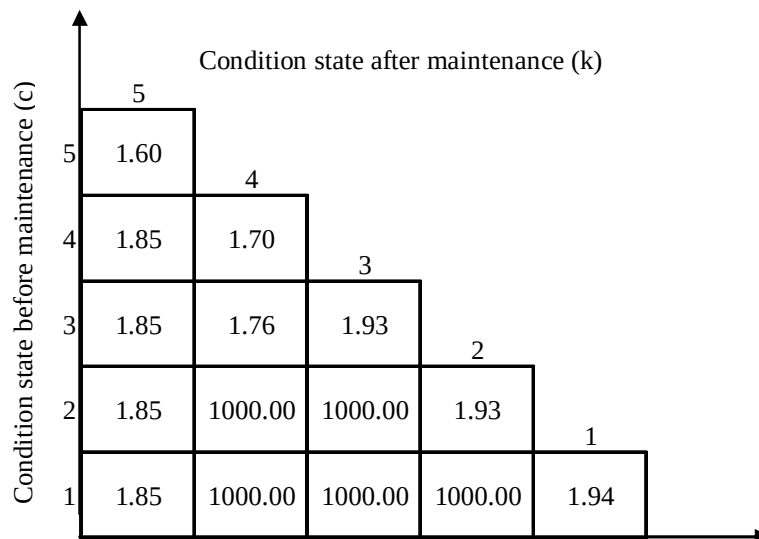


Figure A2.6 Gutter EUAC (\$/LF)

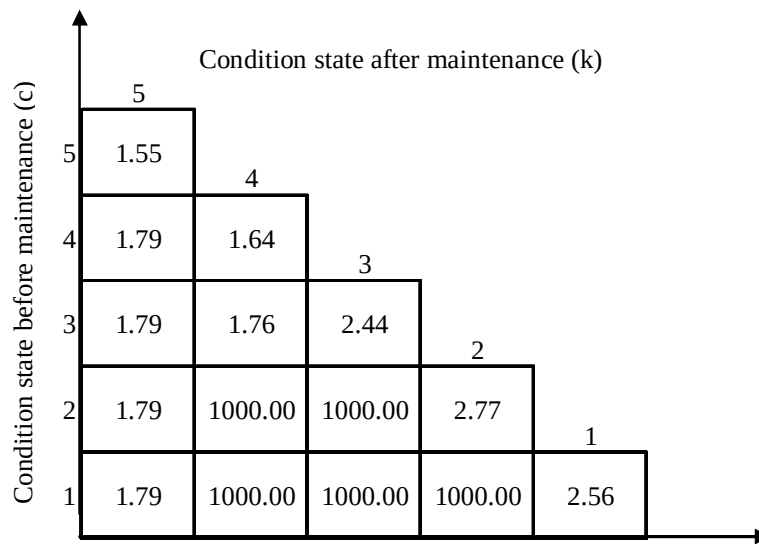


Figure A2.7 Downspout EUAC (\$/LF)

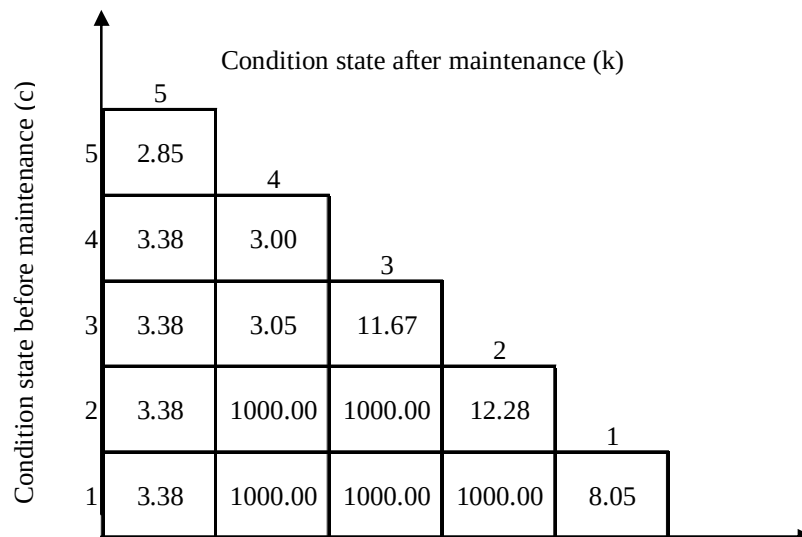


Figure A2.8 Drain strainer EUAC (\$/EA)

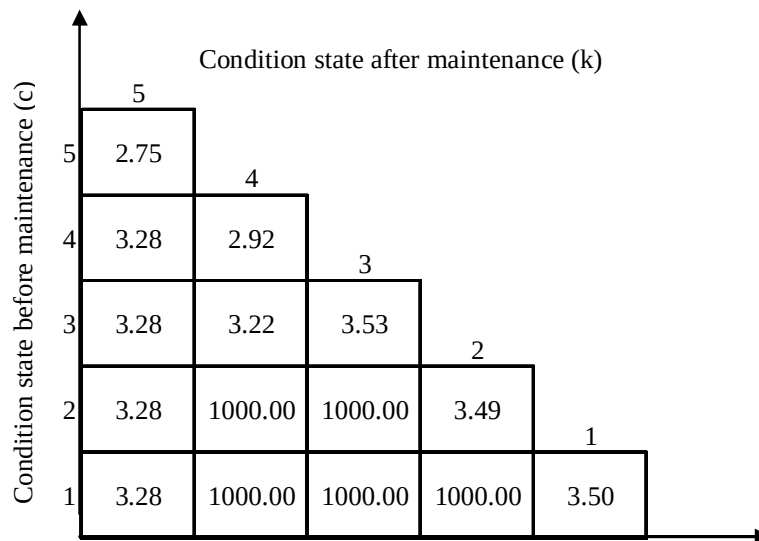


Figure A2.9 Coping cap EUAC (\$/EA)

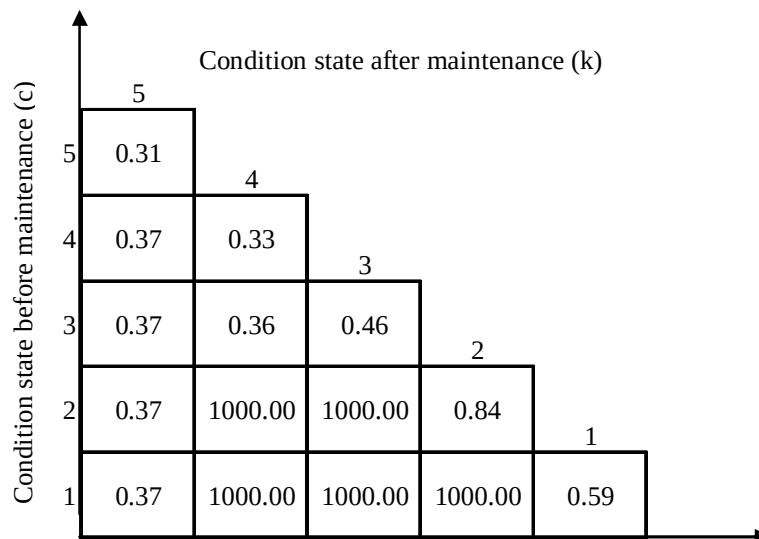


Figure A2.10 Termination bar EUAC (\$/LF)

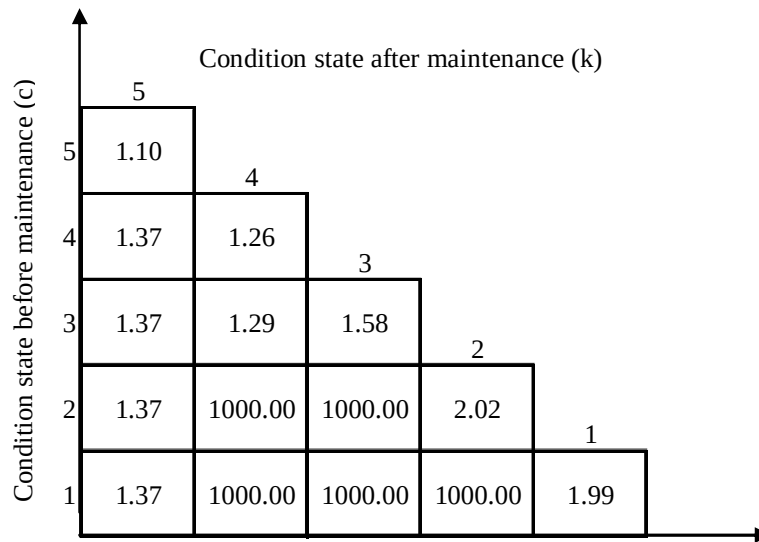


Figure A2.11 Edge metal EUAC (\$/LF)

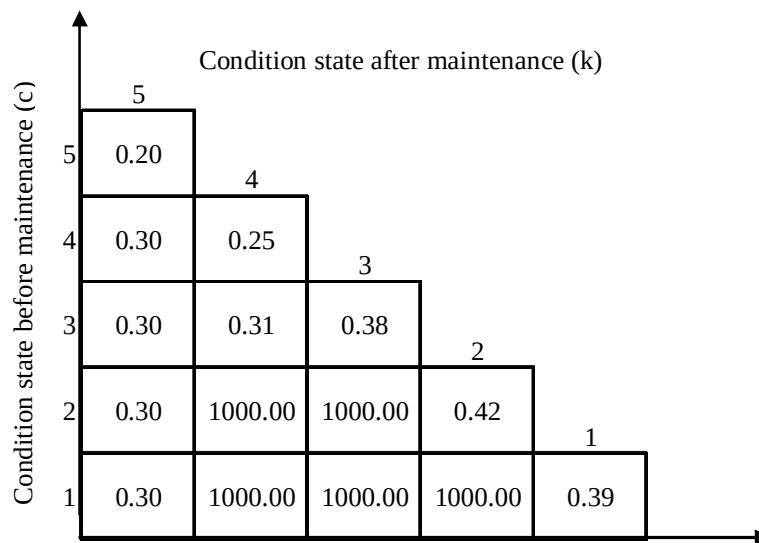


Figure A2.12 Shingle EUAC (\$/SF)

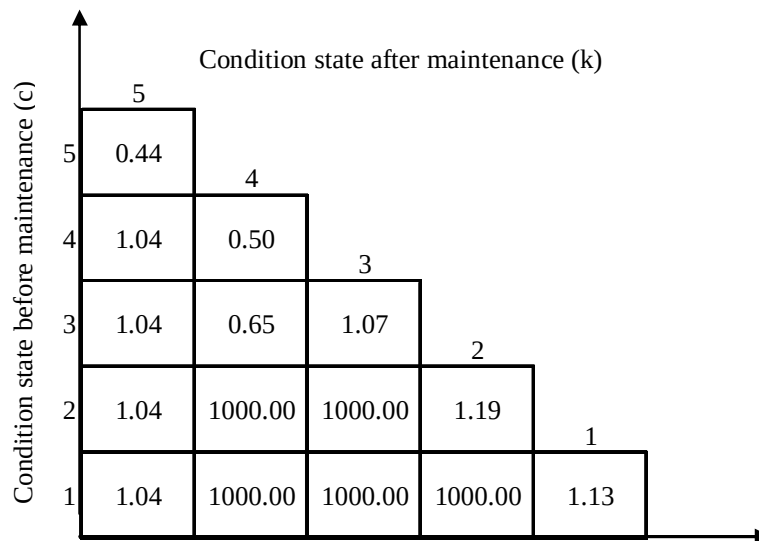


Figure A2.13 Slate EUAC (\$/SF)



Figure A2.14 Hip/Ridge cap EUAC (\$/LF)

APPENDIX B

APPENDIX B1

This appendix includes all the Visual Basic programs used in the analysis:

B1.1 Visual Basic Code (VB)

```
' Keyboard Shortcut: Ctrl+Shift+I
'=====
' The file path where the file will be saved in "k:\outputs\" directory
'=====
Sheets("Initial Building Data").Select
Open "k:\outputs\model_year_1.mod" For Output As #3
Open "k:\outputs\model_year_1.txt" For Output As #4
'=====
' Writing the header of the model file
'=====
Print #3, "set I1;"
Print #3, "set J1;"
Print #3, "set L1;"
Print #3, "set K1;"

Print #4, "set I1;"
Print #4, "set J1;"
Print #4, "set L1;"
Print #4, "set K1;"

msgin:
Budget = InputBox("Please enter the Budget: ", "Budget")
If Trim(Budget) = "" Then GoTo msgin

Print #3, "param budget:= " & Budget & ";"
Print #3, " "
Print #3, "var x{i in I1,j in J1,l in L1,k in K1}binary;"

Print #4, "param budget:= " & Budget & ";"
Print #4, " "
Print #4, "var x{i in I1,j in J1,l in L1,k in K1}binary;"
'=====
' Writing the objective function formula
'=====
txtmsg = "maximize REUAC:"
For i = 2 To 50401
    xx = Trim(Cells(i, 10).Value)
    If Trim(xx) <> "0" And IsNumeric(Trim(xx)) Then
```

```

a = (((i - 2) \ 3600) + 1)
b = (((i - 2) \ 300) + 1) - ((a - 1) * 12)
c = (((i - 2) \ 20) + 1) - ((b - 1) * 15) - ((a - 1) * 180)
d = Trim(Cells(i, 4).Value)
txtmsg = txtmsg & xx & "*x[" & a & "," & b & "," & c & "," & d & "]" + "
End If
Next i
For i = 2 To 50401
xx = Trim(Cells(i, 22).Value)
If Trim(xx) <> "0" And IsNumeric(Trim(xx)) Then
a = (((i - 2 + 50401) \ 3600) + 1)
b = (((i - 2 + 50401) \ 300) + 1) - ((a - 1) * 12)
c = (((i - 2 + 50401) \ 20) + 1) - ((b - 1) * 15) - ((a - 1) * 180)
d = Trim(Cells(i, 16).Value)
txtmsg = txtmsg & xx & "*x[" & a & "," & b & "," & c & "," & d & "]" + "
End If
Next i

Print #3, Mid(Trim(txtmsg), 1, (Len(Trim(txtmsg))) - 1) & ";"
Print #4, Mid(Trim(txtmsg), 1, (Len(Trim(txtmsg))) - 1) & ";"
'=====
' Writing the budget constraint formula
'=====
Print #3, " "
Print #3, " "

Print #4, " "
Print #4, " "

txtmsg = "subject to constraint1 : "
For i = 2 To 50401
xx = Trim(Cells(i, 11).Value)
If Trim(xx) <> "0" And IsNumeric(Trim(xx)) Then
a = (((i - 2) \ 3600) + 1)
b = (((i - 2) \ 300) + 1) - ((a - 1) * 12)
c = (((i - 2) \ 20) + 1) - ((b - 1) * 15) - ((a - 1) * 180)
d = Trim(Cells(i, 4).Value)
txtmsg = txtmsg & xx & "*x[" & a & "," & b & "," & c & "," & d & "]" + "
End If
Next i
For i = 2 To 50401
xx = Trim(Cells(i, 23).Value)
If Trim(xx) <> "0" And IsNumeric(Trim(xx)) Then
a = (((i - 2 + 50401) \ 3600) + 1)
b = (((i - 2 + 50401) \ 300) + 1) - ((a - 1) * 12)
c = (((i - 2 + 50401) \ 20) + 1) - ((b - 1) * 15) - ((a - 1) * 180)

```

```

        d = Trim(Cells(i, 16).Value)
        txtmsg = txtmsg & xx & "*"x[" & a & "," & b & "," & c & "," & d & "]"+"
    End If
Next i

Print #3, Mid(Trim(txtmsg), 1, (Len(Trim(txtmsg))) - 1) & "<=budget;"
Print #4, Mid(Trim(txtmsg), 1, (Len(Trim(txtmsg))) - 1) & "<=budget;"
'=====
' Writing the bottom of the model file
'=====
Print #3, " "
Print #3, " "
Print #4, " "
Print #4, " "

Print#3, "subject to constraint2 {i in I1,j in J1,l in L1}:
        x[i,j,l,10]+x[i,j,l,11]+x[i,j,l,12]+x[i,j,l,13]+x[i,j,l,14]+x[i,j,l,15]=1;"
Print #3, "subject to constraint3 {i in I1,j in J1,l in L1}:
        x[i,j,l,21]+x[i,j,l,22]+x[i,j,l,23]+x[i,j,l,24]+x[i,j,l,25]=1;"
Print #3, "subject to constraint4 {i in I1,j in J1,l in L1}:
        x[i,j,l,32]+x[i,j,l,33]+x[i,j,l,34]+x[i,j,l,35]=1;"
Print #3, "subject to constraint5 {i in I1,j in J1,l in L1}:
        x[i,j,l,43]+x[i,j,l,44]+x[i,j,l,45]=1;"
Print #3, "subject to constraint6 {i in I1,j in J1,l in L1}:
        x[i,j,l,54]+x[i,j,l,55]=1;"

Print #4, "subject to constraint2 {i in I1,j in J1,l in L1}:
        x[i,j,l,10]+x[i,j,l,11]+x[i,j,l,12]+x[i,j,l,13]+x[i,j,l,14]+x[i,j,l,15]=1;"
Print #4, "subject to constraint3 {i in I1,j in J1,l in L1}:
        x[i,j,l,21]+x[i,j,l,22]+x[i,j,l,23]+x[i,j,l,24]+x[i,j,l,25]=1;"
Print #4, "subject to constraint4 {i in I1,j in J1,l in L1}:
        x[i,j,l,32]+x[i,j,l,33]+x[i,j,l,34]+x[i,j,l,35]=1;"
Print #4, "subject to constraint5 {i in I1,j in J1,l in L1}:
        x[i,j,l,43]+x[i,j,l,44]+x[i,j,l,45]=1;"
Print #4, "subject to constraint6 {i in I1,j in J1,l in L1}:
        x[i,j,l,54]+x[i,j,l,55]=1;"
'=====
' Closing the file
'=====
Close #3
Close #4

MsgBox "Creation of the file model is successfully done, The file is saved in K:\outputs\ ",
vbInformation, " Done"

End Sub

```

B1.2 Visual Basic Code 1 (VB1)

```
' Keyboard Shortcut: Ctrl+Shift+B
'=====
' The path where the file will be saved in "K:\outputs\" directory
'=====
Sheets("Building Data").Select
Open "K:\outputs\model_year_.mod" For Output As #3
Open "K:\outputs\model_year_.txt" For Output As #4
'=====
' Writing the header of the model file
'=====
Print #3, "set I1;"
Print #3, "set J1;"
Print #3, "set L1;"
Print #3, "set K1;"

Print #4, "set I1;"
Print #4, "set J1;"
Print #4, "set L1;"
Print #4, "set K1;"

msgin:
Budget = InputBox("Please enter the Budget: ", "Budget")
If Trim(Budget) = "" Then GoTo msgin

Print #3, "param budget:= " & Budget & ";"
Print #3, " "
Print #3, "var x{i in I1,j in J1,l in L1,k in K1} binary;"

Print #4, "param budget:= " & Budget & ";"
Print #4, " "
Print #4, "var x{i in I1,j in J1,l in L1,k in K1} binary;"
'=====
' Writing the objective function formula
'=====
txtmsg = "maximize REUAC:"
For i = 2 To 50401
    xx = Trim(Cells(i, 16).Value)
    If Trim(xx) <> "0" And IsNumeric(Trim(xx)) Then
        a = (((i - 2) \ 3600) + 1)
        b = (((i - 2) \ 300) + 1) - ((a - 1) * 12)
        c = (((i - 2) \ 20) + 1) - ((b - 1) * 15) - ((a - 1) * 180)
        d = Trim(Cells(i, 4).Value)
        txtmsg = txtmsg & xx & "*x[" & a & "," & b & "," & c & "," & d & "]" + "
    End If
```

```

Next i
For i = 2 To 50401
    xx = Trim(Cells(i, 35).Value)
    If Trim(xx) <> "0" And IsNumeric(Trim(xx)) Then
        a = (((i - 2 + 50401) \ 3600) + 1)
        b = (((i - 2 + 50401) \ 300) + 1) - ((a - 1) * 12)
        c = (((i - 2 + 50401) \ 20) + 1) - ((b - 1) * 15) - ((a - 1) * 180)
        d = Trim(Cells(i, 23).Value)
        txtmsg = txtmsg & xx & "*x[" & a & "," & b & "," & c & "," & d & "]"
    End If
Next i

Print #3, Mid(Trim(txtmsg), 1, (Len(Trim(txtmsg))) - 1) & ";"
Print #4, Mid(Trim(txtmsg), 1, (Len(Trim(txtmsg))) - 1) & ";"
'=====
' Writing the budget constraint formula
'=====
Print #3, " "
Print #3, " "

Print #4, " "
Print #4, " "

txtmsg = "subject to constraint1 : "
For i = 2 To 50401
    xx = Trim(Cells(i, 17).Value)
    If Trim(xx) <> "0" And IsNumeric(Trim(xx)) Then
        a = (((i - 2) \ 3600) + 1)
        b = (((i - 2) \ 300) + 1) - ((a - 1) * 12)
        c = (((i - 2) \ 20) + 1) - ((b - 1) * 15) - ((a - 1) * 180)
        d = Trim(Cells(i, 4).Value)
        txtmsg = txtmsg & xx & "*x[" & a & "," & b & "," & c & "," & d & "]"
    End If
Next i
For i = 2 To 50401
    xx = Trim(Cells(i, 36).Value)
    If Trim(xx) <> "0" And IsNumeric(Trim(xx)) Then
        a = (((i - 2 + 50401) \ 3600) + 1)
        b = (((i - 2 + 50401) \ 300) + 1) - ((a - 1) * 12)
        c = (((i - 2 + 50401) \ 20) + 1) - ((b - 1) * 15) - ((a - 1) * 180)
        d = Trim(Cells(i, 23).Value)
        txtmsg = txtmsg & xx & "*x[" & a & "," & b & "," & c & "," & d & "]"
    End If
Next i

Print #3, Mid(Trim(txtmsg), 1, (Len(Trim(txtmsg))) - 1) & "<=budget;"

```

```

Print #4, Mid(Trim(txtmsg), 1, (Len(Trim(txtmsg))) - 1) & "<=budget;"
'=====
' Writing the bottom of the model file
'=====

Print #3, " "
Print #3, " "

Print #4, " "
Print #4, " "

Print #3, "subject to constraint2 {i in I1,j in J1,l in L1}:
      x[i,j,l,10]+x[i,j,l,11]+x[i,j,l,12]+x[i,j,l,13]+x[i,j,l,14]+x[i,j,l,15]=1;"
Print #3, "subject to constraint3 {i in I1,j in J1,l in L1}:
      x[i,j,l,21]+x[i,j,l,22]+x[i,j,l,23]+x[i,j,l,24]+x[i,j,l,25]=1;"
Print #3, "subject to constraint4 {i in I1,j in J1,l in L1}:
      x[i,j,l,32]+x[i,j,l,33]+x[i,j,l,34]+x[i,j,l,35]=1;"
Print #3, "subject to constraint5 {i in I1,j in J1,l in L1}:
      x[i,j,l,43]+x[i,j,l,44]+x[i,j,l,45]=1;"
Print #3, "subject to constraint6 {i in I1,j in J1,l in L1}:
      x[i,j,l,54]+x[i,j,l,55]=1;"

Print #4, "subject to constraint2 {i in I1,j in J1,l in L1}:
      x[i,j,l,10]+x[i,j,l,11]+x[i,j,l,12]+x[i,j,l,13]+x[i,j,l,14]+x[i,j,l,15]=1;"
Print #4, "subject to constraint3 {i in I1,j in J1,l in L1}:
      x[i,j,l,21]+x[i,j,l,22]+x[i,j,l,23]+x[i,j,l,24]+x[i,j,l,25]=1;"
Print #4, "subject to constraint4 {i in I1,j in J1,l in L1}:
      x[i,j,l,32]+x[i,j,l,33]+x[i,j,l,34]+x[i,j,l,35]=1;"
Print #4, "subject to constraint5 {i in I1,j in J1,l in L1}:
      x[i,j,l,43]+x[i,j,l,44]+x[i,j,l,45]=1;"
Print #4, "subject to constraint6 {i in I1,j in J1,l in L1}:
      x[i,j,l,54]+x[i,j,l,55]=1;"
'=====
' Closing the file
'=====

Close #3
Close #4
MsgBox "Creation of the file model is successfully done, The file is saved in K:\outputs\ ",
vbInformation, " Done"

End Sub

```

B1.3 Visual Basic Code 2 (VB2)

```
' Keyboard Shortcut: Ctrl+Shift+R
'=====
' The file path where the file will be saved in "k:\"
' directory
'=====
Sheets("Building Data").Select
Open "K:\output" For Input As #1
'=====
' Read Data character by character from the file and
' store it in jj
'=====
Do Until EOF(1)
    Line Input #1, xx
    For k = 1 To Len(Trim(xx))
        DoEvents
        If Asc(Mid(Trim(xx), k, 1)) = 48 Or Asc(Mid(Trim(xx), k, 1)) = 49 Then
            jj = jj & Chr(Asc(Mid(Trim(xx), k, 1)))
            i = i + 1
        End If
    Next k
Loop
'=====
' Creating CSV file
'=====

Open "k:\ampl_output_1.csv" For Output As #2
    For i = 1 To 50400
        DoEvents

        Print #2, Mid(Trim(jj), i, 1) & "," & Mid(Trim(jj), i + 50400, 1)
    Next i
Close #2
'=====
' Close file and display the Done message
'=====
Close #1
MsgBox "The file is successfully created. The file is named as ampl_output.csv",
vbInformation, " Done"

End Sub
```

B1.4 Visual Basic Code 3 (VB3)

```
' Keyboard Shortcut: Ctrl+Shift+S
'=====
' The file path where the file will be saved in "k:\outputs\" directory
'=====
Sheets("Decision").Select
Open "K:\outputs\reroof_year.mod" For Output As #3
Open "K:\outputs\reroof_year.txt" For Output As #4
'=====
' Writing the header of the model file
'=====
Print #3, "set N1;"

Print #4, "set N1;"

msgin:
Budget = InputBox("Please enter the Budget: ", "Budget")
If Trim(Budget) = "" Then GoTo msgin

Print #3, " "

Print #4, " "

Print #3, "param budget:= " & Budget & ";"
Print #3, " "
Print #3, "var x{n in N1}binary;"

Print #4, "param budget:= " & Budget & ";"
Print #4, " "
Print #4, "var x{n in N1}binary;"

Print #3, " "

Print #4, " "
'=====
' Writing the objective function formula
'=====
txtmsg = "maximize REUAC:"
For i = 586 To 701
    xx = Trim(Cells(i, 4).Value)
    If Trim(xx) <> "0" And IsNumeric(Trim(xx)) Then
        txtmsg = txtmsg & xx & "*x[" & i - 585 & "]" + "
    End If
Next i
```

```

Print #3, Mid(Trim(txtmsg), 1, (Len(Trim(txtmsg))) - 1) & ";"
Print #4, Mid(Trim(txtmsg), 1, (Len(Trim(txtmsg))) - 1) & ";"
'=====
' Writing the budget constraint formula
'=====
Print #3, " "
Print #3, " "

Print #4, " "
Print #4, " "

txtmsg = "subject to constraint1: "
For i = 586 To 701
    xx = Trim(Cells(i, 5).Value)
    If Trim(xx) <> "0" And IsNumeric(Trim(xx)) Then
        txtmsg = txtmsg & xx & "*x[" & i - 585 & "]" + "
    End If
Next i

Print #3, Mid(Trim(txtmsg), 1, (Len(Trim(txtmsg))) - 1) & "<=budget;"
Print #4, Mid(Trim(txtmsg), 1, (Len(Trim(txtmsg))) - 1) & "<=budget;"
'=====
' Closing the file
'=====
Close #3
Close #4
MsgBox "Creation of the file model is successfully done, The file is saved in K:\outputs\ ",
vbInformation, " Done"

End Sub

```

B1.5 Visual Basic Code 4 (VB4)

```
' Keyboard Shortcut: Ctrl+Shift+E
'=====
' The file path where the file will be saved in "k:\"
' directory
'=====
Sheets("Decision").Select
Open "K:\r_output" For Input As #1
'=====
' Read Data character by character from the file and
' store it in jj
'=====
Do Until EOF(1)
    Line Input #1, xx
    For k = 1 To Len(Trim(xx))
        DoEvents
        If Asc(Mid(Trim(xx), k, 1)) = 48 Or Asc(Mid(Trim(xx), k, 1)) = 49 Then
            jj = jj & Chr(Asc(Mid(Trim(xx), k, 1)))
            i = i + 1
        End If
    Next k
Loop
'=====
' Creating CSV file
'=====
Open "k:\ampl_r_output.csv" For Output As #2
    For i = 1 To 116
        DoEvents

        Print #2, Mid(Trim(jj), i, 1)
    Next i
Close #2
'=====
' Close file and display the Done message
'=====
Close #1
MsgBox "The file is successfully created. The file is named as ampl_r_output.csv",
vbInformation, " Done"

End Sub
```

APPENDIX B2

This appendix includes an optimization model created by Visual Basic.

B2.1 Optimization model created by Visual Basic code VB and VB1

```
set I1;  
set J1;  
set L1;  
set K1;  
param budget:= 54000;
```

```
var x{i in I1,j in J1,l in L1,k in K1}binary;
```

```
maximize REUAC:2.396936333333333E-02*x[1,1,4,54]+-  
81.0383281553196*x[1,1,4,55]+2.396936333333333E-02*x[1,1,6,54]+-  
298.56704181053*x[1,1,6,55]+2.396936333333333E-02*x[1,1,7,54]+-  
383.158770098389*x[1,1,7,55]+2.396936333333333E-02*x[1,1,12,54]+-  
264.300954184872*x[1,1,12,55]+0.4871*x[1,1,13,54]+-  
964.181048906213*x[1,1,13,55]+0.2207*x[1,2,1,54]+-  
626.094916916993*x[1,2,1,55]+1.546694333333333E-02*x[1,2,4,54]+-  
52.2923872434826*x[1,2,4,55]+7.733471666666667E-03*x[1,2,6,43]+-  
100.684663094628*x[1,2,6,44]+-113.254222256756*x[1,2,6,45]+7.733471666666667E-  
03*x[1,2,6,54]+-96.3296240426723*x[1,2,6,55]+0.0001*x[1,2,9,43]+-  
3.00300916126406*x[1,2,9,44]+-3.37535811163398*x[1,2,9,45]+0.0003*x[1,2,9,54]+-  
8.53958118936015*x[1,2,9,55]+0.002*x[1,2,12,43]+-25.1562471935999*x[1,2,12,44]+-  
27.4546466219908*x[1,2,12,45]+1.346694333333333E-02*x[1,2,12,54]+-  
148.494806618571*x[1,2,12,55]+0.0368*x[1,3,1,54]+-  
104.396433813074*x[1,3,1,55]+7.583126666666667E-03*x[1,3,4,54]+-  
25.6378902814699*x[1,3,4,55]+0.0004*x[1,3,6,32]+5.37224440921064*x[1,3,6,33]+7.0922  
6575414475*x[1,3,6,34]+6.56212777193392*x[1,3,6,35]+2.394375555555556E-  
03*x[1,3,6,43]+-31.1731789452634*x[1,3,6,44]+-  
35.064865176123*x[1,3,6,45]+4.788751111111111E-03*x[1,3,6,54]+-  
59.649613272081*x[1,3,6,55]+0.0036*x[1,3,8,43]+-59.0788817742593*x[1,3,8,44]+-  
64.3001099560661*x[1,3,8,45]+0.0015*x[1,3,10,43]+-43.7447428447757*x[1,3,10,44]+-  
49.2024820962666*x[1,3,10,45]+6.083126666666667E-03*x[1,3,10,54]+-  
167.24372281349*x[1,3,10,55]+0.005687345*x[1,3,11,43]+-  
18.9362985537847*x[1,3,11,44]+-20.9758590377591*x[1,3,11,45]+1.895781666666667E-  
03*x[1,3,11,54]+-5.95126660615399*x[1,3,11,55]+0.0429*x[2,1,1,43]+-  
153.012128753889*x[2,1,1,44]+-176.436042967313*x[2,1,1,45]+9.864996666666667E-  
03*x[2,1,7,43]+-167.257513868322*x[2,1,7,44]+-  
182.597794701843*x[2,1,7,45]+0.0024*x[2,1,8,43]+-39.3859211828396*x[2,1,8,44]+-
```

42.8667399707107*x[2,1,8,45]+9.86499666666667E-
 03*x[2,1,12,32]+150.760239481251*x[2,1,12,33]+179.265500144647*x[2,1,12,34]+170.88
 8147794775*x[2,1,12,35]+0.3129*x[2,2,1,43]+-1116.02552650564*x[2,2,1,44]+-
 1286.87267702732*x[2,2,1,45]+2.35333333333333E-02*x[2,2,4,43]+-
 98.8224720795283*x[2,2,4,44]+-113.938975647893*x[2,2,4,45]+2.35333333333333E-
 02*x[2,2,6,43]+-306.388364797817*x[2,2,6,44]+-
 344.638149417888*x[2,2,6,45]+2.35333333333333E-02*x[2,2,7,43]+-
 398.999306271216*x[2,2,7,44]+-435.594142993464*x[2,2,7,45]+0.0024*x[2,2,8,43]+-
 39.3859211828396*x[2,2,8,44]+-
 42.8667399707107*x[2,2,8,45]+0.0191*x[2,2,12,32]+291.892706241012*x[2,2,12,33]+347.
 082839301121*x[2,2,12,34]+330.863124759988*x[2,2,12,35]+4.43333333333333E-
 03*x[2,2,12,43]+-55.7630146124797*x[2,2,12,44]+-
 60.8578000120796*x[2,2,12,45]+1.5876*x[2,3,1,43]+-5662.51877878029*x[2,3,1,44]+-
 6529.36740827286*x[2,3,1,45]+0.0573*x[2,3,4,43]+-240.617322244631*x[2,3,4,44]+-
 277.423653171002*x[2,3,4,45]+0.0573*x[2,3,6,43]+-746.007930718765*x[2,3,6,44]+-
 839.140196670466*x[2,3,6,45]+0.0573*x[2,3,12,32]+875.678118723035*x[2,3,12,33]+104
 1.24851790336*x[2,3,12,34]+992.589374279964*x[2,3,12,35]+0.026*x[2,4,1,43]+-
 92.7346234872056*x[2,4,1,44]+-106.930935131705*x[2,4,1,45]+0.05776*x[2,4,4,43]+-
 242.548979630888*x[2,4,4,44]+-279.650788955621*x[2,4,4,45]+0.05776*x[2,4,12,43]+-
 726.512418951164*x[2,4,12,44]+-792.890194443094*x[2,4,12,45]+1.3181*x[3,1,1,54]+-
 3739.2646578536*x[3,1,1,55]+0.0048*x[3,1,4,10]+603.205466247455*x[3,1,4,11]+-
 47354.16*x[3,1,4,12]+-47354.16*x[3,1,4,13]+-
 47354.16*x[3,1,4,14]+622.600322247455*x[3,1,4,15]+0.002*x[3,1,4,21]+85.032085619799
 *x[3,1,4,22]+-19896.5*x[3,1,4,23]+-
 19896.5*x[3,1,4,24]+93.8168009364397*x[3,1,4,25]+3.87102616666667E-02*x[3,1,4,54]+-
 130.876020539064*x[3,1,4,55]+0.0002*x[3,1,9,43]+-6.00601832252813*x[3,1,9,44]+-
 6.75071622326796*x[3,1,9,45]+4.55102616666667E-02*x[3,1,12,43]+-
 572.433696166039*x[3,1,12,44]+-624.734075866334*x[3,1,12,45]+0.9912*x[4,1,1,43]+-
 3535.32918463531*x[4,1,1,44]+-
 4076.53626548253*x[4,1,1,45]+0.0012*x[4,1,4,21]+51.0192513718794*x[4,1,4,22]+-
 11937.9*x[4,1,4,23]+-
 11937.9*x[4,1,4,24]+56.2900805618638*x[4,1,4,25]+3.83333333333333E-02*x[4,1,4,43]+-
 160.971448854756*x[4,1,4,44]+-
 185.594648718239*x[4,1,4,45]+0.0001*x[4,1,9,10]+147.200761888366*x[4,1,9,11]+-
 1844.75*x[4,1,9,12]+-1844.75*x[4,1,9,13]+-
 1844.75*x[4,1,9,14]+151.874641888366*x[4,1,9,15]+0.0001*x[4,1,9,21]+91.224355213843
 *x[4,1,9,22]+-1896.5*x[4,1,9,23]+-
 1896.5*x[4,1,9,24]+100.124641888366*x[4,1,9,25]+3.95333333333333E-02*x[4,1,12,43]+-
 497.255152860157*x[4,1,12,44]+-
 542.686848228018*x[4,1,12,45]+0.0003*x[5,1,1,10]+10.5113727677111*x[5,1,1,11]+-
 2987.8905*x[5,1,1,12]+-2987.8905*x[5,1,1,13]+-
 2987.8905*x[5,1,1,14]+10.8756815177111*x[5,1,1,15]+6.6941*x[5,1,1,43]+-
 23875.9555032963*x[5,1,1,44]+-
 27531.0143409671*x[5,1,1,45]+0.003*x[5,1,4,21]+127.548128429698*x[5,1,4,22]+-
 29844.75*x[5,1,4,23]+-
 29844.75*x[5,1,4,24]+140.72520140466*x[5,1,4,25]+0.03108358*x[5,1,4,43]+-

130.527884561549*x[5,1,4,44]+-150.494246374051*x[5,1,4,45]+0.0006*x[5,1,9,43]+-
 18.0180549675844*x[5,1,9,44]+-
 20.2521486698039*x[5,1,9,45]+0.03408358*x[5,1,12,43]+-
 428.707481861418*x[5,1,12,44]+-467.876322256177*x[5,1,12,45]+0.0008*x[5,2,2,43]+-
 2.86300175028636*x[5,2,2,44]+-
 3.39765529314231*x[5,2,2,45]+0.0005*x[5,2,5,10]+47.3603502295042*x[5,2,5,11]+-
 4948.25*x[5,2,5,12]+-4948.25*x[5,2,5,13]+-
 4948.25*x[5,2,5,14]+48.9148977295042*x[5,2,5,15]+3.19766833333333E-03*x[5,2,5,43]+-
 15.2688197151309*x[5,2,5,44]+-31976.6833333333*x[5,2,5,45]+3.69766833333333E-
 03*x[5,2,6,43]+-48.1411850232863*x[5,2,6,44]+-
 54.1511715960818*x[5,2,6,45]+0.0001*x[5,2,9,43]+-3.00300916126406*x[5,2,9,44]+-
 3.37535811163398*x[5,2,9,45]+0.3406*x[6,1,1,43]+-1214.82356768239*x[6,1,1,44]+-
 1400.79525022533*x[6,1,1,45]+0.032688*x[6,1,4,43]+-137.265253569502*x[6,1,4,44]+-
 158.262205494829*x[6,1,4,45]+0.032688*x[6,1,12,43]+-411.153704132196*x[6,1,12,44]+-
 448.718744389818*x[6,1,12,45]+0.0267*x[6,2,1,43]+-95.2313248887842*x[6,2,1,44]+-
 109.809844923712*x[6,2,1,45]+6.28467333333333E-03*x[6,2,12,43]+-
 79.0493979521793*x[6,2,12,44]+-86.2717427506579*x[6,2,12,45]+0.0192*x[6,3,1,43]+-
 68.4809527290133*x[6,3,1,44]+-
 78.9643828664897*x[6,3,1,45]+0.03982412*x[6,3,12,43]+-
 500.912703493792*x[6,3,12,44]+-546.678570815878*x[6,3,12,45]+0.0191*x[6,4,1,43]+-
 68.1242811002164*x[6,4,1,44]+-78.55311003906*x[6,4,1,45]+0.006005*x[6,4,4,43]+-
 25.2165274010298*x[6,4,4,44]+-29.0738051883397*x[6,4,4,45]+0.006005*x[6,4,7,43]+-
 101.81264167813*x[6,4,7,44]+-111.150545127865*x[6,4,7,45]+0.0048*x[6,4,8,43]+-
 78.7718423656791*x[6,4,8,44]+-85.7334799414214*x[6,4,8,45]+0.006005*x[6,4,11,43]+-
 19.9939467036863*x[6,4,11,44]+-
 22.1474226588581*x[6,4,11,45]+0.006005*x[6,4,12,43]+-
 75.5316321987836*x[6,4,12,44]+-82.4325764825274*x[6,4,12,45]+0.0191*x[6,5,1,43]+-
 68.1242811002164*x[6,5,1,44]+-78.55311003906*x[6,5,1,45]+5.99749333333333E-
 03*x[6,5,4,43]+-25.1850049920888*x[6,5,4,44]+-
 29.0374609145212*x[6,5,4,45]+5.99749333333333E-03*x[6,5,7,43]+-
 101.685368811597*x[6,5,7,44]+-111.011599234094*x[6,5,7,45]+0.0048*x[6,5,8,43]+-
 78.7718423656791*x[6,5,8,44]+-85.7334799414214*x[6,5,8,45]+5.99749333333333E-
 03*x[6,5,11,43]+-19.9689528829943*x[6,5,11,44]+-
 22.1197368437997*x[6,5,11,45]+5.99749333333333E-03*x[6,5,12,43]+-
 75.4372124176503*x[6,5,12,44]+-82.3295300422062*x[6,5,12,45]+0.063017*x[6,6,4,43]+-
 264.624464151656*x[6,6,4,44]+-305.103077694189*x[6,6,4,45]+0.063017*x[6,6,6,43]+-
 820.439472427651*x[6,6,6,44]+-
 922.863835490101*x[6,6,6,45]+0.002*x[6,6,7,21]+2.71935363176898*x[6,6,7,22]+-
 19958.6*x[6,6,7,23]+-
 19958.6*x[6,6,7,24]+4.38066773426655*x[6,6,7,25]+0.061017*x[6,6,7,43]+-
 1034.5215582472*x[6,6,7,44]+-1129.40429842913*x[6,6,7,45]+0.002*x[6,6,8,43]+-
 32.8216009856996*x[6,6,8,44]+-35.7222833089256*x[6,6,8,45]+0.0028*x[6,6,8,54]+-
 43.3588930436228*x[6,6,8,55]+0.063017*x[6,6,12,43]+-792.635614699541*x[6,6,12,44]+-
 865.054733088997*x[6,6,12,45]+0.0015*x[6,6,13,21]+24.8185461791559*x[6,6,13,22]+-
 14968.95*x[6,6,13,23]+-
 14968.95*x[6,6,13,24]+26.5346011970306*x[6,6,13,25]+0.0004*x[6,6,13,32]+2.638548158

62145*x[6,6,13,33]+2.88141890969841*x[6,6,13,34]+2.93589365254149*x[6,6,13,35]+1.0
 858*x[6,6,13,43]+-2764.93836962368*x[6,6,13,44]+-
 3268.54668017611*x[6,6,13,45]+0.0001*x[6,6,15,21]+1.51378879469785*x[6,6,15,22]+-
 997.93*x[6,6,15,23]+-
 997.93*x[6,6,15,24]+1.67986563124971*x[6,6,15,25]+0.062917*x[6,6,15,43]+-
 201.444973094284*x[6,6,15,44]+-245.46084078662*x[6,6,15,45]+1.0273*x[7,1,1,43]+-
 3664.08764263101*x[7,1,1,44]+-4225.00575618463*x[7,1,1,45]+0.0519125*x[7,1,4,43]+-
 217.993834921892*x[7,1,4,44]+-251.339535693537*x[7,1,4,45]+0.0004*x[7,1,9,43]+-
 12.0120366450563*x[7,1,9,44]+-13.5014324465359*x[7,1,9,45]+0.0519125*x[7,1,12,43]+-
 652.961841218877*x[7,1,12,44]+-
 712.619671382049*x[7,1,12,45]+0.9313*x[8,1,2,32]+603.146744651081*x[8,1,2,33]+-
 9308180.5225*x[8,1,2,34]+864.182031870703*x[8,1,2,35]+0.0003*x[8,1,5,10]+28.4162101
 377025*x[8,1,5,11]+-2968.95*x[8,1,5,12]+-2968.95*x[8,1,5,13]+-
 2968.95*x[8,1,5,14]+29.3489386377025*x[8,1,5,15]+0.008*x[8,1,5,21]+410.524509556101
 *x[8,1,5,22]+-79503.2*x[8,1,5,23]+-
 79503.2*x[8,1,5,24]+451.438363672067*x[8,1,5,25]+0.001*x[8,1,5,32]+22.2789299669025
 *x[8,1,5,33]+25.6750145657807*x[8,1,5,34]+25.3797954590084*x[8,1,5,35]+0.029695833
 2*x[8,1,5,43]+-141.797170987006*x[8,1,5,44]+-
 296958.332*x[8,1,5,45]+0.0389958332*x[8,1,6,43]+-507.699840003248*x[8,1,6,44]+-
 571.081520781443*x[8,1,6,45]+0.0004*x[8,1,9,43]+-12.0120366450563*x[8,1,9,44]+-
 13.5014324465359*x[8,1,9,45]+0.0389958332*x[8,1,10,43]+-
 1137.24179690118*x[8,1,10,44]+-
 1279.127856568*x[8,1,10,45]+0.034*x[8,2,2,32]+22.0197458586242*x[8,2,2,33]+-
 339824.05*x[8,2,2,34]+31.5496500414516*x[8,2,2,35]+0.0083125*x[8,2,4,43]+-
 34.9063087462215*x[8,2,4,44]+-
 40.2457961079224*x[8,2,4,45]+0.0001*x[8,2,9,10]+147.200761888366*x[8,2,9,11]+-
 1844.75*x[8,2,9,12]+-1844.75*x[8,2,9,13]+-
 1844.75*x[8,2,9,14]+151.874641888366*x[8,2,9,15]+0.0083125*x[8,2,12,43]+-
 104.555652398399*x[8,2,12,44]+-114.108375022649*x[8,2,12,45]+0.2368*x[9,1,1,43]+-
 844.598416991165*x[9,1,1,44]+-
 973.894055353372*x[9,1,1,45]+0.0016*x[9,1,4,32]+38.016800652608*x[9,1,4,33]+42.4811
 916999754*x[9,1,4,34]+41.9334407491518*x[9,1,4,35]+0.017992*x[9,1,4,43]+-
 75.5529993337765*x[9,1,4,44]+-87.1100587757883*x[9,1,4,45]+0.0002*x[9,1,9,43]+-
 6.00601832252813*x[9,1,9,44]+-
 6.75071622326796*x[9,1,9,45]+0.012*x[9,1,12,21]+1123.91323007913*x[9,1,12,22]+-
 118633.8*x[9,1,12,23]+-
 118633.8*x[9,1,12,24]+1201.47212026806*x[9,1,12,25]+0.007592*x[9,1,12,43]+-
 95.4931143469051*x[9,1,12,44]+-104.217838577077*x[9,1,12,45]+1.7908*x[9,2,1,43]+-
 6387.27552849568*x[9,2,1,44]+-
 7365.07379360988*x[9,2,1,45]+0.0372*x[9,2,4,32]+883.890615173135*x[9,2,4,33]+987.68
 7707024428*x[9,2,4,34]+974.952497417779*x[9,2,4,35]+0.020768*x[9,2,4,43]+-
 87.2101317343191*x[9,2,4,44]+-100.55033907601*x[9,2,4,45]+0.057968*x[9,2,6,43]+-
 754.704846909343*x[9,2,6,44]+-848.922843291337*x[9,2,6,45]+0.001*x[9,2,9,43]+-
 30.0300916126406*x[9,2,9,44]+-
 33.7535811163398*x[9,2,9,45]+0.012*x[9,2,12,21]+1123.91323007913*x[9,2,12,22]+-
 118633.8*x[9,2,12,23]+-

118633.8*x[9,2,12,24]+1201.47212026806*x[9,2,12,25]+0.045968*x[9,2,12,43]+-
 578.191185497699*x[9,2,12,44]+-631.017597959837*x[9,2,12,45]+0.8652*x[10,1,1,43]+-
 3085.92293235116*x[10,1,1,44]+-
 3558.33250292119*x[10,1,1,45]+0.0002*x[10,1,4,10]+25.133561093644*x[10,1,4,11]+-
 1973.09*x[10,1,4,12]+-1973.09*x[10,1,4,13]+-
 1973.09*x[10,1,4,14]+25.941680093644*x[10,1,4,15]+3.95452983333333E-
 02*x[10,1,4,43]+-166.060799168093*x[10,1,4,44]+-
 191.462497894773*x[10,1,4,45]+3.97452983333333E-02*x[10,1,6,43]+-
 517.457378105584*x[10,1,6,44]+-582.057198257586*x[10,1,6,45]+0.001*x[10,1,9,43]+-
 30.0300916126406*x[10,1,9,44]+-33.7535811163398*x[10,1,9,45]+0.0014*x[10,1,10,43]+-
 40.828426655124*x[10,1,10,44]+-45.9223166231821*x[10,1,10,45]+3.83452983333333E-
 02*x[10,1,10,54]+-1054.22931283045*x[10,1,10,55]+0.9279*x[10,2,1,43]+-
 3309.55604360685*x[10,2,1,44]+-
 3816.20056571957*x[10,2,1,45]+0.0001*x[10,2,4,10]+12.566780546822*x[10,2,4,11]+-
 986.545*x[10,2,4,12]+-986.545*x[10,2,4,13]+-
 986.545*x[10,2,4,14]+12.970840046822*x[10,2,4,15]+0.0001*x[10,2,4,21]+4.25160428098
 995*x[10,2,4,22]+-994.825*x[10,2,4,23]+-
 994.825*x[10,2,4,24]+4.69084004682199*x[10,2,4,25]+4.04256141666667E-
 02*x[10,2,4,43]+-169.75746999787*x[10,2,4,44]+-
 195.724634621258*x[10,2,4,45]+4.06256141666667E-02*x[10,2,6,43]+-
 528.918505386631*x[10,2,6,44]+-594.949142437612*x[10,2,6,45]+0.0008*x[10,2,9,43]+-
 24.0240732901125*x[10,2,9,44]+-27.0028648930719*x[10,2,9,45]+4.06256141666667E-
 02*x[10,2,10,43]+-1184.77136308794*x[10,2,10,44]+-
 1332.5873691235*x[10,2,10,45]+0.0804*x[10,3,1,43]+-286.763989552743*x[10,3,1,44]+-
 330.663353253425*x[10,3,1,45]+1.97952966666667E-02*x[10,3,4,43]+-
 83.1255022159057*x[10,3,4,44]+-95.8408990727823*x[10,3,4,45]+1.97952966666667E-
 02*x[10,3,6,43]+-257.721610894661*x[10,3,6,44]+-
 289.89579647499*x[10,3,6,45]+0.0004*x[10,3,9,43]+-12.0120366450563*x[10,3,9,44]+-
 13.5014324465359*x[10,3,9,45]+1.97952966666667E-02*x[10,3,10,43]+-
 577.293441479586*x[10,3,10,44]+-
 649.318486554635*x[10,3,10,45]+0.0301*x[10,4,1,43]+-107.35816026788*x[10,4,1,44]+-
 123.79312105632*x[10,4,1,45]+0.0002*x[10,4,4,10]+25.133561093644*x[10,4,4,11]+-
 1973.09*x[10,4,4,12]+-1973.09*x[10,4,4,13]+-
 1973.09*x[10,4,4,14]+25.941680093644*x[10,4,4,15]+0.0074*x[10,4,4,21]+314.618716793
 256*x[10,4,4,22]+-73617.05*x[10,4,4,23]+-
 73617.05*x[10,4,4,24]+347.122163464827*x[10,4,4,25]+0.008753045*x[10,4,4,43]+-
 36.7562696228054*x[10,4,4,44]+-
 42.3787385736505*x[10,4,4,45]+0.008753045*x[10,4,10,43]+-
 255.2664684225*x[10,4,10,44]+-287.114359933544*x[10,4,10,45]+0.0373*x[10,5,1,43]+-
 133.03851754126*x[10,5,1,44]+-
 153.404764631253*x[10,5,1,45]+0.008*x[10,5,4,21]+340.128342479196*x[10,5,4,22]+-
 79586*x[10,5,4,23]+-
 79586*x[10,5,4,24]+375.267203745759*x[10,5,4,25]+0.008*x[10,5,10,43]+-
 233.305295172137*x[10,5,10,44]+-
 262.413237846755*x[10,5,10,45]+0.0375*x[10,6,1,43]+-133.751860798854*x[10,6,1,44]+-
 154.227310286113*x[10,6,1,45]+0.0085*x[10,6,4,21]+361.386363884146*x[10,6,4,22]+-

84560.125*x[10,6,4,23]+-
 84560.125*x[10,6,4,24]+398.721403979869*x[10,6,4,25]+1.39869333333333E-
 03*x[10,6,4,43]+-5.87347023574316*x[10,6,4,44]+-
 6.77191298777067*x[10,6,4,45]+9.898693333333334E-03*x[10,6,10,43]+-
 288.677196243975*x[10,6,10,44]+-
 324.693521006511*x[10,6,10,45]+0.0218*x[10,7,1,43]+-77.7544150777339*x[10,7,1,44]+-
 89.6574763796601*x[10,7,1,45]+0.0062*x[10,7,4,21]+263.599465421377*x[10,7,4,22]+-
 61679.15*x[10,7,4,23]+-
 61679.15*x[10,7,4,24]+290.832082902963*x[10,7,4,25]+4.59608333333333E-
 04*x[10,7,4,43]+-1.93001267797529*x[10,7,4,44]+-
 2.22523949146891*x[10,7,4,45]+6.65960833333333E-03*x[10,7,10,43]+-
 194.215235992395*x[10,7,10,44]+-
 218.446173192654*x[10,7,10,45]+0.0597*x[10,8,1,43]+-212.932962391776*x[10,8,1,44]+-
 245.529877975491*x[10,8,1,45]+0.0096*x[10,8,4,21]+408.154010975035*x[10,8,4,22]+-
 95503.2*x[10,8,4,23]+-
 95503.2*x[10,8,4,24]+450.320644494911*x[10,8,4,25]+0.0096*x[10,8,10,43]+-
 279.966354206564*x[10,8,10,44]+-
 314.895885416106*x[10,8,10,45]+0.0301*x[10,9,1,43]+-107.35816026788*x[10,9,1,44]+-
 123.79312105632*x[10,9,1,45]+0.0074*x[10,9,4,21]+314.618716793256*x[10,9,4,22]+-
 73617.05*x[10,9,4,23]+-
 73617.05*x[10,9,4,24]+347.122163464827*x[10,9,4,25]+1.16712666666667E-
 03*x[10,9,4,43]+-4.90106270948752*x[10,9,4,44]+-
 5.65075992286145*x[10,9,4,45]+8.56712666666667E-03*x[10,9,10,43]+-
 249.844501967969*x[10,9,10,44]+-
 281.01593095541*x[10,9,10,45]+0.0373*x[10,10,1,43]+-133.03851754126*x[10,10,1,44]+-
 153.404764631253*x[10,10,1,45]+0.0078*x[10,10,4,21]+331.625133917216*x[10,10,4,22]
 +-77596.35*x[10,10,4,23]+-
 77596.35*x[10,10,4,24]+365.885523652115*x[10,10,4,25]+0.0078*x[10,10,10,43]+-
 227.472662792834*x[10,10,10,44]+-
 255.852906900586*x[10,10,10,45]+0.0001*x[11,1,1,10]+3.50379092257037*x[11,1,1,11]+
 -995.9635*x[11,1,1,12]+-995.9635*x[11,1,1,13]+-
 995.9635*x[11,1,1,14]+3.62522717257037*x[11,1,1,15]+1.2073*x[11,1,1,43]+-
 4306.09657446551*x[11,1,1,44]+-
 4965.29684555797*x[11,1,1,45]+0.0001*x[11,1,4,10]+12.566780546822*x[11,1,4,11]+-
 986.545*x[11,1,4,12]+-986.545*x[11,1,4,13]+-
 986.545*x[11,1,4,14]+12.970840046822*x[11,1,4,15]+0.0485*x[11,1,4,21]+2062.02807628
 012*x[11,1,4,22]+-482490.125*x[11,1,4,23]+-
 482490.125*x[11,1,4,24]+2275.05742270866*x[11,1,4,25]+1.18333333333333E-
 03*x[11,1,4,43]+-4.96911863855984*x[11,1,4,44]+-
 5.72922611260649*x[11,1,4,45]+0.0004*x[11,1,9,43]+-12.0120366450563*x[11,1,9,44]+-
 13.5014324465359*x[11,1,9,45]+4.97833333333333E-02*x[11,1,10,43]+-
 1451.83940974828*x[11,1,10,44]+-1632.97571135054*x[11,1,10,45]+4.97833333333333E-
 02*x[11,1,11,43]+-165.756088825731*x[11,1,11,44]+-
 183.609079883456*x[11,1,11,45]+0.0012*x[12,1,1,10]+42.0454910708444*x[12,1,1,11]+-
 11951.562*x[12,1,1,12]+-11951.562*x[12,1,1,13]+-
 11951.562*x[12,1,1,14]+43.5027260708444*x[12,1,1,15]+0.8546*x[12,1,1,32]+2328.93395

644462*x[12,1,1,33]+-
 8539808.423*x[12,1,1,34]+2676.83941678635*x[12,1,1,35]+0.025*x[12,1,4,21]+1062.9010
 7024749*x[12,1,4,22]+-248706.25*x[12,1,4,23]+-
 248706.25*x[12,1,4,24]+1172.7100117055*x[12,1,4,25]+0.0134*x[12,1,4,43]+-
 56.2700195127059*x[12,1,4,44]+-
 64.8774337258539*x[12,1,4,45]+0.0384*x[12,1,6,21]+1283.2531214708*x[12,1,6,22]+-
 382012.8*x[12,1,6,23]+-
 382012.8*x[12,1,6,24]+1424.84426610566*x[12,1,6,25]+0.0008*x[12,1,9,43]+-
 24.0240732901125*x[12,1,9,44]+-27.0028648930719*x[12,1,9,45]+0.0384*x[12,1,10,43]+-
 1119.86541682626*x[12,1,10,44]+-
 1259.58354166442*x[12,1,10,45]+0.2576*x[13,1,1,43]+-918.786115780929*x[13,1,1,44]+-
 1059.43880345874*x[13,1,1,45]+0.026272*x[13,1,4,43]+-
 110.322832286404*x[13,1,4,44]+-
 127.198502898928*x[13,1,4,45]+0.026272*x[13,1,6,43]+-
 342.043985267773*x[13,1,6,44]+-384.74504793938*x[13,1,6,45]+0.0004*x[13,1,9,43]+-
 12.0120366450563*x[13,1,9,44]+-
 13.5014324465359*x[13,1,9,45]+0.026272*x[13,1,10,43]+-
 766.174589345298*x[13,1,10,44]+-
 861.765073088744*x[13,1,10,45]+0.5954*x[13,2,1,43]+-2123.62287785701*x[13,2,1,44]+-
 2448.71841451604*x[13,2,1,45]+0.0339212*x[13,2,4,43]+-
 142.443775066746*x[13,2,4,44]+-
 164.232866037421*x[13,2,4,45]+0.0339212*x[13,2,6,43]+-
 441.631487251263*x[13,2,6,44]+-
 496.765138556688*x[13,2,6,45]+0.0001*x[13,2,9,10]+147.200761888366*x[13,2,9,11]+-
 1844.75*x[13,2,9,12]+-1844.75*x[13,2,9,13]+-
 1844.75*x[13,2,9,14]+151.874641888366*x[13,2,9,15]+0.001*x[13,2,9,43]+-
 30.0300916126406*x[13,2,9,44]+-
 33.7535811163398*x[13,2,9,45]+0.0339212*x[13,2,10,43]+-
 989.249447324137*x[13,2,10,44]+-
 1112.67149045592*x[13,2,10,45]+0.0339212*x[13,2,12,43]+-
 426.66504615177*x[13,2,12,44]+-465.647279496937*x[13,2,12,45]+0.3427*x[13,3,1,43]+-
 1222.31367188713*x[13,3,1,44]+-
 1409.43197960135*x[13,3,1,45]+0.029168*x[13,3,4,43]+-
 122.483875309448*x[13,3,4,44]+-
 141.219775142963*x[13,3,4,45]+0.029168*x[13,3,6,43]+-
 379.747981207765*x[13,3,6,44]+-427.156042870579*x[13,3,6,45]+0.0006*x[13,3,9,43]+-
 18.0180549675844*x[13,3,9,44]+-
 20.2521486698039*x[13,3,9,45]+0.029168*x[13,3,10,43]+-
 850.631106197612*x[13,3,10,44]+-
 956.758665189269*x[13,3,10,45]+0.0459*x[13,4,1,43]+-163.712277617798*x[13,4,1,44]+-
 188.774227790202*x[13,4,1,45]+0.019008*x[13,4,4,43]+-
 79.8194426042921*x[13,4,4,44]+-
 92.0291239000769*x[13,4,4,45]+0.01*x[13,4,6,32]+134.306110230266*x[13,4,6,33]+177.3
 06643853619*x[13,4,6,34]+164.053194298348*x[13,4,6,35]+0.009008*x[13,4,6,43]+-
 117.27817521666*x[13,4,6,44]+-131.919282576048*x[13,4,6,45]+0.0003*x[13,4,9,43]+-
 9.00902748379219*x[13,4,9,44]+-

10.1260743349019*x[13,4,9,45]+0.019008*x[13,4,10,43]+-
 554.333381328998*x[13,4,10,44]+-623.49385312389*x[13,4,10,45]+0.2142*x[13,5,1,43]+-
 763.990628883055*x[13,5,1,44]+-
 880.946396354275*x[13,5,1,45]+0.031266*x[13,5,4,43]+-
 131.293912692855*x[13,5,4,44]+-
 151.377450960638*x[13,5,4,45]+0.031266*x[13,5,6,43]+-
 407.062547327276*x[13,5,6,44]+-457.880582706785*x[13,5,6,45]+0.0004*x[13,5,9,43]+-
 12.0120366450563*x[13,5,9,44]+-
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 393.267612377547*x[13,5,12,44]+-
 429.198490641582*x[13,5,12,45]+0.5281*x[14,1,1,43]+-1883.58287167666*x[14,1,1,44]+-
 2171.9318016559*x[14,1,1,45]+0.0291625*x[14,1,4,43]+-
 122.460779405917*x[14,1,4,44]+-141.193146345538*x[14,1,4,45]+0.0004*x[14,1,9,43]+-
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 13.5014324465359*x[14,1,9,45]+0.0291625*x[14,1,10,43]+-
 850.470708807181*x[14,1,10,44]+-
 956.578256088249*x[14,1,10,45]+0.0059*x[14,2,1,43]+-21.0436260990197*x[14,2,1,44]+-
 24.2650968183484*x[14,2,1,45]+0.00426712*x[14,2,4,43]+-
 17.9187257957506*x[14,2,4,44]+-20.6596861940497*x[14,2,4,45]+0.0001*x[14,2,9,43]+-
 3.00300916126406*x[14,2,9,44]+-
 3.37535811163398*x[14,2,9,45]+0.00426712*x[14,2,10,43]+-
 124.442711391866*x[14,2,10,44]+-
 139.968596935081*x[14,2,10,45]+0.1809*x[14,3,1,43]+-645.218976493673*x[14,3,1,44]+-
 743.992544820207*x[14,3,1,45]+0.01508*x[14,3,4,43]+-63.3247682277317*x[14,3,4,44]+-
 73.0113209392445*x[14,3,4,45]+0.0004*x[14,3,9,43]+-12.0120366450563*x[14,3,9,44]+-
 13.5014324465359*x[14,3,9,45]+0.01508*x[14,3,10,43]+-
 439.780481399478*x[14,3,10,44]+-
 494.648953341133*x[14,3,10,45]+0.0106*x[14,4,1,43]+-37.8071926524761*x[14,4,1,44]+-
 43.5949197075412*x[14,4,1,45]+0.00805716*x[14,4,4,43]+-
 33.8340709266413*x[14,4,4,44]+-
 39.0095420834777*x[14,4,4,45]+0.00805716*x[14,4,6,43]+-
 104.898870140838*x[14,4,6,44]+-117.994534502712*x[14,4,6,45]+0.0001*x[14,4,9,43]+-
 3.00300916126406*x[14,4,9,44]+-
 3.37535811163398*x[14,4,9,45]+0.00805716*x[14,4,10,43]+-
 234.972261506142*x[14,4,10,44]+-264.28818043117*x[14,4,10,45]+0.5087*x[14,5,1,43]+-
 1814.38857569006*x[14,5,1,44]+-
 2092.14487313455*x[14,5,1,45]+0.02980056*x[14,5,4,43]+-
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 977.507679905812*x[14,5,10,45]+0.0506*x[14,6,1,43]+-180.475844171254*x[14,6,1,44]+-
 208.104050679395*x[14,6,1,45]+0.00984525*x[14,6,4,43]+-
 41.3427171348857*x[14,6,4,44]+-
 47.6667577902584*x[14,6,4,45]+0.00984525*x[14,6,6,43]+-
 128.178613960016*x[14,6,6,44]+-144.180541383419*x[14,6,6,45]+0.0004*x[14,6,9,43]+-

12.0120366450563*x[14,6,9,44]+-
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 287.118619661685*x[14,6,10,44]+-
 322.940491238846*x[14,6,10,45]+0.0013*x[14,6,12,32]+19.8670428331579*x[14,6,12,33]
 +23.6234393241601*x[14,6,12,34]+22.519479695706*x[14,6,12,35]+0.00854525*x[14,6,1
 2,43]+-107.483210665555*x[14,6,12,44]+-
 117.303409523283*x[14,6,12,45]+0.1217*x[14,7,1,43]+-434.069372245881*x[14,7,1,44]+-
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 64.4895576316448*x[14,7,4,44]+-74.3542838173697*x[14,7,4,45]+0.0003*x[14,7,9,43]+-
 9.00902748379219*x[14,7,9,44]+-
 10.1260743349019*x[14,7,9,45]+0.01535738*x[14,7,10,43]+-
 447.869759246334*x[14,7,10,44]+-
 503.747476330375*x[14,7,10,45]+0.01535738*x[14,7,12,43]+-
 193.167023763023*x[14,7,12,44]+-
 210.815720469815*x[14,7,12,45]+0.6032*x[15,1,2,32]+390.656197115357*x[15,1,2,33]+-
 6028878.44*x[15,1,2,34]+559.727908970694*x[15,1,2,35]+0.0488*x[15,1,5,43]+-
 233.019289189901*x[15,1,5,44]+-
 488000*x[15,1,5,45]+0.04*x[15,1,12,21]+3746.37743359709*x[15,1,12,22]+-
 395446*x[15,1,12,23]+-
 395446*x[15,1,12,24]+4004.90706756018*x[15,1,12,25]+0.0088*x[15,1,12,43]+-
 110.687487651839*x[15,1,12,44]+-
 120.80044513676*x[15,1,12,45]+1.2611*x[15,2,2,32]+816.738279479736*x[15,2,2,33]+-
 12604473.8075*x[15,2,2,34]+1170.21363727278*x[15,2,2,35]+0.0034*x[15,2,5,21]+174.47
 2916561343*x[15,2,5,22]+-33788.86*x[15,2,5,23]+-
 33788.86*x[15,2,5,24]+191.861304560629*x[15,2,5,25]+0.0264*x[15,2,5,32]+588.1637511
 26225*x[15,2,5,33]+677.820384536611*x[15,2,5,34]+670.026600117822*x[15,2,5,35]+0.0
 34*x[15,2,5,43]+-162.349504763456*x[15,2,5,44]+-
 340000*x[15,2,5,45]+0.0638*x[15,2,6,43]+-830.633612213913*x[15,2,6,44]+-
 934.33062037654*x[15,2,6,45]+0.0048*x[15,2,8,43]+-78.7718423656791*x[15,2,8,44]+-
 85.7334799414214*x[15,2,8,45]+0.001*x[15,2,9,43]+-30.0300916126406*x[15,2,9,44]+-
 33.7535811163398*x[15,2,9,45]+0.04*x[15,2,12,21]+3746.37743359709*x[15,2,12,22]+-
 395446*x[15,2,12,23]+-
 395446*x[15,2,12,24]+4004.90706756018*x[15,2,12,25]+0.0238*x[15,2,12,43]+-
 299.359341603838*x[15,2,12,44]+-
 326.710294801691*x[15,2,12,45]+0.0425*x[15,3,2,32]+27.5246823232803*x[15,3,2,33]+-
 424780.0625*x[15,3,2,34]+39.4370625518145*x[15,3,2,35]+0.0062*x[15,3,5,21]+318.1564
 94905978*x[15,3,5,22]+-61614.98*x[15,3,5,23]+-
 61614.98*x[15,3,5,24]+349.864731845852*x[15,3,5,25]+0.0031*x[15,3,5,32]+69.06468289
 73976*x[15,3,5,33]+79.5925451539202*x[15,3,5,34]+78.6773659229261*x[15,3,5,35]+0.0
 093*x[15,3,6,43]+-121.079821216135*x[15,3,6,44]+-
 136.195529302536*x[15,3,6,45]+0.0048*x[15,3,8,43]+-78.7718423656791*x[15,3,8,44]+-
 85.7334799414214*x[15,3,8,45]+0.0093*x[15,3,10,43]+-
 271.217405637609*x[15,3,10,44]+-
 305.055388996853*x[15,3,10,45]+0.004*x[15,3,12,21]+374.637743359709*x[15,3,12,22]+
 -39544.6*x[15,3,12,23]+-
 39544.6*x[15,3,12,24]+400.490706756018*x[15,3,12,25]+0.0053*x[15,3,12,43]+-

66.6640550630396*x[15,3,12,44]+-
 72.7548135482756*x[15,3,12,45]+1.9199*x[16,1,1,32]+5232.06213781656*x[16,1,1,33]+-
 19185090.3245*x[16,1,1,34]+6013.64848617846*x[16,1,1,35]+0.0696*x[16,1,4,43]+-
 292.26816105107*x[16,1,4,44]+-
 336.975327411898*x[16,1,4,45]+0.035*x[16,1,6,32]+470.071385805931*x[16,1,6,33]+620.
 573253487665*x[16,1,6,34]+574.186180044218*x[16,1,6,35]+0.0346*x[16,1,6,43]+-
 450.46901226648*x[16,1,6,44]+-506.705947727716*x[16,1,6,45]+0.001*x[16,1,9,43]+-
 30.0300916126406*x[16,1,9,44]+-33.7535811163398*x[16,1,9,45]+0.0696*x[16,1,10,43]+-
 2029.75606799759*x[16,1,10,44]+-
 2282.99516926677*x[16,1,10,45]+0.0152*x[16,2,1,32]+41.422649354035*x[16,2,1,33]+-
 151889.876*x[16,2,1,34]+47.6105302306957*x[16,2,1,35]+0.00533332*x[16,2,4,43]+-
 22.3959716766795*x[16,2,4,44]+-
 25.8217996148337*x[16,2,4,45]+0.0052*x[16,2,6,32]+69.8391773197384*x[16,2,6,33]+92.
 1994548038817*x[16,2,6,34]+85.3076610351409*x[16,2,6,35]+1.33320000000001E-
 04*x[16,2,6,43]+-1.73573782414356*x[16,2,6,44]+-
 1.95242881361443*x[16,2,6,45]+0.0001*x[16,2,9,43]+-3.00300916126406*x[16,2,9,44]+-
 3.37535811163398*x[16,2,9,45]+0.00533332*x[16,2,10,43]+-
 155.536474605933*x[16,2,10,44]+-
 174.941721209107*x[16,2,10,45]+0.0433*x[16,3,1,32]+118.000047173008*x[16,3,1,33]+-
 432686.2915*x[16,3,1,34]+135.627365722969*x[16,3,1,35]+0.00876664*x[16,3,4,43]+-
 36.8133584970797*x[16,3,4,44]+-
 42.4445601192851*x[16,3,4,45]+0.0084*x[16,3,6,32]+112.817132593424*x[16,3,6,33]+14
 8.93758083704*x[16,3,6,34]+137.804683210612*x[16,3,6,35]+3.66640000000001E-
 04*x[16,3,6,43]+-4.77340920975094*x[16,3,6,44]+-
 5.36932568424539*x[16,3,6,45]+0.0001*x[16,3,9,43]+-3.00300916126406*x[16,3,9,44]+-
 3.37535811163398*x[16,3,9,45]+0.00876664*x[16,3,10,43]+-
 255.662941608483*x[16,3,10,44]+-
 287.56029842961*x[16,3,10,45]+0.0173*x[16,4,1,32]+47.1455153832108*x[16,4,1,33]+-
 172874.6615*x[16,4,1,34]+54.1883008546734*x[16,4,1,35]+5.59998833333333E-
 03*x[16,4,4,43]+-23.5157800587755*x[16,4,4,44]+-
 27.1129008926409*x[16,4,4,45]+0.0052*x[16,4,6,32]+69.8391773197384*x[16,4,6,33]+92.
 1994548038817*x[16,4,6,34]+85.3076610351409*x[16,4,6,35]+3.99988333333333E-
 04*x[16,4,6,43]+-5.20758235360641*x[16,4,6,44]+-
 5.85770137345943*x[16,4,6,45]+0.0001*x[16,4,9,43]+-3.00300916126406*x[16,4,9,44]+-
 3.37535811163398*x[16,4,9,45]+5.59998833333333E-03*x[16,4,10,43]+-
 163.313366383607*x[16,4,10,44]+-
 183.688883806757*x[16,4,10,45]+0.024*x[16,5,1,32]+65.4041831905815*x[16,5,1,33]+-
 239826.12*x[16,5,1,34]+75.1745214168879*x[16,5,1,35]+0.0076*x[16,5,4,43]+-
 31.9143394251168*x[16,5,4,44]+-
 36.7961564415291*x[16,5,4,45]+0.0046*x[16,5,6,32]+61.7808107059224*x[16,5,6,33]+81.
 5610561726646*x[16,5,6,34]+75.46446937724*x[16,5,6,35]+0.003*x[16,5,6,43]+-
 39.0580068439144*x[16,5,6,44]+-43.9340417104956*x[16,5,6,45]+0.0076*x[16,5,10,43]+-
 221.64003041353*x[16,5,10,44]+-
 249.292575954417*x[16,5,10,45]+0.0076*x[16,5,12,43]+-
 95.5937393356795*x[16,5,12,44]+-
 104.327657163565*x[16,5,12,45]+0.1624*x[16,6,1,32]+442.568306256268*x[16,6,1,33]+-

1622823.412*x[16,6,1,34]+508.680928254275*x[16,6,1,35]+0.0051*x[16,6,4,10]+640.9058
 07887921*x[16,6,4,11]+-50313.795*x[16,6,4,12]+-50313.795*x[16,6,4,13]+-
 50313.795*x[16,6,4,14]+661.512842387921*x[16,6,4,15]+0.0113*x[16,6,4,43]+-
 47.4515836189237*x[16,6,4,44]+-54.7100747091156*x[16,6,4,45]+0.0164*x[16,6,6,43]+-
 213.517104080065*x[16,6,6,44]+-240.172761350709*x[16,6,6,45]+0.0002*x[16,6,9,43]+-
 6.00601832252813*x[16,6,9,44]+-6.75071622326796*x[16,6,9,45]+0.0164*x[16,6,10,43]+-
 478.275855102881*x[16,6,10,44]+-
 537.947137585848*x[16,6,10,45]+0.024*x[16,7,1,32]+65.4041831905815*x[16,7,1,33]+-
 239826.12*x[16,7,1,34]+75.1745214168879*x[16,7,1,35]+0.003*x[16,7,4,10]+377.0034164
 0466*x[16,7,4,11]+-29596.35*x[16,7,4,12]+-29596.35*x[16,7,4,13]+-
 29596.35*x[16,7,4,14]+389.12520140466*x[16,7,4,15]+0.0046*x[16,7,4,43]+-
 19.3165738625707*x[16,7,4,44]+-
 22.2713578461886*x[16,7,4,45]+0.0046*x[16,7,6,32]+61.7808107059224*x[16,7,6,33]+81.
 5610561726646*x[16,7,6,34]+75.46446937724*x[16,7,6,35]+0.003*x[16,7,6,43]+-
 39.0580068439144*x[16,7,6,44]+-43.9340417104956*x[16,7,6,45]+0.0001*x[16,7,9,43]+-
 3.00300916126406*x[16,7,9,44]+-3.37535811163398*x[16,7,9,45]+0.0076*x[16,7,10,43]+-
 221.64003041353*x[16,7,10,44]+-
 249.292575954417*x[16,7,10,45]+0.0076*x[16,7,12,43]+-
 95.5937393356795*x[16,7,12,44]+-
 104.327657163565*x[16,7,12,45]+0.0373*x[16,8,1,43]+-133.03851754126*x[16,8,1,44]+-
 153.404764631253*x[16,8,1,45]+8.06666666666667E-03*x[16,8,4,43]+-
 33.8739918459573*x[16,8,4,44]+-39.0555695563598*x[16,8,4,45]+0.0001*x[16,8,9,43]+-
 3.00300916126406*x[16,8,9,44]+-3.37535811163398*x[16,8,9,45]+8.06666666666667E-
 03*x[16,8,10,43]+-235.249505965238*x[16,8,10,44]+-
 264.600014828811*x[16,8,10,45]+0.0373*x[16,9,1,43]+-133.03851754126*x[16,9,1,44]+-
 153.404764631253*x[16,9,1,45]+8.06666666666667E-03*x[16,9,4,43]+-
 33.8739918459573*x[16,9,4,44]+-39.0555695563598*x[16,9,4,45]+0.0001*x[16,9,9,43]+-
 3.00300916126406*x[16,9,9,44]+-3.37535811163398*x[16,9,9,45]+8.06666666666667E-
 03*x[16,9,10,43]+-235.249505965238*x[16,9,10,44]+-
 264.600014828811*x[16,9,10,45]+1.0971*x[16,10,1,43]+-
 3913.04443953128*x[16,10,1,44]+-
 4512.07418973051*x[16,10,1,45]+0.049272105*x[16,10,4,43]+-
 206.906142521052*x[16,10,4,44]+-
 238.555800497822*x[16,10,4,45]+0.049272105*x[16,10,6,43]+-
 641.49007143469*x[16,10,6,44]+-721.57423874464*x[16,10,6,45]+0.0012*x[16,10,9,43]+-
 36.0361099351688*x[16,10,9,44]+-
 40.5042973396078*x[16,10,9,45]+0.049272105*x[16,10,10,43]+-
 1436.93037509719*x[16,10,10,44]+-
 1616.20657607191*x[16,10,10,45]+0.0379*x[16,11,1,43]+-
 135.178547314042*x[16,11,1,44]+-
 155.872401595831*x[16,11,1,45]+0.00825*x[16,11,4,43]+-
 34.6438552970018*x[16,11,4,44]+-
 39.9431961371862*x[16,11,4,45]+0.0001*x[16,11,9,43]+-
 3.00300916126406*x[16,11,9,44]+-
 3.37535811163398*x[16,11,9,45]+0.00825*x[16,11,10,43]+-
 240.596085646266*x[16,11,10,44]+-

270.613651529466*x[16,11,10,45]+0.0589*x[16,12,1,43]+-
 210.0795893614*x[16,12,1,44]+-242.239695356054*x[16,12,1,45]+1.02321616666667E-
 02*x[16,12,4,43]+-42.9674579582459*x[16,12,4,44]+-
 49.540029134432*x[16,12,4,45]+0.0001*x[16,12,9,43]+-
 3.00300916126406*x[16,12,9,44]+-3.37535811163398*x[16,12,9,45]+1.02321616666667E-
 02*x[16,12,10,43]+-298.402187236337*x[16,12,10,44]+-
 335.631834140181*x[16,12,10,45]+0.0313*x[17,1,2,43]+-
 112.014943479954*x[17,1,2,44]+-132.933263344193*x[17,1,2,45]+0.0074*x[17,1,7,43]+-
 125.464371093782*x[17,1,7,44]+-136.971529383214*x[17,1,7,45]+0.0036*x[17,1,8,43]+-
 59.0788817742593*x[17,1,8,44]+-64.3001099560661*x[17,1,8,45]+0.0074*x[17,1,12,43]+-
 93.0781146163195*x[17,1,12,44]+-
 101.582192501366*x[17,1,12,45]+0.0006*x[17,2,1,10]+21.0227455354222*x[17,2,1,11]+-
 5975.781*x[17,2,1,12]+-5975.781*x[17,2,1,13]+-
 5975.781*x[17,2,1,14]+21.7513630354222*x[17,2,1,15]+1.6625*x[17,2,1,43]+-
 5929.6658287492*x[17,2,1,44]+-
 6837.41075601766*x[17,2,1,45]+0.0615833*x[17,2,4,43]+-
 258.603991989315*x[17,2,4,44]+-
 298.161676445476*x[17,2,4,45]+0.0615833*x[17,2,6,43]+-
 801.773650956945*x[17,2,6,44]+-901.867756956655*x[17,2,6,45]+0.0009*x[17,2,9,43]+-
 27.0270824513766*x[17,2,9,44]+-
 30.3782230047058*x[17,2,9,45]+0.0615833*x[17,2,10,43]+-
 1795.96374802178*x[17,2,10,44]+-
 2020.03414378601*x[17,2,10,45]+0.2967*x[17,3,1,43]+-1058.24472264053*x[17,3,1,44]+-
 1220.24647898372*x[17,3,1,45]+0.021316*x[17,3,4,43]+-
 89.5113235770775*x[17,3,4,44]+-
 103.203535619425*x[17,3,4,45]+0.021316*x[17,3,6,43]+-
 277.520157961626*x[17,3,6,44]+-312.166011033642*x[17,3,6,45]+0.0002*x[17,3,9,43]+-
 6.00601832252813*x[17,3,9,44]+-
 6.75071622326796*x[17,3,9,45]+0.021316*x[17,3,10,43]+-
 621.641958986159*x[17,3,10,44]+-
 699.200072242679*x[17,3,10,45]+1.4863*x[18,1,1,32]+4050.42656150672*x[18,1,1,33]+-
 14852231.7565*x[18,1,1,34]+4655.49546591336*x[18,1,1,35]+0.01*x[18,1,4,10]+1256.678
 0546822*x[18,1,4,11]+-98654.5*x[18,1,4,12]+-98654.5*x[18,1,4,13]+-
 98654.5*x[18,1,4,14]+1297.0840046822*x[18,1,4,15]+0.0061*x[18,1,4,21]+259.347861140
 387*x[18,1,4,22]+-60684.325*x[18,1,4,23]+-
 60684.325*x[18,1,4,24]+286.141242856141*x[18,1,4,25]+0.03876602*x[18,1,4,43]+-
 162.788410584324*x[18,1,4,44]+-187.68954428098*x[18,1,4,45]+0.0008*x[18,1,9,43]+-
 24.0240732901125*x[18,1,9,44]+-
 27.0028648930719*x[18,1,9,45]+0.05486602*x[18,1,10,43]+-
 1600.06662387755*x[18,1,10,44]+-
 1799.6962444956*x[18,1,10,45]+0.0504*x[18,2,1,32]+137.348784700221*x[18,2,1,33]+-
 503634.852*x[18,2,1,34]+157.866494975465*x[18,2,1,35]+0.001*x[18,2,4,10]+125.667805
 46822*x[18,2,4,11]+-9865.45*x[18,2,4,12]+-9865.45*x[18,2,4,13]+-
 9865.45*x[18,2,4,14]+129.70840046822*x[18,2,4,15]+7.06651833333333E-
 03*x[18,2,4,43]+-29.6741137689224*x[18,2,4,44]+-
 34.2132518539824*x[18,2,4,45]+0.0001*x[18,2,9,43]+-3.00300916126406*x[18,2,9,44]+-

3.37535811163398*x[18,2,9,45]+8.066518333333333E-03*x[18,2,10,43]+-
 235.245180096223*x[18,2,10,44]+-
 264.595149250026*x[18,2,10,45]+1.2152*x[19,1,2,32]+787.011622570594*x[19,1,2,33]+-
 12145711.34*x[19,1,2,34]+1127.62160971682*x[19,1,2,35]+0.0003*x[19,1,5,10]+28.41621
 01377025*x[19,1,5,11]+-2968.95*x[19,1,5,12]+-2968.95*x[19,1,5,13]+-
 2968.95*x[19,1,5,14]+29.3489386377025*x[19,1,5,15]+0.0176*x[19,1,5,21]+903.15392102
 3422*x[19,1,5,22]+-174907.04*x[19,1,5,23]+-
 174907.04*x[19,1,5,24]+993.164400078548*x[19,1,5,25]+0.003*x[19,1,5,32]+66.83678990
 07074*x[19,1,5,33]+77.0250436973421*x[19,1,5,34]+76.1393863770253*x[19,1,5,35]+4.5
 3333333333333333E-02*x[19,1,5,43]+-216.466006351275*x[19,1,5,44]+-
 453333.3333333333*x[19,1,5,45]+0.0384*x[19,1,6,32]+515.735463284222*x[19,1,6,33]+68
 0.857512397896*x[19,1,6,34]+629.964266105656*x[19,1,6,35]+2.783333333333333E-
 02*x[19,1,6,43]+-362.371507940762*x[19,1,6,44]+-
 407.610275869598*x[19,1,6,45]+6.623333333333333E-02*x[19,1,7,43]+-
 1122.96263677182*x[19,1,7,44]+-1225.95688686687*x[19,1,7,45]+0.0036*x[19,1,8,43]+-
 59.0788817742593*x[19,1,8,44]+-64.3001099560661*x[19,1,8,45]+6.623333333333333E-
 02*x[19,1,10,43]+-1931.57342294598*x[19,1,10,44]+-
 2172.56293167293*x[19,1,10,45]+6.623333333333333E-02*x[19,1,12,43]+-
 833.091052894716*x[19,1,12,44]+-
 909.206380631595*x[19,1,12,45]+1.8636*x[19,2,2,32]+1206.94112888624*x[19,2,2,33]+-
 18626355.87*x[19,2,2,34]+1729.29199462498*x[19,2,2,35]+0.026*x[19,2,5,21]+1334.2046
 5605733*x[19,2,5,22]+-258385.4*x[19,2,5,23]+-
 258385.4*x[19,2,5,24]+1467.17468193422*x[19,2,5,25]+0.0361*x[19,2,5,43]+-
 172.376974175316*x[19,2,5,44]+-361000*x[19,2,5,45]+0.0621*x[19,2,6,43]+-
 808.500741669028*x[19,2,6,44]+-909.434663407259*x[19,2,6,45]+0.0621*x[19,2,7,43]+-
 1052.88343850322*x[19,2,7,44]+-1149.45026685102*x[19,2,7,45]+0.0036*x[19,2,8,43]+-
 59.0788817742593*x[19,2,8,44]+-64.3001099560661*x[19,2,8,45]+0.0621*x[19,2,10,43]+-
 1811.03235377371*x[19,2,10,44]+-
 2036.98275878544*x[19,2,10,45]+0.0621*x[19,2,12,43]+-
 781.101475361276*x[19,2,12,44]+-
 852.466777612814*x[19,2,12,45]+0.0375*x[19,3,2,32]+24.2864844028944*x[19,3,2,33]+-
 374805.9375*x[19,3,2,34]+34.797408133954*x[19,3,2,35]+0.0077*x[19,3,5,10]+729.34939
 3534365*x[19,3,5,11]+-76203.05*x[19,3,5,12]+-76203.05*x[19,3,5,13]+-
 76203.05*x[19,3,5,14]+753.289425034365*x[19,3,5,15]+0.0077*x[19,3,5,32]+171.5477607
 45149*x[19,3,5,33]+197.697612156511*x[19,3,5,34]+195.424425034365*x[19,3,5,35]+0.0
 077*x[19,3,6,21]+257.318985294927*x[19,3,6,22]+-76601.525*x[19,3,6,23]+-
 76601.525*x[19,3,6,24]+285.710959609728*x[19,3,6,25]+0.0077*x[19,3,6,43]+-
 100.248884232714*x[19,3,6,44]+-112.764040390272*x[19,3,6,45]+0.0154*x[19,3,12,43]+-
 193.703103390719*x[19,3,12,44]+-
 211.400778989329*x[19,3,12,45]+0.2441*x[19,4,2,32]+158.088822473241*x[19,4,2,33]+-
 2439736.7825*x[19,4,2,34]+226.507928679951*x[19,4,2,35]+0.0203895*x[19,4,5,43]+-
 97.3595655110142*x[19,4,5,44]+-
 203895*x[19,4,5,45]+0.007*x[19,4,6,21]+233.926350268115*x[19,4,6,22]+-
 69637.75*x[19,4,6,23]+-
 69637.75*x[19,4,6,24]+259.737236008844*x[19,4,6,25]+0.0133895*x[19,4,6,43]+-
 174.322394212197*x[19,4,6,44]+-196.084950494227*x[19,4,6,45]+0.0002*x[19,4,9,43]+-

6.00601832252813*x[19,4,9,44]+-
 6.75071622326796*x[19,4,9,45]+0.0203895*x[19,4,12,43]+-
 256.461651076952*x[19,4,12,44]+-
 279.893258649541*x[19,4,12,45]+0.1156*x[19,5,1,32]+315.030149034634*x[19,5,1,33]+-
 1155162.478*x[19,5,1,34]+362.090611491344*x[19,5,1,35]+0.0136*x[19,5,4,43]+-
 57.109870550209*x[19,5,4,44]+-
 65.8457536322099*x[19,5,4,45]+0.0018*x[19,5,6,21]+60.152490068944*x[19,5,6,22]+-
 17906.85*x[19,5,6,23]+-
 17906.85*x[19,5,6,24]+66.7895749737026*x[19,5,6,25]+0.0118*x[19,5,6,43]+-
 153.62816025273*x[19,5,6,44]+-172.807230727949*x[19,5,6,45]+0.0036*x[19,5,8,43]+-
 59.0788817742593*x[19,5,8,44]+-64.3001099560661*x[19,5,8,45]+0.0136*x[19,5,12,43]+-
 171.062480916479*x[19,5,12,44]+-186.691597029537*x[19,5,12,45]+0.777*x[20,1,1,43]+-
 2771.33855575226*x[20,1,1,44]+-3195.58986912825*x[20,1,1,45]+4.20833333333333E-
 02*x[20,1,4,43]+-176.718655807938*x[20,1,4,44]+-
 203.750646962414*x[20,1,4,45]+0.0004*x[20,1,9,43]+-12.0120366450563*x[20,1,9,44]+-
 13.5014324465359*x[20,1,9,45]+0.008*x[20,1,12,32]+122.258725127125*x[20,1,12,33]+1
 45.375011225601*x[20,1,12,34]+138.581413512037*x[20,1,12,35]+3.40833333333333E-
 02*x[20,1,12,43]+-428.704379257598*x[20,1,12,44]+-
 467.872936183093*x[20,1,12,45]+0.1071*x[20,2,1,43]+-381.995314441528*x[20,2,1,44]+-
 440.473198177138*x[20,2,1,45]+0.0006*x[20,2,4,21]+25.5096256859397*x[20,2,4,22]+-
 5968.95*x[20,2,4,23]+-
 5968.95*x[20,2,4,24]+28.1450402809319*x[20,2,4,25]+0.02875*x[20,2,4,43]+-
 120.728586641067*x[20,2,4,44]+-139.195986538679*x[20,2,4,45]+0.0002*x[20,2,9,43]+-
 6.00601832252813*x[20,2,9,44]+-
 6.75071622326796*x[20,2,9,45]+0.0006*x[20,2,11,21]+13.5998943724423*x[20,2,11,22]+
 -5981.37*x[20,2,11,23]+-
 5981.37*x[20,2,11,24]+16.4171018159342*x[20,2,11,25]+0.02875*x[20,2,11,43]+-
 95.7245574905882*x[20,2,11,44]+-
 106.034704653151*x[20,2,11,45]+0.0014*x[20,2,12,21]+131.123210175898*x[20,2,12,22]
 +-13840.61*x[20,2,12,23]+-
 13840.61*x[20,2,12,24]+140.171747364606*x[20,2,12,25]+0.0014*x[20,2,12,32]+21.39527
 68972469*x[20,2,12,33]+25.4406269644801*x[20,2,12,34]+24.2517473646064*x[20,2,12,
 35]+0.02655*x[20,2,12,43]+-333.949181495038*x[20,2,12,44]+-
 364.460433906928*x[20,2,12,45]+0.5586*x[20,3,1,43]+-1992.36771845973*x[20,3,1,44]+-
 2297.37001402193*x[20,3,1,45]+0.03495*x[20,3,4,43]+-146.763968803662*x[20,3,4,44]+-
 169.213903635716*x[20,3,4,45]+0.0002*x[20,3,9,43]+-6.00601832252813*x[20,3,9,44]+-
 6.75071622326796*x[20,3,9,45]+0.03495*x[20,3,11,43]+-
 116.367766410298*x[20,3,11,44]+-
 128.90131922183*x[20,3,11,45]+0.002*x[20,3,12,21]+187.318871679854*x[20,3,12,22]+-
 19772.3*x[20,3,12,23]+-
 19772.3*x[20,3,12,24]+200.245353378009*x[20,3,12,25]+0.002*x[20,3,12,32]+30.5646812
 817813*x[20,3,12,33]+36.3437528064001*x[20,3,12,34]+34.6453533780092*x[20,3,12,35]
 +0.03095*x[20,3,12,43]+-389.292925320958*x[20,3,12,44]+-
 424.860656475308*x[20,3,12,45]+0.0806*x[20,4,1,43]+-287.477332810337*x[20,4,1,44]+-
 331.485898908285*x[20,4,1,45]+0.0133*x[20,4,4,43]+-55.8500939939544*x[20,4,4,44]+-
 64.3932737726759*x[20,4,4,45]+0.0001*x[20,4,9,43]+-3.00300916126406*x[20,4,9,44]+-

3.37535811163398*x[20,4,9,45]+0.0133*x[20,4,11,43]+-
 44.2830126826025*x[20,4,11,44]+-
 49.0525764134576*x[20,4,11,45]+0.0008*x[20,4,12,21]+74.9275486719418*x[20,4,12,22]
 +-7908.92*x[20,4,12,23]+-
 7908.92*x[20,4,12,24]+80.0981413512037*x[20,4,12,25]+0.0008*x[20,4,12,32]+12.225872
 5127125*x[20,4,12,33]+14.5375011225601*x[20,4,12,34]+13.8581413512037*x[20,4,12,3
 5]+0.0117*x[20,4,12,43]+-147.164046082559*x[20,4,12,44]+-
 160.609682738646*x[20,4,12,45]+0.2019*x[21,1,2,32]+130.758432025183*x[21,1,2,33]+-
 2017955.1675*x[21,1,2,34]+187.349245393208*x[21,1,2,35]+0.0004*x[21,1,5,10]+37.8882
 801836034*x[21,1,5,11]+-3958.6*x[21,1,5,12]+-3958.6*x[21,1,5,13]+-
 3958.6*x[21,1,5,14]+39.1319181836034*x[21,1,5,15]+0.0025*x[21,1,5,21]+128.288909236
 282*x[21,1,5,22]+-24844.75*x[21,1,5,23]+-
 24844.75*x[21,1,5,24]+141.074488647521*x[21,1,5,25]+0.034*x[21,1,5,32]+757.48361887
 4684*x[21,1,5,33]+872.950495236544*x[21,1,5,34]+862.913045606287*x[21,1,5,35]+0.00
 31*x[21,1,5,43]+-14.8024548460798*x[21,1,5,44]+-
 31000*x[21,1,5,45]+0.0016*x[21,1,6,32]+21.4889776368426*x[21,1,6,33]+28.3690630165
 79*x[21,1,6,34]+26.2485110877357*x[21,1,6,35]+0.0384*x[21,1,6,43]+-
 499.942487602104*x[21,1,6,44]+-562.355733894344*x[21,1,6,45]+0.0002*x[21,1,9,43]+-
 6.00601832252813*x[21,1,9,44]+-6.75071622326796*x[21,1,9,45]+0.04*x[21,1,10,43]+-
 1166.52647586068*x[21,1,10,44]+-
 1312.06618923378*x[21,1,10,45]+0.0229*x[21,2,2,32]+14.8309464753675*x[21,2,2,33]+-
 228881.4925*x[21,2,2,34]+21.2496172338012*x[21,2,2,35]+0.006466*x[21,2,5,43]+-
 30.8750558176619*x[21,2,5,44]+-64660*x[21,2,5,45]+0.006466*x[21,2,6,43]+-
 84.1830240842502*x[21,2,6,44]+-94.6925045666882*x[21,2,6,45]+0.0036*x[21,2,8,43]+-
 59.0788817742593*x[21,2,8,44]+-
 64.3001099560661*x[21,2,8,45]+0.006466*x[21,2,10,43]+-
 188.56900482288*x[21,2,10,44]+-
 212.09549948964*x[21,2,10,45]+1.1706*x[21,3,2,32]+758.126897120751*x[21,3,2,33]+-
 11699942.145*x[21,3,2,34]+1086.23589230951*x[21,3,2,35]+0.0798*x[21,3,5,21]+4094.98
 198282211*x[21,3,5,22]+-793044.42*x[21,3,5,23]+-
 793044.42*x[21,3,5,24]+4503.09767762887*x[21,3,5,25]+0.0798*x[21,3,6,43]+-
 1038.94298204812*x[21,3,6,44]+-1168.64550949918*x[21,3,6,45]+0.0006*x[21,3,9,43]+-
 18.0180549675844*x[21,3,9,44]+-20.2521486698039*x[21,3,9,45]+0.0798*x[21,3,10,43]+-
 2327.22031934207*x[21,3,10,44]+-
 2617.57204752138*x[21,3,10,45]+0.3902*x[21,4,2,32]+252.708965706917*x[21,4,2,33]+-
 3899980.715*x[21,4,2,34]+362.078630769836*x[21,4,2,35]+0.025246*x[21,4,5,43]+-
 120.5492822723*x[21,4,5,44]+-252460*x[21,4,5,45]+0.025246*x[21,4,6,43]+-
 328.686146927154*x[21,4,6,44]+-369.719605674391*x[21,4,6,45]+0.0001*x[21,4,9,43]+-
 3.00300916126406*x[21,4,9,44]+-
 3.37535811163398*x[21,4,9,45]+0.0053*x[21,4,10,10]+198.594976426525*x[21,4,10,11]+
 -52616.015*x[21,4,10,12]+-52616.015*x[21,4,10,13]+-
 52616.015*x[21,4,10,14]+210.136229926525*x[21,4,10,15]+0.019946*x[21,4,10,43]+-
 581.688427187931*x[21,4,10,44]+-
 654.261805261422*x[21,4,10,45]+0.0369*x[21,5,2,32]+23.8979006524481*x[21,5,2,33]+-
 368809.0425*x[21,5,2,34]+34.2406496038107*x[21,5,2,35]+0.0099*x[21,5,5,21]+508.0240
 80575675*x[21,5,5,22]+-98385.21*x[21,5,5,23]+-

98385.21*x[21,5,5,24]+558.654975044184*x[21,5,5,25]+0.008*x[21,5,5,32]+178.23143973
 522*x[21,5,5,33]+205.400116526246*x[21,5,5,34]+203.038363672067*x[21,5,5,35]+0.001
 9*x[21,5,6,10]+220.147644716686*x[21,5,6,11]+-18744.355*x[21,5,6,12]+-
 18744.355*x[21,5,6,13]+-
 18744.355*x[21,5,6,14]+227.820106916686*x[21,5,6,15]+0.0038*x[21,5,6,21]+126.988590
 145548*x[21,5,6,22]+-37803.35*x[21,5,6,23]+-
 37803.35*x[21,5,6,24]+141.000213833372*x[21,5,6,25]+0.0122*x[21,5,6,43]+-
 158.835894498585*x[21,5,6,44]+-178.665102956016*x[21,5,6,45]+0.0001*x[21,5,9,43]+-
 3.00300916126406*x[21,5,9,44]+-3.37535811163398*x[21,5,9,45]+0.0179*x[21,5,10,43]+-
 522.020597947657*x[21,5,10,44]+-
 587.149619682114*x[21,5,10,45]+0.0267*x[21,6,2,32]+17.2919768948608*x[21,6,2,33]+-
 266861.8275*x[21,6,2,34]+24.7757545913752*x[21,6,2,35]+0.0068*x[21,6,5,32]+151.4967
 23774937*x[21,6,5,33]+174.590099047309*x[21,6,5,34]+172.582609121257*x[21,6,5,35]+
 0.0014*x[21,6,6,21]+46.7852700536231*x[21,6,6,22]+-13927.55*x[21,6,6,23]+-
 13927.55*x[21,6,6,24]+51.9474472017687*x[21,6,6,25]+0.0054*x[21,6,6,43]+-
 70.3044123190459*x[21,6,6,44]+-79.0812750788921*x[21,6,6,45]+0.0001*x[21,6,9,43]+-
 3.00300916126406*x[21,6,9,44]+-3.37535811163398*x[21,6,9,45]+0.0068*x[21,6,10,43]+-
 198.309500896316*x[21,6,10,44]+-
 223.051252169742*x[21,6,10,45]+0.0358*x[22,1,7,43]+-606.976281778025*x[22,1,7,44]+-
 662.646047556629*x[22,1,7,45]+0.0048*x[22,1,8,43]+-78.7718423656791*x[22,1,8,44]+-
 85.7334799414214*x[22,1,8,45]+0.597*x[22,1,14,43]+-3006.37381587333*x[22,1,14,44]+-
 6203.5696499677*x[22,1,14,45]+0.1295*x[23,1,2,32]+83.8693261379953*x[23,1,2,33]+-
 1294329.8375*x[23,1,2,34]+120.167049422588*x[23,1,2,35]+0.0001*x[23,1,9,43]+-
 3.00300916126406*x[23,1,9,44]+-
 3.37535811163398*x[23,1,9,45]+0.003*x[23,1,12,21]+280.978307519782*x[23,1,12,22]+-
 29658.45*x[23,1,12,23]+-
 29658.45*x[23,1,12,24]+300.368030067014*x[23,1,12,25]+0.01503034*x[23,1,12,43]+-
 189.053474221926*x[23,1,12,44]+-
 206.326336654187*x[23,1,12,45]+0.8215*x[23,2,2,32]+532.035918319406*x[23,2,2,33]+-
 8210748.7375*x[23,2,2,34]+762.295220854485*x[23,2,2,35]+0.0474597*x[23,2,5,43]+-
 226.619376212417*x[23,2,5,44]+-474597*x[23,2,5,45]+0.0474597*x[23,2,6,43]+-
 617.893762470042*x[23,2,6,44]+-695.03214645587*x[23,2,6,45]+0.0005*x[23,2,9,43]+-
 15.0150458063203*x[23,2,9,44]+-
 16.8767905581699*x[23,2,9,45]+0.0474597*x[23,2,10,43]+-
 1384.07491466013*x[23,2,10,44]+-
 1556.75669302945*x[23,2,10,45]+0.0099*x[23,3,2,32]+6.41163188236412*x[23,3,2,33]+-
 98948.7675*x[23,3,2,34]+9.18651574736385*x[23,3,2,35]+0.004042*x[23,3,5,43]+-
 19.3004911251144*x[23,3,5,44]+-40420*x[23,3,5,45]+0.004042*x[23,3,10,43]+-
 117.877500385722*x[23,3,10,44]+-
 132.584288422073*x[23,3,10,45]+0.004042*x[23,3,12,43]+-
 50.8407755782653*x[23,3,12,44]+-
 55.4858408230434*x[23,3,12,45]+0.0099*x[23,4,2,32]+6.41163188236412*x[23,4,2,33]+-
 98948.7675*x[23,4,2,34]+9.18651574736385*x[23,4,2,35]+0.004042*x[23,4,5,43]+-
 19.3004911251144*x[23,4,5,44]+-40420*x[23,4,5,45]+0.004042*x[23,4,12,43]+-
 50.8407755782653*x[23,4,12,44]+-
 55.4858408230434*x[23,4,12,45]+0.0885*x[23,5,2,32]+57.3161031908308*x[23,5,2,33]+-

884542.0125*x[23,5,2,34]+82.1218831961314*x[23,5,2,35]+0.01383232*x[23,5,5,43]+-
 66.0491265214602*x[23,5,5,44]+-138323.2*x[23,5,5,45]+0.01383232*x[23,5,6,43]+-
 180.087616409071*x[23,5,6,44]+-
 202.569907944308*x[23,5,6,45]+0.01383232*x[23,5,10,43]+-
 403.394187564432*x[23,5,10,44]+-
 453.722984766553*x[23,5,10,45]+0.005*x[24,1,2,10]+174.517841917861*x[24,1,2,11]+-
 49798.175*x[24,1,2,12]+-49798.175*x[24,1,2,13]+-
 49798.175*x[24,1,2,14]+180.589654417861*x[24,1,2,15]+0.1402*x[24,1,2,32]+90.7990696
 876212*x[24,1,2,33]+-
 1401274.465*x[24,1,2,34]+130.095909876809*x[24,1,2,35]+0.01911666*x[24,1,5,43]+-
 91.2817730509226*x[24,1,5,44]+-191166.6*x[24,1,5,45]+0.01911666*x[24,1,6,43]+-
 248.886212370928*x[24,1,6,44]+-
 279.957379268454*x[24,1,6,45]+0.01911666*x[24,1,10,43]+-
 557.502250500673*x[24,1,10,44]+-
 627.058080926944*x[24,1,10,45]+0.005*x[24,2,2,21]+86.2903775708823*x[24,2,2,22]+-
 49886.15*x[24,2,2,23]+-
 49886.15*x[24,2,2,24]+92.6146544178605*x[24,2,2,25]+0.2622*x[24,2,2,32]+169.8110989
 45038*x[24,2,2,33]+-
 2620643.115*x[24,2,2,34]+243.303477672606*x[24,2,2,35]+0.0078*x[24,2,5,10]+738.8214
 63580266*x[24,2,5,11]+-77192.7*x[24,2,5,12]+-77192.7*x[24,2,5,13]+-
 77192.7*x[24,2,5,14]+763.072404580266*x[24,2,5,15]+0.01467036*x[24,2,5,43]+-
 70.0507553147533*x[24,2,5,44]+-146703.6*x[24,2,5,45]+0.02247036*x[24,2,6,43]+-
 292.54915822174*x[24,2,6,44]+-329.071244496618*x[24,2,6,45]+0.0003*x[24,2,9,43]+-
 9.00902748379219*x[24,2,9,44]+-
 10.1260743349019*x[24,2,9,45]+0.02247036*x[24,2,10,43]+-
 655.306746553023*x[24,2,10,44]+-
 737.064990397776*x[24,2,10,45]+0.3047*x[24,3,1,54]+-
 864.391124533792*x[24,3,2,55]+0.02191668*x[24,3,4,43]+-
 92.0337321831143*x[24,3,4,44]+-
 106.111787626175*x[24,3,4,45]+0.02191668*x[24,3,6,43]+-
 285.340612478627*x[24,3,6,44]+-
 320.962777758528*x[24,3,6,45]+0.02191668*x[24,3,7,43]+-
 371.589523332927*x[24,3,7,44]+-405.670429540877*x[24,3,7,45]+0.0024*x[24,3,8,43]+-
 39.3859211828396*x[24,3,8,44]+-42.8667399707107*x[24,3,8,45]+0.0004*x[24,3,9,43]+-
 12.0120366450563*x[24,3,9,44]+-
 13.5014324465359*x[24,3,9,45]+0.02191668*x[24,3,10,43]+-
 639.159687074159*x[24,3,10,44]+-
 718.903370206402*x[24,3,10,45]+0.02191668*x[24,3,12,43]+-
 275.670709871513*x[24,3,12,44]+-
 300.857352263627*x[24,3,12,45]+0.0422*x[24,4,1,32]+115.002355443439*x[24,4,1,33]+-
 421694.261*x[24,4,1,34]+132.181866824695*x[24,4,1,35]+0.0118*x[24,4,4,43]+-
 49.5512112126813*x[24,4,4,44]+-57.1308744750057*x[24,4,4,45]+0.0118*x[24,4,6,43]+-
 153.62816025273*x[24,4,6,44]+-
 172.807230727949*x[24,4,6,45]+0.0093*x[24,5,1,32]+25.3441209863503*x[24,5,1,33]+-
 92932.6215*x[24,5,1,34]+29.1301270490441*x[24,5,1,35]+0.0053*x[24,5,4,43]+-
 22.2560524938314*x[24,5,4,44]+-

25.6604775184347*x[24,5,4,45]+0.0012*x[24,5,6,21]+40.1016600459626*x[24,5,6,22]+-
 11937.9*x[24,5,6,23]+-
 11937.9*x[24,5,6,24]+44.5263833158018*x[24,5,6,25]+0.0041*x[24,5,6,43]+-
 53.3792760200164*x[24,5,6,44]+-
 60.0431903376774*x[24,5,6,45]+0.01*x[25,1,2,21]+172.580755141765*x[25,1,2,22]+-
 99772.3*x[25,1,2,23]+-
 99772.3*x[25,1,2,24]+185.229308835721*x[25,1,2,25]+1.5348*x[25,1,2,43]+-
 5492.66885792439*x[25,1,2,44]+-6518.40167989353*x[25,1,2,45]+6.55219783333333E-
 02*x[25,1,4,43]+-275.14350741252*x[25,1,4,44]+-
 317.231179619975*x[25,1,4,45]+6.55219783333333E-02*x[25,1,6,43]+-
 853.052626056715*x[25,1,6,44]+-959.548443016953*x[25,1,6,45]+0.001*x[25,1,9,43]+-
 30.0300916126406*x[25,1,9,44]+-33.7535811163398*x[25,1,9,45]+6.55219783333333E-
 02*x[25,1,10,43]+-1910.82806191509*x[25,1,10,44]+-
 2149.22931057187*x[25,1,10,45]+0.0276*x[25,2,2,43]+-98.7735603848795*x[25,2,2,44]+-
 117.21910761341*x[25,2,2,45]+0.0082*x[25,2,4,10]+1030.4760048394*x[25,2,4,11]+-
 80896.69*x[25,2,4,12]+-80896.69*x[25,2,4,13]+-
 80896.69*x[25,2,4,14]+1063.6088838394*x[25,2,4,15]+0.0082*x[25,2,6,43]+-
 106.758552040033*x[25,2,6,44]+-120.086380675355*x[25,2,6,45]+0.0001*x[25,2,9,43]+-
 3.00300916126406*x[25,2,9,44]+-3.37535811163398*x[25,2,9,45]+0.0082*x[25,2,10,43]+-
 239.13792755144*x[25,2,10,44]+-268.973568792924*x[25,2,10,45]+0.0123*x[25,3,2,43]+-
 44.0186519106528*x[25,3,2,44]+-
 52.2389501320631*x[25,3,2,45]+0.0044*x[25,3,4,10]+552.938344060167*x[25,3,4,11]+-
 43407.98*x[25,3,4,12]+-43407.98*x[25,3,4,13]+-
 43407.98*x[25,3,4,14]+570.716962060167*x[25,3,4,15]+0.0044*x[25,3,6,43]+-
 57.2850767044078*x[25,3,6,44]+-64.4365945087269*x[25,3,6,45]+0.0001*x[25,3,9,43]+-
 3.00300916126406*x[25,3,9,44]+-3.37535811163398*x[25,3,9,45]+0.0044*x[25,3,10,43]+-
 128.317912344675*x[25,3,10,44]+-
 144.327280815715*x[25,3,10,45]+0.6572*x[25,4,1,43]+-2344.04594445352*x[25,4,1,44]+-
 2702.88502186755*x[25,4,1,45]+0.0001*x[25,4,4,10]+12.566780546822*x[25,4,4,11]+-
 986.545*x[25,4,4,12]+-986.545*x[25,4,4,13]+-
 986.545*x[25,4,4,14]+12.970840046822*x[25,4,4,15]+4.51252666666667E-
 02*x[25,4,4,43]+-189.492510138014*x[25,4,4,44]+-
 218.478469964787*x[25,4,4,45]+0.0006*x[25,4,9,43]+-18.0180549675844*x[25,4,9,44]+-
 20.2521486698039*x[25,4,9,45]+4.52252666666667E-02*x[25,4,10,43]+-
 1318.91177361316*x[25,4,10,44]+-1483.46358231037*x[25,4,10,45]+4.52252666666667E-
 02*x[25,4,11,43]+-150.579778749931*x[25,4,11,44]+-
 166.798184134261*x[25,4,11,45]+1.1252*x[25,5,2,54]+-
 2859.85260848753*x[25,5,3,55]+4.99810766666667E-02*x[25,5,4,43]+-
 209.883295470103*x[25,5,4,44]+-241.988357387201*x[25,5,4,45]+0.0008*x[25,5,9,43]+-
 24.0240732901125*x[25,5,9,44]+-27.0028648930719*x[25,5,9,45]+4.99810766666667E-
 02*x[25,5,10,43]+-1457.60623059223*x[25,5,10,44]+-
 1639.46201989586*x[25,5,10,45]+4.99810766666667E-02*x[25,5,11,43]+-
 166.414485106777*x[25,5,11,44]+-184.33838965553*x[25,5,11,45]+0.006*x[25,6,1,43]+-
 21.4002977278167*x[25,6,1,44]+-24.676369645778*x[25,6,1,45]+4.07790333333333E-
 03*x[25,6,12,43]+-51.2923721424691*x[25,6,12,44]+-
 55.9786974876525*x[25,6,12,45]+0.0091*x[25,7,1,43]+-32.4571182205219*x[25,7,1,44]+-

37.4258272960967*x[25,7,1,45]+0.00442766*x[25,7,12,43]+-
 55.6916547246072*x[25,7,12,44]+-
 60.7799203311619*x[25,7,12,45]+0.0224*x[25,8,2,32]+14.5071266833289*x[25,8,2,33]+-
 223884.08*x[25,8,2,34]+20.7856517920152*x[25,8,2,35]+0.0064*x[25,8,5,10]+606.212482
 937654*x[25,8,5,11]+-63337.6*x[25,8,5,12]+-63337.6*x[25,8,5,13]+-
 63337.6*x[25,8,5,14]+626.110690937654*x[25,8,5,15]+3.86346666666667E-
 04*x[25,8,5,43]+-1.84479970589251*x[25,8,5,44]+-
 3863.46666666667*x[25,8,5,45]+0.0032*x[25,8,6,21]+106.937760122567*x[25,8,6,22]+-
 31834.4*x[25,8,6,23]+-
 31834.4*x[25,8,6,24]+118.737022175471*x[25,8,6,25]+3.58634666666667E-
 03*x[25,8,6,43]+-46.6918508837721*x[25,8,6,44]+-
 52.5209013472101*x[25,8,6,45]+0.0048*x[25,8,8,43]+-78.7718423656791*x[25,8,8,44]+-
 85.7334799414214*x[25,8,8,45]+0.0001*x[25,8,9,43]+-3.00300916126406*x[25,8,9,44]+-
 3.37535811163398*x[25,8,9,45]+6.78634666666667E-03*x[25,8,10,43]+-
 197.911326525889*x[25,8,10,44]+-
 222.603400243817*x[25,8,10,45]+0.0242*x[26,1,1,32]+65.949218050503*x[26,1,1,33]+-
 241824.671*x[26,1,1,34]+75.8009757620287*x[26,1,1,35]+0.3289*x[26,1,1,43]+-
 1173.09298711315*x[26,1,1,44]+-1352.67632941606*x[26,1,1,45]+2.38817666666667E-
 02*x[26,1,4,43]+-100.285632562032*x[26,1,4,44]+-
 115.625950311416*x[26,1,4,45]+2.38817666666667E-02*x[26,1,6,43]+-
 310.924735303811*x[26,1,6,44]+-349.740844284552*x[26,1,6,45]+0.0024*x[26,1,8,43]+-
 39.3859211828396*x[26,1,8,44]+-42.8667399707107*x[26,1,8,45]+2.38817666666667E-
 02*x[26,1,10,43]+-696.467827674846*x[26,1,10,44]+-
 783.361464562588*x[26,1,10,45]+0.0026*x[26,2,1,32]+7.08545317897967*x[26,2,1,33]+-
 25981.163*x[26,2,1,34]+8.14390648682953*x[26,2,1,35]+0.0381*x[26,2,1,43]+-
 135.891890571636*x[26,2,1,44]+-
 156.69494725069*x[26,2,1,45]+0.008015105*x[26,2,4,43]+-
 33.6574712497304*x[26,2,4,44]+-
 38.8059286151687*x[26,2,4,45]+0.008015105*x[26,2,6,43]+-
 104.351341981564*x[26,2,6,44]+-117.378652461334*x[26,2,6,45]+0.0024*x[26,2,8,43]+-
 39.3859211828396*x[26,2,8,44]+-
 42.8667399707107*x[26,2,8,45]+0.008015105*x[26,2,10,43]+-
 233.745804732584*x[26,2,10,44]+-
 262.908706841464*x[26,2,10,45]+1.6079*x[27,1,2,32]+1041.3396872377*x[27,1,2,33]+-
 16070679.1175*x[27,1,2,34]+1492.02006769559*x[27,1,2,35]+0.0188*x[27,1,5,10]+1780.7
 4916862936*x[27,1,5,11]+-186054.2*x[27,1,5,12]+-186054.2*x[27,1,5,13]+-
 186054.2*x[27,1,5,14]+1839.20015462936*x[27,1,5,15]+0.0548*x[27,1,5,32]+1220.885362
 18626*x[27,1,5,33]+1406.99079820478*x[27,1,5,34]+1390.81279115366*x[27,1,5,35]+4.3
 40000000000003E-04*x[27,1,5,43]+-2.07234367845119*x[27,1,5,44]+-
 4340.00000000003*x[27,1,5,45]+0.056*x[27,1,6,21]+1871.41080214492*x[27,1,6,22]+-
 557102*x[27,1,6,23]+-
 557102*x[27,1,6,24]+2077.89788807075*x[27,1,6,25]+0.018034*x[27,1,6,43]+-
 234.790698474384*x[27,1,6,44]+-
 264.102169402359*x[27,1,6,45]+0.074034*x[27,1,7,43]+-
 1255.22016885906*x[27,1,7,44]+-1370.34462248066*x[27,1,7,45]+0.0048*x[27,1,8,43]+-
 78.7718423656791*x[27,1,8,44]+-

85.7334799414214*x[27,1,8,45]+0.074034*x[27,1,10,43]+-
 2159.06552784675*x[27,1,10,44]+-
 2428.43770634333*x[27,1,10,45]+0.0207*x[27,1,12,21]+1938.75032188649*x[27,1,12,22]
 +-204643.305*x[27,1,12,23]+-
 204643.305*x[27,1,12,24]+2072.5394074624*x[27,1,12,25]+0.053334*x[27,1,12,43]+-
 670.841643911728*x[27,1,12,44]+-
 732.133061468629*x[27,1,12,45]+0.5809*x[27,2,2,32]+376.213834390436*x[27,2,2,33]+-
 5805993.8425*x[27,2,2,34]+539.035050267036*x[27,2,2,35]+0.0225*x[27,2,5,32]+501.275
 924255305*x[27,2,5,33]+577.687827730066*x[27,2,5,34]+571.04539782769*x[27,2,5,35]+
 0.011648*x[27,2,5,43]+-55.6190303377863*x[27,2,5,44]+-
 116480*x[27,2,5,45]+0.034148*x[27,2,6,43]+-444.584272568663*x[27,2,6,44]+-
 500.086552110002*x[27,2,6,45]+0.034148*x[27,2,7,43]+-
 578.967208663575*x[27,2,7,44]+-632.068079105133*x[27,2,7,45]+0.0048*x[27,2,8,43]+-
 78.7718423656791*x[27,2,8,44]+-
 85.7334799414214*x[27,2,8,45]+0.034148*x[27,2,10,43]+-
 995.863652442267*x[27,2,10,44]+-
 1120.11090574887*x[27,2,10,45]+0.0126*x[27,2,12,21]+1180.10889158308*x[27,2,12,22]
 +-124565.49*x[27,2,12,23]+-
 124565.49*x[27,2,12,24]+1261.54572628146*x[27,2,12,25]+0.021548*x[27,2,12,43]+-
 271.033407263845*x[27,2,12,44]+-
 295.796362705329*x[27,2,12,45]+0.6991*x[27,3,2,32]+452.764833228359*x[27,3,2,33]+-
 6987382.1575*x[27,3,2,34]+648.716480705259*x[27,3,2,35]+0.012*x[27,3,5,10]+1136.648
 4055081*x[27,3,5,11]+-118758*x[27,3,5,12]+-118758*x[27,3,5,13]+-
 118758*x[27,3,5,14]+1173.9575455081*x[27,3,5,15]+0.0265*x[27,3,5,32]+590.391644122
 915*x[27,3,5,33]+680.387885993189*x[27,3,5,34]+672.564579663723*x[27,3,5,35]+0.024
 *x[27,3,6,21]+802.033200919253*x[27,3,6,22]+-238758*x[27,3,6,23]+-
 238758*x[27,3,6,24]+890.527666316035*x[27,3,6,25]+0.0145*x[27,3,6,43]+-
 188.780366412253*x[27,3,6,44]+-212.347868267396*x[27,3,6,45]+0.0385*x[27,3,7,43]+-
 652.753822582513*x[27,3,7,44]+-712.622146115369*x[27,3,7,45]+0.0048*x[27,3,8,43]+-
 78.7718423656791*x[27,3,8,44]+-85.7334799414214*x[27,3,8,45]+0.0385*x[27,3,10,43]+-
 1122.78173301591*x[27,3,10,44]+-
 1262.86370713751*x[27,3,10,45]+0.0138*x[27,3,12,21]+1292.500214591*x[27,3,12,22]+-
 136428.87*x[27,3,12,23]+-
 136428.87*x[27,3,12,24]+1381.69293830826*x[27,3,12,25]+0.0247*x[27,3,12,43]+-
 310.679652840958*x[27,3,12,44]+-
 339.064885781586*x[27,3,12,45]+0.016*x[27,4,2,32]+10.3622333452349*x[27,4,2,33]+-
 159917.2*x[27,4,2,34]+14.8468941371537*x[27,4,2,35]+0.0048*x[27,4,8,43]+-
 78.7718423656791*x[27,4,8,44]+-
 85.7334799414214*x[27,4,8,45]+0.0051668*x[27,4,12,43]+-
 64.9886489999459*x[27,4,12,44]+-
 70.926334083251*x[27,4,12,45]+0.0535*x[27,5,2,32]+34.6487177481293*x[27,5,2,33]+-
 534723.1375*x[27,5,2,34]+49.6443022711077*x[27,5,2,35]+0.0096*x[27,5,5,43]+-
 45.8398601685052*x[27,5,5,44]+-
 96000*x[27,5,5,45]+0.0096*x[27,5,12,21]+899.130584063301*x[27,5,12,22]+-
 94907.04*x[27,5,12,23]+-
 94907.04*x[27,5,12,24]+961.177696214444*x[27,5,12,25]+0.0227*x[27,6,2,32]+14.701418

5585521*x[27,6,2,33]+-
 226882.5275*x[27,6,2,34]+21.0640310570868*x[27,6,2,35]+0.012725*x[27,6,5,43]+-
 60.7616896504405*x[27,6,5,44]+-
 127250*x[27,6,5,45]+0.001*x[27,6,6,21]+33.4180500383022*x[27,6,6,22]+-
 9948.25*x[27,6,6,23]+-
 9948.25*x[27,6,6,24]+37.1053194298348*x[27,6,6,25]+0.011725*x[27,6,6,43]+-
 152.651710081632*x[27,6,6,44]+-
 171.708879685187*x[27,6,6,45]+0.0001*x[27,6,9,32]+40.0844954306164*x[27,6,9,33]+48.
 7019908387359*x[27,6,9,34]+48.374641888366*x[27,6,9,35]+0.012725*x[27,6,10,43]+-
 371.10123513318*x[27,6,10,44]+-
 417.401056449995*x[27,6,10,45]+0.0241*x[27,7,2,32]+15.6081139762601*x[27,7,2,33]+-
 240875.2825*x[27,7,2,34]+22.3631342940878*x[27,7,2,35]+0.012725*x[27,7,5,43]+-
 60.7616896504405*x[27,7,5,44]+-
 127250*x[27,7,5,45]+0.0055*x[27,7,6,21]+183.799275210662*x[27,7,6,22]+-
 54715.375*x[27,7,6,23]+-
 54715.375*x[27,7,6,24]+204.079256864091*x[27,7,6,25]+0.007225*x[27,7,6,43]+-
 94.0646998157605*x[27,7,6,44]+-105.807817119444*x[27,7,6,45]+0.0001*x[27,7,9,43]+-
 3.00300916126406*x[27,7,9,44]+-
 3.37535811163398*x[27,7,9,45]+0.012725*x[27,7,10,43]+-
 371.10123513318*x[27,7,10,44]+-
 417.401056449995*x[27,7,10,45]+0.0443*x[27,8,2,32]+28.6904335746192*x[27,8,2,33]+-
 442770.7475*x[27,8,2,34]+41.1073381422443*x[27,8,2,35]+0.00806666*x[27,8,5,43]+-
 38.5181840027994*x[27,8,5,44]+-
 80666.6*x[27,8,5,45]+0.004*x[27,8,6,21]+133.672200153209*x[27,8,6,22]+-
 39793*x[27,8,6,23]+-
 39793*x[27,8,6,24]+148.421277719339*x[27,8,6,25]+0.00406666*x[27,8,6,43]+-
 52.9452113706243*x[27,8,6,44]+-59.5549366874681*x[27,8,6,45]+0.0001*x[27,8,9,43]+-
 3.00300916126406*x[27,8,9,44]+-
 3.37535811163398*x[27,8,9,45]+0.00806666*x[27,8,12,43]+-
 101.463446493362*x[27,8,12,44]+-
 110.733649859874*x[27,8,12,45]+2.3197*x[27,9,2,32]+1502.32954318384*x[27,9,2,33]+-
 23184995.5525*x[27,9,2,34]+2152.52127062221*x[27,9,2,35]+0.0644*x[27,9,5,43]+-
 307.509061963722*x[27,9,5,44]+-
 644000*x[27,9,5,45]+0.0644*x[27,9,6,21]+2152.12242246666*x[27,9,6,22]+-
 640667.3*x[27,9,6,23]+-
 640667.3*x[27,9,6,24]+2389.58257128136*x[27,9,6,25]+0.0015*x[27,9,9,43]+-
 45.0451374189609*x[27,9,9,44]+-50.6303716745097*x[27,9,9,45]+0.0644*x[27,9,10,43]+-
 1878.1076261357*x[27,9,10,44]+-
 2112.42656466638*x[27,9,10,45]+0.028*x[27,10,2,32]+18.1339083541611*x[27,10,2,33]+-
 -279855.1*x[27,10,2,34]+25.982064740019*x[27,10,2,35]+0.0067959*x[27,10,5,43]+-
 32.4503235124109*x[27,10,5,44]+-67959*x[27,10,5,45]+0.0067959*x[27,10,6,43]+-
 88.4781029035193*x[27,10,6,44]+-
 99.5237846867857*x[27,10,6,45]+0.0001*x[27,10,9,43]+-
 3.00300916126406*x[27,10,9,44]+-
 3.37535811163398*x[27,10,9,45]+0.0067959*x[27,10,10,43]+-
 198.189931932541*x[27,10,10,44]+-

222.916765385345*x[27,10,10,45]+0.0168*x[27,11,2,32]+10.8803450124967*x[27,11,2,33]
]+-167913.06*x[27,11,2,34]+15.5892388440114*x[27,11,2,35]+0.0065668*x[27,11,5,43]+-
 31.3563743494312*x[27,11,5,44]+-65668*x[27,11,5,45]+0.0065668*x[27,11,6,43]+-
 85.4953731142057*x[27,11,6,44]+-
 96.1686883681609*x[27,11,6,45]+0.0001*x[27,11,9,43]+-
 3.00300916126406*x[27,11,9,44]+-
 3.37535811163398*x[27,11,9,45]+0.0065668*x[27,11,10,43]+-
 191.508651542049*x[27,11,10,44]+-
 215.401906286509*x[27,11,10,45]+0.0676*x[27,12,1,32]+184.221782653471*x[27,12,1,33]
]+-
 675510.238*x[27,12,1,34]+211.741568657568*x[27,12,1,35]+0.0103668*x[27,12,4,43]+-
 43.5328386779343*x[27,12,4,44]+-
 50.1918940260584*x[27,12,4,45]+0.0001*x[27,12,9,43]+-
 3.00300916126406*x[27,12,9,44]+-
 3.37535811163398*x[27,12,9,45]+0.0103668*x[27,12,12,43]+-
 130.394891703306*x[27,12,12,44]+-
 142.308415300427*x[27,12,12,45]+0.2551*x[28,1,2,43]+-
 912.939683122564*x[28,1,2,44]+-1083.42733160076*x[28,1,2,45]+0.01995*x[28,1,4,43]+-
 83.7751409909316*x[28,1,4,44]+-
 96.5899106590138*x[28,1,4,45]+0.01995*x[28,1,12,43]+-
 250.933565756159*x[28,1,12,44]+-
 273.860100054358*x[28,1,12,45]+0.0012*x[28,2,1,10]+42.0454910708444*x[28,2,1,11]+-
 11951.562*x[28,2,1,12]+-11951.562*x[28,2,1,13]+-
 11951.562*x[28,2,1,14]+43.5027260708444*x[28,2,1,15]+3.1436*x[28,2,1,32]+8566.85792
 824634*x[28,2,1,33]+-
 31413224.618*x[28,2,1,34]+9846.60939692204*x[28,2,1,35]+0.05252*x[28,2,4,43]+-
 220.544882448307*x[28,2,4,44]+-
 254.280807409093*x[28,2,4,45]+0.0176*x[28,2,6,21]+588.157680674119*x[28,2,6,22]+-
 175089.2*x[28,2,6,23]+-
 175089.2*x[28,2,6,24]+653.053621965092*x[28,2,6,25]+0.03492*x[28,2,6,43]+-
 454.635199663164*x[28,2,6,44]+-
 511.392245510169*x[28,2,6,45]+0.05252*x[28,2,12,43]+-
 660.603051303932*x[28,2,12,44]+-720.959020293478*x[28,2,12,45];

subject to constraint1

:1160.02075*x[1,2,6,45]+35*x[1,2,9,45]+300*x[1,2,12,45]+4*x[1,3,6,34]+60*x[1,3,6,35]+3
 59.1563333333333*x[1,3,6,45]+432*x[1,3,8,45]+375*x[1,3,10,45]+170.62035*x[1,3,11,45]+
 2359.5*x[2,1,1,45]+1479.7495*x[2,1,7,45]+288*x[2,1,8,45]+98.6499666666667*x[2,1,12,3
 4]+1479.7495*x[2,1,12,35]+17209.5*x[2,2,1,45]+1412*x[2,2,4,45]+3530*x[2,2,6,45]+3530
 *x[2,2,7,45]+288*x[2,2,8,45]+191*x[2,2,12,34]+2865*x[2,2,12,35]+665*x[2,2,12,45]+8731
 8*x[2,3,1,45]+3438*x[2,3,4,45]+8595*x[2,3,6,45]+573*x[2,3,12,34]+8595*x[2,3,12,35]+14
 30*x[2,4,1,45]+3465.6*x[2,4,4,45]+8664*x[2,4,12,45]+4800*x[3,1,4,12]+4800*x[3,1,4,13]
 +4800*x[3,1,4,14]+288*x[3,1,4,15]+2000*x[3,1,4,23]+2000*x[3,1,4,24]+120*x[3,1,4,25]+7
 0*x[3,1,9,45]+6826.53925*x[3,1,12,45]+54516*x[4,1,1,45]+1200*x[4,1,4,23]+1200*x[4,1,
 4,24]+72*x[4,1,4,25]+2300*x[4,1,4,45]+100*x[4,1,9,12]+100*x[4,1,9,13]+100*x[4,1,9,14]
 +35*x[4,1,9,15]+100*x[4,1,9,23]+100*x[4,1,9,24]+35*x[4,1,9,25]+5930*x[4,1,12,45]+300*

$x[5,1,1,12]+300*x[5,1,1,13]+300*x[5,1,1,14]+16.5*x[5,1,1,15]+368175.5*x[5,1,1,45]+3000$
 $*x[5,1,4,23]+3000*x[5,1,4,24]+180*x[5,1,4,25]+1865.0148*x[5,1,4,45]+210*x[5,1,9,45]+5$
 $112.537*x[5,1,12,45]+56*x[5,2,2,45]+500*x[5,2,5,12]+500*x[5,2,5,13]+500*x[5,2,5,14]+3$
 $7.5*x[5,2,5,15]+3197.668333333333*x[5,2,5,45]+554.65025*x[5,2,6,45]+35*x[5,2,9,45]+18$
 $733*x[6,1,1,45]+1961.28*x[6,1,4,45]+4903.2*x[6,1,12,45]+1468.5*x[6,2,1,45]+942.701*x[$
 $6,2,12,45]+1056*x[6,3,1,45]+5973.618*x[6,3,12,45]+1050.5*x[6,4,1,45]+360.3*x[6,4,4,45]$
 $+900.75*x[6,4,7,45]+576*x[6,4,8,45]+180.15*x[6,4,11,45]+900.75*x[6,4,12,45]+1050.5*x[$
 $6,5,1,45]+359.8496*x[6,5,4,45]+899.624*x[6,5,7,45]+576*x[6,5,8,45]+179.9248*x[6,5,11,4$
 $5]+899.624*x[6,5,12,45]+3781.02*x[6,6,4,45]+9452.55*x[6,6,6,45]+2000*x[6,6,7,23]+200$
 $0*x[6,6,7,24]+300*x[6,6,7,25]+9152.55*x[6,6,7,45]+240*x[6,6,8,45]+9452.55*x[6,6,12,45]$
 $+1500*x[6,6,13,23]+1500*x[6,6,13,24]+63*x[6,6,13,25]+8*x[6,6,13,34]+16.8*x[6,6,13,35]$
 $+45603.6*x[6,6,13,45]+100*x[6,6,15,23]+100*x[6,6,15,24]+6*x[6,6,15,25]+3775.02*x[6,6,$
 $15,45]+56501.5*x[7,1,1,45]+3114.75*x[7,1,4,45]+140*x[7,1,9,45]+7786.875*x[7,1,12,45]+$
 $931300*x[8,1,2,34]+65191*x[8,1,2,35]+300*x[8,1,5,12]+300*x[8,1,5,13]+300*x[8,1,5,14]+$
 $22.5*x[8,1,5,15]+8000*x[8,1,5,23]+8000*x[8,1,5,24]+600*x[8,1,5,25]+20*x[8,1,5,34]+75*$
 $x[8,1,5,35]+29695.8332*x[8,1,5,45]+5849.37498*x[8,1,6,45]+140*x[8,1,9,45]+9748.9583*$
 $x[8,1,10,45]+34000*x[8,2,2,34]+2380*x[8,2,2,35]+498.75*x[8,2,4,45]+100*x[8,2,9,12]+10$
 $0*x[8,2,9,13]+100*x[8,2,9,14]+35*x[8,2,9,15]+1246.875*x[8,2,12,45]+13024*x[9,1,1,45]+$
 $16*x[9,1,4,34]+96*x[9,1,4,35]+1079.52*x[9,1,4,45]+70*x[9,1,9,45]+12000*x[9,1,12,23]+1$
 $2000*x[9,1,12,24]+1800*x[9,1,12,25]+1138.8*x[9,1,12,45]+98494*x[9,2,1,45]+372*x[9,2,$
 $4,34]+2232*x[9,2,4,35]+1246.08*x[9,2,4,45]+8695.2*x[9,2,6,45]+350*x[9,2,9,45]+12000*$
 $x[9,2,12,23]+12000*x[9,2,12,24]+1800*x[9,2,12,25]+6895.2*x[9,2,12,45]+47586*x[10,1,1,$
 $45]+200*x[10,1,4,12]+200*x[10,1,4,13]+200*x[10,1,4,14]+12*x[10,1,4,15]+2372.7179*x[1$
 $0,1,4,45]+5961.79475*x[10,1,6,45]+350*x[10,1,9,45]+350*x[10,1,10,45]+51034.5*x[10,2,1$
 $,45]+100*x[10,2,4,12]+100*x[10,2,4,13]+100*x[10,2,4,14]+6*x[10,2,4,15]+100*x[10,2,4,2$
 $3]+100*x[10,2,4,24]+6*x[10,2,4,25]+2425.53685*x[10,2,4,45]+6093.842125*x[10,2,6,45]+$
 $280*x[10,2,9,45]+10156.4035416667*x[10,2,10,45]+4422*x[10,3,1,45]+1187.7178*x[10,3,$
 $4,45]+2969.2945*x[10,3,6,45]+140*x[10,3,9,45]+4948.82416666667*x[10,3,10,45]+1655.5$
 $*x[10,4,1,45]+200*x[10,4,4,12]+200*x[10,4,4,13]+200*x[10,4,4,14]+12*x[10,4,4,15]+7400$
 $*x[10,4,4,23]+7400*x[10,4,4,24]+444*x[10,4,4,25]+525.1827*x[10,4,4,45]+2188.26125*x[$
 $10,4,10,45]+2051.5*x[10,5,1,45]+8000*x[10,5,4,23]+8000*x[10,5,4,24]+480*x[10,5,4,25]+$
 $2000*x[10,5,10,45]+2062.5*x[10,6,1,45]+8500*x[10,6,4,23]+8500*x[10,6,4,24]+510*x[10,$
 $6,4,25]+83.9216*x[10,6,4,45]+2474.673333333333*x[10,6,10,45]+1199*x[10,7,1,45]+6200*$
 $x[10,7,4,23]+6200*x[10,7,4,24]+372*x[10,7,4,25]+27.5765*x[10,7,4,45]+1664.9020833333$
 $3*x[10,7,10,45]+3283.5*x[10,8,1,45]+9600*x[10,8,4,23]+9600*x[10,8,4,24]+576*x[10,8,4,$
 $25]+2400*x[10,8,10,45]+1655.5*x[10,9,1,45]+7400*x[10,9,4,23]+7400*x[10,9,4,24]+444*$
 $x[10,9,4,25]+70.0276*x[10,9,4,45]+2141.78166666667*x[10,9,10,45]+2051.5*x[10,10,1,45]$
 $+7800*x[10,10,4,23]+7800*x[10,10,4,24]+468*x[10,10,4,25]+1950*x[10,10,10,45]+100*x$
 $[11,1,1,12]+100*x[11,1,1,13]+100*x[11,1,1,14]+5.5*x[11,1,1,15]+66401.5*x[11,1,1,45]+10$
 $0*x[11,1,4,12]+100*x[11,1,4,13]+100*x[11,1,4,14]+6*x[11,1,4,15]+48500*x[11,1,4,23]+48$
 $500*x[11,1,4,24]+2910*x[11,1,4,25]+70.99999999999999*x[11,1,4,45]+140*x[11,1,9,45]+1$
 $2445.833333333333*x[11,1,10,45]+1493.5*x[11,1,11,45]+1200*x[12,1,1,12]+1200*x[12,1,1,$
 $13]+1200*x[12,1,1,14]+66*x[12,1,1,15]+854600*x[12,1,1,34]+47003*x[12,1,1,35]+25000*$
 $x[12,1,4,23]+25000*x[12,1,4,24]+1500*x[12,1,4,25]+804*x[12,1,4,45]+38400*x[12,1,6,23]$
 $+38400*x[12,1,6,24]+5760*x[12,1,6,25]+280*x[12,1,9,45]+9600*x[12,1,10,45]+14168*x[1$
 $3,1,1,45]+1576.32*x[13,1,4,45]+3940.8*x[13,1,6,45]+140*x[13,1,9,45]+6568*x[13,1,10,45]$

]+32747*x[13,2,1,45]+2035.272*x[13,2,4,45]+5088.18*x[13,2,6,45]+100*x[13,2,9,12]+100
 *x[13,2,9,13]+100*x[13,2,9,14]+35*x[13,2,9,15]+350*x[13,2,9,45]+8480.3*x[13,2,10,45]+
 5088.18*x[13,2,12,45]+18848.5*x[13,3,1,45]+1750.08*x[13,3,4,45]+4375.2*x[13,3,6,45]+2
 10*x[13,3,9,45]+7292*x[13,3,10,45]+2524.5*x[13,4,1,45]+1140.48*x[13,4,4,45]+100*x[13
 ,4,6,34]+1500*x[13,4,6,35]+1351.2*x[13,4,6,45]+105*x[13,4,9,45]+4752*x[13,4,10,45]+11
 781*x[13,5,1,45]+1875.96*x[13,5,4,45]+4689.9*x[13,5,6,45]+140*x[13,5,9,45]+4689.9*x[
 13,5,12,45]+29045.5*x[14,1,1,45]+1749.75*x[14,1,4,45]+140*x[14,1,9,45]+7290.625*x[14,
 1,10,45]+324.5*x[14,2,1,45]+256.0272*x[14,2,4,45]+35*x[14,2,9,45]+1066.78*x[14,2,10,4
 5]+9949.5*x[14,3,1,45]+904.8*x[14,3,4,45]+140*x[14,3,9,45]+3770*x[14,3,10,45]+583*x[
 14,4,1,45]+483.4296*x[14,4,4,45]+1208.574*x[14,4,6,45]+35*x[14,4,9,45]+2014.29*x[14,4
 ,10,45]+27978.5*x[14,5,1,45]+1788.0336*x[14,5,4,45]+140*x[14,5,9,45]+7450.14*x[14,5,1
 0,45]+2783*x[14,6,1,45]+590.715*x[14,6,4,45]+1476.7875*x[14,6,6,45]+140*x[14,6,9,45]
 +2461.3125*x[14,6,10,45]+13*x[14,6,12,34]+195*x[14,6,12,35]+1281.7875*x[14,6,12,45]+
 6693.5*x[14,7,1,45]+921.4428*x[14,7,4,45]+105*x[14,7,9,45]+3839.345*x[14,7,10,45]+23
 03.607*x[14,7,12,45]+603200*x[15,1,2,34]+42224*x[15,1,2,35]+48800*x[15,1,5,45]+4000
 0*x[15,1,12,23]+40000*x[15,1,12,24]+6000*x[15,1,12,25]+1320*x[15,1,12,45]+1261100*x
 [15,2,2,34]+88277*x[15,2,2,35]+3400*x[15,2,5,23]+3400*x[15,2,5,24]+255*x[15,2,5,25]+5
 28*x[15,2,5,34]+1980*x[15,2,5,35]+34000*x[15,2,5,45]+9570*x[15,2,6,45]+576*x[15,2,8,
 45]+350*x[15,2,9,45]+40000*x[15,2,12,23]+40000*x[15,2,12,24]+6000*x[15,2,12,25]+357
 0*x[15,2,12,45]+42500*x[15,3,2,34]+2975*x[15,3,2,35]+6200*x[15,3,5,23]+6200*x[15,3,5
 ,24]+465*x[15,3,5,25]+62*x[15,3,5,34]+232.5*x[15,3,5,35]+1395*x[15,3,6,45]+576*x[15,3
 ,8,45]+2325*x[15,3,10,45]+4000*x[15,3,12,23]+4000*x[15,3,12,24]+600*x[15,3,12,25]+79
 5*x[15,3,12,45]+1919900*x[16,1,1,34]+105594.5*x[16,1,1,35]+4176*x[16,1,4,45]+350*x[
 16,1,6,34]+5250*x[16,1,6,35]+5190*x[16,1,6,45]+350*x[16,1,9,45]+17400*x[16,1,10,45]+
 15200*x[16,2,1,34]+836*x[16,2,1,35]+319.9992*x[16,2,4,45]+52*x[16,2,6,34]+780*x[16,2
 ,6,35]+19.99800000000001*x[16,2,6,45]+35*x[16,2,9,45]+1333.33*x[16,2,10,45]+43300*x[
 16,3,1,34]+2381.5*x[16,3,1,35]+525.9984*x[16,3,4,45]+84*x[16,3,6,34]+1260*x[16,3,6,35
]+54.99600000000002*x[16,3,6,45]+35*x[16,3,9,45]+2191.66*x[16,3,10,45]+17300*x[16,4,
 1,34]+951.5*x[16,4,1,35]+335.9993*x[16,4,4,45]+52*x[16,4,6,34]+780*x[16,4,6,35]+59.99
 82499999999*x[16,4,6,45]+35*x[16,4,9,45]+1399.99708333333*x[16,4,10,45]+24000*x[1
 6,5,1,34]+1320*x[16,5,1,35]+456*x[16,5,4,45]+46*x[16,5,6,34]+690*x[16,5,6,35]+450*x[1
 6,5,6,45]+1900*x[16,5,10,45]+1140*x[16,5,12,45]+162400*x[16,6,1,34]+8932*x[16,6,1,35
]+5100*x[16,6,4,12]+5100*x[16,6,4,13]+5100*x[16,6,4,14]+306*x[16,6,4,15]+678*x[16,6,
 4,45]+2460*x[16,6,6,45]+70*x[16,6,9,45]+4100*x[16,6,10,45]+24000*x[16,7,1,34]+1320*
 x[16,7,1,35]+3000*x[16,7,4,12]+3000*x[16,7,4,13]+3000*x[16,7,4,14]+180*x[16,7,4,15]+2
 76*x[16,7,4,45]+46*x[16,7,6,34]+690*x[16,7,6,35]+450*x[16,7,6,45]+35*x[16,7,9,45]+190
 0*x[16,7,10,45]+1140*x[16,7,12,45]+2051.5*x[16,8,1,45]+484*x[16,8,4,45]+35*x[16,8,9,4
 5]+2016.66666666667*x[16,8,10,45]+2051.5*x[16,9,1,45]+484*x[16,9,4,45]+35*x[16,9,9,4
 5]+2016.66666666667*x[16,9,10,45]+60340.5*x[16,10,1,45]+2956.3263*x[16,10,4,45]+73
 90.81575*x[16,10,6,45]+420*x[16,10,9,45]+12318.02625*x[16,10,10,45]+2084.5*x[16,11,
 1,45]+495*x[16,11,4,45]+35*x[16,11,9,45]+2062.5*x[16,11,10,45]+3239.5*x[16,12,1,45]+
 613.9297*x[16,12,4,45]+35*x[16,12,9,45]+2558.04041666667*x[16,12,10,45]+2191*x[17,
 1,2,45]+1110*x[17,1,7,45]+432*x[17,1,8,45]+1110*x[17,1,12,45]+600*x[17,2,1,12]+600*x
 [17,2,1,13]+600*x[17,2,1,14]+33*x[17,2,1,15]+91437.5*x[17,2,1,45]+3694.998*x[17,2,4,4
 5]+9237.495*x[17,2,6,45]+315*x[17,2,9,45]+15395.825*x[17,2,10,45]+16318.5*x[17,3,1,4
 5]+1278.96*x[17,3,4,45]+3197.4*x[17,3,6,45]+70*x[17,3,9,45]+5329*x[17,3,10,45]+14863

$00 \cdot x[18,1,1,34] + 81746.5 \cdot x[18,1,1,35] + 10000 \cdot x[18,1,4,12] + 10000 \cdot x[18,1,4,13] + 10000 \cdot x[18,1,4,14] + 600 \cdot x[18,1,4,15] + 6100 \cdot x[18,1,4,23] + 6100 \cdot x[18,1,4,24] + 366 \cdot x[18,1,4,25] + 2325.9612 \cdot x[18,1,4,45] + 280 \cdot x[18,1,9,45] + 13716.505 \cdot x[18,1,10,45] + 50400 \cdot x[18,2,1,34] + 2772 \cdot x[18,2,1,35] + 1000 \cdot x[18,2,4,12] + 1000 \cdot x[18,2,4,13] + 1000 \cdot x[18,2,4,14] + 60 \cdot x[18,2,4,15] + 423.9911 \cdot x[18,2,4,45] + 35 \cdot x[18,2,9,45] + 2016.62958333333 \cdot x[18,2,10,45] + 1215200 \cdot x[19,1,2,34] + 85064 \cdot x[19,1,2,35] + 300 \cdot x[19,1,5,12] + 300 \cdot x[19,1,5,13] + 300 \cdot x[19,1,5,14] + 22.5 \cdot x[19,1,5,15] + 17600 \cdot x[19,1,5,23] + 17600 \cdot x[19,1,5,24] + 1320 \cdot x[19,1,5,25] + 60 \cdot x[19,1,5,34] + 225 \cdot x[19,1,5,35] + 45333.3333333333 \cdot x[19,1,5,45] + 384 \cdot x[19,1,6,34] + 5760 \cdot x[19,1,6,35] + 4175 \cdot x[19,1,6,45] + 9935 \cdot x[19,1,7,45] + 432 \cdot x[19,1,8,45] + 16558.3333333333 \cdot x[19,1,10,45] + 9935 \cdot x[19,1,12,45] + 1863600 \cdot x[19,2,2,34] + 130452 \cdot x[19,2,2,35] + 26000 \cdot x[19,2,5,23] + 26000 \cdot x[19,2,5,24] + 1950 \cdot x[19,2,5,25] + 36100 \cdot x[19,2,5,45] + 9315 \cdot x[19,2,6,45] + 9315 \cdot x[19,2,7,45] + 432 \cdot x[19,2,8,45] + 15525 \cdot x[19,2,10,45] + 9315 \cdot x[19,2,12,45] + 37500 \cdot x[19,3,2,34] + 2625 \cdot x[19,3,2,35] + 7700 \cdot x[19,3,5,12] + 7700 \cdot x[19,3,5,13] + 7700 \cdot x[19,3,5,14] + 577.5 \cdot x[19,3,5,15] + 154 \cdot x[19,3,5,34] + 577.5 \cdot x[19,3,5,35] + 7700 \cdot x[19,3,6,23] + 7700 \cdot x[19,3,6,24] + 1155 \cdot x[19,3,6,25] + 1155 \cdot x[19,3,6,45] + 2310 \cdot x[19,3,12,45] + 244100 \cdot x[19,4,2,34] + 17087 \cdot x[19,4,2,35] + 20389.5 \cdot x[19,4,5,45] + 7000 \cdot x[19,4,6,23] + 7000 \cdot x[19,4,6,24] + 1050 \cdot x[19,4,6,25] + 2008.425 \cdot x[19,4,6,45] + 70 \cdot x[19,4,9,45] + 3058.425 \cdot x[19,4,12,45] + 115600 \cdot x[19,5,1,34] + 6358 \cdot x[19,5,1,35] + 816 \cdot x[19,5,4,45] + 1800 \cdot x[19,5,6,23] + 1800 \cdot x[19,5,6,24] + 270 \cdot x[19,5,6,25] + 1770 \cdot x[19,5,6,45] + 432 \cdot x[19,5,8,45] + 2040 \cdot x[19,5,12,45] + 42735 \cdot x[20,1,1,45] + 2525 \cdot x[20,1,4,45] + 140 \cdot x[20,1,9,45] + 80 \cdot x[20,1,12,34] + 1200 \cdot x[20,1,12,35] + 5112.5 \cdot x[20,1,12,45] + 5890.5 \cdot x[20,2,1,45] + 600 \cdot x[20,2,4,23] + 600 \cdot x[20,2,4,24] + 36 \cdot x[20,2,4,25] + 1725 \cdot x[20,2,4,45] + 70 \cdot x[20,2,9,45] + 600 \cdot x[20,2,11,23] + 600 \cdot x[20,2,11,24] + 18 \cdot x[20,2,11,25] + 862.5 \cdot x[20,2,11,45] + 1400 \cdot x[20,2,12,23] + 1400 \cdot x[20,2,12,24] + 210 \cdot x[20,2,12,25] + 14 \cdot x[20,2,12,34] + 210 \cdot x[20,2,12,35] + 3982.5 \cdot x[20,2,12,45] + 30723 \cdot x[20,3,1,45] + 2097 \cdot x[20,3,4,45] + 70 \cdot x[20,3,9,45] + 1048.5 \cdot x[20,3,11,45] + 2000 \cdot x[20,3,12,23] + 2000 \cdot x[20,3,12,24] + 300 \cdot x[20,3,12,25] + 20 \cdot x[20,3,12,34] + 300 \cdot x[20,3,12,35] + 4642.5 \cdot x[20,3,12,45] + 4433 \cdot x[20,4,1,45] + 798 \cdot x[20,4,4,45] + 35 \cdot x[20,4,9,45] + 399 \cdot x[20,4,11,45] + 800 \cdot x[20,4,12,23] + 800 \cdot x[20,4,12,24] + 120 \cdot x[20,4,12,25] + 8 \cdot x[20,4,12,34] + 120 \cdot x[20,4,12,35] + 1755 \cdot x[20,4,12,45] + 201900 \cdot x[21,1,2,34] + 14133 \cdot x[21,1,2,35] + 400 \cdot x[21,1,5,12] + 400 \cdot x[21,1,5,13] + 400 \cdot x[21,1,5,14] + 30 \cdot x[21,1,5,15] + 2500 \cdot x[21,1,5,23] + 2500 \cdot x[21,1,5,24] + 187.5 \cdot x[21,1,5,25] + 680 \cdot x[21,1,5,34] + 2550 \cdot x[21,1,5,35] + 3100 \cdot x[21,1,5,45] + 16 \cdot x[21,1,6,34] + 240 \cdot x[21,1,6,35] + 5760 \cdot x[21,1,6,45] + 70 \cdot x[21,1,9,45] + 10000 \cdot x[21,1,10,45] + 22900 \cdot x[21,2,2,34] + 1603 \cdot x[21,2,2,35] + 6466 \cdot x[21,2,5,45] + 969.9 \cdot x[21,2,6,45] + 432 \cdot x[21,2,8,45] + 1616.5 \cdot x[21,2,10,45] + 1170600 \cdot x[21,3,2,34] + 81942 \cdot x[21,3,2,35] + 79800 \cdot x[21,3,5,23] + 79800 \cdot x[21,3,5,24] + 5985 \cdot x[21,3,5,25] + 11970 \cdot x[21,3,6,45] + 210 \cdot x[21,3,9,45] + 19950 \cdot x[21,3,10,45] + 390200 \cdot x[21,4,2,34] + 27314 \cdot x[21,4,2,35] + 25246 \cdot x[21,4,5,45] + 3786.9 \cdot x[21,4,6,45] + 35 \cdot x[21,4,9,45] + 5300 \cdot x[21,4,10,12] + 5300 \cdot x[21,4,10,13] + 5300 \cdot x[21,4,10,14] + 1325 \cdot x[21,4,10,15] + 4986.5 \cdot x[21,4,10,45] + 36900 \cdot x[21,5,2,34] + 2583 \cdot x[21,5,2,35] + 9900 \cdot x[21,5,5,23] + 9900 \cdot x[21,5,5,24] + 742.5 \cdot x[21,5,5,25] + 160 \cdot x[21,5,5,34] + 600 \cdot x[21,5,5,35] + 1900 \cdot x[21,5,6,12] + 1900 \cdot x[21,5,6,13] + 1900 \cdot x[21,5,6,14] + 285 \cdot x[21,5,6,15] + 3800 \cdot x[21,5,6,23] + 3800 \cdot x[21,5,6,24] + 570 \cdot x[21,5,6,25] + 1830 \cdot x[21,5,6,45] + 35 \cdot x[21,5,9,45] + 4475 \cdot x[21,5,10,45] + 26700 \cdot x[21,6,2,34] + 1869 \cdot x[21,6,2,35] + 136 \cdot x[21,6,5,34] + 510 \cdot x[21,6,5,35] + 1400 \cdot x[21,6,6,23] + 1400 \cdot x[21,6,6,24] + 210 \cdot x[21,6,6,25] + 810 \cdot x[21,6,6,45] + 35 \cdot x[21,6,9,45] + 1700 \cdot x[21,6,10,45] + 5370 \cdot x[22,1,7,45] + 576 \cdot x[22,1,8,45] + 179100 \cdot x[22,1,14,45] + 129500 \cdot x[23,1,2,34] + 9065 \cdot x[23,1,2,35] + 35 \cdot x[23,1,9,45] + 3000 \cdot x[23,1,12,23] + 3000 \cdot x[23,1,12,24] + 450 \cdot x[23,1,12,25] + 2254.551 \cdot x[23,1,12,45] + 821500 \cdot x[23,2,2,34] + 57505 \cdot x[23,2,2,35] + 47459.7 \cdot x[23,2,5,45] + 7118.955 \cdot x[23,2,6,45] + 175 \cdot x[23,2,9,45] + 11864.925 \cdot x[23,2,10,45] + 990$

$0 \times [23,3,2,34] + 693 \times [23,3,2,35] + 4042 \times [23,3,5,45] + 1010.5 \times [23,3,10,45] + 606.3 \times [23,3,12,45] + 9900 \times [23,4,2,34] + 693 \times [23,4,2,35] + 4042 \times [23,4,5,45] + 606.3 \times [23,4,12,45] + 88500 \times [23,5,2,34] + 6195 \times [23,5,2,35] + 13832.32 \times [23,5,5,45] + 2074.848 \times [23,5,6,45] + 3458.08 \times [23,5,10,45] + 5000 \times [24,1,2,12] + 5000 \times [24,1,2,13] + 5000 \times [24,1,2,14] + 350 \times [24,1,2,15] + 140200 \times [24,1,2,34] + 9814 \times [24,1,2,35] + 19116.66 \times [24,1,5,45] + 2867.499 \times [24,1,6,45] + 4779.165 \times [24,1,10,45] + 5000 \times [24,2,2,23] + 5000 \times [24,2,2,24] + 350 \times [24,2,2,25] + 262200 \times [24,2,2,34] + 18354 \times [24,2,2,35] + 7800 \times [24,2,5,12] + 7800 \times [24,2,5,13] + 7800 \times [24,2,5,14] + 585 \times [24,2,5,15] + 14670.36 \times [24,2,5,45] + 3370.554 \times [24,2,6,45] + 105 \times [24,2,9,45] + 5617.59 \times [24,2,10,45] + 1315.0008 \times [24,3,4,45] + 3287.502 \times [24,3,6,45] + 3287.502 \times [24,3,7,45] + 288 \times [24,3,8,45] + 140 \times [24,3,9,45] + 5479.17 \times [24,3,10,45] + 3287.502 \times [24,3,12,45] + 42200 \times [24,4,1,34] + 2321 \times [24,4,1,35] + 708 \times [24,4,4,45] + 1770 \times [24,4,6,45] + 9300 \times [24,5,1,34] + 511.5 \times [24,5,1,35] + 318 \times [24,5,4,45] + 1200 \times [24,5,6,23] + 1200 \times [24,5,6,24] + 180 \times [24,5,6,25] + 615 \times [24,5,6,45] + 10000 \times [25,1,2,23] + 10000 \times [25,1,2,24] + 700 \times [25,1,2,25] + 107436 \times [25,1,2,45] + 3931.3187 \times [25,1,4,45] + 9828.29675 \times [25,1,6,45] + 350 \times [25,1,9,45] + 16380.4945833333 \times [25,1,10,45] + 1932 \times [25,2,2,45] + 8200 \times [25,2,4,12] + 8200 \times [25,2,4,13] + 8200 \times [25,2,4,14] + 492 \times [25,2,4,15] + 1230 \times [25,2,6,45] + 35 \times [25,2,9,45] + 2050 \times [25,2,10,45] + 861 \times [25,3,2,45] + 4400 \times [25,3,4,12] + 4400 \times [25,3,4,13] + 4400 \times [25,3,4,14] + 264 \times [25,3,4,15] + 660 \times [25,3,6,45] + 35 \times [25,3,9,45] + 1100 \times [25,3,10,45] + 36146 \times [25,4,1,45] + 100 \times [25,4,4,12] + 100 \times [25,4,4,13] + 100 \times [25,4,4,14] + 6 \times [25,4,4,15] + 2707.516 \times [25,4,4,45] + 210 \times [25,4,9,45] + 11306.3166666667 \times [25,4,10,45] + 1356.758 \times [25,4,11,45] + 2998.8646 \times [25,5,4,45] + 280 \times [25,5,9,45] + 12495.2691666667 \times [25,5,10,45] + 1499.4323 \times [25,5,11,45] + 330 \times [25,6,1,45] + 611.6855 \times [25,6,12,45] + 500.5 \times [25,7,1,45] + 664.149 \times [25,7,12,45] + 22400 \times [25,8,2,34] + 1568 \times [25,8,2,35] + 6400 \times [25,8,5,12] + 6400 \times [25,8,5,13] + 6400 \times [25,8,5,14] + 480 \times [25,8,5,15] + 386.346666666667 \times [25,8,5,45] + 3200 \times [25,8,6,23] + 3200 \times [25,8,6,24] + 480 \times [25,8,6,25] + 537.952 \times [25,8,6,45] + 576 \times [25,8,8,45] + 35 \times [25,8,9,45] + 1696.58666666667 \times [25,8,10,45] + 24200 \times [26,1,1,34] + 1331 \times [26,1,1,35] + 18089.5 \times [26,1,1,45] + 1432.906 \times [26,1,4,45] + 3582.265 \times [26,1,6,45] + 288 \times [26,1,8,45] + 5970.44166666667 \times [26,1,10,45] + 2600 \times [26,2,1,34] + 143 \times [26,2,1,35] + 2095.5 \times [26,2,1,45] + 480.9063 \times [26,2,4,45] + 1202.26575 \times [26,2,6,45] + 288 \times [26,2,8,45] + 2003.77625 \times [26,2,10,45] + 1607900 \times [27,1,2,34] + 112553 \times [27,1,2,35] + 18800 \times [27,1,5,12] + 18800 \times [27,1,5,13] + 18800 \times [27,1,5,14] + 1410 \times [27,1,5,15] + 1096 \times [27,1,5,34] + 4110 \times [27,1,5,35] + 434.000000000003 \times [27,1,5,45] + 56000 \times [27,1,6,23] + 56000 \times [27,1,6,24] + 8400 \times [27,1,6,25] + 2705.1 \times [27,1,6,45] + 11105.1 \times [27,1,7,45] + 576 \times [27,1,8,45] + 18508.5 \times [27,1,10,45] + 20700 \times [27,1,12,23] + 20700 \times [27,1,12,24] + 3105 \times [27,1,12,25] + 8000.1 \times [27,1,12,45] + 580900 \times [27,2,2,34] + 40663 \times [27,2,2,35] + 450 \times [27,2,5,34] + 1687.5 \times [27,2,5,35] + 11648 \times [27,2,5,45] + 5122.2 \times [27,2,6,45] + 5122.2 \times [27,2,7,45] + 576 \times [27,2,8,45] + 8537 \times [27,2,10,45] + 12600 \times [27,2,12,23] + 12600 \times [27,2,12,24] + 1890 \times [27,2,12,25] + 3232.2 \times [27,2,12,45] + 699100 \times [27,3,2,34] + 48937 \times [27,3,2,35] + 12000 \times [27,3,5,12] + 12000 \times [27,3,5,13] + 12000 \times [27,3,5,14] + 900 \times [27,3,5,15] + 530 \times [27,3,5,34] + 1987.5 \times [27,3,5,35] + 24000 \times [27,3,6,23] + 24000 \times [27,3,6,24] + 3600 \times [27,3,6,25] + 2175 \times [27,3,6,45] + 5775 \times [27,3,7,45] + 576 \times [27,3,8,45] + 9625 \times [27,3,10,45] + 13800 \times [27,3,12,23] + 13800 \times [27,3,12,24] + 2070 \times [27,3,12,25] + 3705 \times [27,3,12,45] + 16000 \times [27,4,2,34] + 1120 \times [27,4,2,35] + 576 \times [27,4,8,45] + 775.02 \times [27,4,12,45] + 53500 \times [27,5,2,34] + 3745 \times [27,5,2,35] + 9600 \times [27,5,5,45] + 9600 \times [27,5,12,23] + 9600 \times [27,5,12,24] + 1440 \times [27,5,12,25] + 22700 \times [27,6,2,34] + 1589 \times [27,6,2,35] + 12725 \times [27,6,5,45] + 1000 \times [27,6,6,23] + 1000 \times [27,6,6,24] + 150 \times [27,6,6,25] + 1758.75 \times [27,6,6,45] + 1.5 \times [27,6,9,34] + 35 \times [27,6,9,35] + 3181.25 \times [27,6,10,45] + 24100 \times [27,7,2,34] + 1687 \times [27,7,2,35] + 12725 \times [27,7,5,45] + 5500 \times [27,7,6,23] + 55$

$00 * x[27,7,6,24] + 825 * x[27,7,6,25] + 1083.75 * x[27,7,6,45] + 35 * x[27,7,9,45] + 3181.25 * x[27,7,10,45] + 44300 * x[27,8,2,34] + 3101 * x[27,8,2,35] + 8066.66 * x[27,8,5,45] + 4000 * x[27,8,6,23] + 4000 * x[27,8,6,24] + 600 * x[27,8,6,25] + 609.999 * x[27,8,6,45] + 35 * x[27,8,9,45] + 1209.999 * x[27,8,12,45] + 2319700 * x[27,9,2,34] + 162379 * x[27,9,2,35] + 64400 * x[27,9,5,45] + 64400 * x[27,9,6,23] + 64400 * x[27,9,6,24] + 9660 * x[27,9,6,25] + 525 * x[27,9,9,45] + 16100 * x[27,9,10,45] + 28000 * x[27,10,2,34] + 1960 * x[27,10,2,35] + 6795.9 * x[27,10,5,45] + 1019.385 * x[27,10,6,45] + 35 * x[27,10,9,45] + 1698.975 * x[27,10,10,45] + 16800 * x[27,11,2,34] + 1176 * x[27,11,2,35] + 6566.8 * x[27,11,5,45] + 985.02 * x[27,11,6,45] + 35 * x[27,11,9,45] + 1641.7 * x[27,11,10,45] + 67600 * x[27,12,1,34] + 3718 * x[27,12,1,35] + 622.008 * x[27,12,4,45] + 35 * x[27,12,9,45] + 1555.02 * x[27,12,12,45] + 17857 * x[28,1,2,45] + 1197 * x[28,1,4,45] + 2992.5 * x[28,1,12,45] + 1200 * x[28,2,1,12] + 1200 * x[28,2,1,13] + 1200 * x[28,2,1,14] + 66 * x[28,2,1,15] + 3143600 * x[28,2,1,34] + 172898 * x[28,2,1,35] + 3151.2 * x[28,2,4,45] + 17600 * x[28,2,6,23] + 17600 * x[28,2,6,24] + 2640 * x[28,2,6,25] + 5238 * x[28,2,6,45] + 7878 * x[28,2,12,45] \leq \text{budget};$

subject to constraint2 {i in I1,j in J1,l in L1}:

$x[i,j,l,10] + x[i,j,l,11] + x[i,j,l,12] + x[i,j,l,13] + x[i,j,l,14] + x[i,j,l,15] = 1;$

subject to constraint3 {i in I1,j in J1,l in L1}:

$x[i,j,l,21] + x[i,j,l,22] + x[i,j,l,23] + x[i,j,l,24] + x[i,j,l,25] = 1;$

subject to constraint4 {i in I1,j in J1,l in L1}: $x[i,j,l,32] + x[i,j,l,33] + x[i,j,l,34] + x[i,j,l,35] = 1;$

subject to constraint5 {i in I1,j in J1,l in L1}:

$x[i,j,l,43] + x[i,j,l,44] + x[i,j,l,45] = 1;$

subject to constraint6 {i in I1,j in J1,l in L1}:

$x[i,j,l,54] + x[i,j,l,55] = 1;$

B2.2 Optimization model created by Visual Basic code VB3

```
set N1;  
param budget:= 246000;  
var x{n in N1}binary;
```

```
maximize REUAC:-5675.13383863912*x[1]+-1239.89902283163*x[2]+-  
190.323470956974*x[3]+163.944664686934*x[4]+-975.511714775115*x[5]+-  
6567.01800248075*x[6]+-140.876676304214*x[7]+-6247.78415946704*x[8]+-  
4599.50495090298*x[9]+-3537.29990849246*x[10]+84.4818663404727*x[11]+-  
1652.56879687619*x[12]+-129.54664379505*x[13]+-93.1571371110475*x[14]+-  
92.6719436885941*x[15]+-92.6719436885941*x[16]+-12508.1802715824*x[17]+-  
4354.78461469127*x[18]+848.630966993555*x[19]+177.371070415313*x[20]+2228.0709  
8923499*x[21]+-2898.04564391036*x[22]+-3451.96985046589*x[23]+-  
3713.22970093308*x[24]+-  
324.395214953141*x[25]+744.413532710329*x[26]+275.06874600737*x[27]+798.571261  
682304*x[28]+604.892093457979*x[29]+831.924448598228*x[30]+705.503532710329*x[  
31]+721.153214953331*x[32]+823.951724182617*x[33]+10119.7514152376*x[34]+-  
1091.98142387274*x[35]+-2333.68532520897*x[36]+-  
1452.72528711642*x[37]+215.92722299783*x[38]+-908.006292676793*x[39]+-  
2238.64670010557*x[40]+-25.0104441026754*x[41]+-766.845650537963*x[42]+-  
44.934018218366*x[43]+-2156.40896864932*x[44]+-161.13135111786*x[45]+-  
515.893397846712*x[46]+10488.9295819987*x[47]+2058.03372206788*x[48]+2058.0337  
2206788*x[49]+8335.49035248122*x[50]+268.077872471334*x[51]+500.40907092163*x[  
52]+275.623762746979*x[53]+275.06874600737*x[54]+1575.7538479832*x[55]+858.718  
74600737*x[56]+-136.209115580213*x[57]+-136.209115580213*x[58]+-  
4006.30082313811*x[59]+-138.400146930029*x[60]+-215.08624417358*x[61]+-  
186.553229429004*x[62]+-7025.75889612873*x[63]+-  
1257.728604282*x[64]+8461.28229510431*x[65]+392.382878741342*x[66]+4042.389860  
81696*x[67]+3363.56347647999*x[68]+1593.97384959773*x[69]+335.917624280299*x[7  
0]+319.971608221231*x[71]+-2965.34831657268*x[72]+98.416523364659*x[73]+-  
1944.02048598041*x[74]+-81.2821682241689*x[75]+2027.00476767681*x[76]+-  
5.57234182178868*x[77]+10021.4634226967*x[78]+77.6128075655834*x[79]+2189.0123  
3495431*x[80]+398.626039113278*x[81]+-  
34029.7522388905*x[82]+292.496429964206*x[83]+-199.898637842769*x[84]+-  
2.40900366968157*x[85]+-2.40900366968157*x[86]+-  
21.5350328047292*x[87]+101.280843486904*x[88]+757.867296003448*x[89]+-  
1711.81346740734*x[90]+68.6581718917311*x[91]+77.2308293505474*x[92]+-  
4900.64125635664*x[93]+1503.68505264407*x[94]+815.187143026161*x[95]+-  
2162.28106530139*x[96]+-3735.37647698892*x[97]+-30.6437951023183*x[98]+-  
46.4764225718494*x[99]+825.58359190446*x[100]+-996.872772151429*x[101]+-  
116.276599905305*x[102]+12711.7787373252*x[103]+4331.78245134161*x[104]+7307.8  
4040752784*x[105]+-  
3.89333926413184*x[106]+2519.94164683556*x[107]+99.476324919013*x[108]+278.760  
657733401*x[109]+209.999146161217*x[110]+2768.23880681209*x[111]+-
```

6.81334371223073*x[112]+-4.08800622733843*x[113]+253.263403998675*x[114]+-846.866814148484*x[115]+13294.6875925749*x[116];

subject to constraint1:

77936*x[1]+24277*x[2]+4048*x[3]+4719*x[4]+34452*x[5]+174636*x[6]+2600*x[7]+144991*x[8]+99120*x[9]+76384*x[10]+88*x[11]+32357*x[12]+2536.5*x[13]+1824*x[14]+1814.5*x[15]+1814.5*x[16]+174032*x[17]+85265.9*x[18]+95458.25*x[19]+3485*x[20]+19654.4*x[21]+148636.4*x[22]+68350.8*x[23]+73304.1*x[24]+6351.6*x[25]+2377.9*x[26]+2946.7*x[27]+2962.5*x[28]+1722.2*x[29]+4716.3*x[30]+2377.9*x[31]+2946.7*x[32]+90555*x[33]+17645.6*x[34]+21380.8*x[35]+49418.2*x[36]+28444.1*x[37]+3809.7*x[38]+17778.6*x[39]+43832.3*x[40]+489.7*x[41]+15014.7*x[42]+879.8*x[43]+42222.1*x[44]+4199.8*x[45]+10101.1*x[46]+58812*x[47]+122957.25*x[48]+4163.25*x[49]+137272.85*x[50]+1086.8*x[51]+3095.95*x[52]+1236.95*x[53]+1716*x[54]+11611.6*x[55]+1716*x[56]+2666.95*x[57]+2666.95*x[58]+78442.65*x[59]+2709.85*x[60]+4211.35*x[61]+3443*x[62]+138037.3*x[63]+24626.1*x[64]+96609.5*x[65]+3276*x[66]+118482*x[67]+181701*x[68]+3480.75*x[69]+23799.75*x[70]+11271*x[71]+64491*x[72]+8460.9*x[73]+44129.4*x[74]+6367.4*x[75]+22209*x[76]+2290*x[77]+128766*x[78]+42922*x[79]+4059*x[80]+2937*x[81]+192532.5*x[82]+13273.75*x[83]+82150*x[84]+990*x[85]+990*x[86]+8850*x[87]+15246*x[88]+28056*x[89]+33517*x[90]+4642*x[91]+1023*x[92]+100412*x[93]+1794*x[94]+799.5*x[95]+42718*x[96]+73138*x[97]+600*x[98]+910*x[99]+2184*x[100]+22951.5*x[101]+2645.5*x[102]+160790*x[103]+58090*x[104]+69910*x[105]+1600*x[106]+5350*x[107]+2270*x[108]+2410*x[109]+4430*x[110]+231970*x[111]+2800*x[112]+1680*x[113]+4630.6*x[114]+16581.5*x[115]+204412*x[116]<=budget;

APPENDIX B3

This appendix includes the data files required by Ampl to run the optimization models.

B3.1 Data file for maintenance optimization model

```
set I1:=1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28;
```

```
set J1:=1 2 3 4 5 6 7 8 9 10 11 12;
```

```
set L1:=1 2 3 4 5 6 7 8 9 10 11 12 13 14 15;
```

```
set K1:=10 11 12 13 14 15 21 22 23 24 25 32 33 34 35 43 44 45 54 55;
```

B3.2 Data file for re-roofing optimization model

```
set N1:=1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31  
32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61  
62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91  
92 93 94 95 96 97 98 99 100 101 102 103 104 105 106 107 108 109 110 111 112 113 114  
115 116;
```

APPENDIX C

APPENDIX C.1

This appendix includes tables and graphs showing the weighted average condition state sensitivity to budget levels of all maintenance elements.

Table C1.1 Single-ply membrane weighted average condition state sensitivity to budget levels

Year	Weighted average condition state				
	200K	250K	300K	350K	400K
1	3.82	3.82	3.82	3.82	3.82
2	3.93	3.98	4.04	4.09	4.14
3	3.93	3.96	3.99	4.02	4.05
4	3.94	4.02	4.03	4.04	4.06
5	3.94	4.10	4.10	4.11	4.12
6	3.94	4.18	4.18	4.18	4.19
7	3.95	4.26	4.26	4.26	4.26
8	3.98	4.33	4.32	4.32	4.42
9	4.03	4.38	4.37	4.37	4.64
10	4.09	4.42	4.41	4.42	4.71
11	4.18	4.44	4.44	4.60	4.71
12	4.27	4.46	4.46	4.73	4.71
13	4.34	4.48	4.46	4.72	4.71
14	4.47	4.49	4.50	4.72	4.71
15	4.53	4.50	4.63	4.72	4.71

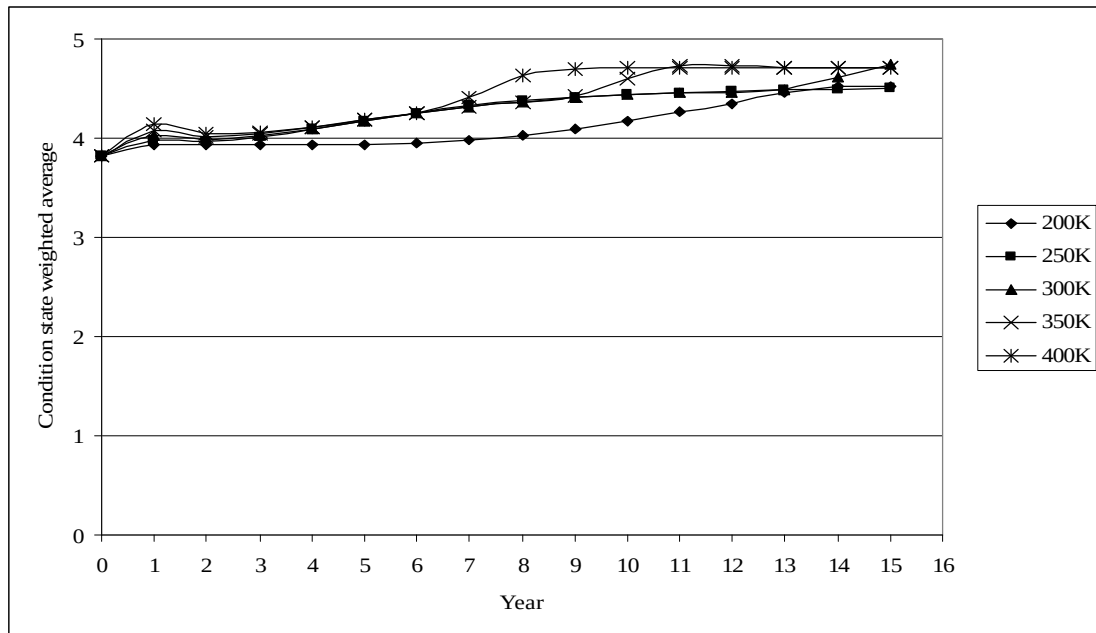


Figure C1.1 Single-ply membrane weighted average condition state sensitivity to budget levels

Table C1.2 Multiple-ply membrane weighted average condition state sensitivity to budget levels

Year	Weighted average condition state				
	200K	250K	300K	350K	400K
1	3.23	3.23	3.23	3.23	3.23
2	3.23	3.23	3.23	3.23	3.23
3	2.82	2.99	3.16	3.33	3.49
4	2.30	2.55	2.91	3.26	3.61
5	1.93	2.22	2.77	3.28	3.81
6	1.70	2.02	2.72	3.42	4.17
7	1.56	1.93	2.78	3.68	4.60
8	1.47	1.92	2.97	4.01	4.74
9	1.41	2.00	3.21	4.38	4.72
10	1.36	2.15	3.48	4.75	4.81
11	1.33	2.33	3.79	4.73	4.80
12	1.30	2.54	4.12	4.77	4.78
13	1.31	2.76	4.50	4.80	4.78
14	1.29	2.99	4.75	4.78	4.77
15	1.31	3.22	4.72	4.77	4.76

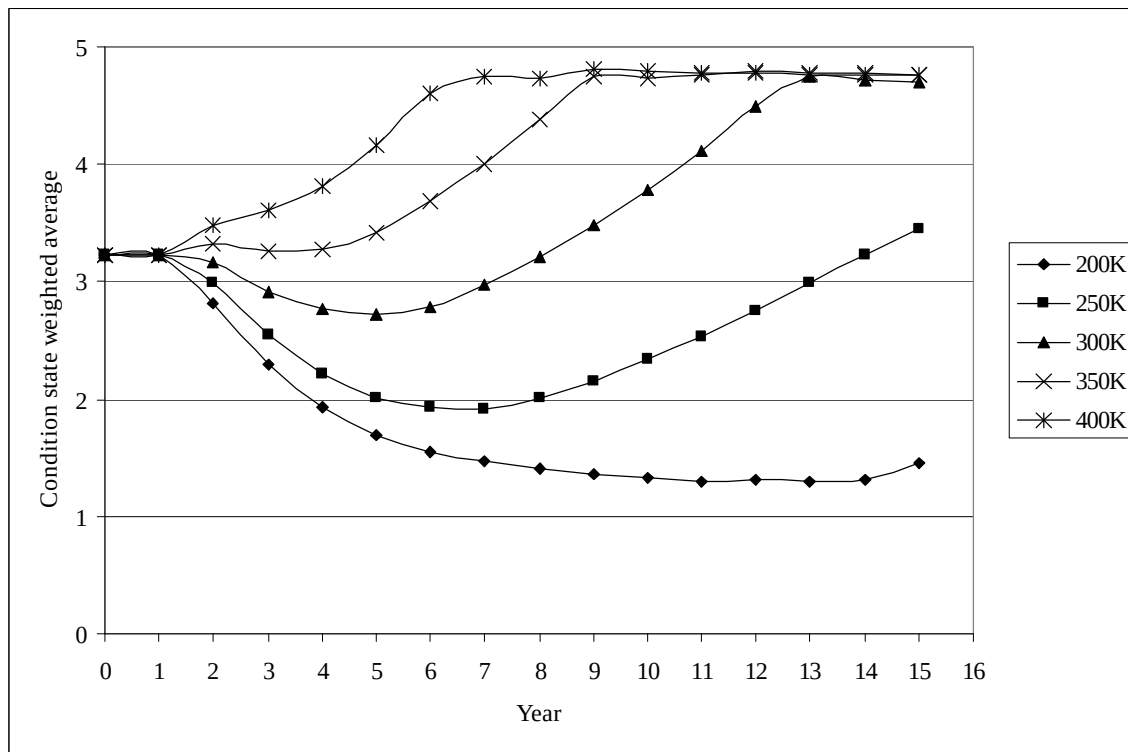


Figure C1.2 Multiple-ply membrane weighted average condition state sensitivity to budget levels

Table C1.3 Single-ply base flashing weighted average condition state sensitivity to budget levels

Year	Weighted average condition state				
	200K	250K	300K	350K	400K
1	3.76	3.76	3.76	3.76	3.76
2	4.14	4.14	4.14	4.14	4.14
3	4.13	4.13	4.13	4.13	4.13
4	4.12	4.12	4.12	4.12	4.12
5	4.10	4.10	4.10	4.10	4.10
6	4.09	4.09	4.09	4.09	4.09
7	4.09	4.09	4.09	4.09	4.09
8	4.08	4.08	4.08	4.08	4.08
9	4.07	4.07	4.07	4.07	4.07
10	4.06	4.06	4.06	4.06	4.06
11	4.06	4.06	4.06	4.06	4.06
12	4.05	4.05	4.05	4.05	4.05
13	4.05	4.05	4.05	4.05	4.05
14	4.04	4.04	4.04	4.04	4.04
15	4.04	4.04	4.04	4.04	4.04

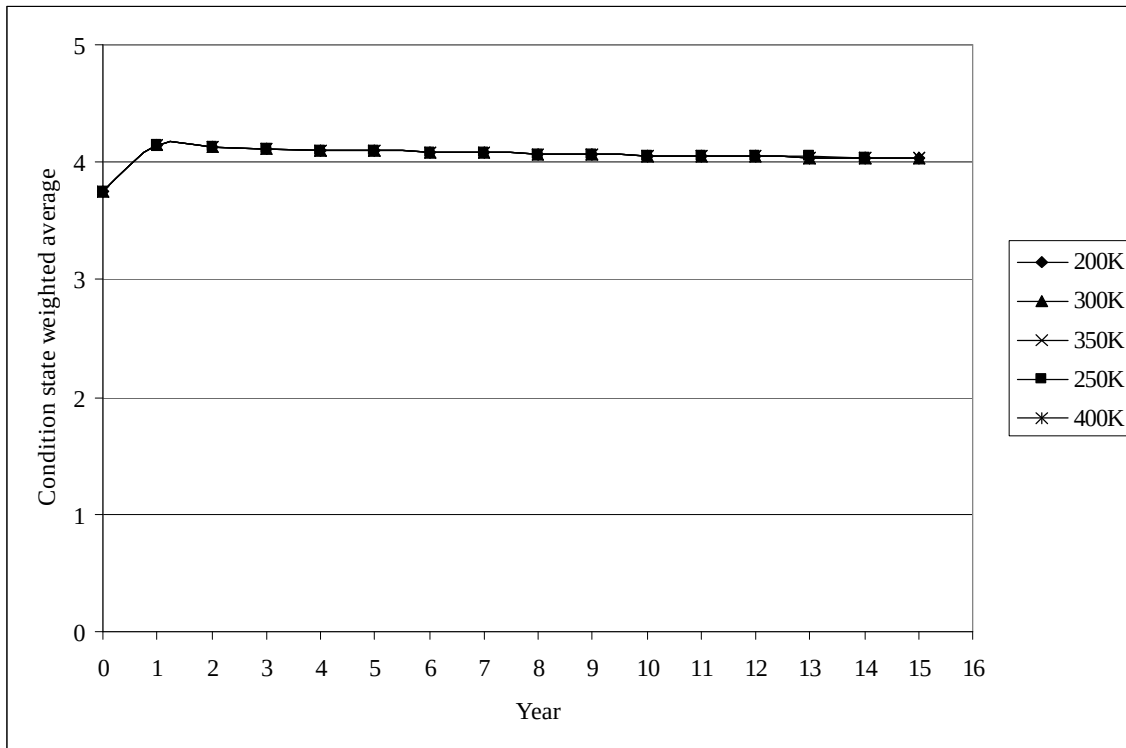


Figure C1.3 Single-ply base flashing weighted average condition state sensitivity to budget levels

Table C1.4 Multiple-ply base flashing weighted average condition state sensitivity to budget levels

Year	Weighted average condition state				
	200K	250K	300K	350K	400K
1	3.26	3.26	3.26	3.26	3.26
2	4.23	4.23	4.23	4.23	4.23
3	4.22	4.22	4.22	4.22	4.22
4	4.20	4.20	4.20	4.20	4.20
5	4.18	4.18	4.18	4.18	4.18
6	4.17	4.17	4.17	4.17	4.17
7	4.16	4.16	4.16	4.16	4.16
8	4.14	4.14	4.14	4.14	4.14
9	4.13	4.13	4.13	4.13	4.13
10	4.12	4.12	4.12	4.12	4.12
11	4.11	4.11	4.11	4.11	4.11
12	4.10	4.10	4.10	4.10	4.10
13	4.10	4.10	4.10	4.10	4.10
14	4.09	4.09	4.09	4.09	4.09
15	4.08	4.08	4.08	4.08	4.08

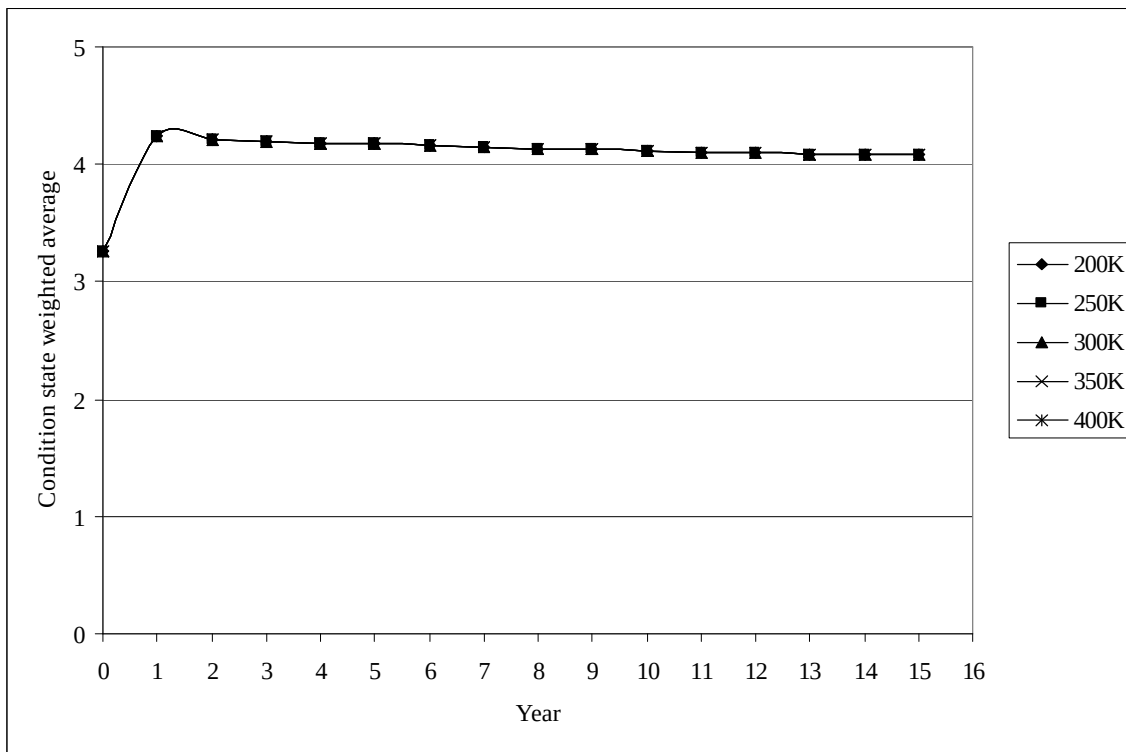


Figure C1.4 Multiple-ply base flashing weighted average condition state sensitivity to budget levels

Table C1.5 Counter flashing weighted average condition state sensitivity to budget levels

Year	Weighted average condition state				
	200K	250K	300K	350K	400K
1	3.68	3.68	3.68	3.68	3.68
2	4.15	4.15	4.15	4.15	4.15
3	4.12	4.12	4.12	4.12	4.12
4	4.10	4.10	4.10	4.10	4.10
5	4.09	4.09	4.09	4.09	4.09
6	4.07	4.07	4.07	4.07	4.07
7	4.06	4.06	4.06	4.06	4.06
8	4.05	4.05	4.05	4.05	4.05
9	4.04	4.04	4.04	4.04	4.04
10	4.04	4.04	4.04	4.04	4.04
11	4.03	4.03	4.03	4.03	4.03
12	4.02	4.02	4.02	4.02	4.02
13	4.02	4.02	4.02	4.02	4.02
14	4.02	4.02	4.02	4.02	4.02
15	4.01	4.01	4.01	4.01	4.01

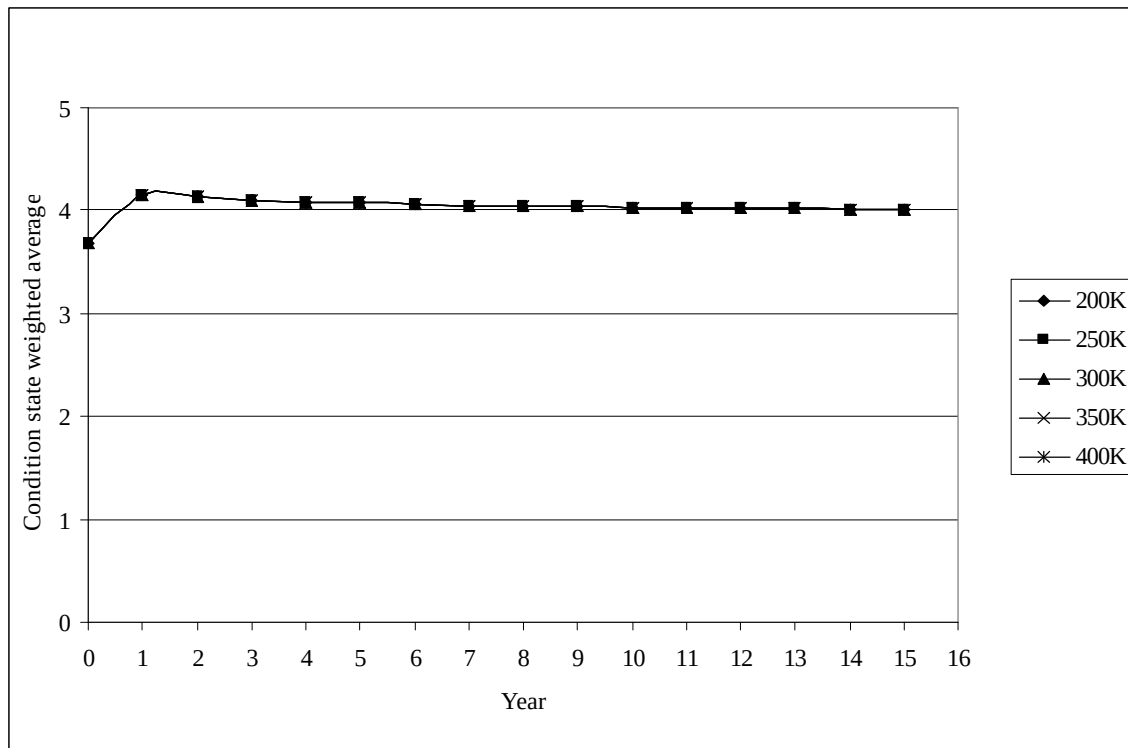


Figure C1.5 Counter flashing weighted average condition state sensitivity to budget levels

Table C1.6 Gutter weighted average condition state sensitivity to budget levels

Year	Weighted average condition state				
	200K	250K	300K	350K	400K
1	4.04	4.04	4.04	4.04	4.04
2	4.05	4.05	4.05	4.05	4.04
3	4.04	4.04	4.04	4.04	4.03
4	4.04	4.04	4.04	4.04	4.03
5	4.03	4.03	4.03	4.03	4.02
6	4.02	4.02	4.02	4.02	4.01
7	4.02	4.02	4.02	4.02	4.01
8	4.02	4.02	4.02	4.02	4.02
9	4.01	4.01	4.01	4.01	4.02
10	4.01	4.01	4.01	4.01	4.01
11	4.01	4.01	4.01	4.01	4.01
12	4.01	4.01	4.01	4.01	4.01
13	4.01	4.01	4.01	4.01	4.01
14	4.00	4.00	4.00	4.00	4.01
15	4.00	4.00	4.00	4.00	4.00

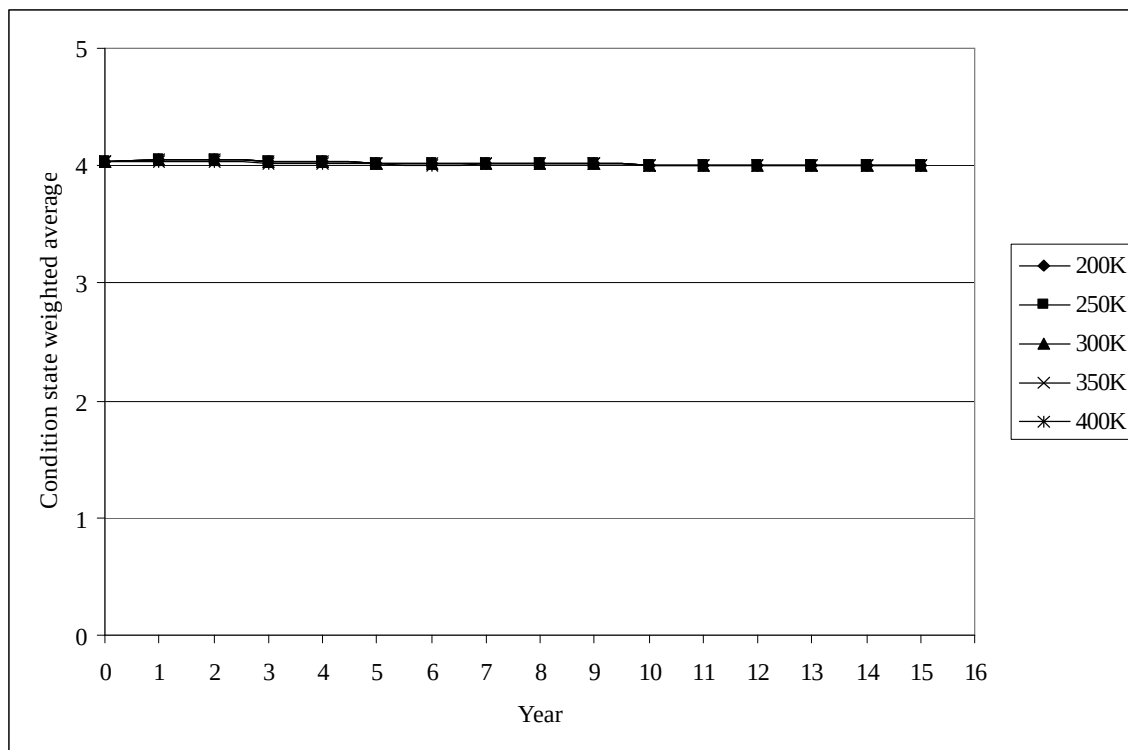


Figure C1.6 Gutter weighted average condition state sensitivity to budget levels

Table C1.7 Downspout weighted average condition state sensitivity to budget levels

Year	Weighted average condition state				
	200K	250K	300K	350K	400K
1	4.03	4.03	4.03	4.03	4.03
2	4.03	4.03	4.03	4.03	4.03
3	4.03	4.03	4.03	4.03	4.03
4	4.02	4.02	4.02	4.02	4.02
5	4.02	4.02	4.02	4.02	4.02
6	4.01	4.01	4.01	4.01	4.01
7	4.01	4.01	4.01	4.01	4.01
8	4.01	4.01	4.01	4.01	4.01
9	4.01	4.01	4.01	4.01	4.01
10	4.01	4.01	4.01	4.01	4.01
11	4.01	4.01	4.01	4.01	4.01
12	4.00	4.00	4.00	4.00	4.00
13	4.00	4.00	4.00	4.00	4.00
14	4.00	4.00	4.00	4.00	4.00
15	4.00	4.00	4.00	4.00	4.00

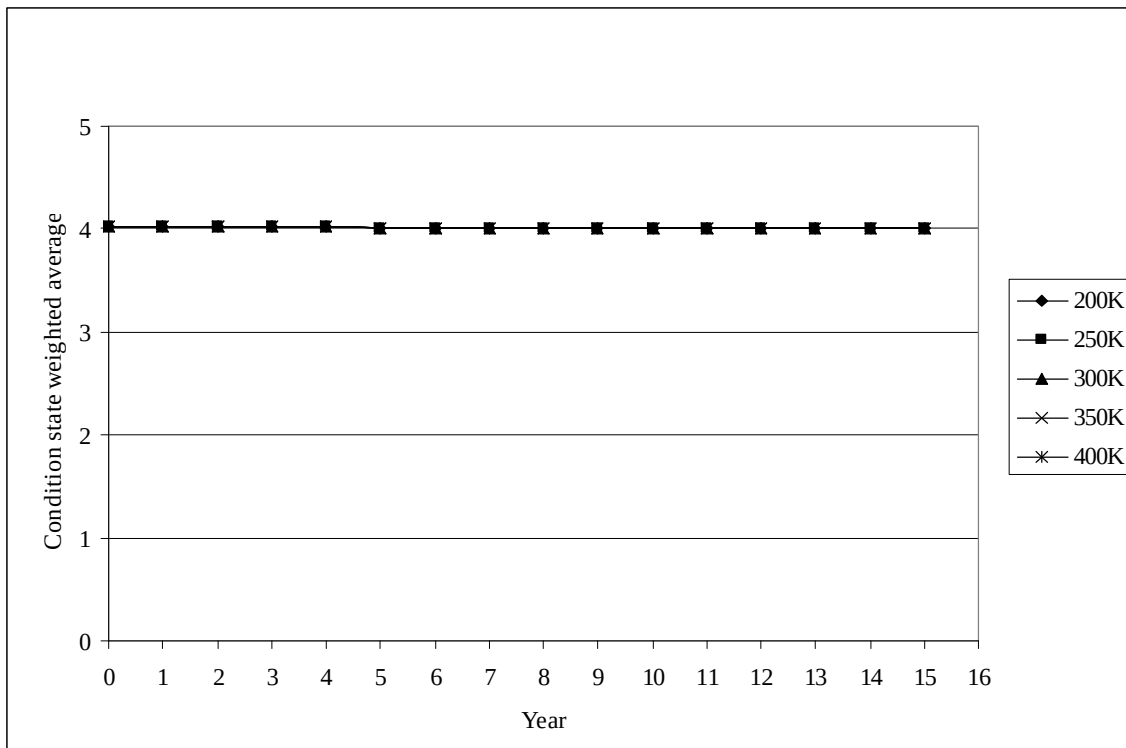


Figure C1.7 Downspout weighted average condition state sensitivity to budget levels

Table C1.8 Drain Strainer weighted average condition state sensitivity to budget levels

Year	Weighted average condition state				
	200K	250K	300K	350K	400K
1	3.97	3.97	3.97	3.97	3.97
2	4.03	4.03	4.03	4.03	4.03
3	4.02	4.02	4.02	4.02	4.02
4	4.02	4.02	4.02	4.02	4.02
5	4.02	4.02	4.02	4.02	4.02
6	4.01	4.01	4.01	4.01	4.01
7	4.01	4.01	4.01	4.01	4.01
8	4.01	4.01	4.01	4.01	4.01
9	4.01	4.01	4.01	4.01	4.01
10	4.01	4.01	4.01	4.01	4.01
11	4.01	4.01	4.01	4.01	4.01
12	4.01	4.01	4.01	4.01	4.01
13	4.01	4.01	4.01	4.01	4.01
14	4.00	4.00	4.00	4.00	4.00
15	4.00	4.00	4.00	4.00	4.00

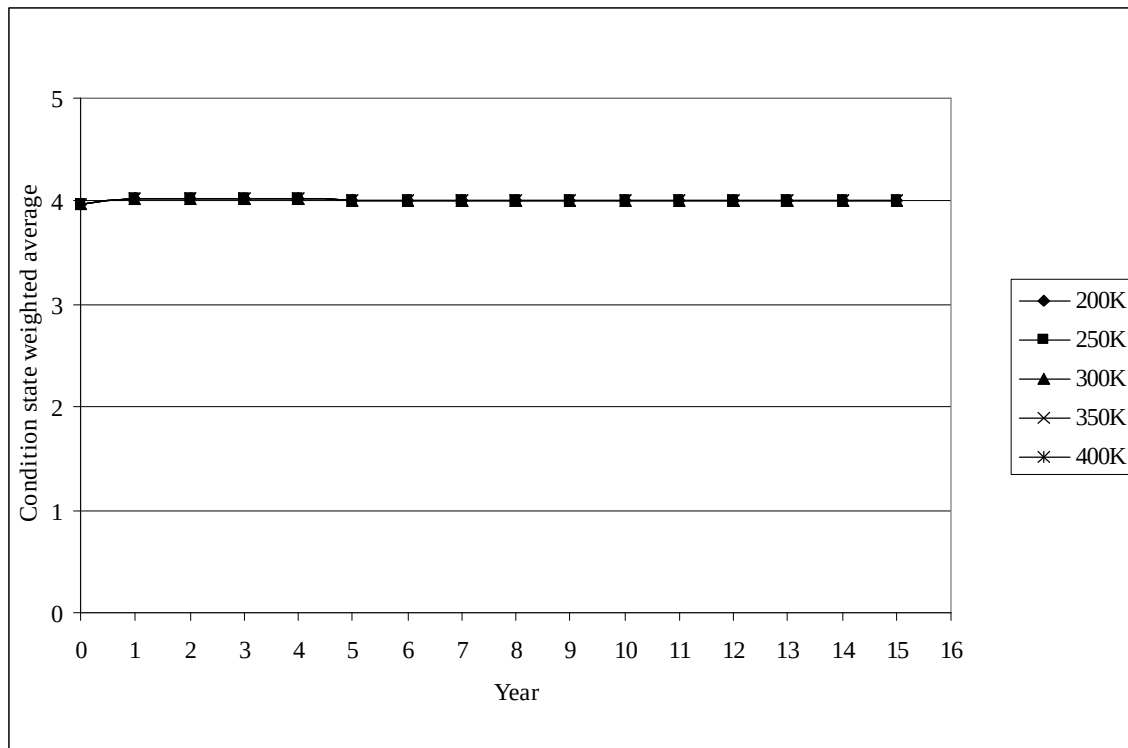


Figure C1.8 Drain Strainer weighted average condition state sensitivity to budget levels

Table C1.9 Coping cap weighted average condition state sensitivity to budget levels

Year	Weighted average condition state				
	200K	250K	300K	350K	400K
1	4.02	4.02	4.02	4.02	4.02
2	4.03	4.03	4.02	4.03	4.03
3	4.02	4.02	4.01	4.02	4.02
4	4.02	4.02	4.01	4.02	4.02
5	4.02	4.02	4.01	4.02	4.02
6	4.01	4.01	4.00	4.01	4.01
7	4.01	4.01	4.00	4.01	4.01
8	4.01	4.01	4.00	4.01	4.01
9	4.01	4.01	4.00	4.01	4.01
10	4.01	4.01	4.00	4.01	4.01
11	4.01	4.01	4.00	4.01	4.01
12	4.01	4.00	4.00	4.01	4.01
13	4.00	4.00	4.00	4.00	4.00
14	4.00	3.99	4.01	4.00	4.00
15	4.00	4.00	4.01	4.00	4.00

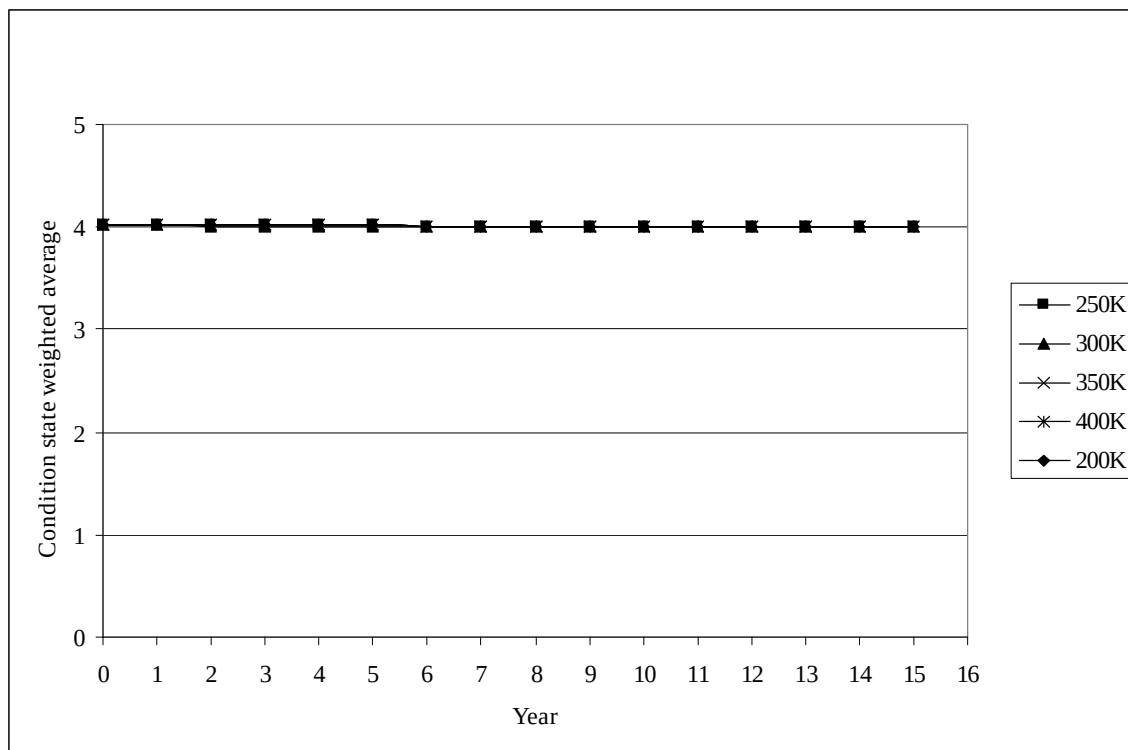


Figure C1.9 Coping cap weighted average condition state sensitivity to budget levels

Table C1.10 Termination bar weighted average condition state sensitivity to budget levels

Year	Weighted average condition state				
	200K	250K	300K	350K	400K
1	4.00	4.00	4.00	4.00	4.00
2	4.01	4.01	4.01	4.01	4.01
3	4.01	4.01	4.01	4.01	4.01
4	4.01	4.01	4.01	4.01	4.01
5	4.01	4.01	4.01	4.01	4.01
6	4.00	4.00	4.00	4.00	4.00
7	4.00	4.00	4.00	4.00	4.00
8	4.00	4.00	4.00	4.00	4.00
9	4.00	4.00	4.00	4.00	4.00
10	4.00	4.00	4.00	4.00	4.00
11	4.00	4.00	4.00	4.00	4.00
12	4.00	4.00	4.00	4.00	4.00
13	4.00	4.00	4.00	4.00	4.00
14	4.00	4.00	4.00	4.00	4.00
15	4.00	4.00	4.00	4.00	4.00

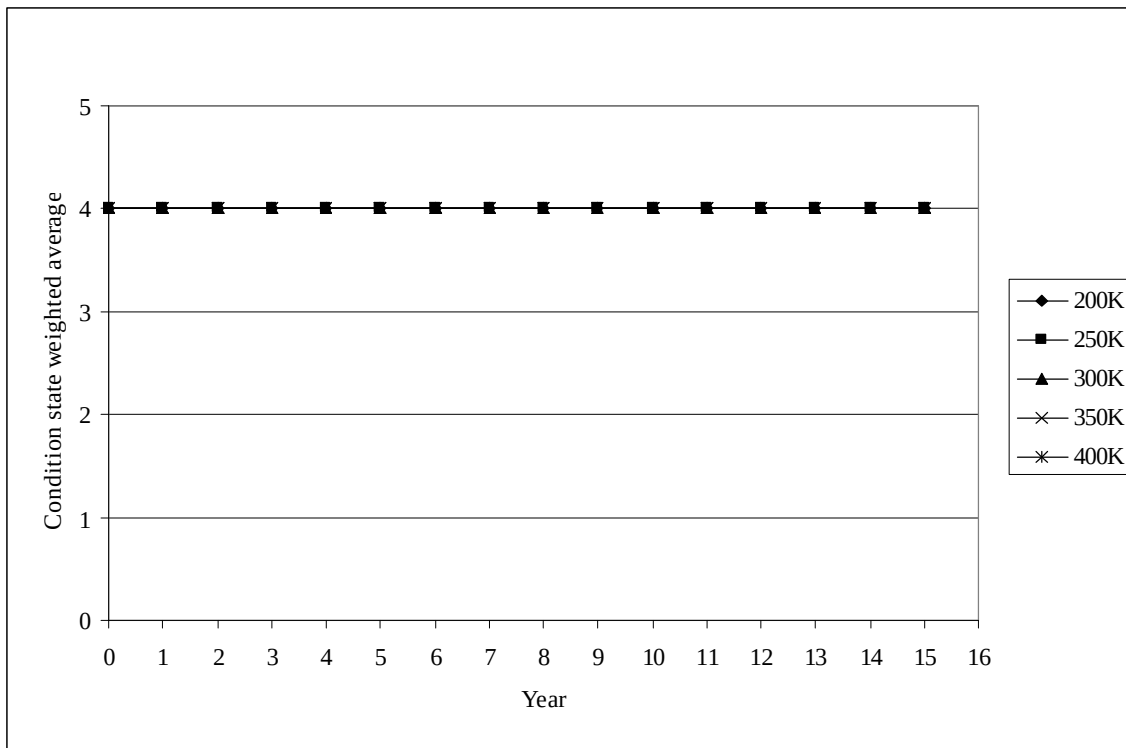


Figure C1.10 Termination bar weighted average condition state sensitivity to budget levels

Table C1.11 Edge metal weighted average condition state sensitivity to budget levels

Year	Weighted average condition state				
	200K	250K	300K	350K	400K
1	3.64	3.64	3.64	3.64	3.64
2	4.15	4.15	4.15	4.15	4.15
3	4.13	4.13	4.13	4.13	4.13
4	4.11	4.11	4.11	4.11	4.11
5	4.10	4.10	4.10	4.10	4.10
6	4.09	4.09	4.09	4.09	4.09
7	4.07	4.07	4.07	4.07	4.07
8	4.07	4.07	4.07	4.07	4.07
9	4.06	4.06	4.06	4.06	4.06
10	4.05	4.05	4.05	4.05	4.05
11	4.04	4.04	4.04	4.04	4.04
12	4.04	4.04	4.04	4.04	4.04
13	4.03	4.03	4.03	4.03	4.03
14	4.03	4.03	4.03	4.03	4.03
15	4.03	4.03	4.03	4.03	4.03

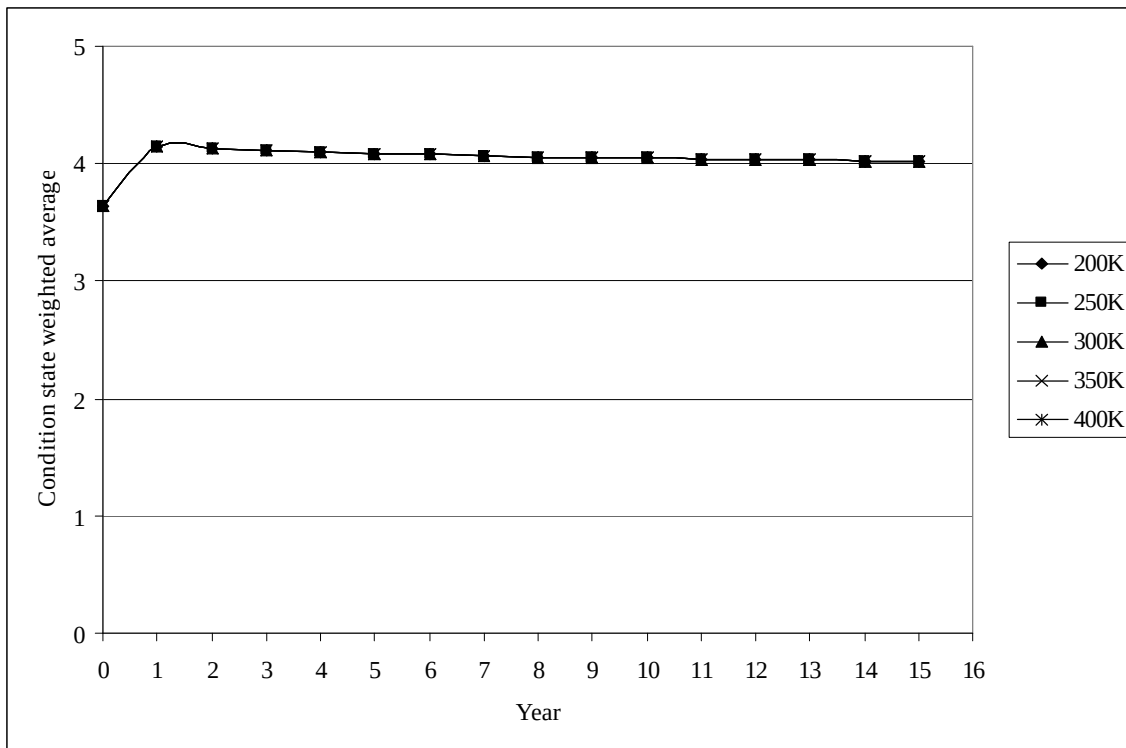


Figure C1.11 Edge metal weighted average condition state sensitivity to budget levels

Table C1.12 Shingle weighted average condition state sensitivity to budget levels

Year	Weighted average condition state				
	200K	250K	300K	350K	400K
1	4.31	4.31	4.31	4.31	4.31
2	4.31	4.31	4.31	4.31	4.31
3	4.29	4.29	4.29	4.29	4.29
4	4.27	4.27	4.27	4.27	4.27
5	4.25	4.26	4.26	4.26	4.26
6	4.25	4.25	4.25	4.25	4.25
7	4.24	4.25	4.25	4.25	4.24
8	4.24	4.24	4.23	4.24	4.34
9	4.23	4.23	4.21	4.23	4.33
10	4.23	4.23	4.22	4.23	4.41
11	4.23	4.23	4.20	4.23	4.48
12	4.22	4.21	4.19	4.33	4.53
13	4.21	4.20	4.18	4.41	4.56
14	4.20	4.20	4.19	4.48	4.60
15	4.20	4.21	4.19	4.53	4.62

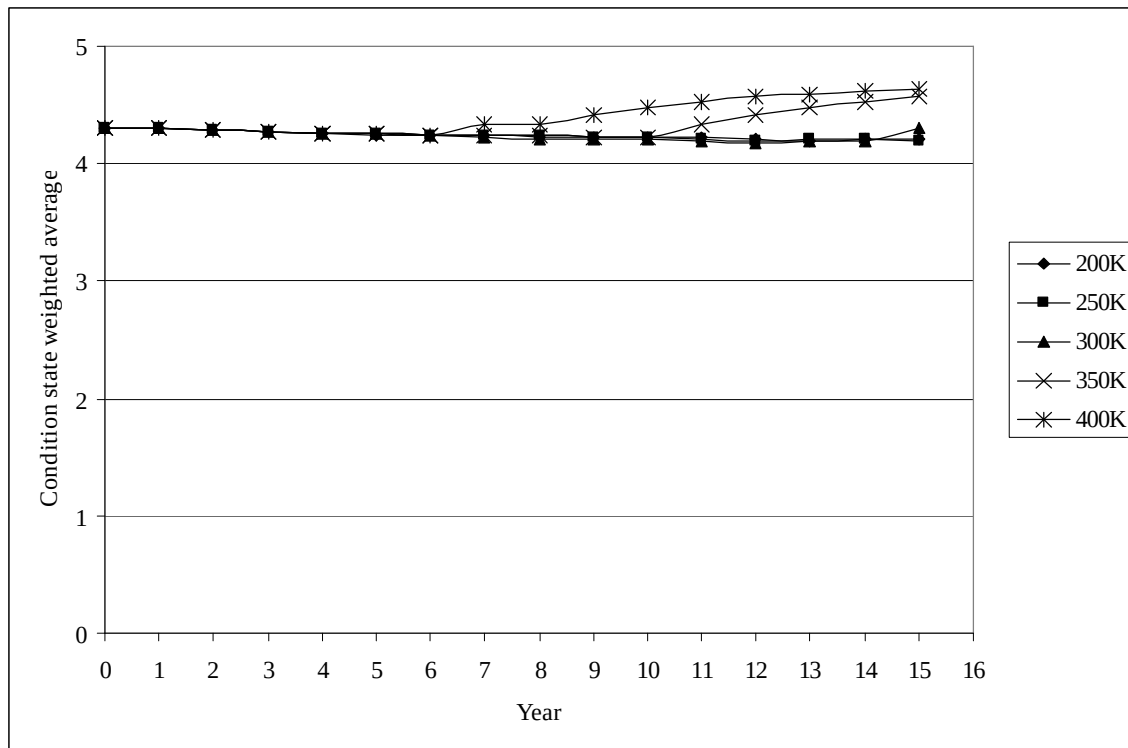


Figure C1.12 Shingle weighted average condition state sensitivity to budget levels

Table C1.13 Slate weighted average condition state sensitivity to budget levels

Year	Weighted average condition state				
	200K	250K	300K	350K	400K
1	4.00	4.00	4.00	4.00	4.00
2	4.00	4.00	4.00	4.00	4.00
3	4.00	4.00	4.00	4.00	4.00
4	4.00	4.00	4.00	4.00	4.00
5	4.00	4.00	4.00	4.00	4.00
6	4.00	4.00	4.00	4.00	4.00
7	4.00	4.00	4.00	4.00	4.00
8	4.00	4.00	4.00	4.00	4.00
9	4.00	4.00	4.00	4.00	4.00
10	4.00	4.00	4.00	4.00	4.00
11	4.00	4.00	4.00	4.00	4.00
12	4.00	4.00	4.00	4.00	4.00
13	4.00	4.00	4.00	4.00	4.00
14	4.00	4.00	4.00	4.00	4.00
15	4.00	4.00	4.00	4.00	4.00

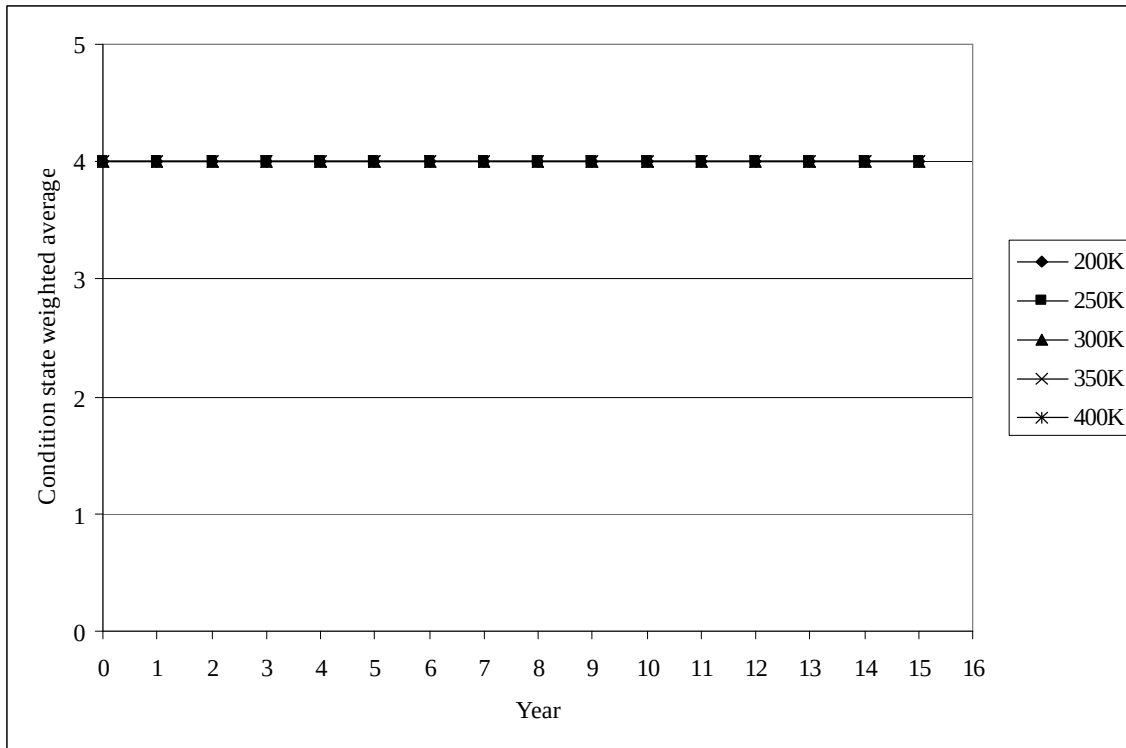


Figure C1.13 Slate weighted average condition state sensitivity to budget levels

Table C1.14 Hip/ridge cap weighted average condition state sensitivity to budget levels

Year	Weighted average condition state				
	200K	250K	300K	350K	400K
1	4.00	4.00	4.00	4.00	4.00
2	4.00	4.00	4.00	4.00	4.00
3	4.00	4.00	4.00	4.00	4.00
4	4.00	4.00	4.00	4.00	4.00
5	4.00	4.00	4.00	4.00	4.00
6	4.00	4.00	4.00	4.00	4.00
7	4.00	4.00	4.00	4.00	4.00
8	4.00	4.00	4.00	4.00	4.00
9	4.00	4.00	4.00	4.00	4.00
10	4.00	4.00	4.00	4.00	4.00
11	4.00	4.00	4.00	4.00	4.00
12	4.00	4.00	4.00	4.00	4.00
13	4.00	4.00	4.00	4.00	4.00
14	4.00	4.00	4.00	4.00	4.00
15	4.00	4.00	4.00	4.00	4.00

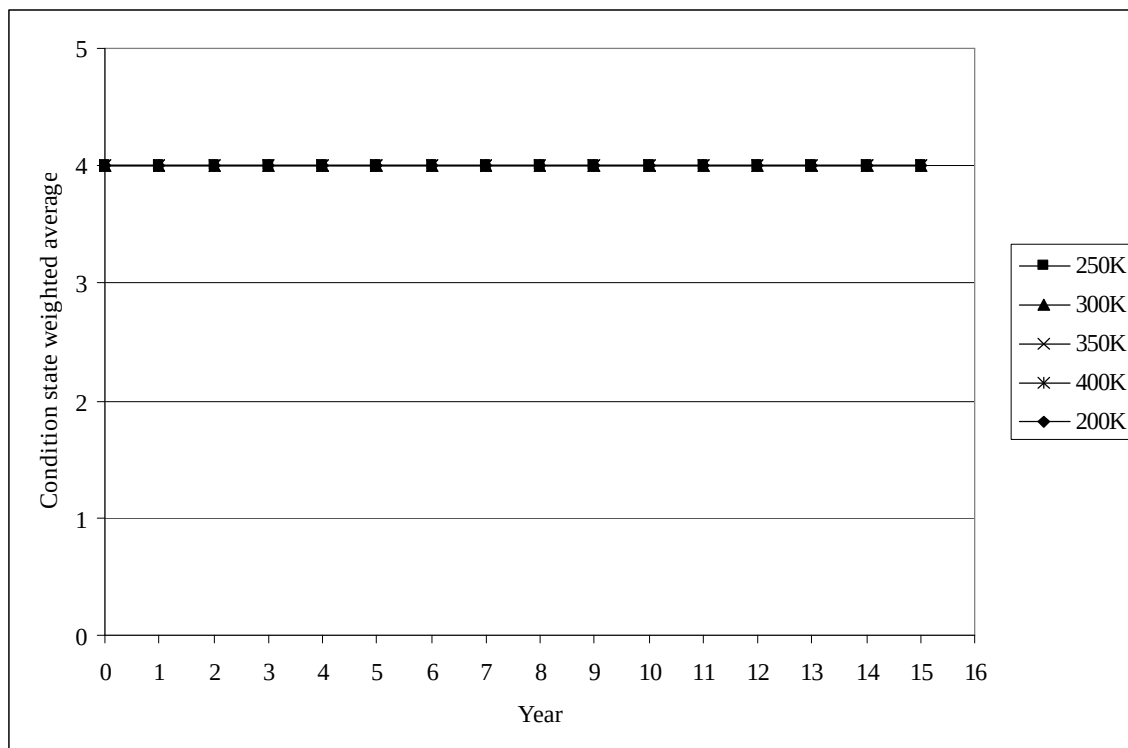


Figure C1.14 Hip/ridge cap weighted average condition state sensitivity to budget levels

APPENDIX C2

This appendix includes tables and graphs showing the weighted average condition state sensitivity to Real Rates of Return of all maintenance elements.

Table C2.1 Single-ply membrane weighted average condition state sensitivity to Real Rate of Return levels

Year	Weighted average condition state				
	1.50%	3.00%	4.50%	6.00%	7.50%
1	3.82	3.82	3.82	3.82	3.82
2	4.04	4.04	4.04	4.04	4.04
3	4.00	3.99	3.99	3.99	3.99
4	4.04	4.03	4.03	4.03	4.03
5	4.11	4.10	4.10	4.10	4.10
6	4.19	4.18	4.18	4.18	4.18
7	4.27	4.26	4.26	4.26	4.26
8	4.33	4.32	4.32	4.32	4.32
9	4.39	4.37	4.37	4.37	4.37
10	4.42	4.41	4.41	4.41	4.41
11	4.44	4.44	4.44	4.44	4.44
12	4.46	4.46	4.46	4.46	4.46
13	4.46	4.46	4.46	4.48	4.48
14	4.50	4.50	4.50	4.50	4.49
15	4.62	4.63	4.63	4.62	4.62

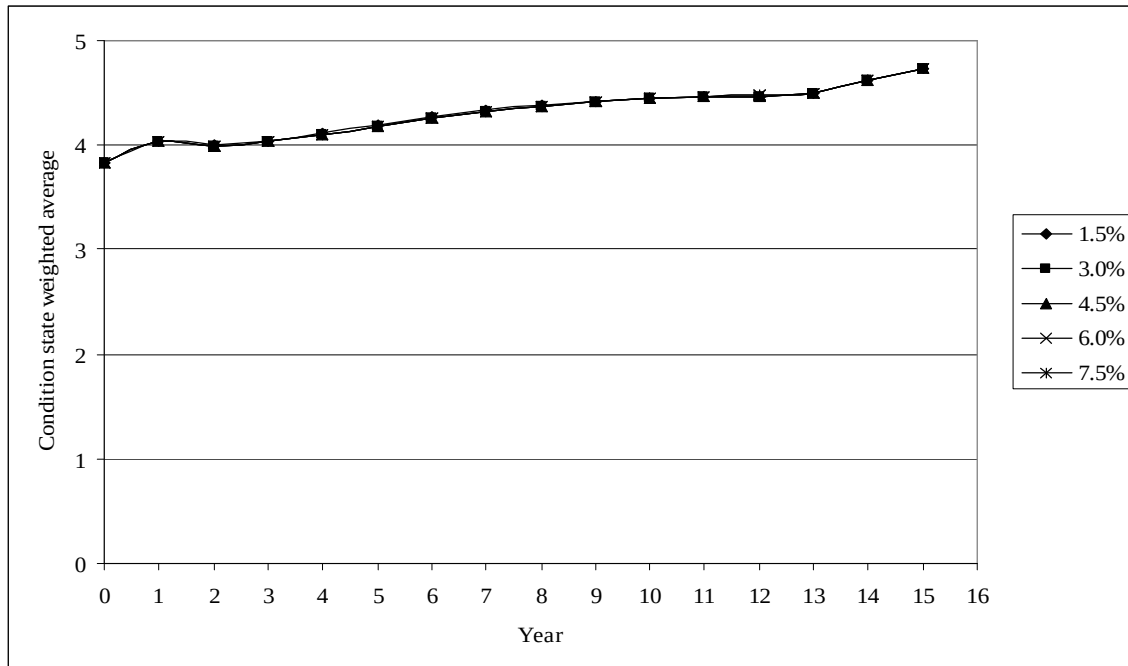


Figure C2.1 Single-ply membrane weighted average condition state sensitivity to Real Rate of Return levels

Table C2.2 Multiple-ply membrane weighted average condition state sensitivity to Real Rate of Return levels

Year	Weighted average condition state				
	1.50%	3.00%	4.50%	6.00%	7.50%
1	3.23	3.23	3.23	3.23	3.23
2	3.23	3.23	3.23	3.23	3.23
3	3.15	3.16	3.16	3.16	3.16
4	2.89	2.91	2.91	2.91	2.98
5	2.72	2.77	2.75	2.75	2.82
6	2.68	2.72	2.71	2.71	2.78
7	2.74	2.78	2.78	2.78	2.86
8	2.92	2.97	2.96	2.97	3.01
9	3.15	3.21	3.20	3.21	3.24
10	3.44	3.48	3.48	3.48	3.51
11	3.76	3.79	3.78	3.79	3.81
12	4.09	4.12	4.10	4.11	4.13
13	4.48	4.50	4.49	4.45	4.56
14	4.75	4.75	4.75	4.75	4.75
15	4.72	4.72	4.72	4.72	4.72

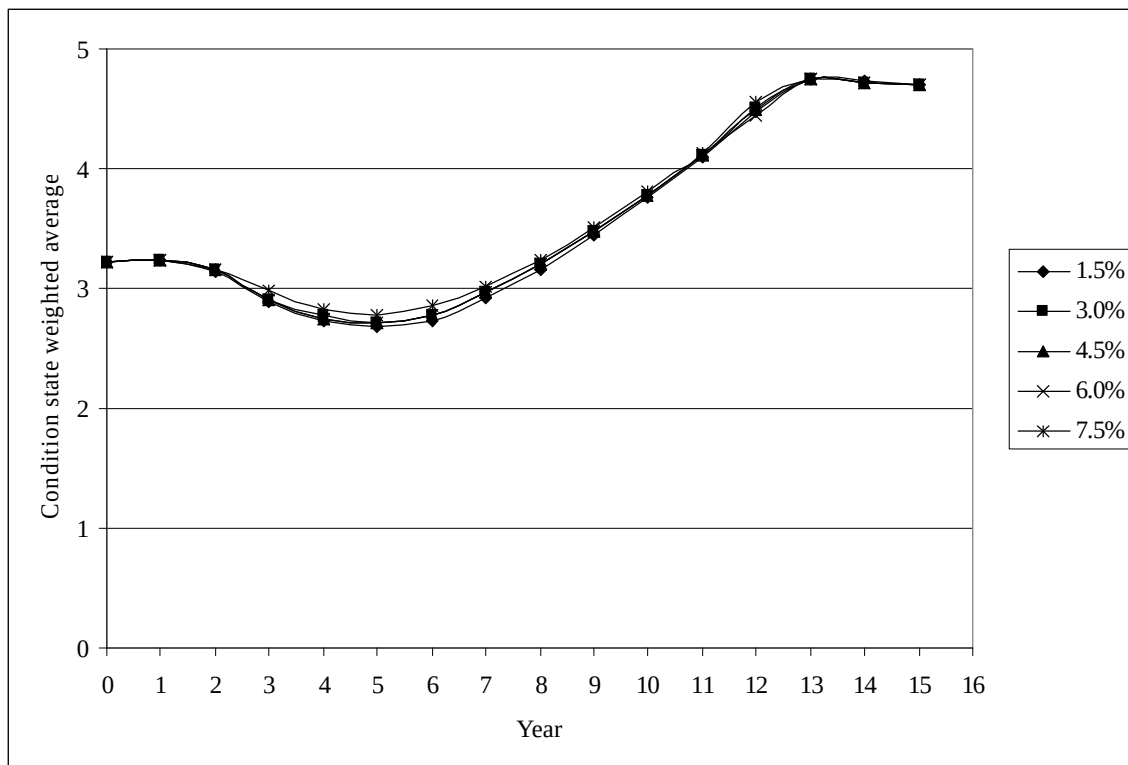


Figure C2.2 Multiple-ply membrane weighted average condition state sensitivity to Real Rate of Return levels

Table C2.3 Single-ply base flashing weighted average condition state sensitivity to Real Rate of Return levels

Year	Weighted average condition state				
	1.50%	3.00%	4.50%	6.00%	7.50%
1	3.76	3.76	3.76	3.76	3.76
2	4.14	4.14	4.14	4.14	4.14
3	4.13	4.13	4.13	4.13	4.13
4	4.12	4.12	4.12	4.12	4.12
5	4.10	4.10	4.10	4.10	4.10
6	4.09	4.09	4.09	4.09	4.09
7	4.09	4.09	4.09	4.09	4.09
8	4.08	4.08	4.08	4.08	4.08
9	4.07	4.07	4.07	4.07	4.07
10	4.06	4.06	4.06	4.06	4.06
11	4.06	4.06	4.06	4.06	4.06
12	4.05	4.05	4.05	4.05	4.05
13	4.05	4.05	4.05	4.05	4.05
14	4.04	4.04	4.04	4.04	4.04
15	4.04	4.04	4.04	4.04	4.04

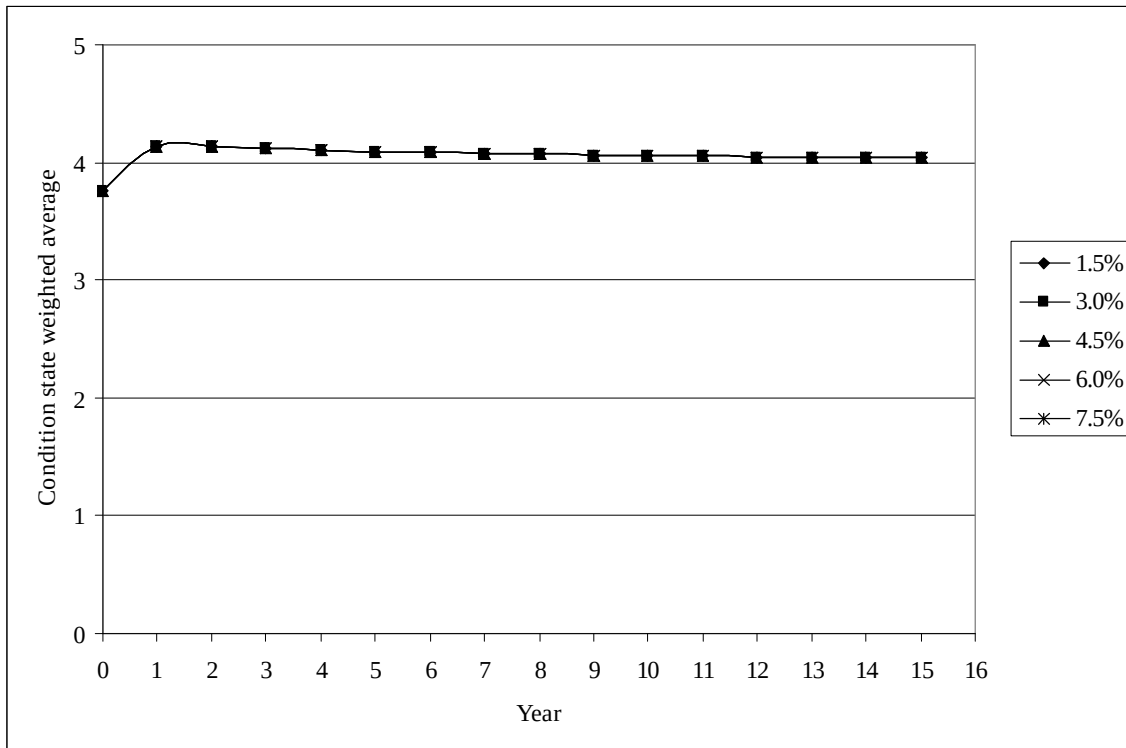


Figure C2.3 Single-ply base flashing weighted average condition state sensitivity to Real Rate of Return levels

Table C2.4 Multiple-ply base flashing weighted average condition state sensitivity to Real Rate of Return levels

Year	Weighted average condition state				
	1.50%	3.00%	4.50%	6.00%	7.50%
1	3.26	3.26	3.26	3.26	3.26
2	4.23	4.23	4.23	4.23	4.23
3	4.22	4.22	4.22	4.22	4.22
4	4.20	4.20	4.20	4.20	4.20
5	4.18	4.18	4.18	4.18	4.18
6	4.17	4.17	4.17	4.17	4.17
7	4.16	4.16	4.16	4.16	4.16
8	4.14	4.14	4.14	4.14	4.14
9	4.13	4.13	4.13	4.13	4.13
10	4.12	4.12	4.12	4.12	4.12
11	4.11	4.11	4.11	4.11	4.11
12	4.10	4.10	4.10	4.10	4.10
13	4.10	4.10	4.10	4.10	4.10
14	4.09	4.09	4.09	4.09	4.09
15	4.08	4.08	4.08	4.08	4.08

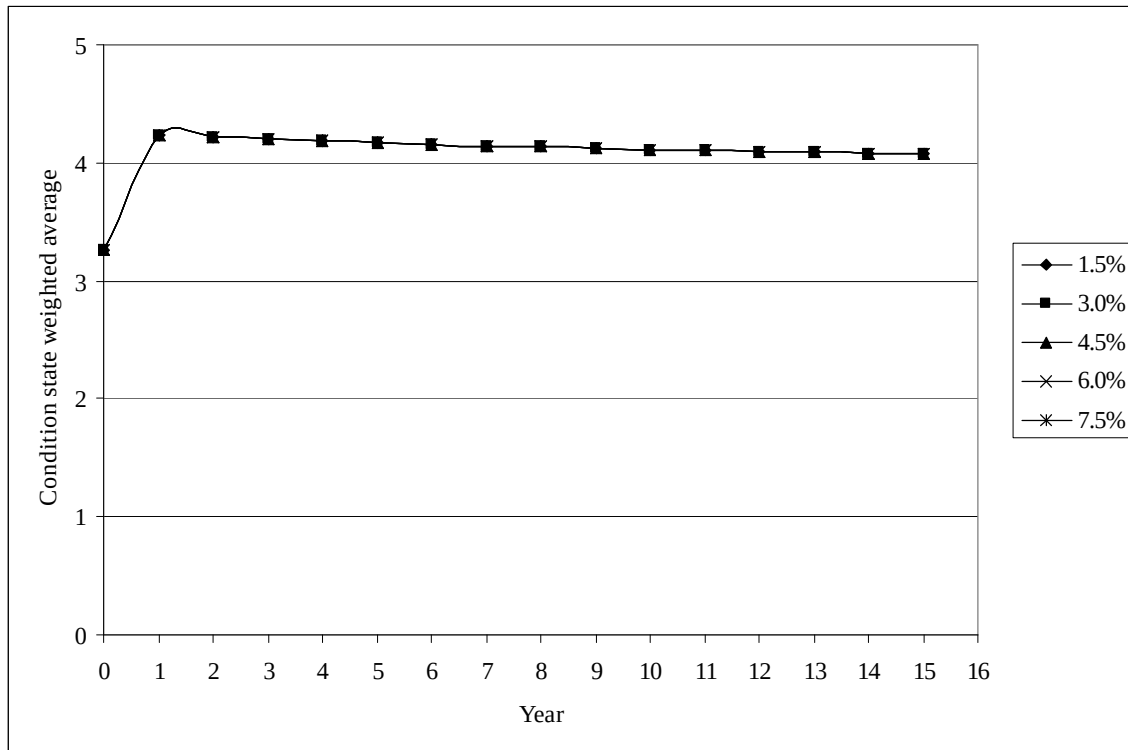


Figure C2.4 Multiple-ply base flashing weighted average condition state sensitivity to Real Rate of Return levels

Table C2.5 Counter flashing weighted average condition state sensitivity to Real Rate of Return levels

Year	Weighted average condition state				
	1.50%	3.00%	4.50%	6.00%	7.50%
1	3.68	3.68	3.68	3.68	3.68
2	4.15	4.15	4.15	4.15	4.15
3	4.12	4.12	4.12	4.12	4.12
4	4.10	4.10	4.10	4.10	4.10
5	4.09	4.09	4.09	4.09	4.09
6	4.07	4.07	4.07	4.07	4.07
7	4.06	4.06	4.06	4.06	4.06
8	4.05	4.05	4.05	4.05	4.05
9	4.04	4.04	4.04	4.04	4.04
10	4.04	4.04	4.04	4.04	4.04
11	4.03	4.03	4.03	4.03	4.03
12	4.02	4.02	4.02	4.02	4.02
13	4.02	4.02	4.02	4.02	4.02
14	4.02	4.02	4.02	4.02	4.02
15	4.01	4.01	4.01	4.01	4.01

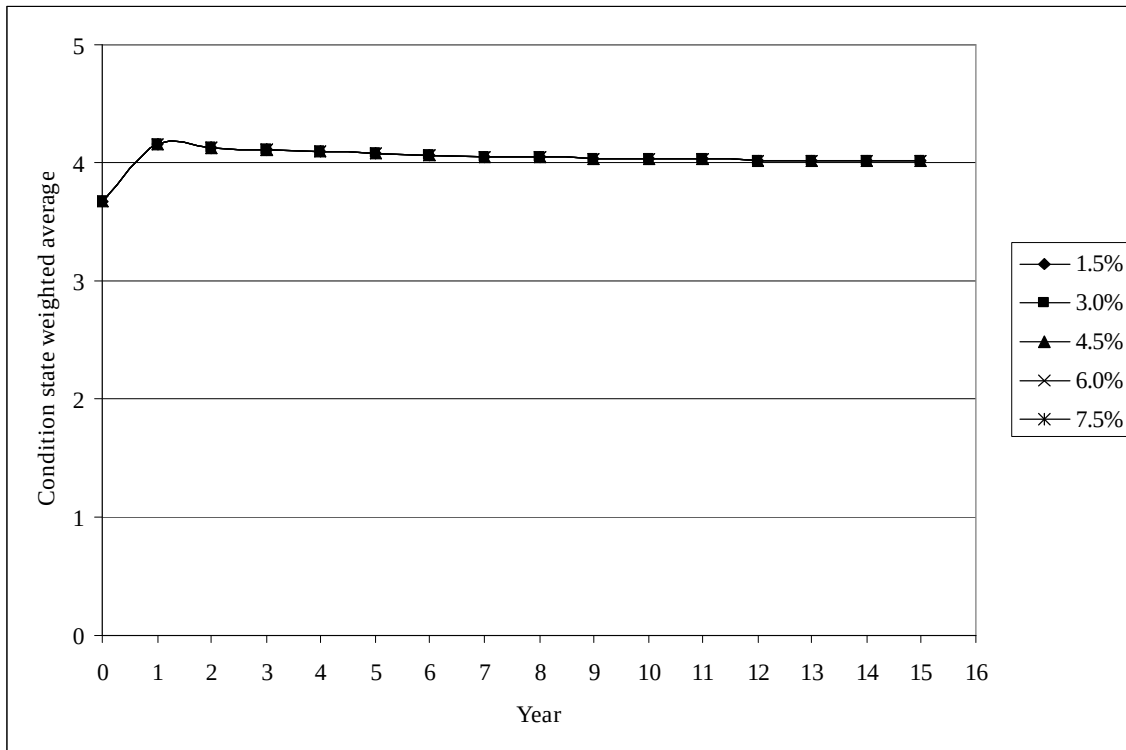


Figure C2.5 Counter flashing weighted average condition state sensitivity to Real Rate of Return levels

Table C2.6 Gutter weighted average condition state sensitivity to Real Rate of Return levels

Year	Weighted average condition state				
	1.50%	3.00%	4.50%	6.00%	7.50%
1	4.04	4.04	4.04	4.04	4.04
2	4.04	4.05	4.05	4.04	4.05
3	4.03	4.04	4.04	4.03	4.04
4	4.03	4.04	4.04	4.02	4.04
5	4.03	4.03	4.03	4.02	4.03
6	4.03	4.02	4.02	4.01	4.02
7	4.02	4.02	4.02	4.01	4.02
8	4.02	4.02	4.02	4.00	4.02
9	4.01	4.01	4.01	4.00	4.01
10	4.01	4.01	4.01	4.00	4.01
11	4.01	4.01	4.01	4.00	4.01
12	4.01	4.01	4.01	3.99	4.01
13	4.01	4.01	4.01	3.99	4.01
14	4.01	4.00	4.00	4.01	4.00
15	4.00	4.00	4.00	4.01	4.00

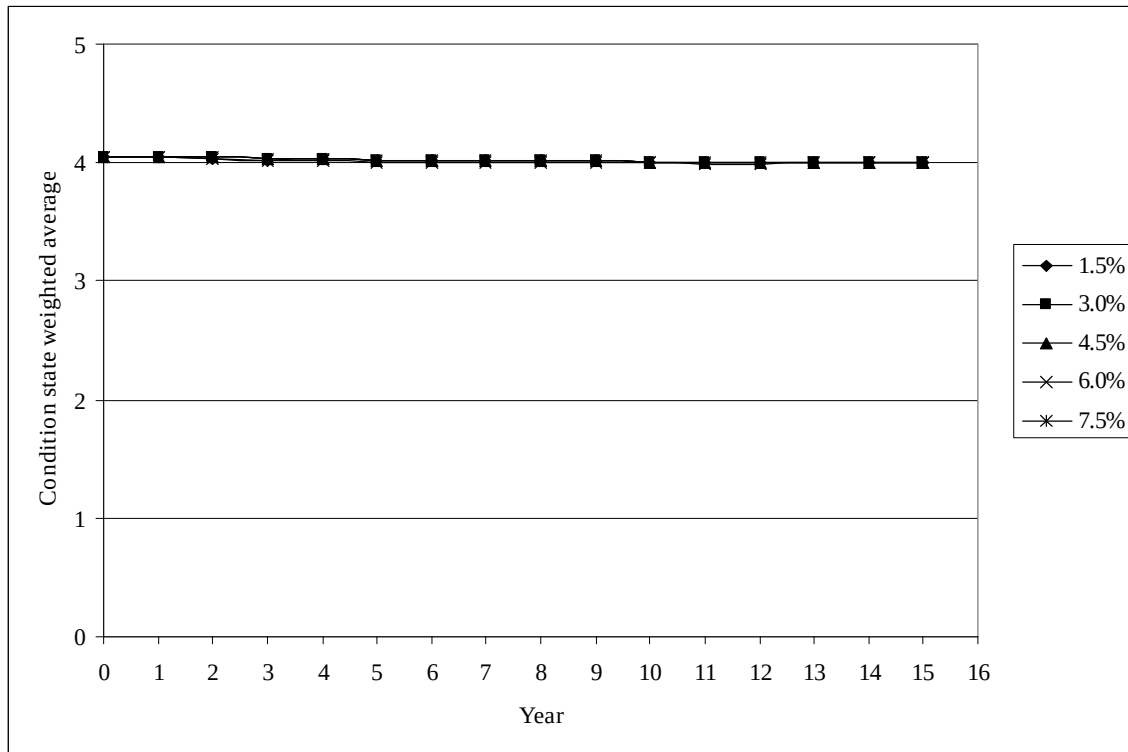


Figure C2.6 Gutter weighted average condition state sensitivity to Real Rate of Return levels

Table C2.7 Downspout weighted average condition state sensitivity to Real Rate of Return levels

Year	Weighted average condition state				
	1.50%	3.00%	4.50%	6.00%	7.50%
1	4.03	4.03	4.03	4.03	4.03
2	4.03	4.03	4.03	4.03	4.03
3	4.03	4.03	4.03	4.03	4.03
4	4.02	4.02	4.02	4.02	4.02
5	4.02	4.02	4.02	4.02	4.02
6	4.01	4.01	4.01	4.01	4.01
7	4.01	4.01	4.01	4.01	4.01
8	4.01	4.01	4.01	4.01	4.01
9	4.01	4.01	4.01	4.01	4.01
10	4.01	4.01	4.01	4.01	4.01
11	4.01	4.01	4.01	4.01	4.01
12	4.00	4.00	4.00	4.00	4.00
13	4.00	4.00	4.00	4.00	4.00
14	4.00	4.00	4.00	4.00	4.00
15	4.00	4.00	4.00	4.00	4.00

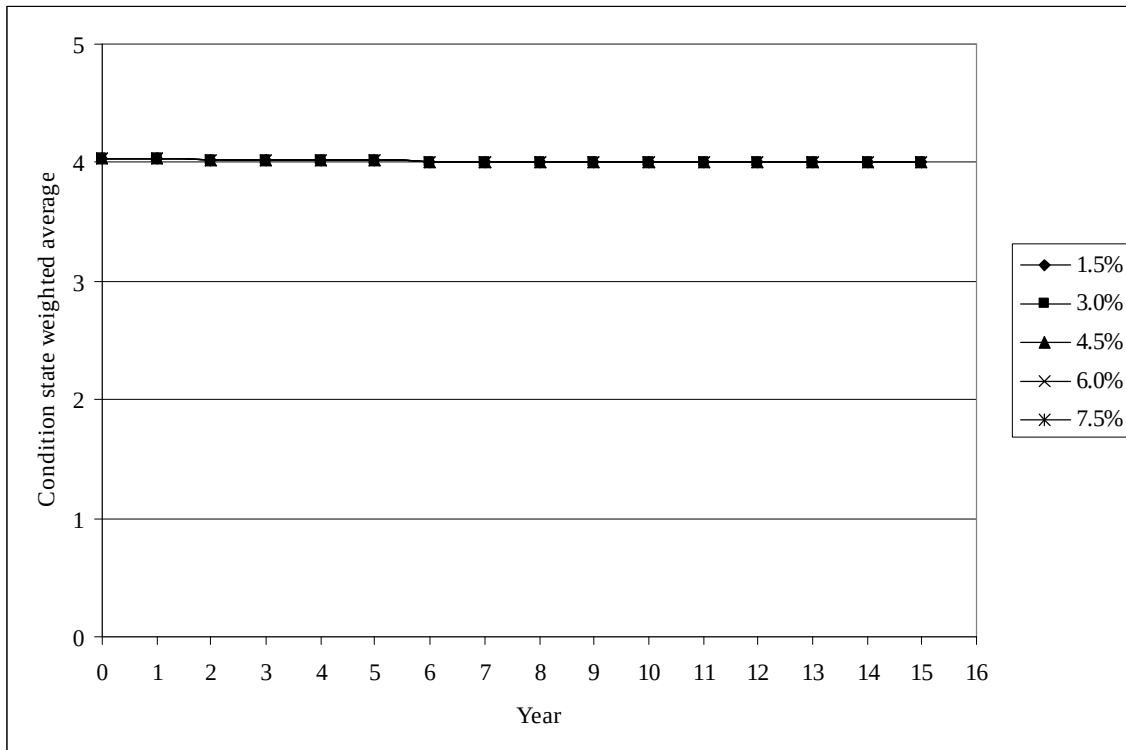


Figure C2.7 Downspout weighted average condition state sensitivity to Real Rate of Return levels

Table C2.8 Drain Strainer weighted average condition state sensitivity to Real Rate of Return levels

Year	Weighted average condition state				
	1.50%	3.00%	4.50%	6.00%	7.50%
1	3.97	3.97	3.97	3.97	3.97
2	4.03	4.03	4.03	4.03	4.03
3	4.02	4.02	4.02	4.02	4.02
4	4.02	4.02	4.02	4.02	4.02
5	4.02	4.02	4.02	4.02	4.02
6	4.01	4.01	4.01	4.01	4.01
7	4.01	4.01	4.01	4.01	4.01
8	4.01	4.01	4.01	4.01	4.01
9	4.01	4.01	4.01	4.01	4.01
10	4.01	4.01	4.01	4.01	4.01
11	4.01	4.01	4.01	4.01	4.01
12	4.01	4.01	4.01	4.01	4.01
13	4.01	4.01	4.01	4.01	4.01
14	4.00	4.00	4.00	4.00	4.00
15	4.00	4.00	4.00	4.00	4.00

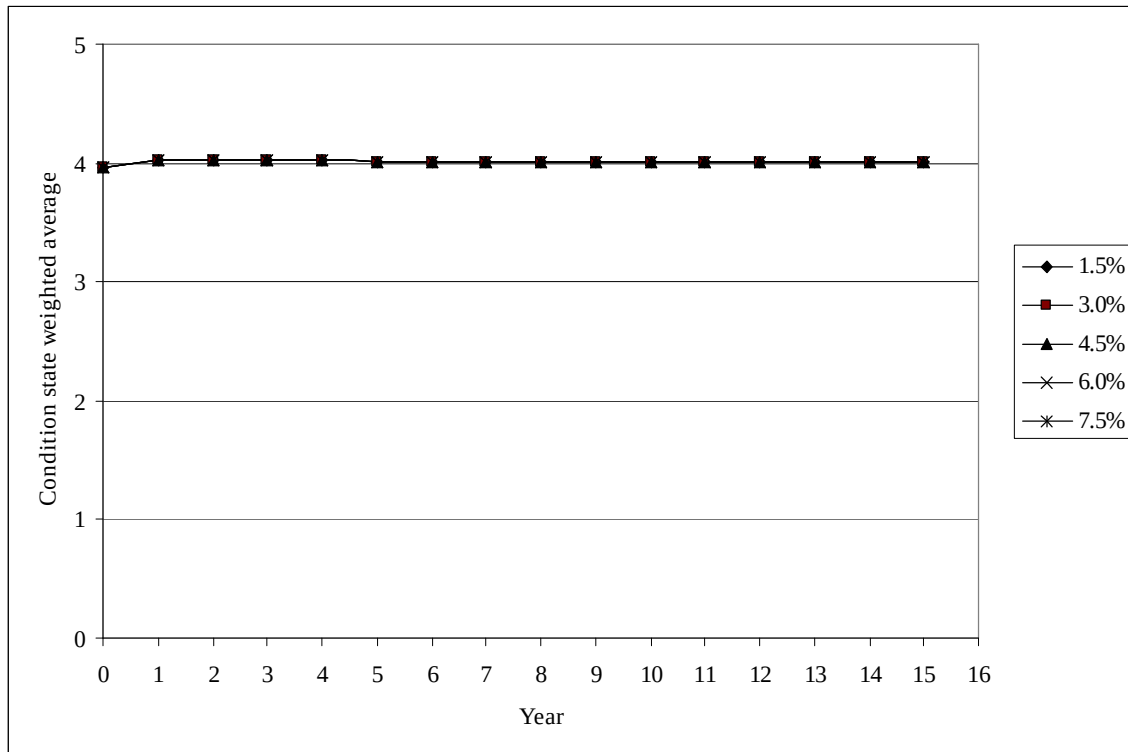


Figure C2.8 Drain Strainer weighted average condition state sensitivity to Real Rate of Return levels

Table C2.9 Coping cap weighted average condition state sensitivity to Real Rate of Return levels

Year	Weighted average condition state				
	1.50%	3.00%	4.50%	6.00%	7.50%
1	4.02	4.02	4.02	4.02	4.02
2	4.03	4.02	4.03	4.03	4.03
3	4.02	4.01	4.02	4.02	4.02
4	4.02	4.01	4.02	4.02	4.02
5	4.02	4.01	4.02	4.02	4.02
6	4.01	4.00	4.01	4.01	4.01
7	4.01	4.00	4.01	4.01	4.01
8	4.01	4.00	4.01	4.01	4.01
9	4.01	4.00	4.01	4.01	4.01
10	4.01	4.00	4.01	4.01	4.01
11	4.01	4.00	4.01	4.01	4.01
12	4.01	4.00	4.01	4.01	4.01
13	4.00	4.00	4.00	4.00	3.85
14	4.00	4.01	4.00	4.00	4.05
15	4.00	4.01	4.00	4.00	4.05

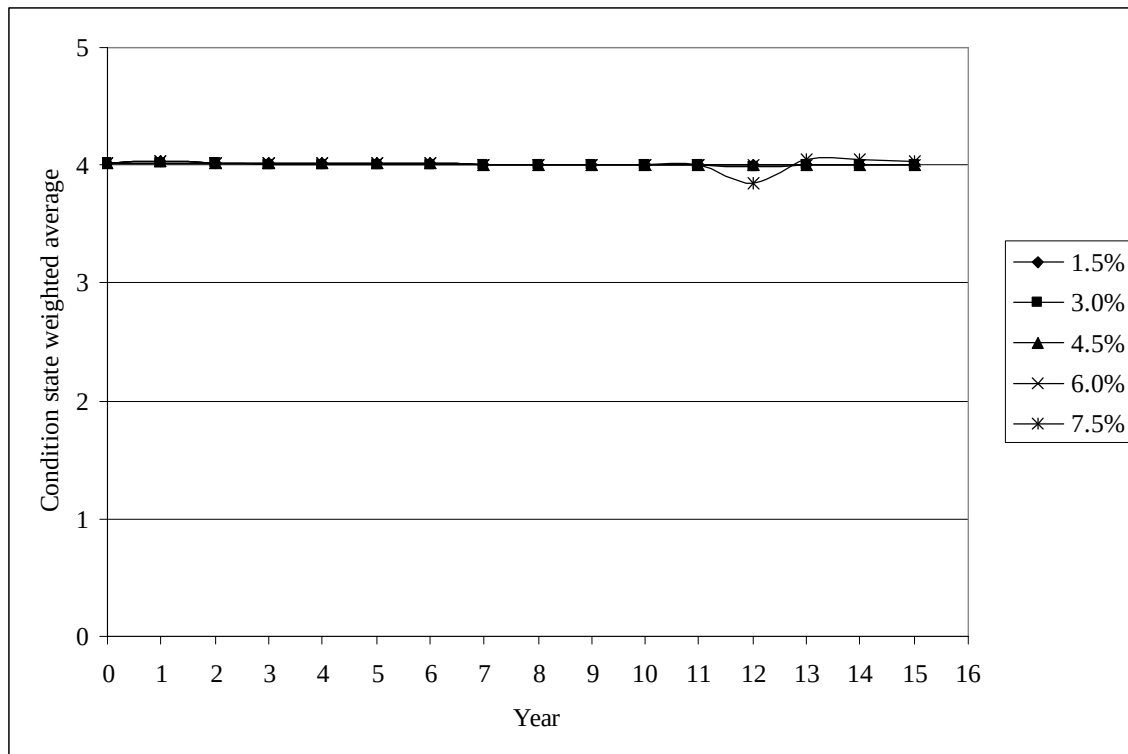


Figure C2.9 Coping cap weighted average condition state sensitivity to Real Rate of Return levels

Table C2.10 Termination bar weighted average condition state sensitivity to Real Rate of Return levels

Year	Weighted average condition state				
	1.50%	3.00%	4.50%	6.00%	7.50%
1	4.00	4.00	4.00	4.00	4.00
2	4.01	4.01	4.01	4.01	4.01
3	4.01	4.01	4.01	4.01	4.01
4	4.01	4.01	4.01	4.01	4.01
5	4.01	4.01	4.01	4.01	4.01
6	4.00	4.00	4.00	4.00	4.00
7	4.00	4.00	4.00	4.00	4.00
8	4.00	4.00	4.00	4.00	4.00
9	4.00	4.00	4.00	4.00	4.00
10	4.00	4.00	4.00	4.00	4.00
11	4.00	4.00	4.00	4.00	4.00
12	4.00	4.00	4.00	4.00	4.00
13	4.00	4.00	4.00	4.00	4.00
14	4.00	4.00	4.00	4.00	4.00
15	4.00	4.00	4.00	4.00	4.00

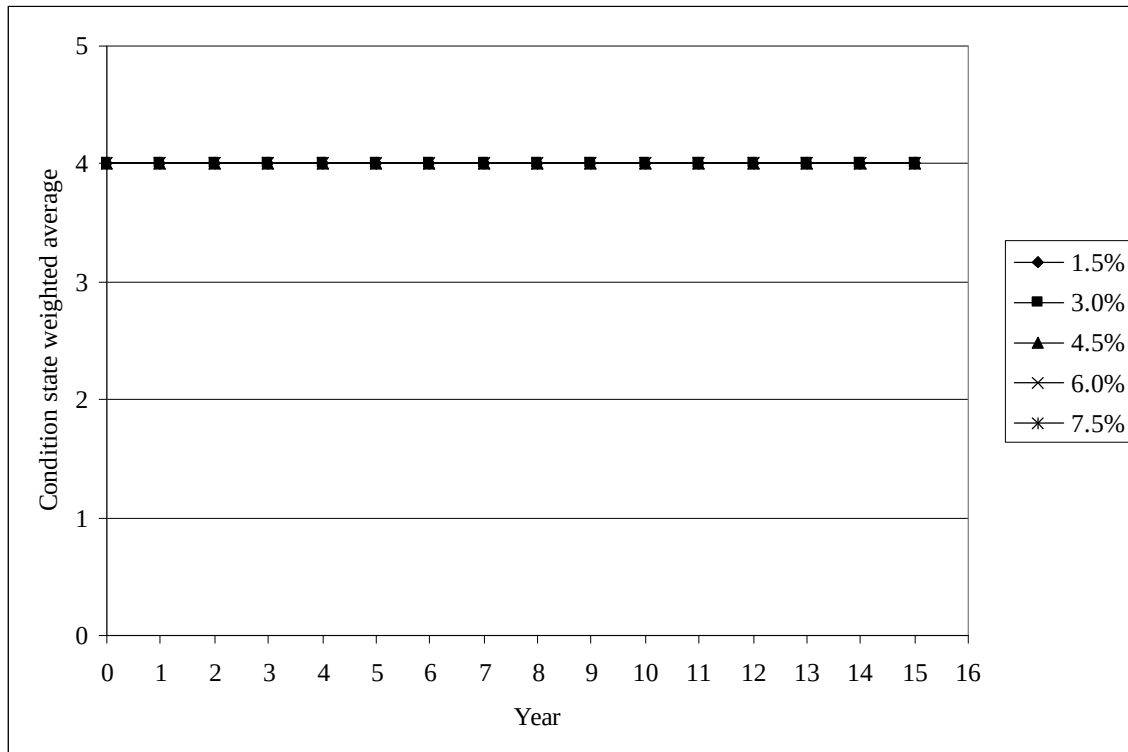


Figure C2.10 Termination bar weighted average condition state sensitivity to Real Rate of Return levels

Table C2.11 Edge metal weighted average condition state sensitivity to Real Rate of Return levels

Year	Weighted average condition state				
	1.50%	3.00%	4.50%	6.00%	7.50%
1	3.64	3.64	3.64	3.64	3.64
2	4.15	4.15	4.15	4.15	4.15
3	4.13	4.13	4.13	4.13	4.13
4	4.11	4.11	4.11	4.11	4.11
5	4.10	4.10	4.10	4.10	4.10
6	4.09	4.09	4.09	4.09	4.09
7	4.07	4.07	4.07	4.07	4.07
8	4.07	4.07	4.07	4.07	4.07
9	4.06	4.06	4.06	4.06	4.06
10	4.05	4.05	4.05	4.05	4.05
11	4.04	4.04	4.04	4.04	4.04
12	4.04	4.04	4.04	4.04	4.04
13	4.03	4.03	4.03	4.03	4.03
14	4.03	4.03	4.03	4.03	4.03
15	4.03	4.03	4.03	4.03	4.03

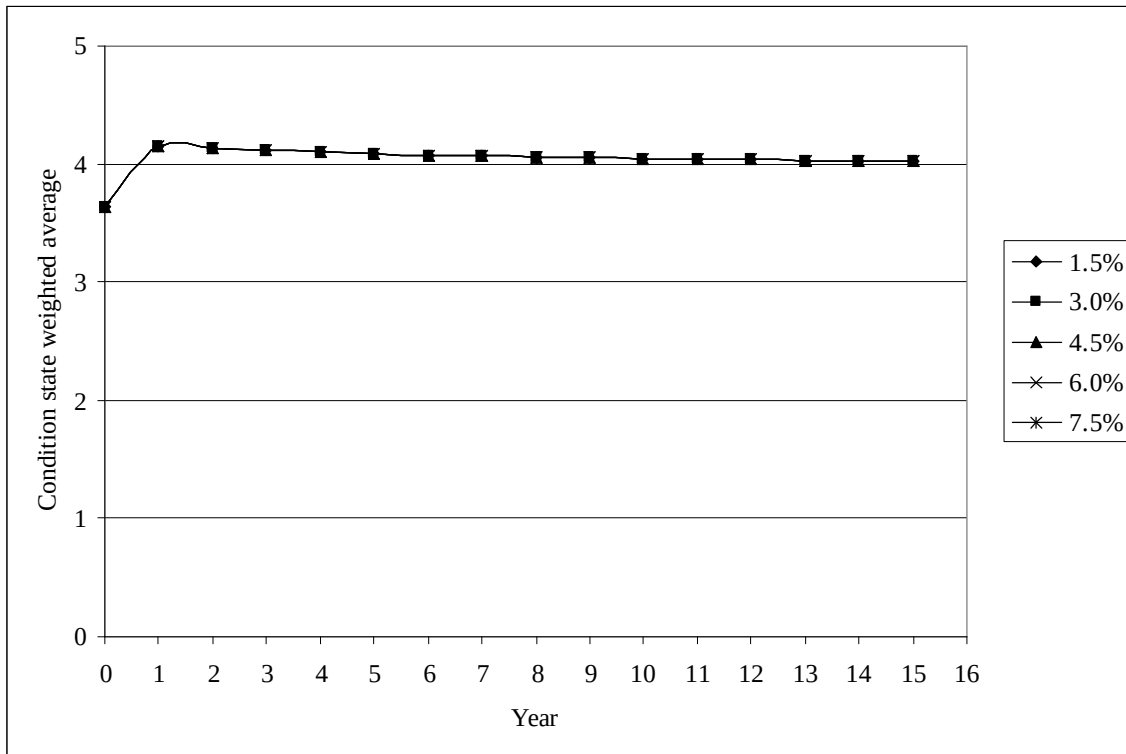


Figure C2.11 Edge metal weighted average condition state sensitivity to Real Rate of Return levels

Table C2.12 Shingle weighted average condition state sensitivity to Real Rate of Return levels

Year	Weighted average condition state				
	1.50%	3.00%	4.50%	6.00%	7.50%
1	4.31	4.31	4.31	4.31	4.31
2	4.31	4.31	4.31	4.31	4.31
3	4.29	4.29	4.29	4.29	4.29
4	4.27	4.27	4.27	4.27	4.27
5	4.26	4.26	4.26	4.26	4.26
6	4.25	4.25	4.25	4.25	4.25
7	4.25	4.25	4.25	4.24	4.25
8	4.24	4.23	4.24	4.22	4.24
9	4.23	4.21	4.22	4.21	4.23
10	4.23	4.22	4.22	4.21	4.22
11	4.22	4.20	4.22	4.20	4.22
12	4.22	4.19	4.22	4.20	4.21
13	4.20	4.18	4.21	4.19	4.19
14	4.21	4.19	4.21	4.20	4.20
15	4.21	4.19	4.20	4.20	4.20

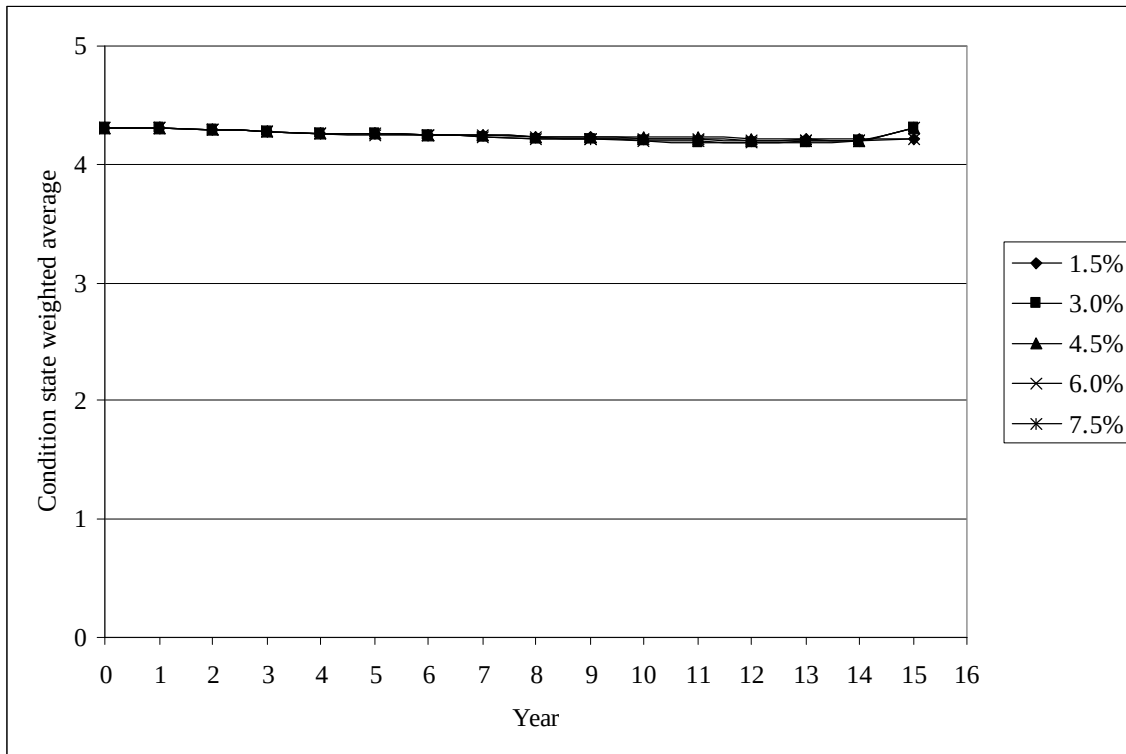


Figure C2.12 Shingle weighted average condition state sensitivity to Real Rate of Return levels

Table C2.13 Slate weighted average condition state sensitivity to Real Rate of Return levels

Year	Weighted average condition state				
	1.50%	3.00%	4.50%	6.00%	7.50%
1	4.00	4.00	4.00	4.00	4.00
2	4.00	4.00	4.00	4.00	4.00
3	4.00	4.00	4.00	4.00	4.00
4	4.00	4.00	4.00	4.00	4.00
5	4.00	4.00	4.00	4.00	4.00
6	4.00	4.00	4.00	4.00	4.00
7	4.00	4.00	4.00	4.00	4.00
8	4.00	4.00	4.00	4.00	4.00
9	4.00	4.00	4.00	4.00	4.00
10	4.00	4.00	4.00	4.00	4.00
11	4.00	4.00	4.00	4.00	4.00
12	4.00	4.00	4.00	4.00	4.00
13	4.00	4.00	4.00	4.00	3.96
14	4.00	4.00	4.00	4.00	3.99
15	4.00	4.00	4.00	4.00	4.00

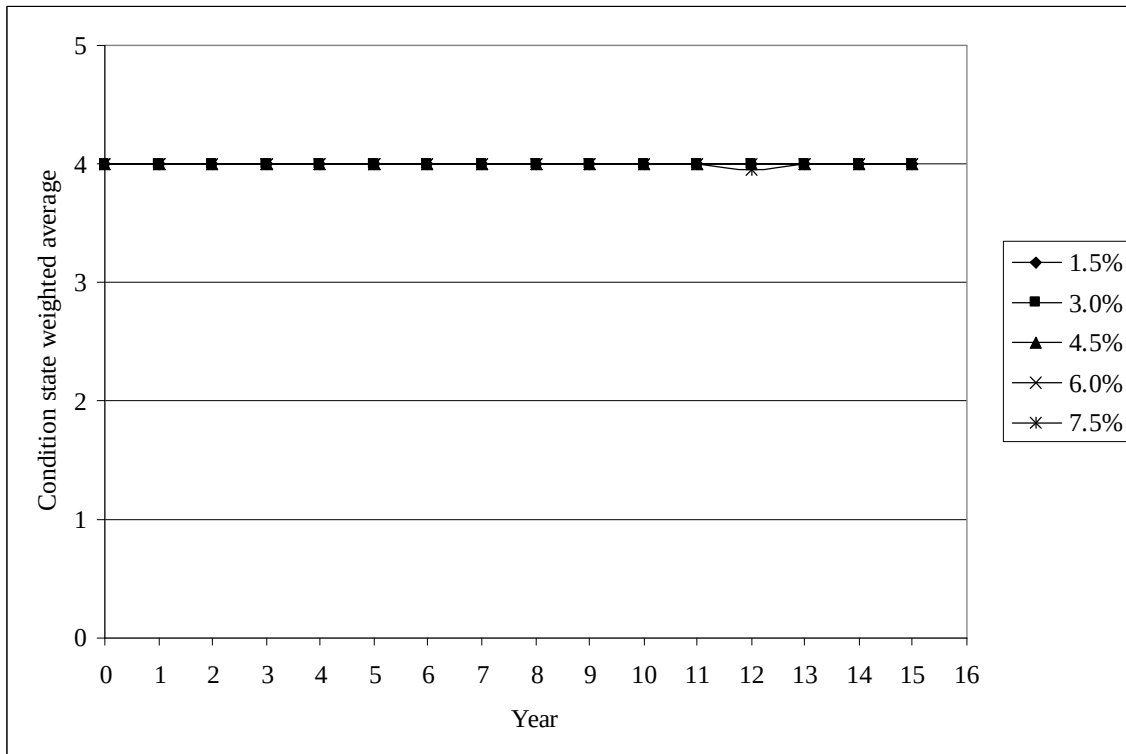


Figure C2.13 Slate weighted average condition state sensitivity to Real Rate of Return levels

Table C2.14 Hip/ridge cap weighted average condition state sensitivity to Real Rate of Return levels

Year	Weighted average condition state				
	1.50%	3.00%	4.50%	6.00%	7.50%
1	4.00	4.00	4.00	4.00	4.00
2	4.00	4.00	4.00	4.00	4.00
3	4.00	4.00	4.00	4.00	4.00
4	4.00	4.00	4.00	4.00	4.00
5	4.00	4.00	4.00	4.00	4.00
6	4.00	4.00	4.00	4.00	4.00
7	4.00	4.00	4.00	4.00	4.00
8	4.00	4.00	4.00	4.00	4.00
9	4.00	4.00	4.00	4.00	4.00
10	4.00	4.00	4.00	4.00	4.00
11	4.00	4.00	4.00	4.00	4.00
12	4.00	4.00	4.00	4.00	4.00
13	4.00	4.00	4.00	4.00	4.00
14	4.00	4.00	4.00	4.00	4.00
15	4.00	4.00	4.00	4.00	4.00

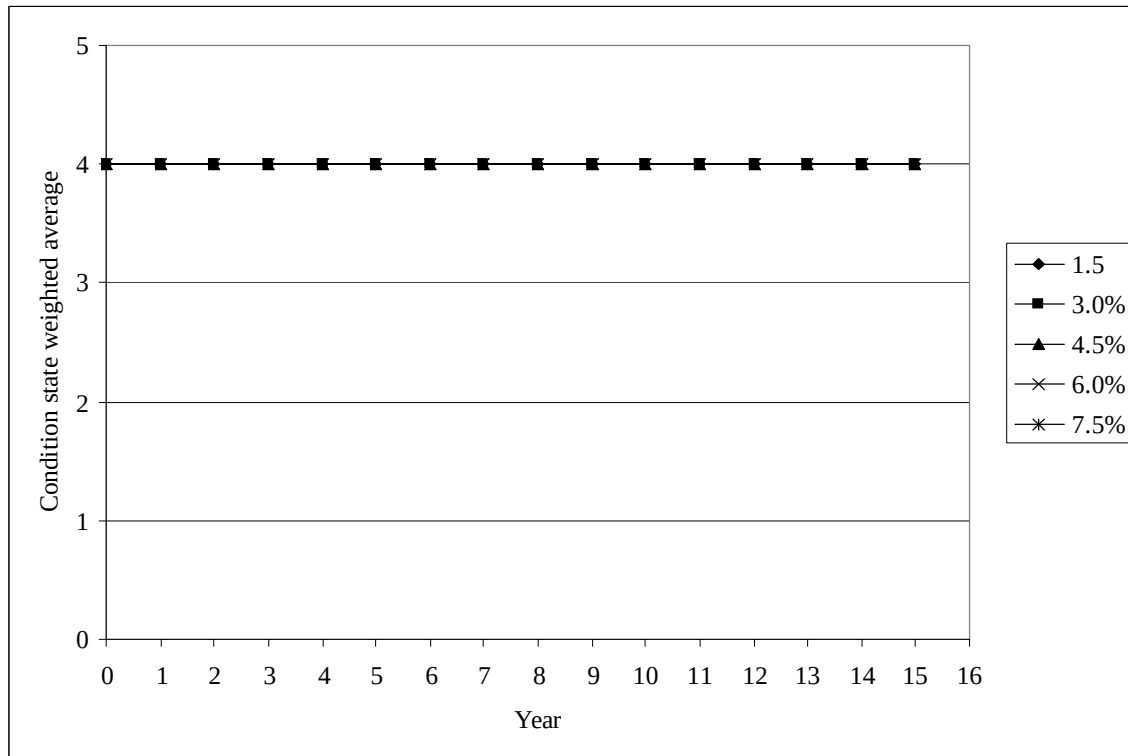


Figure C2.14 Hip/ridge cap weighted average condition state sensitivity to Real Rate of Return levels