

ABSTRACT

YUMOTO, YUJI. Risk Preference, Correlation Choice, Sabotage, and the Design of Promotion Tournaments. (Under the direction of David J. Flath.)

I examine properties of worker behavior under promotion tournaments, and discuss their implications for the design of promotion tournaments. Given their tasks, workers will decide how much effort they exert. In addition to this decision-making, they have the opportunity to decide their approaches to perform their tasks within their delegated authority. I define the term approach as a method, way, procedure, plan or project to perform his task, into which a worker infuses his effort. Through their approach choices, they can control the riskiness and correlation of their performances. It is shown that under the loser-selecting tournament, which means the promotion ratio is more than one-half, workers prefer a low risk approach or a common approach which peers also know well; on the other hand, under the winner-selecting tournament, which means the promotion ratio is less than one-half, they prefer a high risk approach or their own original approaches. These results suggest that the loser-selecting tournament is more efficient than the winner-selecting tournament in terms of the cost for implementing high efforts of risk-averse workers. I rigorously show this in the case of three workers. I argue that the winner-selecting tournament is better suited for the upper job levels or for firms in innovative and immature markets or industries; on the other hand, the loser-selecting tournament is better suited for the lower job levels or for firms in stable, mature, or strictly regulated markets or industries. Furthermore, I investigate properties of sabotage under tournaments. Under the one-winner tournament, each worker attacks his peers so as to minimize the maximal value of their expected performances. The sabotage operates to create more homogeneity. Under the one-loser tournament, each worker intensively attacks one peer so as to minimize the minimal value of his peers' expected performance. One worker is intensively attacked by all peers. I argue that the one-loser tournament is more subject to damage by sabotage than the one-winner tournament.

**RISK PREFERENCE, CORRELATION CHOICE, SABOTAGE, AND
THE DESIGN OF PROMOTION TOURNAMENTS**

by

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To my wife, Junko, and my sons, Masaya and Shinya

Biography

Yuji Yumoto was born in Nagano, Japan in 1962 to Norishige and Sumie Yumoto. In 1981, he graduated from Suzaka High School and enrolled in Kyoto University. In 1985, he earned a Bachelor of Arts in Economics and began his graduate studies in economics at Kyoto University. He received a Master of Arts in Economics in 1987. In 1991, he became an assistant professor at the School of Business Administration at Nanzan University. Getting financial support from Nanzan University, he entered the Graduate School at North Carolina State University to study economics in the fall of 1993. In 1996, he returned to Japan to work at Nanzan University. In 1999, he became an associate professor at Nanzan University.

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Chapter 1

Introduction

Textbooks of human resource management, labor economics, or personnel economics always refer to promotion as a device to induce workers' efforts through competition among them.¹ According to this view, promotion competition among n workers for m vacancies of the next higher rank can be regarded as a simple tournament under which m agents become winners and the others become losers. One of typical examples is the competition for CEO of a corporation: there is just one winner and there are many losers.² Another good example is the military promotion system. Asch and Warner (2001) research the personnel system of the United States Military. For example, officers' promotion decisions over O-3 level are made by centralized promotion boards. These promotions are nationally competitive and based on vacancies to the next higher rank. Officers are considered for promotion by entry-year group and either promoted or not promoted at specific years of service points. The O-4, O-5, O-6, and O-7 promotion rates are about 80%, 60%, 40%, and 7%, respectively.

In this paper I examine properties of workers' behavior under this simple promotion tournament, and discuss their implications for the design of promotion tournaments.

Given their tasks, workers will decide how much effort they make such as in the

¹For example, see Lazear (1998), Ehrenberg and Smith (2000), or Baron and Kreps (1999). Gibbons and Waldman (1999) offer an excellent survey of theory and evidence of careers including promotions in firms.

²To be precise, in the competition for the position of CEO, some contestants are insiders who pursue the promotion to the top and others are outsiders who would join the firm only if they were selected as CEO. See Tsoulouhas, Knoeber, Agrawal (2002) and Chen (1996) for theoretical studies, and Agrawal, Knoeber, Tsoulouhas (2003), and Parrion (1997) for empirical evidence.

standard agency model. In addition to this private decision-making, they have the opportunity to decide how to make efforts to perform their tasks within their delegated authority. I give an example. Consider an ambulance driver who carries an emergency case to a hospital. There are two routes available. One is a direct route, but sometimes has heavy traffic. The other is an indirect route, a long distance, but rarely has heavy traffic. He must decide two things. First, he must choose between the two routes. Second, he must choose how much effort he exerts to drive speedily but carefully. I call the first one the decision on “approach.” I define the term “approach” as a method, way, procedure, plan or project to perform his task, into which a worker infuses his efforts. A salesperson can choose to reinforce the relationship with his current customers, or can try to develop new big customers. Also he can continue to use the sales technique learned in on-the-job or off-the-job training, or can try his new ideas to sell more. A product or brand manager can maintain the current concept, or can develop a new concept. A scholar can choose a reliable research topic, or can try a challenging one... As these examples show, workers can choose the riskiness of their performances, which affects their rewards under the promotion tournament, to a greater or lesser degree, mainly within their authority. In Chapter 3, I examine the properties of their risk choices under promotion tournaments, and argue their implications for the design of promotion tournaments. Their risk choices depend on promotion ratio m/n . When the promotion ratio is less than one-half, *i.e.*, $2 < n/2$, the tournament is called “winner-selecting” because only a small part of the contestants can become winners.³ On the other hand, when the ratio is more than one-half, *i.e.*, $m > n/2$, it is called “loser-selecting” because only a small part of the contestants can become losers.⁴ I show that workers strictly prefer a high risk approach under the winner-selecting tournament, but prefer a low risk approach under the loser-selecting tournament. Their choice of the low risk approach reduces the noise of their performances. The reduction of noise enhances the workers’ incentive to make efforts to perform their tasks under the given spread between the winner-prize and the loser-prize. Consequently, a risk-neutral firm can offer risk-averse workers better insurance against the compensation risk. This suggests that the loser-selecting tournament is more efficient than the winner-selecting tournament in terms of the cost for implementing high efforts of risk-averse workers. I rigorously show this in case of three workers.

³It might also be proper to call it “success-prizing” or “merit-prizing” because a relatively pretty high performance is necessary to become winners. Yun (1997) also calls it “prize-giving.”

⁴Since only very bad outcomes will lead to losers under this type of tournament, “failure-penalizing” or “demerit-penalizing” might be also suitable names. Yun calls it “penalty-giving.”

I argue that the winner-selecting tournament is better suited for firms in innovative and immature markets or industries, but the loser-selecting tournament is better suited for firms in stable, mature, or strictly regulated markets or industries.

In addition to the riskiness, through the choice of their approaches they will also be able to control the correlation of their performances to a limited degree, which also affects their rewards under the promotion tournament. For example, if salespersons use the same sales technique that they learned together in training, they will get outcomes correlated with one another. If marketing planners who compete in the planning of some product or brand use a similar concept, their marketing plans will be similar. In Chapter 4, the properties of their correlation choices under promotion tournaments are investigated, and their implications for the design of tournaments are discussed. Workers prefer their own original approaches under the winner-selecting tournament, but prefer the common approach which they know well under the loser-selecting tournament. I show this perfectly in the model without individual-specific noise, and partially in the model with individual-specific noise. The tournaments filter out common noise. Hence, the loser-selecting tournament is more efficient than the winner-selecting tournament in terms of the cost for implementing high effort of risk-averse workers. I rigorously show this in the case of three workers without individual-specific noise. I argue that the winner-selecting tournament is better suited for the upper job levels, but the loser-selecting is better for the lower job levels.

Furthermore I examine another type of workers' behavior which is totally different from the efforts and the approaches to perform their tasks: sabotage among them. Sabotage is unproductive actions to reduce the peer's performance. They might choose how much effort they exert and how to make effort to reduce their peers' performance without their authority. These negative activities clearly reduce efficiency. In Chapter 5, I investigate properties of sabotage under the one-winner tournament and those under the one-loser tournament, and argue their implications for the tournament design. Under the one-winner tournament, each worker attacks his peers so as to minimize the maximal value of their expected performance. The sabotage among them operates to create more homogeneity in a heterogeneous field of workers. Furthermore, the uniqueness of the equilibrium expected performance vector is shown under moderate assumptions. Under the one-loser tournament, each worker intensively attacks one peer so as to minimize the minimal value of his peers' expected performances. One worker is intensively attacked by all his peers. Furthermore, embedding workers' effort choice into the model in the simplest form, I argue that the

one-loser tournament is more subject to damage by sabotage, *ceteris paribus*, than the one-winner tournament. I show that although heterogeneity increases the relative incentive for sabotage under the one-winner tournament, the sabotage might work like a “handicap system” and restore the incentive of their efforts.

In the next chapter, I first define tournaments, and summarize their advantages and disadvantages over individualistic contracts. Next, I briefly summarize the literature closely related to the scope of this paper. Last, I introduce the empirical study by Hanada and Sekimoto (1985, 1986), by which my research is initially motivated.

Chapter 2

Literature Review

2.1 Tournaments as incentive devices

In the field of economics or contract theory, the (rank-order) tournaments mean contracts in which a contestant's reward depends only on the ranking of his performance in the ordering of the performances of the other contestants. We can often see this type of contract in the real world. Sports competitions are a typical example. School admission examinations, elections, and litigations are also examples familiar to us. Most important applications in the context of firms are promotions: the winners receive the higher position, but the losers do not.

Tournaments give contestants motivation to compete with one another for bigger prizes. Tournament theory predicts that contestants have the stronger incentive to exert effort, the greater the spread between the winner's prize and the loser's prize, the higher the idiosyncratic noise on outcomes, or the smaller the heterogeneity in ability among contestants (Lazear and Rosen, 1982).¹²

¹See McLaughlin (1988) or Rosen (1988) for surveys of tournament theory.

²There are several pieces of empirical evidence about tournament theory. Ehrenberg and Bognanon (1990) use data from the Professional Golfers' Association and relate the performance quality of players to the structure of compensation. They find evidence consistent with prediction from tournament theory that the larger prize gaps induce more effort. Knoeber and Thurman (1994) use data on the performance of the broiler chicken producers to test three predictions from tournament theory: 1) changes in the level of prizes that leave prize differentials unchanged will not affect performance; 2) under mixed tournaments in which players heterogeneous in ability compete, more able players will choose less risky strategies; 3)

However, tournaments are not generally efficient; the optimal contract should depend on all the relevant information, which is rarely summarized in the order of the performances.³ Mookherjee (1984) shows that the optimal tournament for risk-averse agents is typically inferior to the optimal contract.

Despite this shortcoming, tournaments are very often seen in the real world. Lazear and Rosen (1981), Green and Stocky (1983), Nalebuff and Stiglitz (1983), Malcomson (1986) examined the advantages and disadvantages of tournaments over individualistic contracts that depend only on the absolute level of the individual performance. Tournaments have three main advantages over individualistic contracts.

Advantages of tournaments over individualistic contracts

First, tournaments save measurement costs. The only performance information needed in a tournament contract is the relative, ordinal information about who did better, which may be much easier to measure than the absolute, cardinal information about individual performance level.

A second advantage of tournaments arises when there is common noise on contestants' performances. For example, the sales made by individual salesperson may be affected by the same market factors, such as the actions of competitors, the market's acceptance of sales product, and economic climate. Relative comparisons eliminate the effect of common noise on their rewards. Reduction of uncertainty as to the common noise increases the incentive for contestants to make effort under the given spread between the winner's and loser's prize. It follows that the principal can offer risk-averse agents better insurance against the compensation risk. As emphasized by Holmström (1982: p. 325), 'One finds that relative performance evaluation will be valuable if one agent's output provides information about another agent's state uncertainty. Such will be the case if and only if agents face some common uncertainties. Thus, inducing competition among agents by tying their rewards to

tournament organizers will attempt to handicap players of unequal ability or reduce mixing to avoid the disincentive effects of mixed tournaments. The results of the tests support these predictions. Backer and Huselid (1992) use a panel data set from auto racing: NASCAR and IMSA. They also find that the larger prize gaps induce higher performance of players. Bull, Schotter, and Weigelt (1987) conduct experiments using university students as subjects. They find that where players are identical, the Nash equilibrium predicted by the tournament model appears to obtain. In the case of unequal players, however, the results were not entirely consistent with predictions. While the advantaged players choose the predicted effort level, the disadvantaged players choose a higher effort level than the predicted level. Overall, their results were supportive of tournament theory.

³See Holmström (1979,1982) on the nature of information that optimal contracts should depend on.

each other's performance has no intrinsic value. Rather, competition is the consequence of the efficient use of information.'

Third, performance payments under tournaments can avoid a principal's moral hazard problem caused by unverifiability of performance. Under individual contracts, a principal may be tempted to renege on performance payments that have been earned by claiming that performance was lower than it actually was. This will be a serious problem in circumstances in which it is difficult for agents to observe one another's performance and to verify performance for third parties. In these circumstances, it is still possible to verify whether the promised tournaments payments have been earned. Under tournament contracts, the total payment from the principal to all agents is set in advance and is independent from the outcome that occurs. Hence, the principal has no incentive to misrepresent the agents' performance to save total payments. Malcomson (1984) argues that rank-order tournaments can provide performance incentives even in the case in which the performance is observable by the principal but not verifiable.

In addition to these general advantages of tournaments over individualistic contracts, Fairburn and Malcomson (2001) argue an advantage of promotions over monetary bonuses to motivate employees. When performance is unverifiable, use of promotion reduces the incentive for managers to be affected by employees' influence activities that would reduce the effectiveness of monetary bonuses. For example, the managers could make themselves better off by rewarding those employees who offer bribes rather than those who have performed well. However, since promotions affect the future performance of the firm which affects the payoff to the managers, they provide the managers with incentives to promote the most productive employees.

Disadvantages of tournaments over individualistic contracts

Tournaments, however, have two potential disadvantages. First, tournaments are subject to a problem of collusion of contestants to reduce effort. As Mookherjee (1984) and Dye (1984) point out, rewards based only on relative performance generate incentives for contestants to collude and supply less effort than would be supplied in the absence of collusion and simply to split the winner's prize. Collusion is more likely and effective when the number of contestants is small and they can observe one another's effort, performance, and reward.

A second disadvantage of tournaments is a problem of sabotage, in which a con-

testant takes unproductive actions to reduce his peer's performance. This clearly creates inefficiency. Sabotage takes a variety of forms: spreading false gossip about a rival, concealing relevant information, theft, or destruction of rival's output, etc. Lazear (1989) argues that pay compression reduces the levels of negative activities and enhances efficiency.

2.2 The scope of the paper and other related literature

I examine properties of workers' behavior under promotion tournaments, and discuss their implications for the design of promotion tournaments. In Chapter 3, I examine workers' choices of riskiness of their performances under promotion tournaments. This chapter is closely related to the literature on R & D portfolios under patent mechanism such as Bhattacharya and Mookherjee (1986), Dasgupta and Maskin (1987), and Klette and de Meza (1986). The "winner-take-all" patent mechanism is very similar to the "one-winner" tournament. The only difference between the two mechanisms is the winner's reward: under the former the winner's reward depends on his outcome, but under the latter the winner's prize is fixed.

In Chapter 4, workers' choices of correlation of their performances under promotion tournaments are discussed. The literature on R & D portfolios above are also closely related to this chapter. Competing firms dislike the correlation of their performances as workers do under the one-winner tournament. This chapter has much in common with the literature on conformity or conservatism such as Scharfstein and Stein (1990), and Zwiebel (1995). In these papers, conformity is an agency phenomenon such as in my model. Scharfstein and Stein show that a manager may imitate the actions of other managers in order to maintain his reputation in the labor market, ignoring substantive private information. Zwiebel shows that relative performance evaluation, especially fear of firing, may cause managers to choose inferior industry standards.

In Chapter 5, I study properties of sabotage under the one-winner tournament and those under the one-loser tournament and argue their implications for the tournament design. This chapter has much in common with multi-task principal-agency literature, such as Holmström and Milgrom (1991) and Itoh (1991,1992). This literature studies the optimal allocation of an agent's effort when he has more than one task, and the optimal compensation scheme to induce the efficient effort levels. A worker in my model also has

multi-task: a task given by principal and “tasks” not given by her—sabotage of his peers; and the worker decides how much “effort” to be allocated to each task. As far as I know, only Lazear (1989) and Chen (2003) directly examine sabotage within firms. Lazear shows that the optimal prize gap is smaller than it would be in the absence of the possibility of sabotage. This result offers one explanation for the wage compression frequently described by personnel managers. Chen investigates the properties of sabotage among workers in promotion tournaments as I try to do in Chapter 5. He finds that a worker with higher comparative ability in productive activity is subject to more total attack, and a worker with the highest ability in productive activity does not necessarily have the greatest promotion chance. One problem of his analysis, however, is that the uniqueness of equilibrium is not guaranteed in his model. It makes qualifications for the comparative statics results. I examine the properties of sabotage under the one-loser tournament as well as those under the one-winner tournament he also studied, and firstly investigate the properties of allocation of an agent’s sabotage to his peers. In the context of political competition, Skaperdas and Grofman (1995) discuss positive and negative campaigning in electoral contests. Kornad (2000) studies sabotage in lobbying contests. He argues that due to a positive externality, sabotage is a “small number” phenomenon.

2.3 Empirical study of promotion systems by Hanada and Sekimoto

In this section, I introduce the empirical study of promotion systems of Japanese firms by Hanada and Sekimoto, by which my project was initially motivated.

A clear way of presenting a promotion system of an organization is a career tree, which portrays the career paths followed by all individuals during their employment in the organization. Rosembaum (1979, 1984) pioneeringly uses the career tree method for the study on promotion system of a large corporation in the US and proposes a famous “tournament” model which implies employees who are promoted first retain the right to compete for higher positions and those who are promoted late tend to move into the category of loser, from which there is limited opportunity to be promoted. Hanada (1984, 1987, 1989), Hanada and Sekimoto (1985, 1986), Pucik (1985), Wakabayashi (1986) and Nihon Rodo

Kenkyu Kiko (1994) apply this method for the Japanese firms. Especially, Hanada and Sekimoto (1985, 1986) investigate the career trees of eleven companies and find two distinguishable promotion systems in those companies.⁴ One, which they call a “traditional,” “conservative,” or “bureaucratic” promotion system, is characterized by the facts that (1) there is little possibility for any employee whose promotion was delayed once to surpass those whose promotions were not delayed, and (2) for workers whose promotions were not delayed, the promotion ratio in the first selection at each level is fairly high. That is to say, this system emphasizes differentiation of losers.

Figure 2.1 shows the career tree of a Japanese corporation that has adopted such a promotion system. Twenty-seven male college graduates joined this company in 1955. Twenty-four men (89 percent) were promoted to the first level of assistant chief in the fourth year. Further, ten men (42 percent) in this first group of twenty-four were promoted to the second level at the first second-level selection in the thirteenth year. Then eight men (80 percent) out of ten and five men (63 percent) out of eight were promoted to the third and the fourth levels at the first selection, respectively. No person whose promotion was delayed once was promoted to a higher position at the first selection.

Figure 2.2 is another example. Only two workers whose promotions were delayed once were promoted to the higher positions at the first selection (indicated with A). At each level the ratio of those who were promoted in the first selection out of employees whose promotions were not delayed at the lower levels was fairly high, except for the fifth level (potential executives) : 70 percent at the first level, 82 percent at the second level, 78 percent and 90 percent at the third and fourth levels, respectively.

On the other hand, the other promotion system, which they call “innovative,” is characterized by the facts that (1) there is substantive possibility for workers whose promotions were delayed once to surpass those whose promotions were not delayed, and (2) the promotion ratio in the first selection at each level is low. That is, there is a clear road for a second chance for losers in the system. Figure 2.3 shows the career tree of a company that has adopted an “innovative” promotion system. This company hired forty-eight male college graduates in 1960. Ten (21 percent) were promoted to the first level in the fifth year, fourteen (29 percent) in the sixth year, thirteen (27 percent) in the seven year, and eight

⁴First, they published their findings in Hanada and Sekimoto (1985, 1986). Then, based on their findings, Hanada (1987) discussed the principle of competition in personnel system of the Japanese corporations. This paper was translated into English and was published as Hanada (1989).

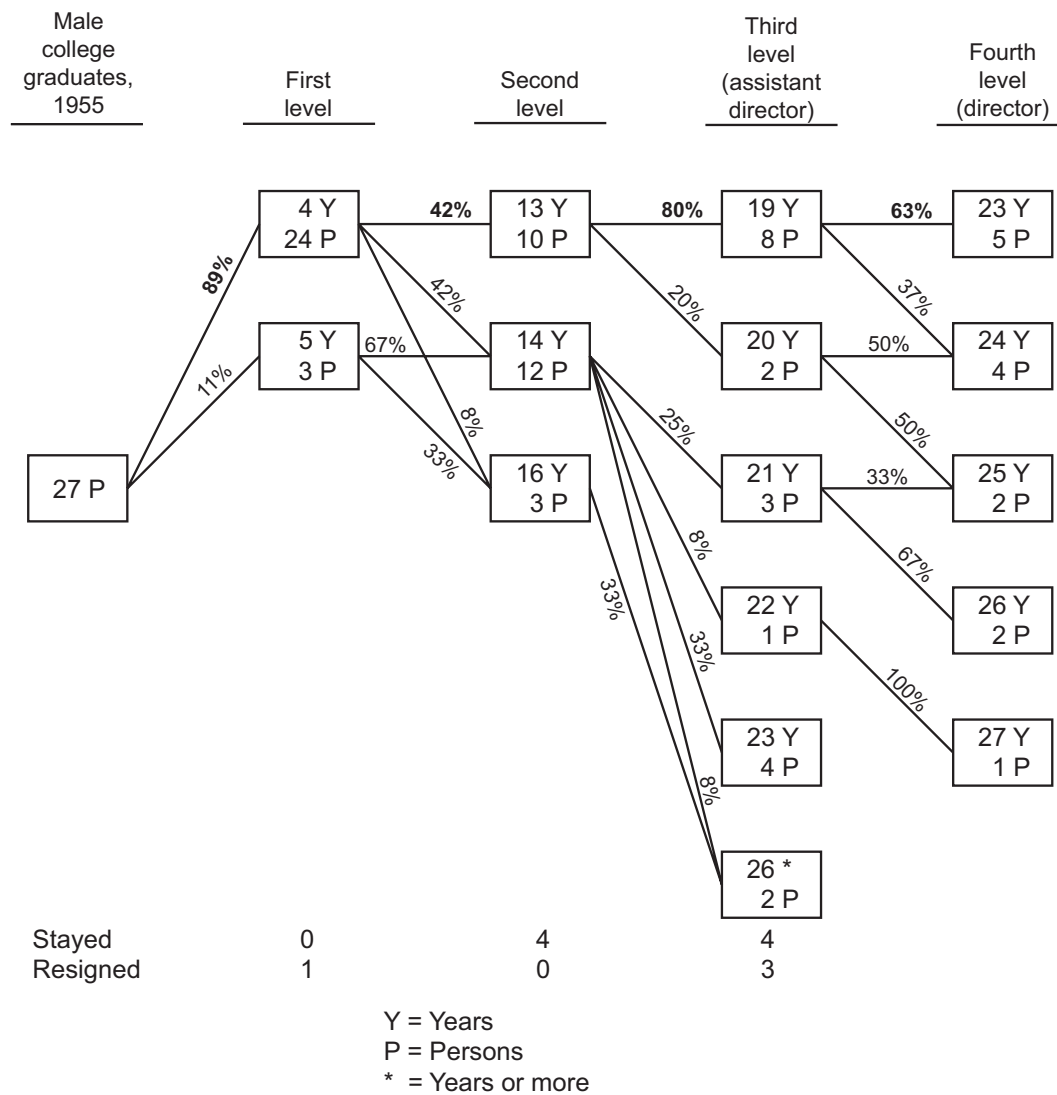


Figure 2.1: Career Tree of a Traditional Conservative Corporation A
SOURCE: Modified from Figure 3 in Hanada (1989)

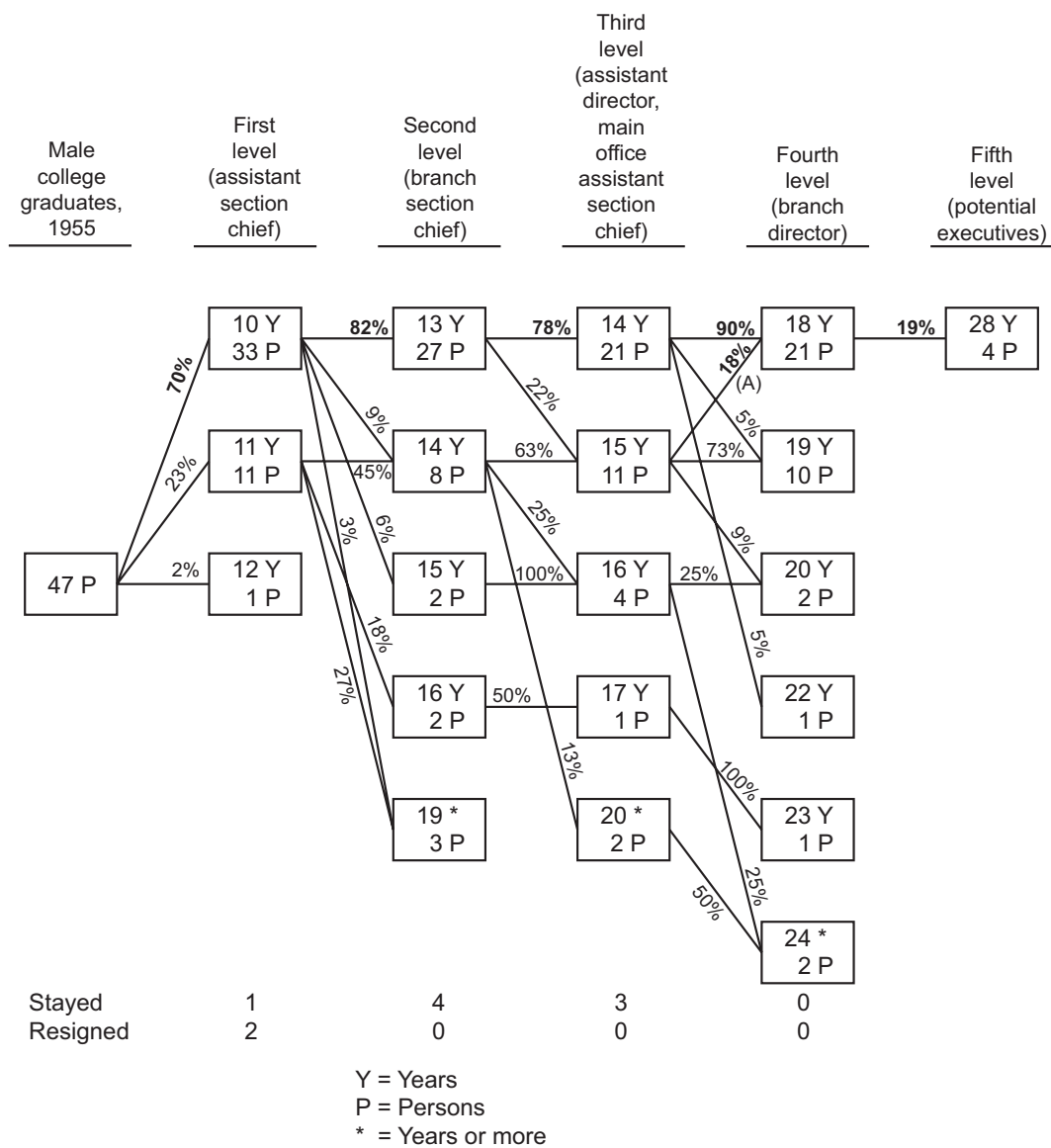


Figure 2.2: Career Tree of a Traditional Conservative Corporation B
SOURCE: Modified from Figure 4 in Hanada (1989)

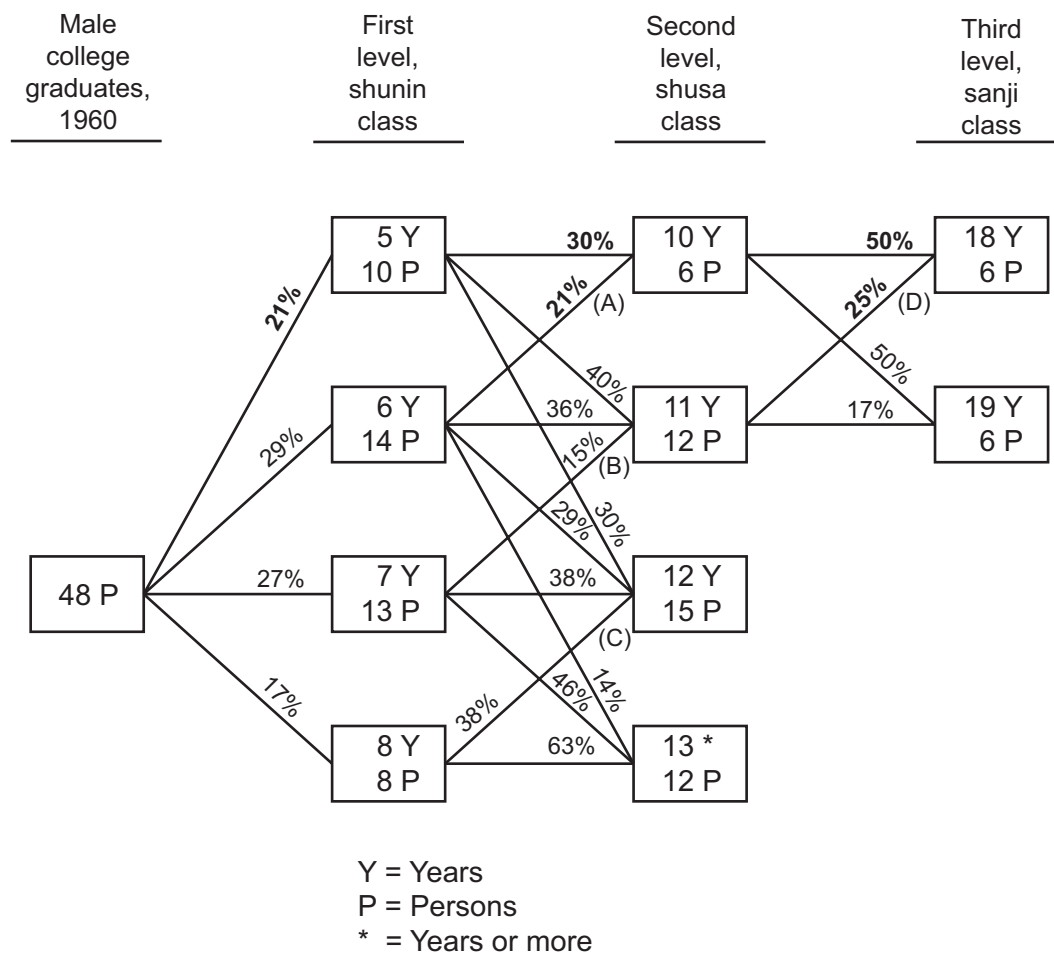


Figure 2.3: Career Tree of an Innovative Corporation C
SOURCE: Modified from Figure 6 in Hanada (1989)

(17 percent) in the eighth year. Further, six were promoted to the second level at the first selection: three (30 percent) from the first group of ten men and three (21 percent) from the second group of fourteen, respectively. Case A, B, C, and D in the figure present examples of a second chance for losers. ‘The management of this company says that it advertises to the outside the fact that here is such a clear road for losers to be given a second chance, and it maintains that the innovativeness of the company is supported by this personnel structure’ (Hanada 1989: p. 15).

In addition to the career trees, Hanada and Sekimoto surveyed the sense of belonging at eleven companies, including three corporations depicted in Figures 2.1–3. They find out there is a clear relationship between the sense of belonging that employees have for companies and the promotion systems. At corporations where the conservative promotion system was adopted, many employees have a sense of belonging, which is simply conformist: individuals do not push forth their sense of value, but they accept the will of the corporation as it is. On the other hand, the sense of belonging in companies with the innovative promotion system is referred to as the self-realization type. Many employees are willing to express their own wishes and desires.

On these findings Hanada argues that there is a strong relationship between promotion systems and organizational climate:

In a corporation where there is no second chance in the promotion and demerits are stressed, there exists a general feeling that outstanding actions will be punished while going along with others becomes important. At least in an organization where one can be promoted with the certainty of 60-80 percent if one does not fail, and can successfully avoid being labeled a loser, there will not be many who will be adventurous if it involves a risk of failing. Consequently, an organizational climate will be formed whereby failure is feared and aggressiveness is inhibited. ... On the other hand, at corporations where the personnel system gives a second chance to losers, and where merits are the major basis for assessing individuals, there is a more positive approach to the improvement of performance of individuals. The organizational climate where “go for broke” is a phrase often mouthed by executives exists at a corporation with such a system of promotion and status acquisition. Higher values are given to more positive and innovative behavior in a corporation where merits, rather than demerits, are emphasized in personal evaluation. Individuals will have a positive approach toward the corporation on their own without being afraid of failure. (Hanada 1989: p. 19)

In Japanese large firms, employees in the same category in terms of year of graduation from college are subject to the same promotion process. As shown in the title of Hanada (1989) ’s paper, it seems that they compete for the limited number of positions of higher ranks: they compete under promotion tournaments. While my paper does not an-

alyze several-round promotion tournaments but rather one-round promotion tournaments, I rigorously show that workers' behavior strongly depends on the promotion rate, which is a key factor of the two promotion systems described above. It is shown that under the loser-selecting tournament, which means the promotion ratio is more than one-half, workers prefer a low risk, common, or conventional approach in the organization they belong to. Consequently, conservatism or conformity dominates in the organization. On the other hand, under the winner-selecting tournament, which means the promotion ratio is less than one-half, they prefer a high risk or their own original approach. As a result, innovativeness or diversity appears.

Though it is not stated, I guess the two conservative firms used as examples belong to the insurance and banking industries, which were strictly regulated at the time; the innovative firm belongs to the electrical appliance industry.⁵ I argue the winner-selecting tournament is better suited for firms in innovative and immature markets or industries; on the other hand, the loser-selecting tournament is better suited for firms in stable, mature, or strictly regulated markets or industries.

⁵Hanada (1989) takes up five companies as examples. 'They belong to finance, insurance, electrical appliance, transportation, and distribution industries, and we can say that they have leadership positions in their respective industries' (p. 17).

Chapter 3

Risk Preference in Promotion Tournaments

3.1 Introduction

In this chapter, I examine the properties of workers' risk choices under promotion tournaments, and discuss their implications for the design of promotion tournament.

As I have already illustrated in the first chapter, workers will have opportunities to choose their approaches, which affects the riskiness of their performances, within their delegated authority. I show that they prefer a high risk approach to a low risk one under the winner-selecting tournament, which means the promotion rate is less than one-half, but prefer the low risk approach under the loser-selecting tournament, which means the promotion rate is more than one-half. The logic of this result is simple. Under the winner-selecting tournament, it will be necessary for a worker to achieve very good performance in order to get the promotion. Increasing the weight in the upper tail of the distribution of his outcome improves his winning probability. Thus, the high risk approach is the better. On the other hand, under the loser-selecting tournament he wants to avoid a very bad outcome. He can reduce his probability of losing by decreasing the weight in the lower tail of the distribution. Hence, the low risk approach is the better.

Their choice of the low risk approach reduces the noise of their performances. The reduction of noise enhances the workers' incentive to make efforts to perform their tasks under the given spread between the winner-prize and the loser-prize. Consequently, a risk-neutral firm can offer the better insurance against the compensation risk to the risk-averse workers. This suggests that the loser-selecting tournament is more efficient than the winner-selecting tournament in terms of the cost for implementing high efforts of risk-averse workers. I rigorously show this in the case of three workers. To put it more precisely, the sum of three effects leads to this superiority of the one-loser tournament. First, even if there were only one available approach, the result really holds because of the larger loss of deviation under the one-loser tournament. I call this the order statistic effect. The second one is the noise reduction effect described above. The third one is the increasing noise deviation effect. Agents can deviate using the high risk approach. This effect is nonpositive, but the magnitude is less than that of the noise reduction effect. Thus the total is positive. When a firm's revenue is given by the sum of workers' performance, as in the standard agency model, the risk-neutral firm prefers the one-loser tournament. When it is given by the maximum value of their performances, as in competitions of new product development, marketing plan, advertising plan in a division or a firm, the firm chooses the tournament comparing the revenue advantage of the one-winner tournament with the cost advantage of the one-loser tournament.

From these results, I argue that the winner-selecting tournament is better suited for firms in innovative and immature markets or industries in which there is a high possibility that good ideas with high risk exist, on the other hand, the loser-selecting tournament is better suited for firms in stable, mature, or strictly regulated markets or industries in which there is a low possibility that good high risk ideas exist.

Furthermore, I examine whether the extension to a continuum of approach will add new insight or not. Although I obtain the result only in the limited case (the number of winners $m = 1, 2, n - 2, n - 1$), it seems to suggest that this extension does not add a new finding, that is, the two-approach model is not restrictive, at least in the case of homogeneous workers.

I also investigate how heterogeneity in ability across workers will have an effect on their approach choices in case of two workers. Under the assumptions that imply the riskier approach has the fatter-tail distribution, the high-ability worker prefers the low risk one in order to defend his advantage over the low-ability worker, but the low-ability worker bets

on the high risk approach in order to overcome his disadvantage. The result suggests that when a firm wants to get high-ability workers to choose the high risk approach, the firm should choose a small winner rate m/n .

The chapter is organized as follows. In Sections 3.2 and 3.3, I present the basic model in which agents choose between the high risk approach and the low risk approach under a promotion tournament, and examine the equilibrium of the model. In Section 3.4, agent's effort is introduced into the basic model. I compare the principal's minimum cost of implementing high effort under the one-winner tournament to that under the one-loser tournament and discuss the design of promotion tournament. In Section 3.5, I examine whether or not the extension to a continuum of approach adds new insight. In Section 3.6, I examine how heterogeneity in ability will affect their risk choices.

3.2 The simple risk choice model

Consider a principal with n agents, indexed by $i = 1, 2, \dots, n$. Considering the advantages of tournaments over individual contracts explained in Chapter 2, she offers them the following simple rank-order tournament contract (m, R_W, R_L) .¹ Each agent is given his task and is paid one of two possible rewards, R_W and R_L , depending on his performance ranking: the top m agents (winners) are paid the high reward R_W respectively, but the other $n - m$ agents (losers) are paid the low reward R_L respectively. I mean that the win corresponds to the promotion to the next upper rank.

The performance of agent i is given by a continuous random variable $\tilde{z}_i(a_i)$ in the following form:

$$\tilde{z}_i(a_i) = \tilde{\gamma} + \tilde{x}_i(a_i) \quad (3.1)$$

where a_i is the agent i 's action explained later on and

$$\tilde{x}_i(a_i) = \mu_i + \tilde{\epsilon}_i(a_i). \quad (3.2)$$

The performance random variable $\tilde{z}_i(a_i)$ consists of the common component $\tilde{\gamma}$ and the individual(and action a_i)-specific component $\tilde{x}_i(a_i)$, and furthermore $\tilde{x}_i(a_i)$ is divided into the $\tilde{x}_i(a_i)$'s mean μ_i and the individual(and action)-specific noise $\tilde{\epsilon}_i(a_i)$. I assume that

¹The reasons why she chooses a class of this simple tournament from possible tournaments is discussed at the end of this section.

$(\tilde{\gamma}, \tilde{\epsilon}_1(a_1), \dots, \tilde{\epsilon}_n(a_n))$ are independently distributed. Also assume that $\mu_i = 0$ for $i = 1, 2, \dots, n$ until Section 3.6, in which I analyze a simple model with heterogeneity in ability across agents.

At this moment, note that the agents' rewards are independent of the common noise in this setting: the order of realized performance (z_1, z_2, \dots, z_n) is exactly the same as that of (x_1, x_2, \dots, x_n) . This is one of the main advantages of tournament contracts over individual contracts described in the previous chapter. However, the purpose of this paper is not the comparison between tournaments and other type of contracts, but the comparison among tournaments. Hereafter I simply assume that the tournaments perfectly filter out the common noise, and so regard $\tilde{x}_i(a_i)$ as the agent i 's performance: $\tilde{z}_i(a_i) = \tilde{x}_i(a_i)$.

Agent i chooses his action a_i , which is named "approach," from his action set $A_i = \{r, s\}$ without his private cost or disutility. Approaches r and s represent the "high" risk and the "low" risk approach in the meaning explained afterward, respectively. I assume that $\tilde{\epsilon}_i(r)$ is independently and identically distributed with the c.d.f. $F_r(\epsilon)$, the p.d.f. $f_r(\epsilon)$, and the support lying in an interval $[-\bar{\epsilon}(r), \bar{\epsilon}(r)]$ across all agents. Also assume that $\tilde{\epsilon}_i(s)$ is independently and identically distributed with the c.d.f. $F_s(\epsilon)$, the p.d.f. $f_s(\epsilon)$, and the support lying in an interval $[-\bar{\epsilon}(s), \bar{\epsilon}(s)]$ across all agents ($0 < \bar{\epsilon}(s) \leq \bar{\epsilon}(r)$). It is allowed that $\bar{\epsilon}(s)$ and $\bar{\epsilon}(r)$ is infinity.

For tractable analysis, I make the following three assumptions regarding the distribution functions F_s and F_r :

Assumption 3.1 (symmetry). For any real value ϵ , it holds that

$$F_r(\epsilon) = 1 - F_r(-\epsilon), F_s(\epsilon) = 1 - F_s(-\epsilon), f_r(\epsilon) = f_r(-\epsilon), f_s(\epsilon) = f_s(-\epsilon).$$

Assumption 3.2 (single-crossing). The two graphs of $F_r(\epsilon)$ and $F_s(\epsilon)$ intersects only once:

$$\begin{cases} F_r(\epsilon) - F_s(\epsilon) \geq 0 & \text{if } \epsilon \in (-\bar{\epsilon}(r), 0), \\ F_r(\epsilon) - F_s(\epsilon) = 0 & \text{if } \epsilon = 0, \epsilon \in (-\infty, -\bar{\epsilon}(r)], \text{ or } \epsilon \in [\bar{\epsilon}(r), +\infty), \\ F_r(\epsilon) - F_s(\epsilon) \leq 0 & \text{if } \epsilon \in (0, \bar{\epsilon}(r)). \end{cases}$$

Assumption 3.3. There exists some open interval $\bar{I} = (0, \bar{d})$ such that

$$F_r(\epsilon) - F_s(\epsilon) < 0, f_r(\epsilon) > 0 \text{ and } f_s(\epsilon) > 0 \text{ for all } x \in \bar{I}.$$

The last assumption is made mainly for easier proof of lemmas and propositions. Note that given Assumption 3.2, approach r is riskier than approach s in the sense of a mean-preserving spread (Rothschild and Stiglitz, 1970), *i.e.*,

$$\int_{-\bar{\epsilon}(r)}^y F_r(x)dx \geq \int_{-\bar{\epsilon}(r)}^y F_s(x)dx \quad \text{for all } y \in [-\bar{\epsilon}(r), \bar{\epsilon}(r)]. \quad (3.3)$$

Note that approaches are not efforts. As described in the first chapter, I define the term “approach” as a method, way, procedure, plan or project to perform his task, into which an agent infuses his efforts. For example, a salesperson can choose to reinforce the relationship with his current customers, or can choose to develop new big customers. Also he can continue to use the sales technique learned in on-the-job or off-the-job training, or can try his new ideas to sell more. A product or brand manager can maintain the current concept, or can develop a new concept.

I consider only pure (nonrandom) strategies.² Thus the agent i ’s strategy is simply given by $a_i \in A_i = \{r, s\}$. A strategy profile (an approach profile) for agents is given by $\mathbf{a} = (a_1, a_2, \dots, a_n) \in A_1 \times A_2 \times \dots \times A_n = A$. I denote by \mathbf{a}_{-i} a strategy profile except agent i .

I treat a tournament contract (m, R_W, R_L) which is offered by the principal, as the given until Section 3.4, in which agents’ effort variables are embedded in a more simplified model. For a while, we concentrate on the analysis of approach choice. The agents choose a_i simultaneously and independently so as to maximize their expected utility. In this model, they do not have any private costs to choose their strategy. It follows that maximizing their expected utility is equivalent to maximizing their winning probability. Thus it is not necessary to specify their utility functions and the principal’s revenue function until Section 3.4. I assume that the structure of game is common knowledge among agents.

Disadvantages of general n -prize tournaments over simple promotion tournaments

²Extension to mixed strategies will not change the main results shown below.

Tournaments with n -prize level have several disadvantages over the simple two-prize level tournaments in the context of promotions when an agent's performance is very difficult to measure with any accuracy and hence is subjectively evaluated in some measure. The first disadvantage is the measurement costs: It is more difficult to rank agents completely than to divide them into winners and losers.

Other disadvantages of general n -prize tournament are mainly related to agents' motivations after the tournament. It will be more difficult to convince them the complete ranking than to convince them who are promoted. The less they feel satisfaction with performance evaluation, the more their morale lessens. Furthermore, when the complete ranking comes into the open, they will strongly feel the heterogeneity in ability among them.³ This feeling will reduce their incentive to make effort under the next round of the promotion tournament. The best agents might become the targets of headhunting or sabotage by their peers. On the other hand, under simple promotion tournament they only know who are promoted. the feeling of the homogeneity among winners will carry on. Since the information of the ranking among winners does not come into the open, the best agents will not necessarily become the targets of headhunting or sabotage.

3.3 The strictly dominant strategy equilibrium

I make the following proposition on the model described above.

Proposition 3.1. Given Assumptions 3.1–3.3, the following hold:

- (i) Under the winner-selecting tournament (i.e., $n > 2m$), the high risk approach r is a strictly dominant strategy for every agent i .
- (ii) If $n = 2m$, then both approaches are indifferent for all agents.
- (iii) Under the loser-selecting tournament (i.e., $n < 2m$), the low risk approach s is a dominant strategy for each agent i .

I prepare three lemmas in order to prove the proposition. Without loss of generality, I consider agent n 's decision making. Let $\tilde{x}_{(n-m)}(l, n-1)$ denote the $n-m$ th order statistic

³Note that principal's moral hazard problem would be caused if the complete ranking did not come into the open.

of performances of $n - 1$ agents except agent n given a strategy profile \mathbf{a}_{-n} in which l peers choose the low risk approach s , and the other $n - l - 1$ peers choose the high risk approach r . Then the density function $\phi_{(n-m)}(x; l, n - 1)$ is represented by:

$$\begin{aligned} & \phi_{(n-m)}(x; l, n - 1) \\ &= \begin{cases} C_1^l H(x; l - 1, m - 1, n - 2) f_s(x) \\ \quad + C_1^{n-1-l} H(x; l, m - 1, n - 2) f_r(x) & \text{if } l = 1, \dots, n - 2, \\ C_1^{n-1} H(x; 0, m - 1, n - 2) f_r(x) & \text{if } l = 0, \\ C_1^{n-1} H(x; n - 2, m - 1, n - 2) f_s(x) & \text{if } l = n - 1. \end{cases} \end{aligned} \quad (3.4)$$

where

$$H(x; a, b, c) = \sum_{i=\max[0, a+b-c]}^{\min[a, b]} C_i^a C_{b-i}^{c-a} \bar{F}_s(x)^i \bar{F}_r(x)^{b-i} F_s(x)^{a-i} F_r(x)^{c-a-b+i},$$

a, b, c are integers that satisfy $0 \leq a \leq c$ and $0 \leq b \leq c$, and $\bar{F}_r(x) = 1 - F_r(x)$, $\bar{F}_s(x) = 1 - F_s(x)$, and C_r^n is the number of combinations of n objects taken r at a time:

$$C_r^n = \binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

Agent n can get the high reward if his performance is better than the top m th performance, which is the realized value of the $n - m$ th order statistic of performances of his peers $\tilde{x}_{(n-m)}(l, n - 1)$. Let $\Pr(\tilde{x}_n(r) > \tilde{x}_{(n-m)}(l, n - 1))$ denote the probability that the realized value of $\tilde{x}_n(r)$ will be bigger than that of $\tilde{x}_{(n-m)}(l, n - 1)$. Similarly, let $\Pr(\tilde{x}_n(s) > \tilde{x}_{(n-m)}(l, n - 1))$ denote the probability that the realized value of $\tilde{x}_n(s)$ will be bigger than that of $\tilde{x}_{(n-m)}(l, n - 1)$. If the former probability is bigger than the latter probability, the best response strategy of agent n will be approach r . If the latter is bigger than the former, his best response strategy will be approach s . I have the following useful lemma to demonstrate that:

Lemma 3.1. Suppose that \tilde{x} is a continuous random variable with a density function $\phi(x)$, which is statistically independent of $\tilde{x}_n(r)$ and $\tilde{x}_n(s)$. Then given Assumptions 3.1–3.3, the following hold:

- (i) If $\phi(x) \geq \phi(-x)$ for any $x > 0$ and at least $\phi(x) > \phi(-x)$ for any $x \in \bar{I}$, then the probability that the realized value of $\tilde{x}_n(r)$ is larger than that of \tilde{x} is greater

than the probability that the realized value of $\tilde{x}_n(s)$ is larger than that of \tilde{x} , that is,
 $\Pr(\tilde{x}_n(r) > \tilde{x}) > \Pr(\tilde{x}_n(s) > \tilde{x})$.

- (ii) If $\phi(x) = \phi(-x)$ for any $x > 0$, then $\Pr(\tilde{x}_n(r) > \tilde{x}) = \Pr(\tilde{x}_n(s) > \tilde{x})$.
- (iii) If $\phi(x) \leq \phi(-x)$ for any $x > 0$ and at least $\phi(x) < \phi(-x)$ for any $x \in \bar{I}$, then
 $\Pr(\tilde{x}_n(r) > \tilde{x}) < \Pr(\tilde{x}_n(s) > \tilde{x})$.

Proof: See Appendix. ■

According to this lemma, if $\phi_{(n-m)}(x; l, n-1) - \phi_{(n-m)}(-x; l, n-1)$ is nonnegative (nonpositive) for any positive x , then the high risk approach r (the low risk approach s) is his best response to his peers' strategy. From the equation (3.4), the $\phi_{(n-m)}(x; l, n-1) - \phi_{(n-m)}(-x; l, n-1)$ is given by:

$$\begin{aligned} & \phi_{(n-m)}(x; l, n-1) - \phi_{(n-m)}(-x; l, n-1) \\ &= \begin{cases} C_1^l \{H(x; l-1, m-1, n-2) - H(-x; l-1, m-1, n-2)\} f_s(x) \\ \quad + C_1^{n-1-l} \{H(x; l, m-1, n-2) - H(-x; l, m-1, n-2)\} f_r(x) \\ \quad \quad \quad \text{if } l = 1, \dots, n-2, \\ C_1^{n-1} \{H(x; 0, m-1, n-2) - H(-x; 0, m-1, n-2)\} f_r(x) & \text{if } l = 0, \\ C_1^{n-1} \{H(x; n-2, m-1, n-2) \\ \quad - H(-x; n-2, m-1, n-2)\} f_s(x) & \text{if } l = n-1. \end{cases} \end{aligned} \tag{3.5}$$

The next Lemma 3.2 tells us that under the winner-selecting tournament, $H(x; \cdot) - H(-x; \cdot)$ in the equation (3.5) is nonnegative for any positive x regardless of his peers' strategy profile. Lemma 3.3 implies that by the symmetry of distributions, $H(x; \cdot) - H(-x; \cdot)$ under the loser-selecting tournament has the opposite sign of $H(x; \cdot) - H(-x; \cdot)$ under the winner-selecting tournament.

Lemma 3.2. Suppose that a, b and c are integers such that $0 \leq a \leq c$, $0 \leq b \leq c$ and $c > 2b$. Given Assumptions 3.1 and 3.2, $H(x; a, b, c) - H(-x; a, b, c)$ is nonnegative for any

$x > 0$, and especially strictly positive for any $x > 0$ such that $\bar{F}_s(x) > 0$ and $\bar{F}_r(x) > 0$.

Proof: See Appendix. ■

Lemma 3.3. Suppose that a, b and c are integers such that $0 \leq a \leq c$ and $0 \leq b \leq c$. Then for any $x > 0$,

$$H(x; a, b, c) = H(-x; a, c - b, c).$$

Proof: See Appendix. ■

Proposition 3.1 can be easily proved by using Lemmas 3.1, 3.2 and 3.3. See Appendix for full details of the proof.

Although the proof is very long (especially the proof of Lemma 3.2!), the logic of the result is simple: under the winner-selecting tournament, it will be necessary for an agent to achieve a very good performance in order to receive the high reward. Increasing the weight in the upper tail of the distribution of his outcome improves his high reward probability. Thus, the high risk approach is better than the low risk one for him. On the other hand, under the loser-selecting tournament he wants to avoid a very bad outcome. He can reduce his low reward probability by decreasing the weight in the lower tail of the distribution. Hence, the low risk approach is better than high risk one.

By the way, someone might guess that claims in Proposition 3.1 would still hold without Assumption 3.2 (single-crossing) if F_r is a mean-preserving spread of F_s . But this guess is not correct. I will show an example. Suppose that there are three agents and the

distributions of $\tilde{x}_i(r)$ and $\tilde{x}_i(s)$ are the following (also see Figure 3.1):

$$F_r(x) = \begin{cases} 0 & \text{if } x < -d - 0.1, \\ x + d + 0.1 & \text{if } -d - 0.1 \leq x < -d, \\ 0.1 & \text{if } -d \leq x < -0.1, \\ 4x + 0.5 & \text{if } -0.1 \leq x < 0.1, \\ 0.9 & \text{if } 0.1 \leq x < d, \\ x - d + 0.9 & \text{if } d \leq x < d + 0.1, \\ 1 & \text{if } d + 0.1 \leq x, \end{cases}$$

where $d > 1$.

$$F_s(x) = \begin{cases} 0 & \text{if } x < -0.5, \\ x + 0.5 & \text{if } -0.5 \leq x < 0.5, \\ 1 & \text{if } 0.5 \leq x. \end{cases}$$

Then the density functions are the following:

$$f_r(x) = \begin{cases} 4 & \text{if } -0.1 \leq x \leq 0.1, \\ 1 & \text{if } d \leq |x| \leq d + 0.1, \\ 0 & \text{otherwise.} \end{cases}$$

$$f_s(x) = \begin{cases} 1 & \text{if } -0.5 \leq x \leq 0.5, \\ 0 & \text{otherwise.} \end{cases}$$

You can easily check that $F_r(\cdot)$ is a mean-preserving spread of $F_s(\cdot)$ when $d > 1$: approach r is riskier than approach s in the sense of a mean-preserving spread. Also note that these distribution functions are satisfied with all assumptions except Assumption 3.2. Actually the graphs of the two distribution functions $F_r(\cdot)$ and $F_s(\cdot)$ intersect three times: at $x = -0.4, 0, +0.4$.

Without loss of generality, I consider agent 3's decision making. First, consider the case of $m = 1$: the winner-selecting tournament. Suppose that the two other agents choose approach r . If agent 3 also chooses r , his winning probability is $1/3$. On the other

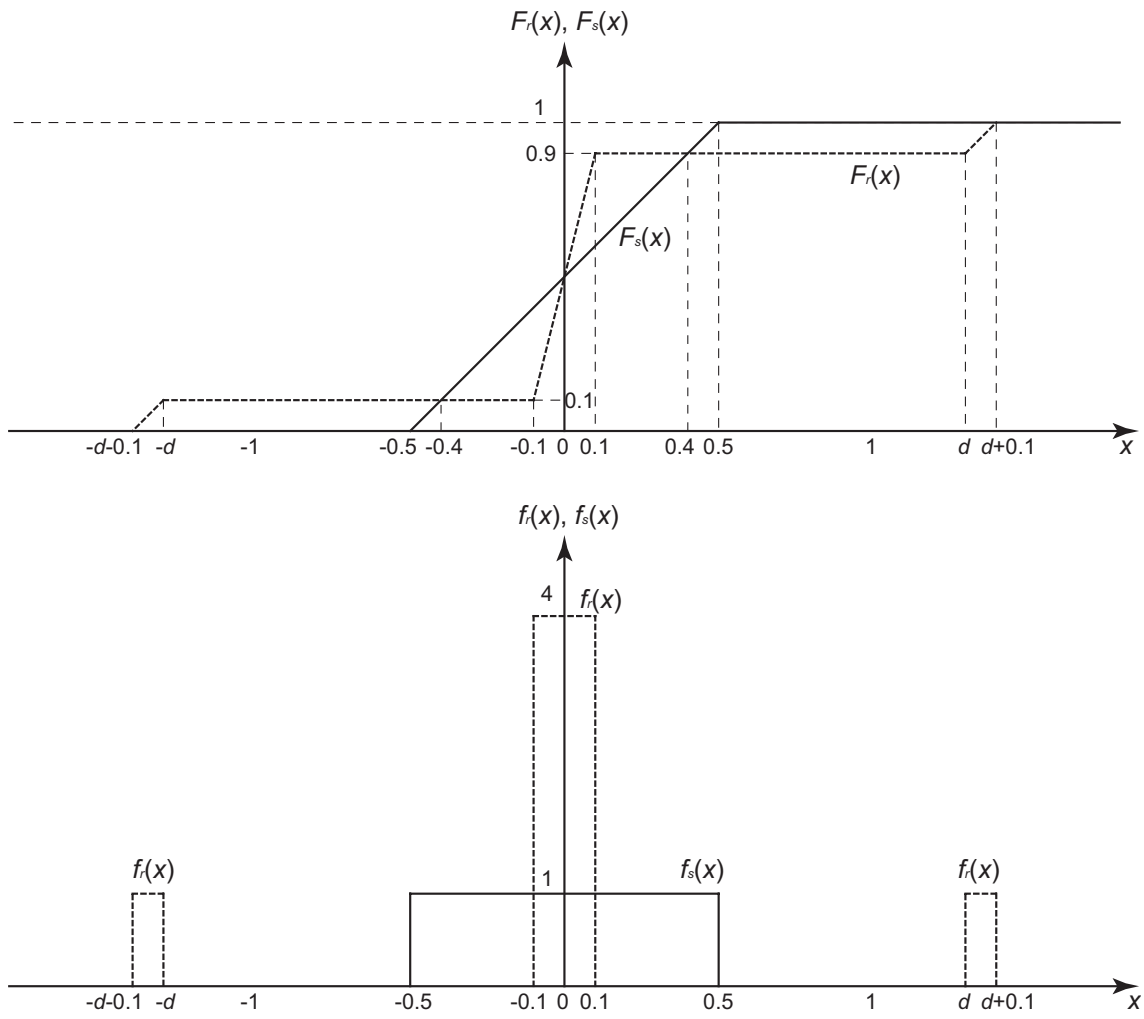


Figure 3.1: An Example Not Satisfying the Single-crossing Condition

Table 3.1: A Numerical Example That Approach s (r) Is a Strictly Dominant Strategy Under the Winner-selecting (Loser-selecting) Tournament

a_3	$(a_1.a_2)$	Agent 3's high reward probability when $m = 1$	Agent 3's high reward probability when $m = 2$
r	(r, r)	$1/3$	$2/3$
s	(r, r)	$1166/3000$	$1834/3000$
r	(r, s) or (s, r)	$917/3000$	$2083/3000$
s	(r, s) or (s, r)	$1046/3000$	$1954/3000$
r	(s, s)	$908/3000$	$2092/3000$
s	(s, s)	$1/3$	$2/3$

hand, if he chooses s , then the probability goes up to $1166/3000$. In the situation that one peer chooses r and the other chooses s , if agent 3 chooses r , then his winning probability is $917/3000$; if he chooses s , then it is $1046/3000$. Last, in the situation that all peers choose s , if agent 3 chooses r , then his high reward probability is $908/3000$; if he chooses s , then it is $1/3$. Thus it follows that the low risk approach s is a strictly dominant strategy for agent 3. This is also true for the other agents: (s, s, s) is a dominant strategy equilibrium.

Next, consider the case of $m = 2$: the loser-selecting tournament. First, suppose that the other two agents choose r . If agent 3 also chooses r , his winning probability is $2/3$. On the other hand, if he chooses s , then it goes down to $1834/3000$. In the situation that one peer chooses r and the other chooses s , if agent 3 chooses r , then his winning probability is $2083/3000$; if he chooses s , then it is $1954/3000$. Last, in the situation that the others choose s , if agent 3 chooses r , his winning probability is $2092/3000$; if he chooses s , then it is $2/3$. Hence, the high risk approach r is a strictly dominant strategy for agent 3. Agents are homogeneous. (r, r, r) is a dominant strategy equilibrium (See Table 3.1). This is because “ $F_r(\cdot)$ is a mean-preserving spread of $F_s(\cdot)$ ” does not generally imply a thicker-tail of distribution, which Assumption 3.2 may secure.

3.4 Effort, risk choice, and design of promotion tournaments

In this section, I embed agents' efforts which enhance their performances in the simple model described in section 3.2 and argue the principal's design problem of tourna-

ments. Since the introduction of agents' efforts into the risk choice model makes it difficult to analyze clearly, I only analyze the case of three agents.

To perform his task, agent i chooses not only his approach a_i from his approach set $A_i = \{r, s\}$, but also his effort level e_i from his effort set $E_i = \{e_L, e_H\}$. For simplicity, I assume that $e_L = 0 < e_H$. I consider only pure strategy choices for agents. Agent i 's strategy is $(a_i, e_i) \in A_i \times E_i$ which is his strategy set: $A_i \times E_i = \{(r, 0), (r, e_H), (s, 0), (s, e_H)\}$. A strategy profile for agents is given by (\mathbf{a}, \mathbf{e}) where an approach profile $\mathbf{a} = (a_1, a_2, a_3) \in A_1 \times A_2 \times A_3$ and an effort profile $\mathbf{e} = (e_1, e_2, e_3) \in E_1 \times E_2 \times E_3$.

His performance $\tilde{x}_i(a_i, e_i)$ is given by a continuous random variable in the following form:

$$\tilde{x}_i(a_i, e_i) = e_i + \tilde{\epsilon}_i(a_i). \quad (3.6)$$

In addition to Assumption 3.1 (symmetry), I make the following assumptions regarding the distribution functions F_r and F_s instead of Assumptions 3.2 and 3.3:

Assumption 3.4 (unimodal distribution). F_r and F_s are unimodal:

$$df_r(\epsilon)/d\epsilon \geq 0 \quad \text{and} \quad df_s(\epsilon)/d\epsilon \geq 0 \quad \text{according as} \quad \epsilon \leq 0.$$

Assumption 3.5. $\tilde{\epsilon}_i(r) = \alpha \tilde{\epsilon}_i(s)$ ($\alpha > 1$). That is, for any real number ϵ ,

$$F_r(\epsilon) = F_s(\epsilon/\alpha), f_r(\epsilon) = (1/\alpha)f_s(\epsilon/\alpha). \quad (3.7)$$

Note that if F_r and F_s satisfy Assumptions 3.1, 3.4, and 3.5, then these distribution functions always satisfy Assumptions 3.1–3.3, but the inverse does not always hold: I make a little more restrictive assumptions than those in the first model.

I assume that all agents have the same von Neumann and Morgenstern utility function:

$$u(R, e) = u(R) - v(e), \quad (3.8)$$

where R represents the reward he receives from the principal, and $v(e)$ measures the cost or disutility of his effort e . Assume that $u' > 0, u'' < 0$ (the agents are risk averse), $v(e_L) = v(0) = 0, v(e_H) > 0$ and $e_H > v(e_H)$. For simple notation, let $v_H = v(e_H)$.

The principal offers the agents a tournament contract (m, R_W, R_L) where m is the number of winners ($m = 1$, or 2), R_W is the reward of (each) winner, and R_L is the reward

of (each) loser. The principal's revenue function is given by $\pi(x_1, x_2, x_3)$. For example, simply $\pi(x_1, x_2, x_3) = x_1 + x_2 + x_3$. Later, I will specify the revenue function and argue about the design of the adequate tournament contract. I assume that the principal is risk neutral and maximizes the expected revenue minus the sum of the total payment.

The timing of the game is as follows:

1. The principal offers the agents a tournament contract (m, R_W, R_L) .
2. The agents decide whether to accept the contract or not. If the contract is accepted, the game goes to the third stage. If either agent rejects the offer, the game ends and all the parties obtain their reservation payoffs. Denote by u^0 every agent's reservation utility. The principal's reservation utility is zero.
3. The agents choose actions (a_i, e_i) simultaneously and independently.
4. The principal observes the outcomes and the payments are made according to the contract in force.

It is assumed that the structure of the game is common knowledge among all players.

I assume that the principal always benefits by having the agents exert effort e_H . Following the Grossman and Hart (1983) procedure, I begin by comparing her minimum cost of implementing an effort profile (e_H, e_H, e_H) under the one-winner tournament ($m = 1$) to that under the one-loser tournament ($m = 2$).

The minimization problem of the implementation cost under the one-winner tournament (the winner-selecting tournament)

First, I identify the following fact:

Corollary 3.1. Make Assumptions 3.1, 3.4 and 3.5 regarding F_r and F_s . When the principal wants to implement the effort profile (e_H, e_H, e_H) using only this one-winner tournament, she cannot implement any approach profile other than (r, r, r) at the same time.

Proof: Result follows straightforwardly from Proposition 3.1. ■

According to this result, the following is the principal's cost minimization problem when she wants to implement the effort profile (e_H, e_H, e_H) under the one-winner tournament.

ment:

$$\min_{R_W, R_L} R_W + 2R_L$$

subject to:

$$D^{1W,r} \{u(R_W) - u(R_L)\} \geq v_H \quad (3.9)$$

$$D^{1W,s} \{u(R_W) - u(R_L)\} \geq v_H \quad (3.10)$$

$$\frac{1}{3}u(R_W) + \frac{2}{3}u(R_L) - v_H \geq u^0 \quad (3.11)$$

where

$$D^{1W,r} = \frac{1}{3} - \int_{-\bar{\epsilon}(r)}^{\bar{\epsilon}(r)} \{F_r(x - e_H)\}^2 f_r(x) dx$$

$$D^{1W,s} = \frac{1}{3} - \int_{-\bar{\epsilon}(s)}^{\bar{\epsilon}(s)} \{F_r(x - e_H)\}^2 f_s(x) dx$$

The first restriction (3.9) is the incentive compatibility constraint on an agent's deviation from a strategy (r, e_H) to a strategy $(r, 0)$. I denote by $D^{1W,r}$ the amount of decrease in his probability of winning caused by this deviation. The second restriction (3.10) is the incentive compatibility constraint on an agent's deviation from a strategy (r, e_H) to $(s, 0)$. Denote by $D^{1W,s}$ the amount of decrease in his probability of winning caused by this deviation. Since we already know that the incentive compatibility constraint on an agent's deviation from a strategy (r, e_H) to (s, e_H) is not binding by Proposition 3.1, it has not been included in the list of constraints above. The third restriction (3.11) is the participation constraint. Note that one of two incentive compatibility constraints is only binding. If $D^{1W,r} < D^{1W,s}$, then only (3.9) is binding. Otherwise, (3.10) is binding. Let D^{1W} denote the smaller one of $D^{1W,r}$ and $D^{1W,s}$:

$$D^{1W} = \min(D^{1W,r}, D^{1W,s}).$$

Then the optimal solution of this cost minimization problem (R_W^{1W*} and R_L^{1W*}) is given by

$$R_W^{1W*} = u^{-1}(u^0 + v_H + \frac{2v_H}{3D^{1W}}),$$

$$R_L^{1W*} = u^{-1}(u^0 + v_H - \frac{v_H}{3D^{1W}}),$$

where u^{-1} represents the inverse function of u .

The minimization problem of the implementation cost under the one-loser tournament (the loser-selecting tournament)

The following result follows from Proposition 3.1:

Corollary 3.2. Make Assumptions 3.1, 3.4 and 3.5 regarding F_r and F_s . When the principal wants to implement an effort profile (e_H, e_H, e_H) using only this one-loser tournament contract $(2, R_W, R_L)$, she cannot implement any approach profile other than (s, s, s) at the same time.

By this result, when the principal wants to implement the effort profile (e_H, e_H, e_H) under the one-loser tournament, the principal's cost minimization problem is given by:

$$\min_{R_W, R_L} \quad 2R_W + R_L$$

subject to:

$$D^{1L,r} \{u(R_W) - u(R_L)\} \geq v_H \quad (3.12)$$

$$D^{1L,s} \{u(R_W) - u(R_L)\} \geq v_H \quad (3.13)$$

$$\frac{2}{3}u(R_W) + \frac{1}{3}u(R_L) - v_H \geq u^0 \quad (3.14)$$

where

$$D^{1L,r} = \frac{2}{3} - \int_{-\bar{e}(r)}^{\bar{e}(r)} [1 - \{1 - F_s(x - e_H)\}^2] f_r(x) dx$$

$$D^{1L,s} = \frac{2}{3} - \int_{-\bar{e}(s)}^{\bar{e}(s)} [1 - \{1 - F_s(x - e_H)\}^2] f_s(x) dx.$$

The first restriction (3.12) is the incentive compatibility constraint on an agent's deviation from a strategy (s, e_H) to $(r, 0)$. The amount of decrease in his probability of winning caused by this deviation is given by $D^{1L,r}$. The second restriction (3.13) is the incentive compatibility constraint on an agent's deviation from a strategy (r, e_H) to $(s, 0)$. The amount of decrease in his probability of winning caused by this deviation is given by $D^{1L,s}$. The third restriction (3.14) is the participation constraint. Note that only one of two incentive compatibility constraints is binding as in the case of the one-winner tournament. Let D^{1L} denote the smaller one of $D^{1L,r}$ and $D^{1L,s}$:

$$D^{1L} = \min(D^{1L,r}, D^{1L,s}).$$

Then the optimal solution of this cost minimization problem (R_W^{1L*} and R_L^{1L*}) is given by

$$\begin{aligned} R_W^{1L*} &= u^{-1}\left(u^0 + v_H + \frac{v_H}{3D^{1L}}\right), \\ R_L^{1L*} &= u^{-1}\left(u^0 + v_H - \frac{2v_H}{3D^{1L}}\right). \end{aligned}$$

Comparison of the minimum implementation cost

Here, I derive the very useful proposition for the comparison of the total implementation costs:

Proposition 3.2. Suppose that $D^{1L} = D^{1W} > 0$. If the agent's utility function on his reward is given by (1) $u(R) = -e^{-r_A R}$ ($r_A > 0$), (2) $u(R) = R^{\frac{1}{n}}$ ($n = 2, 3, \dots$), or (3) $u(R) = \log R$, then the following inequality holds:

$$R_W^{1W*} + 2R_L^{1W*} \geq 2R_W^{1L*} + R_L^{1L*}, \quad (3.15)$$

where the equality holds iff $u(R) = R^{\frac{1}{2}}$.

Proof: See Appendix. ■

The first utility function $u(R) = -e^{-r_A R}$ ($r_A > 0$) exhibits constant absolute risk aversion r_A . The second utility function $u(R) = R^{\frac{1}{n}}$ exhibits constant relative risk aversion $r_R = \frac{n-1}{n}$. The last utility function $u(R) = \log R$ has constant relative risk aversion $r_R = 1$.⁴ These utility functions are those that are most frequently used when the utility function is specified.

See the following lemma:

Lemma 3.4. Make Assumptions 3.1, 3.4 and 3.5 regarding to F_r and F_s . Then, (i) $D^{1W} = D^{1W,r}$, (ii) $D^{1L,s} > D^{1W}$, and (iii) $D^{1L,r} > D^{1W}$. Therefore, it holds that $D^{1L} > D^{1W}$.

⁴Although I have not arrived at the complete proof yet, I confidently predict that the result follows for the utility functions with constant relative risk aversion $r_R \geq 1/2$, but the result is the reverse for those with $0 < r_R < 1/2$.

Proof: See Appendix. ■

Lemma 3.4 tells us that D^{1L} is strictly greater than D^{1W} . Furthermore, we easily can know that $2R_W^{1L*} + R_L^{1L*}$ is decreasing in D^{1L} , and $R_W^{1W*} + 2R_L^{1W*}$ is decreasing in D^{1W} . Thus we have the following result immediately from Proposition 3.2 and Lemma 3.4.

Proposition 3.3. Suppose that the agent's utility function on his reward is given by (1) $u(R) = -e^{-r_A R}$ ($r_A > 0$), (2) $u(R) = R^{\frac{1}{n}}$ ($n = 2, 3, \dots$), or (3) $u(R) = \log R$. Then given Assumptions 3.1, 3.4 and 3.5, the one-loser tournament is more efficient than the one-winner tournament in terms of the cost for implementing high effort of all agents:

$$R_W^{1W*} + 2R_L^{1W*} > 2R_W^{1L*} + R_L^{1L*}.$$

Proof: The result straightforwardly follows from Proposition 3.2 and Lemma 3.4. ■

Given the several assumptions described above, the sum of the three effects leads to this superiority of the one-loser tournament to the one-winner tournament: $D^{1L} > D^{1W}$. First, even if there were only one available approach, the result holds. To be precise, suppose that only high risk approach r were available. Denote by D_r^{1L} the amount of decrease in an agent's winning probability caused by his deviation from a strategy (r, e_H) to a strategy $(r, 0)$ under the one-loser tournament:

$$D_r^{1L} = \frac{2}{3} - \int_{-\bar{\epsilon}(r)}^{\bar{\epsilon}(r)} [1 - \{1 - F_r(x - e_H)\}^2] f_r(x) dx.$$

In the proof of part (iii) of Lemma 3.4, we can see that the inequality $D_r^{1L} - D^{1W,r} > 0$ holds. I call this positive effect on the IC constraint $D_r^{1L} - D^{1W,r}$ the order statistic effect.

Second, by choosing the one-loser tournament, the principal can let agents choose the low risk approach. The diminution of noise increases the incentive for effort under the given spread between the winner's and loser's reward. It follows that the principal can offer the better insurance against the compensation risk to the risk-averse agents. $D^{1L,s} - D_r^{1L} > 0$. I call this positive effect the noise reduction effect.

Third, agents can deviate using the high risk approach r in addition to the low risk approach s . Under the one-loser tournament, agents may prefer a deviation using the high risk approach to a deviation using the low risk approach when e_H is large. I call this nonpositive effect on the IC constraint $\min(0, D^{1L,r} - D^{1L,s})$ the increasing noise deviation

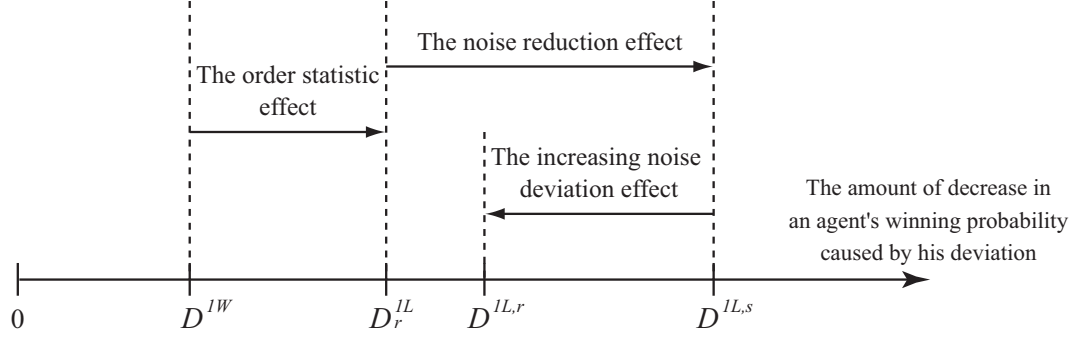


Figure 3.2: Three Effects of Choosing the One-loser Tournament on the IC Constraint (Risk Choice Model)

effect. As shown in the proof of part (iii) of Lemma 3.4, (the absolute value of) the increasing noise deviation effect is smaller than the noise reduction effect. Hence, the sum of these three effects is positive. These effects are illustrated in Figure 3.2.

Taking the agent's effort as a continuous variable, we can easily derive the similar result using the first-order approach.⁵ That is, given the utility function specified in the above form, Assumptions 3.1, 3.4 and 3.5, the one-loser tournament is more efficient than the one-winner tournament in terms of the cost for implementing the given same effort level of all agents $e = (e^+, e^+, e^+)$. Under the one-winner tournament, the IC constraint by the first-order approach is given by

$$D^{1W}\{u(R_W) - u(R_L)\} = v'(e^+). \quad (3.16)$$

where

$$D^{1W} = \int_{-\bar{\epsilon}(r)}^{\bar{\epsilon}(r)} \{f_r(x)\}^2 dx = \frac{1}{\alpha} \int_{-\bar{\epsilon}(s)}^{\bar{\epsilon}(s)} \{f_s(x)\}^2 dx. \quad (3.17)$$

On the other hand, under the one-loser tournament, the IC constraint by the first-order approach is given by

$$D^{1L}\{u(R_W) - u(R_L)\} = v'(e^+). \quad (3.18)$$

⁵For the first-order approach, see textbooks on contract theory, *e.g.*, Macho-Stadler and Perez-Castrillo (2001) and Salanié (1997). Mirrlees (1975) and Rogerson (1985) give us the best known set of sufficient conditions for its validity. It requires that the distribution function of performance is convex in effort and the density of performance with respect to effort has a monotone likelihood ratio. LiCalzi and Spaeter (2003) provide several families of distributions satisfying both properties.

where

$$D^{1L} = \int_{-\bar{\epsilon}(s)}^{\bar{\epsilon}(s)} \{f_s(x)\}^2 dx. \quad (3.19)$$

Since D^{1L} is greater than D^{1W} , the result follows straightforwardly from Proposition 3.2. There are two main differences from the model with two effort levels. First, the order-statistic effect disappears. Second, the increasing noise deviation effect is ignored under the first-order approach.

Principal's revenue function and choice of tournament

To begin with, I discuss the principal's choice problem of tournament under the following two specification of the revenue function:

- (1) $\pi(x_1, x_2, x_3) = \sum_{i=1}^3 x_i$.
- (2) $\pi(x_1, x_2, x_3) = \max\{x_1, x_2, x_3\}$.

(1) This type of revenue function is seen in the standard agency model. Given that all players choose (s, e_H) , the expected revenue $E\{\pi(x_1, x_2, x_3)|s, e_H\}$ is $3e_H$.⁶ Also, given that all agents choose (r, e_H) , the expected revenue $E\{\pi(x_1, x_2, x_3)|r, e_H\}$ is $3e_H$. Thus both tournaments have the same expected revenue. The one-loser tournament, however, is more efficient than the one-winner tournament in terms of implementation cost as shown in Proposition 3.3. Hence, the risk-neutral principal chooses the one-loser tournament.

(2) In this case, it holds that

$$E\{\pi(x_1, x_2, x_3)|r, e_H\} - E\{\pi(x_1, x_2, x_3)|s, e_H\} = (\alpha - 1)E\{\pi(x_1, x_2, x_3)|s, e_H\}. \quad (3.20)$$

Keep in mind that $\tilde{\epsilon}_i(r) = \alpha \tilde{\epsilon}_i(s)$ ($\alpha > 1$). If the difference of expected revenue $(\alpha - 1)E\{\pi(x_1, x_2, x_3)|s, e_H\}$ is larger than the difference of the implementation cost between the one-winner and the one-loser tournament $R_W^{1W*} + 2R_L^{1W*} - 2R_W^{1L*} - R_L^{1L*}$, the risk-neutral principal will choose the one-winner tournament. Otherwise, she will choose the one-loser tournament. This case is applicable to competitions in a division or in a firm of new product development, marketing plan, advertising plan, and so on, in which only the best is adopted. Such competitions are more likely in the environment of an innovative and immature than in a stable and mature business.

⁶E is the expectation operator.

I discuss some other implication of the results in the previous and this sections for the design of promotion tournaments. In innovative and immature markets or industries, there will be often good high risk approaches, but the steady approach will often be out-of-date and poor in terms of expected return. Then, if a firm used the loser-selecting tournament, there would be a possibility that workers choose the poor low risk approach although I have not formally analyzed such a situation. Thus the winner-selecting tournament will be better suited for firms in innovative and immature markets or industries if the difference of expected revenue between the high risk and the low risk approach is larger than difference of the implementation cost.

On the other hand, in stable, mature, or strictly regulated markets or industries, good high risk approaches will rarely exist. If a firm used the winner-selecting tournament, there would be the fear that workers choose the poor high risk approach. The loser-selecting tournament will be better suited.

Firms should also take account of the disadvantages of tournament contracts described in the previous chapter. Sabotage in tournament contracts is the main theme in chapter 5. I will argue that the one-loser tournament is, *ceteris paribus*, more subject to damage from sabotage than the one-winner tournament.

3.5 A continuum of approaches

The purpose of this section is to check whether or not the model in which agents can choose between only two approaches is restrictive in effect. To see this, I assume that agent i chooses an approach a_i from a continuum of approaches labeled by real numbers in the interval $[\underline{a}_i, \bar{a}_i]$, instead of the two approaches r and s . The density function, the cumulative probability function and the support of $\tilde{\epsilon}_i(a_i)$ are represented by $f_i(\cdot; a_i)$, $F_i(\cdot; a_i)$ and $[-\bar{\epsilon}_i(a_i), -\bar{\epsilon}_i(a_i)]$. Assume that $\bar{\epsilon}_i(a_i) \geq \bar{\epsilon}_i(a'_i)$ for any $a_i > a'_i$. Recall that $(\tilde{\epsilon}_1(a_1), \tilde{\epsilon}_2(a_2), \tilde{x}_n(a_n))$ are independently distributed.

According to this extension of the model, I modify assumptions 3.1–3.3:

Assumption 3.1' (symmetry). For any real value ϵ ,

$$F_i(\epsilon; a_i) = 1 - F_i(-\epsilon; a_i) \text{ and } f_i(\epsilon; a_i) = f_i(-\epsilon; a_i).$$

Assumption 3.2'(single-crossing). For any a_i and a'_i such that $a_i > a'_i$, the graph of the distribution function of $\tilde{\epsilon}_i(a_i)$ intersects with that of $\tilde{\epsilon}_i(a'_i)$ only once at $\epsilon = 0$.

$$\begin{cases} F_i(\epsilon; a_i) - F_i(\epsilon; a'_i) \geq 0 & \text{if } \epsilon \in (-\bar{\epsilon}_i(a_i), 0), \\ F_i(\epsilon; a_i) - F_i(\epsilon; a'_i) = 0 & \text{if } \epsilon = 0, \epsilon \in (-\infty, -\bar{\epsilon}_i(a_i)], \text{ or } \epsilon \in [\bar{\epsilon}_i(a_i), +\infty), \\ F_i(\epsilon; a_i) - F_i(\epsilon; a'_i) \leq 0 & \text{if } \epsilon \in (0, \bar{\epsilon}_i(a_i)). \end{cases}$$

Assumption 3.3'. There exists some open interval $\bar{I} = (0, \bar{d})$ such that for any two approaches a_i and a'_i with $a_i > a'_i$ of any agent i

$$F_i(\epsilon; a_i) - F_i(\epsilon; a'_i) < 0, f_i(\epsilon; a_i) > 0 \quad \text{and} \quad f_i(\epsilon; a'_i) > 0 \quad \text{for all } \epsilon \in \bar{I}.$$

Note that given Assumptions 3.1' and 3.2', raising a_i increases the riskiness of agent i 's performance \tilde{x}_i in the sense of a mean-preserving increase in spread:

$$a_i > a'_i \quad \text{implies} \quad \int_{-\bar{\epsilon}_i(a_i)}^y F_i(x_i; a_i) dx_i \geq \int_{-\bar{\epsilon}_i(a_i)}^y F_i(x_i; a'_i) dx_i \quad \text{for all } y \in [\underline{x}, -\bar{\epsilon}_i(a_i)]. \quad (3.21)$$

The minimum risk approach is \underline{a}_i and the maximum risk approach is \bar{a}_i of all approaches available to agent i .

Under this setting, we have the following proposition corresponding to Proposition 3.1.

Proposition 3.4. Given Assumptions 3.1'–3.3', the following hold:

- (i) Case of $m = 1$:
 - (a) If $n = 2$, then all available approaches are indifferent for every agent.
 - (b) If $n \geq 3$ (*i.e.*, winner-selecting tournament), then the maximum risk approach \bar{a}_i is a strictly dominant strategy for every agent.
- (ii) Case of $m = 2$ ($n \geq 3$):

- (a) If $n = 3$ (*i.e.*, loser-selecting tournament), then the minimum risk approach \underline{a}_i is a strictly dominant strategy for every agent.
- (b) If $n = 4$, then all available approaches are indifferent for every agent.
- (c) If $n \geq 5$ (winner-selecting tournaments), then the maximum risk approach \bar{a}_i is a strictly dominant strategy for every agent.

This proposition tells us that when the number of winners is one or two, the maximum risk approach is a strictly dominant strategy under the winner-selecting tournament, while the minimum risk approach is a strictly dominant strategy under the loser-selecting tournament.

I prepare one lemma to prove this proposition. Without loss of generality, I consider agent n 's decision making. Let $\tilde{x}_{(n-m)}(a_1, a_2, \dots, a_{n-1})$ denote the $n - m$ th order statistic of performances of $n - 1$ agents except agent n , $\tilde{x}_1(a_1), \tilde{x}_2(a_2), \dots, \tilde{x}_{n-1}(a_{n-1})$. If agent n 's performance is better than the top m th performance, he will get the high reward. Agent n 's best response on $(a_1, a_2, \dots, a_{n-1})$ is approach a_n which maximizes $\Pr(\tilde{x}_n(a_n) > \tilde{x}_{(n-m)}(a_1, a_2, \dots, a_{n-1}))$, the probability that the realized value of $\tilde{x}_n(a_n)$ is bigger than that of $\tilde{x}_{(n-m)}(a_1, a_2, \dots, a_{n-1})$. The following lemma corresponding to lemma 3.1 in the previous section is a strong tool to find the best response:

Lemma 3.5. Suppose that \tilde{x} is a continuous random variable with a density function $\phi(x)$ and statistically independent of $\tilde{x}_n(a_n)$. Given Assumptions 3.1'–3.3', the following hold:

- (i) If $\phi(x) \geq \phi(-x)$ for any $x > 0$ and at least $\phi(x) > \phi(-x)$ for any $x \in \bar{I}$, then \bar{a}_n is the only approach that maximizes $\Pr(\tilde{x}_n(a_n) > \tilde{x})$ and \underline{a}_n is the only approach that minimizes $\Pr(\tilde{x}_n(a_n) > \tilde{x})$ for all $a_n \in [\underline{a}_n, \bar{a}_n]$.
- (ii) If $\phi(x) = \phi(-x)$ for any $x > 0$, then $\Pr(\tilde{x}_n(a_n) > \tilde{x})$ is constant for all $a_n \in [\underline{a}_n, \bar{a}_n]$.
- (iii) If $\phi(x) \leq \phi(-x)$ for any $x > 0$ and at least $\phi(x) < \phi(-x)$ for any $x \in \bar{I}$, then \underline{a}_n is the only approach that maximizes $\Pr(\tilde{x}_n(a_n) > \tilde{x})$ and \bar{a}_n is the only approach that minimizes $\Pr(\tilde{x}_n(a_n) > \tilde{x})$ for all $a_n \in [\underline{a}_n, \bar{a}_n]$.

Proof: See Appendix. ■

Using Lemma 3.5, it is not very difficult to prove Proposition 3.4 from Lemma 3.5.

See Appendix.

From the symmetry of distribution, Proposition 3.4 can be transformed into the following proposition.

Proposition 3.5 Given Assumptions 3.1'–3.3', the following hold:

- (i) Case of $m = n - 1$:
 - (a) If $n = 2$, then all available approaches are indifferent for every agent i .
 - (b) If $n \geq 3$ (*i.e.*, loser-selecting tournament), then the minimum risk approach \underline{a}_i is a strictly dominant strategy for every agent i .
- (ii) Case of $m = n - 2$ ($n \geq 3$):
 - (a) If $n = 3$ (*i.e.*, winner-selecting tournament), then the maximum risk approach \bar{a}_i is a strictly dominant strategy for every agent i .
 - (b) If $n = 4$, then all available approaches will be indifferent for every agent i .
 - (c) If $n \geq 5$ (*i.e.*, loser-selecting tournament), then the minimum risk approach \underline{a}_i is a strictly dominant strategy for every agent.

Proof: See Appendix for the detail of the proof. ■

These two propositions automatically lead to the following corollary:

Corollary 3.3. Suppose that the number of agents n is equal or less than five. Then given Assumptions 3.1'–3.3', the following hold:

- (i) Under the winner-selecting tournament (*i.e.*, $n > 2m$), then to choose the maximum risk approach \bar{a}_i is a strictly dominant strategy for every agent i .
- (ii) If $n = 2m$, then all available approaches are indifferent for all agents.
- (iii) Under the loser-selecting tournament (*i.e.*, $n < 2m$), then to choose the minimum risk approach \underline{a}_i is a dominant strategy for each agent i .

Proof: The result follows straightforwardly from Propositions 3.4 and 3.5. ■

Although I have obtained the result only in the limited case ($m = 1, 2, n - 2, n - 1$),

it seem to suggest that the extension to a continuum of approach does not add new findings, that is, the two-approach model is not restrictive at least in case of homogeneous agents.

3.6 Heterogeneity in ability across agents and risk choice

Up to now, I have assumed that all agents have the same expected values for their performances. In this section, I relax this assumption and see how heterogeneity in ability will have an effect on their approach choices.

Consider the tournament in which two agents compete for the high reward. As heretofore, each agent i chooses an approach a_i from a continuum of approaches labelled by numbers in the interval $[\underline{a}_i, \bar{a}_i]$. The performance for agent i is given by a real-valued random variable $\tilde{x}_i(a_i)$ in the following form:

$$\tilde{x}_i(a_i) = \mu_i + \tilde{\epsilon}_i(a_i). \quad (3.22)$$

Assume that the abler agent has the higher μ_i . Suppose that agent 1 has higher ability than agent 2: $\mu_1 > \mu_2$. Instead of Assumptions 3.2'–3.3', I make the following two assumptions regarding the family of $F_i(\cdot; a_i)$:

Assumption 3.2'' (strong single-crossing). For any a_i and a'_i such that $a_i > a'_i$, the graph of the distribution function of $\tilde{\epsilon}_i(a_i)$ intersects with that of $\tilde{\epsilon}_i(a'_i)$ only once at $\epsilon = 0$.

$$\begin{cases} F_i(\epsilon; a_i) - F_i(\epsilon; a'_i) > 0 & \text{if } \epsilon \in (-\bar{\epsilon}_i(a_i), 0), \\ F_i(\epsilon; a_i) - F_i(\epsilon; a'_i) = 0 & \text{if } \epsilon = 0, \epsilon \in (-\infty, -\bar{\epsilon}_i(a_i)], \text{ or } \epsilon \in [\bar{\epsilon}_i(a_i), +\infty), \\ F_i(\epsilon; a_i) - F_i(\epsilon; a'_i) < 0 & \text{if } \epsilon \in (0, \bar{\epsilon}_i(a_i)). \end{cases}$$

Assumption 3.4' (strictly unimodal distribution). $F_i(\cdot, a_i)$ is strictly unimodal for any a_i and agent i :

$$\begin{cases} df_i(\epsilon; a_i)/d\epsilon > 0 & \text{if } \epsilon \in (-\bar{\epsilon}_i(a_i), 0), \\ df_i(\epsilon; a_i)/d\epsilon < 0 & \text{if } \epsilon \in (0, \bar{\epsilon}_i(a_i)). \end{cases}$$

Assumption 3.6 (non-triviality). For any (a_1, a_2) , every agent has positive probability to become the winner:

$$\mu_1 - \bar{\epsilon}_1(\underline{a}_1) < \mu_2 + \bar{\epsilon}_2(\underline{a}_2).$$

We have the following result:

Proposition 3.6. Given Assumptions 3.1', 3.2'', 3.4' and 3.6, the following hold under the two-agent tournament described above:

- (i) The minimum risk approach \underline{a}_1 is a strictly dominant strategy for agent 1 (high ability agent).
- (ii) the maximum risk approach \bar{a}_2 is a strictly dominant strategy for agent 2 (low ability agent).

Proof: The result follows from the logic of Lemma 3.1 or 3.4. See Appendix for full details of the proof. ■

The intuition is simple. The assumptions imply that the riskier approach has the fatter-tail distribution. The high ability agent prefers the low risk approach to the high risk one in order to defend his advantage over the low ability agent, but the low ability agent bets on the high risk approach in order to overcome his disadvantage.

The result implies that if a firm wants to get workers with high ability to choose the high risk approach, the firm should choose a small promotion rate m/n .

When the agent's effort as a continuous variable are embedded in the model, there is another setting about the heterogeneity in ability across agents: $v'_H(e) < v'_L(e)$, which implies the high ability agent's marginal cost or disutility of his effort is less than the low ability agent's. Although I do not offer a formal model and the analysis, the result will be the same as the proposition above. That is, given the R_H and R_L , the high ability agent chooses higher effort than the low-ability agent does. This implies the high ability agent's

greater expected performance than the low ability agent's. Thus, the same result as above will follow.

Chapter 4

Correlation Choice in Promotion Tournaments

4.1 Introduction

In addition to the riskiness, through workers' approach choice they will also be able to control the correlation of their performances to a limited degree, which also affects their rewards under their tournament. For example, if salespersons use the same sales technique that they learned in training together, they will get the correlated outcomes with one another. If marketing planners who compete in the planning of some product or brand use a similar concept, their marketing plans will be similar.

Under the winner-selecting tournament, they dislike the correlation of their performances. Under this tournament, they will almost never be able to become winners without good performances. Suppose that a worker chooses the common approach when some peer chooses the common approach. Even if he achieves good performance, then his peer also has a good performance with high probability, which reduces the probability that he may become a winner. Thus, it would be better for him to choose his own original approach, than to choose the common one. On the other hand, under the loser-selecting tournament, they prefer the correlation of their performances. Under this tournament, they will be able

to become winners as long as they do not have very bad performances. Suppose that a worker chooses the common approach when some peer chooses the same one. Even if he has a very bad outcome, his peer also has a very bad outcome with high probability, which reduces the fear he will be a loser. Hence, he will be better to choose the common approach. Explained in another way, the tournament filters out common noise, and hence when some peer chooses the common approach, the common approach and his own original approach correspond to the low risk one and high risk one, respectively. I show this perfectly in the model without individual-specific noise, and partially in the model with individual-specific noise.

The tournament filters out common noise. The reduction of noise enhances the workers' incentive to make efforts to perform their tasks under the given spread between the winner-prize and the loser-prize. Consequently, a risk-neutral firm can offer the better insurance against the compensation risk to the risk-averse workers. This suggests that the loser-selecting tournament is more efficient than the winner-selecting tournament in terms of the cost for implementing high efforts of risk-averse workers. I rigorously show this in the case of three workers without individual-specific noise.

From these results, as in the previous chapter I argue that the winner-selecting tournament is better suited for firms in innovative and immature markets or industries in which there is a high possibility that good original ideas exist, but the loser-selecting tournament is better suited for firms in stable, mature, or strictly regulated markets or industries in which there is a low possibility that there good original ideas exist.

In addition to this point, nature of task is also important to design the appropriate tournament. The loser-selecting tournament will be better suited to get workers voluntarily to improve their skill in the *approach* learned in off-the-job or on-the-job training. On the other hand, the winner-selecting tournament will be better suited to get them to develop good ideas. The lower rank job contains more routine work, in which it is important to improve the skill. On the other hand, the higher rank job contains more non-routine work, in which it is important to get new ideas. Thus it follows that the winner-selecting tournament is better suited for the upper rank, but the loser-selecting is better suited for the lower rank.

I also examine how heterogeneity in ability across workers will affect their approach choices in the case of two workers. I define two kinds of heterogeneity in ability: the *efficiency* ability that enhances performance on the given approach, and the *innovation* ability that finds good new ideas. It is shown that a high innovation-ability worker with

low efficiency-ability will try his new idea, but a high innovation-ability worker with high efficiency-ability will not try his new idea. This result suggest that if a firm wants to enhance the promotion ratio of high innovation-ability workers with high efficiency-ability to low innovation-ability workers with high efficiency-ability, the firm has to use the small promotion rate m/n .

The tournament model with many common approaches is also investigated to see how the ratio of winners m/n affects diversity of their behaviors. It is shown that the level of behavior diversity tends to be greater under the winner-selecting tournament than under the loser-selecting tournament. It is shown, however, that there is a possibility that there might exist some Nash equilibrium other than perfect homogeneity or perfect diversity.

The plan of this chapter is as follows. In Sections 4.2, 4.3 and 4.4, I present the basic model that agents choose between the common approach and his own original approach under promotion tournaments, and derive the equilibrium. In Section 4.5, I introduce individual-specific noise into the model and derive the equilibrium. In Section 4.6, agent's effort is embedded into the basic model (but with three agents). I compare the principal's minimum cost of implementing high effort under the one-winner tournament to that under the one-loser tournament and discuss the design of promotion tournaments. In Section 4.7, I analyze how heterogeneity in ability across agents will affect their approach choices. I define two kinds of heterogeneity in ability and discuss the implication of the result to the design of promotion tournament. The main purpose of Section 4.8 is to see how the ratio of winners m/n affects diversity of their behaviors. To examine this point, I analyze the tournament model with many common approaches.

4.2 The simple correlation choice model

As in the previous chapter, I consider a principal with n agents, indexed by $i = 1, 2, \dots, n$. She offers them the simple rank-order contract (m, R_W, R_L) : depending on their performances ranking, the top m agents (winners) are paid the high reward R_W respectively, but the other $n - m$ agents are paid the low one R_L respectively. I mean that the win corresponds to the promotion to the next upper rank.

The difference with the previous chapter is in the property of approaches that the agents choose. Agent i chooses his approach a_i from his approach set $A_i = \{c, o\}$ without

any private cost to himself. Approach c means a “common” approach that all agents know well. For example, it might be a “conventional” approach in the workplace, division, or firm, which all agents learned in on-the-job or off-the-job training together. On the other hand, approach o is each agent’s “original” one, which is assumed not to be known to his peers. It is assumed that the approaches are not contractible.

Agent i ’s performance or its evaluation is given by the following real-valued continuous random variable $\tilde{x}_i(a_i)$:¹

$$\tilde{x}_i(a_i) = \begin{cases} \tilde{\eta} & \text{if } a_i = c, \\ \tilde{\theta}_i & \text{if } a_i = o. \end{cases}$$

A random variable $\tilde{\eta}$ represents a common performance of agents who choose the “conventional” approach c . I assume that all agents who choose approach c will get the same performance. For example, if salespersons use the same sales technique that they learned in training together, they will get outcomes that correlate one another. If marketing planners who compete in planning use the conventional concept, their marketing plans will be similar. I make this assumption for a tractable and simple analysis, but it might seem to be an extreme assumption. I will relax this assumption later by introducing individual-specific noise. On the other hand, A random variable $\tilde{\theta}_i$ is agent i ’s performance when he choose his own original approach o . I make the following assumption regarding these random variables:

Assumption 4.1. $(\tilde{\eta}, \tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_n)$ are independent and identically distributed continuous random variables.

This assumption implies that the two approaches c and o are the same in regard to the expected value and the risk’s degree of performance. The difference between c and o is a correlation with peers’ performances. If agent i chooses approach o , then his performance is always statistically independent of any peer’s performance. If he chooses approach c , he will have the same performance as any other agents who choose approach c : the perfect correlation with them. It is assumed that when some agents have the same performance

¹This specification does not include the common noise. One of the main advantages of tournament contracts over individual contracts is that they difference out the effect of the common noise on the agents’ rewards. Since the purpose of this paper, however, is not the comparison between tournaments and other types of contracts, but the comparison among tournaments, I do not include the common noise in this specification for simplicity.

level, their rankings are randomly determined.²

I consider only pure strategies.³ Agent i 's strategy is simply given by $a_i \in A_i = \{c, o\}$. A strategy profile (an approach profile) for agents is given by $\mathbf{a} = (a_1, a_2, \dots, a_n) \in A_1 \times A_2 \times \dots \times A_n = A$. I denote by \mathbf{a}_{-i} a strategy profile except agent i .

I treat a tournament contract (m, R_W, R_L) , which is offered by the principal, as the given until Section 4.6 in which agents' effort variables are embedded in a more simplified model. For a while, we concentrate on the analysis of agent's approach choices. The agents choose a_i simultaneously and independently so as to maximize their expected utility. In this model, they do not have any private costs to choose strategy. It follows that maximizing their expected utility is equivalent to maximizing their winning probability. Thus it is not necessary to specify their utility functions until Section 4.6. I assume that the structure of game is common knowledge among agents.

4.3 High-reward probability functions

To derive the equilibrium of this game, we need to know each agent's high-reward probability given any strategy profile. Suppose that k agents choose approach c and the other $n - k$ agents choose o . I denote by $P_W^c(m, n, k)$ high-reward probability of an agent who chooses approach c . In the same way, denote by $P_W^o(m, n, k)$ high-reward probability of an agent who choose approach o .

Obviously, the sum of all agents' high-reward probabilities is always m for any strategy profiles. Hence, the following equations always hold for any n and m .

$$nP_W^c(m, n, n) = nP_W^o(m, n, 0) = m, \quad (4.1)$$

and

$$kP_W^c(m, n, k) + (n - k)P_W^o(m, n, k) = m \text{ for } k = 1, \dots, n - 1. \quad (4.2)$$

The identical equation (4.1) tells us all values of the high-reward probability functions for $k = 0$ or n . For $k = 1, \dots, n - 1$, if we get values of $P_W^c(m, n, k)$, then we will be able to derive easily the values of $P_W^o(m, n, k)$ by using the identical equation (4.2). Hence, first I will

²Imagine the very small individual-specific noise which is a continuous random variables.

³Extension to mixed strategies will not change the main results shown below.

derive the values of $P_W^c(m, n, k)$ under the tournament with $2m \leq n$, and then under one with $2m < n$. Last, I will get $P_W^o(m, n, k)$ using the derived $P_W^c(m, n, k)$ and the identical equation (4.2).

The tournament with $2m \leq n$

Case 1: $k = 1, \dots, m$.

When $n - k$ agents choose their own original approaches o and k agents choose approach c , the statistically independent $n - k + 1$ approaches are executed. Consider placing the approaches in the order of their realized outcomes. By our setting, all places are equally likely in approach c : The probability of i th place is $1/(n - k + 1)$ for $i = 1, 2, \dots, n - k + 1$. If approach c is better than or equal to the $m - k + 1$ th place, all k agents who chose approach c will get the high reward. If it gets $m - k + 1 + i$ th place ($i = 1, \dots, k - 1$), only $k - i$ in k will get the high reward. However, if it is worse than the $m + 1$ th place, no one who chose approach c will get a high reward. Thus, the high-reward probability $P_W^c(m, n, k)$ of an agent who chooses approach c is given by

$$\begin{aligned} P_W^c(m, n, k) &= \frac{m - k + 1}{n - k + 1} + \left(\frac{k - 1}{k} + \dots + \frac{1}{k} \right) \frac{1}{n - k + 1} \\ &= \frac{2m - k + 1}{2(n - k + 1)} \\ &\leq \frac{m}{n}. \text{ (Equality holds iff } k = 1 \text{ or } 2m = n.) \end{aligned}$$

Case 2: $k = m + 1, \dots, n - m - 1$.

If the approach c wins the i th place ($i = 1, \dots, m$), then $m + 1 - i$ in k agents who chose approach c will get the high-reward. If it is worse than the $m + 1$ th place, they will not get it. Hence, we have

$$\begin{aligned} P_W^c(m, n, k) &= \left(\frac{m}{k} + \dots + \frac{1}{k} \right) \frac{1}{n - k + 1} \\ &= \frac{m(m + 1)}{2(n - k + 1)} \\ &\leq \frac{m}{n}. \text{ (Equality holds iff } 2m = n.) \end{aligned}$$

Case 3: $k = n - m, \dots, n - 1$.

In this case, it is much easier to calculate $P_W^o(m, n, k)$ than $P_W^c(m, n, k)$. An agent who chooses his own original approach o will get the high-reward if and only if his outcome is greater than that of approach c . His high-reward probability is just one-half by the model setting. By equation (4.2), we can get the value of $P_W^c(m, n, k)$:

$$\begin{aligned} P_W^c(m, n, k) &= \frac{m - (n - k)(1/2)}{k} \\ &= \frac{2m - n + k}{2k} \\ &\leq \frac{m}{n}. \text{ (Equality holds iff } 2m = n.) \end{aligned}$$

Note that $P_W^c(m, n, k) = m/n = 1/2$ for any k if $2m = n$.

The tournament with $2m > n$

Case 1': $k = 1, \dots, n - m$.

This subcase is exactly the same as case 1 in the tournament with $2m \leq n$.

$$\begin{aligned} P_W^c(m, n, k) &= \frac{2m - k + 1}{2(n - k + 1)} \\ &\geq \frac{m}{n}. \text{ (Equality holds iff } k = 1.) \end{aligned}$$

Case 2': $k = n - m + 1, \dots, m - 1$.

Note that the number of approaches executed by agents $n - k + 1$ is equal or less than m . We have the following value of $P_W^c(m, n, k)$.

$$\begin{aligned} P_W^c(m, n, k) &= \frac{m - k + 1}{n - k + 1} + \left(\frac{k - 1}{k} + \dots + \frac{k - (n - m)}{k} \right) \frac{1}{n - k + 1} \\ &= 1 - \frac{(n - m)(n - m + 1)}{2k(n - k + 1)} \\ &> \frac{m}{n}. \end{aligned}$$

Case 3': $k = m, \dots, n - 1$.

This subcase is the same as case 3 in the tournament with $2m \leq n$. An agent who chooses approach o will have the high-reward if and only if his outcome is greater than that of the common approach c . His high-reward probability is just one-half by the model setting.

Using equation (4.2), we can get the value of $P_W^c(m, n, k)$:

$$\begin{aligned} P_W^c(m, n, k) &= \frac{m - (n - k)(1/2)}{k} \\ &= \frac{2m - n + k}{2k} \\ &> \frac{m}{n}. \end{aligned}$$

We have derived $P_W^c(m, n, k)$ for all cases. We can derive $P_W^o(m, n, k)$ from the identical equation (4.2) and $P_W^c(m, n, k)$. To sum up, we have the following high-reward probability functions:

- Under the winner-selecting tournament ($2m < n$)

$$P_W^c(m, n, k) = \begin{cases} \frac{2m-k+1}{2(n-k+1)} & \text{if } k = 1, \dots, m, \\ \frac{m(m+1)}{2k(n-k+1)} & \text{if } k = m+1, \dots, n-m-1, \\ \frac{2m-n+k}{2k} & \text{if } k = n-m, \dots, n. \end{cases} \quad (4.3)$$

$$P_W^o(m, n, k) = \begin{cases} \frac{2m(n-k+1)-k(2m-k+1)}{2(n-k)(n-k+1)} & \text{if } k = 0, \dots, m, \\ \frac{m(2n-2k+2-m-1)}{2(n-k)(n-k+1)} & \text{if } k = m+1, \dots, n-m-1, \\ \frac{1}{2} & \text{if } k = n-m, \dots, n-1. \end{cases} \quad (4.4)$$

From (4.3) and (4.4), we know:

$$P_W^c(m, n, k) \leq \frac{m}{n}. \quad (\text{Equality holds iff } k = 1 \text{ or } k = n.) \quad (4.5)$$

$$P_W^o(m, n, k) \geq \frac{m}{n}. \quad (\text{Equality holds iff } k = 0 \text{ or } k = 1.) \quad (4.6)$$

- Under the tournament with $2m = n$

$$P_W^c(m, n, k) = P_W^o(m, n, k) = \frac{m}{n} = \frac{1}{2}, \quad \text{for any } k. \quad (4.7)$$

- Under the loser-selecting tournament ($2m > n$)

$$P_W^c(m, n, k) = \begin{cases} \frac{2m-k+1}{2(n-k+1)} & \text{if } k = 1, \dots, n-m, \\ 1 - \frac{(n-m)(n-m+1)}{2k(n-k+1)} & \text{if } k = n-m+1, \dots, m-1, \\ \frac{2m-n+k}{2k} & \text{if } k = m, \dots, n. \end{cases} \quad (4.8)$$

$$P_W^o(m, n, k) = \begin{cases} \frac{2m(n-k+1)-k(2m-k+1)}{2(n-k)(n-k+1)} & \text{if } k = 0, \dots, n-m, \\ \frac{2(m-k)(n-k+1)+(n-m)(n-m+1)}{2(n-k)(n-k+1)} & \text{if } k = n-m+1, \dots, m-1, \\ \frac{1}{2} & \text{if } k = m, \dots, n-1. \end{cases} \quad (4.9)$$

From (4.8) and (4.9), we know:

$$P_W^c(m, n, k) \geq \frac{m}{n}. \quad (\text{Equality holds iff } k = 1 \text{ or } k = n.) \quad (4.10)$$

$$P_W^o(m, n, k) \leq \frac{m}{n}. \quad (\text{Equality holds iff } k = 0 \text{ or } k = 1.) \quad (4.11)$$

4.4 The weakly dominant strategy equilibrium

Now, consider strategy profiles in which l agents in $n-1$ except agent i choose approach c . If agent i also chooses approach c , then his high-reward probability will be $P_W^c(m, n, l+1)$. On the other hand, if he chooses his own original approach, then it will be $P_W^o(m, n, l)$. Under the winner-selecting tournament ($2m < n$), we have from (4.5) and (4.6)

$$P_W^c(m, n, l+1) \leq P_W^o(m, n, l). \quad (\text{Equality holds iff } l = 0.) \quad (4.12)$$

Under the tournament with $2m = n$, we get from (4.7):

$$P_W^c(m, n, l+1) = P_W^o(m, n, l) = \frac{1}{2}. \quad (4.13)$$

On the other hand, under the loser-selecting tournament ($2m > n$), we have from (4.10) and (4.11):

$$P_W^c(m, n, l+1) \geq P_W^o(m, n, l). \quad (\text{Equality will hold with } l = 0) \quad (4.14)$$

Proposition 4.1 briefly summarizes the above discussion:

Proposition 4.1. Given Assumption 4.1, the following hold:

- (i) Under the winner-selecting tournament ($2m < n$), his own original approach o is a weakly dominant strategy for every agent i .
- (ii) Under the tournament with $2m = n$, both approaches are indifferent for every agent i .

- (iii) Under the loser-selecting tournament ($2m > n$), the common approach c is a weakly dominant strategy for every agent i .

The logic for this result is the following. Under the winner-selecting tournament, an agent will almost never be able to get the high reward without a very good performance. Suppose that he chooses the common approach when some peer chooses the common approach. Even if he achieves a good performance, then his peer also has a good performance, which reduces the probability he gets the high reward. Thus, it will be better for him to choose his own original approach than to choose the common one. If no peer chooses the common approach, two approaches are equivalent for him. It follows that the original approach is a weakly dominant strategy.

On the other hand, under the loser-selecting tournament, an agent will be able to get the high reward as long as he does not have a very bad performance. As in the case above, suppose that he chooses the common approach when some peer chooses the same one. Even if he has a very bad outcome, his peer also has the same very bad outcome, which reduces the fear he will be the loser. Hence, it will be better for him to choose the common approach than to choose his own original approach. If all peers choose approach o , obviously two approaches are indifferent for him. Thus the common approach is a weakly dominant strategy under the loser-selecting tournament.

From this result, we know that under the winner-selecting tournament, a strategy profile (o, o, \dots, o) is a weakly dominant strategy equilibrium, and under the loser-selecting tournament, a strategy profile (c, c, \dots, c) is a weakly dominant strategy equilibrium. However, there are other Nash equilibria. In fact, the following is a complete list of Nash equilibria (only pure strategy equilibria):

Under the winner-selecting tournament ($2m < n$),

- a strategy profile (o, o, \dots, o) , which is a weakly dominant strategy equilibrium.
- any strategy profiles in which $n - 1$ agents choose their own approaches o , but the other chooses c .

Under the loser-selecting tournament ($2m > n$)

- a strategy profile (c, c, \dots, c) , which is a weakly dominant strategy equilibrium.

- a strategy profile (o, o, \dots, o) .

The equilibrium other than the weakly dominant strategy equilibrium always includes a weakly dominated strategy. I select the weakly dominant strategy equilibrium profile as the most reasonable one, *i.e.*, as one that rational players will choose. Furthermore, note that this weakly dominant strategy equilibrium is also the only trembling-hand perfect Nash equilibrium, which is the Nash equilibrium that is robust to the possibility that players make mistakes with very small probability.⁴

4.5 Model with individual-specific noise

In this section, I embed the agents' individual-specific noise in the model, and check whether the results are robust or not. Agent i ' performance is given by the following real-valued continuous random variable $\tilde{x}_i(a_i)$

$$\tilde{x}_i(a_i) = \begin{cases} \tilde{\eta} + \tilde{\epsilon}_i & \text{if } a_i = c, \\ \tilde{\theta}_i + \tilde{\epsilon}_i & \text{if } a_i = o. \end{cases} \quad (4.15)$$

I assume that $(\tilde{\eta}, \tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_n)$ are independent and identically and normally distributed random variables with zero mean and variance σ_a^2 . The common cumulative distribution function and the density function are given by $F_a(\cdot)$ and $f_a(\cdot)$, respectively. Random variable, $\tilde{\epsilon}_i$, is the agent i 's individual-specific noise. It can be thought of as pure luck, unknown personal ability, measurement error unknown in advance, or other random factors beyond someone's control. $(\tilde{\epsilon}_1, \tilde{\epsilon}_2, \dots, \tilde{\epsilon}_n)$ are independent and identically and normally distributed random variables with zero mean and variance σ_ϵ^2 . I also denote the common distribution function and the density function by $F_\epsilon(\cdot)$ and $f_\epsilon(\cdot)$, respectively. Furthermore, I assume that $(\tilde{\eta}, \tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_n, \tilde{\epsilon}_1, \tilde{\epsilon}_2, \dots, \tilde{\epsilon}_n)$ are also statistically independent. These are summarized as follows:

Assumption 4.2.

- (i) The random variables $(\tilde{\eta}, \tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_n)$ are independent and identically and normally distributed with zero mean and variance σ_a^2 .

⁴For the formal definition of a trembling-hand perfect Nash equilibrium, see textbooks on game theory, *e.g.*, Osborne and Rubinstein (1994).

- (ii) $(\tilde{\epsilon}_1, \tilde{\epsilon}_2, \dots, \tilde{\epsilon}_n)$ are also independent and identically and normally distributed with zero mean and variance σ_ϵ^2 .
- (iii) $(\tilde{\eta}, \tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_n, \tilde{\epsilon}_1, \tilde{\epsilon}_2, \dots, \tilde{\epsilon}_n)$ are statistically independent.

As in the previous section, I denote by $P_W^c(m, n, k)$ high-reward probability of an agent who chooses approach c given that k agents choose approach c and the other $n - k$ agents o . In the same way, let $P_W^o(m, n, k)$ denote high-reward probability of an agent who chooses approach o given that k agents choose approach c and the other $n - k$ agents o .

Two-agent tournament

First I consider the two-agent-one-winner tournament. When both agents choose the same approach (c, c) or (o, o) , they obviously have the same winning probability:

$$P_W^c(1, 2, 2) = P_W^o(1, 2, 0) = \frac{1}{2}.$$

On the other hand, even if they choose a different approach from each other (c, o) or (o, c) , their performances are statistically independent and identically distributed. Therefore, we have

$$P_W^c(1, 2, 1) = P_W^o(1, 2, 0) = \frac{1}{2}.$$

Thus, both approaches are indifferent for every agent.

Proposition 4.2. Under the two-agent tournament with individual-specific noise, approaches c and o are indifferent for every agent.

Note that the assumption of normal distributions is not necessary to get this result.

Three-agent tournament

Next, I consider the three-agent tournament. At first, I derive the high reward probability function $P_W^c(1, 3, k)$ and $P_W^o(1, 3, k)$ in case of the one-winner tournament ($m = 1$). If all three agents choose the same approach: (c, c, c) or (o, o, o) , they clearly have the same high reward probability:

$$P_W^c(1, 3, 3) = P_W^o(1, 3, 0) = \frac{1}{3}. \quad (4.16)$$

Even if one agent chooses c and the others choose o , their performances are independent and identically distributed random variables. Hence, three agents have the same winning probability:

$$P_W^c(1, 3, 1) = P_W^o(1, 3, 1) = \frac{1}{3}. \quad (4.17)$$

When two agents choose approach c , the situation is more complicated. Without loss of generality, suppose that agent 1 and agent 2 choose approach c . Which approach will agent 3 prefer in terms of his winning probability, c or o ? Define $\tilde{y}_i \equiv \tilde{x}_i - \tilde{\eta}$. Then we have

$$\tilde{y}_1(c) = \tilde{\epsilon}_1, \tilde{y}_2(c) = \tilde{\epsilon}_2,$$

and

$$\tilde{y}_3(a_3) = \begin{cases} \tilde{\epsilon}_3 & \text{if } a_3 = c, \\ \tilde{\theta}_3 - \tilde{\eta} + \tilde{\epsilon}_3 & \text{if } a_3 = o. \end{cases}$$

Note that these random variables $(\tilde{y}_1(c), \tilde{y}_2(c), \tilde{y}_3(a_3))$ have zero mean and are statistically independent. We can apply the results in Chapter 3 to this. Let $\tilde{y}_{(2)}(c, 2)$ be the second order statistic of performances of two agents 1 and 2: $\tilde{y}_{(2)}(c, 2) = \max[\tilde{y}_1(c), \tilde{y}_2(c)]$. Agent 3 can get a high reward when the realized value of his relative performance $\tilde{y}_n(a_n)$ is greater than that of $\tilde{y}_{(2)}(c, 2)$. The density function $\phi_{(2)}(y; c, 2)$ is given by

$$\phi_{(2)}(y; c, 2) = 2F_\epsilon(y)f_\epsilon(y).$$

Since the distribution function of $\tilde{\epsilon}_i$ is symmetric around zero and the density $f_\epsilon(y)$ is always positive, we have

$$\phi_{(2)}(y; c, 2) - \phi_{(2)}(-y; c, 2) = 2(2F_\epsilon(y) - 1)f_\epsilon(y) > 0 \text{ for any } y > 0.$$

On the other hand, $\tilde{y}_3(c)$ is normally distributed with zero mean and variance σ_ϵ^2 , and $\tilde{y}_3(o)$ is normally distributed with zero mean and variance $2\sigma_a^2 + \sigma_\epsilon^2$. We can easily make sure that the family of distribution functions of $\tilde{y}_3(a_3)$ satisfies Assumption 3.2 in the previous chapter: For the riskiness of agent 3's *relative* performance $\tilde{y}_3(a_3)$, approach o is riskier than approach c in the sense of a mean-preserving spread. Obviously the family of distribution functions of $\tilde{y}_3(a_3)$ also satisfies Assumptions 3.1 and 3.3. Thus, part (i) of Lemma 3.1 tells us that agent 3 strictly prefers approach o to approach c when the other two agents choose

c : $P_W^o(1, 3, 2) > P_W^c(1, 3, 3) = \frac{1}{3}$. The sum of all three agents' high-reward probabilities is always one: $2P_W^c(1, 3, 2) + P_W^o(1, 3, 2) = 1$. Hence, we have

$$P_W^c(1, 3, 2) < \frac{1}{3} < P_W^o(1, 3, 2). \quad (4.18)$$

From (4.16) (4.17) and (4.18), we have

$$P_W^c(1, 3, k) \leq P_W^o(1, 3, k + 1), \text{ for } k=0,1,2 \quad (4.19)$$

where strict inequality holds if $k = 1$, and equality holds if $k = 0, 2$. This implies approach o is a weakly dominant strategy for every agent.

Next, consider the two-winner case. By the same logic as the one-winner case, we have

$$P_W^c(2, 3, 3) = P_W^o(2, 3, 0) = \frac{2}{3}. \quad (4.20)$$

$$P_W^c(2, 3, 1) = P_W^o(2, 3, 1) = \frac{2}{3}. \quad (4.21)$$

Suppose that agent 1 and agent 2 choose approach c . Which approach will agent 3 prefer in terms of his high reward probability? Let $\tilde{y}_{(1)}(c, 2)$ be the first order statistic of relative performances of two agents 1 and 2; that is, $\tilde{y}_{(2)}(c, 2) = \min[\tilde{y}_1(c), \tilde{y}_2(c)]$. Then the density function $\phi_{(1)}(y; c, 2)$ is given by

$$\phi_{(1)}(y; c, 2) = 2[1 - F_\epsilon(y)]f_\epsilon(y).$$

Since the distribution function of $\tilde{\epsilon}_i$ is symmetric around zero and the density $f_\epsilon(y)$ is always positive, we have

$$\phi_{(1)}(y; c, 2) - \phi_{(2)}(-y; c, 2) = 2(1 - 2F_\epsilon(y))f_\epsilon(y) < 0 \text{ for any } y > 0.$$

Applying part (iii) of Lemma 3.1, we find that agent 3 strictly prefers approach c to approach o when the other two agents choose c . That is, $P_W^c(2, 3, 2) > P_W^o(2, 3, 3) = \frac{2}{3}$. The sum of all three agents' high-reward probabilities is always two: $2P_W^c(2, 3, 2) + P_W^o(2, 3, 2) = 2$. Hence, we have

$$P_W^c(2, 3, 2) > \frac{2}{3} > P_W^o(2, 3, 2). \quad (4.22)$$

From (4.20) (4.21) and (4.22), we have

$$P_W^c(1, 3, k) \geq P_W^o(1, 3, k + 1), \text{ for } k=0,1,2$$

where strict inequality holds if $k = 1$, and equality holds if $k = 0, 2$. This means that approach c is a weakly dominant strategy for every agent on the two-winner-one-loser tournament. Proposition 4.3 summarizes the above discussion:

Proposition 4.3. Given Assumption 4.2, the following hold under the three-agent tournament with individual-specific noise described above.

- (i) Approach o is a weakly dominant strategy for every agent under the winner-selecting tournament ($m = 1$).
- (ii) Approach c is a weakly dominant strategy for every agent under the loser-selecting tournament ($m = 2$).

The logic of this result of the three-agent model is almost the same as that of the risk choice model in Chapter 3. The tournament filters out common noise. For an agent, the common approach and his own original one correspond to the low risk one and the high risk one, respectively, when his peers choose the common approach. Then he prefers his own original approach (the high risk approach) under the one-winner tournament; and he prefers the common approach (the low risk approach) under the one-loser tournament. When no peer chooses the common approach, both approaches' performances are independent of peers' performances, and therefore indifferent for him. Thus the result follows.

The n -agent tournament

From Propositions 4.2 and 4.3 we know that when the number of agents is two or three, the introduction of the individual-specific noise does not change the result for the approach choice at all. Although I predict the same result holds even if the number of agents is greater than three, the situation is too complicated to easily derive the result. I only present the following proposition:

Proposition 4.4. Given Assumption 4.2, a strategy profile (c, c, \dots, c) is a Nash equilib-

rium if $2m \geq n$, but not if $2m < n$.

Proof: See Appendix. ■

Also note that a strategy profile (o, o, \dots, o) is always a Nash equilibrium for any m and n .

4.6 Effort, correlation choice, and design of promotion tournaments

I introduce the agents' efforts which enhance their performances in the simple model described in section 4.2 and argue the principal's design problem of the tournament contract. As in Chapter 3, I analyze the three-agent tournament.

Agent i chooses not only his approach a_i from his approach set $A_i = \{c, o\}$, but also his effort level e_i from his effort set $E_i = \{e_L, e_H\}$. For simplicity, I assume that $e_L = 0 < e_H$. Since I consider only pure strategy choices, his strategy is represented by $(a_i, e_i) \in A_i \times E_i$ which is his strategy set: $A_i \times E_i = \{(c, 0), (c, e_H), (o, 0), (o, e_H)\}$. A strategy profile for the agents is given by (\mathbf{a}, \mathbf{e}) where an approach profile $\mathbf{a} = (a_1, a_2, a_3) \in A_1 \times A_2 \times A_3$ and an effort profile $\mathbf{e} = (e_1, e_2, e_3) \in E_1 \times E_2 \times E_3$.

His performance $\tilde{x}_i(a_i, e_i)$ is given by a continuous random variable in the following form:

$$\tilde{x}_i(a_i) = \begin{cases} e_i + \tilde{\eta} & \text{if } a_i = c, \\ e_i + \tilde{\theta}_i & \text{if } a_i = o. \end{cases}$$

Assumption 4.3.

- (i) $(\tilde{\eta}, \tilde{\theta}_1, \tilde{\theta}_2, \tilde{\theta}_3)$ are independently and identically distributed with zero mean. The common c.d.f., p.d.f., and support are represented by $F(\cdot)$, $f(\cdot)$, and $[-\bar{\theta}, \bar{\theta}]$.
- (ii) $F(\cdot)$ is symmetric: For any real value ϵ ,

$$F(\epsilon) = 1 - F(-\epsilon), \text{ and } f(\epsilon) = f(-\epsilon).$$

(iii) $F(\cdot)$ is unimodal:

$$df_r(\epsilon)/d\epsilon \gtrless 0 \text{ according as } \epsilon \lesseqgtr 0.$$

I assume that all agents have the same von Neumann and Morgenstern utility function:

$$u(R, e) = u(R) - v(e), \quad (4.23)$$

where R represents the reward he receives, and $v(e)$ measures the cost or disutility of his effort e . Assume that $u' > 0, u'' < 0$, $v(e_L) = v(0) = 0, v(e_H) > 0$ and $e_H > v(e_H)$. For simple notation, let $v_H = v(e_H)$.

The principal offers the agents a tournament contract (m, R_W, R_L) where m is the number of winners ($m = 1$, or 2), R_W is the reward of (each) winner, and R_L is the reward of (each) loser. The principal's revenue function is represented by $\pi(x_1, x_2, x_3)$. Afterward, I will specify the revenue function and argue about the design of adequate tournament contract. I assume that the principal is risk neutral and maximizes the expected revenue minus the sum of the total payment.

The timing of the game is as follows:

1. The principal offers the agents a tournament contract (m, R_W, R_L) .
2. The agents decide whether to accept the contract or not. If the contract is accepted, the game goes to the third stage. If either agent rejects the offer, the game ends and all the parties obtain their reservation payoffs. Denote by u^0 every agent's reservation utility. The principal's reservation utility is zero.
3. The agents choose actions (a_i, e_i) simultaneously and independently.
4. The principal observes the outcomes and the payments are made according to the contract in force.

Assuming that the principal always benefits by having the agents exert effort e_H as in Section 3.4, I begin by comparing her minimum cost of implementing an effort profile (e_H, e_H, e_H) under the one-winner tournament ($m = 1$) to that under the one-loser tournament ($m = 2$).

The minimization problem of the implementation cost under the one-winner tournament (the winner-selecting tournament)

The main result in section 4.4 indicates that given an effort profile (e_H, e_H, e_H) , an approach profile (o, o, o) is a weakly dominant strategy equilibrium under the one-winner tournament. Although there are other Nash equilibria, I assume that when the principal wants to implement the effort profile (e_H, e_H, e_H) using only this one-winner tournament, she cannot implement any approach profile other than (o, o, o) at the same time. Then the principal's cost minimization problem is given by:

$$\min_{R_W, R_L} R_W + 2R_L$$

subject to:

$$D^{1W} \{u(R_W) - u(R_L)\} \geq v_H \quad (4.24)$$

$$\frac{1}{3}u(R_W) + \frac{2}{3}u(R_L) - v_H \geq u^0 \quad (4.25)$$

where

$$D^{1W} = \frac{1}{3} - \int_{-\bar{\theta}}^{\bar{\theta}} \{F(x - e_H)\}^2 f(x) dx.$$

The first restriction (4.24) is the incentive compatibility constraint on an agent's deviation from a strategy (o, e_H) to $(o, 0)$ or $(c, 0)$. I denote by D^{1W} the amount of decrease in his probability of winning caused by this deviation. The second restriction (4.25) is the participation constraint.

Then the optimal solution of this cost minimization problem $(R_W^{1W*}$ and $R_L^{1W*})$ is given by

$$R_W^{1W*} = u^{-1}(u^0 + v_H + \frac{2v_H}{3D^{1W}}), \quad (4.26)$$

$$R_L^{1W*} = u^{-1}(u^0 + v_H - \frac{v_H}{3D^{1W}}), \quad (4.27)$$

The minimization problem of the implementation cost under the one-loser tournament (the loser-selecting tournament)

The main result in section 4.4 indicates that given an effort profile (e_H, e_H, e_H) , an approach profile (c, c, c) is a weakly dominant strategy equilibrium under the one-winner tournament. Although there are other many Nash equilibria shown in section 4.4, I assume that when the principal wants to implement the effort profile (e_H, e_H, e_H) using only this one-winner tournament, she cannot implement any approach profile other than (c, c, c) at the same time. Then the principal's cost minimization problem is given by:⁵

$$\min_{R_W, R_L} \quad 2R_W + R_L$$

subject to:

$$D^{1L}\{u(R_W) - u(R_L)\} \geq v_H \quad (4.28)$$

$$\frac{2}{3}u(R_W) + \frac{1}{3}u(R_L) - v_H \geq u^0 \quad (4.29)$$

where

$$D^{1L} = \frac{2}{3} - \int_{-\bar{\theta}}^{\bar{\theta}} F(x - e_H) f(x) dx.$$

The first restriction (4.28) is the incentive compatibility constraint on an agent's deviation from a strategy (c, e_H) to $(o, 0)$. Denote by D^{1L} the amount of decrease in his probability of winning caused by this deviation. We do not have to include the IC constraint on an agent's deviation from a strategy (c, e_H) to $(c, 0)$ in the list because it would bring him zero winning probability.

The optimal solution of this cost minimization problem (R_W^{1L*} and R_L^{1L*}) is represented by

$$R_W^{1L*} = u^{-1}(u^0 + v_H + \frac{v_H}{3D^{1L}}), \quad (4.30)$$

$$R_L^{1L*} = u^{-1}(u^0 + v_H - \frac{2v_H}{3D^{1L}}). \quad (4.31)$$

Comparison of the minimum implementation cost

I have already had a useful tool to compare the implementation cost: Proposition 3.2 in the previous chapter. The next lemma says that the result is applicable to this case:

Lemma 4.1. Given Assumption 4.3, $D^{1L} - D^{1W} > 1/6$.

⁵We know that if their rewards were equally shared when some agents had the same performance level, the principal would be able to offer full insurance for risk-averse agents. Again imagine the very small individual-specific noise which is a continuous random variable.

Proof: See Appendix. ■

Proposition 4.5. Suppose that the agent's utility function on his reward is given by (1) $u(R) = -e^{-r_A R}$ ($r_A > 0$), (2) $u(R) = R^{\frac{1}{n}}$ ($n = 2, 3, \dots$), or (3) $u(R) = \log R$. Then the one-loser tournament is more efficient than the one-winner tournament in terms of the cost for implementing high effort of all agents:

$$R_W^{1W*} + 2R_L^{1W*} > 2R_W^{1L*} + R_L^{1L*}.$$

Proof: The result automatically follows from Proposition 3.2 and Lemma 4.1. ■

As in the case of risk choice in the previous chapter, given the several assumptions described above, the sum of the same three effects leads to this superiority of the one-loser tournament to the one-winner tournament: $D^{1L} > D^{1W}$. First, even if the available approach were only o , the same result held: the order statistic effect. Suppose that only original approach o were available. Denote by D_o^{1L} the amount of decrease in his probability of winning caused by his deviation from a strategy (o, e_H) to a strategy $(o, 0)$ under the one-loser tournament:

$$D_o^{1L} = \frac{2}{3} - \int_{-\bar{\theta}}^{\bar{\theta}} [1 - \{1 - F(x - e_H)\}^2] f(x) dx.$$

You already know that $D_o^{1L} - D^{1W} > 0$ holds from the discussion of the order statistic effect in Chapter 3.

The second effect is the noise reduction effect. By choosing the one-loser tournament the principal can let agents choose the common approach. The perfect cut of noise increases the incentive for effort under the given spread between the winner's and loser's reward. Let D_c^{1L} the amount of decrease in his probability of winning caused by his deviation from a strategy (c, e_H) to a strategy $(c, 0)$ under the one-loser tournament. Obviously $D_c^{1L} = 2/3$ and therefore $D_c^{1L} - D_o^{1L} > 0$.

The last effect is the increasing noise deviation effect: $D^{1L} - D_c^{1L}$. Agents can deviate using the original approach o in addition to the common approach c . Under the one-loser tournament, agents prefer a deviation using the original approach to using the common approach. You can easily check that (the absolute value of) this negative effect is smaller than the noise reduction effect. Hence, the sum of the three effects is always positive. These three effects are illustrated in Figure 4.1.

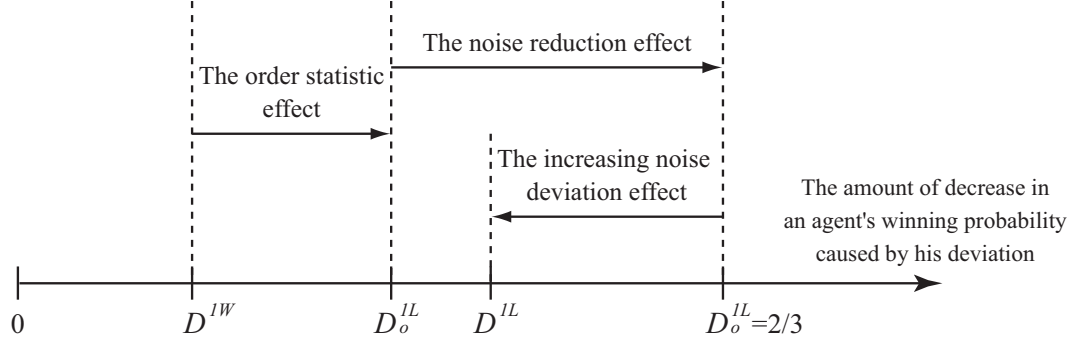


Figure 4.1: Three Effects of Choosing the One-loser Tournament on the IC Constraint (Correlation Choice Model)

Principal's revenue function and choice of tournament

First, I argue about the principal's best choice of tournament under the following form of the revenue function just as in the previous chapter:

$$(1) \pi(x_1, x_2, x_3) = \sum_{i=1}^3 x_i.$$

$$(2) \pi(x_1, x_2, x_3) = \max\{x_1, x_2, x_3\}.$$

(1) Both tournaments have the same expected revenue $3e_H$. The one-loser tournament is strictly more efficient than the one-winner tournament in terms of implementation cost as shown in Proposition 4.5. Hence, the risk-neutral principal chooses the one-loser tournament.

(2) When the revenue is defined as the realized value of the third order statistic of performances of three agents, given that all agents choose (c, e_H) , the expected revenue is equal to e_H , or given that all agents choose (o, e_H) , the expected value is greater than e_H : The principal dislikes the correlation among agents' performances. If the expected revenue advantage of the one-winner tournament is larger than the cost advantage of the one-winner tournament, the principal will choose the one-winner tournament. Otherwise, she will choose the other. As I already stated in the previous chapter, this form of the revenue function typically describes that of competitions or contests of new product development, marketing plan, advertising plan, and so on in a firm or a division. Such competitions are more likely under an innovative and immature business environment than under a more stable and mature one.

I derive some other implication of the results in the last section and this section to the design of promotion tournaments. I argue the circumstances to which each tournament is well suited. In innovative and immature markets or industries, the workers have many opportunities to develop good ideas. If they worked under the loser-selecting tournament, they would not try their ideas for fear of failure or even would not try to think of new ideas and would do their tasks by the conventional approach. In these circumstances, the winner-selecting tournament such as the one-winner tournament will be better. Contrary to such markets or industries, in stable, mature, or regulated markets or industries, good new ideas will rarely come to workers' heads. If a firm used the winner-selecting tournament, there would be the danger that agents bet on his own original approach inferior to the conventional one. In this case, the loser-selecting tournament will be better.

Nature of task is also important for designing appropriate promotion tournaments. The loser-selecting tournament will be better suited to get workers voluntarily to improve their skill in *approach* learned in off-the-job or on-the-job training. On the other hand, the winner-selecting tournament will be better suited to get them to develop good ideas. The lower rank job contains more routine work, in which it is important to improve the skill. On the other hand, the higher rank job contains more non-routine work, in which it is important to develop new ideas. Thus it follows that the winner-selecting tournament is better suited for the upper job levels, but the loser-selecting is better suited for the lower job levels. One example is the military, shown in the first chapter. Asch and Warner (2001) research the personnel system of the United States Military. They report that the promotion rates decline sharply with rank. For example, officers' promotion decisions over O-3 level are made by centralized promotion boards. These promotions are nationally competitive and based on vacancies in the next higher rank. They are considered for promotion by entry-year group and either promoted or not promoted at specific years of service points. The O-4, O-5, O-6, and O-7 promotion rates are about 80%, 60%, 40%, and 7%, respectively.

4.7 Heterogeneity in ability and approach choice

In this section, I analyze how heterogeneity in ability across agents will affect their approach choices and discuss the implication of the result to the design of promotion tournaments. I introduce two kinds of heterogeneity in ability into the model. Consider the

tournament in which two agents 1 and 2 compete for the high reward. There are two types of agents with regard to the set of approaches available: type A and type B . Type A agent has two approaches available: c and o . However, type B agent has only one approach, c . That is to say, type A agent has his own original approach, but type B does not. This is the first heterogeneity in ability. I call this ability the innovation ability. Agent 1 is type A with probability P_{1A} and type B with probability $P_{1B} = 1 - P_{1A}$. Agent 2 is type A with probability P_{2A} and type B with probability $P_{2B} = 1 - P_{1A}$. I assume that $0 < P_{1A} < 1$ and $0 < P_{2A} < 1$. It is assumed that each agent knows his type, but does not know his peer's type and only knows the prior probability of each type.

For the simplicity of expression, I call agent i of type A and agent i of type B , “agent iA ” and “agent iB ,” respectively.

Agent iA 's performance ($i = 1, 2$) is given by the following continuous random variable $\tilde{x}_{iA}(a_{iA})$:

$$\tilde{x}_{iA}(a_{iA}) = \begin{cases} \mu_i + \tilde{\eta} + \tilde{\epsilon}_i & \text{if } a_A = c, \\ \mu_i + \tilde{\theta}_i + \tilde{\epsilon}_i & \text{if } a_A = o. \end{cases}$$

On the other hand, the agent iB 's performance is given by the following continuous random variable \tilde{x}_{iB} :

$$\tilde{x}_{iB} = \mu_i + \tilde{\eta} + \tilde{\epsilon}_i.$$

The random variables $(\tilde{\eta}, \tilde{\theta}_1, \tilde{\theta}_2, \tilde{\epsilon}_1, \tilde{\epsilon}_2)$ satisfy Assumption 4.2.

I assume the expected value of agent i 's performance μ_i is constant regardless of his type or his approach choice. Furthermore, I assume that agent 1 has the higher ability than agent 2: $\mu_1 > \mu_2$. This is the second heterogeneity in ability. I call this ability the efficiency ability because if the agent's effort as a continuous variable is built in the model and the heterogeneity in ability between agents is defined as the difference of marginal cost or disutility of his effort: $v'_H(e) < v'_L(e)$, then it will hold that $\mu_1 > \mu_2$ in the equilibrium.

The agents (of Type A) choose approaches simultaneously and independently. It is assumed that the structure of game is common knowledge.

I have the following result:

Proposition 4.6. Make Assumption 4.2. Then the following hold:

- (i) The common approach c is a strictly dominant strategy for agent 1A (high ability agent with his own original approach).
- (ii) His own original approach o is a strictly dominant strategy for agent 2A (low ability agent with his own original approach).

Proof: See Appendix. ■

The logic of this result is very similar to that of the risk choice model. The tournament filters out common noise. If there is a possibility that the peer chooses the common approach, the common approach and the original approach will correspond to the low risk one and the high risk one, respectively. The high-ability agent 1A prefer the low risk to the high risk approach in order to defend his advantage over the low-ability agent, but the low-ability agent 2A bets on the high risk approach in order to overcome his disadvantage.

The high ability agents in terms of efficiency behave conservatively without respect to their innovative ability. If the principal enhances the promotion ratio of type 1A to type 1B, she will have to use the small promotion rate m/n .

4.8 Model with many common approaches and diversity of agents' behavior

In this section, I analyze the tournament model with many common approaches. The main purpose of this section is to see how the ratio of winners m/n affects diversity of their behaviors. The previous models in this chapter are a little restrictive to see this point because there is only one common approach and agents cannot copy the peers' original approaches.

It is assumed that the possible approach set $A_c = \{c_1, c_2, \dots, c_l\}$ is common to all agents and $n \leq l$. The l common approaches are indexed by j . Since I consider only pure strategy choices, agent i 's strategy set and a strategy profile are A_c and $\mathbf{a} \in A_c \times A_c \times \dots \times A_c$, respectively. Agent i 's performance $\tilde{x}_i(a_i)$ is given by a continuous random variable in the

form:

$$\tilde{x}_i(a_i) = \tilde{\eta}_j + \tilde{\epsilon}_i \text{ if } a_i = c_j. \quad (4.32)$$

As in last section, I put forward the following assumption regarding to these random variables:

Assumption 4.4.

- (i) The random variables $(\tilde{\eta}_1, \tilde{\eta}_2, \dots, \tilde{\eta}_l)$ are independent and identically normally distributed with zero mean and variance σ_a^2 .
- (ii) $(\tilde{\epsilon}_1, \tilde{\epsilon}_2, \dots, \tilde{\epsilon}_n)$ are also normally distributed with zero mean and variance σ_ϵ^2 .
- (iii) $(\tilde{\eta}_1, \tilde{\eta}_2, \dots, \tilde{\eta}_l, \tilde{\epsilon}_1, \tilde{\epsilon}_2, \dots, \tilde{\epsilon}_n)$ are statistically independent.

The agents choose approaches simultaneously and independently. It is assumed that the structure of the game is common knowledge. I use Nash equilibrium as equilibrium concept.

To measure the diversity of agents' actions, I add the following notations and terms:

- I call the set of agents who choose the same approach a *cluster*. The size of a cluster implies the number of agents who make up the cluster.
- $d(\mathbf{a})$: The number of clusters in a strategy profile \mathbf{a} . That is, the number of approaches at least one agent chooses in \mathbf{a} . If all agents choose the same approach, then $d = 1$. If each agent chooses a different approach from one another, then $d = n$. d can be interpreted as a measure of the diversity of their actions.
- $\underline{d}^*(m, n)$: The minimum value of d of Nash equilibrium profiles under the n -agent- m -winner tournament.
- $\bar{d}^*(m, n)$: The maximum value of d of Nash equilibrium profiles under the n -agent- m -winner tournament.
- $n = q_m m + r_m$ (n, q_m, m and r_m are integers. $0 \leq r_m < m$): Let q_m and r_m be the quotient given by dividing n by m and its residue, respectively. I also sometimes denote the residue by $\text{mod}(m, n)$.

Under the two-agent tournament, all strategy profiles are obviously Nash equilibria. The level of diversity is not unique: $\underline{d}^*(1, 2) = 1$ and $\bar{d}^*(1, 2) = 2$.

Three-agent tournament

I have the following proposition corresponding to Proposition 4.3:

Proposition 4.7. Given Assumption 4.4, the following hold:

- (i) Under the three-agent-one-winner tournament, each agent chooses an different approach from one another for any Nash equilibrium: $\underline{d}^*(1, 3) = \bar{d}^*(1, 3) = 3$, full diversity of actions.
- (ii) Under the three-agent-one-loser tournament, all three agents choose the same approach for any Nash equilibrium: $\underline{d}^*(2, 3) = \bar{d}^*(2, 3) = 1$, perfect homogeneity of actions.

Proof: See Appendix. ■

The n -agent tournament

The following result is the counterpart of Proposition 4.4:

Proposition 4.8. Given Assumption 4.4, any strategy profile for which all agents choose the same approach is a Nash equilibrium if $2m \geq n$, but not if $2m < n$. That is to say, $\underline{d}^*(m, n) = 1$ if $2m \geq n$, but $\underline{d}^*(m, n) > 1$ otherwise.

The proof is omitted because it is essentially the same as that of Proposition 4.4.

This result shows the condition in which full homogeneity is an equilibrium. The next proposition shows the condition in which full diversity is an equilibrium:

Proposition 4.9. Given Assumption 4.4, any strategy profile for which each agent chooses a different approach is a Nash equilibrium if $2m \leq n$, but not if $2m > n$. That is, $\bar{d}^*(m, n) =$

n if $2m \leq n$, but $\bar{d}^*(m, n) < n$ otherwise.

Proof: See Appendix. ■

The logic of these results is essentially the same as that of the results in the model with one common approach and original approaches.

The next corollary summarizes the two proposition above in terms of the level of diversity:

Corollary 4.1. Given Assumption 4.4, the following hold:

- (i) Under the winner-selecting tournament, $\underline{d}^*(m, n) > 1$ and $\bar{d}^*(m, n) = n$.
- (ii) Under the tournament with $2m = n$, $\underline{d}^*(m, n) = 1$ and $\bar{d}^*(m, n) = n$.
- (iii) Under the winner-selecting tournament, $\underline{d}^*(m, n) = 1$ and $\bar{d}^*(m, n) < n$.

This result shows that the level of behavior diversity tends to be greater under the winner-selecting tournament than under the loser-selecting tournament. If it held that $\underline{d}^*(m, n) = \bar{d}^*(m, n) = n$ under the winner-selecting tournament and $\underline{d}^*(m, n) = \bar{d}^*(m, n) = 1$ under the loser-selecting tournament, the result would be much clearer, simpler, and stronger. We have this strong result under the three-agent tournament. However, it does not generally hold under the n -agent tournament: There sometimes exist other Nash equilibria. Next, using the model without individual-specific noise as the special case of the model, I examine this point.

A model with many common approaches and without individual-specific noise

Agent i 's performance $\tilde{x}_i(a_i)$ is given by a continuous random variable in the following form:

$$\tilde{x}_i(a_i) = \tilde{\eta}_j \text{ if } a_i = c_j. \quad (4.33)$$

I assume that $(\tilde{\eta}_1, \tilde{\eta}_2, \dots, \tilde{\eta}_l)$ are independently and identically distributed. Furthermore, assume that their orders are randomly determined when some agents have the same performance level.

First, I examine the winner-selecting tournament. For $\underline{d}^*(m, n)$, I get the following result:

Proposition 4.10. If $2m < n$, then it holds that

$$\underline{d}^*(m, n) \geq \max[q_m - 1, 2]. \quad (4.34)$$

Furthermore, if $\underline{d}^*(m, n) = q_m - 1$, then $\text{mod}(m, n) = \text{mod}(q_m - 1, n) = 0$: this Nash equilibrium profile means that $q_m - 1$ approaches are chosen by $n/(q_m - 1)$ agents, respectively.

Proof: See Appendix. ■

This proposition gives us a meaningful lower bound of the diversity under the winner-selecting tournament. Since the quotient q_m decreases as m increases, it shows that there exists a trend that under the winner-selecting tournament $\underline{d}^*(m, n)$ tends to decrease as the number of winners increases.⁶

From Proposition 4.10, we can identify a Nash equilibrium with $d = q_m - 1$. The condition for its existence, however, is less likely satisfied. We can also identify Nash equilibria with $d = q_m$ in the proof of Proposition 4.10. By further scrutiny, I find that Nash equilibria with $d = q_m$ are limited in only the following two types of strategy profiles (the proof is omitted):

- (i) A strategy profile for which each size of cluster is equal to or more than m . Differences of size between any two clusters are one or zero.
- (ii) A strategy profile for which one cluster has the size less than m , the other $q_m - 1$ clusters have the same size which is more than m .

Moreover, the necessary and sufficient condition that such a Nash equilibrium exists is the following (the proof is omitted):

$$\text{mod}(n, q_m) = 0 \text{ or } \text{mod}(n, m) - \text{mod}(n, q_m) + q_m - m \leq 0.$$

While this condition is not always satisfied, it holds more likely than the condition of Nash equilibrium with $d = q_m - 1$. Especially, if $m \geq q_m$, i.e., m/n is rather large, then the condition will often be satisfied.

⁶I do not assert that $\underline{d}^*(m, n)$ always decreases as m increases. I mean just the “downward-sloping trend” by Proposition 4.10.

I do not know whether there exists some Nash equilibrium profile \mathbf{a} with $q_m < d(\mathbf{a}) < n$ or not. I have not found such an equilibrium yet.

Thus, under the winner-selecting tournament, there is a possibility that several clusters will be formed. Then the number of the clusters might be q_m and the sizes might be equal to or slightly bigger than m .

Furthermore, Proposition 4.10 straightforwardly gives us the following corollary.

Corollary 4.2. If $n > 2$ and $m = 1$, then $\underline{d}^*(1, n) = \bar{d}^*(1, n) = n$: full diversity in any Nash equilibrium under the one-winner tournament with more than two agents.

Proof: The result follows automatically from Propositions 4.9 and 4.10. ■

Next, I examine the loser-selecting tournament. Under this type of tournament, $\underline{d}^*(m, n)$ is equal to one, *i.e.*, full homogeneity of behavior. For $\bar{d}^*(m, n)$, I have not been able to find the counterpart of Proposition 4.10. However, I have the following two results to help find out some Nash equilibrium other than full homogeneity:

Proposition 4.11. Under the loser-selecting tournament, any strategy profile \mathbf{a} in which there is a cluster with the size equal to or more than m and $d(\mathbf{a}) \geq 2$ is not a Nash equilibrium.

Proof: See Appendix. ■

Proposition 4.12. Under the loser-selecting tournament, any strategy profile \mathbf{a} in which every cluster has the size equal to or more than $n - m$ and $d(\mathbf{a}) \geq 2$ is not a Nash equilibrium.

Proof: See Appendix. ■

These two propositions straightforwardly lead to the two following corollaries: the first is the counterpart of corollary 4.2. The second offers the somewhat surprising result that there are no Nash equilibria with two clusters under the loser-selecting tournament.

Corollary 4.3. If $n > 2$ and $m = n - 1$, then $\underline{d}^*(1, n) = \bar{d}^*(1, n) = 1$: full homogeneity in

any Nash equilibrium under the one-loser tournament with more than two agents.

Proof: The result automatically follows from Proposition 4.12. ■

Corollary 4.4. Under the loser-selecting tournament ($2m > n$), there do not exist any Nash equilibria with $d(\mathbf{a}) = 2$.

Proof: If $2m > n$, any strategy profiles with $d = 2$ can be classified into the following two types:

- (i) A strategy profile for which one cluster has the size equal to or more than m .
- (ii) A strategy profile for which both sizes of two approaches are more than $n - m$.

The former type of strategy profiles is not a Nash equilibrium by Proposition 4.11. The latter type of strategy profiles is also not a Nash equilibrium by Proposition 4.12. ■

Since there exist no Nash equilibria with $d = 2$ under the loser-selecting tournament, someone might imagine that there also do not exist any Nash equilibria with $d > 2$. Such a equilibrium, however, exists. I have found Nash equilibria with $d = q_{n-m} + 1$, for which all clusters have the same size. In this equilibrium, the size of clusters is a little smaller than $n - m$. For example, in the case where $n = 16$ and $m = 11$, four clusters which have the same size 4 form a Nash equilibrium profile. This type of strategy profile is not a Nash equilibrium when m/n is close one. If this equilibrium (if it exists) gives $\bar{d}^*(m, n)$, I cannot assert that under the loser-selecting tournament there is a trend that $\bar{d}^*(m, n)$ decreases as m increases because q_{n-m} increases as m increases. However, there might be Nash equilibria with more clusters. Further scrutiny is needed for this point.

Chapter 5

Who Is the Target of Sabotage? :

The Dark Side of Tournaments

5.1 Introduction

One of the major disadvantages of tournaments is the problem of sabotage, in which an agent takes unproductive actions to reduce his rival's performance. This clearly creates inefficiency. Sabotage takes a variety of forms: spreading false gossip about a rival, concealing relevant information, theft or destruction of rival's output, etc.

Although this is a very important problem for the design of tournament contracts, there are few theoretical researches on the general properties of sabotage among agents (see Chapter 2).

One of the major reasons is the complicated structure of the problem. Consider the n -agent tournament. Then the natural extension of the standard agency model implies n effort variables e_i , which is agent i 's effort level, and $n(n-1)$ sabotage variables s_{ij} , which is the sabotage level agent i exerts against agent j . In addition to these many variables, the sabotage has the positive externalities: agent j 's performance reduction caused by agent i 's attack is also enjoyed by the other agents under tournaments. Although the model is tractable when $n = 2$, it is difficult to analyze it sharply mainly for the nonuniqueness

problem of the equilibrium when $n \geq 3$.

I take another approach in order to handle this complexity. I conveniently consider the two-step decision making: First, agents simultaneously and independently choose their effort level and total amount of sabotage, which is called “their sabotage resources.” Next, they simultaneously and independently allocate their sabotage resources to their peers.¹ I mainly examine the second step: “the sabotage resource allocation problem.” Equilibrium conditions in the second step game must also be satisfied in the equilibrium of the game in which all variables are simultaneously and independently chosen. Although we can say nothing about effort level or total sabotage level, we can know “who is the target.” I restrict attention to the one-winner tournament and the one-loser tournament. Several properties of sabotage under each tournament are examined given a moderate assumption on individual-specific noise: a log-concavity of c.d.f. F or reliability function $1 - F$.

Under the one-winner tournament, an agent allocates the sabotage resource to his peers so as to minimize the maximal value of their expected performances. The sabotage among agents operates to reduce the heterogeneity in ability. Furthermore, the uniqueness of the equilibrium expected performance vector is shown.

Under the one-loser tournament, agents allocate the resource to his their peers so as to minimize minimal value of their expected performance. One agent is intensively attacked by all his peers.

Furthermore, embedding agents’ effort choice into the model in the simplest form, I argue that the one-loser tournament is more subject to damage of sabotage, *ceteris paribus*, than the one-winner tournament. I show that while heterogeneity among agents will increase the relative incentive of the sabotage under the one-winner tournament, sabotage may work like a “handicap system” and restore the incentive of the effort.

Thus, firms which adopt a very high promotion rate should deal seriously with the sabotage problem. Japanese traditional corporations have such promotion systems as seen in the empirical study by Hanada and Sekimoto introduced in Chapter 2. They seem to cope by using extensive rotation.

This chapter is structured as follows. In Section 5.2, I present the simple sabotage model. In Sections 5.3 and 5.4, Nash equilibria are derived and the properties of sabotage are examined under the one-winner tournament and those under the one-loser tournament,

¹This approach is based on the assumption that agent i ’s disutility function of his sabotage activity can be expressed as $v(\sum_{j \neq i} s_{ij})$.

respectively. In Section 5.5, agents' efforts are built in the model. I compare the two tournaments about vulnerability to sabotage among agents.

5.2 The simple sabotage model

Suppose that n agents, indexed by $i = 1, 2, \dots, n$, compete under the tournament (m, R_W, R_L) : m is the number of winners, R_W and R_L are a winner's reward and a loser's reward, respectively. I restrict attention to a case of one winner ($m = 1$) and that of one loser ($m = n - 1$) through this chapter.

Agent i has his resource t_i to use sabotage against his peers (*e.g.*, time used for sabotage \times his ability in sabotage). Let s_{ij} denote the amount of resource that agent i allocates to attack agent j . Assume that s_{ij} is nonnegative for any i and j . Agent i chooses his sabotage vector $\mathbf{s}_i = (s_{i1}, s_{i2}, \dots, s_{in})$ from the feasible set of his sabotage vector S_i . Since I consider only pure strategies, \mathbf{s}_i and S_i represent agent i 's strategy and his strategy set, respectively:

$$S_i = \{\mathbf{s}_i = (s_{i1}, s_{i2}, \dots, s_{in}) \in \mathbf{R}^n \mid \sum_{j=1}^n s_{ij} \leq t_i, s_{ii} = 0, s_{ij} \geq 0\}.$$

Although self-sabotage s_{ii} is included as one element of an agent's sabotage vector for a simpler notation, I assume that s_{ii} is always zero.² S_i is a compact and convex subset of \mathbf{R}^n . I let $\mathbf{s} = (\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_n)$ and $S = S_1 \times S_2 \times \dots \times S_n$. That is, \mathbf{s} and S represent a strategy profile for agents and the feasible set of strategy profiles, respectively. S is a compact and convex subset of $\mathbf{R}^{n \times n}$ because each S_i is a compact and convex subset of \mathbf{R}^n . I also denote a strategy profile for agent i 's peers by $\mathbf{s}_{-i} = (\mathbf{s}_1, \dots, \mathbf{s}_{i-1}, \mathbf{s}_{i+1}, \dots, \mathbf{s}_n)$. Moreover, for simplicity, I assume that if agents used the resource for other activities, the private benefit they had would be zero.

Next, I denote the total amount of sabotage agent i receives by $d_i(\mathbf{s})$. I refer to $\mathbf{d}(\mathbf{s}) = (d_1(\mathbf{s}), d_2(\mathbf{s}), \dots, d_n(\mathbf{s}))$ as a total damage vector. The total damage vector can be calculated as the sum of all agents' sabotage vectors:

$$\mathbf{d}(\mathbf{s}) = \mathbf{s}_1 + \mathbf{s}_2 + \dots + \mathbf{s}_n = (d_1(\mathbf{s}), d_2(\mathbf{s}), \dots, d_n(\mathbf{s})) = \left(\sum_{j=1}^n s_{j1}, \sum_{j=1}^n s_{j2}, \dots, \sum_{j=1}^n s_{jn} \right).$$

²I discuss the model in which the s_{ii} is his effort level that enhances his own performance in section 5.5.

Agent i 's performance $\tilde{x}_i(\mu_i, \mathbf{s})$ is a continuous random variable and depends on both his ability in effort (\times time used for his effort) μ_i and the total amount of sabotage he receives. I specify $\tilde{x}_i(\mu_i, \mathbf{s})$ in the form:

$$\tilde{x}_i(\mu_i, \mathbf{s}) = \mu_i - d_i(\mathbf{s}) + \tilde{\epsilon}_i$$

where μ_i represents the expected value of agent i 's performance without sabotage, and d_i is the total amount of sabotage agent i receives as mentioned above, and $\tilde{\epsilon}_i$ is an individual-specific noise for agent i , which is a continuous random variable with zero mean. Note that μ_i is not an action variable like t_i for agent i in this model. I assume that $(\tilde{\epsilon}_1, \tilde{\epsilon}_2, \dots, \tilde{\epsilon}_n)$ are independently and identically distributed with the distribution function $F(\cdot)$, the density function $f(\cdot)$, the support $[\underline{\epsilon}, \bar{\epsilon}]$. I also assume that for all y such that $\underline{\epsilon} < y < \bar{\epsilon}$, $f(y) > 0$. I allow that $\underline{\epsilon}$ and $\bar{\epsilon}$ are $-\infty$ and $+\infty$.

I treat the tournament contract (m, R_W, R_L) as the given. The agents choose \mathbf{s}_i simultaneously and independently so as to maximize their expected utility. In this model, they do not have any private costs to choose strategy. It follows that for an agent, maximizing his expected utility is equivalent to maximizing his winning probability. Thus it is not necessary to specify their utility function, R_W and R_L . I assume that μ_i and t_i and the structure of game are the common knowledge among agents.

5.3 Sabotage in the one-winner tournament ($m = 1$)

First, without loss of generality, I consider agent n 's best response strategy to his peers' strategy profile \mathbf{s}_{-n} .

Agent n 's winning probability is given by:

$$\begin{aligned} & \Pr(\tilde{x}_n(\mu_n, \mathbf{s}) > \max[\tilde{x}_1(\mu_1, \mathbf{s}), \tilde{x}_2(\mu_2, \mathbf{s}), \dots, \tilde{x}_{n-1}(\mu_{n-1}, \mathbf{s})]) \\ &= \int_{-\infty}^{+\infty} \left\{ \prod_{i=1}^{n-1} F(x - \mu_i + d_i(\mathbf{s})) \right\} f(x - \mu_n + d_n(\mathbf{s})) dx \end{aligned}$$

Then, his best response strategy to \mathbf{s}_{-n} is given as the solution of the following maximization problem [P1]:

$$\max_{\mathbf{s}_n \in S_n} \int_{-\infty}^{+\infty} \left\{ \prod_{i=1}^{n-1} F(x - \mu_i + d_i(\mathbf{s})) \right\} f(x - \mu_n + d_n(\mathbf{s})) dx. \quad [\text{P1}]$$

Let $B_n^{(1)}(\boldsymbol{\mu}, \mathbf{s}_{-n})$ denote the set of solutions of [P1] where $\boldsymbol{\mu} = (\mu_1, \mu_2, \dots, \mu_n)$.

Hereafter I will often use the following notation:

$$b_i(\mu_i, \mathbf{s}) \equiv \mu_i - d_i(\mathbf{s}) = \mu_i - \sum_{j=1}^n s_{ji},$$

where $b_i(\mu_i, \mathbf{s})$ represents the expected value of agent i 's performance x_i given a strategy profile \mathbf{s} . Let $\mathbf{b}(\boldsymbol{\mu}, \mathbf{s}) = (b_1(\mu_1, \mathbf{s}), b_2(\mu_2, \mathbf{s}), \dots, b_n(\mu_n, \mathbf{s}))$. For the saving of space, hereafter I will sometimes simply write b_i and \mathbf{b} for $b_i(\mu_i, \mathbf{s})$ and $\mathbf{b}(\boldsymbol{\mu}, \mathbf{s})$, respectively, in order to leave room for misunderstanding. Using $b_i(\mu_i, \mathbf{s})$, the maximization problem [P1] can be written in the form:

$$\max_{\mathbf{s}_n \in S_n} \int_{-\infty}^{+\infty} \left\{ \prod_{i=1}^{n-1} F(x - b_i(\mu_i, \mathbf{s})) \right\} f(x - b_n(\mu_n, \mathbf{s})) dx. \quad [\text{P1}]$$

I put the following assumption in order to secure that this maximization problem is not trivial.

Assumption 5.1. For any feasible strategy profile $\mathbf{s} = (s_1, s_2, \dots, s_n) \in S$, every agent has strictly positive probability to become a winner.

Furthermore, for tractable analysis I restrict attention to the cumulative distribution function of $\tilde{\epsilon}_i$ that satisfies the following condition.

Assumption 5.2 (twice-differentiability and log-concavity of cumulative distribution function F).³ The cumulative distribution function F is twice-differentiable on $(\underline{\epsilon}, \bar{\epsilon})$, and the function $\ln F$ is strictly concave on $(\underline{\epsilon}, \bar{\epsilon})$, *i.e.*, $d(f(y)/F(y))/dy$ is negative on $(\underline{\epsilon}, \bar{\epsilon})$.

Assumption 5.2 holds if the F is uniform, normal, logistic, extreme value, chi-squared, exponential, Laplace, log normal, and under some restrictions on the parameters, Weibull, gamma, or beta.⁴ This assumption seems not so restrictive.

It seems difficult to characterize agent n 's best response by solving directly the maximization problem [P1]. Instead, I try to characterize his best response making use of

³The log-concavity of F is often called “the monotone hazard rate property” in the field of the mechanism design or contract theory. For this point, see Laffont and Tirole (1993, Chapter 1).

⁴See Bagnoli and Bergstrom (1989) for a more complete list of distributions whose c.d.f. is log-concave.

the following simple minimization problem [P2]:

$$\min_{\mathbf{s}_n \in S_n} \max(b_1(\mu_1, \mathbf{s}), b_2(\mu_2, \mathbf{s}), \dots, b_{n-1}(\mu_{n-1}, \mathbf{s})). \quad [\text{P2}]$$

The solution for this minimization problem is the agent n 's sabotage vector, which minimizes the maximal value of all peers' expected performance values. Let $B_n^{(2)}(\boldsymbol{\mu}, \mathbf{s}_{-n})$ denote the set of solutions of [P2].

We have the following clear-cut result for the best response strategy:

Proposition 5.1. Given Assumptions 5.1 and 5.2, the following hold:

- (i) $B_n^{(2)}(\boldsymbol{\mu}, \mathbf{s}_{-n})$ is a singleton: the minimization problem [P2] has just one solution.
- (ii) $B_n^{(1)}(\boldsymbol{\mu}, \mathbf{s}_{-n}) = B_n^{(2)}(\boldsymbol{\mu}, \mathbf{s}_{-n})$: the maximization problem [P1] also has just one solution, which is also the unique solution of [P2].

For the proof of Proposition 5.1, I use the following lemma:

Lemma 5.1. Given Assumption 5.2 and any positive constant δ such that $\bar{\epsilon} - \underline{\epsilon} > \delta > 0$, $F(y + \delta)/F(y)$ is continuous in y on $(\underline{\epsilon}, +\infty)$ given, and is strictly increasing in y on $(\underline{\epsilon}, \bar{\epsilon})$, and is always equal to one on $[\bar{\epsilon}, +\infty)$.

Proof: See Appendix. ■

This lemma follows from the log-concavity of F . See Appendix for the proof of Proposition 5.1.

Since agent numbering is arbitrary, every agent i 's best response strategy to his peers' strategy profile \mathbf{s}_{-i} is also unique and it minimizes $\max(b_1(\mu_1, \mathbf{s}), b_2(\mu_2, \mathbf{s}), \dots, b_{i-1}(\mu_{i-1}, \mathbf{s}), b_{i+1}(\mu_{i+1}, \mathbf{s}), \dots, b_n(\mu_n, \mathbf{s})).$

Although the claim of Proposition 5.1 that “to attack the best peer” is the agent's best response strategy is not so surprising itself, it is rather surprising that the assumption of F 's log-concavity perfectly assures this.

Next, I derive the set of Nash equilibria and the features of the equilibria.

Let $b_{\max}(\boldsymbol{\mu}, \mathbf{s})$ denote the maximum of all agents' expected value of performance

under a strategy profile \mathbf{s} :

$$\begin{aligned} b_{\max}(\boldsymbol{\mu}, \mathbf{s}) &= \max(b_1(\mu_1, \mathbf{s}), b_2(\mu_2, \mathbf{s}), \dots, b_n(\mu_n, \mathbf{s})) \\ &= \max(\mu_1 - \sum_{j=1}^n s_{j1}, \mu_2 - \sum_{j=1}^n s_{j2}, \dots, \mu_n - \sum_{j=1}^n s_{jn}). \end{aligned}$$

Furthermore, consider the following minimization problem:

$$\min_{\mathbf{s} \in S} b_{\max}(\boldsymbol{\mu}, \mathbf{s}).$$

Since the domain S is a compact and convex set in $\mathbf{R}^{n \times n}$ and the function $b_{\max}(\boldsymbol{\mu}, \mathbf{s})$ is continuous on S , by Weierstrass's theorem there exists the minimum value and the optional solution(s) for this minimization problem. I denote by \mathbf{s}^* and $b_{\max}^*(\boldsymbol{\mu})$ the optimal solution(s) and the minimal value of the minimization problem, respectively.

This minimization problem can be transformed into the following minimization problem [P5]:

$$b_{\max}^*(\boldsymbol{\mu}) = \arg \min_{d_1, d_2, \dots, d_n} \max(\mu_1 - d_1, \mu_2 - d_2, \dots, \mu_n - d_n) \quad [\text{P5}]$$

subject to

$$\sum_{i=1}^n d_i \leq T, \tag{5.1}$$

$$0 \leq d_i \leq T - t_i, \quad \text{for } i = 1, 2, \dots, n, \tag{5.2}$$

$$\text{where } T = \sum_{i=1}^n t_i.$$

The first constraint (5.1) implies that the total damage of all agents cannot exceed the total amount of sabotage resources of all players. The second constraints (5.2) imply that agent i ' damage cannot exceed the total sabotage resources of all his peers.

Without loss of generality, I assume that

$$\mu_1 \geq \mu_2 \geq \dots \geq \mu_n.$$

I denote by $(d_1^*(\boldsymbol{\mu}), d_2^*(\boldsymbol{\mu}), \dots, d_n^*(\boldsymbol{\mu}))$ the solution of the problem above.

Here, note that the constraints (5.2) of at most two agents can be binding because $2T - t_i - t_j \geq T \geq d_i + d_j$ for all i, j but $i \neq j$. If two agent's constraints (5.2) are binding, the case is trivial: $t_2 = T$, and $t_1 = t_3 = \dots = t_n = 0$. The solution is: $d_1^* = T, d_2^* = \dots = d_n^* = 0$.

If only one agent l 's constraint (5.2) is binding, the solution is: $d_l^* = T - t_l$ but other d_i^* s are indeterminate. If any agent's constraint (5.2) is not binding, then the solution satisfies:

$$d_1^*(\boldsymbol{\mu}) \geq d_2^*(\boldsymbol{\mu}) \geq \dots \geq d_n^*(\boldsymbol{\mu}), \quad (5.3)$$

$$b_{\max}^*(\boldsymbol{\mu}) = \mu_i - d_i^*(\boldsymbol{\mu}) \quad \text{for } i = 1, 2, \dots, k, \quad (5.4)$$

$$\sum_{i=1}^k d_i^*(\boldsymbol{\mu}) = T, \quad (5.5)$$

$$\mu_k > b_{\max}^*(\boldsymbol{\mu}) \geq \mu_{k+1} \text{ (if } k = n, \mu_n > b_{\max}^*(\boldsymbol{\mu})) \quad (5.6)$$

where k is the largest index number such that $d_i^*(\boldsymbol{\mu}) > 0$.

The solution $(d_1^*(\boldsymbol{\mu}), d_2^*(\boldsymbol{\mu}), \dots, d_n^*(\boldsymbol{\mu}))$ can be derived from the two equations (5.4), (5.5) and the inequalities (5.6).⁵ Undoubtedly, given $\boldsymbol{\mu}$, the solution is unique if any agent's constraint (5.2) is not binding. I have the following proposition:

Proposition 5.2. Given Assumptions 5.1 and 5.2, there exists at least one Nash equilibrium in this one-winner game. Moreover, for any Nash equilibrium \mathbf{s}^{NE} , it holds:

$$b_{\max}(\boldsymbol{\mu}, \mathbf{s}^{NE}) = b_{\max}^*(\boldsymbol{\mu}).$$

Proof: See Appendix. ■

By this proposition, we know that given $\boldsymbol{\mu}$, the unique solution of [P5] is also the unique equilibrium damage vector $(d_1(\mathbf{s}^{NE}), d_2(\mathbf{s}^{NE}), \dots, d_n(\mathbf{s}^{NE}))$ if any agent's constraint (5.2) is not binding, or two agent's constraints (5.2) are binding (it's a trivial case). If only one agent l 's constraint (5.2) is binding, agent l 's equilibrium damage d_l^* is $T - t_l$: All his peers allocate their resources to attack him. By Proposition 5.1, agent l 's strategy is also unique. Thus it follows that given $\boldsymbol{\mu}$ the equilibrium damage vector $(d_1(\mathbf{s}^{NE}), d_2(\mathbf{s}^{NE}), \dots, d_n(\mathbf{s}^{NE}))$ is unique. Proposition 5.3 follows straightforwardly from the above discussion and (5.3), (5.4), (5.5) and (5.6) that tell us the properties of the unique solution of [P5]:

Proposition 5.3. Given Assumptions 5.1 and 5.2, Nash equilibria in this one-winner tournament game have the following features:

⁵First, solve for k and $b_{\max}^*(\boldsymbol{\mu})$. Next, use $d_i^*(\boldsymbol{\mu}) = \mu_i - b_{\max}^*(\boldsymbol{\mu})$ for $i = 1, \dots, k$, and $d_{k+1}^* = \dots = d_n^* = 0$.

- (i) Given μ , the equilibrium damage vector $(d_1(\mathbf{s}^{NE}), d_2(\mathbf{s}^{NE}), \dots, d_n(\mathbf{s}^{NE}))$ is unique. This implies that the equilibrium expected performance vector $(b_1(\mathbf{s}^{NE}), b_2(\mathbf{s}^{NE}), \dots, b_n(\mathbf{s}^{NE}))$ is also unique.⁶

(ii) If any agent's constraint (5.2) is not binding, the following hold:

- (a) The higher ability μ_i an agent has, the more he is subject to be sabotaged:

$$\text{If } \mu_1 \geq \mu_2 \geq \dots \geq \mu_n, \text{ then } d_1(\mathbf{s}^{NE}) \geq d_2(\mathbf{s}^{NE}) \geq \dots \geq d_n(\mathbf{s}^{NE}).$$

- (b) An agent with higher ability, however, does not have the lower winning probability than an agent with lower ability:

$$\text{If } \mu_1 \geq \mu_2 \geq \dots \geq \mu_n, \text{ then } b_1(\mathbf{s}^{NE}) \geq b_2(\mathbf{s}^{NE}) \geq \dots \geq b_n(\mathbf{s}^{NE}).$$

- (c) Sabotage always reduces the degree of heterogeneity among agents:

$$\begin{aligned} & \max[b_1(\mathbf{s}^{NE}), b_2(\mathbf{s}^{NE}), \dots, b_n(\mathbf{s}^{NE})] - \min[b_1(\mathbf{s}^{NE}), b_2(\mathbf{s}^{NE}), \dots, b_n(\mathbf{s}^{NE})] \\ & \leq \max[\mu_1, \mu_2, \dots, \mu_n] - \min[\mu_1, \mu_2, \dots, \mu_n]. \end{aligned}$$

where the equality holds if $\max[\mu_1, \mu_2, \dots, \mu_n] - \min[\mu_1, \mu_2, \dots, \mu_n] = 0$: if $\mu_1 = \mu_2 = \dots = \mu_n = \mu$, then it holds that $b_1 = b_2 = \dots = b_n = \mu - \frac{T}{n}$ in all Nash equilibria.

I give two simple examples which violate the above (a), (b) or (c) when some agent's constraint (5.2) is binding:

Example 1. $\mu_1 > \mu_2 = \mu_3 = \dots = \mu_n$, and $T = t_1 > 0$. Then $d_1(\mathbf{s}^{NE}) = 0$ and $d_i(\mathbf{s}^{NE}) = t_1/(n-1)$ for $j = 2, 3, \dots, n$. This case violates (a) and (c).

Example 2. $T = t_2 > \mu_1 - \mu_2$. Then $b_1(\mathbf{s}^{NE}) < b_2(\mathbf{s}^{NE})$. This case violates (b): An agent with the highest ability does not have the highest winning probability.

If there is one agent who has extremely high μ_i , then he will be intensively sabotaged by his peers. As business magazines or general magazines sometimes report, casual observation seems to support this result.⁷

⁶Note that this result does not imply the uniqueness of Nash equilibrium.

⁷For example, see "The front runners in the rat race often lose out (in Japanese)," *AERA*, November 18, (1996). pp. 6-9.

On the other hand, if the heterogeneity among agents is small, sabotage will disperse among many agents. Especially in a case of homogeneous agents all agents will receive the same amount of sabotage.

5.4 Sabotage in the one-loser tournament ($m = n - 1$)

Without loss of generality, I consider agent n 's best response strategy to his peers' strategy profile \mathbf{s}_{-n} . Maximizing his winning probability (high reward probability) is equivalent to minimizing his losing probability (low reward probability). I use his losing probability instead of his winning probability to derive his best response strategy.

Agent n 's losing probability is given by:

$$\begin{aligned} & \Pr(\tilde{x}_n(\mu_n, \mathbf{s}) < \min[\tilde{x}_1(\mu_1, \mathbf{s}), \tilde{x}_2(\mu_2, \mathbf{s}), \dots, \tilde{x}_{n-1}(\mu_{n-1}, \mathbf{s})]) \\ &= \int_{-\infty}^{+\infty} \left\{ \prod_{i=1}^{n-1} \bar{F}(x - b_i(\mu_i, \mathbf{s})) \right\} f(x - b_n(\mu_n, \mathbf{s})) dx. \end{aligned}$$

where $\bar{F}(\cdot) = 1 - F(\cdot)$ which is called the “reliability” function or the “decumulated” distribution function.

His best response strategy to \mathbf{s}_{-n} is given as the solution of the following minimization problem [P3]:

$$\min_{\mathbf{s}_n \in S_n} \int_{-\infty}^{+\infty} \left\{ \prod_{i=1}^{n-1} \bar{F}(x - b_i(\mu_i, \mathbf{s})) \right\} f(x - b_n(\mu_n, \mathbf{s})) dx. \quad [\text{P3}]$$

Let $B_n^{(3)}(\boldsymbol{\mu}, \mathbf{s}_{-n})$ denote the set of the solutions of [P3].

I put the following assumptions similar to those on the one-winner model.

Assumption 5.1’. For any feasible strategy profile $\mathbf{s} = (s_1, s_2, \dots, s_n) \in S$, every agent has strictly positive probability to become the loser.

Assumption 5.2’ (twice-differentiability and log-concavity of reliability function). The reliability function \bar{F} is twice-differentiable on $(\underline{\epsilon}, \bar{\epsilon})$, and the function $\ln \bar{F}$ is strictly concave on $(\underline{\epsilon}, \bar{\epsilon})$: The derivative $d(-f(y)/\bar{F}(y))/dy$ is negative on $(\underline{\epsilon}, \bar{\epsilon})$.

Assumption 5.2’ holds if the c.d.f. F is uniform, normal, Logistic, extreme value,

chi-squared, exponential, Laplace and under some restrictions on the parameters, Weibull, gamma, or beta.⁸

It seems difficult to characterize his best response by solving directly the minimization problem [P3]. As in the case of the one-winner tournament, I try to characterize his best response by making use of the following simple minimization problem [P4]:

$$\min_{\mathbf{s}_n \in S_n} \min(b_1(\mu_1, \mathbf{s}), b_2(\mu_2, \mathbf{s}), \dots, b_{n-1}(\mu_{n-1}, \mathbf{s})). \quad [\text{P4}]$$

The solution for this minimization problem's solution is agent n 's sabotage vector which minimizes the minimal value of all peers' expected performance values. Let $B_n^{(4)}(\boldsymbol{\mu}, \mathbf{s}_{-n})$ denote the set of solutions of [P4]. And let $b_{i,-n}(\mu_i, \mathbf{s}_{-n})$ denote the expected value of agent i 's performance where agent n 's sabotage vector s_n is eliminated:

$$b_{i,-n}(\mu_i, \mathbf{s}_{-n}) \equiv b_i(\mu_i, \mathbf{s}) - s_{ni} = \mu_i - \sum_{j=1}^{n-1} s_{ji}.$$

Obviously, the solution for [P4] allocates all resource to sabotage an agent who has the minimal value of $(b_{1,-n}, b_{2,-n}, \dots, b_{n-1,-n})$. When two or more agents have the same minimal value, the solution implies that agent n selects one of them and attacks only him. In this case, the number of the solutions is the same as that of agents who have the minimal value. These solutions, however, are essentially identical except the sabotaged agent's index number.

The next proposition tells us that this solution set coincides with the set of agent n 's best response:

Proposition 5.4. Given Assumptions 5.1' and 5.2', the following hold:

- (i) $B_n^{(3)}(\boldsymbol{\mu}, \mathbf{s}_{-n})$ is not empty.
- (ii) $B_n^{(3)}(\boldsymbol{\mu}, \mathbf{s}_{-n}) = B_n^{(4)}(\boldsymbol{\mu}, \mathbf{s}_{-n})$: The solution set of [P3] coincide with that of [P4].

For the proof of Proposition 5.4, I use the following lemma:

Lemma 5.2. Given Assumption 5.2' and any positive constant δ such that $\bar{\epsilon} - \underline{\epsilon} > \delta > 0$, $\bar{F}(y + \delta)/\bar{F}(y)$ is continuous in y on $(-\infty, \bar{\epsilon})$, and is always one on $(-\infty, \underline{\epsilon} - \delta)$, strictly

⁸See Bagnoli and Bergstrom (1989) for a more complete list of distributions whose reliability function is log-concave.

decreasing in y on $[\underline{\epsilon} - \delta, \bar{\epsilon} - \delta)$, and is always zero on $[\bar{\epsilon} - \delta, \bar{\epsilon})$.

Proof of Lemma 5.2: See Appendix. ■

See Appendix for the proof of Proposition 5.4.

Proposition 5.4 implies that agent n 's best response strategy to \mathbf{s}_{-n} is a sabotage vector that allocates all resource of sabotage to an agent who has the minimal value of $(b_{1,-n}, b_{2,-n}, \dots, b_{n-1,-n})$ or to one of them if two or more agents have the same minimal value. This strategy means “to sabotage the weakest agent given his peers’ strategy profile.”

Given this nature of best response, it is not so difficult to characterize the properties of the equilibrium. First, an agent with the lowest value of expected performance b_i is one who was sabotaged by someone because if not, any agents could reduce their losing probability by attacking him.

Second, the difference between the lowest value and the second lowest value of agents’ expected performance is larger than or equal to the maximum value of the amount of resource t_i which each agent attacking him has, because if not, any agent who attacked the agent with the lowest expected value could reduce his losing probability by changing the target into the second lowest agent. This also implies just one agent has the lowest value of expected performance.

Third, an agent with the lowest value of expected performance is one who is sabotaged by all his peers because if not, any agent who did not attack him could reduce his losing probability by changing the target into him.

Fourth, just two agents are sabotaged. The other $n - 2$ agents are not at all sabotaged. This follows from the second feature.

Fifth, One of the two agents attacked is always the one with the lowest ability because even if he is not a victim attacked by $n - 1$ agents, he will be sabotaged by the victim attacked by $n - 1$ agents.

Sixth, there might be multiple equilibria. For example, suppose that there are four agents and $\boldsymbol{\mu} = (10.3, 10.2, 10.1, 10)$ and $t_i = 1$ for all agents. Then there exist the following four Nash equilibria: a strategy profile for which agent 4 attacks agent 3, and the other agents attack agent 4; a strategy profile for which agent 3 attacks agent 4, and the other agents attack agent 3; a strategy profile for which agent 2 attacks agent 4, and the other agents attack agent 2; a strategy profile for which agent 1 attacks agent 4, and the

other agents attack agent 1. In the first equilibrium, $\mathbf{b} = \boldsymbol{\mu} - \mathbf{d} = (10.3, 10.2, 9.1, 7)$: an agent with the smallest ability becomes a loser with the highest probability. In the second equilibrium, $\mathbf{b} = (10.3, 10.2, 7.1, 9)$: an agent with the second smallest ability becomes a loser with the highest probability. In the third equilibrium, $\mathbf{b} = (10.3, 7.2, 10.1, 9)$: an agent with the second highest ability becomes a loser with the highest probability. In the fourth equilibrium, $\mathbf{b} = (7.3, 10.2, 10.1, 9)$: an agent with the highest ability becomes a loser with the highest probability!

Seventh, as you have already seen in the example above, an agent with the lowest ability does not always become a loser with the highest probability and an agent with the highest ability does not necessarily become a loser with the smallest probability.

Eighth, except an equilibrium in which an agent with the highest ability is intensively attacked, sabotage among agents always increases heterogeneity among agents:

$$\begin{aligned} & \max[(b_1(\mathbf{s}^{NE}), b_2(\mathbf{s}^{NE}), \dots, b_n(\mathbf{s}^{NE}))] - \min[(b_1(\mathbf{s}^{NE}), b_2(\mathbf{s}^{NE}), \dots, b_n(\mathbf{s}^{NE}))] \\ & > \max(\mu_1, \mu_2, \dots, \mu_n) - \min(\mu_1, \mu_2, \dots, \mu_n). \end{aligned} \quad (5.7)$$

Especially if $\mu_i = \mu$ and $t_i = 1$ for all agents,

$$\max[(b_1(\mathbf{s}^{NE}), b_2(\mathbf{s}^{NE}), \dots, b_n(\mathbf{s}^{NE}))] - \min[(b_1(\mathbf{s}^{NE}), b_2(\mathbf{s}^{NE}), \dots, b_n(\mathbf{s}^{NE}))] = n - 1. \quad (5.8)$$

Ninth, an agent who has much larger sabotage resources than any of his peers have will be less apt to be the victim attacked by all peers because by attacking an peer with the lowest ability, he might lower the peer's expected performance below the expected value of his performance, where he would be intensively sabotaged. Proposition 5.5 summarizes the above discussion:

Proposition 5.5. Given Assumptions 5.1' and 5.2', Nash equilibrium in this one-loser tournament game has the following features:

- (i) There are often multiple Nash equilibria.
- (ii) Just two agents are attacked. The other $n - 2$ agents are not attacked at all. One of the two is always an agent who has the lowest ability μ_i (one of them if two or more agents have the same lowest ability).

- (iii) One of the two is sabotaged by all his peers. The other is attacked by the victim who is sabotaged by $n - 1$ agents.
- (iv) The agent attacked by all his peers $n - 1$ becomes a loser with a probability higher than any other agent. The other agent sabotaged becomes a loser with the second highest probability.
- (v) An agent with the lowest ability does not always become the loser with the highest probability, and an agent with the highest ability does not always become the loser with the smallest probability.
- (vi) Except for an equilibrium for which an agent with the highest ability is intensively attacked, sabotage always increases heterogeneity among agents in the sense that the inequality (5.7) holds. Especially, if $\mu_i = \mu$ and $t_i = 1$ for all i , the inequality (5.8) holds.
- (vii) An agent who has much larger sabotage resources than any of his peers has will be less apt to be the victim sabotaged by all peers.

The features are totally different from those in the one-winner tournament game: the attack always concentrates on one agent. A typical example will be “bullying” in the workplace, which is expected to cut one or more workers by downsizing or some other reason.

If there is one agent who has much lower μ_i than any other agent, there is a possibility that he will always be intensively sabotaged on any Nash equilibria. However, if the differences among agents’ abilities $(\mu_1, \mu_2, \dots, \mu_n)$ are not very large, every agent has the fear of being picked out as a “scapegoat” (that is, to be attacked by all other agents).

5.5 Effort versus sabotage, and the one-winner tournament versus the one-loser tournament

The purpose of this section is to try to give valuable suggestions to a principal who designs a tournament contract from the view of sabotage. An important question for the principal who designs the tournament contract is: what type of tournament is subject

to large damage by sabotage among agents? To answer this question, I modify the model and compare the one-winner tournament and the one-loser tournament. Agent i ' strategy and his strategy set is modified as follows:

$$S_i = \{\mathbf{s}_i = (s_{i1}, s_{i2}, \dots, s_{in}) \in \mathbf{R}^n \mid \sum_{j=1}^n s_{ij} \leq t_i, s_{ij} \geq 0\},$$

where s_{ii} is his effort level. That is, it is assumed that agent i can use the resource not only to attack his peers but also to enhance his own performance. In response to this change, his performance is also modified in the following form:

$$\tilde{x}_i(\mathbf{s}) = \alpha_i s_{ii} - d_i(\mathbf{s}) + \tilde{c}_i,$$

where

$$d_i(\mathbf{s}) = \sum_{j \neq i} s_{ji},$$

and α_i represents his ability in effort.

First, I consider a case of homogeneous agents, which is the simplest but most important case: $\alpha_i = \alpha$ and $t_i = 1$ for every agent i . I denote by α_{1W} the minimum value of α at which a strategy profile for which every agent allocates all his resource for effort is a Nash equilibrium under the one-winner tournament. Similarly, I denote by α_{1L} the minimum value of α at which a strategy profile for which every agent allocates all his resource for effort is a Nash equilibrium under the one-loser tournament. I compare α_{1W} with α_{1L} :

Proposition 5.6. Assume that $\alpha_i = \alpha$ and $t_i = 1$ for every agent i . Then given Assumptions 5.1, 5.2, 5.1' and 5.2', it holds that (i) $\alpha_{1W} = 1/(n-1)$ and (ii) $\alpha_{1L} > 1/(n-1)$.

Proof: See Appendix. ■

The intuition is very simple. For agent i , allocating the amount of resource Δ to the effort and allocating the amount $\alpha\Delta$ to attack each of his peer is equivalent in terms of the relative performance. The total amount of resources used in the latter is $(n-1)\alpha\Delta$. If $\alpha < 1/(n-1)$, player i will strictly prefer the latter to the former. If $\alpha > 1/(n-1)$, he will strictly prefer the former to the latter. If $\alpha = 1/(n-1)$, both will be indifferent for him. Given that agents are homogeneous and that the others use all resource to effort,

his best sabotage action is to attack all peers equally under the one-winner tournament. However, under the one-loser tournament his best sabotage action is to attack only one peer intensively, even given the same condition. Thus it follows that when $\alpha = 1/(n-1)$, a strategy profile such that all players allocate all resource to the effort is a Nash equilibrium under the one-winner tournament, but is not under the one-loser tournament.

This result suggests that relative incentive of the sabotage is stronger under the one-loser tournament than under the one-winner tournament in the case of homogeneous agents.

Next, I consider a case of heterogeneous agents: I mean that $\alpha_i = \alpha$ and $t_i = t$ do not necessarily hold.

Under the one-winner tournament, the heterogeneity in α_i across agents will increase the relative incentive of the sabotage because attacking all his peers equally would not necessarily be his best sabotage action in the case of heterogeneity, even if all his peers allocated their resource to the effort. However, as we see in the previous section, the sabotage among agents reduces the heterogeneity among them. This may enhance the relative incentive of their effort. I give an example. Consider the three-agent-one-winner tournament such that $(\alpha_1, \alpha_2, \alpha_3) = (1, 1/2, 1/2)$, $t_i = 1$ for $i = 1, 2, 3$. You can easily check that the following strategy profile is a Nash equilibrium: $\mathbf{s}_1 = (s_{11}, s_{12}, s_{13}) = (1, 0, 0)$, $\mathbf{s}_2 = (s_{21}, s_{22}, s_{23}) = (1/3, 2/3, 0)$, $\mathbf{s}_3 = (s_{31}, s_{32}, s_{33}) = (1/3, 0, 2/3)$. Then the equilibrium expected performance vector $(E\{x_1(\mu_1, \mathbf{s})\}, E\{x_2(\mu_2, \mathbf{s})\}, E\{x_3(\mu_3, \mathbf{s})\}) = (1/3, 1/3, 1/3)$. Agents 2 and 3 allocate the resource to attack agent 1 until the heterogeneity is canceled out and then allocate the remain of the resource to exert effort. This example suggests the possibility that under the one-winner tournament the sabotage works like a “handicap system,” which is a device to enhance the competition among heterogeneous players under a given prize spread (Lazear and Rosen, 1982). The difference, however, is: a handicap system is not the social cost itself, but sabotage is.

On the other hand, under the one-loser tournament, it is clear that agent 2 or 3 has no incentive to the effort without reference to the degree of heterogeneity because his best sabotage action is always to attack only one peer. In addition to this, under the one-loser tournament the sabotage among agents does not generally reduce the heterogeneity among them, but reinforces it. This may have a negative impact on the incentive for agents with high ability to make effort.

From the above discussion, I argue that the one-loser tournament will suffer, *ceteris*

paribus, more damage from sabotage than the one-winner tournament. Hence, firms which adopt very high promotion rates should deal seriously with the sabotage problem. As seen in the empirical study by Hanada and Sekimoto introduced in Chapter 2, Japanese “traditional” corporations have such promotion systems. They seem to cope with the problem by using extensive rotation. ‘In the Japanese firm employees in the same category in terms of year of graduation from college are subject to the same promotion process. Although I have no data, extensive rotations enable them to work in various departments, regional offices, or factories, and as a result there is little chance that they will work together in the same place, particularly in large firms’ (Itoh 1994: p. 253–254).

Chapter 6

Conclusions

When employees compete for the limited number of positions in higher rank, a promotion rate strongly affects their behavior. I have shown that under the loser-selecting tournament, they prefer a low risk approach or a common approach in the organization they belong to, and as a result conservatism or conformity dominates in the organization. On the other hand, it has been shown that under the winner-selecting tournament, they prefer a high risk approach or their own original approaches, and consequently climate of innovativeness or diversity appears in the organization. These results are consistent with findings of empirical study by Hanada and Sekimoto. Furthermore, it has been shown in the case of three workers that the loser-selecting tournament is more efficient than the winner-selecting tournament in terms of the cost for implementing high efforts of risk-averse workers because of two effects: the order statistic effect and the noise reduction effect. However, I have argued that the one-loser tournament is more subject to damage by sabotage, *ceteris paribus*, than the one-winner tournament.

This paper has analyzed one-period tournament models. Promotion tournaments in real organizations often consist of several rounds. Promotion systems of “traditional” corporations in Hanada and Sekimoto are like elimination tournaments with several rounds.¹ On the other hand, promotion systems of “innovative” corporations in Hanada and Sekimoto

¹Rosen (1986) offers a model of elimination tournaments, which are like “tennis tournaments.” In each round, workers are divided into pairs and they compete in the pair. The winner in one round proceed to the next round.

are like promotion tournaments with several rounds but without elimination.² Examining properties of workers' behavior under promotion tournaments with several rounds waits future research.

Firms should take account of disadvantages of tournaments when they design the promotion tournaments. While I have examined properties of workers' sabotage under promotion tournaments, I have not done those of collusion among workers to reduce effort, which is the other of two main disadvantages of tournaments. Investigating properties of workers' collusive behavior under promotion tournaments also remains the topic of future research.

²Meyer (1992) offers a model of such tournaments in which two workers compete.

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Appendix

Proof of Lemma 3.1:

$$\begin{aligned}
& \Pr(\tilde{x}_n(r) > \tilde{x}) - \Pr(\tilde{x}_n(s) > \tilde{x}) \\
&= \int_{-\infty}^{+\infty} \{1 - F_r(x)\} \phi(x) dx - \int_{-\infty}^{+\infty} \{1 - F_s(x)\} \phi(x) dx \\
&= \int_{-\infty}^{+\infty} \{F_s(x) - F_r(x)\} \phi(x) dx \\
&= \int_0^{+\infty} \{F_s(x) - F_r(x)\} \{\phi(x) - \phi(-x)\} dx. \tag{A.1}
\end{aligned}$$

By Assumption 3.1 (symmetric distribution), we have the last equation (A.1). By Assumption 3.2 (single-crossing), $F_s(x) - F_r(x) \geq 0$ for all $x > 0$. Moreover, by Assumption 3.3, $F_s(x) - F_r(x)$ is strictly positive at least for all $x \in \bar{I}$. Hence, (i) if $\phi(x) \geq \phi(-x)$ for all $x > 0$ and $\phi(x) > \phi(-x)$ for all $x \in \bar{I}$, then the equation (A.1) is positive; (ii) if $\phi(x) = \phi(-x)$ for all $x > 0$, then (A.1) is equal to zero; and (iii) if $\phi(x) \leq \phi(-x)$ for all $x > 0$ and $\phi(x) < \phi(-x)$ for all $x \in \bar{I}$, then (A.1) is negative. ■

Proof of Lemma 3.2: I prove this lemma by dividing into the following cases:

- Case 1: $a = 0, \dots, b$.
 - Case 1.1: a is an odd number in Case 1.
 - Case 1.2: a is an even number in Case 1.
- Case 2: $a = b + 1, \dots, c - b$.
 - Case 2.1: $2b > a$ in Case 2.

- * Case 2.1.1: $2b - a$ is an odd number in Case 2.1.
- * Case 2.1.2: $2b - a$ is an even number in Case 2.1.
- Case 2.2: $2b \leq a$ in Case 2.
- Case 3: $a = c - b + 1, \dots, c$.
 - Case 3.1: $2b > a$ in Case 3.
 - Case 3.2: $2b \leq a$ in Case 3.

Case 1: $a = 0, \dots, b$.

In this case, $H(x; a, b, c)$ is represented by the following equation.

$$H(x; a, b, c) = \sum_{i=0}^a C_i^a C_{b-i}^{c-a} \bar{F}_s(x)^i \bar{F}_r(x)^{b-i} F_s(x)^{a-i} F_r(x)^{c-a-b+i}$$

We can arrange $H(x; a, b, c) - H(-x; a, b, c)$ into the following (A.2) expression. To save of space, hereafter I will often simply write F_r, \bar{F}_r, F_s , and \bar{F}_s for $F_r(x), \bar{F}_r(x), F_s(x)$ and $\bar{F}_s(x)$, respectively.

$$\begin{aligned}
 & H(x; a, b, c) - H(-x; a, b, c) \\
 &= \sum_{i=0}^a C_i^a C_{b-i}^{c-a} \bar{F}_s(x)^i \bar{F}_r(x)^{b-i} F_s(x)^{a-i} F_r(x)^{c-a-b+i} \\
 &\quad - \sum_{i=0}^a C_i^a C_{b-i}^{c-a} \bar{F}_s(-x)^i \bar{F}_r(-x)^{b-i} F_s(-x)^{a-i} F_r(-x)^{c-a-b+i} \\
 &= \sum_{i=0}^a C_i^a C_{b-i}^{c-a} \bar{F}_s(x)^i \bar{F}_r(x)^{b-i} F_s(x)^{a-i} F_r(x)^{c-a-b+i} \\
 &\quad - \sum_{i=0}^a C_i^a C_{b-i}^{c-a} F_s(x)^i F_r(x)^{b-i} \bar{F}_s(x)^{a-i} \bar{F}_r(x)^{c-a-b+i} \\
 &= \sum_{i=0}^a C_i^a C_{b-i}^{c-a} \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{c-a-b+i} - \sum_{j=0}^a C_{a-j}^a C_{b-a+j}^{c-a} F_s^{a-j} F_r^{b-a+j} \bar{F}_s^j \bar{F}_r^{c-b-j} \\
 &= \sum_{i=0}^a \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{b-a+i} \{C_i^a C_{b-i}^{c-a} F_r^{c-2b} - C_{a-i}^a C_{b-a+i}^{c-a} \bar{F}_r^{c-2b}\} \\
 &= \sum_{i=0}^a C_i^a \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{b-a+i} \{C_{b-i}^{c-a} F_r^{c-2b} - C_{b-a+i}^{c-a} \bar{F}_r^{c-2b}\}. \tag{A.2}
 \end{aligned}$$

Here, I divide Case 1 into the two subcases.

Case 1.1: a is an odd number in Case 1.

If a is an odd number, then equation (A.2) can be arranged into the following (A.3):

$$\begin{aligned}
& H(x; a, b, c) - H(-x; a, b, c) \\
&= \sum_{i=0}^{\frac{a-1}{2}} C_i^a \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{b-a+i} \{C_{b-i}^{c-a} F_r^{c-2b} - C_{b-a+i}^{c-a} \bar{F}_r^{c-2b}\} \\
&\quad + \sum_{i=\frac{a-1}{2}+1}^a C_i^a \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{b-a+i} \{C_{b-i}^{c-a} F_r^{c-2b} - C_{b-a+i}^{c-a} \bar{F}_r^{c-2b}\} \\
&= \sum_{i=0}^{\frac{a-1}{2}} C_i^a \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{b-a+i} \{C_{b-i}^{c-a} F_r^{c-2b} - C_{b-a+i}^{c-a} \bar{F}_r^{c-2b}\} \\
&\quad + \sum_{j=0}^{\frac{a-1}{2}} C_{a-j}^a \bar{F}_s^{a-j} \bar{F}_r^{b-a+j} F_s^j F_r^{b-j} \{C_{b-a+j}^{c-a} F_r^{c-2b} - C_{b-j}^{c-a} \bar{F}_r^{c-2b}\} \\
&= \sum_{i=0}^{\frac{a-1}{2}} C_i^a [\bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{b-a+i} \{C_{b-i}^{c-a} F_r^{c-2b} - C_{b-a+i}^{c-a} \bar{F}_r^{c-2b}\} \\
&\quad - \bar{F}_s^{a-i} \bar{F}_r^{b-a+i} F_s^i F_r^{b-i} \{C_{b-i}^{c-a} \bar{F}_r^{c-2b} - C_{b-a+i}^{c-a} F_r^{c-2b}\}]. \tag{A.3}
\end{aligned}$$

Using that $F_r^{c-2b} > \bar{F}_r^{c-2b} \geq 0$ for all $x > 0$ and $C_{b-i}^{c-a} > C_{b-a+i}^{c-a} > 0$ for $i = 0, \dots, (a-1)/2$, we have the following two inequalities:

$$C_{b-i}^{c-a} F_r^{c-2b} - C_{b-a+i}^{c-a} \bar{F}_r^{c-2b} > C_{b-i}^{c-a} \bar{F}_r^{c-2b} - C_{b-a+i}^{c-a} F_r^{c-2b}, \text{ for } i = 0, \dots, (a-1)/2$$

and

$$C_{b-i}^{c-a} F_r^{c-2b} - C_{b-a+i}^{c-a} \bar{F}_r^{c-2b} > 0, \quad \text{for } i = 0, \dots, (a-1)/2.$$

Using these two inequalities and nonnegativity of $\bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{b-a+i}$ and

$\bar{F}_s^{a-i} \bar{F}_r^{b-a+i} F_s^i F_r^{b-i}$ into the equation (A.3), we have following inequality:

$$\begin{aligned}
& H(x; a, b, c) - H(-x; a, b, c) \\
& \geq \sum_{i=0}^{\frac{a-1}{2}} C_i^a \{ \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{b-a+i} - \bar{F}_s^{a-i} \bar{F}_r^{b-a+i} F_s^i F_r^{b-i} \} \{ C_{b-i}^{c-a} F_r^{c-2b} - C_{b-a+i}^{c-a} \bar{F}_r^{c-2b} \} \quad (\text{A.4}) \\
& = \sum_{i=0}^{\frac{a-1}{2}} C_i^a \bar{F}_s^i F_s^i \{ \bar{F}_r^{b-i} F_s^{a-2i} F_r^{b-a+i} - \bar{F}_s^{a-2i} \bar{F}_r^{b-a+i} F_r^{b-i} \} \{ C_{b-i}^{c-a} F_r^{c-2b} - C_{b-a+i}^{c-a} \bar{F}_r^{c-2b} \}.
\end{aligned}$$

Furthermore, using that $F_s^{a-2i} \geq F_r^{a-2i} > 0$ and $0 \leq \bar{F}_s^{a-2i} \leq \bar{F}_r^{a-2i}$

for any $x > 0$ and $i = 0, \dots, (a-1)/2$, we have:

$$\begin{aligned}
& \geq \sum_{i=0}^{\frac{a-1}{2}} C_i^a F_s^i \bar{F}_s^i \{ \bar{F}_r^{b-i} F_r^{a-2i} F_r^{b-a+i} - \bar{F}_r^{a-2i} \bar{F}_r^{b-a+i} F_r^{b-i} \} \{ C_{b-i}^{c-a} F_r^{c-2b} - C_{b-a+i}^{c-a} \bar{F}_r^{c-2b} \} \\
& = \sum_{i=0}^{\frac{a-1}{2}} C_i^a \bar{F}_s^i F_s^i \{ \bar{F}_r^{b-i} F_r^{b-i} - \bar{F}_r^{b-i} F_r^{b-i} \} \{ C_{b-i}^{c-a} F_r^{c-2b} - C_{b-a+i}^{c-a} \bar{F}_r^{c-2b} \} \\
& = 0
\end{aligned}$$

Hence, $H(x; a, b, c) - H(-x; a, b, c)$ is nonnegative for any $x > 0$. Furthermore, $H(x; a, b, c) - H(-x; a, b, c)$ is strictly positive for any $x > 0$ such that $\bar{F}_s(x) > 0$ and $\bar{F}_r(x) > 0$ because strict inequality holds for (A.4).

Case 1.2: a is an even number in Case 1.

If a is zero, then

$$H(x; 0, b, c) - H(-x; 0, b, c) = F_r^b \bar{F}_r^b \{ C_b^c F_r^{c-2b} - C_b^c \bar{F}_r^{c-2b} \} \geq 0 \text{ for any } x > 0.$$

If $\bar{F}_r(x) > 0$, strict inequality holds.

If a is an even number equal or more than two, then the expression (A.2) which

represents $H(x; a, b, c) - H(-x; a, b, c)$ can be arranged into the following (A.5):

$$\begin{aligned}
& H(x; a, b, c) - H(-x; a, b, c) \\
&= \sum_{i=0}^{\frac{a}{2}-1} C_i^a \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{b-a+i} \{C_{b-i}^{c-a} F_r^{c-2b} - C_{b-a+i}^{c-a} \bar{F}_r^{c-2b}\} \\
&\quad + C_{\frac{a}{2}}^a \bar{F}_s^{\frac{a}{2}} \bar{F}_r^{b-\frac{a}{2}} F_s^{\frac{a}{2}} F_r^{b-\frac{a}{2}} \{C_{b-\frac{a}{2}}^{c-a} F_r^{c-2b} - C_{b-\frac{a}{2}}^{c-a} \bar{F}_r^{c-2b}\} \\
&\quad + \sum_{i=\frac{a}{2}+1}^a C_i^a \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{b-a+i} \{C_{b-i}^{c-a} F_r^{c-2b} - C_{b-a+i}^{c-a} \bar{F}_r^{c-2b}\} \\
&= \sum_{i=0}^{\frac{a}{2}-1} C_i^a \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{b-a+i} \{C_{b-i}^{c-a} F_r^{c-2b} - C_{b-a+i}^{c-a} \bar{F}_r^{c-2b}\} \\
&\quad + C_{\frac{a}{2}}^a C_{b-\frac{a}{2}}^{c-a} \bar{F}_s^{\frac{a}{2}} \bar{F}_r^{b-\frac{a}{2}} F_s^{\frac{a}{2}} F_r^{b-\frac{a}{2}} \{F_r^{c-2b} - \bar{F}_r^{c-2b}\} \\
&\quad + \sum_{j=0}^{\frac{a}{2}-1} C_{a-j}^a \bar{F}_s^{a-j} \bar{F}_r^{b-a+j} F_s^j F_r^{b-j} \{C_{b-a+j}^{c-a} F_r^{c-2b} - C_{b-j}^{c-a} \bar{F}_r^{c-2b}\} \\
&= \sum_{i=0}^{\frac{a}{2}-1} C_i^a [\bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{b-a+i} \{C_{b-i}^{c-a} F_r^{c-2b} - C_{b-a+i}^{c-a} \bar{F}_r^{c-2b}\} \\
&\quad - \bar{F}_s^{a-i} \bar{F}_r^{b-a+i} F_s^i F_r^{b-i} \{C_{b-i}^{c-a} \bar{F}_r^{c-2b} - C_{b-a+i}^{c-a} F_r^{c-2b}\}] \\
&\quad + C_{\frac{a}{2}}^a C_{b-\frac{a}{2}}^{c-a} \bar{F}_s^{\frac{a}{2}} \bar{F}_r^{b-\frac{a}{2}} F_s^{\frac{a}{2}} F_r^{b-\frac{a}{2}} \{F_r^{c-2b} - \bar{F}_r^{c-2b}\}. \tag{A.5}
\end{aligned}$$

Since $F_r^{c-2b} > \bar{F}_r^{c-2b} \geq 0$ for any $x > 0$, the second term in the RHS of the equation (A.5) is nonnegative for any $x > 0$, and especially strictly positive for any $x > 0$ such that $\bar{F}_s(x) > 0$ and $\bar{F}_r(x) > 0$. In addition, it holds that $C_{b-i}^{c-a} > C_{b-a+i}^{c-a} > 0$ for $i = 0, \dots, a/2 - 1$. Using these inequalities, I can easily show that the first term in RHS of the equation (A.5) is nonnegative for any $x > 0$, and especially strictly positive for any $x > 0$ such that $\bar{F}_s(x) > 0$ and $\bar{F}_r(x) > 0$ using the same logic as Case 1.1 (the full details are omitted).

Thus, the proof is completed for this subcase.

Case 2: $a = b + 1, \dots, c - b$.

In this case $H(x; a, b, c)$ is the following:

$$H(x; a, b, c) = \sum_{i=0}^b C_i^a C_{b-i}^{c-a} \bar{F}_s(x)^i \bar{F}_r(x)^{b-i} F_s(x)^{a-i} F_r(x)^{c-a-b+i}.$$

Then $H(x; a, b, c) - H(-x; a, b, c)$ is arranged into the following equation (A.6):

$$\begin{aligned}
& H(x; a, b, c) - H(-x; a, b, c) \\
&= \sum_{i=0}^b C_i^a C_{b-i}^{c-a} \bar{F}_s(x)^i \bar{F}_r(x)^{b-i} F_s(x)^{a-i} F_r(x)^{c-a-b+i} \\
&\quad - \sum_{i=0}^b C_i^a C_{b-i}^{c-a} \bar{F}_s(-x)^i \bar{F}_r(-x)^{b-i} F_s(-x)^{a-i} F_r(-x)^{c-a-b+i} \\
&= \sum_{i=0}^b C_i^a C_{b-i}^{c-a} \bar{F}_s(x)^i \bar{F}_r(x)^{b-i} F_s(x)^{a-i} F_r(x)^{c-a-b+i} \\
&\quad - \sum_{i=0}^b C_i^a C_{b-i}^{c-a} F_s(x)^i F_r(x)^{b-i} \bar{F}_s(x)^{a-i} \bar{F}_r(x)^{c-a-b+i} \\
&= \sum_{i=0}^b C_i^a C_{b-i}^{c-a} \{ \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{c-a-b+i} - F_s^i F_r^{b-i} \bar{F}_s^{a-i} \bar{F}_r^{c-a-b+i} \}. \tag{A.6}
\end{aligned}$$

Here, I divide Case 2 into the following two cases.

Case 2.1: $2b > a$ in Case 2.

Arranging the equation (A.6), we have the following equation (A.7):

$$\begin{aligned}
& H(x; a, b, c) - H(-x; a, b, c) \\
&= \sum_{i=0}^{a-b-1} C_i^a C_{b-i}^{c-a} \{ \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{c-a-b+i} - F_s^i F_r^{b-i} \bar{F}_s^{a-i} \bar{F}_r^{c-a-b+i} \} \\
&+ \sum_{i=a-b}^b C_i^a C_{b-i}^{c-a} \{ \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{c-a-b+i} - F_s^i F_r^{b-i} \bar{F}_s^{a-i} \bar{F}_r^{c-a-b+i} \} \\
&= \sum_{i=0}^{a-b-1} C_i^a C_{b-i}^{c-a} \{ \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{c-a-b+i} - F_s^i F_r^{b-i} \bar{F}_s^{a-i} \bar{F}_r^{c-a-b+i} \} \\
&+ \sum_{j=0}^{2b-a} C_{a-b+j}^a C_{2b-a-j}^{c-a} \{ \bar{F}_s^{a-b+j} \bar{F}_r^{2b-a-j} F_s^{b-j} F_r^{c-2b+j} - F_s^{a-b+j} F_r^{2b-a-j} \bar{F}_s^{b-j} \bar{F}_r^{c-2b+j} \} \\
&= \sum_{i=0}^{a-b-1} C_i^a C_{b-i}^{c-a} \{ \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{c-a-b+i} - F_s^i F_r^{b-i} \bar{F}_s^{a-i} \bar{F}_r^{c-a-b+i} \} \\
&+ \sum_{j=0}^{2b-a} C_{a-b+j}^a C_{2b-a-j}^{c-a} \bar{F}_s^{a-b+j} \bar{F}_r^{2b-a-j} F_s^{b-j} F_r^{c-2b+j} \\
&- \sum_{k=0}^{2b-a} C_{b-k}^a C_k^{c-a} F_s^{b-k} F_r^k \bar{F}_s^{a-b+k} \bar{F}_r^{c-a-k} \\
&= \sum_{i=0}^{a-b-1} C_i^a C_{b-i}^{c-a} \{ \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{c-a-b+i} - F_s^i F_r^{b-i} \bar{F}_s^{a-i} \bar{F}_r^{c-a-b+i} \} \\
&+ \sum_{i=0}^{2b-a} \bar{F}_s^{a-b+i} \bar{F}_r^{2b-a-i} F_s^{b-i} F_r^i \{ C_{a-b+i}^a C_{2b-a-i}^{c-a} F_r^{c-2b} - C_{b-i}^a C_i^{c-a} \bar{F}_r^{c-2b} \}. \tag{A.7}
\end{aligned}$$

Furthermore, I divide this subcase into the following two sub-subcases.

Case 2.1.1: $2b - a$ is an odd number in Case 2.1.

Using that $C_{b-i}^a = C_{a-b+i}^a$ to arrange the expression (A.7), we get the following (A.8):

$$\begin{aligned}
& H(x; a, b, c) - H(-x; a, b, c) \\
&= \sum_{i=0}^{a-b-1} C_i^a C_{b-i}^{c-a} \{ \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{c-a-b+i} - F_s^i F_r^{b-i} \bar{F}_s^{a-i} \bar{F}_r^{c-a-b+i} \} \\
&+ \sum_{i=0}^{\frac{2b-a-1}{2}} \bar{F}_s^{a-b+i} \bar{F}_r^{2b-a-i} F_s^{b-i} F_r^i \{ C_{a-b+i}^a C_{2b-a-i}^{c-a} F_r^{c-2b} - C_{b-i}^a C_i^{c-a} \bar{F}_r^{c-2b} \} \\
&+ \sum_{i=\frac{2b-a+1}{2}}^{2b-a} \bar{F}_s^{a-b+i} \bar{F}_r^{2b-a-i} F_s^{b-i} F_r^i \{ C_{a-b+i}^a C_{2b-a-i}^{c-a} F_r^{c-2b} - C_{b-i}^a C_i^{c-a} \bar{F}_r^{c-2b} \} \\
&= \sum_{i=0}^{a-b-1} C_i^a C_{b-i}^{c-a} \{ \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{c-a-b+i} - F_s^i F_r^{b-i} \bar{F}_s^{a-i} \bar{F}_r^{c-a-b+i} \} \\
&+ \sum_{i=0}^{\frac{2b-a-1}{2}} \bar{F}_s^{a-b+i} \bar{F}_r^{2b-a-i} F_s^{b-i} F_r^i \{ C_{a-b+i}^a C_{2b-a-i}^{c-a} F_r^{c-2b} - C_{b-i}^a C_i^{c-a} \bar{F}_r^{c-2b} \} \\
&+ \sum_{j=0}^{\frac{2b-a-1}{2}} \bar{F}_s^{b-j} \bar{F}_r^j F_s^{a-b+j} F_r^{2b-a-j} \{ C_{b-j}^a C_j^{c-a} F_r^{c-2b} - C_{a-b+j}^a C_i^{2b-a-j} \bar{F}_r^{c-2b} \} \\
&= \sum_{i=0}^{a-b-1} C_i^a C_{b-i}^{c-a} \{ \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{c-a-b+i} - F_s^i F_r^{b-i} \bar{F}_s^{a-i} \bar{F}_r^{c-a-b+i} \} \\
&+ \sum_{i=0}^{\frac{2b-a-1}{2}} C_{b-i}^a [\bar{F}_s^{a-b+i} \bar{F}_r^{2b-a-i} F_s^{b-i} F_r^i \{ C_{2b-a-i}^{c-a} F_r^{c-2b} - C_i^{c-a} \bar{F}_r^{c-2b} \} \\
&\quad - \bar{F}_s^{b-i} \bar{F}_r^i F_s^{a-b+i} F_r^{2b-a-i} \{ C_{2b-a-i}^{c-a} \bar{F}_r^{c-2b} - C_i^{c-a} F_r^{c-2b} \}]. \tag{A.8}
\end{aligned}$$

Since $a - 2i$ is positive for $i = 0, \dots, a - b - 1$, we know that $F_s^{a-2i} \geq F_r^{a-2i} > 0$ and $0 \leq \bar{F}_s^{a-2i} \leq \bar{F}_r^{a-2i}$ for any $x > 0$ and $i = 0, \dots, a - b - 1$. Using these inequalities, we can easily see that the first term in RHS of (A.8) is nonnegative for any $x > 0$, and especially

strictly positive for any $x > 0$ such that $\bar{F}_s(x) > 0$ and $\bar{F}_r(x) > 0$. Namely,

$$\begin{aligned}
& \sum_{i=0}^{a-b-1} C_i^a C_{b-i}^{c-a} \{ \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{c-a-b+i} - F_s^i F_r^{b-i} \bar{F}_s^{a-i} \bar{F}_r^{c-a-b+i} \} \\
&= \sum_{i=0}^{a-b-1} C_i^a C_{b-i}^{c-a} \bar{F}_s^i F_s^i \{ \bar{F}_r^{b-i} F_r^{a-2i} F_r^{c-a-b+i} - F_r^{b-i} \bar{F}_s^{a-2i} \bar{F}_r^{c-a-b+i} \} \\
&\geq \sum_{i=0}^{a-b-1} C_i^a C_{b-i}^{c-a} \bar{F}_s^i F_s^i \{ \bar{F}_r^{b-i} F_r^{a-2i} F_r^{c-a-b+i} - F_r^{b-i} \bar{F}_r^{a-2i} \bar{F}_r^{c-a-b+i} \} \\
&= \sum_{i=0}^{a-b-1} C_i^a C_{b-i}^{c-a} \bar{F}_s^i F_s^i F_r^{b-i} \bar{F}_r^{b-i} \{ F_r^{c-2b} - \bar{F}_r^{c-2b} \} \\
&\geq 0. \text{ (Strict inequality holds if } \bar{F}_s > 0 \text{ and } \bar{F}_r > 0.)
\end{aligned}$$

Next, I show that the second term in the RHS of (A.8) is nonnegative for any $x > 0$, and especially strictly positive for any $x > 0$ such that $\bar{F}_s(x) > 0$ and $\bar{F}_r(x) > 0$. Since $C_{2b-a-i}^{c-a} > C_i^{c-a}$ for $i = 0, 1, \dots, (2b-a-1)/2$, we have the following two inequalities for any $x > 0$:

$$C_{2b-a-i}^{c-a} F_r^{c-2b} - C_i^{c-a} \bar{F}_r^{c-2b} > C_{2b-a-i}^{c-a} \bar{F}_r^{c-2b} - C_i^{c-a} F_r^{c-2b}, \text{ for } i = 0, 1, \dots, (2b-a-1)/2.$$

and

$$C_{2b-a-i}^{c-a} F_r^{c-2b} - C_i^{c-a} \bar{F}_r^{c-2b} > 0, \quad \text{for } i = 0, 1, \dots, (2b-a-1)/2.$$

Using these two inequalities, I can show that the second term of RHS of (A.8) is nonnegative

for any $x > 0$:

$$\begin{aligned}
& \sum_{i=0}^{\frac{2b-a-1}{2}} C_{b-i}^a [\bar{F}_s^{a-b+i} \bar{F}_r^{2b-a-i} F_s^{b-i} F_r^i \{C_{2b-a-i}^{c-a} F_r^{c-2b} - C_i^{c-a} \bar{F}_r^{c-2b}\} \\
& \quad - \bar{F}_s^{b-i} \bar{F}_r^i F_s^{a-b+i} F_r^{2b-a-i} \{C_{2b-a-i}^{c-a} \bar{F}_r^{c-2b} - C_i^{c-a} F_r^{c-2b}\}] \\
& \geq \sum_{i=0}^{\frac{2b-a-1}{2}} C_{b-i}^a [\bar{F}_s^{a-b+i} \bar{F}_r^{2b-a-i} F_s^{b-i} F_r^i \{C_{2b-a-i}^{c-a} F_r^{c-2b} - C_i^{c-a} \bar{F}_r^{c-2b}\} \\
& \quad - \bar{F}_s^{b-i} \bar{F}_r^i F_s^{a-b+i} F_r^{2b-a-i} \{C_{2b-a-i}^{c-a} F_r^{c-2b} - C_i^{c-a} \bar{F}_r^{c-2b}\}] \\
& = \sum_{i=0}^{\frac{2b-a-1}{2}} C_{b-i}^a \{C_{2b-a-i}^{c-a} F_r^{c-2b} - C_i^{c-a} \bar{F}_r^{c-2b}\} \\
& \quad \times \bar{F}_s^{a-b+i} F_s^{a-b+i} \{\bar{F}_r^{2b-a-i} F_s^{2b-a-2i} F_r^i - \bar{F}_s^{2b-a-2i} \bar{F}_r^i F_r^{2b-a-i}\} \\
& \geq \sum_{i=0}^{\frac{2b-a-1}{2}} C_{b-i}^a \{C_{2b-a-i}^{c-a} F_r^{c-2b} - C_i^{c-a} \bar{F}_r^{c-2b}\} \\
& \quad \times \bar{F}_s^{a-b+i} F_s^{a-b+i} \{\bar{F}_r^{2b-a-i} F_r^{2b-a-i} - \bar{F}_r^{2b-a-i} F_r^{2b-a-i}\} \\
& = 0.
\end{aligned} \tag{A.9}$$

Note that the second term of RHS of (A.8) is strictly positive for any $x > 0$ such that $\bar{F}_s(x) > 0$ and $\bar{F}_r(x) > 0$ because strict inequality of (A.9) holds.

Thus all terms in the RHS of the equation (A.8) are nonnegative for any $x > 0$, and especially strictly positive for any $x > 0$ such that $\bar{F}_s(x) > 0$ and $\bar{F}_r(x) > 0$. This implies that Lemma 3.2 holds for Case 2.1.1.

Case 2.1.2: $2b - a$ is an even number in Case 2.1.

Arranging $H(x; a, b, c) - H(-x; a, b, c)$ using the same logic as Case 2.1.1, we have the

following equation (A.9):

$$\begin{aligned}
& H(x; a, b, c) - H(-x; a, b, c) \\
&= \sum_{i=0}^{a-b-1} C_i^a C_{b-i}^{c-a} \{ \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{c-a-b+i} - F_s^i F_r^{b-i} \bar{F}_s^{a-i} \bar{F}_r^{c-a-b+i} \} \\
&+ \sum_{i=0}^{\frac{2b-a}{2}-1} C_{b-i}^a [\bar{F}_s^{a-b+i} \bar{F}_r^{2b-a-i} F_s^{b-i} F_r^i \{ C_{2b-a-i}^{c-a} F_r^{c-2b} - C_i^{c-a} \bar{F}_r^{c-2b} \} \\
&\quad - \bar{F}_s^{b-i} \bar{F}_r^i F_s^{a-b+i} F_r^{2b-a-i} \{ C_{2b-a-i}^{c-a} \bar{F}_r^{c-2b} - C_i^{c-a} F_r^{c-2b} \}] \\
&+ C_{\frac{a}{2}}^a C_{\frac{2b-a}{2}}^{c-a} \{ \bar{F}_s^{\frac{a}{2}} \bar{F}_r^{\frac{2b-a}{2}} F_s^{\frac{a}{2}} F_r^{c-b-\frac{a}{2}} - F_s^{\frac{a}{2}} F_r^{\frac{2b-a}{2}} \bar{F}_s^{\frac{a}{2}} \bar{F}_r^{c-b-\frac{a}{2}} \}. \tag{A.10}
\end{aligned}$$

I can easily show that both the first term and the second term on RHS of (A.10) are nonnegative for any $x > 0$, and especially strictly positive for any $x > 0$ such that $\bar{F}_s(x) > 0$ and $\bar{F}_r(x) > 0$ using the same logic as Case 2.1.1 (full details on proof are omitted). I show that the third term is also nonnegative for any $x > 0$, and especially strictly positive for any $x > 0$ such that $\bar{F}_s(x) > 0$ and $\bar{F}_r(x) > 0$ as follows:

$$\begin{aligned}
& C_{\frac{a}{2}}^a C_{\frac{2b-a}{2}}^{c-a} \{ \bar{F}_s^{\frac{a}{2}} \bar{F}_r^{\frac{2b-a}{2}} F_s^{\frac{a}{2}} F_r^{c-b-\frac{a}{2}} - F_s^{\frac{a}{2}} F_r^{\frac{2b-a}{2}} \bar{F}_s^{\frac{a}{2}} \bar{F}_r^{c-b-\frac{a}{2}} \} \\
&= C_{\frac{a}{2}}^a C_{\frac{2b-a}{2}}^{c-a} \bar{F}_s^{\frac{a}{2}} \bar{F}_r^{\frac{2b-a}{2}} F_s^{\frac{a}{2}} F_r^{\frac{2b-a}{2}} \{ F_r^{c-2b} - \bar{F}_r^{c-2b} \} \\
&\geq 0. \text{ (Strict inequality holds if } \bar{F}_s > 0 \text{ and } \bar{F}_r > 0.)
\end{aligned}$$

Thus all terms in the RHS of the equation (A.10) are nonnegative for any $x > 0$, and especially strictly positive for any $x > 0$ such that $\bar{F}_s(x) > 0$ and $\bar{F}_r(x) > 0$. This implies that Lemma 3.2 holds for Case 2.1.2.

Case 2.2: $2b \leq a$ in Case 2.

In this subcase, $a - 2i \geq 0$ for $i = 0, \dots, b$ (equality holds only if either $i = b$ or $2b = a$). Then I can easily show that equation (A.6), which represents $H(x; a, b, c) - H(-x; a, b, c)$, is nonnegative for any $x > 0$, and especially strictly positive for any $x > 0$ such that $\bar{F}_s(x) > 0$

and $\bar{F}_r(x) > 0$:

$$\begin{aligned}
& H(x; a, b, c) - H(-x; a, b, c) \\
&= \sum_{i=0}^b C_i^a C_{b-i}^{c-a} \{ \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{c-a-b+i} - F_s^i F_r^{b-i} \bar{F}_s^{a-i} \bar{F}_r^{c-a-b+i} \} \\
&= \sum_{i=0}^b C_i^a C_{b-i}^{c-a} \bar{F}_s^i F_s^i \{ \bar{F}_r^{b-i} F_r^{a-2i} F_r^{c-a-b+i} - F_r^{b-i} \bar{F}_s^{a-2i} \bar{F}_r^{c-a-b+i} \} \\
&\geq \sum_{i=0}^b C_i^a C_{b-i}^{c-a} \bar{F}_s^i F_s^i \{ \bar{F}_r^{b-i} F_r^{a-2i} F_r^{c-a-b+i} - F_r^{b-i} \bar{F}_r^{a-2i} \bar{F}_r^{c-a-b+i} \} \\
&= \sum_{i=0}^b C_i^a C_{b-i}^{c-a} \bar{F}_s^i F_s^i \{ \bar{F}_r^{b-i} F_r^{c-b-i} - F_r^{b-i} \bar{F}_r^{c-b-i} \} \\
&= \sum_{i=0}^b C_i^a C_{b-i}^{c-a} \bar{F}_s^i \bar{F}_r^{b-i} F_s^i F_r^{b-i} \{ F_r^{c-2b} - \bar{F}_r^{c-2b} \} \\
&\geq 0. \text{ (Strict inequality holds if } \bar{F}_s > 0 \text{ and } \bar{F}_r > 0.)
\end{aligned}$$

Case 3: $a = c - b + 1, \dots, c$.

In this case, $H(x; a, b, c)$ is given by the following:

$$H(x; a, b, c) = \sum_{i=a+b-c}^b C_i^a C_{b-i}^{c-a} \bar{F}_s(x)^i \bar{F}_r(x)^{b-i} F_s(x)^{a-i} F_r(x)^{c-a-b+i}$$

Then $H(x; a, b, c) - H(-x; a, b, c)$ is arranged in the following form:

$$\begin{aligned}
& H(x; a, b, c) - H(-x; a, b, c) \\
&= \sum_{i=a+b-c}^b C_i^a C_{b-i}^{c-a} \bar{F}_s(x)^i \bar{F}_r(x)^{b-i} F_s(x)^{a-i} F_r(x)^{c-a-b+i} \\
&\quad - \sum_{i=a+b-c}^b C_i^a C_{b-i}^{c-a} \bar{F}_s(-x)^i \bar{F}_r(-x)^{b-i} F_s(-x)^{a-i} F_r(-x)^{c-a-b+i} \\
&= \sum_{i=a+b-c}^b C_i^a C_{b-i}^{c-a} \bar{F}_s(x)^i \bar{F}_r(x)^{b-i} F_s(x)^{a-i} F_r(x)^{c-a-b+i} \\
&\quad - \sum_{i=a+b-c}^b C_i^a C_{b-i}^{c-a} F_s(x)^i F_r(x)^{b-i} \bar{F}_s(x)^{a-i} \bar{F}_r(x)^{c-a-b+i} \\
&= \sum_{i=a+b-c}^b C_i^a C_{b-i}^{c-a} \{ \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{c-a-b+i} - F_s^i F_r^{b-i} \bar{F}_s^{a-i} \bar{F}_r^{c-a-b+i} \}. \tag{A.11}
\end{aligned}$$

Here, I divide Case 3 into the following two subcases.

Case 3.1: $2b > a$ in Case 3.

Arranging (A.11), I have the following (A.12):

$$\begin{aligned}
& H(x; a, b, c) - H(-x; a, b, c) \\
&= \sum_{i=a+b-c}^{a-b-1} C_i^a C_{b-i}^{c-a} \{ \bar{F}_s^i \bar{F}_r^{b-i} F_s^{a-i} F_r^{c-a-b+i} - F_s^i F_r^{b-i} \bar{F}_s^{a-i} \bar{F}_r^{c-a-b+i} \} \\
&+ \sum_{i=0}^{2b-a} \bar{F}_s^{a-b+i} \bar{F}_r^{2b-a-i} F_s^{b-i} F_r^i \{ C_{a-b+i}^a C_{2b-a-i}^{c-a} F_r^{c-2b} - C_{b-i}^a C_i^{c-a} \bar{F}_r^{c-2b} \}. \quad (\text{A.12})
\end{aligned}$$

I can easily prove that (A.12) is nonnegative for any $x > 0$, and especially strictly positive for any $x > 0$ such that $\bar{F}_s(x) > 0$ and $\bar{F}_r(x) > 0$ using the same logic as Case 2.1.1 and 2.1.2 (full details are omitted).

Case 3.2: $2b \leq a$ in Case 3.

In this subcase, $a - 2i \geq 0$ for $i = a + b - c, \dots, b$ (equality holds iff either $i = b$ or $2b = a$). Then I can prove that (A.11) nonnegative for any $x > 0$, and especially strictly positive for any $x > 0$ such that $\bar{F}_s(x) > 0$ and $\bar{F}_r(x) > 0$ using the same logic as Case 2.2 (full details are also omitted). Thus, the proof is completed. ■

Proof of Lemma 3.3:

$$\begin{aligned}
& H(-x; a, c-b, c) \\
&= \sum_{i=\max[0, a-b]}^{\min[a, c-b]} C_i^a C_{c-b-i}^{c-a} \bar{F}_s(-x)^i \bar{F}_r(-x)^{c-b-i} F_s(-x)^{a-i} F_r(-x)^{b-a+i} \\
&= \sum_{i=\max[0, a-b]}^{\min[a, c-b]} C_i^a C_{c-b-i}^{c-a} F_s(x)^i F_r(x)^{c-b-i} \bar{F}_s(x)^{a-i} \bar{F}_r(x)^{b-a+i} \\
&= \sum_{i=\max[0, a-b]}^{\min[a, c-b]} C_i^a C_{c-b-i}^{c-a} \bar{F}_s(x)^{a-i} \bar{F}_r(x)^{b-a+i} F_s(x)^i F_r(x)^{c-b-i} \\
&\text{let } j = a - i, \\
&= \sum_{j=a-\min[a, c-b]}^{a-\max[0, a-b]} C_{a-j}^a C_{c-a-b+j}^{c-a} \bar{F}_s(x)^j \bar{F}_r(x)^{b-j} F_s(x)^{a-j} F_r(x)^{c-a-b+j} \\
&\text{using } C_{a-j}^a = C_j^a \text{ and } C_{c-a-b+j}^{c-a} = C_{b-j}^{c-a}, \\
&= \sum_{j=\max[0, a+b-c]}^{\min[a, b]} C_j^a C_{b-j}^{c-a} \bar{F}_s(x)^j \bar{F}_r(x)^{b-j} F_s(x)^{a-j} F_r(x)^{c-a-b+j} \\
&= H(x; a, b, c). \blacksquare
\end{aligned}$$

Proof of Proposition 3.1:

(i) If $n > 2m$, then $\phi_{(n-m)}(x; l, n-1) - \phi_{(n-m)}(-x; l, n-1)$ represented by (3.5) is nonnegative for all $x > 0$ and especially strictly positive for all $x \in \bar{I}$ by Lemma 3.2 and Assumption 3.3. Thus, applying part (i) of Lemma 3.1, we immediately find that agent n will strictly prefer the riskier approach r to the low risk approach s without regard to the other agents' choices, *i.e.*, the high risk approach r is a strictly dominant strategy for agent n . The model is symmetric for every agent. Hence, the high risk approach r is a strictly dominant strategy for every agent.

(ii) If $n = 2m$, then by Lemma 3.3, we have:

$$\begin{aligned}
& H(x; l, m-1, 2m-2) - H(-x; l, m-1, 2m-2) \\
&= H(x; l, m-1, 2m-2) - H(-x; l, 2m-2-(m-1), 2m-2) = 0,
\end{aligned}$$

for any $l = 0, \dots, n-1$ and any x . Hence $\phi_{(n-m)}(x; l, n-1) - \phi_{(n-m)}(-x; l, n-1)$ is also always zero for any x . Then part (ii) of Lemma 3.1 indicates that the two approaches are indifferent for every agent without regard to the other agents' choices.

(iii) Using Lemma 3.3, we have:

$$\begin{aligned} & H(x; l, m-1, n-2) - H(-x; l, m-1, n-2) \\ &= -\{H(x; l, n-m-1, n-2) - H(-x; l, n-m-1, n-2)\}. \end{aligned} \quad (\text{A.13})$$

If $n < 2m$, then $n > 2(n-m)$. The RHS of equation (A.13) is nonpositive for any $x > 0$ and particularly strictly negative for any $x > 0$ such that $\bar{F}_s(x) > 0$ and $\bar{F}_r(x) > 0$ by Lemma 3.2. On the other hand, the density functions $f_r(x)$ and $f_s(x)$ are nonnegative for all $x > 0$, and especially positive for any $x \in \bar{I}$ by Assumption 3.3. Hence $\phi_{(n-m)}(x; l, n-1) - \phi_{(n-m)}(-x; l, n-1)$ is nonpositive for all $x > 0$ and especially strictly negative for all $x \in \bar{I}$. Thus, by part (iii) of Lemma 3.1, we can immediately see that approach s is a strictly dominant strategy for agent n . ■

Proof of Proposition 3.2: Let $\Delta = \frac{v_H}{3D^{1W}} (= \frac{v_H}{3D^{1L}})$ and $G = u^0 + v_H$.

(1) $u(R) = -e^{-r_A R}$ ($r_A > 0$). we straightforwardly have the result:

$$\begin{aligned} & R_W^{1W*} + 2R_L^{1W*} - 2R_W^{1L*} - R_L^{1L*} \\ &= -\frac{1}{r_A} \log\{(-G - 2\Delta)(-G + \Delta)^2\} + \frac{1}{r_A} \log\{(-G - \Delta)^2(-G + 2\Delta)\} \\ &= -\frac{1}{r_A} \log\left\{\frac{-G^3 + 3G\Delta - 2\Delta^3}{-G^3 + 3G\Delta + 2\Delta^3}\right\} \\ &> 0. \end{aligned}$$

(2) $u(R) = R^{\frac{1}{n}}$ ($n = 2, 3, \dots$). By using the binomial formula, we have the result:

$$\begin{aligned} & R_W^{1W*} + 2R_L^{1W*} - 2R_W^{1L*} - R_L^{1L*} \\ &= (G + 2\Delta)^n + 2(G - \Delta)^n - 2(G + \Delta)^n - (G - 2\Delta)^n \\ &= \sum_{i=0}^n C_i^n G^{n-i} \{(2\Delta)^i + 2(-\Delta)^i - 2(\Delta)^i - (-2\Delta)^i\} \\ &\geq 0 \quad (\text{equality holds iff } n = 2) \end{aligned}$$

because $(2\Delta)^i + 2(-\Delta)^i - 2(\Delta)^i - (-2\Delta)^i = 0$ when i is zero or an even number, and $(2\Delta)^i + 2(-\Delta)^i - 2(\Delta)^i - (-2\Delta)^i = 2(2\Delta)^i - 4\Delta^i \geq 0$ when i is an odd number.

(3) $u(R) = \log R$. By using the Maclaurin series for exponential function and part (2) of this proof, we have the result:

$$\begin{aligned}
& R_W^{1W*} + 2R_L^{1W*} - 2R_W^{1L*} - R_L^{1L*} \\
&= e^{G+2\Delta} + 2e^{G-\Delta} - 2e^{G+\Delta} - e^{G-2\Delta} \\
&= \left\{ 1 + \sum_{n=1}^{+\infty} \frac{(G+2\Delta)^n}{n!} \right\} + 2 \left\{ 1 + \sum_{n=1}^{+\infty} \frac{(G-\Delta)^n}{n!} \right\} \\
&\quad - 2 \left\{ 1 + \sum_{n=1}^{+\infty} \frac{(G+\Delta)^n}{n!} \right\} - \left\{ 1 + \sum_{n=1}^{+\infty} \frac{(G-2\Delta)^n}{n!} \right\} \\
&= \sum_{n=1}^{+\infty} \frac{(G+2\Delta)^n + 2(G-\Delta)^n - 2(G+\Delta)^n - (G-2\Delta)^n}{n!} \\
&> 0,
\end{aligned}$$

because $(G+2\Delta)^n + 2(G-\Delta)^n - 2(G+\Delta)^n - (G-2\Delta)^n > 0$ for any integer greater than 2 by part (2) of this proof, $(G+2\Delta)^2 + 2(G-\Delta)^2 - 2(G+\Delta)^2 - (G-2\Delta)^2 = 0$, and $(G+2\Delta) + 2(G-\Delta) - 2(G+\Delta) - (G-2\Delta) = 0$. ■

Proof of Lemma 3.4: (i) The second order statistic of their performances on two agents who choose the high effort and high risk approach r has the following density function:

$$\phi^{(2)}(x) = f_r(x - e_H)F_r(x - e_H).$$

For any $x > 0$, $F_r(x - e_H) > F_r(-x - e_H)$ and $f_r(x - e_H) \geq f_r(-x - e_H)$ (by the unimodal distribution). Therefore, it holds that $\phi^{(2)}(x) - \phi^{(2)}(-x)$ is positive for any x such that $0 < x < \bar{\epsilon}(r) + e_H$, or is zero for any $x \geq \bar{\epsilon}(r) + e_H$. The result follows from Lemma 3.1.

(ii) We have:

$$\begin{aligned}
D^{1L,s} - D^{1W} &= \frac{2}{3} - \int_{-\bar{\epsilon}(s)}^{\bar{\epsilon}(s)} [1 - \{1 - F_s(x - e_H)\}^2] f_s(x) dx \\
&\quad - \left(\frac{1}{3} - \int_{-\bar{\epsilon}(r)}^{\bar{\epsilon}(r)} \{F_r(x - e_H)\}^2 f_r(x) dx \right) \\
&= \frac{1}{3} - \int_{-\bar{\epsilon}(s)}^{\bar{\epsilon}(s)} ([1 - \{1 - F_s(x - e_H)\}^2] - \{F_s(x - \frac{e_H}{\alpha})\}^2) f_s(x) dx \\
&\geq \frac{1}{3} - \int_{-\bar{\epsilon}(s)}^{\bar{\epsilon}(s)} ([1 - \{1 - F_s(x - e_H)\}^2] - \{F_s(x - e_H)\}^2) f_s(x) dx, \quad (\text{A.14})
\end{aligned}$$

where the equality holds iff $e_H = 0$. When $e_H = 0$, obviously $D^{1L,s} - D^{1W} = 0$. When $e_H \geq 2\bar{\epsilon}(s)$, the value of (A.14) is $\frac{1}{3}$. The result will follow by showing that the expression (A.14) is strictly increasing in $e_H \in [0, 2\bar{\epsilon}(s)]$.

$$\begin{aligned}
&\frac{d \left(\frac{1}{3} - \int_{-\bar{\epsilon}(s)}^{\bar{\epsilon}(s)} ([1 - \{1 - F_s(x - e_H)\}^2] - \{F_s(x - e_H)\}^2) f_s(x) dx \right)}{de_H} \\
&= 2 \int_{-\bar{\epsilon}(s)}^{\bar{\epsilon}(s)} \{1 - 2F_s(x - e_H)\} f_s(x - e_H) f_s(x) dx \\
&= 2 \int_{-\bar{\epsilon}(s) - e_H}^{\bar{\epsilon}(s) - e_H} \{1 - 2F_s(y)\} f_s(y) f_s(y + e_H) dy. \quad (\text{A.15})
\end{aligned}$$

When $\bar{\epsilon}(s) \leq e_H < 2\bar{\epsilon}(s)$, the sign of (A.15) is positive because $\{1 - 2F_s(y)\} f_s(y) f_s(y + e_H)$ is positive for any y such that $-\bar{\epsilon}(s) < y < \bar{\epsilon}(s) - e_H$, and is zero for any y such that $-\bar{\epsilon}(s) - e_H \leq y \leq -\bar{\epsilon}(s)$. When $0 \leq e_H < \bar{\epsilon}(s)$, we have

$$\begin{aligned}
&2 \int_{-\bar{\epsilon}(s) - e_H}^{\bar{\epsilon}(s) - e_H} \{1 - 2F_s(y)\} f_s(y) f_s(y + e_H) dy \\
&= 2 \left[\int_0^{\bar{\epsilon}(s) - e_H} \{1 - 2F_s(y)\} f_s(y) f_s(y + e_H) dy + \int_{-\bar{\epsilon}(s) - e_H}^0 \{1 - 2F_s(y)\} f_s(y) f_s(y + e_H) dy \right]
\end{aligned}$$

Notice that $\{1 - 2F_s(y)\} f_s(y) f_s(y + e_H)$ is negative for any y such that $\bar{\epsilon}(s) - e_H \geq y > 0$, is positive for any y such that $-\bar{\epsilon}(s) \leq y < 0$, or is zero for any y such that $-\bar{\epsilon}(s) - e_H \leq y < -\bar{\epsilon}(s)$. Also notice that $f_s(y + e_H) \geq f_s(y - e_H)$ when $-\bar{\epsilon}(s) - e_H \leq y \leq 0$ since the

distribution function is unimodal. Using these facts, we have:

$$\begin{aligned}
& 2 \left[\int_0^{\bar{\epsilon}(s)-e_H} \{1 - 2F_s(y)\} f_s(y) f_s(y + e_H) dy + \int_{-\bar{\epsilon}(s)-e_H}^0 \{1 - 2F_s(y)\} f_s(y) f_s(y + e_H) dy \right] \\
& > 2 \left[\int_0^{\bar{\epsilon}(s)-e_H} \{1 - 2F_s(y)\} f_s(y) f_s(y + e_H) dy + \int_{-\bar{\epsilon}(s)+e_H}^0 \{1 - 2F_s(y)\} f_s(y) f_s(y - e_H) dy \right] \\
& = 2 \int_0^{\bar{\epsilon}(s)-e_H} [\{1 - 2F_s(y)\} f_s(y) f_s(y + e_H) + \{1 - 2F_s(-y)\} f_s(-y) f_s(-y - e_H)] dy \\
& = 0. \quad (\text{by the symmetry of distribution})
\end{aligned}$$

(iii) I show that for any $e_H > 0$, it holds that

$$D^{1L,r} \geq \frac{2}{3} - \int_{-\bar{\epsilon}(r)}^{\bar{\epsilon}(r)} [1 - \{1 - F_r(x - e_H)\}^2] f_r(x) dx > D^{1W,r}$$

(a) Proof of part of $D^{1L,r} \geq \frac{2}{3} - \int_{-\bar{\epsilon}(r)}^{\bar{\epsilon}(r)} [1 - \{1 - F_r(x - e_H)\}^2] f_r(x) dx$:

$$\begin{aligned}
& D^{1L,r} - \left(\frac{2}{3} - \int_{-\bar{\epsilon}(r)}^{\bar{\epsilon}(r)} [1 - \{1 - F_r(x - e_H)\}^2] f_r(x) dx \right) \\
& = \int_{-\bar{\epsilon}(r)}^{\bar{\epsilon}(r)} \{F_r(x - e_H) - F_s(x - e_H)\} \{2 - F_r(x - e_H) - F_s(x - e_H)\} f_r(x) dx \\
& = \int_{-\bar{\epsilon}(r)-e_H}^{\bar{\epsilon}(r)-e_H} \{F_r(y) - F_s(y)\} \{2 - F_r(y) - F_s(y)\} f_r(y + e_H) dy \tag{A.16}
\end{aligned}$$

When $e_H \geq 2\bar{\epsilon}(r)$, obviously (A.16) is equal to zero. When $2\bar{\epsilon}(r) > e_H \geq \bar{\epsilon}(r)$, the value of (A.16) is nonnegative since $\{F_r(y) - F_s(y)\} \{2 - F_r(y) - F_s(y)\} f_r(y + e_H)$ is nonnegative for any y such that $\bar{\epsilon}(r) - e_H \geq y \geq -\bar{\epsilon}(r) - e_H$. When $\bar{\epsilon}(r) > e_H > 0$, (A.16) is given by:

$$\begin{aligned}
& \int_{-\bar{\epsilon}(r)-e_H}^{\bar{\epsilon}(r)-e_H} \{F_r(y) - F_s(y)\} \{2 - F_r(y) - F_s(y)\} f_r(y + e_H) dy \\
& = \int_0^{\bar{\epsilon}(r)-e_H} \{F_r(y) - F_s(y)\} \{2 - F_r(y) - F_s(y)\} f_r(y + e_H) dy \\
& \quad + \int_{-\bar{\epsilon}(r)-e_H}^0 \{F_r(y) - F_s(y)\} \{2 - F_r(y) - F_s(y)\} f_r(y + e_H) dy.
\end{aligned}$$

Notice that $\{F_r(y) - F_s(y)\} \{2 - F_r(y) - F_s(y)\} f_r(y + e_H)$ is negative for any y such that $\bar{\epsilon}(r) - e_H \geq y > 0$, is positive for any y such that $-\bar{\epsilon}(r) \leq y < 0$, or is zero for any y such that $-\bar{\epsilon}(r) - e_H \leq y < -\bar{\epsilon}(r)$. Also notice that $f_r(y + e_H) \geq f_r(y - e_H)$ when $-\bar{\epsilon}(s) - e_H \leq y \leq 0$

since the distribution function is unimodal. Using these facts, we have:

$$\begin{aligned}
& \int_0^{\bar{\epsilon}(r)-e_H} \{F_r(y) - F_s(y)\} \{2 - F_r(y) - F_s(y)\} f_r(y + e_H) dy \\
& \quad \int_{-\bar{\epsilon}(r)-e_H}^0 \{F_r(y) - F_s(y)\} \{2 - F_r(y) - F_s(y)\} f_r(y + e_H) dy. \\
& > \int_0^{\bar{\epsilon}(r)-e_H} \{F_r(y) - F_s(y)\} \{2 - F_r(y) - F_s(y)\} f_r(y + e_H) dy \\
& \quad + \int_{-\bar{\epsilon}(r)+e_H}^0 \{F_r(y) - F_s(y)\} \{2 - F_r(y) - F_s(y)\} f_r(y - e_H) dy. \\
& = \int_0^{\bar{\epsilon}(r)-e_H} \left[\{F_r(y) - F_s(y)\} \{2 - F_r(y) - F_s(y)\} f_r(y - e_H) \right. \\
& \quad \left. + \{F_r(-y) - F_s(-y)\} \{2 - F_r(-y) - F_s(-y)\} f_r(-y - e_H) \right] dy. \\
& = 2 \int_0^{\bar{\epsilon}(r)-e_H} \{F_r(y) - F_s(y)\} \{1 - F_r(y) - F_s(y)\} f_r(y + e_H) dy \\
& > 0, \quad \text{for any } e_H \text{ such that } \bar{\epsilon}(r) > e_H > 0.
\end{aligned} \tag{A.17}$$

(b) Proof of $\frac{2}{3} - \int_{-\bar{\epsilon}(r)}^{\bar{\epsilon}(r)} [1 - \{1 - F_r(x - e_H)\}^2] f_r(x) dx > D^{1W,r}$ for any $e_H > 0$:

$$\begin{aligned}
& \frac{2}{3} - \int_{-\bar{\epsilon}(r)}^{\bar{\epsilon}(r)} [1 - \{1 - F_r(x - e_H)\}^2] f_r(x) dx - D^{1W,r} \\
& = \frac{1}{3} - 2 \int_{-\bar{\epsilon}(r)}^{\bar{\epsilon}(r)} [F_r(x - e_H) - \{F_r(x - e_H)\}^2] f_r(x) dx
\end{aligned} \tag{A.18}$$

When $e_H = 0$, the value of (A.18) is $1/3 - 2(1/2 - 1/3) = 0$. On the other hand, when $e_H \geq 2\bar{\epsilon}(r)$, the value of (A.18) is $1/3$. The result will follow by showing that the expression (A.18) is strictly increasing in $e_H \in [0, 2\bar{\epsilon}(r)]$:

$$\begin{aligned}
& \frac{d \left(\frac{1}{3} - 2 \int_{-\bar{\epsilon}(r)}^{\bar{\epsilon}(r)} [F_r(x - e_H) - \{F_r(x - e_H)\}^2] f_r(x) dx \right)}{de_H} \\
& = 2 \int_{-\bar{\epsilon}(r)}^{\bar{\epsilon}(r)} \{1 - 2F_r(x - e_H)\} f_r(x - e_H) f(x) dx \\
& = 2 \int_{-\bar{\epsilon}(r)-e_H}^{\bar{\epsilon}(r)-e_H} \{1 - 2F_r(y)\} f_r(y) f(y + e_H) dy.
\end{aligned} \tag{A.19}$$

When $\bar{\epsilon}(r) \leq e_H < 2\bar{\epsilon}(r)$, the sign of (A.19) is positive because $\{1 - 2F_r(y)\} f_r(y) f(y + e_H)$ is positive for any y such that $-\bar{\epsilon}(r) < y < \bar{\epsilon}(r) - e_H$, and is zero for any y such that $-\bar{\epsilon}(r) - e_H \leq y \leq -\bar{\epsilon}(r)$. When $0 \leq e_H < \bar{\epsilon}(r)$, we have

$$\begin{aligned}
& 2 \int_{-\bar{\epsilon}(r)-e_H}^{\bar{\epsilon}(r)-e_H} \{1 - 2F_r(y)\} f_r(y) f_r(y + e_H) dy \\
&= 2 \left[\int_0^{\bar{\epsilon}(r)-e_H} \{1 - 2F_r(y)\} f_r(y) f_r(y + e_H) dy + \int_{-\bar{\epsilon}(r)-e_H}^0 \{1 - 2F_r(y)\} f_r(y) f_r(y + e_H) dy \right].
\end{aligned}$$

Notice that $\{1 - 2F_r(y)\} f_r(y) f_r(y + e_H)$ is negative for any y such that $\bar{\epsilon}(r) - e_H \geq y > 0$, is positive for any y such that $-\bar{\epsilon}(r) \leq y < 0$, or is zero for any y such that $-\bar{\epsilon}(r) - e_H \leq y < -\bar{\epsilon}(r)$. Also notice that $f_r(y + e_H) \geq f_r(y - e_H)$ when $-\bar{\epsilon}(r) - e_H \leq y \leq 0$ because the distribution function is unimodal. Using these facts, we have:

$$\begin{aligned}
& 2 \left[\int_0^{\bar{\epsilon}(r)-e_H} \{1 - 2F_r(y)\} f_r(y) f_r(y + e_H) dy + \int_{-\bar{\epsilon}(r)-e_H}^0 \{1 - 2F_r(y)\} f_r(y) f_r(y + e_H) dy \right] \\
&> 2 \left[\int_0^{\bar{\epsilon}(r)-e_H} \{1 - 2F_r(y)\} f_r(y) f_r(y + e_H) dy + \int_{-\bar{\epsilon}(r)+e_H}^0 \{1 - 2F_r(y)\} f_r(y) f_r(y - e_H) dy \right] \\
&= 2 \int_0^{\bar{\epsilon}(r)-e_H} [\{1 - 2F_r(y)\} f_r(y) f_r(y + e_H) + \{1 - 2F_r(-y)\} f_r(-y) f_r(-y - e_H)] dy \\
&= 0. \quad (\text{By the symmetry of distribution.}) \quad \blacksquare
\end{aligned}$$

Proof of Lemma 3.5: (i) For any two approaches a_n and $a'_n \in [\underline{a}_n, \bar{a}_n]$ such that $a_n > a'_n$, we know that

$$\Pr(\tilde{x}_n(a_n) > \tilde{x}) > \Pr(\tilde{x}_n(a'_n) > \tilde{x})$$

by applying part (i) of Lemma 3.1. Hence, it is clear that \bar{a}_n is the only approach that maximizes $\Pr(\tilde{x}_n(a_n) > \tilde{x})$ and \underline{a}_n is the only approach that minimizes $\Pr(\tilde{x}_n(a_n) > \tilde{x})$ for all $a_n \in [\underline{a}_n, \bar{a}_n]$.

(ii) For any two different approaches a_n and a'_n , by applying part (ii) of Lemma 3.1, we have

$$\Pr(\tilde{x}_n(a_n) > \tilde{x}) = \Pr(\tilde{x}_n(a'_n) > \tilde{x})$$

Thus, $\Pr(\tilde{x}_n(a_n) > \tilde{x})$ has the same value for all $a_n \in [\underline{a}_n, \bar{a}_n]$.

(iii) For any two approaches a_n and $a'_n \in [\underline{a}_n, \bar{a}_n]$ such that $a_n > a'_n$, we have that

$$\Pr(\tilde{x}_n(a_n) > \tilde{x}) < \Pr(\tilde{x}_n(a'_n) > \tilde{x})$$

by applying part (iii) of Lemma 3.1. Obviously, \underline{a}_n is the only approach that maximizes $\Pr(\tilde{x}_n(a_n) > \tilde{x})$ and \bar{a}_n is the only approach that minimizes $\Pr(\tilde{x}_n(a_n) > \tilde{x})$ for all $a_n \in [\underline{a}_n, \bar{a}_n]$. ■

Proof of Proposition 3.4:

(i) Case of $m = 1$.

Let $\phi_{(n-1)}(x; a_1, a_2, \dots, a_{n-1})$ denote the density function of the $n - 1$ th order statistic $\tilde{x}_{(n-1)}(a_1, a_2, \dots, a_{n-1})$ of $n - 1$ agents' performances $x_1(a_1), x_2(a_2), \dots, x_{n-1}(a_{n-1})$. This density function is expressed in the following form:

$$\begin{aligned} & \phi_{(n-1)}(x; a_1, a_2, \dots, a_{n-1}) \\ &= \begin{cases} f_1(x; a_1), & \text{if } n = 2, \\ \sum_{i=1}^{n-1} f_i(x; a_i) [\prod_{j \neq i}^{n-1} F_j(x; a_j)], & \text{if } n \geq 3. \end{cases} \end{aligned} \quad (\text{A.20})$$

(a) If $n = 2$, then from Assumption 3.1' (symmetric distribution) we have:

$$\phi_{(1)}(x; a_1) - \phi_{(1)}(-x; a_1) = f_1(x; a_1) - f_1(-x; a_1) = 0.$$

Thus, by part (ii) of Lemma 3.5, I see that agent n (agent 2) gets the high reward with the same probability without regard to his approach choice. Numbering of agents is arbitrary. Hence, agent 1 also gets the high reward with the same probability without regard to his approach choice.

(b) If $n \geq 3$, then we will get

$$\begin{aligned} & \phi_{(n-1)}(x; a_1, a_2, \dots, a_{n-1}) - \phi_{(n-1)}(-x; a_1, a_2, \dots, a_{n-1}) \\ &= \sum_{i=1}^{n-1} f_i(x; a_i) [\prod_{j \neq i}^{n-1} F_j(x; a_j)] - \sum_{i=1}^{n-1} f_i(-x; a_i) [\prod_{j \neq i}^{n-1} F_j(-x; a_j)] \\ &= \sum_{i=1}^{n-1} f_i(x; a_i) [\prod_{j \neq i}^{n-1} F_j(x; a_j) - \prod_{j \neq i}^{n-1} F_j(-x; a_j)] \\ &= \sum_{i=1}^{n-1} f_i(x; a_i) [\prod_{j \neq i}^{n-1} F_j(x; a_j) - \prod_{j \neq i}^{n-1} \bar{F}_j(x; a_j)] \\ &\geq 0 \quad \text{for all } x > 0. \end{aligned} \quad (\text{A.21})$$

By Assumption 3.3', (A.21) is strictly positive for all $x \in \bar{I}$. Applying part (i) of Lemma 3.5, we see that the maximum risk approach \bar{a}_n is a strictly dominant strategy for agent n . Since numbering is arbitrary, \bar{a}_i is a strictly dominant strategy for every agent i .

(ii) Case of $m = 2$.

The density function of the $n - 2$ th order statistic $\tilde{x}_{(n-1)}(a_1, a_2, \dots, a_{n-1})$ of $n - 1$ agents' performances $x_1(a_1), x_2(a_2), \dots, x_{n-1}(a_{n-1})$ is given by

$$\begin{aligned} \phi_{(n-2)}(x; a_1, a_2, \dots, a_{n-1}) & \quad (A.22) \\ &= \begin{cases} f_1(x; a_1)\bar{F}_2(x; a_2) + f_2(x; a_2)\bar{F}_1(x; a_1), & \text{if } n = 3, \\ \sum_{i=1}^{n-1} \sum_{j \neq i}^{n-1} f_i(x; a_i)\bar{F}_j(x; a_j) \prod_k^{n-1} F_k(x; a_k), & \text{if } n \geq 4. \end{cases} \end{aligned}$$

(a) If $n = 3$, then

$$\begin{aligned} & \phi_{(1)}(x; a_1, a_2) - \phi_{(1)}(-x; a_1, a_2) \\ &= f_1(x; a_1)\bar{F}_2(x; a_2) + f_2(x; a_2)\bar{F}_1(x; a_1) - f_1(-x; a_1)\bar{F}_2(-x; a_2) + f_2(-x; a_2)\bar{F}_1(-x; a_1) \\ &= f_1(x; a_1)\bar{F}_2(x; a_2) + f_2(x; a_2)\bar{F}_1(x; a_1) - f_1(x; a_1)F_2(x; a_2) + f_2(x; a_2)F_1(x; a_1) \\ &= f_1(x; a_1)[\bar{F}_2(x; a_2) - F_2(x; a_2)] + f_2(x; a_2)[\bar{F}_1(x; a_1) - F_1(x; a_1)] \quad (A.23) \\ &\leq 0 \quad \text{for all } x > 0. \end{aligned}$$

From Assumption 3.3', we know that (A.23) is strictly negative for any $x \in \bar{I}$. Applying part of (iii) of Lemma 3.5, we immediately find that the \underline{a}_n is a strictly dominant strategy for agent n (agent 3). Because all agents are assumed to be essentially homogeneous, \underline{a}_i is a strictly dominant strategy for every agent i .

(b) If $n = 4$, I have

$$\begin{aligned}
& \phi_{(2)}(x; a_1, a_2, a_3) - \phi_{(2)}(-x; a_1, a_2, a_3) \\
&= \sum_{i=1}^3 \sum_{j \neq i}^3 f_i(x; a_i) \bar{F}_j(x; a_j) F_k(x; a_k) - \sum_{i=1}^3 \sum_{j \neq i}^3 f_i(-x; a_i) \bar{F}_j(-x; a_j) F_k(-x; a_k) \quad k \neq i, j. \\
&= \sum_{i=1}^3 \sum_{j \neq i}^3 f_i(x; a_i) \bar{F}_j(x; a_j) F_k(x; a_k) - \sum_{i=1}^3 \sum_{j \neq i}^3 f_i(x; a_i) F_j(x; a_j) \bar{F}_k(x; a_k) \quad k \neq i, j. \\
&= \sum_{i=1}^3 f_i(x; a_i) \left[\sum_{j \neq i}^3 \{ \bar{F}_j(x; a_j) F_k(x; a_k) - F_j(x; a_j) \bar{F}_k(x; a_k) \} \right] \quad k \neq i, j. \\
&= 0.
\end{aligned}$$

By part (ii) of Lemma 3.5, agent 4 will get the high reward with the same probability without regard to his choice. All agents are assumed to be essentially homogeneous. Hence, every agent also gets the high reward with the same probability without regard to his approach choice.

(c) If $n \geq 5$, then

$$\begin{aligned}
& \phi_{(n-2)}(x; a_1, a_2, a_3, \dots, a_{n-1}) - \phi_{(n-2)}(-x; a_1, a_2, a_3, \dots, a_{n-1}) \\
&= \sum_{i=1}^{n-1} \sum_{j \neq i}^{n-1} f_i(x; a_i) \bar{F}_j(x; a_j) \prod_{k \neq i, j}^{n-1} F_k(x; a_k) - \sum_{i=1}^{n-1} \sum_{j \neq i}^{n-1} f_i(-x; a_i) \bar{F}_j(-x; a_j) \prod_{k \neq i, j}^{n-1} F_k(-x; a_k) \\
&= \sum_{i=1}^{n-1} \sum_{j \neq i}^{n-1} f_i(x; a_i) \bar{F}_j(x; a_j) \prod_{k \neq i, j}^{n-1} F_k(x; a_k) - \sum_{i=1}^{n-1} \sum_{j \neq i}^{n-1} f_i(x; a_i) F_j(x; a_j) \prod_{k \neq i, j}^{n-1} \bar{F}_k(x; a_k) \\
&= \sum_{i=1}^{n-1} f_i(x; a_i) \left[\sum_{j \neq i}^{n-1} \bar{F}_j(x; a_j) \prod_{k \neq i, j}^{n-1} F_k(x; a_k) - \sum_{j \neq i}^{n-1} F_j(x; a_j) \prod_{k \neq i, j}^{n-1} \bar{F}_k(x; a_k) \right] \\
&= \sum_{i=1}^{n-1} f_i(x; a_i) \left[\sum_{j \neq i}^{n-1} \bar{F}_j(x; a_j) F_{k(-i, j+1)}(x; a_{k(-i, j+1)}) \right. \\
&\quad \left. \times \left\{ \prod_{l \neq i, j, k(-i, j+1)}^{n-1} F_l(x; a_l) - \prod_{l \neq i, j, k(-i, j+1)}^{n-1} \bar{F}_l(x; a_l) \right\} \right] \tag{A.24} \\
&\geq 0 \quad \text{for all } x > 0,
\end{aligned}$$

where $k(-i, j+1)$ is defined as the smallest integer of all integers from $j+1$ to $n-1$ except i , but if $j+1$ is larger than $n-1$ or $j+1 = i = n-1$, $k(-i, j+1)$ is the smallest positive

integer except i . That is,

$$k(-i, j+1) = \begin{cases} j+1, & \text{if } j+1 \neq n, i, \\ j+2, & \text{if } j+1 = i \text{ and } i < n-1, \\ 1, & \text{if } j+1 = n \text{ and } i > 1, \\ 1, & \text{if } j+1 = i = n-1, \\ 2, & \text{if } j+1 = n \text{ and } i = 1. \end{cases}$$

Since (A.24) is strictly positive for any $x \in \bar{I}$, the maximum risk approach \bar{a}_n is a strictly dominant strategy for agent n by part (i) of Lemma 3.5. Numbering of agents is arbitrary. Therefore, \bar{a}_i is a strictly dominant strategy for every agent i . ■

Proof of Proposition 3.5: Set $m = k$. Then by Assumption 3.1' (symmetry), we have:

$$\Pr(\tilde{x}_n(a_n) > \tilde{x}_{(n-k)}(a_1, a_2, \dots, a_{n-1})) = \Pr(\tilde{x}_n(a_n) < \tilde{x}_{(k)}(a_1, a_2, \dots, a_{n-1})) \quad (\text{A.25})$$

Substituting $\Pr(\tilde{x}_n(a_n) < \tilde{x}_{(k)}(a_1, a_2, \dots, a_{n-1})) = 1 - \Pr(\tilde{x}_n(a_n) > \tilde{x}_{(k)}(a_1, a_2, \dots, a_{n-1}))$ into (A.25), we get,

$$\Pr(\tilde{x}_{n-k}(a_n) > \tilde{x}_{(k)}(a_1, a_2, \dots, a_{n-1})) = 1 - \Pr(\tilde{x}_k(a_n) > \tilde{x}_{(n-k)}(a_1, a_2, \dots, a_{n-1})). \quad (\text{A.26})$$

From the equation (A.26), we find that the approach which maximizes (minimizes) the probability that agent n will get the high reward when $m = k$ minimizes (maximizes) the probability when $m = n - k$. Using this fact in addition to Proposition 3.4 and Lemma 3.5, we can straightforwardly prove this proposition.

(i) Case of $m = n - 1$.

(a) If $n = 2$, then part (i)-(a) of Proposition 3.5 is equivalent to part (i)-(a) of Proposition 3.4. Hence, all available approaches will be indifferent for every agent i .

(b) Using part (i) of Lemma 3.5, I already proved in part (i)-(b) of proposition 3.4 that approach \bar{a}_n is agent n 's only approach that maximizes his high reward probability if $m = 1$ and $n \geq 3$. It is also clear that \underline{a}_n is the only approach which minimizes his high reward probability if $m = 1$ and $n \geq 3$ by part of (i) of Lemma 3.5. Then from the fact explained above, we know that \underline{a}_n is the only approach which maximizes agent n 's high

reward probability if $m = n - 1$ and $n \geq 3$. That is to say, \underline{a}_n is a strictly dominant strategy for agent n if $m = n - 1$ and $n \geq 3$. Numbering of agents is arbitrary. Therefore, \underline{a}_i is a strictly dominant strategy for every agent i if $m = n - 1$ and $n \geq 3$.

(ii) The remains of this proposition can be easily proved using the same logic as part (i). The details of proof are omitted. ■

Proof of Proposition 3.6: (i) For any feasible approaches $a'_1 \neq \underline{a}_1$ and a_2 , we have

$$\begin{aligned}
& \Pr(\tilde{x}_1(\underline{a}_1) > \tilde{x}_2(a_2)) - \Pr(\tilde{x}_1(a'_1) > \tilde{x}_2(a_2)) \\
&= \int_{\mu_2 - \bar{\epsilon}_2(a_2)}^{\mu_2 + \bar{\epsilon}_2(a_2)} \{1 - F_1(x - \mu_1; \underline{a}_1)\} f_2(x - \mu_2; a_2) dx \\
&\quad - \int_{\mu_2 - \bar{\epsilon}_2(a_2)}^{\mu_2 + \bar{\epsilon}_2(a_2)} \{1 - F_1(x - \mu_1; a'_1)\} f_2(x - \mu_2; a_2) dx \\
&= \int_{\mu_2 - \bar{\epsilon}_2(a_2)}^{\mu_2 + \bar{\epsilon}_2(a_2)} \{F_1(x - \mu_1; a'_1) - F_1(x - \mu_1; \underline{a}_1)\} f_2(x - \mu_2; a_2) dx. \tag{A.27}
\end{aligned}$$

When $\mu_1 \geq \mu_2 + \bar{\epsilon}_2(a_2)$, $\{F_1(x - \mu_1; a'_1) - F_1(x - \mu_1; \underline{a}_1)\}$ is positive for any x such that $\mu_1 - \bar{\epsilon}_1(a'_1) < x < \mu_2 + \bar{\epsilon}_2(a_2)$ by Assumptions 3.2'' and 3.6, or zero for any $x < \mu_1 - \bar{\epsilon}_1(a'_1)$. Then it follows that (A.27) is positive. On the other hand, when $\mu_1 < \mu_2 + \bar{\epsilon}_2(a_2)$, (A.27) is arranged in the following form:

$$\begin{aligned}
& \int_{\mu_2 - \bar{\epsilon}_2(a_2)}^{\mu_2 + \bar{\epsilon}_2(a_2)} \{F_1(x - \mu_1; a'_1) - F_1(x - \mu_1; \underline{a}_1)\} f_2(x - \mu_2; a_2) dx \\
&= \int_{\mu_1}^{\mu_2 + \bar{\epsilon}_2(a_2)} \{F_1(x - \mu_1; a'_1) - F_1(x - \mu_1; \underline{a}_1)\} f_2(x - \mu_2; a_2) dx \\
&\quad + \int_{2\mu_1 - \mu_2 - \bar{\epsilon}_2(a_2)}^{\mu_1} \{F_1(x - \mu_1; a'_1) - F_1(x - \mu_1; \underline{a}_1)\} f_2(x - \mu_2; a_2) dx \\
&\quad + \int_{\mu_2 - \bar{\epsilon}_2(a_2)}^{2\mu_1 - \mu_2 - \bar{\epsilon}_2(a_2)} \{F_1(x - \mu_1; a'_1) - F_1(x - \mu_1; \underline{a}_1)\} f_2(x - \mu_2; a_2) dx \\
&= \int_{\mu_1}^{\mu_2 + \bar{\epsilon}_2(a_2)} \{F_1(x - \mu_1; a'_1) - F_1(x - \mu_1; \underline{a}_1)\} \{f_2(x - \mu_2; a_2) - f_2(2\mu_1 - \mu_2 - x; a_2)\} dx \\
&\quad + \int_{\mu_2 - \bar{\epsilon}_2(a_2)}^{2\mu_1 - \mu_2 - \bar{\epsilon}_2(a_2)} \{F_1(x - \mu_1; a'_1) - F_1(x - \mu_1; \underline{a}_1)\} f_2(x - \mu_2; a_2) dx \tag{A.28}
\end{aligned}$$

By Assumption 3.1' (symmetric distribution), we have the last equation (A.28). The first term of (A.28) is positive by Assumptions 3.1', 3.2'' (single crossing) and , 3.4' (strictly unimodal distribution). The second term is nonnegative by Assumption 3.2''. It follows

that (A.28) is positive. These results mean that agent 1 strictly prefers \underline{a}_1 to any $a'_1 \neq \underline{a}_1$ regardless of agent 2's strategy.

(ii) Since the procedure of the proof is the same as that of part (i), the proof is omitted. ■

Proof of Proposition 4.4: I prove this proposition using Lemma 3.1 in Chapter 3. Without loss of generality, suppose that agents $1, 2, \dots, n-1$ choose approach c . Let $\tilde{y}_{(n-m)}(c, n-1)$ be the $n-m$ th order statistic of relative performances of $n-1$ agents, $(\tilde{y}_1(c), \tilde{y}_2(c), \dots, \tilde{y}_{n-1}(c))$ where $\tilde{y}_i(a_i) = \tilde{x}_i - \tilde{\eta}$. Agent n can get the high reward when the realized value of his relative performance $\tilde{y}_n(a_n)$ is greater than that of $\tilde{y}_{(n-m)}(c, n-1)$. Which approach will agent n prefer in terms of his high reward probability, c or o ? The density function $\phi_{(n-m)}(y; c, n-1)$ is given by

$$\phi_{(n-m)}(y; c, n-1) = \frac{(n-1)!}{(m-1)!(n-m-1)!} [1 - F_\epsilon(y)]^{m-1} F_\epsilon(y)^{n-m-1} f_\epsilon(y).$$

Because the distribution function of $\tilde{\epsilon}_i$ is symmetric around zero and the density $f_\epsilon(y)$ is always positive, we can arrange $\phi_{(n-m)}(y; c, n-1) - \phi_{(n-m)}(-y; c, n-1)$ into the following form:

$$\begin{aligned} & \phi_{(n-m)}(y; c, n-1) - \phi_{(n-m)}(-y; c, n-1) \\ &= \frac{(n-1)! [1 - F_\epsilon(y)]^{m-1} F_\epsilon(y)^{m-1}}{(m-1)!(n-m-1)!} [F_\epsilon(y)^{n-2m} - \{1 - F_\epsilon(y)\}^{n-2m}] f_\epsilon(y). \end{aligned}$$

Clearly, it holds that

$$\phi_{(n-m)}(y; c, n-1) \begin{cases} \leq \\ \geq \end{cases} \phi_{(n-m)}(-y; c, n-1) \quad \text{according as } 2m \begin{cases} \geq \\ \leq \end{cases} n \text{ for any } y > 0. \quad (\text{A.29})$$

For the riskiness of agent n 's relative performance $\tilde{y}_n(a_n)$, approach o is riskier than approach c in the sense of a mean-preserving spread. Obviously the family of the distribution functions of $\tilde{y}_n(a_n)$ also satisfies Assumptions 3.1, 3.2 and 3.3. Thus Lemma 3.1 implies that if all his peers choose c , agent n 's best response strategy is (i) approach o when $2m < n$, (ii) any one when $2m = n$, and (iii) approach c when $2m > n$. Thus it follows that a strategy profile for which all agents choose approach c is a Nash equilibrium if $2m \geq n$, but not if $2m < n$. ■

Proof of Lemma 4.1: When $e_H = 0$, $D^{1L} - D^{1W} = 1/6$. On the other hand, when $e_H \geq 2\bar{\theta}$, $D^{1L} - D^{1W} = 1/3$. The result will follow by showing that $D^{1L} - D^{1W}$ is strictly

increasing in $e_H \in [0, 2\bar{\theta}]$:

$$\begin{aligned}
& \frac{d(D^{1L} - D^{1W})}{de_H} \\
&= \int_{-\bar{\theta}}^{\bar{\theta}} \{1 - 2F(x - e_H)\} f(x - e_H) f(x) dx \\
&= \int_{-\bar{\theta} - e_H}^{\bar{\theta} - e_H} \{1 - 2F(y)\} f(y) f(y + e_H) dy.
\end{aligned} \tag{A.30}$$

When $\bar{\theta} \leq e_H < 2\bar{\theta}$, the sign of (A.30) is positive because $\{1 - 2F(y)\} f(y) f(y + e_H)$ is positive for any y such that $-\bar{\theta} < y < \bar{\theta} - e_H$, and is zero for any y such that $-\bar{\theta} - e_H \leq y \leq -\bar{\theta}$.

When $0 \leq e_H < \bar{\theta}$, we have

$$\begin{aligned}
& \int_{-\bar{\theta} - e_H}^{\bar{\theta} - e_H} \{1 - 2F(y)\} f(y) f(y + e_H) dy \\
&= \left[\int_0^{\bar{\theta} - e_H} \{1 - 2F(y)\} f(y) f(y + e_H) dy + \int_{-\bar{\theta} - e_H}^0 \{1 - 2F(y)\} f(y) f(y + e_H) dy \right].
\end{aligned} \tag{A.31}$$

Notice that $\{1 - 2F(y)\} f(y) f(y + e_H)$ is negative for any y such that $\bar{\theta} - e_H \geq y > 0$, is positive for any y such that $-\bar{\theta} \leq y < 0$, or zero for any y such that $-\bar{\theta} - e_H \leq y < -\bar{\theta}$. Also notice that $f(y + e_H) \geq f(y - e_H)$ when $-\bar{\theta} - e_H \leq y \leq 0$ since the distribution function is unimodal. Using these facts, we have:

$$\begin{aligned}
& \left[\int_0^{\bar{\theta} - e_H} \{1 - 2F(y)\} f(y) f(y + e_H) dy + \int_{-\bar{\theta} - e_H}^0 \{1 - 2F(y)\} f(y) f(y + e_H) dy \right] \\
&> \left[\int_0^{\bar{\theta} - e_H} \{1 - 2F(y)\} f(y) f(y + e_H) dy + \int_{-\bar{\theta} + e_H}^0 \{1 - 2F(y)\} f(y) f(y - e_H) dy \right] \\
&= \int_0^{\bar{\theta} - e_H} [\{1 - 2F(y)\} f(y) f(y + e_H) + \{1 - 2F(-y)\} f(-y) f(-y - e_H)] dy \\
&= 0. \quad (\text{By the symmetry of distribution.}) \quad \blacksquare
\end{aligned}$$

Proof of Proposition 4.6: (i) Suppose that agent 2A chooses approach c . Then the winning probability of agent 1A who also chooses approach c is given by the following

(A.32):

$$\begin{aligned}
& P_{2A}\Pr(\tilde{x}_{1A}(c) > \tilde{x}_{2A}(c)) + P_{2B}\Pr(\tilde{x}_{1A}(c) > \tilde{x}_{2B}) \\
&= P_{2A}\Pr(\mu_1 + \tilde{\eta} + \tilde{\epsilon}_1 > \mu_2 + \tilde{\eta} + \tilde{\epsilon}_2) + P_{2B}\Pr(\mu_1 + \tilde{\eta} + \tilde{\epsilon}_1 > \mu_2 + \tilde{\eta} + \tilde{\epsilon}_2) \\
&= \Pr(\tilde{\epsilon}_1 - \tilde{\epsilon}_2 > \mu_2 - \mu_1) \\
&= P\left(\tilde{z} > -\frac{\mu_1 - \mu_2}{\sqrt{2\sigma_\epsilon^2}}\right) \tag{A.32}
\end{aligned}$$

where \tilde{z} is a standardized normal random variable with mean 0 and variance 1. On the other hand, the winning probability of agent 1A who chooses approach o is given by the following (A.33):

$$\begin{aligned}
& P_{2A}\Pr(\tilde{x}_{1A}(o) > \tilde{x}_{2A}(c)) + P_{2B}\Pr(\tilde{x}_{1A}(o) > \tilde{x}_{2B}) \\
&= P_{2A}\Pr(\mu_1 + \tilde{\theta}_1 + \tilde{\epsilon}_1 > \mu_2 + \tilde{\eta} + \tilde{\epsilon}_2) + P_{2B}\Pr(\mu_1 + \tilde{\theta}_1 + \tilde{\epsilon}_1 > \mu_2 + \tilde{\eta} + \tilde{\epsilon}_2) \\
&= \Pr(\tilde{\theta}_1 + \tilde{\epsilon}_1 - \tilde{\eta} - \tilde{\epsilon}_2 > \mu_2 - \mu_1) \\
&= P\left(\tilde{z} > -\frac{\mu_1 - \mu_2}{\sqrt{2(\sigma_a^2 + \sigma_\epsilon^2)}}\right) \tag{A.33}
\end{aligned}$$

Because the probability (A.32) is greater than the probability (A.33), agent 1A's best response to agent 2A's choice c is c .

Next, suppose that agent 2A chooses approach o . Then the winning probability of agent 1A who chooses approach c is arranged into the following expression (A.34):

$$\begin{aligned}
& P_{2A}\Pr(\tilde{x}_{1A}(c) > \tilde{x}_{2A}(o)) + P_{2B}\Pr(\tilde{x}_{1A}(c) > \tilde{x}_{2B}) \\
&= P_{2A}\Pr(\mu_1 + \tilde{\eta} + \tilde{\epsilon}_1 > \mu_2 + \tilde{\theta}_2 + \tilde{\epsilon}_2) + P_{2B}\Pr(\mu_1 + \tilde{\eta} + \tilde{\epsilon}_1 > \mu_2 + \tilde{\eta} + \tilde{\epsilon}_2) \\
&= P_{2A}\Pr(\tilde{\eta} + \tilde{\epsilon}_1 - \tilde{\theta}_2 - \tilde{\epsilon}_2 > \mu_2 - \mu_1) + P_{2B}\Pr(\tilde{\epsilon}_1 - \tilde{\epsilon}_2 > \mu_2 - \mu_1) \\
&= P_{2A}P\left(\tilde{z} > -\frac{\mu_1 - \mu_2}{\sqrt{2(\sigma_a^2 + \sigma_\epsilon^2)}}\right) + P_{2B}P\left(\tilde{z} > -\frac{\mu_1 - \mu_2}{\sqrt{2\sigma_\epsilon^2}}\right). \tag{A.34}
\end{aligned}$$

On the other hand, the winning probability of agent 1A who chooses approach o is arranged

into the following expression (A.35):

$$\begin{aligned}
& P_{2A}\Pr(\tilde{x}_{1A}(o) > \tilde{x}_{2A}(o)) + P_{2B}\Pr(\tilde{x}_{1A}(o) > \tilde{x}_{2B}) \\
&= P_{2A}\Pr(\mu_1 + \tilde{\theta}_1 + \tilde{\epsilon}_1 > \mu_2 + \tilde{\theta}_2 + \tilde{\epsilon}_2) + P_{2B}\Pr(\mu_1 + \tilde{\theta}_1 + \tilde{\epsilon}_1 > \mu_2 + \tilde{\eta} + \tilde{\epsilon}_2) \\
&= P_{2A}\Pr(\tilde{\theta}_1 + \tilde{\epsilon}_1 - \tilde{\theta}_2 - \tilde{\epsilon}_2 > \mu_2 - \mu_1) + P_{2B}\Pr(\tilde{\theta}_1 + \tilde{\epsilon}_1 - \tilde{\eta} - \tilde{\epsilon}_2 > \mu_2 - \mu_1) \\
&= P_{2A}P\left(\tilde{z} > -\frac{\mu_1 - \mu_2}{\sqrt{2(\sigma_a^2 + \sigma_\epsilon^2)}}\right) + P_{2B}P\left(\tilde{z} > -\frac{\mu_1 - \mu_2}{\sqrt{2(\sigma_a^2 + \sigma_\epsilon^2)}}\right) \\
&= P\left(\tilde{z} > -\frac{\mu_1 - \mu_2}{\sqrt{2(\sigma_a^2 + \sigma_\epsilon^2)}}\right). \tag{A.35}
\end{aligned}$$

Since probability (A.34) is bigger than probability (A.35), agent 1A's best response to agent 2A's choice o is c . Thus approach c is always agent 1A's best response without regard to agent 2A's strategy: The common approach c is a strictly dominant strategy for agent 1A.

(ii) Suppose that agent 1A chooses approach c . Then the winning probability of agent 2A who also chooses approach c is given by the following (A.36):

$$\begin{aligned}
& P_{1A}\Pr(\tilde{x}_{2A}(c) > \tilde{x}_{1A}(c)) + P_{1B}\Pr(\tilde{x}_{2A}(c) > \tilde{x}_{1B}) \\
&= P_{1A}\Pr(\mu_2 + \tilde{\eta} + \tilde{\epsilon}_2 > \mu_1 + \tilde{\eta} + \tilde{\epsilon}_1) + P_{1B}\Pr(\mu_2 + \tilde{\eta} + \tilde{\epsilon}_2 > \mu_1 + \tilde{\eta} + \tilde{\epsilon}_1) \\
&= \Pr(\tilde{\epsilon}_2 - \tilde{\epsilon}_1 > \mu_1 - \mu_2) \\
&= P\left(\tilde{z} > \frac{\mu_1 - \mu_2}{\sqrt{2\sigma_\epsilon^2}}\right). \tag{A.36}
\end{aligned}$$

On the other hand, the winning probability of agent 2A who chooses approach o is given by the following (A.37)

$$\begin{aligned}
& P_{1A}\Pr(\tilde{x}_{2A}(o) > \tilde{x}_{1A}(c)) + P_{1B}\Pr(\tilde{x}_{2A}(o) > \tilde{x}_{1B}) \\
&= P_{1A}\Pr(\mu_2 + \tilde{\theta}_2 + \tilde{\epsilon}_2 > \mu_1 + \tilde{\eta} + \tilde{\epsilon}_1) + P_{1B}\Pr(\mu_2 + \tilde{\theta}_2 + \tilde{\epsilon}_2 > \mu_1 + \tilde{\eta} + \tilde{\epsilon}_1) \\
&= \Pr(\tilde{\theta}_2 + \tilde{\epsilon}_2 - \tilde{\eta} - \tilde{\epsilon}_1 > \mu_1 - \mu_2) \\
&= P\left(\tilde{z} > \frac{\mu_1 - \mu_2}{\sqrt{2(\sigma_a^2 + \sigma_\epsilon^2)}}\right). \tag{A.37}
\end{aligned}$$

Because probability (A.37) is strictly greater than probability (A.36), agent 2A's best response to agent 1A's choice c is o .

Next, suppose that the agent 1A chooses approach o . Then the winning probability of agent 2A who chooses approach c is arranged into the following expression (A.38):

$$\begin{aligned}
& P_{1A}\Pr(\tilde{x}_{2A}(c) > \tilde{x}_{1A}(o)) + P_{1B}\Pr(\tilde{x}_{2A}(c) > \tilde{x}_{1B}) \\
&= P_{1A}\Pr(\mu_2 + \tilde{\eta} + \tilde{\epsilon}_2 > \mu_1 + \tilde{\theta}_1 + \tilde{\epsilon}_1) + P_{1B}\Pr(\mu_2 + \tilde{\eta} + \tilde{\epsilon}_2 > \mu_1 + \tilde{\eta} + \tilde{\epsilon}_1) \\
&= P_{1A}\Pr(\tilde{\eta} + \tilde{\epsilon}_2 - \tilde{\theta}_1 - \tilde{\epsilon}_1 > \mu_1 - \mu_2) + P_{1B}\Pr(\tilde{\epsilon}_2 - \tilde{\epsilon}_1 > \mu_1 - \mu_2) \\
&= P_{1A}P\left(\tilde{z} > \frac{\mu_1 - \mu_2}{\sqrt{2(\sigma_a^2 + \sigma_\epsilon^2)}}\right) + P_{1B}P\left(\tilde{z} > \frac{\mu_1 - \mu_2}{\sqrt{2\sigma_\epsilon^2}}\right). \tag{A.38}
\end{aligned}$$

On the other hand, the winning probability of agent 2A who chooses approach o is arranged into the following expression (A.39):

$$\begin{aligned}
& P_{1A}\Pr(\tilde{x}_{2A}(o) > \tilde{x}_{1A}(o)) + P_{1B}\Pr(\tilde{x}_{2A}(o) > \tilde{x}_{1B}) \\
&= P_{1A}\Pr(\mu_2 + \tilde{\theta}_2 + \tilde{\epsilon}_2 > \mu_1 + \tilde{\theta}_1 + \tilde{\epsilon}_1) + P_{1B}\Pr(\mu_2 + \tilde{\theta}_2 + \tilde{\epsilon}_2 > \mu_1 + \tilde{\eta} + \tilde{\epsilon}_1) \\
&= P_{1A}\Pr(\tilde{\theta}_2 + \tilde{\epsilon}_2 - \tilde{\theta}_1 - \tilde{\epsilon}_1 > \mu_1 - \mu_2) + P_{1B}\Pr(\tilde{\theta}_2 + \tilde{\epsilon}_2 - \tilde{\eta} - \tilde{\epsilon}_1 > \mu_1 - \mu_2) \\
&= P_{1A}P\left(\tilde{z} > \frac{\mu_1 - \mu_2}{\sqrt{2(\sigma_a^2 + \sigma_\epsilon^2)}}\right) + P_{1B}P\left(\tilde{z} > \frac{\mu_1 - \mu_2}{\sqrt{2(\sigma_a^2 + \sigma_\epsilon^2)}}\right) \\
&= P\left(\tilde{z} > \frac{\mu_1 - \mu_2}{\sqrt{2(\sigma_a^2 + \sigma_\epsilon^2)}}\right). \tag{A.39}
\end{aligned}$$

Since probability (A.39) is bigger than probability (A.38), agent 2A's best response to agent 1A's choice o is o . Thus approach o is always agent 2A's best response without regard to agent 1A's strategy: His own original approach o is a strictly dominant strategy for agent 2A. ■

Proof of proposition 4.7: (i) Consider the three-agent-one-winner tournament. Without loss of generality, suppose that agent 1 and agent 2 choose the same approach. Then if agent 3 also chooses the same approach, his winning probability is 1/3. If agent 3 choose

the different approach, his winning probability is greater than $1/3$ because the probability is the same value as $P_W^o(1, 3, 2)$ in section 4.4 (see (4.18)). Hence, any strategy profile for which all three agents choose the same approach is not a Nash equilibrium in the three-agent-one-winner tournament. Next, suppose that agent 1 and agent 2 choose a different approach from each other. If agent 3 also chooses the different approach, his winning probability is $1/3$. If he chooses the same approach as one of his peers, the winning probability is less than $1/3$ because the winning probability is the same value as $P_W^c(1, 3, 2)$ in section 4.4 (see (4.18)). Hence, any strategy profile for which just two agents choose the same approach is not a Nash equilibrium, but any strategy profile for which three agents choose a different approach from one another is a Nash equilibrium in the three-agent-one-winner tournament. Thus, all three agents choose a different approach from one another in any Nash equilibrium of the three-agent-one-winner tournament.

(ii) Consider the three-agent-two-winner tournament. Without loss of generality, suppose that agent 1 and agent 2 choose a different approach from each other. Then if agent 3 also chooses a different approach, his winning probability is $2/3$. If agent 3 chooses the same approach as one peer, his winning probability is greater than $2/3$ because the probability is the same value as $P_W^c(2, 3, 2)$ in section 4.4 (see (4.22)). Hence, any strategy profile for which all three agents choose a different approach from one another is not a Nash equilibrium in the three-agent-two-winner tournament. Next, suppose that agent 1 and agent 2 choose the same approach. If agent 3 also chooses the same approach, his winning probability is $2/3$. If he choose a different approach, his winning probability is less than $2/3$ because the probability is the same value as $P_W^o(2, 3, 2)$ in section 4.4 (see (4.22)). Hence, any strategy profile for which just two agents choose the same approach is not a Nash equilibrium, but any strategy profile for which three agents choose the same approach is Nash equilibrium in the three-agent-two-winner tournament. Thus, all three agents choose the same approach in any Nash equilibrium of the three-agent-two-winner tournament. ■

Proof of Proposition 4.9: Without loss of generality, suppose that agents $2, 3, \dots, n$ choose different approaches; say, approaches c_2, c_3, \dots, c_n , respectively. If agent 1 chooses a different approach than any other agents choose, say approach c_1 , then the probability that he will

get the high reward is obviously m/n .

Now suppose that agent 1 chooses the same approach that another agent chooses, say approach c_2 . Then we have

$$\tilde{x}_1(c_2) = \tilde{\eta}_2 + \tilde{\epsilon}_1,$$

$$\tilde{x}_2(c_2) = \tilde{\eta}_2 + \tilde{\epsilon}_1,$$

$$\tilde{x}_i(c_i) = \tilde{\eta}_i + \tilde{\epsilon}_i, \quad i = 3, 4, \dots, n.$$

By Assumption 4.2, the random variable $\tilde{x}_i(c_j)$ is normally distributed with zero mean and variance $\sigma_a^2 + \sigma_\epsilon^2$. I denote the distribution function and the density function by $G(\cdot)$ and $g(\cdot)$, respectively.

Suppose that agent 1's realized outcome is given by x . Then the conditional probability that the realized outcome of agent i other than agent 1 and 2 will be equal or less than x is given by $G(x)$ because of the independence among random variables; *i.e.*, for $i = 3, \dots, n$,

$$\Pr(\tilde{x}_i(c_i) \leq x | x_1(c_2) = x) = \Pr(\tilde{x}_i(c_i) \leq x) = G(x) = P\left(\tilde{z} \leq \frac{x}{\sqrt{\sigma_a^2 + \sigma_\epsilon^2}}\right). \quad (\text{A.40})$$

On the other hand, $\tilde{x}_2(c_2)|_{x_1(c_2)=x}$, agent 2's performance given $x_1(c_2) = x$, is normally distributed with mean $(\sigma_a^2/(\sigma_a^2 + \sigma_\epsilon^2))x$ and variance $\sigma_\epsilon^2(2\sigma_a^2 + \sigma_\epsilon^2)/(\sigma_a^2 + \sigma_\epsilon^2)$. Then we have:

$$\Pr(\tilde{x}_2(c_2) \leq x | x_1(c_2) = x) = P\left(\tilde{z} \leq \frac{\sqrt{\frac{\sigma_\epsilon^2}{2\sigma_a^2 + \sigma_\epsilon^2}}}{\sqrt{\sigma_a^2 + \sigma_\epsilon^2}}x\right). \quad (\text{A.41})$$

By comparing (A.40) with (A.41), we have

$$G(x) \underset{\leq}{\overset{\geq}{\geq}} \Pr(\tilde{x}_2(c_2) \leq x | x_1(c_2) = x) \text{ according as } x \underset{\leq}{\overset{\geq}{\geq}} 0.$$

Moreover, it holds:

$$\Pr(\tilde{x}_2(c_2) \leq x | x_1(c_2) = x) = 1 - \Pr(\tilde{x}_2(c_2) \leq -x | x_1(c_2) = -x).$$

It is very convenient to define that $J(x) \equiv G(x) - \Pr(\tilde{x}_2(c_2) \leq x | x_1(c_2) = x)$. Then,

$$J(x) \underset{\leq}{\overset{\geq}{\geq}} 0 \text{ according as } x \underset{\leq}{\overset{\geq}{\geq}} 0, \text{ and } J(x) = -J(-x). \quad (\text{A.42})$$

First, I calculate the conditional probability that agent 1 will get the high reward given that his outcome is x , and next I derive his high reward probability. Given that $x_1(c_2) = x$, the conditional probability that he will get the first place is given by

$$\Pr(\tilde{x}_2(c_2) \leq x | x_1(c_2) = x) G(x)^{n-2} = \{G(x) - J(x)\} G(x)^{n-2}.$$

The conditional probability that he will get the second place is given by

$$C_1^{n-1} \{1 - G(x)\} \{G(x) - J(x)\} G(x)^{n-3} + [1 - \{G(x) - J(x)\}] G(x)^{n-2}.$$

Similarly, the conditional probability that he will get the m th place is given by

$$\begin{aligned} & C_{m-1}^{n-2} \{1 - G(x)\}^{m-1} \{G(x) - J(x)\} G(x)^{n-m-1} \\ & + C_{m-2}^{n-2} \{1 - G(x)\}^{m-2} [1 - \{G(x) - J(x)\}] G(x)^{n-m}. \end{aligned}$$

By adding these conditional probabilities from the first place until m th place, we have the conditional probability that agent 1 will get the high reward, given that he chooses approach c_2 and his realized outcome is x , which is arranged in the following expression (A.43):

$$\begin{aligned} & \sum_{k=0}^{m-1} C_k^{n-2} \{1 - G(x)\}^k \{G(x) - J(x)\} G(x)^{n-k-2} \\ & + \sum_{k=0}^{m-2} C_k^{n-2} \{1 - G(x)\}^k [1 - \{G(x) - J(x)\}] G(x)^{n-k-2}, \\ & = \sum_{k=0}^{m-1} C_k^{n-1} \{1 - G(x)\}^k G(x)^{n-k-1} \\ & - C_{m-1}^{n-2} \{1 - G(x)\}^{m-1} G(x)^{n-m-1} J(x). \end{aligned} \tag{A.43}$$

where C_k^n is the number of combinations of n objects taken k at a time:

$$C_k^n = \binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

Note that the first term in the right hand side of (A.43) is the same as the conditional probability that agent 1 will get the high reward given that he chooses a different approach (here, approach c_1) than any other agents choose and that his realized outcome is x .³

³If he chose approach c_1 , then all agents' outcomes were independently and identically distributed with the distribution function $G(\cdot)$. Given that agent 1's realized outcome is x , the conditional probability that

Using (A.43), we have the probability of his high reward when he chooses approach c_2 , $P^H(\tilde{x}_1(c_2))$:

$$\begin{aligned} P^H(\tilde{x}_1(c_2)) &= \int_{-\infty}^{+\infty} \sum_{k=0}^{m-1} C_k^{n-1} \{1 - G(x)\}^k G(x)^{n-k-1} g(x) dx \\ &\quad - C_{m-1}^{n-2} \int_{-\infty}^{+\infty} \{1 - G(x)\}^{m-1} G(x)^{n-m-1} J(x) g(x) dx. \end{aligned} \quad (\text{A.44})$$

Since the first term in the right hand side of (A.44) is the same as the probability that agent 1 would get the high reward if he chose a different approach (here, approach c_1) than any other agents chose, it is obviously m/n . Furthermore, (A.44) is arranged into the following form from the symmetry of the distributions around zero:

$$\begin{aligned} P^H(\tilde{x}_1(c_2)) &= \frac{m}{n} - C_{m-1}^{n-2} \int_{-\infty}^{+\infty} \{1 - G(x)\}^{m-1} G(x)^{n-m-1} J(x) g(x) dx \\ &= \frac{m}{n} - C_{m-1}^{n-2} \\ &\quad \times \int_0^{+\infty} [\{1 - G(x)\}^{m-1} G(x)^{n-m-1} - G(x)^{m-1} \{1 - G(x)\}^{n-m-1}] J(x) g(x) dx \\ &= \frac{m}{n} - C_{m-1}^{n-2} \\ &\quad \times \int_0^{+\infty} \{1 - G(x)\}^{m-1} G(x)^{m-1} [G(x)^{n-2m} - \{1 - G(x)\}^{n-2m}] J(x) g(x) dx \end{aligned} \quad (\text{A.45})$$

For any $x > 0$, It holds that

$$G(x)^{n-2m} - \{1 - G(x)\}^{n-2m} \gtrless 0 \text{ according as } 2m \lessgtr n.$$

he would get the first place is given by

$$G(x)^{n-1}.$$

The conditional probability that he would get the second place is given by

$$C_1^{n-1} \{1 - G(x)\} G(x)^{n-2}.$$

Similarly, the conditional probability that he would get the m th place is given by

$$C_{m-1}^{n-1} \{1 - G(x)\}^{m-1} G(x)^{n-m}.$$

By adding these conditional probabilities from the first place until m th place, we can get the conditional probability that agent 1 will get the high reward, given that he chooses approach c_1 and his realized outcome is x :

$$\sum_{k=0}^{m-1} C_k^{n-1} \{1 - G(x)\}^k G(x)^{n-k-1},$$

which is the same as the first term in the right hand side of (A.43).

Besides, since $J(x)$ and $G(x)$ are nonnegative for any $x > 0$, the second term in the right hand side of (A.45) is positive if $2m < n$, zero if $2m = n$, or negative if $2m > n$. Thus it follows that

$$P^H(\tilde{x}_1(c_2)) \begin{matrix} \geq \\ \leq \end{matrix} \frac{m}{n} \text{ according as } 2m \begin{matrix} \geq \\ \leq \end{matrix} n,$$

which implies that

- (i) if $2m = n$, agent 1 is indifferent between approach c_1 and approach c_2 ,
- (ii) if $2m < n$, he prefers approach c_1 to approach c_2 ,
- (iii) if $2m > n$, he prefers approach c_2 to approach c_1 .

That is to say, he has any positive incentive to choose the same approach that another agent chooses if $2m \leq n$, but has if $2m > n$. Numbering of agents is arbitrary. It follows that a strategy profile for which all agents choose a different approach from one another is a Nash equilibrium if $2m \leq n$, but not if $2m > n$. ■

Proof of Proposition 4.10: I divide the set of the Nash equilibria in the model into the following two subsets:

- (i) Subset 1: The set of Nash equilibria in which the size of each clusters is equal to or more than m .
- (ii) Subset 2: The set of Nash equilibria in which at least one cluster has the size less than m .

We start from the former subset.

- (i) Subset 1

For the member of this subset, d is equal to or less than q_m . Furthermore, it is clear that the smaller the size of a cluster is, the higher the probability that agents of the cluster get high reward. Then it can be shown that the definition of Nash equilibrium that every agent does not have an incentive to deviate to any other strategies is translated into the following two conditions in this case (the full detail of the proof is omitted):

Condition 1: Differences in size between any two clusters are one or zero.

Condition 2: No agent can increase his high reward probability by switching to another approach which no one chooses.

By *condition 1*, if $\text{mod}(d, n) = 0$, then each size of cluster must be the same in the equilibrium. To begin with, we treat this case.

(a) The case that $\text{mod}(d, n) = 0$

In this case, every size of cluster is n/d . If $n/d = m$, then the probability that each agent gets the high reward is $1/d = m/n$. Even if an agent deviates to another approach which no one chooses, then his high reward probability will be also $1/(d+1) + \{1/(d+1)\} \times 1/d = 1/d = m/n$. Therefore, this combination of agents' strategies is a Nash equilibrium. Then $d = n/m = q_m$.

On the other hand, if $n/d > m$, the probability that each agent gets high reward is $(1/d) \times m/(n/d) = m/n$. If an agent deviates to another approach which no one chooses, then the probability that he gets high reward will be $1/(d+1)$. Hence, this combination will be a Nash equilibrium if and only if

$$\frac{m}{n} \geq \frac{1}{d+1}. \quad (\text{A.46})$$

Substituting $n = q_m m + r_m$ into (A.46), we have

$$q_m - 1 + \frac{r_m}{m} \leq d. \quad (\text{A.47})$$

Note that when each cluster has the size equal to or more than m , d is equal to or less than q_m . If $r_m = \text{mod}(m, n) \neq 0$, then $0 < r_m/m < 1$. For this case, (A.47) implies that this type of a strategy profile is a Nash equilibrium only if $d = q_m$. On the other hand, if $r_m = \text{mod}(m, n) = 0$, it will be a Nash equilibrium only if $d = q_m$ or $d = q_m - 1$. Note that $q_m - 1$ must be equal to or greater than two in a Nash equilibrium because there exists no Nash equilibrium with $d = 1$ under the winner-selecting tournament by the proposition 4.8'.

(b) The case that $\text{mod}(d, n) \neq 0$

I show that there is no Nash equilibrium with $d \leq q_m - 1$ in this case. Let q_d and r_d denote the quotient given by dividing n by d and its residue, respectively. Then *condition 1* implies that if a strategy profile is a Nash equilibrium, it will be composed of r_d clusters with size

$q_d + 1$ and $d - r_d$ clusters with size q_d . Now consider an agent who chooses a cluster with size $q_d + 1$. He will have a probability of $m/[d(q_d - 1)]$ of getting a high reward. If he deviates to another approach which no one chooses, then his high reward probability will be $1/(d + 1)$. Then the condition for him not to deviate is

$$\frac{m}{d(q_d - 1)} \geq \frac{1}{d + 1}.$$

Rearranging it by using $n = q_d d + r_d$, we have

$$\frac{n}{m} + \frac{d - r_d}{m} \leq d + 1. \quad (\text{A.48})$$

Since the second term in the left side is always positive, the following condition is necessary for (A.48) to hold.

$$\frac{n}{m} < d + 1.$$

Substituting $n = q_m m + r_m$ into this, we have

$$q_m - 1 + \frac{r_m}{m} < d. \quad (\text{A.49})$$

Clearly, there is no $d \leq q_m - 1$ satisfying (A.49). Therefore, there is no Nash equilibrium with $d \leq q_m - 1$ in this case.

(ii) Subset 2

I show by contradiction that there is no Nash equilibrium with $d \leq q_m - 1$ in this case. Suppose that such a Nash equilibrium exists. Then note that there is at least one cluster whose size is bigger than m . Obviously an agent who chooses an approach whose size is equal to or less than m gets high reward with probability equal to or more than $1/d$. On the other hand, an agent who chooses an approach whose size is more than m could get high reward with probability equal to or more than $1/d$ by deviating to another cluster whose size is less than m . Hence, for this type of strategy profile to be a Nash equilibrium, it is necessary that his high reward probability be at least $1/d$. Then the sum of the probability that each agent gets high reward is at least n/d . $n/d \geq n/(q_m - 1) > m$. However, this sum must always be equal to m , which is the contradiction. Therefore, there is no Nash equilibrium for which $d \leq q_m - 1$ in this case. ■

Proof of Proposition 4.11: I prove this proposition by contradiction. Suppose that such a Nash equilibrium exists. Every agent who chooses any cluster other than the biggest size

cluster will get high reward with probability $1/2$ because he will get high reward if and only if his outcome is larger than that of the biggest cluster. Let n_r be the number of agents who do not choose the biggest cluster. Suppose that an agent deviates to the biggest cluster. We have his high reward probability P^H from the fact that the sum of all agents' high reward probabilities is always m :

$$\begin{aligned} P^H &= \frac{m - \frac{1}{2}(n_r - 1)}{n - n_r + 1} \\ &= \frac{1}{2} + \frac{2m - n}{2(n - n_r + 1)} \\ &> \frac{1}{2}. \end{aligned}$$

He can increase his high reward probability by deviating from his equilibrium strategy! This contradicts the definition of Nash equilibrium. Therefore, such a Nash equilibrium does not exist. ■

Proof of Proposition 4.12: I divide the set of feasible strategy profiles into the following three subsets:

- (i) Subset 1: the set of strategy profiles for which the size of each cluster is more than $n - m$.
- (ii) Subset 2: the set of strategy profiles for which every cluster has the same size, $n - m$.
- (iii) Subset 3: the set of strategy profiles which belong to either subset 1 or subset 2.

I prove the proposition in this order.

- (i) Subset 1.

Suppose that a Nash equilibrium exists in this subset. However, there clearly exists some agent who can decrease his low reward probability by deviating to the biggest cluster, which contradicts the definition of Nash equilibrium. Therefore, such a Nash equilibrium does not exist.

- (ii) Subset 2.

If $n - m = 1$, then any strategy profile in this subset is not a Nash equilibrium by proposition 4.9'. Consider the case of $n - m \geq 2$. Suppose that a Nash equilibrium exists in this subset.

Then the probability that each agent gets low reward is obviously $1/d$. If an agent deviates to another cluster, then his low reward probability P^L is given by:

$$\begin{aligned} P^L &= \frac{1}{d} \cdot \frac{n-m}{n-m+1} + \frac{1}{d} \cdot \frac{1}{d-1} \cdot \frac{1}{n-m+1} \\ &= \frac{1}{d} \cdot \frac{n-m + \frac{1}{d-1}}{n-m+1} \\ &< \frac{1}{d} \quad (\text{by } d > 2.) \end{aligned}$$

which implies that he can strictly decrease his low reward probability by deviating from his equilibrium strategy. This contradicts the definition of Nash equilibrium. Hence, such a Nash equilibrium does not exist.

(iii) Subset 3.

Suppose that there exists a Nash equilibrium in this case. Let $n-m+a$ ($a \geq 1$) be the biggest size in all clusters. Note that every agent who chooses a clusters with the size $n-m$ gets low reward with probability $1/d$. If he deviates to the biggest cluster, the probability that he will get low reward P^L is given by:

$$\begin{aligned} \text{If } m = n-1, \text{ then } P^L &= \frac{1}{d-1} \cdot \frac{1}{1+a+1} < \frac{1}{d}. \\ \text{If } \frac{n}{2} < m \leq n-2, \text{ then } P &= \frac{1}{d} \cdot \frac{n-m}{n-m+a+1} + \frac{1}{d} \cdot \frac{1}{d-1} \cdot \frac{1}{n-m+a+1} \\ &= \frac{1}{d} \cdot \frac{(n-m)(d-1) + 1}{(n-m)(d-1) + (a+1)(d-1)} \\ &\leq \frac{1}{d} \cdot \frac{(n-m)(d-1) + 1}{(n-m)(d-1) + a+1} \\ &< \frac{1}{d}. \end{aligned}$$

Clearly, he can strictly decrease his low reward probability by deviating to the biggest cluster, which contradicts the definition of Nash equilibrium. Therefore, such a Nash equilibrium does not exist. ■

Proof of Lemma 5.1: If y satisfies $\underline{\epsilon} < y < \bar{\epsilon} - \delta$, we have

$$\begin{aligned} \frac{d(F(y+\delta)/F(y))}{dx} &= \frac{F(y)f(y+\delta) - f(y)F(y+\delta)}{F(y)^2} \\ &= \frac{F(y)F(y+\delta)\{f(y+\delta)/F(y+\delta) - f(y)/F(y)\}}{F(y)^2} < 0 \end{aligned}$$

because the numerator of the last equation is negative by F 's log-concavity (Assumption 5.2) and the denominator is positive.

If y satisfies $\bar{\epsilon} - \delta \leq y < \bar{\epsilon}$, then

$$\begin{aligned} \frac{d(F(y + \delta)/F(y))}{dy} &= \frac{d(1/F(y))}{dy} \\ &= -\frac{f(y)}{F(y)^2} < 0. \end{aligned}$$

Hence, $F(y + \delta)/F(y)$ is a strictly increasing function of y on $(\underline{\epsilon}, \bar{\epsilon})$

If y is equal to or greater than $\bar{\epsilon}$, then $F(y + \delta)/F(y) = 1$ because $F(y + \delta) = F(y) = 1$.

Last, because both $F(y + \delta)$ and $F(y)$ are positive and continuous in y on $(\underline{\epsilon}, +\infty)$, $F(y + \delta)/F(y)$ is also continuous in y on $(\underline{\epsilon}, +\infty)$. ■

Proof of Proposition 5.1:

(i) In the minimization problem [P2], the domain S_n is a compact and convex set of \mathbf{R}^n . Besides, the objective function $\max(b_1(\mu_1, \mathbf{s}), b_2(\mu_2, \mathbf{s}), \dots, b_{n-1}(\mu_{n-1}, \mathbf{s}))$ is obviously continuous on S_n . It follows that by Weierstrass's theorem (or the extreme value theorem) the solution set $B_n^{(2)}(\boldsymbol{\mu}, \mathbf{s}_{-n})$ is not empty. Let $M(\boldsymbol{\mu}, S_{-n})$ be the value function of [P2], that is, $M(\boldsymbol{\mu}, S_{-n}) = \min\{\max(b_1(\mu_1, \mathbf{s}), b_2(\mu_2, \mathbf{s}), \dots, b_{n-1}(\mu_{n-1}, \mathbf{s})) : \mathbf{s}_n \in S_n\}$. Then the solution $\mathbf{s}_n = (s_{n1}, s_{n2}, \dots, s_{nn}) \in B_n^{(2)}(\boldsymbol{\mu}, \mathbf{s}_{-n})$ is uniquely determined in the following:

$$s_{nk} = \begin{cases} 0 & \text{if } \mu_k - \sum_{i=1}^{n-1} s_{ik} \leq M, \text{ or } k = n, \\ \mu_k - \sum_{i=1}^{n-1} s_{ik} - M & \text{if } \mu_k - \sum_{i=1}^{n-1} s_{ik} > M, \end{cases}$$

for $k = 1, \dots, n$.

(ii) In the maximization problem [P1], the domain S_n is also a compact and convex set of \mathbf{R}^n and the objective function $\int_{-\infty}^{+\infty} \{\prod_{i=1}^{n-1} F(x - b_i(\mu_i, \mathbf{s}))\} f(x - b_n(\mu_n, \mathbf{s})) dx$ is also continuous on S_n . Thus, by Weierstrass's theorem, the solution set $B_n^{(1)}(\boldsymbol{\mu}, \mathbf{s}_{-n})$ is not empty. Now consider any sabotage vector of agent n which is not a solution of [P2]: $\mathbf{s}_n \notin B_n^{(2)}(\boldsymbol{\mu}, \mathbf{s}_{-n})$. Then there exists at least one pair of agents j and k such that $b_j(\mu_j, \mathbf{s}) > b_k(\mu_k, \mathbf{s})$ and $s_{nk} > 0$. Agent n can increase his winning probability by changing his strategy in such a way decreasing the sabotage for agent k and increasing the sabotage for agent j by δ such that $b_j - \delta \geq b_k + \delta$ and $s_{nk} \geq \delta > 0$. To show this is true, I show the following (A.50)

is positive, which represents the variation of agent n 's winning probability by changing his strategy:

$$\begin{aligned}
& \int_{-\infty}^{+\infty} F(x - b_j + \delta) F(x - b_k - \delta) \left\{ \prod_{i \neq j, k}^{n-1} F(x - b_i) \right\} f(x - b_n) dx \\
& - \int_{-\infty}^{+\infty} \left\{ \prod_{i=1}^{n-1} F(x - b_i) \right\} f(x - b_n) dx \\
& = \int_{-\infty}^{+\infty} H(x) \left\{ \prod_{i \neq j, k}^{n-1} F(x - b_i) \right\} f(x - b_n) dx
\end{aligned} \tag{A.50}$$

where

$$H(x) = F(x - b_j + \delta) F(x - b_k - \delta) - F(x - b_j) F(x - b_k).$$

If $x \leq b_j - \delta + \underline{\epsilon}$, then obviously $H(x) = 0$. If x satisfies that $b_j - \delta + \underline{\epsilon} < x \leq b_j + \underline{\epsilon}$, $H(x)$ is positive because $H(x) = F(x - b_j + \delta) F(x - b_k - \delta) - 0 \cdot F(x - b_i)$. Next, I show $H(x)$ is positive if x satisfies $b_j + \underline{\epsilon} \leq x < b_j + \bar{\epsilon}$. Divide $H(x)$ by $F(x - b_j) F(x - b_k - \delta)$. By Lemma 5.1 and $x - b_j < x - b_k - \delta$, we know the RHS of the following equation is positive. Thus, $H(x)$ is positive.

$$\frac{H(x)}{F(x - b_j) F(x - b_k - \delta)} = \frac{F(x - b_j + \delta)}{F(x - b_j)} - \frac{F(x - b_k)}{F(x - b_k - \delta)} (> 0).$$

Finally, if $x \geq b_j + \bar{\epsilon}$, then $H(x) = 1 \cdot 1 - 1 \cdot 1 = 0$. In sum, we have:

$$H(x) > 0 \text{ if } b_j - \delta + \underline{\epsilon} < x < b_j + \bar{\epsilon},$$

$$= 0 \text{ otherwise.}$$

Hence, (A.50) is always positive, given Assumptions 5.1 and 5.2. It follows that any \mathbf{s}_n that is not a solution of [P2] is also not a solution of [P1]. This implies $B_n^{(1)}(\boldsymbol{\mu}, \mathbf{s}_{-n})$ is empty or equal to $B_n^{(2)}(\boldsymbol{\mu}, \mathbf{s}_{-n})$ because $B_n^{(2)}(\boldsymbol{\mu}, \mathbf{s}_{-n})$ is a singleton. As already proved above, however, $B_n^{(1)}(\boldsymbol{\mu}, \mathbf{s}_{-n})$ is not empty. Therefore, $B_n^{(1)}(\boldsymbol{\mu}, \mathbf{s}_{-n}) = B_n^{(2)}(\boldsymbol{\mu}, \mathbf{s}_{-n})$. ■

Proof of Proposition 5.2: Let $PM(\mu_i, \mathbf{s})$ denote the set of agents who satisfy $b_i(\boldsymbol{\mu}, \mathbf{s}) = b_{\max}(\boldsymbol{\mu}, \mathbf{s})$:

$$PM(\boldsymbol{\mu}, \mathbf{s}) = \{\text{agent } i; b_i(\mu_i, \mathbf{s}) = b_{\max}(\boldsymbol{\mu}, \mathbf{s})\}.$$

Furthermore, let $n(PM(\boldsymbol{\mu}, \mathbf{s}))$ be the number of the elements (agents) of the set $PM(\boldsymbol{\mu}, \mathbf{s})$.

Case one: $n(PM(\boldsymbol{\mu}, \mathbf{s}^*)) = 1$.

In this case, only one agent satisfies $b_i(\mu_i, \mathbf{s}^*) = b_{\max}^*(\boldsymbol{\mu})$. Without loss of generality, assume that agent 1 is the only member of $PM(\boldsymbol{\mu}, \mathbf{s}^*)$.⁴ Note that \mathbf{s}^* implies that all agents other than agent 1 allocate all sabotage resource to agent 1. Then it is clear that a strategy profile for which all agents other than agent 1 allocate all sabotage resource to agent 1 and agent 1 allocates his resource so as to minimize $\max(b_2, b_3, \dots, b_n)$ is a Nash equilibrium. Thus there exists at least one Nash equilibrium \mathbf{s}^{NE} such that $b_{\max}(\boldsymbol{\mu}, \mathbf{s}^{NE}) = b_{\max}^*(\boldsymbol{\mu})$ in this case.

Next, I show that there does not exist any Nash equilibrium \mathbf{s}^{NE} for which $b_{\max}(\boldsymbol{\mu}, \mathbf{s}^{NE}) > b_{\max}^*(\boldsymbol{\mu})$ by the reductive absurdity. Suppose that there exists such a Nash equilibrium \mathbf{s}' . Obviously, $PM(\boldsymbol{\mu}, \mathbf{s}') = \{1\}$ and $b_1(\mu_1, \mathbf{s}') > b_{\max}^*(\boldsymbol{\mu})$ and all agents use all sabotage resource in the equilibrium. Then there certainly exists some agent (say agent j , $j \neq 1$) who allocates some or all of his sabotage resource to some peer (say agent k) other than agent 1. However, as I shown in the proof of part (ii) of Proposition 5.1, agent j can raise his winning probability by increasing his sabotage for agent 1 instead of decreasing his sabotage for agent k by a small amount. This is inconsistent with the assumption that \mathbf{s}' is a Nash equilibrium. Therefore, there does not exist any Nash equilibrium \mathbf{s}^{NE} in which $b_{\max}(\boldsymbol{\mu}, \mathbf{s}^{NE}) > b_{\max}^*(\boldsymbol{\mu})$.

Case two: $2 \leq n(PM(\boldsymbol{\mu}, \mathbf{s}^*)) \leq n$.

In this case, note that any agent other than the members of the set $PM(\boldsymbol{\mu}, \mathbf{s}^*)$ is not sabotaged in \mathbf{s}^* . It is clear that any \mathbf{s}^* is a Nash equilibrium because \mathbf{s}_i^* minimizes $\max(b_1, b_2, \dots, b_{i-1}, b_{i+1}, \dots, b_n)$ given \mathbf{s}_{-i}^* in this case.

Next, I show that there does not exist any Nash equilibrium \mathbf{s}^{NE} for which $b_{\max}(\boldsymbol{\mu}, \mathbf{s}^{NE}) > b_{\max}^*(\boldsymbol{\mu})$ in this case by the reductive absurdity. Suppose that there exists such a Nash equilibrium \mathbf{s}' . Obviously, $PM(\boldsymbol{\mu}, \mathbf{s}') \subseteq PM(\boldsymbol{\mu}, \mathbf{s}^*)$ ($PM(\boldsymbol{\mu}, \mathbf{s}') \subset PM(\boldsymbol{\mu}, \mathbf{s}^*)$) but $PM(\boldsymbol{\mu}, \mathbf{s}') \neq PM(\boldsymbol{\mu}, \mathbf{s}^*)$ if $n(PM(\boldsymbol{\mu}, \mathbf{s}^*)) = n$ and all agents use all sabotage resource in Nash equilibrium.

Subcase (a): $2 \leq n(PM(\boldsymbol{\mu}, \mathbf{s}')) \leq n - 1$.

In this subcase, there certainly exists some agent (say agent j) who allocates some or all of his sabotage resource to some peer who is the non-member of the set $PM(\boldsymbol{\mu}, \mathbf{s}')$ (say

⁴Note that the discussion below is free from the assumption $\mu_1 \geq \mu_2 \geq \dots \geq \mu_n$.

agent k). However, as I shown in the proof of part (ii) of Proposition 5.1, agent j can raise his winning probability by increasing his sabotage for one member of $PM(\boldsymbol{\mu}, \mathbf{s}')$ instead of decreasing that for agent k by a small amount. This contradicts the definition of a Nash equilibrium. Therefore, there does not exist any Nash equilibrium \mathbf{s}^{NE} in which $b_{\max}(\boldsymbol{\mu}, \mathbf{s}^{NE}) > b_{\max}^*(\boldsymbol{\mu})$ in this subcase.

Subcase (b): $n(PM(\boldsymbol{\mu}, \mathbf{s}')) = 1$.

There exists some non-member agent of $PM(\boldsymbol{\mu}, \mathbf{s}')$ (say agent j) who allocates some or all of his sabotage resource to some peer who is not the member of the set $PM(\boldsymbol{\mu}, \mathbf{s}')$ (say agent k). However, as I shown in the proof of part (ii) of Proposition 5.1, agent j can raise his winning probability by increasing his sabotage for one member of $PM(\boldsymbol{\mu}, \mathbf{s}')$ instead of decreasing that for agent k by a small amount. This contradicts the definition of a Nash equilibrium. Therefore, there also does not exist any Nash equilibrium \mathbf{s}^{NE} for which $b_{\max}(\boldsymbol{\mu}, \mathbf{s}^{NE}) > b_{\max}^*(\boldsymbol{\mu})$ in this subcase.

I have shown that for all cases there exists at least one Nash equilibrium for which $b_{\max}(\boldsymbol{\mu}, \mathbf{s}^{NE}) = b_{\max}^*(\boldsymbol{\mu})$ and furthermore there does not exist any Nash equilibrium \mathbf{s}^{NE} for which $b_{\max}(\boldsymbol{\mu}, \mathbf{s}^{NE}) > b_{\max}^*(\boldsymbol{\mu})$. It is obvious that there does not exist any feasible strategy profile \mathbf{s} such that $b_{\max}(\boldsymbol{\mu}, \mathbf{s}) < b_{\max}^*(\boldsymbol{\mu})$ by the definition of $b_{\max}^*(\boldsymbol{\mu})$. Thus, the proof has been completed. ■

Proof of Lemma 5.2: If y is smaller than $\underline{\epsilon} - \delta$, then $\bar{F}(y + \delta)/\bar{F}(y) = 1$ because $\bar{F}(y + \delta) = \bar{F}(y) = 1$.

If y satisfies $\underline{\epsilon} - \delta \leq y < \underline{\epsilon}$, then we have:

$$\begin{aligned} \frac{d(\bar{F}(y + \delta)/\bar{F}(y))}{dy} &= \frac{d\bar{F}(y + \delta)}{dy} \\ &= -f(y + \delta) < 0. \end{aligned}$$

If y satisfies $\underline{\epsilon} \leq y < \bar{\epsilon} - \delta$, then we have:

$$\begin{aligned} \frac{d(\bar{F}(y + \delta)/\bar{F}(y))}{dy} &= \frac{-\bar{F}(y)f(y + \delta) + f(y)\bar{F}(y + \delta)}{\bar{F}(y)^2} \\ &= \frac{\bar{F}(y)\bar{F}(y + \delta)\{-f(y + \delta)/\bar{F}(y + \delta) + f(y)/\bar{F}(y)\}}{\bar{F}(y)^2} < 0 \end{aligned}$$

because the numerator of the last equation above is negative by \bar{F} 's log-concavity (Assumption 5.2') and the denominator is positive.

If y satisfies $\bar{\epsilon} - \delta \leq y < \bar{\epsilon}$, then $\bar{F}(y + \delta)/\bar{F}(y) = 0$ because $\bar{F}(y + \delta) = 0$ and $\bar{F}(y) > 0$.

Finally, because the functions $\bar{F}(y + \delta)$ and $\bar{F}(y)$ are positive and continuous in y on $(-\infty, \bar{\epsilon})$, $\bar{F}(y + \delta)/\bar{F}(y)$ is also continuous in y on $(-\infty, \bar{\epsilon})$. ■

Proof of Proposition 5.4:

(i) Because the object function of the minimization problem [P3] is obviously continuous on S_n which is a nonempty, compact and convex set of \mathbf{R}^n , the solution set [P3] is not empty by the Weierstrass's theorem.

(ii) (a) The proof of “If \mathbf{s}_n is a solution of [P3], then it is also a solution of [P4]: $B_n^{(3)}(\boldsymbol{\mu}, \mathbf{s}_{-n}) \subset B_n^{(4)}(\boldsymbol{\mu}, \mathbf{s}_{-n})$.

I prove the contraposition: “If \mathbf{s}_n is not a solution of [P4], it is also not a solution of [P3]. Consider any agent n 's sabotage vector $\mathbf{s}_n \notin B_n^{(4)}(\boldsymbol{\mu}, \mathbf{s}_{-n})$. Given $\mathbf{s}_n \notin B_n^{(4)}(\boldsymbol{\mu}, \mathbf{s}_{-n})$, there exists j, k and δ such that $b_j(\mathbf{s}) - \delta < b_k(\mathbf{s})$ and $s_{nk} \geq \delta > 0$. Then agent n can decrease his losing probability by changing his strategy such as decreasing his sabotage for agent k by δ and increasing that for agent j by δ . To show this is true, I prove the following (A.51) is negative, which represents the variation of agent n 's losing probability by changing his strategy:

$$\begin{aligned} & \int_{-\infty}^{+\infty} \bar{F}(x - b_j + \delta) \bar{F}(x - b_k - \delta) \left\{ \prod_{i \neq j, k}^{n-1} \bar{F}(x - b_i) \right\} f(x - b_n) dx \\ & - \int_{-\infty}^{+\infty} \left\{ \prod_{i=1}^{n-1} \bar{F}(x - b_i) \right\} f(x - b_n) dx \\ & = \int_{-\infty}^{+\infty} \bar{H}(x) \left\{ \prod_{i \neq j, k}^{n-1} \bar{F}(x - b_i) \right\} f(x - b_n) dx, \end{aligned} \quad (\text{A.51})$$

where

$$\bar{H}(x) = \bar{F}(x - b_j + \delta) \bar{F}(x - b_k - \delta) - \bar{F}(x - b_j) \bar{F}(x - b_k).$$

At first, if $x \leq b_j - \delta + \underline{\epsilon}$, then $\bar{H}(x) = 1 \cdot 1 - 1 \cdot 1 = 0$. Second, I show $\bar{H}(x)$ is negative if x satisfies $b_j - \delta + \underline{\epsilon} < x < b_j - \delta + \bar{\epsilon}$. Divide $\bar{H}(x)$ by $\bar{F}(x - b_j) \bar{F}(x - b_k - \delta)$ which is positive. Then by Lemma 5.2 and $x - b_j > x - b_k - \delta$, the LHS of the following equation is negative. Thus $\bar{H}(x)$ is negative if x satisfies $b_j - \delta + \underline{\epsilon} < x < b_j - \delta + \bar{\epsilon}$.

$$\frac{\bar{H}(x)}{\bar{F}(x - b_j) \bar{F}(x - b_k - \delta)} = \frac{\bar{F}(x - b_j + \delta)}{\bar{F}(x - b_j)} - \frac{\bar{F}(x - b_k)}{\bar{F}(x - b_k - \delta)} < 0.$$

Next, if x satisfies $b_j - \delta + \bar{\epsilon} \leq x < \min(b_j, b_k) + \bar{\epsilon}$, then $\bar{H}(x) = 0 \cdot \bar{F}(x - b_k - \delta) - \bar{F}(x - b_j)\bar{F}(x - b_k) = -\bar{F}(x - b_j)\bar{F}(x - b_k) > 0$. Last, if x satisfies $\min(b_j, b_k) + \bar{\epsilon} \leq x$, then $\bar{H}(x) = 0 \cdot 0 - 0 \cdot 0 = 0$. To sum up, we have

$$\begin{aligned} \bar{H}(x) &< 0 && \text{if } b_j - \delta + \underline{\epsilon} < x < \min(b_j, b_k) + \bar{\epsilon} \\ &= 0 && \text{otherwise.} \end{aligned}$$

Giving consideration to Assumption 5.1' in addition to this, it is clear that the expression (A.51) is always negative. It follows that any \mathbf{s}_n that is not a solution of [P4] is also not a solution of [P3].

(b) The proof of "If \mathbf{s}_n is a solution of [P4], then it is also a solution of [P3]: $B_n^{(3)}(\boldsymbol{\mu}, \mathbf{s}_{-n}) \supset B_n^{(4)}(\boldsymbol{\mu}, \mathbf{s}_{-n})$.

I prove this by reductive absurdity. Suppose that some solution of [P4] is not a solution of [P3]. As stated above, even if there exist two or more solutions on [P4], these solutions are identical except the agent code sabotaged by agent n . Hence, that some solution of [P4] is not a solution of [P3] implies that any solution of [P4] is not a solution [P3]. From part (a) above, we also know that any \mathbf{s}_n that is not a solution of [P4] is also not a solution of [P3]. These two things imply that the solution set of [P3] must be empty. However, the proof of part (i) shows the set is not empty. This is a contradiction. Thus, it has been shown that $B_n^{(3)}(\boldsymbol{\mu}, \mathbf{s}_{-n}) \supset B_n^{(4)}(\boldsymbol{\mu}, \mathbf{s}_{-n})$.

The result follows from the parts (a) and (b). ■

Proof of Proposition 5.6:

(i) Suppose that in the one-winner tournament every agent except a given agent n allocates all his resource to the efforts. Then their expected performance is α . Also suppose that agent n allocates his resource to the efforts by p and to sabotage activities by $1 - p$ ($0 \leq p \leq 1$). By Proposition 5.1, his best way to allocate $1 - p$ is to attack all his peers equally. Then his and his peers' expected performance are $p\alpha$ and $\alpha - \frac{1-p}{n-1}$, respectively. The difference between them is $(1 - p)(\frac{1}{n-1} - \alpha)$. It follows that his best choice for p is $p = 0$ if $\alpha < \frac{1}{n-1}$, indifferent for any p if $\alpha = \frac{1}{n-1}$, or $p = 1$ if $\alpha > \frac{1}{n-1}$, which completes the proof of part (i).

(ii) I show that when $\alpha \leq \frac{1}{n-1}$, a strategy profile for which all agents allocate all their resource to the efforts is not a Nash equilibrium in the one-loser tournament setting.

Suppose that all agents except agent n allocate all resource to the efforts. If agent n allocates all his resource to the efforts, every agent's expected performance is the same: α . If he attacks his all peers equally, his and peers' expected performances will be zero and $\alpha - \frac{1}{n-1}$, respectively. If $\alpha < \frac{1}{n-1}$, the latter action (sabotage action) is the better for him. If $\alpha = \frac{1}{n-1}$, these two actions are indifferent for him. By Proposition 5.4, however, it is better for him to attack one of his peers intensively than to attack all peers equally. Thus the proof is completed. ■