ABSTRACT

WU, KEHANG

Flow Aggregation based Lagrangian Relaxation with

Applications to Capacity Planning of IP Networks with Multiple Classes of Service

(Under the direction of Professor Douglas S. Reeves)

Nonlinear multicommodity flow problem with integer constraints is an important subject with a wide variety of application contexts. Given the inherent difficulty of solving nonlinear integer programming problems and the lack of efficient systematic approaches, research has primarily been focused on heuristics, which may be inadequate in many cases.

In this dissertation, we present a novel Lagrangian Relaxation based approach to attack the nonlinear multicommodity flow problem with integer constraints. Unlike other methods based on the relaxation of integer constraints or nonlinear constraints, our approach relaxes the flow aggregation equation, or the relationship between individual flows and the variable representing the total amount of traffic in each link. The relaxation of the flow aggregation equation makes the resulting Lagrangian dual problem separable to simpler subproblems, which is not possible if the problem is

relaxed otherwise. The subgradient method is used to find the optimal Lagrangian multipliers.

The aforementioned method is subsequently applied to three closely related capacity planning problems to verify its effectiveness. We address the problems of link dimensioning and routing optimization for IP networks supporting DiffServ Expedited Forwarding (EF) and best effort (BE) traffic classes. We first study capacity planning in the context of legacy IP networks, where only non-bifurcated routing is allowed for traffic with the same source and destination. In the second problem, we assume the use of Multiprotocal Label Switching (MPLS), and therefore allow bifurcated routing. The third problem deals with the design of survivable MPLS networks, where spare capacity is required.

We investigated experimentally the solution quality and running time of this approach. The results from our experiments indicate that our method produces solutions that are within a few percent of the optimal solution, while the running time remains reasonable on practical-sized networks. This represents the first work for capacity planning of multi-class IP networks with non-linear performance constraints and discrete link capacity constraints. We also expect the solution method to be a promising approach for other non-linear multicommodity flow problems with integer

constraints.

**Flow Aggregation based Lagrangian Relaxation with
Applications to Capacity Planning of IP Networks with Multiple Classes
of Service**

by

Kehang Wu

A dissertation submitted to the Graduate Faculty of
North Carolina State University
in partial satisfaction of the
requirements for the Degree of
Doctor of Philosophy

Department of Electrical and Computer Engineering

Raleigh

2004

Approved By:

<table>
<tr><td>Dr. Douglas S. Reeves
Chair of Advisory Committee</td><td>Dr. Arne A. Nilsson</td></tr>
<tr><td>Dr. Mo-Yuen Chow</td><td>Dr. Mladen A. Vouk</td></tr>
</table>

To my wife Yang Tang

for her love and support

Biography

Kehang Wu was born in Fujian, P.R. China. He received his B.E degree in Electronic Engineering from Tsinghua University, P.R. China in July, 1996. Upon graduation, he worked for China Telecom. He was admitted to North Carolina State University, Department of Electrical and Computer Engineering, in the Fall of 1998 as a Master student. After finishing his Master's degree in December 1999, he went on to pursue the Ph.D degree under the supervision of Dr. Douglas S. Reeves. His research interests include network design and optimization.

Acknowledgements

First of all, I would like to express my deep appreciation to my Ph.D. advisor, Dr. Douglas S. Reeves. This dissertation would not be possible without the guidance, encouragement and support from Dr. Reeves. His vision has led me through my graduate research.

I would like to thank Dr. Arne A. Nilsson, Dr. Mo-Yuen Chow, and Dr. Mladen A. Vouk for serving on my advisory committee. Special thanks to Dr. Nilsson for insightful advice and his excellent lectures on telecommunication networks design.

Thanks also to Dr. Micha Pioro (Warsaw Institute of Technology, Poland & Lund University, Sweden) and Dr. J. Cole Smith (University of Arizona), for their valuable discussions and comments on various papers.

Many thanks go to all the former members of the ARQoS research project and the current members of the Cyber Defense Laboratory, Errin Fulp, Zhaoning Li, Vinay Mahadik, Hui Gao, Khurram Khan, Nipul Shah, Zhi Fu, Qingling Jiang, Fang Feng, *etc.* I have enjoyed the collaboration in our group very much.

I would like to thank my wife, Yang Tang, and my mother, Jiequn Chen, for their love and support.

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Chapter 1

Introduction

1.1 Multicommodity flow problem and its applications

The classic multicommodity flow (MCF) problems play a central role in the area of combinatorial optimization. The MCF problems generalize the well-known single-commodity flow (SCF) problems in that flows with different nature, each governed by its own flow constraints, coexist in the same network. Different flows compete for the network resource, such as the link capacity. Multiple flows couple though link capacity, cost function, or any other constraints that relate more than multiple flows.

MCF problems share many characteristics of the SCF variants, but the coupling of multiple flows often invalidates the theoretical basis of the SCF problems, and there-fore reduces the usefulness of some well-developed algorithms for the SCF problems. For example, the max-flow min-cut theorem of Ford and Fulkerson [33] in SCF prob-

lems guarantees that the maximum flow is equal to the minimum cut. Furthermore, in SCF problems with integral arc capacities and node demands, the maximum flow is guaranteed to be integral [2]. None of these properties can be extended to MCF problems, except for several special planar graphs [72] [60], or two-commodity flows with even integral demands and capacities [47] [77]. Many SCF algorithms exploit the single commodity flow property in which flows in the opposition direction on an arc can be cancelled out [2]. In MCF problems, flows do not cancel if they are different commodities.

Hence, most of the techniques developed over the years for continuous MCF problems are essentially based on the solution methods for the large scale linear or nonlinear problems, while the integer MCF problems, which are considered among the most difficult NP-hard problems, call for approximations or heuristic approaches[14].

Since 1960s, MCF has motivated many research topics. For example, column generation by Ford and Fulkerson [34] was originally designed to solve MCF problems, and is still a common technique for solving large-scale linear programming problems. Seymour [80] proposed the max-flow min-cut matroid and several important matroid theorems by studying multicommodity flow. The nonintegrality property of MCF has spurred much research in integer linear programming related to matroids, algorithmic

complexity, and polyhedral combinatorics [43]. The block-angular constraint structure of the MCF constraints serves as a "best practice" for decomposition techniques such as Dantzig-Wolfe decomposition [25] and basis partitioning methods, such as generalized upper bounding [58].

From an application viewpoint, the min-cost MCF problem and its variants are models of a wide variety of telecommunication, transportation, and scheduling problems. Most of them are network routing and network design problems. Listed below are a few applications of the MCF problem.

- **Network routing**

    – *Message or packet routing in telecommunication and computer networks:*
      Consider each requested OD pair to be a commodity. The problem is to
      find a min-cost flow routing for all demands of requested OD pairs while
      satisfying the arc capacities. This appears often in telecommunication and
      computer networks. For example, message routing for multiple OD pairs
      [11], packet routing on virtual circuit data network [66], or routing on a
      ring network [81] are all min-cost MCF problems.

    – *Scheduling and routing in logistics and transportation:*

In logistics or transportation problems, commodities may be objects such as products, cargo, or even personnel. The commodity scheduling and routing problem is often modeled as a MCF problem on a time-space network where a commodity may be a tanker [12], aircraft [45], crew [16], or rail freight [8]. Golden [42] gives a MCF model for port planning that seeks optimal simultaneous routing where commodities are export/import cargo, nodes are foreign ports (as origins), domestic hinterlands (as destinations) and US ports (as transshipment nodes), and arcs are possible routes for cargo traffic. Similar problems appear in grain shipment network [5]. A disaster relief management problem is formulated as a multicommodity multimodal network flow problem with time windows by Haghani and Oh [44] where commodities (food, medical supplies, machinery, and personnel) from different areas are to be shipped via many modes of transportation (car, ship, helicopter,...,etc.) in the most efficient manner to minimize the loss of life and maximize the efficiency of the rescue operations.

– *Production scheduling and planning:*

Jewell [50] solves a warehousing and distribution problem for seasonal

products by formulating it as a MCF model where each time period is a transshipment node, one dummy source and one dummy sink node exist for each product, and arcs connect from source nodes to transshipment nodes, earlier transshipment nodes to later transshipment nodes, and transshipment nodes to sink nodes. Commodities are products to be shipped from sources to sinks with minimum cost. D'Amours *et al.* [24] solve a planning and scheduling problem in a Symbiotic Manufacturing Network (SMN) for a multiproduct order. A broker receives an order of products in which different parts of the products may be manufactured by different manufacturing firms, stored by some storage firms, and shipped by a few transportation firms between manufacturing firms, storage firms and customers. The problem is to design a planning and scheduling bidding scheme for the broker to make decisions on when the bids should be assigned and who they should be assigned to, such that the total cost is minimized. They propose a MCF model where each firm (manufacturing or storage) at each period represents a node, an arc is either a possible transportation link or possible manufacturing (storage) decision, and a commodity represents an order for different product.

– *VLSI design:*

Global routing in VLSI design can be modeled as an origin-destination MCF problem where nodes represent collections of terminals to be wired, arcs correspond to the channels through which the wires run, and commodities are OD pairs to be routed [17][75].

• **Network design**

– *Capacitated telecommunication network design*

Given a graph G, a set of commodities K to be routed according to known demands, and a set of facilities L that can be installed on each arc, the capacitated network design problem is to route flows and install facilities at minimum cost. This is a MCF problem, which involves flow conservation and bundle constraints plus some side constraints related to the installation of the new facilities. The objective function may be nonlinear or general discontinuous step-increasing.

Problems such as the design of a network where the service quality and survivability constraints are met with minimum cost in telecommunication networks appear often in the literatures [37]. We investigated the issues of

capacity planning of multi-class IP networks in [89][90][88]. Both routing variables and link capacities are subject to optimization.

– *Transportation network design*

Similar problems also appear in transportation networks, such as locating vehicle depots in a freight transportation system so that the client demands for empty vehicles are satisfied while the depot opening operating costs and other transportation costs are minimized [3][23]. Crainic also has a survey paper [22] about service network design in freight transportation.

1.2 Capacity planning of multiclass IP networks

In this dissertation, we are interested in the problem of capacity planning of multiclass IP networks. Capacity planning is the process of designing and dimensioning networks to meet the expected demands of users. If networks need to stay ahead of the growth of user demand while still being able to provide a satisfactory service, capacity planning is indispensable. Recent years have witnessed the spectacular growth of the Internet traffic. The nature of offering only best effort (BE) service in the Internet has made capacity planning a straightforward matter [52].

Quality of service (QoS) is the ability of a network element to have some level of assurance that its traffic and service requirements can be satisfied. With the popularizing of e-commerce and new value-added services over IP, like Voice over IP, QoS has become a must. Capacity planning will be an imperative part of IP network management to support various qualities of service.

MPLS and Differentiated Service [78][71] are regarded as two key components of QoS.

MPLS uses a short, fixed-length, locally significant label in the packet header to switch the packets. The initial label is chosen and inserted by the ingress node of a MPLS domain, based on the information in the IP header, associated QoS, or any other policies in effect. The intermediate nodes use the label as an index to find the next hop and the corresponding new label. A label distribution protocol (LDP) propagates label bindings among the nodes in order to establish and tear down the label switched path (LSP). The power of MPLS lies in the fact that the mappings between the packet flows and the LDPs are flexible, which enables an IP network to be traffic engineered. Packets with the same source and destination addresses, which will inevitably follow the same path in the traditional IP network, can be assigned different labels and subsequently be sent to separate LSPs. LSPs can be

setup explicitly to optimize the resource utilization. The use of MPLS labels may also provide faster switching than the normal IP forwarding algorithm.

The essence of DiffServ is prioritization. In traditional IP networks, the DiffServ Code Point (DSCP) field in the headers of IP packets is marked at the edge of the network. Routers within the core of the network forward packets using different predefined per-hop behaviors (PHBs), according to their DSCP field. In MPLS networks, the DSCP field is not visible to the core LSRs. The label value and EXP field (3 bits experimental field) are used instead to determine the PHB scheduling class (PSC) associated with packets [91]. Note that even with MPLS, the signaling and traffic control are still at the level of flow aggregation. Since there is no per-flow signaling or control, accurate dimensioning of the network is particularly important for achieving performance guarantees. To prepare for the deployment of DiffServ in MPLS network, it is necessary to study the capacity planning problem in the context of multiple class-of-service networks.

The IETF DiffServ working group has standardized two PHBs: Expedited Forwarding (EF) and Assured Forwarding (AF). The EF PHB [26] is defined as being such that the EF packets are guaranteed to receive service at or above a configured rate. The EF PHB can be used to build a low loss, low latency, low jitter, assured

bandwidth, end-to-end service, through a DiffServ Domain. As has been discussed in [18], three expected major initial applications of QoS in the IP network are: 1) to distinguish "mission critical" or preferred customers; 2) to provide voice over IP service; 3) to enable services competitive with leased lines. It can be easily seen that the services based on the EF PHB are ideal for all three of these applications. Because of its great value, the EF PHB is very likely to be the first PHB to be put into action. The priority queue is widely considered to be the canonical way to implement the EF PHB, due to its ability to offer a tighter delay bound and smoother service over relatively short time scales [26]. AF PHBs are designed to realize different forwarding assurances, or dropping preferences, for IP packets. AF PHBs are considered useful to differentiate TCP traffic, where the performance is sensitive to the packet loss rate. However, simulations showed that the standard traffic control methods of routers, such as RED(Random Early Detection), do not satisfactorily differentiate between AF PHBs and best effort traffic [7]. Since our approach requires a precise performance model for optimization, we do not include the AF traffic classes in this paper, due to the lack of a consensus on the implementation of the AF PHB.

The problem of link dimensioning and path optimization of networks providing DiffServ EF and BE traffic classes, with or without the present of MPLS capability, is

a capacitated network design problem. We formulate the problem as a MCF problem. The novel aspect of our problem, unlike most of other capacity planning problems, is the fact that two traffic classes, EF and BE, with independent performance and survivability requirements, share the same capacity resource, which results in a complex non-linear performance constraint. In addition to the nonlinear performance constraints, non-bifurcated routing and discrete link capacity constraints dramatically increase the degree of difficulty, and significantly limit the viable solution approaches.

1.3   Contributions of the dissertation

In the practical environment, a realistic modeling of the MCN problem often turns out to be nonlinear integer programming problem. As evidenced in the capacity planning of multiclass IP network, the interaction of BE and EF traffic results in nonlinear performance constraints, while non-bifurcated routing and discrete link capacity yield integer constraints. Due to the inherent difficulty of solving nonlinear integer programming problems, one may prefer to settle for an approximate solution, obtained through some heuristics. There are a broad variety of heuristic procedures that are based on integer or side constraint relaxation. Some of these heuristics can be very sophisticated, and depending on the target problem, they may provide

satisfactory solutions. But in most of the cases, a heuristic may be inadequate and there may be need for a more systematic procedure.

Lagrangian relaxation is a general method. It has been successfully applied to linear MCF problems and integer MCF problems by relaxing bundle constraints or the integer constraints. But those two relaxation approaches do not work well for nonlinear integer MCF problem, primarily because the resulting Lagrangian dual problem is not separable unless the nonlinear function is separable, which does not hold in most instances.

In this dissertation, we propose a new Lagrangian relaxation based approach. Our approach relaxes the flow aggregation equation, or the relationship between individual flows and the variable representing the total amount of traffic in each link. The relaxation of flow aggregation equation makes the resulting Lagrangian dual problem separable to simpler subproblems, which will not be possible if the problem is relaxed otherwise. The subgradient method is used to find out the optimal Lagrangian multipliers.

We then apply our method to the problems of link dimensioning and routing optimization for IP networks supporting DiffServ Expedited Forwarding (EF) and BE traffic classes. We first study the capacity planning in the context of legacy IP

networks, where only non-bifurcated routing is allowed for the traffic with the same source and destination. In the second problem, we assume the presence of MPLS, and therefore allow bifurcated routing. The third problem deals with the design of survivable MPLS networks, where spare capacity is required. We investigated experimentally the solution quality and running time of our proposed approach. The results from our experiments indicate that our method produces solutions that are within a few percent of the optimal solution, and that the running time remains reasonable on practical-sized networks. This srepresents the first work for capacity planning of multi-class IP networks with non-linear performance constraints and discrete link capacity constraints. We also expect the solution method to be a promising approach for other non-linear multicommodity flow problem with integer constraints.

1.4 Outline of the Dissertation

Chapter 2 surveys MCF solution techniques from the literature. Chapter 3 describes the flow aggregation based Lagrangian relaxation method in detail. Chapter 4 overviews the problem formulation of capacity planning of multiclass IP networks and how the proposed method can be applied. Chapter 5 shows the solution method and experimental results of the capacity planning problem in the context of legacy IP

networks. Chapter 6 studies the problem in MPLS networks. Chapter 7 looks into the problem of survivable MPLS network design. This dissertation is concluded in Chapter 8.

Chapter 2

Previous Work on Multicommodity Flow

Problems

In this chapter, we will summarize the formulations and most of the solution techniques that have appeared in the literature for solving the MCF problems. We briefly touched on the differences of the multicommodity flow problems and single commodity flow problems in Section 1.1. As to the MCF problems themselves, different solution techniques may be advantageous for different MCF problems depending on the problem characteristics, such as the linearity of the cost function and constraints, whether there are discrete variables or constraints, the number of flows with respect to the number of nodes, *etc.* This dissertation focuses on solving the class of nonlinear MCF problems with integer constraints. But the method we proposed is actually an extension of the price-directive method developed for the linear continuous MCF

problems.

2.1   Comparison of formulations

- **Arc-based formulation**

  Let N denote the set of all nodes in the network G, L the set of all arcs, and K the set of all commodity OD pairs. Each individual link is defined as (i, j), where i is the starting node of the link while j is the terminating node. For commodity k with origin s_k and destination t_k, let α_k be the total demand. b_k^i is the supply/demand at node i. $b_k^i = 1$ if $i = s_k$, -1 if $i = t_k$, and 0 otherwise. x_k^{ij} is defined as the decision variable, a value between 0 and 1 representing the portion of demand k that is assigned to link (i,j). Let $\tilde{\psi}_{ij}$ be the total capacity on link (i,j), c_{ij} represents the unit cost of arc (i,j).

  The arc-based formulation of MCF problem is a direct extension of the conventional SCF formulation. A simple linear MCF problem can be formulated as follows:

  $$\min \sum_{(i,j) \in L} c_{ij} \tilde{\psi}_{ij} \tag{2.1}$$

subject to:

$$\sum_{(i,j)\in L} x_k^{ij} - \sum_{(j,i)\in L} x_k^{ji} = b_k^i, \forall i \in N, \forall k \in K \tag{2.2}$$

$$\sum_{k\in K} \alpha_k x_k^{ij} \leq \tilde{\psi}_{ij}, \forall (i,j) \in L \tag{2.3}$$

(2.2) represents flow balance constraint. (2.3) is the bundle constraint. It has

$|K||L|$ variables and $|N||K| + |L|$ nontrivial constraints.

- **Path-based formulation**

First suggested by Tomlin [84], the path-based form of the MCF problem is

based on the fact that any network flow solution can be decomposed into path

flows and cycle flows. Under the assumption that no cycle has negative cost,

any arc flow vector can be expressed optimally by simple path flows.

For commodity k, let J_k denote the set of all possible paths from s_k to t_k. Each

individual link is defined by l, $l \in L$. x_k^j is defined as the decision variable, a

value between 0 and 1 representing the portion of demand k that is assigned to

candidate path j, $j \in J_k$. δ_j^l is a binary indicator which equals to 1 if path j

passes through link l, and 0 otherwise. Let $\tilde{\psi}_l$ be the total capacity on link l, c_l

represents the unit cost of link l,

$$\min \sum_{l \in L} c_l \tilde{\psi}_l \tag{2.4}$$

subject to:

$$\sum_{j \in J_k} x_k^j = 1, \forall k \in K \tag{2.5}$$

$$\sum_{k \in K} x_k^j \delta_j^l \alpha_k \le \tilde{\psi}_l, \forall l \in L \tag{2.6}$$

(2.5) represents flow balance constraint, while (2.6) is the bundle constraint. The above formulation has $\sum_{k \in K} |J_k|$ variables and $|K| + |L|$ nontrivial constraints.

When commodities are OD pairs, $|K|$ may be $O(|N|^2)$ in the worst case. In such case, the arc-based form may have $O(|N|^3)$ constraints which make its computations more difficult and memory management less efficient.

The path-based formulation, on the other hand, has at most $O(|N|^2)$ constraints but exponentially many variables in the worst case. The problem caused by a huge number of variables can be resolved by column-generation techniques or by limiting the number of candidate paths for each OD. The path-based formulation is particularly advantageous when the number of OD is vary large with respect to the number of nodes.

Knowing this fact, our solution method will use path-based techniques in this dissertation.

2.2 Linear continuous multicommodity flow problems

Due to the special structure of linear MCF problems, many special solution methods have been suggested in the literature. Basis partitioning methods partition the basis in a way such that computing the inverse of the basis is faster. Resource-directive methods seek optimal capacity allocations for commodities. Price-directive methods try to find the optimal penalty prices (dual variables) for violations of the bundle constraints. Primal-dual methods raise the dual objective while maintaining complementary slackness conditions. In the last two decades, many new methods such as approximation methods, interior-point methods and their parallelization have been proposed. For a complete treatment of the issue, please refer to [2].

2.2.1 Basic partition methods

The underlying idea of basis partitioning methods is to partition the basis so that the special network structure can be maintained and exploited as much as possible to make the inversion of the basis more efficient. Hartman and Lasdon [46] propose a

Generalized Upper Bounding (GUB) technique which is a specialized simplex method

whose only non-graphic operations involve the inversion of a working basis with di-

mension equal to the number of currently saturated arcs. Other basis partitioning

methods have also been suggested by McCallum [65] and Maier [62].

2.2.2 Resource-directive methods

Suppose on each arc (i,j) we have assigned capacity r_{ij}^k for each commodity k

so that $\sum_{k \in K} \alpha_k x_k^{ij} \leq \tilde{\psi}_{ij}$ is satisfied. Then the original problem is equivalent to the

following resource allocation problem (RAP) that has a simple constraint structure

but complex objective function:

$$\min \sum_{k \in K} z^k(r^k) \tag{2.7}$$

$$s.t. \ \sum_{k \in K} r_{ij}^k \leq \tilde{\psi}_{ij}, \forall (i,j) \in L \tag{2.8}$$

$$0 \leq r_k^{ij} \leq \tilde{\psi}_{ij}, \forall (i,j) \in L, \forall k \in K \tag{2.9}$$

For each commodity k, $z^k(r^k)$ can be obtained by solving the following single

commodity min-cost network flow subproblem.

$$\min \sum_{(i,j)\in L} c_{ij}x_k^{ij} = z^k(r^k) \qquad (2.10)$$

$$s.t. \sum_{(i,j)\in L} x_k^{ij} - \sum_{(ji)\in L} x_k^{ji} = b_k^i, \forall i \in N, \forall k \in K \qquad (2.11)$$

$$0 \le x_k^{ij}\alpha_k \le r_k^{ij}, \forall (i,j) \in L \qquad (2.12)$$

It can be shown that the objective function $z(r)$ is piecewise linear on the feasible

set of the capacity allocation vector r. There are several methods in the literature to

solve the RAP such as tangential approximation [38], feasible directions [51], and the

subgradient method [38]. Shetty and Muthukrishnan [82] give a parallel projection

algorithm which parallelizes the procedure of projecting new resource allocations in

the resource-directive methods.

2.2.3   Price-directive method

Price-directive methods are based on the idea that by associating the bundle

constraints with "correct" penalty functions (dual prices, or Lagrange multipliers), a

hard MCF problem can be decomposed into k easy SCF problems.

Price-directive methods includes Lagrangian relaxation, Dantzig-Wolfe decompo-

sition, key variable decomposition, etc. We only introduce the Lagrangian relaxation

method, which is related to our method. Please refer to [2] for more information about other methods.

Lagrange relaxation dualizes the bundle constraints using a Lagrangian multiplier λ_l so that the remaining problem can be decomposed into smaller min-cost network flow problems [2]. In particular,

$$
\begin{aligned}
h(\lambda_l) &= \min[\sum_{l \in L} c_l \tilde{\psi}_l + \sum_{l \in L} \lambda_l (\sum_{k \in K} x_k^j \delta_j^l \alpha_k - \tilde{\psi}_l)] & (2.13) \\
&= \sum_{l \in L} \min (c_l \tilde{\psi}_l - \lambda_l \tilde{\psi}_l) + \sum_{k \in K} \min(\sum_{l \in L} \lambda_l x_k^j \delta_j^l \alpha_k) & (2.14)
\end{aligned}
$$

Note that since the bundle constraint is relaxed and the variable x_k^j becomes independent of each other, therefore (3.8) is separable in K.

The Lagrangian dual problem seeks the optimal Lagrangian multiplier λ_l for $L^* = \max h(\lambda_l)$. This is a non-differentiable optimization problem. Subgradient methods [38][2] are common techniques for determining the Lagrange multipliers. They are easy to implement but have slow convergence rates. They usually do not have a good stopping criterion, and in practice usually terminate when a predetermined number of iterations or nonimproving steps is reached.

More recently, the bundle method [35] has been proposed to solved the Lagrangian dual problem.

2.3 Nonlinear multicommodity flow problems

Nonlinear multicommodity flow problems refer to the type of MCF problems with nonlinear cost function or nonlinear constraint.

2.3.1 Proximal point methods

Ibaraki and Fukushima [48] apply a primal-dual proximal point method to solve the convex MCF problems. The method identifies an approximate saddle point of the augmented Lagrangian at each iteration and guarantees the convergence of these points. Similar methods are also developed by Chi2et et al. [21] and Mahey et al. [61].

2.3.2 Alternating direction methods

DeLeone et al. [27] propose three variants of alternating direction methods [57], which may be viewed as block Gauss-Seidel variants of augmented Lagrangian approaches that take advantage of block-angular structure and parallelization. These methods take alternating steps in both the primal and the dual space to achieve optimality.

2.3.3 Other nonlinear programming methods

Ouorou and Mahey [73] use a minimum mean cycle canceling based algorithm to solve a MCF problem with a nonlinear separable convex cost function. Nagamochi et al. [69] use the relaxation method of Bertsekas' algorithm [13] to solve strictly convex MCNF problems. Bertsekas [14] (section 8.2) surveys techniques for the nonlinear MCF problem.

2.4 Multicommodity flow problems with integer constraints

All the algorithms introduced before this section are fractional solution methods. Adding the integer constraints makes the MCNF problems much harder. Much research has been done regarding the characterization of integral MCF problems. For example, Evans et al. [32] have given a necessary and sufficient condition for unimodularity in some classes of MCF problems. Evans makes the transformation from some MCF problems to their equivalent uncapacitated SCNF problems, and proposes a specialized simplex algorithm and heuristics for certain MCF problems. Truemper and Soun [85] further investigate the topic and obtain additional results about unimodularity and total unimodularity for general MCF problems. Other research about

the characteristics of quarter-integral, half-integral and integral solutions of MCNF problems can be found in [79][60].

The general integral MCF algorithms can also be based on LP-based branch-and-bound schemes such as branch-and-cut or branch-and-price [14]. In branch-and-cut, cuts are dynamically generated throughout the branch-and-bound search tree to cut off fractional solutions. On the other hand, branch-and-price generates variables that are used in conjunction with column generation to strengthen the LP relaxation and resolve the symmetry effect due to formulations with few variables. Barnhart et al. [11] apply branch-and-price-and-cut which generates both variables and cuts during the branch-and-bound procedures to solve binary MCF problems in which integral flow must be shipped along one path for each OD pair. The branch-and-bound schemes are capable of obtaining the optimal solution at the expense of large running time. It is only applicable to problems with small number of integer valuables or constraints.

2.5 Nonlinear multicommodity flow problems with integer constraints

Given the inherent difficulties of the nonlinear integer MCF problems, one may prefer to settle for an approximate solution, obtained through some heuristics. Even though some heuristics may provide satisfactory solutions, it is generally difficult to evaluate the solution quality with respect to the real optimal solution from the heuristics alone.

2.5.1 Simple heuristics

Two of the simplest and most often used general approaches are the following:

- Discard the integer constraints, solve the resulting problem as a nonlinear MCF problem, and use an ad hoc method to round the solution to an integer.

- Approximate the nonlinear function to a piece-wise linear function, discard the complicating side constraints, and solve the problem as a integer programming problem, then use some heuristic to correct this solution and verify its feasibility. This is a less desirable approach since the resulting integer programming problem may very well be quite difficult.

2.5.2   Local search methods

Local search methods are a broad and important class of heuristics for discrete optimization. They apply to the general problem of minimizing a function over a finite set of solutions. In principal, the integer programming problem can be solved by enumerating of the entire set of solutions, which is what branch-and-bound does. A local search method tries to save computation time by iteratively searching through the neighbors of the current solution for a better solution.

More recently, local search methods which can be loosely classified as intelligent, like genetic algorithms, tabu search, and simulated annealing, are gaining more and more attention. Genetic algorithms locate the optimal solution using processes similar to those in natural selection and genetics. Tabu search is a heuristic procedure that employs dynamically generated constraints (or tabus) to avoid getting trapped at a poor local minimum. Simulated annealing finds optima in a way analogous to reaching of minimum energy configurations in metal annealing. See [14] by Bertsekas for more information on those methods.

Chapter 3

Lagrangian Relaxation of Flow

Aggregation Equation

In this chapter, we show how we can effectively apply the Lagrangian relaxation method to a class of nonlinear integer MCF problems.

3.1 Basics of Lagrangian relaxation

Lagrangian relaxation is a flexible solution strategy. The basics of Lagrangian relaxation is to relax the complicated constraints, placing them in the objective function with associated multipliers. For a given optimization problem, there are usually multiple ways to relax the problem. Usually, we choose the constraints to relax so that the Lagrangian dual problem is much easier to solve than the original problem. Ideally, we choose the constraints that couple otherwise independent subproblems.

In these instances, Lagrangian relaxation permits us to decompose a problem into smaller, more tractable subproblems. For this reason, Lagrangian relaxation is also referred to as a decomposition technique.

Since the Lagrangian dual problem is a relaxation of the original problem, key principal of Lagrangian relaxation is that the optimal solution of the dual problem is always a lower bound on the optimal objective value of the primary problem. It provides means to evaluate the solution quality. To obtain the best lower bound, we want to find the Lagrangian multiplier that maximizes the optimal value of the Lagrangian dual problem, which is called the Lagrangian multiplier problem. As mentioned in section 2.2.3, the subgradient method is the most common technique for determining the Lagrange multipliers.

3.2 Bundle constraint based Lagrangian relaxation and linear MCF problems

We will first review how the Lagrangian relaxation works for linear MCF problems. Recall that the path-based formulation of the linear MCF problems is defined as

follows:

$$\min \sum_{l \in L} c_l \tilde{\psi}_l \tag{3.1}$$

$$s.t. \sum_{j \in J_k} x_k^j = 1, \forall k \in K \tag{3.2}$$

$$\sum_{k \in K} x_k^j \delta_j^l \alpha_k \leq \tilde{\psi}_l, \forall l \in L \tag{3.3}$$

Note that the bundle constraint represented by (3.3) is the only thing that couples

the variable x_k^j of different commodities. If we choose (3.3) as the target constraint

to relax, we have the Lagrangian dual problem as follows:

$$
\begin{aligned}
h(\lambda_l) &= \min[\sum_{l \in L} c_l \tilde{\psi}_l + \sum_{l \in L} \lambda_l (\sum_{k \in K} x_k^j \delta_j^l \alpha_k - \tilde{\psi}_l)] \tag{3.4} \\
&= \sum_{l \in L} \min \left(c_l \tilde{\psi}_l - \lambda_l \tilde{\psi}_l \right) + \sum_{k \in K} \min(\sum_{l \in L} \lambda_l x_k^j \delta_j^l \alpha_k) \tag{3.5} \\
s.t. & \quad (3.2)
\end{aligned}
$$

The major benefit of the relaxation is the the problem becomes separable in k,

which significantly lower the computation cost.

3.3 Lagrangian relaxation of the flow aggregation equation

Now we will consider at a simple nonlinear MCF problem.

Let

$$\beta_l = \sum_{k \in K} x_k^j \delta_j^l \alpha_k \tag{3.6}$$

which is the total amount of flow in link l. We call (3.6) the flow aggregation equation.

If we replace the constraint (3.3) with a nonlinear constraint:

$$f(\beta_l) \leq \tilde{\psi}_l, \quad \forall l \in L \tag{3.7}$$

where $f(\beta_l)$ is a nonlinear function of β_l, the original problem becomes a nonlinear MCF problem.

Now, in addition to the bundle constraint, x_k^j is also coupled through the nonlinear function $f()$. It is clear that if we still relax the problem as we did in the previous section, we will have the dual problem:

$$\min[\sum_{l \in L} c_l \tilde{\psi}_l + \sum_{l \in L} \lambda_l (f(\beta_l) - \tilde{\psi}_l)] \tag{3.8}$$

which is neither separable in k, nor in l.

But if we treat the flow aggregation equation (3.6) as a constraint, and choose it as the target of Lagrangian relaxation, we have a Lagrangian dual problem:

$$\min[\sum_{l \in L} c_l \tilde{\psi}_l + \sum_{l \in L} \lambda_l (\beta_l - \sum_{k \in K} x_k^j \delta_j^l \alpha_k)] \tag{3.9}$$

$$s.t. \quad (3.2)(3.3)$$

Note that β_l and x_k^j become independent variables, and it is clear that (3.9) is separable.

A welcome outcome is that the dual problem is separable regardless of the form of the cost function and the additional integer constraint on x_k^j. As long as the problem is separable, the dual problem is relatively easy to solve.

So far we assume that the bundle constraint is a function of the total flow aggregation, β_l. We can further generalize the formulation to support multiple classes of traffic. Assume there are I different types of traffic. β_l^i represents the total amount of traffic type i in link l. Replace the bundle constraint 3.7 with the following:

$$f(\beta_l^1, ..., \beta_l^I) \leq \tilde{\psi}_l, \quad \forall l \in L \tag{3.10}$$

We can treat the problem as same before and relax all the flow aggregation equations.

From what we described above, the flow aggregation based relaxation is able to effectively decompose the complex nonlinear integer MCF problem. But an important question remains: will it yield high quality solutions? Since it is not possible to theoretically analyse the solution quality, we apply the method to capacity planning of multiclass IP network, and evaluate it with large number of test cases. Our exper-

iments will show consistently that the proposed method is able to produce solutions

within a satisfying few percent of the optimal solution.

Chapter 4

Capacity Planning of MultiClass IP

Networks

In this chapter, we will overview three closely related problem that will be investigated, and show how the flow aggregation based Lagrangian relaxation can be applied.

4.1 Overview

As mentioned in section 1.2, the nature of offering only best effort (BE) service has made capacity planning a straightforward matter[52] in the current Internet. Multiprotocol label switching (MPLS) [31] [76] and Differentiated services (DiffServ) [78] [71] are regarded as two key components for providing QoS in the Internet.

We will study three closely related capacity planning problems for IP networks

providing DiffServ BE and EF traffic classes. We do not include the AF traffic classes in this thesis, due to the lack of a consensus on the implementation of the AF PHB.

In the first problem, we study capacity planning of legacy IP networks, where only non-bifurcated routing is allowed for the traffic with the same destination. In the second problem, we assume the presence of MPLS, therefore the allowance of bifurcated routing. The third problem deals with the design of survivable MPLS networks, where spare capacity is required. Each problem also differs slightly in one or more of the following aspects: the choice of the cost function (linear or discrete), the definition of BE and EF traffic demand (OD based or link based,), how EF traffic is multiplexed, and how BE traffic is routed. Please see Chapters 5-7 for details. We will only depict the common assumptions and models used by those three problem in this chapter.

4.2 Previous work

Although there is no previous work specifically targeting the dimensioning and routing problems for IP networks supporting Diffserv other than our own works [89][90][88], there are many papers addressing the issues of QoS routing in general. An extensive survey can be found in [20]. But because the routing and link dimensioning

problems are closely related to each other, it is inappropriate to separate them.

There is extensive research dealing with traffic engineering issues in MPLS networks, such as [29] [56], or the LSP setup and dimensioning problem [4]. However, in those works the link capacity is fixed and not something to be optimized.

Papers where the routing and capacity assignment problems are treated simultaneously include [39] [37] [40][70] [66] [67] [6] [63]. Gerla and Kleinrock [39] presented heuristic methods based on the flow deviation algorithm [36]. Gavish and Neuman [37] proposed a Lagrangian relaxation based approach. [63] studied the network with elastic traffic and approximated the non-linear cost function by a piece-wise linear function. The networks studied in [39] [37] [63] only include one traffic class, though. Medhi and Tipper [67] proposed four approaches for reconfigurable ATM networks based on the Virtual Path concept. Even though ATM networks include multiple traffic classes, Medhi proposed a model that assumes the deterministic multiplexing of different virtual paths, which results in linear performance constraints.

Compared with previous work in the area of IP network dimensioning, the novel aspect of our problem is the fact that two traffic classes, EF and BE, with independent performance and survivability requirements, sharing the same capacity resource, which results in a complex non-linear performance constraint.

4.3   Models and assumptions

DiffServ is still under active research. Performance issues related to traffic aggre-

gation and the interaction between multiple classes are not fully understood. Those

factors impose challenges on the formulation of capacity planning problems. Given

the urgency for the need of capacity planning in DiffServ network on the one hand,

and the difficulty and complexity of the problem on the other hand, we have to resort

to simplifications and approximations.

In the capacity planning problem, we are given a network, which is defined by a set

of links L, candidate paths J, and link-path indicators $\{\delta_j^l\}$. We are also given the EF

and BE user demands. Since the purpose of EF PHB is to build a leased line type of

service, the projected EF user demands are defined in the form of *origin-destination*

(O-D) pairs (i.e., the origin and destination of each traffic source is given). Note that

what we are studying in this paper is the core network. Therefore, when we speak of

O-D pairs, we are referring to the traffic between pairs of edge routers, not between

individual hosts.

For each OD pair, we allow multiple EF demands to be coexist. For an EF

demand pair k, we differentiate between the average arrival rate, α_k, and the requested

bandwidth, ρ_k. ρ_k is usually a value between the average arrival rate and the peak rate. EF and BE demand on link l is characterized by β_{ef}^l and β_{be}^l, the average arrival rate. User demand estimation is out of the scope of this paper. We would like to point out, however, that even if demand estimation is inaccurate, a high-quality solution method for the capacity planning problem, given the estimated demands, is still very desirable.

The goal of the capacity planning problem is to find the minimum cost network that satisfies the projected user demands of both EF and BE user classes. In the case of survivable network design, the network has to have spare capacity to accommodate the traffic that is rerouted due the failure. The variables here are the path routing variable of EF traffic, and depending on the problem, the routing variable of BE traffic, x_{kj}^{class}, which shows whether OD pair k will use the candidate path j., and the discrete link capacity, $\{u_l\}$. The non-bifurcated routing model is used for EF class, where the traffic from a single EF demand pair will follow the same path between the origin and the destination. According to [26], the non-bifurcated routing assumption is necessary to ensure no packets are delivered out of order.

For each OD pair, we allow multiple EF demands to be coexist. But for

It is assumed that the requested bandwidth of a EF user demand is defined in a

way such that the performance of an EF demand will be guaranteed if it is given the requested bandwidth.

$$\eta_l \leqslant \tilde{\psi}_l, \quad \forall l \in L \tag{4.1}$$

where η_l is the total amount of EF demand, $\tilde{\psi}_l$ is the link capacity.

There are many discussions regarding the limits on the link utilization of EF user demands. EF PHB was initially defined in RFC2598 [87]. Unfortunately, [19] shows that the worst case delay jitter can be made arbitrarily large using a FIFO queue unless the amount of EF traffic was limited to a very small fraction of the total capacity. Subsequently, an alternative EF definition, RFC3246 [26], was proposed. The revised EF PHB introduces an error term E_a for the treatment of the EF aggregate, which represents the allowed worst case deviation between the actual EF packet departure time and the ideal departure time of the same packet. This revision makes it possible for the EF utilization to go up as high as 100% [94]. We assume that there is no additional constraints on the EF utilization.

How to define the performance requirement of BE traffic in the service level agreement (SLA) remains an active research topic. [64] suggests using the latency averaged over a large time scale as the primary criteria for the performance of BE traffic in

IP network service level agreements (SLAs). We pick the average delay as the sole performance measurement for BE traffic in this paper. We evaluate the performance of BE traffic on a per-link basis (i.e., not end-to-end). The value $\frac{\tilde{y}}{\psi_l}$ stands for the average transmission delay of packets. We use $\frac{\tilde{y}}{\psi_l}$ as the basis for the delay bound. Let $d_{lmax} = g_l \frac{\tilde{y}}{\psi_l}$, where g_l is a parameter defined by the network designer. The larger the value of g_l, the more bandwidth is required for link l, therefore the lower the link utilization. We assume that the performance of BE traffic is satisfactory if $d_l \leqslant d_{lmax}$. Note that the delay bound d_{lmax} remains the same under the normal condition and the failure scenarios.

Every router is modeled as a M/G/1 system with Poisson packet arrivals and an arbitrary packet length distribution. While it has been suggested that the Internet traffic is long-range dependent [74] and thus bursty, a recent work [68] shows that the network traffic can be smooth and "Poisson-like". [83] concludes, through both simulation and analytic study, that even though the traffic exhibits bursty behavior at certain time scales, the relationship between the variance and the mean is approximately linear if they are measured over larger time scales, where the traffic can be treated as if it were smooth. Our choice of the Poisson arrival model is justified because we are more concerned about the average BE performance over a large time

scale for capacity planning purposes.

From the average queueing delay formula of the priority queue [53], we obtain the performance constraint for BE traffic:

$$
\begin{aligned}
d_l &= \frac{\tilde{y}}{\tilde{\psi}_l} + \frac{\widetilde{y^2}}{2\tilde{y}} \frac{\beta_l^{ef} + \beta_l^{be}}{(\tilde{\psi}_l - \beta_l^{ef})(\tilde{\psi}_l - \beta_l^{ef} - \beta_l^{be})} \\
&\leqslant g_l \frac{\tilde{y}}{\tilde{\psi}_l}
\end{aligned}
\tag{4.2}
$$

With some rearrangement, (4.2) yields

$$
\tilde{\psi}_l \geqslant f(\beta_l^{ef}, \beta_l^{be})
\tag{4.3}
$$

where

$$
f(\beta_l^{ef}, \beta_l^{be}) = \beta_l^{ef} + \frac{\beta_l^{be}}{2} + \frac{\widetilde{y^2}(\beta_l^{ef} + \beta_l^{be})}{4(\tilde{y})^2(g_l - 1)} + \frac{1}{2}\sqrt{(2\beta_l^{ef} + \beta_l^{be} + \frac{\widetilde{y^2}(\beta_l^{ef} + \beta_l^{be})}{2(\tilde{y})^2(g_l - 1)})^2 - 4\beta_l^{be}(\beta_l^{be} + \beta_l^{ef})}.
$$

In order to have a meaningful solution for constraint (4.3), $\tilde{\psi}_l > \beta_l^{ef} + \beta_l^{be}$ is required.

4.4   Solution methods

The basic solution approaches is to apply flow aggregation based Lagrangian relaxation as shown in section 3.3. Due to the different in the structure of three problem, please refer to their corresponding chapter for details in the way the problem is relaxed,

We pick the most commonly used subgradient method to solve the Lagrangian multiplier problem. For the detailed description of the subgradient method and choice of parameters, please refer to the book [2] by Ahuja, Magnanti ,and Orlin.

At each iteration, the solution of path routing variables for the primal problem can be generated from the solution of dual problem. Subsequently he value of link capacity can be computed, the primary objective function can be obtained. As the iteration proceeds, we store the best solution found so far for the primal problem (P). In this way, we are always able to obtain a feasible solution. We set the maximum iterations to 400 in the implementation.

As mentioned earlier, the benefit of Lagrangian optimization procedures is that the solution of the dual problem provides a lower bound for the primal problem. Therefore, the solution quality can be assessed by the duality gap, which is the difference between the solutions of problem (P) and problem (D). Note that because the duality gap is always no smaller than the actual difference between the obtained feasible solution and the optimal solution, it is a conservative estimate of the solution quality.

Chapter 5

Legacy IP Networks Supporting DiffServ

EF and BE Classes

5.1   Models and formulation

In this chapter, we address the problem of capacity planning for DiffServ networks with only EF and best effort traffic classes. The problem is formulated as an optimization problem, where we jointly select the route for each EF user demand pair, and assign a discrete capacity value for each link to minimize the total link cost, subject to the performance constraints of both EF and BE classes. The non-bifurcated routing model is used for EF class, where the traffic from a single EF demand pair will follow the same path between the origin and the destination. While the performance constraint of EF traffic is only represented by a bandwidth requirement, the

performance constraint of the BE class is characterized by the average delay in each

link. Queueing is modeled as M/G/1 strict priority queues.

In this problem we take the approach of defining BE user demands for each *link*

instead of OD based.

We assume the cost function to be linear in this problem. Also assume that the

link capacity if much larger than the EF demand, so that there is no concern about

the EF performance.

The formal problem definition is presented below.

$$\min(\sum_{l \in L} c_l u_l \gamma) \tag{5.1}$$

where u_l is the number of units of capacity needed on link $l \in L$. u_l is integer. γ is

the size of a unit capacity.

Subject to:

$$u_l \gamma \geqslant f(\beta_{ef}^l), \ \forall l \in L \tag{5.2}$$

$$\text{where } \beta_{ef}^l = \sum_{k \in K} \alpha_k \sum_{j \in J} \delta_j^l x_{kj} \tag{5.3}$$

$$u_l \geqslant 0 \text{ and integer}, \ \forall l \in L \tag{5.4}$$

$$\sum_{j \in J} x_{kj} = 1, \ \forall k \in K \tag{5.5}$$

$$x_{kj} = 0/1, \ \forall j \in J, \ k \in K \tag{5.6}$$

Constraint (5.2) ensures the performance of EF and BE traffic. (5.4) imposes a discrete constraint on the link capacities. (5.5) and (5.6) ensure that all traffic from one EF O-D pair will follow one single path.

The fact that the function $f(\beta_{ef}^l)$ is increasing in β_{ef}^l enables us to rewrite the constraint (5.3). The resulting problem is:

$$\min(\sum_{l \in L} c_l u_l \gamma) \tag{5.7}$$

Subject to:

$$\beta_{ef}^l \geqslant \sum_{k \in K} \alpha_k \sum_{j \in J} \delta_j^l x_{kj} \tag{5.8}$$

$$(5.2), \ (5.4), \ (5.5), \ and \ (5.6)$$

We refer to the problem defined by (5.7,5.8,5.2,5.4,5.5,5.6) as problem (P).

5.2 Solution method

Let $\lambda = (\lambda_l)$ be the dual multiplier associated with the flow aggregation constraint (5.8). Then the Lagrangian can be expressed as

$$
\begin{aligned}
L(x, u, \lambda) &= \sum_{l \in L} c_l u_l \gamma + \sum_{l \in L} \lambda_l (\beta_{ef}^l - \sum_{k \in K} \alpha_k \sum_{j \in J} \delta_j^l x_{kj}) \\
&= \sum_{l \in L} (c_l u_l \gamma + \lambda_l \beta_{ef}^l) + \sum_{k \in K} \sum_{l \in L} -\alpha_k \lambda_l \sum_{j \in J} \delta_j^l x_{kj}
\end{aligned} \tag{5.9}
$$

The Lagrangian dual problem (D) is then:

$$\max_{\lambda \leqslant 0} h(\lambda) \tag{5.10}$$

$$\text{where } h(\lambda) = \min_{x,u} L(x, u, \lambda) \tag{5.11}$$

Subject to the constraint:

$$u_l \gamma \geqslant f(\beta_{ef}^l), \ \forall l \in L \tag{5.12}$$

$$u_l \geqslant 0 \text{ and integer}, \ \forall l \in L \tag{5.13}$$

$$\sum_{j \in J} x_{kj} = 1, \ \forall k \in K \tag{5.14}$$

$$x_{kj} = 0/1, \ \forall j \in J, \ k \in K \tag{5.15}$$

For a given λ, the Lagrangian is separable in x and u. (5.11) is reduced to solving

two independent subproblems,

$$\min_{x,u} L(x, u, \lambda) = \min_u L_1(u, \lambda) + \min_x L_2(x, \lambda) \tag{5.16}$$

Subproblem (D1):

$$\min_u L_1(u, \lambda) = \min_u \{ \sum_{l \in L} (c_l u_l \gamma + \lambda_l \beta_{ef}^l) \} \tag{5.17}$$

Subject to the constraint:

$$u_l \gamma \geqslant f(\beta_{ef}^l), \ \forall l \in L \tag{5.18}$$

$$u_l \geqslant 0 \text{ and integer}, \ \forall l \in L \tag{5.19}$$

Subproblem (D2):

$$\min_{x} L_1(x, \lambda) = \min_{x} (\sum_{k \in K} \sum_{l \in L} -\alpha_k \lambda_l \sum_{j \in J} \delta^l_j x_{kj}) \tag{5.20}$$

Subject to the constraint:

$$\sum_{j \in J} x_{kj} = 1, \ \forall k \in K \tag{5.21}$$

$$x_{kj} = 0/1, \ \forall j \in J, \ k \in K \tag{5.22}$$

Subproblem (D1) can be separated into L problems, one for each link. The problem for each link l can be expressed as:

$$\min_{u} (c_l u_l \gamma + \lambda_l \beta^l_{ef}) \tag{5.23}$$

Subject to the constraint:

$$u_l \gamma \geqslant f(\beta^l_{ef}) \tag{5.24}$$

$$u_l \geqslant 0 \text{ and integer,} \tag{5.25}$$

Because $f(\beta^l_{ef})$ is increasing in β^l_{ef} and $\lambda_l \leqslant 0$, the problem (5.23) can be simply rewritten as:

$$\min_{u} (c_l u_l \gamma + \lambda_l f^{-1}(u_l \gamma)), \ u_l \geqslant 0 \text{ and integer} \tag{5.26}$$

The value of u_l^*, which satisfies the equation (5.27) below, can be obtained numerically through any one-dimensional optimization method, such as Newton's method [10].

$$\frac{dz(u_l)}{du_l} = 0, u_l \geqslant 0 \qquad (5.27)$$

where $z(u_l) = c_l u_l \gamma + \lambda_l f^{-1}(u_l \gamma)$

Let u_{l1}^* and u_{l2}^* be the two non-negative integers closest to u_l^*. We know that the solution of u_l to the problem (5.26) is either u_{l1}^* or u_{l2}^*. Among them, the one that minimizes (5.26) is picked as the final solution.

Subproblem (D2) can be separated into K subproblems, one for each O-D pair. The problem for each k is:

$$\min_x (\sum_{l \in L} -\alpha_k \lambda_l \sum_{j \in J} \delta_j^l x_{kj}) \qquad (5.28)$$

Subject to the constraint:

$$\sum_{j \in J} x_{kj} = 1 \qquad (5.29)$$

$$x_{kj} = 0/1, \ \forall j \in J \qquad (5.30)$$

The solution of subproblem (D2) is then easily obtained by setting $x_{kj^*} = 1$ for j^*

satisfying:

$$P(j^*) = \min_j(P(j)) \tag{5.31}$$

$$\text{where } P(j) = \sum_{l \in L} -\alpha_k \lambda_l \delta_j^l x_{kj}$$

For a given initial λ, once we solve the problem (D), a dual subgradient is computed

as follows:

$$\omega_l = \beta_{ef}^l - \sum_{k \in K} \alpha_k \sum_{j \in J} \delta_j^l x_{kj}, \forall l \in L \tag{5.32}$$

The subsequent values of the Lagrangian multipliers are updated:

$$\lambda_l \leftarrow \min(0, \lambda_l + t\omega_l), \forall l \in L \tag{5.33}$$

where the step size, t, is defined by:

$$t = \phi \frac{h^*(u) - h(u)}{||\omega||^2} \tag{5.34}$$

where $h^*(u)$ is the value of the best feasible solution found so far, and ϕ is a scalar

between 0 and 2. ϕ is set to 2 initially in our study and is halved if the solution does

not improve in 10 iterations.

5.3   Experimental results

In this section, we present numerical results based on experimentation. The ob-

jective of our experiment is to evaluate the solution quality and running time of

the algorithm. The program is implemented in C and the computational work is performed on a Pentium IV 2.4GHz PC with 512M memory, running Redhat Linux 7.2.

The network topologies are generated using the Georgia Tech Internetwork Topology Models(GT-IMT) [95]. Link cost is set to be proportional to its length.

The locations of originations and destinations are randomly selected. For each O-D pair, 10 candidate paths are calculated using Yen's K-shortest path algorithm [93]. According to our experimental results, more than 99% of the time, the final solution is chosen among the 5 shortest candidate paths. Therefore, 10 candidate paths are considered adequate. Having more than 10 candidate paths will have minimal impact on the solution quality, while significantly increases the running time.

If not specified, EF demands are randomly generated with a uniform distribution from 0 Mbps to 10 Mbps, while the average BE traffic load of each link is also uniformly distributed from 30 Mbps to 100 Mbps. The link unit γ is set to be 45 Mbps. We use $\tilde{y} = 4396$ (bits) and $\tilde{y^2} = 22790170$ (bits^2) for all the test cases. They are calculated based on a traffic trace (AIX-1014985286-1) from the NLANR Passive Measurement and Analysis project [1].

The BE delay bound factor, g_l, is set to 2 for all links in our experiments. In prac-

tice, g_l should be carefully chosen to reflect the desired BE performance and the link

utilization. Figure (5.1) shows the value of $f(\beta_{ef}^l)$ with respect to β_{ef}^l, when g_l=1.5,

2, 4, and 8. As can be seen from the figure, the link utilization varies significantly as

g_l changes.
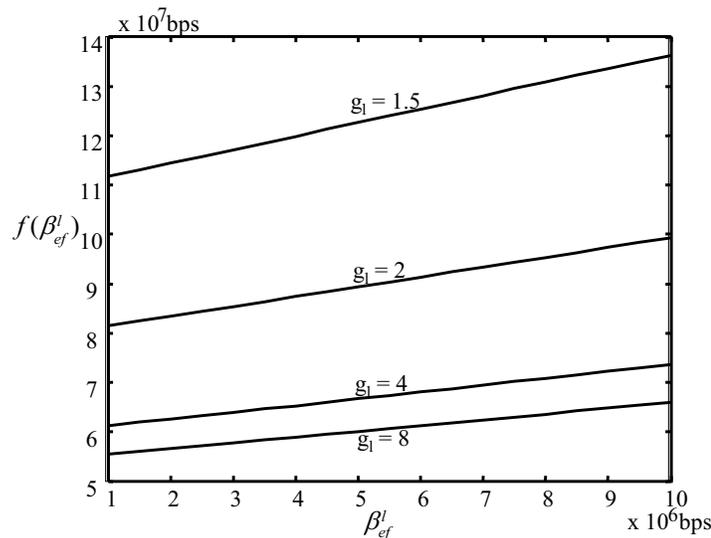
Figure 5.1: $f(\beta_{ef}^l)$ vs. β_{ef}^l, $(\beta_{be}^l = 5 \times 10^7$ bps$)$

The algorithm was tested on 8 different sizes of networks, ranging from 10 nodes

to 1000 nodes. Some details of the network topologies are listed in Table 6.1. To

obtain confidence intervals, we generate 30 different topologies for each network size,

with the same number of nodes, links, and O-D pairs.

The solution quality is represented by the percentage difference between the solu-

tion of the primal problem and the dual problem.

$$\text{Solution Quality} \;=\; \left| \frac{s_p - s_d}{s_d} \right| \tag{5.35}$$

where s_p and s_d are the solutions of primal problem and dual problem respectively.

Table 5.1: Network topology information and experimental results

Node Number	Link Number	O-D Number	Solution(%) Quality (%)	Running Time (sec)
10	25	30	(0.25, 5.24)	(0.99, 1.16)
20	50	90	(0, 5.04)	(4.79, 5.60)
50	125	350	(0.13, 4.13)	(23.88, 27.59)
100	250	1000	(0.16, 5.49)	(57.08, 68.14)
200	500	3000	(0, 4.51)	(491.90, 562.88)
500	1250	12000	(0, 4.12)	(5000.45, 6360.11)
700	1750	20000	(0, 3.28)	(10367.61, 13217.79)
1000	2500	40000	(0, 3.91)	(27905.69, 35644.40)

Table 6.1 shows the running time and solution quality of various network sizes with 95% confidence intervals. In all 240 test cases, the algorithm converges without difficulty. It is easy to see from the table that the Lagrangian Relaxation together with the subgradient method produces reasonable results as the duality gap is bounded by no more than 6%. Given the large number of networks being tested, we are confident that the solution should have good quality for other sizes of networks.

Because capacity planning is usually performed on the time scale of weeks to months, the running time of the algorithm is not the most critical factor. But it is
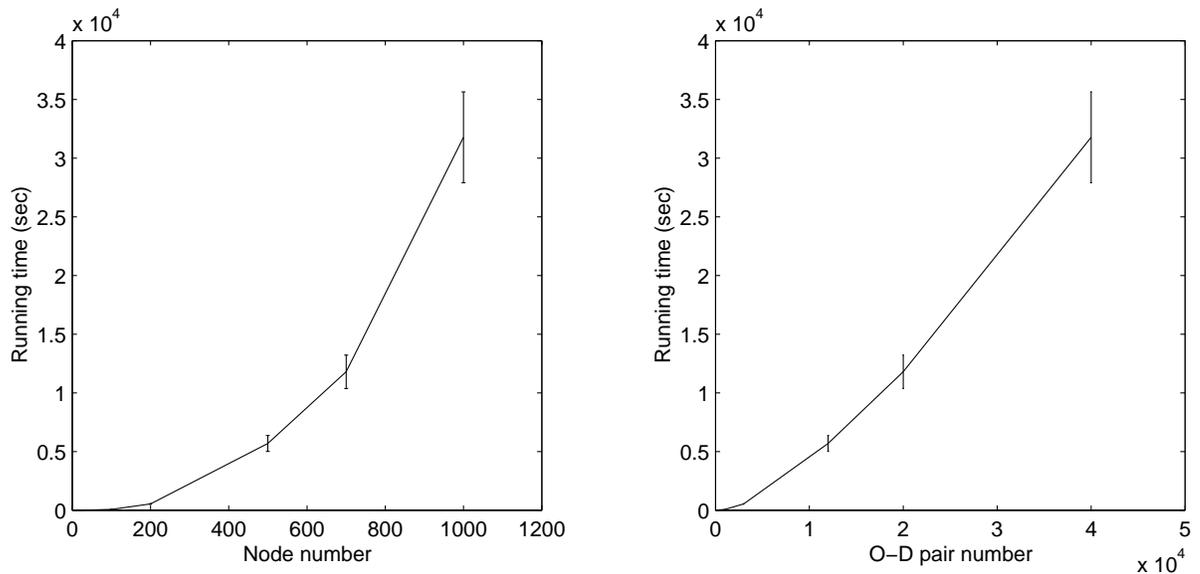
Figure 5.2: Running Time

still desirable to know how the running time scales up with respect to the network

size. As can be seen from the Figure (5.2), while the running time goes up more

quickly than the number of nodes, it increases linearly with respect to the number of

O-D pairs in our test cases. This is understandable; since $K >> L$, when the network

size increases, the dominant portion of the running time is spent on the subproblem

(D2), which in turn has K subproblems. The running time of each one of those K

subproblems is insensitive to the size of the network, due to the fixed number of

candidate paths and the reuse of the link cost throughout all iterations. The size of

the largest network evaluated in this paper is representative of a large network, and

is much larger than the test cases used in most work on capacity planning. It is fair to predict that the running time of the algorithm will stay reasonable for practical sized networks.

Chapter 6

MPLS Networks supporting Diffserv EF and BE classes

6.1 Models and formulation

In this chapter, we address the problem of link dimensioning and path optimization for MPLS networks providing DiffServ EF and BE traffic classes. The problem is formulated as an optimization problem, where we jointly select the routes for edge to edge EF and BE user demand pairs, and assign a discrete capacity value for each link. The goal is to minimize the total link cost, subject to the performance constraints of both EF and BE classes. The non-bifurcated routing model is used for the EF class as required by [26], so that the traffic from a single EF demand pair will follow the same LSP between the origin and the destination. Traffic in the BE class is allowed to

be split across multiple LSPs. While the performance constraint of EF traffic is only

represented by a bandwidth requirement, the performance constraint of the BE class

is characterized by the average delay in each link. Queueing is modeled as M/G/1

strict priority queues.

We assume the cost function to be discrete. let T_l be the index of link type for

link $l \in L$, u_{lt} be the link type decision variable, $u_{lt} = 1$ if link type t is used for link

$l \in L$, 0 other wise. Only one link type can be chosen. ψ_{lt} is defined as the size of

the capacity of link type t, $\tilde{\psi}_l$ as the total capacity of link l.

Let M_k be the set of EF demands for the same O-D pair k. For an EF demand

$m, m \in M_k$. The EF path routing variable is defined by x_{kmj}^{ef}.

We have the flow aggregation equation:

$$\beta_l^{ef} = \sum_{k \in K} \sum_{m \in M_k} \alpha_{km}^{ef} \sum_{j \in J_k} x_{kmj}^{ef} \delta_j^l \tag{6.1}$$

For each O-D pair k , we define one BE demand pair. Like before, we allow an

arbitrary portion of the BE demand to route through any candidate LSP. Because

of the connectionless nature of IP traffic, it is unlikely that all the BE demand can

be clearly mapped to specific O-D pairs. We introduce another variable γ_l, which is

the average arrival rate of extra BE traffic in link l besides that from the BE demand

pairs $\{k : k \in K\}$. Thus the total BE load (average arrival rate) on link l is:

$$\beta_l^{be} = \gamma_l + \sum_{k \in K} \alpha_k^{be} \sum_{j \in J_k} x_{kj}^{be} \delta_j^l \tag{6.2}$$

The capacity and the cost of link l, $\tilde{\psi}_l$ and $\tilde{C}_l$ respectively, are:

$$\tilde{\psi}_l = \sum_{t \in T} u_{lt} \psi_{lt}, \qquad \tilde{C}_l = \sum_{t \in T} u_{lt} C_{lt} \tag{6.3}$$

There is no linear relationship assumed between C_{lt} and ψ_{lt}, therefore $\tilde{C}_l$ is not

necessarily a linear function of $\tilde{\psi}_l$.

The formal problem definition is presented below.

Given: $K, M_k, L, J_k, \alpha_{km}^{ef}, \rho_{km}^{ef}, \alpha_k^{be}, \gamma_l, \delta_j^l, T, \psi_t, C_{lt}$

Variable: $x_{kmj}^{ef}, x_{kj}^{be}, u_{lt}$

Goal:

$$\min \sum_{l \in L} \tilde{C}_l \tag{6.4}$$

Subject to:

$$\tilde{\psi}_l \geqslant f(\beta_{ef}^l) \tag{6.5}$$

$$x_{kmj}^{ef} = 0/1, \sum_{j \in J_k} x_{kmj}^{ef} = 1 \tag{6.6}$$

$$\sum_{j \in J_k} x_{kj}^{be} = 1 \tag{6.7}$$

$$u_{lt} = 0/1, \sum_{t \in T_l} u_{lt} = 1 \tag{6.8}$$

Constraint (6.5) ensures the performance of BE traffic. (6.8) imposes a discrete constraint on the link capacities. (6.6) ensures that all traffic from one EF O-D pair will follow one single path.

Because C_l is a non-decreasing function of β_l^{ef} and β_l^{be}, this problem can be reformulated as:

$$\min \sum_{l \in L} \tilde{C}_l \tag{6.9}$$

Subject to (6.5) (6.6) (6.7) (6.8) and:

$$\beta_l^{ef} \geqslant \sum_{k \in K} \sum_{m \in M_k} \alpha_{km}^{ef} \sum_{j \in J_k} x_{kmj}^{ef} \delta_j^l \tag{6.10}$$

$$\beta_l^{be} \geqslant \gamma_l + \sum_{k \in K} \alpha_k^{be} \sum_{j \in J_k} x_{kj}^{be} \delta_j^l \tag{6.11}$$

We refer to the problem defined by (7.8, 6.5, 6.6, 6.7, 6.8, 6.10, 6.11) as problem (P) in the rest of this chapter.

6.2 Solution method

Like what we did in the previous chapter, we relax the flow aggregation constraint, (6.10) and (6.11), and we have the Lagrangian as:

$$L(x_{kmj}^{ef}, x_{kj}^{be}, u_{lt}, \lambda_l^{ef}, \lambda_l^{be}) = \sum_l \tilde{C}_l - \sum_l \lambda_l^{ef} (\beta_l^{ef} - \sum_{k \in K} \sum_{m \in M_k} \alpha_{km}^{ef} \sum_{j \in J_k} x_{kmj}^{ef} \delta_j^l)$$

$$-\sum_l \lambda_l^{be}(\beta_l^{be} - \gamma_l - \sum_{k \in K} \alpha_k^{be} \sum_{j \in J_k} x_{kj}^{be} \delta_j^l) \tag{6.12}$$

The Lagrangian dual problem (D) is then:

$$\max_{\lambda_l^{ef}, \lambda_l^{be} \geqslant 0} h(\lambda_l^{ef}, \lambda_l^{be}) \tag{6.13}$$

where:

$$h(\lambda_l^{ef}, \lambda_l^{be}) = \min_{x_{kmj}^{ef}, x_{kj}^{be}, u_{lt}} L(x_{kmj}^{ef}, x_{kj}^{be}, u_{lt}, \lambda_l^{ef}, \lambda_l^{be}) \tag{6.14}$$

Since $\beta_l^{ef}, \beta_l^{be}$ and $x_{kmj}^{ef}, x_{kj}^{be}$ are independent variables,

$$
\begin{aligned}
\min L &= \min \sum_l [\tilde{C}_l - \lambda_l^{ef} \beta_l^{ef} - \lambda_l^{be}(\beta_l^{be} - \gamma_l)] + \min \sum_{k \in K} \sum_{m \in M_k} \alpha_{km}^{ef} \sum_{j \in J_k} \sum_l \lambda_l^{ef} x_{kmj}^{ef} \delta_j^l \\
&\quad + \min \sum_{k \in K} \alpha_k^{be} \sum_{j \in J_k} \sum_l \lambda_l^{be} x_{kj}^{be} \delta_j^l \tag{6.15} \\
&= \sum_l \min[\tilde{C}_l - \lambda_l^{ef} \beta_l^{ef} - \lambda_l^{be}(\beta_l^{be} - \gamma_l)] + \sum_{k \in K} \sum_{m \in M_k} \min(\alpha_{km}^{ef} \sum_{j \in J_k} \sum_l \lambda_l^{ef} x_{kmj}^{ef} \delta_j^l) \\
&\quad + \sum_{k \in K} \min(\alpha_k^{be} \sum_{j \in J_k} \sum_l \lambda_l^{be} x_{kj}^{be} \delta_j^l) \tag{6.16}
\end{aligned}
$$

Equation (6.16) shows that the problem (6.12) can be separated into the following three subproblems:

Subproblem (i):

$$\min[\tilde{C}_l - \lambda_l^{ef} \beta_l^{ef} - \lambda_l^{be}(\beta_l^{be} - \gamma_l)] \tag{6.17}$$

$$\text{Subject to: } f(\beta_l^{ef}, \beta_l^{be}, \tilde{\psi}_l) \leqslant d_l^{max} \tag{6.18}$$

Subproblem (i) can be solved by the gradient projection method [15].

Subproblem (ii):

$$\min(\alpha_{km}^{ef} \sum_{j \in J_k} \sum_l \lambda_l^{ef} x_{kmj}^{ef} \delta_j^l) \tag{6.19}$$

This is simply a shortest path problem where the cost of link l is set to λ_l^{ef}. The solution is to let $x_{kmj^*}^{ef} = 1$ for j^* satisfying:

$$P(j^*) = \min_j(P(j)) \tag{6.20}$$

where $P(j) = \sum_l \lambda_l^{ef} x_{kmj}^{ef} \delta_j^l$

Subproblem (iii):

$$\min(\alpha_k^{be} \sum_{j \in J_k} \sum_l \lambda_l^{be} x_{kj}^{be} \delta_j^l) \tag{6.21}$$

Similar to Subproblem (ii), the solution is to set $x_{kj^*}^{be}$ to 1 for j^* satisfying:

$$Q(j^*) = \min_j(Q(j)) \tag{6.22}$$

where $Q(j) = \sum_l \lambda_l^{be} x_{kj}^{be} \delta_j^l$

The subgradient method is used to update λ_l^{ef} and λ_l^{be}. Please referred to the section 5.2 for a detailed description of the procedure and the choices of parameters.

At each iteration, the solution of $x_{kmj^*}^{ef}$ and $x_{kj^*}^{be}$ for the primal problem (P) can be obtained from the solution of subproblem (ii) and(iii). The link capacity $\tilde{\psi}_l$ can

be computed according to (6.5). Consequently, the primary objective function can be derived. As the iteration proceeds, we store the best solution found so far for the primal problem (P). In this way, we are always able to obtain a feasible solution. The maximum number of iterations is set to 400 in the implementation.

6.3   Experimental results

In this section, we present numerical results based on experimentation. Just like previous chapter, the program is implemented in C and the computation is performed on a Pentium IV 2.4GHz PC with 512M memory, running Redhat Linux 7.2.

The network topologies are generated using the Georgia Tech Internetwork Topology Models (GT-ITM) [95]. The locations of origins and destinations are randomly selected. For each O-D pair, 10 candidate paths are calculated using Yen's K-shortest path algorithm [93].

If not specified, EF and BE demand pairs are randomly generated with a uniform distribution from 0 Mbps to 10 Mbps, while the average BE traffic load of each link is also uniformly distributed from 30 Mbps to 100 Mbps. The number of EF demands for the same O-D pair is uniformly distributed from 1 to 10. The number of candidate link types for link l is uniformly distributed from 5 to 10. The capacities of link types

are set to be multiples of 45Mbps, while the costs of the link types for link l are randomly generated in such a way that the cost goes higher and the unit cost per Mbps goes down as the link capacity increases. We use average packet size $\tilde{y} = 4396$ bits and second moment of packet size $\tilde{y^2} = 22790170$ bits^2 for all the test cases.

g_l is set to 2 for all links in our experiments. The average link utilization is about 60% when g_l equals 2.

The algorithm was tested on 8 different sizes of networks, ranging from 10 nodes to 1000 nodes. Some details of the network topologies are listed in Table 6.1. Note that the O-D demand number shown in Table 6.1 includes both EF and BE demands. To obtain confidence intervals, we generate 30 different topologies for each network size, with the same number of nodes, links, and O-D pairs.

Table 6.1: Network topology information and experimental results

Node Number	Link Number	O-D Number	Solution(%) Quality (%)	Running Time (sec)
10	25	30	(0.15, 2.24)	(3.79, 5.20)
20	50	90	(0.01, 3.96)	(15.84, 22.68)
50	125	350	(0, 2.78)	(75.40, 98.38)
100	250	1000	(0.16, 3.51)	(207.55, 241.49)
200	500	3000	(0, 2.15)	(1430.79, 1935.30)
500	1250	12000	(0, 3.02)	(8895.36, 12237.93)
700	1750	20000	(0, 2.41)	(18470.24, 23918.52)
1000	2500	40000	(0, 2.93)	(34807.64, 44630.93)

Table 6.1 shows the running time and Duality Gap (where a value of 0 means optimal quality) of various network sizes, expressed in terms of the 95% confidence intervals. In all 240 test cases, the algorithm converges without difficulty. It is easy to see from the table that the Lagrangian Relaxation together with the subgradient method produces reasonable results as the duality gap is bounded by no more than 4%. Note the primal problem itself is approximated when reducing the size of candidate path set for all possible path set. But according to our experimental results, more than 99% of the time, the final solution is chosen among the 5 shortest candidate paths. Therefore, 10 candidate paths are considered adequate. Having more than 10 candidate paths will have minimal impact on the solution quality, while significantly increasing the running time. Given the large number of networks being tested, we are confident that the solution should have good quality for other sizes of networks.

Chapter 7

Survivable MPLS Networks Supporting

DiffServ EF and BE Classes

7.1 Overview of survivable network design

The last few years witnessed the explosive growth of mission-critical data traffic

carried by the Internet, and the emergence of general societal dependencies on the

Internet. We are now almost as dependent on the availability of the communication

networks as on other basic infrastructures, such as roads, water, and power. With

up to 100 terabits per second of data traffic carried in a single fiber with DWDM,

failure of a single fiber could be catastrophic. Despite efforts to physically protect the

fiber optic cables, cable cuts are surprisingly frequent according to FCC. Survivable

network design, which refers to the incorporation of survivability strategies to mitigate

the impact of the given failure scenarios, has become a vital part of network design, not an afterthought.

Current network architecture is moving toward a two layers structure where IP/MPLS [31] [76] is on top of optical WDM networks. Traditional, network resilience primarily relies upon the functionalities provided by SONET and ATM [92][86]. Both of those are gradually being phasing out of the network. As mentioned in [41], most ISPs rely on the IP layer when failure occurred.

MPLS-based recovery has become a viable solution because of the faster restoration time than IP rerouting. There are many works that investigated the recovery schemes in MPLS networks [30]. In the MPLS layer protection, the failure is detected either by the MPLS layer detection mechanism such as "exchange of Hello message", or the signaling message propagated from the lower layer [30]. Once a failure is detected, protection switching based recovery mechanisms are used. A LSP is protected by a pre-established recovery path or path segment. The resource allocated to the recovery path, or so called spare capacity, may be fully available to preemptible low priority traffic. The spare capacity can be shared by multiple recovery paths corresponding to different failure scenarios, to further reduce the resource [49]redundancy. With MPLS-based recovery,it is possible to differentiate the level of protection for dif-

ferent classes of service [9]. For example, Expedited Forwarding (EF) based service, such as virtual lease line service, can be supported by link protection with reserve path that offers 100% restorability. Best effort (BE) traffic, on the other hand, may not be fully restorable or simply rely on IP rerouting without assigning any spare capacity to the network.

The cost of redundancy for survivability can be very high, compared to a corresponding network designed only to serve the working demands under nominal conditions. One of the major interests in survivable network design has been providing the cost-efficient resource allocation.

Unfortunately, majority works of the survivable network design have been concentrated on the classic spare capacity allocation problem (SCA) [28][59][54][55]. The SCA problem assumes that the information of an operational network is given, the attention is given only to the calculation of backup paths and spare capacities, not the primary path and total capacity. But it has been shown in [49] that even though optimizing working path and backup path together is computationally more expensive, it has significant advantage on the total capacity savings. Since capacity planning is usually performed on the time scale of weeks to months, we can afford extra running time in exchange of the lower network cost.

With the popularization of e-commerce and new value-added service over IP, it is imperative for the next generation Internet to offer Quality of Service (QoS), the ability to support multiple traffic classes with different levels of performance assurance. MPLS and Differentiated Service [78][71] are regarded as two key components of QoS. There are works of survivable network design touched on the issues of multi-classes traffic, such as the architecture aspect of differentiated survivability [9]. But no quantitative treatment of the subject has been done so far.

7.2 Models and formulation

In this chapter, we study the issued of capacity planning of survivable MPLS networks. The targeted MPLS networks will be providing DiffServ EF and BE traffic classes.

We focus on the problem of resource dimensioning and traffic routing and assume that the network topology is given. The problem is formulated as an optimization problem, where we jointly select the primary and backup LSPs for edge to edge EF and BE user demand pairs, and assign a discrete capacity value for each link. The goal is to minimize the total link cost, subject to the performance and survivability constraints of both EF and BE classes. The non-bifurcated (i.e., single-path) routing

model is used for the EF class as required by [26], so that the traffic from a single EF demand pair will follow the same LSP between the origin and the destination. Traffic in the BE class is allowed to be split across multiple LSPs. The performance constraint of EF traffic is only represented by a bandwidth requirement as specified in [26]. Only single link failure case is studied in this chapter. We assume that each EF user demand is assigned a primary LSP as well as a backup LSP with enough preempliable bandwidth, therefore it will not be effected by any single failure. The performance as well as survivability constraints of the BE class is characterized by the upper limit on the average delay in each link under any single failure. Queueing is modeled as M/G/1 strict priority queues. Rest of the assumptions are the same as Chapter 6.

Compared with the problem presented in Chapter 6, several new notations are introduced to represent the variables relates to the backup paths. B_{kj} is the set of candidate backup path, which are link disjoint from j. δ_b^l represents the link-path indicator. $\delta_b^l = 1$ if backup path b uses link l, 0 otherwise. $\hat{x}_{jkb}$ is the backup path routing variable, 1 if backup path $b \in B_{kj}$ is used by path $j \in J_k$. r is defined as the BE traffic restoration level.

The formal problem definition is presented below.

Given: $L, K, T_l, \psi_{lt}, C_{lt}, J_k, \delta_j^l, B_{kj}, \delta_b^l, M_k, \alpha_{km}^{ef}, \rho_{km}^{ef}, \alpha_k^{be}, d_{lmax}, r, g_l, \tilde{y}, \widetilde{y^2}$

Variable: $u_{lt}, x_{kmj}^{ef}, x_{kj}^{be}, \hat{x}_{jbk}^{ef}, \hat{x}_{jbk}^{be}$

Goal:

$$\min \sum_{l \in L} \tilde{C}_{lt}$$

Subject to:

$$u_{lt} = 0/1, \sum_{t \in T_l} u_{lt} = 1 \tag{7.1}$$

$$x_{kmj}^{ef} = 0/1, \sum_{j \in J_k} x_{kmj}^{ef} = 1 \tag{7.2}$$

$$\sum_{j \in J_k} x_{kj}^{be} = 1 \tag{7.3}$$

$$\hat{x}_{jkb}^{ef} = 0/1, \sum_{b \in B_{kj}} \hat{x}_{jkb}^{ef} = 1 \tag{7.4}$$

$$\hat{x}_{jkb}^{be} = 0/1, \sum_{b \in B_{kj}} \hat{x}_{jkb}^{be} = 1 \tag{7.5}$$

$$\tilde{\psi}_l \geqslant \eta_l \tag{7.6}$$

$$\tilde{\psi}_l \geqslant f(\beta_l^{ef}, \beta_l^{be}) \tag{7.7}$$

(7.1) imposes a discrete constraint on the link capacities. Constraint (7.2) ensures that all traffic from one EF O-D pair will follow one single path. (7.4)(7.5) denotes that only one backup path will be used for each working path. Constraint (7.6)(7.7) ensures the performance of EF and BE traffic perspectively.

Because $\tilde{C}_l$ is non-decreasing with respect to β_l^{ef}, η_l, and β_l^{be}, this problem can be

reformulated as:

$$\min \sum_{l \in L} \tilde{C}_l \tag{7.8}$$

Subject to (7.1)(7.2)(7.3)(7.4)(7.5) (7.6)(7.7) and:

$$\eta_l \;\geqslant\; \sum_{k \in K} \sum_{m \in M_k} \rho_{km}^{ef} \sum_{j \in J_k} (x_{kmj}^{ef} \delta_j^l + \sum_{b \in B_{kj}} x_{kmj}^{ef} \delta_j^F \hat{x}_{jkb}^{ef} \delta_b^l) \tag{7.9}$$

$$\beta_l^{ef} \;\geqslant\; \sum_{k \in K} \sum_{m \in M_k} \alpha_{km}^{ef} \sum_{j \in J_k} (x_{kmj}^{ef} \delta_j^l + \sum_{b \in B_{kj}} x_{kmj}^{ef} \delta_j^F \hat{x}_{jkb}^{ef} \delta_b^l) \tag{7.10}$$

$$\beta_l^{be} \;\geqslant\; \sum_{k \in K} \alpha_k^{be} \sum_{j \in J_k} (x_{kj}^{be} \delta_j^l + r \sum_{b \in B_{kj}} x_{kj}^{be} \delta_j^F \hat{x}_{jkb}^{be} \delta_b^l) \tag{7.11}$$

We refer to the problem defined by (7.8,7.1,7.2,7.3,7.4,7.5,7.6,7.7,7.9, 7.10, 7.11)

as problem (P) in the rest of this chapter.

7.3 Solution method

Apply the flow aggregation based relaxation, relax (7.9)(7.10) and (7.11), then we

have the Lagrangian as:

$$L(x_{kmj}^{ef}, x_{kj}^{be}, \hat{x}_{jbk}^{ef}, \hat{x}_{jbk}^{be}, u_{lt}, \lambda_l^{ef}, \lambda_l^{\eta}, \lambda_l^{be})$$

$$= \sum_l \tilde{C}_l - \sum_l \lambda_l^{\eta} [\eta_l - \sum_{k \in K} \sum_{m \in M_k} \rho_{km}^{ef} \sum_{j \in J_k} (x_{kmj}^{ef} \delta_j^l + \sum_{b \in B_{kj}} x_{kmj}^{ef} \delta_j^F \hat{x}_{jkb}^{ef} \delta_b^l)]$$

$$- \sum_l \lambda_l^{ef} [\beta_l^{ef} - \sum_{k \in K} \sum_{m \in M_k} \alpha_{km}^{ef} \sum_{j \in J_k} (x_{kmj}^{ef} \delta_j^l + \sum_{b \in B_{kj}} x_{kmj}^{ef} \delta_j^F \hat{x}_{jkb}^{ef} \delta_b^l)]$$

$$- \sum_l \lambda_l^{be} [\beta_l^{be} - \sum_{k \in K} \alpha_k^{be} \sum_{j \in J_k} (x_{kj}^{be} \delta_j^l + r \sum_{b \in B_{kj}} x_{kj}^{be} \delta_j^F \hat{x}_{jkb}^{be} \delta_b^l)]$$

$$
= \sum_l (\tilde{C}_l - \lambda_l^\eta \eta_l - \lambda_l^{ef} \beta_l^{ef} - \lambda_l^{be} \beta_l^{be})
$$

$$
+ \sum_{k \in K} \sum_{m \in M_k} \sum_{j \in J_k} x_{kmj}^{ef} \sum_{l \in L} (\lambda_l^\eta \rho_{kj}^{ef} + \lambda_l^{ef} \alpha_{km}^{ef})(\delta_j^l + \sum_{b \in B_{kj}} \delta_j^F \hat{x}_{jkb}^{ef} \delta_b^l)
$$

$$
+ \sum_{k \in K} \sum_{j \in J_k} x_{kj}^{be} \sum_{l \in L} \lambda_l^{be} \alpha_k^{ef} (\delta_j^l + r \sum_{b \in B_{kj}} \delta_j^F \hat{x}_{jkb}^{be} \delta_b^l) \tag{7.12}
$$

The Lagrangian dual problem (D) is then:

$$
\max_{\lambda_l^\eta, \lambda_l^{ef}, \lambda_l^{be} \geqslant 0} h(\lambda_l^\eta, \lambda_l^{ef}, \lambda_l^{be}) \tag{7.13}
$$

where:

$$
h(\lambda_l^\eta, \lambda_l^{ef}, \lambda_l^{be}) = \min_{x_{kmj}^{ef}, x_{kj}^{be}, \hat{x}_{jbk}^{ef}, \hat{x}_{jbk}^{be}, u_{lt}} L(x_{kmj}^{ef}, x_{kj}^{be}, \hat{x}_{jbk}^{ef}, \hat{x}_{jbk}^{be}, u_{lt}, \lambda_l^{ef}, \lambda_l^\eta, \lambda_l^{be}) \tag{7.14}
$$

Since $\eta_l, \beta_l^{ef}, \beta_l^{be}$ and $x_{kmj}^{ef}, x_{kj}^{be}, \hat{x}_{jkb}^{ef}, \hat{x}_{jkb}^{be}$ are independent variables,

$$
\min L = \min \sum_l (\tilde{C}_l - \lambda_l^\eta \eta_l - \lambda_l^{ef} \beta_l^{ef} - \lambda_l^{be} \beta_l^{be})
$$

$$
+ \min \sum_{k \in K} \sum_{m \in M_k} \sum_{j \in J_k} x_{kmj}^{ef} \sum_{l \in L} (\lambda_l^\eta \rho_{kj}^{ef} + \lambda_l^{ef} \alpha_{km}^{ef})(\delta_j^l + \sum_{b \in B_{kj}} \delta_j^F \hat{x}_{jkb}^{ef} \delta_b^l)
$$

$$
+ \min \sum_{k \in K} \sum_{j \in J_k} x_{kj}^{be} \sum_{l \in L} \lambda_l^{be} \alpha_k^{ef} (\delta_j^l + r \sum_{b \in B_{kj}} \delta_j^F \hat{x}_{jkb}^{be} \delta_b^l)
$$

$$
= \sum_l \min(\tilde{C}_l - \lambda_l^\eta \eta_l - \lambda_l^{ef} \beta_l^{ef} - \lambda_l^{be} \beta_l^{be})
$$

$$
+ \sum_{k \in K} \sum_{m \in M_k} \min[\sum_{j \in J_k} x_{kmj}^{ef} \sum_{l \in L} (\lambda_l^\eta \rho_{kj}^{ef} + \lambda_l^{ef} \alpha_{km}^{ef})(\delta_j^l + \sum_{b \in B_{kj}} \delta_j^F \hat{x}_{jkb}^{ef} \delta_b^l)]
$$

$$
+ \sum_{k \in K} \min[\sum_{j \in J_k} x_{kj}^{be} \sum_{l \in L} \lambda_l^{be} \alpha_k^{ef} (\delta_j^l + r \sum_{b \in B_{kj}} \delta_j^F \hat{x}_{jkb}^{be} \delta_b^l)] \tag{7.15}
$$

Equation (7.15) shows that the problem (7.14) can be separated into the following

three set of subproblems:

Subproblem (i):

$$\min(\tilde{C}_l - \lambda_l^{\eta}\eta_l - \lambda_l^{ef}\beta_l^{ef} - \lambda_l^{be}\beta_l^{be}) \tag{7.16}$$

$$\text{Subject to: } (7.1)(7.6)(7.7) \tag{7.17}$$

Subproblem (i) can be solved by the gradient projection method [15].

Subproblem (ii):

$$\min[\sum_{j \in J_k} x_{kmj}^{ef} \sum_{l \in L}(\lambda_l^{\eta}\rho_{kj}^{ef} + \lambda_l^{ef}\alpha_{km}^{ef})(\delta_j^l + \sum_{b \in B_{kj}} \delta_j^F \hat{x}_{jkb}^{ef}\delta_b^l)] \tag{7.18}$$

Since we have a small set of candidate path for each O-D, we can simply set the link

cost to $(\lambda_l^{\eta}\rho_{kj}^{ef} + \lambda_l^{ef}\alpha_{km}^{ef})$, and then enumerate the choice of working paths and backup

paths, pick the combination of working and backup paths with the lest total cost.

The resulting choice of working path and backup path is cached. In the subsequent

iteration, only paths whose cost is changed will be compared to the previous solution,

and hence save the running time.

Subproblem (iii):

$$\min[\sum_{j \in J_k} x_{kj}^{be} \sum_{l \in L} \lambda_l^{be}\alpha_k^{ef}(\delta_j^l + r \sum_{b \in B_{kj}} \delta_j^F \hat{x}_{jkb}^{be}\delta_b^l)] \tag{7.19}$$

Similar to Subproblem (ii), the solution is to enumerate the choice of working

path and backup path, then choose the one with the lest combined cost. The results

are similarly cached to speedup the program.

Given the initial values of λ_l^η, λ_l^{ef} and λ_l^{be}, once we solve the problem (D), sub-gradients for the three set of Lagrangeans are computed as follows:

$$\omega_l^\eta \;=\; \eta_l - \sum_{k \in K} \sum_{m \in M_k} \rho_{km}^{ef} \sum_{j \in J_k} (x_{kmj}^{ef} \delta_j^l + \sum_{b \in B_{kj}} x_{kmj}^{ef} \delta_j^F \hat{x}_{jkb}^{ef} \delta_b^l) \tag{7.20}$$

$$\omega_l^{ef} \;=\; \beta_l^{ef} - \sum_{k \in K} \sum_{m \in M_k} \alpha_{km}^{ef} \sum_{j \in J_k} (x_{kmj}^{ef} \delta_j^l + \sum_{b \in B_{kj}} x_{kmj}^{ef} \delta_j^F \hat{x}_{jkb}^{ef} \delta_b^l) \tag{7.21}$$

$$\omega_l^{be} \;=\; \beta_l^{be} - \sum_{k \in K} \alpha_k^{be} \sum_{j \in J_k} (x_{kj}^{be} \delta_j^l + r \sum_{b \in B_{kj}} x_{kj}^{be} \delta_j^F \hat{x}_{jkb}^{be} \delta_b^l) \tag{7.22}$$

To improve the convergence speed, surrogate subgradient method is used, where only one of the three subproblems is solved at a time. The subsequent Lagrangian multipliers are updated using the latest value.

$$\lambda_l^\eta \leftarrow \min(0, \lambda_l^\eta + t\omega_l^\eta), \forall l \in L \tag{7.23}$$

$$\lambda_l^{ef} \leftarrow \min(0, \lambda_l^{ef} + t\omega_l^{ef}), \forall l \in L \tag{7.24}$$

$$\lambda_l^{be} \leftarrow \min(0, \lambda_l^{be} + t\omega_l^{be}), \forall l \in L \tag{7.25}$$

where the step size, t, is defined by:

$$t = \phi \frac{h^* - h}{||\omega_l^{eta}||^2 + ||\omega_l^{ef}||^2 + ||\omega_l^{be}||^2} \tag{7.26}$$

where h^* is the value of the best feasible solution found so far, and ϕ is a scalar between 0 and 2. ϕ is set to 2 initially in our study and is halved if the solution does not improve in 10 iterations. For the detailed description of the subgradient method

and its variants, please refer to the book [13].

At each iteration, the solution of $x^{ef}_{kmj*}, x^{be}_{kj*}, \hat{x}^{ef}_{jkb}$, and $\hat{x}^{ef}_{jkb}$ for the primal problem (P) can be obtained from the solution of subproblem (ii) and(iii). The link capacity $\tilde{\psi}_l$ is computed according to (7.6). Consequently, the primary objective function can be derived. As the iteration proceeds, we store the best solution found so far for the primal problem (P). In this way, we are always able to obtain a feasible solution. The maximum number of iterations is set to 400 in the implementation.

7.4   Experimental results

The network topologies, link cost, and OD pairs are generated the same way as section 6.

If not specified, EF demand pairs are randomly generated with the average rate uniformly distributed from 0 Mbps to 10 Mbps. The requested bandwidth of a EF demand pair is a value between the average rate and the peak rate. It is randomly specified to be 150%-300% of the average rate in our experiments. The average rate of BE demand is uniformly distributed from 10Mbps to 50Mbps. The number of EF demands for the same O-D pair is uniformly distributed from 1 to 10. The number of candidate link types for link l is uniformly distributed from 5 to 10. The capacities of

link types are set to be multiples of 45Mbps, while the costs of the link types for link l are randomly generated in such a way that the cost goes higher and the unit cost per Mbps goes down as the link capacity increases. We use average packet size $\tilde{y} = 4396$ bits and second moment of packet size $\tilde{y^2} = 22790170$ bits^2 for all the test cases. The restoration level of BE traffic is determined by the network designer depending on the budget and desired BE class resilience. It is set to 0.5 in all of the experiments. The BE delay bound factor, g_l, is set to 2 for all links in our experiments.

Two additional greedy type heuristics were implemented for the purpose of comparison. The first heuristic method starts with one O-D pair k, finds out both the best working path j and backup path b for both EF and BE demand among all candidate working and backup paths that would result in the least total cost, and then repeats the process for all $k \in K$. The second heuristic is an iterative method inspired by the heuristic used in [59]. The primary difference here is that [59] deals only with the spare capacity planning problem, while we have to find out the working path first. In the first iteration, for each O-D pair k, the algorithm finds out only the best working paths j for both EF and BE demands among candidate working paths that would result in the least total cost. After all the working paths have been determined, the algorithm looks for the best backup paths b for both EF and BE demands among

Node	Link	O-D	Duality Gap (%)		
Number	Number	Number	LR	H	S
10	25	30	0.12-3.24	3.82-21.89	0-3.40
20	50	90	0.01-2.96	0.2-15.21	0-2.3
50	125	350	0.17-1.78	1.7-13.70	0-1.87
100	250	1000	0.16-1.90	0-8.36	0-1.4
200	500	3000	0-1.64	0-10.21	0-1.55
500	1250	12000	0-2.59	0-11.47	
700	1750	20000	0.2.82	0-9.35	
1000	2500	40000	0-1.97	0-15.09	

Table 7.1: Network topology information and experimental results

candidate backup paths. In the subsequent iteration, the algorithm goes through each O-D pairs to see whether there is any change of working path or backup path that will lower the total cost. The iteration will continue as long as there is at least one adjustment of path in the current iteration. We will call these two methods (H) and (S) respectively.

The algorithms were tested on 8 different sizes of networks, ranging from 10 nodes to 1000 nodes. Some details of the network topologies are listed in Table 6.1. Note that the O-D demand number shown in Table 6.1 includes both EF and BE demands. To obtain confidence intervals, we generated 30 different topologies for each network size, with the same number of nodes, links, and O-D pairs.

Table 7.1 shows the Duality Gaps of the three solution methods, expressed in terms of the 95% confidence intervals. It is easy to see from the table that the
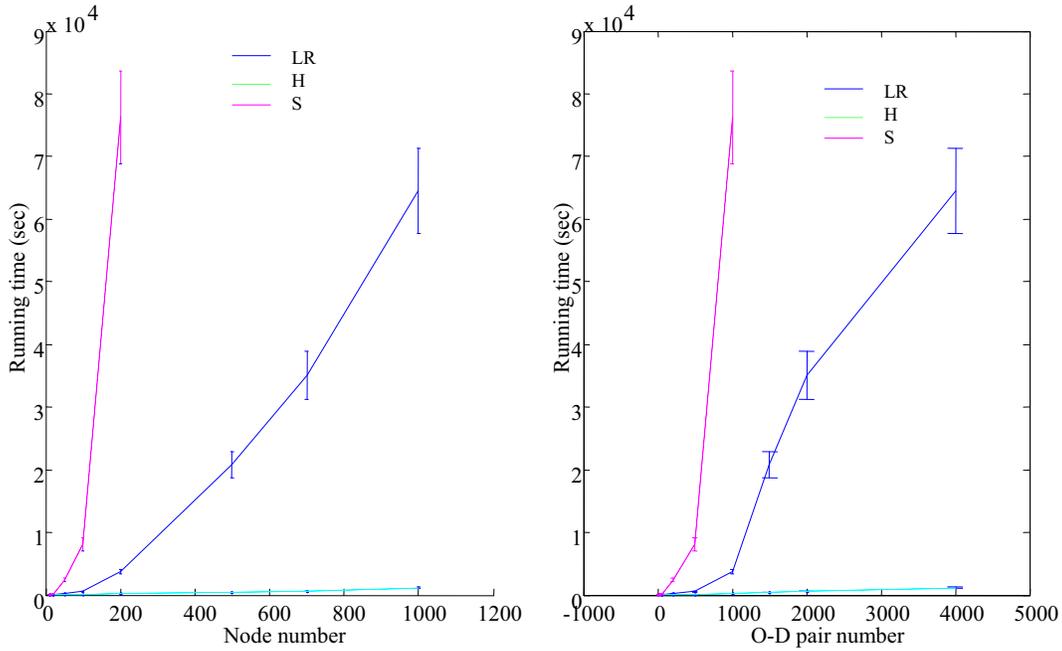
Figure 7.1: Running Time vs. Network Size

Lagrangian Relaxation together with the subgradient method produces reasonable results as the duality gap is bounded by no more than 3.3%. Note the primal problem itself is approximated when reducing the size of candidate path set for all possible path set. But according to our experimental results, more than 97% of the time, the final solution is chosen among the 5 shortest candidate paths. Therefore, 10 candidate paths are considered adequate. Having more than 10 candidate paths will have minimal impact on the solution quality, while significantly increasing the running time. Given the large number of networks being tested, we are confident that the solution should have good quality for other sizes of networks. Heuristic (S) achieves comparable solution quality in networks sized up to 200 nodes, while method

(H) yields solutions with a duality gap of more than 20%, which is not very desirable.

Figure 7.1 shows the running time with respect to the network size. In all 240 test cases of Lagrangian Relaxation based method, the algorithm converges without difficulty. As can be seen from the figure, heuristic (S) takes much longer time than the other two methods.

From the experiments shown above, Lagrangian Relaxation based approach demonstrates a good trade-off between the solution quality and the running time. The size of the largest network evaluated in this paper is representative of a large network, and is much larger than the test cases used in most work on network design. It is fair to predict that the running time of the algorithm will stay reasonable for practical sized networks.

Chapter 8

Conclusions and Future Work

In this dissertation, we presented a flow aggregation based Lagrangian method to attack the nonlinear MCF problem with integer constraints. The nonlinear multi-commodity flow problem with integer constraints is an important subject with a wide variety of application contexts. But because of the inherent difficulty of solving non-linear integer programming problems and the lack of efficient systematic approaches, heuristics have been the prevalent solution choice, which may be inadequate in many cases.

We propose a Lagrangian relaxation based method to extend the often used bundle constraint based relaxation. The method relaxes the relationship between individual flows and the variable representing the total amount of traffic in each link. The relaxation of the flow aggregation equation makes the resulting Lagrangian dual problem separable to simpler subproblems, which is not possible if the problem is relaxed otherwise. The subgradient method is used to find out the optimal Lagrangian multipliers.

We study the effectiveness of the proposed method by applying it to three closely

related capacity planning problems to verify its effectiveness. We address the problems of link dimensioning and routing optimization for IP networks supporting Diff-Serv EF and BE traffic classes. We first study the capacity planning in the context of legacy IP networks, where only non-bifurcated routing is allowed for the traffic with the same destination. In the second problem, we assume the presence of MPLS, therefore the allowance of bifurcated routing. The third problem deals with the design of survivable MPLS networks, where spare capacity is required. A large number of test cases for the three problems shows that the flow aggregation based method is capable of producing solutions that are within a few percent of the optimal solution, and that the running time remains reasonable on practical-sized networks. This represents the first work for capacity planning of multi-class IP networks with non-linear performance constraints and discrete link capacity constraints.

There are several directions for the future research.

- Although the flow aggregation based Lagrangian relaxation has been shown to be effective, other research has shown that intelligent local search methods, such as genetic algorithms, could be very effective, especially when used together with other relaxation methods [67]. It would be interesting to implement those methods and compare their performance.

- There are some better alternatives to the popular subgradient method, such as the bundle method [35] which converges faster and is more robust than the subgradient method. It would be desirable to replace the subgradient method

with those methods.

- The flow aggregation based relaxation can only be applied to those MCN problem where the bundle constraint is the function of the variable(s) representing the flow aggregations. We would like to study how and to what extend the method can be applied to more general MCN problems.

- We studied the multiclass IP network supporting DiffServ BE and EF traffic, but did not include the AF traffic classes in this paper, due to the lack of a consensus on the implementation of the AF PHB. We should pay close attention to the development of the AF PHB, and include the AF traffic class when it permits.

- We applied our methods to IP networks. It is natural to seek opportunities to try out the method on other types of networking design problems, such as design of optical networks and overlay networks.

- Better and more realistic analytical or experimental performance model could be used to formulate the capacity planning problem to improve the usefulness of the problem.

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