ABSTRACT

CONNER, CRISTIN ALANNA. A Model to Project Height, USDA Grade, and Economic Value for Fraser fir Christmas Trees. (Under the direction of Drs. Bronson P. Bullock and John Frampton.)

Fraser fir (*Abies fraseri* [Pursh] Poir.) is a highly valued fresh cut Christmas tree species in the United States. It draws a high price from consumers and is an economically important species in the western portion of North Carolina. North Carolina ranks first in the nation in Christmas tree sales and second in the number of Christmas trees produced, most of which are Fraser fir. Tree quality can be assessed by standards developed by the United States Department of Agriculture (USDA), which divides trees into four grades (premium, 1, 2 and cull) based on factors such as crown density and the number and distribution of defects. Although Fraser fir Christmas trees are economically important, there are currently no models to aid in stand management decisions.

In this research, the two-parameter Weibull distribution was fit to Fraser fir Christmas tree height data for whole plots and individual USDA grades within plots over a range of ages and sites. The derived parameters from the two-parameter Weibull distribution were then modeled from individual stand characteristics (e.g. soil series, elevation, slope, aspect, and stand age) using both parameter recovery and parameter prediction methods. Parameter recovery techniques of estimation were preferred over parameter prediction methods as determined by sum of squares differences. Final parameter estimation equations were developed for whole plots and individual USDA grades.

The estimated parameters from the two-parameter Weibull distribution were then used to determine the proportion of trees in each height class and USDA grade combination. The proportions of trees in each USDA grade were modeled from stand characteristics using

a logistic regression model. These probabilities were multiplied by the appropriate proportions of trees in each USDA grade and height class combination to obtain relative frequencies of trees for each combination. An interface was developed in Microsoft Excel that allows a user to input various stand information and expected prices; the output contains numbers of trees in each USDA grade and height class combination as well as expected revenue for each combination. Models were developed to predict mean height for each USDA grade and were utilized in this spreadsheet. An application of the parameter recovery equations and the USDA grade probability equation was presented to demonstrate how relative frequencies were obtained from the estimated parameters and stand characteristics.

A Model to Project Height, USDA Grade, and Economic Value for
Fraser fir Christmas Trees

by
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INTRODUCTION

The Christmas Tree Industry

Christmas trees are grown commercially in all 50 states. The six states producing the greatest number of Christmas trees are Oregon, North Carolina, Michigan, Pennsylvania, Wisconsin and Washington, respectively (NCTA, 2007). There are approximately 21,000 Christmas tree growers in the United States employing over 100,000 people full or part-time to support the industry. Approximately 500,000 acres of land are used for Christmas tree production in the United States, yielding approximately 35 million trees each year (NCTA, 2007).

There are over 1600 growers of Fraser fir (*Abies fraseri* [Pursh] Poir.) in North Carolina producing approximately 50 million trees on more than 25,000 acres (NCCTA, 2007). The Fraser fir Christmas tree industry in North Carolina is located in the western portion of the state where the species is grown in plantations mostly above 3000 feet in elevation. Fraser fir accounts for over 95% of all Christmas trees produced in North Carolina (NCCTA, 2007). Fraser fir trees grown in North Carolina are shipped to every state in the U.S. and to countries around the world. Other Christmas tree species grown in the state include Virginia pine (*Pinus virginiana* Mill.), eastern white pine (*Pinus strobus* L.), Leyland cypress (x *Cupressocyparis leylandii* Dall. + Jacks.) and eastern redcedar (*Juniperus virginiana* L.). Annual sales of North Carolina Christmas trees were approximately 134 million dollars in 2006 (NCDA&CS, 2007).

Fraser fir is found naturally in disjunct stands at high elevations in the southern Appalachian Mountains of western North Carolina, southern Virginia and eastern Tennessee

(Beck, 1990). Fraser fir is the only fir species native to the southern Appalachian Mountains. In natural stands, the species grows at elevations ranging from 4500 to 6684 feet. The uppermost elevation at which Fraser fir is found is at the top of Mount Mitchell, the highest point in eastern North America (Beck, 1990). Fraser fir thrives in a cool, moist climate with annual precipitation of 75 to 100 inches and average summer temperatures not exceeding 60 °F (Beck, 1990).

Fraser fir Christmas trees are prized by consumers because of their natural conical shape, pleasant aroma, strong branches, blue-green foliage and excellent needle retention after harvest (Hinesley et al., 1995). Hence, Fraser fir Christmas trees draw a high price from consumers, making it an economically important species in North Carolina.

Christmas tree quality for both wholesale and retail sales is defined by United States Department of Agriculture (USDA) grades (USDA, 1989). This system divides trees into grades based upon factors such as density, shape and crown defects. These grades allow buyers and sellers to easily communicate about quality factors. There are four grade categories for Christmas trees: premium, grade #1, grade #2 and cull. Premium grade trees must be well shaped, have dense needle coverage, and have only one minor defect (USDA, 1989). Grade #1 trees may be less dense with one noticeable defect. Grade #2 trees may have a light density with two noticeable defects. Cull grade trees do not meet the requirements of the grade #2 trees; these trees may be used for greenery and wreaths or may be sold in low-priced markets.

Growers often categorize trees based on one-foot height classes and USDA grade. Even when trees are not categorized by USDA grade, the grades have an impact on production and trade terminology (Arnold et al., 1995). USDA Christmas tree grade and height are the most important factors in the determination of a tree's wholesale value (Arnold et al., 1994a; Arnold et al., 1994b). However, wholesale values are not well correlated with retail values. Average retail values of premium, grade #1 and grade #2 trees have not been found to differ significantly from each other; only cull grade trees were found to be significantly different from the superior grade trees (Arnold et al., 1995). This was a result of the large range of retail values found within height and USDA grade combinations.

Growth and Yield Modeling

Although the Christmas tree industry is economically important, management decisions are not currently based on empirical growth predictions. In order to obtain estimates of future Christmas tree yield, it is advantageous to base management decisions upon empirical growth predictions. Future yield projections can be used to alter current management techniques in order to produce a greater quantity of a valuable product class.

Growth and yield prediction models are used to estimate the future growth and final timber yield of forest stands. The purpose of these models is to capture stand dynamics using mathematical equations (Amateis et al., 1996). The forest products industry has traditionally made use of growth and yield models to aid in long-term forest management decisions. Tree diameter is often well correlated with stem volume and thus, economic value. Understanding the relationship between diameter and site and stand conditions is useful for management purposes. Typically, volume predictions are used to better plan when to thin, fertilize and harvest a stand of timber. Diameter distribution-type growth and yield models predict changes in the distribution of tree diameters with stand age. An appropriate statistical distribution, such as the normal, Weibull, lognormal, or beta distribution, is fit to the stand diameter data via maximum likelihood estimation techniques (Bailey, 1980). The distributional parameters are then regressed upon site and stand characteristics (e.g. stand density, age, average height and site index). The end result is a system of equations that are used to predict diameter distributions from stand-level characteristics.

The Weibull distribution probability density function (pdf) has been used extensively in the modeling of diameter distributions in forest stands and was originally proposed by Bailey and Dell (1973) for use in forestry. Advantages of the Weibull distribution include its flexibility in shape and ease of mathematical derivations. The two-parameter form of the Weibull distribution uses a shape parameter to describe the curve and a scale parameter to describe the range of the data. The three-parameter form of the Weibull distribution uses an additional location parameter that estimates the minimum observation, rather than assuming it to be zero as in the case of the two-parameter Weibull (Shiver, 1988). The flexibility of curve shapes allowed by the Weibull distribution is a key advantage; it can assume symmetric, skewed and inverse-J shapes.

Two methods, parameter recovery and parameter prediction, have been used extensively to estimate the parameters of the Weibull distribution in the forestry literature (Bailey and Dell, 1973; Feduccia et al., 1979; Burk and Newberry, 1984; McTague and Bailey, 1987; Cao, 2004; Bullock and Burkhart, 2005, and others). Parameter prediction methods predict the parameters of the distribution directly from stand level characteristics; parameter recovery methods estimate the parameters from moments or percentiles of the distribution. Parameter recovery techniques have been found to be superior to parameter prediction techniques when estimating parameters of the Weibull distribution for loblolly pine (*Pinus taeda* L.) tree diameters (Cao, 2004). The moment based method of parameter recovery estimates noncentral (not centered around the mean) moments of the distribution from stand level characteristics and then solves for the parameters using the predicted moments (Burk and Newberry, 1984) as follows. First, regression equations are developed to predict the first two noncentral moments (mean and quadratic mean squared, respectively) from stand characteristics. The mathematical relationship between these moments and the parameters of the Weibull distribution can be rearranged to obtain first the shape parameter and then the scale parameter.

The percentile method of parameter recovery estimates percentiles of the distribution (instead of moments) from various stand level characteristics; the estimated percentiles are then used to predict the Weibull shape and scale parameters. A comparison of moment estimation and percentile estimation techniques by Liu et al. (2004) revealed that the percentile based parameter recovery technique was superior in terms of producing the smallest prediction error for stem diameter in unthinned black spruce (*Picea mariana* (Mill.) B.S.P.) plantations. Combined moment-percentile methods have been utilized based on the prediction of two or more percentiles and the second noncentral moment of the diameter distribution, which is the quadratic mean diameter squared (Baldwin and Feduccia, 1987; McTague and Bailey, 1987; Brooks et al., 1992; Bullock and Burkhart, 2005). Another study used this combined method to establish a system of equations that incorporated a fertilization regime into timber yield prediction (Bailey et al., 1989). Neither method of parameter estimation has been investigated for Christmas tree height distributions.

Objectives

The objectives of this study are threefold. The first objective is to derive stand level height distributions for Fraser fir Christmas trees over a range of ages and sites. The second objective is to characterize the height distributions for each of the four USDA grades for stands of Fraser fir Christmas trees. The final objective is to develop a user interface that will allow Fraser fir growers to make more informed decisions regarding the management of their plantations and to predict the economic value based upon site characteristics and appropriate prices.

METHODS

Data

Fraser fir Christmas trees were sampled on five farms operated by a Fraser fir Christmas tree production company based in Avery County, NC that sells wholesale to chain store customers (Fig. 1). Three of the farms are located in Avery County, NC. Another farm is located in Watauga County, NC, and the fifth farm is located in Carter County, TN. Only Fraser fir Christmas trees that were of sufficient size to be harvested in 2006 or 2007 were sampled. All trees were planted on a 4 x 4 foot spacing (equivalent to 2722 trees per acre, excluding access roads). Since all farms sampled were managed by the same grower, the cultural methods of growing the trees were similar across all farms. This allowed for the comparison of growth and quality of trees across site conditions without introducing differences due to shearing methods, fertilizer and herbicide application rates and timing, and spacing of trees.

All farms were visited prior to sampling in the summer of 2006 in order to determine the variety of site conditions that existed across the five farms. Slope and aspect were obtained from 7.5 minute topographic maps (scale of 1:24,000); these measures were confirmed in the field using a clinometer and compass. Slopes were grouped into six classes based on percent slope (0-5, 6-10, 11-15, 16-20, 21-25, 26-30). Stand elevations were obtained from the same topographic maps (contour intervals = 40 feet) and ranged from 3380 to 4460 feet. The elevations for stands were then grouped into nine 120-foot elevation classes for modeling purposes (Table 1). The soil series on each farm were obtained from the Natural Resources Conservation Service (NRCS) Web Soil Survey (NRCS, 2006).

Once this information was compiled, site combinations were developed from soil series, slope, aspect and elevation information. This resulted in 102 unique combinations of site conditions on the farms sampled in this study. Soil series observed included Porters, Saunook, and Edneyville. Two different Saunook soils were observed; the difference between them is that one (labelled *Sb*) is very stony. Porters soils are found along mountain ridges between 3600 and 4500 feet. These soils are deep (40 to 60 inches to bedrock), well drained and loamy. Saunook soils are found in coves, colluvial fans (sediments that have been transported from uplands and deposited in valleys) and bench terraces between 2200 and 4000 feet. Saunook soils are very deep (more than 60 inches to bedrock), well drained and loamy. Edneyville soils are found on mountain slopes and ridges between 2400 and 4200 feet. These soils are very deep (60 inches or more to bedrock), well drained and loamy. Fraser fir production is reported as a potential land use for all three soil types, but it is a dominant land use on Porters and Saunook soils (Tuttle, 2005). Slopes ranged from 0% to 30% slope, measured in 5% increments. In the stands sampled, all eight aspects were observed (north, northeast, east, southeast, south, southwest, west, and northwest).

Sampling began after the trees were sheared in August 2006. The crews sheared trees operationally to an approximate 18-inch leader; sampling was delayed until after shearing so that each tree would be measured at its harvested height. This post-sheared height is one factor that the grower uses to determine selling price. One sample plot was randomly chosen within each field of the 102 site combinations and 144 trees (12 x 12 tree block) were measured. If any trees in the 12 x 12 grid were missing, this was noted but no additional trees were sampled. At each sampled site, stand GPS coordinates were recorded adjacent to the first tree in the sample plot at the downslope left corner. The starting point for each sample plot was randomly located along an edge in each plot to be sure that sufficient edge trees were sampled. Christmas tree production usually has a high percentage of edge trees due to the many access roads into the stands. The random locations were chosen using a random number generator. The first 0, 1, 2, or 3 in the random number string assigned the side of the field where the plot began. If the first number was 0 then the sample began along the lower edge, if it was a 1 it began along the left edge, if it was a 2 it began along the top edge and if it was a 3 it began along the right edge, as the sampler faces the field looking upslope. The second digit (whether even or odd) determined if the sample would begin from the left or right side of the row of trees. The third digit assigned how many trees from the left or right the sampler would travel to establish the first tree in the plot.

For each individual tree in the sample, data collected included tree height measured to the top of the sheared leader to the nearest inch with a telescoping height pole and USDA grade (premium, 1, 2, or cull). This method of measuring tree height is different from the USDA protocol. Tree height using the USDA method is obtained by measuring from the base of the stump to the top of the leader, excluding any portion of the leader that extends more than four inches beyond the apex of the cone of the tree's taper.

In June 2006, data collectors were trained by Mr. Michael Fagan of the North Carolina Department of Agriculture and Consumer Services on the USDA method of grading Christmas trees on the stump. Occular estimation techniques were used to determine which trees were in each of the four USDA grades (USDA, 1989). Premium trees must have normal taper, be fresh, clean, healthy and well shaped, have heavy density (70% or more of the main stem covered by branches), and one face may have no more than one minor defect (USDA, 1989). Grade #1 trees are the same except they may have medium density (50% to 70% of the main stem covered) and one face with one minor defect and the remaining faces with no more than two minor defects. Grade #2 trees may have light density (40% to 50% of the main stem covered) and may have two adjacent faces with no more than three minor defects. The remaining two faces may have no more than two noticeable defects. Culls do not meet the requirements of the grade #2 trees. Defects include uneven density, curvature in the stem, insect or disease damage, broken branches, multiple leaders, holes or gaps, goosenecks and needle loss (USDA, 1989). Minor defects are generally slight imperfections while noticeable defects are more serious versions of those defects.

The Two-Parameter Weibull Distribution

The dataset from all Fraser fir stands sampled over all five Christmas tree farms contained individual tree and stand-level information. Each observation contained the following: plot number, planting year, aspect, slope, soil series, elevation class, row and column numbers (for spatial location within the plots), tree number (1 through 144), total height in feet and USDA grade (n=4426 tagged trees; grower provided tag color definitions). Of the 14,688 individual tree observations, 956 were missing (dead or previously harvested) and were removed from the dataset used in this analysis. The 13,732 remaining observations were sorted by plot, then by grade. The sample size, mean height in feet, and percentage of live trees in each USDA grade for each plot were obtained.

The pdf for the two-parameter Weibull distribution of a random variable x is:

$$f(x) = \frac{c}{b}\left(\frac{x}{b}\right)^{c-1} \exp\left[-\left(\frac{x}{b}\right)^{c}\right] \quad x \geq 0, b > 0, c > 0 \tag{1}$$

where, b is the scale parameter, c is the shape parameter, and exp[·] is the exponential function. In this study, the continuous random variable x is tree height. The two-parameter Weibull distribution was fit to each plot's height data (pooling trees of all grades) using SAS Proc Capability (SAS Institute Inc., 2003). The capability procedure constructs the empirical cumulative distribution function (cdf) and estimates the shape and scale parameters of the two-parameter Weibull distribution using maximum likelihood estimation techniques.

Anderson Darling and Cramer von Mises goodness-of-fit statistics were examined in order to determine how well the two-parameter Weibull distribution fit the empirical height data. The null hypothesis for these goodness-of-fit tests assumes that the empirical height data are generated from a two-parameter Weibull distribution while the alternative hypothesis states that the data do not come from a two-parameter Weibull distribution. A larger p-value indicates a good fit of the Weibull distribution to the height data, while a small p-value indicates a poor fit to the data. The Anderson Darling and Cramer von Mises goodness-of-fit statistics and p-values were placed into four p-value categories. The four categories were created such that the p-values were (1) less than 0.05, (2) greater than or equal to 0.05 and less than 0.1, (3) greater than or equal to 0.1 and less than 0.15, and (4) greater than or equal to 0.15.

Next, a separate two-parameter Weibull distribution function was fit to each individual USDA grade within each plot. Due to an insufficient sample size in five stands for premium grade trees, the Weibull distribution could not be fit; these 17 height values were discarded. The two-parameter Weibull scale and shape parameter estimates were obtained for each grade within each plot. The Anderson Darling and Cramer von Mises goodness-of-fit statistics and p-values were placed into the same four categories as listed previously.

The 1st, 5th, 10th, 15th, 25th, 50th, 75th, 90th, 95th, 97th and 99th percentiles of each grade in each plot were calculated from the empirical distribution functions. SAS Proc GLM was used to regress the percentiles on the site variables: slope, aspect, elevation class, soil series and age (SAS Institute Inc., 2003). The full general linear model is:

$$H_i = \beta_0 + \beta_1(SL)_{ij} + \beta_2(AS)_{ik} + \beta_3(EC)_{il} + \beta_4(SO)_{im} + \beta_5(A)_i + \varepsilon_i \qquad (2)$$

where,

H_i = the percentile of interest in the i^{th} plot

B = parameters to be estimated

SL_{ij} = percent slope (indicator variable), j = 1,…, 6 slope categories

AS_{ik} = aspect (indicator variable), k = 1,…, 8 aspects

EC_{il} = elevation class (indicator variable), l = 1,…, 9 elevation classes

SO_{im} = soil series (indicator variable), m = 1,…, 4 soil series

A_i = age in years since planting

ε_i = error term

All of the independent variables are categorical variables except for age, which is a continuous variable. Equation (2) was fit separately for each of the percentiles mentioned previously.

Parameter Prediction

Parameter prediction techniques were used for comparitive purposes with parameter recovery techniques. This method has been used in previous work to predict Weibull distributional parameters for diameter distributions (Feduccia et al., 1979; Cao, 2004). General linear models were used to predict the scale and shape parameters using aspect, slope, elevation class, soil series and age as the independent variables. In this study, the Weibull scale and shape parameters were predicted directly from the site variables using ordinary least squares analysis.

Parameter Recovery

In order to estimate the two-parameter Weibull distribution shape and scale parameters using parameter recovery techniques, the percentiles of the empirical height distribution were modeled as a function of stand characteristics (aspect, slope, elevation class, soil series and age). General linear models were used to predict the percentiles.

The Weibull shape parameter model predicts shape from two different percentiles of the height distribution. To develop the model for the Weibull shape parameter, a percentile equation that calculates the percentile of a distribution from the Weibull parameters was used (Murthy et al., 2004):

$$H_p = b\left[-\ln\left(\frac{100-p}{100}\right)\right]^{1/c} \tag{3}$$

where, H_p = the p^{th} percentile of the height distribution, b = scale parameter and c = shape parameter. This equation is rearranged, as shown below, into the prediction equation for the shape parameter:

$$c = \frac{k}{\ln(H_U) - \ln(H_L)} \tag{4}$$

where,

H_U = selected upper percentile of the height distribution

H_L = selected lower percentile of the height distribution

k = calculated constant

$$\ln(H_U) = \frac{1}{c}\left[\ln\left(b\left(-\ln\left(\frac{100-U}{100}\right)\right)\right)\right]$$

$$\ln(H_U) = \frac{1}{c}\left[\ln(b) + \ln\left(-\ln\left(\frac{100-U}{100}\right)\right)\right]$$

$$\ln(H_U) = \frac{1}{c}\left(\ln(b)\right) + \frac{1}{c}\left(\ln\left(-\ln\left(\frac{100-U}{100}\right)\right)\right)$$

$$\ln(H_L) = \frac{1}{c}\left(\ln(b)\right) + \frac{1}{c}\left(\ln\left(-\ln\left(\frac{100-L}{100}\right)\right)\right)$$

$$\ln(H_U) - \ln(H_L) = \frac{1}{c}\left[\left(\ln\left(-\ln\left(\frac{100-U}{100}\right)\right)\right) - \left(\ln\left(-\ln\left(\frac{100-L}{100}\right)\right)\right)\right]$$

$$c = \frac{\ln\left(-\ln\left(\frac{100-U}{100}\right)\right) - \ln\left(-\ln\left(\frac{100-L}{100}\right)\right)}{\ln(H_U) - \ln(H_L)}$$

While the denominator of this model for the shape parameter (Equation (4)) contains the two percentiles, the numerator is a constant which comes from calculations using this percentile equation. The constant, *k*, needed in the numerator of the model to estimate the shape parameter was calculated by solving Equation (3) for the Weibull scale parameter using two different percentiles. The constant was calculated for various combinations of two percentiles from the height distribution; these percentile combinations were then used to estimate the shape parameter. The combination that provided the lowest SSD was chosen to be used in the model for the shape parameter.

Quadratic mean height, the square root of the average squared height, was utilized in the Weibull scale parameter prediction model. The equation for the scale parameter uses the second noncentral moment of the distribution and the shape parameter in its prediction. The second noncentral moment of the Weibull distribution is also the quadratic mean of a random variable squared; since height distributions were utilized, quadratic mean heights were calculated for each grade within each plot and for each whole plot:

$$\bar{H}_q = \sqrt{\frac{\sum_{i=1}^{n} H_i^2}{n}} \tag{5}$$

where, $H_i = i^{\text{th}}$ height observation and n = sample size. This calculation allows the scale parameter prediction using the quadratic mean heights and the predicted shape parameters. The relationship between the estimated scale parameter, b, and the k^{th} noncentral moment, M_k, is given by (Murthy et al., 2004):

$$M_k = b^k \Gamma\left(1 + \frac{k}{c}\right) \tag{6}$$

where, $\Gamma(\cdot)$ is the gamma function. Therefore, the second noncentral moment (k=2) is:

$$M_2 = b^2 \Gamma\left(1 + \frac{2}{c}\right) \tag{7}$$

The second noncentral moment is also the quadratic mean height squared. Substituting the estimated shape parameter $\hat{c}$ in Equation (7) gives the prediction equation for the scale parameter:

$$b = \sqrt{\frac{\bar{H}_q^2}{\Gamma\left(1 + \frac{2}{\hat{c}}\right)}} \tag{8}$$

Once the prediction equations for the shape and scale parameters were obtained for whole plots, similar prediction equations were constructed for each of the four USDA grades.

The cdf for the two-parameter Weibull distribution is given by:

$$F(x) = 1 - \exp\left[-\left(\frac{x}{b}\right)^c\right] \tag{9}$$

To derive the expected proportion of trees in each height class under a two-parameter Weibull model, the proportion for each height class was calculated by subtracting the lower height limit from the upper height limit of the two-parameter Weibull cdf. This was done for each grade within each plot using the scale and shape parameters that were estimated using the parameter recovery method.

Sum of Squares Differences

The estimated shape and scale parameters from both parameter prediction and parameter recovery models were compared to the maximum likelihood estimation (MLE) parameters from each plot's empirical cdf in order to evaluate model fit. The differences were squared and summed, and the model with the smallest sum of squares differences (SSD) was chosen as the best fitting model. The equation for SSD for the shape parameter follows:

$$SSD = \sum_{i=1}^{n} (\hat{c}_{MLE_i} - \hat{c}_{Ri})^2 \tag{10}$$

where,

SSD = Sum of squares differences

n = number of plots

$\hat{c}_{MLE_i}$ = shape parameter obtained from empirical cdf of i^{th} plot

$\hat{c}_{Ri}$ = shape parameter estimated from regression (parameter prediction or parameter recovery)

SSDs were also calculated in order to evaluate fit for the scale parameter models.

USDA Grade Probability Modeling

Next, a model was developed to predict the proportions of trees in each USDA grade (pooling all height classes together) from the site conditions. The grade probabilities were modeled using the logistic procedure in SAS, which fits a logistic regression model using maximum likelihood estimation techniques (SAS Institute Inc., 2003). The four levels of grade (premium, one, two, cull) were predicted simultaneously using the following model:

$$\log\left(\frac{F_{hi}}{1-F_{hi}}\right) = \beta_{0h} + \beta_1 (SL)_{hj} + \beta_2 (AS)_{hk} + \beta_3 (EC)_{hl} + \beta_4 (SO)_{hm} + \beta_5 (A)_h \qquad (11)$$

i = 1, …, 3 USDA grades

h = 1, …, 13,715 trees

Definitions of terms are as follows:

$F_{hi} = \sum_{m=1}^{i} P_{hm}$ = the probability that tree h is in the i^{th} USDA grade or lower

All other variables are as defined previously.

Model for Mean Height by USDA Grade

In developing the Excel spreadsheet for Christmas tree growers, an issue arose in the calculation of the scale parameter. The prediction equation for the scale parameter uses quadratic mean height; however, using this term would necessitate that the grower perform a sample of tree heights in each USDA grade in order to calculate quadratic mean height for each grade. If the difference between quadratic mean height and mean height were negligible, then mean height could be substituted into the prediction equation for the scale parameter. Differences between mean height and quadratic mean height were examined for each grade.

Mean heights for each USDA grade were modeled from site conditions since this term was substituted for quadratic mean height in the equation for the scale parameter in the Excel spreadsheet. The mean heights for each of the four USDA grades were predicted using the following model:

$$\bar{H}_i = \beta_0 + \beta_1(SL)_{ij} + \beta_2(AS)_{ik} + \beta_3(EC)_{il} + \beta_4(SO)_{im} + \beta_5(A)_i + \varepsilon_i \tag{12}$$

where, $\bar{H}_i$ = the mean height of the i^{th} USDA grade and all other variables are as defined previously.

RESULTS AND DISCUSSION

The two-parameter Weibull distribution was fit to Fraser fir Christmas tree height data collected from farms in western NC and eastern TN using maximum likelihood estimation techniques. Only 6.5% of individual tree observations recorded over all plots were either dead or harvested prior to data collection, leaving 93.5% of the observations available for analysis. Mean heights increased with USDA grade quality. The percentage of trees in USDA grades one, two and cull were very similar to each other, ranging from approximately 29-32%, while the percentage of premium grade trees was much lower (8%) than the percentage of lower grade trees (Table 2).

MLE Fits - Whole Plot Height Data

The two-parameter Weibull distribution was fit to all the height data for each of the 102 plots (over all grades). The number of plots and the percent of observations in each of the four p-value categories for the Anderson Darling and Cramer von Mises goodness-of-fit statistics are shown in Table 3. The evaluation of fit for the whole plot data revealed that approximately half of the plots fit well ($p \geq 0.05$), while half did not. The null hypothesis states that the empirical height data are generated from a two-parameter Weibull distribution while the alternative hypothesis states that the data do not come from a two-parameter Weibull distribution. An example of fitting the two-parameter Weibull distribution to a plot's height data is shown for a selected plot (Fig. 2).

Parameter Prediction - Whole Plot Height Data

Parameter prediction techniques were used to predict the Weibull shape and scale parameters for whole plots (all grades combined). Both parameters were predicted directly from the site characteristics using the full general linear model (Equation (2)). For both scale and shape parameters, the elevation class and age terms were significant ($\alpha = 0.05$ for all parameter tests) in the models; the slope of the plot was significant only in the shape prediction model. The following models were obtained:

$$\hat{b} = \hat{\beta}_0 + \sum_{l=1}^{9} \hat{\beta}_l EC_l + \hat{\beta}_{10} A \tag{13}$$

$$\hat{c} = \hat{\beta}_0 + \sum_{j=1}^{6} \hat{\beta}_j SL_j + \sum_{l=1}^{9} \hat{\beta}_l EC_l + \hat{\beta}_{16} A \tag{14}$$

Table 4 presents the parameter estimates for Equations (13) and (14). The last level of any categorical variable is compared against all other levels of the same variable. Since elevation class and slope were utilized as categorical variables in these models, the last levels (EC_9 and SL_{30}) have parameter estimates of zero. This pertains to categorical variables used in all models developed in this study. The ANOVA tables may be seen in Table 5. The prediction model for the scale parameter had a R^2 value of 0.62 and a SSD of 10.27. The shape parameter model had a R^2 value of 0.43 and a SSD of 355.04.

Parameter Recovery - Whole Plot Height Data

Parameters were estimated for a system of two equations that utilize two different percentiles of the height distribution. First, the full general linear model (Equation (2)) with all site characteristics was used to predict the percentiles of the Weibull distribution for each plot. The percent slope and soil series were nonsignificant for all percentile combinations. The aspect was significant for the following percentiles: 75th and 90th. The elevation class was significant for the following percentiles: 1st, 50th, 75th, 90th, 95th, 97th and 99th. Age was significant for all percentiles.

After trying numerous combinations of percentiles, the 99th and 15th percentiles were found to be superior for predicting the shape parameter for whole plots after examing the SSDs (Equation (10)). The complete set of percentile combinations and their respective sum of squares differences are shown in Table 6. The following equation for the shape parameter was obtained using the 15th and 99th percentiles:

$$\hat{c} = \frac{3.3441}{\ln(\hat{H}_{99}) - \ln(\hat{H}_{15})} \tag{15}$$

The constant was derived from the percentile equation as shown previously.

The prediction equations for the 15th and 99th percentiles of the empirical height distribution are as follows:

$$\hat{H}_{15} = \hat{\beta}_0 + \sum_{j=1}^{6} \hat{\beta}_j SL_j + \hat{\beta}_7 A \tag{16}$$

$$\hat{H}_{99} = \hat{\beta}_0 + \sum_{l=1}^{9} \hat{\beta}_l EC_l + \hat{\beta}_{10} A \tag{17}$$

The parameter estimates for Equations (16) and (17) are presented in Table 7 and the ANOVA tables for the same linear models are presented in Table 8. The slope and age terms were significant in the linear model for the 15th percentile with a resulting R^2 value of 0.43. The elevation class and age terms were significant in the linear model for the 99th percentile with a R^2 value of 0.58. Equation (15) resulted in a SSD of 323.74 for the shape parameter for whole plots. The scale parameter model for whole plots had a SSD of 0.09 using parameter recovery techniques.

Comparison of SSDs - Whole Plot Height Data

SSDs quantify the differences between actual data and estimations and so the estimation models with smaller SSDs are superior in terms of prediction. The SSDs for both scale and shape parameters were smaller for whole plot models generated from parameter recovery techniques (Table 9). The parameter recovery estimates of the scale parameter are very similar to the MLE fitted scale parameters for whole plots (Fig. 3). However, the estimated shape parameters when plotted against the MLE fitted shape parameters exhibit more variation (Fig. 4). This method of parameter estimation over-estimates the shape parameter at small values and under-estimates it at large values. The parameter recovery technique provided better models to predict the shape and scale parameters for whole plots compared to the parameter prediction method.

MLE Fits - USDA Grades Within Plots Height Data

The two-parameter Weibull distribution was fit to the height data by grade within each plot. The frequency of grades within plots and the percentage of observations in the same four p-value categories as used for the whole plot data are shown in Table 10 for the Anderson Darling and Cramer von Mises goodness-of-fit statistics. The Anderson Darling statistics revealed that 77.2% of grades had a good fit ($p \geq 0.05$) to the two-parameter Weibull distribution while 22.8% did not. The Cramer von Mises statistics resulted in 78.5% of grades in plots that had a good fit ($p \geq 0.05$) to the two-parameter Weibull distribution, while 21.5% of grades did not have a good fit. Examples of fitting the two-parameter Weibull distribution to the premium grade (Fig. 5), grade #1 (Fig. 6), grade #2 (Fig. 7), and cull grade (Fig. 8) empirical height distributions are presented for the same selected plot.

The average sample size of trees for whole plots over all p-value groups was large, while grouping the plot data into individual grades resulted in the average sample size of trees decreasing (Table 3, Table 10). Therefore, there was less information available to reject the null hypothesis, which stated that the empirical data were generated from a two-parameter Weibull distribution. This may have inflated the number of grades that failed to reject the null hypothesis, resulting in the fits appearing better than they are in reality.

Parameter Prediction - USDA Grades Within Plots Height Data

Premium Grade

As in the whole plot analysis, the Weibull shape and scale parameters for premium grades were predicted directly from the site characteristics. There were no significant terms among all site variables used as regressors upon the shape parameter, resulting in a mean model. The following scale and shape parameter models were obtained:

$$\hat{b}_P = \hat{\beta}_0 + \sum_{l=1}^{9} \hat{\beta}_l EC_l + \hat{\beta}_{10} A \tag{18}$$

$$\hat{c}_P = \hat{\beta}_0 \tag{19}$$

The parameter estimates for Equations (18) and (19) are presented in Table 11, while the ANOVA tables are shown in Table 12. The prediction model for the scale parameter for premium plots had a SSD of 9.35 and a R^2 value of 0.74. The shape parameter model resulted in a R^2 value of 0 and a SSD of 108794.54.

Grade #1

The elevation class and age terms were significant in the model for the scale parameter for grade #1s, while only age was significant in the shape parameter model. The prediction models for the scale and shape parameters for grade #1s are as follows:

$$\hat{b}_1 = \hat{\beta}_0 + \sum_{l=1}^{9} \hat{\beta}_l EC_l + \hat{\beta}_{10} A \tag{20}$$

$$\hat{c}_1 = \hat{\beta}_0 + \hat{\beta}_1 A \tag{21}$$

Table 13 contains the parameter estimates for Equations (20) and (21). The ANOVA tables for the same equations are presented in Table 14. The general linear model for the scale parameter for grade #1s had a SSD of 9.14 and a R^2 value of 0.73. The shape parameter model for grade #1s resulted in a SSD of 2416.13 with a R^2 value of 0.13.

Grade #2

The prediction model for the scale parameter for grade #2s had three significant terms: slope, elevation class and age. Only elevation class was significant in the model for the shape parameter. The following models were obtained:

$$\hat{b}_2 = \hat{\beta}_0 + \sum_{j=1}^{6} \hat{\beta}_j SL_j + \sum_{l=1}^{9} \hat{\beta}_l EC_l + \hat{\beta}_{16} A \tag{22}$$

$$\hat{c}_2 = \hat{\beta}_0 + \sum_{l=1}^{9} \hat{\beta}_l EC_l \tag{23}$$

The parameter estimates for Equations (22) and (23) are presented in Table 15. Table 16 contains the ANOVA tables for these same equations. The SSD for the scale parameter for grade #2 plots was 9.85 with a R^2 value of 0.66. The SSD for the shape parameter for grade #2 plots was 732.03 and the R^2 value was 0.39.

Cull Grade

The slope, elevation class and age were significant in the prediction model for the scale parameter for cull grades. The aspect was the only significant term in the prediction model for the shape parameter. The parameter prediction models for cull grades are as follows:

$$\hat{b}_C = \hat{\beta}_0 + \sum_{j=1}^{6} \hat{\beta}_j SL_j + \sum_{l=1}^{9} \hat{\beta}_l EC_l + \hat{\beta}_{16} A \tag{24}$$

$$\hat{c}_C = \hat{\beta}_0 + \sum_{k=1}^{8} \hat{\beta}_k AS_k \tag{25}$$

Table 17 contains the parameter estimates for Equations (24) and (25), and the ANOVA tables are presented in Table 18. The prediction model for the scale parameter for cull plots resulted in a SSD of 8.47 and a R^2 value of 0.62. The SSD for the shape parameter for cull plots was 1305.85 with a R^2 value of 0.18.

Parameter Recovery-USDA Grades Within Plots Height Data

Premium Grade

The full general linear model (Equation (2)) was used to predict the percentiles of the Weibull distribution for premium USDA grade trees. Five premium grade distributions had a shape parameter larger than 40 as a result of having too few observations. An estimated shape parameter of this magnitude was considered biologically unrealistic and they were removed from the analysis dataset. Slope, aspect and soil series were nonsignificant at all percentiles. Elevation class and age were significant for all percentiles. After trying numerous combinations of percentiles, the 90^{th} and 15^{th} percentiles were found to be superior in terms of SSD for predicting the shape parameter for premium grades. The following model for the shape parameter was obtained:

$$\hat{c}_P = \frac{2.6510}{\ln(\hat{H}_{90}) - \ln(\hat{H}_{15})} \tag{26}$$

The constant in the numerator of Equation (26) was calculated as shown previously. The complete set of percentile combinations and their respective sum of squares differences for the premium USDA grade are shown in Table 19.

The prediction equations for the 15^{th} and 90^{th} percentiles of the empirical height distribution are as follows:

$$\hat{H}_{15} = \hat{\beta}_0 + \sum_{l=1}^{9} \hat{\beta}_l EC_l + \hat{\beta}_{10} A \tag{27}$$

$$\hat{H}_{90} = \hat{\beta}_0 + \sum_{l=1}^{9} \hat{\beta}_l EC_l + \hat{\beta}_{10} A \tag{28}$$

Table 20 contains the parameter estimates for Equations (27) and (28). The ANOVA tables for the same linear models may be seen in Table 21. Elevation class and age were significant in the linear model for the 15th percentile with a resulting R^2 value of 0.63. These same terms were also significant in the linear model for the 90th percentile resulting in a R^2 value of 0.62. Equation (26) resulted in a SSD of 1824.42 for the shape parameter for premium plots while the scale parameter model had a SSD of 0.18.

Grade #1

When the full general linear model (Equation (2)) was used to predict the percentiles of the two-parameter Weibull distribution for USDA grade #1 trees, the soil term was significant for the 1st percentile and the aspect was significant for the 75th and 90th percentiles. Elevation class was significant for all percentiles except the 10th, while age was significant for all percentiles. Slope was nonsignificant for all percentiles.

One distribution for grade #1 trees had a shape parameter larger than 40 as a result of having few observations and was removed from the dataset in order to improve the percentile model fit. After trying many combinations of percentiles, the 99th and 15th percentiles were found to be superior for predicting the shape parameter for grade #1s. The following model for the shape parameter was obtained:

$$\hat{c}_1 = \frac{3.3441}{\ln(\hat{H}_{99}) - \ln(\hat{H}_{15})} \tag{29}$$

The numerator was calculated as shown previously. The complete set of percentile combinations and their respective sum of squares differences for the USDA grade #1 are shown in Table 22.

The prediction equations for the 15th and 99th percentiles of the empirical height distribution are as follows:

$$\hat{H}_{15} = \hat{\beta}_0 + \sum_{l=1}^{9} \hat{\beta}_l EC_l + \hat{\beta}_{10} A \tag{30}$$

$$\hat{H}_{99} = \hat{\beta}_0 + \sum_{l=1}^{9} \hat{\beta}_l EC_l + \hat{\beta}_{10} A \tag{31}$$

Table 23 contains the parameter estimates for Equations (30) and (31). The ANOVA tables for these percentile prediction models are shown in Table 24. The R^2 values for the 15th and 99th percentiles were 0.66 and 0.48, respectively. The SSD that resulted for the shape parameter model was 1007.83 while the SSD for the scale parameter was 0.16.

Grade #2

When the full general linear model was used to predict the percentiles of the two-parameter Weibull distribution for USDA grade #2 trees, the slope was significant for the 50th, 75th, 90th and 95th percentiles. Both elevation class and age were significant for all percentiles, while the aspect and soil terms were nonsignificant for all percentiles.

The combination of the 97th and 5th percentiles was superior for predicting the shape parameter for grade #2. The following model for the shape parameter was obtained:

$$\hat{c}_2 = \frac{4.2248}{\ln(\hat{H}_{97}) - \ln(\hat{H}_5)} \tag{32}$$

The numerator for the shape parameter model was calculated as shown previously. The complete set of percentile combinations and their respective sum of squares differences for the USDA grade #2 are shown in Table 25.

The prediction equations for the 5^{th} and 97^{th} percentile models for the empirical height distribution are presented:

$$\hat{H}_5 = \hat{\beta}_0 + \sum_{l=1}^{9} \hat{\beta}_l EC_l + \hat{\beta}_{10} A \tag{33}$$

$$\hat{H}_{97} = \hat{\beta}_0 + \sum_{l=1}^{9} \hat{\beta}_l EC_l + \hat{\beta}_{10} A \tag{34}$$

The parameter estimates for Equations (33) and (34) are shown in Table 26 and the ANOVA tables for the same equations are presented in Table 27. The R^2 value for the 5^{th} percentile model was 0.62 and the R^2 value for the 97^{th} percentile model was 0.48. The SSDs for the shape and scale parameter models were 680.96 and 0.13, respectively.

Cull Grade

Results for cull grade trees showed that slope, aspect and soil were nonsignificant for all percentiles. Elevation class was significant for the 25^{th}, 50^{th}, 75^{th}, 90^{th}, 95^{th}, 97^{th} and 99^{th} percentiles. The age term was significant for all percentiles except the 1^{st}. The 99^{th} and 15^{th} percentiles were found to be superior for predicting the shape parameter for cull grades. The following model for the shape parameter was obtained:

$$\hat{c}_C = \frac{3.3441}{\ln(\hat{H}_{99}) - \ln(\hat{H}_{15})} \tag{35}$$

The numerator of Equation (35) was calculated as shown previously. The complete set of percentile combinations and their respective sum of squares differences for the cull USDA grade are shown in Table 28.

The prediction equations for the 15th and 99th percentiles of the empirical height distribution are as follows:

$$\hat{H}_{15} = \hat{\beta}_0 + \sum_{j=1}^{6} \hat{\beta}_j SL_j + \sum_{l=1}^{9} \hat{\beta}_l EC_l + \hat{\beta}_{16} A \tag{36}$$

$$\hat{H}_{99} = \hat{\beta}_0 + \sum_{l=1}^{9} \hat{\beta}_l EC_l + \hat{\beta}_{10} A \tag{37}$$

The parameter estimates for the above equations are shown in Table 29 and their ANOVA tables are presented in Table 30. The R^2 value for the 15th percentile model was 0.41 while the 99th percentile model resulted in a R^2 value of 0.55. The SSD for the shape parameter model was 1079.06 and the SSD for the scale parameter model was 0.15.

Comparison of SSDs - USDA Grades Within Plots Height Data

The SSDs for both scale and shape parameters were smaller for all individual USDA grade models generated from parameter recovery techniques (Table 9). Thus, the parameter recovery technique provided superior models to predict the shape and scale parameters compared to the parameter prediction method. The parameter recovery estimates and the MLE fitted parameters were similar for the scale parameters (Fig. 9) while the shape parameters (Fig. 10) showed more variation. As in whole plots, parameter recovery over-estimated the shape parameter for low values and under-estimated it at large values (regresses the shape parameter estimates toward their mean).

USDA Grade Probability Modeling

The cumulative logistic regression model was used to predict the proportions of trees in each grade from the site characteristics (Equation (11)). All independent variables were significant in the grade prediction model. The logistic procedure in SAS fits a common slopes model to the data. This is a parallel lines regression model based on the cumulative probabilities of the different ordinal response categories rather than on their individual probabilities (SAS Institute Inc., 2003). The cumulative logistic model assumes that the influence of the explanatory variables is the same for each different level of the response variable (Allison, 1999). Since the model's intercept provides the only difference between grade predictions within a plot, only one model is presented for grade:

$$\log\left(\frac{\hat{F}_{hi}}{1-\hat{F}_{hi}}\right) = \hat{\beta}_{0_i} + \sum_{j=1}^{6}\hat{\beta}_j SL_j + \sum_{k=1}^{8}\hat{\beta}_k AS_k + \sum_{l=1}^{9}\hat{\beta}_l EC_l + \sum_{m=1}^{4}\hat{\beta}_m SO_m + \hat{\beta}_{28} A \qquad (38)$$

All terms in the model are as defined previously in Equation **(11**). The parameter estimates for Equation (38) are presented in Table 31. A generalized R^2 value of 0.10 was obtained.

The individual USDA grade probability prediction model's (Equation (38)) score test for the proportional odds assumption had a chi-square value of 278.35 and a very low p-value ($p < 0.01$), resulting in the rejection of the null hypothesis that the coefficients are the same for all four USDA grades. This could mean that the cumulative logistic model is not appropriate; however, others have found that using many independent variables and a large sample size often results in a score test with a low p-value, and rejection of the model is not necessary (Allison, 1999). The USDA grade probability prediction model used five independent variables and 13,715 observations to predict the levels of the USDA grade; with so much information it was very likely that the ordinal assumption would be rejected. This may not, however, result in an invalid model.

Models for Mean Height by USDA Grade

General linear models were used to predict the mean height for each of the four USDA grades (Equation (12)). Elevation class and age were significant in the prediction model for mean height of premium USDA grade trees:

$$\hat{\bar{H}}_P = \hat{\beta}_0 + \sum_{l=1}^{9} \hat{\beta}_l EC_l + \hat{\beta}_{10} A \tag{39}$$

The R^2 value for Equation (39) was 0.67; the parameter estimates are presented in Table **32** and the ANOVA table is shown in Table 33.

Age and elevation class were also the significant terms in the model for mean height of USDA grade #1 trees:

$$\hat{\bar{H}}_1 = \hat{\beta}_0 + \sum_{l=1}^{9} \hat{\beta}_l EC_l + \hat{\beta}_{10} A \tag{40}$$

The parameter estimates and the ANOVA table for Equation (40) are presented in Table 34 and Table 35, respectively. The R^2 value was 0.73.

Slope, elevation class, soil series and age were significant in the prediction model for the mean height for the USDA grade #2:

$$\hat{\hat{H}}_2 = \hat{\beta}_0 + \sum_{j=1}^{6} \hat{\beta}_j SL_j + \sum_{l=1}^{9} \hat{\beta}_l EC_l + \sum_{m=1}^{4} \hat{\beta}_m SO_m + \hat{\beta}_{20} A \tag{41}$$

The R^2 value for Equation (41) was 0.68, the parameter estimates are shown in Table 36 and the ANOVA table is presented in

Table **37**.

Slope, elevation class and age were the significant terms in the model for mean height for the cull USDA grade:

$$\hat{\hat{H}}_C = \hat{\beta}_0 + \sum_{j=1}^{6} \hat{\beta}_j SL_j + \sum_{l=1}^{9} \hat{\beta}_l EC_l + \hat{\beta}_{16} A \tag{42}$$

The parameter estimates and ANOVA table for Equation (42) are presented in Table 38 and Table 39, respectively. The R^2 value was 0.59.

User Interface

A spreadsheet was created using Microsoft Excel that allows a user to input Fraser fir Christmas tree stand information and expected prices per tree and then returns the number of trees in different height class and grade combinations, as well as expected revenue for each combination and over the entire stand. The user inputs the following information: aspect, slope, elevation, soil series, age, number of trees, and expected prices for each height class and grade combination. The aspect, slope, elevation class and soil series must be chosen from drop-down menus in order to limit the choices to those observed in this research, as these were all used as categorical variables in the models developed in this study.

The two-parameter Weibull shape and scale parameters are calculated using the models developed from parameter recovery techniques. The only deviation in the terms used in calculations was the use of mean height rather than quadratic mean height in the equations for the scale parameter. Using quadratic mean height would require the user to perform a sample of tree heights in each grade to be used in a calculation of the quadratic mean height for each grade. The differences between observed mean height and quadratic mean height were negligible in this study (Table 40); therefore, mean height was used in the Excel spreadsheet for ease of use. The linear models for mean height of each USDA grade were incorporated into the spreadsheet for use in calculating the scale parameter.

Next, the parameters are used to calculate the relative frequencies of trees in each height class and grade combination using the cdf of the two-parameter Weibull distribution (Equation (9)). The actual height classes used in the cdf had 0.25 feet subtracted from the upper and lower bounds in order to account for stump height. This shifted the distribution 0.25 feet to the left.

The predicted probabilities of each USDA grade are calculated from the logistic regression model that was developed in this study. The relative frequencies in each column for USDA grade are multiplied by the corresponding predicted probability for that grade in order to obtain the proportion of trees that fall into each height class and grade combination. These proportions are then multiplied by the total number of trees in the stand to calculate the number of trees in each height class and grade combination. The tree numbers in each cell were rounded to whole numbers rather than having fractions of trees, which resulted from multiplying total tree number by the proportions. Therefore, under some combinations of site conditions, the number of trees may be slightly less or slightly more than the number of total trees entered by the user due to rounding. Finally, the expected prices are multiplied by the number of trees to obtain revenue in each height class and grade combination, and for the entire stand. An example of the user interface is displayed in Fig. 11.

Relationship Between Mean Heights and Site Characteristics

Effective modeling of the height distributions necessitated awareness of the relationship between the predictor variables and the percentiles of the height distributions. The relationship between the age since planting and mean heights for USDA grades was linear (Fig. 12). The linear trend was positive for mean height over age. For the last age observed (9 years in the field), mean height appeared to increase less than in previous years. However, there are fewer observations for trees at age 9 and so the sample size may be insufficient. Alternately, sampled trees that had been left to grow until age 9 could have grown poorly, necessitating greater degrees of shearing to allow the trees to fill in and become denser, and therefore more attractive to consumers. The age term was highly significant in every model for individual USDA grades.

The elevation class predictor variable was also highly significant in all models for individual USDA grades. Elevation class and mean heights for individual USDA grades were plotted (Fig. 13). There seems to be no trend for mean heights among elevation classes; however, within the elevation classes, mean heights increase with increasing USDA grade.

Comparison of Actual and Predicted Median Heights

In order to evaluate how consistently the predicted USDA grade height distributions obtained from parameter recovery techniques match actual height distributions, predicted median heights were plotted against actual median heights for each USDA grade within each plot (Fig. 14). Predicted distributions with smaller actual median heights were over-predicted, indicating a right shift of the predicted height distributions. Predicted distributions with larger actual median heights were under-predicted, indicating a left shift of the predicted height distributions.

Model Limitations

The models developed in this study are limited in application because the data used to construct these models were obtained from a single Fraser fir Christmas tree grower. These models may not be applicable for other shearing regimes, tree spacings, herbicide/insecticide application rates, or other cultural practices that can differ drastically among Christmas tree growers. Height growth is partly determined by the method of shearing and the length of the post-shear leader; most genetic expression of growth is lost due to the intensive height and crown density management of these trees.

APPLICATION

The application of estimated height distributions provides information about the relative frequencies of trees in different height classes. These proportions can be obtained for a stand of known age, aspect, slope, elevation and soil series using the system of equations developed by the parameter recovery method. The information required for the utilization of these equations may be obtained from topographic and soil maps with much greater ease than the effort that would be required to assemble an empirical height distribution.

Relative frequencies may be calculated from the stand characteristics listed previously. For this example, a seven year old stand will be examined that has a west aspect, 10% slope, 3500 feet elevation and is grown on Porters soil. The appropriate elevation class is one for a stand located at 3500 feet (Table 1). The height class width is one foot. Calculations will be shown for the premium USDA grade with the understanding that calculations are similar for the other three USDA grades when utilizing the appropriate equations.

Equations (27) and (28) are applied to predict the 15th and 90th percentiles of the height distribution for premium USDA grades as shown in Equations (43) and (44) for the stand characteristics listed previously.

$$\begin{aligned}\hat{H}_{15-P} &= 3.218430860 + 0.771885720(1) + 0.494950014(7) \\ &= 7.5 \text{ ft}\end{aligned} \tag{43}$$

$$\hat{H}_{90-P} = 4.703899005 + 1.156132208(1) + 0.454117879(7)$$
$$= 9.0 \text{ ft} \quad (44)$$

These percentiles are substituted in Equation (26) for the predicted shape parameter as shown in Equation (45).

$$\hat{c}_P = \frac{2.6510}{\ln(9.0) - \ln(7.5)}$$
$$= 14.5402 \quad (45)$$

The scale parameter model (Equation (8)) utilizes quadratic mean height (Equation (5)). Calculation of quadratic mean height requires a sample of trees and since differences between quadratic mean heights and mean heights were negligible in this study, mean heights are substituted in the equation for the scale parameter.

Mean height for the premium USDA grade may be obtained from the model developed and presented in Equation (39). This calculation is shown in Equation (46):

$$\hat{\bar{H}}_P = 3.540379244 + 1.097270044(1) + 0.523121395(7)$$
$$= 8.3 \text{ ft} \quad (46)$$

Substituting the mean height for premium grade trees as listed previously, as well as the estimated shape parameter, results in the scale parameter estimate shown in Equation (47).

$$\hat{b}_P = \sqrt{\frac{(8.3)^2}{\Gamma(1 + 2/14.5402)}}$$
$$= 8.5733 \quad (47)$$

The cdf of the two-parameter Weibull distribution (Equation (9)) permits the calculation of the relative frequency of trees in a particular height class of interest. By substituting appropriate values into the cdf, the proportion of trees below a specified height can be determined. Calculating the proportions for the upper and lower bounds of a height class and then finding the difference between the proportions determines the percentage of trees within that height class. This method is shown for the six to seven foot height class utilizing the estimated shape and scale parameters obtained for this example.

$$\begin{aligned} F(7) &= 1-\exp\left[-\left(\frac{7}{8.5733}\right)^{14.5402}\right] \\ &= 0.051 \end{aligned} \tag{48}$$

$$\begin{aligned} F(6) &= 1-\exp\left[-\left(\frac{6}{8.5733}\right)^{14.5402}\right] \\ &= 0.006 \end{aligned} \tag{49}$$

$$\begin{aligned} F(7)-F(6) &= 0.051-0.006 \\ &= 0.045 \end{aligned} \tag{50}$$

This means that 4.5% of premium USDA grade trees fall into the six to seven foot height class on the site conditions specified. These calculations need to be performed for each size class until nearly 100% of the distribution is represented. The same process must be performed for the other USDA grades utilizing the appropriate parameter recovery models.

Next, the estimated proportion of premium USDA trees must be obtained using the linear model developed using logistic regression (Equation (38)).

$$\begin{aligned} \log\left(\frac{\hat{F}_{hP}}{1-\hat{F}_{hP}}\right) &= -1.8002-0.1451(1)+0(1)-0.1931(1)-0.0386(1)-0.1318(7) \\ &= -3.0996 \end{aligned} \tag{51}$$

The result is a linear predictor (denoted $\hat{n}$ below); it must be back-transformed using a formula (SAS Institute Inc., 2003) to obtain the predicted probability ($\hat{p}$) as shown:

$$\begin{aligned}\hat{p} &= \frac{1}{1+e^{-\hat{n}}} \\ &= 0.043\end{aligned} \tag{52}$$

This means that 4.3% of trees in the stand are in the premium grade. To find the probability for the grade #1, the proportion from the premium grade must be subtracted from the cumulative probability of the premium and grade #1s. To find the probability for the grade #2, the proportions of premiums and grade #1s must be subtracted from the cumulative probability of all three grades. To find the probability for culls, all three higher grade probabilities are subtracted from one.

Once the predicted probabilities are found for each USDA grade, they are multiplied by each of the relative frequencies within the appropriate grade to obtain the frequency of trees in each grade and height class combination over the entire stand. The result is a table of proportions of trees in each grade and height class as shown in Table 41.

In order to determine the number of trees within each USDA grade and height class combination, the total number of trees can be multiplied by each relative frequency. The expected revenue for each USDA grade and height class can then by obtained by multiplying the number of trees in each combination by the expected price per tree for each combination. Finally, the expected revenue for the stand is calculated by summing the expected revenues for all USDA grade and height class combinations.

CONCLUSIONS

Fraser fir is a commercially important Christmas tree species in North Carolina. The ability to predict height distributions on different site characteristics will aid Christmas tree land managers in making management decisions. A two-parameter Weibull distribution was fit to height data for whole plots and individual USDA grades within plots collected from several Fraser fir Christmas tree farms in western North Carolina and eastern Tennessee. Stand age and elevation were the most important predictors of the two-parameter Weibull shape and scale parameter estimates.

Parameter recovery and parameter prediction techniques were used to estimate the shape and scale parameters of the two-parameter Weibull height distributions for Fraser fir Christmas trees. Parameter recovery techniques for parameter estimation performed better than parameter prediction methods as measured by the sum of squares differences for both shape and scale parameter models. Models for both the scale and shape parameters generated by parameter recovery techniques consistently had lower sum of squares differences than models obtained by parameter prediction; therefore, parameter recovery is the preferred method of parameter estimation.

An interface was created in Microsoft Excel that utilizes the final parameter recovery models, the logistic regression model that predicts probabilities of USDA grades, and the mean height models for USDA grades. This spreadsheet allows the Christmas tree grower to input site information and expected prices, and returns the number of trees in each USDA grade and height combination, as well as the expected revenue for the stand.

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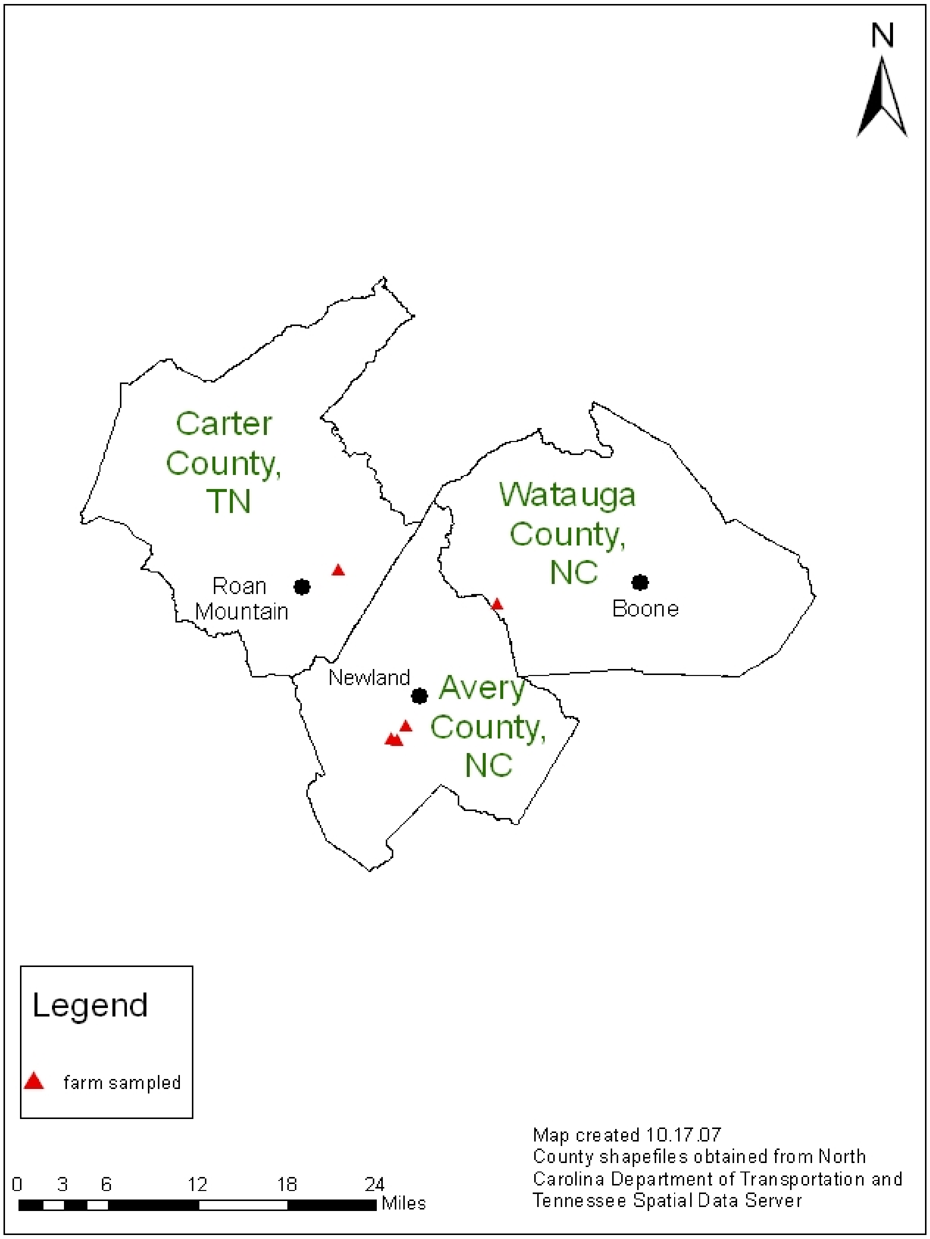


Fig. 1: Location of Christmas tree farms sampled in western North Carolina and eastern Tennessee.

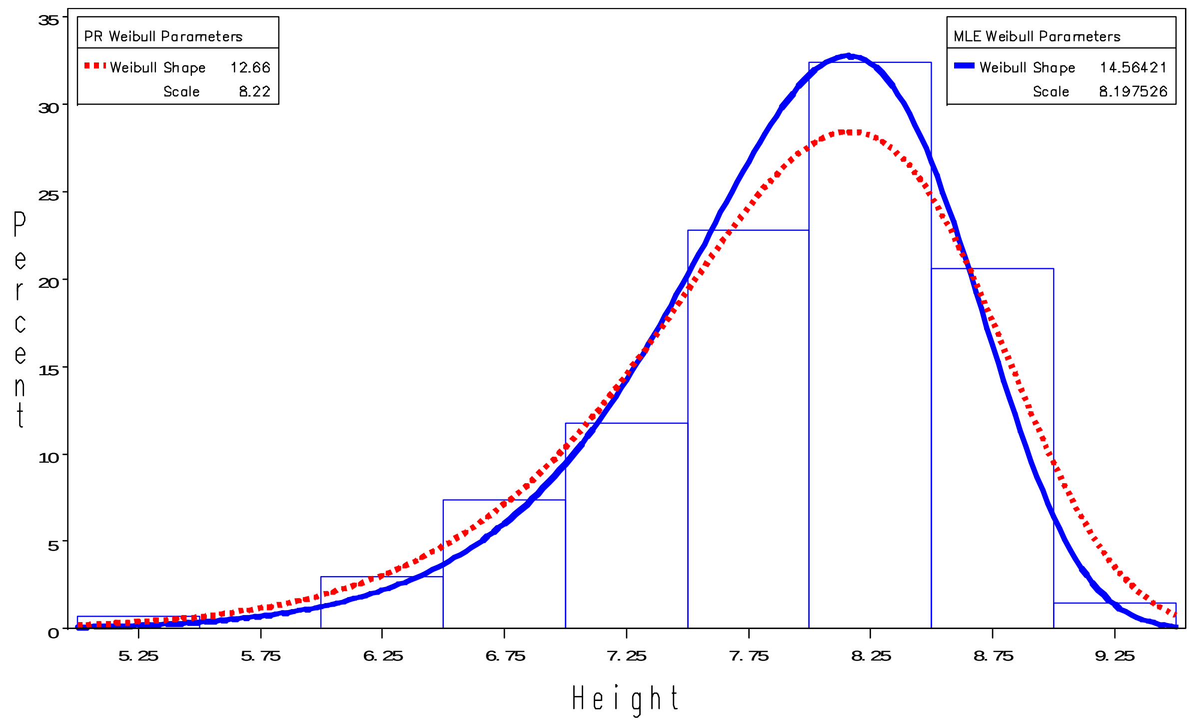


Fig. 2: Example of the MLE (solid) and PR (dashed) fits of the two-parameter Weibull distribution to the height data from a selected plot (plot 21; n=136 trees).

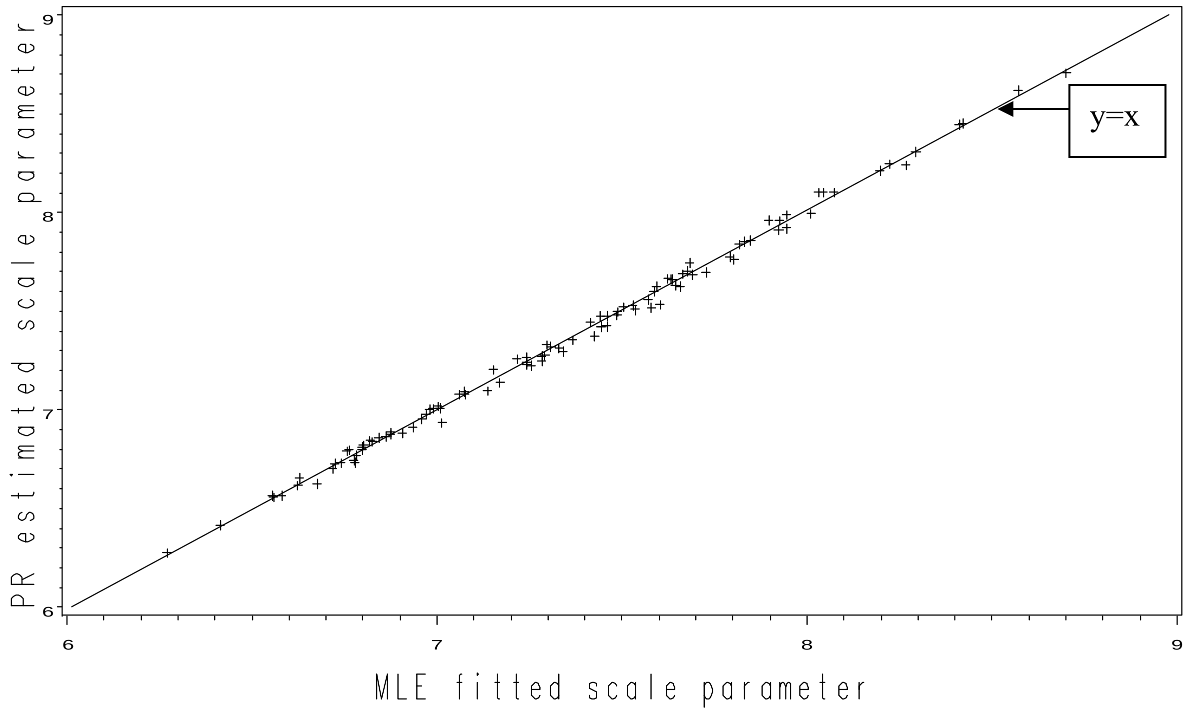


Fig. 3: Scale parameter estimated from parameter recovery (PR) method vs the scale parameter of the MLE fitted two-parameter Weibull height distribution for whole plots (n=102 plots).

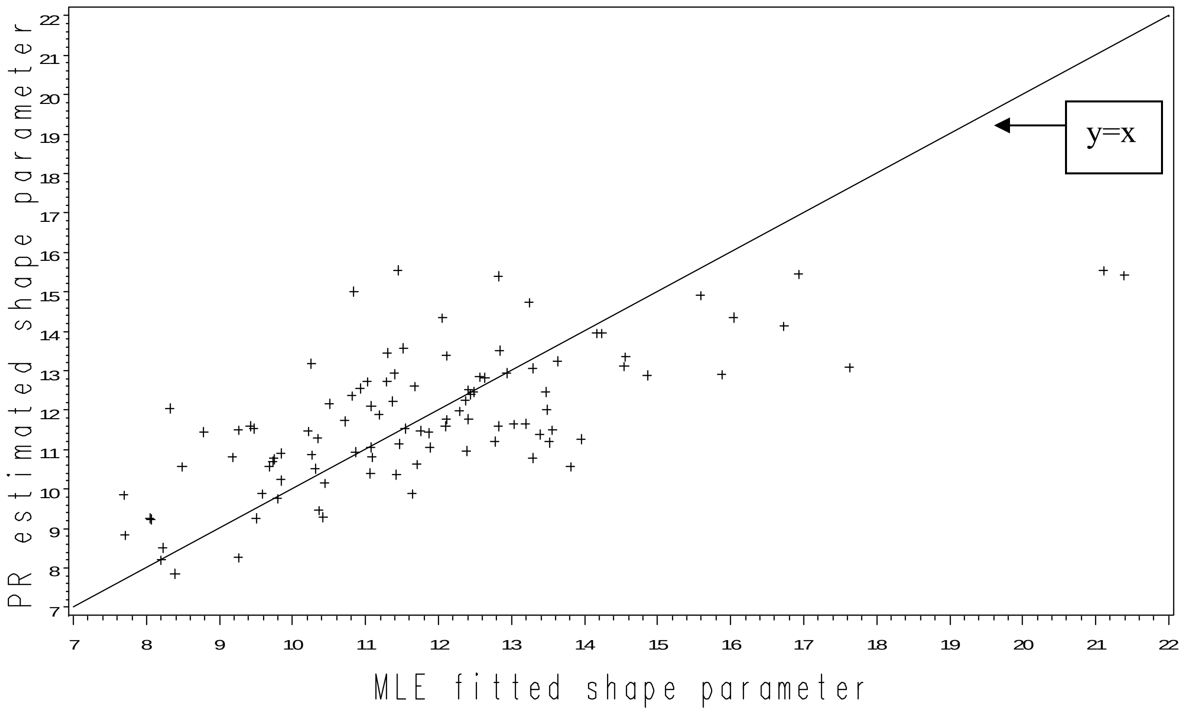


Fig. 4: Shape parameter estimated from parameter recovery (PR) method vs the shape parameter of the MLE fitted two-parameter Weibull height distribution for whole plots (n=102 plots).

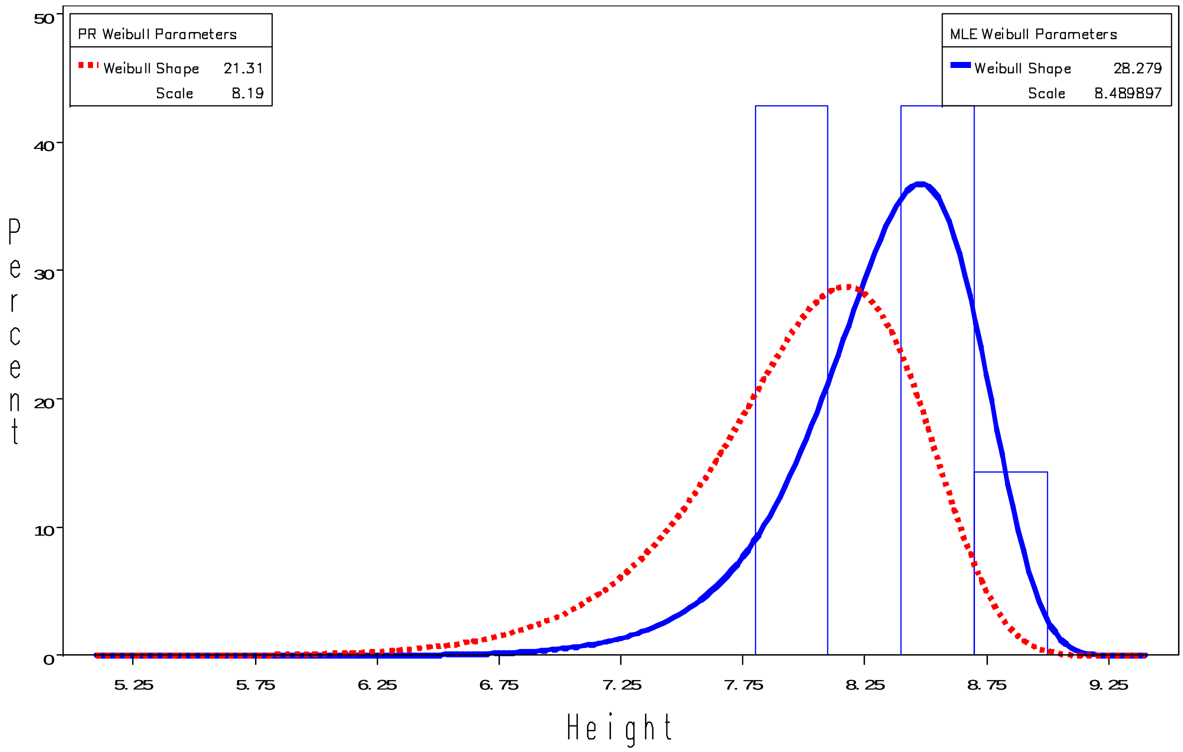


Fig. 5: Example of the MLE (solid) and PR (dashed) fits of the two-parameter Weibull distribution to the height data from the premium grade within a selected plot (plot 21; n=7 trees).

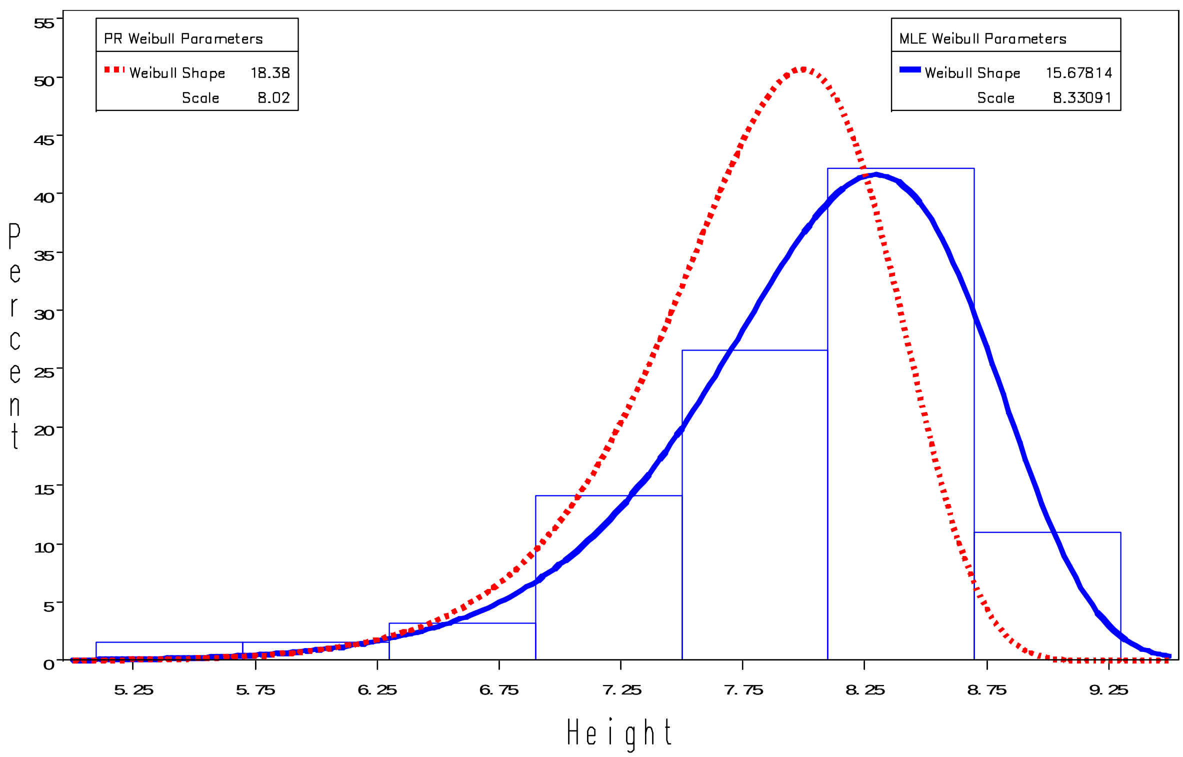


Fig. 6: Example of the MLE (solid) and PR (dashed) fits of the two-parameter Weibull distribution to the height data from the grade #1 within a selected plot (plot 21; n=64 trees).

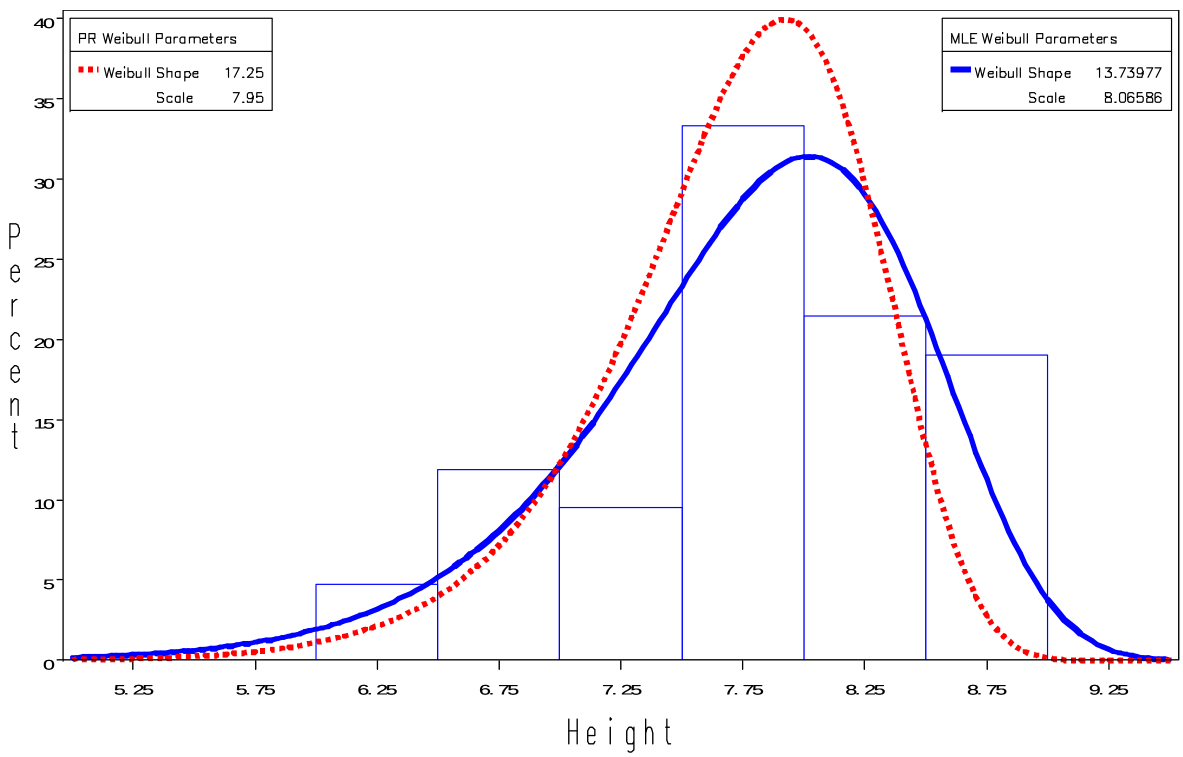


Fig. 7: Example of the MLE (solid) and PR (dashed) fits of the two-parameter Weibull distribution to the height data from the grade #2 within a selected plot (plot 21; n=42 trees).

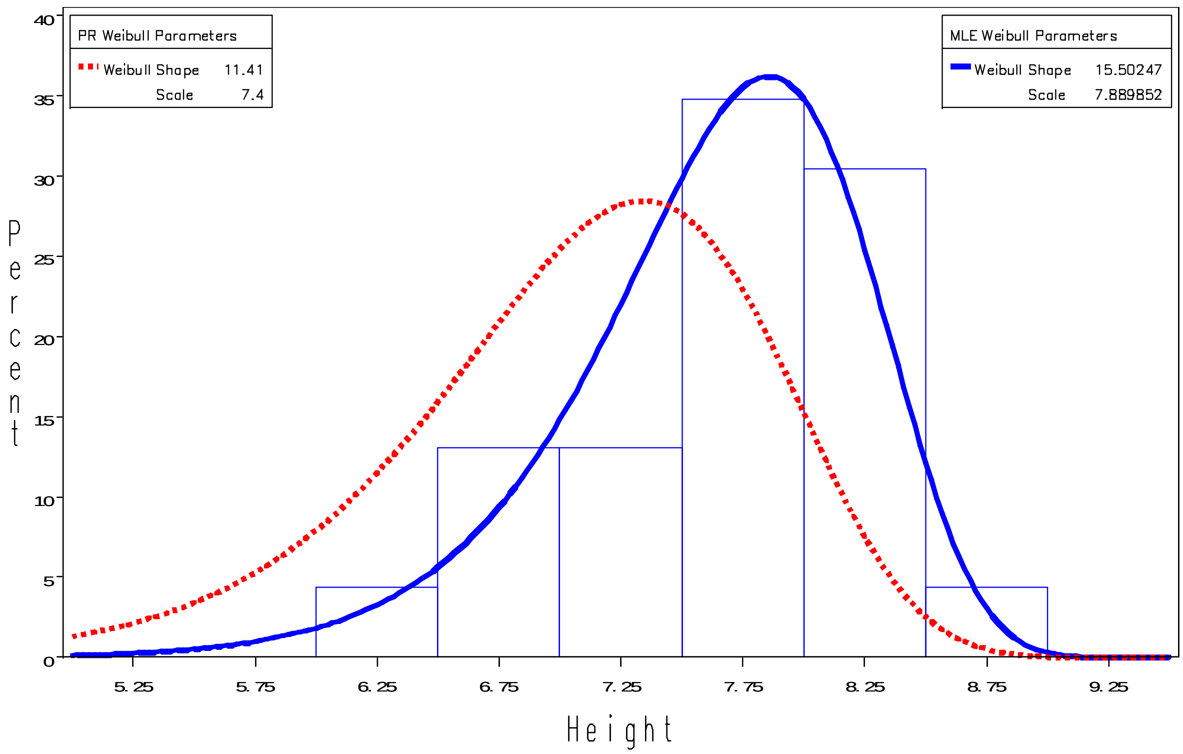


Fig. 8: Example of the MLE (solid) and PR (dashed) fits of the two-parameter Weibull distribution to the height data from the cull grade within a selected plot (plot 21; n=23 trees).

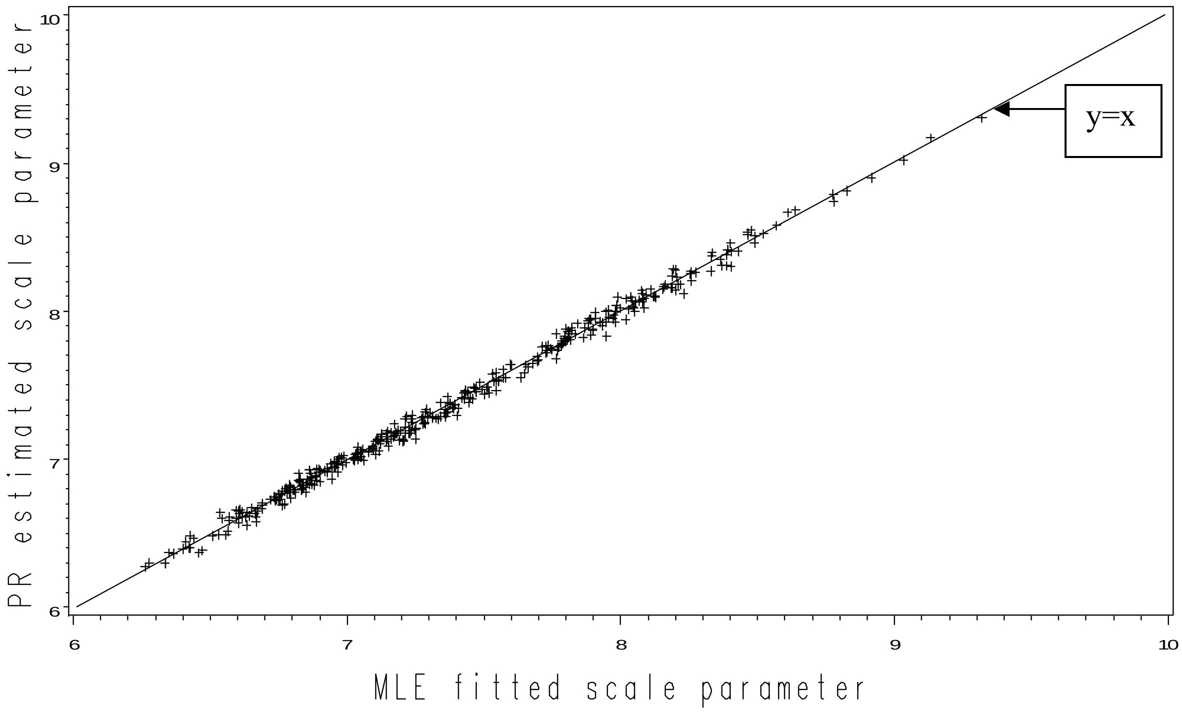


Fig. 9: Scale parameter estimated from parameter recovery (PR) method vs the scale parameter of the MLE fitted two-parameter Weibull height distribution for individual grades (n=390 grades).

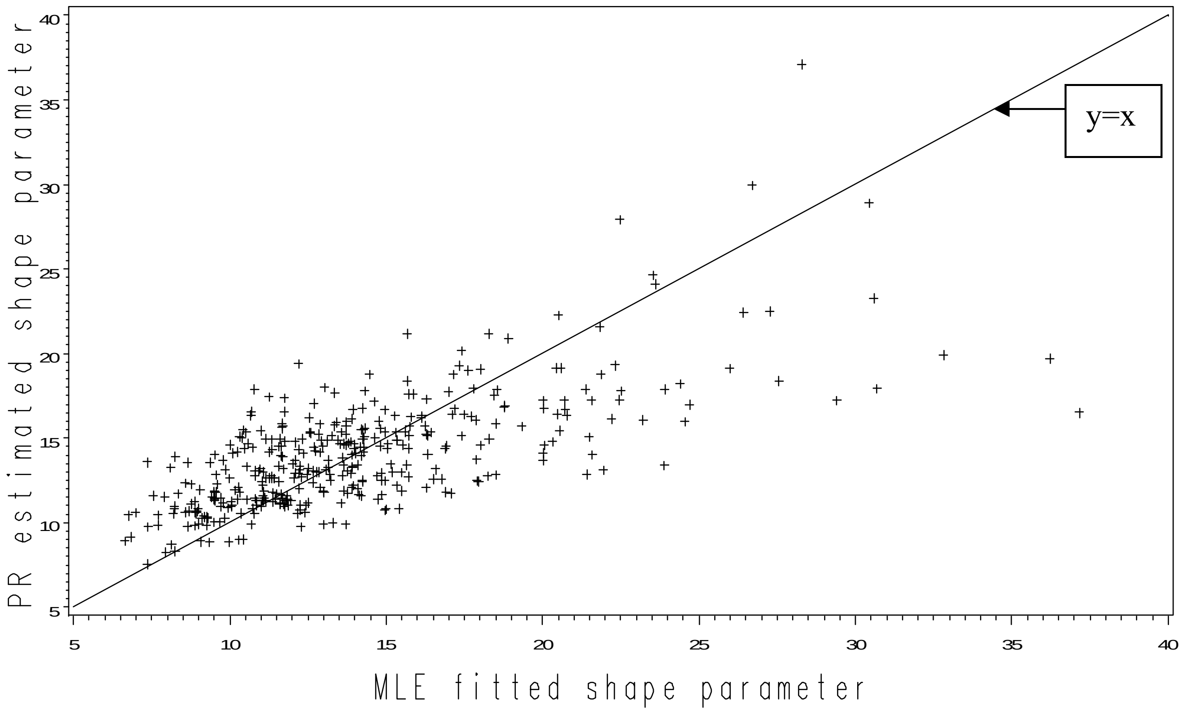
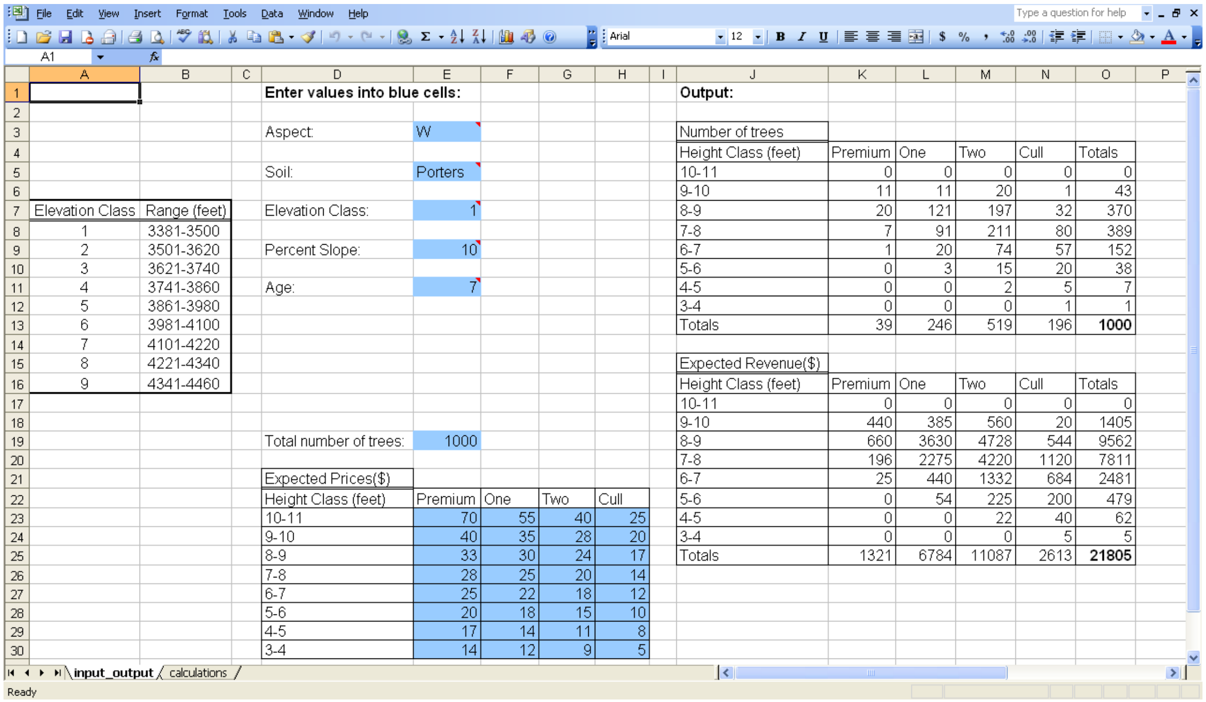


Fig. 10: Shape parameter estimated from parameter recovery (PR) method vs the shape parameter of the MLE fitted two-parameter Weibull height distribution for individual grades (n=390 grades).



Enter values into blue cells:

Aspect:	W
Soil:	Porters
Elevation Class:	1
Percent Slope:	10
Age:	7
Total number of trees:	1000

Elevation Class	Range (feet)
1	3381-3500
2	3501-3620
3	3621-3740
4	3741-3860
5	3861-3980
6	3981-4100
7	4101-4220
8	4221-4340
9	4341-4460

Expected Prices($)

Height Class (feet)	Premium	One	Two	Cull
10-11	70	55	40	25
9-10	40	35	28	20
8-9	33	30	24	17
7-8	28	25	20	14
6-7	25	22	18	12
5-6	20	18	15	10
4-5	17	14	11	8
3-4	14	12	9	5

Output:

Number of trees

Height Class (feet)	Premium	One	Two	Cull	Totals
10-11	0	0	0	0	0
9-10	11	11	20	1	43
8-9	20	121	197	32	370
7-8	7	91	211	80	389
6-7	1	20	74	57	152
5-6	0	3	15	20	38
4-5	0	0	2	5	7
3-4	0	0	0	1	1
Totals	39	246	519	196	**1000**

Expected Revenue($)

Height Class (feet)	Premium	One	Two	Cull	Totals
10-11	0	0	0	0	0
9-10	440	385	560	20	1405
8-9	660	3630	4728	544	9562
7-8	196	2275	4220	1120	7811
6-7	25	440	1332	684	2481
5-6	0	54	225	200	479
4-5	0	0	22	40	62
3-4	0	0	0	5	5
Totals	1321	6784	11087	2613	**21805**

Fig. 11: Excel user interface.

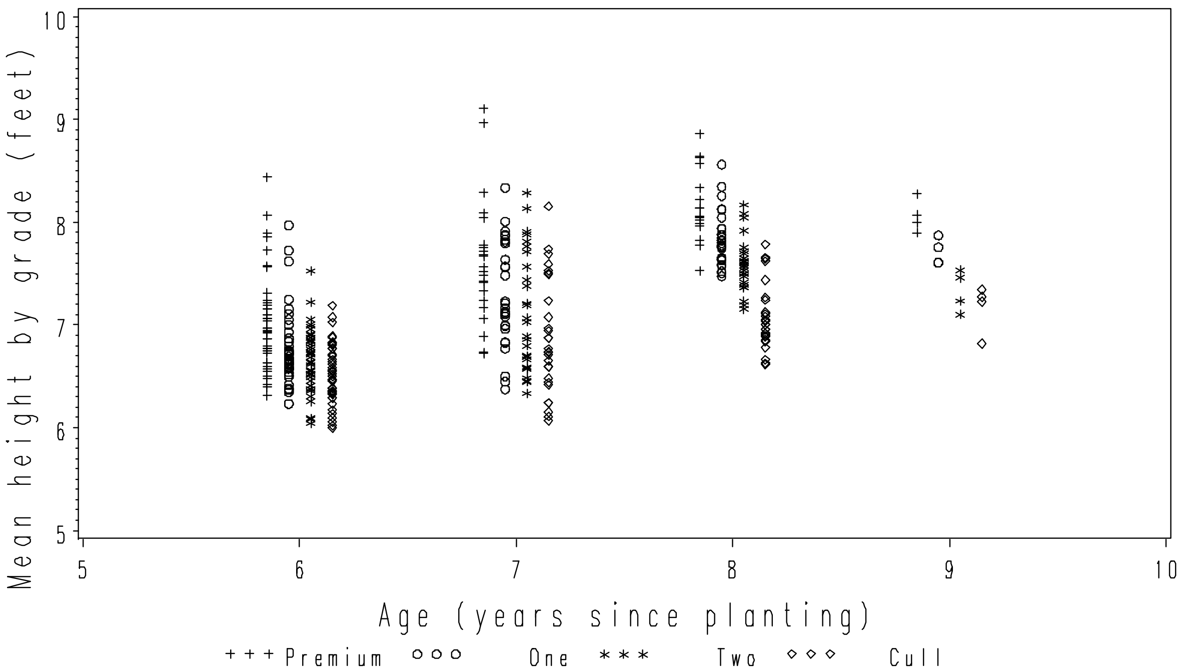


Fig. 12: Mean heights for each grade plotted against age since planting for all plots (n=390 grades).

Note: grades are staggered around ages for visual differentiation.

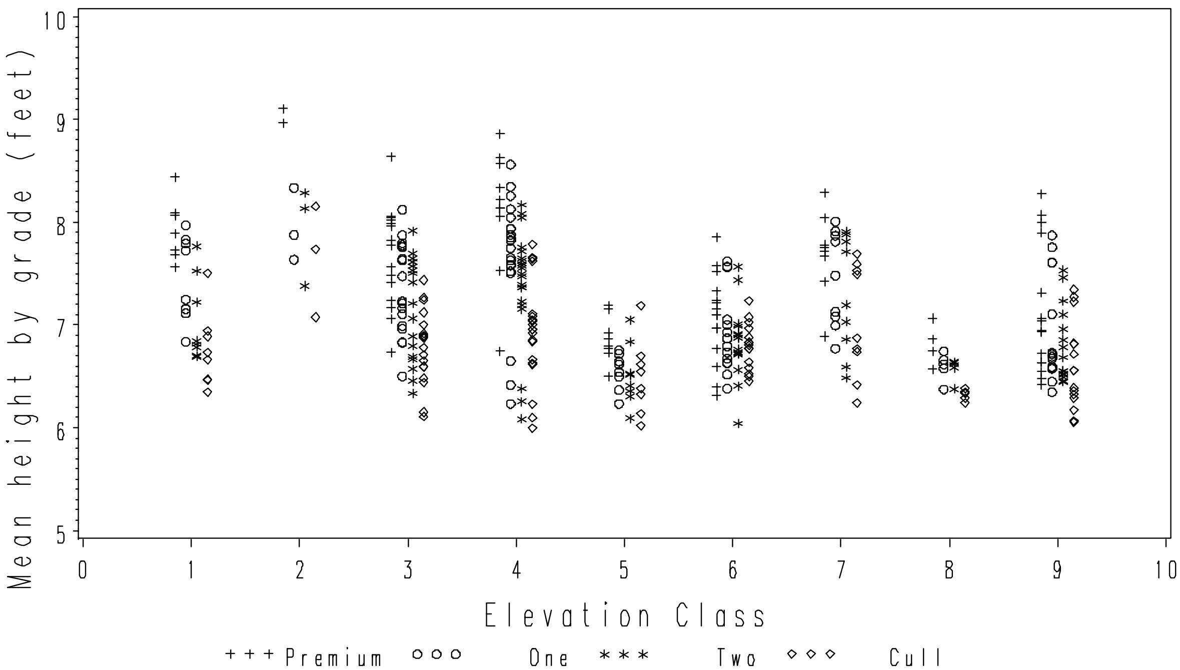


Fig. 13: Mean heights for each grade plotted against elevation class for all plots (n=390 grades).

Note: grades are staggered around elevation classes for visual differentiation.

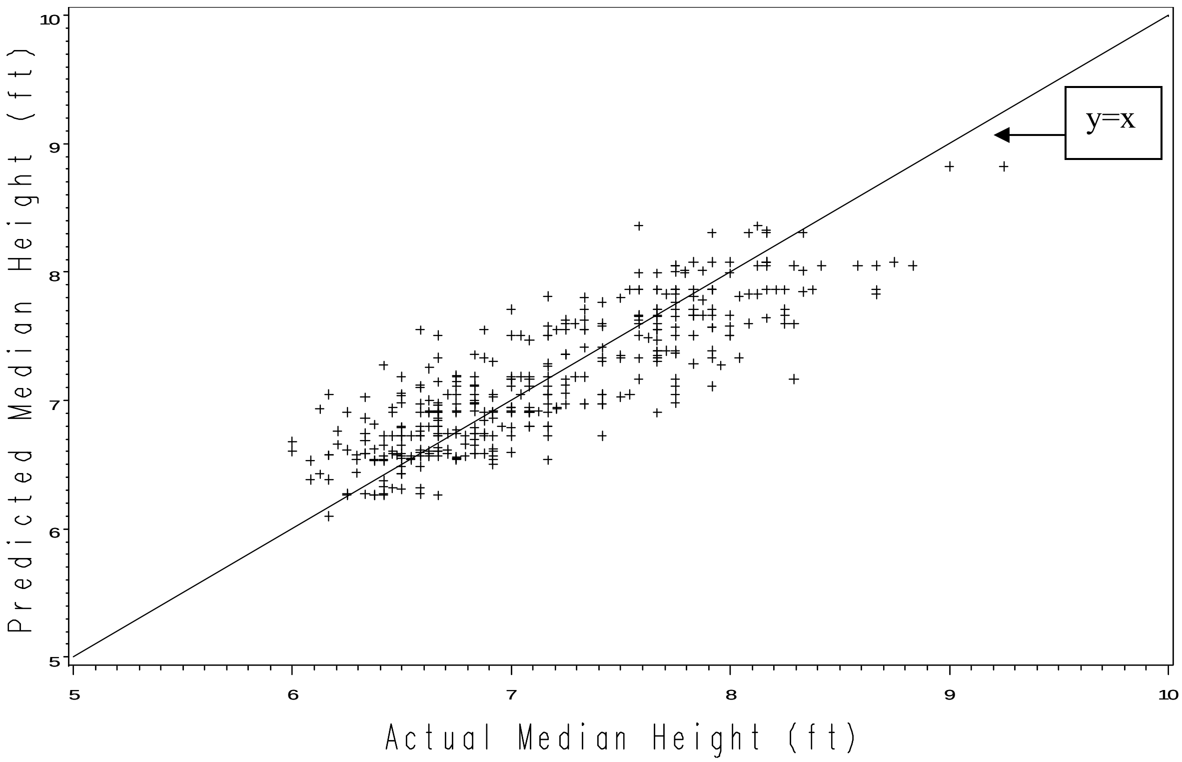


Fig. 14: Predicted median heights for each grade from final parameter recovery models plotted against actual median heights (n=390 grades).

Table 1: Elevation classes used to group stands with similar elevations.

Elevation Class	Elevation Range Sampled (ft)
1	3381-3500
2	3501-3620
3	3621-3740
4	3741-3860
5	3861-3980
6	3981-4100
7	4101-4220
8	4221-4340
9	4341-4460

Table 2: Sample size, percentage of trees and height summary statistics for each USDA grade over all plots.

			Tree Height (ft)			
USDA Grade	*n*	Percentage of trees	Mean	Std. Dev.	Min	Max
Premium	1120	8.2	7.46	0.90	4.25	9.83
One	4302	31.3	7.13	0.86	4.00	9.67
Two	3924	28.6	7.07	0.85	2.50	9.75
Cull	4386	31.9	6.79	0.87	3.00	9.67
Overall	13732	100.0	7.03	0.89	2.50	9.83

Table 3: Distribution of plots into four p-value categories over all height and grade data for Anderson Darling (A-D) and Cramer von Mises (CvM) goodness-of-fit statistics.

p-value group	Average number of trees	A-D frequency of plots	A-D percentage of plots	CvM frequency of plots	CvM percentage of plots
$p \geq 0.15$	132	34	33.3	34	33.3
$0.10 \leq p < 0.15$	133	5	4.9	8	7.8
$0.05 \leq p < 0.10$	134	12	11.8	11	10.8
$p < 0.05$	136	51	50.0	49	48.1

Table 4: Regression parameter estimates from general linear models (Equations (13) and (14)) for scale (R^2=0.62; SSD=10.27) and shape (R^2=0.43; SSD=355.04) parameters for whole plots using parameter prediction techniques.

	Scale (b) parameter			Shape (c) parameter		
Parameter	Estimate	Std. Err.	p-value	Estimate	Std. Err.	p-value
Intercept	4.1805	0.35	<.01	7.7300	2.35	<.01
SL_5	-	-	-	0.4189	0.99	0.67
SL_{10}	-	-	-	-0.2970	0.84	0.72
SL_{15}	-	-	-	0.3164	0.79	0.69
SL_{20}	-	-	-	0.6686	0.87	0.44
SL_{25}	-	-	-	2.4471	0.96	0.01
SL_{30}	-	-	-	0.0000	-	-
EC_1	0.5983	0.15	<.01	-4.1481	0.90	<.01
EC_2	0.9917	0.21	<.01	-2.3985	1.30	0.07
EC_3	0.1420	0.12	0.24	-1.9789	0.75	0.01
EC_4	0.1514	0.12	0.21	-0.9418	0.75	0.21
EC_5	0.1456	0.15	0.34	-1.2943	0.96	0.18
EC_6	0.4495	0.13	<.01	-1.6879	0.80	0.04
EC_7	0.3850	0.14	0.01	0.5951	0.87	0.49
EC_8	-0.0653	0.18	0.71	-0.1382	1.08	0.90
EC_9	0.0000	-	-	0.0000	-	-
Age	0.4239	0.05	<.01	0.7064	0.31	0.03

Table 5: Analysis of variance for the general linear models (Equations (13) and (14)) of the scale and shape parameters for whole plots obtained from parameter prediction techniques.

Source	DF	F-statistic	p-value
Scale			
Elevation Class	8	5.71	<.01
Age	1	72.30	<.01
Shape			
Slope	5	2.50	0.04
Elevation Class	8	4.35	<.01
Age	1	5.09	0.03

Table 6: Percentile combinations and sum of squares differences (SSD) between the MLE shape parameter and the predicted shape parameter over all plots generated from parameter recovery techniques.

Percentile 1	Percentile 2	SSD
H_{99}	H_{1}	409.84
H_{99}	H_{5}	356.67
H_{99}	H_{10}	328.52
H_{99}	**H_{15}**	**323.74**
H_{99}	H_{25}	323.94
H_{97}	H_{1}	421.03
H_{97}	H_{5}	369.50
H_{97}	H_{10}	348.64
H_{97}	H_{15}	342.28
H_{97}	H_{25}	343.80
H_{95}	H_{1}	428.79
H_{95}	H_{5}	382.51
H_{95}	H_{10}	360.58
H_{95}	H_{15}	351.24
H_{95}	H_{25}	345.68
H_{90}	H_{1}	437.05
H_{90}	H_{5}	396.58
H_{90}	H_{10}	377.31
H_{90}	H_{15}	366.50
H_{90}	H_{25}	355.43
H_{75}	H_{1}	454.62
H_{75}	H_{5}	424.51
H_{75}	H_{10}	411.51
H_{75}	H_{15}	405.48
H_{75}	H_{25}	393.29

Note: The combination of the 99th and 15th percentiles had the lowest SSD (bolded above).

Table 7: Regression parameter estimates from general linear models (Equations (16) and (17)) for 15th percentile (R^2=0.43) and 99th percentile (R^2=0.58) for whole plots using parameter recovery techniques.

	15th percentile			99th percentile		
Parameter	**Estimate**	**Std. Err.**	**p-value**	**Estimate**	**Std. Err.**	**p-value**
Intercept	3.7090	0.35	<.01	5.1736	0.41	<.01
Sl_5	0.0593	0.19	0.75	-	-	-
Sl_{10}	0.1050	0.16	0.52	-	-	-
Sl_{15}	0.1654	0.15	0.29	-	-	-
Sl_{20}	0.1779	0.17	0.30	-	-	-
Sl_{25}	0.5767	0.19	<.01	-	-	-
Sl_{30}	0.0000	-	-	-	-	-
EC_1	-	-	-	0.9685	0.17	<.01
EC_2	-	-	-	1.2275	0.24	<.01
EC_3	-	-	-	0.3004	0.14	0.03
EC_4	-	-	-	0.1902	0.14	0.18
EC_5	-	-	-	0.3026	0.18	0.09
EC_6	-	-	-	0.6850	0.15	<.01
EC_7	-	-	-	0.4775	0.16	<.01
EC_8	-	-	-	-0.0041	0.20	0.98
EC_9	-	-	-	0.0000	-	-
Age	0.3506	0.05	<.01	0.4189	0.06	<.01

Table 8: Analysis of variance for the general linear models (Equations (16) and (17)) of the 15th and 99th percentiles of the empirical height distribution for whole plots.

Source	DF	F-statistic	p-value
15th percentile			
Slope	5	2.73	0.02
Age	1	60.61	<.01
99th percentile			
Elevation class	8	7.97	<.01
Age	1	52.69	<.01

Table 9: Comparison of sum of squares differences (SSD) for shape and scale parameter models obtained from parameter prediction (PP) and parameter recovery (PR) techniques.

	SSD - Scale parameter		**SSD - Shape parameter**	
Method	*PP*	*PR*	*PP*	*PR*
Whole plot	10.27	0.09	355.04	323.74
USDA Premium	9.35	0.18	108794.54	1824.42
USDA One	9.14	0.16	2416.13	1007.83
USDA Two	9.85	0.13	732.03	680.96
USDA Cull	8.47	0.15	1305.85	1079.06

Table 10: Distribution of USDA grades within plots into four p-value categories for Anderson Darling (A-D) and Cramer von Mises (CvM) goodness-of-fit statistics.

p-value group	Average number of trees	A-D frequency of grades	A-D percentage of grades	CvM frequency of grades	CvM percentage of grades
$p \geq 0.15$	32	251	64.4	253	64.9
$0.10 \leq p < 0.15$	37	13	3.3	14	3.6
$0.05 \leq p < 0.10$	37	37	9.5	39	10.0
$p < 0.05$	44	89	22.8	84	21.5

Table 11: Regression parameter estimates from the general linear models (Equations (18) and (19)) for the scale (R^2=0.74; SSD=9.35) and shape (R^2=0; SSD=108794.54) parameters for premium grades using parameter prediction techniques.

	Scale (*b*) parameter			Shape (*c*) parameter		
Parameter	**Estimate**	**Std. Err.**	**p-value**	**Estimate**	**Std. Err.**	**p-value**
Intercept	4.1555	0.42	<.01	23.4055	3.95	<.01
EC_1	1.0998	0.17	<.01			
EC_2	1.8116	0.27	<.01			
EC_3	0.2720	0.14	0.05			
EC_4	0.5068	0.16	<.01			
EC_5	0.1776	0.16	0.28			
EC_6	0.3854	0.14	<.01			
EC_7	0.4926	0.16	<.01			
EC_8	0.1010	0.21	0.63			
EC_9	0.0000	-	-			
Age	0.4656	0.06	<.01			

Table 12: Analysis of variance for the general linear model (Equations (18) and (19)) of the scale parameter for premium grades obtained from parameter prediction techniques.

Source	DF	F-statistic	p-value
Scale			
Elevation Class	8	10.85	<.01
Age	1	62.14	<.01
Shape			
Intercept	1	35.11	<.01

Table 13: Regression parameter estimates from the general linear models (Equations (20) and (21)) for scale (R^2=0.73; SSD=9.14) and shape (R^2=0.13; SSD=2416.13) parameters for grade #1s using parameter prediction techniques.

	Scale (*b*) parameter			Shape (*c*) parameter		
Parameter	**Estimate**	**Std. Err.**	**p-value**	**Estimate**	**Std. Err.**	**p-value**
Intercept	3.9390	0.33	<.01	0.8934	3.71	0.81
EC_1	0.8334	0.14	<.01	-	-	-
EC_2	1.0303	0.20	<.01	-	-	-
EC_3	0.1861	0.11	0.11	-	-	-
EC_4	0.3452	0.11	<.01	-	-	-
EC_5	0.1022	0.14	0.45	-	-	-
EC_6	0.4222	0.12	<.01	-	-	-
EC_7	0.4768	0.13	<.01	-	-	-
EC_8	0.0002	0.17	1.00	-	-	-
EC_9	0.0000	-	-	-	-	-
Age	0.4698	0.05	<.01	2.0132	0.53	<.01

Table 14: Analysis of variance for the general linear models (Equations (20) and (21)) of the scale and shape parameters for grade #1s obtained from parameter prediction techniques.

Source	DF	F-statistic	p-value
Scale			
Elevation Class	8	8.48	<.01
Age	1	99.81	<.01
Shape			
Age	1	14.32	<.01

Table 15: Regression parameter estimates from the general linear models (Equations (22) and (23)) for scale (R^2=0.66; SSD=9.85) and shape (R^2=0.39; SSD=732.03) parameters for grade #2s using parameter prediction techniques.

	Scale (*b*) parameter			Shape (*c*) parameter		
Parameter	**Estimate**	**Std. Err.**	**p-value**	**Estimate**	**Std. Err.**	**p-value**
Intercept	3.8159	0.39	<.01	14.4800	0.72	<.01
Sl_5	0.2275	0.17	0.17	-	-	-
Sl_{10}	0.3112	0.14	0.03	-	-	-
Sl_{15}	0.1222	0.13	0.36	-	-	-
Sl_{20}	0.2750	0.14	0.06	-	-	-
Sl_{25}	0.4410	0.16	0.01	-	-	-
Sl_{30}	0.0000	-	-	-	-	-
EC_1	0.5898	0.15	<.01	-5.3271	1.23	<.01
EC_2	1.1334	0.22	<.01	2.7319	1.77	0.13
EC_3	0.1416	0.12	0.26	-1.4952	0.98	0.13
EC_4	0.3090	0.12	0.02	0.5376	0.95	0.57
EC_5	0.1897	0.16	0.24	-3.9088	1.23	<.01
EC_6	0.4723	0.13	0.00	-3.3431	1.02	<.01
EC_7	0.3698	0.14	0.01	1.0596	1.18	0.37
EC_8	0.0357	0.18	0.84	0.6645	1.44	0.65
EC_9	0.0000	-	-	0.0000	-	-
Age	0.4358	0.05	<.01	-	-	-

Table 16: Analysis of variance for the general linear models (Equations (22) and (23)) of the scale and shape parameters for grade #2s obtained from parameter prediction techniques.

Source	DF	F-statistic	p-value
Scale			
Slope	5	2.42	0.04
Elevation Class	8	5.50	<.01
Age	1	69.85	<.01
Shape			
Elevation Class	8	7.30	<.01

Table 17: Regression parameter estimates from the general linear models (Equations (24) and (25)) for scale (R^2=0.62; SSD=8.47) and shape (R^2=0.18; SSD=1305.85) parameters for cull grades using parameter prediction techniques.

	Scale (*b*) parameter				Shape (*c*) parameter		
Parameter	**Estimate**	**Std. Err.**	**p-value**	**Parameter**	**Estimate**	**Std. Err.**	**p-value**
Intercept	4.0126	0.36	<.01	Intercept	9.5077	1.00	<.01
Sl_5	0.1995	0.15	0.20	AS_E	7.1053	1.82	<.01
Sl_{10}	0.1918	0.13	0.14	AS_N	4.8289	2.11	0.02
Sl_{15}	0.1517	0.12	0.22	AS_{NE}	1.7473	1.59	0.28
Sl_{20}	0.2066	0.13	0.13	AS_{NW}	1.6616	1.39	0.23
Sl_{25}	0.5009	0.15	<.01	AS_S	3.7689	1.30	<.01
Sl_{30}	0.0000	-	-	AS_{SE}	3.5058	1.50	0.02
EC_1	0.4594	0.14	<.01	AS_{SW}	2.5021	1.26	0.05
EC_2	1.1403	0.20	<.01	AS_W	0.0000	-	-
EC_3	0.0165	0.12	0.89				
EC_4	0.1737	0.12	0.14				
EC_5	0.2985	0.15	0.05				
EC_6	0.5888	0.12	<.01				
EC_7	0.3647	0.13	0.01				
EC_8	0.0022	0.17	0.99				
EC_9	0.0000	-	-				
Age	0.3806	0.05	<.01				

Table 18: Analysis of variance for the general linear models (Equations (24) and (25)) of the scale and shape parameters for cull grades obtained from parameter prediction techniques.

Source	DF	F-statistic	p-value
Scale			
Slope	5	2.56	0.03
Elevation Class	8	7.38	<.01
Age	1	61.95	<.01
Shape			
Aspect	7	3.00	0.01

Table 19: Percentile combinations and sum of squares differences (SSD) between the MLE shape parameter and the predicted shape parameter for the premium USDA grade generated from parameter recovery techniques.

Percentile 1	Percentile 2	SSD
H_{99}	H_1	2155.16
H_{99}	H_5	2090.77
H_{99}	H_{10}	1873.11
H_{99}	H_{15}	2031.79
H_{99}	H_{25}	1993.43
H_{97}	H_1	2135.98
H_{97}	H_5	2081.89
H_{97}	H_{10}	1848.89
H_{97}	H_{15}	2009.86
H_{97}	H_{25}	1988.04
H_{95}	H_1	2031.77
H_{95}	H_5	2011.43
H_{95}	H_{10}	1872.36
H_{95}	H_{15}	1917.89
H_{95}	H_{25}	1945.71
H_{90}	H_1	1975.94
H_{90}	H_5	1994.46
H_{90}	H_{10}	1894.74
H_{90}	**H_{15}**	**1824.42**
H_{90}	H_{25}	1984.11
H_{75}	H_1	2140.47
H_{75}	H_5	2235.33
H_{75}	H_{10}	2200.90
H_{75}	H_{15}	2022.57
H_{75}	H_{25}	2792.51

Note: The combination of the 90^{th} and 15^{th} percentiles had the lowest SSD (bolded above).

Table 20: Parameter estimates from general linear model for the 15th percentile (R^2=0.63) and the 90th percentile (R^2=0.62) for USDA premium grade.

	15th percentile			90th percentile		
Parameter	**Estimate**	**Std. Err.**	**p-value**	**Estimate**	**Std. Err.**	**p-value**
Intercept	3.2184	0.63	<.01	4.7039	0.55	<.01
EC_1	0.7719	0.25	<.01	1.1561	0.22	<.01
EC_2	1.9419	0.39	<.01	1.7006	0.34	<.01
EC_3	0.0519	0.21	0.80	0.3042	0.18	0.10
EC_4	0.5801	0.24	0.02	0.4493	0.21	0.04
EC_5	0.0671	0.23	0.78	0.2537	0.21	0.22
EC_6	0.0070	0.20	0.97	0.5164	0.17	<.01
EC_7	0.3288	0.24	0.17	0.5399	0.21	0.01
EC_8	0.1869	0.30	0.53	0.1756	0.26	0.50
EC_9	0.0000	-	-	0.0000	-	-
Age	0.4950	0.09	<.01	0.4541	0.08	<.01

Table 21: Analysis of variance for the general linear models of the 15^{th} and 90^{th} percentiles of the empirical height distribution for the premium grade.

Source	DF	F-statistic	p-value
15^{th} percentile			
Elevation class	8	5.17	<.01
Age	1	29.32	<.01
90^{th} percentile			
Elevation class	8	6.18	<.01
Age	1	31.91	<.01

Table 22: Percentile combinations and sum of squares differences (SSD) between the MLE shape parameter and the predicted shape parameter for the USDA grade #1 generated from parameter recovery techniques.

Percentile 1	Percentile 2	SSD
H_{99}	H_1	1128.42
H_{99}	H_5	1110.06
H_{99}	H_{10}	1092.73
H_{99}	**H_{15}**	**1007.83**
H_{99}	H_{25}	1033.65
H_{97}	H_1	1157.68
H_{97}	H_5	1147.84
H_{97}	H_{10}	1116.91
H_{97}	H_{15}	1022.41
H_{97}	H_{25}	1016.61
H_{95}	H_1	1203.83
H_{95}	H_5	1186.11
H_{95}	H_{10}	1157.37
H_{95}	H_{15}	1026.96
H_{95}	H_{25}	1050.29
H_{90}	H_1	1235.05
H_{90}	H_5	1214.41
H_{90}	H_{10}	1202.72
H_{90}	H_{15}	1080.91
H_{90}	H_{25}	1138.72
H_{75}	H_1	1326.65
H_{75}	H_5	1322.54
H_{75}	H_{10}	1312.11
H_{75}	H_{15}	1155.41
H_{75}	H_{25}	1193.99

Note: The combination of the 99^{th} and 15^{th} percentiles had the lowest SSD (bolded above).

Table 23: Parameter estimates from general linear model for the 15th percentile (R^2=0.66) and the 99th percentile (R^2=0.48) for USDA grade #1.

	15th percentile			99th percentile		
Parameter	**Estimate**	**Std. Err.**	**p-value**	**Estimate**	**Std. Err.**	**p-value**
Intercept	2.7211	0.45	<.01	5.1032	0.47	<.01
EC_1	0.4426	0.20	0.03	0.7824	0.20	<.01
EC_2	0.6896	0.27	0.01	1.0481	0.28	<.01
EC_3	0.0278	0.15	0.86	0.2031	0.16	0.21
EC_4	0.2408	0.15	0.12	0.1531	0.16	0.34
EC_5	-0.1801	0.20	0.36	0.2289	0.20	0.26
EC_6	0.0713	0.16	0.66	0.5715	0.17	<.01
EC_7	0.4210	0.18	0.02	0.5203	0.19	0.01
EC_8	-0.0312	0.22	0.89	0.2761	0.23	0.24
EC_9	0.0000	-	-	0.0000	-	-
Age	0.5366	0.06	<.01	0.4308	0.07	<.01

Table 24: Analysis of variance for the general linear models of the 15th and 99th percentiles of the empirical height distribution for the USDA grade #1.

Source	DF	F-statistic	p-value
15th percentile			
Elevation class	8	2.64	0.01
Age	1	70.48	<.01
99th percentile			
Elevation class	8	3.99	<.01
Age	1	42.58	<.01

Table 25: Percentile combinations and sum of squares differences (SSD) between the MLE shape parameter and the predicted shape parameter for the USDA grade #2 generated from parameter recovery techniques.

Percentile 1	Percentile 2	SSD
H_{99}	H_{1}	728.89
H_{99}	H_{5}	708.40
H_{99}	H_{10}	708.42
H_{99}	H_{15}	703.74
H_{99}	H_{25}	689.13
H_{97}	H_{1}	700.57
H_{97}	**H_{5}**	**680.96**
H_{97}	H_{10}	706.73
H_{97}	H_{15}	711.29
H_{97}	H_{25}	696.32
H_{95}	H_{1}	710.30
H_{95}	H_{5}	684.80
H_{95}	H_{10}	702.90
H_{95}	H_{15}	711.25
H_{95}	H_{25}	713.95
H_{90}	H_{1}	759.60
H_{90}	H_{5}	710.30
H_{90}	H_{10}	741.41
H_{90}	H_{15}	745.58
H_{90}	H_{25}	760.75
H_{75}	H_{1}	801.41
H_{75}	H_{5}	753.94
H_{75}	H_{10}	798.67
H_{75}	H_{15}	818.09
H_{75}	H_{25}	847.83

Note: The combination of the 97^{th} and 5^{th} percentiles had the lowest SSD (bolded above).

Table 26: Parameter estimates from general linear models for the 5th percentile (R^2=0.62) and the 97th percentile (R^2=0.48) for USDA grade #2.

	5th percentile			97th percentile		
Parameter	**Estimate**	**Std. Err.**	**p-value**	**Estimate**	**Std. Err.**	**p-value**
Intercept	2.9311	0.47	<.01	5.1032	0.47	<.01
EC_1	-0.0694	0.21	0.74	0.7824	0.20	<.01
EC_2	0.4813	0.28	0.09	1.0481	0.28	<.01
EC_3	-0.1341	0.16	0.41	0.2031	0.16	0.21
EC_4	0.2569	0.16	0.12	0.1531	0.16	0.34
EC_5	-0.3991	0.20	0.05	0.2289	0.20	0.26
EC_6	-0.0706	0.17	0.68	0.5715	0.17	<.01
EC_7	0.4535	0.19	0.02	0.5203	0.19	0.01
EC_8	-0.0230	0.24	0.92	0.2761	0.23	0.24
EC_9	0.0000	-	-	0.0000	-	-
Age	0.4530	0.07	<.01	0.4308	0.07	<.01

Table 27: Analysis of variance for the general linear models of the 5th and 97th percentiles of the empirical height distribution for the USDA grade #2.

Source	DF	F-statistic	p-value
5th percentile			
Elevation class	8	3.03	<.01
Age	1	45.81	<.01
97th percentile			
Elevation class	8	3.99	<.01
Age	1	42.58	<.01

Table 28: Percentile combinations and sum of squares differences (SSD) between the MLE shape parameter and the predicted shape parameter for the USDA cull grade generated from parameter recovery techniques.

Percentile 1	Percentile 2	SSD
H_{99}	H_{1}	1261.59
H_{99}	H_{5}	1234.41
H_{99}	H_{10}	1150.09
$\mathbf{H_{99}}$	$\mathbf{H_{15}}$	**1079.06**
H_{99}	H_{25}	1092.25
H_{97}	H_{1}	1285.31
H_{97}	H_{5}	1270.38
H_{97}	H_{10}	1168.80
H_{97}	H_{15}	1123.12
H_{97}	H_{25}	1142.42
H_{95}	H_{1}	1299.01
H_{95}	H_{5}	1307.20
H_{95}	H_{10}	1207.04
H_{95}	H_{15}	1154.61
H_{95}	H_{25}	1173.61
H_{90}	H_{1}	1315.47
H_{90}	H_{5}	1315.39
H_{90}	H_{10}	1226.56
H_{90}	H_{15}	1168.63
H_{90}	H_{25}	1181.34
H_{75}	H_{1}	1353.33
H_{75}	H_{5}	1415.19
H_{75}	H_{10}	1318.45
H_{75}	H_{15}	1231.68
H_{75}	H_{25}	1275.57

Note: The combination of the 99th and 15th percentiles had the lowest SSD (bolded above).

Table 29: Parameter estimates from general linear model for the 15th percentile (R^2=0.41) and 99th percentile (R^2=0.55) for USDA cull grade.

	15th percentile			99th percentile		
Parameter	**Estimate**	**Std. Err.**	**p-value**	**Estimate**	**Std. Err.**	**p-value**
Intercept	3.7345	0.46	<.01	4.7719	0.46	<.01
Sl_5	0.0696	0.20	0.72	-	-	-
Sl_{10}	0.0959	0.16	0.56	-	-	-
Sl_{15}	0.2257	0.16	0.15	-	-	-
Sl_{20}	0.2264	0.17	0.19	-	-	-
Sl_{25}	0.5761	0.19	<.01	-	-	-
Sl_{30}	0.0000	-	-	-	-	-
EC_1	-0.0039	0.18	0.98	0.9528	0.19	<.01
EC_2	0.6540	0.26	0.01	1.4422	0.27	<.01
EC_3	-0.1193	0.15	0.42	0.0309	0.16	0.84
EC_4	-0.0074	0.15	0.96	0.2673	0.16	0.09
EC_5	0.2107	0.19	0.27	0.3030	0.20	0.13
EC_6	0.3103	0.16	0.05	0.5458	0.16	<.01
EC_7	0.3455	0.17	0.05	0.4237	0.18	0.02
EC_8	0.0084	0.21	0.97	0.0225	0.23	0.92
EC_9	0.0000	-	-	0.0000	-	-
Age	0.2992	0.06	<.01	0.4337	0.06	<.01

Table 30: Analysis of variance for the general linear models of the 15th percentile and the 99th percentile of the empirical height distribution for the USDA cull grade.

Source	DF	F-statistic	p-value
15th percentile			
Slope	5	2.67	0.03
Elevation class	8	2.35	0.02
Age	1	23.61	<.01
99th percentile			
Elevation class	8	7.17	<.01
Age	1	45.20	<.01

Table 31: Parameter estimates from the logistic regression model for USDA grade probabilities (R^2=0.10).

Parameter	Estimate	Std. Err.	p-value	Parameter	Estimate	Std. Err.	p-value
$\text{Intercept}_{\text{Premium}}$	-1.8002	0.22	<.01	EC_4	-0.3015	0.06	<.01
$\text{Intercept}_{\text{One}}$	0.3055	0.22	0.16	EC_5	0.6231	0.06	<.01
$\text{Intercept}_{\text{Two}}$	1.5862	0.22	<.01	EC_6	0.7018	0.05	<.01
SL_5	-0.1175	0.05	0.03	EC_7	0.3279	0.06	<.01
SL_{10}	-0.1451	0.04	<.01	EC_8	-0.1792	0.07	0.01
SL_{15}	0.3318	0.03	<.01	EC_9	0.0000	-	-
SL_{20}	0.2483	0.04	<.01	SO_{Ev}	0.4199	0.11	<.01
SL_{25}	-0.1499	0.05	<.01	SO_{Pu}	-0.0386	0.04	0.37
SL_{30}	0.0000	-	-	SO_{Sa}	-0.2023	0.07	<.01
AS_E	-0.2042	0.07	<.01	SO_{Sb}	0.0000	-	-
AS_N	0.5214	0.09	<.01	Age	-0.1318	0.03	<.01
AS_{NE}	0.4762	0.05	<.01				
AS_{NW}	0.3565	0.05	<.01				
AS_S	-0.3607	0.04	<.01				
AS_{SE}	-0.3350	0.06	<.01				
AS_{SW}	-0.2457	0.04	<.01				
AS_W	0.0000	-	-				
EC_1	-0.1931	0.07	<.01				
EC_2	-1.9595	0.14	<.01				
EC_3	0.5818	0.06	<.01				

Table 32: Parameter estimates from the general linear model for the mean height of the premium USDA grade (R^2=0.67).

Parameter	Estimate	Std. Err.	p-value
Intercept	3.5404	0.45	<.01
EC_1	1.0973	0.20	<.01
EC_2	1.5701	0.27	<.01
EC_3	0.2955	0.16	0.06
EC_4	0.2765	0.16	0.08
EC_5	0.1879	0.19	0.33
EC_6	0.3256	0.16	0.05
EC_7	0.4531	0.18	0.01
EC_8	-0.0015	0.22	0.99
EC_9	0.0000	-	-
Age	0.5231	0.06	<.01

Table 33: Analysis of variance for the general linear model for mean height of the premium USDA grade.

Source	DF	F-statistic	p-value
Elevation class	8	7.86	<.01
Age	1	67.04	<.01

Table 34: Parameter estimates from the general linear model for the mean height of the USDA grade #1 (R^2=0.73).

Parameter	Estimate	Std. Err.	p-value
Intercept	3.6724	0.34	<.01
EC_1	0.7661	0.14	<.01
EC_2	0.9606	0.21	<.01
EC_3	0.1440	0.12	0.22
EC_4	0.3444	0.12	<.01
EC_5	0.0313	0.15	0.83
EC_6	0.3440	0.12	0.01
EC_7	0.4624	0.14	<.01
EC_8	-0.0152	0.17	0.93
EC_9	0.0000	-	-
Age	0.4741	0.05	<.01

Table 35: Analysis of variance for the general linear model for mean height of the USDA grade #1.

Source	DF	F-statistic	p-value
Elevation class	8	7.34	<.01
Age	1	96.19	<.01

Table 36: Parameter estimates from the general linear model for the mean height of the USDA grade #2 (R^2=0.68).

Parameter	Estimate	Std. Err.	p-value
Intercept	3.8070	0.44	<.01
SL_5	0.2689	0.18	0.13
SL_{10}	0.2110	0.14	0.14
SL_{15}	0.0451	0.13	0.74
SL_{20}	0.2305	0.15	0.12
SL_{25}	0.5120	0.16	<.01
SL_{30}	0.0000	-	-
EC_1	0.6645	0.18	<.01
EC_2	1.7001	0.34	<.01
EC_3	0.1082	0.14	0.45
EC_4	0.2178	0.14	0.12
EC_5	0.1160	0.16	0.48
EC_6	0.4293	0.13	<.01
EC_7	0.3415	0.14	0.02
EC_8	0.0588	0.18	0.75
EC_9	0.0000	-	-
SO_{Ev}	-0.7992	0.30	0.01
SO_{Pu}	-0.1866	0.13	0.15
SO_{Sa}	-0.3259	0.18	0.08
SO_{Sb}	0.0000	-	-
Age	0.4346	0.06	<.01

Table 37: Analysis of variance for the general linear model for mean height of the USDA grade #2.

Source	DF	F-statistic	p-value
Slope	5	3.24	0.01
Elevation class	8	4.49	<.01
Soil	3	2.92	0.04
Age	1	62.16	<.01

Table 38: Parameter estimates from the general linear model for the mean height of the cull USDA grade (R^2=0.59).

Parameter	Estimate	Std. Err.	p-value
Intercept	3.8787	0.36	<.01
SL_5	0.1668	0.15	0.28
SL_{10}	0.1654	0.13	0.21
SL_{15}	0.1609	0.12	0.19
SL_{20}	0.2129	0.14	0.12
SL_{25}	0.5123	0.15	<.01
SL_{30}	0.0000	-	-
EC_1	0.3241	0.14	0.02
EC_2	1.0137	0.20	<.01
EC_3	-0.0206	0.12	0.86
EC_4	0.1220	0.12	0.30
EC_5	0.2710	0.15	0.07
EC_6	0.5302	0.12	<.01
EC_7	0.3479	0.13	0.01
EC_8	0.0101	0.17	0.95
EC_9	0.0000	-	-
Age	0.3629	0.05	<.01

Table 39: Analysis of variance for the general linear model for mean height of the cull USDA grade.

Source	DF	F-statistic	p-value
Slope	5	2.68	0.03
Elevation class	8	5.96	<.01
Age	1	55.64	<.01

Table 40: Comparison of mean heights and quadratic mean heights by USDA grade.

USDA Grade	Mean Height (ft)	Std. Dev.	Quadratic Mean Height (ft)	Std. Dev.
Premium	7.44	0.68	7.46	0.68
One	7.23	0.59	7.26	0.59
Two	7.06	0.54	7.09	0.54
Cull	6.79	0.45	6.83	0.46

Table 41: Relative frequencies (%) for a 7 year old Fraser fir Christmas tree stand with west aspect, Porters soil, 10% slope and 3500 ft elevation (developed for Application example).

Height Class (ft)	Premium (%)	Grade #1 (%)	Grade #2 (%)	Cull (%)
9 to 10	1.0	1.4	1.8	0.1
8 to 9	2.1	12.4	19.0	3.2
7 to 8	0.7	8.6	21.4	7.8
6 to 7	0.1	1.9	7.9	5.7
5 to 6	0.0	0.3	1.7	2.1
4 to 5	0.0	0.0	0.2	0.5
3 to 4	0.0	0.0	0.0	0.1
Total	3.9	24.6	52.0	19.5