ABSTRACT

HUANG, SHU. Traffic Grooming in Wavelength Routed Path Networks. (Under the direction of Assistant Professor Rudra Dutta).

Recent increase in bandwidth and development of wavelength routing techniques has prompted the study of traffic grooming in wide area wavelength routed optical networks. These networks are widely expected to form the high speed high performance backbone networks of the future. We have studied the grooming problem as applied to the path network, where the physical topology of fibers simply forms a path. Path networks are important in themselves, because they arise naturally in different realistic scenarios, but also because they are simple elemental topologies which can contribute to the understanding of more complex topologies.

We show that the problem of traffic grooming is NP-Complete in both unidirectional and bidirectional path networks under the objective of minimizing network-wide amount of electronic switching, whether bifurcation of traffic components into integral subcomponents is allowed or not. These results are somewhat surprising in such a simple topology as the path, and underline the inherent complexity of the grooming problem. Our results have implications for grooming problems with other topologies, which we explore. We also explore the approximability of the problem in path networks. We prove that there is no such a polynomial approximation algorithm that it can guarantee an approximation ratio less than infinity, unless $P = NP$.

We propose a heuristic approach to solve the problem practically. Our approach is loosely based on the idea of flow deviation. After defining the deviation framework in the context of our problem, we show that it is a family of algorithms rather than a single one, the different members of the family obtained by choosing different candidate approaches to two key subtasks. Some of these members possess practically important performance guarantees, which we define. We present numerical results obtained by applying our technique to traffic instances of various patterns to validate our theoretical claims.

Traffic Grooming in Wavelength Routed Path Networks

by

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A thesis submitted to the Graduate Faculty of
North Carolina State University
in partial satisfaction of the
requirements for the Degree of
Master of Science in Computer Networking

Department of Computer Science

Raleigh

2002

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To my parents, for their everlasting love and support.

Biography

Shu Huang was born on may 13, 1973 in ShangHai, China. He completed his undergraduate studies in electronic engineering at Beijing University of Posts and Telecommunications, China. He received a Bachelor of Science in 1995.

He spend the next several years in China Academy of Telecommunictions as an engineer with increasing level of experience and responsibility.

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Acknowledgements

The process of finishing this thesis would not have been possible without the guidance, support, coaching and mentoring of my advisor Dr.Rudra Dutta. I would like to express my gratitude to his sincere tremendous effort in guiding and correcting the writing of this thesis.

I would also like to extend my appreciation to committee members, Drs. George Rouskas and Matthias Stallmann.

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Chapter 1

Introduction

In this information age, we have witnessed the relentless demand for computer networks of higher bandwidth and lower costs. In order to fulfill this requirement, copper cables have been replaced by optic fibers in the core network for the much larger bandwidth optic fibers provide. It is well understood that optic fibers will eventually extend to individual homes (FTTH or Fiber To The Home) as a result of higher capacity demands of personal usage and lower costs of optic fibers and optical equipment. To fully utilize the large capacity of optic fibers, different modulation schemes have been proposed. Among them, a two-dimensional approach in both time domain and frequency domain multiplexing has been studied and widely accepted as likely to be dominant in the near future. The first generation of optical networks, those using optical fiber and communication at the physical layer, but not exploiting optical speed and reliability in switching or routing, have been deployed for some years. These networks typcially employ only time division multiplexing, and operate at typical speeds of 2.5 Gbit/s for the fiber. New optical technology has opened up the possibility of much higher bandwidths on a fiber, all the way up to the Tbit/s level. It is generally accepted that networks utilizing such technology and such bandwidth will prove indispensable for the commerce, business, education, and critical infrastructure of the coming decades, and will form the backbone of the next generation of wide area optical networks [7, 20, 22]. However, these advances also give rise to a more complicated network and many new problems previously not considered by network engineers.

With the advance of *Wavelength Division Multiplexing* (WDM) in optical networks, it is possible to multiplex several wavelength channels on a single optical fiber. The number of such wavelengths that can be multiplexed on a fiber now stands at about 200. Each wavelength on a fiber in such a WDM system can be viewed as a channel that provides an optical connection between two nodes. We call such a channel a *lightpath*. Traffic demands are aggregated into data frames by conventional TDM methods, which traverse lightpaths. Multiple lightpaths are now multiplexed on a fiber link using WDM. Lightpaths are originated from electronic traffic streams at the network node originating the traffic and terminated into electronic traffic streams at the node for which the traffic is destined by electro-optical Line Terminating Equipment (LTE), which are among the costliest of the network components used in such an architecture.

In the architecture of first generation optical networks, each wavelength channel would need to be electronically terminated at each intermediate node as well, and reconverted into an optical signal after being forwarded electronically. This so called OEO routing (Opto-Electro-Optic) would not only require many more costly LTEs, but interpose electronic processing and buffering in between optical transport segments, and hence increase delay, variability of delay, and the possibility of loss due to buffer overflow tremendously. However, advancement in the technology of *wavelength routing* gives us the capability of selectively bypassing some lightpaths optically, instead of converting every lightpath coming in on a single optical fiber to electronic signals. That is, a lightpath can traverse one or more intermediate nodes optically without the need of LTEs.

A lightpath thus becomes a more general transport mechanism for the next generation of optical networks, and it is possible for traffic demands at higher layers to be routed entirely on lightpaths, without reference to the physical fiber links in the network. The set of all such lightpaths therefore forms a topology which appears as the network topology to the higher layers. To distinguish it from the physical topology of the network formed by the fiber links, this topology has been variously called the *optical, logical,* or *virtual topology* [8, references thereof]. The problem of designing such virtual topologies may have different objectives, such as delay or hop count minimization, or throughput maximization. In recent times, it has become clear that the increasing bandwidth available even on a single wavelength will force successful network architectures to combine slower speed traffic streams with different source and destination nodes on a single lightpath using TDM techniques. In such a case, some OEO routing and LTE cost is unavoidable. The class of problems in which a virtual topology is designed and network traffic routed on the virtual topology with the objective of minimizing the electronic routing load on the network has been named *traffic grooming* [7, references thereof].

Network design problems related to virtual topology and, more recently, traffic grooming, have been addressed in literature. However, much of the grooming study has focused on ring networks (networks in which the physical topology of fiber links is a ring), because of the existence of legacy high-speed networking protocols such as SONET designed specifically for ring networks. Topologies both simpler and more complex than rings have been neglected in literature until comparatively recently. In this thesis, we focus on an extremely simple network topology, namely that of the unidirectional path (sometimes called a tandem network). It may appear that the path network is too simple to be worth the attention; however, as we show, the grooming problem is sufficiently complex that it is not straightforward even in path networks. Path networks are worth studying for themselves because the path is a natural simple topology that arises in many contexts. Even more importantly, path networks can arise as subnetworks into which more complicated network topologies can be decomposed.

The rest of this thesis is arranged as follows. In the next chapter, we provide a context for the study of path networks by describing the elements of network architecture for the type of networks we consider, and the areas of virtual topology design and traffic grooming in more detail, as well as briefly surveying the literature on the subject. Our main original contributions are in the next two chapters. In Chapter 3 we present original theoretical contributions that determine the intractability of various path network related grooming problems. In Chapter 4 we go on to present a family of heuristics that have good performance characteristics in a practical sense. Finally, we present numerical results validating the heuristic algorithms, and conclude the thesis.

Chapter 2

Context

2.1 Architectural Context

The basic structure of a *wide area optical transport network* consists of a physical topology of optical links interconnecting network nodes which are traffic routers or switches. The physical links are either single fibers or multifiber bundles. They are usually directional, but are often laid in directed pairs so that the network topology can often be appropriately considered to use bidirectional links. The switches that switch traffic operate either entirely in the eletronic domain (*i.e.* they are special purpose electronic computers, like today's Internet routers, or may operate partly in the optical domain, forwarding optical signals without attempting to extract the traffic contained in those signals. Such hybrid optical-electronic routing has been viewed as the defining characteristic of the so called second generation of optical networks, sometimes also called "all-optical" or "almost all-optical" networks. Because the key ability of the network is to switch whole wavelength channels optically, thus performing the equivalent of a large amount of traffic routing without the OEO overhead, these networks have also been known as "wavelength routed" networks. Of late, because of the very high bandwidth these networks can deliver, there has also been a trend towards considering these networks primarily as candidate solutions for *wide area* networking, also called *backbone, core,* or *long-haul* networks. Typically, the nodes of such a network would be connected by other interfaces to lower speed networks such as access networks of metropolitan or campus scope. The access networks inject traffic into and extract traffic from the backbone network. To the backbone network, this traffic is seen to be originated or terminated by the backbone node to which the access network is connected.

First generation optical networks have successfully demonstrated the large bandwidth capabilities of a fiber network. For a backbone network, the availability of the network is critical. As a 2-connected graph, the ring topology has been widely deployed with several varieties. These varieties can be classified by if the ring is directed or undirected (in the former case, it forms a unidirectional ring; while in the later case, it forms a bi-directional ring), path protected or line protected and the number of fibers used to form a protected system. As a mature protocol developed specifically for ring networks, and possessing excellent fault tolerance and self-healing characteristics, the *Synchronous Optical NETwork* (SONET)/ *Synchronous Digital Hierarchy* (SDH) systems has been widely deployed and is considered a good comparison baseline for optical networks. Detailed discussion of these systems can be found in [16]. We are seeing trend of upgrading the current 2.5Gbit/s SONET/SDH systems (OC-48) to 10Gbit/s (OC-192) and even 40Gbit/s systems. In addition, more and more number of wavelengths has been multiplexed onto a single fiber. These have been operated as multiple SONET rings utilizing the same fiber medium. In such a network, traffic flows from access networks are aggregated into SONET/SDH frames at some backbone node (source), converted to optical signals (EO conversion) and transmitted along the ring towards their destination nodes. At each intermediate node, traffic demands undergo OEO routing, until at the destination node traffic flows are dropped and routed locally. At intermediate nodes, all the traffic on a wavelength is electronically processed by a *Sonet Add-Drop Multiplexer* (SADM), which electronically switches some traffic flows, while dropping some for local delivery or adding some originated locally. More complex traffic routing can be realized by interconnecting rings using *Digital Cross Connects* (DCS), which can switch lower-speed streams.

In second generation optical networks, backbone nodes have some capability of optically switching aggregated traffic flows. This gives rise to lightpaths, the optical transport mechanism we mentioned before. Equipment that can add/drop a whole lightpath while bypassing some other lightpaths is called an *Optical Add-Drop Multiplexer* (OADM) when incoming fiber or fibers to a node are from a unique other node, and similarly outgoing fibers are to a unique other node (such as in the ring topology). An OADM is thus usually equipped with two ports connecting to the ring (as well as some local ports connecting

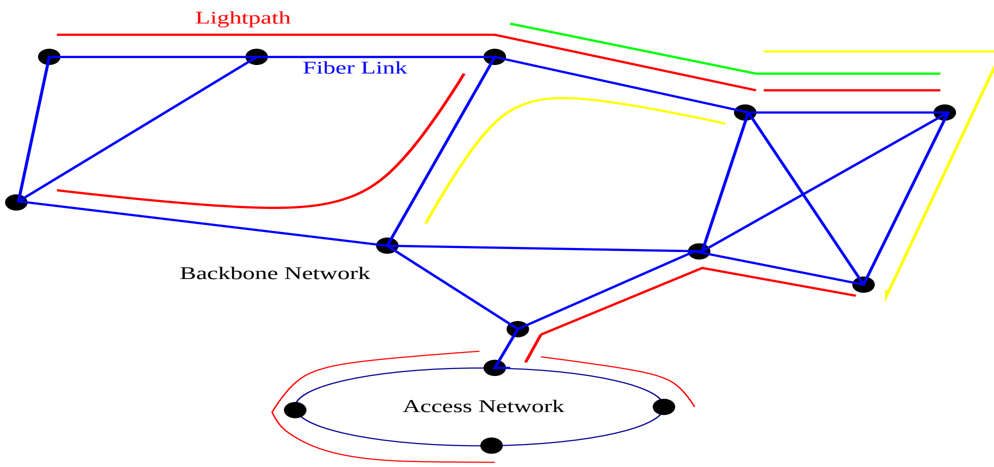

Figure 2.1: A Backbone Optical Network

to the access network, where low speed traffic is added and/or dropped). At each local port, optical signals are converted to electronic domain, i.e., a SADM must be equipped to process it electronically. Therefore, the overall costs of such an optical network is a combination of the costs of OADMs, SADMs and physical links. Since each lightpath to be electronically processed requires a SADM, the cost of SADMs can be dominant when the number of lightpaths is large. In more general topologies, such a piece of equipment is called an *Optical Cross Connect* (OXC). It has also been called a transceiver in some cases. If a node processes all the traffic demands electronically, i.e., no lightpath can bypass it optically, we call it an *opaque node*. An opaque node is thus very costly because it requires the maximum number of SADMs.

In addition, although there are optical converters, which can convert a lightpath from one wavelength to another without a conversion of OEO, available, their high costs prevent them from being widely deployed. Thus, one lightpath is associated with one wavelength (color). This constraint is named wavelength continuity constraint. As in many previous work, we assume that the wavelength conversion is not available in our study. Similar to DCSs in SONET/SDH, all-optical OXCs (Optical Cross Connect) cross connect different optical fibers, i.e., move optical signals in one optical fiber to another, without the need for conversion to electronic signals. An OXC can cross connect a fair large number of optical fibers, therefore, more complicated network topologies can be implemented by OXCs. Arbitrary topologies (sometimes called mesh topologies) have gained increasing attention in literature recently. It's well understood that arbitrary topologies will be prevailing in the optical backbones of the future. However, simple topologies such as paths, stars, rings, spiders or trees continue to be of importance because they arise naturally in many networking situations, as well as possibly contributing to the understanding of arbitrary topologies.

2.2 Virtual Topology Design

From a graph-theory point of view, a network can be viewed as a graph, which is either directed or undirected with network equipment such as routers, switch, etc., as vertices and physical links as edges (or arcs if directed). As mentioned above, in WDM networks, an additional virtual topology consisting of the set of lightpaths is introduced. The aim is to combine the best features of both optics and electronics and provide users with a virtual layer. Thus, from the user's point of view, the network graph is formed by network equipment and lightpaths, and the details of the physical network are hidden. Therefore, a major concern of service providers is how to map traffic from upper layers onto lightpaths.

Since the number of wavelength is limited, we cannot assign every traffic demand from users a lightpath end-to-end so that no eletronic routing is involved at any intermediate nodes. Rather, a virtual topology has to be designed subject to the physical constraints such as the number of fiber links, the number of wavelenths and the number of LTEs, etc. that will *minimize* OEO, even if it cannot be *eliminated*. More specifically, the following four subproblems are to be solved.

1. *Virtual topology design.* As we mentioned, a virtual topology is a digraph $G(V, A)$, where vertices correspond to equipment such as OADMs, OXCs, etc., and there is an edge between two vertices *iff* there is a lightpath established between the corresponding equipment. An example of path networks is shown below. In some work, it was also formulated as undirected graphs by assuming intermediate node as well, and reconverted into an optical signal that traffic demands are symmetric. Therefore, an edge can represent two arcs with opposite directions. However, notice that the underneath physical graph can also be directed or undirected. All four combinations

of directed/undirected lightpaths and physical links are possible.

2. *Routing lightpaths on physical links.* This is a general routing problem which has been studied extensively for decades. However, in this case, lightpaths are to be routed instead of traffic demands. The routing problem has gained intensive attention in the context of traffic engineering (TE). Different constraints are considered and added to the old version routing protocols such as OSPF and IS-IS. In addition, some new constraints and objective functions are of interest in WDM optical networks. As an example, in order to minimize the electronic processing cost (the cost of SADMs as well as traffic delay caused by OEO), a lightpath is expected to bypass as many intermediate nodes as possible. On the other hand, some physical impairments such as loss, chromatic dispersion and nonlinear effects constrain the length of a lightpath so that optical signals can be converted to electronic domain and 3Red (Retimed, Reshaped and Retransmitted). In general case, the constrained routing problem is NP-Hard [1]. Even in bi-directional rings where traffic can only be routed clockwise or counterclockwise, it is still a hard problem.

3. *WLA (Wavelength Assignment problem).* Wavelengths are one of the critical resources in optical networks. The number of wavelengths that can be multiplexed onto a single fiber is limited. The high costs of DWDM (Dense Wavelength Division Multiplexing) has given rise to CWDM (Coarse Wavelength Division Multiplexing). Therefore, it's natural to consider the problem that how to assign wavelengths for traffic demands such that the number of wavelengths is minimized. This problem is named wavelength assignment problem. It turns out that the WLA problem is closely related to the graph coloring problem. In [5], it's been proved that the general WLA problem is NP-Complete by a reduction from the classical graph coloring problem. However, there are some special cases for which fast exact algorithms are available. A path network is one of them.

4. *Traffic Routing.* Once the virtual topology has been realized, the actual node-to-node traffic flows have to be routed over this topology. This is again the general routing problem mentioned above.

The objective of the design of virtual topologies has often been standard network performance characteristics, such as minimization of total delay, hop count, maximization of throughput, etc.,. The second and thirds subproblems together form the *Routing and Wavelength Assignment* (RWA) problem, which has received considerable attention in literature. In RWA problems, it is assumed that the virtual topology to be realized is already given, usually some regular topology. The objectives of the RWA problem are similar to those of the more general virtual topology problem.

2.3 Traffic Grooming

Because optical networks are envisaged to be used in core networks, and also because when virtual topology problems were first considered it was not realized exactly how large the bandwidth of optical networks was going to be, it was deemed a reasonable assumption by most authors that each traffic demand from users would occupy one or several whole lightpath(s). Thus, the basic unit to be assigned to users was assumed to be the whole lightpath. However, it has become clear now that this granularity is too coarse, with the enormous bandwidth now available from the fiber or even in a single wavelength channel. This motivates the problem of traffic grooming, which we discuss next.

When end-to-end traffic demands are significantly less than the bandwidth of a lightpath, several of them must be multiplexed (using electronic TDM methods) if the optical bandwidth is to be well utilized. In such a case, the individual node-to-node traffic demands are said to be *sub-wavelength* in nature. Network design with sub-wavelength traffic demands has grown more and more worthy of consideration because of the following reasons:

1. Optical networks are being extended more and more close to end users, where more flexibilities to set up and tear down low speed traffic commands are required. In the framework of GMPLS, signaling and routing protocols used in MPLS networks have already been extended to other networks including time-division, spatial and wavelength switching networks. LSPs (label switched paths) in lower hierarchy are allowed to be tunneled through LSPs in higher hierarchy. In such a scenario, when traffic demands (a setup signaling message) reaches the edge of the optical network, the GMPLS-aware OXC has to assign them lightpaths and route them by some kind of traffic grooming algorithm.

2. Because of the emerging optical technologies, the bandwidth available on a single

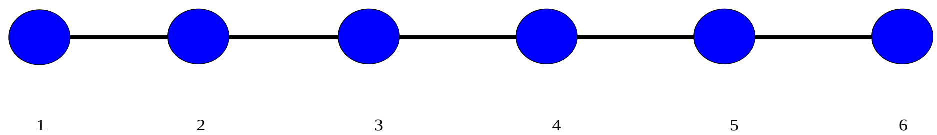

Figure 2.2: A Path network

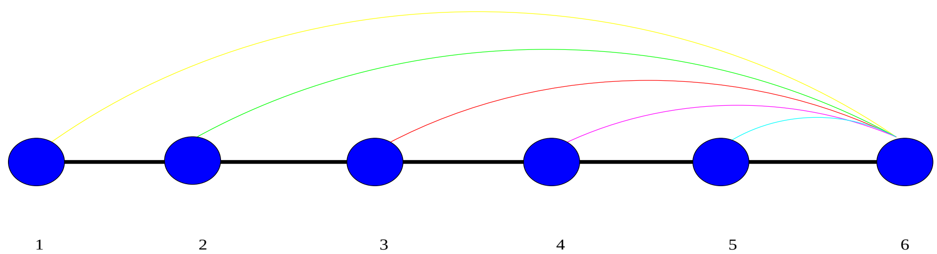

Figure 2.3: An “Egress” virtual topology on the path network

wavelength is increasing, from 2.5Gbit/s to 10Gbit/s, or even higher.

3. In addition, although the number of wavelengths available on a single fiber is increasing very fast, it is still one of the major limits in optical networks. Thus assigning whole lightpaths to small sub-wavelength traffic demands will result in severe underutilization of optical backbones.

In the literature, the optical network design problem is decomposed into sub-problems similar to the ones we presented above because of its complexity. However, these subproblems are tightly coupled so that the optimal solution can only be obtained by an integrated approach. Furthermore, when sub-wavelength traffic demands are concerned, the fourth subproblem becomes of paramount importance, and is called the traffic grooming problem.

Grooming traffic on lightpaths. Instead of routing traffic on physical links directly, a virtual topology formed by lightpaths is considered to be the network on which traffic demands are to be routed. The fact that the virtual topology can be very complicated even when the underneath physical topology is simple (path networks for example) makes the whole problem even harder. On a virtual topology of a network, a typical scenario is that, a low speed traffic demand is groomed with some other demands onto a lightpath and optically transported to an end node, then electronically processed and groomed onto another lightpath until it reaches the destination and dropped there. Grooming strategies, unlike virtual topology problem, usually try to minimize the OEO routing overhead which is the inevitable consequence of sub-wavelength traffic routing.

2.4 Grooming Literature Survey

In this section we briefly discuss recent literature in the traffic grooming area with a view to putting out original work in the proper perspective.

2.4.1 Approaches in Literature

Over the last few years, the traffic grooming problem has been a subject of intensive research. A common thread in current literature is the use of heuristic approaches to solving the grooming problem. The four subproblems identified at the end of the last section are tightly coupled so that solving them separately typically does not lead to a global optimal solution. In the context of traffic grooming, all these sub problems are to be considered so that the all over costs of the optimal solution is minimized. In [7], an ILP (Integer Linear Programming) formulation has been provided to describe this optimization problem. Each sub problem casts some constraints on the optimal solution. However, the complexity of this formulation shows that it is not practical to find an exact optimal solution by solving this ILP problem. Moreover, some of the subproblems have been shown to be individually intractable in computational terms for general cases of the grooming problems. To address this problem, different approaches have been proposed in literature. We categorize them as follows:

1. *Heuristics [2, 4, 6, 15, 18, 26, 28, 29] and approximation algorithms [25].* A number of heuristics have been proposed for each subproblem in literature. Their performances were analyzed and compared either with other heuristics or with some known bounds under certain traffic models. In [25], the authors proposed a two-phase approach with various approximation algorithms for both phases. The performance ratios are also analyzed. However, the overall exact approximation ratio remains unknown.

2. *Divide-and-Conquer.* Although solving the four sub problems separately may lead to a

local optimal, this approach is still of interest, if the local optimal is not to far from the global one. In addition, it is shown that there exist some cases where some constraints can be relaxed, or in some cases eliminated. For example, in a unidirectional ring network, traffic is always routed on the clockwise/counter-clockwise direction along the ring, thus, the routing constraints are eliminated. In star networks, the wavelength assignment problem is equivalent to the minimum edge coloring problem in bipartite graph which can be solved in polynomial time [7].

3. *General schemes and algorithms.* In the context of traffic grooming, some well-known algorithms have also been adapted, such as genetic algorithms, simulated annealing, flow deviation, etc. In [26], the ILP formulation is provided and the computational complexity is shown. Then the authors proposed a simulated-annealing-based algorithm adopting a two-step strategy. Unlike a greedy one, this approach may jump out of traps by giving a chance to change the solutions given by the first step. Thus, it is more likely to find a global optimum. In [24], a genetic algorithm was proposed. According to the simulation, the authors conclude that this approach is superior to other existing heuristics so far.

As an optimization problem, the goal is to minimize the overall cost. In the context of traffic grooming, several different cost functions have been studied:

1. *The number of LTEs.* In the context of virtual topology and RWA (Routing and Wavelength Assignment) problems, some previous work concerned with an objective of minimizing the number of wavelengths. However, in [13], the authors argued that the first-order goal of WLA (wavelength assignment) problem should be to minimize the number of ADMs instead of the number of wavelengths, in terms of the overall cost. As we have mentioned, in the context of traffic grooming, this objective seems to be a better reflection of the real costs of optical networks. Although, in some cases, minimizing the number of ADMs is equivalent to minimizing the number of wavelengths (e g. ,the scenario described in [4], an unidirectional ring with uniform traffic demands sourced from every node to an egress destination node), generally, these two objectives can not be achieved simultaneously [13]. In [4], heuristics to minimizing the number of ADMs subject to the minimum number of wavelengths were proposed. As another variant, the problem of minimizing the number of lightpaths is

studied. This approach is in fact integrated with the virtual topology design problem where the objective function is to minimize the cardinality of the virtual topology matrix. This approach implicitly results in a small number of ADMs since each lightpath requires exact one ADM at each end. It is shown in [13] that subject to the same number of lightpaths, the number of ADMs of feasible solutions can be very different.

2. *The overall electronic routing involved [7,9].* Instead of describing the cost of transceivers directly, this model focuses on the cost of electronic processing including the cost of the local tributary interfaces as well as that of switch fabric. This model provides a finer granularity to capture the overall cost of the network. In addition, from the quality of service point of view (QOS), it provides some kind of real measurement of traffic flows presented in upper layers.

3. *The network throughput or minimum blocking rate [30].* The maximum network throughput problem and minimum network blocking rate problem are dual problems which have been studied extensively in the data network arenas. However, in the context of traffic grooming in optical networks, this model was challenged because that traffic demands are expected to be aggregate high-speed connections in the backbone network, where it is reasonable to assume that the fluctuation caused by individual traffic demands arriving or leaving are "smoothed out", and the aggregate traffic is relatively static. In that case it makes more sense for service providers to optimize the utilization and/or increase the capacity of the network rather than blocking them. However, depending on the time-varying characteristics of aggregated traffic demands of end-users, it may be of importance in the future.

4. *The maximum number of lightpaths originating/terminating at a transparent node.* This model is in fact a min-max approach which, instead of considering the total number of lightpaths, tries to relieve the congestion of the network. This approach does not necessarily lead to a global minimum number of lightpaths; instead the electronic routing load is more fairly distributed among the nodes. This model has been studied extensively in the context of traffic flow problems. From service providers' point of view, this approach may be more practical than considering the number of lightpaths as a whole, because it bounds the ratio of the amount of the electronically

routing traffic to the amount of the pass-through traffic at each node. On the other hand, networks with hub nodes (a node that terminates and retransmits connections between all source and destination pairs) are of interest for the flexibility and low cost in terms of the number of ADMs. In [4], it was shown that any traffic grooming scheme that does not use hub nodes can be transformed into one that uses a hub node without adding any ADMs. However, the importance of this objective remains if distributing the switching capabilities among the network nodes is desirable.

2.4.2 Detailed Discussion

As we mentioned, a vast majority of current deployed transmission systems are based on a physical topology of rings for its simplicity and ability of self-healing. Consequently, most previous work on traffic grooming consider a ring topology. However, this simple physical topology does not make the problem significantly easier. A reduction from the path coloring problem, which was proved in [11] to be NP-Hard in both unidirectional and directional rings, shows the hardness of the problem with an objective to minimize the number of wavelengths. By relaxing the wavelength continuity constraints, wavelength converters are helpful to reduce the number of wavelengths. However, polynomial algorithms have been proposed if wavelength conversion capability is available even at a single node [22,27]. It is also been proved that the grooming problem in rings with an objective to minimize the number of ADMs is NP-Complete [4], though the scenario is rather restricted since it is assumed that the number of wavelengths is assumed to be unbounded and traffic is not allowed to be electronically routed between lightpaths unless the lightpaths are of the same wavelengths. In addition, the authors showed that wavelength converters do not help reduce the number of ADMs. Most work on ring networks consider both the grooming and wavelength assignment problems and make an option or tradeoff between minimizing the number of wavelengths and the number of ADMs. Since both of them are hard problems, some heuristics and approximation algorithms are of interest.

In [13], 2 heuristics, named cut-first and assign-first respectively, were proposed based on the observation that two lightpaths may share one ADM if they have one end point in common. Therefore, splitting lightpaths may help in reducing the number of ADMs. The cut-first algorithm is similar to the cut-and-color heuristic in [23].

In [19], a randomized approximation scheme was provided by breaking every lightpath traversing the most congested link (the link that has the maximal link load) into two "fake" lightpaths and rewiring the ring to a line. Then, the problem was treated as an integer multicommodity flow problem. By relaxing the integer constraints, the average worst case ratio was derived. The authors used a two-phase approach, where the first phase is to minimize the number of noncircular segments (hence, the number of ADMs) and the second phase is to minimize the number of wavelengths. Three heuristics were provided for the first phase.

Unidirectional ring networks were studied in [4] to minimize the number of ADMs under different assumptions of traffic patterns. Heuristics to minimize the number of ADMs for all-to-one, all-to-all uniform and distance dependent traffic patterns were provided. Serving as a metric, the authors derived lower bounds on the number of ADMs required for these traffic patterns. The last part of the paper proved a useful fact that using a hub with a SONET cross connect can reduce the number of ADMs significantly.

The author of [17] extended the work in [4]. Uniform all-to-all traffic in unidirectional ring networks was studied. It was conjectured in [4] that the minimum number of ADMs can be achieved simultaneously with the minimum number of wavelengths for all-to-all uniform traffic. Two cases of g, the grooming factor (the capacity of a lightpath divided by the basic rate of a connection), which is equal to 4 and 16 respectively, were studied. The author proved the conjecture is true for $g = 4$. It also proved that the minimum number of ADMs can be achieved with ADMs uniformly placed among nodes. These two optimal solutions are usually different. For the case that $g = 16$, there are some issues still open.

In [29], the authors extended their previous work on scheduling to the traffic grooming problem. Their approach is generic in that it can be applied to both bi-directional and unidirectional SONET/WDM ring networks under arbitrary traffic and with an arbitrary grooming factor. For uniform traffic, three algorithms proposed in some previous work were applied to construct as few circles (a set of disjoint connections along the ring) as possible for unidirectional rings, bi-directional rings with odd and even number of nodes, respectively. A heuristic for both unidirectional and bi-directional rings with non-uniform traffic was provided thereafter. The heuristic tries to fit connections into circles without creating an additional gap (an interval in a ring that is not occupied by any traffic connections). If a connection overlaps with some connection in every existing circle, a new circle is constructed. If it is a gap maker (an additional gap will be created fitting it in any existing circles), put it into a list. Then, it has two options, one is to minimize the number of circles (wavelength, implicitly), the other is to minimize the number of end nodes (ADMs, implicitly). Then, lower bounds were derived for different traffic patterns. At the second step, after the circle construction, an algorithm to groom circles onto wavelengths by overlapping as many end nodes as possible was proposed.

In the literature, bounds on grooming gained great interest. A tight bound provides a good metric to which the performance of the algorithms are compared. In addition, it also gives us some intrinsic understanding of the problem. In terms of the minimum number of ADMs, a number of bounds on ring networks have been proposed in literatures. A simple lower bound is given as $\sum_i \lceil \max \sum_j t^{ij}, \sum_j t^{ji} \rceil / C$, where t^{ij} is the traffic demand for node i to j, and C is the capacity of a lightpath. The rational behind this is that each node must have a number of ADMs to add the traffic coming into and drop the traffic coming out of the network. This bound is tight only in the all-optical virtual topology and thus very loose in general. For the uniform traffic model some improved lower bounds have been proposed in ring networks. The authors of [4] derived a tighter bound by finding the maximum average number of circuits that can be supported by an ADM. In [14], improved bounds were derived for BLSR/2 and UPSR with a hub node (a DCS) by considering the maximum amount of traffic circuits that can be supported by a lightpath. A tighter bound was given in [6] for uniform traffic in bi-directional ring networks. It is shown that this bound is better than the bound in [4] when the network scale is large. In [9], a general approach to derive a sequence of lower and upper bounds was proposed. For ring networks, an ILP formulation was presented first. The ring network is decomposed into path segments where the ILP formulation can be simplified. By decomposing the ring into larger path segments, this approach generates a sequence of bounds, both upper and lower, in which a successive bound is guaranteed to be at least as strong as the previous one, at the cost of the increasing running time required.

A mesh network topology has gained increasing attention recently. It's well understood that this topology will be prevailing in the next generation optical networks. In [30], traffic grooming in mesh networks is studied. The authors proposed comprehensive ILP (Integer Linear Programming) formulations to the problem with an objective to maximize the "total successfully-routed low-speed traffic" (hence, minimize the blocking rate) for both multi-hop and single-hop traffic grooming. Several heuristics were then proposed and evaluated. Specifically, two heuristics for traffic grooming, namely, MST (Maximizing Single-Hop Traffic) and MRU (Maximizing Resource Utilization), were employed together with some well-known routing (shortest path routing) and wavelength assignment (first-fit) algorithms to provide an integral solution to the traffic grooming problem. As the name suggests, MST sorts all traffic demands according to the amount that is not carried yet (wavelength(s) assigned and routed), then try to assign a direct lightpath for traffic demands as many as possible. MRU is the same except that sorting is based on resource utilization values. The authors also extended the ILP formulation to the cost model.

The authors of [18] presented a heuristic for traffic grooming to minimize the number of transceivers without considering the physical topology. The problem was formulated as an integer multicommodity flow problem which is well-known to be NP-Hard in general case with an objective to minimize the cardinality of the virtual topology matrix. A dual problem was then presented and used to solve the problem heuristically. The main idea is to find a large initial virtual topology(by assigning dedicated lightpath(s) to each source-destination pair), then try to remove lightpaths as many as possible. The rational behind this is that the removal of a lightpath between source node i and destination node j is feasible if and only if a traffic demand of C, the capacity of a lightpath, can be routed through the surplus capacities (the residue network, in the terminology of network flows). The heuristic enumerates all the source-destination pair and solves a minimum cost flow problem for each of them, then picks the minimum cost pair to eliminate. We noticed that there are some similarities between this approach and ours.

In addition to the static traffic model, dynamic traffic model is also studied. Traditionally, optical networks were deployed as the backbone network, where traffic are static and/or predictable. Therefore, some off-line algorithms were used to configure the network statically. Traffic grooming in rings with dynamically changing traffic is studied in [2]. Instead of a single traffic matrix, a set of allowable traffic is considered. The objective is to minimize the number of electronic multiplexing costs (ADMs, specifically), while satisfying a set of allowable traffic matrix. In particular, t-allowable traffic model was proposed, where each node can source at most t duplex circuits at any time. A lower bound was derived and a necessary and sufficient condition was proved by formulating it as a bipartite matching problem. Several algorithms for removing ADMs were proposed. ADMs are removed while keeping the condition satisfied. Thus, any t-allowable traffic can be satisfied by the remaining ADMs. Then, blocking properties have been presented. An upper bound on the number of ADMs that can be removed for a wide-sense nonblocking ring with t-allowable traffic was provided.

Dynamic traffic models are also studied in [15], where six different virtual topologies, namely, Fully Optical, Single-Hub, Double-Hub, PPWDM (point-to-point WDM), Hierarchical, Incremental, ring networks were presented. Costs in terms of the number of wavelengths, the number of transceivers per node and the maximum hop length required to satisfy the traffic models were compared. Traffic models under study were static, dynamic traffic with a upper bound on traffic streams that can be terminated at each node (assumption A) and dynamic traffic that is further bounded by the link load (assumption B), respectively. In addition to the costs, the switching capability was also studied in terms of wide-sense nonblocking (a new traffic demand can be accommodated without disturbing existing traffic demands) and rearrangeably nonblocking (a new traffic demand can be accommodated, however, some existing traffic demands must be rearranged). The only rearrangeably nonblocking topology under traffic assumption B was the Double-Hub ring topology. This was proved by transforming a Double-Hub ring into a three-stage Clos network, which was known to be rearrangeably nonblocking. The authors also introduced an incremental traffic model, where once a traffic connection is setup, it will never be released. This assumption is reasonable in current backbone optical networks where the traffic is less flexible and connections remain for a long time. An architecture for the incremental ring network was proposed and proved to be wide-sense nonblocking.

2.4.3 Summary

There has been considerable interest in the area of traffic grooming recently. Much of the literature has focused on physical topologies that are of practical interest. The largest volume of literature is concerned with ring networks, and some literature exists on completely arbitrary topologies. However, other topologies, both simpler than the ring and more (or equally) complex ones such as trees or spiders have not been adequately addressed. Since such topologies are very probably going to be of practical interest in the near future, both in themselves and as building blocks of more general topologies, research in these topologies are likely to grow and indeed, some such studies are currently appearing (such as our present work).

Another trend in the literature is the application of heuristics, since grooming related problems are sometimes known to be, and more often conjectured to be, computationally intractable. The theoretical results regarding such intractability are not complete.

The grooming problem is known to be NP-complete for ring networks, but this is a consequence of the fact that the wavelength assignment problem is known to be NPC for ring networks. There is no theoretical result in the literature specifically the grooming nature of the problem, except some results that are quite constrained (for example, in the case where no traffic can be exchanged between lightpaths of different wavelengths even when they have been electronically terminated, or the number of wavelengths is unbounded [4], which is of limited practical interest). Such a result is likely to become quite valuable in the future since wavelength converters could become cheap and easily available, thus rendering the wavelength assignment problem moot. The question then would be whether the grooming problem is tractable or not in itself. For topologies other than rings, often the problem is conjectured to be NP-complete because of its relation of virtual topology problems, some of which are known to be (or in their turn conjectured to be) NP-complete.

Thus a need exists both for the examination of specific topologies other than rings, and a rigorous treatment of the complexity issue for them. If a problem turns out to be NP-complete, there is obviously the need for good heuristic algorithms to practically address such a problem. In the next chapters, we present our original contribution to these needs.

Chapter 3

Complexity Results

In this chapter, we present some complexity results of the traffic grooming problem with an objective to minimize the amount of electronic routing. Our main contribution is to prove that traffic grooming problems on unidirectional path networks are NP-Complete, and is presented in Section 3.1. Propositions 3.1 and 3.1 settle the question of whether traffic grooming is tractable in a network topology as simple as the path, which has been open till now. Further, Corollary 3.2.1 shows that for the interesting case when traffic is allowed to be bifurcated to base rate components, an approximation algorithm also cannot be hoped for. In Section 3.2 we explore the consequences of these results for some other topologies of importance. In particular, Propositions 3.4 and 3.5 settle the question of the inherent difficulty of the grooming problem (as opposed to the wavelength assignment subproblem) in ring networks when the total electronic routing cost metric is used, which has been an open question. First, we introduce some general notation and terminology.

We adopt a mathematical modeling of the grooming problem following the one given in [7]. For ease of reference we reproduce some of the essential notation here. In the description of the traffic grooming problem a traffic matrix $T = [t^{(sd)}]$, where $t^{(sd)}$ is the amount of traffic between source node s and destination node d, is given. It is assumed that each traffic demand is in units of a basic traffic rate (OC-3, e.g.), which is not allowed to be bifurcated (routed through different lightpaths), and the number of connection may be larger than the grooming factor C, the capacity of a lightpath (OC-48, e.g.) divided by the traffic rate of a connection, which is also given as part of the problem instance. We call such a $t^{(sd)}$ a traffic component or demand. We define the load L as the maximum amount of traffic traversing any physical link. Each fiber can support a given number W of wavelength channels. The network physical topology is given as a graph $G(V, E)$, with $N = |V|$.

By OEO or electronic routing, we mean traffic that is electronically dropped and added at an intermediate node (a node that is neither the source nor the destination). In practical terms, the problem of interest is the optimization problem of finding the grooming solution such that the total electronic routing over all network nodes is minimized. Instead of considering the optimization problems, we consider the simple decision versions of them. That is, given an instance of traffic grooming, the problem is whether there is a feasible traffic grooming solution such that the total amount of electronic routing is less than or equal to a given number R. Clearly, the simple decision problems are no harder than the corresponding optimization problems, since an answer to the optimization problem translates in a straightforward manner to an answer for the decision problem. On the other hand, if the decision problems are in P, we can simply use a binary search algorithm (note that the amount of electronic routing of any given instance is bounded by $||T||O(N)$, where T is the traffic matrix given, N is the number of nodes in the path network, $||T|| = \sum_{i=1}^{N} \sum_{j=1}^{N} t^{(sd)}$) to find the minimum amount of electronic routing (a Turing reduction). Thus, the optimization problems are NP-easy [12].

3.1 Unidirectional Path Networks

Proposition 3.1 *The decision version of the traffic grooming problem in unidirectional path networks (bifurcated routing of traffic not allowed) is NP-Complete.*

An instance of the problem is provided by specifying a number N of nodes forming a network with directed connections from each node i to $i+1$, $i \in \{1, 2, \ldots N-1\}$, a traffic matrix $T = [t^{(sd)}]$, a grooming factor C, and a number of wavelengths supported by each link W as described in [7], and a goal R. The problem asks whether a valid virtual topology may be formed on the path and all traffic in T assigned to the virtual topology so that the total electronic routing is less than or equal to R over all the nodes. Each source-destination traffic component $t^{(sd)}$ must be routed over a unique sequence of lightpaths from node s to node d. In this context, two lightpaths which have the same source and destination

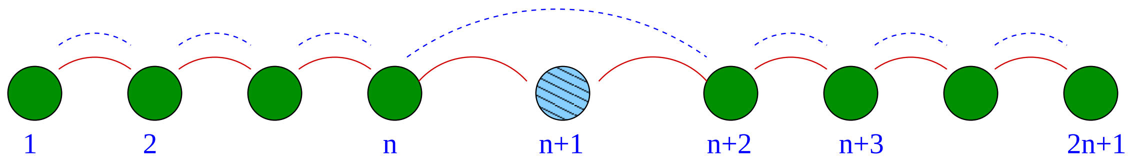

Figure 3.1: Example of path construction of Proposition 3.1

nodes and follow the same physical route over the physical topology, differing only in the wavelength used, are considered the same lightpath (in keeping with [7]), and splitting of the same traffic component over two such lightpaths is not considered bifurcated routing. This provision is required to feasibly route traffic components which are larger than C. In such cases the split must be into integer sub-components. It can be easily shown that requiring no more than one such sub-component to be strictly less than C does not impose a stronger constraint.

Proof: The reduction is from the Subsets Sum problem [12]. Given an instance of the Subsets Sum problem with n elements of size $s_i \in Z^+ \forall i \in \{1, 2, \ldots n\}$, and a goal sum B. Let $B_1 = \max\{B, \sum_i s_i - B\}$. (For the purpose of the Subsets Sum problem, posing the instance with B or B_1 is equivalent.) Construct a path network using the following transformation: $N = 2n + 1$, $W = 2$, $C = \sum_i s_i + 1$, $t^{(i,i+1)} = C + 1, \forall i \in \{1, 2, \ldots n-1\} \cup \{n+1, n+2, \ldots 2n\}$, $t^{(n,n+1)} = t^{(n+1,n+2)} = B_1 + 1$, $t^{(n,n+2)} = C - B_1$, $t^{(i,i+n+1)} = s_i, \forall i \in \{1, 2, \ldots n\}$, all other $t^{(sd)} = 0$, $R = n \sum_i s_i - B_1$. An example is shown in Figure 3.1.

Due to the traffic components of magnitude $C+1$, both wavelengths must be used to form single hop lightpaths over all physical links except the two central ones. Over the two central ones, at least one single-hop lightpath each must be formed due to the traffic components of magnitude $B_1 + 1$ (this quantity is less than C for $0 < B < \sum_i s_i$, *i.e.* non-triviality of the Subsets Sum instance, and hence can always fit in one wavelength); the other wavelength may be used to form two single-hop lightpaths over these two links, or a single two-hop lightpath over them. The electronic routing in the former case is at least as large as in the latter case, thus it suffices to consider the latter case. Thus the virtual topology may be considered forced on us by the construction. On this virtual topology, each of the traffic components corresponding to the object sizes of the Subsets Sum problem must be electronically routed exactly $n - 1$ times at all nodes other than node

$n+1$. At node $n+1$, at most C units of traffic can be optically routed, since only lightpath optically passes through. The $C - B_1$ units of $t^{(n,n+2)}$ must be routed on the wavelength than bypasses node $n+1$, since traffic cannot be bifurcated and the other wavelength does not have enough room for it. Thus there remains room for at most B_1 units of the traffic corresponding to the object sizes of the Subsets Sum problem to optically bypass node $n+1$. To satisfy the electronic routing goal, at least this much traffic must be optically passed through node $n+1$, and because traffic cannot be bifurcated, the electronic routing goal can be satisfied *iff* there is a subset of objects of the Subsets Sum problem instance whose sizes total to B_1, that is, *iff* the Subsets Sum problem instance can be satisfied. Since deciding the satisfiability of the Subsets Sum problem is NP-Complete, the proposition is proved. ■

Corollary 3.1.1 *The decision version of the traffic grooming problem in unidirectional path networks (bifurcated routing of traffic not allowed) is NP-Complete even when a candidate virtual topology is provided.* An instance of the problem is provided as above together with a valid virtual topology on the path, and the question is whether a particular value of electronic routing or lower can be achieved for the given traffic on that topology.

Proof: The proof follows the same lines as that for Proposition 3.1. When constructing an instance of the grooming problem, it is now unnecessary to specify the traffic components of magnitude $C+1$, instead, the virtual topology that was forced is specified as the candidate virtual topology. Then the grooming instance is again shown to be satisfiable *iff* the Subsets Sum instance is. ■

Note: Because of the construction in the proof, the only feasible assignment of the traffic to the virtual topology is the one that satisfies the grooming goal. Thus it is also shown that the value of R can be assigned a larger value without affecting the satisfiability of the instance. In particular, $R = n\sum_i s_i + C - B_1$ will have the same result. Since this is the maximum electronic routing that can occur for the problem instance (every traffic component is electronically routed at every intermediate node), it is also proved that the problem of deciding whether a given virtual topology admits of any feasible routing of traffic at all is also NP-Complete.

In the above proposition, we presented that the traffic grooming problem in unidirectional path networks with traffic bifurcation not allowed is NP-Complete by the reduc-

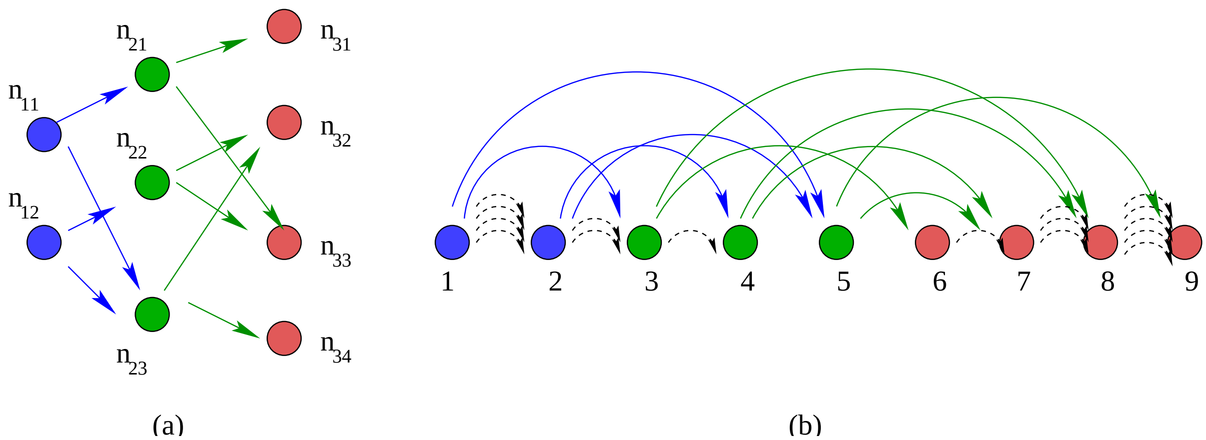

Figure 3.2: Example of path construction of Proposition 3.2: (a) 3-stage network instance, (b) Path instance constructed together with virtual topology forced ($W = 6$ in this instance)

tion from the Subset Sum problem. However, it's known that the Subset Sum problem is not NP-Complete in strong sense [12]. Using a simple dynamic programming approach, a pseduo-polynomial algorithm can be employed to find an optimal solution. If the grooming goal R is bounded by a constant, which is the case in our construction, the algorithm is attractive. Unfortunately, when we eliminate the constraint that traffic components are not allowed to be routed on different lightpaths, the problem is still NP-Complete, now in the strong sense.

Proposition 3.2 *The decision version of the traffic grooming problem in unidirectional path networks (bifurcated routing of traffic allowed) is NP-Complete.*

An instance of the problem is provided exactly as for the case where bifurcation is not allowed, but now it is allowed to bifurcate each traffic component $t^{(sd)}$ in various subcomponents which may follow different routes from source to destination. The bifurcation is restricted to integer subcomponents.

Proof: The reduction is from the Multi-Commodity Flow in three stage networks with three nodes in the second stage, which has been proved NP-Complete in [10]. An instance of the problem consists of a network consisting of three sets $\mathcal{N}_1, \mathcal{N}_2, \mathcal{N}_3$ of nodes forming the first, second and thirds stage of a simple staged network (with $|\mathcal{N}_2| = 3$), a set of directed arcs $E \subset (\mathcal{N}_1 \times \mathcal{N}_2) \cup (\mathcal{N}_2 \times \mathcal{N}_3)$, each of unit capacity, a set of flow requirements $Q \subset (\mathcal{N}_1 \times \mathcal{N}_3)$, each of unit magnitude. The question is whether a feasible flow assignment satisfying the flow requirements exists.

We construct a path network with as many nodes as the three stage network, with a one-to-one correspondence between the nodes of the stage network and the path, as illustrated in Figure 3.2. We define the following quantities. Let A be the set of all ordered node pairs (s, d) of the path network such that $(i, j) \in E$ for the staged network, where s is the node corresponding to i and d is the node corresponding to j. Similarly, let B be the set of all ordered node pairs (s, d) of the path network such that $(i, j) \in Q$ for the staged network. For the link from node i to $i + 1$ $(i \in \{1, 2, \ldots N - 1\})$, let $w_i = |\{(s, d) : (s, d) \in A, s \leq i, d \geq i + 1\}|$. That is, w_i is the number of arcs that would cross link i if the arcs of the staged network were drawn between corresponding nodes of the path network. Construct a path network using the following transformation: $N = |\mathcal{N}_1| + 3 + |\mathcal{N}_3|$, $C = |Q| + 2$, $W = \max_i\{w_i\}$, $t^{(sd)} = C - 1, (s, d) \in A$, $t^{(sd)} = 1, (s, d) \in B$, $t^{(i,i+1)} = (W - w_i)C, \forall i \in \{1, 2, \ldots N - 1\}$. That is, each arc of the staged network generates a traffic component of magnitude $C - 1$, and each flow requirement of the staged network generates a traffic component of magnitude 1, between the corresponding nodes of the path network. All other traffic components are zero, and $R = |Q|$.

Because the magnitude of the traffic components corresponding to the arcs of the staged network are each $Q + 1$, the goal cannot be achieved if even one of these traffic components is completely electronically routed. Thus at least one unit of traffic for such a traffic component must be optically routed, and this is true of every such traffic component. Hence for the goal to be achieved the virtual topology must include at least one direct lightpath for each of these traffic components. That is, any virtual topology satisfying the goal must include at least one lightpath with source node s and d for each node pair $(s, d) \in B$. However, exactly one such, together with the $W - w_i$ single-hop lightpaths that must be formed over each link from node i to $i + 1$ to carry the single-hop traffic, will occupy every wavelength on every link; thus a complete virtual topology is forced. On this virtual topology, the single-hop lightpaths are completely occupied with the single-hop traffic. Each of the lightpaths $(s, d) \in B$ must carry the entire traffic from s to d, since there are only lightpaths from one "stage" to another in the path, and so are the traffic components. The remaining bandwidth and remaining traffic components are exactly the ones corresponding to the arc capacities and flow requirements of the MCF 3-stage problem instance. Every possible path for every traffic component involves exactly one intermediate node from source to destination, thus if it is at all feasible to route the traffic, the electronic routing goal will be met. Thus the path network grooming problem instance is satisfiable *iff* the MCF problem instance is. Since the MCF problem is known to be NP-Complete, so is the path grooming problem. ■

We can show that this problem is NP-Complete in the strong sense. Following the same terminology in [12], let I be the instance of the traffic grooming problem transformed from the instance of the MCF problem. The instance is described by C, W, N and T. We take the $Length(I)$ to be $N + \lceil \log_2 W \rceil + \lceil \log_2 C \rceil + \lceil \log_2 R \rceil + \sum_{sd} \lceil \log_2 t^{(sd)} \rceil$. Because the number of wavelength and the capacity of a lightpath are constrained in practice, we can take the $Max(I)$ to be the larger of R and $\max t^{(sd)}$. From the reduction, we have $R = |Q|$, where $|Q| \leq |\mathcal{N}_1||\mathcal{N}_3| \leq |\mathcal{N}|^2$, and $\max t^{(sd)} \leq WC$. Therefore, there exists a polynomial $p()$ such that $Max(I) \leq p(Length(I))$. Since we have shown that this problem is NP-Complete, it follows that no pseudo-polynomial time algorithm exists for the traffic grooming problem, thus it is NP-Complete in the strong sense.

Combined with the observation that the goal R is bounded by a polynomial of n and $\max_{ij} t_{ij}$, we can rule out the possibility of a fully polynomial time approximation scheme (FPTAS) unless $P = NP$. Furthermore, since the solution values are integers, it is easy to see that for a given instance I, no polynomial time approximation algorithm A satisfying $R_A = \frac{A(I)}{OPT(I)} < 1 + \frac{1}{R}$ exists (otherwise, it implies that $1 < A(I) - OPT(I) < OPT(I)/R$, i.e., it's a polynomial algorithm that gives a YES answer to the corresponding decision problem, which contradicts the NP-Completeness proof). Thus, we also rule out the possibility of a polynomial time approximation scheme (PTAS) unless $P = NP$. However, it might be hoped that approximation algorithms may exist for some useful approximation ratios. We show below in the next corollary that this is not true.

Corollary 3.2.1 *Approximating the optimization version of the unidirectional path network grooming problem (bifurcated routing of traffic allowed) is NP-hard, unless P=NP.*

An instance of the problem is provided exactly as for the proposition for the decision problem of traffic grooming with bifurcation allowed. Now the problem is to find the grooming solution which produces the minimum amount of electronic routing R_o.

Proof: The reduction is from the same Multi-Commodity Flow used in the proof of Proposition 3.2.

Suppose that we have a polynomial time approximation algorithm which has $R_M(I) \leq \infty$, for the traffic grooming problem instance I with $OPT(I) > 0$, where $OPT(I)$

is the optimal value. (Excluding the cases where $OPT(I) = 0$ does not change the intractability of the problem, since those cases are trivially solvable.) It implies that there would be a polynomial time algorithm M satisfying $R_M(I) = \frac{M(I)}{OPT(I)} \leq K$, where, $M(I)$ is the result returned by the algorithm M, for some positive integer K. Then construct the instance I as follows: For any given instance I^{MCF} of the MCF problem, we add $K|Q|$ dummy nodes to $\mathcal{N}_2$. We name them as D=$\{D_1, \dots D_{K|Q|}\}$. First we define the following sets: A and B are sets of node pairs exactly as before. Let H be the set of all ordered node pairs (s, d) of the path network such that either ($s \in \mathcal{N}_1$ and $d = D_1$) or ($s = D_{K|Q|}$ and $d \in \mathcal{N}_3$). Let L be the set of all ordered node pairs (s, d) of the path network such that $s = D_i, d = D_{i+1}, \forall i \in 1, 2, \dots K|Q| - 1$.

Then we construct a path network with as many nodes as the three stage network, exactly as in the proof of Proposition 3.2, with the following additions.

For the link from node i to $i+1$ ($i \in \{1, 2, \dots D, \dots N-1\}$), let $w_i = |\{(s, d) : (s, d) \in A \cup H \cup L, s \leq i, d \geq i+1\}|$. Construct a path network using the following transformation: $N = |\mathcal{N}_1| + 3 + K|Q| + |\mathcal{N}_3|$, $C = K|Q| + 2$, $W = \max_i\{w_i\}$,

$$t^{(sd)} = \begin{cases} C - 1, (s, d) \in A \cup H \\ 1, (s, d) \in B \\ (W - w_i)C + 2, (s, d) \in L \\ (W - w_i)C, \{(s, d) | s \in \{1, 2, \dots N-1, d = s+1, (s, d) \notin L\} \end{cases}$$

All other traffic components are zero, and $R = |Q|$. Since K is independent on I, this construction is in polynomial time. Since the traffic $t^{(sd)} = 1, (s, d) \in B$, can always be routed as $\{s, D_1, \dots D_{K|Q|}, d\}$, $M(I)$ will always return a feasible solution.

If $M(I) \leq K|Q|$, it implies that none of the traffic components of magnitude $K|Q| + 1$ is completely electronically routed. Thus at least one unit of traffic for such a traffic component must be optically routed, and this is true of every such traffic component. Hence the virtual topology must include at least one direct lightpath for each of these traffic components. That is, any virtual topology satisfying the goal must include at least one lightpath with source node s and d for each node pair $(s, d) \in A \cup H \cup L$. However, exactly one such, together with the $W - w_i$ single-hop lightpaths that must be formed over each link from node i to $i+1$ to carry the single-hop traffic, will occupy every wavelength on every link; thus a complete virtual topology is forced.

Furthermore, if even one traffic component in B is routed through the route

$\{s, D_1, \dots D_{K|Q|}, d\}$, it will introduce exactly an amount of electronic routing $K|Q|$. Since it is reasonable to assume that $|Q| > 2$ (again we exclude only trivial cases), we have $K|Q| > 2$. Therefore the amount of electronic routing is at least $K|Q| + |Q| - 1$, which is larger than $K|Q|$. Therefore, $M(I) \leq K|Q|$ implies that no traffic component in B is routed by the route $\{s, D_1, \dots D_{K|Q|}, d\}$.

Thus, it implies that the instance I^{MCF} is satisfiable.

If $M(I) > K|Q|$, since we have $\frac{M(I)}{OPT(I)} \leq K$, it implies that $|Q| < OPT(I)$, which implies the instance of I^{MCF} is not satisfiable.

Thus, using the algorithm M, we can solve the MCF problem in polynomial time. Obviously, if we assume that $P \neq NP$, then M cannot exist. ■

Next, we prove the parallel result to Corollary 3.1.1 for bifurcated routing paths, that even with the virtual topology is given, the decision problem is still NP-Complete.

Corollary 3.2.2 *The decision version of the traffic grooming problem in unidirectional path networks (bifurcated routing of traffic allowed) is NP-Complete even when a candidate virtual topology is provided.*

Proof: This result follows from Proposition 3.2 in the same way as the proof for Corollary 3.1.1 does from Proposition 3.1. In the reduction, we construct the path network as above, but do not include the traffic components introduced to force the virtual topology; instead, the virtual topology which was forced is specified as part of the instance. The rest of the proof follows as before.

Again, by the nature of the proof, it is also proved that the problem of deciding whether a given virtual topology admits of any feasible routing of traffic at all is also NP-Complete. ■

The above also showed that the feasibility check problem, i.e., the problem that given a virtual topology, the question is that if it admits the given traffic matrix, is NP-Complete. The feasibility problem in data networks has been studied for decades. Generally, it is formulated as a multicommodity flow problem which is known to be much harder than single-commodity flow problems. In the single-commodity flow problem, the Max-Flow-Min-Cut theorem is necessary and sufficient to prove the feasibility. However, this good property does not always hold for multicommodity flow problems. One reason is that flows on a same edge do not sum up or cancel each other if they are different commodities. Obviously, in the multicommodity case, the Max-Flow-Min-Cut theorem (the max-flow is defined as the sum of multicommodity requests with source and destination separated by the cut, which is slightly different from the single-commodity case) is still necessary. Otherwise, the traffic matrix can not be satisfied. However, it is not sufficient any more. Extensive research have been reported in literature on the graphs for which the sufficiency holds. For example, in [21], the authors proved that if the graph is planar and all the sources and destinations of multicommodities can be drawn on the boundary of a infinite region, the sufficiency holds. These observation has been used on ring networks. However, our results show that the feasibility problem is intractable in even the path network because of the complexity of the virtual topology it may bring.

3.2 Other Topologies

In this section we explore some of the more pertinent consequences of the propositions proved in the last section. Specifically, we show that the more general case of bidirectional path networks is also intractable. We also show that ring network grooming problems are also intractable independent of the wavelength assignment problem complexity. This is an important result since ring networks are of current and lasting practical importance.

Some other interesting network topologies may also be seen to fall in the intractable class as a result of the path network intractability. These include the *spider networks* and *Manhattan street* or *grid networks*. Since these follow trivially from the path network results, we do not formally state them, nor include the demonstrations.

Proposition 3.3 *The decision version of the traffic grooming problem in bi-directional path networks (in both the cases of bifurcated routing of traffic allowed and not allowed) is NP-Complete.*

An instance of the problem is provided in the same way as that for the corresponding unidirectional path. We assume that between every two adjacent nodes there are now two links, each carrying W wavelengths, in opposite directions. Traffic components are now allowed from every node to every other node. A traffic component is allowed to be carried from source node s to destination node d on a sequence of lightpaths some of which are in one direction and some in the reverse direction; thus a traffic component may traverse the same link multiple times in either direction. However, all of a traffic component must traverse the outgoing link from node s in the direction in which node d lies at least once, and the incoming link to node d from the direction in which node s lies at least once. This forms the basis of the restriction proof below.

Proof: We restrict the problem to the instances in which the *reverse* (decreasing node numbering) single hop traffic components $t^{(i,i-1)} = WC, \forall i \in \{2, 3, \ldots N\}$, and all other traffic components in the reverse direction are zero. For these instances, any feasible solution must include exactly W single hop lightpaths on each of the reverse links, since this is the only way in which the reverse links can provide enough bandwidth to carry the reverse single hop traffic components. The problem instance then is equivalent to the unidirectional path problem instance which is specified by the forward traffic components. Since the construction does not allow any of the reverse traffic components to be routed except uniquely over the single hop lightpaths, they cannot be bifurcated. Thus this restriction can be used for bidirectional path problem instances with both bifurcation allowed and not allowed to obtain instances of the corresponding unidirectional problems, both of which are already known to be NP-Complete; thus both kinds of bidirectional paths are NP-Complete as well.

■

Proposition 3.4 *The decision version of the traffic grooming problem in unidirectional ring networks (in both the cases of bifurcated routing of traffic allowed and not allowed) is NP-Complete, even when every node has full wavelength conversion capability.*

An instance of the problem is provided by specifying the same quantities as above, as described in [7]; now each node i has a single unidirectional link to the node $i \oplus 1$ (this indicates the next node from i in the direction of transmission, and is equal to $i+1$ for $i = 1, 2, \ldots N-1$, and is equal to 1 for $i = N$). No traffic component is allowed to be routed around the entire ring before being delivered to the destination. It has been shown by different authorities that wavelength assignment poses an NP-Complete problem in ring networks. However, it is possible that wavelength converters will become common in the future and wavelength assignment will be irrelevant. This result shows that grooming remains a difficult problem.

Proof: We restrict the problem to instances where the traffic matrix specified is such that no traffic flows over the link from some node i to the next node. Then the instance is equivalent to the unidirectional path network instance obtained by removing that link from the ring. This restriction does not make any assumption regarding whether traffic is bifurcated. Thus this restriction can be used for unidirectional ring problem instances with both bifurcation allowed and not allowed to obtain instances of the corresponding path problems, both of which are already known to be NP-Complete; thus both kinds of unidirectional rings are NP-Complete as well. ■

Proposition 3.5 *The decision version of the traffic grooming problem in bi-directional ring networks (in both the cases of bifurcated routing of traffic allowed and not allowed) is NP-Complete, even when every node has full wavelength conversion capability.*

An instance of the problem is specified as above, now each node i has one unidirectional link to $i \oplus 1$ and one to $i \ominus 1$.

Proof: We restrict the problem to instances where the traffic matrix specified is such that for some node i, $t^{(i,i\oplus 1)}$ and $t^{(i,i\ominus 1)}$ are both equal to WC. Then the two links originating at node i must carry $2W$ lightpaths originating at that node to carry this traffic. Moreover, consider the following assignment of traffic: the W lightpaths on the link from i to $i \oplus 1$ carry the traffic to node $i \oplus 1$ and terminate at that node; similarly the traffic to node $i \ominus 1$. Any other assignment of traffic will result in electronic routing of some or all of this traffic at least once, and reduced bandwidth in the rest of the ring. Thus an optimal solution will include of the $2W$ lightpaths identified above. But in that case none of the remaining traffic in the ring can either optically or electronically routed through node i. Thus the rest of the problem is equivalent to a bi-directional path, which is known to be NP-Complete. Once again, we made no assumption regarding bifurcation of traffic and thus the result applies to both kinds of bi-directional rings. Note that the proof is also free from assumptions about whether traffic can be routed back and forth over the same section of the ring many times, or completely around the ring once or more, before being delivered. Thus all these cases are proved NP-Complete.

Chapter 4

Heuristic Approaches

We have proved the NP-Completeness of the traffic grooming problem. Moreover, we know that even an approximation algorithm with some certain approximation ratio is intractable. We consider heuristics for the general case where bifurcation of traffic is allowed. This case is interesting since we assume that the speed of a connection, which serves as a unit of traffic demands, is high (OC-3, for example), thus it is reasonable to allow traffic bifurcations and allow the consequent network optimization which may be obtained. In additions, there are some concerns on protection and restoration issues that may prefer bifurcated traffic so that they do not share a SRLG (Shared Risk Link Groups). In this section, we concentrate on a family of heuristic algorithms for the bifurcated routing case that we have developed. Some algorithms in this family have good performance guarantees in a practical sense.

In our work, electronic routing minimization is taken as the objective. Instead of minimizing the number of wavelengths, we take the number of wavelengths, W as a constraint. Any feasible solution has to have a number of lightpaths traversing any physical link less or equal to W. In unidirectional path networks, the traffic routing problem is eliminated for the simplicity of the physical topology. There is only way to route a lightpath onto physical links (we do not consider the multiple-fiber link situation). Another good property is that the wavelength assignment problem can be eliminated as well. On path networks, the WLS problem can be viewed as an interval coloring problem [22]. There exists a simple greedy algorithm which assigns a minimum number of colors (wavelengths) to lightpaths in linear time such that every pair of overlapping lightpaths does not share a same color (we call this a feasible assignment). Obviously, $\lceil L/g \rceil$, where L is the load of the network, g is the grooming factor, serves as a lower bound of the number of wavelengths in order to support the traffic matrix. On the other hand, the lower bound is achievable by a simple point-to-point network (i.e. , a lightpath is dropped and electronically processed at every intermediate node). Thus, given at most W wavelengths, if $W \geq \lceil L/g \rceil$, there is always a feasible assignment. Therefore, we do not need to consider the WLS problem in our case. For the remaining two sub problems, virtual topology design and traffic grooming respectively, we follow a combined approach.

4.1 Flow Deviation

Flow deviation method is a widely used generic approach in network flow arenas. Although this method, due to Fratta, Gerla and Kleinrock, has been proposed for decades, it has been shown provably good approximation schemes until recently [3]. It has been shown very useful in solving various optimization problems arising in telecommunications. As an example, in [3], the authors provided an asymptotic analysis of the flow deviation method for the maximum concurrent flow problem and showed that it was a fully polynomial-time approximation scheme (FPTAS). It can be seen as Lagrangian relaxation schemes where, some constraints are relaxed. And a potential function is chosen such that the its gradient is taken as the objective function of the simpler linear programming with relaxed constraints, and therefore, a direction towards feasible solutions. In each iteration, the linear programming is solved and the infeasibility is driven down until it reaches a feasible solution.

4.2 Deviation Grooming

The original flow deviation method is more suited to packet-switched networks in which statistical multiplexing of packets creates aggregate load in different links of the network. In virtual circuit- switched networks such as tomorrow's optical backbones are likely to form and which we have considered, this method is not directly applicable. In addition, we have shown that there are no approximation schemes available to our traffic grooming problem. The reason is partly because of that, even we relax the constraints of the number of wavelengths, the linear programming is still intractable because that traffic flows are required to be integers and the optimal without integer requirements may be fractional. However, we can still adopt a similar idea. Our approach is similar to the flow deviation method in the sense that we try to deviate traffic from one lightpath to several lightpaths so that they can be groomed with other traffic components. By relaxing the constraint of the maximum number of wavelength, we find an initial infeasible solution. From this solution, we deviate some packet flows from one route to another to relieve congestion. We choose a lightpath and break it at the most congested link and thus give lightpaths a chance to be groomed onto a less number of wavelengths. This way, it always has a potential to relieve the most congested link until a feasible solution is received.

This approach is greedy in the sense that when a lightpath is broken, these shorter lightpaths formed will never be merged into some longer one.

Briefly, the basic algorithm runs as follows:

```
Create an initial (optimistic) topology by
assigning lightpaths to each traffic demand.

loop  {

   If the current topology is feasible, end.

   Among the non-zero traffic components, pick one to deviate
   using some criterion such as smallest, longest, etc. that
   pass over congested links

   Deviate this traffic component: add it to other s-d pairs
   that form a path from its source to destination using some
   strategy, and set it to zero.

   Assign lightpaths to each remaining traffic demand.

}
```

Next, we examine the approach in more details.

First, we check if the given traffic matrix can be satisfied by the number of wavelengths. Then, we start our algorithm from an initial solution. This solution is given by assigning each traffic component dedicated lightpath(s), so that no electronic routing occurs. If this solution is feasible, it's optimal in terms of the electronic routing minimization. If not, a traffic component which contributes to the violation (the lightpath assigned for it traversing a link on which the constraint of maximum number of wavelengths is violated) is picked and rerouted on residue network by removing the lightpaths dedicated for it. We number the nodes along the path from 1 to N, where N is the number of nodes. A node is opaque, if all the traffic components traversing it are electronically routed at this node, i.e. , it has LTEs for every wavelength. If a node has an OADM and some lightpaths bypassing it optically, we call it translucent.

We explored several different greedy approaches to pick a traffic component as follows:

1. *Pick the least traffic component contributing to the violation.* Among all violating traffic components, the one with minimum traffic demand is picked. This approach is based on the assumption that a smaller traffic demand would introduce a smaller amount of electronic routing.

2. *Pick the longest traffic component contributing to the violation.* Among all violating traffic components, the one with the longest distance (in physical hops) is picked. This approach is based on the observation that a longer traffic component is more likely to be electronically routed at some intermediate nodes.

3. *Pick the traffic component contributing to the violation with the maximum slack (capacity of a lightpath that is not occupied by traffic) times distance (in physical hops).*

4. *Pick the longest traffic component.* The traffic components with longest distance is picked; break ties by picking smaller components first; break further ties arbitrarily.

5. *Pick the longest traffic component that violates the 2-hop opaque topology.* First, a 2-hop topology is to be decided with either odd nodes being opaque or even nodes being opaque. Then, put all the traffic components that violate this 2-hop topology, i.e. , traversing some opaque node, into a list. Among them, pick the longest traffic component as shown above.

Our experimental experience with the approaches to picking a traffic component showed that there are no big difference between their performance. However, picking the longest components first, as the last two methods do, allows us to argue a well-defined computational complexity for the algorithm. For this reason, we have chosen to use these two approaches in most of our experimentation and in the results we present. We shall refer to one of these two approaches only when we refer to "picking a traffic component" in what follows.

When a traffic component is picked, it's electronically routed at some node (we say that the lightpath carrying this traffic component is broken at some node). Thus, when a lightpath is deleted, at least two additional lightpaths are created and put into the candidates list. However, the number of lightpaths may not increase (and may actually decrease because the resulting traffic may get groomed with existing traffic to and from the node at which we break the lightpath. In the next run, another traffic component is picked in the list and broken. This procedure is repeated until the solution is feasible. Because the longest traffic component is picked and deviated, it is broken into at least two lightpaths with shorter distance. the algorithm is guaranteed to terminate in a polynomial number of picking procedure. In other words, no lightpath that is broken in one step can get formed in a later step. Thus, the approach to picking the longest traffic component is used.

There are several different ways that the picked traffic component is broken.

1. *Break it at both ends of every violating link.* A violating link is a link that has a number of lightpaths more than the number of wavelengths available traversing it. By breaking a lightpath at both ends, it provides a slack for further grooming other traffic components such that the number of lightpaths traversing it can be reduced. In this approach, a lightpath is picked by any of the first 3 pick methods.

2. *Break it at both ends of the most violating link.* Every link is given a score that is calculated as the number of lightpaths traversing it divided by the wavelength limit. We break the picked lightpath at both ends of it in order to release the violation less greedily than the first approach.

3. *Groom it on the residue network first.* We try to reroute the picked lightpath on the residue network without traffic bifurcation along the shortest path. The shortest path is measured in number of lightpaths (hops). The well-known dijkstra's algorithm is used. If it fails, the first approach is used. That is, we try to groom the picked traffic with existing traffic without introducing any extra lightpaths and ADMs.

The next two breaking methods call upon the "opaque nodes decomposition" method introduced in [9]. This method decomposes a path network into several shorter segments by making the end nodes of each segment opaque (processing all traversing traffic components electronically). We use the 2-hop opaque topology (in which every alternate node is opaque) so that the routing of traffic inbetween opaque nodes is trivially simple (either route optically through the transparent node inbetween two opaque nodes, or electronically). The approach to picking the longest traffic component that violates the 2-hop opaque topology (i.e., routed optically through some node that would be opaque in the best 2-hop opaque topology) is used for the following deviation methods. In what follows, we use the term "opaque node" rather loosely, to mean "node which would be opaque in the 2-hop opaque topology".

4. *Break it at node either i, the head of the most violating link or $i-1$, and either $i+2$ or $i+1$ whichever are opaque.* This deviation method has two stages. First, a 2-hop topology with either odd or even nodes being opaque is pre-decided. After a lightpath is picked, it is broken into 3 lightpaths (2 lightpaths, occasionally, refer to the algorithm for detail) and twice the amount of traffic demand carried on the lightpath of electronic routing is introduced. It provides a slack on which other traffic demands can be groomed so that the violation can be relieved. If it generates a feasible solution, the program stops here. Otherwise, after the first stage, if it is still infeasible, the first deviation method is used to further break violating lightpaths. However, in this stage, no traffic component can be longer than 2 hops. Through extensive experimentation, we observed that this stage took place rarely.

5. *Break it at the head node of the most violating link if the node is opaque or the node in front of the head otherwise.* This approach is similar with the previous one, except that a lightpath is broken into 2 lightpaths at each run. Obviously, this the least greedy algorithm so far because it introduces an amount of electronic routing which is equal to the traffic component picked. Again, after the 2 stage, if the solution is still infeasible, the first deviation method is used.

The procedure of pick-and-break is repeated until the result is feasible. Our algorithm can always find a feasible solution because that if the traffic matrix given can be admitted by the network, our algorithm at least terminates with the no-grooming solution (i.e. every node is opaque), which is always feasible. By picking the longest traffic component, we showed that the algorithm terminates in a finite number of steps. Specifically, there are $O(N^2)$ number of traffic components in the given traffic matrix, thus the picking and deviating routines are called at most $O(N^2)$ times. Each traffic component picked is and deviated involves several tasks each of which requires $O(N)$ time. Thus the running time of our algorithms is $O(N^3)$. Next we provide a detailed description of our algorithm in which this claim can be verified. The two approaches to picking a lightpath and the five approaches to deviating them are presented as subroutines. The first three deviation methods can be used with the first picking method, and the last two deviation methods is appropriately used with the second picking method. (The other combinations can also be used though they are not very sensible.) Thus what we present is a family of algorithms rather than a single one. Moreover, other picking and deviating methods can be easily created by extension of these ideas, though we choose not to present any more variations.

4.3 Detailed Algorithm Description

Given: C, W, N, $T = \left[t^{(s,d)}\right]$.

Define: Flow deviated traffic matrix $\left[\tau^{(s,d)}\right]$, candidate lightpath matrix $B = [b_{ij}]$, available wavelength count W_l for each link l from node l to node $l+1$, lightpath routing set for each link $R(l) = \{\ (i,j) \mid$ lightpath (i,j), if it existed, would pass through link $l\ \}$, a violation score $v(\tau^{(s,d)})$ for each $\tau^{(s,d)}$.

procedure **groom**

    // Initialize

    $\tau^{(s,d)} \leftarrow t^{(s,d)}, \forall s, d$

    $W_l \leftarrow W, \forall l$

    number nodes as 0,1,. . . N-1

    number link from node i to $i+1$ as i

    //If select pick_lightpath_d,initialize the 2-hop topology

    init()

    repeat forever

        // recanonize traffic

foreach $\tau^{(s,d)} \geq C$

$\tau^{(s,d)} \leftarrow \tau^{(s,d)} - (C\lceil \tau^{(s,d)}/C \rceil)$

foreach link l traversed by $\tau^{(s,d)}$

$W_l \leftarrow W_l - \lceil \tau^{(s,d)}/C \rceil$

// Try assigning direct lightpaths to deviated subwavelength traffic

foreach (i, j)

$b_{ij} \leftarrow \lceil \tau^{(ij)}/C \rceil$

if ($\sum_{(i,j) \in R(l)} b_{ij} \leq W_l, \forall l$)

return B

// Eliminate a single lightpath and deviate traffic

reset each $v(\tau^{(s,d)})$ to 0

foreach link l

$score_l \longleftarrow \sum_{(i,j) \in R(l)} b_{ij}/W_l \ \forall l$

if (wavelength limit is violated on l)

set $v(\tau^{(s,d)})$ to 1 for each $\tau^{(s,d)}$ traversing l except $\tau^{(l,l+1)}$

// Pick candidate lightpaths

$\tau_1 \leftarrow$ pick_lightpath()

deviate_traffic(τ_1)

end procedure

procedure **pick_lightpath_a**

// Greedily pick the longest traffic contributing to violation

let Sbe the set of the longest traffic components,

i.e. for every $\tau^{(s,d)} \in S, (d - s) \geq (j - i) \ \forall v(\tau^{(i,j)}) > 0$

pick $\tau^{(s,d)}$ s.t. $\tau^{(s,d)} \leq \tau^{(s',d')} \forall \tau^{(s',d')} \in S$

break further ties arbitrarily.

return $\tau^{(s,d)}$

end procedure

procedure **init**

let $\phi_1^{(i)}$ be the minimum electronic routing on node i,

where both $i - 1$ and $i + 1$ are opaque nodes i.e. $\phi_1^i = t^{(i-1,i+1)} mod[W - \max(\lceil t^{(i-1,i}/C\rceil, \lceil t^{(i,i+1)}/C\rceil)]C$

where $t^{(i-1,i+1)} = \sum_{s<i,d>i} \tau^{(s,d)},\ t^{(i-1,i)} = \sum_{s<i} \tau^{(s,i)},\ t^{(i,i+1)} = \sum_{d>i} \tau^{(i,d)}$

let ψ_i be the electronic routing on opaque node i; i.e. $\psi_i = t^{(i-1,i+1)}$

first, let all odd nodes be opaque, let R_1 be the minimum total electronic routing,i.e. ,

$R_1 = \psi_1 + \phi_1^{(2)} + \psi_3 + \phi_1^{(4)} \ldots = \sum_{i<N-1,i\in Odd}(\psi_i) + \sum_{i<N,i\in Even}(\phi_1^{(i)})$

then, let all even nodes be opaque, let R_2 be the minimum total electronic routing,i.e. ,

$R_2 = \phi_1^{(1)} + \psi_2 + \phi_1^{(3)} + \psi_4 \ldots = \sum_{i<N-1,i\in Even}(\psi_i) + \sum_{i<N,i\in Odd}(\phi_1^{(i)})$

if $R_2 \geq R_1$,make all even nodes opaque

else all odd nodes are opaque

end procedure

procedure **pick_lightpath_b**

// pick the least traffic violating the 2-hop topology

let L be the set of all traffic components traversing some opaque node optically

pick $\tau^{(s,d)}, \tau^{(s,d)} \in L$, using pick_lightpath_a, if $L \neq \Phi$

pick $\tau^{(s,d)}$ using pick_lightpath_a, otherwise

return $\tau^{(s,d)}$

end procedure

procedure **deviate_traffic_a** ($\tau^{(s,d)}$)

build the ordered list of nodes $r_1, r_2, \ldots r_n$ between s and d s. t.

each violating link has either s, d, or some r_i as head and tail nodes

increment each of $\tau^{(s,r_1)}, \tau^{(r_1,r_2)}, \ldots \tau^{(r_n,d)}$ by $\tau^{(s,d)}$

$\tau^{(s,d)} \leftarrow 0$

end procedure

procedure **deviate_traffic_b** ($\tau^{(s,d)}$)

let E be the set of physical links between s and d

let l be the link s. t. $score_l = \max_i score_i,\ \forall i \in E$

let r_h be the head node of l

increment

$$\tau^{(s,r_h)}, \tau^{(r_h,r_{h+1})}, and\ \tau^{(r_{h+1},d)} \text{ if } r_h \neq s, and, r_{h+1} \neq d$$

$\tau^{(s,r_{h+1})}, and\ \tau^{(r_{h+1},d)}$ if $r_h = s$

$\tau^{(s,r_h)}, and\ \tau^{(r_h,d)}$ if $r_{h+1} = d$

by $\tau^{(s,d)}$

$\tau^{(s,d)} \leftarrow 0$

end procedure

procedure **deviate_traffic_c** ($\tau^{(s,d)}$)

define connection matrix $K = [k_{ij}]$

$k_{ij} \longleftarrow 1\ if\ \tau^{i,j} < C - \tau^{s,d}$

$k_{ij} \longleftarrow 0$ otherwise

$k_{sd} \longleftarrow 0$

define the distance of k_{ij} as $d(k_{ij})$

$d(k_{ij}) \longleftarrow 1\ \forall k_{ij} = 1$

find the shortest path from s to d

if succeeded

let the shortest path be $s, r_1, r_2, \ldots r_n, d$

increment each of $\tau^{(s,r_1)}, \tau^{(r_1,r_2)}, \ldots \tau^{(r_n,d)}$ by $\tau^{(s,d)}$

$\tau^{(s,d)} \longleftarrow 0$

else

//use other methods to deviate traffic

deviate_traffic($\tau^{(s,d)}$)

end procedure

procedure **deviate_opaque_a** ($\tau^{(s,d)}$)

//$\tau^{(s,d)}$ is picked by pick_lightpath_b

if $\tau^{(s,d)}$ does not traverse any opaque node

use other methods to deviate traffic

else

let E be the set of physical links between s and d

let $O = \{o_1, o_2, \ldots, o_n\}$ be the opaque nodes between s and d

let l be the link s. t. $score_l = \max_i score_i,\ \forall i \in E$

let h be the head node of link l

if h is opaque $r_h \longleftarrow h$

else

if $h = s$, $r_h \longleftarrow h + 1$

else $r_h \longleftarrow h - 1$

increment

$\tau^{(s,r_h)}, \tau^{(r_h,r_{h+2})}, and\ \tau^{(r_{h+2},d)}$ if $r_h \neq s, and, r_{h+2} \neq d$

$\tau^{(s,r_{h+2})}, and\ \tau^{(r_{h+2},d)}$ if $r_h = s$

$\tau^{(s,r_h)}, and\ \tau^{(r_h,d)}$ if $r_{h+2} = d$

by $\tau^{(s,d)}$

$\tau^{(s,d)} \leftarrow 0$

end procedure

procedure **deviate_opaque_b** $(\tau^{(s,d)})$

$//\tau^{(s,d)}$ is picked by pick_lightpath_b

if $\tau^{(s,d)}$ does not traverse any opaque node

use other methods to deviate traffic

else

let E be the set of physical links between s and d

let $O = \{o_1, o_2, \ldots, o_n\}$ be the opaque nodes between s and d

let l be the link s. t. $score_l = \max_i score_i,\ \forall i \in E$

let h be the head node of link l

if h is opaque $r_h \longleftarrow h$

else

if $h = s$, $r_h \longleftarrow h + 1$

else $r_h \longleftarrow h - 1$

increment

$\tau^{(s,r_h)}, \tau^{(r_h,d)}$ if $r_h \neq s$

$\tau^{(s,r_{h+2})}\ and\ \tau^{(r_{h+2},d)}$, otherwise

by $\tau^{(s,d)}$

$\tau^{(s,d)} \leftarrow 0$

end procedure

4.4 Performance Guarantees

We have seen as a consequence of Corollary 3.2.1 that it is not possible to find an approximation algorithm for our problem. However, in practical terms, often the network designer is interested in the incremental benefit to be obtained by adopting a grooming approach, that is, by using partial optical bypass for some traffic instead of complete electronic routing after the manner of first generation optical networks. In this sense, our algorithm does in fact provide some performance guarantees, and is very useful in a practical sense. In what follows, we consider the flavor of the algorithm obtained by using one of the last two deviation methods.

We only consider the case when the given instance is feasible. Let R^* be the minimum electronic routing. Obviously,

$$R^* \leq \sum_{0 \leq i \leq N-1} \psi_i \tag{4.1}$$

A solution that has a electronic routing of $\phi_1(i)$ on every node $i, 0 \leq i \leq N-1$ may not be feasible. However, it serves as a lower bound, i.e.,

$$\sum_{0 \leq i \leq N-1} \phi_1(i) \leq R^* \tag{4.2}$$

Let R_1 and R_2 be the objective values obtained by the 2-hop opaque strategy by designating the odd nodes as opaque and the even nodes as opaque nodes, respectively. Then we have:

$$R_1 + R_2 = (\psi_1 + \phi_1^{(2)} + \psi_3 + \phi_1^{(4)} \ldots) + (\phi_1^{(1)} + \psi_2 + \phi_1^{(3)} + \psi_4 \ldots) = \sum_{0<i<N-1} \phi_1^{(i)} + \sum_{0<i<N-1} \psi_i \tag{4.3}$$

Let δ_i denote $\psi_i - \phi_1^{(i)}$, obviously $\delta_i \geq 0$. Because both R_1 and R_2 are feasible, we have

$$\sum_{0 \leq i \leq N-1} \phi_1(i) \leq \frac{R_1 + R_2}{2} \leq \sum_{0 \leq i \leq N-1} \psi_i = \sum_{0 \leq i \leq N-1} \phi_1(i) + \frac{1}{2} \sum_{0 \leq i \leq N-1} \delta_i \tag{4.4}$$

Let R denote $\min(R_1, R_2)$. We have $R \geq R^*$, and $R - R^* \leq \frac{1}{2} \sum_{0 \leq i \leq N-1} \delta_i$.

Next we prove that our algorithm always gives a feasible solution with electronic routing $R_e \leq R$. At the first stage, we pick traffic components traversing some opaque node and break it at an opaque node. After this stage is finished, there is no such violating traffic components existing. Since we have never broken a traffic component at a translucent node, the electronic routing at any translucent node is zero. Every time we break a traffic component, it introduces some electronic routing, thus, if at any stop before this stage is finished, we have a solution with less electronic routing. If it's still infeasible after this stage, we resort to other methods given above to further deviate the traffic matrix. At this time, each traffic component has a distance of at most 2 physical hops. Suppose a violating traffic component $\tau^{(s,s+2)}$ is picked, $\tau^{(s,s+2)} < C$ after recanonizing, routing it electronically at node $s+1$ gives a electronic routing of $R_e^{(s+1)} = \tau^{(s,s+2)}$, obviously, $R_e^{(s+1)} = \phi_1^{(s+1)}$. Because all the violating 2-hop traffic components are non-overlapping, they are picked and deviated independently. Therefore, $R_e = \sum_{0<i<N-1} R_e^{(i)} \leq R = \min(R_1, R_2)$ and,

$$R_e - R^* \leq \frac{1}{2} \sum_{0 \leq i \leq N-1} \delta_i \tag{4.5}$$

That is, our algorithm is guaranteed to reduce the electronic routing value to *more than halfway to the optimal value*, starting from the base case of total electronic routing (no optical bypassing).

Chapter 5

Numerical Results

5.1 Traffic Models

In our work, we do not have any requirement on the traffic model. Therefore, we expect that our algorithm can be applied to arbitrary traffic patterns. However, in order to compare the performance of our algorithms, different traffic patterns, uniform, distance dependent (rising and falling with distance) are tested. To make the traffic patterns more realistic, we let every traffic component be a random variable with Gaussian distributions. In order to increase the randomness, we have an option to make the deviation of a traffic component a Gaussian random variable as well.

In Figure 5.1, an uniform traffic pattern with constant traffic demands is shown. Let the uniform traffic demand be t, load on a link i be L_i, we have $L_i = (N-i)it$. By differencing it, we have the maximal load $L = (N^2/4)t$ on link $i = N/2$. That is the link in the middle if N is even, or the links in the middle if N is odd. Accordingly, the minimum number of wavelengths required is $W = \lceil L/C \rceil$.

We also investigated t-hop-limited traffic pattern, where traffic demands sourced from any node are destined to a node within t hops. However, to make the load less jagged, we allow a relatively small amount of uniform traffic sourced from each node to nodes beyond t hops. The link load is shown in Figure 5.2, where $t = 4$. It shows that this pattern has a more flattened load distribution. We expect that, to find a feasible solution, more

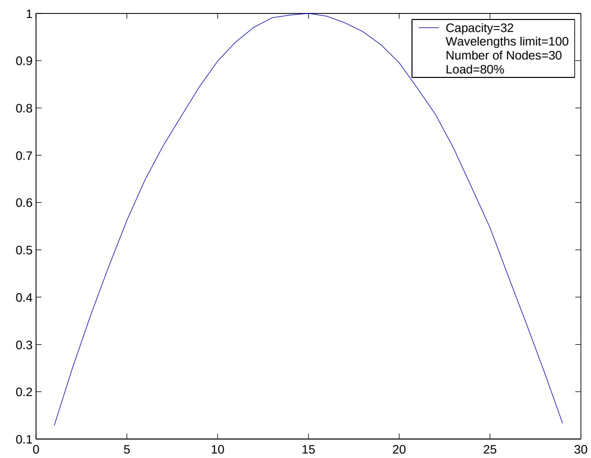

Figure 5.1: Example of Uniform Traffic Model

number of iterations are required.

5.2 Discussion

We adopt the same method of expressing the electronic routing for a grooming solution as in [7], called "grooming effectiveness". This metric is defined as the amount of electronic routing, expressed as a fraction of that of the "no grooming" scenario. The grooming effectiveness also allows us to easily examine the solutions in the light of the claim made in Section 4.4.

For different traffic patterns, some numerical results are shown. In each figure, the grooming effectiveness obtained by the different algorithms on 50 different random instances of a given class of path problem are plotted. As we can see, using different approaches for traffic deviation, the amount of electronic routing differs significantly. The amount of electronic routing for different approaches are shown in figures, which is expressed in terms of the grooming effectiveness. The traffic demands were randomly generated. In Figure 5.3 and 5.4, we investigated the cases with different number of wavelengths available when traffic load is high (80%, in particular) using the uniform traffic pattern. Since the load of the path is heavy, the wavelength margin at the most heavily loaded link is narrow. We assume that every link has a same number of wavelengths. Consequently, the links in the

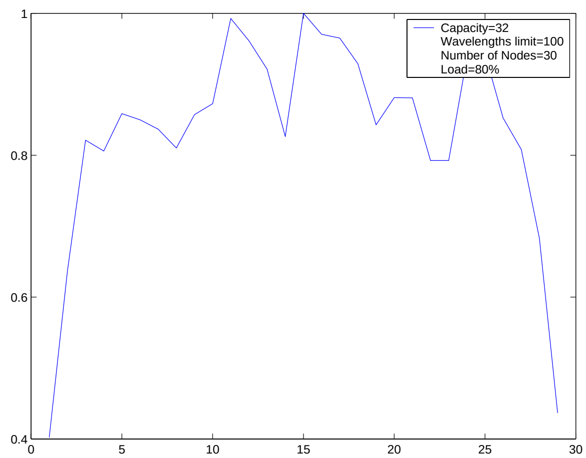

Figure 5.2: A 4-hop-limited Traffic Model

middle are more congested that those near the both ends. As it shows, the heavy load implies that more traffic components are needed to be broken and groomed, thus a larger amount of electronic routing. However, when the number of wavelengths available is large (the case $W = 80$), the second deviation method for 2-hop opaque topology has a better performance. The reason is that it performs a smaller step towards the optimal in each run. We notice that, both opaque_a and opaque_b are guaranteed to be bounded by half of the amount of electronic routing provided by the point-to-point virtual topology. The numerical results show this fact. Moreover, they actually do much better than the upper bound derived, especially in the case when the number of wavelengths is large.

In Figure 5.5 and 5.6, the capacity of a lightpath is reduced while keeping other parameters unchanged. This lower capacity implies a smaller traffic grooming factor, which plays an important role in the performance of our algorithms. Specifically, a larger grooming factor will lead to a larger amount of electronic routing. The reason is that a larger grooming factor will have more lightpaths electronically processed and groomed onto a relatively small number of wavelengths.

In Figure 5.7 and 5.8, we lower the traffic load and show the effect of traffic load. Our numerical results show that the effect on the solutions solved by 2-hop opaque topology deviation methods is not significant. However, it is interesting to notice that the deviate_traffic_c performs better than the deviate_traffic_b. The reason is that, the lighter the traffic load is, more likely deviate_traffic_c will find a shortestpath on the residue network.

We also examine the effect of both smaller capacity of a lightpath and lighter traffic of the network, in Figure 5.9 and 5.10.

Then, we examine the t-hop-limited traffic pattern. Following the same lines, we the effect of the number of wavelengths in Figure 5.11 and 5.12 with a relatively heavier traffic load and smaller capacity of lightpaths. As shown in Figure 5.2, the link load curve is flattened. Because there are more congested links in this traffic pattern than that of the uniform distribution, the running time is also longer. However, it shows that our approach can find a fairly good solution in both cases.

In Figure 5.13 and 5.14, we investigated the cases with a larger capacity of lightpaths. The numerical results show that the capacity does not affect our algorithm significantly. Again, our algorithms lead to some fairly good solutions.

Then, we reduced the traffic load to 45% and examine our algorithms in such a less loaded network. Intuitively, it has a better chance to find a shortestpath on the residue network on which a picked traffic component can be admitted. Therefore, the difference between methods of deviate_b and deviate_c is significant. Our numerical results show this fact. Again, it shows that when the number of wavelengths is larger, our algorithms will lead to some better results in terms of the grooming effectiveness.

We note from the results that the different algorithms are quite successful in grooming the traffic on the path network. The opaque node decomposition based deviation methods appear to perform not only according to the claim made in Section 4.4, but actually perform the best in most cases. This is to be expected since they invests much more computation in each case and are less greedy. The other methods are sometimes good but seem to perform poorly for lower number of wavelengths. The first deviation method, which is the greediest one and deviates each lightpath to produce the maximum electronic routing, performs poorly in most cases, as expected.

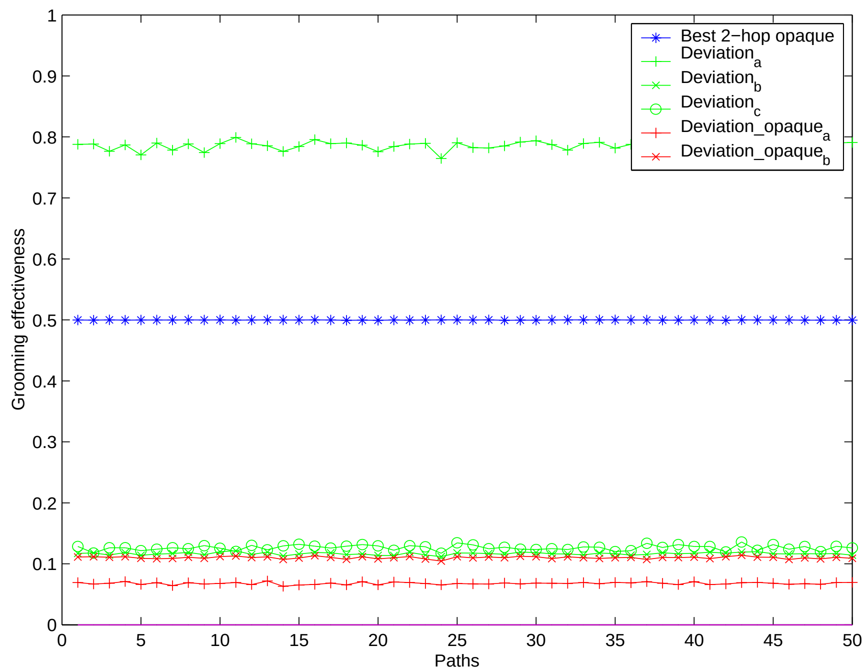

Figure 5.3: Uniform pattern, $N = 30, W = 80, C = 64, L = 80\%$

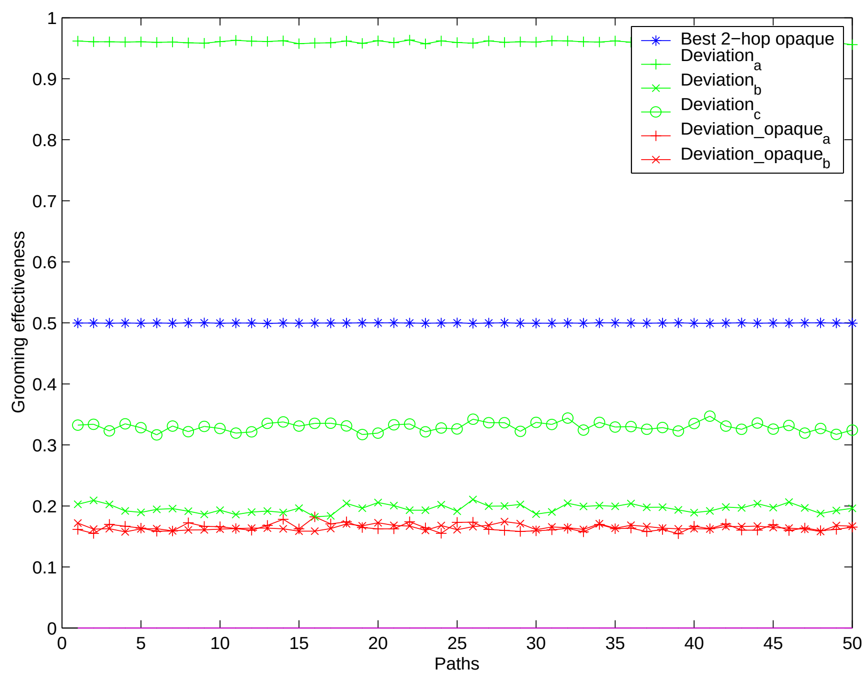

Figure 5.4: Uniform pattern, $N = 30, W = 20, C = 64, L = 80\%$

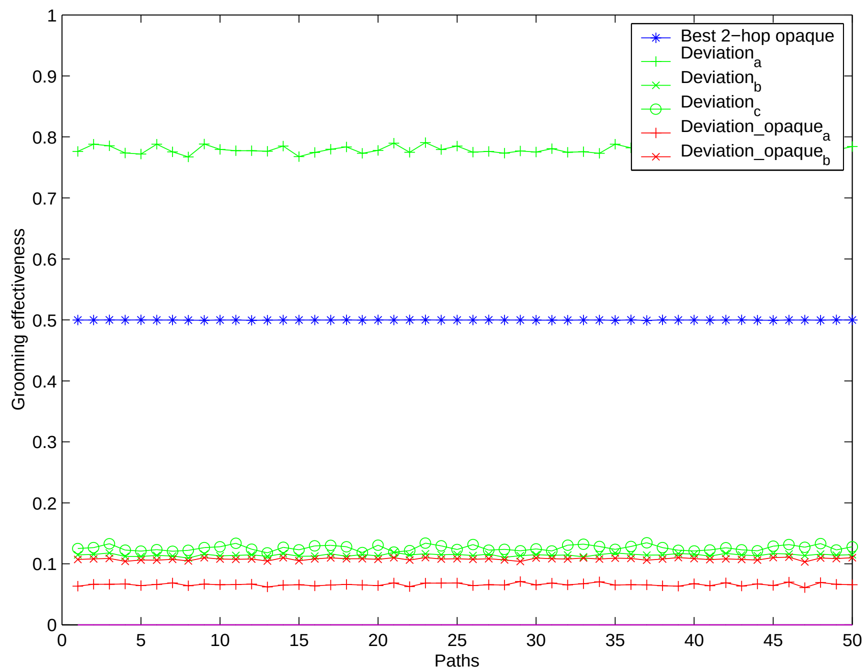

Figure 5.5: Uniform pattern, $N = 30, W = 80, C = 32, L = 80\%$

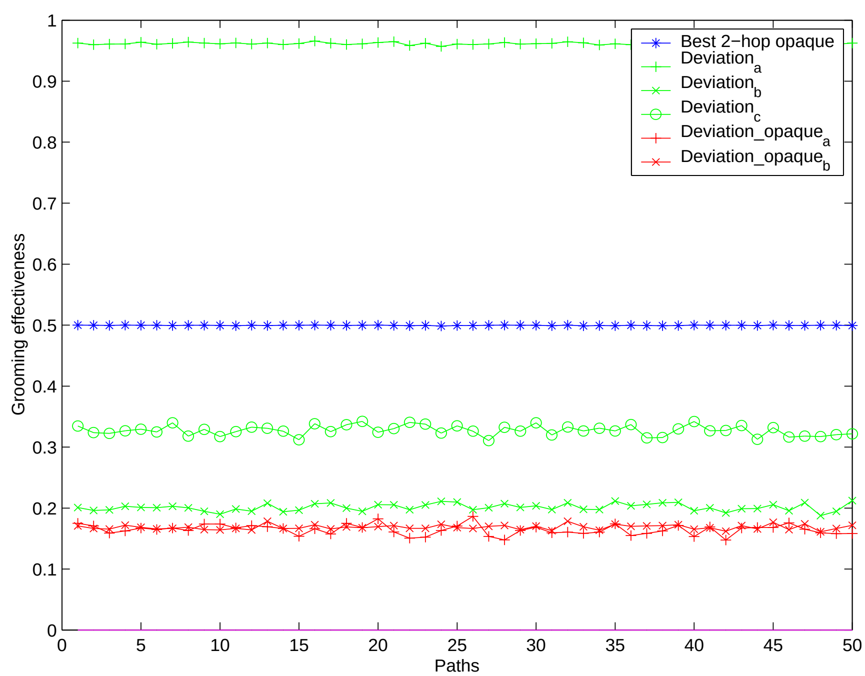

Figure 5.6: Uniform pattern, $N = 30, W = 20, C = 32, L = 80\%$

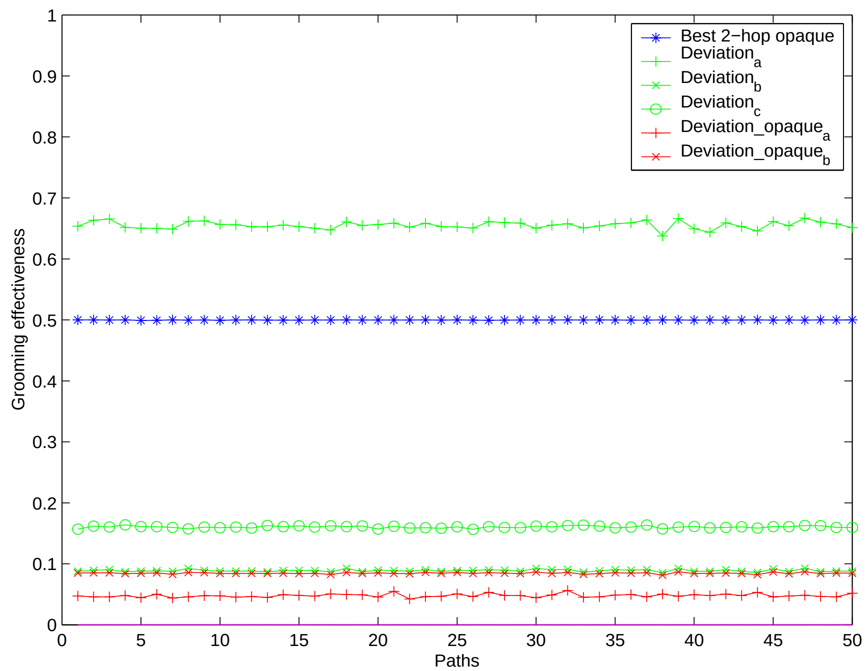

Figure 5.7: Uniform pattern, $N = 30, W = 80, C = 64, L = 40\%$

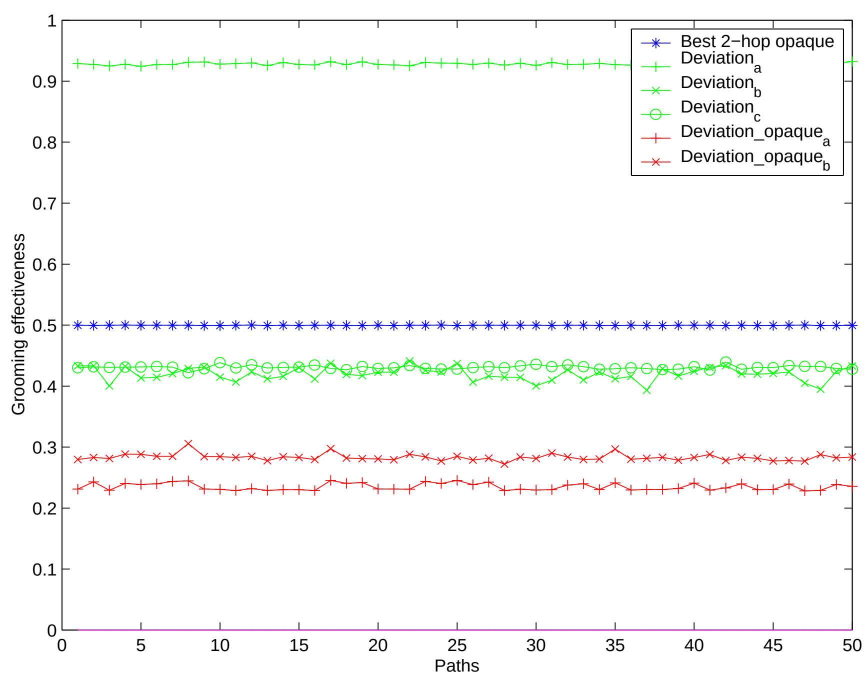

Figure 5.8: Uniform pattern, $N = 30, W = 20, C = 64, L = 40\%$

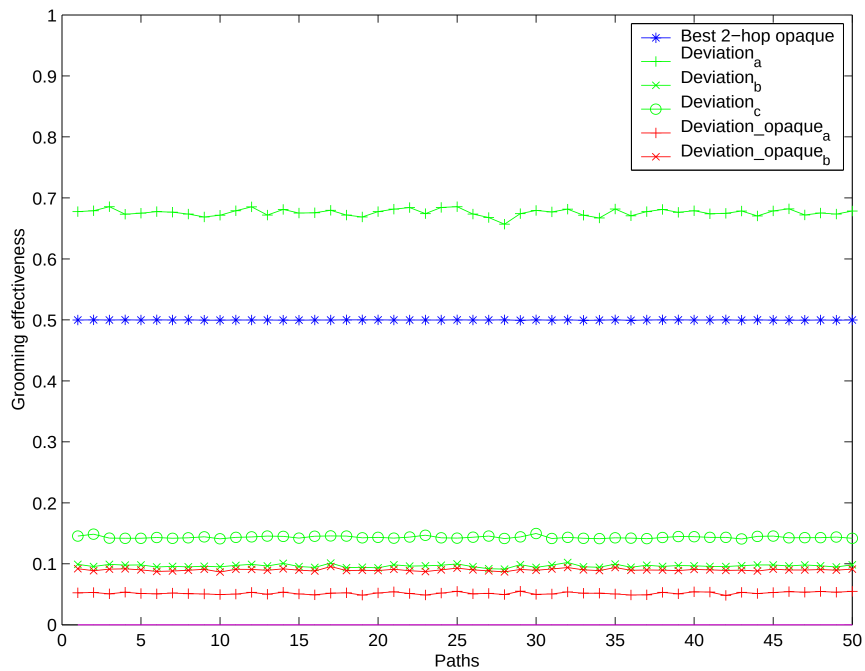

Figure 5.9: Uniform pattern, $N = 30, W = 80, C = 32, L = 40\%$

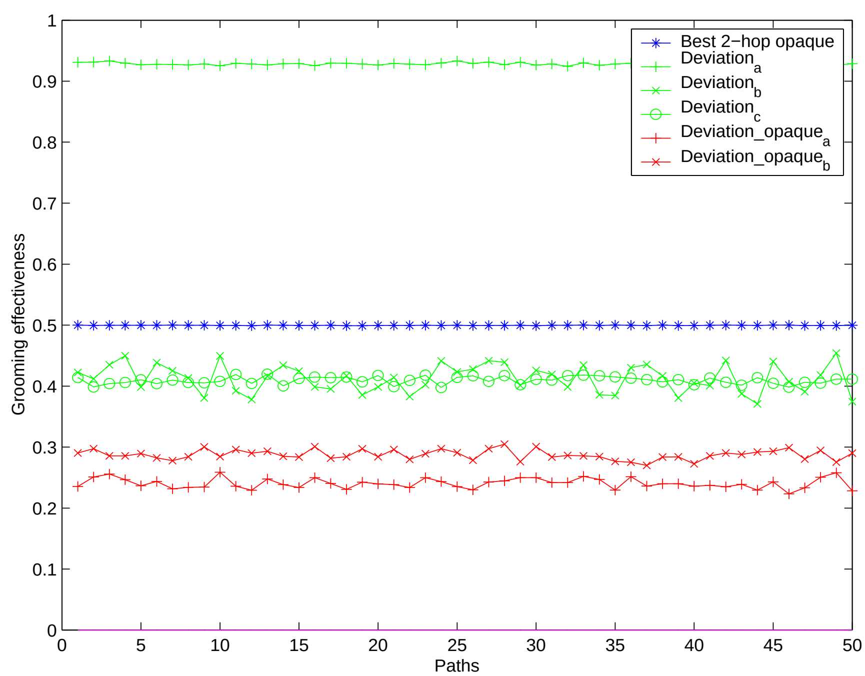

Figure 5.10: Uniform pattern, $N = 30, W = 20, C = 32, L = 40\%$

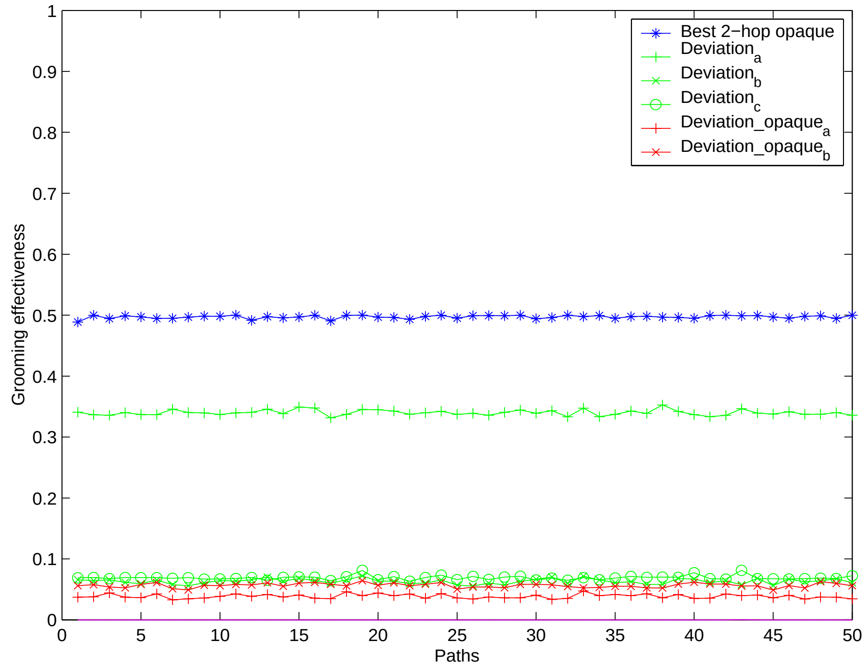

Figure 5.11: 4-hop-limited pattern, $N = 30, W = 100, C = 32, L = 90\%$

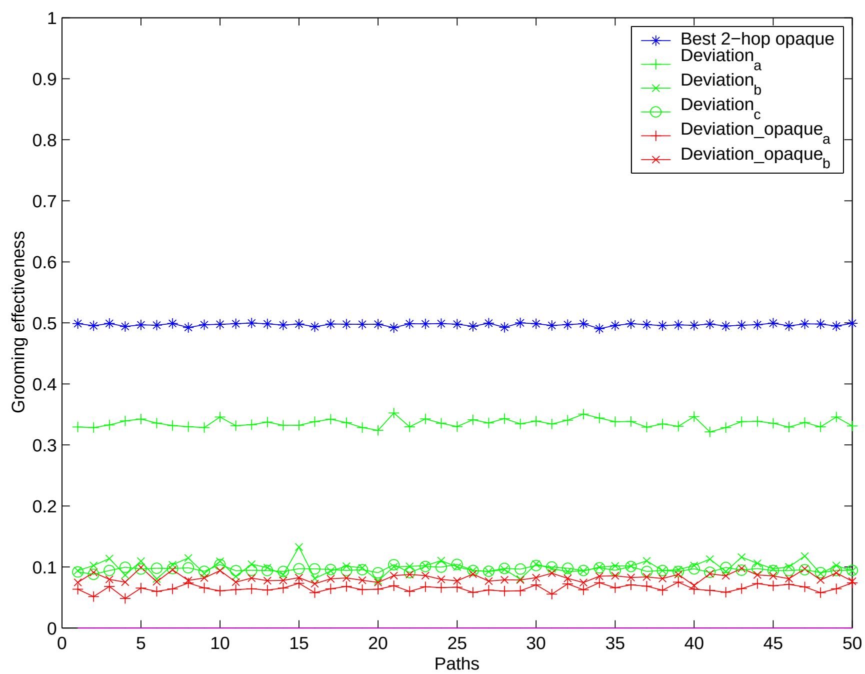

Figure 5.12: 4-hop-limited pattern, $N = 30, W = 50, C = 32, L = 90\%$

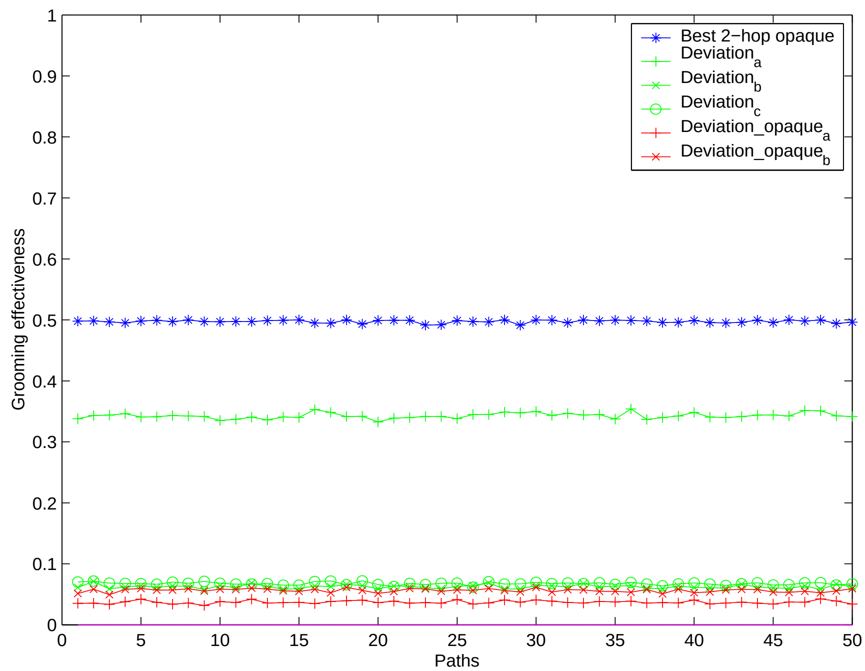

Figure 5.13: 4-hop-limited pattern, $N = 30, W = 100, C = 64, L = 90\%$

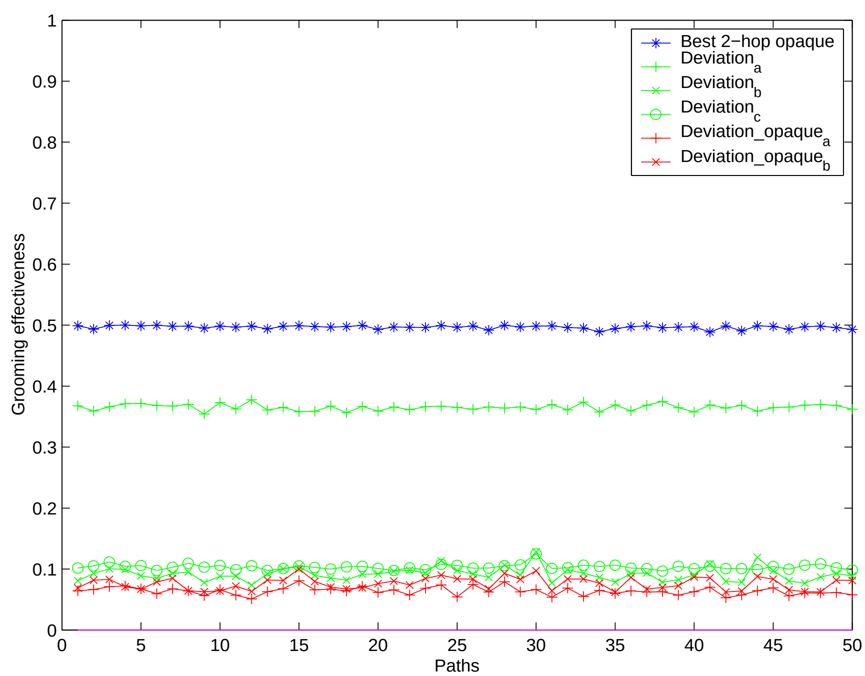

Figure 5.14: 4-hop-limited pattern, $N = 30, W = 50, C = 64, L = 90\%$

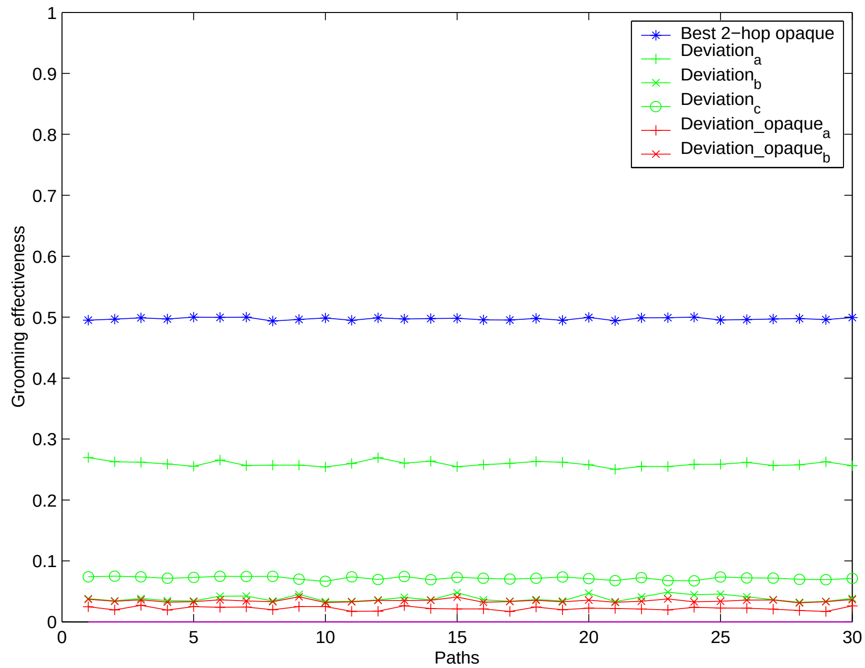

Figure 5.15: 4-hop-limited pattern, $N = 30, W = 100, C = 64, L = 45\%$

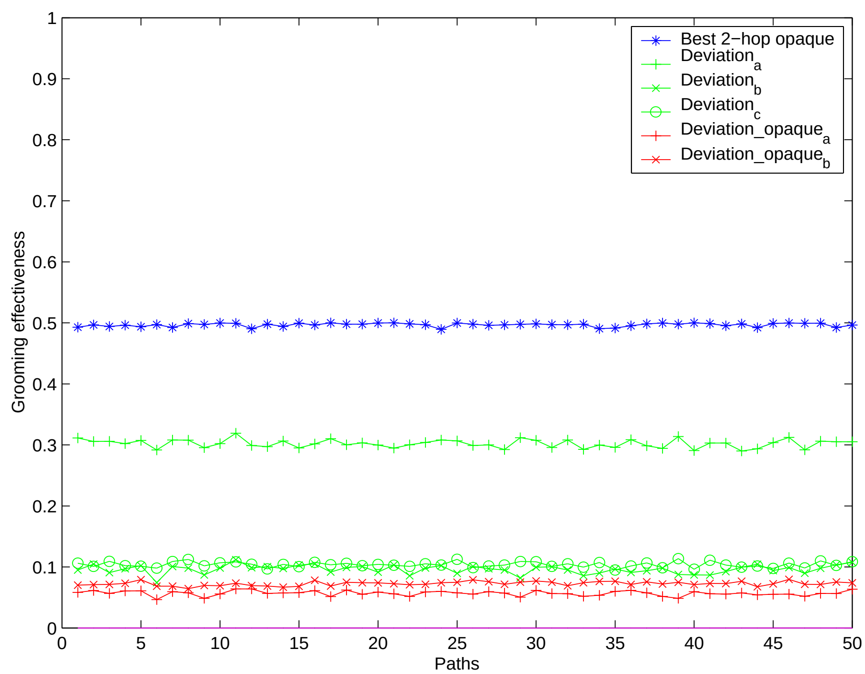

Figure 5.16: 4-hop-limited pattern, $N = 30, W = 50, C = 64, L = 45\%$

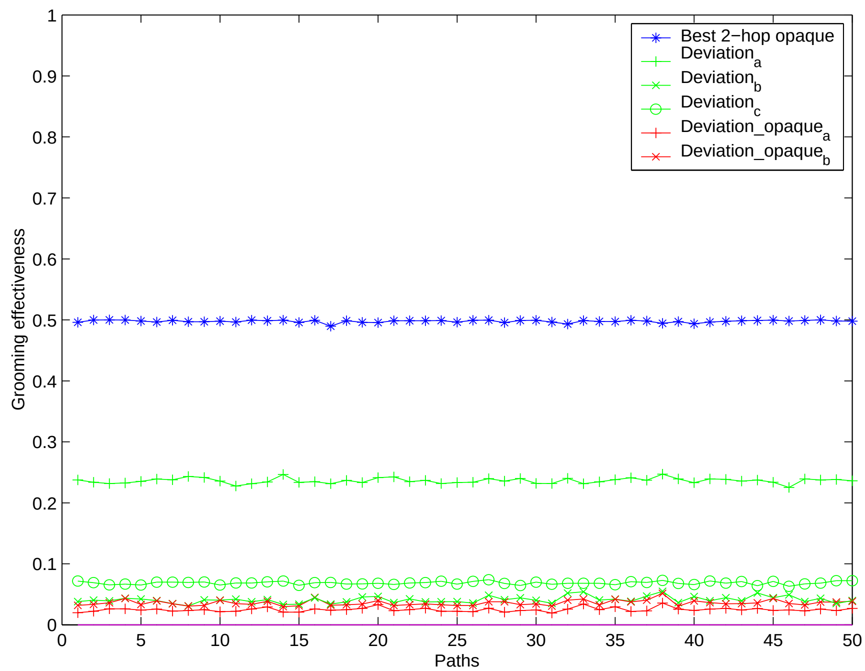

Figure 5.17: 4-hop-limited pattern, $N = 30, W = 100, C = 32, L = 45\%$

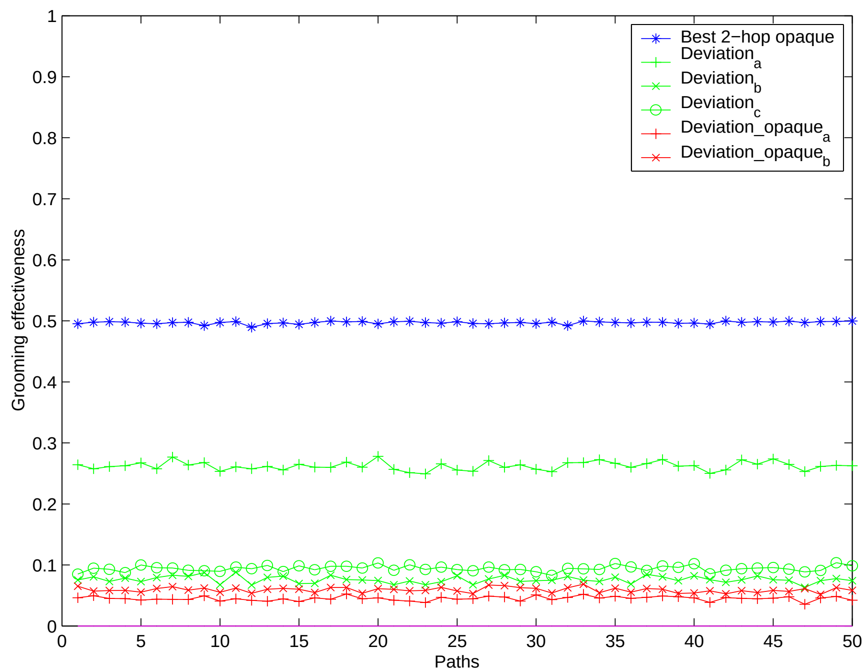

Figure 5.18: 4-hop-limited pattern, $N = 30, W = 50, C = 32, L = 45\%$

Chapter 6

Conclusion

The traffic grooming problem is optical networks is one of the critical problems to be solved in the emergence of next generation optical networks. We have shown the hardness of the problem even in path networks. Consequently, heuristics with modest computational effort are of importance in practice. We propose a simple heuristic for unidirectional path networks with performance guarantees to minimize the amount of electronic routing for static traffic patterns. We have investigated it with extensive numerical experiment. A large path network can be solved in fairly small amount of time and the guarantee is always met.

6.1 Future Work

The systematic approach we have taken to the path network grooming problem can be extended and expanded in many ways. There is need of research in other topologies more complex than the path but not necessarily arbitrary topologies. It would be desirable to follow the same approach of determining whether approximate solutions can be expected. If approximation algorithms exist for some class of problems, it would be desirable to find them; otherwise, heuristic approaches which do not provide approximations but nevertheless possess good performance guarantees in practical terms, such as the one we provide, are required.

The grooming problem should be examined with respect to the other cost functions we mentioned, it is expected that the problem will be intractable with respect to them as well but a rigorous demonstration is still not available to the best of our knowledge even for path networks. Similarly, good heuristic approaches to deal with these different cost functions are required.

The heuristic approach we have proposed can be applied with comparatively simple modifications to some other network topologies such as a bidirectional path or an MSN topology. We expect that the same basic approach, but with more modifications, will be possible to apply in very different topologies. It may also be the case that a decomposition approach can be taken to some topologies yielding subnetworks with path topologies so that our results can be directly applied.

Thus a large body of research remains to be performed in the traffic grooming area which will benefit from our contribution in this thesis.

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