ABSTRACT

MOHAN, RAMYA . Integration of Interconnect Models in a Circuit Simulator. (Under the direction of Dr. Michael B. Steer.)

A novel approach is implemented for synthesizing equivalent circuits. The approach called the Foster's approach finds its application in the transient simulation of distributed structures. The implementation is analogous to that of a Voltage Controlled Current Source, as it is a natural way to handle Admittance Matrix. The two main features of this method are its guaranteed causality and good numerical stability. The method is tested by simulating a six port power/ground plane and comparing the results with measurements. Also, different analyses types are compared and conclusions are made.

INTEGRATION OF INTERCONNECT MODELS IN A CIRCUIT SIMULATOR

by

RAMYA MOHAN

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To my parents who have been the most inspiring force of my life.

BIOGRAPHY

Ramya Mohan was born in Coimbatore, India on 8th February, 1979. She received the Electronics and Communication Engineering degree in June 2000 from Amrita Institute of Technology and Science, affiliated to Bharathiar University, Coimbatore, India. While pursuing her B. Engg. degree, she worked as a summer intern at PRICOL (Premier Instruments and Controls Limited), Coimbatore, India, in the field of Control Systems.

In Fall 2000 she was admitted into the masters program at North Carolina State University. While working towards her masters degree she held a research assistantship with the Electronics Research Laboratory in the Department of Electrical and Computer Engineering. Her research interests include Computer Aided Circuit Simulation, analog and RF Integrated Circuit Design. She is a member of the Institute of Electrical and Electronic Engineers.

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Chapter 1

Introduction

1.1 Motivations and Objectives of This Study

Many techniques have been investigated for the transient simulation of distributed structures as these are best modeled in the frequency domain where dispersive, radiative and skin effects can be captured. Transient analysis based on frequency-domain characterizations has been challenging because of convergence problems, non causality in the transient simulation compliant transform of the frequency-domain characterization, aliasing problems in the conversion, the lengthy convolution and nonlinear iterations and numerical ill-conditioning. Furthermore transient simulators have significant problems with accuracy. It is not surprising therefore that harmonic balance analysis has been critical to the modeling of microwave circuits with the dynamic range required to model distortion with sufficient accuracy. Nevertheless, transient analysis is important when analyzing large RF circuits, transient behavior including electro-thermal effects are important and with the potential for oscillation and chaotic behavior.

In this work, necessary developments are reported that make the simulation of N number of ports with m poles numerically stable. The issue of robustness is addressed by:

- Handling of local reference terminals.

- Formulation of Modified Nodal Admittance Matrix Stamp.
- A methodology that uses Foster's canonical form which facilitates the direct synthesis of an equivalent circuit representation of the power distribution networks.

1.2 Organization of Thesis

There is a considerable amount of research related to the numerical stability of distributed circuits.

- Chapter 2 presents a review of previously published material on local reference terminal and modified nodal admittance matrix formulation, with emphasis on the recent developments.
- Chapter 3 presents the theory behind the application of Foster's form and the technical approach that was used in the implementation of the algorithm along with an example.
- Chapter 4 presents the simulation results for a power/ground plane pair.
- Chapter 5 focuses on the direction of future research after the conclusions.

Chapter 2

Literature Review

2.1 Introduction

Many techniques have been explored for incorporating distributed structure characterizations including developing the impulse response and then convolution iterative techniques. Asymptotic Waveform Evaluation (AWE) and Laplace Inversion are powerful, but have limitations in applications. In addition to all of this Spice has a very limited dynamic range corresponding to roughly a 1% accuracy or 20 dB dynamic range. The most common SPICE transient analysis might also have simulation errors as described in [7, 9].

2.1.1 Asymptotic Waveform Evaluation

As the network size gets larger conventional transient analysis techniques become less efficient in producing results and AWE methods come into the scene with the sacrifice of accuracy. AWE method reduces the dimension of the system of equations. According to [1], AWE is about 2 orders of magnitude faster than the regular transient analysis methods. Modifications to AWE have been made for nonlinear circuits [1, 2]. Numerical inversion, convolution and piecewise linearization methods have been introduced to use the AWE with nonlinear circuits. AWE technique can extract the low frequency poles because the moments only carry information about the low frequency characteristics of the circuit. The higher frequency circuits cannot be modeled well using AWE due to the infinite number of poles and a solution is described in [8]. Most of the AWE methods use the Padé approximation and this and other approximations used have stability problems as indicated in [6].

2.1.2 Convolution Based on Impulse Response

Unfortunately this technique suffers from two major limitations. One of these is the aliasing problem associated with the inverse Fourier transform operation required to extract the impulse response from frequency domain characterization. Many schemes have been developed for extending the dynamic range but this has proved difficult to apply in general. Causality has been a long running problem but has been alleviated recently [3]. Even if the aliasing problem could be avoided, the convolution approach suffers from excessive run times. The convolution integral, which becomes a convolution sum for the computer simulations, is $O(N_T^2)$ when it is implemented (N_T is the total number of discrete time points used to divide the continuous time) [4].

2.1.3 Numerical Inversion of Laplace Transform Technique

This technique does not have aliasing problems since it does not assume that the function is periodic. The inverse transform exists for both periodic and non-periodic functions. There is no causality problem for double sided Laplace Transforms, either. Unlike FFT methods, the desired part of the response can be achieved without doing tedious and unnecessary calculations for the other parts of the response. Laplace techniques suffer from the series approximations and the nonlinear iterations involved. The advantages and the limitations of the Inverse Laplace Methods are discussed in detail in [5].

2.2 Local Reference Terminal Concept

2.2.1 Introduction

The distributed nature of many microwave and millimeter-wave circuits necessitates electromagnetic modeling. Thus the full application of Computer Aided Engineering (CAE) to these circuits requires the integration of electromagnetic models of distributed structures with conventional circuit analysis. In the formative stages of the CAE of microwave circuits, ports were used in specifying the connectivity of networks. The utilization of ports avoided the issue of specifying reference terminals. In analysis, using matrix manipulations or signal flow graphs, one of the terminals of a port was used implicitly as a reference terminal and generally ignored in formulating the mathematical model. The connection of discrete elements is specified nodally but at the highest level of the hierarchy port-based descriptions are used. While it is possible to analyze any circuit with this arrangement it becomes increasingly difficult to specify the connections of large spatially distributed circuits, and also to specify and extract desired output quantities. The alternative to using port-only descriptions is to exclusively use the terminal connectivity description; the only method used in general purpose circuit simulators. The conventional terminal-based specification enables circuit elements to be connected in any possible combination and only one reference terminal (commonly called the global reference terminal or simply ground) is used [14].

2.2.2 Background of Modified Nodal Admittance Matrix

Modified Nodal Admittance(MNA) Matrix analysis was developed to handle elements that do not have nodal admittance descriptions. For each such element one or more additional equations are added to the nodal admittance equations and these equations become additional rows and columns in the evolving matrix system of equations. A similar approach can be followed for the electromagnetic elements. The process is a little more sophisticated, as it is no longer sufficient to add additional rows. Instead the concept of local reference terminals [12] was developed as a generalization of the compression matrix approach. This concept provides another way to incorporate alternate equations in the evolving MNA matrix. However, rather than adding additional constitutive relations, the local reference terminal concept changes the way the port-based parameters are used. Figure 2.1 shows a spatially distributed circuit with local reference terminals indicated by the diagonal symbol. In a conventional circuit only one reference terminal (ground) is possible so that application of KCL to the global reference node introduces just one additional redundant row and column in the indefinite form of the MNA matrix. For a spatially distributed circuit, KCL is applied to each locally referenced group one at a time, as the local reference terminals are electrically connected only through a spatially distributed element, each application results in a redundant row and column in the MNA matrix. This is a particular property of the port-based characterization of the spatially distributed element.

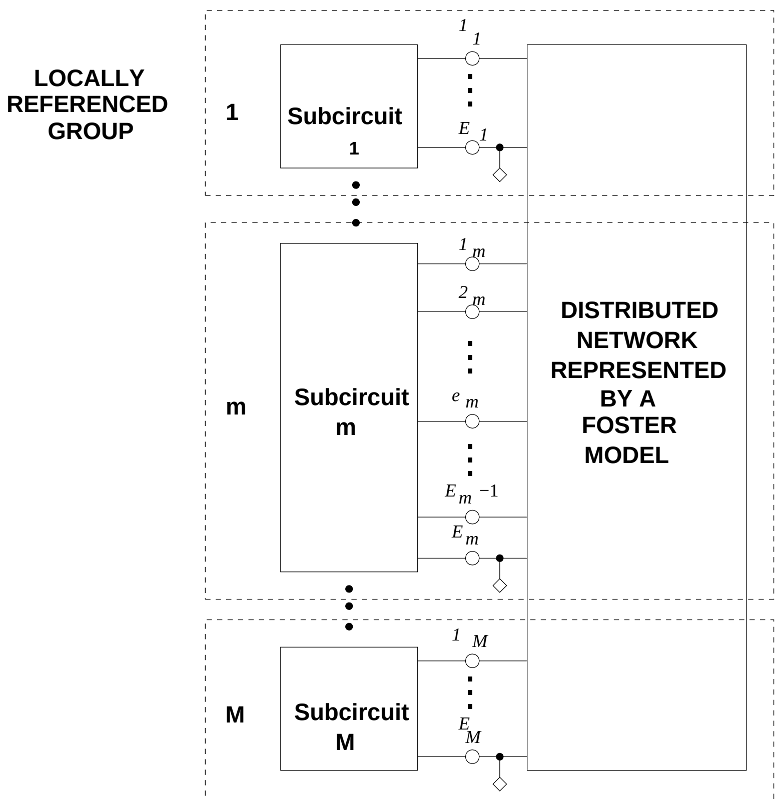

Figure 2.1: Generic spatially distributed circuit.

2.2.3 Local Reference Terminal

The basis of this approach is that in a multi-port element, the terminals of ports with different local reference nodes can be considered isolated. For example, if we write the equivalent circuit of a two-element derived from the port-based admittance (y) parameters,

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \tag{2.1}$$

we obtain the equivalent circuit in Figure 2.2,

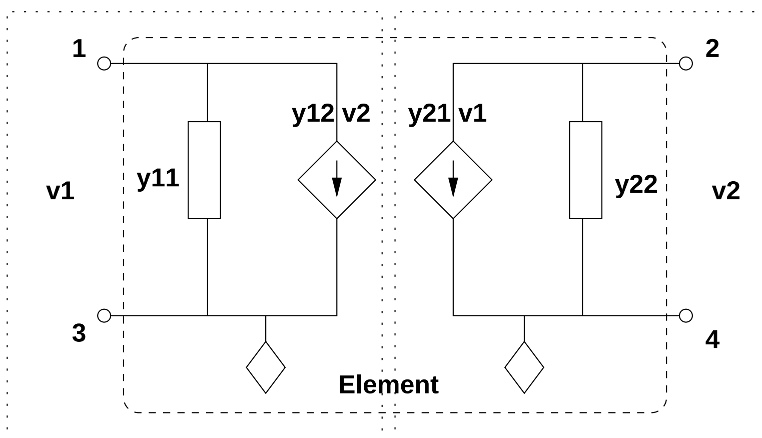

Figure 2.2: Equivalent circuit of a two-port distributed element.

Note that no current can flow between the two ports. This type of circuit model can be generalized for a multi-port element where external and internal local reference nodes are defined. The local reference shown in the Figure 2.2, are internal because they are used internally in the element to measure the voltage at its ports. An external local reference terminal, on the other hand, is an arbitrary chosen terminal from a locally referenced group. In general, for any spatially distributed circuit, Figure 2.1, there is no current between port groups inside the spatially-distributed element. Thus the circuit can be divided into subcircuits, each with a local reference terminal, as shown in the Figure 2.3. Each sub-circuit is isolated with respect to the others, so there is no change in the circuit behavior if all the reference nodes are connected together. The difference is that now there is only one global reference node for all the circuit, and standard model methods can be used to formulate the circuit equations.

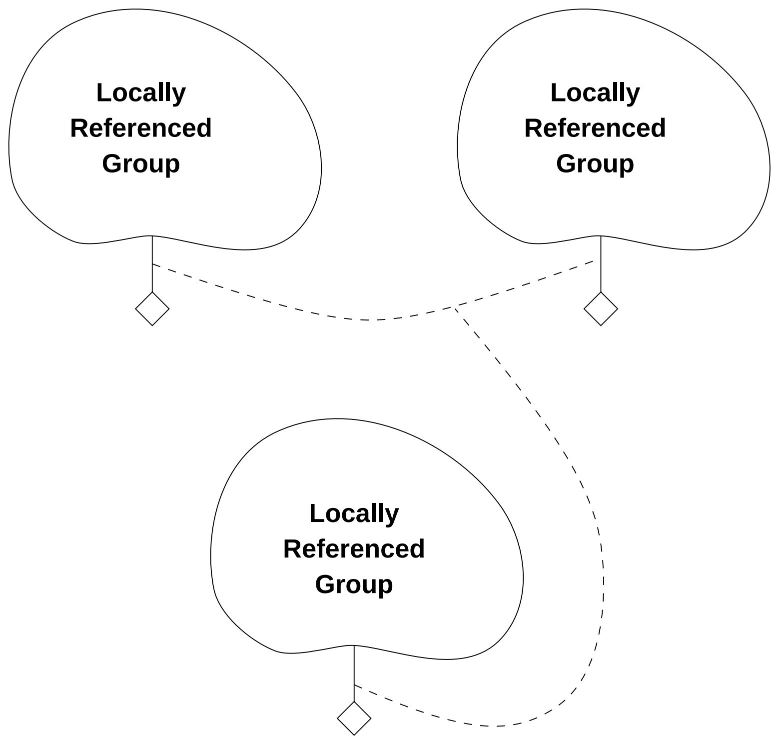

Figure 2.3: Locally referenced groups that are isolated.

The problem becomes that of detecting violations to the assumption that each locally referenced group is isolated from the others and that there is exactly one reference terminal for each one. Such a situation is shown in the figure 2.4. The effect of the connection of the lumped element between the two locally referenced groups in the figure is the creation of a current loop that is non-physical since the two circuits cannot be connected instantaneously. On the other hand, it is valid for two subcircuits to be connected by more than one spatially distributed element. It is also possible for two ports of a spatially distributed element to be connected to the same locally referenced group (for example a delay line). In this case, the external local reference teminal would be the same for both ports of the line, but internally, the line still has two local reference terminals. After all the checking is done, all the local reference terminals are merged into a single global reference terminal, and the solution of the circuit is found by standard procedures. The terminal voltages found are coincident with the voltages referred to each local reference terminal.

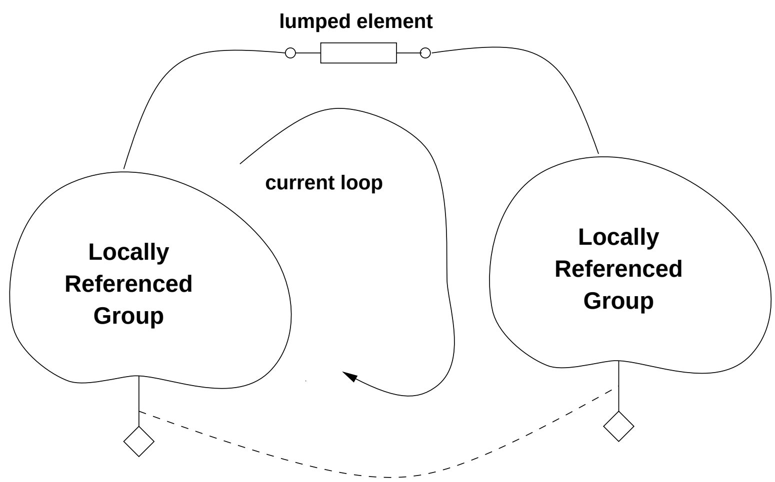

Figure 2.4: Illegal connection between groups.

2.2.4 Modified Nodal Admittance Matrix (MNAM)

The MNAM of the linear sub-circuit is formulated as follows: Define two matrices $\mathbf{G}$ and $\mathbf{C}$ of equal size n_m, where n_m is equal to the number of non-reference nodes in the circuit plus the number of additional required variables. Define a vector $\mathbf{s}$ of size n_m for the right hand side of the system. The contributions of the fixed sources and the non-linear elements (which depend on the time t) will be entered in this vector. All conductors and frequency-independent MNAM stamps arising in the formulation will be entered in $\mathbf{G}$, whereas capacitor and inductor values and other values that are associated with dynamic elements will be stored in matrix $\mathbf{C}$. The linear system obtained is the following:

$$\mathbf{G}\mathbf{u}t + \mathbf{C}\frac{d\mathbf{u}(t)}{dt} = \mathbf{s}(t) \tag{2.2}$$

and the system matrix obtained for any s, is,

$$\mathbf{T} = \mathbf{G} + s\mathbf{C}. \tag{2.3}$$

For instance, consider a voltage source as in Figure 2.5. The *nodes* are denoted by N_+ and N_-, and the voltage source enforces the condition,

$$V_{N+} - V_{N-} = E. \tag{2.4}$$

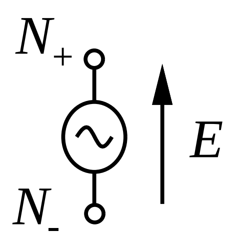

Figure 2.5: Voltage source.

This constitutive equation is in terms of the *terminal voltages* and is appended to the set of previously defined equations. In addition, a current I will flow between the terminals N_+ and N_-. To unify the notation, the currents at either of the terminals will be considered to be positive; thus for the voltage source $I_{N+} = I$, $I_{N-} = -I$. They are taken into account in the KCL as a new variable: I in the N_+ th row and $-I$ in the N_- th row. The matrix will have the following form:

$$\left[\begin{array}{cc|c} & & 1 \\ & & -1 \\ \hline 1 & -1 & \end{array}\right]. \tag{2.5}$$

The increased size of the matrix is seen clearly from the extra row and column.

Chapter 3

Implementation of Foster's Model

3.1 Introduction

Foster's representation of distributed circuits is adopted because its convenient to synthesize equivalent circuits and it is also the output form of VectFit (Vector Fitting Algorithm). Foster's canonical representation is given as,

$$H(s) = \sum_{j=1}^{m} \left(\frac{k_j}{s - p_j} \right) + \sum_{j=1}^{m} \left(\frac{a_j}{s - b_j} + \frac{a_j^*}{s - b_j^*} \right) \tag{3.1}$$

where $k_j/(s - p_j)$ represents the real pole and $a_j/(s - b_j)$ and $a_j^*/(s - b_j^*)$ together represents the complex conjugate pairs. This form is guaranteed to be causal and circumvents the main problem with problem in implementing reduced order models as was indicated in the literature at the beginning of this paper. Since in linear analysis we want to use the Modified Nodal Admittance matrix technique we take the Foster's model as an N-port defined by an admittance matrix.

This is a methodology that facilitates the direct synthesis of an equivalent circuit representation of the power distribution network in terms of lumped circuit elements e.g., resistors, inductors, capacitors, dependent sources, etc and is demonstrated using the approach of Joon and Cangellaris [13] and is described in the following pages. The methodology begins with the extraction from the discrete model of a frequency-dependent multi-port admittance representation of the power distribution network. The ports of the power distribution network are defined as the physical locations at which the power distribution connects to the voltage regulator, the power and ground pin connections at the die, as well as the pins at which decoupling capacitance will be connected. As discussed in detail in [10], through the application of the passive order reduction process [11], the closed-form admittance matrix representation of a power distribution network with N ports can be written in pole-residue form as,

$$Y(s) = \begin{bmatrix} \sum_{k=1}^{M} \frac{R_k^{11}}{s-P_k} & \cdots & \sum_{k=1}^{M} \frac{R_k^{1N}}{s-P_k} \\ \vdots & \sum_{k=1}^{M} \frac{R_k^{ii}}{s-P_k} & \vdots \\ \sum_{k=1}^{M} \frac{R_k^{N1}}{s-P_k} & \cdots & \sum_{k=1}^{M} \frac{R_k^{NN}}{s-P_k} \end{bmatrix} \tag{3.2}$$

where all the elements share the same set of M poles, P_1, P_2, ... , P_M. The poles are in general complex and due to the passivity of the generated reduced model, are all stable. Since complex poles occur in complex conjugate pairs, with their corresponding residues being complex conjugates also, the expression for the current at the ith port in terms of the N port voltages may be cast in the form,

$$\begin{aligned} I^i = & \left[\sum_{k=1}^{M_C} \frac{\left(R_{ck}^{i1} + \bar{R}_{ck}^{i1}\right) s - \left(R_{ck}^{i1}\bar{P}_{ck} + \bar{R}_{ck}^{i1}P_{ck}\right)}{s^2 - \left(P_{ck} + \bar{P}_{ck}\right) s + |P_{ck}|^2} + \sum_{k=1}^{M_R} \frac{R_{rk}^{i1}}{s - P_{rk}}\right] V^1 + \cdots \\ & \left[\sum_{k=1}^{M_C} \frac{\left(R_{ck}^{ii} + \bar{R}_{ck}^{ii}\right) s - \left(R_{ck}^{ii}\bar{P}_{ck} + \bar{R}_{ck}^{ii}P_{ck}\right)}{s^2 - \left(P_{ck} + \bar{P}_{ck}\right) s + |P_{ck}|^2} + \sum_{k=1}^{M_R} \frac{R_{rk}^{ii}}{s - P_{rk}}\right] V^i + \cdots \\ & \left[\sum_{k=1}^{M_C} \frac{\left(R_{ck}^{iN} + \bar{R}_{ck}^{iN}\right) s - \left(R_{ck}^{iN}\bar{P}_{ck} + \bar{R}_{ck}^{iN}P_{ck}\right)}{s^2 - \left(P_{ck} + \bar{P}_{ck}\right) s + |P_{ck}|^2} + \sum_{k=1}^{M_R} \frac{R_{rk}^{iN}}{s - P_{rk}}\right] V^N \end{aligned} \tag{3.3}$$

where complex conjugation is indicated with the over bar, M_C is the number of pairs of complex poles and M_R is the number of real poles. The synthesis methodology is then based on the interpretation of each one of the terms in the equation above in terms of an equivalent circuit.

Starting with the self-admittance term for the kth complex pole pair,

$$I_{ck}^{ii} = \left[\frac{\left(R_{ck}^{ii} + \bar{R}_{ck}^{ii}\right) s - \left(R_{ck}^{ii}\bar{P}_{ck} + \bar{R}_{ck}^{ii}P_{ck}\right)}{s^2 - \left(P_{ck} + \bar{P}_{ck}\right) s + |P_{ck}|^2}\right] V^i. \tag{3.4}$$

Its equivalent circuit realization is shown in the Figure 3.1, For the kth real pole,

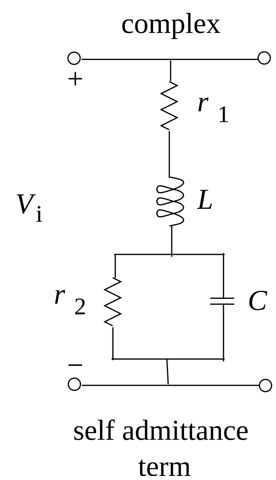

Figure 3.1: Equivalent circuit realization for a complex pole pair in the self-admittance term.

$$\frac{R_k^{ii}}{s - P_k} V^i \tag{3.5}$$

The equivalent circuit realization for the real pole is shown in the Figure 3.2, For the

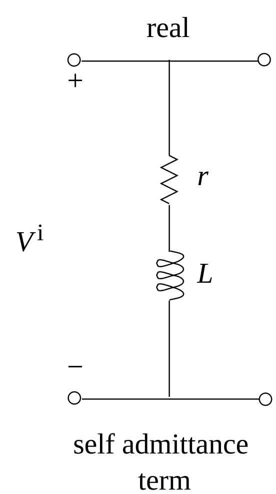

Figure 3.2: Equivalent circuit realization of a real pole term in the self-admittance.

equivalent circuit realization of trans-admittances it is noted that the contribution of the jth port voltage to the current of the ith port through the kth complex pole pair, I_{ck}^{ij}, may be expressed in terms of the self-contribution of the jth port voltage to its current through the same (kth) complex pole pair, I_{ck}^{ij}, as

$$I_{ck}^{ij} = \frac{\left(R_{ck}^{ij} + \bar{R}_{ck}^{ij}\right) s - \left(R_{ck}^{ij}\bar{P}_{ck} + \bar{R}_{ck}^{ij}P_{ck}\right)}{\left(R_{ck}^{ij} + \bar{R}_{ck}^{jj}\right) s - \left(R_{ck}^{jj}\bar{P}_{ck} + \bar{R}_{ck}^{jj}P_{ck}\right)} I_{ck}^{ij}. \tag{3.6}$$

This form suggests an equivalent circuit realization in terms of a current-controlled current source in the Figure 3.3, For the real pole it is very similar to the above and the

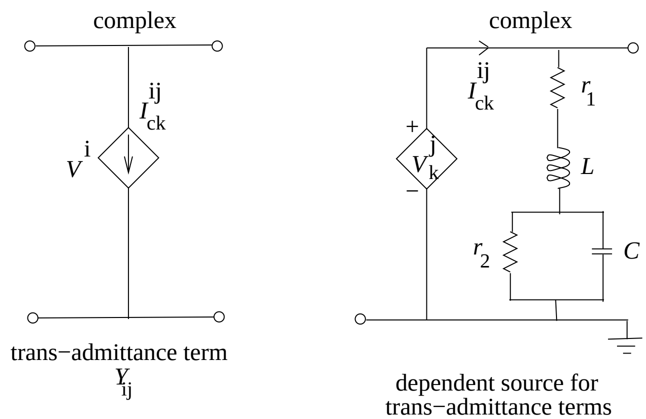

Figure 3.3: Equivalent circuit realization of the contribution of a complex pole pair to the trans-admittance term between ports i and j.

resulting equivalent is shown in Figure 3.4, In this way the equivalent circuit for the

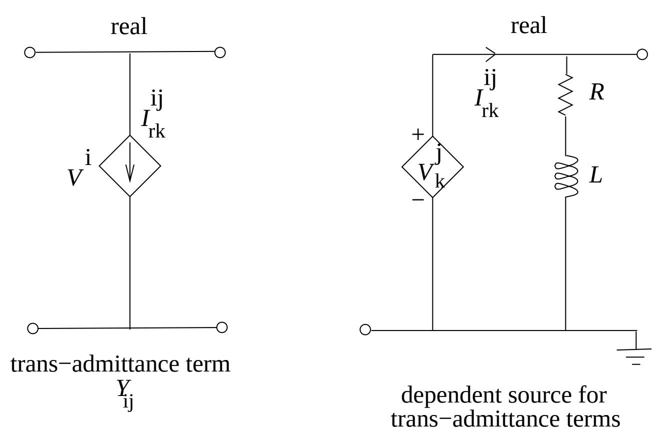

Figure 3.4: Equivalent circuit realization of the contribution of a real pole to the trans-admittance term between ports i and j.

ith port resulting from the integration of all the equivalent sub-circuit realizations of each of the terms in the pole-residue representations of the self-admittance and trans-admittance terms can be derived.

3.2 Implementation in fREEDA

The N-port Foster's model is directly incorporated in the Modified Nodal Admittance Matrix (MNAM) in the circuit simulator (fREEDA). The implementation is analogous to that of a multi-terminal linear Voltage Controlled Current Sources (VCCSs) although a direct implementation is preferred for simulation speed and robustness as well as netlist robustness (that is specifying a single element rather than a complex circuit of VCCSs).

The method followed here is the $Pole - Residual$ method as it has good numerical stability.

3.2.1 Technical Approach

Foster's model describes an admittance matrix wherein each element in the multi-port Admittance Matrix is represented as a rational function in $pole - residue$ format. In this format, different elements of the Admittance matrix may have different values for the poles. However all the elements in the admittance matrix must have the same set or number of poles although there can be any number of N-port Foster models, each one having a different number of poles. The restriction on the number of poles of the admittance matrix elements being the same comes about because time-domain analysis requires derivatives of the modified nodal admittance matrix. (If only steady-state analysis, as in Harmonic Balance analysis, then there would not be this pole restriction but the key guiding principal we have followed is to use the same model in all circuit analyses). The current widely accepted practice for incorporating models in a simulator is to develop a stamp which in this case is the sub-matrix entry in the MNA matrix of the linear network.

3.2.2 Modified Nodal Admittance Matrix Representation

For the MNAMs, the Sparse matrix package $Sparse1.3$ is used. It is a flexible package of subroutines written in C that quickly and accurately solves large sparse systems of linear equations. It also provides utilities such as MNAM reordering and other utilities suited to circuit analysis. There is one sparse matrix per frequency. The linear system solving capability is used to calculate the matrices. The SuperLU package (C) is used for general sparse matrix handling.

3.2.3 Filling of the MNA Matrix

The number of ports and number of poles (and also the data file set) are taken as the required input parameters in the netlist.

Suppose we have a 2-port network, there could be either 1 or 4 instances of the given 'element' (NPortFoster), that is, if

$$Y(s) = [H_{11}(s)\ H_{12}(s); H_{21}(s)\ H_{22}(s)] \tag{3.7}$$

then each $H_{ii}(s)$ could be represented as an instance by this element, depending on the way it is connected in the network.

In the data set, there is a real pole-residue value and a complex pole-residue value. The complex pole-residue value is converted to real pole residue format and then inserted in the matrix. In this way the *element* is created for each transfer function and connected in the circuitry.

The is done using the function called `fillMNAM` function in fREEDA which fills the modified nodal admittance matrix with the calculated transfer function values.

Six Port Network

Consider for example a six-port network where there can be any number of terminals between 7 and 12 depending on the number of LRGs. For illustrative purposes, consider that there is just one $Local\ Reference\ Terminal$. Thus there are 7 terminals numbered from 0 to 6, with the terminal number 6 taken as the reference. A loop is put with respect to the `fillMNAM` function. For each iteration of the loop, the transfer function is evaluated and inserted in the MNAM. Thus for a six port representation in the frequency domain, the transfer function matrix would resemble the following,

$$\begin{array}{cc} & i \\ & \rightarrow \\ j \;\; \downarrow & \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{16} \\ Y_{21} & Y_{22} & \dots & Y_{26} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{61} & Y_{62} & \dots & Y_{66} \end{bmatrix} \end{array}. \tag{3.8}$$

Here Y_{11} to Y_{66} each would contain the given data set, meaning the number of complex pole pairs and the real poles.

The representation for a N-terminal device defined by,

$$I(s) = H(s)V(s) \tag{3.9}$$

and is shown in the Figure 3.5.

The transfer function that is calculated above for each real pole and complex pole is filled in the MNAM using the `setQuad` function given by,

```
mnam -> setQuad ( getTerminal(i)->getRC(),
            getTerminal(ports)->getRC(),
            getTerminal(j)->getRC(),
            getTerminal(ports)->getRC(),g)
```

wherein each admittance matrix element Y_{ij} has the Admittance matrix stamp

$$\begin{bmatrix} g & -g \\ -g & g \end{bmatrix}. \tag{3.10}$$

Netlist

⟨`element name`⟩:⟨`instance name`⟩ ⟨⟨`port specification`⟩ local reference group count as a vector⟩ filename = ⟨`filename`⟩ ports = ⟨`number of ports`⟩ poles = ⟨`number of poles`⟩.

For example, for the multiport, multiple reference group, the format is,

`NPortFoster`:f1 1 0 2 0 3 0 4 0 5 0 6 0 filename = “transimtest.dat” ports = 6 poles = 36

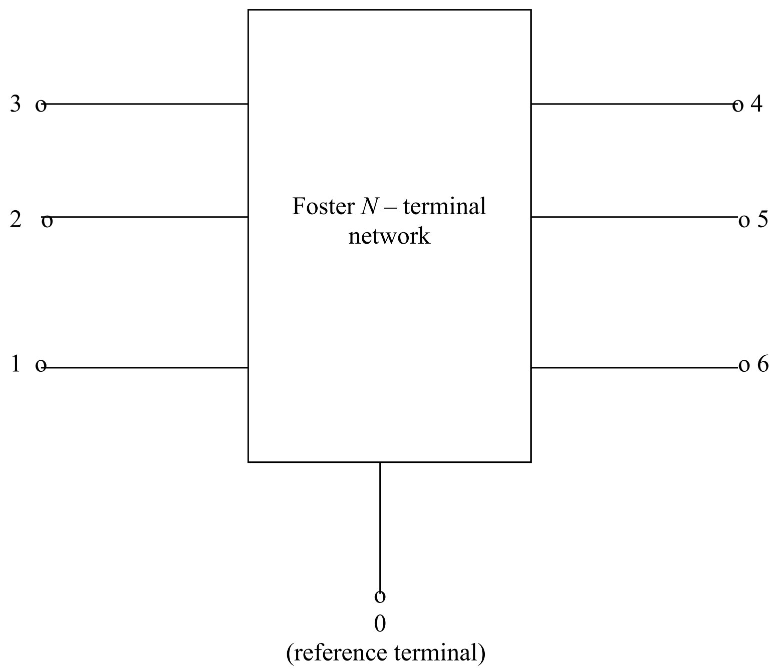

Figure 3.5: Foster N-terminal network.

*Local reference terminal

.ref 0

The algorithm is summarized in the flowchart, given in Figure 3.6.

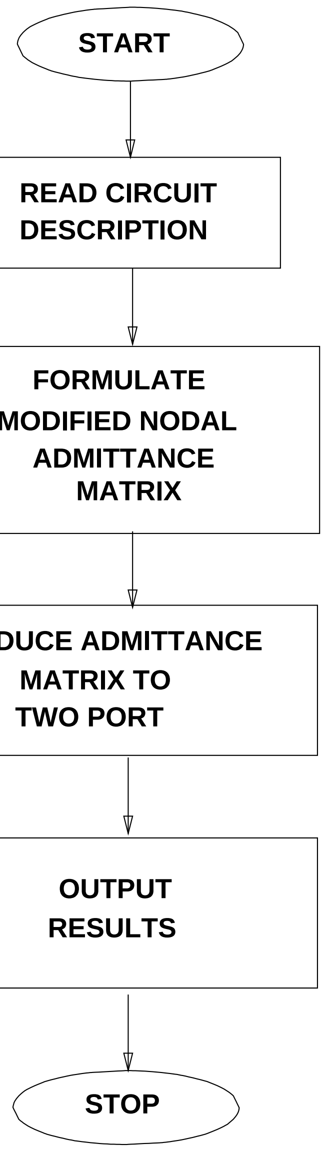

Figure 3.6: Flow of linear circuit analysis program.

3.2.4 Development of the MNA stamp

In this section the transfer function based on Foster's model is solved and the MNA stamp for the model developed. Time domain analysis involves complexity in the calculation of the Modified Nodal Admittance Matrix as it involves the derivatives.

The transfer function $H(s)$, voltage $V(s)$ and the current $I(s)$ are related as:

$$I(s) = H(s)V(s) \tag{3.11}$$

and

$$H(s) = \left(\frac{k_j}{s - p_j}\right) + \left(\frac{a_j s + b_j}{s^2 + c_j s + d_j}\right) \tag{3.12}$$

where j varies from 1 to M.

The MNA stamp is built from stamps for the individual poles. Consider the real pole, $(k_j/(s - p_j))$, first. Then

$$I(s) = \left(\frac{k_j}{s - p_j}\right) V(s). \tag{3.13}$$

Taking the inverse Laplace transform and rearranging,

$$i + \left(\frac{k_j}{p_j}\right) v - \left(\frac{1}{p_j}\right) \frac{di}{dt} \tag{3.14}$$

where $v = v_m - v_n$, the voltage difference between nodes m and n. The real pole adds one extra row and column.

Then the MNA matrix stamp is

$$x : \left[\begin{array}{cc|c} & & 1 \\ & & -1 \\ \hline k_j & -k_j & p_j \end{array}\right] \tag{3.15}$$

and its first derivative is

$$\frac{dx}{dt} : \left[\begin{array}{c|c} & \\ \hline & -1 \end{array}\right]. \tag{3.16}$$

Next consider the complex conjugate pole pair in the term

$$\left(\frac{a_j}{s - b_j}\right) + \left(\frac{a_j^*}{s - b_j^*}\right). \tag{3.17}$$

When multiplied out this yields the real term

$$\left(a_j s + \frac{b_j}{s^2} + c_j s + d_j\right) \tag{3.18}$$

and so

$$I(s) = \left(a_j s + \frac{b_j}{s^2} + c_j s + d_j\right) V(s). \tag{3.19}$$

Taking the inverse Laplace transform and rearranging,

$$\frac{d^2 i}{dt^2} + c_j \frac{di}{dt} + d_i i - a_j \frac{dv}{dt - b_j v} = 0 \tag{3.20}$$

Since this involves second derivative terms, we take an auxiliary variable, say x, and define it as:

$$x = \frac{di}{dt} \tag{3.21}$$

and so

$$x = \frac{d^2 i}{dt^2} \tag{3.22}$$

Then the MNA stamp is

$$x : \left[\begin{array}{cc||c|c} & & 1 & \\ & & -1 & \\ \hline\hline -b_j & b_j & d_j & \\ \hline & & 1 & \end{array}\right] \tag{3.23}$$

and its first derivative is

$$\frac{dx}{dt} : \left[\begin{array}{cc||c|c} & & & \\ & & & \\ \hline\hline -a_j & a_j & c_j & 1 \\ \hline & & & 1 \end{array}\right] \tag{3.24}$$

The number of extra rows and columns in the time domain implementation (`fillMNAM`) function) analysis is given by,

$$\text{Extra rows and columns} = size\ of\ real\ pole + 2 \times size\ of\ complex\ pole$$

The factor of two is present here because of the complex conjugate pairs.

3.3 fREEDA Netlist format

With respect to the Figure 2.1, for $[(E_1-1)+(E_2-1)+\ldots+(E_m-1)+\ldots+(E_M-1)]$ ports and p poles, the netlist format is,

`NPortFoster`:$\langle$`instance name`$\rangle$

$+\ 1_1\ E_1\ 2_1\ E_1\ \ldots\ (E-1)_1\ E_1$

$+\ \ldots\ 1_m\ E_m\ 2_m\ E_m\ \ldots\ e_m\ E_m\ \ldots\ (E-1)_m\ E_m$

$+\ \ldots\ 1_M\ E_M\ 2_M\ E_M\ \ldots\ e_M\ E_M\ \ldots\ (E-1)_M\ E_M$

+ `filename` $=$ "$inputfile.dat''$

+ `ports` $=\ [(E_1-1)+(E_2-1)+\ldots+(E_m-1)+\ldots+(E_M-1)]$ `poles` $=\ p$

Chapter 4

Simulations and Analysis

4.1 Introduction

In this chapter we first describe a 6-port interconnect (power/ground plane). Then we show the results of simulation and measurements on that system. Towards the end, we compare the performance of the different types of transient analysis simulation using different time stepping algorithms to solve the equations.

The structure involves the simple configuration of a 6-pin power/ground plane pair as shown in the figure 4.1 The length of the planes is 8 cm, the width 4 cm

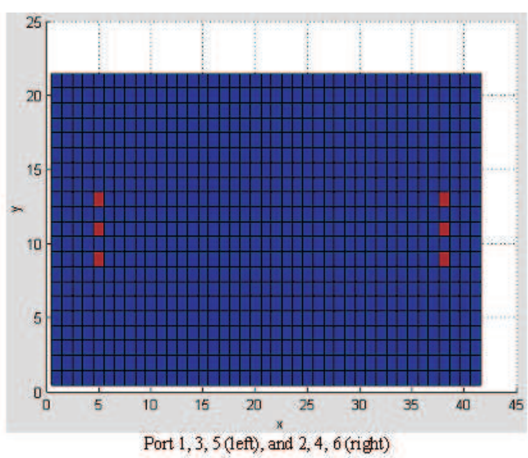

Figure 4.1: Structure of a 6-port interconnect.

and the plane separation is 2 mm. The insulating material between the planes has a relative dielectric constant of 2.33. The frequency bandwidth of interest is 4.5GHz.

The top plane is ground plane and the bottom plane is power plane. The six red squares represent six power pins. The gap between the power pin and ground plane is the port where external circuits are connected.

4.2 Simulation and Measurement Results

The following two figures are the result of the comparison between Spice and tran2 (Time-Marching Transient analysis) and tran3 (Time-Marching Transient analysis with variable time step) analyses of fREEDA.

This result is that of a 6 - port network with 36 * 36 real poles and 36 * 36 complex conjugate pairs.

4.2.1 AC Analysis

AC Analysis was carried out at a frequency of 4GHz with a 1V ac input in both SPICE and fREEDA (for tran2-Time Marching Transient Analysis and tran3-Time Marching Transient Analysis with variable Time step). The Figures 4.2, 4.3, 4.4, 4.5, 4.6, 4.7 show this comparison.

4.2.2 Transient Analysis

A square wave input of pulse width 100e-9 seconds with a rise and fall time of 0.1e-9 seconds was applied to the power pin. The Figures 4.8, 4.9, 4.10, 4.11, 4.12, 4.13 show the comparison between SPICE, $tran2$ and $tran3$ analysis of fREEDA at all the ports.

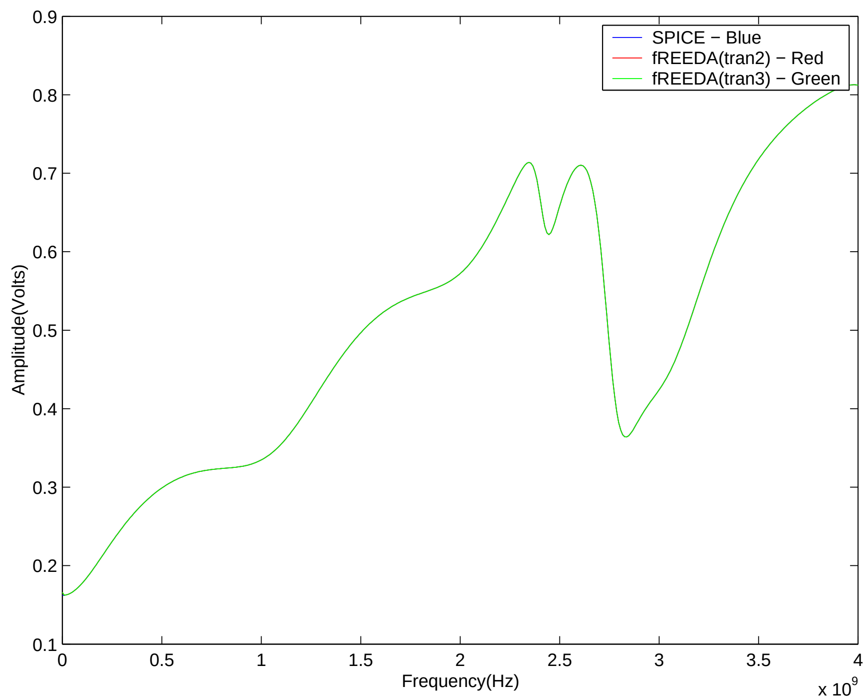

Figure 4.2: AC analysis comparison at port 1.

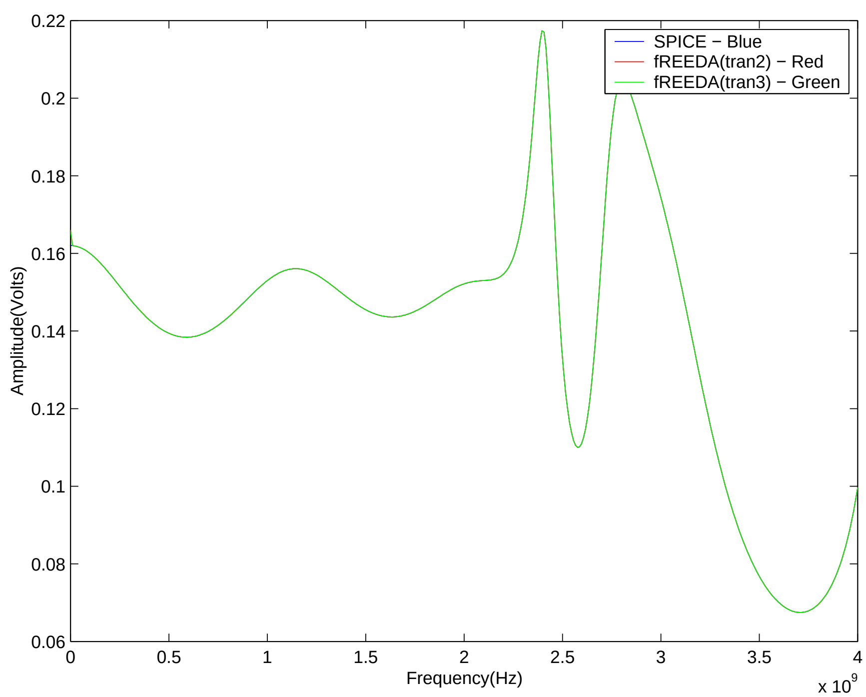

Figure 4.3: AC analysis comparison at port 2.

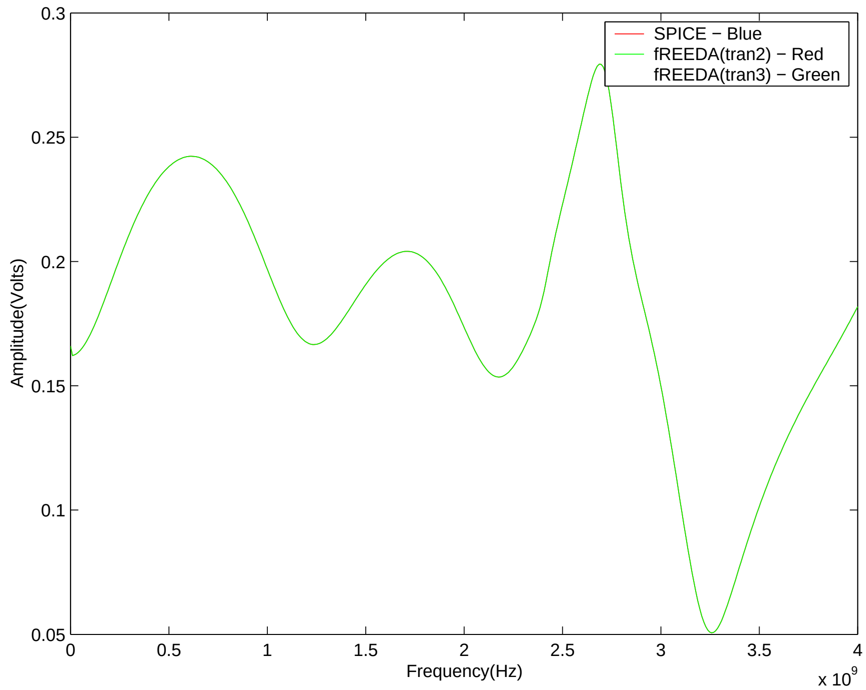

Figure 4.4: AC analysis comparison at port 3.

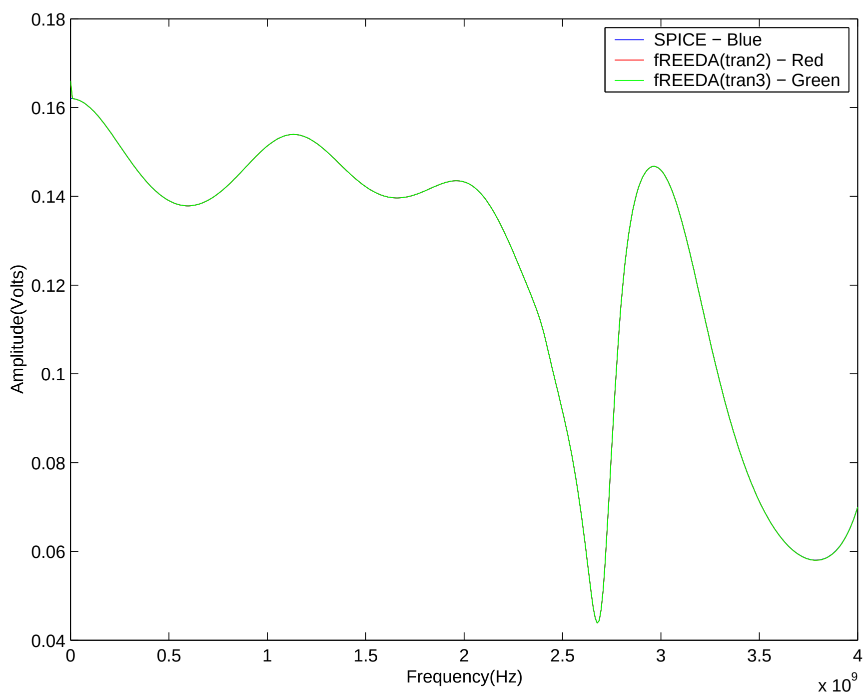

Figure 4.5: AC analysis comparison at port 4.

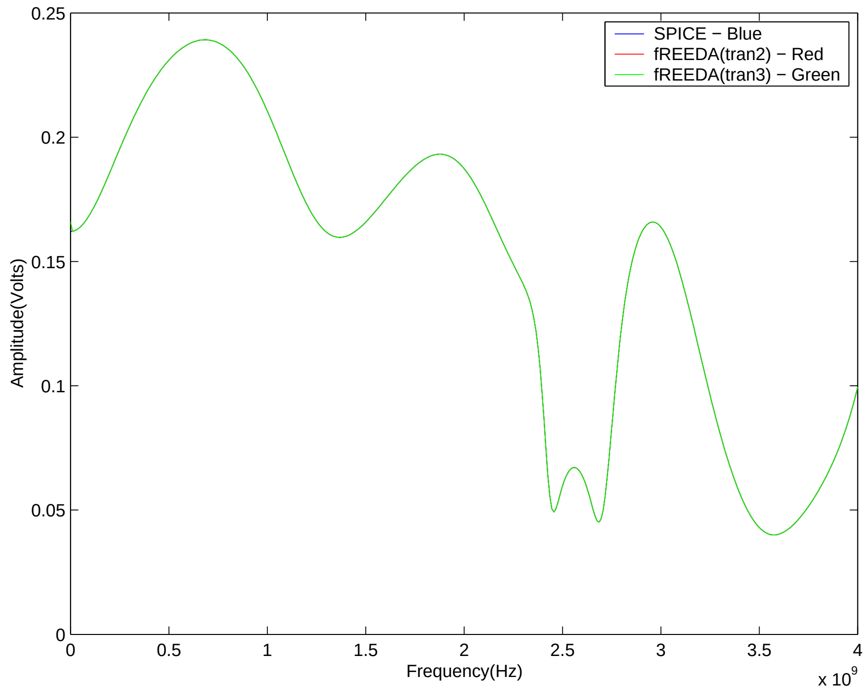

Figure 4.6: AC analysis comparison at port 5.

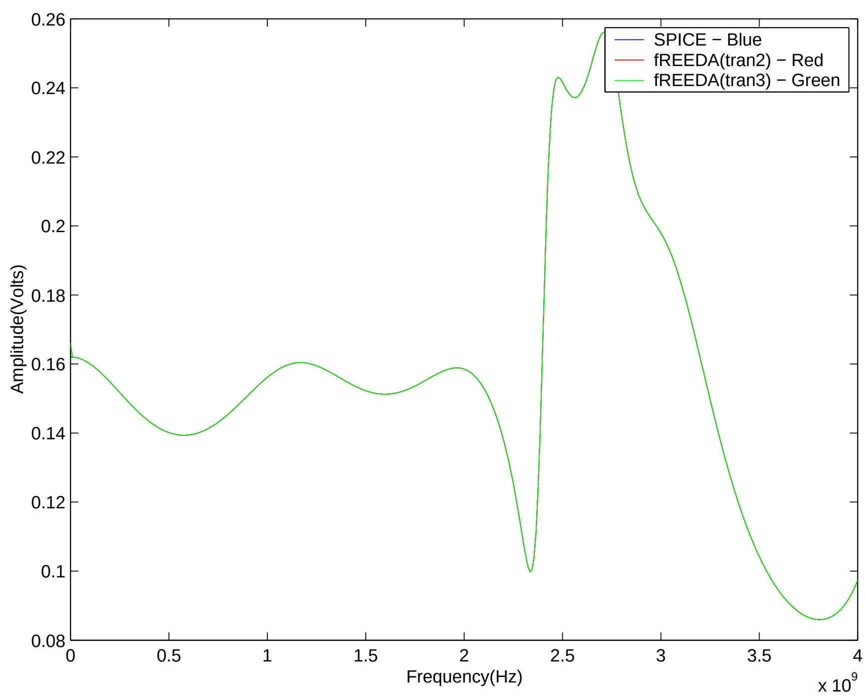

Figure 4.7: AC analysis comparison at port 6.

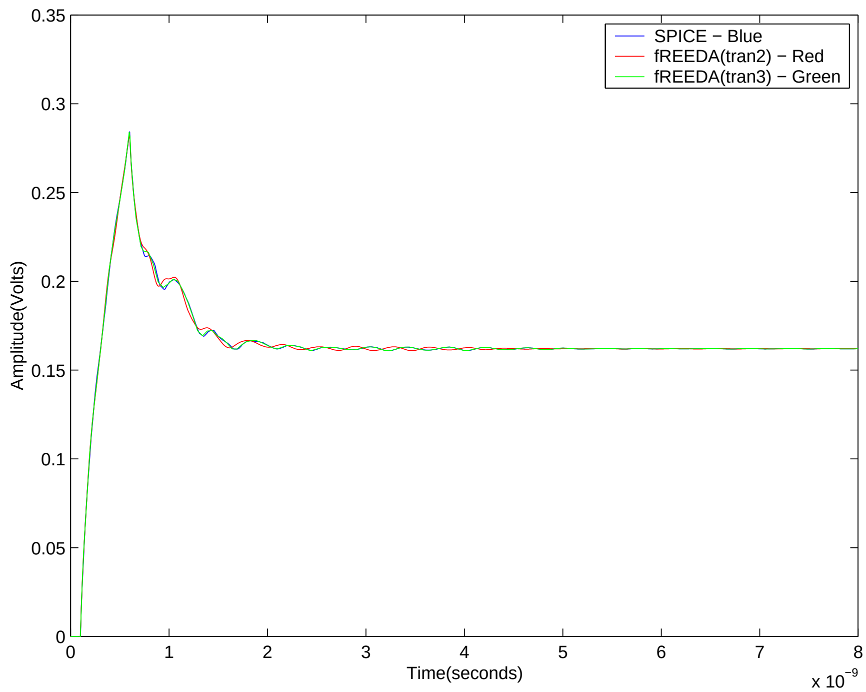

Figure 4.8: Transient analysis comparison at port 1.

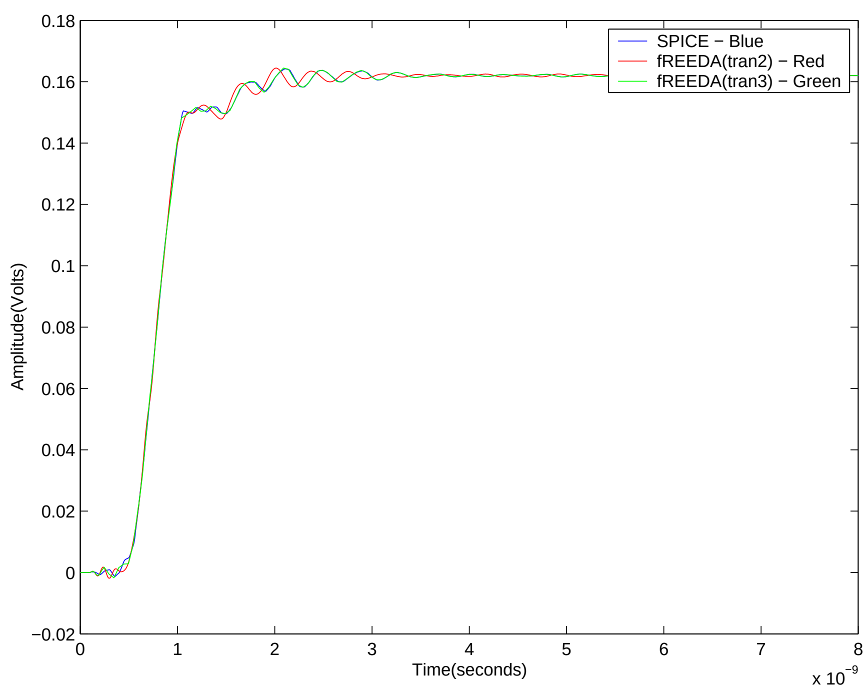

Figure 4.9: Transient analysis comparison at port 2.

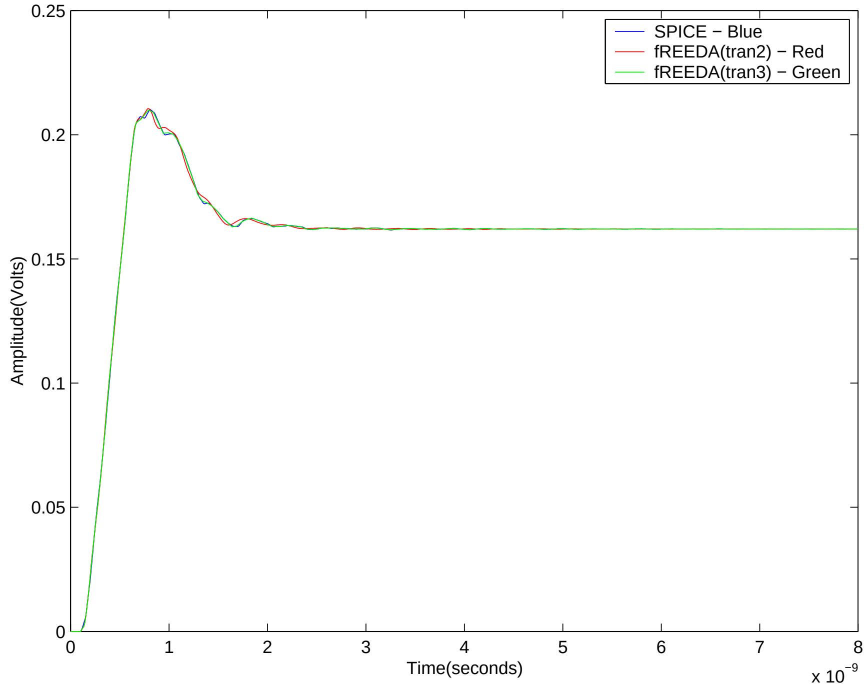

Figure 4.10: Transient analysis comparison at port 3.

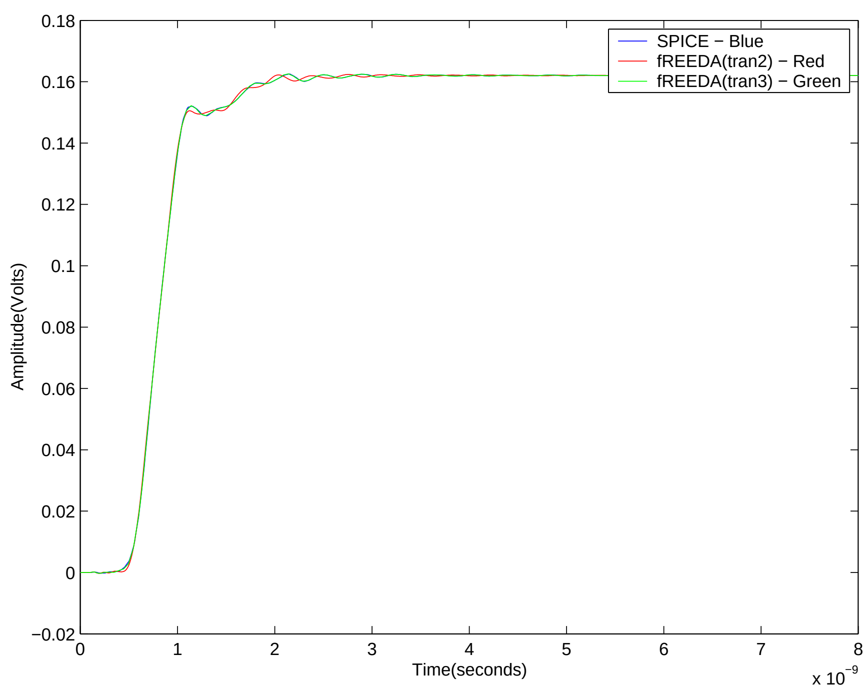

Figure 4.11: Transient analysis comparison at port 4.

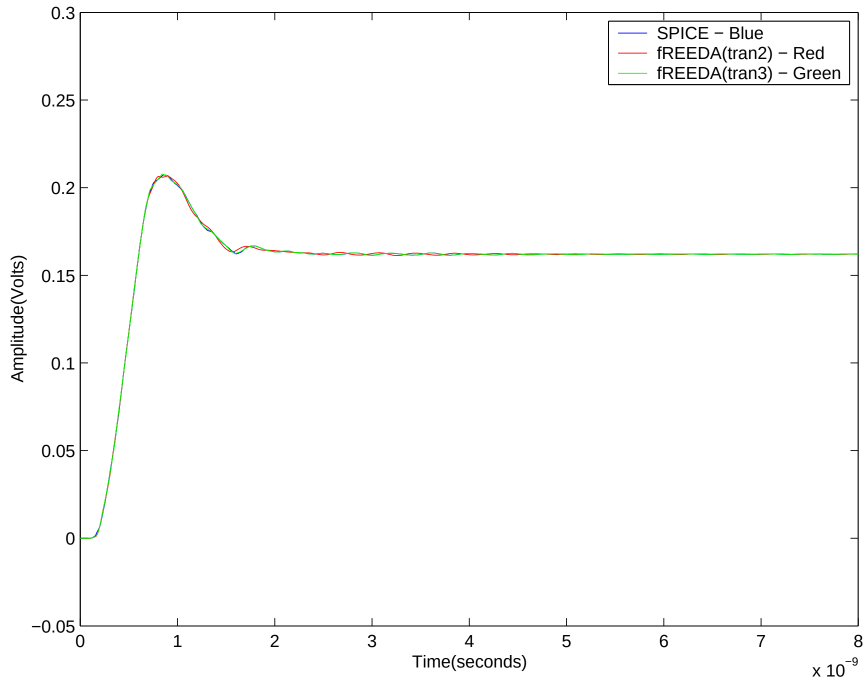

Figure 4.12: Transient analysis comparison at port 5.

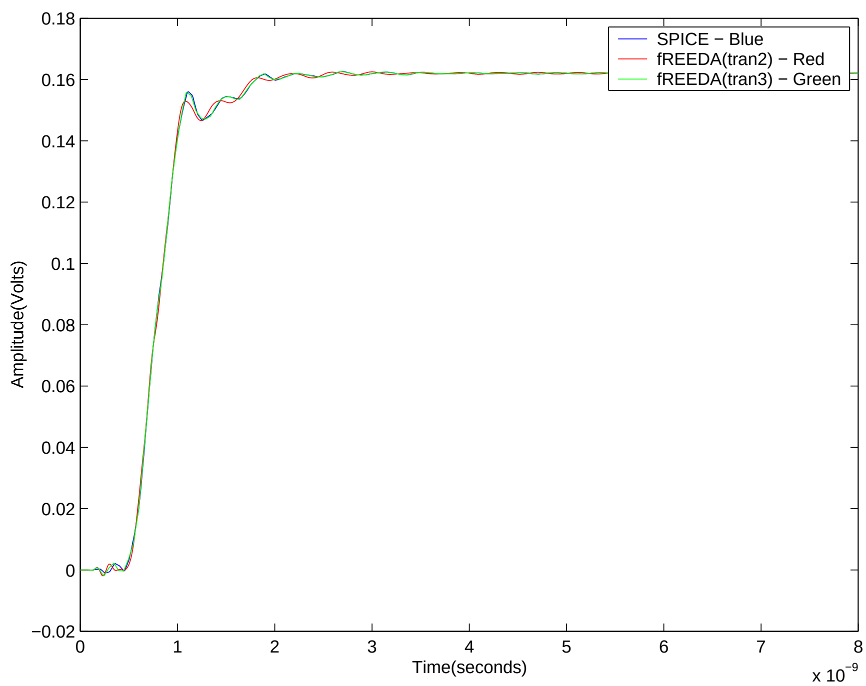

Figure 4.13: Transient analysis comparison at port 6.

Chapter 5

Conclusion

5.1 Conclusion

Foster's canonical representation of the transfer characteristic of linear systems was developed as it is the key to the causal, fully convergent, incorporation of distributed structures in transient circuit simulators.

The main conclusion of this $Thesis$ are:

- The model developed here is passive and hence is stable.
- $Pole - Residual$ method followed here gives excellent numerical stability.
- The method serves as a good choice when robustness is given importance, as it facilitates the direct synthesis of an equivalent circuit representation of the power distribution networks and hence do not slow down the analysis.

This form is guaranteed to be causal and circumvents the main problem in implementing reduced order models of distributed structures including interconnects and electromagnetic modeled spatially distributed structures.

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