ABSTRACT

MOUZZON, MELINDA PETREE. The Effectiveness of Cooperative Learning Strategies in Helping Low Achieving Students Master Systems of Linear Equations. (Under the direction of Dr. Lee V. Stiff.)

The effectiveness of a cooperative learning approach in the teaching and learning of systems of linear equations was the focus of this study. The study analyzed the effects of the mode of instruction over a ten-day study on students' ability to solve systems of linear equations. The students involved in this study were all enrolled in a high school Technical Math course. Students were separated into two groups. In the Test Group, seven of the ten days were devoted were devoted to learning experiences that implemented cooperative learning strategies to facilitate mastery of systems of equations. During the same seven days, the two Control Groups received normal instruction and only used cooperative learning groups for the Cooperative Assessment. The fourth day of the unit was devoted to reviewing solutions to systems of linear equations and for students to complete an individual quiz on graphical methods for solving systems. The ninth day was a review of the concepts discussed in the unit including a Cooperative Assessment and the tenth day was used to administer the unit test. Both the Cooperative Assessment and Unit Test were graded according to an established rubric of guidelines for mastering solving systems of linear equations. A two-sample test indicated that the Test Group scored significantly higher than the Control Groups on the Cooperative Assessment. One of the Control groups outperformed both the other Control group and the Test Group on the Unit Test. An analysis revealed that tenth graders outperformed twelfth graders and Hispanic students surpassed African-American students.

The Effectiveness of Cooperative Learning Strategies in Helping Low-Achieving Students Master Systems of Linear Equations

by

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DEDICATION

Completing the course work required for this thesis was made possible with the support of my dearest friend, Mrs. Darlene Jeffreys. I can never say thank you enough for the care you gave to Lavonte' and Melicia, the joys of my life, during my evening classes. I would like to give my heartfelt appreciation to my husband and best friend, Mr. Lawrence L. Mouzzon, for his patience and support throughout this monumental journey. This thesis is dedicated to my husband, Lawrence and my lifelong friend, Darlene.

BIOGRAPHY

For thirteen years, teaching high school mathematics and coaching varsity women's basketball were my love and my passion. Advancing my career as an educator was seemingly an impossible dream because of my numerous personal responsibilities and other interest. Despite a very demanding schedule, I successfully completed the National Board Certification process in Adolescent and Young Adult Mathematics in 1997. The many challenges this strenuous process entailed helped me gain much insight into my teaching style as well as how my instruction affects the learning of my students. Though I wanted to earn an advanced degree, giving up my love for coaching was the most significant obstacle to pursuing this life-long goal. However, my commitment to maintain high standards for my own expertise and knowledge of mathematics compelled me to seek a Master's Degree in Mathematics Education from my alma mater. I am most grateful for the learning opportunities afforded to me and for the knowledge gained during this three and a half year educational experience in the Graduate School of North Carolina State University.

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CHAPTER 1
THE PROBLEM

Introduction

The learning environment in mathematics classrooms typically vary according to the teaching style of the instructor. “Educators should understand that the content of the mathematics curriculum and the instructional methods impact on each other [when] instructional style requires students to do, think, discuss, and interact” (Driscoll, p. 57). Teachers who are skilled in promoting risk-taking in their classroom structure typically yield students who easily adapt to changes in instructional presentation and level of difficulty in course content. Teachers who are inept in facilitating learning through a variety of formats are more likely to produce students who are unwilling or incapable of adjusting to variations in curriculum and learning experiences. “Cooperative learning methods, in which students work in small, heterogeneous learning groups and are rewarded based on the learning performance of the group members, have been found (in several dozen field experiments) to increase student achievement” (Slavin, Leavey, and Madden, 1984, p. 411).

Many mathematical concepts are difficult for low-achieving students to master and understand. In particular, graphing and solving systems of linear equations are problematic concepts for students in Technical Math courses to consistently grasp. “A major concern among mathematics educators, in addition to promoting students’ activeness, is meeting low-achieving students’ needs for help in the course of learning mathematics” (Leikin and Zaslavsky, 1997, p. 334). Therefore, high school mathematics instruction must be presented in ways that help lower achieving students to master these critical concepts. Leikin and Zaslavsky (1997) argue that the focus of cooperative learning must be to improve task-related interactions that promote learning. Using cooperative learning strategies can aid teachers in increasing students comfort level with graphing linear equations and solving systems of linear equations.

The lessons designed for inclusion in this study encouraged students to learn the terminology and procedures related to solving systems of linear equations as well as the usefulness of different ways to solve systems. The purpose of this study was to determine the effectiveness of cooperative learning strategies on students' mastery of solving systems of linear equations. Moreover, the study analyzed student results on a Cooperative Assessment and a traditional Unit Test on systems of equation.

CHAPTER II
REVIEW OF THE LITERATURE

Cooperative Learning Approach

Cooperative learning strategies can be incorporated into instruction in a variety of ways. Often, teachers place students into cooperative groups to facilitate learning but the results do no always make for a successful learning experience. Therefore, organizing students into groups is not sufficient to develop a cooperative learning approach to instruction. The teacher must purposefully design task in which all participants are actively engaged in the learning process, encourage the exchange of ideas about the task, and opportunities to share these thoughts with the class. Leikin and Zaslavsky (1999) believe a [cooperative] learning environment must offer all members of the group an equal opportunity to interact with one another while encouraging them to communicate their ideas in various ways. Establishing guidelines for how students are to participate during the learning experience is just as important as the tasks they are to complete and often determines the effectiveness of collaboration in mathematics.

Cooperative learning experiences require more thoughtful and goal specific planning than the traditional lecturing presentation of mathematical concepts. A student increases his or her understanding the more frequently they are required to explain their solutions to others (Weissglass, 1993). In order for students to learn from each other, the instructor must be knowledgeable of facilitating instruction in a role where instruction is often indirect and student-led. Research indicates “The way in which the learning material is presented to the students and the way in which a teacher communicates with students during the group work influence students’ learning interactions” (Leikin and

Zaslavsky, 1999, p. 245). Learning in small groups enables students to ask task specific questions and to receive more individualized assistance from their peers as well as the instructor. More importantly, the size of the group is not as significant as the interactions amongst the members and the task designed to meet specific learning objectives. A carefully designed small group format for instruction can provide appropriate assistance when students have questions (Leikin and Zaslavsky, 1997). The educational benefits of small groups in a cooperative learning environment are more meaningful student interactions.

"While learning mathematics in some cooperative-learning settings, students often improve their problem-solving abilities, solve more abstract mathematical problems, and develop their mathematical understanding". (Leikin and Zaslavsky, 1999, p. 245). The cooperative learning activities included in this study were purposefully designed to solicit meaningful collaboration and discussion within each group. Weissglass (1993) argues that mathematics teachers must provide opportunities for more students to expand comprehension and to improve their skill in communicating about mathematics through the use of effective, small group settings and other alternatives to lecturing. Students must collaborate on mathematical tasks which often require extensive verbal dialogue with their classmates (Shachar and Sharan, 1994). Likewise, the reasons for verbalizing processes or solutions are more important than the process of verbalizing a solution in itself (Webb, 1982). Besides improving students' collaboration within their groups, increased understandings of the mathematical concepts as well as more accountability for learning are viable goals for students in the cooperative learning mode of instruction. Slavin's research (1988) found students took more responsibility for their and their classmates learning as well as improved their understanding of mathematical concepts and applications when taught in a cooperative learning setting.

Heterogeneous Group Interaction

To provide the best cooperative learning environment, students should be placed into heterogeneous groups of varying ability levels. The study completed by Leikin and Zaslavsky (1999) stated the structure of a cooperative group is determined by both the number of participants in the group as well as the ability level of the students of the group. Therefore some attention must be given to the organization of the cooperative learning groups to account for the varying abilities of the students in a high school classroom. Students of higher ability prefer to work in homogeneous groups whereas students of lower ability prefer to work with students who can help them during the learning process (Leikin and Zaslavsky, 1999). However, Linchevski and Kutscher (1998) determined that the decrease in achievement of students of higher ability was minimal whereas the increase in achievement of average and lower achieving students was quite significant in heterogeneous group settings. The goal to improve students understanding and mastery of systems of linear equations would be more effectively accomplished by heterogeneous grouping.

Leikin and Zaslavsky (1997) research study claimed to improve the communication of lower achieving students in the mathematics classroom by using small, cooperative learning groups in which students exchange knowledge. In addition to grouping students by varying ability, the task must require thoughtful discussion of meaningful mathematics. Furthermore, heterogeneous groups followed by homogeneous groups for high achieving students provide the most powerful interaction of student dialogue and explanations (Leikin and Zaslavsky, 1997). In this study, heterogeneous grouping was used initially to facilitate increased understanding for the students of lower ability. However, homogeneous grouping proved more appropriate for enhancing the conceptual understanding of more capable learners. Student ability and assessment structure had the most consistent effect on group interaction (Webb, 1982). The format of the task designed and the purpose for the learning experience are important considerations when grouping students according to ability. Therefore, student ability as well as organization within groupings is an essential aspect of cooperative learning.

Mastery Level of Learning

How to determine when a student has mastered a concept is often as challenging as developing an assessment to examine what a student has learned. Numerous studies attest to students recalling and applying more of what they have taught to others than what they have heard, seen, or even practiced. Therefore, more opportunities for students to share their knowledge, understanding, and even questions about mathematics will improve their mastering of learning objectives. Assessments of student learning are challenging in all fields of education, but especially difficult for the various levels of mathematics. Standardized test have been developed for college prepatory courses including Geometry, Algebra I and Algebra II to assess student mastery. Although the concepts taught in Technical Math courses are not assessed using a standardized test, determining the level of student learning is not easy. Webb (1982) believed students who provide explanations of how to complete a task show higher achievement than students who did not actively engage in group interaction, even when the groups were homogeneous in ability level. Meaningful discussion is a vital aspect of the learning and mastering of mathematics. Furthermore, receiving and giving help have a positive correlation on student achievement whereas off-task or passive behaviors negatively affect achievement (Webb, 1982). Working in cooperative groups should encourage students to discuss their results, provide a unified solution and a descriptive procedure for each problem. However, test and quizzes are still the primary form of assessment used to measure student learning and constitutes the largest percentage of the student's grade in a course (Senk, Beckmann, and Thompson, 1997). Assessments that are ongoing and focused on student learning are powerful for promoting success in mathematics.

Methods for Solving Systems of Linear Equations

There are three well-researched methods for solving systems of linear equations: graphical, substitution, and elimination methods. Driscoll and Moyer (2001) urge teachers to "look for activities that allow for multiple approaches as well as permitting extensions to related kinds of algebraic thinking" (p. 284). Introducing systems of linear equations using a graphical approach addresses both visual and kinesthetic learning styles. Using this method to introduce students to systems of linear equations reinforces the importance of understanding the graphs of linear equations while relating the concept of determining a solution to a system. The National Council of Teachers of Mathematics (NCTM, 2000) indicates that students should understand the meaning of and be able to write equivalent forms of expressions, equations, inequalities, and relations. Learning to solve systems by substitution requires students to understand equivalent forms of equations to determine an appropriate solution. In addition, the North Carolina Department of Instruction (NCDPI, 2003) identifies using algebraic expressions and linear functions to model and solve problems as prerequisite skills for students enrolled in Technical Math I. Solving systems of linear equations require students to understand linear functions and models as well as to apply them to find the solution.

Teaching students to solve a system of equations using a graphing calculator in addition to graphing them by hand is advantageous to developing a more complete understanding of this concept. Maddox (1984) emphasized graphical methods should be mastered not in place of or independent from analytical methods, but to provide meaning and interest to them. However, some linear equations are not easily graphed on a graphing calculator such as equations of the form $x = a$, where "a" is a constant. These graphs are vertical lines and can not be graphed in standard form on a graphing calculator because they are typically expressed as y as a function of x. This type of linear equation is a catalyst for helping students appreciate other methods for solving systems of linear equations besides using the calculator, including graphing by hand and substitution. NCTM (2000) standards indicate that students should be able to judge the meaning, utility, and reasonableness of symbol manipulations, including those carried out by technology.

Using a system of equations in which all of the coefficients are not one, is a good motivation for students to learn the elimination method for solving systems of equations as opposed to the other forms. Expressing these equations in the form y as a function of x is both time-consuming and tedious which is a requirement for both of the methods, graphing and substitution. Maddox (1984) argued that monotonous arithmetic is a sure way to lessen the value of the instruction of graphical methods in solving systems of equations. Such equations are helpful in promoting the recognition of the most appropriate method for solving a system of linear equations. The main purpose of graphical methods is to promote students visualization of and appreciation for the harmony between algebra and geometry (Maddox, 1984). More importantly, cooperative learning strategies should encourage students to not only solve various systems of linear equations but to justify their method for solving as well. Maddox emphasizes developing opportunities for students to recognize the usefulness of the intercepts over slope-intercept for graphing systems of equations expressed as Ax + By = C. Exposing students to graphical methods can improve their understanding of solving systems while developing an appreciation for various methods.

CHAPTER III
THE METHOD

The purpose of the study was to determine the effectiveness of cooperative learning strategies on students' mastery of solving systems of linear equations. The variables involved in the study were student performance on a Cooperative Assessment and Unit Test on systems of linear equations. Student mastery was measured by the performance of their group on an assessment given at the completion of the study as well as an individual unit test on the concepts discussed.

The null hypotheses were as follows:

Null Hypothesis 1: There is no significant relationship between the students' performance on the Unit Test and their grade on the Cooperative Assessment given at the completion of the unit on systems of linear equations.

Null Hypothesis 2: There is no significant difference between students' mastery of solving systems of linear equations in a cooperative learning environment to those in a traditional lecture format.

Null Hypothesis 3: Student performance on the Cooperative Assessment is unaffected by ethnicity, gender, grade level, or socioeconomic status.

<u>Subjects</u>

The target population was low-achieving students who have habitually experienced difficulty with succeeding in mathematics. The accessible population included in this study was three different classes of students currently enrolled in a high school Technical Math course. The purpose of this course is two-fold in scope. One goal was to develop students' knowledge of the skills required for a career in a technical field of study while another was to refine students' basic and analytical skills required for completing geometry. The instructors agreed that the dual-nature of this course makes it extremely difficult to meet the instructional needs of all students. More importantly, these students are traditionally among the lowest achieving students in the school and least prepared for success in mathematics. Another important attribute of the classes in this study are the varying grade levels within each class. The three classes were separated: one class was the Test Group and the other two classes were the Control Groups. Students in the Test Group ranged in grade levels from 9th to 11th. Students in the Control Groups ranged in grade levels from 9th to 12th. The mathematical abilities of the students in these classes showed very similar patterns despite the difference in ages.

The Test Group was one of the two classes of students enrolled in the researcher's Technical Math I classes. The class chosen to be the experimental group was selected because it had the lowest class average after the first quarter despite having the same student enrollment as Control Group 1. There were 22 students enrolled in the class used as the Test Group at the start of the study. The class size typically has a tremendous effect on the success for low ability students in mathematics. Students who are weak in mathematical skills often give up in whole class settings, but will continue working in a cooperative group (Felder and Brent, 1994). Furthermore, the Test Group was more evenly distributed by gender with 12 males and 10 females. Felder and Brent believe female ideas and contributions are often undervalued or overlooked in mixed gender groups and females often take passive roles in group interactions in which they are outnumbered (1994). Control Group 1, also had 22 students, of which 13 were males and

9 where females. Moreover, Control Group 2 had only 20 students, 16 males and only 4 females. The researcher believed it would be more difficult to form groups without gender bias in the two Control Groups because of the lower number of females in these classes.

Prior to the study, the students had completed the first quarter of the Technical Math course. The number of students and their first quarter average were used to determine how to form heterogeneous groups of approximately the size. To facilitate meaningful discussion and cohesiveness within the Test Group, the students were divided into five groups, Groups A - D with four students and Group E with five students. Each group was purposefully created to include one student who was performing well in the course and one student who was struggling in the course. The other two students or in Group E, three students, varied in ability but were not considered the strongest nor the weakest in ability in their group. The level of success in the course was based on the students test average and overall grade in the course at the time of the study. The test average was included as a second measure of success in the course because fifty percent of a student's overall grade in this course is determined by his or her test performance.

Control Groups 1 and 2 each contained twenty-two students. The Test Group and Control Group 1 were taught by the researcher; Control Group 2 was taught by another experienced teacher. Control Group 1 and 2 each had six students on free or reduced lunch. The control groups consisted of students of similar age and ability levels as the Test Group. The teacher of the Test Group and Control Group 1 has sixteen years of teaching experience, is National Board Certified in Adolescent and Young Adult

Mathematics, and currently pursuing a Master of Science Degree in Mathematics Education. The teacher of Control Group 2 has nineteen years of teaching experience and holds a Master of Arts Degree in Mathematics Education. Both teachers currently participate in a professional learning community of teachers providing quality instruction in Technical Math courses. The researcher believed that the teaching experience of the instructors would not have an effect on the results of the study.

It is noteworthy to state that little consideration was given to the ethnicity of group members when developing the heterogeneous groups used for this study. A study conducted by Shachar and Sholomo (1994) found using cooperative learning strategies were more effective in improving student learning despite ethnic group. Therefore more attention was devoted to devising groups of varying ability as opposed to diverse ethnicity. However, Webb (1982) found “multiracial groups tended to inhibit the participation of minority students, but this effect was overcome by manipulating students’ expectations about each others’ competence” (p. 438). Grouping students without regards to race would support the hypothesis that student performance on a Cooperative Assessment is unaffected by the ethnicity of the participants.

Treatment

The unit on systems of linear equations was established to take a maximum of two weeks or ten school days for completion, two of which would be devoted to review and a unit test. Understanding equivalent forms of expressions was a prerequisite skill reviewed through simplifying like terms for this unit as well as an essential part of solving systems using the substitution method. The cooperative learning activities in this unit on systems of linear equations employ these skills as a part of the standard course of study for this class. All three methods, graphical, substitution, and elimination methods, are included in the lessons designed for this study to facilitate students understanding of the appropriateness and limitations of the method chosen to solve different systems. The students were taught how to graph systems of linear equations by hand on a coordinate plane and then a follow-up lesson was provided on using a calculator to graph and solve a system of equations. The lessons following solving a system by graphing encouraged students to recognize the relationship between the algebraic method of solving systems and the graphical method by using the calculator to verify their solutions. Using multiple representations help students successfully convey and solve word problems as well as translate problems into tables and graphs (Brenner, Mayer, Mosely, Brar, Durán, Reed, and Webb, 1997). Detailed explanations and multiple practice opportunities were provided for students to master the use of the graphing calculator in solving systems of linear equations in connection with the algebraic methods. The lessons designed for the Test Group connected the graphical method of solving systems of linear equations to the analytical methods discussed thereafter.

Only eight of the days were devoted to instruction of which seven of the learning experiences in the Test Group implemented cooperative learning strategies to facilitate student mastery of the concepts taught. The activities include in the Test Group promoted learning a variety of methods to solve systems of linear equations as well as to make connections between these procedures. Cooperative learning strategies were employed to review a variety of prerequisite skills including identifying equivalent expressions and graphing linear equations. In addition, newly described concepts such as the methods for solving systems of equations and determining the solution of a system was introduced using the cooperative learning. To insure all students were held accountable for learning within their cooperative group, consensus on the answers and process to each problem was a requirement for the cooperative learning activities. Furthermore, justifications for a chosen method of simplifying or solving a problem had to be discussed with the members in each group prior to presenting this information to the entire class.

Day four in the Test Group and Control Group 1, which were taught by the researcher, was designated as a day to review solutions to systems of linear equations and for students to complete an individual quiz on using graphing to solve systems. Control Group 2 was also given a quiz during this unit. However, it was on a different day and covered more concepts than graphical methods of solving systems of linear equations. Therefore, the results on these quizzes were not compared. Table 1 on the following page highlights the mode of instruction used for each lesson in the unit on systems of linear equations.

Table 1
Mode of Instruction Used for Teaching Test Group Unit on Systems of Linear Equations

Day of Week	Mode of Instruction	Lesson Topic
1	Cooperative	Solutions to Systems of Linear Equations
2	Cooperative/Discovery	Graphical Methods for Solving Systems of Linear Equations
3	Cooperative	Using Graphing Calculator to Solve Systems of Linear Equations
4	Traditional	Review of Solutions to Systems of Linear Equations and Quiz on Graphical Methods
5	Cooperative	Solving Systems of Linear Equations Using Substitution
6	Cooperative	Solving Systems of Linear Equations Using Elimination
7	Cooperative	Summary of Methods for Solving Systems of Linear Equations
8	Cooperative/Discovery	Problem Solving Using Systems of Linear Equations
9	Cooperative	Review for Test on Systems of Linear Equations
10	Traditional	Unit 5 Test on Systems of Linear Equations

A Cooperative Assessment was given at the completion of the unit in which all three classes were organized into groups of four or five students. The purpose of which was to determine if the mode of teaching influenced the students' ability to solve systems of linear equations in a cooperative group. During the Cooperative Assessment, the teacher of the Test Group and Control Group 1 purposefully assigned students of varying ability to small groups to determine the effectiveness of heterogeneous grouping on mastering systems of linear equations. However, the students in the Test Group were rearranged so that the groups were different from the ones used throughout the learning experiences during the unit on systems of linear equations. The main reason for this modification was to eliminate the effect familiarity of the group members may have had on the collaborative effort in the Test Group when compared to the two Control Groups. Since the Cooperative Assessment required students to discuss each question, form a unified response to each problem and submit a single activity sheet representative of their group's solutions, it was important to make the testing environment as similar as possible to that of the two Control Groups for more reliable results. The instructors of the three Technical Math classes established 80% as the minimum score for students to master the concepts included in the Cooperative Assessment and the Unit Test. Though 70% represents a passing score in this course, this is not considered to show mastery of the material presented.

In addition, the Unit Test of each student in the Test Group were averaged within their group and compared to the two Control Groups test and overall averages, respectively. The Unit Test was completed individually without any collaboration or discussion. The results on this test would reveal any differences in students' individual performance in comparison to their collaborative efforts.

The 42 students in the two Control Groups were taught the same concepts as the students in the Test Group but a traditional lecture and practice format was the main mode of instruction throughout the unit. Two different teachers of comparable abilities and years of teaching experience were used in the study to offset the possibility that the teacher's pedagogical knowledge may be the cause of the difference in the Test Group and the two Control Groups. However, note that the teacher of Control Group 2 did not use any specific grouping strategy to conduct the Cooperative Assessment. In contrast, the cooperative groups formed in Control Group 1 were organized heterogeneously in a similar fashion as the Test Group for this assessment.

Statistics

A two sample t-test was used to assess any difference between the groups' performance on the Unit Test and Cooperative Assessment. An analysis of variance test was used to determine if there was any significance between students' performance on the Unit Test and Cooperative Assessment between the Test Group and the two Control Groups. Finally, an analysis of variance test was used to determine if ethnicity, gender, grade level, or socioeconomic status significantly affected student performance on the Cooperative Assessment.

CHAPTER IV
RESULTS AND DISCUSSION

Results

There were 22 students in the Test Group and 42 in the two Control Groups. 20 of the students in the Test Group and all of the students in the Control Groups were present to complete the Cooperative Assessment whereas all of the students completed the Unit Test. The two sample t-test verified that the students in the test group performed better on the Cooperative Assessment in comparison to the two control groups with an average of 80%, a t-statistic of 2.1554 and a p-value of 0.0252. The results are highlighted in Table 2 below.

Table 2
Comparison of Scores on Cooperative Assessment by Group

Cooperative Group	Test Group	Control Group 1	Control Group 2	Two-Sample Test on Cooperative Assessment
A	73	82	72	t = 2.1554
B	93	64	26	p-value = 0.0252
C	62	38	45	df = 13
D	83	65	30	Test Group Mean. = 79.6
E	95	89	68	Control Groups Mean = 58.0
Total Students	20	22	20	62
Mean Score	79.6	66.9	48.2	

There were no significant differences between student performance on the Unit Test though the students in the Test Group with an average score of 64% scored lower on the individual Unit Test in comparison to Control Groups 1 and 2 with a combined average score of 69%, with a t-statistic of -1.2601 and a p-value of 0.8851. Table 3 on the next page shows the details of the comparison of the Unit Test scores for each group.

Table 3
Comparison of Unit Test Scores by Group

Cooperative Group	Test Group	Control Group 1	Control Group 2	Two-Sample Test on Unit Test Scores by Group
A	61.25	71.6	78.75	t = -1.2601
B	73.25	54	80.5	p-value = 0.8851
C	61.5	70.25	68	df = 13
D	56.5	64.25	81.333	Test Group Mean. = 63.7
E	65.8	59.25	66	Control Groups Mean = 69.4
Total Students	20	22	20	62
Mean Score	63.7	63.9	74.9	

Results from the analysis of variance test showed several interesting differences between the Test Group and Control Groups when comparing scores by students' ethnicity, gender, grade level, and socioeconomic status. Hispanic students performed significantly better when compared to African-American in all three groups with a p-value of 0.0274. Table 4 below shows the results of these comparisons.

Table 4
Results on Cooperative Assessment by Ethnicity

Ethnic Groups	Number of Students	Test Group Mean Scores	Number of Students	Control Group 1 Mean Scores	Number of Students	Control Group 2 Mean Scores	Total Students by Ethnic Group	Ethnic Group Mean Score
Black (B)	12	77.1	13	58.1	13	52.2	38	57.1
Hispanic (H)	4	94	3	78.3	2	47	9	73
White (W)	4	72.5	5	83.4	5	41.2	14	65.6
Multi (M)	0	n/a	1	65	0	n/a	1	65
Totals	20	79.6	22	66.9	20	48.2	62	_____

ANOVA test Results	Groups Compared					
	B vs H	B vs W	B vs M	H vs W	All Four Groups	
p-value	0.0274	0.7773	0.5516	0.1121	p-value	0.0824
f-stat	2.2785	0.2844	5.178	1.6582	f-stat	2.8452

The ANOVA test comparing results on the Cooperative Assessment based on gender revealed a significantly higher average for females with a p-value of 0.0193 and a mean score of 74%. The difference in the average scores for the females in the Test Group did not vary as much as the differences in the females' averages in the two Control Groups when compared to the males in each class. Table 5 provides details of these comparisons.

Table 5
Results on Cooperative Assessment by Gender

Gender	Number of Students	Test Group	Number of Students	Control Group 1	Number of Students	Control Group 2	Mean Scores	Total Students
Female	9	79.7	8	69.6	4	69.0	73.8	21
Male	11	79.5	14	65.4	16	43.9	60.8	41
Totals	20	79.6	22	66.9	20	48.2	——	62

ANOVA test by Gender

Treatments	1	2357.41493	2357.41493	p-value	0.019292
Error	60	24462.2625	407.704375	f-stat	5.782167
Total	61	26819.67743			

The ANOVA test comparing results on the Cooperative Assessment by grade levels showed the students' performance varied significantly according to their grade level with a p-value of 0.0385. The most significantly different results occurred when comparing the tenth graders to the twelfth graders, which had a p-value of 0.0207. Surprisingly, the twelfth graders and the ninth graders had the lowest averages of the four grade levels with mean scores of 43% and 49%, respectively. The tenth graders had the highest average of the grade levels with a mean score of 71%. The details of these comparisons are shown in Table 6 on the following page.

Table 6

Results on Cooperative Assessment by Grade Level

Grade Levels	Number of Students	Test Group	Number of Students	Control Group 1	Number of Students	Control Group 2	Total Students	Mean Scores
9th	0	n/a	0	n/a	2	49	2	49
10th	15	79.7	13	70.7	8	53.75	36	70.683333
11th	5	79.0	7	64.4	9	47.2	21	60.5047619
12th	0	n/a	2	51	1	26	2	42.666667
Totals	20	79.6	22	66.9	20	48.2	62	n/a

ANOVA test Results	Grade Level Comparisons						
	9 vs 10	9 vs 11	9 vs 12	10 vs 11	10 vs 12	11 vs 12	All four
p-value	0.1356	0.5382	0.775	0.0683	0.0207	0.1781	0.0385
f-stat	2.3379	0.47127	0.3127	1.8598	2.4176	1.3912	2.3633

The ANOVA test comparing students' performance based on socioeconomic status showed no significant difference as proposed by third null hypothesis. Therefore, students performed comparably on the Cooperative Assessment regardless of whether or not they where receiving free or reduced lunch. The averages are almost identical as shown in Table 7 below.

Table 7

Results on Cooperative Assessment by Socioeconomic Status (SES)

SES	Number of Students	Test Group	Number of Students	Control Group 1	Number of Students	Control Group 2	Mean Score
Free/Reduced (FR)	8	79.7	5	67.6	4	43.5	67.6
None (N)	12	79.5	17	66.7	16	50.375	64.3
Totals	20	79.6	22	66.9	20	48.2	------

Total Students By SES		ANOVA test by Socioeconomic Status					
Free/Reduced (FR)	17	Treatments	1	50.653595	50.653595	p-value	0.7394
None (N)	45	Error	60	27209.346	453.48911	f-stat	.1117
Total	62	Total	61	27259.996			

Discussions

The 20 students in the Test Group had the highest average on the Cooperative Assessment with a mean score of 80%. This is significantly higher than the mean score of the two Control Groups of 58%. The fact that the Test Group performed higher on a cooperative learning assignment is not as surprising as the similarity between the students' in the control groups poor performance on the Cooperative Assessment with averages of 68% and 48%, respectively. Though all of the classes discussed warm-up problems and often shared their thoughts and ideas with the whole class, the two Control Groups struggled with completing task in which they had to provide a unified solution and descriptive explanations for their group's findings. The fact that the students did not perform well in either control group is powerful evidence that the mode of instruction is the more important factor in student performance on Cooperative Assessment than the teacher involved.

It is important to note that only three of the cooperative groups, B, D and E in Test Group, actually met the level of 80% for mastery and cooperative group C failed the assignment. It is noteworthy to point out, cooperative group C, also had the most members with five. The larger group size may not be as significant a factor to this lower score as one of the members enrolling in the course during the weeks in which this study was conducted. The larger group size may not be as significant factor to this lower score as the additional member enrolling in the course during the weeks in which this study was conducted. The time for this new student to become familiar with class expectations neither sufficient nor was she acquainted with the other students prior to this two week study. Furthermore, only three of the members of cooperative group E of the Test Group were present to complete the Cooperative Assessment. However, they had the best performance on the Cooperative Assessment with a score of 95%. The two previous statements show group size as a contributing factor to students' success on a cooperative learning activity.

More importantly, the grading of the Cooperative Assessment may have some effect on the difference in the students' performance on the grouped assessment in comparison to the results on the Unit Test. The researcher used an established grading rubric to score all of the Cooperative Assessments. One issue recognized with the grading of the Cooperative Assessment was that several groups were penalized for not using the method stated in the instructions. For example, cooperative group A in Control Group 1 lost four points for failing to use the method stated to solve a given system, which effectively dropped their grade by eight percentage points. Furthermore, cooperative group E in the Test Group did not attempt to solve any of the systems requiring the elimination method. More significantly, the majority of the cooperative groups in Control Group 2 used graphical methods to solve all of the systems of linear equations and thus lost numerous points for not using the method specified in the instructions. When this was discussed with the other instructor, students had been allowed to solve each equation for y and graph the two equations to locate the point of intersection. This accounts for some of the lack of evidence of the students in Control Group 2 mastering the other methods of solving systems of linear equations on the Cooperative Assessment.

In addition, the majority of the cooperative groups in the two Control Groups expressed concerns with not completing the assessment though they were given the entire class period, if needed. The reality that they had not worked together in groups on a common assessment may be a primary cause for the vastly different averages in the two Control Groups in comparison to the Test Group. Several students were absent on the day the classes completed the Cooperative Assessment. Two of the students on free or reduced lunch in the Test Group did not participate in this activity, which may have influenced the results when comparing scores by socioeconomic status.

Overall, the students on free and reduced lunch averaged higher, though not significantly than those who do not receive this benefit in the Test Group and Control Group 1, but this was not the case in Control Group 2. This is an interesting observation since Control Group 2 actually had the lowest number of students on free or reduced lunch with only four but averaged only 44% whereas the Test Group had the most with eight but their average was the highest of all groups at 80%. It is noteworthy to say that the performance of students of higher socioeconomic status where comparable in all three groups though the cooperative groups in the Test Group still had the highest averages.

The fact that the twelfth and nine graders scored the lowest on average on the Cooperative Assessment leads to question whether their abilities are the main reason for this lack of performance. The twelfth graders have the most experience in the high school setting whereas the ninth graders have the least, but they both scored significantly lower than the other two grade levels on average. It is also noticeable that these are also the two smallest groups of students in the three classes with only two ninth graders and three twelfth graders. Therefore, a closer look at specific skill levels may be a valuable topic of research for these two grade levels in particular.

Considering the number of females in the three groups is almost half the number of males, suggest there is some value in comparing the results based on gender. Since the females averaged 73.8% on the Cooperative Assessment while the males averaged on 60.8%, prompts more research into whether student ability or cooperative skills have more influence on a collaborative assessment. The results do not indicate whether it was the collective nature of the assessment or whether stronger mathematical skills that helped the females perform better on the Cooperative Assessment. Comparing the test scores by gender may share some insight into this question.

Lastly, Control Group 2 did not complete the assessment on the same day as the Test Group and Control Group 1 because they had completed the unit several days before the other classes had finished their instruction. In fact, Control Group 2 had already completed the unit test on systems of linear equations prior to attempting the Unit Test for this study. Though neither teacher advocates learning for the test only, the fact that Control Group 2 had already completed the unit test may have contributed to their inability to recall the skills needed to complete the Cooperative Assessment.

Since each teacher scored the Unit Test on systems of linear equations individually, this may have contributed to the Control Group 2 performing higher on average than the Test Group and Control Group 1. The researcher scored the Unit Test averages for Test Group and Control Group 1 with the expectation that understanding of the different methods for solving systems would be an indicator of students' mastery of the material in this unit. The results of the students' performance were similar for these two groups. (See Table 3 to review these results). In contrast, the teacher of Control Group 2 allowed students to use graphical methods for solving all of the systems even in the sections on substitution and elimination methods during the unit test, which may account for their higher scores on average than the other two groups.

Conclusions

More information relating to the effectiveness on teaching systems of linear equations using a cooperative learning format is needed. In addition, how teachers assess students' mastery of solving systems of linear equations is another topic of interest for further research. Senk, Beckmann, and Thompson (1997) advocate for more discussion and research on possible variations in assessment strategies that are applicable in a typical classroom. Though more teachers are using technology and alternative methods for instruction, the methods in which student learning is assessed remains widely based on paper and pencil test, quizzes, and homework assignments.

Control Group 1 and 2 performed significantly below the Test Group on the Cooperative Assessment on average with a p-value of 0.0252. This significant difference occurred despite the fact that two of the cooperative groups in Control Group 1, A and E specifically, did meet the mastery level established for the activity. Control Group 2 had the lowest mean score at 48% on the Cooperative Assessment and none of the groups in this class meet the mastery level of 80%. One reason for this difference may be the fact that the teacher of Control Group 2 did not form heterogeneous groups for the Cooperative Assessment whereas Control Group 1 and the Test Group were arranged to include students of varying ability. The two control groups expressed concern with not having time to complete the activity. Forty-five minutes was allotted for the completion of the Cooperative Assessment activity, which was thought to be enough time. To determine the effect collaboration may have had on student learning, the Unit Test average on systems of linear equations was compared for the three groups as well. The Unit Test average on systems of linear equations appears significantly lower than the students' performance on the Cooperative Assessment for the majority of the students in the Test Group and Control Group 1. The exceptions are cooperative group C of the Test Group whose performance is low on both assessments and cooperative group C of Control Group 1 who actually performed better on the Unit Test than on the Cooperative Assessment. This suggests that students' struggle with systems of linear equations still exists, even when learning in a cooperative instructional mode. In contrast, the majority of the students in Control Group 2 performed significantly better on the Unit Test in comparison with Cooperative Assessment. The only exception to this pattern of success on the individual assessment was with group D of Control Group 2 who actually scored two percentage points lower on the unit test than on their Cooperative Assessment.

The cooperative learning approach to teaching students systems of linear equations appears to have helped students learn the concepts based on their much higher results on the group activity on systems. The results on the Unit Test on systems of linear equations did not provide a similar outcome, since the Test Group scored lower on average than Control Groups 1 and 2, though not significantly. Students did not seem to perform as well on demonstrating their knowledge of systems of linear equations when asked to complete an individual assessment. The significantly lower average on the test on systems in comparison to the group activity completed by the test group is evidence to this conclusion.

The only significant difference by ethnicity in the results of measuring student learning on the Cooperative Assessment was among the Hispanic students who performed better in the Test Group than in Control Group 2 when compared to the Black students with a p-value of 0.0274. The heterogeneous grouping used by the researcher in the Test Group and Control Group 1 may be an indicator of why this difference was significant in one control group but not the other. When comparing results on the Cooperative Assessment by grade level there was a significant difference with a p-value of 0.0385 and the most substantial of the grade level differences was between the tenth and twelfth graders with a p-value of 0.0207. Moreover, the student's results on the Cooperative Assessment varied with respect to gender since the females averaged 74% on this activity while the males' average was only 61%, with a p-value of 0.0193. Comparing the results of the Unit Test by gender would be an interesting topic and would lead to discussion of whether the gap in the level of understanding of mathematics between females and males is closing. Lastly, the socioeconomic status did not affect the results of the study since the p-value of the ANOVA test is 0.7394 indicating no significant difference. Students on free or reduced lunch seemed to perform comparably to the other students on the Cooperative Assessment on average. Analyzing the Unit Test results based on socioeconomic status to determine if this pattern of comparable performance is a worthwhile objective.

In addition, comparing the unit test average on systems of linear equations to the overall average in the course to determine any significant differences in students' performance may prove beneficial to review. This will provide a stimulus for using cooperative learning strategies for teaching other mathematical concepts when students have traditional struggled in math. Research on the effectiveness of cooperative learning strategies in teaching other challenging concepts to low-achieving students is a worthwhile pursuit. Possible topics may include graphing linear equations, surface area and volume of three-dimensional objects, and quadratic functions. These are other areas in which students in Technical Math courses have difficult and may benefit from the additional structure and assistance the cooperative learning approach offers. It is imperative that some attention be given to helping students retain the knowledge gained in such a way that they can perform well on individual assessment despite having learned a mathematical concept in a cooperative environment.

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APPENDICES

Appendix A

DAY #1 Lesson Topic: Solutions to a System of Linear Equations

UNIT #5: Systems of Linear Equations

Textbook Used: ***N/A***

Required Materials: Student handouts, homework sheets, PowerPoint of lesson notes, One set of "How You Like Me Now?" game cards and one number cube for each group	**Related skills:** Identifying like terms, Adding and subtracting like terms

Lesson Objectives: To define a system of equations and to determine if an ordered pair is a solution to a system or just one equation.

Instructional Presentation: Cooperative learning/Discovery Format

NCTM Standards to Address: Interpret representations of functions of two variables; Use the language of mathematics to express mathematical ideas precisely; Understand the meaning of equivalent forms of expressions and equations

NCSCOS Objectives to Address: Use algebraic expressions to model and solve problems; define and use linear functions to model and solve problems

Targeted Audience: Students enrolled in Technical Math I course

Time Period: 90-minute blocked scheduled will be most appropriate for completing this activity in one setting

Review/Focus: (3 min.)	PowerPoint will contain the groups in which students are placed for the cooperative learning activities. Instruct students to find their name and sit with the members of their group. Pass out the handout on Systems of Linear Equations. (Groups should be heterogeneous with students of varying abilities – low, average, and high – are included in each group of 4 or 5 students.) Students will collaboratively complete the following Warm-up Activity on simplifying like terms. Simplify each expression and express in standard form. 1. $x - x^2 - 2x + 4x^2$ 2. $3x^3 - 4x^2 + 3x - x^3 + 2x^2$ 3. $-5x + 2x^2 + 3x - 4x^2 + x^3$ The lesson objectives will be stated as well as the following questions: **Guiding Question(s):** What does it mean to simplify an expression? How do you know when an expression is written in standard form?

Teacher Input: **(3 min. for instructing whole class on purpose of cooperative group and lesson objectives and 50-65 min. as needed throughout the cooperative learning activity)**	**Guiding Question(s):** Did everyone find his/her correct group and receive a handout? Give student 5 minutes to discuss the Warm-up Activity within their group and come to a consensus on the correct answers.
Guided Practice: (50-65 min. will be used for completing the various activities on the handout)	Display key word in the PowerPoint presentation: Like terms After students discuss appropriate definition of this term, ask for examples and non-examples of like terms. Give groups 3 minutes to discuss the definition of Like terms and come to a consensus with other members of their group. Inform groups to be prepared to share your definitions with the class and to reconcile any differences in their groups' definitions and the one given during the class discussion. **Guiding Question:** How can you determine whether two terms are alike? (Ask for a volunteer to share definition with the class. Determine whether other groups agree or disagree with proposed definition. Address any misconceptions in the definition before moving on to simplifying like terms) Accurate definition should be displayed in PowerPoint. Give groups 3 minutes to discuss how to simplify an expression with like terms. Remind each group to be prepared to share their reasoning with the class and to reconcile any differences in their groups' explanation and the ones given during the class discussion **Guiding Question:** What does it mean to simplify like terms? (Ask for a volunteer to share suggestion for simplifying like terms with the class. Determine whether other groups agree or disagree with proposed method. **Guiding Question:** How do you simplify an expression with more than one pair like terms? (Reconcile in misconceptions in the process proposed by the class.) Display the steps for simplifying an expression with like terms and have each student record them on their notes handout. Give groups 5 minutes to complete example 1 on identifying the like terms and simplifying each expression. Remind groups to discuss their simplified answers with the other members of their group and to come to an agreement on the correct answer to each problem. Give groups 5 minutes to discuss the following questions.

Guided Practice: (50-65 min. will be used for completing the various activities on the handout)	**Discussion questions:** When have you heard the term system in a real-life situation? Have you used the term system of equations before? What did it mean to solve an equation in the unit of study we just completed? What does it mean to solve a system of equations? (For each question, ask for a volunteer to share their group's response. Determine whether other groups agree or disagree with responses and address any misconceptions in students thinking.) Display key words in PowerPoint: System of Linear Equations and Solution to a system of Equations. Give groups 5 minutes to discuss the definitions of the terms shown. Remind them they must come to an agreement on the best definition of each term and reconcile any difference in their group's definitions and the ones provided during the class discussion. **Guiding Question:** How is a system of equations different from a linear equation? What does it mean for an ordered pair to be a solution to a system? An equation in a system? (For each definition, ask for a volunteer to share their groups definition. Determine whether other groups agree or disagree with proposed definitions. Address any misconceptions in the definitions offered and display accurate definitions in the PowerPoint after the class discussion).
Independent Practice: (10-15 min. of the activity will be used for comparing results and making conjectures about the results in each activity)	Instruct students to follow the directions for complete the problems shown. Determine if the ordered pair is a solution to the system or just an equation. Come to a consensus on what whether the ordered pair is or is not a solution to the system or just one of its equations. Show your work and be prepared to explain your reasoning. 1) $2x - y = 5$ (-1,-7) 2) $y - 2x = 4$ (-2,0) 3) $\frac{1}{2}x + y = -2$ (-8,2) $x + y = -8$ $x - 2y = 2$ $x + \frac{1}{2}y = 7$ **Guiding Question(s):** How did you determine whether each ordered pair was a solution? When is an ordered pair a solution to one of the equations but not the system? (These types of questions will be asked as I observe the work of each group.) Give each group a set of "playing cards". Display the following instructions in the PowerPoint and allow groups play until the last 5-10 minutes of class:

Independent Practice: (10-15 min. of the activity will be used for comparing results and making conjectures about the results in each activity)	"How You Like me, Now?" Matching Game Instructions • Your group of 4 or 5 has been given a set of "playing cards". Place them faced down. • Roll the die to determine who will select first. The highest number rolled goes first. • Continue clockwise until everyone has had a turn • For the "like" terms that are selected, you can ADD them correctly to keep the matching pair and earn 1 pt. • For the "like" terms that are selected, you must SUBTRACT them correctly to keep the matching pair and earn 2 pts. • Continue play until all cards or matched or time is called. • The group with the most points from correctly matching like terms will earn 5 extra credit points.
Closure: (3-5 min. will be necessary for students to summary their knowledge and understanding of solutions to systems of equations and simplifying expressions)	Ask students to stack their cards and put the rubber band around them to keep them in place. Pass out the students homework sheet. **Guiding Question(s):** Can I have a group volunteer to explain how a system of linear equations relates to an equation? (**Expected answer:** A system must have two or more equations.) Can I have a group volunteer to explain how to determine when an ordered pair is a solution to a system of equations? (**Expected answer**: When the ordered pair satisfies all of the equations in the system.) Can I have a group volunteer explain when an ordered pair is only a solution to an equation in a system? (**Expected answer**: When the ordered pair satisfies one of the equations in the system but not the other(s).) Why are identifying like terms important to determining solutions to a system? (**Expected answer**: The coordinates of the ordered pair must be substituted into the correct places for x and y in each equation in the system to correctly determine solutions.) Tomorrow we will begin our study of solving systems of equations. Any questions? (Address as needed)
Homework: Assignment: (1-2 min. to point out assignment as shown on homework assignment sheet)	All problems on the worksheet on solutions to systems of linear equations and simplifying expressions are due tomorrow. (PowerPoint should include this homework assignment).

DAY #2 Lesson Topic: Graphical Methods for Solving Systems of Linear Equations

UNIT #5: Systems of Linear Equations

Textbook Used: ***N/A***

Required Materials: Student handouts, homework sheets, PowerPoint of lesson notes, One set of “How You Like Me Now?” game cards and one number cube for each group, rulers, graph paper, and graphing calculator (Day #3 only) for each student	**Related skills:** Slope-intercept form of a linear equation, Graphing linear equations, solutions to a system of equations, and parallel lines

Lesson Objectives: To review prerequisite skills of expressing an equation in slope-intercept form and finding the x- and y-intercepts. To solve a system of equations by graphing and locating the point of intersection of the lines.

Instructional Presentation: Cooperative learning/Discovery Format

NCTM Standards to Address: Interpret representations of functions of two variables; Use the language of mathematics to express mathematical ideas precisely; Recognize and use connections among mathematical ideas; Understand how mathematical ideas interconnect and build on one another to produce a coherent whole; Write and understand the meaning of equivalent forms of equations.

NCSCOS Objectives to Address: Use algebraic expressions to model and solve problems; define and use linear functions to model and solve problems

Targeted Audience: Students enrolled in Technical Math I course

Time Period: 90-minute blocked scheduled will be most appropriate for completing this activity in one setting

Review/Focus: (15-20 min.)	PowerPoint will contain the groups in which students are placed for the cooperative learning activities. Instruct students to find their name and sit with the members of their group. Pass out the handout on Systems of Linear Equations. (Groups should be heterogeneous with students of varying abilities – low, average, and high – are included in each group of 4 or 5 students.) Groups should remain unchanged unless productivity is lacking in the group, which would require reorganizing the groups to improve on-task behavior. Students will collaboratively complete the following Warm-up Activity on solutions to systems. G**uiding Question(s):** Did everyone find his/her correct group and receive a handout? Give student 5 minutes to discuss the Warm-up Activity within their group and come to a consensus on the correct answers. Determine if the ordered pair is a solution to the system. 1. $2x - 3y = 5$ (1,-1) 2. $2x - 2y = -6$ (1,4) $y + 2x = 1$ $x + \frac{1}{2}y = 3$ Solve each equation for the specified variable: 1. $y + 2x = 1$ for y 2. $x + \frac{1}{2}y = 3$ for x The lesson objectives will be stated as well as the following questions: **Guiding Question(s):** How did you determine which ordered pair was a solution to the stated system? What does it mean to solve an equation for a specific variable? (Ask for volunteers to explain each problem in the Warm-up Activity and address any mistakes or misconceptions during discussion.) Display the directions below to complete homework check on previous night's assignment. ▪ Write your answers to the homework questions below: ▪ #2, 5, 8, 12, 15 ▪ You have 3 minutes. ▪ Exchange papers for grading. ▪ Give a grade out of 5 and initial. ▪ Return to owner to review. ▪ Pass to your right my left for collecting. ▪ Questions on the homework? Allow volunteers to put up solutions to the problems on which students have questions. (Give out participation stickers to volunteers after they explain their problem).

Teacher Input: **(3 min. for instructing whole class on purpose of cooperative group and lesson objectives and 50-65 min. as needed throughout the cooperative learning activity)**	Have groups fill in the blanks on their notes sheets provided. **Review Questions:** A linear equation expressed in the form y = mx + b where m is the ____________ and b is the _____________ is written in ______________ form. (Ask for a volunteer to complete the statement shown. Determine whether the other groups agree or disagree with these responses. Address any misconceptions in the class discussion) Give groups 5 minutes to complete Ex 1 shown. Ex1 Identify the slope and y-intercept. y = -2x + 1 y – x = 3 2y = 2x – 3 2x + 3y = 6 **Guiding Questions:** Which equations are expressed in slope-intercept form? How do you change an equation into slope-intercept form? (These questions will be asked while groups are working on the example).

Guided Practice: (50-65 min. will be used for completing the various examples on the handout)	Display key words in the PowerPoint presentation: X-intercept and Y-intercept After students discuss appropriate definition of these terms. Give groups 3 minutes to discuss the definitions of the x- and y-intercepts and come to a consensus with other members of their group. Inform groups to be prepared to share your definitions with the class and to reconcile any differences in their groups' definitions and the one given during the class discussion. **Guiding Question:** How can you find the x-intercept? The y-intercept? (Ask for a volunteer to share definition with the class. Determine whether other groups agree or disagree with proposed definition. Address any misconceptions in the definitions before moving on to how to find these intercepts.) Accurate definition should be displayed in PowerPoint. Give groups 3 minutes to discuss how to find the x- and y-intercepts. Remind each group to be prepared to share their reasoning with the class and to reconcile any differences in their groups' explanation and the ones given during the class discussion **Guiding Question:** Where in the coordinate plane does the x-intercept of an equation lie? The y-intercept? (Ask for a volunteer to share suggestion for simplifying like terms with the class. Determine whether other groups agree or disagree with proposed method. **Guiding Question:** How many points do you need to graph a line? (Reconcile in misconceptions in the process proposed by the class.) Display the steps for finding the x- and y-intercept and have each student record them on their notes handout. Give groups 5 minutes to discuss the following questions. **Discussion questions:** How do you graph a line using the x- and y-intercepts? The slope-intercept form of a line? (For each question, ask for a volunteer to share their group's response. Determine whether other groups agree or disagree with responses and address any misconceptions in students thinking.) Display and explain the steps for solving a system by graphing. Complete examples #2 with the class. (A consistent, inconsistent, and dependent system is included in this example) Address any misconceptions in the definitions offered and display accurate step in the PowerPoint after the class discussion). Display the definitions for consistent, inconsistent, and dependent systems and instruct students to include these on their notes sheets. What do you notice about the graphs of the consistent system? The inconsistent system? The dependent system?

Independent Practice: (10-15 min. of the activity will be used for completing the activity at each station and to collaborate on the correct answer to each problem.)	Place activity box on each groups desk prior to beginning the cooperative learning stations and rotating so that each group can complete all of the activities. Instruct students to follow the directions displayed at each of the four cooperative activity stations: 1) Simplifying Like Terms 2) Ordered Pair Solutions of Linear Systems 3) Solving Systems by Graphing Intercepts 4) Solving Systems by Graphing Slope-Intercept Form. Remind them that each person must complete one of the problems in each activity and explain their solution to the other members of the group. Work must be shown and the group must agree on the correct answer to each question. Be prepared to explain your reasoning. Each activity contains a card with a problem related to the concepts shown above for each member of the group. **Guiding Question(s):** What are like terms? How do you simplify an expression? Does an expression contain an equal sign? How did you determine whether each ordered pair was a solution? When is an ordered pair a solution to one of the equations but not the system? How did you graph each line? How do you determine if the system has a solution and is consistent, has a solution and is dependent, or does not have a solution and is therefore inconsistent? (These types of questions will be asked as I observe the work of each group.) Rotate groups every 10-15 minutes depending on time remaining in class. (Try to divide the time each group spends at each cooperative learning station is approximately the same.)

Closure: **(3-5 min. will be necessary for students to summary their knowledge and understanding of solutions to systems of equations and simplifying expressions)**	Ask students to leave any extra handouts at the last station and place inside the activity box. Pass out the students homework sheet. **Guiding Question(s):** Can I have a group volunteer to explain how to determine when an ordered pair is a solution to a system of equations? (**Expected answer**: When the ordered pair satisfies all of the equations in the system.) Can I have a group volunteer explain when an ordered pair is only a solution to an equation in a system? (**Expected answer**: When the ordered pair satisfies one of the equations in the system but not the other(s).) What two ways can you solve a system of equations graphically? (**Expected answer**: By graphing the lines in the system using the x- and y-intercepts or by expressing each equation in the system in slope-intercept form.) Can I have a volunteer explain the difference between consistent, dependent, and inconsistent systems? (**Expected answer:** Consistent systems have one ordered pair solution which is the point of the intersection of the two lines, dependent systems have infinitely many solutions because the lines are the same, but inconsistent systems do not have a solution because the lines are parallel.) Which method of graphing a system to determine its solution do you prefer? Tomorrow we will use the graphing calculator to solve systems of linear equations. Any questions? (Address as needed)
Homework: **Assignment:** **(1-2 min. to point out assignment as shown on homework assignment sheet)**	All problems on the worksheet on x- and y-intercepts to solve systems are due tomorrow. (PowerPoint should contain this homework assignment).

DAY #3 Lesson Topic: Using Graphing Calculator to Solve Systems of Linear Equations

UNIT #5: Systems of Linear Equations

Textbook Used: ***N/A***

Required Materials: Student handouts, homework sheets, PowerPoint of lesson notes, rulers, graph paper, and graphing calculator for each student	**Related skills:** Slope-intercept form of a linear equation, Graphing linear equations, solutions to a system of equations, and parallel lines

Lesson Objectives: To review graphing lines using slope-intercept form and the x- and y-intercepts. To use a graphing calculator to solve a system of equations by graphing and calculating the point of intersection.

Instructional Presentation: Cooperative learning/Discovery Format

NCTM Standards to Address: Interpret representations of functions of two variables; Use the language of mathematics to express mathematical ideas precisely; Recognize and use connections among mathematical ideas; Understand how mathematical ideas interconnect and build on one another to produce a coherent whole; Write and understand the meaning of equivalent forms of equations; judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

NCSCOS Objectives to Address: Use algebraic expressions to model and solve problems; define and use linear functions to model and solve problems

Targeted Audience: Students enrolled in Technical Math I course

Time Period: 90-minute blocked scheduled will be most appropriate for completing this activity in one setting

Review/Focus: (15-20 min.)	PowerPoint will contain the groups in which students are placed for the cooperative learning activities. Instruct students to pick up their graphing calculator, find their name and sit with the members of their group. Pass out the handout on Using a Graphing Calculator to Solve Systems of Linear Equations. (Groups should be heterogeneous with students of varying abilities – low, average, and high – are included in each group of 4 or 5 students.) Groups should remain unchanged unless productivity is lacking in the group, which would require reorganizing the groups to improve on-task behavior. The lesson objectives will be stated before students begin cooperative learning station rotations. Display the directions below to complete homework check on previous night's assignment. ▪ Write your answers to the homework questions below: ▪ #3, 7, 8, 9, 14 ▪ You have 3 minutes. ▪ Exchange papers for grading. ▪ Give a grade out of 5 and initial. ▪ Return to owner to review. ▪ Pass to your right my left for collecting. ▪ Questions on the homework? Allow volunteers to put up solutions to the problems on which students have questions. (Give out participation stickers to volunteers after they explain their problem).

Teacher Input: **(3 min. for instructing whole class on purpose of cooperative group and lesson objectives and 50-65 min. as needed throughout the cooperative learning activity)**	Display the steps for graphing on the TI-83/84 Plus calculator. Emphasize the importance of slope-intercept form in graphing on the calculator. Model these steps for each group using the systems in Ex 1 shown. Ex1 Determine whether the system is consistent, inconsistent or dependent. If consistent, state the solution to the system as an ordered pair. $y = -2x + 1$ $\quad\quad$ $y - x = 3$ $2y = 2x - 3$ $\quad\quad$ $2x + 3y = 6$ **Guiding Questions:** Which equations are expressed in slope-intercept form? How do you change an equation into slope-intercept form? (These questions will be asked while groups are working on the example).

Guided Practice: (50-65 min. will be used for completing the various activities at each cooperative learning station)	Display key words in the PowerPoint presentation: X-intercept and Y-intercept After students discuss appropriate definition of these terms. Give groups 3 minutes to discuss the definitions of the x- and y-intercepts and come to a consensus with other members of their group. Inform groups to be prepared to share your definitions with the class and to reconcile any differences in their groups' definitions and the one given during the class discussion. **Guiding Question:** How can you find the x-intercept? The y-intercept? (Ask for a volunteer to share definition with the class. Determine whether other groups agree or disagree with proposed definition. Address any misconceptions in the definitions before moving on to how to find these intercepts.) Accurate definition should be displayed in PowerPoint. Give groups 3 minutes to discuss how to find the x- and y-intercepts. Remind each group to be prepared to share their reasoning with the class and to reconcile any differences in their groups' explanation and the ones given during the class discussion **Guiding Question:** Where in the coordinate plane does the x-intercept of an equation lie? The y-intercept? (Ask for a volunteer to share suggestion for simplifying like terms with the class. Determine whether other groups agree or disagree with proposed method. **Guiding Question:** How many points do you need to graph a line? (Reconcile any misconceptions in the process proposed by the class.) Display the steps for finding the x- and y-intercept and have each student record them on their notes handout. Give groups 5 minutes to discuss the following questions. **Discussion questions:** How do you graph a line using the x- and y-intercepts? The slope-intercept form of a line? (For each question, ask for a volunteer to share their group's response. Determine whether other groups agree or disagree with responses and address any misconceptions in students thinking.) Display and explain the steps for solving a system by graphing. Complete examples #2 with the class. (A consistent, inconsistent, and dependent system is included in this example) Address any misconceptions in the definitions offered and display accurate step in the PowerPoint after the class discussion). Display the definitions for consistent, inconsistent, and dependent systems and instruct students to include these on their notes sheets. What do you notice about the graphs of the consistent system? The inconsistent system? The dependent system?

Independent Practice: (10-15 min. of the activity will be used for completing the activity at each station and to collaborate on the correct answer to each problem.)	Place activity box on each groups desk prior to beginning the cooperative learning stations and rotating so that each group can complete all of the activities. Instruct students to follow the directions displayed at each of the four cooperative activity stations: 1) Expression or Equation 2) Ordered Pair Solutions of Linear Systems 3) Solving Systems by Graphing by Hand 4) Solving Systems by Graphing on the Calculator. Remind them that each person must complete one of the problems in each activity and explain their solution to the other members of the group. Work must be shown and the group must agree on the correct answer to each question. Be prepared to explain your reasoning. Each activity contains a card with a problem related to the concepts shown above for each member of the group. **Guiding Question(s):** How is an expression different from an equation? How did you determine whether each ordered pair was a solution? When is an ordered pair a solution to one of the equations but not the system? Which method did you use to graph each line by hand? How do you determine if the system has a solution and is consistent, has a solution and is dependent, or does not have a solution and is therefore inconsistent? (These types of questions will be asked as I observe the work of each group.) Rotate groups every 10-15 minutes depending on time remaining in class. (Try to divide the time each group spends at each cooperative learning station is approximately the same.)

Closure: (3-5 min. will be necessary for students to summary their knowledge and understanding of solutions to systems of equations and simplifying expressions)	Ask students to leave any extra handouts at the last station and place inside the activity box. Pass out the students homework sheet. **Guiding Question(s):** Can I have a group volunteer to explain the difference between an equation and an expression? (**Expected answer**: An equation has an equal sign between two expressions whereas an expression does NOT contain an equal sign.) Can I have a group volunteer explain when an ordered pair is only a solution to an equation in a system? (**Expected answer**: When the ordered pair satisfies one of the equations in the system but not the other(s).) What two ways can you solve a system of equations graphically? (**Expected answer**: By graphing the lines in the system using the x- and y-intercepts or by expressing each equation in the system in slope-intercept form.) Can I have a volunteer explain the difference between consistent, dependent, and inconsistent systems? (**Expected answer:** Consistent systems have one ordered pair solution which is the point of the intersection of the two lines, dependent systems have infinitely many solutions because the lines are the same, but inconsistent systems do not have a solution because the lines are parallel.) In what form must the equations be expressed to solve a system using the graphing calculator? Which method of graphing a system to determine its solution do you prefer, by hand or using the calculator? Tomorrow we will begin our study of algebraic methods to solve systems of linear equations. Any questions? (Address as needed)
Homework: Assignment: (1-2 min. to point out assignment as shown on homework assignment sheet)	All problems on the worksheet on solving systems by graphing are due tomorrow. (PowerPoint should include this homework assignment)

DAY #5 Lesson Topic: Solving Systems of Linear Equations Using Substitution

UNIT #5: Systems of Linear Equations

Textbook Used: N/A

Required Materials: Student handouts, homework sheets, PowerPoint of lesson notes, rulers, graph paper, and graphing calculator for each student	**Related skills:** Slope-intercept form of a linear equation, graphing linear equations, solutions to a system of equations, and parallel lines, coefficients of terms, the distributive property and inverse operations.

Lesson Objectives: To review graphing lines using slope-intercept form. To use the substitution method to determine the solution to a system of linear equations and whether system is consistent, inconsistent, or dependent. To connect the graph of a system to the solution determined using substitution.

Instructional Presentation: Cooperative learning/Discovery Format

NCTM Standards to Address: Interpret representations of functions of two variables; Use the language of mathematics to express mathematical ideas precisely; Recognize and use connections among mathematical ideas; Understand how mathematical ideas interconnect and build on one another to produce a coherent whole; Write and understand the meaning of equivalent forms of equations; judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

NCSCOS Objectives to Address: Use algebraic expressions to model and solve problems; define and use linear functions to model and solve problems

Targeted Audience: Students enrolled in Technical Math I course

Time Period: 90-minute blocked scheduled will be most appropriate for completing this activity in one setting

Review/Focus: (15-20 min.)	PowerPoint will contain the groups in which students are placed for the cooperative learning activities. Instruct students to pick up their graphing calculator, find their name and sit with the members of their group. (Groups should be heterogeneous with students of varying abilities – low, average, and high – are included in each group of 4 or 5 students.) Groups should remain unchanged unless productivity is lacking in the group, which would require reorganizing the groups to improve on-task behavior. The PowerPoint should include the following warm-up activity 1. What does it mean to solve an equation? 2. Solve each equation for the stated variable. y – x = 3 for y 2x – y = 5 for y x + 2y = 6 for x x – 2y = 4 for x 3. Why were the first two equations solved for y but the last two for x? 4. If the two equations solved for y represent a system. Graph these two equations to determine a solution to the system. Is the system consistent, inconsistent, or dependent? Explain your reasoning. The lesson objectives will be stated before students begin cooperative learning station rotations. Display the directions below to complete homework check on previous night's assignment. ▪ Write your answers to the homework questions below: ▪ #2, 5, 7, 10, 13 ▪ You have 10 minutes. ▪ Exchange papers for grading. ▪ Give a grade out of 10 and initial. ▪ Return to owner to review. ▪ Pass to your right my left for collecting. ▪ Questions on the homework? Allow volunteers to put up solutions to the problems on which students have questions. (Give out participation stickers to volunteers after they explain their problem).

Teacher Input: **(5 min. for explaining the steps to solving systems by substitution and guiding questions throughout the 50-65 min. cooperative group learning activity)**	Ask for volunteers to recall when a system was called consistent, inconsistent, and dependent based on the graphs of the lines. Explain the following steps of solving systems of equations using substitution. (PowerPoint should contain the following steps). Steps for solving systems by substitution: 1) Look for a variable with a coefficient of 1. 2) Solve the equation for this variable. 3) Replace the chosen variable with its equivalent expression into the other equation 4) Solve for the remaining variable using opposite operations. 5) Substitute the value found in step 4 back into the equation found in step 2. 6) Write solution to system as (x,y). **Guiding Questions:** What are the coefficients in an equation? Recall in warm-up activity to solve an equation meant to get the indicated variable on one side of the equal sign by itself. (This question will be asked while groups are discussing the steps for solving systems using substitution and the statement will be made after they have completed their discussion).

<table>
<tr>
<td>Guided Practice: (50-65 min. will be used for completing the various activities at each cooperative learning station)</td>
<td>Give students the following examples to complete within their group. Each student must attempt to solve two of the equations using substitution and share their solution with the other group members. In addition, the group must come to a consensus on the solution to each system, if any, and prepare to justify their results with the class.

1) -3x + y = -2
y = x + 6
2) y + 4 = x
-2x + y = 8
3) y – 2 = x
-x = y
4) 6y + 4x = 12
-6x + y = -8
5) 3x + y = 5
2x – 5y = 9
6) x + 4y = 5
4x – 2y = 11
7) 2y – 3x = 4
x = -2
8) 3y + x = -1
x = 3y
9) 2x + y = -1
6x = 3y - 3

Guiding Question: While the groups are completing the examples above ask students for which variable will you solve? Explain your reasoning to your group. Be sure to show your substitution step as well as your simplifying steps to find the variable of each variable. How should you express your solution to the system? (Ask for a volunteer to share their solution to each system with the class.) While groups are working, remind students to be prepared to share their reasoning with the class and to reconcile any differences in their groups' explanation and the ones given during the class discussion Guiding Question: Did all of the systems have the same type of solution? What were some of the differences? For systems with no solution or infinitely many solutions have students express each equation in slope-intercept form and enter on their graphing calculator. What do you notice about the graphs of the systems with no solution? Infinitely many solutions? (Ask for a volunteer to share their groups' responses with the class) Determine whether other groups agree or disagree with proposed method. Guiding Question: What do you think the graphs of the systems with one solution will have in common? (Reconcile any misconceptions in student responses.) Display the steps for finding the x- and y-intercept and have each student record them on their notes handout. Determine Address any mistakes in determining each solution before moving on to the next example. Give groups 5 minutes to discuss the following questions. Discussion questions: How can you use the results of the substitution method to determine when a system is consistent,</td>
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</table>

Guided Practice: (50-65 min. will be used for completing the various activities at each cooperative learning station)	inconsistent, or dependent? How does this connect to the results the graphical methods revealed about these outcomes? (For each question, ask for a volunteer to share their group's response. **Expected response**: No solution, if system results in unequal results in identical quantities such as 2 = 2 or x = x. One solution when a single ordered pair (x,y) is obtained. Determine whether other groups agree or disagree with responses and address any misconceptions in students thinking.)
Independent Practice: (10-15 min. of the activity will be used for completing the activity at each station and to collaborate on the correct answer to each problem.)	Pass out the Handout on Solving Systems using substitution. Remind students all work must be shown and the group must agree on the correct answer to each question. Be prepared to explain your reasoning. Each student should complete their own handout out thought they are allowed to discuss the problems with the other members of their group. **Guiding Question(s):** Is it easier for you to determine if the system has a solution and is consistent, has a solution and is dependent, or does not have a solution and is therefore inconsistent, using a graphical method or the substitution method? Justify your answer. (This question will be asked as I observe the work of each group.)
Closure: (3-5 min. will be necessary for students to summary their knowledge and understanding of solutions to systems of equations and simplifying expressions)	**Guiding Question(s):** Can I have a group volunteer to explain for which variable should an equation of a system be solved? (**Expected answer**: Any variable with a coefficient of 1.) Which method do you prefer, graphing systems or solving using substitution? (Answers will vary). Tomorrow we will discuss solving solve systems of linear equations using the eliminations method.
Homework: Assignment: (1-2 min. to point out assignment as shown on homework assignment sheet)	All problems on the worksheet on solving systems by substitution are due tomorrow. (PowerPoint should include this homework assignment)

DAY #6 Lesson Topic: Solving Systems of Linear Equations Using Elimination

UNIT #5: Systems of Linear Equations

Textbook Used: ***N/A***

Required Materials: Student handouts, homework sheets, PowerPoint of lesson notes, rulers, graph paper, and graphing calculator for each student	**Related skills:** Slope-intercept form of a linear equation, Graphing linear equations, solutions to a system of equations, and parallel lines, definition of opposites and inverse operations

Lesson Objectives: To review consistent, inconsistent, and dependent systems. To use the elimination method to determine the solution to a system of linear equations and whether system is consistent, inconsistent, or dependent. To connect the graph of a system to the solution determined using elimination.

Instructional Presentation: Cooperative learning/Discovery Format

NCTM Standards to Address: Interpret representations of functions of two variables; Use the language of mathematics to express mathematical ideas precisely; Recognize and use connections among mathematical ideas; Understand how mathematical ideas interconnect and build on one another to produce a coherent whole; Write and understand the meaning of equivalent forms of equations; judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

NCSCOS Objectives to Address: Use algebraic expressions to model and solve problems; define and use linear functions to model and solve problems

Targeted Audience: Students enrolled in Technical Math I course

Time Period: 90-minute blocked scheduled will be most appropriate for completing this activity in one setting

Review/Focus: (15-20 min.)	PowerPoint will contain the groups in which students are placed for the cooperative learning activities. Instruct students to pick up their graphing calculator, find their name and sit with the members of their group. (Groups should be heterogeneous with students of varying abilities – low, average, and high – are included in each group of 4 or 5 students.) Groups should remain unchanged unless productivity is lacking in the group, which would require reorganizing the groups to improve on-task behavior. Pass out the Reteaching Handout on Solving Systems Using Substitution. The PowerPoint should include the following warm-up activity Use the substitution method to determine the solution to each system of linear equations. Discuss problems 2-10 on the handout on substitution. #1 has been done already for an Example. Each group should be prepared to put up the solutions to two of these systems on the board and explain your reasoning. 2) $-x + y = 3$, $2x - 5y = 9$ 3) $x + 4y = 5$, $x - 2y = 11$ 4) $y - 2 = x$, $-x = y$ 5) $2x + 4y = 4$, $y + x = 2$ 6) $3x + y = 5$, $2x - 5y = 9$ 7) $x + 4y = 5$, $4x - 2y = 11$ 8) $2y - 3x = 4$, $x = -2$ 9) $3y + x = -1$, $x = 3y$ 10) $2x + y = -1$, $6x = 3y - 3$ You have 10 minutes, so divide and conquer. (Be sure to explain your solution to your group and to come to a consensus on the correct solution) Ask for Volunteers, to put solutions on the board? ☺ The lesson objectives will be stated before students begin cooperative learning station rotations. Display the directions below to complete homework check on previous night's assignment. ▪ Write your answers to the homework questions below: ▪ #3,6,8,12,15 ▪ You have 5 minutes. ▪ Exchange papers for grading. ▪ Give a grade out of 10 and initial.

	▪ Return to owner to review. ▪ Pass to your right my left for collecting. ▪ Questions on the homework? Allow volunteers to put up solutions to the problems on which students have questions. (Give out participation stickers to volunteers after they explain their problem).

Teacher Input: **(5 min. for explaining the steps to solving systems by substitution and guiding questions throughout the 50-65 min. cooperative group learning activity)**	Ask for volunteers to recall when a system was called consistent, inconsistent, and dependent based on the graphs of the lines. Put up an example where all of the coefficients are not one and are also not multiples of each other. Give the groups about 5 minutes to attempt to solve the following system. Find the solution to system: $2x + 3y = 5$ $-2x + 3y = 1$ (Ask for a volunteer to share their work with the class.) **Guiding Question:** What was the most difficult aspect of using substitution for this system? Did anyone solve using graphing? What was the challenge of using a graphical method to solve this system? Inform class that there is a less complicated method of solving systems in which none of the coefficients are 1. Ask what do you notice about the coefficients on the x terms in the system? What would happen to this variable if we added the two equations? What would you get for the value of y? How could you find the value of x? Inform class the process just completed is another method for solving system called elimination. Ask based on what we did, why is this process so named elimination? Explain the following steps of solving systems of equations using elimination. (PowerPoint should contain the following steps). Steps for solving systems by elimination: 7) Look for a variable with the same (or opposite) coefficient in the system. 8) Eliminate this variable by subtracting (or adding) the two equations together. 9) Solve the resulting equation for the remaining variable. 10) Substitute the value found in step 3 back into one of the original equations of the system. 11) Write solution to system as (x,y). Note: If all of the coefficients in the system are distinct then create same (or opposite) coefficients by multiplying one (or sometimes both) equations by a number.

Teacher Input: **(5 min. for explaining the steps to solving systems by substitution and guiding questions throughout the 50-65 min. cooperative group learning activity)**	**Guiding Questions:** What happens when you multiply an equation by a number? Does this change the solution to the equation? The coefficients change but the solution to the equation is the same. (Give a simple example such as x = 4 and multiply by 3 to get 3x = 12 and ask students to solve again!!) Explain that by multiplying an equation by a number to create the same (or opposite) coefficient does not change the solution to the system.
Guided Practice: (50-65 min. will be used for completing the various activities at each cooperative learning station)	Give students the following examples to complete within their group. Each student must attempt to solve two of the equations using elimination and share their solution with the other group members. In addition, the group must come to a consensus on the solution to each system, if any, and prepare to justify their results with the class. (Identify the coefficient in each system. Recall a consistent system has one unique solution, an inconsistent system does not have a solution, and a dependent system has infinitely many solutions, specifically, any point on the linear equation in the system. Keep these criteria in mind as you attempt to solve systems using the elimination method. Encourage students to use the graph of the system to confirm their solutions.) 1) $-3x + y = -2$, $y = x + 6$ 2) $y + 4 = x$, $-2x + y = 8$ 3) $y - 2 = x$, $-x = y$ 4) $6y + 4x = 12$, $-6x + y = -8$ 5) $3x + y = 5$, $2x - 5y = 9$ 6) $x + 4y = 5$, $4x - 2y = 11$ 7) $2y - 3x = 4$, $x = -2$ 8) $3y + x = -1$, $x = 3y$ 9) $2x + y = -1$, $6x = 3y - 3$

Guided Practice: (50-65 min. will be used for completing the various activities at each cooperative learning station)	**Guiding Question:** While the groups are completing the examples above ask students, “Which variable will you eliminate first”? Explain your reasoning to your group. Be sure to show your work while finding the value of each variable. How should you express your solution to the system? (Ask for a volunteer to share their solution to each system with the class.) While groups are working, remind students to be prepared to share their reasoning with the class and to reconcile any differences in their groups’ explanation and the ones given during the class discussion. Address any mistakes in determining each solution before moving on to the next example. Give groups 5 minutes to discuss the following questions. **Discussion questions:** How can you use the results of the elimination method to determine when a system is consistent, inconsistent, or dependent? How does this connect to the results the graphical methods displayed about these outcomes? (For each question, ask for a volunteer to share their group’s response. **Expected response**: No solution, if system results in unequal quantities such as $5 \neq -6$. Infinitely many solutions, if system results in identical quantities such as $2 = 2$ or $x = x$. One solution when a single ordered pair (x,y) is obtained. (Determine whether other groups agree or disagree with responses and address any misconceptions in students thinking.) How do these forms of solutions to systems compare to the forms of solutions using substitution? (They are the same)
Independent Practice: (10-15 min. of the activity will be used for completing the activity at each station and to collaborate on the correct answer to each problem.)	Pass out the Handout on Solving Systems using elimination and instruct groups to complete problems 1-4 before leaving class. Remind students all work must be shown and the group must agree on the correct answer to each question. Be prepared to explain your reasoning. Each student should solve one of the systems (for groups with 5 members, two students must work together to solve the system. However, all solutions must be discussed with and explained to the other members of the group. Each student has to write the group’s solution to these four problems on their individual paper. **Guiding Question:** Are you explaining your solution to the other members of your group? Ask each other for assistance with solving your system before inquiring from me.

Closure: (3-5 min. will be necessary for students to summary their knowledge and understanding of solutions to systems of equations and simplifying expressions)	**Guiding Question(s):** Can I have a group volunteer to explain which variable to eliminate first in a system of equations? (**Expected answer**: Any variable with a coefficient the same (or opposite.) When should you add the two equations to eliminate a variable? (**Expected answer:** When the coefficients are already opposite) When should you subtract the two equations to eliminate a variable? (**Expected answer:** When the coefficients are the same) Which algebraic method do you prefer, solving using substitution or elimination? (Answers will vary). **Guiding Question(s):** Is it easier for you to determine if the system has a solution and is consistent, has a solution and is dependent, or does not have a solution and is therefore inconsistent, using a graphical method or the algebraic methods of substitution or elimination? Explain. Tomorrow we will again discuss solving solve systems of linear equations using the eliminations method.
Homework: Assignment: (1-2 min. to point out assignment as shown on homework assignment sheet)	All of the multiples of five on the handout on elimination are due tomorrow. (PowerPoint should include this homework assignment).

DAY #7 Lesson Topic: Summary of Methods to Solve Systems of Linear Equations UNIT #5: Systems of Linear Equations	
Textbook Used: ***N/A***	
Required Materials: Student handouts, homework sheets, PowerPoint of lesson notes, rulers, graph paper, and graphing calculator for each student	**Related skills:** Slope-intercept form of a linear equation, graphing linear equations, solutions to a system of equations, and parallel lines, definition of opposites and inverse operations
Lesson Objectives: To review consistent, inconsistent, and dependent systems. To use each of the methods, graphing by hand and on the calculator, substitution, and elimination to determine the solution to a system of linear equations and whether system is consistent, inconsistent, or dependent. To connect the form of the equations in a system to the most efficient method for solving the system.	
Instructional Presentation: Cooperative learning/Discovery Format	
NCTM Standards to Address: Interpret representations of functions of two variables; Use the language of mathematics to express mathematical ideas precisely; Recognize and use connections among mathematical ideas; Understand how mathematical ideas interconnect and build on one another to produce a coherent whole; Write and understand the meaning of equivalent forms of equations; judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology. **NCSCOS Objectives to Address:** Use algebraic expressions to model and solve problems; define and use linear functions to model and solve problems	
Targeted Audience: Students enrolled in Technical Math I course	
Time Period: 90-minute blocked scheduled will be most appropriate for completing this activity in one setting	

Review/Focus: (15-20 min.)	PowerPoint will contain the groups in which students are placed for the cooperative learning activities. Instruct students to pick up their graphing calculator, find their name and sit with the members of their group. (Groups should be heterogeneous with students of varying abilities – low, average, and high – are included in each group of 4 or 5 students.) Groups should remain unchanged unless productivity is lacking in the group, which would require reorganizing the groups to improve on-task behavior. Display the directions below to complete homework check on previous night's assignment. ▪ Write your solutions to the homework questions below: ▪ #5,15,20,35,40 ▪ You have 5 minutes. ▪ Exchange papers for grading. ▪ Give a grade out of 10 and initial. ▪ Return to owner to review. ▪ Pass to your right my left for collecting. ▪ Questions on the homework? Allow volunteers to put up solutions to the problems on which students have questions. (Give out participation stickers to volunteers after they explain their problem).

Teacher Input: **(5 min. for explaining the steps to solving systems by substitution and guiding questions throughout the 50-65 min. cooperative group learning activity)**	Place activity box on each groups desk prior to beginning the cooperative learning activity. Instruct students to follow the directions provided in their activity box. The objectives to today's lesson will be stated prior to students starting the cooperative learning activity The directions in each box state for the students to solve the system of equations using the described method. Remind students that each person must complete one of the problems in each activity and explain their solution to the other members of the group. Work must be shown and the group must agree on the correct answer to each question. Be prepared to explain your reasoning. Each activity contains a card with a problem related to the concepts shown above for each member of the group. The systems included in today's cooperative learning activities are the exact same systems but they must be solved using the method specified. **Guiding Question(s):** What where the two way to solve a system by graphing? Where is the solution to the system in your graph? In what form must a linear equation be expressed in order to enter it into your graphing calculator? How do you find the solution to a system of equations on the graphing calculator? For which variable should you solve to use the substitution method? How can you find the value of the remaining variable? Which variable is the easiest to eliminate? How can you get the coefficients to be the same? The opposite? (These types of questions will be asked as I observe the work of each group.) Rotate groups every 10-15 minutes depending on time remaining in class. (Try to divide the time each group spends at each cooperative learning station is approximately the same.)

Guided Practice: (50-65 min. will be used for completing the various activities at each cooperative learning station)	Each student must attempt to solve at least one of the equations using the method stated at their cooperative learning station and share their solution with the other group members. In addition, the group must come to a consensus on the solution to each system, if any, and prepare to justify their results with the class. (Identify the coefficient in each system. Recall a consistent system has one unique solution, an inconsistent system does not have a solution, and a dependent system has infinitely many solutions, specifically, any point on the linear equation in the system. Keep these criteria in mind as you attempt to solve systems using the specified method! Encourage students to use the graph of the system to confirm their solutions.) The cards placed at each table will include the following systems: 1) $-3x + y = -2$, $y = x + 6$ 2) $y + 4 = x$, $-2x + y = 8$ 3) $y - 2 = x$, $-x = y$ 4) $6y + 4x = 12$, $-6x + y = -8$ 5) $3x + y = 5$, $2x - 5y = 9$ 6) $x + 4y = 5$, $4x - 2y = 11$ 7) $2y - 3x = 4$, $x = -2$ 8) $3y + x = -1$, $-3y + x = 0$ 9) $2x + y = -1$, $6x = 3y - 3$ 10) $y = x - 1$, $y = 2x + 2$ Placing the procedures for each method in the activity box may be beneficial to help groups recall the methods learned earlier in the unit. Inform students that they will not have time to complete all of the problems but each person will have time to complete and explain the solution to at least one of the systems before moving on to the next station. **Guiding Question:** Did you explain your solution to the other members of your group? Did you show all of your work? Does everyone in your group agree on the solutions to the systems you have solved? While groups are working, remind students to be prepared to share their reasoning with the class and to reconcile any differences in their groups' explanation and the ones given during the class discussion. Address any mistakes in determining each solution before moving on to the next example. Select a person from each group to put up one of the solution to the system of equations shown above. They must use the method describe at the table where their group is currently. (Determine whether other

Guided Practice: (50-65 min. will be used for completing the various activities at each cooperative learning station)	groups agree or disagree with responses and address any misconceptions in students thinking.) After a group has explained their solution, ask the class did anyone else solve this system? Did their solution, ask the class did anyone else solve this system? Did you use the same or a different method? If different methods were used, ask which method was easier to use to solve this system? Explain your reasoning. How does the form of the equations in the system help
Independent Practice: (10-15 min. of the activity will be used for completing the activity at each station and to collaborate on the correct answer to each problem.)	Pass out handout with four additional problems for all groups to complete before leaving class. Remind students all work must be shown and the group must agree on the correct answer to each question. Be prepared to explain your reasoning. Each student should solve one of the systems (for groups with 5 members, two students must work together to solve the system. However, all solutions must be discussed with and explained to the other members of the group. Each student has to write the group's solution to these four problems on their individual paper. **Guiding Question:** Are you explaining your solution to the other members of your group? Ask each other for assistance with solving your system before inquiring from me.
Closure: (3-5 min. will be necessary for students to summary their knowledge and understanding of solutions to systems of equations and simplifying expressions)	**Guiding Question(s):** As we have progressed through this unit, what methods for solving systems of linear equations have you used? (**Expected answer:** Graphing by hand (and on the calculator), substitution, and elimination.) Which do you prefer? Explain your reasoning. Does the method you choose affect the solution to the system? (**Expected answer:** No, but the method chosen may make your work easier or more difficult. Tomorrow we will use systems of linear equations to solve real-life problems.
Homework: Assignment: (1-2 min. to point out assignment as shown on homework assignment sheet)	Study over all of the terms and methods for solving systems in preparation for a quiz tomorrow. (PowerPoint should include this reminder).

DAY #8 Lesson Topic: Using Systems of Linear Equations to Solve Real-life Problems UNIT #5: Systems of Linear Equations	
Textbook Used: ***N/A***	
Required Materials: Student handouts, homework sheets, PowerPoint of lesson notes, number cubes, rulers, graph paper, and graphing calculator for each student	**Related skills:** Slope-intercept form of a linear equation, Graphing linear equations, solutions to a system of equations, and parallel lines, definition of opposites and inverse operations
Lesson Objectives: To review the four-step problem solving plan. To write a system of equations for a real-life problem and to select the most appropriate method of solving the system to find the solution to the problem. To connect the form of the equations in a system to the most efficient method for solving the system.	
Instructional Presentation: Cooperative learning/Discovery Format	
NCTM Standards to Address: Interpret representations of functions of two variables; Use the language of mathematics to express mathematical ideas precisely; Recognize and use connections among mathematical ideas; Understand how mathematical ideas interconnect and build on one another to produce a coherent whole; Write and understand the meaning of equivalent forms of equations; judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology. **NCSCOS Objectives to Address:** Use algebraic expressions to model and solve problems; define and use linear functions to model and solve problems	
Targeted Audience: Students enrolled in Technical Math I course	
Time Period: 90-minute blocked scheduled will be most appropriate for completing this activity in one setting	
Review/Focus: (15-20 min.)	PowerPoint will contain the groups in which students are placed for the cooperative learning activities. Instruct students to pick up their graphing calculator, find their name and sit with the members of their group. (Groups should be heterogeneous with students of varying abilities – low, average, and high – are included in each group of 4 or 5 students.) Groups should remain unchanged unless productivity is lacking in the group, which would require reorganizing the groups to improve on-task behavior. Pass out the Quiz on Solving Systems of Equations. You have 20 minutes to complete the quiz. This is an individual effort, so you should NOT discuss the quiz with your neighbors.

Teacher Input: **(5 min. for explaining the steps to solving systems by substitution and guiding questions throughout the 50-65 min. cooperative group learning activity)**	The objectives to today's lesson will be stated prior to students starting the cooperative learning activity **Guiding Question:** When is graphing an appropriate method for solving a system of equations? (**Expected answer:** When the equations are already expressed in slope-intercept form or to check your solution to a system of equations found using another method) When is substitution an appropriate method for solving a system of equations? (**Expected answer:** When at least one of the coefficients of the variables is a 1). When is elimination an appropriate method for solving a system of equations? (**Expected answer:** When all of the coefficients are different from 1 and neither equation is expressed in slope-intercept form) Does anyone recall our four step problem solving plan? (**Expected answer:** Understand the problem, devise a plan, carry out the plan, and look back) Review the key processes for completing each step of the problem plan in the context of the unit solving systems of linear equations: 1) Understand the problem – identify the key words, numerical information, and what you are asked to find 2) Devise a plan – select variables that relate to what you are asked to find; use the key words and the numerical information in one sentence to write an equation using the variables identified; use the key words and the numerical information in another sentence to write a second equations; this is the system you must solve to answer the question 3) Carry out the plan – identify the most appropriate method for solving the system; solve the system using this method 4) Look back – be sure to answer the question asked in the context of the problem; Is your answer reasonable?; use the calculator to graph the system or substitute solution back into system to check accuracy. Place activity box on each groups desk prior to beginning the cooperative learning activity. Instruct students to follow the directions provided in their activity box.

Guided Practice: (50-65 min. will be used for completing the various activities at each cooperative learning station)	Work through the following example with the students to model the four-step problem solving plan.
Independent Practice: (10-15 min. of the activity will be used for completing the activity at each station and to collaborate on the correct answer to each problem.)	The directions in each box state for the students instruct students to underline the key words in each problem and to circle the numerical information. In addition, each student must write out the system of equations for at least one of the problems and select the most appropriate method to use to solve the system. (The group must come to a consensus on the appropriateness of the method chosen as well as the correctness of the solution). Work must be shown and the group must agree on the correct answer to each question. Be prepared to explain your reasoning. Once the groups have completed all of the problems in their activity box, ask for volunteers to put their solutions on the board. (Determine if the other groups agree or disagree with the system of equations written as well as the method chosen to solve the system). Reconcile in mistakes in equations or misconceptions in the appropriate method to use to solve each system. Be sure that the groups answer the question asked in the problem with units included in their answer. **Guiding Question:** Are you explaining your solution to the other members of your group? Ask each other for assistance with solving your system before inquiring from me.
Closure: (3-5 min. will be necessary for students to summary their knowledge and understanding of solutions to systems of equations and simplifying expressions)	**Guiding Question(s):** Can I have a group volunteer to explain how to determine the most efficient method to solving a real-life problem using systems of linear equations? (**Expected answer**: If both equations are expressed in slope-intercept form, you can use graphing or substitution. If one of the coefficients of the equations is a 1, then substitution is most efficient to solving the system. When all of the coefficients are different, then elimination is the most appropriate method for solving the system). Tomorrow we will review for our test on solving systems of linear equations. Any questions? (Address any questions)

Homework: Assignment: (1-2 min. to point out assignment as shown on homework assignment sheet)	All of the word problems on the handout provided are due tomorrow. Begin reviewing for our test on systems of linear equations. (PowerPoint should include this homework assignment).

Appendix B

Review Activity on Systems of Linear Equations

Group Members: Grading Rubric ______________________________________ Date:10-19-06

I. Simplify each expression.

1. $3x^2 + 4x - 2x^2 - 2x$ 2. $4a - 3b + 2b - 5a$

2 pts ea. -1 incorrect coefficient or incorrect variable or exponent

II. Determine if the ordered pair is a solution to the system. Show all work!!

1. $y = x + 6$ (1,6)
$y = 2x - 7$

2. $2x - y = 3$ (-2,-7)
$-x + y = -5$

2 pts ea.
-1 incorrect solution
-1 computational error in work
-2 no work shown but correct answer

III. Solve each equation for y. Graph each line and find the solution to the system of equations. Write the solution, inconsistent, or dependent. (Check with calculator!)

1. $y = 3x - 1$
$y = -2x + 4$

2. $4x + y = 6$
$y = -4x - 1$

3. $y = -3x - 4$
$3x + y = -4$

4 pts ea.
-1 solution not written as ordered pair
-1 incorrect solution/did not state inconsistent or dependent
-1 error in work
-1 line not graphed/incorrect graph
-2 no work shown but correct solution

IV. Solve each system by substitution. Write the solution, inconsistent, or dependent.

1. $y = x + 4$
$y = 3x$

2. $4x + 2y = 8$
$y = -2x + 4$

3. $y = 3x + 5$
$-3x + y = -6$

4 pts ea.
-1 solution not written as ordered pair
-1 incorrect solution/did not state inconsistent or dependent
-1 used different method
-1 error in work
-2 no work shown but correct solution

V. Solve each system by elimination. Write the solution, inconsistent, or dependent.

1.	$x + 2y = 7$ $3x - 2y = -3$	2.	$5x + 7y = 77$ $5x + 3y = 53$	3.	$2x - 3y = -11$ $3x + 2y = 29$

4 pts ea.
- -1 solution not written as ordered pair
- -1 incorrect solution/did not state inconsistent or dependent
- -1 used different method
- -1 error in work
- -2 no work shown but correct solution

VI. Write a system of equations for the problem below. Identify your variables and be sure to answer the question asked!!

5 pts ea.
- -1 solution not stated in context of problem
- -1 variables not identified
- -1 incorrect equation in system
- -1 error in work
- -3 no work shown but correct solution

1) The sum of two numbers is 5 but there difference is only 3. Find the two numbers.

2) You purchased four hot dogs and two drinks for $8.00 at the last football game. This week you bought three hot dogs and one drink for $5.50. How much was each hot dog and drink?

<table>
<tr><td colspan="26">When finished with this activity, each member rates ALL other group members in the categories below using a scale of 0-4. (0→ did nothing to →4 very helpful, accurate, etc.) (-1 no ratings for individual students)</td></tr>
<tr><td>List names to the right →</td><td colspan="5"></td><td colspan="5"></td><td colspan="5"></td><td colspan="5"></td><td colspan="5"></td></tr>
<tr><td>Cooperated with group & displayed a positive attitude</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Knowledgeable of concepts & Understands procedures</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Performed work accurately</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Participated consistently & asked/answered questions</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
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