

ABSTRACT

JUNG, JAE SUNG. Branch Current State Estimation Method for Power Distribution Systems. (Under the direction of. Mesut E Baran).

Effective management of distribution systems requires analysis tools that can estimate the state of the system (the operating condition). This thesis aims at development of new analysis tools for this purpose. The main tool is the state estimator that will use historical data and the real-time data to estimate the state of the system determined by voltage at all of the nodes of a distribution feeder.

This thesis considers the incorporation of voltage measurements in a branch-current-based state estimation (BCSE) program. Original BCSE is designed to include only power and current measurements. The motivation for enhancing BCSE is that with the adoption of large scale automated meter infrastructure (AMI) technologies, voltage measurements will be available at the distribution level. Hence, including these measurements has the potential to improve the accuracy of state estimation.

Furthermore, this thesis presents a statistical technique for assessing the BCSE performance. For statistical analysis, 300 Monte Carlo simulations are performed. The overall performance including bias, consistency and quality of estimates is evaluated in order to see the effectiveness of the BCSE method. These concepts of statistical technique are illustrated and tested in this thesis.

Finally, since correct connectivity is critical in system operations, topology estimation is expected to become a standard Energy Management System (EMS) function. Hence, two types algorithm are presented for detection and identification of topology error in BCSE. The

first approach uses the idea that when the switch status changes, it will affect the measurements. The second approach is based on changing the on/off status of branches one after the other and performing a state estimation in each case. The effectiveness of the proposed approaches is demonstrated. In addition, topology detection results obtained by the two proposed methods are also compared.

For testing the revised BCSE, a reduced version of the IEEE 34 node radial test feeder is used. The simulation platform used in this study is developed using C language on Microsoft Visual Studio .NET 2003.

Branch Current State Estimation Method for Power Distribution Systems

by
Jae Sung Jung

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APPROVED BY:

Dr. Subhashish Bhattacharya

Dr. Winser E. Alexander

Dr. Sujit Ghosh

Dr. Mesut E. Baran
Chair of Advisory Committee

DEDICATION

The thesis is dedicated to my father who has inspired me. He always taught me that the most successful persons are those who retain enthusiasm for the mission, and are able to do their best without regret. It is also dedicated to my mother who supported me from the beginning of my studies and offered unconditional love and encouragement throughout the course of this thesis. Whenever I felt frustrated, she urged me on. As a result, I finally have a chance to introduce my thesis. I also dedicate this work to my brother. He took care of my family in my stead so that I could concentrate on my studies.

BIOGRAPHY

Jaesung Jung was born in Daejeon, South Korea on August 5, 1979. He grew up and attended school in Daejeon and graduated in 1998 from Dong-Daejeon High School. He entered ChungNam National University in Daejeon, South Korea that spring as an Electrical Engineering major. While at college, he was exposed to the system engineering laboratory led by Dr. Kim, Kurn Jung, an expert in the power system field. he became intensely interested in this field. He then decided to pursue a second major by adding computer engineering, in order to combine his study of power systems with computer systems. He earned his Bachelor degree in Electrical Engineering and Computer Engineering in February, 2006. After he graduated, he worked as an electrical engineer, managing equipment for BOSCH Corporation for 1 year. This experience solidified his aspirations and he decided to study abroad. He entered North Carolina State University (NCSU) in Raleigh, NC as a master student with a Korea Electric Power Research Institute (KEPRI) Scholarship.

He is currently a master student in NCSU and will earn a Master of Science in Electrical and Computer Engineering in May 2009. He is working at the Semiconductor Power Electronics Center (SPEC) at NCSU focused on state estimation in distribution systems with Dr. Mesut E. Baran, director of power systems in SPEC. His research interests are in the area of power system dynamics and computer simulations in power systems and power system wide area monitoring and control.

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Chapter 1

Introduction

1.1 Background

Our literature indicates that utilities have been improving their means of monitoring their distribution systems mainly to improve service reliability [1-6, 10, 12]. Recently, there has been an additional incentive to advance the monitoring of feeders – the improvement of efficiency by adopting advanced functions such as voltage control for demand management. Effective management of distribution systems requires analysis tools that can estimate the state of the system (the operating condition) and predict the response of the system to changing load and weather conditions. The main tool used for system analysis is power flow analysis. But this tool is not very suitable for real-time monitoring as it requires accurate load and system data.

In order to better monitor the system operating conditions for system management, some utilities have begun the installation of limited Supervisory Control And Data Acquisition (SCADA) systems at the distribution level. Additionally, some utilities have deployed large scale Advanced Metering Infrastructures (AMI). With the availability of real-time measurements, new methods are proposed for monitoring the operating point (state) of

distribution system. One of the approaches is power flow based [4-6, 12] and the others [2, 3] are extensions of the conventional state estimation (SE) method for three-phase analysis. Although SE is preferred over the power flow approach, its computational complexity may prevent its use in practical applications.

In this thesis, a branch-current-based three-phase SE (BCSE) method [1] is considered, as it is computationally more efficient and less sensitive to line parameters than the conventional node-voltage-based SE methods. BCSE is very efficient in handling line-flow and power-injection measurements for radial networks. In the original algorithm, the voltage measurements have not been available [1]. But with AMI, voltage measurements will be readily available. In this thesis, we show the enhancement of the original BCSE to include more accurate voltage measurements.

Computer simulations such as power flow studies give exact answers, but in reality we never know the absolutely true state of a physical operating system. Even when great care is taken to ensure accuracy, unavoidable random noise enters into the measurement process to distort more or less the physical results. However, repeated measurements of the same quantity under carefully controlled conditions reveal certain statistical properties from which the true value can be estimated. Thus, statistical technique is necessary to the process of determining the correctness of an SE implementation [16, 17].

In this thesis, some statistical techniques are presented for assessing the BCSE performance. Because of the statistical nature of pseudo measurements, the performance of the SE needs to be assessed through statistical measurements. In this manner, it is essential to have performance measures for assessing the quality of state estimation.

The SE application relies on the basic assumption that the topology of the system is known beyond any doubt [21-24]. However, in most of the real world situations, the state of some switching devices is unknown or, for some reason, the current value in the database is under suspicion. If a circuit breaker or a switch is a part of the modeled network but is not monitored by SCADA, its open/close position in the database is updated manually by the power system dispatchers. In many system maintenance jobs, after a series of manually directed switching operations, the dispatcher often forgets to update the open/close positions of these switches. The result of this situation is a topology error in the network. Model topology errors can also occur when the telemetered circuit breaker ON/OFF status is incorrect.

Correct connectivity in power network modeling is so critical to modern market and security operations that topology estimation is expected to become a standard EMS function. Therefore, topology error identification and detection algorithm using BCSE method is proposed in this thesis.

1.2 Thesis Objective

The main objective of this thesis is to improve upon BCSE method using the availability of voltage measurements with the adoption of AMI technologies in the distribution system. Original BCSE method is a suitable algorithm to solve distribution state estimation and is designed to include only power and current measurements. However, because of its complexity, voltage measurement has been ignored. Thus, the enhancing of BCSE extended to include voltage measurements in the estimation is proposed. Four combinations of

different measurement are considered to assess the impact of voltage measurements on the BCSE method.

Furthermore, some statistical techniques are presented using the results obtained from the enhancement of the BCSE method. Performance measures are adopted to evaluate the enhanced BCSE method. For these measures, Monte Carlo simulation is performed in enhancing BCSE method to compute their results by repetitive random sampling.

The third objective is to describe an approach by which the circuit breaker status error can be detected and identified in the presence of analog measurement error using the BCSE method. The use of normalized residuals from the result of the BCSE method is proposed for the detection of topology errors. Two types of topology error are considered : switching device error and shunt capacitor bank error.

For testing the revised BCSE, a reduced version of IEEE 34 node radial test feeder is used. The simulation platform used in this study is developed using C language on Microsoft Visual Studio .NET 2003.

1.3 Related Work

Distribution systems consist mainly of feeders. The feeder has characteristics such as radial, weekly meshed etc. Although the main feeder is the three-phase backbone of the circuit, branching from the mains are one or more lateral branches which can be single and two-phase rather than three-phase. Furthermore, the loads on the feeders can be single and two-phase for residential service and three-phase for industrial service. Unbalanced representation of both networks and loads is needed [7].

Distribution systems provide very few real-time measurements. The most common measurement type is power which is available only at the substation and only measures current magnitude. While few voltage magnitude measurements are obtained, they are more accurate. Because of a lack of real-time data, pseudo measurements, primarily obtained from historical load data and customer billing data are substituted. Therefore, performing high efficiency SE with minimal real-time data is a challenging task [2, 4].

A Distribution SE has a critical role in the Distribution Management System (DMS) for estimating unknown states which provide limited measurement information. For reliable and optimal DMS control, several DSE methods have been proposed. There are two main approaches in SE. one is the algorithm based on power flow [4-6] and the others are Weighted Least Square (WLS) based [1-3]. Because of the complexity of computation in the power flow approach, its use may be unwieldy in the practical power flow approach, and it may prevent use in practical power systems. In this paper, for efficient calculation, BCSE method is used [1].

BCSE is very efficient in handling line-flow and power-injection measurements for radial networks. In the original algorithm the voltage measurements were not available. But with AMI, voltage measurements will be readily available. In paper [9], the method for handling voltage measurements in BCSE method is introduced. This method converts the voltage measurements into equivalent measurements. In addition, in paper [11], an algorithm for treating power and current measurements with the BCSE method is proposed using the same technique as the voltage measurements treatment.

Statistical technique is the suggested process for determining the correctness of an SE implementation. For this purpose, Monte Carlo simulation is one of the key devices to assess SE simulation. However, the disadvantage is the computer time required to achieve the acceptable accuracy of the estimator. In general, the Monte Carlo process is repeated until interesting quantities are within the range expected. In papers [13, 19, 20], an approximate procedure that is able to determine, with very few iterations, the total number of runs required to obtain desired accuracy.

Furthermore, in [18], the statistical test used for hypothesis testing to make statistical decisions is introduced. The goal of this statistical test is to ensure the accuracy of the quality of state estimation. This paper examines a number of hypothesis testing problem settings for multivariate data. For the parametric test, Hotelling's T^2 test is used while for the nonparametric test, multivariate sign test and sign rank test are employed.

It is important to have a test method that gives assurances that the measurement increase or the type of measurement in the state estimation reflects true improvement in the performance. Regarding this issue, the paper [16, 17] introduced performance evaluation to assess the effectiveness of WLS through certain statistical measures including bias, consistency and quality of the estimates.

The SE application relies on the basic assumption that the topology of the system is known beyond any doubt. Since most of the switching in a distribution system is done manually and not telemetered, SE can help the dispatchers keep the network topology information up-to-date by detecting status changes in switches [24]. In this paper, the direct algorithm to monitor topology error is introduced.

As another approach, the use of normalized residuals from the result of the SE method is proposed for the detection of topology errors. When a topology error happens, the bus/branch model generated by the topology process is locally incorrect, causing a topological error. Unlike the parameter errors where threshold is exceeded, topology errors usually cause the state estimate to be significantly biased. As a result, the bad data detection & identification routine may erroneously eliminate several analog measurements which appear as interacting bad data, finally yielding an unacceptable state. Therefore there is a need to develop effective mechanisms intended to detect and identify these types of gross errors [25].

1.4 Thesis Outline

- **Chapter 2**

This Chapter considers the incorporation of voltage measurements in Branch Current State Estimation (BCSE) programs. Originally, the BCSE was designed to include only power and current measurements. The motivation for enhancing BCSE is that with the adoption of large scale automated meter infrastructure (AMI) technologies, voltage measurements will be available at the distribution level. Including these measurements has the potential to improve the accuracy of the state estimation. The Chapter elaborates the technical approach taken to accomplish this task, and the test results for assessment.

- **Chapter 3**

This chapter presents a statistical technique to assess the BCSE performance. Because of the statistical nature of pseudo measurements, the performance of the BCSE needs to be

assessed through statistical measurements. Some statistical measures quantified in terms of bias, consistency, and quality are adopted to evaluate the enhanced BCSE method. For statistical analysis, 300 Monte Carlo simulations are performed.

- **Chapter 4**

This Chapter describes the topology error identification algorithm in a Branch Current State Estimation (BCSE) program. BCSE application relies on the basic assumption that the topology of the system is known beyond any doubt. However, in most real world situations, the state of some switching devices is unknown or, for some other reason, the current value in the database is under suspicion. In this Chapter, two approaches are described to address topology identification problem in the scope of state estimation.

- **Chapter 5**

Overall conclusion and future work.

1.5 Glossary

In this master thesis, the following abbreviations and terms are used :

AMI : Automated Meter Infrastructure is an intelligent technology that includes metering systems capable of recording and reporting energy consumption and other measurements at more frequent intervals than the customer's billing cycle.

BCSE : Branch Current State Estimation algorithm, like conventional node-voltage-based SE methods, is based on the weighted least square approach. Rather than using the node voltages as the system state, the method used the branch currents as the state.

CB : Circuit Breaker is an automatically-operated electrical switch designed to protect an electrical circuit from damage caused by overload or short circuit.

EMS : Energy Management System, which is used in the monitoring and control of the power generation and transmission.

IEEE : Institute of Electrical and Electronics Engineering

LF : Load Flow analysis is in the planning the future expansion of power systems as well as in determining the best operation of existing systems. The principal information obtained from the load flow study is the magnitude and phase angle of the voltage at each bus and the real and reactive power flowing in each line.

P.U. : Per Unit system is the expression of system quantities as fractions of a defined base unit quantity in electrical engineering in the field of power transmission.

SCADA : Supervisory Control And Data Acquisition systems are used to monitor and control power system in a wide range of applications like power station control, transmission, distribution automation.

SCB : Shunt Capacitor Banks are mainly installed to provide capacitive reactive compensation/power factor correction.

SE : State Estimation as a mathematical analysis tool acts as a noise filter to eliminate errors in data. In practices, other conveniently measured quantities such as P, Q line flows are

available, but they cannot be used in conventional power-flow calculations. these limitations can be removed by state estimation.

SW : Switch is an electrical component which can break an electrical circuit, interrupting the current or diverting it from one conductor to another.

WLS : Weighted Least Square state estimation algorithm. Commonly, this algorithm is based on the assumption that the measurement errors have normal distributed noises with known variances and zero means.

Chapter 2

Including Voltage Measurements in Branch Current State Estimation for Distribution Systems

2.1 Overview

BCSE is tailored to perform state estimation on distribution networks. There are a number of significant differences in the characteristics of typical distribution networks compared to typical transmission networks.

Distribution systems consist mainly of feeders. Feeders are mainly radial, but have laterals that can be single or two-phase rather than three-phase. Furthermore, loads on the feeders are more distributed than that of the transmission and these loads can be single and two-phase (for residential service) or three phase (for commercial and industrial service). Therefore, distribution systems are unbalanced in nature. Also, feeder line sections are usually short, un-transposed, and have high r/x ratios.

So, to obtain the consistent and accurate data, new methods are proposed for monitoring and operation of distribution system. One of the approaches is power flow based [2] and the others [3,4] are extensions of the conventional state estimation (SE) method for three-phase analysis. Although SE is preferred over the power flow approach, its computational complexity may prevent its use in practical applications. In this paper, for efficient calculation A branch-current-based three-phase SE (BCSE) method [1] is used.

The method is computationally more efficient and more insensitive to line parameters than the conventional node-voltage-based SE methods. The method has superior performance both in terms of computational speed and memory requirements. Furthermore, the method is insensitive to line parameters, which improves both its convergence and bad data handling performance. The BCSE method, like conventional node-voltage-based SE methods, is based on the weighted least square (WLS) approach.

BCSE is very efficient in handling line-flow and power-injection measurements for radial networks. However, handling voltage measurements increases the complexity of the algorithm, as using the branch currents as state variables makes the treatment of voltage measurements difficult.

Power system state estimation relies on measurement data obtained from substations and on topological model. A practical SE must possess the ability to handle power, current and voltage measurements efficiently. Although a distribution system does not have an overwhelming number of voltage measurements, they are often found in the telemetry of a distribution system and are more accurate than the other available real or pseudo

measurements. So, one of the main focus of this project is to develop a BCSE method with power, current magnitude and voltage magnitude measurements.

2.2 State Estimation

State estimation (SE) as a mathematical analysis tool acts as a noise filter to eliminate errors in data. The acquired data always contains inaccuracies which are unavoidable since physical measurements cannot be entirely free of random errors or noise. Because of noise, the true values of physical quantities are never known and we have to consider how to calculate the best possible estimates of the unknown quantities. The method of least squares is often used to “best fit” measured data relating two or more quantities.

2.2.1 The Weighted Least Square (WLS) Approach

The SE method is based on the weighted least square (WLS) approach. WLS is useful for estimating the values of model parameters when the response values have differing degrees of variability over the combinations of the predictor values. Mathematically, WLS find the best estimates which are chosen as those which minimize the weighted sum of the squares of the measurements errors. WLS state estimation tries to find a system state, represented by \hat{x} , by solving the following optimization problem:

$$f = \min_x J(x) = \sum_{i=1}^m w_i (z_i - h_i(x))^2 = [z - h(x)]^T W [z - h(x)] \quad (2.1)$$

where w_i and h_i represent the weight and the measurement function associated with measurement z_i respectively. For the solution of this optimization problem gives the estimated state \hat{x} which must satisfy the following optimality condition:

$$\Delta f(x) = \frac{\partial f}{\partial x_i} = 0 \quad (2.2)$$

$$\begin{aligned} \frac{\partial f}{\partial x_i} &= 2 \cdot \sum_{i=1}^m w_i (z_i - h_i(x)) \cdot \frac{\partial h_i(x)}{\partial x_i} = 0 \\ &= \sum_{i=1}^m w_i (z_i - h_i(x)) \cdot \frac{\partial h_i(x)}{\partial x_i} = [H^T] [W] [z - h(x)] = 0 \end{aligned} \quad (2.3)$$

where $H(x) = \frac{\partial h(x)}{\partial x}$ is the Jacobian matrix of the measurement function $h(x)$. Since $h(x)$ is

usually non-linear, the solution is obtained by an iterative method. The iterative method involves solving the linear equation of the following type at each iteration to compute the correction $x^{k+1} = x^k + \Delta x^k$.

$$\Rightarrow G(\hat{x}^{(k+1)} - \hat{x}^{(k)}) = H^T W [z - h(x^{(k)})] \quad (2.4)$$

where $\frac{\partial d(\hat{x})}{\partial x} = -H^T W H = -G$

is the jacobian of the optimality condition equation :

$$d(\hat{x}) = H^T W [z - h(x)] \quad (2.5)$$

One of the main challenges in implementing this approach for SE in distribution feeders is incorporating the unbalanced nature of distribution feeders into the problem. The most important of these issues is the representation of feeders which will be discussed in the next subsection.

2.2.2 Feeder Representation

In general, main feeders are three-phase, however some laterals can be two-phase or single-phase. The lines are usually short and un-transposed. Loads can be three-phase, two-phase or single-phase (like residential customers). Therefore it is desirable to use a three phase model as also recommended for power flow analysis of feeders. A three-phase line model takes into account the magnetic coupling between the phases in lines, which for a line section $l, l = 1 \cdots b$, such as the one shown in Fig.1, is of the following form

$$\begin{bmatrix} V_{r,1} \\ V_{r,2} \\ V_{r,3} \end{bmatrix} = \begin{bmatrix} V_{S,1} \\ V_{S,2} \\ V_{S,3} \end{bmatrix} - l \begin{bmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{31} & z_{32} & z_{33} \end{bmatrix} \begin{bmatrix} I_{l,1} \\ I_{l,2} \\ I_{l,3} \end{bmatrix} \quad (2.6)$$

or $V_r = V_s - Z_l I_l$

where $Z_l = g_l Z$ is the line impedance matrix and g_l is the line length. Note that this equation is written for the assumed branch current direction shown in Fig 1, and the phases are numbered as $\varphi = 1, 2, 3$ rather than labeled as a,b,c.

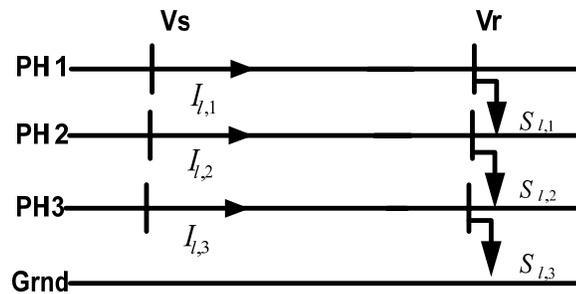


Figure 1. A three-phase line section

2.2.3 Branch-Current-Based on State Estimation

The branch-current-based SE method, like conventional node-voltage-based SE methods, is based on the weighted least square (WLS) approach. Rather than using the node voltages as the system state x , the method uses the branch currents and solves the following WLS problem to obtain an estimate of the system operating point defined by the system state x :

$$\min_x J(x) = \sum_{i=1}^m w_i (z_i - h_i(x))^2 = [z - h(x)]^T W [z - h(x)] \quad (2.7)$$

where w_i and $h_i(x)$ represent the weight and the measurements function associated with measurement z_i respectively. For the solution of this problem the conventional iterative method is adapted by solving following normal equations at each iteration to compute the correcting $x^{k+1} = x^k + \Delta x^k$

$$[G(x^k)]\Delta x^k = H^T(x^k)W[z - h(x^k)] \quad (2.8)$$

where

$$G(x) = H^T(x)WH(x) \quad (2.9)$$

is the gain matrix and H is the Jacobian of the measurement function $h(x)$.

Hence the only difference between the node voltage based SE and BCSE is the measurement functions associated with the type of measurements to be processed. To illustrate these functions for BCSE, consider two cases

Case 1: power flow (P , Q) or current magnitude (I) measurements on a line section of a feeder

Case2: voltage measurement (V) at a node of a feeder.

Case 1 – power flow (P, Q) or current magnitude (I) measurements:

Power measurements in BCSE are converted to equivalent complex current measurement by using the current estimate of the node voltage:

$$I_r^m = \frac{P^m V_r + Q^m V_x}{V_r^2 + V_x^2}, \quad I_x^m = \frac{P^m V_x - Q^m V_r}{V_r^2 + V_x^2} \quad (2.10)$$

Hence the resulting measurement functions are linear as the state variables are the complex branch currents,

$$I_l = I_{lr} + jI_{lx}, \quad l=1..n \quad (2.11)$$

The current magnitude measurements, on the other hand are non-linear, as

$$I_l = \sqrt{I_{lr}^2 + I_{lx}^2} \quad (2.12)$$

The current magnitude measurements introduce coupling terms between the real and imaginary parts. For example, the current measurement I_l^m introduces the following non-zero elements into the measurement Jacobian H

$$\frac{\partial h_l^m}{\partial I_r} = \cos \phi, \quad \frac{\partial h_l^m}{\partial I_x} = \sin \phi \quad (2.13)$$

where $\phi_{l,\varphi} = \tan^{-1}(I_{xl,\varphi} / I_{rl,\varphi})$

Case 2 –Voltage magnitude (V) measurements:

A voltage at the node t of a radial feeder Vt is the voltage at the substation minus the voltage drop on the line sections between the substation and this node, and hence, the measurement function for the voltage measurement Vt can be written in terms of the branch currents as:

$$V_t^m = V_S - \sum Z_l I_l \quad (2.14)$$

The voltage magnitude measurements introduce coupling terms between the phases of branch currents and the real and imaginary parts of branch currents. The voltage measurement V_t introduces the following non-zero elements into the measurement Jacobian H

$$\frac{\partial h_{V_l, \phi}^m}{\partial I_{rl}} = X_l \sin \phi_{l, \phi} - R_l \cos \phi_{l, \phi}, \quad \frac{\partial h_{V_l, \phi}^m}{\partial I_{xl}} = -R_l \sin \phi_{l, \phi} - X_l \cos \phi_{l, \phi} \quad (2.15)$$

where $Z_l = R_l + jX_l$ is line impedance and $V_S = V_{SR} + jV_{SX}$ is substation voltage

$$\phi = \text{Tan}^{-1}(V_{SR} - \text{real}(\sum_{j=1}^m Z_j I_j) / V_{SX} - \text{imag}(\sum_{j=1}^m Z_j I_j)) \quad (2.16)$$

Hence, both the Jacobian H and the gain matrix G are revised to include voltage measurements in BCSE.

2.2.4 BCSE Algorithm

BCSE constructs the Jacobian and gain matrices and solves the update equations of () iteratively. The algorithm involves the following steps at each iteration k :

Step 1 - Given the node voltage V^{k-1} , convert power measurements into equivalent current measurements

Step 2 - Use current measurements to obtain an estimate of branch currents $\hat{x}_\phi^k = [\hat{I}_{r, \phi}^k \quad \hat{I}_{x, \phi}^k]$

by solving the update equations (1) for each phase $\phi = 1, 2, 3$

Step 3: Given the branch currents, update the node voltages V^k by the forward sweep procedure.

Step 4: Check for convergence; if two successive updates of branch currents are less than a convergence tolerance then stop, otherwise go to step 1

2.3 State Estimation Test Results

For testing the revised BCSE with voltage measurements, a test feeder is used. The test feeder is a 34 bus, 23kV, 3-phase radial IEEE test feeder [5]. A reduced version of this test feeder is used to facilitate debugging and assessment. A one-line diagram of the feeder is given in Fig. 2 with the nodes renumbered to make the illustration of the results easier. The feeder is predominantly three-phase with some single-phase laterals and has both spot and distributed loads. For test purpose, distributed line section loads are lumped equally at terminal nodes of the line section. The nominal load data is taken as the actual load and the power flow results are used to determine the correct measurements for this load. The minimum voltage for this loading is

$$V_{\min} = V_{21,a} = 0.9402 \angle -3.057$$

which indicates a heavy loading condition on the feeder. The line data used is given in [6] with line r/x ratios varying between 0.57 and 1.37.

For SE, the available measurements assumed are given in the figure also: voltage and power flow at the substation, current measurements on branches 6-7, and voltage measurements on nodes 8, 10 and 17.

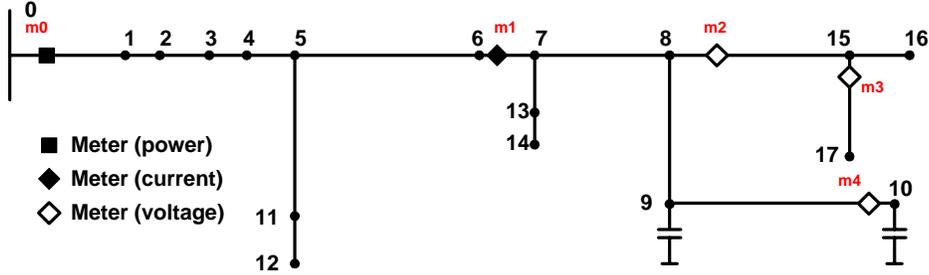


Figure 2. One-line diagram of reduced feeder

To generate measurement data for testing purpose, first the actual measurements have been obtained by running a power flow for the given load. Then measurement error was added to the actual measurements.

$$Z = Z^a \pm e_z \quad (2.17)$$

where Z^a is actual data and e_z is the measurement error. The forecasted load data is created by perturbing the actual load data by adding error of 30%. The power and current magnitude measurement errors are selected from Normal distribution with a standard deviation σ of 0.0233 (accuracy is 7% of their measured values). The voltage measurements data are generated by adding measurement error with a standard deviation σ of 0.0067 (2% measurement error). The weights are obtained by using standard deviation σ of measurements error.

$$w_i = \frac{1}{\sigma_i^2} \quad (2.18)$$

where w_i and σ_i represent weight associated with measurement z_i and standard deviation of measurement error, respectively. The revised algorithm was implemented using the C language on Microsoft Visual Studio .NET 2003. To assess the impact of voltage measurements on the state estimation, four cases have been considered:

Case 1: forecasted load data.

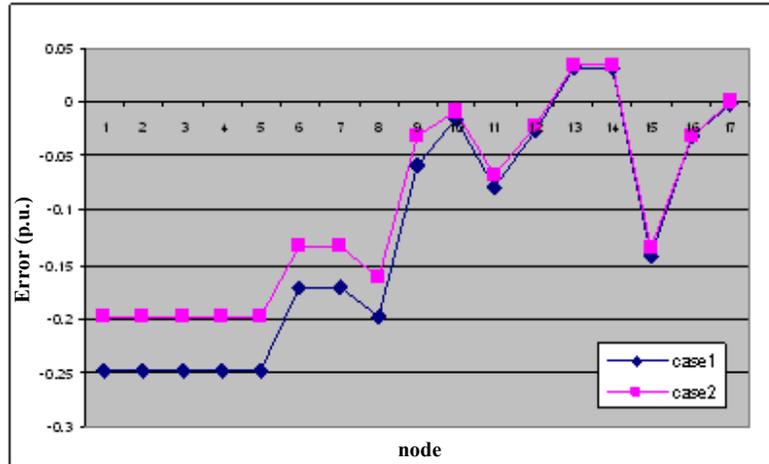
Case 2: The same measurements as in Case 1 plus three voltage measurements m2-m4 from the feeder nodes.

Case 3: Power measurement at the substation (both real and reactive), indicated as m_0 in Fig. 2, plus a current measurement on the feeder, m_1 , and forecasted load data.

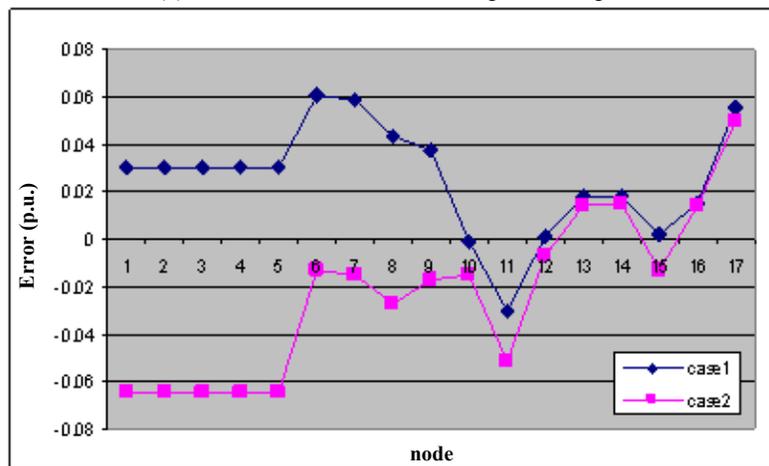
Case 4: The same measurements as in Case 3 plus three voltage measurements m2-m4 from the feeder nodes.

The enhanced BCSE has run for these four cases. For these simulations, the addition of voltage measurements do not affect the convergence of the method; it takes about 3-6 iterations for the solution to converge. This indicates that BCSE's computational performance does not degrade with the addition of voltage measurements.

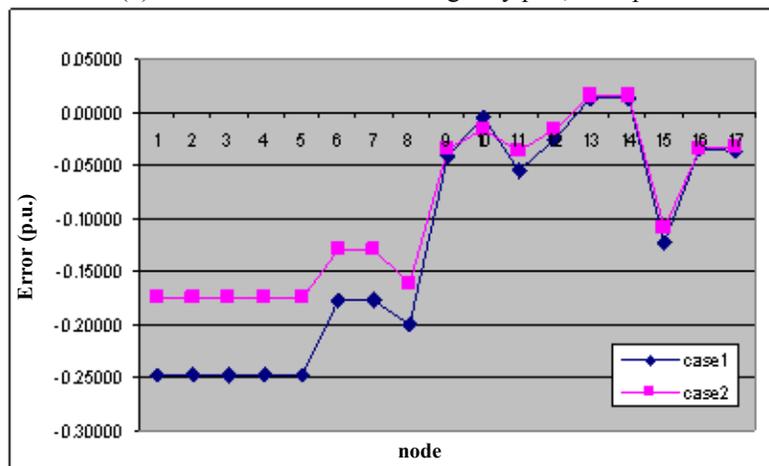
Test results are given in the Appendix I. A summary of the results are given in Fig. 3-4 for the four cases considered. The figures show the error in the estimated state ($x = [I_r, I_x]$) of phase a branch currents on the feeder. Fig. 3 compares the results for Case 1 and Case 2. These results indicate that adding voltage measurements decrease the error in branch current estimates. Note that the improvement in the estimation is more on the real part of the current than the imaginary part. Since, in this case the currents have small imaginary component, the overall reduction of the error in the current magnitude is considerable especially towards the substation end of the feeder. Hence, these results indicate that having voltage measurements helps improve the estimation over the conventional one that is based on the forecasted load only (case 1).



(a) Error in branch current real part, I_r in p.u.

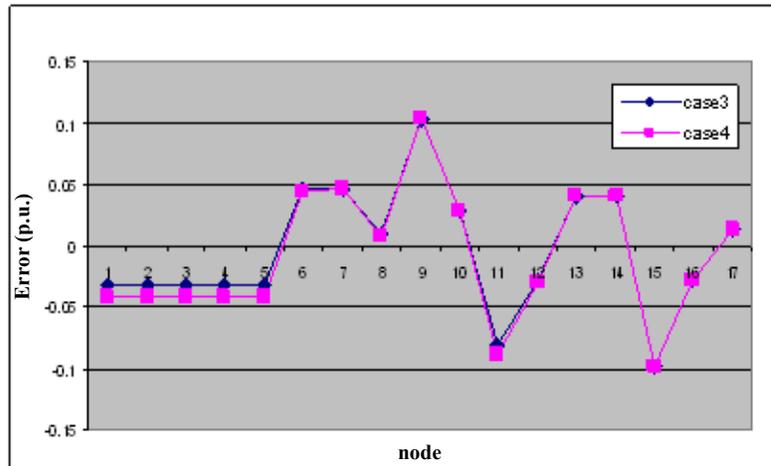


(b) Error in branch current imaginary part, I_x in p.u.

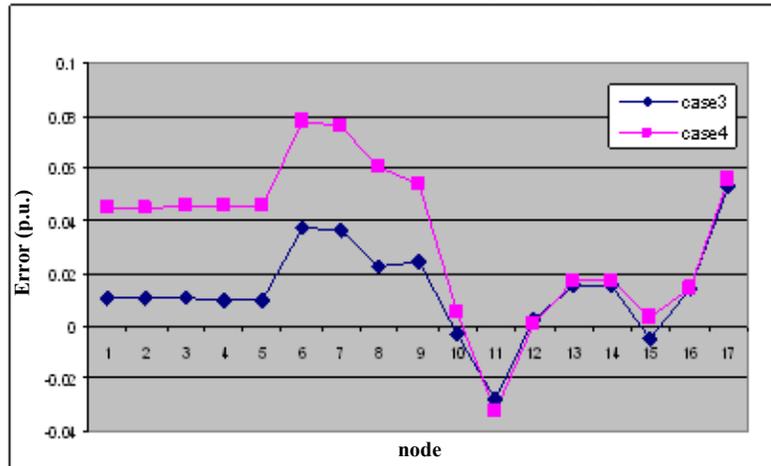


(c) Error in branch current magnitude in p.u.

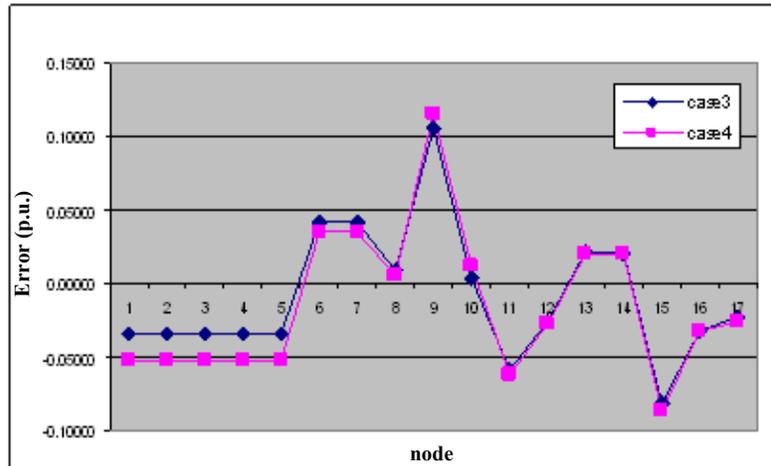
Figure 3. Branch Current Estimation for Case 1 and Case 2



(a) Error in branch current real part, I_r in p.u.



(b) Error in branch current imaginary part, I_x in p.u.



(c) Error in branch current magnitude in p.u.

Figure 4. Branch Current Estimation for Case 3 and Case 4

Figure 4 compares the results for Case 3 and Case 4. Note that these two cases illustrate the effect of adding voltage measurements to a system which has some limited power and current measurement from the feeder (Case 3). The results indicate that in this case adding voltage measurements does not improve the estimation as much as it did in the previous case. The differences between the two cases are not statistically significant.

2.4 Conclusions

This chapter shows that the basic BCSE can be extended to include voltage measurements in the estimation. The initial test results indicate that the impacts of the voltage measurements on the estimated values are marginal. These results are based on the simulated measurement with assumed accuracy on especially the load forecast values. The method however now allows including the voltage measurements in actual applications in which the load data may not be as accurate as assumed.

Chapter 3

Performance of Branch Current State

Estimation Method

3.1 Overview

In the previous chapter, the enhancement of the original BCSE to include voltage measurements is described. Because of the statistical nature of pseudo measurements, the performance of the BCSE needs to be assessed through statistical measurements. Thus, this chapter looks at the performance of enhanced BCSE method in the presence of measurement noises through Monte Carlo simulation.

Singh and others have proposed some statistic measures quantified in terms of bias, consistency, and quality through Monte Carlo simulation in [17] for assessing the quality of state estimation. In this chapter, these performance measures are adopted to evaluate the enhanced BCSE method.

3.2 Monte Carlo Simulation

Monte Carlo simulation is used to estimate expected values of random variables when it is infeasible or impossible to compute an exact result with a deterministic algorithm. Monte

Carlo methods are a class of computational algorithms that rely on repeated random sampling to compute their results. Monte Carlo methods are often used when simulating physical and mathematical systems. Because of their reliance on repeated computation and random or pseudo-random numbers, Monte Carlo methods are most suited to calculation by a computer. Monte Carlo methods tend to be used when it is infeasible or impossible to compute an exact result with a deterministic algorithm.

Monte Carlo simulation methods are especially useful in studying systems with a large number of coupled degrees of freedom. Furthermore, Monte Carlo methods are useful for modeling phenomena with significant uncertainty in inputs, such as the calculation of risk in business. Furthermore, the basic characteristics of Monte Carlo simulation is described :

- Monte Carlo simulation allows several inputs to be used at the same time to create the probability distribution of one or more outputs.
- Different types of probability distributions can be assigned to the input of the model. When the distribution is unknown, the one that represents the best fit could be chosen.
- The use of random numbers characterized Monte Carlo simulation as a stochastic method. The random numbers have to be independent; no correlation should exist between them.
- Monte Carlo simulations generate the output as a range instead of a fixed value and shows how likely the output value is to occur in the range.

In general, the Monte Carlo simulation involves the following series of steps [15].

Step 1 – Construct a simulated “universe” of some randomizing mechanism whose composition is similar to the universe whose behavior we wish to describe and investigate. The term “universe” refers to the system that is relevant for a single simple event.

Step 2 – specify the procedure that produces a pseudo-sample which simulates the real-life sample in which we are interested. That is, specify the procedural rules by which the sample is drawn from the simulated universe. These rules must correspond to the behavior of the real universe in which we are interested. To put it another way, the simulation procedure must produce simple experimental events with the same probabilities that the simple events have in the real world.

Step 3 – If several simple events must be combined into a composite event, and if the composite event was not described in the procedure in step 2, describe it now.

Step 4 – Calculate the probability of interest from the tabulation of outcomes of the resampling trials.

3.2.1 Monte Carlo Simulation Sample Size [15]

Monte Carlo simulation is used to estimate quantities in such as :

- The bias and variance of an estimator
- The percentiles of a test statistic or pivotal quantity
- The power function of a hypothesis test
- The mean length and coverage probability of a confidence interval

In this manner, Monte Carlo simulation sample size can be defined depending on which quantities are interested [15].

- Bias Estimation

If the N observations in a sample are denoted by x_1, x_2, \dots, x_N , the sample mean is

$$\bar{x} = \frac{\sum_{i=1}^N x_i}{N} \quad (3.1)$$

and the standard deviation of sample is $\frac{\sigma}{\sqrt{N}}$ where $\sigma^2 = \text{var}(x)$. So given a guess of σ , say $\tilde{\sigma}$, and an acceptable value, say d , for the standard deviation of our estimate, the sample size for bias estimation is defined as :

$$N = \frac{\tilde{\sigma}^2}{d^2} \quad (3.2)$$

- Variance Estimation

The sample variance is :

$$S_{N-1}^2 = \frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N-1} \quad (3.3)$$

For large N, the approximate variance of the sample variance will be close to $\sigma^4(Kurt(x)-1)/N$, where $\sigma^2 = \text{var}(x)$ and $Kurt(x)$ is the kurtosis of the distribution of x . Many estimators are approximately normal provided the sample size is not too small. So, the approximate standard deviation of the variance estimation is $\sqrt{2/N} \cdot \sigma^2$. For acceptable d , the sample size for variance estimation is defined as :

$$N = \frac{2\sigma^4}{d^2} \quad (3.4)$$

- Power Estimation

For a new test procedure of the form “reject the null hypothesis if $T > c_\alpha$ ”, the power at a particular alternative by

$$\widehat{pow} = \frac{1}{N} \sum_{i=1}^N I(T_i > c_\alpha) \quad (3.5)$$

where T_i is the test statistic for the i -th Monte Carlo sample, c_α is a given critical values, and I is the indicator function having value 1 if $T_i > c_\alpha$ and 0 otherwise. This is binomial sampling, and the worst variance of our estimate (occurring at power $1/2$) is given by $1/(4N)$.

Setting $d = 1/(2\sqrt{N})$ yields

$$N = \frac{1}{4d^2} \quad (3.6)$$

- Confidence Intervals

Coverage probability and average confidence interval length are both important quantities that should be reported whenever studying confidence intervals. Obviously we would like intervals that achieve the nominal $1-\alpha$ coverage (like 95%) and are short on average. For sample size considerations, we need a preliminary estimate $\tilde{\sigma}$ of the standard deviation of the lengths and an acceptable value d for the standard error of our estimate of average length.

Then just Eq. 3.3 is obtained. For coverage estimation or one-sided error estimation, it is inverted to $d = \sqrt{\frac{\alpha(1-\alpha)}{N}}$ where d is the acceptable standard deviation for coverage estimate,

to get

$$N = \frac{\alpha(1-\alpha)}{d^2} \quad (3.7)$$

3.2.2 Monte Carlo Simulations on BCSE Method

Monte Carlo simulation procedure is described in BCSE Method below :

Step 1 – BCSE method tries to find a system state, represented by \hat{x} , by minimizing the weighted sum of the squares of the measurements errors.

Step 2 – First the actual measurements have been obtained by running a power flow for the given load. Then measurement error obtained from random generator based on Normal distribution was added to the actual measurements.

$$Z = Z^a \pm e_z \quad (3.8)$$

where Z^a is actual data and e_z is the measurement error. The forecasted load data is created by perturbing the actual load data by adding error of 30%. The power and current magnitude measurement errors are selected from Normal distribution with a standard deviation σ of 0.0233 (accuracy is 7% of their measured values). The voltage measurements data are generated by adding measurement error with a standard deviation σ of 0.0067 (2% measurement error).

Step 3 – The sample size is defined in order to achieve acceptable results. In this thesis, 300 Monte Carlo simulations have been chosen.

Step 4 – To calculate the probability of the interest, the estimated states ($x = [I_r, I_x]$) of branch currents are obtained and then, hypothesis testing is adopted to test the bias in estimated state variables. Furthermore, the performance measures which are bias, consistency and overall quality are examined.

In order to achieve acceptable results obtained from Monte Carlo simulation, the sample size is defined in the previous section. However, this determination can be only adopted in univariate statistics. Since we have N state ($x = [I_r, I_x]$) of branch currents in

BCSE method, we need another sample size determination. In BCSE method, we performed both parametric and nonparametric test for the bias test. For parametric tests, Hotelling's T^2 test is used and as with the nonparametric test, multivariate sign test and sign rank tests are applied. In Hotelling's T^2 test, sample size is enough if the following condition is satisfied :

$$\left| \frac{T^2}{p} - 1 \right| \approx 0 \quad (3.9)$$

where T^2 and p is Hotelling's T^2 test statistics value and number of state, respectively.

Similarly, the enough sample size can be defined in multivariate sign test and sign rank tests to achieve acceptable accuracy of the result.

For the multivariate sign test, the following condition is satisfied.

$$\left| \frac{Q^2}{p} - 1 \right| \approx 0 \quad (3.10)$$

where Q^2 and p is sign test statistics value and number of state, respectively.

For the multivariate sign rank test, the following condition is used to check the suitable sample size.

$$\left| \frac{U^2}{p} - 1 \right| \approx 0 \quad (3.11)$$

where U^2 and p is sign rank test statistics value and number of state, respectively.

3.3 Bias in BCSE Method

Since we have N state ($x = [I_r, I_x]$) of branch currents, we need to perform multivariate tests in order to ascertain the quality of BCSE method. Oja and Randles have examined a

number of hypothesis testing problem settings for multivariate data in [18]. In this chapter, hypothesis testing is adopted to test the bias in estimated state variables obtained from BCSE method.

3.3.1 Multivariate Statistical Tests

One of the most important problems in the area of multivariate analysis is to obtain the mean vector of the given sample. After that, we can get rough information about the population where the sample is surveyed. Often people may hope to test the hypothesis whether the sample mean vector equals to the specified value in advance. For this, there are two typical approaches, parametric test and nonparametric test. Most hypothesis tests are based on the assumption that random samples are from normal populations. This is called a parametric test because it is based on a particular parametric family of distributions. Alternately, these procedures are not distribution-free because they depend on the assumption of normality. The primary advantage of the parametric test is that it has greater statistical power to detect differences. The other approach is called a nonparametric test which has no assumptions about the distribution of the underlying population other than that it is continuous. One of the advantages is that the data need not be quantitative but can be categorical or rank data. In SE problems, the assumptions for the parametric test may be difficult or impossible to justify so that both parametric and nonparametric tests are performed. For parametric tests, Hotelling's T^2 test is used and as with the nonparametric test, multivariate sign test and sign rank tests are applied.

3.3.2 Multivariate Parametric Tests [18]

Let x_1, x_2, \dots, x_N be independent and identically distributed from $F(x - \theta)$, where $F(\cdot)$ represents a continuous p -dimensional distribution “located” at the vector parameter $\theta = (\theta_1, \theta_2, \dots, \theta_p)^T$. The hypothesis whether the sample mean vector equals to the vector specified in advance is :

$$H_0 : \theta = \mu \qquad H_a : \theta \neq \mu \qquad (3.12)$$

Note the above hypothesis is equivalent to :

$$H_0 : \theta - \mu = 0 \qquad H_a : \theta - \mu \neq 0 \qquad (3.13)$$

Hotelling's T^2 test statistics involves the following calculations.

$$T^2 = N(\bar{x} - \mu)' S^{-1} (\bar{x} - \mu) \qquad (3.14)$$

where

\bar{x} : the mean vector of sample, $ave\{x_i\}$

N : the sample size.

S : the sample covariance matrix, $ave\{(x_i - \bar{x})(x_i - \bar{x})^T\}$

μ : the vector given in the hypothesis.

$\frac{N-1}{N-p} T^2$ has F distribution with degrees of freedom, p and $N-p$. Thus given significant

level is α , the null hypothesis is rejected when :

$$T^2 \geq \frac{N-1}{N-p} F_{p, N-p}(\alpha) \qquad (3.15)$$

where $F_{v_1, v_2}(\alpha)$ is the upper α th quantile of an F distribution with v_1 and v_2 degrees of freedom.

Moreover, the p-value for this test statistics is :

$$p - value = 1 - F\left(\frac{N - p}{(N - 1)p} \cdot T^2, p, N - p\right) \quad (3.16)$$

3.3.3 Multivariate Nonparametric Tests [18]

For the nonparametric test, multivariate sign test and signed rank test are used. As for the sign test, spatial sign function is defined as :

$$S_i = S(A_x x_i) \text{ for } i = 1, \dots, N \quad (3.17)$$

where A_x is transformation proposed by Tyler. Basically, Tyler's shape matrix V_x is positive definite symmetric $p \times p$ with trace equals p . Thus, for any A_x with $A_x^T A_x = V_x^{-1}$

$$p \cdot \text{ave}\{S_i S_i^T\} = I_p \quad (3.18)$$

Tyler's transformation matrix A_x makes the sign covariance matrix equal to $\frac{1}{p} I_p$, the variance-covariance matrix of a vector that is uniformly distributed on the unit p sphere. Due to the fact that S_i and $-S_i$ give the same contribution to the sample covariance matrix, A_x could be considered as the method to make the direction of transformed data points $\pm A_x x_i$.

The sign test rejects null hypothesis for large

$$Q^2 = Np \bar{S}^T \bar{S} = Np \|\bar{S}\|^2 \quad (3.19)$$

For large sample size, underlying distribution is directionally symmetric and null hypothesis holds, Q^2 has approximate χ_p^2 . Therefore, we should reject null hypothesis in favor of alternative hypothesis with the condition :

$$Q^2 \geq \chi_p^2 \quad (3.20)$$

For the multivariate rank test, we first use the signs of transformed differences :

$$S_{ij} = S(A_x(x_i - x_j)) \quad (3.21)$$

This makes the centered rank :

$$R_i = ave_j(S_{ij}) \quad (3.22)$$

and the average of R_i is zero. In multivariate case, the data based transformation A_x is selected to make the rank procedure affine invariant. So the transformation is chosen to satisfy the follow :

$$p \cdot ave(R_i R_i^T) = ave(R_i^T R_i) I_p \quad (3.23)$$

This transformation then leads the rank covariance matrix equivalent to a number times the identity matrix, that is :

$$ave(R_i R_i^T) = \left(\frac{c_x^2}{p} \right) I_p \quad (3.24)$$

Through the theoretical analysis, we get the test statistics as :

$$U^2 = \frac{Np}{4c_x^2} \left\| ave(S(A_x(x_i + x_j))) \right\|^2 \quad (3.25)$$

If random sample is form and elliptically symmetric distribution with symmetry center $\theta = 0$, then we have U^2 has an approximate chi-square distribution with degree of freedom p . So we would reject the null hypothesis if

$$U^2 \geq \chi_p^2(\alpha) \quad (3.26)$$

3.4 Performance of BCSE Method

The performance measures are bias, consistency and overall quality. These concepts of statistical techniques are illustrated and used in this chapter [17].

3.4.1 Bias

State vector, \hat{x} is an unbiased estimate of true state vector, x_t if the expected value of \hat{x} is equal to x_t .

$$E(\hat{x}) = x_t \quad (3.27)$$

This is equivalent to saying that the mean of the probability distribution of \hat{x} is equal to x_t .

3.4.2 Consistency

If \hat{x}_N is an estimator of x_t based on a random sample of N observations, \hat{x}_N is consistent for x_t in this condition :

$$\lim_{N \rightarrow \infty} P(|\hat{x}_N - x_t| < \varepsilon) = 1 \quad (3.28)$$

Thus, consistency is a large-sample property, describing the limiting behavior of \hat{x}_N as N tends to infinity. It is usually difficult to prove consistency using the above definition. Since there are no well established tests for consistency, we have not performed the test in BCSE.

3.4.3 Quality

In the previous two measures, the mean of the estimate is the interesting quantity. On the other hand, quality measures the degree of its variance. In other words, if the variance is large, it means poor quality. Quality is defined as :

$$Q_{trace} = \ln\left(\frac{1}{tr(P_x)}\right) \quad (3.29)$$

where $P_x = E[(x_t - \hat{x})(x_t - \hat{x})^T]$ is state error covariance matrix. The trace of P_x is the numerical sum of the variance of estimates.

3.5 Test Results

For testing the revised BCSE with voltage measurements, a test feeder is used. The test feeder is a 34 bus, 23kV, 3-phase radial IEEE test feeder [5]. A reduced version of this test feeder is used to facilitate debugging and assessment. A one-line diagram of the feeder is given in Fig. 5 with the nodes renumbered to make illustration of the results easier. The feeder is predominantly three-phase with some single-phase laterals and has both spot and distributed loads. For test purposes, distributed line section loads are lumped equally at terminal nodes of the line section. The nominal load data are taken as the actual load and the

power flow results are used to determine the correct measurements for this load. The minimum voltage for this loading is

$$V_{\min} = V_{21,a} = 0.9402 \angle -3.057$$

which indicates a heavy loading condition on the feeder. The line data used are given in [6] with line r/x ratios varying between 0.57 and 1.37.

For SE, the available measurements assumed are given in the figure also: voltage and power flow at the substation, current measurements on branches 6-7, and voltage measurements on nodes 8, 10 and 17.

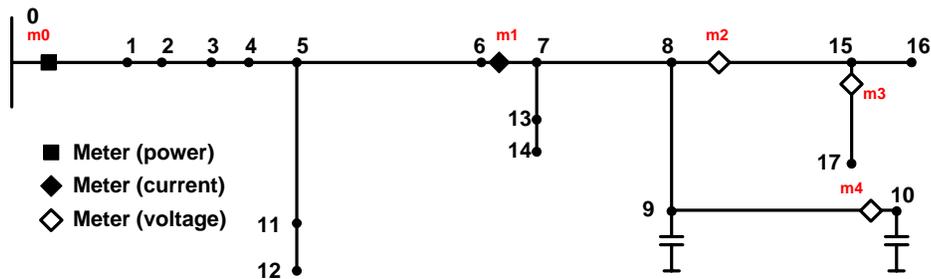


Figure 5. One-line diagram of reduced feeder

The revised algorithm was implemented using C language on Microsoft Visual Studio .NET 2003 and for result analysis using MATLAB 7.0. Convergence tolerance is 10^{-3} and maximum iteration is 10. For testing, four different cases were considered :

Case 1: forecasted load data.

Case 2: The same measurements as in Case 1 plus three voltage measurements m2-m4 from the feeder nodes.

Case 3: Power measurement at the substation (both real and reactive), indicated as m0 in Fig. 5, plus a current measurement on the feeder, m1, and forecasted load data.

Case 4: The same measurements as in Case 3 plus three voltage measurements m2-m4 from the feeder nodes.

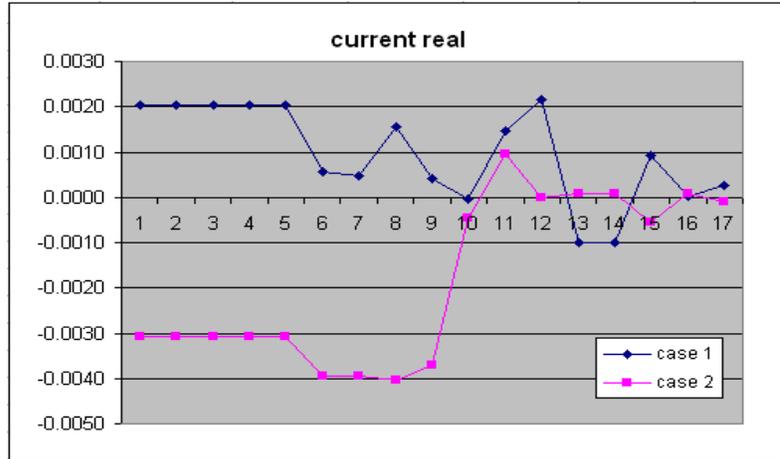
The enhanced BCSE has run for these four cases. For these simulations, the addition of voltage measurements do not affect the convergence of the method; it takes about 3-6 iterations for the solution to converge. This indicates that BCSE's computational performance does not degrade with the addition of voltage measurements.

Table 1. A summary of branch current real part, Ir in p.u

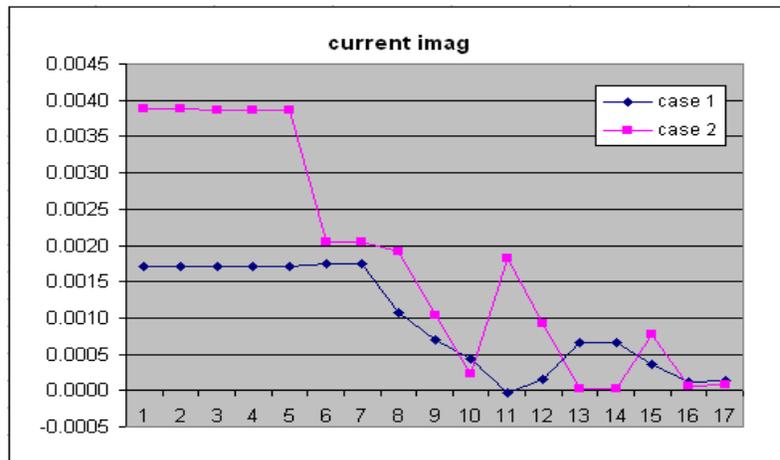
In	case1			case2			case3			case4		
	Mean	Std	Error									
1	5.0987	0.2256	0.0046	5.1079	0.2013	-0.0046	5.0974	0.0909	0.0059	5.1042	0.0881	-0.0009
2	5.0987	0.2256	0.0046	5.1079	0.2013	-0.0046	5.0974	0.0909	0.0059	5.1041	0.0882	-0.0008
3	5.0987	0.2256	0.0046	5.1079	0.2013	-0.0046	5.0974	0.0909	0.0059	5.104	0.0882	-0.0007
4	5.0987	0.2256	0.0046	5.1079	0.2013	-0.0046	5.0974	0.0909	0.0059	5.1039	0.0883	-0.0006
5	5.0987	0.2256	0.0046	5.1079	0.2013	-0.0046	5.0974	0.0909	0.0059	5.1038	0.0883	-0.0005
6	3.3206	0.1766	-0.0031	3.3192	0.1611	-0.0017	3.3131	0.0654	0.0044	3.3182	0.0632	-0.0007
7	3.2893	0.1764	-0.0032	3.2878	0.1611	-0.0017	3.2816	0.0649	0.0045	3.2869	0.0636	-0.0008
8	2.9025	0.1727	-0.0039	2.8994	0.1594	-0.0008	2.8931	0.073	0.0055	2.8963	0.0684	0.0023
9	1.7116	0.1615	-0.0049	1.7016	0.1426	0.0051	1.6951	0.09	0.0116	1.7034	0.0855	0.0033
10	0.2356	0.021	-0.0015	0.2328	0.0215	0.0013	0.2343	0.0371	-0.0002	0.2397	0.0363	-0.0056
11	1.7468	0.1331	0.0077	1.7572	0.1234	-0.0027	1.7527	0.0914	0.0018	1.7543	0.0877	0.0002
12	0.8005	0.0816	0.0058	0.8101	0.0754	-0.0038	0.8089	0.0705	-0.0026	0.8054	0.0673	0.0009
13	0.3498	0.0361	0.0007	0.351	0.0341	-0.0005	0.3511	0.0337	-0.0006	0.3534	0.0368	-0.0029
14	0.3498	0.0361	0.0007	0.351	0.034	-0.0005	0.3511	0.0336	-0.0006	0.3533	0.0368	-0.0028
15	1.1051	0.0695	0.0007	1.1116	0.0739	-0.0058	1.1117	0.0666	-0.0059	1.1069	0.0659	-0.0011
16	0.1279	0.0135	0.0002	0.1283	0.0138	-0.0002	0.1284	0.0137	-0.0003	0.1291	0.013	-0.001
17	0.4247	0.0419	0.0057	0.4333	0.0459	-0.0029	0.4334	0.0447	-0.003	0.4284	0.0443	0.002

Table 2. A summary of branch current imag part, Ix in p.u

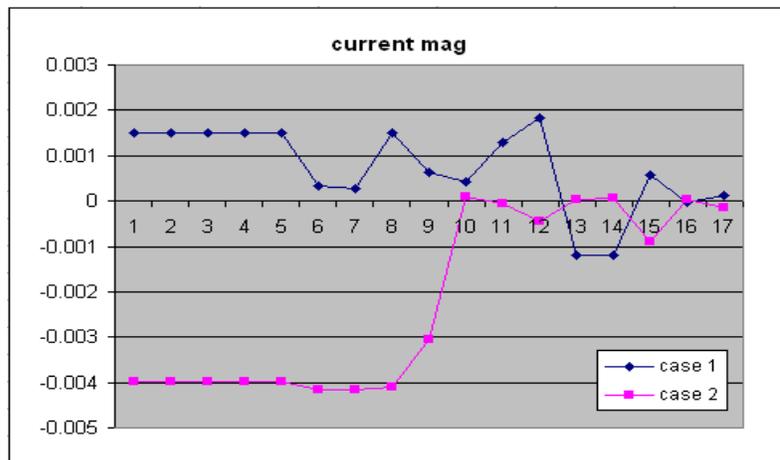
In	case1			case2			case3			case4		
	Mean	Std	Error									
1	-1.3809	0.1402	-0.002	-1.3913	0.144	0.0084	-1.3848	0.0402	0.0019	-1.3863	0.0431	0.0034
2	-1.3809	0.1402	-0.002	-1.3913	0.144	0.0084	-1.3848	0.0403	0.0019	-1.3864	0.0433	0.0035
3	-1.3809	0.1402	-0.002	-1.3913	0.1439	0.0084	-1.3848	0.0405	0.0019	-1.3864	0.0434	0.0035
4	-1.3809	0.1402	-0.002	-1.3913	0.1439	0.0084	-1.3848	0.0406	0.0019	-1.3864	0.0436	0.0035
5	-1.3809	0.1402	-0.002	-1.3913	0.1438	0.0084	-1.3848	0.0407	0.0019	-1.3864	0.0437	0.0035
6	-0.4006	0.1241	0.0047	-0.4046	0.1248	0.0087	-0.4014	0.0756	0.0055	-0.3969	0.0717	0.001
7	-0.3829	0.1242	0.0047	-0.3869	0.1248	0.0087	-0.3838	0.0757	0.0056	-0.3791	0.0718	0.0009
8	-0.0791	0.1211	0.0037	-0.0867	0.1199	0.0113	-0.0839	0.0772	0.0085	-0.0745	0.0717	-0.0009
9	0.6891	0.1108	0.0036	0.6841	0.1123	0.0086	0.6859	0.0808	0.0068	0.6952	0.0732	-0.0025
10	0.8667	0.0165	0.0004	0.8671	0.0231	0	0.8678	0.0224	-0.0007	0.8663	0.0234	0.0008
11	-0.9632	0.061	-0.0066	-0.9697	0.0693	-1E-04	-0.9663	0.0637	-0.0035	-0.9723	0.0572	0.0025
12	-0.4421	0.0407	-0.002	-0.4439	0.0447	-0.0002	-0.4426	0.0416	-0.0015	-0.4488	0.0427	0.0047
13	-0.2746	0.0253	0.0008	-0.2713	0.0264	-0.0025	-0.271	0.0255	-0.0028	-0.2755	0.0263	0.0017
14	-0.2746	0.0253	0.0008	-0.2713	0.0264	-0.0025	-0.271	0.0255	-0.0028	-0.2755	0.0263	0.0017
15	-0.7114	0.0438	0.0004	-0.7132	0.0395	0.0022	-0.7122	0.0375	0.0012	-0.7124	0.0414	0.0014
16	-0.0746	0.0065	0.0012	-0.0738	0.0074	0.0004	-0.0737	0.0073	0.0003	-0.074	0.0069	0.0006
17	-0.3199	0.032	0.0001	-0.3187	0.0275	-0.0011	-0.3183	0.0267	-0.0015	-0.32	0.0313	0.0002



(a) Error in branch current real part, I_r in p.u.

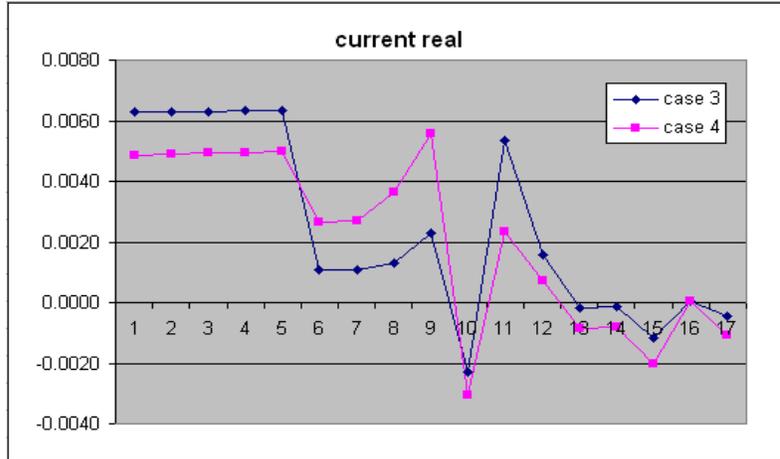


(b) Error in branch current imaginary part, I_x in p.u.

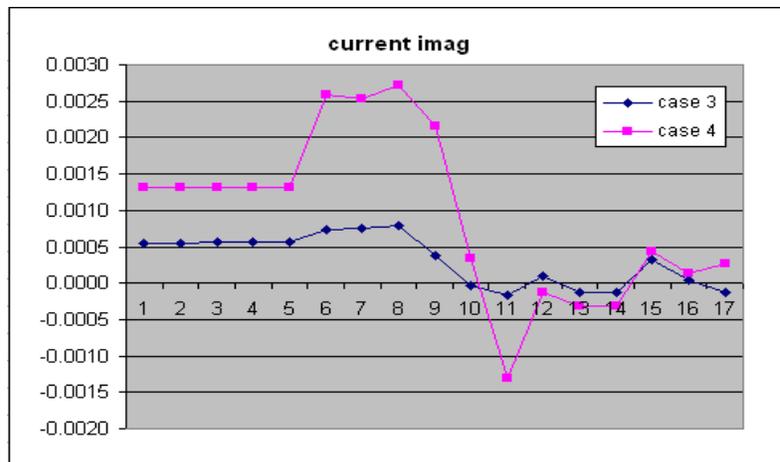


(c) Error in branch current magnitude part in p.u.

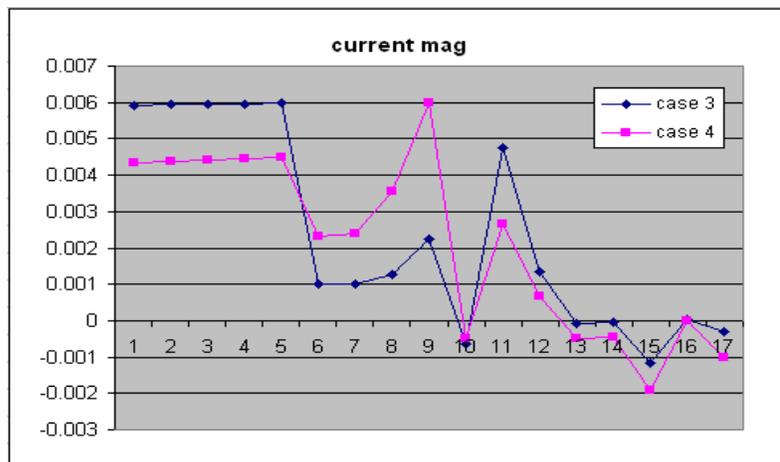
Figure 6. Currents error for Case 1 and Case 2 from Monte Carlo simulation



(a) Error in branch current real part, I_r in p.u.



(b) Error in branch current imaginary part, I_x in p.u.



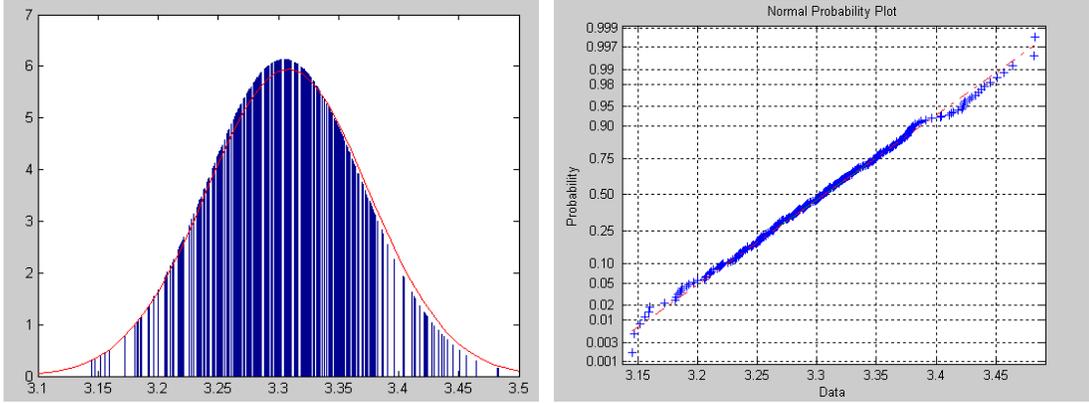
(c) Error in branch current magnitude in p.u.

Figure 7. Currents error for Case 3 and Case 4 from Monte Carlo simulation

To test the performance of BCSE, 300 Monte Carlo simulations have been performed and the results are given in the Appendix II. A summary of test results are given in table 1, 2 and figures 6-7. Phase a is only given as the typical example. Tables 1, 2 show the details of the mean, standard deviation and error of the estimated state ($x = [I_r, I_x]$) of branch currents in 300 Monte Carlo simulations. Figures 6-7 show the error in the estimated state ($x = [I_r, I_x]$) of branch currents of the feeder. As these figures indicate, the error in the estimated state ($x = [I_r, I_x]$) is not zero. Hence, we have adapted the bias test to determine if this error is significant.

3.5.1 Bias Test

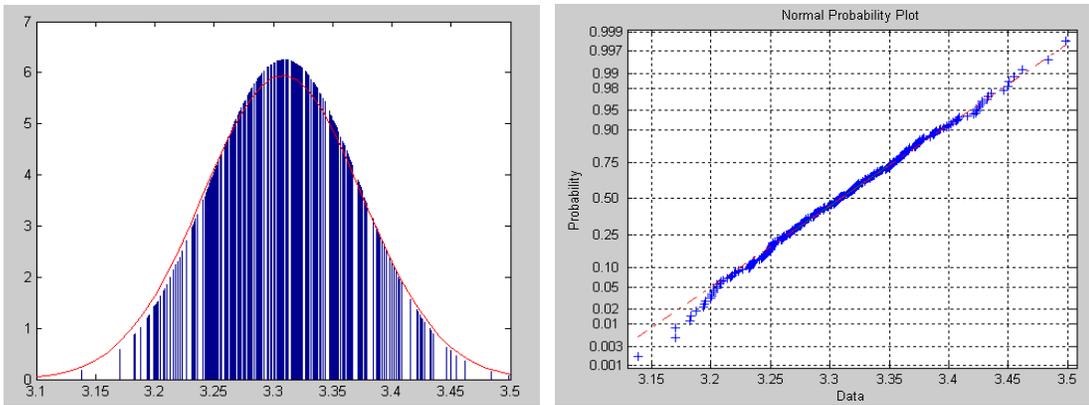
Many bias tests depend on the assumption of normality. Thus, we check the normality of the result given in Figure 8-9 before the bias test. To check normality, histogram and normal probability plot using MATLAB is given in these figures. The purpose of a normal probability plot is to graphically assess whether the data in x could come from a normal distribution. If the data are normal, the plot will be linear. Other distribution types will introduce curvature in the plot. Seventh current magnitude in Phase a is only given as the typical example in case 3 and case 4. These figures show the histogram and normal probability plot of the result obtained from 300 Monte Carlo simulations.



(a) Histogram of I7 magnitude

(b) Normal probability plot of I7 magnitude

Figure 8. Normality check of I7 magnitude for case 3



(a) Histogram of I7 magnitude

(b) Normal probability plot of I7 magnitude

Figure 9. Normality check of I7 magnitude for case 4

If the data are normal, the plot will be close to red line which indicates the data come from normal distribution. These figures indicate that the results are close to red line, thus, the results have the normality. Furthermore, we have N state ($x = [I_r, I_x]$) of branch currents, we need to check all states to ascertain the normality of BCSE method. However, we have the sample of 300 observations and 102 states. It is too many to check the normality of all states and thus, the interesting quantities are chose to define the normality of BCSE method. Figure

10-13 show the histogram and normal probability plot of the interesting quantities in case 3 and phase a is given as the typical example.

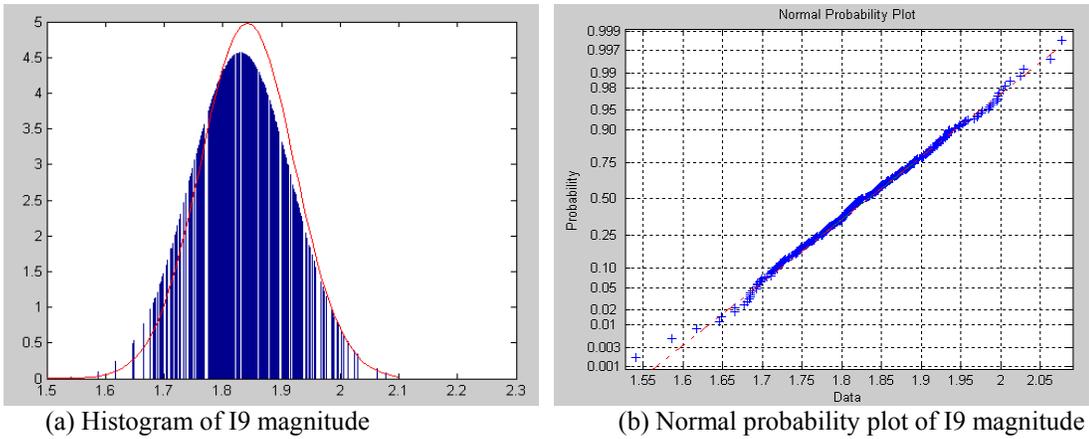


Figure 10. Normality check of I9 magnitude for case 3

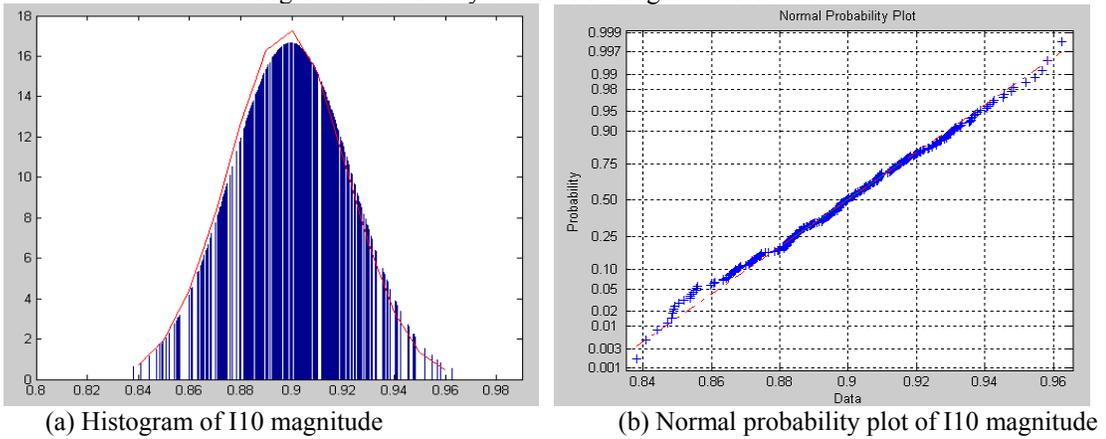


Figure 11. Normality check of I10 magnitude for case 3

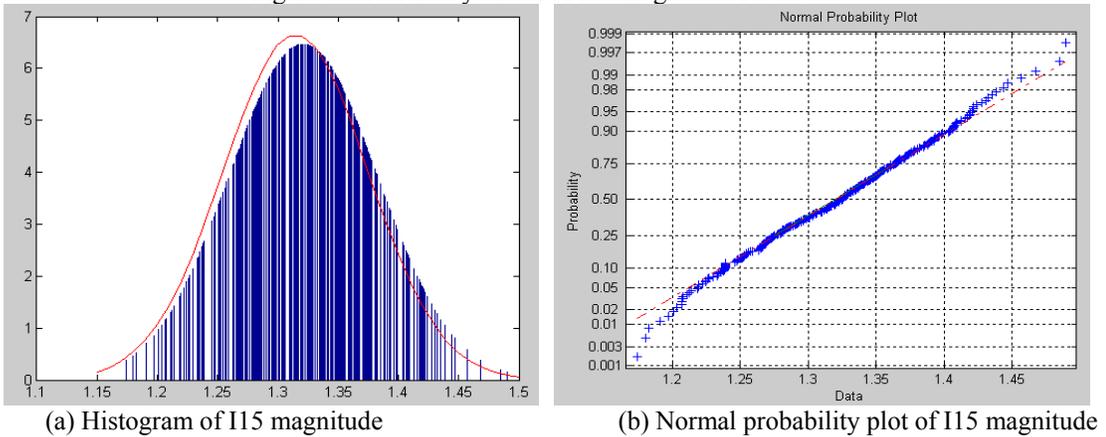


Figure 12. Normality check of I15 magnitude for case 3

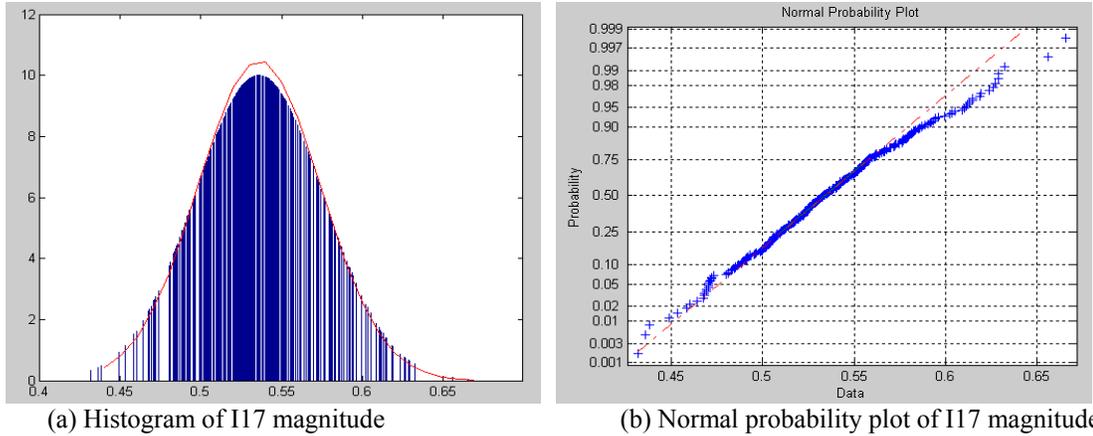


Figure 13. Normality check of I17 magnitude for case 3

These figures indicate that the results are close to red line, thus, the results have the normality. For the bias test, both parametric and nonparametric tests are performed. Given the sample of 300 observations and 102 parameters, we first use the Hotelling's T^2 test. We calculated the mean vector by R, statistical software, then use the mean vector to subtract the vector given in the alternative hypothesis. We would like to test whether the difference vector is equal to zero vector or not. Next we derived the sample covariance matrix. After that, we calculated the sample T^2 value based on the above Eq 3.14. Since we found out T^2 value from this sample is 237.1137 which is greater than $\frac{299}{198} F_{102,198}(0.05) = 1.9923$, we arrive at the conclusion to reject the null hypothesis since the significant level is 0.05. That is, the mean vector is significantly different than the alternative. The p-value for this test is 0.005140386, which supported our conclusion. Since Hotelling's T^2 test is based on the known distribution, we could use a nonparametric test to verify the conclusion that we obtained from the parametric case. To do the nonparametric test, Dong Wang in the NCSU Statistics Department has done the tests using R-code composed by Sejia Sirkia. The results

are that the Q^2 is 131.2375 and the p-value as 0.027 in the multivariate sign test and U^2 is 230.6029 and the p-value is 5.97e-12. Thus, we are in favor of the alternative hypothesis based on both of the nonparametric tests. We arrived at the same conclusion that the population mean vector is not equal to that in the alternative hypothesis regardless of the test methods we used.

3.5.2 Effect of voltage measurements on the BCSE

Figure 14 shows the sum of square error plot in four cases. These results indicate that in the bias measure, case 1 (0.0017) is compared with case 2 (0.0045) which includes voltage measurements. Similarly, case 3 (0.0016) and case 4 (0.0012) are also compared to illustrate the effect of adding voltage measurements. However, as previous bias test indicates, the bias measures in four cases are not zero. Thus, we can't measure performance of BCSE using bias.

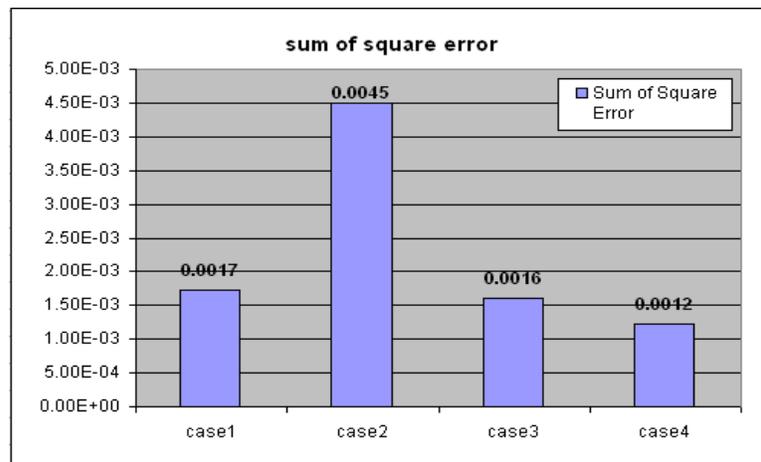


Figure 14. Sum of Square Error Plot in four cases

Quality is chosen to assess the performance of BCSE method. The results are given in figure 15. Note that case 1 and case 2 can be used to assess the impact of voltage

measurements to the BCSE with only load data. As these figure indicate, case 2 (0.6885) is slightly better than case 1 (0.6598) in quality. Hence, these results indicate that having voltage measurements helps improve the estimation over the conventional one that is based on forecasted load only (case 1).

Similarly, case 3 and case 4 are compared. Note that theses two cases illustrate the effect of adding voltage measurements to a system which has somewhat limited power and current measurement from the feeder (Case 3). As these figure indicate, the quality of case 4 (3.5075) is slightly better than that of case 3 (3.4447). Hence, the results indicate that adding voltage measurements improve the estimation as it did in the previous case.



Figure 15. Quality of the estimates in four cases

3.5.3 Further bias study

Although quality is chosen to assess the performance of BCSE method, the bias measure is still important. Thus, further bias measure is studied. To generate forecasted load data in previous test, it is created by perturbing the actual load data by adding error of 30% to both real and imaginary load independently. In this section, real load data is generated as previous

test as :

$$Z_p = Z_p^a \pm e_p \quad (3.30)$$

where Z_p^a is real load actual data and e_p is the measurement error (30%) for real load and

then for generating imaginary load data, power factor is used as :

$$Z_Q = Z_p \cdot \tan(Z_\theta) \quad (3.31)$$

where $Z_\theta = \cos^{-1}(Z_{pf}^a + e_{pf})$ is the angle and Z_{pf}^a is power factor actual data and e_{pf} is the measurement error (10%) for power factor. For the bias test, basic T test is performed in case 1 and case 4. P-value is calculated and selected less than 0.05 as a bias point which means the simulation is biased.

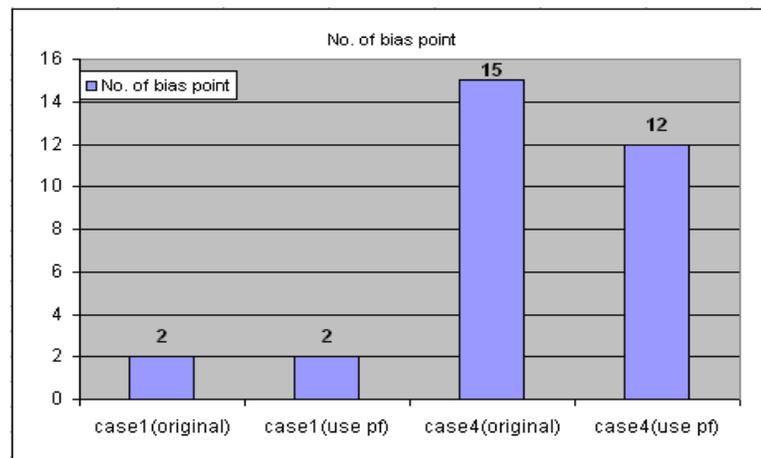


Figure 16. No. of bias point in case 1 and case 4

These results indicate that the simulation with measurement load data using power factor is still biased but it reduced the bias point in case 4. Furthermore, the quality is measured to assess the performance of BCSE method. The results are given in figure 17. As this figure indicates, generating imaginary load data using power factor doesn't help improve the estimation in either cases.

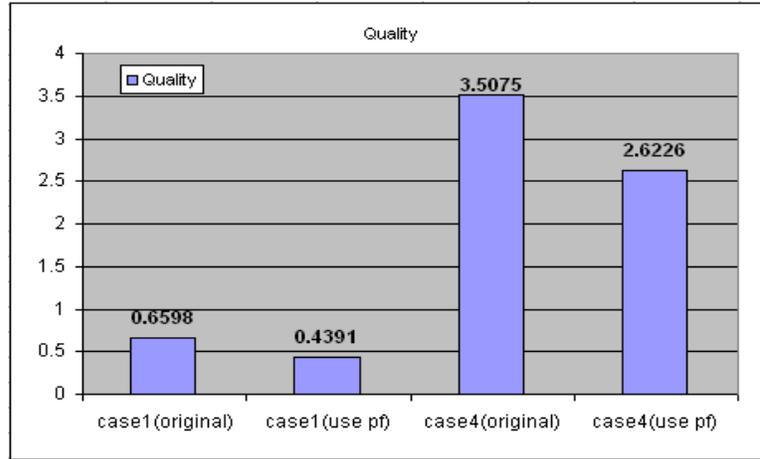


Figure 17. Quality of the estimates in case 1 and case 4

Another reason for the bias of the simulation can be that the sample size is not large enough. Thus, 5000 Monte Carlo simulations are performed as a further study. The results are given in Figure 18-19. Figure 18 shows the sum of square error plot in four cases. These results indicate that in the bias measure, case 1 (0.00054) is compared with case 2 (0.00052) which includes voltage measurements. Similarly, case 3 (0.00045) and case 4 (0.00044) are also compared to illustrate the effect of adding voltage measurements. This figure indicates that the sum of square is much smaller than 300 Monte Carlo simulations results however, as previous bias measure, the bias measures in four cases are not zero.

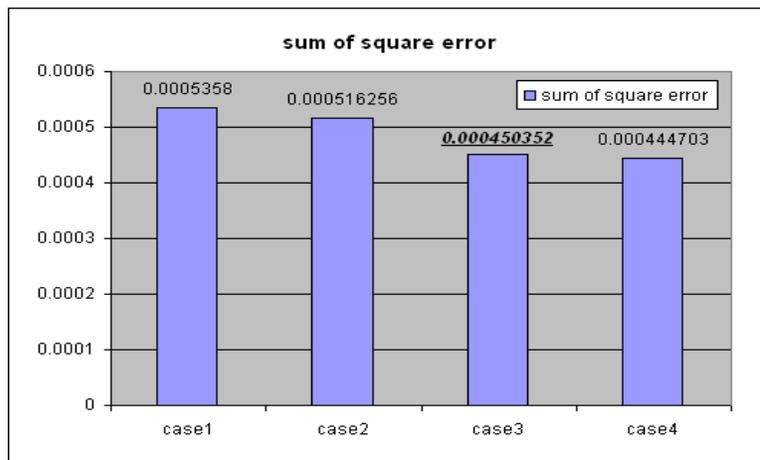


Figure 18 Sum of Square Error in four cases in 5000 Monte Carlo simulations

Furthermore, the quality is measured to assess the performance of BCSE method in 5000 Monte Carlo simulation. The results are given in figure 18. As these figure indicate, case 2 (0.6865) is slightly better than case 1 (0.6575) in quality. Hence, these results indicate that having voltage measurements helps improve the estimation over the conventional one that is based on forecasted load only (case 1) which is same result in 300 Monte Carlo simulations.

Similarly, case 3 and case 4 are compared. As these figure indicate, the quality of case 3 (3.4542) is slightly better than that of case 4 (3.3414). Hence, the results indicate that adding voltage measurements doesn't improve the estimation which is different results in 300 Monte Carlo simulations.



Figure 19 Quality in four cases in 5000 Monte Carlo simulations

3.6 Conclusion

This chapter adopts statistical techniques to assessing the BCSE performance. For statistical analysis, 300 Monte Carlo simulations are performed. The initial tests indicate that the error in the estimated state ($x = [I_r, I_x]$) is not zero. Hence, we have adapted the bias test to test if this error is significant. Multivariate parametric and nonparametric tests are

performed to make statistical decision using BCSE results. It was found that the population mean vector is significantly different from the actual mean vector in both tests. In addition, the overall performance of the estimate, quantified in terms of bias, consistency, and quality, is evaluated through Monte Carlo simulation for assessing the quality of BCSE method. Among these measures, quality is chosen to evaluate enhanced BCSE method. The result indicates that the voltage measurements on the estimated values are helpful in improving the estimation.

Chapter 4

Topology Error Identification Using Branch Current State Estimation

4.1 Overview

State estimation (SE) procedures became one of the most frequently used power system applications in control centers. SE can be understood in a broader sense as including a set of applications globally aimed at identifying the state of the system given that, at a certain moment, there are a number of telemetered or manually entered values. In this broader sense, SE can include:

- Topology Processor, which processes switching device status to obtain a simplified one-line diagram of the system;
- Component Model and Measurement Allocation, whose purposes are to identify the mathematical model of the components, to get the required data from the system database and to allocate the available measurements to the devices preserved in the simplified model;

- Observability Analysis, which analyzes the available measurements to check if they carry enough information to estimate the state of the system. It can also provide information about places where new measurement devices should be located;
- State Estimation Procedure, which traditionally assumes that the topology of the network is fixed, and aims at identifying the values of the state variables that, according to some criterion, more adequately explain addressed as the redundancy level increases in order to deal with measurement errors, absence of measurements in certain areas or time-skew problems;
- Bad Data Analysis, which evaluates the effects of large errors on measurements in order to detect and identify them.

The SE relies on the basic assumption that the topology of the system is known beyond any doubt. However, in most of the real world situations, the status of some switching devices is unknown or, for some reason, the current value in the database is under suspicion. If a circuit breaker or a switch is a part of the modeled network but is not monitored by SCADA, its open/close position in the database is updated manually by the power system dispatchers. In many system maintenance jobs, after a series of manually directed switching operations, the dispatcher often forgets to update the open/close positions of these switches. The result of this situation is a topology error in the network. Model topology error can also occur when the telemetered circuit breaker ON/OFF status is incorrect.

4.2 Topology Error Identification

To ensure and accurate state estimation (SE), the accuracy of measured data should be validated since any bad analogue data or network topology error can reduce the dependability of the database needed for power system operation and control.

4.2.1 Proposed Method I

To illustrate the process, consider the small radial feeder in Fig. 20, we can define a zone for each switch in a radial feeder: it is the part of the feeder between the switch and its down stream neighbors. Similarly, meters placed in a feeder divide the feeder into meter zones. As illustrated in Fig. 20 also. By using SE, the total load in each switch zone can be estimated with a certain accuracy, for example by 30%. When the status of a switch changes, for example SW2 in Fig.20 opens, the measurements from m_0 will change and this change will be approximately equal to the total load in switch zone SW2. Since we have a good estimation of this load from SE performed before switching, by comparing this value with the change in the measurements we can detect that indeed it was the switching of SW2 that caused the change in measurement m_0 . This type of consistency check can easily be generalized for detecting topology error in switch status in a given feeder.

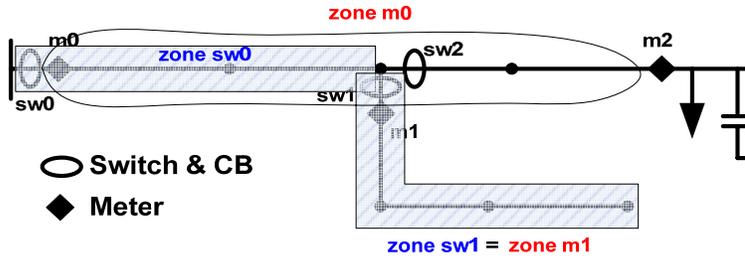


Figure 20. Zone defined by switches and meters in a feeder

4.2.1.1 Topology Error Processing

Based on proposed method, the basic flowchart is illustrated in Fig. 21. In this algorithm two type of input data is used for simulation: input1 which is normal input data when there is no switch open as a reference and input2 which is input data when some switch status is changed. SE is performed by each input data. For detecting topology error, current magnitude and power measurements are used. The accuracy for comparing the value between load and meter change relies on the variance obtained from 300 times Monte-Carlo simulation ($a_i = 3\sigma_i^2$).

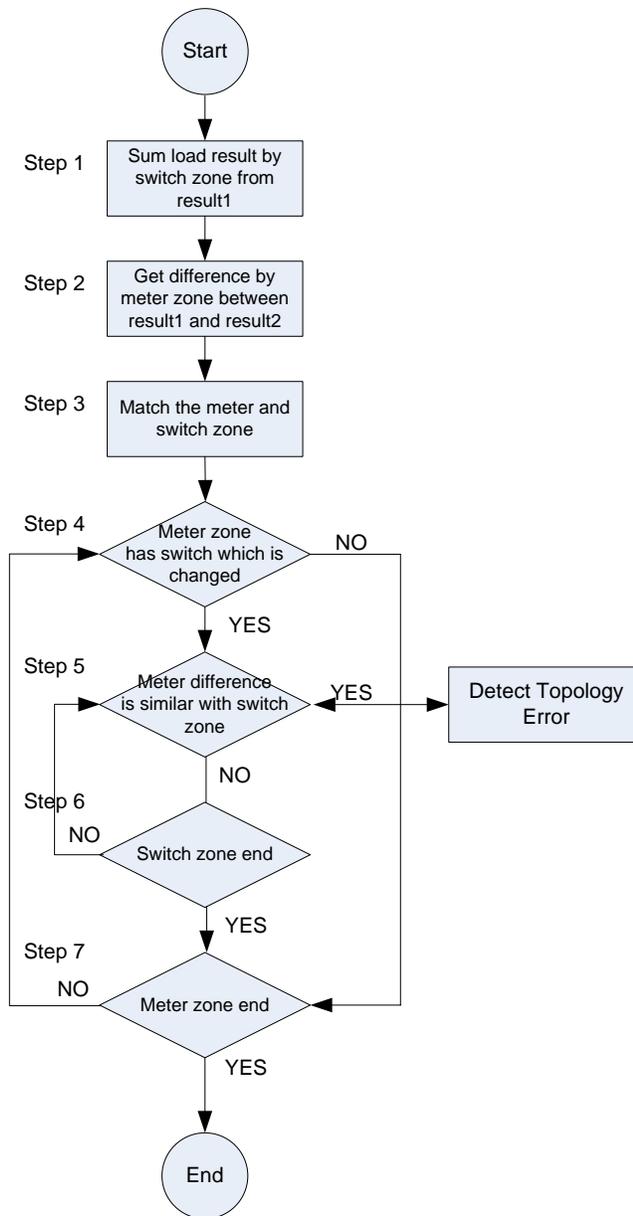


Figure 21. Flowchart of Method I Algorithm

Step 1 - Define a zone for each switch in a radial feeder and Estimate the total load in each switch zone by using SE performed by using input1 data.

Step 2 - Define a meter zones and get measurement difference by each meter zone between input1 and input2.

Step 3 - Match the switch zone and meter zone. In other words, figure out how many switch zone is included in meter zone.

Step 4 - Define which meter zone status is changed. If measurements difference, represented by Δm_k , is similar with next meter difference, represented by Δm_{k+1} , with a certain accuracy 30% ($\Delta m_k \approx \Delta m_{k+1}$, when Δm_k is within $\Delta m_{k+1}(1 \pm 0.3)$), the switch status is changed in next meter zone. When there is no status change in meter zone, go step 7.

Step 5 - Compare the value between meter difference, Δm_k and summation of load, Δy_i in switch zone. If a change in meter (Δm_k) falls within the range of one of the estimated load values of switch zone ($\Delta y_i(1 \pm a_i)$) then it will indicate a change in the status of the corresponding switch. If the meter type is current measurement, compare the power magnitude (S). Otherwise if the meter type is power measurement, compare the real and reactive power (P,Q). Although find the topology error, it's possible to find more accurate value ($\Delta m_k - \Delta y_i \approx 0$) so run the process up to switch zone end.

Step 6 - When the switch zone is end in same meter zone, go next step. Otherwise go back to step 5.

Step 7 - When meter zone is end, go next step. Otherwise go back to step 5.

4.2.2 Proposed Method II

In the result of method I, if the switch zone which has similar load exists, this algorithm doesn't detect topology error correctly. For accurate topology error detection, enhanced algorithm is needed. The correct statuses of all circuit breakers (CB) in the system are known all the time. However, in some rare cases, the assumed status of certain CBs may be wrong. A common situation is when the topology process encounters a CB whose status is unknown. In cases like this, the topology process must decide on the most likely CB status, for which it uses the status history for the same breaker and/or the values of related measurements as a guide. Hence, the risk of assuming the wrong status for the CB still will not be completely avoided. When this happens, the bus/branch model generated by the topology process is locally incorrect, leading to a topological error. Unlike the parameter errors, topology errors usually cause the state estimate to be significantly biased. As a result, the bad data detection & identification routine may erroneously eliminate several analog measurements which appear as interacting bad data, finally yielding an unacceptable state. It is also possible for the SE process to diverge, or have serious convergence problems, in the presence of topology errors. Therefore there is a need to develop effective mechanisms intended to detect and identify this kind of gross errors. The aim of this method is to present approaches to deal with topological errors and related matters using bad data detection & identification routine.

4.2.2.1 Overview of Bad Data Detection & Identification

When the system model is correct and the measurements are accurate, there is good reason to accept the state estimates calculated by the WLS approach. But if a measurement is

grossly erroneous or bad, it should be detected and then identified so that it can be removed from the estimator calculations. This is called bad data detection & identification in state estimation procedure [25]. The bad data detection procedure is described below :

Step 1 - Calculate the estimated errors $\hat{e}_j = z_j - \hat{z}_j$ after running state estimation program.

Step 2 - Evaluate the weighted sum of squares $\hat{f} = \sum_{j=1}^{N_m} \hat{e}_j^2 / \sigma_j^2$.

Step 3 - For the appropriate number of degrees of freedom $k = (N_m - N_s)$ and a specified probability α , determine whether or not the value of \hat{f} is less than the critical value corresponding to α . In practice, this means we check that the inequality $\hat{f} < \chi_{k,\alpha}^2$ is satisfied.

If it is, then the measured raw data and the state estimates are accepted as being accurate.

Step 4 - when the requirement of inequality is not met, there is reason to suspect the presence of at least one bad data measurement. Upon such detection, omit the measurement corresponding to the largest standardized error, namely, $(z_j - \hat{z}_j) / \sqrt{R_{jj}^i}$ and reevaluate the state estimates along with the sum of squares \hat{f} . If the new value of \hat{f} satisfies the chi-square test of inequality, then the omitted measurement has been successfully identified as the bad data point.

4.2.2.2 Topology Error Processing

In this topology detecting method, changing the on/off status of branches on after the other is based. And state estimation is performed in each case. If after reversing a branch status, residues in one of the estimation runs are within threshold values then the original

branch status is declared false. Based on proposed method, the basic flowchart is illustrated in Fig. 22.

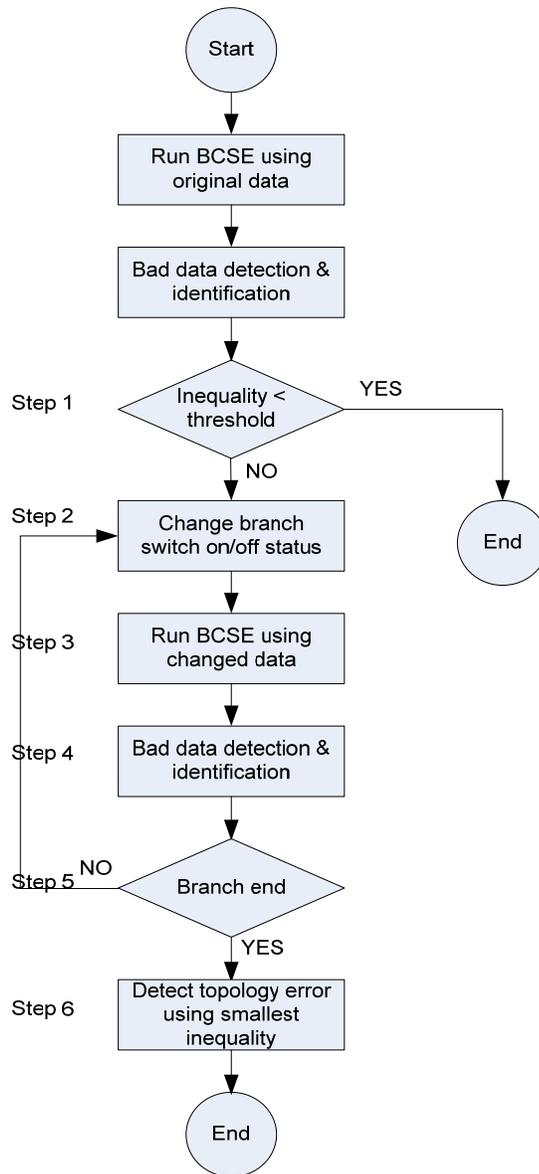


Figure 22. Flowchart of Method II Algorithm

Step 1 - Detect a topology error using bad data detection & identification. When the topology error occurs, it will be detected in the weighted sum of squares, called inequality.

Step 2 - When detect topology error, change the switch on/off status. This change is performed sequentially by switch information in input data.

Step 3 - Perform the BCSE using the input data which change switch on/off status.

Step 4 - Run bad data detection & identification using BCSE result and save the inequality value in every case.

Step 5 - If the branch having switch is end, go next step otherwise, go step 1.

Step 6 - Define which switch status is wrong. Using saved inequality value, smallest value represents no topology error. So the switch status having smallest inequality value is right but current known status is wrong.

4.3 Topology Error Identification Test Result

Same IEEE test feeder is used on previous test. The test feeder is assumed to have three line switches, three fuses and two tie switches as shown in Fig. 23. Measurements assumed are given in the figure also: voltage and power flow at the substation, current measurements on branches 6-7, and voltage measurements on node 8, 10 and 17.

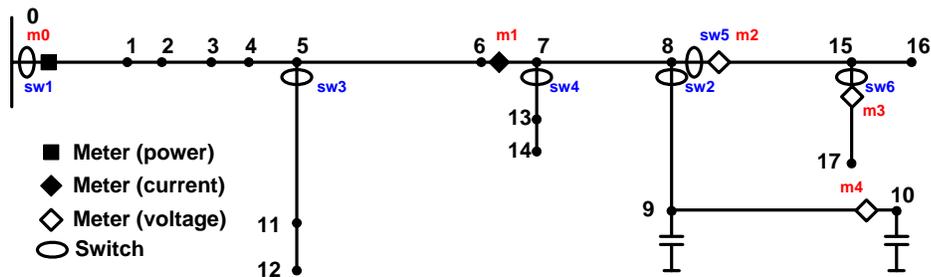


Figure 23. One-line diagram of the test feeder with switch

4.3.1 Method I Result

For testing topology error detection, only two measurements (m_0, m_1) is used and two different case were considered :

Case 1 : simulation with voltage measurement

Case 2 : simulation without voltage measurement

Each case is simulated when m_1 is power measurement or when m_1 is current measurement is included. By switch location, these different cases are tested. To compare the results of those cases, a load-flow program was used to provide the basic measurements data. The LF solution can be considered as an exact solution of the distribution system. If a case can obtain a solution closer to the LF solution, it is obviously a better case and has more accurate solution. So each case was compared with this exact solution as a reference. To generate measurements data for testing purposes, measurement error was added to the actual measurements.

$$Z = Z^a \pm e_z$$

where Z^a is actual data and e_z is generated by each data accuracy.

Test results are given in the Appendix III. A summary of the results is given in table 1-12. Only one case when node 9 open is given as the typical example. In this case, the switch zone 2 is open in fig. 23. Summation of load result table shows estimated load values of switch zone. Meter difference table shows change in meter zone. By comparing these values, final result is represented in Topology error detection table.

- **Case 1-1** : simulation with voltage & power (m_0) & current measurement (m_1)

Table 3. Case1-1 summation of load in switch zone

sw_zo	SL1	PL1	QL1	SL2	PL2	QL2	SL3	PL3	QL3
1	0.266	0.182	0.182	1.484	1.045	1.045	1.352	0.947	0.947
2	2.122	1.5	1.5	2.784	1.969	1.969	2.623	1.854	1.854
3	2.272	1.607	1.607	0.003	-0.002	-0.002	0.003	-0.002	-0.002
4	0.434	0.307	0.307	0.556	0.392	0.392	0.507	0.359	0.359
5	1.046	0.74	0.74	0.996	0.704	0.704	1.702	1.203	1.203
6	0.601	0.425	0.425	0	0	0	0.001	0	0
SUM	6.742	4.761	4.761	5.824	4.108	4.108	6.187	4.361	4.361

Table 4. Case 1-1 meter difference in meter zone

me_zo	mtyp	Pm1	Qm1	Pm2	Qm2	Pm3	Qm3
1	s	1.698	1.387	2.07	1.546	1.808	1.447
2	i	1.337	0	1.536	0	1.722	0

Table 5. Case 1-1 topology error detection result

me_zo	sw_zo	sw1	sw2	ph	P1(S1)	Q1	P2(S2)	Q2	P3(S3)	Q3
1	1	ON	ON	3						
	3	ON	ON	1						
2	2	ON	ON	3						
	4	ON	ON	3						
	5	ON	ON	3	err				err	
	6	ON	ON	1						

Although it has to detect switch zone 2 as a topology error, this case fails to detect the all phase error and correct switch zone.

- **Case 1-2** : simulation with voltage & power measurement (m_0, m_1)

Table 6. Case 1-2 summation of load in switch zone

sw_zo	SL1	PL1	QL1	SL2	PL2	QL2	SL3	PL3	QL3
1	0.266	0.182	0.182	1.483	1.044	1.044	1.352	0.947	0.947
2	2.114	1.495	1.495	2.789	1.972	1.972	2.627	1.858	1.858
3	2.277	1.61	1.61	0.003	-0.002	-0.002	0.003	-0.002	-0.002
4	0.434	0.307	0.307	0.556	0.393	0.393	0.508	0.359	0.359
5	1.045	0.739	0.739	0.997	0.705	0.705	1.704	1.205	1.205
6	0.6	0.424	0.424	0	0	0	0.001	0	0
SUM	6.736	4.757	4.757	5.828	4.111	4.111	6.195	4.367	4.367

Table 7. Case 1-2 meter difference in meter zone

me_zo	mtyp	Pm1	Qm1	Pm2	Qm2	Pm3	Qm3
1	s	1.698	1.387	2.07	1.546	1.808	1.447
2	s	1.622	1.274	2.004	1.471	1.754	1.345

Table 8. Case 1-2 topology error detection result

me_zo	sw_zo	sw1	sw2	ph	P1(S1)	Q1	P2(S2)	Q2	P3(S3)	Q3
1	1	ON	ON	3						
	3	ON	ON	1						
2	2	ON	ON	3	err		err	err	err	
	4	ON	ON	3						
	5	ON	ON	3		err				err
	6	ON	ON	1						

This result indicates that using power measurement helps to detect topology error in every phase and it detects right switch zone (switch zone 2) but some wrong switch zones (switch zone 5) are detected as previous case.

- **Case 2-1** : simulation without voltage & with power (m_0) & current measurement (m_1)

Table 9. Case 2-1 summation of load in switch zone

sw_zo	SL1	PL1	QL1	SL2	PL2	QL2	SL3	PL3	QL3
1	0.266	0.182	0.182	1.483	1.044	1.044	1.352	0.947	0.947
2	2.119	1.498	1.498	2.78	1.965	1.965	2.619	1.852	1.852
3	2.266	1.602	1.602	0.003	-0.002	-0.002	0.003	-0.002	-0.002
4	0.434	0.307	0.307	0.556	0.393	0.393	0.508	0.359	0.359
5	1.046	0.739	0.739	0.995	0.704	0.704	1.699	1.201	1.201
6	0.601	0.425	0.425	0	0	0	0.001	0	0
SUM	6.731	4.753	4.753	5.817	4.103	4.103	6.181	4.357	4.357

Table 10. Case 2-1 meter difference in meter zone

me_zo	mtyp	Pm1	Qm1	Pm2	Qm2	Pm3	Qm3
1	s	1.698	1.387	2.07	1.546	1.808	1.447
2	i	1.335	0	1.537	0	1.723	0

Table 11. Case 2-1 topology detection result

me_zo	sw_zo	sw1	sw2	ph	P1(S1)	Q1	P2(S2)	Q2	P3(S3)	Q3
1	1	ON	ON	3						
	3	ON	ON	1						
2	2	ON	ON	3						
	4	ON	ON	3						
	5	ON	ON	3	err				err	
	6	ON	ON	1						

This result shows the impact of voltage measurements by comparing with case 1-1 in topology detection. However, the topology detection result is same with case 1-1.

- **Case 2-2** : simulation without voltage & with power measurement (m_0, m_1)

Table 12. Case 2-2 summation of load in switch zone

sw_zo	SL1	PL1	QL1	SL2	PL2	QL2	SL3	PL3	QL3
1	0.266	0.182	0.182	1.482	1.043	1.043	1.352	0.947	0.947
2	2.11	1.492	1.492	2.782	1.967	1.967	2.621	1.854	1.854
3	2.272	1.606	1.606	0.003	-0.002	-0.002	0.003	-0.002	-0.002
4	0.434	0.307	0.307	0.556	0.393	0.393	0.508	0.359	0.359
5	1.044	0.738	0.738	0.995	0.704	0.704	1.7	1.202	1.202
6	0.6	0.424	0.424	0	0	0	0.001	0	0
SUM	6.724	4.749	4.749	5.82	4.104	4.104	6.186	4.36	4.36

Table 13. Case 2-2 meter difference in meter zone

me_zo	mtyp	Pm1	Qm1	Pm2	Qm2	Pm3	Qm3
1	s	1.698	1.387	2.07	1.546	1.808	1.447
2	s	1.622	1.274	2.004	1.471	1.754	1.345

Table 14. Case 2-2 topology error detection result

me_zo	sw_zo	sw1	sw2	ph	P1(S1)	Q1	P2(S2)	Q2	P3(S3)	Q3
1	1	ON	ON	3						
	3	ON	ON	1						
2	2	ON	ON	3	err		err	err	err	
	4	ON	ON	3						
	5	ON	ON	3		err				err
	6	ON	ON	1						

This result also shows the impact of voltage measurements by comparing with case 1-2 in topology detection. However, the topology detection result is same with case 1-2 and

although it detects a few wrong switch zones, using power measurement is helps to detect topology error rather than current measurements.

In summary, note that case 1 and case 2 can be used to assess the impact of voltage measurements to the topology error detection. As these tables indicate, same result of topology error detection is obtained. The results for these two cases indicate that the impact of voltage measurements is marginal as SE result in previous chapter. Case 1-1 and Case 1-2 show the impact of power and current measurements for topology error detection. As these tables indicate, better result is obtained when using power measurements. Similarly, case 2-1 and case 2-2 show same result in topology error detection. In every case, Topology error detection doesn't work correct. The result in case 1-1 and case 2-1 using current measurements fails to detect all phase error and correct switch zone. In case 1-2 and case 2-1 using power measurements, it detects all phase error but detect wrong switch zone as previous case. The summation of load result in switch zone indicates that the loads are similar in the switch zone 2 and switch zone 5, 6 ($sw2 \approx sw5 + sw6$). Therefore, if the switch zone which has similar load exists, this algorithm doesn't detect topology error correctly. Another simulation results in Appendix III also show example to fail to detect the topology error like this case. For accurate topology error detection, enhanced algorithm is needed.

4.3.2 Method II Result

4.3.2.1 Switch Error Detection

For testing topology error detection, all measurements (m_0, m_1, m_2, m_3, m_4) is used and two different case were considered :

Case 1 : simulation with power and current measurements

Case 2 : simulation with power, current and voltage measurements

To compare the results of those cases, a load-flow program was used to provide the basic measurements data. The LF solution can be considered as an exact solution of the distribution system. If a case can obtain a solution closer to the LF solution, it is obviously a better case and has more accurate solution. So each case was compared with this exact solution as a reference. To generate measurements data for testing purposes, measurement error was added to the actual measurements.

$$Z = Z^a \pm e_z$$

where Z^a is actual data and e_z is generated by each data accuracy.

Test results are given in the Appendix IV. A summary of the results are given. Only one case when node 9 open is given as the typical example. In this case, the switch 2 is open. Topology error detection table shows weighted sum of squares values in each case. By checking the smallest value, final result is also represented in Topology error detection table.

- **Case 1 :** simulation with power and current measurements (when node 9 open)

Table 15. Case 1 topology error detection result when node 9 open

```
***** BCSE TOPOLOGY DETECTION RESULT *****
inequality = 243.00038
***** SIMULATION *****
sw 1 op sw 9 op sw 11 op sw 13 op sw 15 op sw 17 op
2464.36816 6.66763 222.10188 176.60193 138.00252 193.49509
*****
Smallest inequality is 6.66763
Detect switch 9 Open
*****
```

- **Case 2** : simulation with power, current and voltage measurements (when node 9 open)

Table 16. Case 2 topology error detection result when node 9 open

```

***** BCSE TOPOLOGY DETECTION RESULT *****
inequality = 243.57712

***** SIMULATION *****
sw 1 op sw 9 op sw 11 op sw 13 op sw 15 op sw 17 op
2500.58667 3.86066 248.46680 159.69975 125.27337 196.52029
*****

Smallest inequality is 3.86066
Detect Switch 9 Open

*****

```

Note that case 1 and case 2 can be used to assess the impact of voltage measurements to the topology error detection. As these tables indicate, same result of topology error detection is obtained. The results for these two cases indicate that the impact of voltage measurements are marginal as SE. In every case, this algorithm detects topology error correctly. Another simulation results in Appendix IV also show example to success to detect the topology error like this case.

4.3.2.2 Shunt Capacitor Bank Detection

For testing shunt capacitor bank (SCB) error detection, same IEEE test feeder is used with previous test. The test feeder is assumed to have two same SCB on node 9, 10. All simulation is same with previous test. SCB is mainly installed to provide capacitive reactive compensation/ power factor correction. Thus, if SCB error occurs, it affects reactive power. By using this idea, switch and SCB error is classified. When the inequality value of reactive power is much bigger than the one of real power ($\hat{f}_Q \gg \hat{f}_P$), SCB error occurs.

Test results are given in the Appendix IV. A summary of the results are given. Only one case when node 10 SCB open is given as the typical example. Topology error detection table shows weighted sum of squares values in each case. By checking the smallest value, final result is also represented in Topology error detection table.

- **Case 1** : simulation with power and current measurements (when SCB 10 open)

Table 17. Case 1 SCB error detection when SCB 10 open

```

***** BCSE TOPOLOGY DETECTION RESULT *****

inequality = 55.12613

***** SIMULATION (CAP error) *****
CAP 9 op CAP 10 op
3.59224 4.67353
*****

Smallest inequality is 3.59224
Detect CAP 9 Open

```

- **Case 2** : simulation with power, current and voltage measurements (when SCB 10 open)

Table 18. Case 2 SCB error detection when SCB 10 open

```

***** BCSE TOPOLOGY DETECTION RESULT *****

inequality = 63.64521

***** SIMULATION (CAP error) *****
CAP 9 op CAP 10 op
9.86627 16.76312
*****

Smallest inequality is 9.86627
Detect CAP 9 Open

```

Note that case 1 and case 2 can be used to assess the impact of voltage measurements to the topology error detection. As these tables indicate, same result of topology error detection is obtained. The results for these two cases indicate that the impact of voltage measurements are marginal as SE. In this case, Topology error detection doesn't work correct. In this

simulation model, the SCB exist at a short distance and have same capacity. Furthermore, the SCB exists in same meter zone. Therefore, if the SCB which has similar capacity and closed exists, this algorithm doesn't detect topology error correctly. For accurate SCB error detection, enhanced algorithm is needed.

4.4 Conclusion

Correct connectivity in power network modeling is so critical to modern market and security operations that topology estimation is expected to become a standard EMS function. In this manner two type algorithms are proposed for topology error detection in this chapter. First method gets an idea that when the switch status changes, it will affect the measurement. But this algorithm fails to detect the topology error in every case. The problem is that if switch zone which has similar load exists, this algorithm doesn't detect topology error correctly. On the other hand, second method is capable of topology error detection. This algorithm is based on changing the on/off status of branches one after the other and performing a state estimation in each case. If after reversing a branch status, residues in one of the estimation runs are within threshold values then the original branch status is declared false. This algorithm detects topology error correctly in every case successfully. However, in SCB error detection, When the SCB which has similar capacity and closed each other, this algorithm doesn't work correct. For accurate SCB error detection, enhanced algorithm is needed.

Chapter 5

Conclusion and Future Work

5.1 Conclusion

This thesis shows that the basic BCSE can be extended to include voltage measurements in the estimation. The revised algorithm was implemented using the C language on Microsoft Visual Studio .NET 2003. To assess the impact of voltage measurements on the BCSE method, four cases having different measurements are considered. These test results indicate that the impact of voltage measurements on the estimation depends upon the existence of the other measurements from the feeder. These results are based on the simulated measurements with assumed accuracy especially on the load forecast value.

Furthermore, some statistical techniques are adapted to assessing the BCSE performance. For statistical analysis, 300 Monte Carlo simulations are performed. These results indicate that the error in the estimate state is not zero. Thus, we have adapted the bias test to determine whether this error is significant. It was found that the population mean vector is significantly different from the actual mean vector. In addition, quality is chosen for assessing the quality of the BCSE method. The result indicates that the voltage measurements of the estimated values are helpful in improving the estimation.

For topology error identification, two types of algorithms are proposed in this thesis. The first method uses idea that when the switch status changes, it will affect measurement. These test results indicate that this algorithm fails to detect the topology error. The second method is based on changing the on/off status of branches successively and performing an SE in each case. Its result shows that the second method is suitable for topology error detection. However, in SCB error detection, neither method works correctly.

5.2 Future Work

Several topics of interest arise from the completion of this study. Areas that would help progress toward a better understanding of the feasibility and implementation of BCSE are as follows:

- In this thesis, we got statistical decisions that population mean vector is not equal to that in the alternative hypothesis regardless of the test methods meant for bias estimation. Thus, as further work, the reason why the simulation is biased needs to be understood and then a statistical method such as Jackknife can be used to correct the average bias of the biased components.
- In this thesis, when the SCB has similar capacity and is located at a short distance, the proposed algorithm doesn't work correctly. For accurate SCB error detection, enhanced an algorithm is needed.

- Measurements that are inaccurate due to meter, telemetry, or other types of errors will deteriorate the SE if they are not detected, identified, and eliminated. Thus, Bad data detection and identification in SE has a crucial role to ensure the quality of SE results.
- Synchronized phasor measurements obtained by phasor measurement units (PMU) are rapidly populating power systems. Thus, the BCSE method will be extended to include phasor measurements in the estimation.
- This thesis' results are based on simulated measurements with assumed accuracy especially on the load forecast values. Thus, the results depend on a load modeling procedure that provides an estimate of load demands at all nodes. Future work can determine if such a load modeling technique can be combined with the BCSE method.
- Error analysis in numerical analysis can be used to determine the effectiveness of the SE results obtained by the BCSE method.
- For testing the revised BCSE, a test feeder at a heavy loading condition is used. However, the feeder in practical world is under any load condition, from light to heavy loading. Thus, the revised BCSE needs to test the feeder at light loading condition.
- In this thesis, it was assumed that the test feeder has only shunt capacitor bank for reactive power injection. As a future work, the revised BCSE needs to test when there is real power injection on the feeder.

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APPENDICES

APPENDIX I - State Estimation Test Result

- Common input data (load measurements and impedance)

reduced 3-PH IEEE TEST FEEDER with mod load with 4 meas													
vo	ang	bkva/3p	bkv/ln	eps	mxitr	prnt	dcpl	itgn					
24.9	0.	300.	24.9	.1	5	0	0	2					
ln	sbs	rbs	sec	cod	lng	dty	P1	Q1	P2	Q2	P3	Q3	QC/Ph
1	0	2	0	1	4310	1	0.00	0.00	32.87	16.99	32.77	16.95	0.
2	2	3	0	1	32230	1	0.00	0.00	15.42	7.97	0.00	0.00	0.
3	3	5	0	1	66730	1	0.00	0.00	0.00	0.00	0.00	0.00	0.
4	5	6	0	1	10	1	0.00	0.00	0.00	0.00	0.00	0.00	0.
5	6	7	0	1	310	1	2.86	1.48	41.30	21.35	2.73	1.40	0.
6	7	12	0	1	67840	1	3.02	1.56	3.80	1.97	0.00	0.00	0.
7	12	13	0	1	10	1	4.08	2.98	4.31	1.92	7.74	3.72	0.
8	13	15	0	1	11930	1	9.86	6.03	8.70	6.23	58.69	32.38	0.
9	15	17	0	1	2030	1	132.84	103.93	115.88	89.99	136.00	105.93	100.
10	17	19	0	1	3442	1	19.73	15.79	57.57	35.50	32.48	22.67	100.
LAT													
11	7	22	0	3	49860	1	93.42	48.47	0.00	0.00	0.00	0.00	0.
12	22	23	0	3	11760	1	80.66	41.53	0.00	0.00	0.00	0.00	0.
LAT													
13	13	26	0	9	5280	1	0.00	0.00	0.00	0.00	0.00	0.00	0.
14	26	27	0	1	10560	1	32.84	22.82	39.67	26.74	40.05	27.00	0.
LAT													
15	15	30	0	1	4700	1	62.27	33.13	35.53	19.05	96.26	50.51	0.
16	30	31	0	1	860	1	11.46	6.01	29.07	22.19	17.41	15.78	0.
LAT													
17	30	33	0	3	2140	1	43.28	29.73	0.00	0.00	0.00	0.00	0.
ZLN													
lid	r11	x11	r12	x12	r13	x13	r22	x22	r23	x23	r33	x33	
1	0.368	0.685	0.017	0.150	0.015	0.110	0.375	0.678	0.019	0.207	0.372	0.678	
2	0.977	0.871	0.016	0.170	0.015	0.126	0.984	0.865	0.018	0.227	0.981	0.865	
3	1.928	1.419	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
4	0.	0.	0.	0.	0.	0.	1.928	1.419	0.	0.	0.	0.	
5	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.928	1.419	
6	0.977	0.871	0.	0.	0.015	0.126	0.	0.	0.	0.	0.981	0.868	
7	0.977	0.871	0.015	0.126	0.	0.	0.981	0.868	0.	0.	0.	0.	
8	0.0	0.	0.	0.	0.	0.	0.977	0.871	0.015	0.126	0.981	0.868	
9	78.53	168.64	0.	0.	0.	0.	78.53	168.64	0.	0.	78.53	168.64	
XTR													
xid	lid	sr	amn	amx	ztr	ztx	zsr	zsx	vdes				
1	3	100.	0.9	1.1	0.0	0.01	0.2	0.2	1.0				

Actual : Actual load data

- Result

In	nd	v1m	v1a	v2m	v2a+120	v3m	v3a-120
		Phase 1		Phase 2		Phase 3	
1	2	0.9989	-0.0548	0.9992	-0.0396	0.9993	-0.044
2	3	0.9907	-0.4757	0.994	-0.3206	0.9944	-0.3531
3	5	0.9741	-1.3782	0.984	-0.8839	0.984	-1.0023
4	6	0.9741	-1.3783	0.984	-0.884	0.984	-1.0024
5	7	0.974	-1.3826	0.9839	-0.8867	0.984	-1.0054
6	12	0.9656	-2.019	0.9739	-1.3883	0.9737	-1.7484
7	13	0.9656	-2.0191	0.9739	-1.3884	0.9737	-1.7485
8	15	0.9645	-2.1247	0.9726	-1.4707	0.9722	-1.8703
9	17	0.9644	-2.1376	0.9724	-1.4848	0.9722	-1.8828
10	19	0.9645	-2.145	0.9725	-1.496	0.9723	-1.8902
11	22	0.9523	-1.5845	0.9839	-0.8867	0.984	-1.0054
12	23	0.95	-1.6071	0.9839	-0.8867	0.984	-1.0054
13	26	0.9308	-3.216	0.9359	-2.726	0.9357	-3.0866
14	27	0.9305	-3.2218	0.9356	-2.7314	0.9354	-3.0951
15	30	0.9641	-2.1357	0.9723	-1.4724	0.9718	-1.8826
16	31	0.9641	-2.1357	0.9723	-1.4727	0.9718	-1.8829
17	33	0.9639	-2.1362	0.9723	-1.4724	0.9718	-1.8826

<Voltage result>

In	nd	P1	Q1	P2	Q2	P3	Q3
		Phase 1		Phase 2		Phase 3	
1	2	510.33	138.29	425.59	86.64	441.01	99.52
2	3	509.9	137.65	394.09	70.14	414.7	85.63
3	5	506.72	132.8	376.49	59.64	413.11	82.98
4	6	500.19	122.7	373.27	55.37	409.72	77.48
5	7	500.19	122.7	373.27	55.37	409.72	77.48
6	12	323.95	30.75	330.92	33.47	406.9	76.02
7	13	318.38	25.32	324.13	28.34	403.59	70
8	15	279.96	-2.59	281.76	-0.78	357.96	39.19
9	17	162.01	-72.87	200.13	-53.25	175.09	-66.06
10	19	19.44	-84.41	54.36	-66.47	30.55	-78.66
11	22	173.12	90.3	0	0	0	0
12	23	77.93	40.15	0	0	0	0
13	26	34.75	25.23	38.92	27.58	38.91	27.57
14	27	34	23.61	38.01	25.62	38	25.62
15	30	109.12	64.57	71.59	45.16	126.4	73.54
16	31	12.6	6.61	29.77	22.72	18.86	17.09
17	33	42.62	29.27	0	0	0	0

< Power result>

In	nd	I _{mag}	I _{real}	I _{mag}	I _{mag}	I _{real}	I _{mag}	I _{mag}	I _{real}	I _{mag}
		Phase 1			Phase 2			Phase 3		
1	2	5.287351	5.1033	-1.3829	4.343132	-2.8782	-3.2525	4.521041	-1.3432	4.3169
2	3	5.287351	5.1033	-1.3829	4.006006	-2.582	-3.0629	4.237561	-1.3298	4.0235
3	5	5.287351	5.1033	-1.3829	3.834931	-2.4301	-2.9667	4.237561	-1.3298	4.0235
4	6	5.287351	5.1033	-1.3829	3.834931	-2.4301	-2.9667	4.237561	-1.3298	4.0235
5	7	5.287351	5.1033	-1.3829	3.834931	-2.4301	-2.9667	4.237561	-1.3298	4.0235
6	12	3.341039	3.3175	-0.3959	3.380342	-2.0184	-2.7116	4.206873	-1.3288	3.9915
7	13	3.307792	3.2861	-0.3782	3.3408	-1.9818	-2.6895	4.206873	-1.3288	3.9915
8	15	2.899581	2.8986	-0.0754	2.893041	-1.5	-2.4738	3.698276	-1.3856	3.4289
9	17	1.841917	1.7067	0.6927	2.129339	-0.6073	-2.0409	1.924875	-1.4483	1.2679
10	19	0.898145	0.2341	0.8671	0.882994	0.2909	-0.8337	0.867977	-0.8617	-0.1042
11	22	2.00469	1.7545	-0.9698	0	0	0	0	0	0
12	23	0.920513	0.8063	-0.4441	0	0	0	0	0	0
13	26	0.444766	0.3505	-0.2738	0.489695	-0.4498	-0.1936	0.489826	0.0603	0.4861
14	27	0.444766	0.3505	-0.2738	0.489695	-0.4498	-0.1936	0.489826	0.0603	0.4861
15	30	1.314654	1.1058	-0.711	0.870422	-0.7804	-0.3855	1.504173	0.0541	1.5032
16	31	0.147639	0.1281	-0.0734	0.385099	-0.3591	-0.1391	0.261841	0.0636	0.254
17	33	0.536205	0.4304	-0.3198	0	0	0	0	0	0

< Current Result>

Case 1 : forecasted load data

- Input data (measurements data)

MSM
sbus ldbus mtyp Pm1 Qm1 Pm2 Qm2 Pm3 Qm3 Wmsm
END

- Result

In	nd	v1m	v1a	v2m	v2a+120	v3m	v3a-120	P1	Q1	P2	Q2	P3	Q3
		Phase 1			Phase 2								
1	2	0.9989	-0.0574	0.9992	-0.0421	0.9993	-0.0452	535.28	141.42	445.59	94.19	445.58	87.72
2	3	0.9903	-0.498	0.9936	-0.3458	0.9947	-0.3635	534.81	140.73	416.68	77.93	418.8	71.66
3	5	0.9728	-1.4435	0.9829	-0.9588	0.9851	-1.0311	531.28	135.44	399.16	66.55	417.26	69.01
4	6	0.9728	-1.4436	0.9828	-0.9589	0.9851	-1.0312	524.02	124.42	395.53	61.6	413.97	63.52
5	7	0.9727	-1.4481	0.9828	-0.9618	0.985	-1.0343	524.02	124.42	395.53	61.6	413.97	63.52
6	12	0.9635	-2.1086	0.9722	-1.5129	0.9754	-1.7941	340.43	35.9	349.26	37.06	410.73	62.25
7	13	0.9635	-2.1087	0.9722	-1.513	0.9754	-1.7942	334.5	29.91	342.59	31.28	407.51	56.24
8	15	0.9623	-2.2201	0.9707	-1.6032	0.9741	-1.9203	298.76	0.46	300.11	4.51	364.54	25.73
9	17	0.9623	-2.233	0.9706	-1.6191	0.9741	-1.9329	167.42	-69.56	222.57	-48.34	175.33	-75.77
10	19	0.9624	-2.2406	0.9706	-1.6314	0.9742	-1.9406	20.74	-84.47	60.7	-69.98	33.05	-81.66
11	22	0.9506	-1.6998	0.9828	-0.9618	0.985	-1.0343	180.63	86.83	0	0	0	0
12	23	0.9482	-1.7252	0.9828	-0.9618	0.985	-1.0343	80.48	40	0	0	0	0
13	26	0.9283	-3.1119	0.9361	-2.9093	0.9386	-3.0002	31.75	26.98	38.91	25.19	36.37	27.39
14	27	0.928	-3.1163	0.9358	-2.9157	0.9383	-3.0072	31.04	25.45	38.05	23.34	35.54	25.61
15	30	0.962	-2.2337	0.9705	-1.604	0.9737	-1.9335	122.71	63.97	69.6	45.28	133.23	74.86
16	31	0.962	-2.2338	0.9705	-1.6044	0.9736	-1.9339	15.63	7.87	34.81	24.56	24.12	17.14
17	33	0.9617	-2.2331	0.9705	-1.604	0.9737	-1.9335	43.07	34.49	0	0	0	0

<Voltage result>

< Power result>

In	nd	Imag	Ireal	Imag	Imag	Ireal	Imag	Imag	Ireal	Imag
		Phase 1			Phase 2			Phase 3		
1	2	5.536464	5.3528	-1.4142	4.55441	-3.0437	-3.388	4.541378	-1.4682	4.2975
2	3	5.536464	5.3528	-1.4142	4.242575	-2.7629	-3.2196	4.251687	-1.4713	3.989
3	5	5.536464	5.3528	-1.4142	4.072802	-2.6076	-3.1286	4.251687	-1.4713	3.989
4	6	5.536464	5.3528	-1.4142	4.072802	-2.6076	-3.1286	4.251687	-1.4713	3.989
5	7	5.536464	5.3528	-1.4142	4.072802	-2.6076	-3.1286	4.251687	-1.4713	3.989
6	12	3.519251	3.4894	-0.4574	3.573752	-2.1517	-2.8534	4.217348	-1.4664	3.9542
7	13	3.48543	3.4578	-0.438	3.538531	-2.1162	-2.836	4.217348	-1.4664	3.9542
8	15	3.100581	3.0983	-0.1189	3.087276	-1.6531	-2.6074	3.746499	-1.5339	3.4181
9	17	1.883896	1.7664	0.6549	2.346235	-0.7774	-2.2137	1.960921	-1.5336	1.222
10	19	0.903847	0.2496	0.8687	0.954392	0.2861	-0.9105	0.904503	-0.8995	-0.095
11	22	2.060366	1.8338	-0.9393	0	0	0	0	0	0
12	23	0.945406	0.8337	-0.4458	0	0	0	0	0	0
13	26	0.432396	0.319	-0.2919	0.476801	-0.4301	-0.2058	0.466664	0.0712	0.4612
14	27	0.432396	0.319	-0.2919	0.476801	-0.4301	-0.2058	0.466664	0.0712	0.4612
15	30	1.438096	1.2485	-0.7137	0.855445	-0.7731	-0.3662	1.568873	0.0342	1.5685
16	31	0.181864	0.1591	-0.0881	0.438984	-0.4035	-0.1729	0.303887	0.0388	0.3014
17	33	0.573573	0.4334	-0.3757	0	0	0	0	0	0

< Current Result>

Case 2 : forecasted load data, voltage measurements

- Input data (measurements data)

MSM									
sbus	ldbus	mtyp	Pm1	Qm1	Pm2	Qm2	Pm3	Qm3	Wmsm
13	15	v	23.78	0.00	24.49	0.00	24.17	0.00	50.0
17	19	v	24.01	0.00	24.32	0.00	24.26	0.00	50.0
30	33	v	23.92	0.00	24.44	0.00	23.93	0.00	50.0
END									

- Result

ln	nd	v1m	v1a	v2m	v2a+120	v3m	v3a-120	P1	Q1	P2	Q2	P3	Q3
		Phase 1			Phase 2		Phase 3						
1	2	0.9989	-0.0573	0.9992	-0.0426	0.9994	-0.0453	530.15	131.85	443.87	91.17	438.13	74.43
2	3	0.9905	-0.4968	0.9936	-0.3498	0.9951	-0.3646	529.68	131.18	414.96	74.91	411.36	58.39
3	5	0.9735	-1.4396	0.983	-0.9703	0.9861	-1.0336	526.24	126.05	397.45	63.54	409.91	55.86
4	6	0.9735	-1.4397	0.983	-0.9704	0.9861	-1.0337	519.15	115.36	393.85	58.6	406.84	50.62
5	7	0.9734	-1.4442	0.9829	-0.9733	0.9861	-1.0368	519.16	115.37	393.85	58.6	406.84	50.64
6	12	0.9645	-2.1016	0.9724	-1.5328	0.9772	-1.7961	336.64	28.82	347.64	34.17	403.62	49.41
7	13	0.9645	-2.1017	0.9724	-1.5329	0.9772	-1.7962	330.79	22.98	341	28.4	400.6	43.68
8	15	0.9634	-2.2125	0.9709	-1.6245	0.9759	-1.922	295.21	-6.16	298.6	1.77	357.92	13.68
9	17	0.9633	-2.2253	0.9708	-1.6406	0.9759	-1.9347	164.6	-74.8	221.17	-50.87	171.5	-82.77
10	19	0.9634	-2.2329	0.9708	-1.6529	0.9761	-1.9423	20	-85.85	60.33	-70.64	31.99	-83.61
11	22	0.9515	-1.7018	0.9829	-0.9733	0.9861	-1.0368	179.56	84.88	0	-0.01	-0.02	-0.03
12	23	0.9491	-1.7278	0.9829	-0.9733	0.9861	-1.0368	80.06	39.22	0	0	-0.01	-0.01
13	26	0.9296	-3.1012	0.9365	-2.9277	0.9409	-2.9946	31.59	26.68	38.83	25.06	36.09	26.89
14	27	0.9294	-3.1055	0.9362	-2.9341	0.9406	-3.0017	30.9	25.2	37.98	23.22	35.29	25.18
15	30	0.963	-2.226	0.9707	-1.6256	0.9755	-1.9352	122	62.62	69.49	45.08	130.99	70.77
16	31	0.963	-2.2261	0.9707	-1.626	0.9755	-1.9357	15.6	7.82	34.76	24.46	24	16.92
17	33	0.9627	-2.2254	0.9707	-1.6256	0.9755	-1.9352	42.8	33.98	0	-0.01	-0.01	-0.03

<Voltage result>

< Power result>

ln	nd	I _{mag}	I _{real}	I _{imag}	I _{mag}	I _{real}	I _{imag}	I _{mag}	I _{real}	I _{imag}
		Phase 1			Phase 2			Phase 3		
1	2	5.462998	5.3015	-1.3185	4.531302	-3.0089	-3.3881	4.444079	-1.546	4.1665
2	3	5.462998	5.3015	-1.3185	4.220212	-2.7282	-3.2198	4.157351	-1.549	3.858
3	5	5.463022	5.3015	-1.3186	4.050828	-2.5729	-3.1288	4.157444	-1.549	3.8581
4	6	5.463046	5.3015	-1.3187	4.050891	-2.573	-3.1288	4.157499	-1.5489	3.8582
5	7	5.463167	5.3016	-1.3188	4.050891	-2.573	-3.1288	4.157647	-1.5488	3.8584
6	12	3.471106	3.4499	-0.3831	3.553903	-2.1183	-2.8536	4.123869	-1.5438	3.824
7	13	3.437803	3.4185	-0.3638	3.518878	-2.0829	-2.8362	4.123979	-1.5436	3.8242
8	15	3.061383	3.061	-0.0484	3.070811	-1.6215	-2.6078	3.665577	-1.6077	3.2942
9	17	1.876744	1.7373	0.7099	2.337417	-0.7483	-2.2144	1.95125	-1.5754	1.1513
10	19	0.915009	0.2421	0.8824	0.956891	0.2934	-0.9108	0.917299	-0.9102	-0.1139
11	22	2.040467	1.8222	-0.9182	0.0001	0.0001	0	0.000361	-0.0002	-0.0003
12	23	0.936952	0.8288	-0.437	0	0	0	0.000141	-0.0001	-0.0001
13	26	0.428775	0.3172	-0.2885	0.475272	-0.4285	-0.2056	0.460548	0.068	0.4555
14	27	0.428984	0.3173	-0.2887	0.475362	-0.4286	-0.2056	0.460874	0.0682	0.4558
15	30	1.423463	1.2403	-0.6985	0.853191	-0.7707	-0.366	1.525521	0.008	1.5255
16	31	0.181175	0.1587	-0.0874	0.437841	-0.4023	-0.1728	0.300921	0.0373	0.2986
17	33	0.567447	0.4304	-0.3698	0.0001	0.0001	0	0.000283	-0.0002	-0.0002

< Current Result>

Case 3 : forecasted load data, power, current measurements

- Input data (measurements data)

MSM									
sbus	ldbus	mtyp	Pm1	Qm1	Pm2	Qm2	Pm3	Qm3	Wmsm
0	2	s	498.97	135.21	433.31	88.21	458.41	103.45	50.0
12	13	i	23.41	0.00	23.51	0.00	29.21	0.00	50.0
END									

- Result

In	nd	v1m	v1a	v2m	v2a+120	v3m	v3a-120
		Phase 1		Phase 2		Phase 3	
1	2	0.9989	-0.0551	0.9992	-0.0379	0.9993	-0.0467
2	3	0.9907	-0.4781	0.9939	-0.3093	0.9942	-0.3769
3	5	0.9741	-1.385	0.9838	-0.8533	0.9835	-1.0705
4	6	0.9741	-1.3852	0.9838	-0.8534	0.9835	-1.0706
5	7	0.974	-1.3894	0.9838	-0.8559	0.9834	-1.0739
6	12	0.9657	-2.01	0.9738	-1.3285	0.9728	-1.8634
7	13	0.9657	-2.0101	0.9738	-1.3286	0.9728	-1.8635
8	15	0.9646	-2.1146	0.9725	-1.4055	0.9712	-1.9948
9	17	0.9646	-2.1265	0.9723	-1.4195	0.9712	-2.0081
10	19	0.9647	-2.1337	0.9724	-1.431	0.9714	-2.016
11	22	0.9518	-1.6387	0.9838	-0.8559	0.9834	-1.0739
12	23	0.9494	-1.6638	0.9838	-0.8559	0.9834	-1.0739
13	26	0.931	-2.964	0.9366	-2.6497	0.9352	-3.0904
14	27	0.9307	-2.9682	0.9383	-2.6554	0.935	-3.0977
15	30	0.9643	-2.1275	0.9722	-1.4057	0.9708	-2.0085
16	31	0.9643	-2.1276	0.9722	-1.406	0.9708	-2.009
17	33	0.964	-2.1266	0.9722	-1.4057	0.9708	-2.0085

<Voltage result>

In	nd	P1	Q1	P2	Q2	P3	Q3
		Phase 1		Phase 2		Phase 3	
1	2	513.69	139.4	419.18	88.47	458.16	100.17
2	3	513.26	138.75	389.44	72.19	431.27	83.88
3	5	510.07	133.86	371.86	61.21	429.53	80.96
4	6	503.51	123.65	368.56	57.09	425.85	74.92
5	7	503.51	123.64	368.48	57.08	425.84	74.9
6	12	319.59	34.55	319.9	32.41	422.57	73.57
7	13	314.11	29.06	313.6	27.41	418.95	66.99
8	15	279.3	-0.31	272.67	0.92	375.48	35.99
9	17	152.19	-70.11	197.77	-51.49	181.69	-69.83
10	19	16.81	-84.61	54.66	-70.74	34.82	-80.02
11	22	180.97	87.41	0.16	0.01	0.02	0.04
12	23	80.61	40.24	0.08	0	0.01	0.02
13	26	30.87	26.9	37.43	24.93	36.84	27.85
14	27	30.24	25.43	36.69	23.2	35.96	25.99
15	30	118.6	63.82	67.1	44.97	136.91	78.29
16	31	15.47	7.87	33.54	24.4	24.32	17.32
17	33	41.49	34.43	-0.07	-0.01	0.02	0.02

< Power result>

In	nd	I _{mag}	I _{real}	I _{mag}	I _{mag}	I _{real}	I _{mag}	I _{mag}	I _{real}	I _{mag}
		Phase 1			Phase 2			Phase 3		
1	2	5.322685	5.1369	-1.394	4.284128	-2.862	-3.1879	4.689764	-1.4232	4.4686
2	3	5.322589	5.1368	-1.394	3.963951	-2.5765	-3.0124	4.396817	-1.4276	4.1586
3	5	5.322589	5.1368	-1.394	3.791723	-2.4198	-2.9192	4.39666	-1.4277	4.1584
4	6	5.322562	5.1368	-1.3939	3.790928	-2.4194	-2.9185	4.396503	-1.4278	4.1582
5	7	5.322562	5.1368	-1.3939	3.79007	-2.4189	-2.9178	4.396474	-1.428	4.1581
6	12	3.300287	3.2716	-0.4342	3.268386	-1.9506	-2.6225	4.36157	-1.4236	4.1227
7	13	3.266444	3.24	-0.4148	3.232547	-1.9148	-2.6044	4.361508	-1.4237	4.1226
8	15	2.892168	2.8905	-0.0982	2.799967	-1.4639	-2.3868	3.87757	-1.494	3.5782
9	17	1.737023	1.6034	0.6681	2.101513	-0.6078	-2.0117	2.004154	-1.5132	1.3141
10	19	0.894315	0.2067	0.8701	0.919456	0.3279	-0.859	0.898546	-0.8958	-0.0702
11	22	2.063291	1.8356	-0.9422	0.001581	-0.0009	-0.0013	0.000424	0.0003	0.0003
12	23	0.946495	0.8344	-0.4468	0.00086	-0.0005	-0.0007	0.000224	0.0001	0.0002
13	26	0.424007	0.3097	-0.2896	0.461785	-0.4185	-0.1952	0.474786	0.0739	0.469
14	27	0.424373	0.3102	-0.2896	0.462492	-0.419	-0.1958	0.474475	0.0738	0.4687
15	30	1.396238	1.2043	-0.7065	0.830615	-0.7542	-0.348	1.623867	0.0499	1.6231
16	31	0.179999	0.1573	-0.0875	0.426616	-0.394	-0.1636	0.307513	0.04	0.3049
17	33	0.559123	0.4167	-0.3728	0.000781	0.0005	0.0006	0.000316	0.0001	0.0003

< Current Result>

Case 4 : forecasted load data, power, current, and voltage measurements

- Input data (measurements data)

MSM									
sbus	ldbus	mtyp	Pm1	Qm1	Pm2	Qm2	Pm3	Qm3	Wmsm
0	2	s	498.97	135.21	433.31	88.21	458.41	103.45	50.0
12	13	i	23.41	0.00	23.51	0.00	29.21	0.00	50.0
13	15	v	23.78	0.00	24.49	0.00	24.17	0.00	50.0
17	19	v	24.01	0.00	24.32	0.00	24.26	0.00	50.0
30	33	v	23.92	0.00	24.44	0.00	23.93	0.00	50.0
END									

- Result

In	nd	v1m	v1a	v2m	v2a+120	v3m	v3a-120
		Phase 1		Phase 2		Phase 3	
1	2	0.9989	-0.0548	0.9992	-0.038	0.9993	-0.0469
2	3	0.9907	-0.4757	0.9939	-0.3098	0.9942	-0.3781
3	5	0.9738	-1.3785	0.9836	-0.8548	0.9837	-1.0739
4	6	0.9738	-1.3786	0.9836	-0.8549	0.9837	-1.074
5	7	0.9738	-1.3829	0.9836	-0.8575	0.9836	-1.0773
6	12	0.9653	-1.9966	0.9735	-1.3311	0.9732	-1.869
7	13	0.9653	-1.9967	0.9735	-1.3311	0.9732	-1.8692
8	15	0.9642	-2.1001	0.9721	-1.4082	0.9716	-2.0008
9	17	0.9641	-2.1118	0.972	-1.4222	0.9716	-2.0141
10	19	0.9642	-2.1189	0.972	-1.4337	0.9717	-2.0221
11	22	0.9516	-1.6377	0.9836	-0.8575	0.9836	-1.0773
12	23	0.9491	-1.6634	0.9836	-0.8575	0.9836	-1.0773
13	26	0.9304	-2.9478	0.9382	-2.6506	0.9357	-3.0966
14	27	0.9301	-2.9519	0.9379	-2.6564	0.9354	-3.1039
15	30	0.9638	-2.1129	0.9719	-1.4084	0.9712	-2.0146
16	31	0.9638	-2.113	0.9719	-1.4088	0.9712	-2.015
17	33	0.9636	-2.1119	0.9719	-1.4084	0.9712	-2.0146

<Voltage result>

In	nd	P1	Q1	P2	Q2	P3	Q3
		Phase 1		Phase 2		Phase 3	
1	2	514.61	142.87	419.64	90.84	457.72	97.95
2	3	514.18	142.24	389.94	74.8	430.81	81.49
3	5	510.96	137.33	372.36	63.85	429.09	78.57
4	6	504.35	127.07	369.03	59.68	425.45	72.55
5	7	504.34	127.07	368.95	59.68	425.44	72.53
6	12	319.73	38.47	320.22	35.15	422.18	71.18
7	13	314.22	32.97	313.89	30.1	418.61	64.63
8	15	279.42	3.46	272.97	3.51	375.14	33.7
9	17	152.28	-67.15	198.03	-49.17	181.49	-71.19
10	19	16.84	-83.83	54.72	-70.17	34.75	-80.41
11	22	181.65	86.91	0.17	0	0.01	0.05
12	23	80.89	40.03	0.08	0	0	0.02
13	26	30.86	27.04	37.43	25.03	36.84	27.79
14	27	30.22	25.56	36.88	23.28	35.97	25.94
15	30	118.63	64.62	67.14	45.21	136.8	77.54
16	31	15.47	7.9	33.55	24.53	24.31	17.28
17	33	41.5	34.74	-0.07	0	0.02	0.02

< Power result>

In	nd	Imag	Ireal	Imag	Imag	Ireal	Imag	Ireal	Imag	
		Phase 1			Phase 2			Phase 3		
1	2	5.340742	5.1461	-1.4287	4.293605	-2.8849	-3.18	4.680897	-1.4403	4.4538
2	3	5.340796	5.1461	-1.4289	3.973731	-2.6016	-3.0037	4.387716	-1.446	4.1426
3	5	5.340823	5.1461	-1.429	3.801293	-2.4454	-2.9103	4.387499	-1.4462	4.1423
4	6	5.340823	5.1461	-1.429	3.8005	-2.445	-2.9096	4.38747	-1.4464	4.1422
5	7	5.340726	5.146	-1.429	3.79963	-2.4446	-2.9088	4.387347	-1.4466	4.142
6	12	3.307173	3.273	-0.4742	3.275192	-1.9766	-2.6115	4.35261	-1.4423	4.1067
7	13	3.273038	3.2413	-0.4547	3.239002	-1.9406	-2.5933	4.352582	-1.4425	4.1066
8	15	2.894925	2.8917	-0.1366	2.804109	-1.4887	-2.3763	3.870438	-1.5122	3.5628
9	17	1.726115	1.6038	0.6382	2.098852	-0.6299	-2.0021	2.006433	-1.5238	1.3053
10	19	0.886899	0.2066	0.8625	0.915485	0.3225	-0.8568	0.90162	-0.8987	-0.0725
11	22	2.068007	1.8434	-0.9373	0.001749	-0.0009	-0.0015	0.0005	0.0004	0.0003
12	23	0.948466	0.8377	-0.4448	0.000806	-0.0004	-0.0007	0.000283	0.0002	0.0002
13	26	0.425033	0.3097	-0.2911	0.462481	-0.4195	-0.1947	0.474199	0.0733	0.4685
14	27	0.425402	0.3103	-0.291	0.463096	-0.4199	-0.1953	0.473903	0.0733	0.4682
15	30	1.401109	1.205	-0.7149	0.83265	-0.7569	-0.347	1.618387	0.0436	1.6178
16	31	0.180232	0.1574	-0.0878	0.427626	-0.3953	-0.1631	0.307077	0.0397	0.3045
17	33	0.561552	0.417	-0.3781	0.000721	0.0004	0.0006	0.0003	0	0.0003

< Current Result>

APPENDIX II - State Estimation Test Result (300 Monte-Carlo Simulation)

Case 1 : forecasted load data

- Current Real

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	5,0987	0,2256	0,0046	-2,8866	0,1532	0,0084	-1,3437	0,1438	0,0005
2	5,0987	0,2256	0,0046	-2,5924	0,1513	0,0104	-1,3305	0,1408	0,0007
3	5,0987	0,2256	0,0046	-2,4406	0,1509	0,0105	-1,3305	0,1408	0,0007
4	5,0987	0,2256	0,0046	-2,4406	0,1509	0,0105	-1,3305	0,1408	0,0007
5	5,0987	0,2256	0,0046	-2,4406	0,1509	0,0105	-1,3305	0,1408	0,0007
6	3,3206	0,1766	-0,0031	-2,0285	0,1451	0,0101	-1,3293	0,1408	0,0005
7	3,2893	0,1764	-0,0032	-1,9917	0,145	0,0099	-1,3293	0,1408	0,0005
8	2,9025	0,1727	-0,0039	-1,5098	0,141	0,0098	-1,3869	0,14	0,0013
9	1,7116	0,1615	-0,0049	-0,6188	0,1333	0,0115	-1,4526	0,1215	0,0043
10	0,2356	0,021	-0,0015	0,2889	0,0376	0,002	-0,862	0,026	0,0003
11	1,7468	0,1331	0,0077	0	0	0	0	0	0
12	0,8005	0,0816	0,0058	0	0	0	0	0	0
13	0,3498	0,0361	0,0007	-0,4499	0,0342	0,0001	0,0608	0,027	-0,0005
14	0,3498	0,0361	0,0007	-0,4499	0,0342	0,0001	0,0608	0,027	-0,0005
15	1,1051	0,0695	0,0007	-0,7782	0,0383	-0,0022	0,0563	0,0778	-0,0022
16	0,1279	0,0135	0,0002	-0,3576	0,0248	-0,0015	0,0637	0,0183	-0,0001
17	0,4247	0,0419	0,0057	0	0	0	0	0	0

- Current Imaginary

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	-1,3809	0,1402	-0,002	-3,2443	0,1638	-0,0082	4,3176	0,1913	-0,0007
2	-1,3809	0,1402	-0,002	-3,0572	0,1624	-0,0057	4,023	0,1904	0,0005
3	-1,3809	0,1402	-0,002	-2,9613	0,1619	-0,0054	4,023	0,1904	0,0005
4	-1,3809	0,1402	-0,002	-2,9613	0,1619	-0,0054	4,023	0,1904	0,0005
5	-1,3809	0,1402	-0,002	-2,9613	0,1619	-0,0054	4,023	0,1904	0,0005
6	-0,4006	0,1241	0,0047	-2,7062	0,1569	-0,0054	3,9907	0,1902	0,0008
7	-0,3829	0,1242	0,0047	-2,684	0,1569	-0,0055	3,9907	0,1902	0,0008
8	-0,0791	0,1211	0,0037	-2,4684	0,1538	-0,0054	3,4276	0,1872	0,0013
9	0,6891	0,1108	0,0036	-2,0385	0,147	-0,0024	1,2677	0,1465	0,0002
10	0,8667	0,0165	0,0004	-0,838	0,0491	0,0043	-0,1028	0,0302	-0,0014
11	-0,9632	0,061	-0,0066	0	0	0	0	0	0
12	-0,4421	0,0407	-0,002	0	0	0	0	0	0
13	-0,2746	0,0253	0,0008	-0,1937	0,0355	0,0001	0,4868	0,0399	-0,0007
14	-0,2746	0,0253	0,0008	-0,1937	0,0355	0,0001	0,4868	0,0399	-0,0007
15	-0,7114	0,0438	0,0004	-0,3824	0,0507	-0,0031	1,5049	0,1014	-0,0017
16	-0,0746	0,0065	0,0012	-0,1371	0,0295	-0,002	0,254	0,0204	0
17	-0,3199	0,032	0,0001	0	0	0	0	0	0

- Current Magnitude

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	5,2844	0,223	0,00295	4,3445	0,1823	-0,00137	4,5238	0,2	-0,00276
2	5,2844	0,223	0,00295	4,0105	0,1798	-0,00449	4,2392	0,1989	-0,00164
3	5,2844	0,223	0,00295	3,8396	0,1795	-0,00467	4,2392	0,1989	-0,00164
4	5,2844	0,223	0,00295	3,8396	0,1795	-0,00467	4,2392	0,1989	-0,00164
5	5,2844	0,223	0,00295	3,8396	0,1795	-0,00467	4,2392	0,1989	-0,00164
6	3,347	0,177	-0,00596	3,3845	0,1707	-0,00416	4,2082	0,1988	-0,00133
7	3,3138	0,1768	-0,00601	3,3447	0,1707	-0,00390	4,2082	0,1988	-0,00133
8	2,9061	0,1725	-0,00652	2,8962	0,1679	-0,00316	3,6997	0,1975	-0,00142
9	1,8488	0,1566	-0,00688	2,1339	0,1553	-0,00456	1,9315	0,1496	-0,00663
10	0,8984	0,0172	-0,00025	0,8875	0,0428	-0,00451	0,8686	0,0251	-0,00062
11	1,9964	0,1218	0,00829	0	0	0,00000	0	0	0
12	0,9159	0,0756	0,00461	0	0	0,00000	0	0	0
13	0,4456	0,0345	-0,00083	0,4908	0,0383	-0,00111	0,4914	0,0389	-0,00157
14	0,4456	0,0345	-0,00083	0,4908	0,0383	-0,00111	0,4914	0,0389	-0,00157
15	1,3152	0,0655	-0,00055	0,8681	0,0483	0,00232	1,508	0,0998	-0,00383
16	0,1483	0,0124	-0,00066	0,384	0,0272	0,00110	0,2625	0,0202	-0,00066
17	0,5329	0,0386	0,00331	0	0	0,00000	0	0	0

- Voltage Magnitude

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	0.9989	0.0001	0	0.9992	0.0001	0	0.9993	0.0001	0
2	0.9907	0.0004	0	0.994	0.0004	0	0.9944	0.0004	0
3	0.9741	0.0013	0	0.9839	0.0012	1E-04	0.984	0.0012	0
4	0.9741	0.0013	0	0.9839	0.0012	1E-04	0.984	0.0012	0
5	0.974	0.0013	0	0.9839	0.0012	0	0.984	0.0012	0
6	0.9656	0.0019	0	0.9738	0.0019	1E-04	0.9737	0.002	0
7	0.9656	0.0019	0	0.9738	0.0019	1E-04	0.9737	0.002	0
8	0.9645	0.002	0	0.9724	0.002	0.0002	0.9723	0.0021	-0.0001
9	0.9644	0.002	0	0.9723	0.002	1E-04	0.9722	0.0021	0
10	0.9646	0.002	-1E-04	0.9723	0.002	0.0002	0.9724	0.0021	-1E-04
11	0.9525	0.0021	-0.0002	0.9839	0.0012	0	0.984	0.0012	0
12	0.9501	0.0022	-1E-04	0.9839	0.0012	0	0.984	0.0012	0
13	0.9307	0.0036	1E-04	0.9358	0.0039	1E-04	0.9356	0.0039	1E-04
14	0.9305	0.0037	0	0.9355	0.0039	1E-04	0.9354	0.0039	0
15	0.9641	0.002	0	0.9722	0.002	0.0001	0.9719	0.0021	-1E-04
16	0.9641	0.002	0	0.9722	0.002	0.0001	0.9719	0.0021	-1E-04
17	0.9639	0.002	0	0.9722	0.002	0.0001	0.9719	0.0021	-1E-04

- Power Real

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	5.0987	0.2256	0.0046	4.2529	0.1814	0.003	4.411	0.2003	-0.0009
2	5.0945	0.2253	0.0045	3.9411	0.1788	-0.0002	4.147	0.1984	0
3	5.0626	0.2228	0.0046	3.7654	0.177	-0.0005	4.131	0.1968	1E-04
4	4.9973	0.2177	0.0046	3.7329	0.174	-0.0002	4.0971	0.1994	1E-04
5	4.9973	0.2177	0.0046	3.7329	0.174	-0.0002	4.097	0.1934	0.0002
6	3.2427	0.1704	-0.0032	3.3093	0.1658	-1E-04	4.0685	0.1933	0.0005
7	3.1868	0.1676	-0.003	3.241	0.163	0.0003	4.0353	0.1899	0.0006
8	2.8032	0.1638	-0.0036	2.8174	0.1606	0.0002	3.5789	0.1887	0.0007
9	1.6248	0.1525	-0.0047	2.0046	0.1506	-0.0033	1.7525	0.1478	-0.0016
10	0.196	0.0201	-0.0016	0.5481	0.0513	-0.0045	0.3067	0.0309	-0.0012
11	1.7234	0.1298	0.0078	0	0	0	0	0	0
12	0.7737	0.0774	0.0056	0	0	0	0	0	0
13	0.3469	0.035	0.0006	0.3892	0.0388	0	0.3895	0.039	-0.0004
14	0.3393	0.0339	0.0007	0.38	0.0376	1E-04	0.3802	0.0376	-0.0002
15	1.0904	0.0669	0.0008	0.7121	0.0533	0.0038	1.2644	0.1096	-0.0004
16	0.1259	0.013	1E-04	0.2952	0.0297	0.0025	0.1886	0.0195	0
17	0.4206	0.0404	0.0056	0	0	0	0	0	0

- Power Imaginary

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	1.3809	0.1402	0.002	0.8777	0.1319	-0.0113	0.9951	0.131	1E-04
2	1.3745	0.14	0.002	0.7132	0.1311	-0.0118	0.8555	0.1289	0.0008
3	1.3261	0.1391	0.0019	0.608	0.1301	-0.0116	0.8288	0.1283	0.001
4	1.2251	0.1377	0.0019	0.5651	0.1294	-0.0114	0.7735	0.1273	0.0013
5	1.225	0.1377	0.002	0.5651	0.1294	-0.0114	0.7735	0.1273	0.0013
6	0.3118	0.1216	-0.0043	0.3458	0.1279	-0.0111	0.7588	0.1272	0.0014
7	0.2572	0.1213	-0.004	0.2941	0.1276	-0.0107	0.6984	0.1265	0.0016
8	-0.0228	0.1188	-0.0031	0.0029	0.1225	-0.0107	0.3895	0.1233	0.0024
9	-0.7257	0.1094	-0.003	-0.522	0.1189	-0.0105	-0.6649	0.1093	0.0043
10	-0.8437	0.0161	-0.0004	-0.6651	0.0307	0.0004	-0.7863	0.0231	-0.0003
11	0.8968	0.0594	0.0062	0	0	0	0	0	0
12	0.3998	0.039	0.0017	0	0	0	0	0	0
13	0.2531	0.0241	-0.0008	0.2758	0.0282	0	0.2765	0.0261	-0.0008
14	0.2368	0.0229	-0.0007	0.256	0.0267	0.0002	0.2567	0.0244	-0.0005
15	0.6461	0.0419	-0.0004	0.4513	0.0312	0.0003	0.7379	0.0586	-0.0025
16	0.0672	0.0063	-0.0011	0.227	0.023	0.0002	0.1709	0.0181	0
17	0.293	0.0308	-0.0003	0	0	0	0	0	0

Case 2 : forecasted load data, voltage measurements

- Current Real

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	5,1079	0,2013	-0,00460	-2,8829	0,1381	0,0047	-1,3595	0,1536	0,0163
2	5,1079	0,2013	-0,00460	-2,5878	0,1375	0,0058	-1,3444	0,1531	0,0146
3	5,1079	0,2013	-0,00460	-2,436	0,1359	0,0059	-1,3444	0,1531	0,0146
4	5,1079	0,2013	-0,00460	-2,436	0,1358	0,0059	-1,3444	0,1531	0,0146
5	5,1079	0,2013	-0,00460	-2,436	0,1358	0,0059	-1,3444	0,153	0,0146
6	3,3192	0,1611	-0,00170	-2,0252	0,134	0,0068	-1,3433	0,1531	0,0145
7	3,2878	0,1611	-0,00170	-1,9866	0,134	0,0068	-1,3433	0,153	0,0145
8	2,8994	0,1594	-0,00080	-1,5088	0,1309	0,0088	-1,402	0,1511	0,0164
9	1,7016	0,1426	0,00510	-0,6154	0,122	0,0081	-1,4657	0,1269	0,0174
10	0,2328	0,0215	0,00130	0,292	0,0452	-0,0011	-0,8622	0,0269	0,0005
11	1,7572	0,1234	-0,00270	0	0,0003	0	0	0,0002	0
12	0,8101	0,0754	-0,00380	0	0,0002	0	0	0,0001	0
13	0,351	0,0341	-0,00050	-0,4477	0,035	-0,0021	0,0622	0,0312	-0,0019
14	0,351	0,034	-0,00050	-0,4477	0,035	-0,0021	0,0622	0,0311	-0,0019
15	1,1116	0,0739	-0,00580	-0,7813	0,0391	0,0009	0,0598	0,0736	-0,0057
16	0,1283	0,0138	-0,00020	-0,358	0,0256	-0,0011	0,0636	0,0187	0
17	0,4333	0,0459	-0,00290	0	0,0003	0	0	0,0001	0

- Current Imaginary

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	-1,3809	0,1402	-0,002	-3,2443	0,1638	-0,0082	4,3176	0,1913	-0,0007
2	-1,3809	0,1402	-0,002	-3,0572	0,1624	-0,0057	4,023	0,1904	0,0005
3	-1,3809	0,1402	-0,002	-2,9613	0,1619	-0,0054	4,023	0,1904	0,0005
4	-1,3809	0,1402	-0,002	-2,9613	0,1619	-0,0054	4,023	0,1904	0,0005
5	-1,3809	0,1402	-0,002	-2,9613	0,1619	-0,0054	4,023	0,1904	0,0005
6	-0,4006	0,1241	0,0047	-2,7062	0,1569	-0,0054	3,9907	0,1902	0,0008
7	-0,3829	0,1242	0,0047	-2,684	0,1569	-0,0055	3,9907	0,1902	0,0008
8	-0,0791	0,1211	0,0037	-2,4684	0,1538	-0,0054	3,4276	0,1872	0,0013
9	0,6891	0,1108	0,0036	-2,0385	0,147	-0,0024	1,2677	0,1465	0,0002
10	0,8667	0,0165	0,0004	-0,838	0,0491	0,0043	-0,1028	0,0302	-0,0014
11	-0,9632	0,061	-0,0066	0	0	0	0	0	0
12	-0,4421	0,0407	-0,002	0	0	0	0	0	0
13	-0,2746	0,0253	0,0008	-0,1937	0,0355	0,0001	0,4868	0,0399	-0,0007
14	-0,2746	0,0253	0,0008	-0,1937	0,0355	0,0001	0,4868	0,0399	-0,0007
15	-0,7114	0,0438	0,0004	-0,3824	0,0507	-0,0031	1,5049	0,1014	-0,0017
16	-0,0746	0,0065	0,0012	-0,1371	0,0295	-0,002	0,254	0,0204	0
17	-0,3199	0,032	0,0001	0	0	0	0	0	0

- Current Magnitude

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	5,296	0,2004	-0,00865	4,3503	0,1675	-0,00717	4,5383	0,2059	-0,01726
2	5,296	0,2004	-0,00865	4,0129	0,1654	-0,00689	4,2525	0,2049	-0,01494
3	5,296	0,2004	-0,00865	3,8415	0,1642	-0,00657	4,2525	0,2049	-0,01494
4	5,296	0,2004	-0,00865	3,8415	0,1642	-0,00657	4,2525	0,2049	-0,01494
5	5,296	0,2004	-0,00865	3,8415	0,1642	-0,00657	4,2525	0,2049	-0,01494
6	3,3462	0,1602	-0,00516	3,3872	0,1617	-0,00686	4,2217	0,2048	-0,01483
7	3,3128	0,1603	-0,00501	3,3477	0,1617	-0,00690	4,2217	0,2048	-0,01483
8	2,9032	0,1589	-0,00362	2,9048	0,1578	-0,01176	3,7117	0,2033	-0,01342
9	1,8376	0,1401	0,00432	2,1385	0,1467	-0,00916	1,9506	0,1563	-0,02573
10	0,8981	0,0211	0,00005	0,8871	0,0494	-0,00411	0,8689	0,0265	-0,00092
11	2,0087	0,1144	-0,00401	0,0002	0,0002	-0,00020	0,0002	0,0002	-0,0002
12	0,925	0,0735	-0,00449	0,0001	0,0001	-0,00010	0,0001	0,0001	-0,0001
13	0,4445	0,0321	0,00027	0,4868	0,0402	0,00289	0,4947	0,0378	-0,00487
14	0,4445	0,0321	0,00027	0,4868	0,0402	0,00289	0,4947	0,0378	-0,00487
15	1,3216	0,0684	-0,00695	0,8727	0,0466	-0,00228	1,4949	0,1014	0,009273
16	0,1483	0,0124	-0,00066	0,3849	0,0282	0,00020	0,2629	0,0175	-0,00106
17	0,5389	0,0412	-0,00269	0,0002	0,0002	-0,00020	0,0002	0,0002	-0,0002

- Voltage Magnitude

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	0,9989	0,0001	0	0,9992	0,0001	0	0,9993	0,0001	0
2	0,9907	0,0004	0	0,994	0,0004	0	0,9944	0,0004	0
3	0,974	0,0012	1E-04	0,9839	0,0011	1E-04	0,984	0,0012	0
4	0,974	0,0012	1E-04	0,9839	0,0011	1E-04	0,984	0,0012	0
5	0,9739	0,0012	1E-04	0,9839	0,0011	0	0,984	0,0012	0
6	0,9655	0,0018	1E-04	0,9738	0,0017	1E-04	0,9737	0,002	0
7	0,9655	0,0018	1E-04	0,9738	0,0017	1E-04	0,9737	0,002	0
8	0,9644	0,0019	1E-04	0,9724	0,0019	0,0002	0,9723	0,0021	-0,0001
9	0,9643	0,0019	1E-04	0,9723	0,0019	1E-04	0,9722	0,0021	0
10	0,9644	0,0019	1E-04	0,9723	0,0019	0,0002	0,9724	0,0021	-1E-04
11	0,9522	0,0021	1E-04	0,9839	0,0011	0	0,984	0,0012	0
12	0,9499	0,0022	1E-04	0,9839	0,0011	0	0,984	0,0012	0
13	0,9309	0,0035	-1E-04	0,9359	0,0037	0	0,9353	0,0038	0,0004
14	0,9306	0,0035	-1E-04	0,9356	0,0037	0	0,9351	0,0038	0,0003
15	0,964	0,0019	1E-04	0,9722	0,0019	0,0001	0,9719	0,0021	-1E-04
16	0,964	0,0019	1E-04	0,9722	0,0019	0,0001	0,9719	0,0021	-1E-04
17	0,9638	0,0019	1E-04	0,9722	0,0019	0,0001	0,9719	0,0021	-1E-04

- Power Real

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	5,1079	0,2013	-0,0046	4,2607	0,1689	-0,0048	4,4275	0,2058	-0,0174
2	5,1037	0,201	-0,0047	3,945	0,1661	-0,0041	4,1617	0,2044	-0,0147
3	5,0717	0,1988	-0,0045	3,7686	0,1636	-0,0037	4,1456	0,2028	-0,0145
4	5,0061	0,1941	-0,0042	3,7361	0,1609	-0,0034	4,1115	0,1994	-0,0143
5	5,0061	0,1941	-0,0042	3,7361	0,1609	-0,0034	4,1114	0,1994	-0,0142
6	3,2411	0,1552	-0,0016	3,3127	0,158	-0,0035	4,0831	0,1993	-0,0141
7	3,1852	0,1529	-0,0014	3,2446	0,1553	-0,0033	4,0497	0,1959	-0,0138
8	2,8003	0,1511	-0,0007	2,826	0,1511	-0,0084	3,5914	0,1942	-0,0118
9	1,6152	0,1348	0,0049	2,008	0,1405	-0,0067	1,7702	0,1533	-0,0193
10	0,1932	0,0211	0,0012	0,5448	0,0577	-0,0012	0,3074	0,0325	-0,0019
11	1,7335	0,1201	-0,0023	0	0,0001	0	0	0,0001	0
12	0,7827	0,0716	-0,0034	0	0,0001	0	0	0,0001	0
13	0,3479	0,0329	-0,0004	0,3841	0,0407	0,0051	0,3914	0,0396	-0,0023
14	0,3403	0,032	-0,0003	0,375	0,0393	0,0051	0,3821	0,0383	-0,0021
15	1,0967	0,0713	-0,0055	0,7177	0,0489	-0,0018	1,2517	0,1069	0,0123
16	0,1263	0,0133	-0,0003	0,2968	0,0302	0,0009	0,189	0,0189	-0,0004
17	0,4288	0,0443	-0,0026	0	0,0001	0	0	0,0001	0

- Power Imaginary

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	1,3913	0,144	-0,0084	0,869	0,1267	-0,0026	0,9864	0,1423	0,0088
2	1,3849	0,1438	-0,0084	0,7057	0,126	-0,0043	0,8478	0,1421	0,0085
3	1,3362	0,1428	-0,0082	0,6009	0,1254	-0,0045	0,821	0,1415	0,0088
4	1,2348	0,1412	-0,0078	0,5579	0,1251	-0,0042	0,7654	0,1405	0,0094
5	1,2348	0,1411	-0,0078	0,5579	0,125	-0,0042	0,7654	0,1404	0,0094
6	0,3157	0,1229	-0,0082	0,3399	0,1239	-0,0052	0,7507	0,1401	0,0095
7	0,2612	0,1229	-0,008	0,2882	0,1236	-0,0048	0,6898	0,1394	0,0102
8	-0,0152	0,1184	-0,0107	-0,0037	0,122	-0,0041	0,3789	0,1363	0,013
9	-0,7204	0,1108	-0,0083	-0,5281	0,1153	-0,0044	-0,6701	0,1133	0,0095
10	-0,844	0,0224	-1E-04	-0,6668	0,0388	0,0021	-0,7862	0,0267	-0,0004
11	0,9027	0,0672	0,0003	0	0,0003	0	0	0,0003	0
12	0,4011	0,0422	0,0004	0	0,0001	0	0	0,0001	0
13	0,2498	0,0254	0,0025	0,2764	0,0268	-0,0006	0,279	0,0281	-0,0033
14	0,2336	0,0243	0,0025	0,2569	0,0251	-0,0007	0,2589	0,0268	-0,0027
15	0,6476	0,038	-0,0019	0,4514	0,0319	0,0002	0,7347	0,0606	0,0007
16	0,0665	0,0071	-0,0004	0,2263	0,0225	0,0009	0,171	0,0168	-0,0001
17	0,2914	0,0264	-0,2914	0	0,0002	0	0	0,0002	0

Case 3 : forecasted load data, power, current measurements

- Current Real

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	5,0974	0,0909	0,0059	-2,8775	0,0435	-0,00070	-1,3435	0,046	0,0003
2	5,0974	0,0909	0,0059	-2,5824	0,0444	0,00040	-1,3285	0,0477	-0,0013
3	5,0974	0,0909	0,0059	-2,4306	0,0448	0,00050	-1,3285	0,0478	-0,0013
4	5,0974	0,0909	0,0059	-2,4305	0,0449	0,00040	-1,3285	0,0478	-0,0013
5	5,0974	0,0909	0,0059	-2,4305	0,045	0,00040	-1,3285	0,0478	-0,0013
6	3,3131	0,0654	0,0044	-2,0197	0,0496	0,00130	-1,3273	0,0478	-0,0015
7	3,2816	0,0649	0,0045	-1,983	0,0497	0,00120	-1,3273	0,0478	-0,0015
8	2,8931	0,073	0,0055	-1,5034	0,0597	0,00340	-1,3863	0,0558	0,0007
9	1,6951	0,09	0,0116	-0,6105	0,0666	0,00320	-1,455	0,0832	0,0067
10	0,2343	0,0371	-0,0002	0,2921	0,0428	-0,00120	-0,8609	0,0283	-0,0008
11	1,7527	0,0914	0,0018	0	0,0008	0,00000	0	0,0006	0
12	0,8089	0,0705	-0,0026	0	0,0004	0,00000	0	0,0003	0
13	0,3511	0,0337	-0,0006	-0,4475	0,0343	-0,00230	0,0625	0,0308	-0,0022
14	0,3511	0,0336	-0,0006	-0,4475	0,0343	-0,00230	0,0625	0,0308	-0,0022
15	1,1117	0,0666	-0,0059	-0,7809	0,0366	0,00050	0,0643	0,0636	-0,0102
16	0,1284	0,0137	-0,0003	-0,3578	0,0248	-0,00130	0,0636	0,0186	0
17	0,4334	0,0447	-0,0003	0	0,0003	0,00000	0	0,0003	0

- Current Imaginary

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	-1,3848	0,0402	0,0019	-3,2489	0,0608	-0,0036	4,316	0,0644	0,0009
2	-1,3848	0,0403	0,0019	-3,0576	0,0582	-0,0053	4,0199	0,0656	0,0036
3	-1,3848	0,0405	0,0019	-2,9607	0,0583	-0,006	4,0198	0,0656	0,0037
4	-1,3848	0,0406	0,0019	-2,9607	0,0583	-0,006	4,0197	0,0656	0,0038
5	-1,3848	0,0407	0,0019	-2,9606	0,0584	-0,0061	4,0196	0,0657	0,0039
6	-0,4014	0,0756	0,0055	-2,7047	0,0587	-0,0069	3,9873	0,066	0,0042
7	-0,3838	0,0757	0,0056	-2,6826	0,0585	-0,0069	3,9872	0,066	0,0043
8	-0,0839	0,0772	0,0085	-2,4715	0,0683	-0,0023	3,4214	0,0713	0,0075
9	0,6859	0,0808	0,0068	-2,0367	0,0767	-0,0042	1,2722	0,0991	-0,0043
10	0,8678	0,0224	-0,0007	-0,8358	0,0512	0,0021	-0,1019	0,037	-0,0023
11	-0,9663	0,0637	-0,0035	-0,0001	0,0011	0,0001	0,0001	0,0012	-0,0001
12	-0,4426	0,0416	-0,0015	0	0,0006	0	0,0001	0,0006	-0,0001
13	-0,271	0,0255	-0,0028	-0,189	0,0347	-0,0046	0,4898	0,0388	-0,0037
14	-0,271	0,0255	-0,0028	-0,189	0,0347	-0,0046	0,4898	0,0387	-0,0037
15	-0,7122	0,0375	0,0012	-0,3874	0,0435	0,0019	1,4887	0,0863	0,0145
16	-0,0737	0,0073	0,0003	-0,1389	0,0283	-0,0002	0,2544	0,0181	-0,0004
17	-0,3183	0,0267	-0,0015	0	0,0004	0	0	0,0004	0

- Current Magnitude

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	5,2824	0,0885	0,00495	4,3401	0,0674	0,00303	4,5204	0,0708	0,000641
2	5,2824	0,0885	0,00495	4,0024	0,0635	0,00361	4,2339	0,0714	0,003661
3	5,2824	0,0885	0,00495	3,8308	0,0633	0,00413	4,2338	0,0714	0,003761
4	5,2823	0,0885	0,00505	3,8307	0,0633	0,00423	4,2337	0,0714	0,003861
5	5,2823	0,0885	0,00505	3,8307	0,0633	0,00423	4,2337	0,0714	0,003861
6	3,3382	0,0655	0,00284	3,3758	0,0621	0,00454	4,2026	0,0715	0,004273
7	3,3049	0,0651	0,00289	3,3363	0,0617	0,00450	4,2025	0,0715	0,004373
8	2,8953	0,073	0,00428	2,8934	0,0732	-0,00036	3,6919	0,0776	0,006376
9	1,8306	0,0874	0,01132	2,1272	0,08	0,00214	1,9343	0,104	-0,00943
10	0,8996	0,0227	-0,00145	0,8868	0,0447	-0,00381	0,8678	0,0265	0,000177
11	2,0027	0,0872	0,00199	0,0012	0,0008	-0,00120	0,0012	0,0008	-0,0012
12	0,9231	0,0693	-0,00259	0,0006	0,0004	-0,00060	0,0006	0,0004	-0,0006
13	0,4444	0,0312	0,00037	0,4866	0,0391	0,00309	0,4948	0,0375	-0,00497
14	0,4444	0,0312	0,00037	0,4866	0,0391	0,00309	0,4948	0,0375	-0,00497
15	1,3211	0,0617	-0,00645	0,8725	0,0439	-0,00208	1,4915	0,085	0,012673
16	0,1483	0,0123	-0,00066	0,3847	0,0273	0,00040	0,2629	0,0174	-0,00106
17	0,5388	0,0398	-0,00259	0,0005	0,0003	-0,00050	0,0004	0,0002	-0,0004

- Voltage Magnitude

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	0.9989	0	0	0.9992	0	0	0.9993	0	0
2	0.9907	0.0002	0	0.994	0.0001	0	0.9944	0.0001	0
3	0.9741	0.0004	0	0.984	0.0004	0	0.984	0.0004	0
4	0.9741	0.0004	0	0.984	0.0004	0	0.984	0.0004	0
5	0.974	0.0004	0	0.9839	0.0004	0	0.984	0.0004	0
6	0.9656	0.0006	0	0.9739	0.0006	0	0.9737	0.0006	0
7	0.9656	0.0006	0	0.9739	0.0006	0	0.9737	0.0006	0
8	0.9645	0.0007	0	0.9725	0.0006	1E-04	0.9723	0.0006	-0.0001
9	0.9644	0.0007	0	0.9724	0.0006	0	0.9722	0.0007	0
10	0.9645	0.0007	0	0.9725	0.0006	0	0.9724	0.0006	-1E-04
11	0.9524	0.0012	-1E-04	0.9839	0.0004	0	0.984	0.0004	0
12	0.95	0.0013	0	0.9839	0.0004	0	0.984	0.0004	0
13	0.931	0.0026	-0.0002	0.936	0.003	-0.0001	0.9353	0.0029	0.0004
14	0.9307	0.0026	-0.0002	0.9357	0.003	-1E-04	0.9351	0.003	0.0003
15	0.9641	0.0007	0	0.9723	0.0006	0	0.9719	0.0006	-1E-04
16	0.9641	0.0007	0	0.9723	0.0006	0	0.9719	0.0006	-1E-04
17	0.9639	0.0007	0	0.9723	0.0006	0	0.9719	0.0006	-1E-04

- Power Real

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	5.0974	0.0909	0.0059	4.2524	0.0684	0.0035	4.4095	0.0724	0.0006
2	5.0932	0.0908	0.0058	3.9365	0.0641	0.0044	4.1433	0.0724	0.0037
3	5.0614	0.0898	0.0058	3.76	0.0633	0.0049	4.1273	0.0718	0.0038
4	4.9962	0.0877	0.0057	3.7278	0.0623	0.0049	4.0934	0.0706	0.0038
5	4.9961	0.0877	0.0058	3.7277	0.0623	0.005	4.0933	0.0705	0.0039
6	3.2354	0.0634	0.0041	3.3039	0.0609	0.0053	4.0648	0.0705	0.0042
7	3.1797	0.062	0.0041	3.2361	0.0596	0.0052	4.0317	0.0693	0.0042
8	2.7946	0.0699	0.005	2.8174	0.0706	0.0002	3.5735	0.0751	0.0061
9	1.6093	0.0863	0.0108	1.9994	0.078	0.0019	1.7577	0.1034	-0.0068
10	0.1946	0.0358	-0.0002	0.5448	0.0548	-0.0012	0.3071	0.0388	-0.0016
11	1.7293	0.0891	0.0019	0.0001	0.0013	-0.0001	0.0001	0.0013	-0.0001
12	0.7817	0.0671	-0.0024	0.0001	0.0007	-0.0001	0.0001	0.0006	-0.0001
13	0.348	0.0325	-0.0005	0.3841	0.0394	0.0051	0.3913	0.0395	-0.0022
14	0.3405	0.0316	-0.0005	0.375	0.0381	0.0051	0.3819	0.0382	-0.0019
15	1.0969	0.0644	-0.0057	0.7178	0.0465	-0.0019	1.2469	0.0898	0.0171
16	0.1263	0.0132	-0.0003	0.2969	0.0293	0.0008	0.1889	0.0188	-0.0003
17	0.4289	0.0432	-0.0027	0	0.0004	0	0	0.0004	0

- Power Imaginary

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	1.3848	0.0402	-0.0019	0.8675	0.0301	-0.0011	0.9945	0.032	0.0007
2	1.3784	0.0403	-0.0019	0.7043	0.0352	-0.0029	0.8556	0.0364	0.0007
3	1.3301	0.0402	-0.0021	0.5998	0.0362	-0.0034	0.8291	0.0365	0.0007
4	1.2293	0.0402	-0.0023	0.5572	0.0363	-0.0035	0.7742	0.0366	0.0006
5	1.2293	0.0403	-0.0023	0.5572	0.0365	-0.0035	0.7741	0.0367	0.0007
6	0.3131	0.074	-0.0056	0.3394	0.0445	-0.0047	0.7594	0.0371	0.0008
7	0.2589	0.0739	-0.0057	0.288	0.0447	-0.0046	0.6993	0.0373	0.0007
8	-0.0173	0.0752	-0.0086	-0.0036	0.0528	-0.0042	0.3879	0.0452	0.004
9	-0.7216	0.0786	-0.0071	-0.5277	0.0603	-0.0048	-0.6644	0.0716	0.0038
10	-0.8447	0.0213	0.0006	-0.6667	0.0346	0.002	-0.7849	0.0231	-0.0017
11	0.8998	0.0619	0.0032	0	0.0005	0	0.0001	0.0005	-0.0001
12	0.4	0.0393	0.0015	0	0.0003	0	0	0.0002	0
13	0.2496	0.0246	0.0027	0.2762	0.0264	-0.0004	0.2793	0.0276	-0.0036
14	0.2334	0.0236	0.0027	0.2567	0.0248	-0.0005	0.2592	0.0263	-0.003
15	0.6468	0.036	-0.0011	0.4511	0.03	0.0005	0.7376	0.0531	-0.0022
16	0.0665	0.0071	-0.0004	0.2262	0.0219	0.001	0.1711	0.0168	-0.0002
17	0.2911	0.0257	0.0016	0	0.0003	0	0	0.0002	0

Case 4 : forecasted load data, power, current, and voltage measurements

• **Current Real**

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	5,1042	0,0881	-0,00090	-2,8799	0,0478	0,0017	-1,3366	0,0464	-0,0066
2	5,1041	0,0882	-0,00080	-2,5855	0,0496	0,0035	-1,3242	0,0484	-0,0056
3	5,104	0,0882	-0,00070	-2,433	0,051	0,0029	-1,3241	0,0484	-0,0057
4	5,1039	0,0883	-0,00060	-2,433	0,0512	0,0029	-1,3241	0,0485	-0,0057
5	5,1038	0,0883	-0,00050	-2,433	0,0514	0,0029	-1,324	0,0485	-0,0058
6	3,3182	0,0632	-0,00070	-2,0204	0,0546	0,002	-1,3229	0,0484	-0,0059
7	3,2869	0,0636	-0,00080	-1,9838	0,0549	0,002	-1,3229	0,0484	-0,0059
8	2,8963	0,0684	0,00230	-1,5	0,0626	0	-1,3831	0,0531	-0,0025
9	1,7034	0,0855	0,00330	-0,6064	0,0725	-0,0009	-1,4523	0,0814	0,004
10	0,2397	0,0363	-0,00560	0,2913	0,0422	-0,0004	-0,8627	0,0304	0,001
11	1,7543	0,0877	0,00020	0	0,0009	0	-0,0001	0,0007	0,0001
12	0,8054	0,0673	0,00090	0	0,0005	0	0	0,0003	0
13	0,3534	0,0368	-0,00290	-0,4517	0,0317	0,0019	0,0636	0,0284	-0,0033
14	0,3533	0,0368	-0,00280	-0,4517	0,0317	0,0019	0,0637	0,0284	-0,0034
15	1,1069	0,0659	-0,00110	-0,781	0,0394	0,0006	0,0596	0,0689	-0,0055
16	0,1291	0,013	-0,00100	-0,3579	0,0247	-0,0012	0,0639	0,0189	-0,0003
17	0,4284	0,0443	0,00200	0	0,0003	0	0	0,0003	0

• **Current Imaginary**

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	-1,3863	0,0431	0,0034	-3,2465	0,0581	-0,006	4,3103	0,0623	0,0066
2	-1,3864	0,0433	0,0035	-3,0587	0,0583	-0,0042	4,02	0,0611	0,0035
3	-1,3864	0,0434	0,0035	-2,9628	0,0598	-0,0039	4,02	0,0612	0,0035
4	-1,3864	0,0436	0,0035	-2,9627	0,0597	-0,004	4,0199	0,0612	0,0036
5	-1,3864	0,0437	0,0035	-2,9627	0,0596	-0,004	4,0199	0,0612	0,0036
6	-0,3969	0,0717	0,001	-2,7048	0,0601	-0,0068	3,9878	0,0612	0,0037
7	-0,3791	0,0718	0,0009	-2,6827	0,0602	-0,0068	3,9878	0,0612	0,0037
8	-0,0745	0,0717	-0,0009	-2,4644	0,0708	-0,0094	3,4249	0,0703	0,004
9	0,6952	0,0732	-0,0025	-2,0281	0,0793	-0,0128	1,2704	0,0938	-0,0025
10	0,8663	0,0234	0,0008	-0,8358	0,0541	0,0021	-0,1029	0,0365	-0,0013
11	-0,9723	0,0572	0,0025	-0,0001	0,0012	0,0001	0,0001	0,0012	-0,0001
12	-0,4488	0,0427	0,0047	-0,0001	0,0006	0,0001	0,0001	0,0006	-0,0001
13	-0,2755	0,0263	0,0017	-0,196	0,0327	0,0024	0,4862	0,04	-0,0001
14	-0,2755	0,0263	0,0017	-0,1959	0,0327	0,0023	0,4862	0,0401	-0,0001
15	-0,7124	0,0414	0,0014	-0,3885	0,0459	0,003	1,4964	0,0864	0,0068
16	-0,074	0,0069	0,0006	-0,141	0,0285	0,0019	0,2527	0,02	0,0013
17	-0,32	0,0313	0,0002	-0,0001	0,0004	0,0001	0	0,0004	0

• **Current Magnitude**

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	5,2893	0,086	-0,00195	4,34	0,0633	0,00313	4,5129	0,0673	0,008141
2	5,2892	0,086	-0,00185	4,0053	0,0624	0,00071	4,2327	0,0663	0,004861
3	5,2892	0,0861	-0,00185	3,834	0,0636	0,00093	4,2327	0,0662	0,004861
4	5,2891	0,0861	-0,00175	3,834	0,0635	0,00093	4,2326	0,0662	0,004961
5	5,289	0,0862	-0,00165	3,8339	0,0635	0,00103	4,2325	0,0662	0,005061
6	3,3426	0,0634	-0,00156	3,3765	0,0611	0,00384	4,2018	0,0659	0,005073
7	3,3094	0,0638	-0,00161	3,3369	0,0614	0,00390	4,2017	0,0659	0,005173
8	2,8981	0,0684	0,00148	2,8857	0,0725	0,00734	3,6939	0,0754	0,004376
9	1,8413	0,0841	0,00062	2,118	0,0824	0,01134	1,931	0,0984	-0,00613
10	0,8996	0,024	-0,00145	0,8864	0,0489	-0,00341	0,8697	0,0291	-0,00172
11	2,0068	0,0815	-0,00211	0,0013	0,0008	-0,00130	0,0012	0,0008	-0,0012
12	0,9232	0,0635	-0,00269	0,0007	0,0004	-0,00070	0,0006	0,0004	-0,0006
13	0,4491	0,0344	-0,00433	0,4932	0,0353	-0,00351	0,4913	0,0385	-0,00147
14	0,449	0,0344	-0,00423	0,4932	0,0353	-0,00351	0,4913	0,0386	-0,00147
15	1,3172	0,0611	-0,00255	0,873	0,0483	-0,00258	1,4992	0,085	0,004973
16	0,1491	0,012	-0,00146	0,3855	0,0281	-0,00040	0,2613	0,0195	0,000541
17	0,5359	0,0399	0,00031	0,0005	0,0003	-0,00050	0,0004	0,0003	-0,0004

- Voltage Magnitude

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	0.9989	0	0	0.9992	0	0	0.9993	0	0
2	0.9907	0.0002	0	0.994	0.0001	0	0.9944	0.0001	0
3	0.974	0.0004	1E-04	0.984	0.0004	0	0.984	0.0004	0
4	0.974	0.0004	1E-04	0.984	0.0004	0	0.984	0.0004	0
5	0.974	0.0004	0	0.9839	0.0004	0	0.984	0.0004	0
6	0.9655	0.0007	1E-04	0.9739	0.0006	0	0.9737	0.0006	0
7	0.9655	0.0007	1E-04	0.9739	0.0006	0	0.9737	0.0006	0
8	0.9644	0.0007	1E-04	0.9725	0.0007	1E-04	0.9722	0.0007	0
9	0.9644	0.0007	0	0.9724	0.0007	0	0.9722	0.0007	0
10	0.9645	0.0007	0	0.9725	0.0007	0	0.9723	0.0007	0
11	0.9523	0.0011	0	0.9839	0.0004	0	0.984	0.0004	0
12	0.9499	0.0012	1E-04	0.9839	0.0004	0	0.984	0.0004	0
13	0.9305	0.0028	0.0003	0.9357	0.0028	0.0002	0.9355	0.0029	0.0002
14	0.9302	0.0028	0.0003	0.9354	0.0028	0.0002	0.9352	0.0029	0.0002
15	0.9641	0.0007	0	0.9723	0.0007	0	0.9718	0.0007	0
16	0.9641	0.0007	0	0.9723	0.0007	0	0.9718	0.0007	0
17	0.9639	0.0007	0	0.9723	0.0007	0	0.9718	0.0007	0

- Power Real

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	5.1042	0.0881	-0.0009	4.2515	0.0638	0.0044	4.4011	0.0684	0.009
2	5.0998	0.088	-0.0008	3.939	0.0627	0.0019	4.1413	0.0673	0.0057
3	5.0679	0.0871	-0.0007	3.763	0.0634	0.0019	4.1253	0.0667	0.0058
4	5.0024	0.0851	-0.0005	3.7308	0.0623	0.0019	4.0914	0.0656	0.0058
5	5.0023	0.0852	-0.0004	3.7307	0.0623	0.002	4.0913	0.0655	0.0059
6	3.2402	0.0613	-0.0007	3.3044	0.06	0.0048	4.0631	0.0651	0.0059
7	3.1845	0.0608	-0.0007	3.2366	0.0592	0.0047	4.03	0.064	0.0059
8	2.7972	0.0654	0.0024	2.8097	0.07	0.0079	3.5749	0.0727	0.0047
9	1.6168	0.082	0.0033	1.9902	0.0807	0.0111	1.7549	0.0974	-0.004
10	0.1998	0.0351	-0.0054	0.5452	0.0551	-0.0016	0.3071	0.0381	-0.0016
11	1.7309	0.0854	0.0003	0.0001	0.0014	-0.0001	0.0002	0.0013	-0.0002
12	0.7784	0.064	0.0009	0	0.0007	0	0.0001	0.0006	-0.0001
13	0.3503	0.0356	-0.0028	0.392	0.0355	-0.0028	0.3877	0.0404	0.0014
14	0.3426	0.0346	-0.0026	0.3827	0.0343	-0.0026	0.3784	0.0391	0.0016
15	1.0922	0.0636	-0.001	0.7188	0.0505	-0.0029	1.2556	0.0918	0.0084
16	0.1271	0.0125	-0.0011	0.2986	0.0301	-0.0009	0.1873	0.0196	0.0013
17	0.4242	0.0427	0.002	0	0.0004	0	0	0.0004	0

- Power Imaginary

In	phase1			phase2			phase3		
	Mean	Std	Error	Mean	Std	Error	Mean	Std	Error
1	1.3863	0.0431	-0.0034	0.8708	0.0399	-0.0044	0.9976	0.0368	-0.0024
2	1.38	0.0433	-0.0035	0.7065	0.0437	-0.0051	0.8594	0.0392	-0.0031
3	1.3315	0.0432	-0.0035	0.6008	0.0455	-0.0044	0.833	0.0394	-0.0032
4	1.2304	0.0431	-0.0034	0.5582	0.0456	-0.0045	0.7782	0.0395	-0.0034
5	1.2304	0.0432	-0.0034	0.5582	0.0458	-0.0045	0.7782	0.0397	-0.0034
6	0.3085	0.07	-0.001	0.3398	0.0527	-0.0051	0.7636	0.0401	-0.0034
7	0.254	0.07	-0.0008	0.2886	0.0528	-0.0052	0.7035	0.0403	-0.0035
8	-0.0267	0.0698	0.0008	-0.0029	0.0598	-0.0049	0.3924	0.0449	-0.0005
9	-0.731	0.0713	0.0023	-0.5267	0.0663	-0.0058	-0.6627	0.0714	0.0021
10	-0.8435	0.0224	-0.0006	-0.666	0.0376	0.0013	-0.7869	0.0259	0.0003
11	0.9055	0.0559	-0.0025	-0.0001	0.0006	0.0001	0	0.0005	0
12	0.406	0.0408	-0.0045	0	0.0003	0	0	0.0002	0
13	0.2539	0.0253	-0.0016	0.2762	0.0266	-0.0004	0.2787	0.0255	-0.0003
14	0.2373	0.0241	-0.0012	0.2562	0.0251	0	0.2589	0.0241	-0.0027
15	0.647	0.0398	-0.0013	0.4506	0.0303	0.001	0.7371	0.0558	-0.0017
16	0.0667	0.0066	-0.0006	0.2252	0.021	0.002	0.1705	0.0163	0.0004
17	0.2929	0.0303	-0.0002	0	0.0003	0	0	0.0002	0

APPENDIX III – Topology Error Detection Result using method I

1. When node 11 open (switch 3 open)

- **Case 1-1** : simulation with voltage measurement using current measurement (m_1)

- Summation of load in switch zone & Meter difference in meter zone

sw_zo	SL1	PL1	QL1	SL2	PL2	QL2	SL3	PL3	QL3
1	0.266	0.182	0.182	1.484	1.045	1.045	1.352	0.947	0.947
2	2.122	1.5	1.5	2.784	1.969	1.969	2.623	1.854	1.854
3	2.272	1.607	1.607	0.003	-0.002	-0.002	0.003	-0.002	-0.002
4	0.434	0.307	0.307	0.556	0.392	0.392	0.507	0.359	0.359
5	1.046	0.74	0.74	0.996	0.704	0.704	1.702	1.203	1.203
6	0.601	0.425	0.425	0	0	0	0.001	0	0
SUM	6.742	4.761	4.761	5.824	4.108	4.108	6.187	4.361	4.361

me_zo	mtyp	Pm1	Qm1	Pm2	Qm2	Pm3	Qm3
1	s	1.795	1.005	-0.012	-0.002	0.006	-0.007
2	i	0.011	0	-0.002	0	-0.001	0

- Topology error detection result

me_zo	sw_zo	sw1	sw2	ph	P1(S1)	Q1	P2(S2)	Q2	P3(S3)	Q3
1	1	ON	ON	3						
	3	ON	ON	1	err					
2	2	ON	ON	3						
	4	ON	ON	3						
	5	ON	ON	3						
	6	ON	ON	1						

- **Case 2-1** : simulation without voltage measurement using current measurement (m_1)

- Summation of load in switch zone & Meter difference in meter zone

sw_zo	SL1	PL1	QL1	SL2	PL2	QL2	SL3	PL3	QL3
1	0.266	0.182	0.182	1.483	1.044	1.044	1.352	0.947	0.947
2	2.119	1.498	1.498	2.78	1.965	1.965	2.619	1.852	1.852
3	2.266	1.602	1.602	0.003	-0.002	-0.002	0.003	-0.002	-0.002
4	0.434	0.307	0.307	0.556	0.393	0.393	0.508	0.359	0.359
5	1.046	0.739	0.739	0.995	0.704	0.704	1.699	1.201	1.201
6	0.601	0.425	0.425	0	0	0	0.001	0	0
SUM	6.731	4.753	4.753	5.817	4.103	4.103	6.181	4.357	4.357

me_zo	mtyp	Pm1	Qm1	Pm2	Qm2	Pm3	Qm3
1	s	1.795	1.005	-0.012	-0.002	0.006	-0.007
2	s	0.006	0	-0.003	0	0.003	0

- Topology error detection result

me_zo	sw_zo	sw1	sw2	ph	P1(S1)	Q1	P2(S2)	Q2	P3(S3)	Q3
1	1	ON	ON	3						
	3	ON	ON	1	err					
2	2	ON	ON	3						
	4	ON	ON	3						
	5	ON	ON	3						
	6	ON	ON	1						

2. When node 13 open (switch 4 open)

- **Case 1-1** : simulation with voltage measurement using current measurement (m_1)

- Summation of load in switch zone & Meter difference in meter zone

sw_zo	SL1	PL1	QL1	SL2	PL2	QL2	SL3	PL3	QL3
1	0.266	0.182	0.182	1.484	1.045	1.045	1.352	0.947	0.947
2	2.122	1.5	1.5	2.784	1.969	1.969	2.623	1.854	1.854
3	2.272	1.607	1.607	0.003	-0.002	-0.002	0.003	-0.002	-0.002
4	0.434	0.307	0.307	0.556	0.392	0.392	0.507	0.359	0.359
5	1.046	0.74	0.74	0.996	0.704	0.704	1.702	1.203	1.203
6	0.601	0.425	0.425	0	0	0	0.001	0	0
SUM	6.742	4.761	4.761	5.824	4.108	4.108	6.187	4.361	4.361

me_zo	mtyp	Pm1	Qm1	Pm2	Qm2	Pm3	Qm3
1	s	0.369	0.284	0.409	0.299	0.405	0.306
2	i	0.37	0	0.399	0	0.423	0

- Topology error detection result

me_zo	sw_zo	sw1	sw2	ph	P1(S1)	Q1	P2(S2)	Q2	P3(S3)	Q3
1	1	ON	ON	3						
	3	ON	ON	1						
2	2	ON	ON	3						
	4	ON	ON	3	err		err		err	
	5	ON	ON	3						
	6	ON	ON	1						

- **Case 1-2** : simulation with voltage measurement using power measurement (m_1)

- Summation of load in switch zone & Meter difference in meter zone

sw_zo	SL1	PL1	QL1	SL2	PL2	QL2	SL3	PL3	QL3
1	0.266	0.182	0.182	1.483	1.044	1.044	1.352	0.947	0.947
2	2.114	1.495	1.495	2.789	1.972	1.972	2.627	1.858	1.858
3	2.277	1.61	1.61	0.003	-0.002	-0.002	0.003	-0.002	-0.002
4	0.434	0.307	0.307	0.556	0.393	0.393	0.508	0.359	0.359
5	1.045	0.739	0.739	0.997	0.705	0.705	1.704	1.205	1.205
6	0.6	0.424	0.424	0	0	0	0.001	0	0
SUM	6.736	4.757	4.757	5.828	4.111	4.111	6.195	4.367	4.367

me_zo	mtyp	Pm1	Qm1	Pm2	Qm2	Pm3	Qm3
1	s	0.369	0.284	0.409	0.299	0.405	0.306
2	s	0.347	0.252	0.389	0.276	0.389	0.276

- Topology error detection result

me_zo	sw_zo	sw1	sw2	ph	P1(S1)	Q1	P2(S2)	Q2	P3(S3)	Q3
1	1	ON	ON	3						
	3	ON	ON	1						
2	2	ON	ON	3						
	4	ON	ON	3	err	err	err	err	err	err
	5	ON	ON	3						
	6	ON	ON	1						

- **Case 2-1** : simulation without voltage measurement using current measurement (m_1)

- Summation of load in switch zone & Meter difference in meter zone

sw_zo	SL1	PL1	QL1	SL2	PL2	QL2	SL3	PL3	QL3
1	0.266	0.182	0.182	1.483	1.044	1.044	1.352	0.947	0.947
2	2.119	1.498	1.498	2.78	1.965	1.965	2.619	1.852	1.852
3	2.266	1.602	1.602	0.003	-0.002	-0.002	0.003	-0.002	-0.002
4	0.434	0.307	0.307	0.556	0.393	0.393	0.508	0.359	0.359
5	1.046	0.739	0.739	0.995	0.704	0.704	1.699	1.201	1.201
6	0.601	0.425	0.425	0	0	0	0.001	0	0
SUM	6.731	4.753	4.753	5.817	4.103	4.103	6.181	4.357	4.357

me_zo	mtyp	Pm1	Qm1	Pm2	Qm2	Pm3	Qm3
1	s	0.369	0.284	0.409	0.299	0.405	0.306
2	i	0.362	0	0.4	0	0.426	0

- Topology error detection result

me_zo	sw_zo	sw1	sw2	ph	P1(S1)	Q1	P2(S2)	Q2	P3(S3)	Q3
1	1	ON	ON	3						
	3	ON	ON	1						
2	2	ON	ON	3						
	4	ON	ON	3	err		err		err	
	5	ON	ON	3						
	6	ON	ON	1						

- **Case 2-2** : simulation without voltage measurement using power measurement (m_1)

- Summation of load in switch zone & Meter difference in meter zone

sw_zo	SL1	PL1	QL1	SL2	PL2	QL2	SL3	PL3	QL3
1	0.266	0.182	0.182	1.482	1.043	1.043	1.352	0.947	0.947
2	2.11	1.492	1.492	2.782	1.967	1.967	2.621	1.854	1.854
3	2.272	1.606	1.606	0.003	-0.002	-0.002	0.003	-0.002	-0.002
4	0.434	0.307	0.307	0.556	0.393	0.393	0.508	0.359	0.359
5	1.044	0.738	0.738	0.995	0.704	0.704	1.7	1.202	1.202
6	0.6	0.424	0.424	0	0	0	0.001	0	0
SUM	6.724	4.749	4.749	5.82	4.104	4.104	6.186	4.36	4.36

me_zo	mtyp	Pm1	Qm1	Pm2	Qm2	Pm3	Qm3
1	s	0.369	0.284	0.409	0.299	0.405	0.306
2	s	0.347	0.252	0.389	0.276	0.389	0.276

- Topology error detection result

me_zo	sw_zo	sw1	sw2	ph	P1(S1)	Q1	P2(S2)	Q2	P3(S3)	Q3
1	1	ON	ON	3						
	3	ON	ON	1						
2	2	ON	ON	3						
	4	ON	ON	3	err	err	err	err	err	err
	5	ON	ON	3						
	6	ON	ON	1						

3. When node 15 open (switch 5 open)

- **Case 1-1** : simulation with voltage measurement using current measurement (m_1)

- Summation of load in switch zone & Meter difference in meter zone

sw_zo	SL1	PL1	QL1	SL2	PL2	QL2	SL3	PL3	QL3
1	0.266	0.182	0.182	1.484	1.045	1.045	1.352	0.947	0.947
2	2.122	1.5	1.5	2.784	1.969	1.969	2.623	1.854	1.854
3	2.272	1.607	1.607	0.003	-0.002	-0.002	0.003	-0.002	-0.002
4	0.434	0.307	0.307	0.556	0.392	0.392	0.507	0.359	0.359
5	1.046	0.74	0.74	0.996	0.704	0.704	1.702	1.203	1.203
6	0.601	0.425	0.425	0	0	0	0.001	0	0
SUM	6.742	4.761	4.761	5.824	4.108	4.108	6.187	4.361	4.361

me_zo	mtyp	Pm1	Qm1	Pm2	Qm2	Pm3	Qm3
1	s	1.148	0.739	0.75	0.487	1.318	0.823
2	i	1.077	0	0.722	0	1.323	0

- Topology error detection result

me_zo	sw_zo	sw1	sw2	ph	P1(S1)	Q1	P2(S2)	Q2	P3(S3)	Q3
1	1	ON	ON	3						
	3	ON	ON	1						
2	2	ON	ON	3						
	4	ON	ON	3			err			
	5	ON	ON	3					err	
	6	ON	ON	1						

- **Case 1-2** : simulation with voltage measurement using power measurement (m_1)

- Summation of load in switch zone & Meter difference in meter zone

sw_zo	SL1	PL1	QL1	SL2	PL2	QL2	SL3	PL3	QL3
1	0.266	0.182	0.182	1.483	1.044	1.044	1.352	0.947	0.947
2	2.114	1.495	1.495	2.789	1.972	1.972	2.627	1.858	1.858
3	2.277	1.61	1.61	0.003	-0.002	-0.002	0.003	-0.002	-0.002
4	0.434	0.307	0.307	0.556	0.393	0.393	0.508	0.359	0.359
5	1.045	0.739	0.739	0.997	0.705	0.705	1.704	1.205	1.205
6	0.6	0.424	0.424	0	0	0	0.001	0	0
SUM	6.736	4.757	4.757	5.828	4.111	4.111	6.195	4.367	4.367

me_zo	mtyp	Pm1	Qm1	Pm2	Qm2	Pm3	Qm3
1	s	1.148	0.739	0.75	0.487	1.318	0.823
2	s	1.093	0.649	0.718	0.454	1.267	0.741

- Topology error detection result

me_zo	sw_zo	sw1	sw2	ph	P1(S1)	Q1	P2(S2)	Q2	P3(S3)	Q3
1	1	ON	ON	3						
	3	ON	ON	1						
2	2	ON	ON	3						
	4	ON	ON	3				err		
	5	ON	ON	3	err		err		err	
	6	ON	ON	1						

- **Case 2-1** : simulation without voltage measurement using current measurement (m_1)

- Summation of load in switch zone & Meter difference in meter zone

sw_zo	SL1	PL1	QL1	SL2	PL2	QL2	SL3	PL3	QL3
1	0.266	0.182	0.182	1.483	1.044	1.044	1.352	0.947	0.947
2	2.119	1.498	1.498	2.78	1.965	1.965	2.619	1.852	1.852
3	2.266	1.602	1.602	0.003	-0.002	-0.002	0.003	-0.002	-0.002
4	0.434	0.307	0.307	0.556	0.393	0.393	0.508	0.359	0.359
5	1.046	0.739	0.739	0.995	0.704	0.704	1.699	1.201	1.201
6	0.601	0.425	0.425	0	0	0	0.001	0	0
SUM	6.731	4.753	4.753	5.817	4.103	4.103	6.181	4.357	4.357

me_zo	mtyp	Pm1	Qm1	Pm2	Qm2	Pm3	Qm3
1	s	1.148	0.739	0.75	0.487	1.318	0.823
2	i	1.073	0	0.723	0	1.327	0

- Topology error detection result

me_zo	sw_zo	sw1	sw2	ph	P1(S1)	Q1	P2(S2)	Q2	P3(S3)	Q3
1	1	ON	ON	3						
	3	ON	ON	1						
2	2	ON	ON	3						
	4	ON	ON	3			err			
	5	ON	ON	3					err	
	6	ON	ON	1						

- **Case 2-2** : simulation without voltage measurement using power measurement (m_1)

- Summation of load in switch zone & Meter difference in meter zone

sw_zo	SL1	PL1	QL1	SL2	PL2	QL2	SL3	PL3	QL3
1	0.266	0.182	0.182	1.482	1.043	1.043	1.352	0.947	0.947
2	2.11	1.492	1.492	2.782	1.967	1.967	2.621	1.854	1.854
3	2.272	1.606	1.606	0.003	-0.002	-0.002	0.003	-0.002	-0.002
4	0.434	0.307	0.307	0.556	0.393	0.393	0.508	0.359	0.359
5	1.044	0.738	0.738	0.995	0.704	0.704	1.7	1.202	1.202
6	0.6	0.424	0.424	0	0	0	0.001	0	0
SUM	6.724	4.749	4.749	5.82	4.104	4.104	6.186	4.36	4.36

me_zo	mtyp	Pm1	Qm1	Pm2	Qm2	Pm3	Qm3
1	s	1.148	0.739	0.75	0.487	1.318	0.823
2	s	1.093	0.649	0.718	0.454	1.267	0.741

- Topology error detection result

me_zo	sw_zo	sw1	sw2	ph	P1(S1)	Q1	P2(S2)	Q2	P3(S3)	Q3
1	1	ON	ON	3						
	3	ON	ON	1						
2	2	ON	ON	3						
	4	ON	ON	3				err		
	5	ON	ON	3	err		err		err	
	6	ON	ON	1						

4. When node 17 open (switch 6 open)

- **Case 1-1** : simulation with voltage measurement using current measurement (m_1)

- Summation of load in switch zone & Meter difference in meter zone

sw_zo	SL1	PL1	QL1	SL2	PL2	QL2	SL3	PL3	QL3
1	0.266	0.182	0.182	1.484	1.045	1.045	1.352	0.947	0.947
2	2.122	1.5	1.5	2.784	1.969	1.969	2.623	1.854	1.854
3	2.272	1.607	1.607	0.003	-0.002	-0.002	0.003	-0.002	-0.002
4	0.434	0.307	0.307	0.556	0.392	0.392	0.507	0.359	0.359
5	1.046	0.74	0.74	0.996	0.704	0.704	1.702	1.203	1.203
6	0.601	0.425	0.425	0	0	0	0.001	0	0
SUM	6.742	4.761	4.761	5.824	4.108	4.108	6.187	4.361	4.361

me_zo	mtyp	Pm1	Qm1	Pm2	Qm2	Pm3	Qm3
1	s	0.455	0.338	-0.005	0	0.002	-0.004
2	i	0.448	0	-0.002	0	-0.001	0

- Topology error detection result

me_zo	sw_zo	sw1	sw2	ph	P1(S1)	Q1	P2(S2)	Q2	P3(S3)	Q3
1	1	ON	ON	3						
	3	ON	ON	1						
2	2	ON	ON	3						
	4	ON	ON	3	err					
	5	ON	ON	3						
	6	ON	ON	1						

- **Case 1-2** : simulation with voltage measurement using power measurement (m_1)

- Summation of load in switch zone & Meter difference in meter zone

sw_zo	SL1	PL1	QL1	SL2	PL2	QL2	SL3	PL3	QL3
1	0.266	0.182	0.182	1.483	1.044	1.044	1.352	0.947	0.947
2	2.114	1.495	1.495	2.789	1.972	1.972	2.627	1.858	1.858
3	2.277	1.61	1.61	0.003	-0.002	-0.002	0.003	-0.002	-0.002
4	0.434	0.307	0.307	0.556	0.393	0.393	0.508	0.359	0.359
5	1.045	0.739	0.739	0.997	0.705	0.705	1.704	1.205	1.205
6	0.6	0.424	0.424	0	0	0	0.001	0	0
SUM	6.736	4.757	4.757	5.828	4.111	4.111	6.195	4.367	4.367

me_zo	mtyp	Pm1	Qm1	Pm2	Qm2	Pm3	Qm3
1	s	0.455	0.338	-0.005	0	0.002	-0.004
2	s	0.427	0.295	0	0	0	0

- Topology error detection result

me_zo	sw_zo	sw1	sw2	ph	P1(S1)	Q1	P2(S2)	Q2	P3(S3)	Q3
1	1	ON	ON	3						
	3	ON	ON	1						
2	2	ON	ON	3						
	4	ON	ON	3		err				
	5	ON	ON	3						
	6	ON	ON	1	err					

Case 2-1 : simulation without voltage measurement using current measurement (m_1)

- Summation of load in switch zone & Meter difference in meter zone

sw_zo	SL1	PL1	QL1	SL2	PL2	QL2	SL3	PL3	QL3
1	0.266	0.182	0.182	1.483	1.044	1.044	1.352	0.947	0.947
2	2.119	1.498	1.498	2.78	1.965	1.965	2.619	1.852	1.852
3	2.266	1.602	1.602	0.003	-0.002	-0.002	0.003	-0.002	-0.002
4	0.434	0.307	0.307	0.556	0.393	0.393	0.508	0.359	0.359
5	1.046	0.739	0.739	0.995	0.704	0.704	1.699	1.201	1.201
6	0.601	0.425	0.425	0	0	0	0.001	0	0
SUM	6.731	4.753	4.753	5.817	4.103	4.103	6.181	4.357	4.357

me_zo	mtyp	Pm1	Qm1	Pm2	Qm2	Pm3	Qm3
1	s	0.455	0.338	-0.005	0	0.002	-0.004
2	i	0.442	0	-0.003	0	0.003	0

- Topology error detection result

me_zo	sw_zo	sw1	sw2	ph	P1(S1)	Q1	P2(S2)	Q2	P3(S3)	Q3
1	1	ON	ON	3						
	3	ON	ON	1						
2	2	ON	ON	3						
	4	ON	ON	3	err					
	5	ON	ON	3						
	6	ON	ON	1						

• **Case 2-2** : simulation without voltage measurement using power measurement (m_1)

- Summation of load in switch zone & Meter difference in meter zone

sw_zo	SL1	PL1	QL1	SL2	PL2	QL2	SL3	PL3	QL3
1	0.266	0.182	0.182	1.482	1.043	1.043	1.352	0.947	0.947
2	2.11	1.492	1.492	2.782	1.967	1.967	2.621	1.854	1.854
3	2.272	1.606	1.606	0.003	-0.002	-0.002	0.003	-0.002	-0.002
4	0.434	0.307	0.307	0.556	0.393	0.393	0.508	0.359	0.359
5	1.044	0.738	0.738	0.995	0.704	0.704	1.7	1.202	1.202
6	0.6	0.424	0.424	0	0	0	0.001	0	0
SUM	6.724	4.749	4.749	5.82	4.104	4.104	6.186	4.36	4.36

me_zo	mtyp	Pm1	Qm1	Pm2	Qm2	Pm3	Qm3
1	s	0.455	0.338	-0.005	0	0.002	-0.004
2	s	0.427	0.295	0	0	0	0

- Topology error detection result

me_zo	sw_zo	sw1	sw2	ph	P1(S1)	Q1	P2(S2)	Q2	P3(S3)	Q3
1	1	ON	ON	3						
	3	ON	ON	1						
2	2	ON	ON	3						
	4	ON	ON	3		err				
	5	ON	ON	3						
	6	ON	ON	1	err					

APPENDIX IV – Topology Error Detection Result using method II

♣ Switch Error Detection

1. When node 11 open (switch 3 open)

- **Case 1** : simulation with power and current measurements (when node 9 open)

- Topology error detection result

```
***** BCSE TOPOLOGY DETECTION RESULT *****
inequality = 80.34628
***** SIMULATION *****
sw 1 op sw 9 op sw 11 op sw 13 op sw 15 op sw 17 op
1845.58813 471.69302 2.61440 66.09863 142.10568 73.17811
*****
Smallest inequality is 2.61440
Detect Switch 11 Open
*****
```

- **Case 2** : simulation with power, current and voltage measurements (when node 9 open)

- Topology error detection result

```
***** BCSE TOPOLOGY DETECTION RESULT *****
inequality = 79.12688
***** SIMULATION *****
sw 1 op sw 9 op sw 11 op sw 13 op sw 15 op sw 17 op
1863.40198 410.27487 1.62064 67.51985 163.71422 58.31192
*****
Smallest inequality is 1.62064
Detect Switch 11 Open
*****
```

2. When node 13 open (switch 4 open)

- **Case 1** : simulation with power and current measurements (when node 9 open)

- Topology error detection result

```
***** BCSE TOPOLOGY DETECTION RESULT *****  
  
inequality = 21.19109  
  
***** SIMULATION *****  
sw 1 op sw 9 op sw 11 op sw 13 op sw 15 op sw 17 op  
1917.07788 341.05899 22.00794 2.38132 57.18052 9.58974  
*****  
  
Smallest inequality is 2.38132  
Detect Switch 13 open  
  
*****
```

- **Case 2** : simulation with power, current and voltage measurements (when node 9 open)

- Topology error detection result

```
***** BCSE TOPOLOGY DETECTION RESULT *****  
  
inequality = 20.85036  
  
***** SIMULATION *****  
sw 1 op sw 9 op sw 11 op sw 13 op sw 15 op sw 17 op  
1931.28247 291.50647 20.82707 3.84644 62.74544 13.29288  
*****  
  
Smallest inequality is 3.84644  
Detect Switch 13 open  
  
*****
```

3. When node 15 open (switch 5 open)

- **Case 1** : simulation with power and current measurements (when node 9 open)

- Topology error detection result

```
***** BCSE TOPOLOGY DETECTION RESULT *****  
  
inequality = 99.63425  
  
***** SIMULATION *****  
sw 1 op sw 9 op sw 11 op sw 13 op sw 15 op sw 17 op  
2322.92554 259.37488 75.93719 43.13559 3.00320 69.16583  
*****  
  
Smallest inequality is 3.00320  
Detect Switch 15 Open  
  
*****
```

- **Case 2** : simulation with power, current and voltage measurements (when node 9 open)

- Topology error detection result

```
***** BCSE TOPOLOGY DETECTION RESULT *****  
  
inequality = 99.54684  
  
***** SIMULATION *****  
sw 1 op sw 9 op sw 11 op sw 13 op sw 15 op sw 17 op  
2316.33008 242.14455 73.37872 49.93416 5.98718 60.49139  
*****  
  
Smallest inequality is 5.98718  
Detect Switch 15 Open  
  
*****
```

4. When node 17 open (switch 6 open)

- **Case 1** : simulation with power and current measurements (when node 9 open)

- Topology error detection result

```
***** BCSE TOPOLOGY DETECTION RESULT *****
inequality = 12.85155
***** SIMULATION *****
sw 1 op sw 9 op sw 11 op sw 13 op sw 15 op sw 17 op
1773.92627 399.38525 12.57243 11.26142 99.41380 1.67434
*****
Smallest inequality is 1.67434
Detect Switch 17 Open
*****
```

- **Case 2** : simulation with power, current and voltage measurements (when node 9 open)

- Topology error detection result

```
***** BCSE TOPOLOGY DETECTION RESULT *****
inequality = 12.50052
***** SIMULATION *****
sw 1 op sw 9 op sw 11 op sw 13 op sw 15 op sw 17 op
1795.81323 335.28391 14.38356 3.62456 106.57454 2.53852
*****
Smallest inequality is 2.53852
Detect Switch 17 Open
*****
```

♣ Shunt Capacitor Bank Error Detection

1. When node SCB 9 open

- **Case 1** : simulation with power and current measurements (when SCB 9 open)

- Topology error detection result

```
***** BCSE TOPOLOGY DETECTION RESULT *****
inequality = 55.13657
***** SIMULATION (CAP error) *****
CAP 9 op CAP 10 op
3.59280 4.67284
*****
Smallest inequality is 3.59280
Detect CAP 9 open
```

- **Case 2** : simulation with power, current and voltage measurements (when SCB 9 open)

- Topology error detection result

```
***** BCSE TOPOLOGY DETECTION RESULT *****
inequality = 63.65241
***** SIMULATION (CAP error) *****
CAP 9 op CAP 10 op
9.92943 16.83188
*****
Smallest inequality is 9.92943
Detect CAP 9 open
```